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Dissertation

Implied Volatility Indices and their Properties

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Section 1
Introduction

Implied Volatility Indices have received an increasing attention since 1993, when VIX was first introduced at the Chicago Board of Options Exchange (CBOE). In the years that followed, all the major Exchanges introduced Implied Volatility Indices for their most representative Stock Indices, constantly renewing the methodologies of their calculation and at cases also introducing derivatives on them. We focus on the eight most important implied volatility indices currently trading at the CBOE, at EUREX and at EURONEXT, namely the American VIX, VXO, VXN and VXD, the German VDAX, the French VX1 and VX6 and the European VSTOXX. The purpose of this study is to examine their properties from the econometric aspect and for risk management purposes.

We begin by the definition of implied volatility to continue with a review of the evolution of implied volatility indices. Implied volatility is the volatility of an underlying asset implied by the market prices of options written on it. Implied volatility indices are indices disseminated by Option Exchanges that express the market consensus of the near term volatility of traded stock indices. Since volatility often signifies financial turmoil, the implied volatility indices are often referred to as the "investor fear gauge". This means that they reach their highest levels during times of financial turmoil and investor fear. As markets recover and investor fear subsides, implied volatility index levels tend to drop. Implied volatility indices are used as possible trading signals to identify buying/selling opportunities in the market, as a forecast tool of the future return volatility over the remaining life of the relevant option, for risk management purposes in the calculation of VaR and most recently as the underlying to volatility derivatives[1].

The idea of developing a volatility index was first suggested by Brenner and Galai in 1989. Fleming, Ostdiek and Whaley in 1993 describe the construction of an implied volatility index (the VIX) originally based on S&P100 options. The French VX1 and VX6 were developed according to Brenner and Galai. In 2004 Skiadopoulos developed a methodology for the construction of GVIX, the Greek market volatility index based on FTSE/ASE-20 option series.

After the introduction in 1993 of VIX with S&P100 as underlying, in 2003 the CBOE revised the methodology of its calculation to incorporate a broader range of expiries and make it independent of an option pricing model, unlike the old VIX which used Black and Scholes.[2] The new VIX was based on the primary US stock market benchmark, S&P500.

[1] On volatility derivatives see Whaley (1993). The idea of trading volatility is also developed in Carr and Madan (2002), and in Brenner et al. (2001).

[2] Daily asset returns violate the B-S model assumption of normal distribution, with implied volatility varying across strike prices and maturities. As a result, options of different strike prices yield different volatilities for the underlying. It has been observed that OTM options minimize the effect of measurement errors on the calculation of implied volatilities. See Panigirzoglou and Skiadopoulos (2004).

The old VIX continued to be disseminated under a new ticker name, VXO, based on S&P100. At the same time, the CBOE applied the same calculation methodology to VXN which was introduced in 1997 and was based on the Nasdaq-100 options. It decided, though, not to back-calculate VXN[3], making available to the public a VXN price history beginning as late as 2001. VX1 and VX6 were created by the Marché des Options Négociables de Paris (MONEP) in 1997 and were based on implied volatilities of around at-the-money CAC40 Index call option of 1 month and 6 months respectively. Euronext – LIFFE (London International Financial Futures and Options Exchange) issued another four implied volatility indices based on CAC 40 options: V1X (V6X) based on CAC40 put 1 month (6 months) and MX1 (MX6) based on CAC40 average call/put 1 month (6 months). VDAX, was introduced by Deutsche Börse, in 1994, to measure the expected price fluctuation of the DAX-30 Index, based on around at-the-money DAX options. In April 2005, the CBOE introduced VXD, the Implied Volatility Index of the Dow Jones Industrial Average Index (DJIA), calculated with the 2003 methodology. One month later, in May 2005, EUREX began dissemination of VSTOXX, the Implied Volatility Index of DJEUROSTOXX50, VSMI, the Implied Volatility Index of the Swiss Market Index SMI, and VDAX NEW, the revisited VDAX, applying a calculation method similar to that implemented by CBOE in 2003. The basic principles of the construction of the eight implied volatility indices under study are presented in Table 1.

1.1. Literature

Implied Volatility measures have been the focus of analysis in a wide range of studies over the last years. These include for example Whaley (1993, 2000), Aboura and Villa (1999), Gemmill and Kamiyama (2000), Giot (2005a), Skiadopoulos (2004), Wagner and Szimayer (2004). Giot (2005a) reports a negative and statistically significant relationship for the period 1995-2002 between the levels of S&P100 and NASDAQ100 and their implied volatility indices: positive stock index returns lead to decreased implied volatility levels, while negative returns lead to higher implied volatility levels. He also finds that this relationship is asymmetric in the sense that negative stock index returns yield bigger proportional changes in implied volatility measures than do positive returns. Whaley (2000) observes an asymmetric negative relationship between weekly changes of the old VIX and weekly returns of S&P100 over the period 1995-2000. Skiadopoulos (2004) also reports the existence of an asymmetric leverage effect[4] between returns of GVIX and changes of it's underlying FTSE/ASE-20 and finds a contemporaneous spillover of implied volatility change between GVIX and VXO/ VXN. Gemmill and Kamiyama (2000) examine whether there are spillovers of implied volatility and implied skewness across time zones, using daily data of the index-option markets of the US, Japan and UK. They find that the level of implied volatility spills across markets but the

[3] Information provided after communication with CBOE officials.
[4] The "leverage effect" refers to the negative relationship between stock returns and volatility: volatility increases when the stock prices fall. It is attributed to the effect that a change in the market valuation of a firm's equity has on the degree of leverage in its capital structure.

skewness of the volatility smile is a local phenomenon. Aboura and Villa (1999) report on the volatility transmission between VX1, VDAX and VIX that the intra-regional correlation is not stronger that the multi-national correlation contrary to the returns process. They also observed that US market volatility index is the most influential.

Several papers have been written on the information content and the predictive power of implied volatility index prices as a forecast of future realized volatility. Fleming-Ostdiek-Whaley (1995) found that implied volatilities contained substantial information for future volatility. Moraux, Navatte and Villa (1999) showed that VX1 can be used to generate volatility forecasts over different horizons and that these forecasts are reasonably accurate predictors of future realized volatility. Giot (2005a) found that VIX and VXN provide accurate and meaningful information as to future volatility forecasts. Fleming et al. (1995) concluded that implied volatility (VIX 1986-1992) is an upwardly biased estimator of future volatility even if the magnitude of the bias is not economically significant. They also concluded that implied volatility dominates past volatility as a forecast of future volatility. Jorion (1995) found that implied volatility is an efficient indicator of future return volatility for foreign currency futures. In line with the above, Aboura and Villa (1999) showed for VIX, VDAX and VX1 that past implied volatility informs more about future implied volatility than past realized volatility. Malz (2000) found that implied volatility contains information regarding future large-magnitude returns, which is not contained in other risk measures, and this fact can help risk managers posture themselves for stress events. On the other hand, Canina and Figlewski (1993), reported for S&P100 (1983-1987) that implied volatilities have little predictive power for future volatility – in fact implied volatility has no correlation with future return volatility- and therefore they are significantly biased forecasts. Figlewski (2004) concluded that even though implied volatility contains significant information about future volatility, it does not pass the test of forecast rationality and is not necessarily a more accurate forecast of future volatility than historical volatility. He also showed that the historic volatility forecasts more accurately for large samples and long rather than short forecasting horizons. On a more general framework, Christoffersen and Diebold (2000) find that volatility forecastability, although clearly of relevance for risk management at the very short horizons, may not be important for risk management more generally, since it decays quickly across horizons.

Furthermore, Carr and Madan (2002) develop the idea of trading volatility, suggesting a strategy of combining static positions in options with dynamic trading in futures, in order to create payoffs related to realized volatility. In the same line, Brenner et al. (2001) introduce a new volatility instrument, an option on a straddle, which can be used to hedge volatility risk.

There is also a growing literature on the use of implied volatilities as inputs in VaR models in a risk management context. Bluhm and Yu (2001), for example, compared two basic approaches, time series techniques and volatility implied in option prices, to forecast volatility in the German stock market and for VaR calculation purposes. They concluded that when option pricing is the primary interest implied volatility should be used, whereas when VaR is the concern, ARCH-type models are more useful. Giot (2005b) assessed the information content of implied volatility indices in a Value-at-Risk framework for VIX and VXN for the period 1994-2000. He concluded that volatility forecasts based on the VIX/VXN indices are meaningful inputs in VaR models as the number of VaR violations is correctly modeled in most cases. He also showed that they perform equally well during “difficult” market conditions. In a more general framework, Christoffersen, Hahn and Inoue (2000) developed a methodology for the proper specification of a VaR measure. They calculated, tested and compared competing VaR measures from either historical or option-price based volatility measures in an application to daily returns on the S&P500 index.

1.2. Contribution to existing literature

We have seen that the informational content of implied volatility has become an important research topic in the academic literature. Although previous studies examined the properties of VDAX, VXN, VX1 and the old VIX (with S&P100 as underlying), there is a literature gap as far as the VX6 and the new VIX (with S&P500 as underlying) are concerned. International transmission of implied volatility has been studied for VX1-VIX-VDAX (Aboura and Villa, 1999), for GVIX-VXO/VXN (Skiadopoulos, 2004), and for S&P500-FTSE100-NK225 options (Gemmill and Kamiyama, 2000) but not for VIX-VXO-VXN-VDAX-VX1-VX6. The efficiency of implied volatility as measure of the stock index volatility for VaR calculation has been previously studied for DAX (Bluhm and Yu, 2000), S&P100 and Nasdaq-100 (Giot, 2005b). Only Giot (2005b), however, uses directly an implied volatility index in a VaR context. We will test the efficiency of the VIX comparative with other measures as an input in a VaR model for S&P500. The present study provides for the first time a comparative analysis of the properties of implied volatility indices, based on an extended data set, incorporating the recently introduced VXD and VSTOXX.

1.3. Structure of the present study

The rest of the study is structured as follows. First we describe the data set and present the descriptive statistics. We proceed with the study of the informational content of Implied Volatility Indices daily changes for the daily returns of the underlying Stock Indices, investigating the existence of a leverage effect, its symmetry and stability over time. In the same section we apply Granger Causality tests to see whether the time series of the Implied Volatility Indices could help forecast Stock indices returns and vice versa. The international transmission of implied volatility movements will be examined in a Vector Autoregression framework. The second part of our research focuses on the construction and backtesting of Value-at-Risk models for a virtual long position on the S&P500. Results from the Delta Normal Variance-Covariance method are tested against results from the Historical Simulation method using established Backtesting criteria. We particularly concentrate on the performance of the VIX as variance input in the Delta Normal method.

Table 1
Calculation Methodologies

Implied Volatility Index	**Option Pricing model**	**Options used**	**Represents**
VXO	Black-Scholes (1973) and Merton (1973)	4 puts and calls of 2 nearest to 30 days expiries, with 2 strikes around an ATM point.	The implied volatility of an ATM option with constant 30 calendar days to expiry.
VIX, VXN, VXD	Independent of model	OTM puts and calls of 2 nearest to 30 days expiries, covering a wide range of strikes.	The square root of implied variance across options of all strikes, with constant 30 calendar days to expiry.
$VDAX$	Black's model (1976)	8 pairs of puts and calls of 2 nearest to 45 days expiries, with 4 strikes around an ATM point.	The implied volatility of an ATM option with constant 45 calendar days to expiry.
$VDAX \quad NEW$	Independent of model	OTM puts and calls of 2 nearest to 30 days expiries, covering a wide range of strikes.	The square root of implied variance across options of all strikes, with constant 30 calendar days to expiry.
$VX1, VX6$	Black-Scholes (1973)	4 calls of 2 nearest to 31 (185) days expiries, with 2 strikes around an ATM point.	The implied volatility of an ATM option with a constant 31 (VX1) and 185 (VX6) calendar days to expiry.
$VSTOXX$	Independent of model	OTM puts and calls of 2 nearest to 30 days expiries, covering a wide range of strikes.	The square root of implied variance across options of all strikes, with constant 30 calendar days to expiry.

Table 1: Outline of the methodologies used for the calculation of the Implied Volatility Indices.

Section 2
The Dataset

Our sample comprises of eight Implied Volatility Indices which are the American VIX, VXO, VXN and VXD, the German VDAX, the French VX1 and VX6, the European VSTOXX and their underlying stock indices. We provide a brief description of the indices and then continue to the technical details of the time series construction.

The CBOE Volatility Index VIX is based on real-time S&P 500 (SPX) index option bid/ask quotes. The underlying S&P 500 Composite Stock Price Index is a market-value-weighted index of 500 stocks that are traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the Nasdaq National Market System. The companies chosen for inclusion in the Index are leading companies in leading industries within the U.S. economy. The S&P 500 is used by 97% of U.S. money managers and pension plan sponsors. About $626 billion is indexed to the S&P 500. VXO is based on S&P100 (OEX) options also traded at the CBOE. The underlying Standard & Poor's 100 Stock Index measures large company U.S. stock market performance. This market capitalization-weighted index is made up of 100 major, blue chip stocks across diverse industry groups.

The Nasdaq Volatility Index is based on Nasdaq-100 index options (NDX). The underlying Nasdaq-100 index, disseminated by the Nasdaq Stock Market, includes 100 of the largest domestic and international non-financial companies listed on the NASDAQ Stock Market based on market capitalization. The Index reflects companies across major industry groups including computer hardware and software, telecommunications, retail/wholesale trade and biotechnology. It does not contain financial companies including investment companies. The CBOE DJIA Volatility Index (VXD) is based on the Dow Jones Industrial Average Index Options (DJX) traded at the CBOE. The Dow Jones Industrial Average a price-weighted index composed of 30 of the largest, most liquid NYSE and NASDAQ listed stocks. These 30 stocks represent about a fifth of the $8 trillion-plus market value of all U.S. stocks and about a fourth of the value of stocks listed on the New York Stock Exchange.

VDAX is based on the DAX option contract (ODAX), one of the products of EUREX Derivatives Exchange, with the highest trading volume. The underlying stock index DAX is a blue chip index that comprises the 30 largest German shares with the highest turnover, representing about 70% of the overall market capitalization of domestic listed companies. The trading in these shares accounts for more than 80% of Germany's exchange-traded equity volumes. The French VX1 and VX6 are based on the CAC40 Index Options Contract PXL (to be gradually replaced by the CAC40 Index Options Contract PXA introduced on May 2005), traded at EURONEXT – LIFFE. The underlying CAC 40 index consists of the 40 stocks that are most representative of the economic sectors quoted on the Eurolist market operated by Euronext Paris. Dow Jones EURO STOXX 50 Volatility Index (VSTOXX), disseminated by STOXX Limited, is calculated by using the Dow Jones EURO STOXX 50 Index Options (OESX) traded in Eurex. The underlying Dow Jones EURO STOXX 50, disseminated by

STOXX Limited and Deutsche Boerse Group, is a European blue-chip index, representing the 50 leading shares in Eurozone. It includes sector leaders in Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. The DJ EUROSTOXX50 captures approximately 60% of the free-float market capitalization of the Dow Jones EURO STOXX Index, which in turn covers approximately 95% of the free-float market capitalization of the represented Eurozone countries. All options used for the calculation of the specific Implied Volatility Indices are European Style except the S&P100 (OEX) options which are American Style.

Sixteen time series of daily closing prices constitute our dataset. Holidays are discarded from the samples, taking into account only trading days. The sources for historical data are CBOE for the American Implied Volatility indices, EURONEXT for the French IV indices, BLOOMBERG for VDAX, and EUREX for VSTOXX and DJEUROSTOXX50.[5] Time series of the American, the French and the German underlying Stock Indices have been downloaded from DATASTREAM. The Stock Indices samples have been adjusted to IV indices samples, so that data for each IV index and its underlyer are completely synchronized.

Time series start from the date of available historical data for each implied volatility index and end at 3/17/2005 for all indices. In this way, the largest sample is that of VXO time series, starting at 1/2/1986 with 4841 observations. Of the rest American IV indices, VIX starts at 1/2/1990 with 3833 observations, VXN at 2/2/2001 with 1033 observations and VXD at 10/6/1997 with 1872 observations. Of the European indices, VDAX starts at 1/2/1992 with 3322 observations, VX1 and VX6 both start at 10/14/1997 with 1871 and 1866 observations respectively and VSTOXX starts at 1/4/1999 with 1579 observations. Although the French indices span exactly the same time period, the sample of VX6 consists of five observations less than VX1, due to the fact that EURONEXT did not provide data for certain dates.

[5] We have avoided DATASTREAM as a reference data source, since it fills in holidays with the previous day's price quote, resulting in a sample bias when used for inference.

Figure 1

S&P 500 - VIX

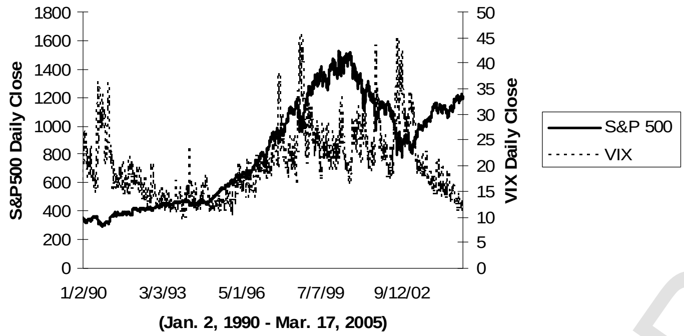


Figure 2

S&P 100 - VXO

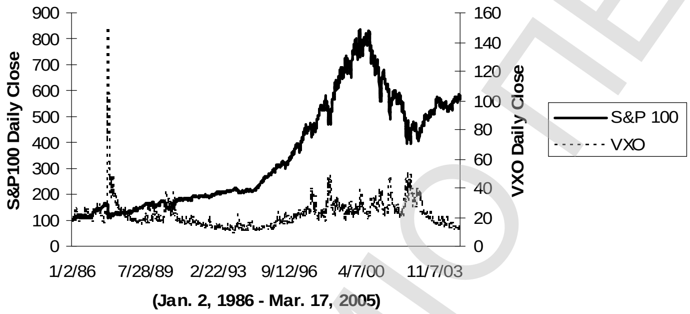


Figure 3

Nasdaq100 - VXN

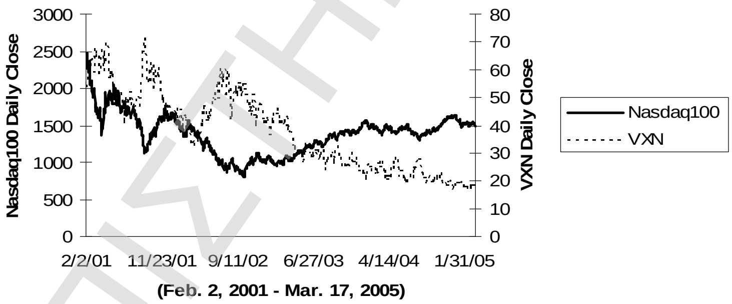
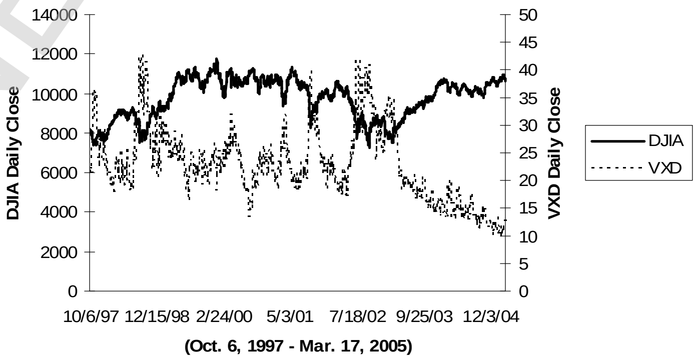


Figure 4

DJIA - VXD



Figures 1-4: Daily Closing levels of the American Implied Volatility Indices against daily closing levels of their underlying stock indices.

Figure 5

DAX - VDAX

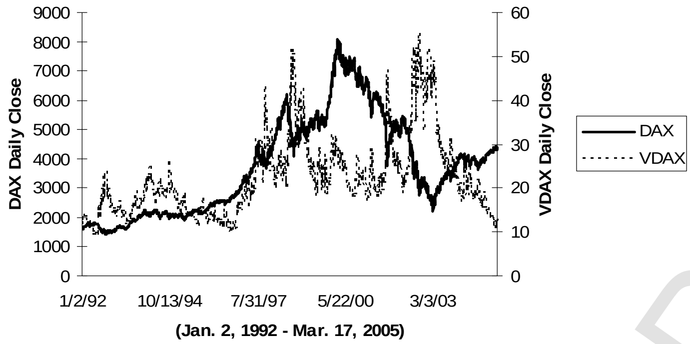


Figure 6

CAC40 - VX1

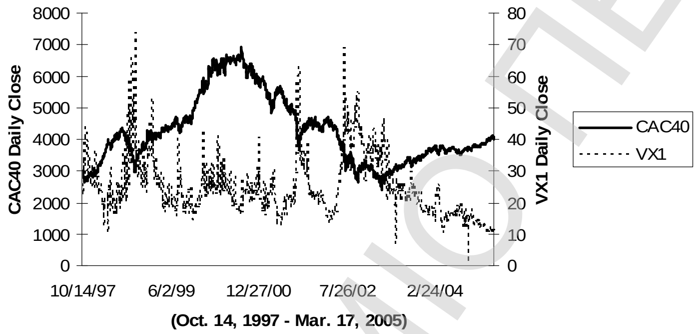


Figure 7

CAC40 - VX6

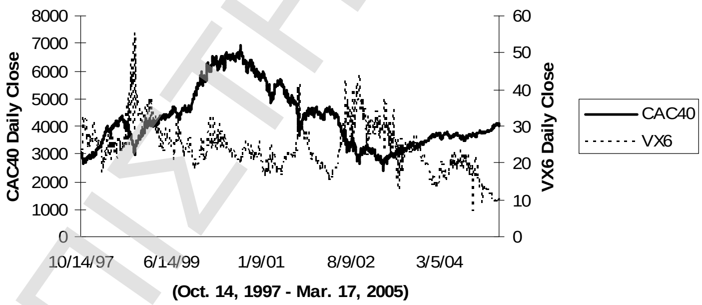
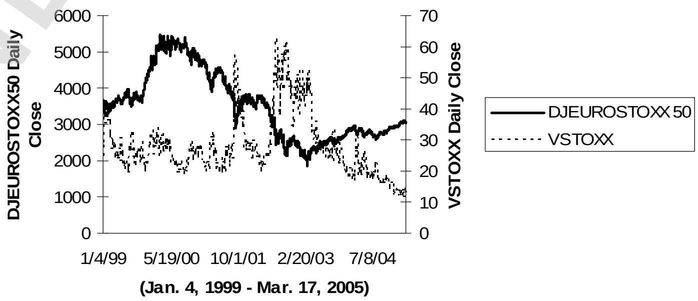


Figure 8

DJEUROSTOXX 50 - VSTOXX



Figures 5-8: Daily Closing levels of the European Implied Volatility Indices against daily closing levels of their underlying stock indices.

2.1. Descriptive Statistics

Figures 1 to 8 plot the evolution of the implied volatility indices against the evolution of their underlying stock indices. In each figure we observe that when the stock index spikes upward the implied volatility index spikes downward and vice versa. There are cases when the implied volatility index overreacts to downward movements of the underlying stock index. This for example is evident during the October 1987 market crash when VXO reached the unprecedented level of 150.19 units. We will analyze, in the process, the observed negative relationship between implied volatility indices and stock indices, as well as its symmetry.

Table 2 presents the descriptive statistics of the indices daily closing levels. Table 3 presents the descriptive statistics for the daily returns of the Stock Indices and the daily changes of Implied Volatility Index Levels. We use continuously compounded returns, R_t calculated as the log differences of the daily closing prices S_t of the stock indices, and daily changes of the implied volatility indices, Δ_t^{IV} , calculated as the first differences of their daily closing prices S^{IV} .

$$R_t = \ln S_t - \ln S_{t-1} = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

$$\Delta IV_t = S_t^{IV} - S_{t-1}^{IV}$$

For each time series we report the mean, median, maximum, minimum, the standard deviation, skewness, kurtosis and the Jarque-Bera statistic. We also report autocorrelations and partial autocorrelations up to the third lag, as well as the Ljung-Box Q-statistic for 12 lags. For the levels of Stock Indices, the mean varies from 350.21 for S&P100 to 9758.65 for the DJIA, with standard deviations varying from 98.36 for S&P100 to 7286.27 for DJIA. The average mean for the implied volatility indices is around the 25 level, with standard deviations varying between 6.21 for VX6 and 13.98 for VXN. Stock indices as well as IV indices are all positively skewed indicating a long right tail in their distributions, except for DJIA which is negatively skewed. The distributions of the IV indices are peaked (leptokurtic) relative to the normal distribution, since their kurtosis values exceed the value of 3. An exception seems to be VXN with a kurtosis of 1.90. On the contrary all distributions of the stock indices, except that of Nasdaq-100, are more flat relative to normal, with reported kurtosis less than 3. The Jarque-Bera statistics, all significant at the 1% significance level, indicate rejection of the null hypothesis of a normal distribution for the levels of both the IV and the Stock Indices.

Autocorrelations up to the third lag are all significant at the 1% level and close to 1, with an average value of 0.98 for the Stock indices, and 0.94 for the IV indices. Partial autocorrelations, fall from an average of 0.98 (stock indices) and 0.94 (IV indices) at the first lag, to an average of 0.01 (stock indices) and 0.05 (IV indices) at the second and third lag. This indicates that autocorrelations between two and three time lags are mostly attributed to the first order autocorrelation. VX1 and VX6 are an exception since they display partial second order autocorrelations as high as 0.34 and 0.35. The Ljung-Box statistic significant at 1% significance level indicates the existence of autocorrelation up to the twelfth lag.

As presented in Table 3, stock index returns as well as the changes of IV indices are almost zero mean. Maximum values for the daily stock returns vary from 6% for S&P500 to 10% for Nasdaq100, with minimums averaging around -8% with an exception of S&P100 which has a minimum value of -24%, reported during the October 1987 market crash. Maximum values for the daily changes of IV indices vary from 8.43 for VXD to 30.31 for VX1. Only VXO displays a maximum of 113.82. Stock Index returns report a standard deviation of around 0.01 whereas changes of IV indices a standard deviation varying from 1.17 for VDAX to 3.77 for VX1. The Jarque-Bera statistic significant at the 1% indicates rejection of the null hypothesis of a normal distribution for both stock index returns and IV indices changes. With reported kurtosis over the value of 3, the distributions of stock index returns as well as IV changes are peaked (leptokurtic) relative to the normal. Distributions of IV changes are all positively skewed whereas distributions of stock index returns are negatively skewed except for the Nasdaq100 returns which display positive skewness.

Stock index returns do not display significant autocorrelations up to the third lag except for the S&P100 returns with a first order autocorrelation of -0.025 significant at 1%. On the contrary, all IV indices, except VXN, display in their daily changes low but significantly different from zero autocorrelation up to the third lag. First order significant autocorrelations are all negative varying from -0.375 for VX6 to -0.036 for VIX daily changes. Finally, the null hypothesis of no autocorrelation up to the twelfth lag is not rejected by the Ljung-Box Q-statistic either for the stock indices returns or the IV indices changes.

2.2. Unit Root Tests

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests test the validity of the null hypothesis that a series is unit-root non-stationary (integrated of order one), against the alternative hypothesis that the series is (trend-) stationary. We perform ADF and PP tests for the IV indices and the underlying stock indices. T-statistics as well as the MacKinnon lower-tail critical values at 1%, 5% and 10% level are reported in Table 4. In the Phillips – Perron test, brandwidth is selected according to the Newey-West brandwidth parameter method, whereas in the ADF test, lag length is selected according to the Schwartz Information Criterion, so that the number of lags is sufficient to remove serial correlation in the residuals. Results from the tests show that all series of stock indices, except Nasdaq-100 series, are unit-root non-stationary at 1% significance level. VXO is stationary in all cases. The rest of the IV indices are unit-root non-stationary if no constant or trend is included in the test equation.

Since most series are unit-root non-stationary, we will difference once to ensure that the series we will use for inference are stationary. In particular, we will take first differences of the implied volatility indices and log differences of the underlying stock indices.

Table 2
Descriptive Statistics of Indices Levels

Panel A: Levels of Stock Indices							
	$S\&P500$	$S\&P100$	$Nasdaq100$	$DJIA$	DAX	$CAC40$	$DJEUROSTOXX50$
Mean	820.77	350.21	1381.77	9758.65	3742.91	4254.38	3513.25
Median	845.13	265.11	1415.30	10022.16	3651.94	4010.36	3495.43
Maximum	1527.46	832.65	2473.24	11722.98	8064.97	6922.33	5464.43
Minimum	295.46	98.36	804.65	7286.27	1402.34	2403.04	1849.64
Std. Dev.	365.04	207.50	273.96	1015.97	1674.77	1073.25	936.22
Skewness	0.15	0.55	0.36	-0.50	0.51	0.72	0.46
Kurtosis	1.60	1.95	3.79	2.12	2.26	2.53	2.04
Jarque-Bera	329.11*	466.09*	48.73*	138.12*	223.09*	177.19*	115.57*
AC(1)	0.999*	0.999*	0.985*	0.993*	0.999*	0.998*	0.998*
AC(2)	0.999*	0.999*	0.970*	0.986*	0.998*	0.996*	0.997*
AC(3)	0.998*	0.998*	0.957*	0.979*	0.997*	0.994*	0.995*
Partial AC(1)	0.999*	0.999*	0.985*	0.993*	0.999*	0.998*	0.998*
Partial AC(2)	0.005*	0.014*	-0.006*	0.004*	0.008*	-0.011*	0.006*
Partial AC(3)	0.012*	0.013*	0.037*	0.012*	0.010*	0.035*	0.036*
Ljung-Box Q(12)	45680*	57805*	10546*	20663*	39495*	21983*	18695*

Panel B: Levels of Implied Volatility Indices								
	VIX	VXO	VXN	VXD	$VDAX$	$VX1$	$VX6$	$VSTOXX$
Mean	19.79	21.29	37.19	22.53	22.07	24.32	23.83	27.63
Median	18.86	20.07	37.43	22.11	20.35	22.34	23.29	25.53
Maximum	45.74	150.19	71.72	42.95	55.42	73.79	55.47	62.73
Date of max.	10/8/98	10/19/87	9/20/01	9/10/98	9/24/02	10/8/98	10/8/98	7/24/02
Minimum	9.31	9.04	16.80	9.94	9.36	1.22	7.02	11.60
Std. Dev.	6.37	8.11	13.98	6.35	8.80	8.91	6.21	9.70
Skewness	0.92	3.29	0.30	0.48	1.23	1.25	0.58	1.19
Kurtosis	3.73	35.72	1.90	3.11	4.34	5.14	4.28	4.17
Jarque-Bera	623.77*	224701.50*	68.19*	72.53*	1080.84*	843.62*	232.52*	464.32*
AC(1)	0.981*	0.954*	0.993*	0.981*	0.991*	0.910*	0.941*	0.984*
AC(2)	0.964*	0.916*	0.985*	0.962*	0.983*	0.886*	0.926*	0.971*
AC(3)	0.950*	0.901*	0.978*	0.945*	0.976*	0.884*	0.914*	0.958*
Partial AC(1)	0.981*	0.954*	0.993*	0.981*	0.991*	0.910*	0.941*	0.984*
Partial AC(2)	0.027*	0.053*	-0.031*	0.014*	0.055*	0.337*	0.348*	0.063*
Partial AC(3)	0.085*	0.251*	0.043*	0.026*	0.036*	0.279*	0.189*	0.037*
Ljung-Box Q(12)	38874*	41213*	11450*	18559*	36378*	15723*	17662*	16192*

Table 2: Descriptive statistics for the daily closing levels of the implied volatility indices and the underlying stock indices. We report autocorrelations (AC) and partial autocorrelations of 1, 2 and 3 lags. For the Jarque-Bera statistic, the Ljung-Box statistic and autocorrelation values, one asterisk denotes significance of the reported value at 1% level, two asterisks significance at 5% and three asterisks significance at 10%.

Table 3
Descriptive Statistics of Indices Returns & Changes

Panel A: Returns of Stock Indices							
	$R^{S\&P500}$	$R^{S\&P100}$	$R^{Nasdaq100}$	R^{DJIA}	R^{DAX}	R^{CAC40}	$R^{DJEUROSTOXX50}$
Mean	0.0003	0.0004	-0.0005	0.0001	0.0003	0.0002	-0.0001
Median	0.0004	0.0006	0.0005	0.0003	0.0007	0.0004	0.0003
Maximum	0.06	0.09	0.10	0.06	0.08	0.07	0.07
Minimum	-0.07	-0.24	-0.09	-0.07	-0.09	-0.08	-0.07
Std. Dev.	0.01	0.01	0.02	0.01	0.01	0.02	0.02
Skewness	-0.10	-1.91	0.15	-0.11	-0.17	-0.09	-0.04
Kurtosis	6.69	42.98	4.73	5.97	6.09	5.12	5.11
Jarque-Bera	2181.64*	325272.10*	132.78*	691.23*	1341.21*	352.90*	293.81*
AC(1)	0.000	-0.025*	-0.026	-0.005	-0.026	0.008	-0.015
AC(2)	-0.024	-0.050*	-0.074**	-0.026	-0.006	-0.034	-0.028
AC(3)	-0.033***	-0.029***	0.003***	-0.010	-0.028	-0.051***	-0.061***
Partial AC(1)	0.000	-0.025*	-0.026	-0.005	-0.026	0.008	-0.015
Partial AC(2)	-0.024	-0.050*	-0.075**	-0.026	-0.007	-0.034	-0.028
Partial AC(3)	-0.033***	-0.031***	-0.001***	-0.010	-0.028	-0.051***	-0.062***
Ljung-Box Q(12)	33.253*	30.956*	18.902***	19.526***	32.819*	19.166***	40.192*

Panel B: Changes of Implied Volatility Indices								
	ΔVIX	ΔVXO	ΔVXN	ΔVXD	$\Delta VDAX$	$\Delta VX1$	$\Delta VX6$	$\Delta VSTOXX$
Mean	-0.001	-0.001	-0.035	-0.005	0.000	-0.007	-0.007	-0.003
Median	-0.040	-0.030	-0.060	-0.040	-0.040	-0.110	-0.070	-0.090
Maximum	9.92	113.82	10.58	8.43	9.22	30.31	21.36	19.18
Minimum	-7.80	-66.09	-7.60	-6.58	-5.64	-27.31	-14.70	-9.42
Std. Dev.	1.23	2.44	1.49	1.22	1.17	3.77	2.11	1.67
Skewness	0.56	17.09	0.20	0.58	0.77	0.61	0.69	1.43
Kurtosis	9.03	1088.38	7.94	7.34	10.68	20.50	19.95	18.65
Jarque-Bera	6010.41*	238000000*	1055.11*	1571.90*	8493.61*	23968.91*	22480.43*	16634.44*
AC(1)	-0.036**	-0.075*	0.031	-0.023	-0.060*	-0.369*	-0.375*	-0.046***
AC(2)	-0.093*	-0.260*	-0.046	-0.039	-0.042*	-0.122*	-0.040*	-0.058**
AC(3)	-0.061*	0.033*	0.023	-0.084*	-0.042*	0.091*	-0.051*	0.003**
Partial AC(1)	-0.036**	-0.075*	0.031	-0.023	-0.060*	-0.369*	-0.375*	-0.046***
Partial AC(2)	-0.094*	-0.267*	-0.047	-0.039	-0.045*	-0.299*	-0.211*	-0.060**
Partial AC(3)	-0.069*	-0.013*	0.026	-0.086*	-0.048*	-0.098*	-0.180*	-0.002**
Ljung-Box Q(12)	116.98*	425.19*	41.003*	42.817*	47.445*	323.72*	294.72*	30.688*

Table 3: Descriptive statistics for the daily changes of the implied volatility indices and the daily returns of the underlying stock indices. We report autocorrelations (AC) and partial autocorrelations of 1, 2 and 3 lags. For the Jarque-Bera statistic, the Ljung-Box statistic and autocorrelation values, one asterisk denotes significance of the reported value at 1% level, two asterisks significance at 5% and three asterisks significance at 10%.

Table 4
Unit Root Tests

Panel A: Stock Indices Levels

	*S & P*500	*S & P*100	*Nasdaq*100	*DJIA*	*DAX*	*CAC*40	*DJESTOXX* 50
ADF							
None	0.763	0.670	-1.691***	0.246	0.100	-0.021	-0.506
Constant	-1.039	-0.938	-3.992*	-2.602***	-1.432	-1.552	-1.081
Constant & trend	-1.536	-1.624	-3.718**	-2.593	-1.227	-1.884	-1.902
Phillips – Perron							
None	0.980	0.856	-1.778***	0.290	0.127	0.041	-0.500
Constant	-0.948	-0.856	-4.021*	-2.518	-1.413	-1.461	-0.939
Constant & trend	-1.323	-1.441	-3.723**	-2.507	-1.187	-1.811	-1.761

Panel B: Implied Volatility Indices Levels

	VIX	*VXO*	*VXN*	*VXD*	*VDAX*	*VX*1	*VX* 6	*VSTOXX*
ADF								
None	-1.333	-2.037**	-1.305	-1.261	-1.333	-1.404	-0.967	-1.167
Constant	-3.786*	-5.915*	-1.681	-4.034*	-3.601*	-3.702*	-2.657***	-3.335**
Constant & trend	-3.965*	-5.931*	-4.214*	-4.705*	-3.818**	-3.935**	-3.328***	-3.464**
Phillips – Perron								
None	-1.184	-2.270**	-1.373	-1.017	-1.149	-1.770***	-1.097	-1.002
Constant	-5.154*	-8.977*	-1.352	-3.493*	-3.272**	-9.628*	-5.129*	-3.034**
Constant & trend	-5.435*	-9.039*	-3.930**	-4.235*	-3.461**	-10.238*	-6.909*	-3.186***

MacKinnon (1996) lower tail critical values			
ADF/ Phillips – Perron Test Exogenous included in test equation:	1% level	5% level	10% level
None	-2.566	-1.941	-1.617
Constant	-3.432	-2.862	-2.567
Constant & Linear Trend	-3.960	-3.411	-3.127

Table 4: Augmented Dickey – Fuller (ADF) and Philips – Perron Unit Root Tests for the daily closing levels of the Stock Indices and the Implied Volatility Indices. Reported values are t-statistics. We test against MacKinnon (1996) lower tail critical values. One asterisk denotes rejection of the null hypothesis of a unit root at 1% level, two asterisks rejection at 5% level, three asterisks rejection at 10% level.

2.3. ARCH Test

Stock returns often display the phenomenon of volatility clustering. This means that returns have periods of low volatility and other periods of high volatility, that is volatility does not remain constant over time. This is also referred to as heteroscedasticity. We perform Engle's test for the detection of Autoregressive Conditional Heteroscedasticity in the Stock indices returns. The test is implemented in Matlab with "archtest" function. The null hypothesis is that a time series of returns is a random sequence of Gaussian disturbances displaying no ARCH effects. The t-statistic under the null hypothesis is asymptotically Chi-Squared distributed.

Performing the ARCH test we extract the sample mean from the actual returns. This is consistent with the definition of the conditional mean equation for returns, in which the innovations process is $e_t = R_t - c$, and c is the mean of R_t. We test for heteroscedasticity up to 10, 15 and 20 lags at the 5% significance level. The critical values are 18.3070, 24.9958 and 31.4104 for 10, 15 and 20 lags. Results from the ARCH test for the Stock Indices returns as displayed in Table 5, show that all test statistics values are significant at the 5% level, providing evidence in support of ARCH effects.

2.4. Concurrent Correlations

Concurrent Correlations between the returns of the Stock Indices and correlations between the IV indices daily changes are presented in Table 6. Values have been estimated from a sample of synchronized data of the period 2/2/2001 – 3/17/2005. Results for the stock indices returns provide evidence in support of regional correlation. We observe that markets which are geographically close and also happen to be integrated are more correlated. We can see that returns of the DAX, CAC40 and DJEUROSTOXX50, which is a Eurozone blue-chip Index, are more correlated with one another than with the American indices.

This is not exactly true for the correlation of daily movements of Implied Volatility Indices across markets. Although there is a higher correlation between the IV indices changes in integrated markets, this is not so pronounced as with the stock index returns. Furthermore VDAX seems to be more correlated with the American IV indices than with the French IV indices. Yet it is with VSTOXX that VDAX displays the highest correlation. The international transmission of Implied Volatility movements will be the focus of section 2.5 where we examine the existence of spillover effects.

Table 5
ARCH Tests

Stock Indices Returns								
	Critical Values	$R_t^{S\&P500}$	$R_t^{S\&P100}$	$R_t^{Nasdaq100}$	R_t^{DJIA}	R_t^{DAX}	R_t^{CAC40}	$R_t^{DJEUROSTOXX50}$
10 lags	18.3070	426.794*	274.178*	177.407*	206.297*	708.227*	296.508*	345.264*
15 lags	24.9958	441.458*	275.524*	189.499*	213.464*	718.078*	326.838*	366.449*
20 lags	31.4104	468.854*	278.103*	192.172*	222.987*	721.871*	335.637*	370.552*

Table 5: Engle's ARCH test for the Stock Indices daily returns. We perform the test for 10, 15 and 20 lags. One asterisk denotes rejection of the null hypothesis of no heteroscedasticity at the 5% significance level.

Table 6
Concurrent Correlations

Panel A: Stock Indices Returns							
	$R_t^{S\&P500}$	$R_t^{S\&P100}$	$R_t^{Nasdaq100}$	R_t^{DJIA}	R_t^{DAX}	R_t^{CAC40}	$R_t^{DJEUROSTOXX50}$
$R_t^{S\&P500}$	1	0.99	0.84	0.97	0.66	0.57	0.60
$R_t^{S\&P100}$	0.99	1	0.84	0.97	0.65	0.56	0.59
$R_t^{Nasdaq100}$	0.84	0.84	1	0.76	0.53	0.43	0.46
R_t^{DJIA}	0.97	0.97	0.76	1	0.65	0.57	0.60
R_t^{DAX}	0.66	0.65	0.53	0.65	1	0.88	0.92
R_t^{CAC40}	0.57	0.56	0.43	0.57	0.88	1	0.98
$R_t^{DJEUROSTOXX50}$	0.60	0.59	0.46	0.60	0.92	0.98	1

Panel B: Implied Volatility Changes								
	ΔVIX_t	ΔVXO_t	ΔVXN_t	ΔVXD_t	$\Delta VDAX_t$	$\Delta VX1_t$	$\Delta VX6_t$	$\Delta VSTOXX_t$
ΔVIX_t	1	0.92	0.67	0.86	0.62	0.28	0.28	0.58
ΔVXO_t	0.92	1	0.65	0.82	0.61	0.26	0.31	0.57
ΔVXN_t	0.67	0.65	1	0.65	0.52	0.30	0.24	0.50
ΔVXD_t	0.86	0.82	0.65	1	0.60	0.25	0.25	0.58
$\Delta VDAX_t$	0.62	0.61	0.52	0.60	1	0.48	0.46	0.87
$\Delta VX1_t$	0.28	0.26	0.30	0.25	0.48	1	0.60	0.45
$\Delta VX6_t$	0.28	0.31	0.24	0.25	0.46	0.60	1	0.46
$\Delta VSTOXX_t$	0.58	0.57	0.50	0.58	0.87	0.45	0.46	1

Table 6: Pairwise correlations between the contemporaneous changes of the Implied Volatility Indices and correlations between the contemporaneous returns of the underlying Stock Indices. Values are estimated from the sample of synchronized data of the period 2/2/2001 – 3/17/2005.

Section 3
Leverage Effect

The negative correlation between stock index movements and implied volatility movements is often attributed to the leverage effect. When the stock price of a company falls, with dept remaining at the same levels, the company becomes more levered and more risk is associated with stock holding. This explanation of the phenomenon of volatility increasing as the stock prices fall, applied to individual stocks, is theoretically extended to Stock Indexes.

The “leverage effect” explanation has been called into question since several anomalies have been associated with it. For example the effect is different for implied and historical volatilities. It is also different for falling versus rising markets, in that it is much weaker or nonexistent when positive stock returns reduce leverage. As Figlewski observes, “A fall in the market price for the stock should increase its subsequent volatility, and a price rise of the same magnitude should reduce volatility by a comparable amount. However, the existence of a "leverage effect" is most commonly associated with falling, rather than rising, stock prices. This raises the question of whether it may be an asymmetrical phenomenon more closely related to negative returns than to leverage per se.”[6] Figlewski concludes that the “leverage effect" is really a "down market effect" that may have little direct connection to firm leverage.”

3.1. Testing the leverage effect

Giot (2005a) examines the existence of a leverage effect for the Nasdaq100 index and VXN and the S&P100 and (the old) VIX, by regressing the stock index log differences on log differences of the relative implied volatility index. We follow Whaley’s (2000) and Skiadopoulos (2004) methodology in that we regress the log returns of the stock indices on the changes of the volatility indices. We test the asymmetry of the leverage effect by adding the positive changes of the IV index as a second regressor.

$$R_t = a_1 \Delta IV_t + a_2 \Delta IV_t^+ + e_t \tag{1}$$

In the above equation, R_t denotes the stock index daily returns, ΔIV_t the daily changes of the relevant implied volatility index and ΔIV_t^+ the positive changes of the IV index. This means that $\Delta IV^+ = \Delta IV$ if ΔIV >0 and $\Delta IV^+ = 0$, otherwise.

Results from Least Squares regressions are presented in Table 7. For all regressions, we have tested the existence of serial correlation and heteroscedasticity of some unknown form in the residuals, using the Breusch-Godfrey LM test and White’s test respectively. We found significant t-statistics for both tests. For this reason, the significance of the estimated coefficients has been tested using Newey-West autocorrelation and

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[6] See Figlewski and Wang (2000).

heteroscedasticity consistent standard errors. As shown in Table 7, the adjusted R-squared are quite high varying from 26% for the VX1 regression to 55% for the VSTOXX regression. The coefficients of the IV changes are all negative and significant at the 1% significance level, verifying, in line with the related literature, the existence of the leverage effect. The leverage effect however seems to be asymmetric only for the French indices. In particular the coefficient of the VX1 positive changes is found significant at the 1%, whereas that of VX6 positive changes is significant at the 10% level. They are both negative indicating an intensification of the negative relationship between stock returns and IV changes, in the case of an upward movement of the implied volatility index. The general non-verification of asymmetry in the leverage effect is in contrast to Whaley (2000) who reports a stronger reaction of VIX changes to a negative return of S&P100 than to positive one. Our results do not support the characterization neither of the American IV indices nor of VDAX and VSTOXX as investor's fear gauges.

3.2. Stability of the Leverage effect

3.2.1. Testing for a structural break at the 11th September 2001

First we test the stability of the leverage effect assuming a possible structural break at the time of the 11th September 2001 attack. For this purpose, we add a dummy variable D in the regression we used to test the leverage effect. The dummy variable takes the value of one if $t > 9/11/2001$ and the value of zero otherwise.

$$R_t = a_1 \Delta IV_t + a_2 \Delta IV_t^+ + b_1 D \Delta IV_t + b_2 D \Delta IV_t^+ + e_t \quad (2)$$

Table 8 presents the estimated coefficients, the t-statistics, and adjusted R-squared and the Durbin-Watson statistics for each stock index regression. T-statistics have been computed with Newey-West autocorrelation and heteroscedasticity consistent standard errors. Although the Durbin Watson statistic does not indicate first order autocorrelation in the residuals, we take into account autocorrelations of higher order.

The adjusted R-squared statistics indicate a fit similar to that found in the regressions of the leverage effect alone. Only for the S&P100 returns, there is a notable increase from 45% to 49%. The coefficients of the IV changes after 9/11/2001 are found significant at the 1% level for the S&P500, the S&P100 and the DJIA returns and at 5% for the DAX returns. The negative sign of these coefficients indicates an intensification of the leverage effect after the 11th September attack, for the three American indices and the German index. This means that 1% raise of the implied volatility index after the 11th September, is associated with a larger drop of the underlying stock index, than it did before. On the contrary, the risk-return relationship for the French indices, for DJEUROSTOXX and for Nasdaq-100 has not been affected. All coefficients of positive IV changes after 9/11/2001 have been found insignificant, indicating that asymmetric features in the leverage effect have not been developed or intensified after 9/11/2001.

Table 7
Leverage Effect

Dependent Variable ↓	ΔIV_t	ΔIV_t^+	Adj. R^2	D-W
$R_t^{S\&P500}$	-0.0062*	0.0003	0.53	2.08
	(-29.45)	(1.10)		
$R_t^{S\&P100}$	-0.0034*	0.0003	0.45	2.02
	(-3.41)	(0.45)		
$R_t^{Nasdaq100}$	-0.0087*	-0.0011	0.36	2.11
	(-12.54)	(-1.20)		
R_t^{DJIA}	-0.0073*	0.0007	0.51	2.15
	(-24.82)	(1.41)		
R_t^{DAX}	-0.0082*	0.0004	0.41	2.16
	(-16.77)	(0.66)		
$R_t^{CAC40}(VX1)$	-0.0017*	-0.0008*	0.26	2.03
	(-6.03)	(-3.23)		
$R_t^{CAC40}(VX6)$	-0.0037*	-0.0009***	0.32	1.97
	(-8.71)	(-1.954)		
R_t^{STOXX}	-0.0076*	0.0010	0.55	1.92
	(-19.15)	(1.59)		

Table 7: Test for the existence of an asymmetric leverage effect. Reported values are the estimated coefficients from the regression $R_t = a_1 \Delta IV_t + a_2 \Delta IV_t^+ + e_t$, where ΔIV_t and ΔIV_t^+ denote the changes and the positive changes respectively of the Implied Volatility Index and R_t the returns of the underlying Stock Index at time t. Values in parentheses are t-statistics computed with Newey-West autocorrelation and heteroscedasticity consistent standard errors. The Adjusted R-squared (Adj. R^2) and the Durbin-Watson statistics are also reported. One asterisk denotes rejection of the null hypothesis of a zero coefficient at the 1% significance level, two asterisks rejection of the null at the 5% level and three asterisks rejection of the null at the 10% level.

Table 8
Impact of the September 11th, 2001, on the leverage effect

Dependent Variable ↓	ΔIV_t	ΔIV_t^+	$D\Delta IV_t$	$D\Delta IV_t^+$	Adj. R^2	D-W
$R_t^{S\&P500}$	-0.0058*	0.0002	-0.0019*	0.0002	0.54	2.07
	(-25.15)	(0.73)	(-4.21)	(0.44)		
$R_t^{S\&P100}$	-0.0030*	0.0001	-0.0042*	0.0003	0.49	2.01
	(-3.41)	(0.22)	(-4.33)	(0.49)		
$R_t^{Nasdaq100}$	-0.0081*	-0.0037***	-0.0007	0.0035	0.36	2.11
	(-5.72)	(-1.648)	(-0.43)	(1.42)		
R_t^{DJIA}	-0.0068*	0.0006	-0.0014**	0.00003	0.51	2.14
	(-21.40)	(1.22)	(-2.26)	(0.03)		
R_t^{DAX}	-0.0066*	0.0002	-0.0040*	0.0003	0.43	2.12
	(-12.88)	(0.31)	(-4.75)	(0.18)		
$R_t^{CAC40}(VX1)$	-0.0015*	-0.0008*	-0.0006	-0.0003	0.27	2.03
	(-5.77)	(-3.00)	(-0.78)	(-0.46)		
$R_t^{CAC40}(VX6)$	-0.0039*	-0.0009	0.0003	0.0001	0.32	1.98
	(-5.49)	(-1.31)	(0.33)	(0.12)		
$R_t^{DJESTOXX50}$	-0.0083*	0.0031*	0.0009	-0.0035	0.56	1.93
	(-18.24)	(2.93)	(1.27)	(-2.61)		

Table 8: Test for the impact of the September 11th, 2001 attack on the leverage effect. Reported values are the estimated coefficients from the regression $R_t = a_1 \Delta IV_t + a_2 \Delta IV_t^+ + b_1 D\Delta IV_t + b_2 D\Delta IV_t^+ + e_t$, where ΔIV_t and ΔIV_t^+ denote the changes and the positive changes respectively of the Implied Volatility Index and R_t the returns of the underlying Stock Index at time t. D is a dummy variable which takes the value of one if $t > 9/11/2001$ and the value of zero otherwise. Values in parentheses are t-statistics computed with Newey-West autocorrelation and heteroscedasticity consistent standard errors. The Adjusted R-squared (Adj. R^2) and the Durbin-Watson (D-W) statistics are also reported. One asterisk denotes rejection of the null hypothesis of a zero coefficient at the 1% significance level, two asterisks rejection of the null at the 5% level and three asterisks rejection of the null at the 10% level.

3.2.2. CUSUM Test

We also test the stability of the leverage effect applying the CUSUM test, at the 5% significance level, for the coefficients of both regression (1) and regression (2). The CUSUM test is based on the cumulative sum of the recursive residuals, and is appropriate when there is uncertainty about when a structural change might have taken place. The t recursive residual is the scaled one step ahead forecast error for the dependent variable in period t, using the vector of coefficients estimated by the first t-1 observations. The one step ahead forecast error is defined as:

$$e_t = y_t - x_t^{'} b_{t-1}$$

where y_t is the observation for the dependent variable in period t, x_t the vector of observations for the regressors in period t, and b_{t-1} the estimated coefficient vector using the $t-1$ observations. Its forecast variance is given by:

$$s_{ft}^{2} = s^{2}\left[1 + x_t^{'}\left(X_{t-1}^{'} X_{t-1}\right)^{-1} x_t\right]$$

where X_{t-1} is the matrix of regressors from period 1 to period t-1. The recursive residual w_t is defined as:

$$w_t = \frac{e_t}{\sqrt{1 + x_t^{'}\left(X_{t-1}^{'} X_{t-1}\right)^{-1} x_t}}$$

Under the null hypothesis that the coefficients of the regression remain constant over the sample period, the recursive residuals will be independently and normally distributed with zero mean and constant variance, $w_t \sim N\left(0, s^{2}\right)$. We could test the stability of coefficients if we plot $w_t / \hat{s}$, where

$$\hat{s}^{2} = \frac{\sum_{t=k+1}^{T} \left(w_t - \overline{w}\right)^2}{T-k-1}$$

T is the total number of observations and k the number of regressors, which is also the number of observations used to form the first estimate of the coefficient vector. Under the null hypothesis, $\left(w_t / \hat{s}\right) \sim N\left(0,1\right)$.The CUSUM test is based on the statistic:

$$W_t = \sum_{t=k+1}^{t} w_t / \hat{s}$$

Under the null hypothesis that the vector of coefficients remains stable over time, $W_t \sim N(0, T-k)$. That is, its variance is approximately equal to the number of residuals being summed since each has variance 1 and they are independent. The test plots W_t against t. For stability of coefficients, W_t should not significantly depart from the zero line.

Confidence bounds are obtained by a pair of 5% significance lines, which are found by connecting the points $\left[k, \pm -0.948\sqrt{T-k}\right]$ and $\left[T, \pm 3\times 0.948\sqrt{T-k}\right]$. If W_t strays outside these boundaries, then the null is rejected at the 5% level.

According to CUSUM test results, presented in Figures 9-24, the coefficients for the Nasdaq-100 – VXN, the DJIA – VXD and the CAC40 – VX1 and VX6 relationships remain stable over the entire sample period. On the contrary, the coefficients for S&P500 – VIX, the S&P100 – VXO and the DJEUROSTOXX50 – VSTOXX relationship are stabilized after the 11th September 2001.

Figures 9 - 16
Stability of the Leverage Effect for the American Indices

Fig. 9: S&P500 – VIX
(1/2/1990 – 3/17/2005)

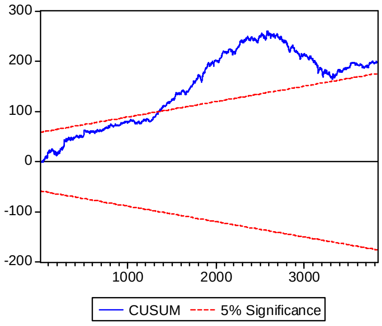


Fig. 10: S&P500 – VIX
(9/11/2001 - 3/17/2005)

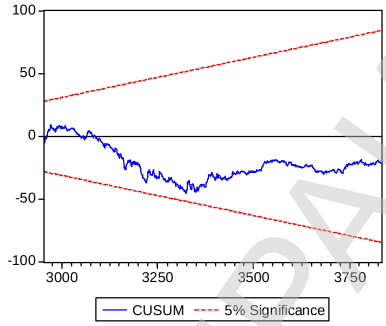


Fig. 11: S&P100 – VXO
(1/2/1986 – 3/17/2005)

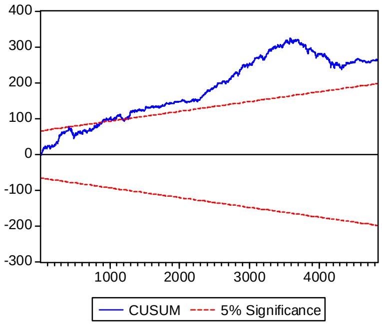


Fig. 12: S&P100 – VXO
(9/11/2001 - 3/17/2005)

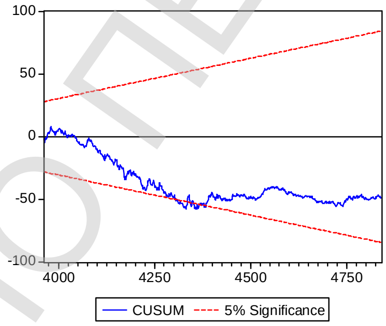


Fig. 13: Nasdaq100 – VXN
(2/2/2001 – 3/17/2005)

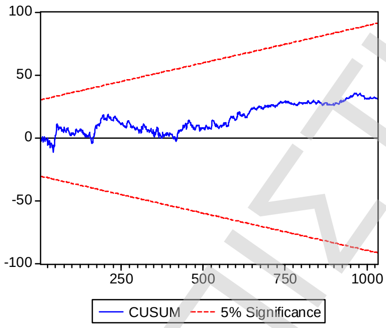


Fig. 14: Nasdaq100 – VXN
(9/11/2001 - 3/17/2005)

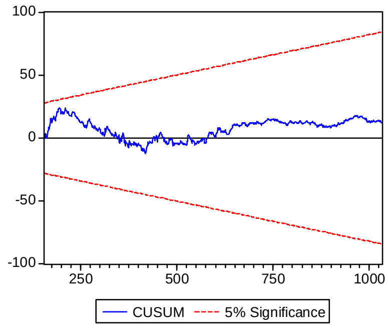


Fig. 15: DJIA – VXD
(10/6/1997 – 3/17/2005)

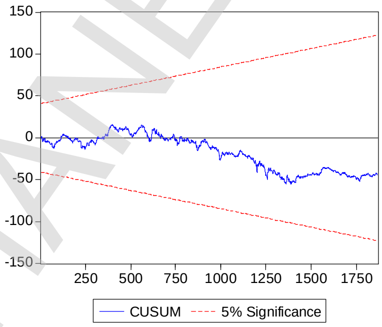
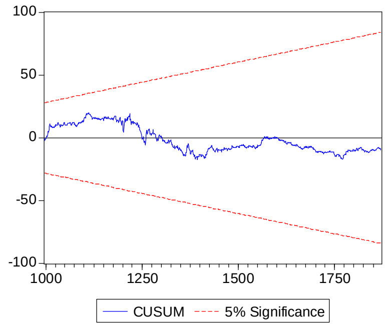


Fig. 16: DJIA – VXD
(9/11/2001 - 3/17/2005)



Figures 6 – 16: CUSUM Test of the stability of the leverage effect over time for the American Indices.

Figures 17 - 24
Stability of the Leverage Effect for the European Indices

Fig. 17: DAX – VDAX
(1/2/1992 – 3/17/2005)

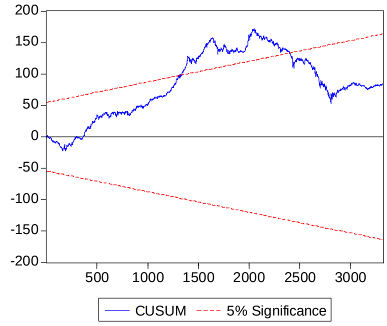


Fig. 18: DAX – VDAX
(9/11/2001 - 3/17/2005)

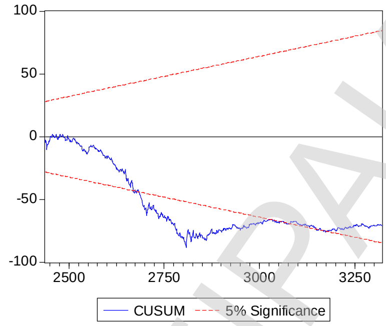


Fig. 19: CAC40 – VX1
(10/14/1997 – 3/17/2005)

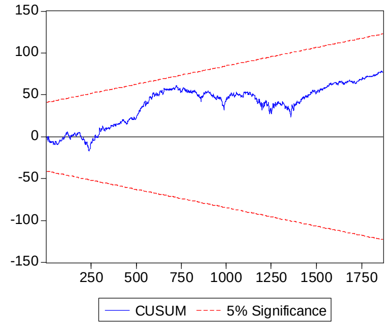


Fig. 20: CAC40 – VX1
(9/11/2001 - 3/17/2005)

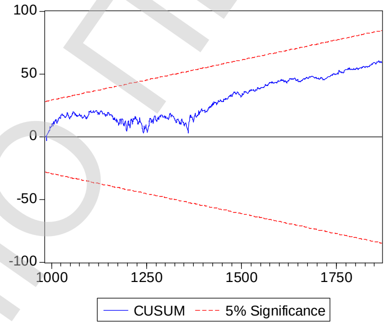


Fig. 21: CAC40 – VX6
(10/14/1997 – 3/17/2005)

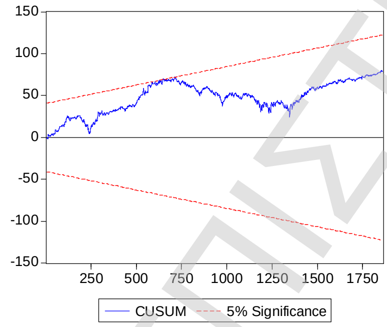


Fig. 22: CAC40 – VX6
(9/11/2001 - 3/17/2005)

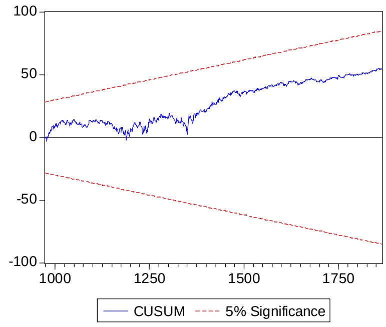


Fig. 23: DJ EUROSTOXX 50 – VSTOXX
(1/4/1999 – 3/17/2005)

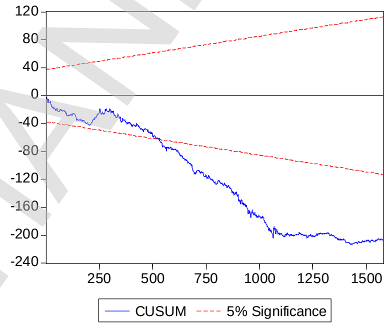
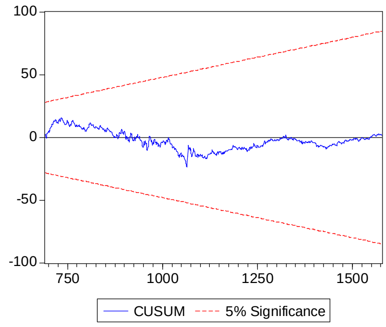


Fig. 24: DJ EUROSTOXX 50 – VSTOXX
(9/11/2001 - 3/17/2005)



Figures 17-24: CUSUM Test of the stability of the leverage effect over time, for the European Indices.

Section 4
Granger Causality

In this section we examine whether the movements of implied volatility indices could be used to forecast the future movements of the underlying stock indices, and vice versa. A proper tool to test this, are Granger Causality (GC) tests. Causality in the sense defined by Granger (1969) and Sims (1972) is inferred when lagged values of a variable, x_t, have explanatory power in a regression of a variable y_t on lagged values of y_t and x_t. Granger Causality implies precedence and information content and not causation in the common use of the term. Granger non-causality is another name for strong exogeneity. That is, lagged values of x do not provide information about the conditional mean of y once lagged values of y itself are accounted for:

$$E\left[y_t \middle| y_{t-1}, x_{t-1}, x_{t-2}, ...\right] = E\left[y_t \middle| y_{t-1}\right]$$

The null hypothesis that is tested is that "x does not Granger cause y". We run Pairwise Granger Causality tests for the Implied Volatility indices daily changes, ΔIV, and the stock indices returns, R. This means we test two hypotheses, namely that "ΔIV do not Granger cause R" and that "R do not Granger cause ΔIV". The Pairwise test is based on bivariate regressions of the form:

$$R_t = c + \sum_{i=1}^{l} a_i R_{t-i} + \sum_{i=1}^{l} b_i \Delta IV_{t-i} + e_{1t}$$

$$\Delta IV_t = c + \sum_{i=1}^{l} a_i \Delta IV_{t-i} + \sum_{i=1}^{l} b_i R_{t-i} + e_{2t}$$

An F-statistic is reported for each equation. It is Wald statistic for the joint hypothesis:

$$b_1 = b_2 = ... = b_l = 0$$

l is the lag length that corresponds to reasonable beliefs about the longest time over which the variables could help predict one another.

The methodology we use is to split the sample in two. The first part will be used for testing the existence of granger causality, and for in-sample fitting of the parameters, and the second part will be used for out-of-sample forecasting evaluation. We choose to leave the last year of daily observations (3/18/2004 – 3/17/2005) for out-of-sample forecasting. That way we have at least three years (around 1000 daily observations) for in-sample fitting. The in-sample period varies from approximately 3 years (2/2/2001 – 3/17/2004) for VXN, to 18 years (1/2/1986 – 3/17/2004) for VXO.

4.1. Pairwise Granger Causality Tests

First we run the Pairwise Granger causality tests using various lag orders from 2 to 100 lags and test the null at the 1% and 5% significance level. Table 9 presents the F-statistics as well as the corresponding probabilities from the tests. First we focus on the power of lagged values of stock returns to predict the next day's change of the relevant IV index. A general observation is that lagged values of the returns of the European stock indices and the returns of S&P100 are informative of next day's IV index change at all lag orders. On the contrary, the returns of Nasdaq100 do not Granger cause VXN changes, at least up to the 12th lag. The same applies for the returns of S&P500 and DJIA up to the 4th lag.

Turning to the predictive power of IV changes for next day's stock index returns, we observe that VXN and VXD changes do not Granger cause the underlying index returns at no lag order. The same almost applies to VX6 changes for which Granger non-causality is rejected only at the 8th and 10th lag order. The changes of VDAX and VSTOXX do not Granger cause the underlying stock index returns at least up to the 8th lag order, whereas changes of VIX and VX1 do not provide information for next day's returns at least up to the 5th and 4th lag, respectively. Only the changes of VXO seem to be informative of next day's returns of S&P100 at all lag orders.

The drawback of the Granger Causality Test is that it tends to reject the null hypothesis of Granger non-causality. A finding of causal effects might result from the omission of an intervening variable that is correlated with both of (or all) the left-hand-side variables. For this reason we will verify results from GC tests in a Least Squares regression framework.

4.2. Information Criteria for lag length specification

One question that rises is how many lags, l, to use in order to test in a regression framework the results of Granger Causality tests. We follow the general to specific approach.[7] We suppose there is an appropriate "true" value of l, that we seek. A general-to-simple approach would begin from a model that contains more than l lagged values – it is assumed that though the precise value of l, is unknown, one can posit a maintained value that should be larger than l,. So, if some maximum L is known, then $l < L$ can be chosen to minimize some measure for assessing "out of sample" prediction properties, as the Akaike information criterion, AIC(l), of the Schwartz criterion, SC(l). We must bear in mind the AIC has been seen to overfitting, whereas SC to underfitting in some finite sample cases. For this reason, we will weight the results of the two criteria to select an optimal lag order. We posit 100 days as a maximum lag order.

[7] Greene, 2003, pp. 564 – 565.

Table 9
Granger Causality Tests

Panel A: American Indices																
Null Hypothesis	Lags	2	3	4	5	6	7	8	9	10	12	15	20	40	60	100
$R^{S\&P500}$ does not	F-Statistic	0.61	1.17	1.58	4.01*	2.83**	2.58**	2.11**	1.91	1.82	2.86*	3.38*	3.09*	2.04*	1.88*	1.55*
G.C. ΔVIX	Prob.	(0.55)	(0.32)	(0.18)	(0.00)	(0.01)	(0.01)	(0.03)	(0.05)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ΔVIX does not G.C.	F-Statistic	3.60**	2.35	1.87	2.02	2.47**	2.70**	2.46**	2.25**	2.01**	2.04**	2.01**	2.14*	1.41	1.25	1.28**
$R^{S\&P500}$	Prob.	(0.03)	(0.07)	(0.11)	(0.07)	(0.02)	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)	(0.01)	(0.00)	(0.05)	(0.10)	(0.04)
$R^{S\&P100}$ does not	F-Statistic	11.23*	8.46*	10.78*	14.79*	12.38*	10.15*	10.76*	10.57*	10.40*	9.26*	7.82*	6.87*	3.99*	3.24*	2.23*
G.C. ΔVXO	Prob.	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ΔVXO does not G.C.	F-Statistic	19.40*	17.55*	14.16*	14.20*	11.82*	10.14*	10.15*	9.92*	10.28*	8.66*	7.18*	5.68*	4.22*	3.70*	2.74*
$R^{S\&P100}$	Prob.	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$R^{Nasdaq100}$ does not	F-Statistic	1.31	1.23	1.18	0.92	0.74	0.91	0.93	1.60	1.42	1.87**	1.68	2.87*	2.13*	1.44**	1.54*
G.C. ΔVXN	Prob.	(0.27)	(0.30)	(0.32)	(0.47)	(0.62)	(0.50)	(0.49)	(0.11)	(0.17)	(0.04)	(0.05)	(0.00)	(0.00)	(0.02)	(0.00)
ΔVXN does not G.C.	F-Statistic	1.98	1.36	1.73	1.84	1.53	1.30	1.11	1.29	1.09	0.86	0.81	1.30	1.35	0.97	1.15
$R^{Nasdaq100}$	Prob.	(0.14)	(0.25)	(0.14)	(0.10)	(0.16)	(0.25)	(0.36)	(0.24)	(0.37)	(0.59)	(0.67)	(0.17)	(0.08)	(0.55)	(0.17)
R^{DJIA} does not G.C.	F-Statistic	2.17	1.63	1.76	2.65**	2.12	2.16**	1.90	2.12**	2.03**	2.11**	1.97**	2.34*	1.48**	1.26	1.13
ΔVXD	Prob.	(0.12)	(0.18)	(0.14)	(0.02)	(0.05)	(0.03)	(0.06)	(0.02)	(0.03)	(0.01)	(0.02)	(0.00)	(0.03)	(0.09)	(0.19)
ΔVXD does not G.C.	F-Statistic	0.85	1.05	1.14	1.20	1.64	1.59	1.35	1.29	1.14	1.17	1.28	1.82**	1.40	1.23	0.97
R^{DJIA}	Prob.	(0.43)	(0.37)	(0.34)	(0.31)	(0.13)	(0.13)	(0.21)	(0.24)	(0.33)	(0.30)	(0.21)	(0.02)	(0.05)	(0.12)	(0.57)

Table 9: Pairwise Granger Causality Tests between the Stock Indices Returns and the Implied Volatility Indices Changes, using various lag orders from 2 to 100 lags. Results are based on the samples of the indices, after we have excluded the last year of observations (3/18/04 – 3/17/05) for out-of-sample forecasting. Reported values are F-statistics whereas numbers in parentheses are p-values. One asterisk denotes rejection of the null hypothesis of no Granger Causality at the 1% significance level and two asterisks rejection at the 5% significance.

Table 9 (Continued)
Granger Causality Tests

Null Hypothesis	Lags	2	3	4	5	6	7	8	9	10	12	15	20	40	60	100
Panel B: European Indices																
R^{DAX} does not G.C. $\Delta VDAX$	F-Statistic	15.59*	10.46*	7.85*	6.52*	6.40*	5.73*	5.10*	5.58*	5.31*	4.63*	3.85*	3.11*	2.32*	2.43*	1.81*
	Prob.	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$\Delta VDAX$ does not G.C. R^{DAX}	F-Statistic	2.11	1.44	1.98	1.89	1.48	1.79	1.86	2.82*	2.59*	2.21**	2.11**	2.13*	2.14*	2.21*	1.68*
	Prob.	(0.12)	(0.23)	(0.09)	(0.09)	(0.18)	(0.08)	(0.06)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)
R^{CAC40} does not G.C. $\Delta VX1$	F-Statistic	6.51*	7.74*	6.30*	5.84*	6.13*	5.98*	4.91*	4.52*	4.19*	3.66*	2.98*	2.33*	1.58**	1.24	1.03
	Prob.	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.11)	(0.40)
$\Delta VX1$ does not G.C. R^{CAC40}	F-Statistic	1.49	1.68	2.20	2.83**	2.92**	4.04*	4.13*	3.70*	3.41*	2.94*	2.84*	2.73*	2.04*	1.61*	1.38**
	Prob.	(0.23)	(0.17)	(0.07)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
R^{CAC40} does not G.C. $\Delta VX6$	F-Statistic	3.93**	4.45*	5.68*	6.48*	5.52*	4.51*	3.93*	3.68*	3.19*	3.01*	2.89*	2.46*	2.00*	1.52**	1.35**
	Prob.	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)
$\Delta VX6$ does not G.C. R^{CAC40}	F-Statistic	1.09	2.21	2.06	2.25	2.32**	1.93	1.99**	1.74	1.61	1.48	1.40	1.32	1.05	0.96	0.92
	Prob.	(0.34)	(0.08)	(0.08)	(0.05)	(0.03)	(0.06)	(0.04)	(0.08)	(0.10)	(0.13)	(0.14)	(0.16)	(0.38)	(0.56)	(0.69)
$R^{DJESTOXX50}$ does not G.C. $\Delta VSTOXX$	F-Statistic	0.69	3.59**	3.01**	4.38*	3.98*	3.76*	3.79*	3.61*	3.30*	2.84*	2.35*	1.89**	1.54**	1.19	1.05
	Prob.	(0.50)	(0.01)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)	(0.16)	(0.35)
$\Delta VSTOXX$ does not G.C. $R^{DJESTOXX50}$	F-Statistic	0.55	1.87	1.50	1.40	1.78	1.36	1.75	2.61**	2.18**	1.75	1.81**	1.40	1.80*	1.54**	1.20
	Prob.	(0.56)	(0.13)	(0.20)	(0.22)	(0.10)	(0.22)	(0.08)	(0.01)	(0.02)	(0.05)	(0.03)	(0.11)	(0.00)	(0.01)	(0.10)

Table 9: Pairwise Granger Causality Tests between the Stock Indices Returns and the Implied Volatility Indices Changes, using various lag orders from 2 to 100 lags. Results are based on the samples of the indices, after we have excluded the last year of observations (3/18/04 – 3/17/05) for out-of-sample forecasting. Reported values are F-statistics whereas numbers in parentheses are p-values. One asterisk denotes rejection of the null hypothesis of no Granger Causality at the 1% significance level and two asterisks rejection at the 5% significance level.

Table 10 shows the values of the two information criteria as well as the adjusted R-squared statistics for the purpose of lag length specification in the Granger causality regressions. The information criteria and the adjusted R-squared have been obtained by running 10 bivariate Vector Autoregressions (VAR) for each pair of indices, each of a different lag order, namely for $l = 2, 4, 6, 8, 10, 15, 20, 40, 60, 100$.

The VAR specification is of the form:

$$Y_t = C + \sum_{i=1}^{l} A_i Y_{t-i} + e_t$$

where $Y_t = \begin{bmatrix} R_t \\ \Delta IV_t \end{bmatrix}$, C is a (2×1) vector of constants, and A_i a (1×2) vector of coefficients.

Values of the Akaike and Schwarz Information Criteria as well as the adjusted R-squared statistics reported in a VAR framework for each equation are the same as the ones reported in a single equations framework. However, a VAR framework is convenient only at this stage. After we will have chosen the optimal lag order, we will test the significance of each coefficient in a single equation framework. A single equation framework is more flexible, especially if Granger causality runs one way only, which is the case, as we have seen from the results of Granger causality tests.

Results for lag length selection, as presented in Table 10, show in the first place that the adjusted R-squared statistics for the regressions with the stock index returns as dependent variable are very low varying from 0% to 6%. We obtain significantly better results for regressions with the IV index changes as dependent, especially for VXO, VX1 and VX6 changes for which the adjusted R-squared statistics average at 11%, 25% and 23% respectively. However, the adjusted R-squared for the regressions of the rest of the IV indices as dependents remain at the level of around 3%. Another important remark is that the adjusted R-squared statistics increase only very slightly as the lag order increases, or remain invariable. This means that the addition of lags does not increase the explanatory power of the regressors. An exception seems to be VXN for which the statistic takes the value of zero up to the 4th lag order and reaches the level of 12% at the 40th order and the level of 17% with a specification of 100 lags.

In the same table, we see that the Schwarz Information Criterion generally takes its minimum at the 2nd lag order, except in the regressions with the VX1, the VX6 and the VSTOXX changes as dependents, where it chooses as optimal the 4th lag order. The Akaike IC in most cases chooses between the 8th and the 20th lag order. AIC takes its minimum in a lower order in the cases of DJIA returns and DAX returns, where it chooses the 2nd and the 4th lag order respectively. An order of 40 lags is chosen by the AIC for the VXN changes.

Table 10
Information Criteria for Lag Length specification in Granger Causality regressions

Panel A: American Indices											
Dependent Variable	**Lags**	**2**	**4**	**6**	**8**	**10**	**15**	**20**	**40**	**60**	**100**
ΔVIX	Adj. R^2	0.01	0.02	0.02	0.03	0.03	0.04	0.05	0.05	0.06	0.06
	AIC	3.30	3.29	3.29	3.28	3.28	3.26*	3.26	3.27	3.27	3.29
	SIC	3.30*	3.31	3.31	3.31	3.32	3.32	3.33	3.41	3.49	3.64
$R^{S\&P500}$	Adj. R^2	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.02	0.02
	AIC	-6.276	-6.275	-6.276	-6.279*	-6.277	-6.278	-6.277	-6.260	-6.250	-6.230
	SIC	-6.27*	-6.26	-6.25	-6.25	-6.24	-6.23	-6.20	-6.12	-6.04	-5.87
ΔVXO	Adj. R^2	0.08	0.09	0.10	0.11	0.12	0.12	0.13	0.13	0.14	0.14
	AIC	4.59	4.58	4.57	4.56	4.55*	4.55	4.55	4.56	4.56	4.59
	SIC	4.60	4.60	4.59	4.59	4.58*	4.60	4.61	4.67	4.73	4.87
$R^{S\&P100}$	Adj. R^2	0.01	0.02	0.02	0.02	0.02	0.02	0.03	0.04	0.05	0.05
	AIC	-6.030	-6.038	-6.039	-6.039	-6.042*	-6.041	-6.039	-6.040	-6.040	-6.020
	SIC	-6.03*	-6.03	-6.02	-6.02	-6.01	-6.00	-5.98	-5.93	-5.87	-5.73
ΔVXN	Adj. R^2	0.00	0.00	0.02	0.02	0.03	0.04	0.09	0.12	0.10	0.17
	AIC	3.84	3.85	3.84	3.84	3.84	3.84	3.81	3.79*	3.83	3.79
	SIC	3.87*	3.90	3.92	3.94	3.97	4.03	4.06	4.29	4.59	5.13
$R^{Nasdaq100}$	Adj. R^2	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.03	0.01	0.06
	AIC	-4.51	-4.50	-4.50	-4.49	-4.49	-4.49	-4.50	-4.52	-4.54*	-4.54
	SIC	-4.48*	-4.45	-4.42	-4.39	-4.37	-4.30	-4.26	-4.01	-3.78	-3.20
ΔVXD	Adj. R^2	0.00	0.01	0.02	0.02	0.02	0.03	0.05	0.06	0.06	0.05
	AIC	3.34	3.34	3.33	3.34	3.34	3.31	3.29*	3.30	3.32	3.38
	SIC	3.36*	3.37	3.38	3.39	3.41	3.41	3.43	3.57	3.73	4.09
R^{DJIA}	Adj. R^2	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.02	0.01
	AIC	-5.91*	-5.90	-5.90	-5.90	-5.90	-5.90	-5.90	-5.88	-5.85	-5.78
	SIC	-5.89*	-5.87	-5.86	-5.84	-5.82	-5.79	-5.76	-5.61	-5.44	-5.08

Table 10: Information Criteria for lag length specification in the Granger Causality regressions. We report the Akaike (AIC) and the Schwarz (SIC) Information Criteria as well as the Adjusted R-squared (Adj. R^2) from the regressions: $\Delta IV_t = c + \sum_{i=1}^{l} a_i \Delta IV_{t-i} + \sum_{i=1}^{l} b_i R_{t-i} + e_t$ and $R_t = c + \sum_{i=1}^{l} a_i R_{t-i} + \sum_{i=1}^{l} b_i \Delta IV_{t-i} + e_t$, where R denotes the Stock Index daily returns, and ΔIV the Implied Volatility Index daily changes. Regressions are run in a bivariate Vector Autoregression Setting, using various lag orders from 2 to 100 lags. One asterisk denotes lag order selection defined by the minimum of the information criterion. Results are based on the samples of the indices, after we have excluded the last year of observations (3/18/04 – 3/17/05) for out-of-sample forecasting.

Table 10 (Continued)
Information Criteria for Lag Length specification in Granger Causality regressions

Panel B: European Indices											
Dependent Variable	**Lags**	**2**	**4**	**6**	**8**	**10**	**15**	**20**	**40**	**60**	**100**
$\Delta VDAX$	Adj. R^2	0.01	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.06	0.06
	AIC	3.179	3.180	3.176	3.178	3.175*	3.179	3.183	3.190	3.190	3.230
	SIC	3.19*	3.20	3.20	3.21	3.22	3.24	3.26	3.35	3.43	3.64
R^{DAX}	Adj. R^2	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.03	0.05	0.05
	AIC	-5.57	-5.60*	-5.57	-5.58	-5.58	-5.57	-5.57	-5.57	-5.56	-5.53
	SIC	-5.56*	-5.55	-5.55	-5.54	-5.53	-5.51	-5.49	-5.41	-5.32	-5.13
$\Delta VX1$	Adj. R^2	0.22	0.25	0.25	0.25	0.26	0.26	0.27	0.27	0.27	0.27
	AIC	5.361	5.337	5.335	5.333	5.333	5.329	5.325*	5.360	5.390	5.460
	SIC	5.38	5.37*	5.38	5.39	5.40	5.43	5.46	5.63	5.81	6.17
R^{CAC40}	Adj. R^2	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.04	0.04	0.03
$(\Delta VX1)$	AIC	-5.36	-5.36	-5.36	-5.37	-5.37	-5.38	-5.39*	-5.36	-5.33	-5.26
	SIC	-5.34*	-5.33	-5.32	-5.32	-5.30	-5.28	-5.25	-5.08	-4.91	-4.55
$\Delta VX6$	Adj. R^2	0.19	0.23	0.23	0.23	0.24	0.24	0.25	0.26	0.25	0.26
	AIC	4.21	4.16	4.16	4.16	4.16	4.15*	4.16	4.17	4.21	4.28
	SIC	4.22	4.19*	4.20	4.22	4.23	4.26	4.30	4.45	4.63	4.99
R^{CAC40}	Adj. R^2	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.01
$(\Delta VX6)$	AIC	-5.36	-5.36	-5.36	-5.36	-5.36	-5.38*	-5.37	-5.34	-5.31	-5.24
	SIC	-5.34*	-5.34	-5.32	-5.30	-5.29	-5.27	-5.23	-5.06	-4.89	-4.53
$\Delta VSTOXX$											
	Adj. R^2	0.00	0.01	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05
	AIC	3.94	3.92	3.91	3.90*	3.90	3.91	3.91	3.94	3.98	4.05
	SIC	3.96	3.95*	3.96	3.97	3.99	4.03	4.08	4.26	4.47	4.90
$R^{DJESTOXX\,50}$											
	Adj. R^2	0.00	0.00	0.02	0.02	0.03	0.03	0.03	0.05	0.06	0.04
	AIC	-5.30	-5.31	-5.32	-5.32	-5.33*	-5.32	-5.31	-5.30	-5.27	-5.17
	SIC	-5.29*	-5.27	-5.27	-5.26	-5.24	-5.20	-5.15	-4.97	-4.78	-4.33

Table 10: Information Criteria for lag length specification in the Granger Causality regressions. We report the Akaike (AIC) and the Schwarz (SIC) Information Criteria as well as the Adjusted R-squared (Adj. R^2) from the regressions: $\Delta IV_t = c + \sum_{i=1}^{l} a_i \Delta IV_{t-i} + \sum_{i=1}^{l} b_i R_{t-i} + e_t$ and $R_t = c + \sum_{i=1}^{l} a_i R_{t-i} + \sum_{i=1}^{l} b_i \Delta IV_{t-i} + e_t$, where R denotes the Stock Index daily returns, ΔIV the Implied Volatility Index daily changes, and l the lag order. Regressions are run in a bivariate Vector Autoregression Setting, using various lag orders from 2 to 100 lags. One asterisk denotes lag order selection defined by the minimum of the information criterion. Results are based on the samples of the indices, after we have excluded the last year of observations (3/18/04 – 3/17/05) for out-of-sample forecasting.

4.3. Testing Granger Causality results in a regression setting

Following the above results, we choose to verify the findings from the Granger causality tests in a single regressions setting using 2, 4 and 6 lags. Thus, we use specifications of the form:

$$R_t = c + \sum_{i=1}^{l} a_i R_{t-i} + \sum_{i=1}^{l} b_i \Delta IV_{t-i} + e_t \qquad (3)$$

$$\Delta IV_t = c + \sum_{i=1}^{l} a_i \Delta IV_{t-i} + \sum_{i=1}^{l} b_i R_{t-i} + e_t \qquad (4)$$

For each regression we have performed the Breusch-Godfrey Serial Correlation LM Test and we have found evidence of autocorrelation, at the 5% level, for the VIX, the VXO, and the VX1 changes as well as the DAX, and the CAC40 returns. Hence, we test the significance of the coefficients using Newey-West autocorrelation and heteroscedasticity consistent standard errors.

Table 11 presents the results for regression (3). We observe that the adjusted R^2 are in most cases below 1% and coefficients are found non-significant even at the 10% significance level. The highest adjusted R-squared is reported for the S&P100 returns, namely 2%, with the IV coefficients significant at 1% for the first lag, and at 10% for the second and third lag. However, the explanatory power of the regressors is two small to be used for prediction. The results of regression (3) verify the Granger causality tests, according which, IV indices do not Granger cause stock index returns.

Results from regression (4) are presented in Table 12. It is notable that coefficients of the lagged values of the dependent are found significant for all IV changes, even in cases when lagged stock index returns are found insignificant. For all IV indices, an order of 6 lags yields better results that the orders of 2 and 4 lags. The highest adjusted R-squared are 10% for VXO changes, and 23% and 25% for the changes of VX6 and VX1 respectively. However the coefficients of the S&P100 returns are found insignificant at least up to the third lag. This means that only lagged values of VXO could be used to predict its own movement one day ahead. On the contrary today's return of DAX could have information content for tomorrow's change of VDAX. But that would be only when used with today's VDAX changes and their predictive power would be limited to 1%. The results imply that only the CAC40 returns have important information content for the VX1 and VX6 future changes and this does not extend over one day.

We have also regressed the IV changes on lagged stock index returns, and the stock index returns on lagged IV changes, for 2, 4 and 6 lags, without including lagged values of the dependent in the equations.

$$R_t = c + \sum_{i=1}^{l} a_i \Delta IV_{t-i} + e_t$$

$$\Delta IV_t = c + \sum_{i=1}^{l} a_i R_{t-i} + e_t$$

However, we observed that the adjusted R-squared fell dramatically when lagged values of the dependent were not included in regressions. An implication is that, in the cases where lagged values of the stock returns could help predict next day's movements of the corresponding IV index, that would be possible only in conjunction with lagged changes of the IV index. This applies notably to the use of CAC40 returns for next day's prediction of VX1 and VX6 movements.

Since the results we obtained for the French indices cannot be generalized, we do not proceed to out-of-sample forecasting.

Table 11
Test of Granger Causality in a regression setting
Information content of the history of IV changes for the underlying Stock Indices Returns

Dependent Variable	Lags	c	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6	Adj. R^2
$R_t^{S\&P500}$	2	0.0003**	-0.040	0.002					-0.0005	0.0003					0.001
	4	0.0004**	-0.039	0.0008	-0.043	-0.005			-0.0004	0.0003	-0.0001	-0.0001			0.001
	6	0.0003***	-0.038	0.007	-0.035	0.003	0.002	0.016	-0.0004***	0.0003	0.0000	0.0000	0.0004***	0.0004***	0.004
$R_t^{S\&P100}$	2	0.0004**	0.037	0.001					0.0005*	0.0004					0.01
	4	0.0004**	0.051	-0.0005	-0.002	-0.047			0.0006*	0.0004***	0.0003	-0.0001			0.02
	6	0.0004**	0.048	0.004	-0.011	-0.046	-0.062**	-0.008	0.0006*	0.0004***	0.0002***	-0.0001	-0.0004***	0.000	0.02
$R_t^{Nasdaq100}$	2	-0.0008	-0.017	-0.025					0.0003	0.001					0.007
	4	-0.0007	-0.016	-0.019	0.015	-0.044			0.0003	0.001	0.0002	-0.001			0.005
	6	-0.0006	-0.010	-0.021	0.005	-0.050	0.003	0.006	0.0005	0.001	0.0000	-0.001	0.0001	0.0002	0.004
R_t^{DJIA}	2	0.0002	-0.039	-0.037					-0.0004	-0.0002					-0.0008
	4	0.0002	-0.038	-0.033	0.028	0.050			-0.0004	-0.0001	0.0005	0.0004			-0.001
	6	0.0002	-0.040	-0.027	0.034	0.054	-0.014	0.035	-0.0004	-0.0000	0.0006	0.0004	0.0005	0.0007***	0.002
R_t^{DAX}	2	0.0003	-0.006	0.018					0.0004	0.0005					0.0008
	4	0.0003	-0.006	0.015	-0.033	-0.009			0.0004	0.0004	-0.0001	-0.0006			0.002
	6	0.0003	-0.008	0.015	-0.031	-0.011	-0.058***	-0.051***	0.0004	0.0004	0.0000	-0.0006	-0.0003	0.0000	0.005
R_t^{CAC40}	2	0.0001	0.013	-0.011					0.000	0.0002					0.0009
$(\Delta VX1)$	4	0.0001	0.020	0.004	-0.049	0.018			0.000	0.0004***	0.0001	0.0003			0.004
	6	0.0002	0.015	-0.001	-0.059	0.007	-0.091**	-0.068***	0.000	0.0003	0.000	0.0001	-0.0003**	-0.0003***	0.01
R_t^{CAC40}	2	0.0001	0.037	-0.026					0.0003	0.0003					0.0004
$(\Delta VX6)$	4	0.0001	0.048	0.003	-0.034	0.008			0.0006***	0.0008**	0.0006	0.0003			0.004
	6	0.0002	0.047	-0.004	-0.041	0.006	-0.080**	-0.063***	0.0006***	0.0007**	0.0005	0.0002	-0.0005	-0.0005	0.009
$R_t^{DJESTOXX50}$	2	-0.0002	-0.011	0.001					0.000	0.0004					-0.001
	4	-0.0002	-0.012	0.017	-0.126**	0.019			0.000	0.0006	-0.0008	0.000			0.004
	6	-0.0002	-0.012	0.020	-0.131**	0.038	-0.079***	-0.153*	0.000	0.0007	-0.0008	0.0002	-0.0002	-0.0008	0.02

Table 11: Test of Granger Causality in a regression setting using 2,4 and 6 lags. We run regressions of the form $R_t = c + \sum_{i=1}^{l} a_i R_{t-i} + \sum_{i=1}^{l} b_i \Delta IV_{t-i} + e_t$ for $l = 2,4,6$, where R denotes the stock index returns and ΔIV the changes of implied volatility indices. Reported values are the estimated coefficients from the samples of indices after exclusion of the period 3/18/04 – 3/17/05. Significance is tested using t-statistics from Newey – West Heteroscedasticity-consistent standard errors. One asterisk denotes significance at 1%, two asterisks significance at 5% and three asterisks significance at 10%.

Table 12
Test of Granger Causality in a regression setting
Information content of the history of Stock Indices Returns for the changes of the IV Indices

Dependent Variable	Lags	c	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6	Adj. R^2
ΔVIX_t	2	-0.0006	-0.048	-0.080**					-1.462	2.813					0.01
	4	-0.003	-0.064	-0.095*	-0.041	-0.005			-2.464	1.587	4.367	5.345***			0.02
	6	-0.006	-0.069***	-0.104*	-0.056	-0.016	0.030	-0.070*	-2.615	0.974	3.350	4.879	10.155*	-2.036	0.02
ΔVXO_t	2	0.010	-0.144**	-0.307**					-14.486	-12.489					0.08
	4	0.003	-0.149*	-0.331**	-0.003	-0.023			-14.049	-14.594	5.969	15.399**			0.09
	6	-0.005	-0.141*	-0.339**	0.037	-0.023	0.142**	-0.034	-12.788	-16.767	10.469	14.258**	20.905*	-3.266	0.10
ΔVXN_t	2	-0.040	-0.008	-0.030					-4.331	1.612					0.001
	4	-0.036	-0.006	-0.029	0.070	-0.046			-4.463	2.091	3.265	2.459			0.004
	6	-0.042	0.006	-0.052	0.079	-0.041	0.025	-0.148*	-3.247	0.894	3.616	3.602	1.750	-0.119	0.02
ΔVXD_t	2	-0.003	-0.059	-0.012					-6.117	3.936					0.002
	4	-0.002	-0.070***	-0.022	-0.109**	-0.074			-6.684	2.945	-3.385	-4.950			0.009
	6	-0.004	-0.071***	-0.030	-0.121**	-0.074	0.018	-0.059	-6.626	2.351	-3.888	-4.805	8.415**	0.295	0.02
$\Delta VDAX_t$	2	0.006	-0.130**	-0.023					-9.395**	3.572					0.01
	4	0.006	-0.132**	-0.023	-0.040	0.007			-9.446**	3.631	0.586	-0.795			0.02
	6	0.004	-0.133**	-0.025	-0.046	0.0009	-0.002	-0.029	-9.335**	3.409	0.409	-0.945	1.964	4.694***	0.02
$\Delta VX1_t$	2	0.001	-0.545*	-0.321*					-22.849*	2.782					0.22
	4	0.003	-0.612*	-0.463*	-0.186*	-0.139**			-29.697*	-11.195	3.566	-6.800			0.25
	6	0.0007	-0.615*	-0.467*	-0.201*	-0.158*	-0.002	0.043	-28.838*	-11.334	3.093	-8.391	11.291***	16.661**	0.25
$\Delta VX6_t$	2	-0.005	-0.513*	-0.211*					-9.758***	3.985					0.19
	4	-0.003	-0.602*	-0.397*	-0.298*	-0.157**			-14.533*	-6.541	-6.543	-5.225			0.23
	6	-0.005	-0.601*	-0.393*	-0.299*	-0.163**	0.039	0.012	-14.265*	-5.940	-6.353	-5.682	10.897**	4.187	0.23
$\Delta VSTOXX_t$	2	0.001	0.013	-0.060					4.793	1.471					0.004
	4	-0.004	0.034	-0.127***	0.105	-0.045			5.647	-4.019	12.780**	-3.561			0.01
	6	-0.006	0.040	-0.159**	0.120	-0.102***	0.067	-0.055	6.210	-5.764	13.938**	-7.650	11.134**	2.980	0.03

Table 12: Test of Granger Causality in a regression setting using 2, 4 and 6 lags. We run regressions of the form $\Delta IV_t = c + \sum_{i=1}^{l} a_i \Delta IV_{t-i} + \sum_{i=1}^{l} b_i R_{t-i} + e_t$ for $l = 2,4,6$, where R denotes the stock index returns and ΔIV the changes of implied volatility indices. Reported values are the estimated coefficients from the samples of indices after exclusion of the period 3/18/04 – 3/17/05. Significance is tested using t-statistics from Newey – West Heteroscedasticity-consistent standard errors. One asterisk denotes significance at 1%, two asterisks significance at 5% and three asterisks significance at 10%.

Section 5
Spillover Effects

In this section we expect to capture spillover effects between the daily movements of implied volatility indices. For trading purposes, we are only interested in lagged spillover effects. Consequently we are not examining the contemporaneous relationship. The methodological framework that is going to be used is an unrestricted Vector Autoregression. Vector Autoregressions have been popularized by the work of Sims (1980) and have been extensively used for analyzing the dynamics of macroeconomic systems and particularly for tracing the effects of policy changes in the economy. The main question addressed by VARs is about the effect of a shock in a variable on other variables of the model and the time-path of this effect. The purpose of a VAR model is to summarize all of the dynamic interrelations between the variables, as well as their projection in time.

The VARs advantage over a single equations framework is that they permit us to study jointly multiple series, detecting correlations between the disturbances across equations. Cross-equation correlations of disturbances may reflect factors common to all equations or cross-equations restrictions. Estimating the equations separately would waste information of the impact of random disturbances on the system. The VARs advantage over systems of structural equations is that they do not predetermine the structure of the dynamic interrelations between variables of interest. The structure of the model is identified by testing exclusion restrictions. Researchers however are inclined to pose as few restrictions as possible, since structural VARs have been debated on the basis of the simultaneous equations bias. That is, an observed correlation structure between variables may imply more than one 'real' structure between variables.

We will analyze the spillover effects between the Implied Volatility Indices movements in the framework of an unrestricted VAR. This is appropriate since we do not want to predetermine the structure or direction of their interrelationships.

5.1. Specification of the model and parameter estimation

A multivariate time series Y_t is a VAR process of order p, or a VAR (p) if it follows the model:

$$Y_t = \Phi_0 X_t + \sum_{i=1}^{p} \Phi_i Y_{t-i} + e_t \tag{5}$$

$$e_t \sim N(0, \Omega)$$

where Y_t is a vector of endogenous variables, X_t a vector of exogenous variables, $\Phi_o, \Phi_1, ..., \Phi_p$ are matrices of coefficients to be estimated, and e_t is a vector of innovations that may be contemporaneously correlated but are uncorrelated with their own lagged values and with all of the right-hand side variables. They have mean zero, covariance matrix Ω, and are assumed normal. Variables in vector Y_t are all treated as endogenous and each is assumed to be a function of the lagged values of all of the endogenous variables in the system. Treating a vector of variables X_t as exogenous with respect to the other system variables means that it is assumed to be determined outside the system. This is equivalent to stating that other variables in the system are not informative about future values of X_t or that there is no feedback relationship. In this framework, OLS yields consistent and efficient estimates, even though innovations may be contemporaneously correlated.

To build the VAR model, first we define the endogenous and exogenous variables to be included in the model. The vector (8×1) of endogenous variables is:

$$Y_t = \left[\Delta VIX_t \quad \Delta VXO_t \quad \Delta VXN_t \quad \Delta VXD_t \quad \Delta VDAX_t \quad \Delta VX1_t \quad \Delta VX6_t \quad \Delta VSTOXX_t\right]'$$

An (8×1) vector C will be included as the set of exogenous variables.

Next we have to identify the lag order p. Towards this end, we specify a maximum lag of 15, 10 and 5 days to test for, and observe the values of the information criteria. The Schwarz information criterion invariably takes its minimum value (22.97) at the 1st lag, whereas the Hannan-Quinn information criterion takes its minimum at 22.38, thereby selecting the 3rd lag. We also perform a VAR Lag Exclusion Wald test for a maximum of 5 lags. The test shows that lags up to 3 days are, for all variables but ΔVIX, statistically significant at the 5% significance level. In accordance with the test results, we specify a VAR of order 3.

$$Y_t = C + \sum_{i=1}^{3} \Phi_i Y_{t-i} + e_t \tag{6}$$

$$e_t \sim N(0, \Omega)$$

The VAR model will be estimated on a common sample of the Implied Volatility Indices. Since the first observation for VXN is for 2/2/2001, the common sample consists of around four years of daily observations from 2/2/2001 until 3/17/2005. After synchronization of the data, the total number of observations that are used for the VAR estimation is 1000.

Table 13 presents the results from a VAR(3) with the representation of equation (6). We report the estimated coefficients as well as the corresponding t-statistics and the adjusted R-squared. The significance of coefficients is tested at 1%, 5% and 10% level. Constants are all found insignificant. The highest adjusted R-squared statistics are 25% and 32% reported for VX1 and VX6 changes respectively, whereas the lowest are 3% and 4% reported for VIX and VXN changes. The range of the model's explanatory power for the other IV-indices-changes is between 7% and 10%. This means that the European IV indices, notably the French indices, are more affected by implied volatility movements in other markets than the American indices, especially VIX and VXN.

Table 13
Spillovers of Implied Volatility Changes under a VAR(3) model

Dependent Variable à	ΔVIX_t	ΔVXO_t	ΔVXN_t	ΔVXD_t	$\Delta VDAX_t$	$\Delta VX1_t$	$\Delta VX6_t$	$\Delta VSTOXX_t$
ΔVIX(-1)	0.175***	0.672*	0.288**	0.422*	0.351*	0.452**	0.213	0.440*
(t-statistic)	(1.730)	(5.806)	(2.411)	(5.027)	(3.259)	(2.253)	(1.559)	(3.293)
ΔVIX(-2)	0.057	0.356*	0.083	0.235***	0.097	0.147	-0.108	0.244***
	(0.533)	(2.922)	(0.661)	(2.648)	(0.853)	(0.694)	(-0.748)	(1.733)
ΔVIX(-3)	-0.047	0.285**	0.084	-0.016	-0.033	0.523	0.118	-0.014
	(-0.469)	(2.457)	(0.700)	(-0.188)	(-0.307)	(2.604)	(0.865)	(-0.103)
ΔVXO(-1)	-0.005	-0.450*	-0.015	0.056	0.087	0.120	0.224**	0.118
	(-0.064)	(-5.041)	(-0.164)	(0.872)	(1.046)	(0.779)	(2.125)	(1.149)
ΔVXO(-2)	0.105	-0.175***	0.213**	0.095	0.151***	0.165	0.261**	0.159
	(1.258)	(-1.839)	(2.171)	(1.371)	(1.711)	(1.000)	(2.323)	(1.449)
ΔVXO(-3)	0.138***	-0.141	0.147	0.119***	0.170**	0.176	0.085	0.180***
	(1.760)	(-1.573)	(1.595)	(1.839)	(2.049)	(1.133)	(0.810)	(1.745)
ΔVXN(-1)	-0.026	-0.036	-0.107**	-0.029	-0.131*	-0.163	-0.021	-0.093***
	(-0.686)	(-0.820)	(-2.394)	(-0.912)	(-3.234)	(-2.161)	(-0.413)	(-1.851)
ΔVXN(-2)	-0.008	-0.024	-0.067	-0.032	-0.017	0.058	0.041	-0.053
	(-0.199)	(-0.558)	(-1.488)	(-1.000)	(-0.416)	(0.766)	(0.789)	(-1.052)
ΔVXN(-3)	0.021	0.026	0.000	0.051	0.014	0.007	0.072	0.030
	(0.569)	(0.602)	(0.006)	(1.621)	(0.343)	(0.100)	(1.412)	(0.609)
ΔVXD(-1)	-0.160**	-0.152	-0.030	-0.447*	-0.015	-0.013	-0.174	-0.096
	(-1.964)	(-1.630)	(-0.309)	(-6.603)	(-0.168)	(-0.081)	(-1.583)	(-0.887)
ΔVXD(-2)	-0.213**	-0.218**	-0.237**	-0.349*	-0.071	-0.277***	-0.084	-0.194***
	(-2.518)	(-2.252)	(-2.367)	(-4.960)	(-0.792)	(-1.652)	(-0.738)	(-1.735)
ΔVXD(-3)	-0.224*	-0.283*	-0.355*	-0.260*	-0.059	-0.669*	-0.188***	-0.069
	(-2.817)	(-3.105)	(-3.780)	(-3.938)	(-0.702)	(-4.245)	(-1.747)	(-0.656)
ΔVDAX(-1)	-0.111***	-0.127***	-0.056	-0.086	-0.404*	-0.285**	-0.228**	-0.037
	(-1.663)	(-1.665)	(-0.715)	(-1.554)	(-5.705)	(-2.166)	(-2.537)	(-0.418)
ΔVDAX(-2)	-0.194*	-0.252*	-0.134	-0.108***	-0.378*	-0.072	-0.280*	-0.221**
	(-2.805)	(-3.187)	(-1.640)	(-1.881)	(-5.136)	(-0.525)	(-2.995)	(-2.414)
ΔVDAX(-3)	-0.093	-0.142***	-0.031	0.007	-0.252*	0.275**	-0.125	-0.318*
	(-1.410)	(-1.872)	(-0.398)	(0.125)	(-3.579)	(2.096)	(-1.398)	(-3.643)
ΔVX1(-1)	0.060*	0.098*	0.060**	0.080*	0.054**	-0.557*	0.138*	0.093*
	(2.709)	(3.873)	(2.276)	(4.365)	(2.311)	(-12.689)	(4.620)	(3.164)
ΔVX1(-2)	0.031	0.054***	0.046	0.049**	0.024	-0.357*	0.112*	0.063***
	(1.217)	(1.890)	(1.562)	(2.340)	(0.893)	(-7.196)	(3.316)	(1.888)
ΔVX1(-3)	0.000	0.011	0.023	0.021	0.012	-0.133*	0.100*	0.013
	(-0.009)	(0.418)	(0.891)	(1.154)	(0.503)	(-3.017)	(3.316)	(0.443)
ΔVX6(-1)	-0.030	-0.054	-0.063***	-0.035	-0.067**	-0.026	-0.800*	-0.109*
	(-0.953)	(-1.487)	(-1.699)	(-1.346)	(-1.991)	(-0.408)	(-18.734)	(-2.616)
ΔVX6(-2)	0.001	-0.018	-0.082***	-0.027	-0.041	-0.065	-0.556*	-0.074
	(0.033)	(-0.429)	(-1.894)	(-0.880)	(-1.045)	(-0.905)	(-11.293)	(-1.538)
ΔVX6(-3)	-0.036	-0.055	-0.079**	-0.047***	-0.074**	-0.209*	-0.384*	-0.082**
	(-1.172)	(-1.558)	(-2.168)	(-1.822)	(-2.232)	(-3.414)	(-9.197)	(-2.014)
ΔVSTOXX(-1)	0.003	0.009	-0.017	-0.008	0.157*	0.388*	0.169**	-0.193*
	(0.059)	(0.151)	(-0.273)	(-0.187)	(2.833)	(3.747)	(2.402)	(-2.795)
ΔVSTOXX(-2)	0.118**	0.182*	0.103	0.082***	0.188*	0.346*	0.255*	0.024
	(2.155)	(2.915)	(1.605)	(1.801)	(3.232)	(3.202)	(3.465)	(0.330)
ΔVSTOXX(-3)	0.114**	0.163*	0.100	0.050	0.197*	-0.008	0.071	0.192*
	(2.186)	(2.737)	(1.630)	(1.148)	(3.555)	(-0.078)	(1.015)	(2.793)
C	-0.009	-0.013	-0.040	-0.006	-0.005	-0.016	-0.011	-0.007
	(-0.235)	(-0.297)	(-0.847)	(-0.188)	(-0.118)	(-0.199)	(-0.196)	(-0.134)
Adj. R-squared	**0.03**	**0.07**	**0.04**	**0.10**	**0.09**	**0.25**	**0.32**	**0.09**

Table 13: Spillovers of the Implied Volatility Indices daily changes with three time lags. The regression setting is a VAR(3): $Y_t = C + \sum_{i=1}^{3} \Phi_i Y_{t-i} + e_t$ where Y_t is a (8×1) vector of the eight Implied Volatility Indices daily changes at time t, Y_{t-i} is an (8×1) vector of the eight IV Indices daily changes at time $t-i$, C an (8×1) vector of constants included as exogenous variables, and Φ_i (1×8) vectors of coefficients to be estimated. Reported values are the estimated coefficients and values in parentheses are t-statistics. One asterisk denotes significance at the 1% level, two asterisks at the 5% level and three asterisks at the 10% level. The Adj. R^2 for each regression is also reported. The sample used spans the period 2/2/2001-3/17/2005.

Results from the parameter estimation justify the choice of a VAR(3) model since lagged values at the third lag were found significant for all variables. The only exception is VXN whose effect on the other IV indices is limited to one period. Another general observation is that for each IV index-changes its own lagged values are found significant most at the 1% level. This extends to the third lag, although gradually fading out, for all European indices and for VXD. Only VIX seems to satisfy the efficient market hypothesis, since there is significance of its own lagged values only at the first lag and at the 10% level.

A spillover effect is detected at first sight from VIX-changes to other indices with coefficients as high as 0.67 for VXO, around 0.44 for VXD, VX1 and VSTOXX, and 0.29 for VXN. The effect of VIX extends to the second day for VXO, VXD and VSTOXX and also to the third day for VXO. VXO spills over to VX6 extending to two time periods, and also to VXN (with a coefficient of 0.21) and VDAX (with coefficients around 0.15) with two days difference. On the other hand there is no important spillover of VXN changes except over one period for VDAX (-0.13) at the 1% level, and VSTOXX (-0.9) at the 10% level. More influential is VXD whose values up to the 3rd lag are significant for VIX, VXO, VXN, and to a smaller extend for the European VX1, VX6 and VSTOXX. What is notable about VXD spillover effect is that the coefficients of the third lag are larger than coefficients at the second lag, and these larger than coefficients at the first lag. This could mean that the information content of VXD is not immediately incorporated to the other indices movements. This is observed also in other indices when coefficients are insignificant at the first lag and significant at the second or third lag.

Continuing at table 13, we will examine the transmission of implied volatility movements from the European markets towards the other markets. VDAX is quite influential since it significantly affects all the other indices with the exception of VXN. Coefficients estimates show that current changes of the IV indices reflect less the information content of VDAX changes at the first lag than that at the second lag. Hence VDAX changes at the 2nd lag affect VIX by -0.19, VXO by -0.25, VXD by -0.11, VX6 by -0.28 and VSTOXX by -0.22. This effect of VDAX extends to the 3rd time period, in the cases of VXO , VX1 and VSTOXX. Almost the same picture of better results at the second lag, is presented by VSTOXX. VSTOXX at the first lag affects VDAX (0.16), VX1 (0.39) and VX6 (0.17). This effect persists up to the second lag for the French indices and up to the third lag for VDAX. There is also a spillover effect from VSTOXX to VIX and VXO appearing at the second lag (with coefficients 0.12 and 0.18) and at the third lag (0.11 and 0.16). Information about VX1 changes seems to be immediately incorporated to the movements of the other IV indices, since coefficients are higher at the first lag than at the other lags. Lagged values of VX1 at the first period are significant for the current changes of all IV indices. VX1 spills over mainly to VX6 (0.14), but also to VSTOXX (0.09), VXO (0.10), VXD (0.08), VIX (0.06), VXN (0.06) and VDAX (0.05). In some cases the effect extends to the second and third lag. VX6 is notably less influential than VX1. Still, its first lagged value affects VSTOXX by -0.10, VXN by -0.06 and VDAX by -0.07.

There is also some information transmission about VX6 changes towards VX1 and VXD, which takes place with a three lags difference.

After estimating the parameters of the VAR, we test the stability of the model. The estimated VAR is stable (stationary) if all the inverse roots of the characteristic AR polynomial have modulus less than one and lie inside the unit circle. If the VAR is not stable, certain results (such as impulse response standard errors) are not valid. Results from the VAR Stability Condition Check show that no root lies outside the unit circle, therefore the model satisfies the stability condition.

5.2. Impulse Response Functions

Next we want to trace the effect of a one-time shock to the j th variable endogenous variable on future values of all the endogenous variables. Equation (5) can be written in the form:

$$Y_t = C + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \dots + \Phi_p Y_{t-p} + e_t$$

A vector MA(∞) representation of this VAR is:

$$Y_t = m + e_t + \Gamma_{t-1} e_{t-1} + \Gamma_{t-2} e_{t-2} + \dots \qquad (7)$$

where m is the mean or equilibrium of the process, and

$$\Gamma_s = \frac{\partial Y_{t+s}}{\partial e_t'}. \qquad (8)$$

If we define the g_{ij} element of Γ_s as $g_{ij} = \dfrac{\partial y_{i,t+s}}{\partial e_{jt}}$, then the impulse response function of the VAR is the plot of the g_{ij} elements of Γ_s as a function of s. The impulse response function describes the response of $y_{i,t+s}$ to one-time impulse in y_{jt} with all other variables at time t or earlier held constant. g_{ij} is viewed as the dynamic multiplier that identifies the consequences of a one-unit increase in the j th variable's innovation at time t, (e_{jt}), for the value of the i th variable at time $t+s$, $(y_{i,t+s})$, holding all other innovations at all dates constant.

In a VAR, the innovations e_t are assumed uncorrelated with their own lags and with lags of the endogenous variables but are allowed to be contemporaneously correlated. If e_t are contemporaneously correlated then the innovation of the first variable at time t, e_{1t}, provides information about the values of innovations of the other variables at the same time t, e_{2t}, $e_{3t} \dots e_{nt}$. Hence the innovation of the j th variable, e_{jt}, cannot be explicitly associated with the j th variable, since it has a common component with the concurrent innovations of the other variables. To sidestep this problem, the variance-covariance matrix of e_t, Ω, is often transformed so that innovations become contemporaneously uncorrelated. A transformation is the following:

$$\Omega = ADA'$$

where A is a unique lower triangular matrix with 1s along the principal diagonal and D is a unique diagonal matrix with positive entries along the principal diagonal. The vector u_t of orthogonalized residuals is given by:

$$u_t \equiv A^{-1}e_t \tag{9}$$

The elements of u_t are contemporaneously uncorrelated since $E\left(u_t u_t'\right) = D$, where D is a diagonal matrix whose (j, j) element is the variance of u_{jt}. The vector u_t instead of e_t is then used to calculate the dynamic multipliers for the impulse responses:

$$\frac{\partial Y_{t+s}}{\partial u_{jt}} = \Gamma_s a_j \tag{10}$$

where a_j is the j th column of the matrix A.

Another common transformation of Ω is the Cholesky decomposition, where Ω is decomposed as:

$$\Omega = A\sqrt{D}\sqrt{D}A' = PP'$$

where $P \equiv A\sqrt{D}$ is the Cholesky factor with the standard deviation of u_t along its principal diagonal, and $\sqrt{D}$ is the diagonal matrix whose (j, j) element is the standard deviation of u_{jt}. This means that:

$$p_j = a_j\sqrt{Var\left(u_{jt}\right)} \tag{11}$$

where p_j is the j the column of the matrix P.The Cholesky orthogonalized residuals are given by:

$$v_t \equiv P^{-1}e_t = D^{-1/2}A^{-1}e_t = D^{-1/2}u_t = \frac{u_t}{\sqrt{D}}$$

We can see that v_t is the standardized u_t, meaning than the impulse in this case is defined as one standard deviation in u_t. Under Cholesky decomposition, the vector v_t is used instead of e_t in the impulse response function, as:

$$\frac{\partial Y_{t+s}}{\partial v_{jt}} = \Gamma_s p_j \tag{12}$$

The drawback with the use of orthogonalized residuals in the impulse response function is that an ordering of variables is imposed. Changing the recursive ordering of the variables leads to different dynamic multipliers. Since we do not to impose any theoretical assumptions about the relative importance of each variable, we choose to plot the impulse responses as defined in equation (8) and not as defined in equations (10) or (12). However, we have described the process of Cholesky factorization, since it will be used for the variance decomposition.

Figure 25 plots the impulse responses of the changes of each IV index to one unit Innovations in changes of all the IV indices. The vertical axis measures $\frac{\partial \Delta IV_t}{\partial e_{\Delta IV, t+1-i}}$ for the time periods $i = 1, ..., 6$. The first period refers to the concurrent response whereas periods 2 to 6 refer to the response to past innovations of 1 to 5 lags. We have chosen to present the impulse responses up to 6 periods, since the responses of the IV-changes to past innovations after the 5^{th} lag converge to zero. As expected, the concurrent response of the changes of each IV index to one unit of its own innovation is one unit, whereas to innovations of the other IV-changes is zero. We can observe the mean reverting behavior of the responses, which is an indication of the stability of the VAR, or its tendency to return to its equilibrium after a disturbance of the system.

Examining the main responses of the indices one by one, we can see that VIX responds only to VXD and VDAX and VSTOXX innovations of the previous two days with a magnitude of response of about 0.20 units for 1 unit innovation of VXD and VDAX and with a response of 0.10 units for 1 unit innovation of VSTOXX. VSTOXX but especially VDAX are the only European indices which seem to provoke a response to the American indices. The less affected is VXD who responses mainly to the previous day's VIX 1 unit innovation by 0.40 units and to VIX innovation two days before, by 0.23 units. Similar is the response of the other American indices to VIX lagged innovations, with VXO's magnitude of response to the previous day's VIX 1 unit innovation reaching 0.67 units. VXO and VXN also respond slightly to VXD.

Turning our attention to the European indices, a first observation is that the responses of VX1 and VX6 to their own lagged innovations as well as to those of the other indices persist over a longer time period. In the case of VX1 this stands for its response to all indices except VXO and VX6, whereas VX6 has an almost zero response to VXN and VX1 past innovations. It is interesting that the French indices do not significantly respond to one another's past innovations. The magnitude of their response to the other indices 1 unit innovations, extended over 6 periods, does not exceed the 0.40 units. VDAX seems to respond mainly to VIX previous day's 1 unit innovation by almost 0.40 units. The magnitude of VSTOXX' s response to the same impulse is a little larger, around 0.45 units and persists one day longer. VSTOXX responds to all other indices past innovations, albeit to a lesser extend. It is notable that it takes two days for VSTOXX to respond to VXD and VDAX 1 unit innovations, with a magnitude of response around 0.20 units.

Figure 25
Spillovers of Implied Volatility Changes under a VAR model
Impulse Responses to One Unit Innovations

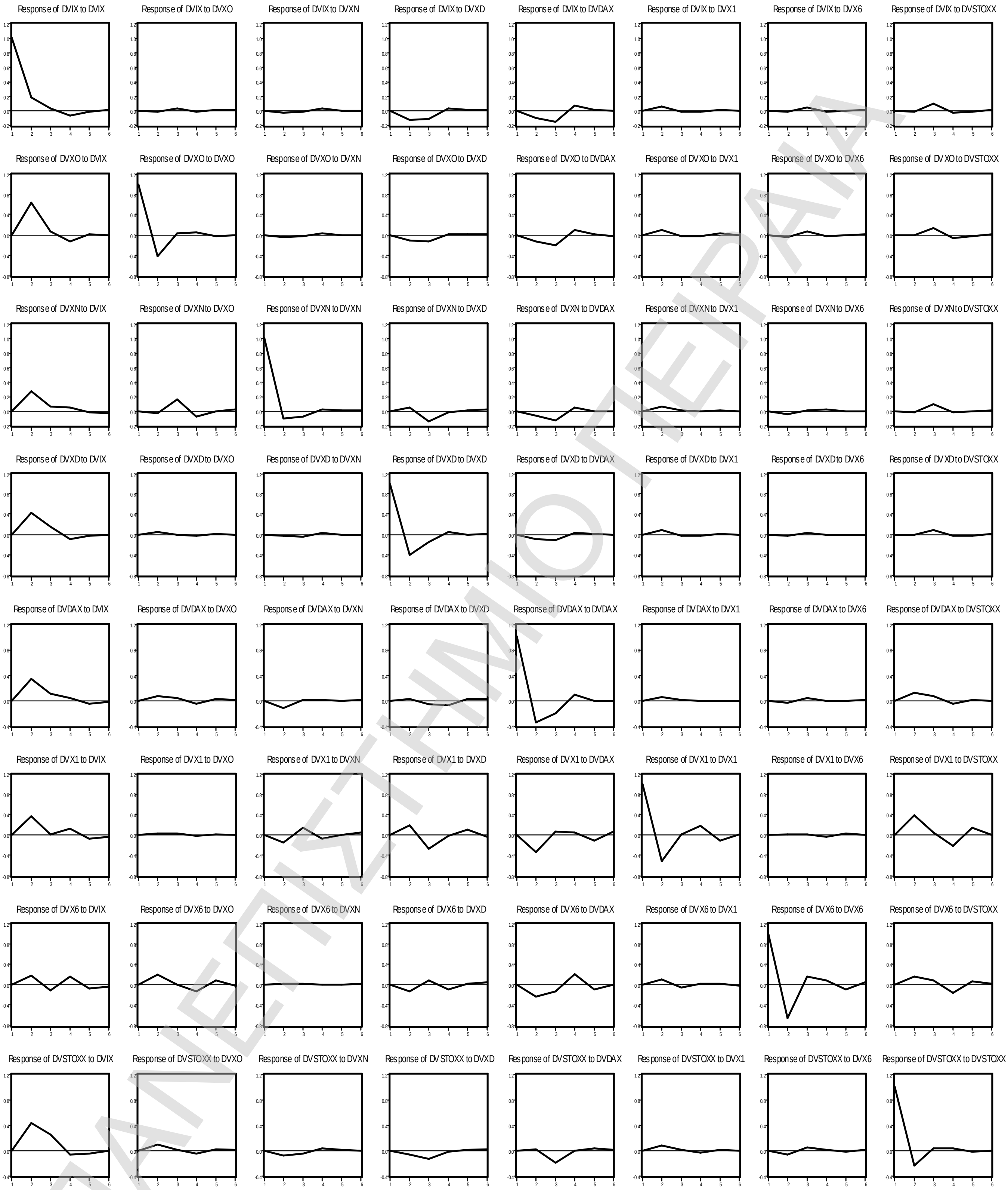


Figure 25: Non-factorized Impulse Responses of the changes of each Implied Volatility Index to one unit innovations on the changes of the other Implied Volatility Indices. The vertical axis measures $\frac{\partial \Delta IV_t}{\partial e_{\Delta IV,t+1-i}}$ for time periods $i = 1, \ldots, 6$.

The first period refers to the concurrent response whereas periods 2 to 6 refer to the response to past innovations of 1 to 5 lags.

5.3. Variance Decomposition

While impulse response functions trace the effects of a shock to one endogenous variable to future values of her variables in the VAR, variance decomposition separates the variation in an endogenous variable into the component shocks to the VAR.

Equation (7) can be written as the error in forecasting Y, s periods into the future:

$$Y_{t+s} - \hat{Y}_{t+s} = e_{t+s} + \Gamma_1 e_{t+s-1} + \Gamma_2 e_{t+s-2} + \ldots + \Gamma_{s-1} e_{t+1}$$

The mean square error (MSE) of this forecast is:

$$MSE\left(\hat{Y}_{t+s|t}\right) = E\left[\left(Y_{t+s} - \hat{Y}_{t+s|t}\right)\left(Y_{t+s} - \hat{Y}_{t+s|t}\right)'\right] = \Omega + \Gamma_1 \Omega \Gamma_1' + \Gamma_2 \Omega \Gamma_2' + \ldots + \Gamma_{s-1} \Omega \Gamma_{s-1}'$$

where $\Omega = \mathrm{E}\left(e_t e_t'\right)$. Using equation (9) we can write e_t in terms of u_t:

$$e_t = Au_t = a_1 u_{1t} + a_2 u_{2t} + \ldots + a_n u_{nt}$$

Then $\Omega = \mathrm{E}\left(e_t e_t'\right) = a_1 a_1' Var\left(u_{1t}\right) + a_2 a_2' Var\left(u_{2t}\right) + \ldots + a_n a_n' Var\left(u_{nt}\right)$, and the MSE can be written as:

$$MSE\left(\hat{Y}_{t+s|t}\right) = \sum_{j=1}^{n} \left\{ Var\left(u_{jt}\right)\left[a_j a_j' + \Gamma_1 a_j a_j' \Gamma_1' + \Gamma_2 a_j a_j' \Gamma_2' + \ldots + \Gamma_{s-1} a_j a_j' \Gamma_{s-1}'\right]\right\}$$

where n is the number of endogenous variables of the VAR. Using equation (11) we can express the MSE in terms of the j th column of the Cholesky factor P:

$$MSE\left(\hat{Y}_{t+s|t}\right) = \sum_{j=1}^{n} \left[p_j p_j' + \Gamma_1 p_j p_j' \Gamma_1' + \Gamma_2 p_j p_j' \Gamma_2' + \ldots + \Gamma_{s-1} p_j p_j' \Gamma_{s-1}'\right]$$

Each term $\left\{p_j p_j' + \Gamma_1 p_j p_j' \Gamma_1' + \Gamma_2 p_j p_j' \Gamma_2' + \ldots + \Gamma_{s-1} p_j p_j' \Gamma_{s-1}'\right\}$ of the sum expresses the contribution of orthogonalized disturbance of the variable j to the MSE of the s-period-ahead forecast of Y. As $s \to \infty$, the $MSE\left(\hat{Y}_{t+s|t}\right)$ converges to the unconditional variance of Y_t. Therefore, for large s, each term of the sum expresses the portion of the total variance of Y_t that is due to the innovation u_j.

A factorization of Ω is necessary in Variance Decomposition, so that variance portions will add up to one (100%). This requires, as we have seen in the impulse response function, an ordering of the variables. We choose to give precedence to VIX since the estimation of the VAR(3) model indicated that it has the highest spillover effect on the other indices. The ordering of the variables is the following:

$$\left\{\Delta VIX_t, \Delta VXO_t, \Delta VXN_t, \Delta VXD_t, \Delta VDAX_t, \Delta VX1_t, \Delta VX6_t, \Delta VSTOXX_t\right\}$$

Figure 26 presents for each IV index daily changes, the decomposition of its forecast variance into the component past innovations to all the IV indices. As a consequence of the

VAR ordering assumed, the first period variance for the first variable, ΔVIX , is completely due to its own innovation. From the second period the forecast variance of VIX changes is decomposed to 98.4% of its own past innovations and around 0.20% of the innovations on the other IV changes. Most of the forecast variance of VXO, VXN, VXD and VDAX changes is decomposed into their own past innovations and the past innovations of VIX changes. The contribution of VIX in these cases is stabilized after the second period to approximately 83% for VXO, 74% for VXD, 46% for VXN and 40% for VDAX. The forecast variance decomposition for VSTOXX changes is stabilized after the third period to 21% of its own past innovations, 36% of the innovations on VIX changes and 38% of the past innovations on VDAX changes. The same picture of main components is presented for the French VX1, with its forecast variance comprising of around 72% of its own past innovations, 14% of the past disturbances on VDAX changes and 10% of innovations on VIX changes. Finally, the forecast variance decomposition of VX6 changes is stabilized after the third period to 57% of its own past disturbances, 17% of VX1 past innovations, 16% of innovations hitting VDAX changes, and 7% of the past disturbances on VIX changes.

Figure 26
Spillovers of Implied Volatility Changes under a VAR model
Variance Decomposition

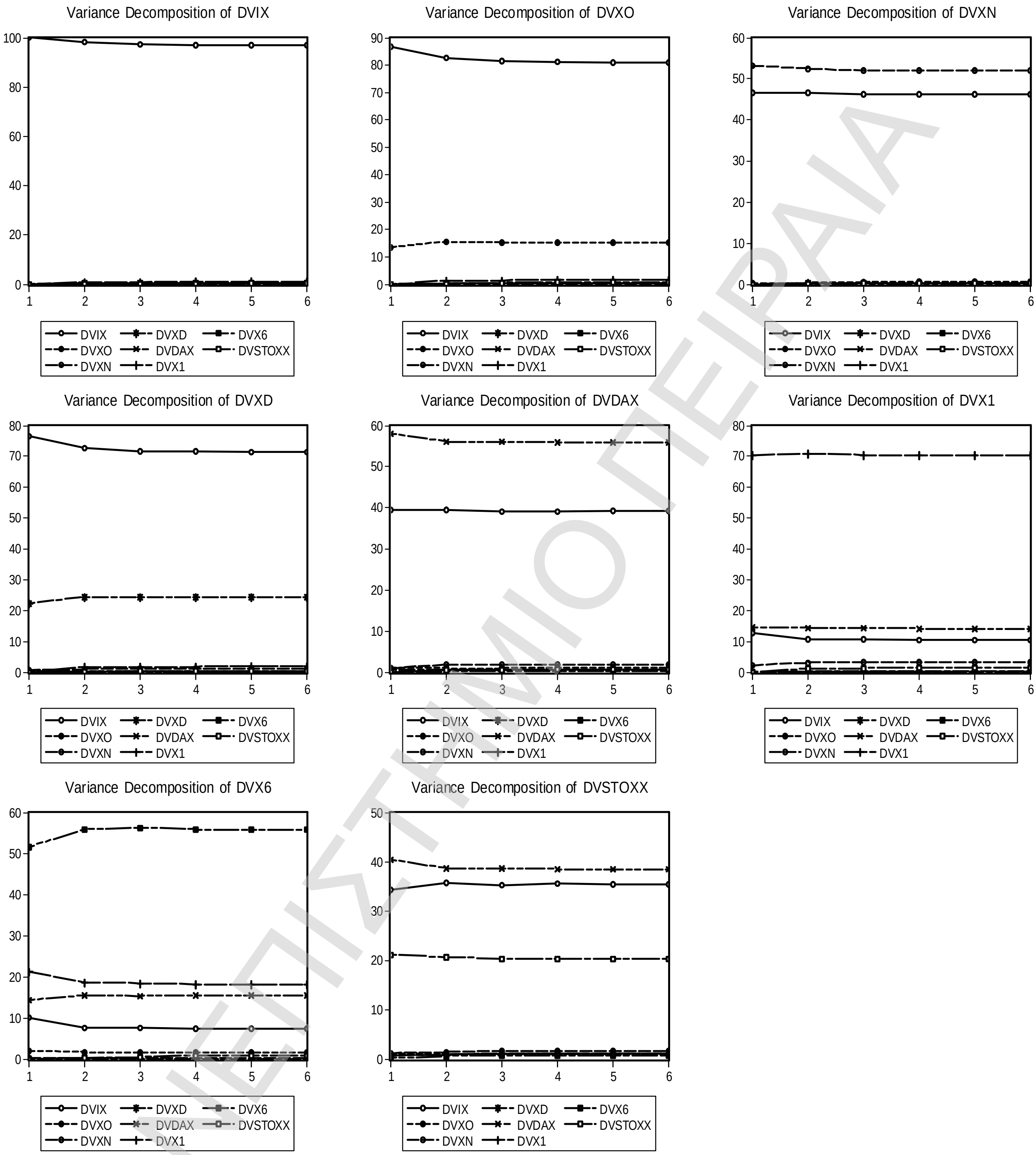


Figure 26: Variance Decomposition of the changes of each Implied Volatility Index into the component shocks to the VAR model. The vertical axis in each graph reports the percentage of the forecast variance due to past innovations on each IV index changes, over 6 periods. Variance Decomposition is based on Cholesky factorization with the following ordering of the variables: $\Delta VIX, \Delta VXO, \Delta VXN, \Delta VXD, \Delta VDAX, \Delta VX1, \Delta VX6, \Delta VSTOXX$.

Section 6
Calculating Value-at-Risk

In 1996 JP Morgan introduced RiskMetrics a methodology of computing simple measures of market risk for a given portfolio of assets. An important contribution of the RiskMetrics (1996) methodology is the introduction of the Value- at-Risk (VaR) concept which collapses the entire distribution of the portfolio returns into a single number which investors have found useful and easily interpreted as a measure of market risk. Value at Risk (VaR) is a widely used risk measure that answers the question "What is the $value such that a portfolio loss over the next time horizon T exceeds this value only $p \times 100\%$ of the times?" The definition of $VaR is:

$$\Pr(\$Loss_{t+T} > \$VaR_{t+T}) = p$$

There are three main competing models of measuring VaR, parametric models: variance – covariance approach and Monte Carlo (MC) Simulation methods (MC methods assume normality), and non-parametric models: Historical simulation. In the Variance-Covariance approach we use either the Delta Normal method in the case of linear portfolios (stocks, commodities) or the Delta Gamma method for non-linear portfolios (options, interest rates, bonds). There is a growing interest in testing and comparing various VaR methods[8], since VaR is, among others, an approved regulatory methodology in the market risk component in Basel I and is expected to remain so in Basel II.

In this section we will proceed as follows. Using our dataset, we will calculate different VaR measures for 1-day-ahead forecast period at the 1% and 5% significance level, for a long position on the S&P500 index. We have chosen to use the variance – covariance approach and Historical Simulation. Since our portfolio is linear we will use the Delta Normal method in which we need an estimate of volatility. As inputs for the delta normal method we will use Historical volatility, volatility derived from an Exponentially Weighted Moving Average Model (EWMA), an Exponential GARCH model and as a third input the implied volatility index for the S&P500 stock index, VIX. Our aim is to test the performance in the calculation of VaR of the three volatility imputs in the variance – covariance approach and compare it with the performance of VaR calculated with historical simulation. For this purpose we will use Kupiec Test as a Backtesting method. The fundamental question we will address is: Can an implied volatility index (VIX) be used as an efficient input in the calculation of VaR for its underlying stock index (S&P500), and how well does it perform compared to other VaR measures?

We will follow Skiadopoulos et al. (2004) in the methodology of historical simulation, and Figlewski (2004) in calculating historical volatility. We will model volatility by an EWMA according the RiskMetrics model of J.P.Morgan (1996) and Christoffersen (2003) 20-23. We will follow Giot (2005b) in the use of implied volatility as an input in VaR.

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[8] Several tests for comparing different VaR models are discussed in Christoffersen, Hann and Inoue (2001).

The sample for the VaR calculation begins at 2/2/2001 and ends 3/17/2005. The first 500 observations (2/2/2001 – 2/28/2003) will be used for the EGARCH parameter estimation. The first variance forecast as well as the first VaR forecast will be for 3/3/2003. Hence, the Backtesting period will be from 3/3/2003 until 3/17/2005, and will include 499 observations.

6.1. Variance – Covariance Method

For long position in a portfolio, VaR is the left percentile of the cumulative density function of the distribution of its future returns. Since not only the future returns but also the distribution of future returns is uncertain, VaR is itself a random variable. The Delta – Normal method for the calculation of VaR requires an explicit assumption about the distribution of future returns. Under the assumption of a normal distribution and zero mean for the daily returns, VaR is given by:

$$VaR_{t+T}^{p} = Z_{p}^{-1} s_{t+T}$$

where $Z(\cdot)$ is the cumulative density function of $N(0,1)$ distribution and s_{t+T} is the standard deviation of the returns. $Z(w)$ gives the probability that z_{t+T} is equal or less to the number w, whereas the inverse, Z_{p}^{-1}, gives the number w such that p*100% of the probability mass is below that number. Z_{p}^{-1} is 1,645 for a 5% significance level and 2,326 for a significance level of 1%. Next we define the three estimates that are going to be used for the standard deviation, s_{t+1}, of the returns, the Historical Moving Average, the Risk Metrics Exponentially Weighted Moving Average, the Exponential GARCH and the VIX.

<u>Historical Moving Average</u>

The first input for variance is a Historical Moving Average of past squared returns calculated as:

$$s_{t+1}^{2} = \frac{1}{n-1} \sum_{i=t+1-n}^{t} \left(R_i - \overline{R} \right)^2$$

, where $\overline{R} = \frac{1}{n} \sum_{i=t+1-n}^{t} R_i$, $\overline{R} = 0$ and n is 100 and 250 days. As we move on to the next day, the oldest observation drops out of the n-day measurement sample and a new observation is taken into account.

Risk Metrics

Risk Metrics model for the forecast of tomorrow's variance is a weighted average of today's variance and today's squared return. The unknown parameter l contained in the model, was estimated by Risk Metrics for a large number of assets, at 0.94. We use the same estimate.

$$s_{t+1}^{2} = l\, s_{t}^{2} + (1 - l) R_t^2$$

$$l = 0.94$$

The initial forecast for variance will be a historical a historical average of squared returns of the previous 252 days.

EGARCH

GARCH models are designed to capture certain characteristics that are commonly associated with financial time series, that is fat tails, volatility clustering and leverage effects. The leverage effect, refers to the empirical observation that asset returns are negatively correlated with changes is volatility. That is, volatility tends to rise in response to lower than expected returns and to fall in response to higher than expected returns. Asymmetric GARCH models capture this effect. The EGARCH model was introduced by Nelson (1991). In contrast to GARCH models, Nelson's exponential GARCH, brings forward the asymmetric relation between returns and changes in volatility[9]. A negative value of L_j indicates that volatility tends to rise (fall) when news affecting returns are negative (positive). In other words, downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. This is also highlighted by Engle (1993) in the news impact curve with asymmetric response to good and bad news.

We will use an EGARCH (p,q) model to forecast conditional variance with conditional mean equation:

$$R_t = \overline{R} + e_t = \overline{R} + s_t z_t$$

$$z_t = \frac{e_t}{s_t}\,,\ z_t \sim i.i.d.N(0,1)$$

Imposing a zero value for the mean return, we have $R_t = e_t = s_t z_t$. The specification of the conditional variance equation is:

$$\ln s_t^{2} = k + \sum_{i=1}^{p} G_i \ln s_{t-i}^{2} + \sum_{j=1}^{q} A_j \left| z_{t-j} \right| + \sum_{j=1}^{q} L_j z_{t-j}$$

Since $R_t = s_t z_t$ we approximate z_{t-1} by the standardized returns:

$$z_{t-1} = \frac{R_{t-1}}{s_{t-1}}\ ,\ \left| z_{t-1} \right| = \frac{\left| R_{t-1} \right|}{s_{t-1}}$$

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[9] See Nelson (1991), pp. 349-351.

Estimation of the parameters of the EGARCH model will be based on a sample of 500 observations from 2/2/2001 until 2/28/2003. Assuming that the estimated model remains stable over time, we will use the in-sample estimated parameters to derive out-of-sample forecasts for the conditional variance.

We have chosen to fit an EGARCH(1,2) for S&P500 returns' variance, after several diagnostic tests for the optimal Garch (p) an Arch(q) order. The ARCH LM test and the Correlogram of squared standardized residuals show that there does not remain any non-modeled heteroscedasticity and autocorrelation. Statistical significance of coefficients is tested using Bollerslev – Wooldridge Standard Errors. Convergence is achieved with the standard Marquardt optimization algorithm with maximum 500 iterations. Table 14-17 present the parameter estimation as well as the diagnostic tests for the model.

The estimated variance equation for S&P 500 returns is the following:

$$\ln s^2_{t,R_t^{S\&P500}} = -0.1444 + 0.9872 \ln s^2_{t-1} - 0.2770\left|z_{t-1}\right| + 0.3025\left|z_{t-2}\right| - 0.2012 z_{t-1}$$

As initial values for variance in order to feed in the first two values of the GARCH, the ARCH and the Leverage terms, we use historical averages of squared returns from the previous 500 observations.

<u>VIX</u>

Since the VIX refers to volatility per year, we need to apply the following transformation in order to derive one-day-ahead variance estimates. Assuming 252 trading days we have:

$$s_t = VIX_t \sqrt{\frac{1}{252}}\ .$$

6.2. Historical Simulation

The Historical Simulation is a non-parametric method to calculate VaR, in that we don't have to make assumptions for the stochastic model underlying the daily returns, as with the variance-covariance approach. Yet the assumption that underlies Historical Simulation is that tomorrow's return will follow the distribution of the returns of the N-days previous period. For 1-day-ahead p% VaR with an N-day rolling window, the 1-day VaR estimate is the left p percentile of the cumulative distribution of the previous N daily returns. For each VaR forecast, we use a window of 100 and 250 previous observations, rolling it over as a new observation becomes available and the oldest drops out of the window. The drawback of historical simulation is that it is very slow at updating VaR forecasts.

Table 14
Variance Estimation using an EGARCH (1,2) model for S&P500 daily log Returns

Dependent Variable: $\log s_t^2$	Constant	GARCH Coefficient	ARCH Coefficients		LEVERAGE Coefficients	
Dependent in the mean equation –	k	G_1	A_1	A_2	L_1	L_2
$R_t^{S\&P500}$	-0.1444*	0.9872*	-0.2770*	0.3025*	-0.2012*	0.0399
(z-statistic)	(-2.734)	(178.937)	(-3.009)	(3.284)	(-3.568)	(0.713)

Table 14: Estimation of the conditional variance for the S&P500 Returns using an EGARCH (2,1) model. Parameter estimation is based on the sample period 2/2/2001 – 2/28/2003. The mean equation is specified as $R_t = e_t = z_t s_t$, whereas the conditional variance equation is specified as: $\ln s_t^2 = k + \sum_{i=1}^{p} G_i \ln s_{t-i}^2 + \sum_{j=1}^{q} A_j \left|z_{t-j}\right| + \sum_{j=1}^{q} L_j z_{t-j}$ where $z_{t-j} = \frac{e_{t-j}}{s_{t-j}}$. Reported values are coefficient estimates and values in parentheses t-statistics. One asterisk denotes rejection of the null hypothesis of a zero coefficient at the 1% significance level.

Table 15

EGARCH (1,2) Diagnostic Test I

Autocorrelations of the Standardized Squared Residuals	
AC(1)	0.009
AC(2)	-0.007
AC(3)	0.016
Ljung-Box Q(12)	6.9835

Table 15: Diagnostic test for remaining autocorrelation of the Standardized Squared Residuals in the EGARCH model. We report autocorrelations up to the third lag as well as the Ljung-Box statistic for autocorrelation up to the 12th lag. Autocorrelations are all insignificant even at the 10% level.

Table 16

EGARCH (1,2) Diagnostic Test II

ARCH Test (for 2 lags) on the Standardized Squared Residuals	
F-statistic	0.032
Obs*R-squared	0.064

Table 16: Diagnostic test for remaining heteroscedasticity on the squared standardized residuals in the EGARCHmodel. The reported ARCH test statistics, F-statistic and Observations*R-squared, are insignificant up to the second lag, even at the 10% significance level.

Section 7
Backtesting

The success of a VaR model depends on two aspects. First, it is the ability to accurately forecast VaR violations over a given time horizon and with a given probability. Second, it is the ability to produce low VaR estimates so that low regulatory capital is imposed. Backtesting is the procedure for assessing the forecasting performance of a VaR model. It tests whether the proportion of actual VaR violations in a sample is significantly different from the given confidence level. We employ two backtesting procedures, Basle Traffic Light and Kupiec Test to assess the performance of the VaR models discussed in the previous section.

7.1. Basle traffic light

Since 1996, the Basle Committee on Banking Supervision enforces banks, and other financial institutions like insurance companies, to hold regulatory capital against their market risk exposure. The capital rules cover all assets in a bank's trading account as well as all foreign exchange and commodity positions and are applicable to any bank or other financial institution whose trading activity accounts for more than 10% of its total assets or is more than $ 1 billion. These capital charges are based on VaR estimates generated by the banks' internal VaR models and are imposed according to a multiplication factor derived by the classification of the VaR model into the green, yellow or zone. This is called the Traffic Light concept. The benchmark is 1-day-ahead 1% VaR, and the classification depends on the frequency of VaR violations over a period of 250 trading days. 1% VaR corresponds to 2 or 3 VaR violations over 250 trading days. However, regulatory authorities allow for up to 4 VaR violations in 250 days in order to classify a VaR model in the green zone. This corresponds to the minimum multiplication factor of 3. Necessary capital reserves are assigned by multiplying the 1% daily VaR with the multiplication factor. Models which produce between 5 and 9 VaR violations over a year are classified in the yellow zone. 5, 6, 7, 8 and 9 violations over a year correspond to a multiplication factor of 3.40, 3.50, 3.65, 3.75 and 3.85 respectively. When 10 or more violations a year are observed, the model is classified in the red zone and a multiplication factor of 4 is imposed. Since holding regulatory capital is costly, a VaR model falling into the red zone is usually rejected.

Basel's Traffic Light VaR Model Classification

	Green Zone	Yellow Zone					Red Zone
VaR violations over 250 days	0-4	5	6	7	8	9	10 or more
Multiplication Factor for Regulatory Capital	3	3.40	3.50	3.65	3.75	3.85	4

7.2. Kupiec Test

Kupiec Test is a test of unconditional coverage in that it tests the null hypothesis that a proportion of violations of each day's VaR forecast over a given sample is not significantly different than the significance level of VaR. Chistoffersen has proposed a test of conditional coverage, in that it also tests the independence of violations, or the violations' clustering[10]. Next we give a brief description of Kupiec Test. The test is based on the comparison between daily realized returns and the corresponding VaR forecasts, conceptualized as a binomial experiment. For each day two outcomes are possible, the forecasted VaR would have either understated or overstated the realized return. For each day, the outcome of the binomial experiment I_t could be one, corresponding to a violation of the VaR limit, that is a loss exceeding the VaR forecast, or zero, for a nonviolation, that is when the realized return is covered by the VaR forecast. Thus a series of ones and zeros, of violations and nonviolations, is constructed. The Sequence of VaR violations is defined as:

$$I_{t+1} = \begin{cases} 1 & \text{, if } R_{t+1} < -VaR_{t+1} \\ 0 & \text{, if the above statement is false.} \end{cases}$$

If the sequence of violations should be completely unpredictable, then the series of outcomes should be distributed as a series of draws from an independent Bernoulli distribution:

$$H_0 : I_{t+1} \sim i.i.d. \text{ Bernoulli (p)}$$

The Bernoulli distribution function is:

$$f\left(I_{t-1}; p\right) = (1-p)^{1-I_{t+1}} p^{I_{t+1}}$$

Unconditional coverage tests the null hypothesis that the proportion of actual VaR violations, p , is not significantly different from the given probability level, p , $(H_0 : p = p)$. To perform the test we use the likelihood of an $i.i.d.$ Bernoulli sequence:

$$L(p) = \prod_{t=1}^{T} (1-p)^{1-I_{t+1}} p^{I_{t+1}} = (1-p)^{T_0} p^{T_1}$$

where T_0 is the number of VaR nonviolations in the sample, and T_1 is the number of VaR violations in the sample. The likelihood ratio used for unconditional testing is the following:

$$LR_{uc} = -2\ln\left[\frac{L(p)}{L(p)}\right] \sim X_1^2$$

It is asymptotically distributed Chi-squared with one degree of freedom. By substitution we have:

[10] See Christoffersen (2003), pp. 181-189.

$$LR_{uc} = -2\ln\left[\frac{L(p)}{L(p)}\right] = -2\left[\ln L(p) - \ln L(p)\right] = -2\left\{\ln\left[(1-p)^{T_0}\, p^{T_1}\right] - \ln\left[(1-p)^{T_0}\, p^{T_1}\right]\right\}$$

$$= -2\left[T_0 \ln(1-p) + T_1 \ln p - T_0 \ln(1-p) - T_1 \ln p\right] = -2\left[T_0 \ln\left(\frac{1-p}{1-p}\right) + T_1 \ln\left(\frac{p}{p}\right)\right] \sim X_1^2$$

The critical value for Kupiec Test at the 5% is 3.8415. When the estimated value for the likelihood ratio exceeds the critical value, then we reject the null hypothesis of an adequate VaR model.

7.3. Backtesting Results

The following table presents the results from backtesting 1-day-ahead 99% VaR and 95% VaR of S&P500 daily returns calculated with the variance- covariance approach using four different estimates for variance, and with Historical Simulation with a rolling window of 100 and 250 past observations.

Table 17
Backtesting VaR for S&P500

		Variance – Covariance Delta Normal Method					**Historical Simulation**	
		Historical MA(100)	**Historical MA(250)**	**Risk Metrics**	**E-GARCH**	**VIX**	**HS(100)**	**HS(250)**
99% VaR								
Exceptions	**T_1**	1	1	4	18	1	8	2
Ratio of exceptions	p	0,2%	0,2%	0,8%	3,6%	0,2%	1,6%	0,4%
Kupiec test	**LR_{uc}**	4,7973*	4,7973*	0,2129	20,5114*	4,7973*	1,5505	2,3409
Basel Classification		Green zone	Green Zone	Green Zone	Yellow Zone	Green Zone	Green Zone	Green Zone

		Variance – Covariance Delta Normal Method					**Historical Simulation**	
		Historical MA(100)	**Historical MA(250)**	**Risk Metrics**	**E-GARCH**	**VIX**	**HS(100)**	**HS(250)**
95% VaR								
Exceptions	**T_1**	19	9	23	34	3	22	14
Ratio of exceptions	p	3,8%	1,8%	4,6%	6,8%	0,6%	4,4%	2,8%
Kupiec test	**LR_{uc}**	1,6218	14,0771*	0,1645	3,1190	32,1915*	0,3817	5,9721*

Table 17: Backtesting 1-day-ahead 99% and 95% VaR for a long position on S&P500. The backtesting period is 3/32003 – 3/17/2005 and consists of 499 daily observations. Kupiec Test is performed at the 5% level with critical value 3.8415. An asterisk denotes rejection of the VaR model. Classification of the models for the 99% VaR, according to Basel Traffic Light System, is also presented. The model is classified into the green zone if the VaR violations over a horizon of 500 trading days are not over 9, into the yellow zone if the violations reported are between 10 and 19, whereas a number of exceptions higher than 20 results in the red zone classification.

Results from the backtesting of S&P500 daily VaR models show initially that all models are classified into the green zone except for the EGARCH based model which is classified into the yellow zone. This is according to Basel Traffic Light system which is purely based on the proportion of exceptions in a given sample. Historical Moving Averages as well as the Implied Volatility index VIX in the Variance –Covariance approach, for the one-day-ahead 99% VaR are rejected by the Kupiec's Test on the basis of producing very few violations of the VaR estimates. E-GARCH on the contrary is rejected on the basis of producing too many violations. The test rejects the null hypothesis that the ratio of violations is equal to the coverage rate of the VaR model, not only when too many VaR violations have occurred but also when we have too few violations in the sample[11]. Too few violations imply an overstatement of VaR and hence the commitment of excess capital. For the 95% VaR the models that have performed better according to Kupiec test are Historical MA(100), Historical Simulation (100), Risk Metrics and E-GARCH. These models are not rejected although their ratio of observed exceptions differs significantly from the expected probability of violations. VIX on the contrary is rejected for having a very low ratio of exceptions relative to the 5%. Consequently the use of VIX in an 1-day-ahead VaR model would result in the acceptance of the model according to Basel traffic light but also in a commitment of excess regulatory capital since it overstates daily VaR estimates.

[11] See Benos, Angelidis (2004), pp. 6-7.

Section 8
Conclusions

The purpose of this dissertation was to examine the properties of the Implied Volatility Indices. We verified in line with current literature the existence of the Leverage effect for all indices although it was found asymmetric only for the French indices. This means that the negative relationship between stock returns and IV changes is intensified in the case of a downward movement of returns than in the case of an upward movement. The general non-verification of asymmetry in the leverage effect is in contrast to existing literature that reports IV indices as investor's fear gauges. We have also examined the stability of the leverage effect assuming a structural break at the 11th September 2001 attack. We have found an intensification of the leverage effect after the 11th September attack, for the three American indices S&P500, S&P100 and DJIA and the German index DAX. This means that a raise of the implied volatility index after the 11th September, is associated with a larger drop of the underlying stock index, than it did before. On the contrary, the risk-return relationship for the French indices, for DJEUROSTOXX and for Nasdaq-100 has not been affected. There were no asymmetric features developed of intensified after the 9/11/2001. The CUSUM test indicates stability of coefficients for all indices relationship except S&P500-VIX, S&P100-VXO and DJEUROSTOXX50 – VSTOXX stabilized after the 11th September 2001.

We tested whether IV indices past prices helped the prediction of today's stock returns, and if returns could be used to forecast future prices of IV indices. Granger Causality tests for different lag orders showed that only returns Granger cause IV indices changes, especially returns of the European indices and S&P100. For the other American indices, there is no Granger causality neither way at least at the first lag orders. We checked these results, using regression analysis. For various lag orders, R-squared for the regressions with the stock index returns as dependent variable are very low varying from 0% to 6% whereas regressions with IV index changes as dependent averaged at 3% with the exceptions of VXO (11%), VX1(25%) and VX6 (23%). Regressions for 2,4,6 lags verified Granger causality results, and further indicated that only the CAC40 past returns could be used to predict VX1 and VX6 future changes but this does not extend over one day, and it could be done only in conjunction with lagged values of the IV indices themselves.

We used a Vector Autoregression of order 3 to examine the spillovers between IV indices' movements. With a VaR(4) the model was punished with lower adjusted R-squared. However the French IV indices seem to incorporate information from the other IV indices for a longer time period, around 5 days. Regression results showed that in line with the current literature, there is a high and significant spillover from the VIX towards all the other indices which persists to a second day for VXO, VXD and VSTOXX. From the other American indices, there is a regional spillover to the American market, whereas VXN doesn't seem to have any spillover effects at all. The highest spillover of the European indices is coming from VDAX and VSTOXX. These indices affect very strongly the other European indices at the first lag, but they present a strong spillover also to VIX and VXO appearing at the second lag and persisting to the third lag. VX1 affects mainly VX6 but also the American indices to a lesser extent than VSTOXX and VDAX. In general, VXN and VX6 are the less influential. Impulse response analysis of the spillovers showed that responses generally fade to zero after the 5th period.

Applying Cholesky factorization to the system of IV-indices disturbances, we also decomposed the forecast variance of each IV index changes into their own past innovations and the past innovations of the changes of the other IV indices. The findings indicated a high contribution of VIX in the variance forecast of the other indices, namely 83% for VXO, 74% for VXD, 46% for VXN and 40% for VDAX. VSTOXX changes were decomposed into 21% of its own past innovations, 36% of the innovations on VIX changes and 38% of the past innovations on VDAX changes. The VX1 forecast variance comprised of around 72% of its own past innovations, 14% of VDAX and 10% VIX innovations. We also found that VX1, VDAX and to a smaller extend VIX past innovations contribute considerably to VX6 forecast variance.

Finally we calculated and back-tested S&P500 daily 99% and 95% VaR models with the historical simulation (HS) and the variance covariance approach using as inputs for variance two historical moving averages, Risk metrics estimates, EGARCH estimates and VIX time series. Basel Traffic Light and Kupiec test were used as backtesting criteria. Results showed that HS(100) and HS(250) outperformed the other models. The VIX generally overstated the expected VaR resulting in an acceptance by Basle criteria of proportion of exceptions, but a rejection by Kupiec test.

Further research could examine the performance of the other Implied Volatility indices in VaR models for their respective underlying index, extending the backtesting methodology to independence and conditional coverage tests. A recently introduced implied volatility index, VSMI, based on the Swiss Market Index (SMI) options, could also be the subject of future analysis.

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