University of Piraeus
Department:
Financial Management and Banking

Testing the presence of polynomial way trend in the conditional variance of stationary macroeconomic and financial time series.

Student: Antypas Antonios (0302)
Supervisor: Nikitas Pittis

Dedicated

Piraeus 2005

Contents

Introduction

Most data in macroeconomics and finance come in the form of time series, a set of repeated observations of the same variable. Time series analysis has a fundamental difference with structural econometric analysis. In the latter case the object of interest is the effect of a set of explanatory variables $\mathrm{X}:\{x_1, x_2, x_3, ...\}$ on a dependent variable Y, where changes in X are only associated with a particular structural event.

On the other hand, time series analysis involves stochastic processes, **indexed** collections of random variables $\mathrm{X}:\{x_{1t}, x_{2t}, x_{3t}, ...\}$, creating an ordering among the observations. In this case changes in X are expected whenever the index, usually time, evolves. Ordering is very important in time series, because it indicates the dependence of our data from one period to another.

In this paper, we consider stationary first order autoregressive models assuming that the errors that generate our process exhibits polynomial trend in their variance. We organize this work into three major sections: theory, Monte Carlo simulations and empirical work.

In the first part, basic theoretical concepts of first order autoregressive models are presented. Limiting results, statistical inference and forecasting using the ordinary least squares estimator in AR (1) regressions are discussed. We continue with a discussion on heteroskedasticity. We describe it in details by defining it, presenting the generated difficulties and suggesting existing methods for dealing with this violation of the classical linear regression assumptions. Next, we focus on our hypotheses for the conditional variance of the error. The exact required properties of the error driving the model are stated and discuss the limiting behavior of our series under the existence of polynomial trend in the second moment of the AR (1) errors. Then we make some preliminary comments on how these theoretical results is expected to affect the discussed concepts (statistical inference, forecasting and limiting results) of AR(1) models.

In the next section we present the results of Monte Carlo simulations. We conducted these experiments in order to demonstrate the effect of polynomial trend in the variance of the AR(1) disturbances in the least squares test statistic and compare alternative methods of correcting the suggested kind of heteroskedasticity.

The last section of this paper demonstrates empirical results. For the purposes of this work, we created a database which consists of a variety of macroeconomic and financial data. These time series are tested for the presence of a polynomial way trend in the variance of the fitted residuals by an AR(1) regression. We aim in locating whether real data exhibits or not such a behavior in their disturbances. Afterwards, we conclude with a discussion on how our results changes existing results on the efficiency of the tested markets.

PART I

1. The first order autoregressive process

A first order autoregression model, denoted AR (1), satisfies the following difference equation:

$$y_t = c + r\, y_{t-1} + e_t$$

The process e_t is assumed to be a white noise process, which is historically, the most common building block in time series analysis. A white noise process is quite restrictive, has mean zero, finite variance s^2 and errors uncorrelated across time:

$$E(e_t) = 0$$
$$E(e_t^2) = s^2$$
$$E(e_t e_t) = 0 \quad for \quad t \neq t$$

In this paper we are particularly interested in stationary first autoregressive processes which indicates that |ρ|<1.

Under these assumptions, we can see that the unconditional mean of a stationary AR (1) process is:

$$m = \frac{c}{1-r},$$

the unconditional variance is:

,

while the j-th autocovariance is:

$$g_j = \frac{r^j}{1-r^2} s^2.$$

We can see that a positive value of ρ implies positive correlation between y_t and y_{t-1}, while negative value of ρ implies negative correlation. The absolute value of ρ, indicates us the size of information we can get from previous values of y_t in order to create an expectation for future prices of y_t: the bigger the value of |ρ| the more information we get about the value of the next observation.

1.1 Estimating coefficients for AR (1) model

A first order autoregression has the form of the classical standard regression model

$$y_t = X_t^{'} b + u_t$$

with $X_t^{'} = (1, y_{t-1})$, β = ρ and $u_t = e_t$. The assumption that e_t is a white noise process indicates that e_t is uncorrelated to y_t but this will no be the case for lagged values of y_t, that is $E[e_t \mid y_{t-1}] \neq 0$. Without this independence the ordinary least squares coefficient

$$\hat{r}_T = \frac{\sum_{t=1}^{T} y_{t-1} y_t}{\sum_{t=1}^{T} y_{t-1}^2}$$

gives a biased estimate of ρ for an autoregression in small samples.

However, we can derive asymptotic results for the OLS estimator, even under the presence of such dependence. If the regression model is the stationary AR (1) model, with |ρ|<1, e_t is a white noise process with finite fourth moment m_4 then:

$$\sqrt{\mathrm{T}}(\hat{r}_T - r_T) \xrightarrow{L} N(0, 1 - r^2)$$

and

$$\sqrt{\mathrm{T}}(S_T^2 - s^2) \xrightarrow{L} N(0, m_4 - s^4)$$

1.2 Statistical Inference in AR (1) model

In order to be able to compile hypotheses testing in our model, we need the appropriate test statistics and their distributions. Because of the dependence between e_t and lagged values of y_t, the standard t and F statistics can only be justified asymptotically. In classical linear regression we know that t statistic is distributed as *t (T-k)*, where k: number of explanatory variables and F statistic is distributed as *F (m, T-k).* To justify the usual OLS inference rules, we have to appeal to asymptotic results, for which they take under consideration the mentioned dependence. In the case of first order autoregressive model t and F statistic convergence only in law and changes in asymptotic criteria are required:

$$t_T \xrightarrow{L} N(0,1)$$

and

$$mF_T \xrightarrow{L} c^2(m) .$$

1.3 Forecasting an AR (1) process

In time series analysis we are interested in forecasting the value of a variable Y_{t+1} based on a set of variables X_t observed at date t. In AR (1) models we want to forecast y_t based on its most recent value y_{t-1}. To evaluate the usefulness of this forecast we need to specify a loss function, a summary of how concerned we are if our forecast is off by a particular amount. For this reason a measure is defined as the mean squared error associated with our forecast as:

$$MSE(Y^F_{t+1|t}) = E(Y_{t+1} - Y^F_{t+1|t})^2$$

The forecast with the smallest mean squared errors turns out to be the expectation of y_{t+1} conditional on y_t.

A very common forecast method is based on linear projection; we define the forecasted variable y_{t+1} to be a linear function of y_t, meaning $y^*_{t+1|t} = a' y_t$. The linear projection that produces the smallest mean squared error among the class of linear forecasting rules is the one such that the forecast error $(y^*_{t+1|t} - a' y_t)$ is uncorrelated with y_t.

In order to obtain the forecast of y_t as a function of its first lagged value y_{t-1} we use the Wiener-Kolmogorov prediction formula:

$$\hat{E}\left[Y_{t+s} \mid Y_t, Y_{t-1,\dots}\right] = m + \left[\frac{y\,(L)}{L^s}\right]_+ \frac{1}{y\,(L)}(Y_t - m)\,,$$

where L is the lag operator and ψ(L) is a polynomial of lag operators.

For the covariance-stationary AR (1) process of our concern we have:

$$y\,(L) = \frac{1}{1 - r\,L} = 1 + r\,L + r^2 L^2 + r^3 L^3 + \dots$$

and

$$\left[\frac{y\,(L)}{L^s}\right]_+ = r^s + r^{s+1} L^1 + r^{s+2} L^2 + \dots = \frac{r^s}{1 - r\,L}\,.$$

Substituting the above results in the Wiener-Kolmogorov prediction formula, we get the optimal linear one s-period-ahead forecast for a stationary AR (1) process:

$$\hat{E}\left[Y_{t+s} \mid Y_t, Y_{t-1,\dots}\right] = m + \frac{r^s}{1 - r\,L}(1 - r\,L)(Y_t - m) = m + r^s (Y_t - m)$$

The forecast decays geometrically from $(Y_t - m)$ toward μ as the forecast horizon s increases, because |ρ|<1. The mean squared s-period-ahead forecast error is:

$$\left[1 + r^2 + r^4 + \dots + r^{2(s-1)}\right] s^2$$

We can see that the MSE grows with s and asymptotically approaches

$$\frac{s^2}{1 - r^2}\,,$$

the unconditional variance of y_t.

From what we have seen so far, the assumption of homoskedasticity plays a crucial role in the structure of the AR (1) model. It is used in order to obtain the asymptotic behavior of both the ordinary least squared coefficient t and F-statistic and we have seen that the mean squared s-period-ahead forecast error of

y_t is a function of s^2. However, the assumption of homoskedasticity is very restrictive and unreal for macroeconomic and financial data.

Changes in the variance is a very common increment of time series data and are quite important for understanding financial markets since investors require higher expected returns as compensation for holding riskier assets. A variance that changes over time has major implications for the validity and efficiency of statistical inference about the parameters that describe the dynamics of the level of y_t and in addition forecasting future values of y_t.

We proceed now by trying to describe in general the concept of heteroskedasticity. We will demonstrate the existence of heteroskedasticity in the generalized regression model and modify the results of the classical model. We will consider the consequences for the least squares estimator and examine alternative estimation approaches that can make better use of the characteristics of the model.

2. A general discussion on heteroskedasticity

2.1 Introduction

Under classical assumptions of homoskedasticity, no serial correlation among the disturbances and regressors either fixed in repeated samples or stochastic but uncorrelated with the disturbances , the OLS estimator is Best, Linear, Unbiased (BLU), consistent and asymptotically normally distributed (CAN) and if disturbances are in addition normally distributed, asymptotically efficient among all CAN estimators. In the following table we present the finite and large-sample –properties of the least squares estimator under classical assumptions:

	Finite Sample Properties	Large Sample Properties
Disturbances are normally distributes	$b \mid X \sim N[b, s^{2}(X'X)^{-1}]$	$b \xrightarrow{a} N\left[b, \frac{s^{2}}{n} Q^{-1}\right]$ where
Distribution of disturbances is unknown	E[b|X]=E[b]=β Var[b|X]=σ^2(X´X)$^{-1}$	$\lim_{n\to\infty} \frac{1}{n}(X'X)^{-1} \xrightarrow{p} Q$ and Q a positive defined matrix

The limiting distribution of the least squares estimator for an AR (1) model under more general assumptions on the error of the regression and alternative values of ρ has attracted a lot of research efforts during the last thirty five years or so. All initial studies were restricted by the assumption of i.i.d. errors, or further that is $e_t \sim i.i.d.N(0, s^2)$ therefore assuming that the disturbances of the regression have a constant variance over time.

2.2 The generalized regression model

The violation of this assumption arise the problem of heteroskedasticity. We need to specify how heteroskedasticity affects the properties of OLS estimator. We use the generalized linear regression model:

$$y = Xb + e$$
$$\mathrm{E}[e \mid \mathrm{X}] = 0$$
$$\mathrm{E}[ee' \mid \mathrm{X}] = s^{2}\Omega = \Sigma$$

where Ω is a positive definite matrix. We can see that for Ω=I we get the classical model as a special case of the generalized linear regression model. This kind of assumptions allows both heteroskedasticity and disturbances to be correlated across observations, therefore $s^{2}\Omega$ would be:

$$\sigma_i^2\Omega = \Sigma = \begin{bmatrix} \sigma_1^2 & \varepsilon_1\varepsilon_2 & \cdots & \varepsilon_1\varepsilon_n \\ \varepsilon_2\varepsilon_1 & \sigma_2^2 & \cdots & \varepsilon_2\varepsilon_n \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_n\varepsilon_1 & \varepsilon_n\varepsilon_2 & \cdots & \sigma_n^2 \end{bmatrix}$$

If the regressors and disturbances are uncorrelated, then the unbiased-ness of least squares in finite samples is unaffected by the presence of heteroskedasticity:

$$E\left[\hat{\beta} \mid \mathrm{X}\right] = E\left[(X'X)^{-1}X'Y \mid \mathrm{X}\right] = E\left[(X'X)^{-1}X'\left(X\beta + \varepsilon\right) \mid \mathrm{X}\right] =$$
$$= E\left[((X'X)^{-1}X'X\beta + (X'X)^{-1}X'\varepsilon) \mid \mathrm{X}\right] = \beta + (X'X)^{-1}X'E(\varepsilon \mid \mathrm{X}) = \beta$$

We have already seen that the former cannot be the case for an autoregressive model since the vector X of the regressors contains lagged values of Y where we saw that though, under our stated assumptions ε_t is independent of y_t, it will no be the case that ε_t is independent for lagged values of y_t.

The finite sample variance of the generalized least squares estimator is:

$$\mathrm{var}\left[\hat{\beta} \mid \mathrm{X}\right] = E\left[\left(\hat{\beta} - \beta\right)\left(\hat{\beta} - \beta\right)' \mid \mathrm{X}\right] =$$
$$= E\left[\left((X'X)^{-1}X'Y - \beta\right)\left((X'X)^{-1}X'Y - \beta\right)' \mid \mathrm{X}\right] =$$
$$= E\left[\left((X'X)^{-1}X'\left(X\beta + \varepsilon\right) - \beta\right)\left((X'X)^{-1}X'\left(X\beta + \varepsilon\right) - \beta\right)' \mid \mathrm{X}\right] =$$
$$= E\left[\left((X'X)^{-1}X'\varepsilon\right)\left((X'X)^{-1}X'\varepsilon\right)' \mid \mathrm{X}\right] =$$
$$= E\left[(X'X)^{-1}X'\varepsilon\varepsilon'X(X'X)^{-1} \mid \mathrm{X}\right] =$$
$$= \sigma^2(X'X)^{-1}(X'\Omega X)(X'X)^{-1}$$

If the regressors are stochastic, then the unconditional variance is:

$$E\left[\mathrm{var}\left[\hat{\beta} \mid \mathrm{X}\right]\right]$$

The LS estimator is a linear function of ε. Therefore, if ε is in addition normally distributed, then:

$$\hat{\beta} \mid \mathrm{X} \sim \mathrm{N}\left[\beta, \sigma^2(X'X)^{-1}(X'\Omega X)(X'X)^{-1}\right]$$

Regarding the asymptotic properties of generalized least squares, the existing literature demands that the regressors are sufficiently well behaved, that is:

$$\lim_{n\to\infty}\left(\frac{X'X}{n}\right)\xrightarrow{p}Q$$
$$\lim_{n\to\infty}\left(\frac{X'\Omega X}{n}\right)\xrightarrow{p}V$$

where Q and V are both finite positive definite matrices, then the generalized least squares
estimator is asymptotically distributed with mean β and covariance matrix $\frac{s^2}{n}Q^{-1}VQ^{-1}$:

$$\hat{b}\xrightarrow{a}\mathrm{N}\left(b,\frac{s^2}{n}Q^{-1}VQ^{-1}\right).$$

From what we have seen so far, unless V=I, this is not the same covariance matrix of the OLS estimator derived in the classical linear regression model, so the t statistic associated with this estimator will not have the same interpretation as a Gaussian variable divided by an estimate of its standard deviation. Thus, the OLS statistic for testing the null hypothesis $\mathrm{H}_0:\hat{b}=b$:

$$t=\frac{\hat{b}-b}{\hat{s}_{\hat{b}}}$$

will not have a t(T-k) distribution in small samples, nor will it even asymptotically be N(0,1), since the standardization will not be the appropriate one any more.

However, the choice whether we should use or not the ordinary least squares estimator under heteroskedasticity is not so clear. Our final decision depends on the knowledge or ignorance of Ω matrix. Certainly if Ω is known then the generalized least squares method which we will describe now gives an efficient estimator of the regression.

2.3 The generalized least squares method

Since Ω is a positive definite matrix, it can be factored into:

$$\Omega = C\Lambda C'$$

where the columns of C are the characteristic vectors of Ω and the characteristic roots of Ω are arrayed in the diagonal matrix Λ. Let $\Lambda^{1/2}$ be the diagonal matrix with ith diagonal element $\sqrt{l_i}$, and let $T = C\Lambda^{1/2}$. Then Ω=TT'. Also let $P' = C\Lambda^{1/2}$, so $\Omega^{-1} = P'P$. Premultiple the general regression model by P we obtain:

$$Py = PXb + Pe$$

or

$$y_* = X_* b + e_*$$

Then, the variance of e_* is:

$$E\left[e_* e_*'\right] = Ps^2 \Omega P' = s^2 I$$

so the classical regression model applies to this transformed model. Since Ω is known, y_* and X_* are observed data.

The generalized least squares estimator will be:

$$\begin{aligned} \hat{b} &= (X_*' X_*)^{-1} X_*' y_* = \\ &= (X' P' P X)^{-1} X' P' P Y = \\ &= (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \end{aligned}$$

and will be efficient.

2.4 The feasible generalized least squares method

Even if the matrix Ω is not known, but we have knowledge of its structure and can estimate it using sample data, then we can use feasible generalized least squares method, which is similar with the generalized least squares method with the only difference of using the estimated matrix $\hat{\Omega}$. In this case the feasible generalized least squares estimator can be denoted:

$$\hat{b} = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}Y \ .$$

The third possibility is that Ω is completely unknown, both as to its structure and the specific values of its elements. In this situation least squares may be the only estimator available, and as such, the only available strategy is to try to devise an estimator for the appropriate asymptotic covariance matrix.

2.5 White's heteroskedasticity consistent estimator

Halbert White (1980) presented a covariance matrix estimator which is consistent in the presence of heteroskedasticity, but does not depend on supposed specific formal model of the structure of heteroskedasticity. Nevertheless, White does not relaxes the assumption of no serial correlation among the disturbances. We saw that the asymptotic variance of the least squares estimator in the generalized regression model is $\frac{s^2}{n}Q^{-1}VQ^{-1}$ where $Q = \frac{1}{n}(X'X) = \frac{1}{n}\sum_{i=1}^{n} E(x_i x_i')$ and $V = \frac{1}{n}(X'\Sigma X) = \frac{1}{n}\sum_{i=1}^{n} E(s_i^2 x_i x_i')$. Under his stated assumptions, White concludes that we can calculate consistent estimators of V from the sample observations using the OLS fitted residuals:

$$\hat{V} = \frac{1}{n}\sum_{i=1}^{n} \hat{e}_i^2 x_i x_i'$$

and therefore extract an estimation for the asymptotic covariance matrix of the OLS estimator:

White heteroskedasticity consistent estimator: $n(X'X)^{-1}\hat{V}(X'X)^{-1}$

This result is extremely important and useful. It implies that without actually specifying the type of heteroskedasticity, we can still make appropriate inferences based on the results of least squares.

2.6 Newey-West autocorrelation consistent covariance estimator

The next step would be to extend White's result to the more general case where the disturbances would be both heteroskedastic and serial correlated. This application is more likely to arise in time series models. In this case we would need an estimator for:

$$V_* = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij} x_i x_i'$$

Whitney K. Newey and Kenneth D. West (1987a) proposed a heteroskedasticity and autocorrelation consistent covariance matrix:

$$\hat{V}_* = \hat{V} + \frac{1}{n}\sum_{l=1}^{L}\sum_{t=l+1}^{n} w_l \hat{e}_t \hat{e}_{t-l}(x_t x_{t-l}' + x_{t-l} x_t')$$

$$w_l = 1 - \frac{1}{(L+1)}$$

This formulation of the weights of the off the diagonal elements of $\hat{V}_*$, makes them smaller as we move away from the diagonal. The only question needed to be answered is to determine in advance how large L is to be.

2.7 Modeling heteroskedasticity

A different approach for dealing with heteroskedasticity was introduced by Robert Engel (1982) who suggested a modeling path for the conditional variance of the disturbances. The increased importance played by risk and uncertainty considerations in modern economic theory, however, has necessitated the development of new econometric time series techniques that allow for the modeling of time varying variances and covariances. The autoregressive conditional heteroskedastic (ARCH) class of models introduced by Engle supports that the conditional variance of the errors of time series models is a function of the variance of the previous period. The ARCH(r) regression model is obtained by assuming that the mean of the dependent variable y_t is given as $x_t b$, a linear combination of lagged endogenous and exogenous variables in the information set y_{t-1} with β a vector of unknown parameters. The parameter r indicates the number of observations in h_t. Formally,

$$\begin{aligned} & y_t \mid y_{t-1} \sim N(x_t b, h_t) \\ & h_t = h(e_{t-1}, e_{t-2}, \ldots, e_{t-p}, a) \\ & e_t = y_t - x_t b \end{aligned}$$

The ordinary least squares estimator of β is consistent under these assumptions as x and ε are uncorrelated through the definition of the regression as a conditional expectation. If the x's where to be treated as fixed constants then the least squares standard errors would be correct. However, in time series analysis, there are lagged dependent variables in x_t and the standard errors as conventionally computed will not be consistent, since the squares of the disturbances will be correlated squares of the x's.

In 1986 Bollerslev proposed the generalized autoregressive conditional heteroskedasticity model denoted $u_t \sim GARCH(r, m)$. GARCH models are en extension of the ARCH models where the conditional variance of y_t is not only one function of h_t but in addition it depends on "lagged" functions of h_t, that is

$$y_t \mid y_{t-1} \sim N(x_t b, f(h_t, h_{t-1}, h_{t-2}, \ldots, h_{t-m}))$$

It is obvious that ARCH(r) models are a special case of GARCH(r, m) models for m=1

Volatility clustering is the optical indicator in order to assume GARCH models in our series and is immediately apparent when time series are plotted through time. Figure 1 plots daily returns of the one month forward rate among Germany and New Zealand.

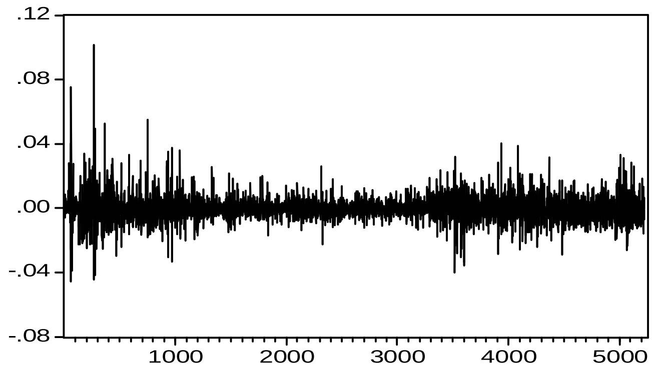


Figure 1

It is clear from visual inspection of the figure, and any reasonable statistical test, that the returns are not i.i.d. through time. In this figure we can observe three different periods based on the volatility, a high volatility period at the beginning, then a decrease and then again high volatility.

2.8 A new approach on dealing with heteroskedasticity

In this paper, following suggestions from a paper by Kourogenis N. and Pittis N. (2005) we are studying series which are heteroskedastic in a specific way. The variance of the disturbances appears to grow without a limit in a polynomial way. Like the case of GARCH models, a trending variance can easily be detected from the t-plot of the time series data we are observing. Figure 2 plots daily returns of the foreign exchange rate between Canadian and US dollar for the last thirty five years.

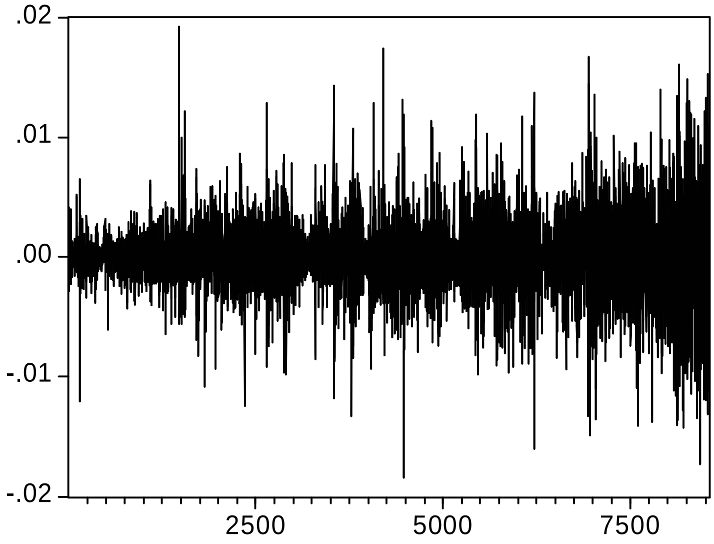


Figure 2

We can see that the volatility of returns increase as we proceed in time.

In the next section we proceed by presenting the exact assumptions of the innovation sequence e_t of our AR (1) model and the concluding limiting distribution for both the least squares estimator of ρ and the associated t statistic for the stable case $|\rho|<1$. We then make a reference on how these results can differentiate currently inference results about this category of time series data.

3. Asymptotic theory for stationary first-order autoregressions with trending variance.

3.1 Stating the model

In this section we present the work of Nikolaos Kourogenis and Nikitas Pittis in their paper "Asymptotic theory for first-order autoregressions with asymptotically unbounded error variance" (2005). All assumptions, propositions and theorems are adopted from their work.

Our model of interested is stationary AR (1) models. That is

$$y_t = c + r\, y_{t-1} + u_t \text{ with } |\rho|<1.$$

For this case, we assume that u_t is a stochastic process that satisfies the following assumption:

Assumption U_1:

a. $u_t = \sqrt{f(t)}v_t$, where $f(t) = b_k t^k + g(t) > 0$ with $g(t) = O(t^{k-1})$ if $k \geq 1$ and $g(t) = 0$ if $0 \leq k < 1$;

and $\{v_t\}_{t\geq 1}$ is a stochastic process such that:

b. $E[v_t \mid X_{t-1}] = 0$ a.s., where X_{t-1} is the σ-algebra generated by $y_0, v_1, v_2, ..., v_{t-1}$

c. $E\left[v_t^2 \mid X_{t-1}\right] = s^2 < \infty$ and

d. $\sup_t E\left[|v_t|^{2r}\right] \leq \mathrm{B} < \infty, r > 2$

Assumption U1 can be simplified as follows: Let $v'(t) = \sqrt{b_k v_t}$. Consider that for a well defined setting b_k must be assumed positive. Then if we set $f_1(t) = t^k + g_1(t)$ where $g_1(t) = \frac{1}{b_k} g\ (t)$, we have $ut = \sqrt{f_1(t)}v_t'$. Moreover

$$\frac{1}{T}\sum_{t=1}^{T} v_t'^2 = \frac{b_k}{T}\sum_{t=1}^{T} u_t^2 \xrightarrow{p} b_k s_v^2 = s_{v'}^2$$

and

$$\frac{E\left[\left(\sum_{t=1}^{T} v_t'\right)^2\right]}{T} = \frac{E\left[b_k\left(\sum_{t=1}^{T} v_t\right)^2\right]}{T} \longrightarrow b_k s^2 = s'^2$$

Therefore, without any loss of generality and for the rest of the paper we can assume that in assumption U_1 we have $b_k = 1$.

This particular set of assumptions regarding u_t does not allow v_t being conditionally heteroskedastic. Higher unconditional moments of v_t are assumed to be bounded although not necessarily identical.

Finally the assumption on the initial value of the process y_0 is the following:

Assumption S_1: y_0 is an arbitrary constant or a random variable with $E\left[|y_0|^{2r}\right] \le B$, where, without any loss of generality the constants r and B are the ones defined in assumption U_1.

Under these assumptions, the matrix Σ defined in the generalized regression model will have the follow structure:

$$\sigma_i^2 \Omega = \Sigma = \begin{bmatrix} 1^k \sigma^2 & \varepsilon_1 \varepsilon_2 & \cdots & \varepsilon_1 \varepsilon_t \\ \varepsilon_2 \varepsilon_1 & 2^k \sigma^2 & \cdots & \varepsilon_2 \varepsilon_t \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_t \varepsilon_1 & \varepsilon_t \varepsilon_2 & \cdots & t^k \sigma^2 \end{bmatrix}$$

For the case of f(t)=1, i.e. k=0, $\beta_0=1$, that is when the variance of the innovations $u_t\,(=v_t)$ is bounded, the asymptotic behavior is well know under both ρ=1 and |ρ|<1, that is

$$\sqrt{T}\,(\hat{\rho}_{\mathrm{T}} - \rho) \xrightarrow{L} N(0,(1-\rho^2)$$

and

$$t_\rho \xrightarrow{L} N\left(0, \frac{1+\rho}{1-\rho}\right)$$

as $\mathrm{T} \to \infty$.

In the next section we generalize the existing result by allowing for asymptotically unbounded m-th moment of u_t, with m>1.

3.2 Asymptotic results for the stable root case, |ρ|<1

When |ρ|<1 our set of assumptions allows sequence u_t to be a martingale difference sequence.

Assumptions U1(b),(c) and (d) can be found in Davidson (2000). These assumptions are sufficient for obtaining limiting results for the OLS estimator $\hat{r}_T$, when |ρ|<1 and k=0. Moreover, if u_t satisfies assumption U1(a), then the requirements of the following proposition are satisfied:

Proposition 1: If $\{u_t\}_{t\geq 1}$ satisfies assumption U1 (a) and (b) then:

$$\frac{1}{T^{k+1}}\sum_{t=1}^{T}u_t^2 \xrightarrow{p} \frac{s_v^2}{k+1}.$$

The previous result is necessary in order to obtain the asymptotic behaviour of the OLS estimator $\hat{r}$.

Before presenting the main result for this case, we need to take an intermediate step, stated in the form of the following proposition:

Proposition 2: If |ρ|<1, y_0 satisfies assumption S_1 and $\{u_t\}_{t\geq 1}$ satisfies assumption U_1 then

$$\frac{1}{\mathrm{T}^{k+1/2}}\sum_{t=1}^{T}u_t y_{t-1} \xrightarrow{L} N(0,\frac{s^4}{(2k+1)(1-r^2)})$$

The result of the previous proposition is based on the fact that the series $\sum_{j=1}^{\infty} r^{2j} j^{k-d}$ converges for every d. In fact, the increase of the variance of u_t is eliminated by the continuous multiplication by ρ, whose absolute value is strictly less than unity.

Now we are ready to state the main result of this section:

Theorem 2: If |ρ|<1, y_0 satisfies assumption S_1 and $\{u_t\}_{t\geq 1}$ satisfies assumption U_1 then

$$\sqrt{T}\,(\hat{r}_{\mathrm{T}} - r) \xrightarrow{L} N(0,\frac{(k+1)^2(1-r^2)}{(2k+1)})$$

and

$$t_r \xrightarrow{L} N\left(0,\frac{(k+1)^2(1+r)}{(2k+1)(1-r)}\right)$$

as $\mathrm{T}\rightarrow\infty$.

The estimator of $\hat{k}$ is given by: $\left[\frac{1}{\ln 2} * \ln \frac{\sum_{t=1}^{T} \hat{u}_t^2}{\sum_{t=1}^{T/2} \hat{u}_t^2}\right] - 1$

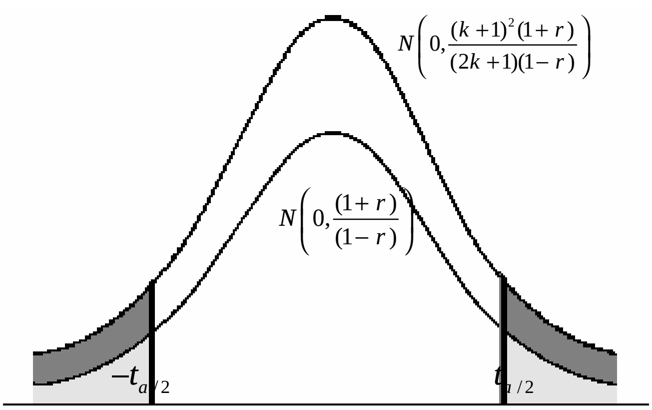
It can been seen that for the bounded variance case, k=0, the above relationships become $\sqrt{T}\,(\hat{r}_{\mathrm{T}} - r) \xrightarrow{L} N(0,(1-r^2)$ and $t_r \xrightarrow{L} N\left(0, \frac{1+r}{1-r}\right)$ already shown from the existing literature. Again, the more general theory developed for this case accommodates the existing results as special cases. As far as the effects of k on the limiting distributions are concerned, it is clear that their variances are proportional to the ratio $\frac{(k+1)^2}{(2k+1)}$ which is an increasing function of k, k>0.

3.3 A preliminary discussion on the new approach.

By comparing the above limiting results for test statistic of the OLS coefficient estimator, $t_r \xrightarrow{L} N\left(0, \frac{(k+1)^2(1+r)}{(2k+1)(1-r)}\right)$ and

$$t_r \xrightarrow{L} N\left(0, \frac{1+r}{1-r}\right),$$

in combination with the fact that $(k+1)^2 = k^2 + 2k + 1 > 2k + 1$, for k>0, we can conclude that the asymptotic variance of the t-statistic when we adjust for polynomial trend will be bigger than the asymptotic variance of the t-statistic in the known boundary case. This conclusion is presented in the following figure:



If we had the critical value t_a for a statistical level of significance α for the test statistic $t_r \xrightarrow{L} N\left(0, \frac{1+r}{1-r}\right)$, this would define a rejection area of α% for the null hypothesis (the sum of the light grey areas). We can see that this critical value $t_{a/2}$ corresponds to a wider rejection area of the null hypothesis in the case there is polynomial trend of k order. Therefore, if we do not detect polynomial trend while this exists, our hypothesis testing results will be misleading since our statistical level of significance won't be the desired one.

Suppose we want to test the statistical significance of the OLS coefficient estimator $\hat{r}_{\mathrm{T}}$ of our AR (1) model, that is:

$$H_0 : \hat{r}_{\mathrm{T}} = 0$$

Since the asymptotic distribution for both cases (with or with no trend) is the mean zero Normal distribution, we have to standardize appropriately these statistics in order to converge to N (0, 1). Therefore:

$$\frac{t_{\hat{r}}}{\sqrt{\frac{1+r}{1-r}}} \xrightarrow{L} N(0,1) \text{ and}$$

$$t_{\hat{r}}^* = \frac{t_{\hat{r}}}{(\frac{k+1}{\sqrt{2k+1}})\sqrt{\frac{1+r}{1-r}}} \xrightarrow{L} N(0,1)$$

Since $\frac{k+1}{\sqrt{2k+1}} > 1$, we can say that $t_{\hat{r}} > t_{\hat{r}}^*$ for k>0. If now we define our desired level of significance α according to which we obtain the critical value t_a of the test statistic from the tables of standard Normal distribution, we can reach three different results:

- $t_{\hat{r}} > t_{\hat{r}}^* \geq t_a$
- $t_a \geq t_{\hat{r}} > t_{\hat{r}}^*$
- $t_{\hat{r}} \geq t_a \geq t_{\hat{r}}^*$

In the first two cases, the finding of polynomial trend in the conditional variance of time series will no affect our conclusions in statistical inference. In the first case, both test statistics suggest us to reject the null hypothesis, while in the second case both test statistics indicate the opposite.

The case that we are interested in is the third one. If this is the case we will have conflicting results in our hypothesis testing, i.e. we may decide no to reject the null due to the presence of polynomial trend. Because of this, we can say that if we do not correct our test for the presence or trend, there is a possibility to over reject the null hypothesis.

Another point we wish to emphasize, is the relationship between $t_{\hat{r}}$ and the estimate of kappa. As we have seen, calculating the adjusted to kappa test statistic $t_{\hat{r}}^*$ we have to divide $t_{\hat{r}}$ by the ratio $(\frac{k+1}{\sqrt{2k+1}})$, that is:

$$t_{\hat{r}}^* = \frac{t_{\hat{r}}}{(\frac{k+1}{\sqrt{2k+1}})}$$

or

$$t_{\hat{r}} = t_{\hat{r}}^* \times (\frac{k+1}{\sqrt{2k+1}})$$

Therefore, we expect to locate in our macroeconomic and financial time series a positive relationship between $t_{\hat{r}}$ and kappa, since the ratio $(\frac{k+1}{\sqrt{2k+1}})$ increases with k and $t_{\hat{r}}$ is influenced positively by the ratio.

We continue now to the second part of this paper, the Monte Carlo simulations. We could say that a Monte Carlo simulation is for econometricians what a scientific lab is for physics and chemistry researches. It gives us the opportunity to verify different aspects of random variables. At first, we describe the design of the experiment and then present and comment its results.

PART II

4. Monte Carlo Simulation

4.1 Design of the experiments

Our first set of results suggests that our data generating process will be a stationary driftless first order autoregressive model with errors which exhibits polynomial trend in their variance, that is:

$$y_t = r\, y_{t-1} + u_t \text{ and } u_t = \sqrt{f(t) + g(t)}v_t .$$

Without loss of generality we set ρ=0.5 and set y_0 to be drawn from a Normal distribution $y_0 \sim N\left(0, \frac{1}{1-r}\right) \Rightarrow y_0 \sim N(0,2)$. We choose functions f and g to be $f(t) = \frac{1}{9}t^k$ for k>0 and $g(t) = t^{k-1} * \cos(\frac{t}{3}) + 2$ for $k \geq 1$ and $g(t) = 0$ for $0 \leq k < 1$. Process v_t is assumed to be drawn from a Normal distribution $v_t \sim N(0,1)$. We run the experiments for different values of the sample size T and values of the order k of the polynomial of f.

We use values of T of 25, 50, and 100 and of k of 0, 1, 2, 3. We wish to examine if the presence of polynomial trend in the variance of the errors has fast influence that can affect our results in finite samples and if this velocity depends on the value of k. All simulations were based on 10,000 replications. Computations were done the econometric program EVIEWS 4.0, using the random seed generator 21.

For each replication we stored the OLS t-statistic, the t-statistic given by the weighted least squares estimator with weight being the series $w = \frac{1}{\sqrt{t^k}}$, the t-statistic using White's heteroskedasticity-consistent covariance matrix, the t-statistic adjusted to the known k and the t-statistic adjusted to the estimated k.

At the end of all executions, we calculated the percentiles of the t-statistics:

1%	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**

for each of the above results to be able to compare them with the Normal distribution and also the empirical size of rejections at the nominal 5% level.

Apart from the above set of experiments we have also conducted simulation for first order autoregressive models where the variance of the errors exhibits similar behavior as the studied case of polynomial trend, but is caused by a different structure of heteroskedasticity.

In the first case, we allow one break in the variance structure. We let T=100, ρ=0.5, $y_0 \sim N(0,2)$ and set the errors of the regression for the first half of the sample to be $u_t \sim N(0,1)$ and for the remaining sample to be $u_t \sim N(0,2)$, $u_t \sim N(0,10)$ and $u_t \sim N(0,100)$.

In the second case, we allow two breaks in the variance structure. We let T=120, ρ=0.5, $y_0 \sim N(0,2)$ and set the errors of the regression for the first 40 observations of the sample to be $u_t \sim N(0,1)$, for the next 40 observations to be $u_t \sim N(0,3)$, and for the last 40 observations to be $u_t \sim N(0,6)$.

In the third case, we allow five breaks in the variance structure. We let T=100, ρ=0.5, $y_0 \sim N(0,2)$ and set the errors of the regression for the first 20 observations of the sample to be $u_t \sim N(0,1)$. Then for the next 20 observations the variance increases by one unit, concluding to be $u_t \sim N(0,6)$ for the last 20 observations.

Finally, we let the errors to be an exponential function of time of the form: $f_1(t) = 0.1 * e^{\frac{t}{5}}$, that is $u_t = \sqrt{0.1 * e^{\frac{t}{5}}} v_t$, for T=50,100,200,500 and ρ=0.5.

The aim of the additional experiments is to test how the test statistic adjusted to the estimated k is better or not than the competitive t-statistics in cases where although we do not have polynomial trend in the variance, the variance appears to increase in a different way as time evolves.

4.2 Results of the experiments

Our first set of experiments varies the sample size T and the value of k. In tables 1A and 1B we have the results for T=25, in tables 2A and 2B for T=50 and in tables 3A and 3B for T=100. The indicator A refers to the tables of percentiles and the indicator B to the tables of empirical sizes of the test statistics.

An important first result to be stated is that the OLS t-statistic appears to give very misleading results as k increases for all three sample sizes. Therefore, from the start we get evidence for the need of correction heteroskedasticity.

Weighted least squares t-statistic in the small sample T=25 does not work at all. For T=50 and 100 it can "catch" heteroskedasticity and correct it in a small percent only for k>1. Nevertheless, it does not seem to be able to remove all the effect of heteroskedasticity.

White's t-statistic appears to approximate Normal distribution much more than the OLS and WLS t-statistic. An expected result is that it works more appropriate as the sample size increases since the squared OLS residuals tend to underestimate the squares of the true disturbances (MacKinnon and White (1985)). The end result is that in small samples, the White estimator is a bit too consistent; the matrix is a bit too small, so asymptotic t ratios are a little too large, driving us to over-rejections.

The "best" t-statistic appears to be the one which is adjusted to the estimated k that approximates very closely the Normal distribution regardless the sample size or the value of k. We also observe that for $k \neq 0$, we locate significant differences between the t-statistic adjusted to known k and the t-statistic adjusted to the estimated k. The reason why this is happening is that in small samples the real k does not have the time to dominate the errors of the regression, while the estimated k, which is smaller than the known k, can describe better the polynomial trend effect in the variance of the errors.

TABLE 1A
Percentiles of the Empirical Distributions of t Statistics.
T=25.

	k=0							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-2.480	-2.074	-1.749	-1.382	1.198	1.593	1.914	2.363
WLS	-2.936	-2.327	-1.907	-1.491	1.324	1.800	2.252	2.791
White	-3.262	-2.632	-2.114	-1.599	1.333	1.786	2.229	2.717
t_ρ^* (k-Known)	-2.480	-2.074	-1.749	-1.382	1.198	1.593	1.914	2.363
t_ρ^* (k-Estimated)	-2.434	-2.041	-1.711	-1.349	1.170	1.557	1.895	2.331
	k=1							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-2.607	-2.202	-1.831	-1.444	1.228	1.603	1.991	2.420
WLS	-2.826	-2.296	-1.877	-1.470	1.262	1.687	2.084	2.531
White	-3.084	-2.551	-2.093	-1.594	1.311	1.748	2.153	2.637
t_ρ^* (k-Known)	-2.258	-1.907	-1.586	-1.251	1.064	1.388	1.724	2.095
t_ρ^* (k-Estimated)	-2.502	-2.060	-1.721	-1.339	1.137	1.488	1.836	2.243
	k=2							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-3.426	-2.865	-2.393	-1.877	1.530	2.073	2.559	3.178
WLS	-6.272	-4.145	-3.099	-2.179	1.891	2.666	3.566	4.878
White	-3.583	-2.928	-2.387	-1.800	1.354	1.870	2.308	2.899
t_ρ^* (k-Known)	-2.553	-2.136	-1.783	-1.399	1.140	1.545	1.907	2.369
t_ρ^* (k-Estimated)	-2.618	-2.156	-1.795	-1.362	1.107	1.505	1.880	2.391
	k=3							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-3.824	-3.262	-2.691	-2.095	1.691	2.317	2.873	3.615
WLS	-10.391	-6.569	-4.487	-2.961	2.585	3.893	5.442	7.671
White	-3.977	-3.090	-2.505	-1.861	1.388	1.943	2.419	3.095
t_ρ^* (k-Known)	-2.530	-2.157	-1.780	-1.385	1.118	1.532	1.901	2.391
t_ρ^* (k-Estimated)	-2.497	-2.078	-1.702	-1.287	1.028	1.414	1.808	2.313

TABLE 1B
Empirical Sizes of t Statistics (nominal size =0.05).
T=25.

t-statistics:	k=0	k=1	k=2	k=3
OLS	5.61	6.67	14.67	19.22
WLS	8.48	7.33	21.33	31.76
White	10.03	9.49	12.68	14.02
t_ρ^* (k-Known)	5.61	3.68	5.85	5.86
t_ρ^* (k-Estimated)	5.14	5.25	5.92	4.83

TABLE 2A
Percentiles of the Empirical Distributions of t Statistics.
T=50.

	k=0							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-2.483	-2.107	-1.741	-1.353	1.199	1.572	1.904	2.279
WLS	-2.759	-2.283	-1.908	-1.437	1.277	1.685	2.095	2.604
White	-2.869	-2.350	-1.912	-1.457	1.290	1.699	2.062	2.507
t_ρ^* (k-Known)	-2.483	-2.107	-1.741	-1.353	1.199	1.572	1.904	2.279
t_ρ^* (k-Estimated)	-2.444	-2.080	-1.720	-1.338	1.182	1.550	1.878	2.245
	k=1							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-2.577	-2.162	-1.829	-1.445	1.242	1.673	2.037	2.376
WLS	-2.597	-2.210	-1.849	-1.451	1.260	1.668	2.011	2.454
White	-2.780	-2.318	-1.917	-1.483	1.272	1.676	2.052	2.433
t_ρ^* (k-Known)	-2.232	-1.872	-1.584	-1.251	1.075	1.449	1.764	2.058
t_ρ^* (k-Estimated)	-2.446	-2.033	-1.741	-1.373	1.172	1.585	1.912	2.273
	k=2							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-3.151	-2.631	-2.232	-1.780	1.532	2.048	2.524	3.058
WLS	-3.629	-2.680	-2.189	-1.679	1.480	1.992	2.479	3.160
White	-2.968	-2.428	-2.035	-1.563	1.306	1.752	2.138	2.622
t_ρ^* (k-Known)	-2.349	-1.961	-1.663	-1.327	1.142	1.526	1.881	2.280
t_ρ^* (k-Estimated)	-2.507	-2.094	-1.761	-1.391	1.190	1.604	1.998	2.406
	k=3							
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-3.541	-2.960	-2.508	-1.992	1.715	2.301	2.818	3.403
WLS	-3.614	-2.762	-2.201	-1.705	1.488	2.012	2.498	3.137
White	-3.130	-2.547	-2.105	-1.619	1.330	1.776	2.193	2.735
t_ρ^* (k-Known)	-2.342	-1.958	-1.659	-1.318	1.135	1.522	1.864	2.251
t_ρ^* (k-Estimated)	-2.531	-2.081	-1.757	-1.384	1.189	1.591	1.971	2.438

TABLE 2B
Empirical Sizes of t Statistics (nominal size =0.05).
T=50.

	T=50			
t-statistics:	k=0	k=1	k=2	k=3
OLS	5.50	6.74	13.23	17.63
WLS	7.68	6.89	12.18	12.58
White	7.67	7.74	9.10	10.16
t_ρ^* (k-Known)	5.50	3.42	4.56	4.50
t_ρ^* (k-Estimated)	5.17	5.23	5.98	5.94

TABLE 3A
Percentiles of the Empirical Distributions of t Statistics.
T=100.

	k=0							
t-statistics:	**0.01**	**0.025**	**0.05**	**0.1**	**0.9**	**0.95**	**0.975**	**0.99**
OLS	-2.424	-2.016	-1.718	-1.338	1.220	1.581	1.896	2.297
WLS	-2.547	-2.131	-1.782	-1.399	1.266	1.665	1.994	2.446
White	-2.558	-2.184	-1.807	-1.395	1.266	1.643	1.999	2.421
t_ρ^* (k-Known)	-2.424	-2.016	-1.718	-1.338	1.220	1.581	1.896	2.297
t_ρ^* (k-Estimated)	-2.396	-2.001	-1.704	-1.329	1.206	1.569	1.870	2.256
	k=1							
t-statistics:	**0.01**	**0.025**	**0.05**	**0.1**	**0.9**	**0.95**	**0.975**	**0.99**
OLS	-2.585	-2.159	-1.813	-1.444	1.304	1.750	2.102	2.569
WLS	-2.603	-2.204	-1.828	-1.428	1.322	1.750	2.118	2.577
White	-2.567	-2.133	-1.790	-1.397	1.239	1.660	2.001	2.504
t_ρ^* (k-Known)	-2.239	-1.870	-1.570	-1.250	1.129	1.516	1.820	2.225
t_ρ^* (k-Estimated)	-2.375	-1.984	-1.650	-1.315	1.189	1.598	1.932	2.371
	k=2							
t-statistics:	**0.01**	**0.025**	**0.05**	**0.1**	**0.9**	**0.95**	**0.975**	**0.99**
OLS	-3.134	-2.602	-2.244	-1.778	1.607	2.130	2.611	3.189
WLS	-3.968	-2.870	-2.242	-1.686	1.495	2.063	2.645	3.577
White	-2.644	-2.218	-1.867	-1.454	1.275	1.691	2.074	2.512
t_ρ^* (k-Known)	-2.336	-1.939	-1.672	-1.325	1.197	1.588	1.946	2.377
t_ρ^* (k-Estimated)	-2.372	-1.967	-1.689	-1.343	1.197	1.597	1.959	2.410
	k=3							
t-statistics:	**0.01**	**0.025**	**0.05**	**0.1**	**0.9**	**0.95**	**0.975**	**0.99**
OLS	-3.519	-2.940	-2.531	-2.021	1.796	2.400	2.899	3.559
WLS	-3.873	-2.746	-2.162	-1.634	1.466	2.001	2.557	3.549
White	-2.723	-2.266	-1.909	-1.502	1.282	1.719	2.109	2.531
t_ρ^* (k-Known)	-2.328	-1.944	-1.674	-1.337	1.188	1.587	1.918	2.354
t_ρ^* (k-Estimated)	-2.381	-1.984	-1.698	-1.343	1.194	1.599	1.937	2.383

TABLE 3B
Empirical Sizes of t Statistics (nominal size =0.05).
T=100.

t-statistics:	**k=0**	**k=1**	**k=2**	**k=3**
OLS	5.01	7.11	14.35	19.16
WLS	6.16	7.56	12.65	12.07
White	6.49	6.31	7.36	7.83
t_ρ^* (k-Known)	5.01	3.82	4.75	4.76
t_ρ^* (k-Estimated)	4.87	5.00	5.03	5.02

Next, we present the rest of the results created by different assumptions for the variance structure.

In the case where variance changes only once (Tables 4 to 6), the OLS t-statistic becomes less appropriate the bigger the increase in the variance. Although no very close to Normal distribution, White's test statistic seems to correct efficiently the heteroskedasticity problem. Its ability to perform well seems to decrease but in a smaller rank as the OLS t-statistic while the jump in the variance is bigger. The t-statistic adjusted to the estimated k under-rejects the null hypothesis and the problem gets bigger as variance gets also bigger. This result was expected as the estimator of k uses the squared residuals of two samples: the full sample and the first half sample, therefore creating in this case unexpected large estimates of k.

In the next two cases, where the variance breaks in two and five points respectively (Tables 7 and 8), t-statistic adjusted to estimated k seems to be the most appropriate way to conduct statistical inference. The difference we the previous case is that now, we imposed a structure of increasing variance more smoothly and therefore approximating more our basic form of heteroskedasticity.

Finally, in Tables 9A and 9B we present the results of the experiment where the errors were allowed to increase by time in an exponential way. We can see that as T gets larger the problem with the OLS t-statistic also gets less appropriate. However White's t-statistic and t-statistic adjusted to estimated k seems to stay unaffected by the sample size. Even though, both of them fail to correct the presence of heteroskedasticity, the latter seems to be more appropriate for hypothesis testing.

In the appendix, there is table which presents all the "pseudo" estimations of k which we get for all the above cases.

TABLE 4
Percentiles of the Empirical Distributions and Empirical Sizes of t Statistics.
Variance Shifts. T=100

Subperiods	Variance
[1,50]	1
[51,100]	2

t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	SIZE
OLS	-2.516	-2.091	-1.761	-1.395	1.266	1.683	2.015	2.431	6.08
WLS	-3.009	-2.432	-2.014	-1.544	1.415	1.890	2.353	2.954	10.02
White	-2.585	-2.132	-1.775	-1.383	1.237	1.652	1.985	2.492	6.09
t_ρ^* (k-Estimated)	-2.353	-1.941	-1.633	-1.290	1.172	1.550	1.876	2.284	4.41

TABLE 5
Percentiles of the Empirical Distributions and Empirical Sizes of t Statistics.
Variance Shifts. T=100

Subperiods	Variance
[1,50]	1
[51,100]	10

t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	SIZE
OLS	-3.058	-2.563	-2.190	-1.717	1.538	2.045	2.471	2.987	12.74
WLS	-17.634	-13.353	-10.313	-7.157	5.556	7.937	10.285	13.325	64.39
White	-2.589	-2.202	-1.853	-1.438	1.259	1.666	2.042	2.517	6.96
t_ρ^* (k-Estimated)	-2.196	-1.859	-1.563	-1.217	1.088	1.454	1.780	2.151	3.65

TABLE 6
Percentiles of the Empirical Distributions and Empirical Sizes of t Statistics.
Variance Shifts. T=100

Subperiods	Variance
[1,50]	1
[51,100]	100

t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	SIZE
OLS	-3.322	-2.857	-2.400	-1.897	1.665	2.231	2.663	3.244	16.23
WLS	-87.262	-58.851	-42.194	-28.053	18.220	25.856	34.531	46.057	89.14
White	-2.626	-2.224	-1.858	-1.460	1.260	1.676	2.047	2.509	7.22
t_ρ^* (k-Estimated)	-1.804	-1.532	-1.281	-1.004	0.883	1.191	1.428	1.739	1.02

TABLE 7
Percentiles of the Empirical Distributions and Empirical Sizes of t Statistics.
Variance Shifts. T=120

Subperiods	Variance
[1,40]	1
[41,80]	3
[81,120]	6

t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	SIZE
OLS	-2.669	-2.326	-1.952	-1.539	1.421	1.856	2.236	2.735	9.16
WLS	-3.995	-3.091	-2.415	-1.793	1.672	2.252	2.974	3.957	15.46
White	-2.508	-2.133	-1.775	-1.387	1.271	1.669	1.995	2.475	6.24
t_ρ^* (k-Estimated)	-2.316	-2.038	-1.700	-1.335	1.239	1.620	1.955	2.388	5.40

TABLE 8

Percentiles of the Empirical Distributions and Empirical Sizes of t Statistics. Variance Shifts. T=120

Subperiods	Variance
[1,20]	1
[21,40]	2
[41,60]	3
[61,80]	4
[81,100]	5
[101,120]	6

t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	SIZE
OLS	-2.577	-2.214	-1.880	-1.499	1.340	1.770	2.183	2.562	7.96
WLS	-3.054	-2.403	-1.986	-1.524	1.438	1.890	2.265	2.865	9.71
White	-2.457	-2.098	-1.791	-1.411	1.255	1.687	2.027	2.406	6.24
t_ρ^* (k-Estimated)	-2.319	-1.983	-1.690	-1.338	1.193	1.601	1.954	2.304	5.14

TABLE 9A

Percentiles of the Empirical Distributions of t Statistics.

$$u = \sqrt{0.1 * e^{t/5}} * v$$

				T=50				
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-5.372	-4.442	-3.652	-2.889	2.506	3.455	4.345	5.447
White	-4.010	-3.086	-2.469	-1.874	1.490	2.070	2.674	3.386
t_ρ^* (k-Estimated)	-2.959	-2.396	-1.945	-1.514	1.301	1.804	2.286	2.970
				T=100				
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-7.617	-6.194	-5.180	-4.069	3.603	4.929	6.315	7.883
White	-3.892	-3.090	-2.469	-1.873	1.499	2.076	2.641	3.537
t_ρ^* (k-Estimated)	-2.895	-2.331	-1.929	-1.502	1.328	1.828	2.367	2.965
				T=200				
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-10.672	-8.910	-7.463	-5.870	4.965	6.855	8.759	11.107
White	-3.962	-3.075	-2.478	-1.882	1.443	2.028	2.584	3.428
t_ρ^* (k-Estimated)	-2.841	-2.353	-1.970	-1.537	1.297	1.807	2.314	2.930
				T=500				
t-statistics:	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
OLS	-17.272	-13.962	-11.432	-8.933	8.078	11.168	14.335	17.768
White	-4.072	-3.098	-2.431	-1.832	1.509	2.124	2.736	3.496
t_ρ^* (k-Estimated)	-2.884	-2.332	-1.902	-1.488	1.345	1.860	2.384	2.968

TABLE 9B

Empirical Sizes of t Statistics (nominal size =0.05).

$$u = \sqrt{0.1 * e^{t/5}} * v$$

t-statistics:	T=50	T=100	T=200	T=500
OLS	34.68	50.16	64.12	76.63
White	14.58	14.73	14.52	14.78
t_ρ^* (k-Estimated)	8.95	8.82	9.06	8.92

PART III

5. The construction of our database

We proceed this study to its final part which is an empirical work over the discussed theory so far. At the beginning, we give a full description of the database we constructed in order to test the so far theory and then describe briefly the history of the efficient market hypothesis (EMH).

In this paper we are interested in macroeconomic and financial time series data. We aimed in collecting a wide range of data in order to correspond to a wide field of economists concerns. We searched for series that theory allows us to impose at them a first order autoregressive model.

From the field of finance we have focused our research on stock market indices. We examined both emerging and developed markets. We use yearly, daily and intraday data (every 10 seconds) to observe the behavior of these series.

From the field of macroeconomics we examined the behavior of foreign exchange markets, interest rates and prices of commodities. For these series we use yearly and daily data.

In order to have stationary processes, for level time series we generated their returns from one period to the other, by taking logarithmic differences. For interest rates we observed their level changes from one period to the other by taking differences in the level of the series.

Our concern was to emphasize in these series that exhibited a growing variance as time evolves with lack of outlays which could lead us to bad estimations of the parameter k. In table 10 we show the full collection of our chosen time series to apply the new approach discussed in this paper while in appendix there is a table of all data we have collected.

Table 10

Selected data

DESCRIPTION	MNEMONIC	SOURCE	FREQUENCY	STARTING DATE	OBSERVATIONS
Australia ASX All-Ordinaries (w/GFD extension)	_aordd	Global Financial Data	Year	1875	130
UK FT-Actuaries All-Share Index (w/GFD extension)	_ftsad	Global Financial Data	Year	1800	205
Australia Commonwealth 10-year Bonds	igaus10d	Global Financial Data	Year	1858	147
Canadian Government Bonds 10+ Years Maturity	igcand	Global Financial Data	Year	1855	150
UK 2 1/2% Consol Yield	iggbrcw	Global Financial Data	Year	1800	205
Japan 7-year Government Bond Yield	igjpn7d	Global Financial Data	Year	1870	135
USA 10-year Bond Constant Maturity Yield	igusa10d	Global Financial Data	Year	1800	205
Moody's Corporate AAA Yield	mocaaad	Global Financial Data	Year	1857	148
France SBF-250 Index (w/GFD extension)	_sbf250d	Global Financial Data	Year	1856	149
Silver Cash Price (US$/Ounce)	__xag_hd	Global Financial Data	Year	1800	205
USA 10-year Government Bond Total Return Index	trusg10m	Global Financial Data	Year	1800	205
Foreign Exchange Rates Canada $ to U.S. $	dexcaus	Federal Reserve Bank of St. Louis	Daily	4/1/1971	8551
Foreign Exchange Rates Hong Kong $ To Australian $	hkaudsp	Datastream	Daily	31/5/1993	3025
Stock Market: Cyprus Datastream Index	totmkcp	Datastream	Daily	23/12/1992	3141
Stock Market: Finland Datastream Index	totmkfn	Datastream	Daily	25/3/1988	4381
Stock Market: Dow Jones Industrial Index	djindus	Datastream	Daily	1/1/1980	6331
Stock Market: Dow Jones Industrial	d&j-ind	Dukascopy Trading Technologies	Intraday(10sec)	22/3/2005	2340
Stock Market: Dow Jones Industrial	d&j-ind	Dukascopy Trading Technologies	Intraday(10sec)	12/4/2005	2295
Stock Market: Dow Jones Industrial	d&j-ind	Dukascopy Trading Technologies	Intraday(10sec)	3/5/2005	2340
Stock Market: UK FTSE-100	*futsee-100*	Dukascopy Trading Technologies	Intraday(10sec)	1/4/2005	3000
Stock Market: UK FTSE-100	*futsee-100*	Dukascopy Trading Technologies	Intraday(10sec)	19/4/2005	3000
Stock Market: UK FTSE-100	*futsee-100*	Dukascopy Trading Technologies	Intraday(10sec)	21/4/2005	3000
Stock Market: UK FTSE-100	*futsee-100*	Dukascopy Trading Technologies	Intraday(10sec)	16/5/2005	3000

6. A review on efficiency market hypothesis

One of the earliest and most enduring questions of financial economics is whether future financial assets prices can be forecasted. The concept of efficient market hypothesis which asserts that the asset price changes are unpredictable can be traced back at least as far as the pioneering theoretical contribution of Bachelier (and the empirical research of Cowles). Bachelier (1900) was the first who attempted to formulate as a model the changes of stock exchange rates. He recognized the necessity of dependence / heterogeneity restrictions and should be credited with the first formulation of the stochastic process known today as a Brownian motion.

The modern literature on financial market efficiency begins with Samuelson (1965) who in his landmark article tried to prove why properly anticipated prices fluctuate randomly. In a informationally efficient market price changes must be unforecastable if they are properly anticipated, i.e. if they fully incorporate the expectations and information of all market participants.

Fama (1970) summarizes this idea by stating that:" A market in which prices always *fully reflect* available information is called *efficient.* His work divides market efficiency into three categories. In the first category, weak form tests, he is interested in how accurate can past returns predict future returns. Then, semi-strong form tests in which the concern is whether prices efficiently adjust to other information that is obviously publicly available. Finally, strong form tests which tests if any investors have private information that is not fully reflected in market prices.

Jensen (1978) gave a weaker and economically more sensible version of the efficient hypothesis by saying that prices fully reflect information to the point where the marginal benefits of acting on information, that is the profits to made, do not exceed the marginal costs.

Grossman and Stiglitz (1980) defined market efficiency as a status where information and trading costs, the costs of getting prices to reflect information, are always zero.

More recently, Malkiel (1992) has given the definition of informational efficiency more explicitly, the economic implication of which is that is impossible to make economic profits by trading on the basis of the given information set.

In the following section we demonstrate empirical results over the selected time series. We use econometric program Eviews 4.0 for our statistical calculations. We divide the following section into two parts: first we give a summary regarding all data we used and make general comments. Then we take the case of the France SBF 250 index and make a more detailed analysis.

7. Empirical Findings

7.1 Summary Results

In this section we describe the approach to be followed along with the tests to be used in our study. Our primary target is to locate the influence of the polynomial fashion trend in the variance of the fitted residuals that generates the regression..

We will check how the presence of polynomial trend can affect the final structure of our first order autoregressive model that is whether we should consider as statistical significant or not the coefficient of our regression. As we have already demonstrated, we know there is a tendency in the t-statistic of the OLS coefficient to approach zero, after adjusting it for the presence of a positive no zero kappa, where k is the grade of the deterministic polynomial trend that drives the variance. Therefore taking under consideration the presence of kappa, helps in a facing the discussed problem of over rejecting the null hypothesis of zero coefficients. If this is the case, we can say that under the presence of polynomial trend in the variance of the error of the regression, we may have indication of market inefficiency, while the previous results suggested the opposite. For this purpose we will compare statistical inference results using the OLS t-statistic, the White's corrected for heteroskedasticity t-statistic and the t-statistic adjusted to estimated k.

Table 11 shows our empirical findings according the discussed approach:

Table 11

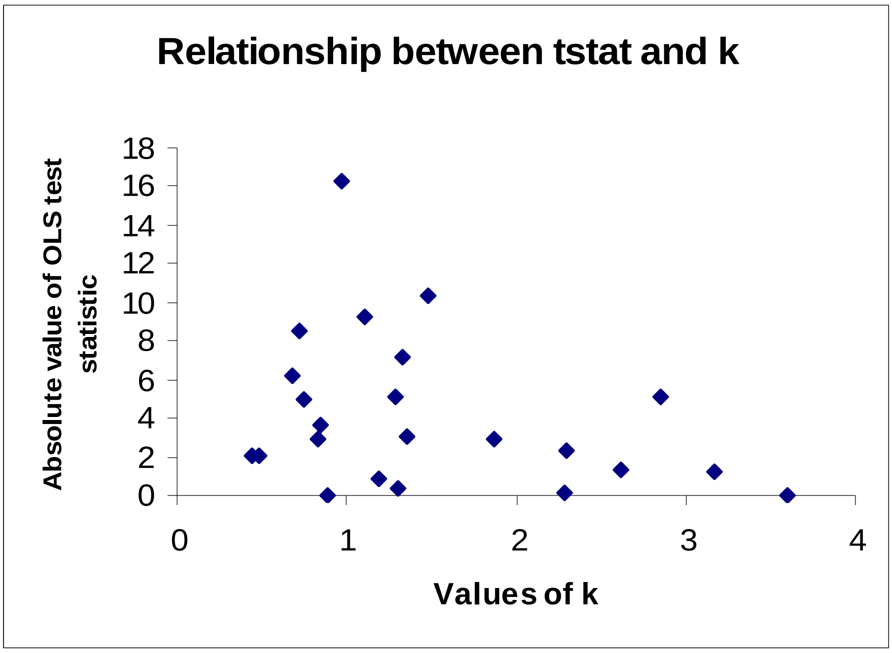
Summary Results

Series	$\hat{r}_{LS}$	$\hat{k}$	t_{LS}	t_{White}	$t^*_{\hat{k}}$
_aordd	0.079	1.193	0.897	0.765	0.753
_ftsad	0.001	0.886	0.016	0.026	0.014
igaus10d	-0.014	2.283	-0.170	-0.096	-0.122
igcand	-0.112	2.616	-1.362	-0.637	-0.940
iggbrcw	-0.089	3.164	-1.273	-0.524	-0.828
igjpn7d	-0.405	2.852	-5.060	-2.924	-3.401
igusa10d	-0.203	1.869	-2.952	**-1.543***	-2.240
mocaaad	0.031	1.297	0.385	0.253	0.318
_sbf250d	0.191	2.295	2.346	2.779	**1.683***
__xag_hd	0.002	3.595	0.036	0.041	0.022
trusg10m	0.451	1.333	7.205	4.746	5.913
dexcaus	0.032	0.836	2.977	1.976	2.650
hkaudsp	-0.090	0.749	-5.014	**-1.540***	-4.531
totmkcp	0.182	1.480	10.373	4.416	8.323
totmkfn	0.046	1.355	3.087	**1.956***	2.525
djindus	0.063	1.286	5.083	2.361	4.202
d&j-ind22mar	0.018	0.718	8.487	5.930	7.710
d&j-ind12apr	0.077	0.846	3.597	**1.495***	3.197
d&j-ind3may	0.329	0.970	16.309	10.851	14.195
futsee-1001apr	0.038	0.441	2.040	**1.463***	**1.942***
futsee-10019apr	-0.114	0.681	-6.166	-2.695	-5.637
futsee-10021apr	-0.175	1.103	-9.253	-2.888	-7.878
futsee-10016may	-0.038	0.484	-2.060	-2.235	**-1.947***

As we can see in seven out of the total 23 series tested the OLS t-statistic gives conflicting results in comparison with t_{White} and $t^*_{\hat{k}}$ while it rejects the null hypothesis while the other two tests do the opposite. We can see that in four cases t_{White} conflicts with OLS t-statistic, while $t^*_{\hat{k}}$ conflicts with OLS t-statistic two times. Another interested results is that t_{White} and $t^*_{\hat{k}}$ works the same only in one case, and that for the cases that $t^*_{\hat{k}}$ does not reject the null, White's t-statistic gets bigger than OLS t-statistic.

A final result we wish to present is the relationship between the OLS test statistic and the value of the estimation of k. Figure 3 demonstrates the scatter plot between these data:

Figure 3



We can see that for values of k less than two there is a positive relationship between the value of k and the test statistic of OLS estimator, while in general view this relationship is not that clear.

7.2 The case of France SBF-250 Index (w/GFD extension)

In figures 4 and 5 we can see the t-plot of _sbf250d and the residuals series for an AR (1) fitted model:

Figure 4

_sbf250d

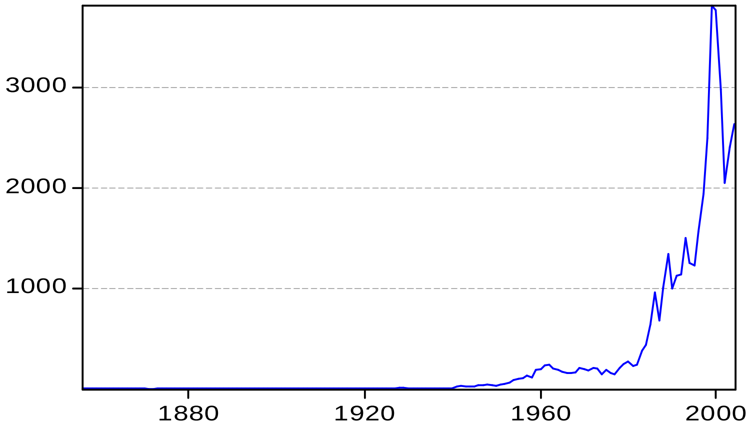
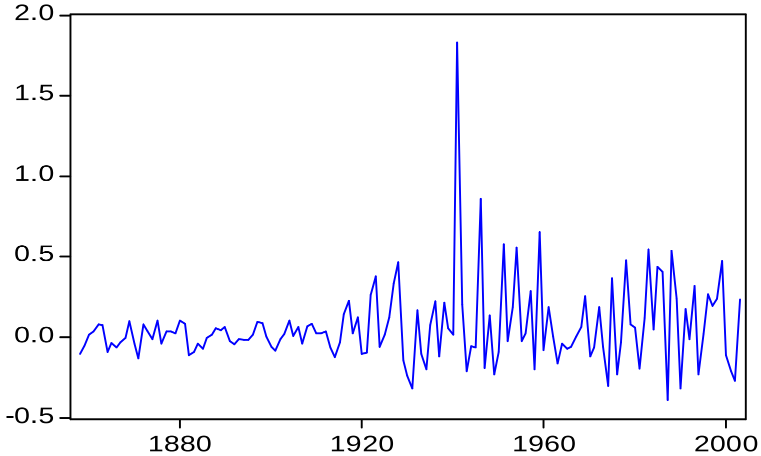


Figure 5

Residuals series



We can observe that the variance of the fitted residuals appears to grow as time passes. Test statistics that we have presented so far, tested the null hypothesis $H_0 : \hat{r}_{LS} = 0$. For _sbf250d we have found that the OLS and White's t-statistic rejects the null. This result informs the investors that there the market is inefficient since from such a model he can get information about tomorrow's price based on today's. However, $t_{\hat{k}}^{*}$ suggests the opposite that the market is efficient and the appropriate decision for the investor should be not to take under consideration yesterday's prices.

In addition to testing the null hypothesis $H_0 : \hat{r}_{LS} = 0$, someone might be interested in testing a different null of the form: $H_0 : \hat{r}_{LS} = r_0$. We will demonstrate below, that also in this case we will have conflicting results in statistical inference. Tables 12 and 13 shows OLS and $t^*_{\hat{k}}$ tests statistics for alternative values of r_0:

Table 12

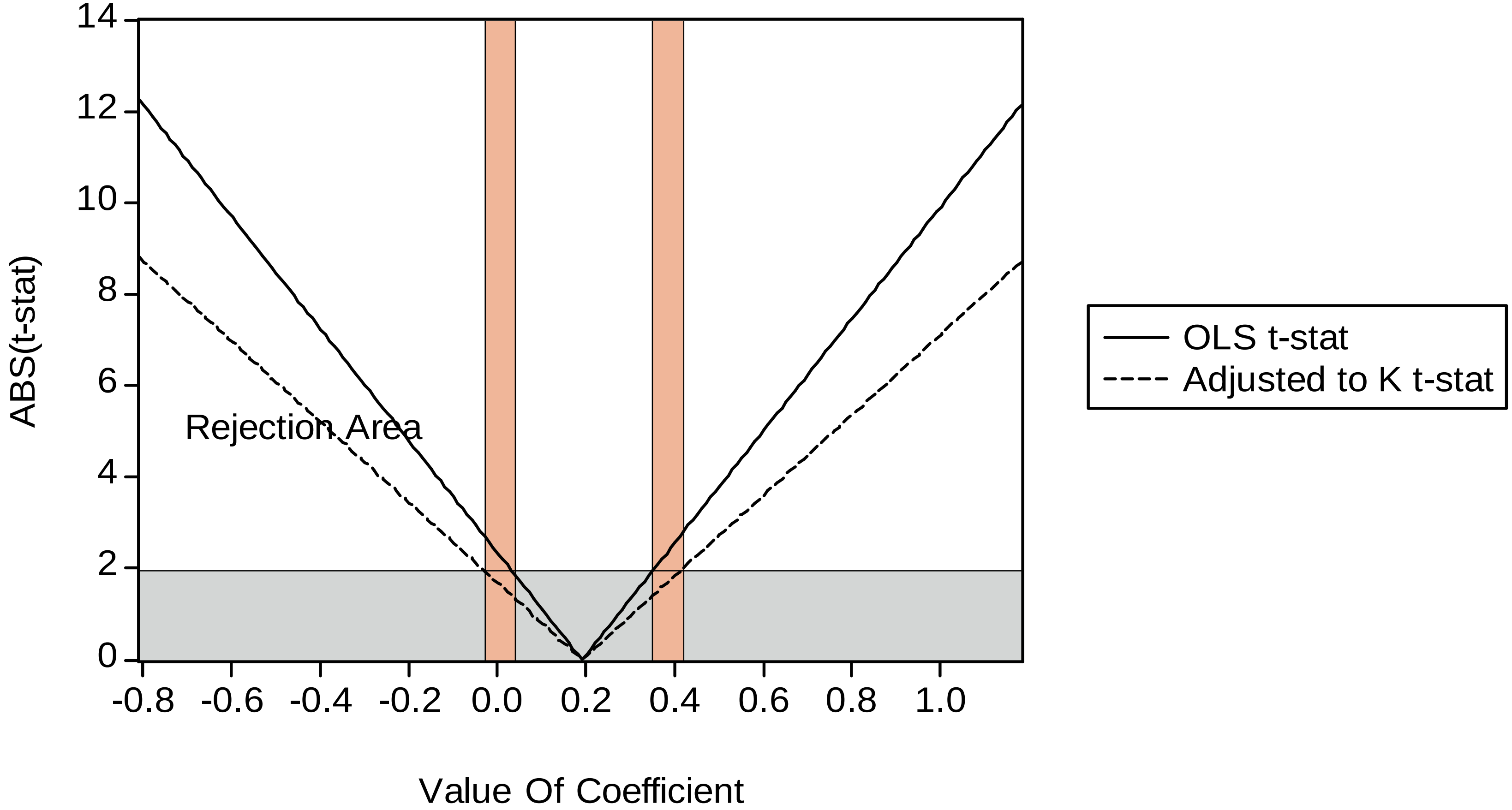
$H_0 : \hat{r}_{LS} = r_0$	r_0 **-0.04**	r_0 **-0.03**	r_0 **-0.02**	r_0 **-0.01**	r_0 **0**	r_0 **0.01**	r_0 **0.02**	r_0 **0.03**	r_0 **0.04**
t_{LS}	2.836	2.714	2.591	2.469	2.346	2.224	2.101	1.979	1.856
t_{White}	3.341	3.197	3.053	2.908	2.764	2.620	2.475	2.331	2.187
$t^*_{\hat{k}}$	2.035	1.947	1.859	1.772	1.684	1.596	1.508	1.420	1.332

Table 13

$H_0 : \hat{r}_{LS} = r_0$	r_0 0.35	r_0 0.36	r_0 0.37	r_0 0.38	r_0 0.39	r_0 0.40	r_0 0.41	r_0 0.42	r_0 0.43
t_{LS}	-1.942	-2.064	-2.187	-2.309	-2.432	-2.554	-2.677	-2.799	-2.922
t_{White}	-2.287	-2.432	-2.576	-2.720	-2.865	-3.009	-3.153	-3.298	-3.442
$t^*_{\hat{k}}$	-1.393	-1.481	-1.569	-1.657	-1.745	-1.833	-1.921	-2.009	-2.097

As we can observe $t^*_{\hat{k}}$ is possible to lead us in different inference results if we have knowledge of the real value of r_0 and we wish to test the equality $H_0 : \hat{r}_{LS} = r_0$. Figure 5 demonstrates all values of r_0 for which t_{LS} and $t^*_{\hat{k}}$ give different results:

Figure 5


14
12
10
8
6
4
2
0
ABS(t-stat)
Rejection Area
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
Value Of Coefficient
OLS t-stat
Adjusted to K t-stat

8. Conclusions

This paper has studied a new approach on heteroskedasticity and the asymptotic properties of the stationary first order autoregressive model. The new theory has rested the assumption that the variances of the innovations driving the model are bounded, thus precluding trending moments. Innovations variance was allowed to be asymptotically unbounded, evolving in a polynomial-like fashion.

Monte Carlo simulations have shown that the new approach is able to correct statistical inferences when the disturbances of the regression exhibits polynomial trend in their variance. We also saw evidence that the new theory is appropriate to approach alternative forms of growing variance.

Empirical findings have supported the existence of macroeconomic and financial time series that includes the discussed behavior. In these time series we have encountered conflicting results in statistical inference between classical test statistics and the new test statistic adjusted to estimated k.

References

Major paper

Nikolaos Kourogenis; Nikitas Pittis: "Asymptotic Theory for First-Order Autoregressions with Asymptotically Unbounded Error Variance." (2005)

Halbert White: "A Heteroskedasticty- Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity", Econometrica (Vol. 48, 1980)

James G. MacKinnon; Halbert White: "Some Heteroskedasticity- Consistent Covariance Matrix Estimators With Improved Finite Sample Properties", Journal of Econometrics (Vol. 29, 1985)

Fama F. Eugene: "Efficient Capital Markets: A Review of Theory and Empirical Work.", The Journal of Finance (Vol. 25, 1970b)

Fama F. Eugene: "Efficient Capital Markets II", The Journal of Finance (Vol.XLVI,1991)

Robert F. Engle: "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation", Econometrica (Vol.50, 1982)

Robert F. Engle: "The use of ARCH/ GARCH Models In Applied Econometrics.", The Journal of Economic Perspectives (Vol. 15,2001)

Tim Bollerslev: "Generalized Autoregressive Conditional Heteroskedasticity", Journal of Econometrics (Vol. 31, 1986)

Whitney K. Newey; Kenneth D. West: "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix" Econometrica (Vol. 55, 1987)

APPENDIX

TABLE A
"Pseudo" Kappa's

Variance Shifts	Estimated "Pseudo" Kappa
T=100	
$(s_1^2, s_2^2) = (1,2)$	0.605
$(s_1^2, s_2^2) = (1,10)$	2.448
$(s_1^2, s_2^2) = (1,100)$	5.631
T=120	
$(s_1^2, s_2^2, s_3^2) = (1,3,6)$	1.007
$(s_1^2, s_2^2, s_3^2, s_4^2, s_5^2, s_6^2) = (1,2,3,4,5,6)$	0.819
Exponential growing	
T= 50	5.978
T=100	13.201
T=200	27.626
T=500	70.897

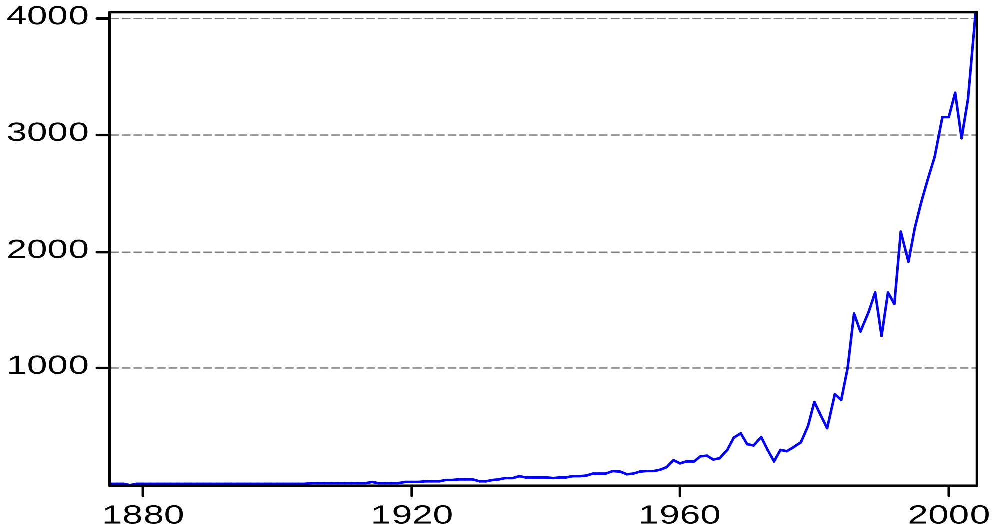
TABLE B
Complete Database

Source - Database	Mnemonic	Description
Daily Data: Foreign Exchange Rates		
Federal Reserve Bank of St. Louis	DEXCAUS	Canada / U.S.
Federal Reserve Bank of St. Louis	DEXINUS	India / U.S.
Federal Reserve Bank of St. Louis	DEXJPUS	Japan / U.S.
Federal Reserve Bank of St. Louis	DEXNOUS	Norway / U.S.
Federal Reserve Bank of St. Louis	DEXSDUS	Sweden / U.S.
Federal Reserve Bank of St. Louis	DEXSFUS	South Africa / U.S.
Federal Reserve Bank of St. Louis	DEXSIUS	Singapore / U.S.
Federal Reserve Bank of St. Louis	DEXSZUS	Switzerland / U.S.
Federal Reserve Bank of St. Louis	DEXTHUS	Thailand / U.S.
Federal Reserve Bank of St. Louis	DEXUSAL	U.S. / Australia
Federal Reserve Bank of St. Louis	DEXUSNZ	U.S. / New Zealand
Federal Reserve Bank of St. Louis	DEXUSUK	U.S. / U.K.
Federal Reserve Bank of St. Louis	DEXTAUS	Taiwan / U.S.
Federal Reserve Bank of St. Louis	DEXKOUS	South Korea / U.S.
Federal Reserve Bank of St. Louis	DEXCHUS	China / U.S.
Datastream	ARGUK	Argentina / U.S.
Datastream	CHILEUK	Chile / U.K.
Datastream	HKUK	Hong Kong / U.K.
Datastream	KORUK	Korea / U.K.
Datastream	MALUK	Malaysia / U.K.
Datastream	SINGUK	Singapore / U.K.
Datastream	DENUK	Denmark / U.K.
Datastream	INDUK	India / U.K.
Datastream	NORUK	Norway / U.K.
Datastream	PHILUK	Philippines / U.K.
Datastream	CANUK	Canada / U.K.
Datastream	DENCAN	Denmark / Canada
Datastream	DENJAP	Denmark / Japan
Datastream	ARCADSP	Canada / Argentine
Datastream	ARHKDSP	Hong Kong / Argentine
Datastream	ARJPYSP	Japan / Argentine
Datastream	ARNZDSP	Argentine / New Zealand
Datastream	ARSEKSP	Sweden / Argentine
Datastream	ARZARSP	South Africa / Argentine
Datastream	BRAUDSP	Brazil / Australia
Datastream	HKAUDSP	Hong Kong / Australia
Datastream	HKCADSP	Hong Kong / Canada
Datastream	HKCGFSP	Hong Kong / Switzerland
Datastream	MXDEMSP	Mexico / Germany
Datastream	MXFRFSP	Mexico / France

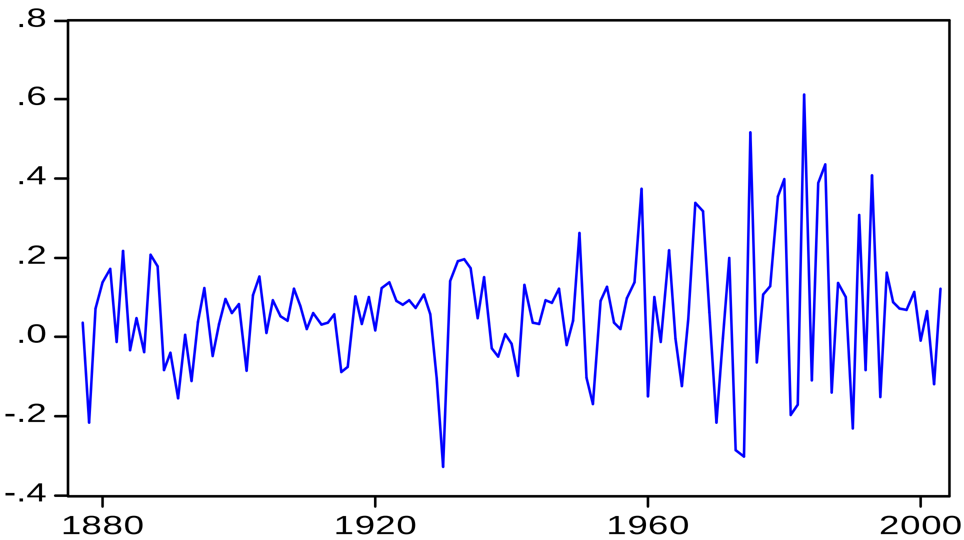
Daily Data:Stock Price Indexes		
Datastream	AMSTEOE	AEX INDEX (AEX)
Datastream	AUSTOLD	ASX ALL ORDINARIES 1971
Datastream	DAXINDX	DAX (Germany)
Datastream	FTSE100	FTSE 100 (England)
Datastream	HNGKNGI	Hang Seng NGI
Datastream	IFGMAR$	S&P/IFCG M ARGENTINA
Datastream	IFGWJO$	S&P/IFCG W JORDAN
Datastream	ISEQUIT	RELAND SE OVERALL (ISEQ)
Datastream	JAPDOWA	NIKKEI 225 STOCK AVERAGE
Datastream	PSECOMP	PHILIPPINES SE COMPOSITE
Datastream	WIEIREL	FTSE W IRELAND
Datastream	BEL	BEL 20 (Belgium)
Datastream	US_S_P50001	S&P 500 (U.S.)
Datastream	DJ_TRSPT	Dow Jones Transportation (U.S.)
Datastream	DJ_UTILS	Dow Jones Utilities (U.S.)
Datastream	IT_30	MILAN MIB 30
Datastream	JP_NIKKEI	Nikeei (Japan)
Datastream	NASCOMP	NASDAQ COMPOSITE
Datastream	NYSE_ALL	New York Stock Exchange All
Datastream	SNGPORI	SINGAPORE STRAITS TIMES
Datastream	TOTMKAR	Total Market: Argentina
Datastream	TOTMKAU	Total Market: Australia
Datastream	TOTMKBR	Total Market: Brazil
Datastream	TOTMKCA	Total Market: China A
Datastream	TOTMKCH	Total Market: China
Datastream	TOTMKCN	Total Market: Canada
Intraday Data		
Dukascopy Tradind Technologies	Intraday data (10 sec) on stock market indices: Dow Jones Industrial - FTSE 100 UK – NYSE	
Yearly Data		
Global Financial Data	aordd	Australia ASX All-Ordinaries (w/GFD extension)
Global Financial Data	bbkad	Germany All Government Securities
Ohio State University	britann	Share Price
Ohio State University	britann1	Share Price and Interest Rate
Ohio State University	britmon	Share Price
Global Financial Data	corn	Corn Spot Price (US$/Bushel)
Global Financial Data	crb	CRB Commodity Index
Ohio State University	dow1900	Dow Jones Industrial Average
Ohio State University	dowdaily	Dow Jones Industrial Average
Global Financial Data	ftsad	UK FT-Actuaries All-Share Index (w/GFD extension)
Global Financial Data	fwbxxd	Germany CDAX Composite Index (w/GFD extension)
Global Financial Data	gold	Gold Bullion Price-New York (US$/Ounce)
Global Financial Data	gsptsed	Canada S&P/TSX 300 Composite (w/GFD extension)
Global Financial Data	ibusa3d	USA 3-month EuroDollar Deposits

Global Financial Data	idusad	USA Federal Reserve Bank of NY Discount Rate
Global Financial Data	igaus10d	Australia Commonwealth 10-year Bonds
Global Financial Data	igcand	Canadian Government Bonds 10+ Years Maturity
Global Financial Data	igfra10d	France 10-year Government Bond Yield
Global Financial Data	iggbrcw	UK 2 1/2% Consol Yield
Global Financial Data	igjpn7d	Japan 7-year Government Bond Yield
Global Financial Data	igusa10d	USA 10-year Bond Constant Maturity Yield
Global Financial Data	inf_aus	Australia Consumer Price Index
Global Financial Data	inf_can	Canada Consumer Price Index
Global Financial Data	inf_ger	Germany Consumer Price Index
Global Financial Data	inf_jp	Japan Consumer Price Index
Global Financial Data	inf_uk	UK Retail Price Index
Global Financial Data	inf_usa	USA BLS Consumer Price Index
Global Financial Data	mocaaad	Moody's Corporate AAA Yield
Global Financial Data	nasdaq	NASDAQ Composite Index (w/NQB extension)
Global Financial Data	nikkei225	Japan Nikkei 225 Stock Average (w/GFD extension)
Global Financial Data	nyse	NYSE Composite
British Petroleum	oilprices	CRUDE OIL PRICES 1861 - 1999
Global Financial Data	sbf250d	France SBF-250 Index (w/GFD extension)
Global Financial Data	silver	Silver Cash Price (US$/Ounce)
Global Financial Data	trusabim	USA Total Return Commercial/T-Bill Index
Global Financial Data	trusg10m	USA 10-year Government Bond Total Return Index
Global Financial Data	wheat	Wheat #2 Cash Price (US Dollars/Bushel)

SUMMARY RESULTS FOR SELECTED MACROECONOMIC AND FINANCIAL TIME SERIES DATA

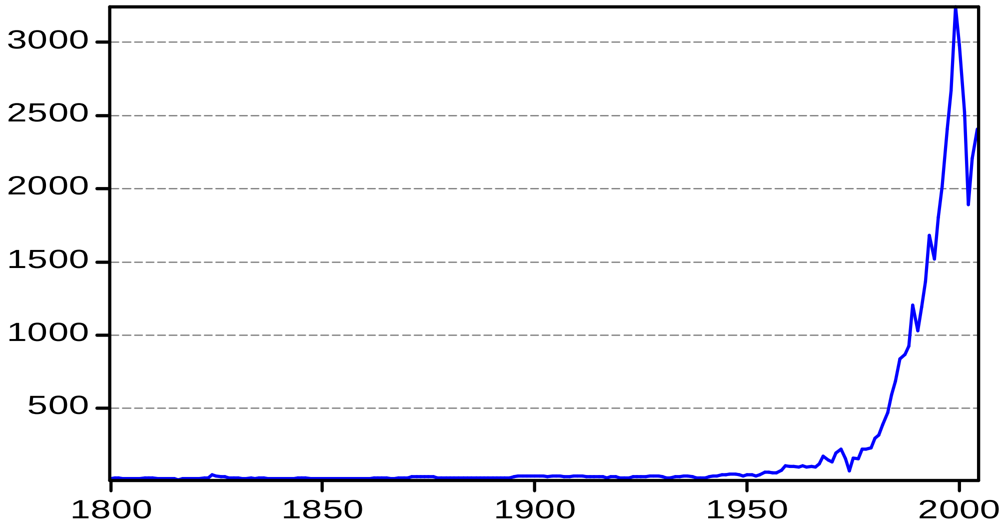


Australia ASX All-Ordinaries (w/GFD extension)

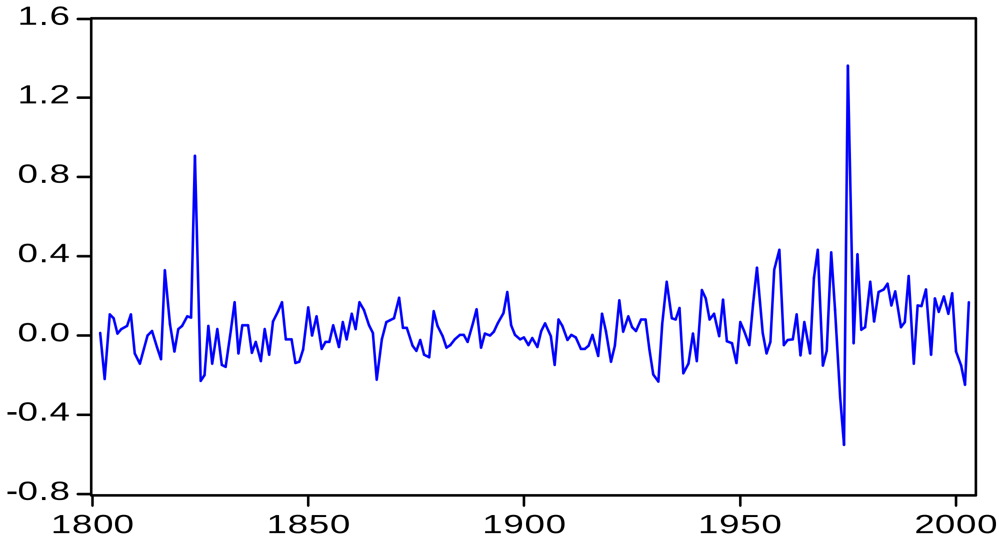


Fitted Residuals of AR(1) model over the returns

Description	*Australia ASX All-Ordinaries (w/GFD extension)*				
Source	*Global Financial Data*			***Mnemonic***	*aordd*
Starting Date	*1875*	***Observations***	*130*	***Frequency***	*Years*
Estimated K	*1.193*	***Value of ρ***	*0.079648*	***t-stat***	*0.897645*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.120,-0.090] U [0.259,0.289]*	

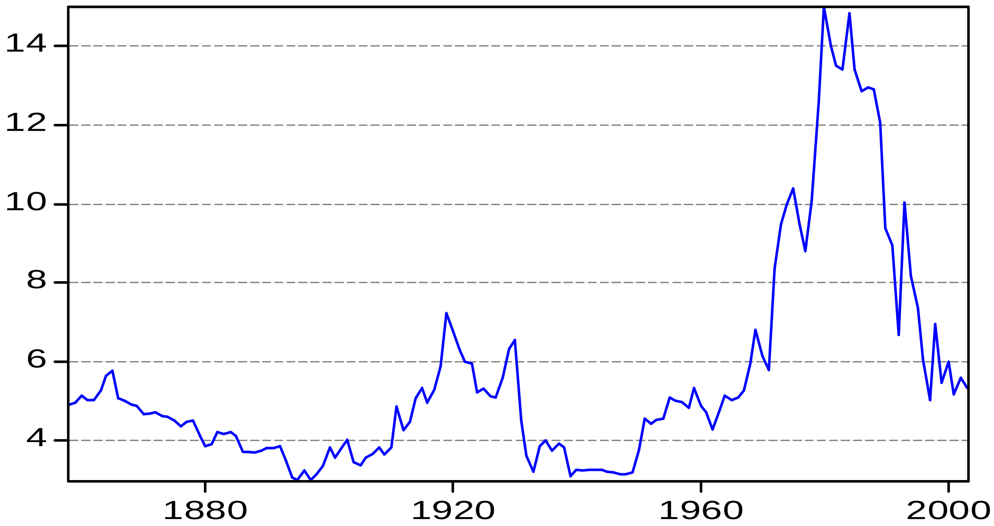


UK FT-Actuaries All-Share Index (w/GFD extension)

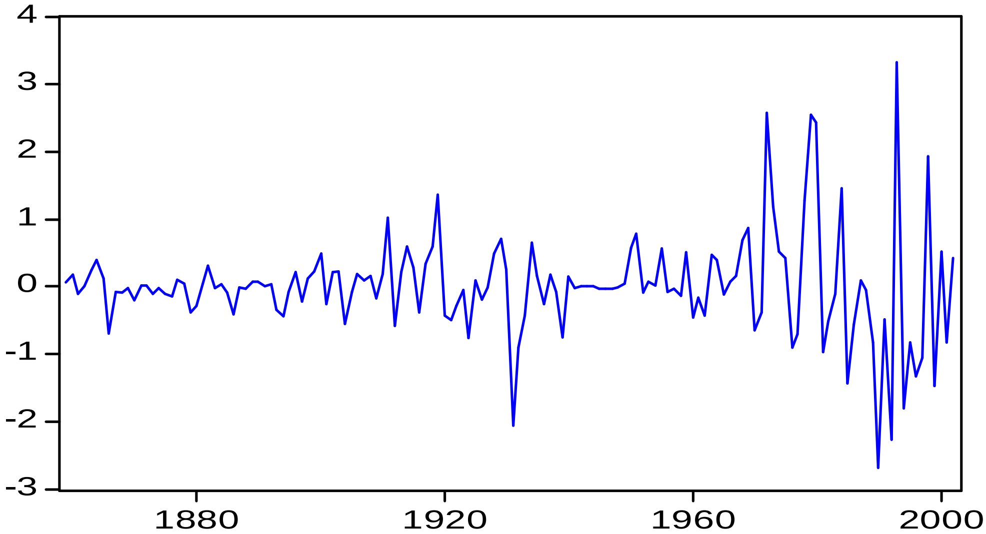


Fitted Residuals of AR(1) model over the returns

Description	*UK FT-Actuaries All-Share Index (w/GFD extension)*				
Source	*Global Financial Data*			**Mnemonic**	*ftsad*
Starting Date	*1800*	**Observations**	*205*	**Frequency**	*Years*
Estimated K	*0.886*	**Value of ρ**	*0.0011155*	**t-stat**	*0.0157964*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.148,-0.128] U [0.141,0.161]*	

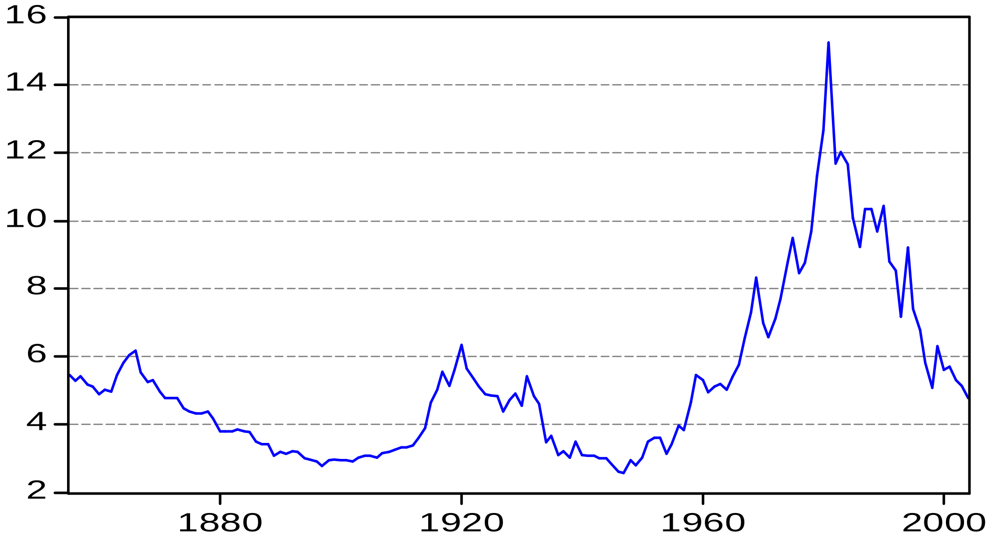


Australia Commonwealth 10-year Bonds

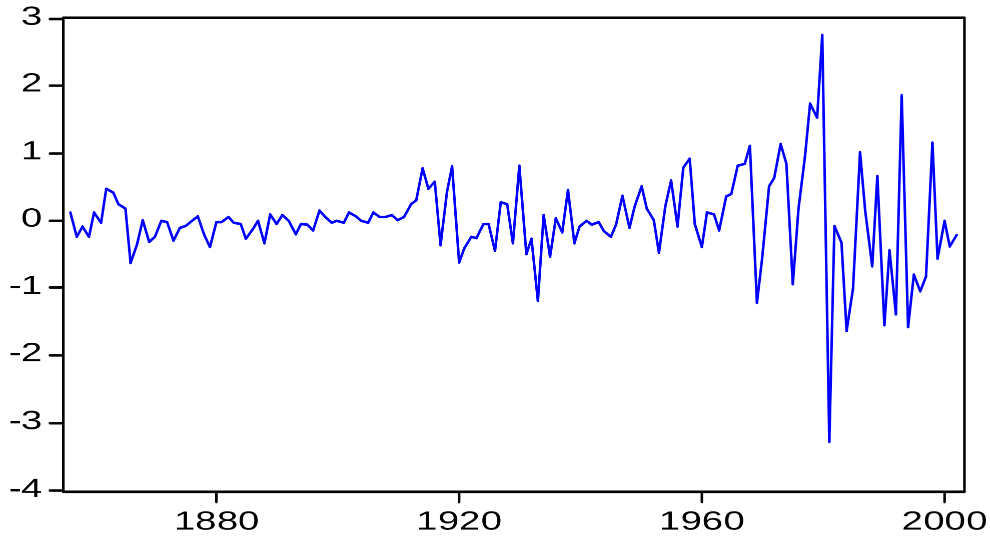


Fitted Residuals of AR(1) model over the yearly changes of the rates

Description	*Australia Commonwealth 10-year Bonds*				
Source	*Global Financial Data*			***Mnemonic***	*igaus10d*
Starting Date	*1858*	***Observations***	*147*	***Frequency***	*Years*
Estimated K	*2.283*	***Value of ρ***	*-0.0142437*	***t-stat***	*-0.1701741*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.234,-0.174] U [0.155,0.215]*	

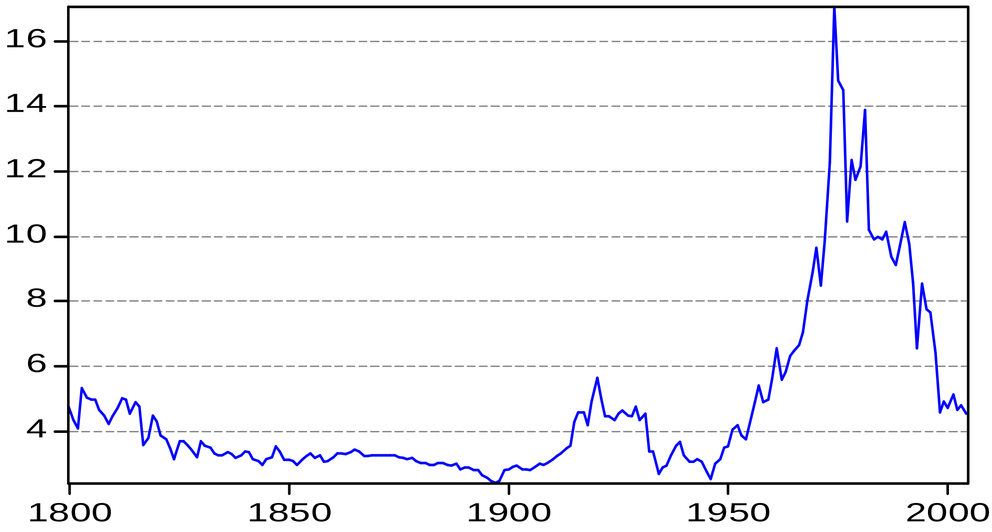


Canadian Government Bonds 10+ Years Maturity

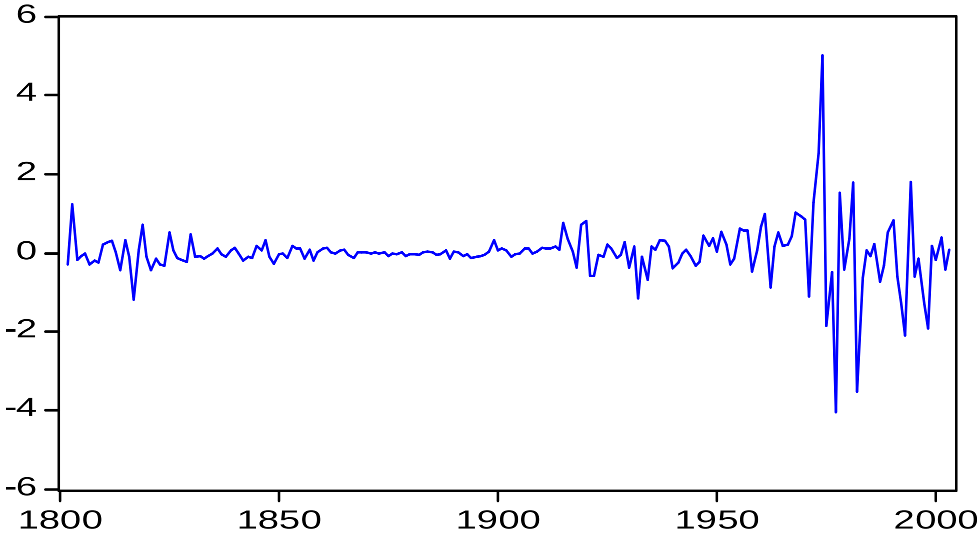


Fitted Residuals of AR(1) model over the yearly changes of the rates

Description	*Canadian Government Bonds 10+ Years Maturity*				
Source	*Global Financial Data*			***Mnemonic***	*igcand*
Starting Date	*1855*	***Observations***	*150*	***Frequency***	*Years*
Estimated K	*2.616*	***Value of ρ***	*-0.112012*	***t-stat***	*-1.3620109*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.342,-0.272] U [0.057,0.127]*	

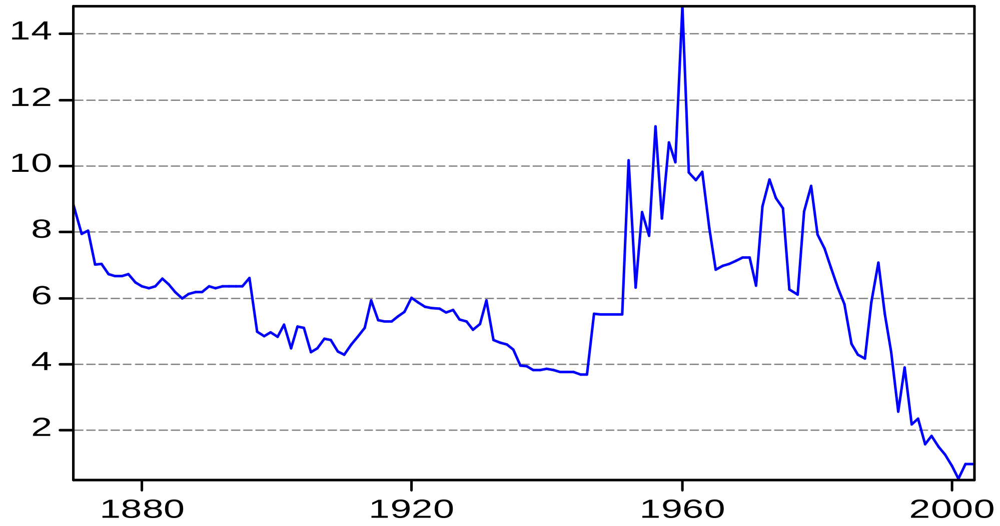


UK 2 1/2% Consol Yield

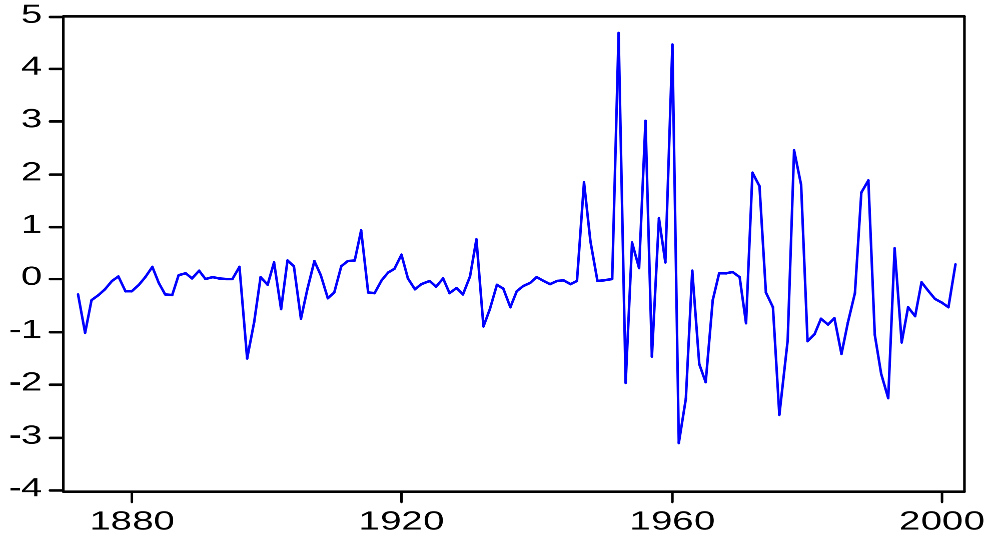


Fitted Residuals of AR(1) model over the yearly changes of the rates

Description	*UK 2 1/2% Consol Yield*				
Source	*Global Financial Data*			***Mnemonic***	*iggbrcw*
Starting Date	*1800*	***Observations***	*205*	***Frequency***	*Years*
Estimated K	*3.164*	***Value of ρ***	*-0.0894311*	***t-stat***	*-1.2739115*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.299,-0.219] U [0.050,0.130]*	

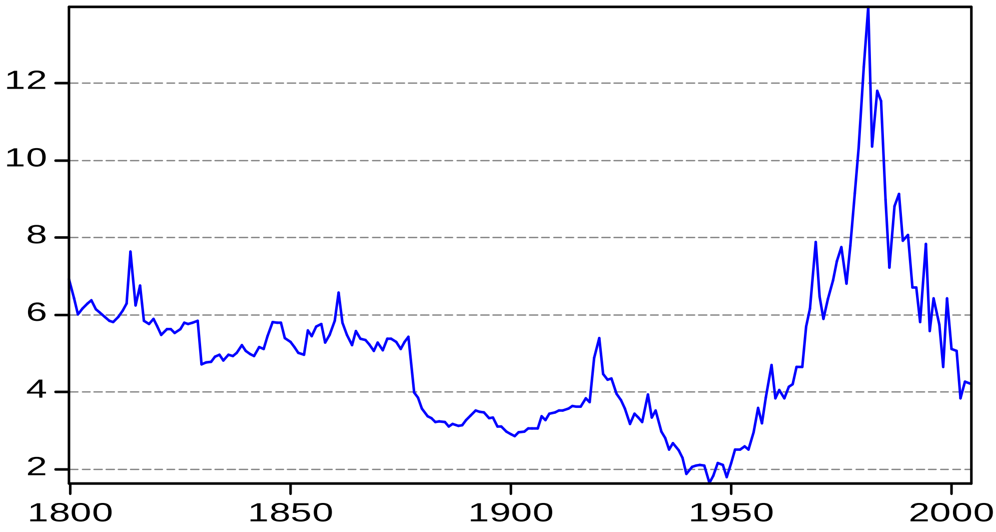


Japan 7-year Government Bond Yield

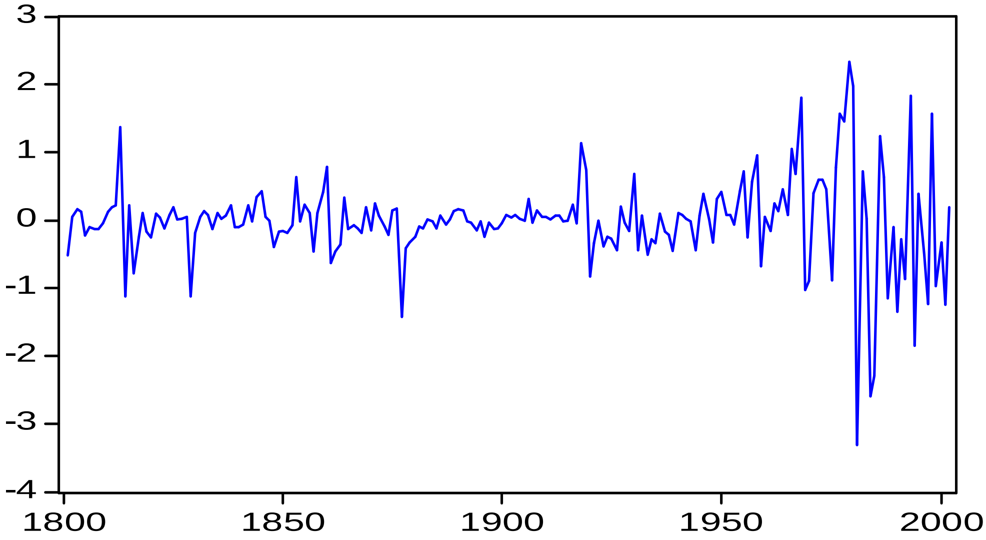


Fitted Residuals of AR(1) model over the yearly changes of the rates

Description	*Japan 7-year Government Bond Yield*				
Source	*Global Financial Data*			***Mnemonic***	*igjpn7d*
Starting Date	*1870*	***Observations***	*134*	***Frequency***	*Years*
Estimated K	*2.851*	***Value of ρ***	*-0.405055*	***t-stat***	*-5.0604942*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.635,-0.555] U [-0.245,-0.165]*	

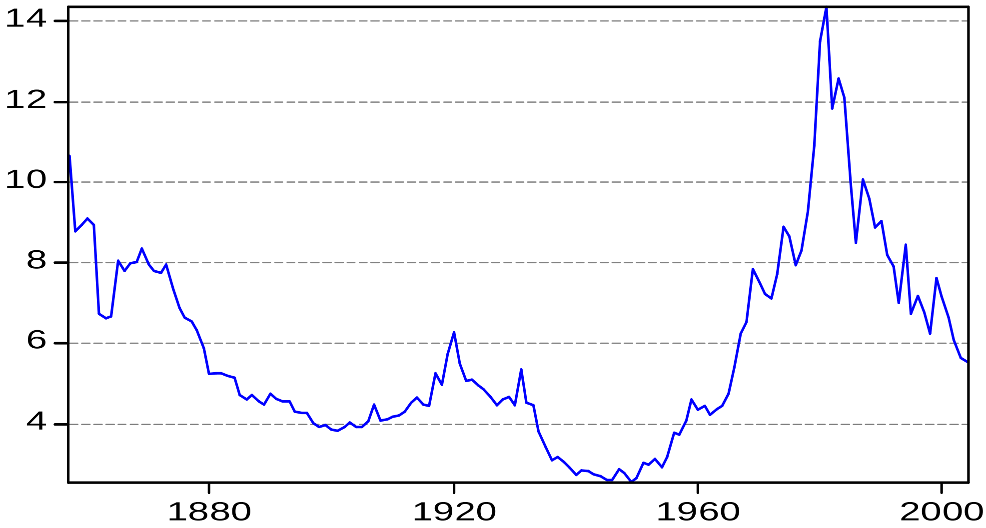


USA 10-year Bond Constant Maturity Yield

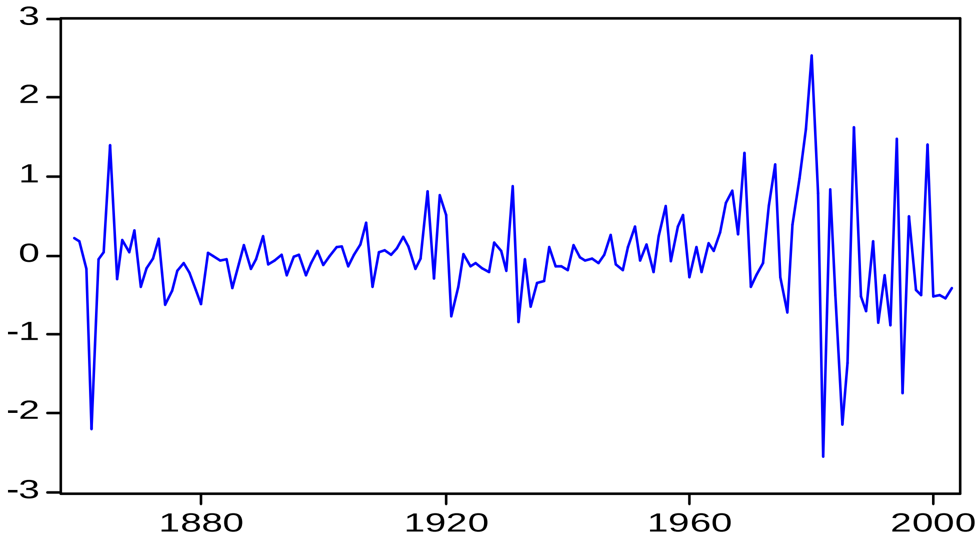


Fitted Residuals of AR(1) model over the yearly changes of the rates

Description	*USA 10-year Bond Constant Maturity Yield*				
Source	*Global Financial Data*			***Mnemonic***	*igusa10d*
Starting Date	*1800*	***Observations***	*205*	***Frequency***	*Years*
Estimated K	*1.869*	***Value of ρ***	*-0.2038043*	***t-stat***	*-2.9523305*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.373,-0.333] U [-0.063,-0.023]*	

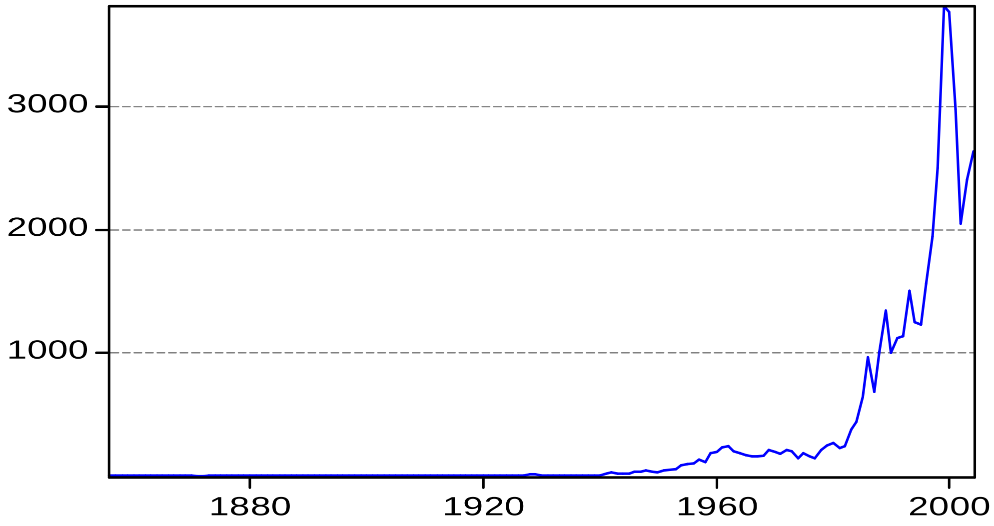


Moody's Corporate AAA Yield

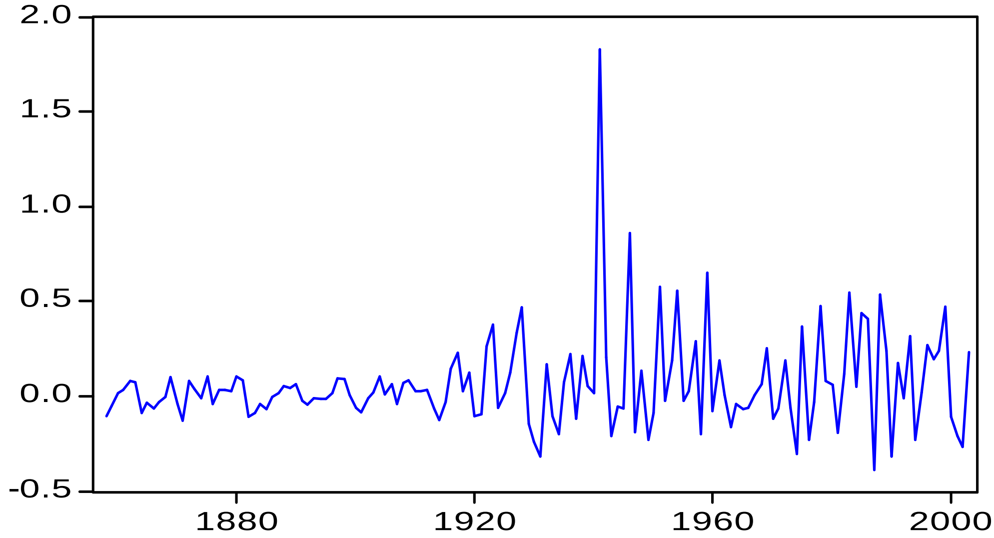


Fitted Residuals of AR(1) model over the yearly changes of the rates

Description	*Moody's Corporate AAA Yield*				
Source	*Global Financial Data*			***Mnemonic***	*mocaaad*
Starting Date	*1857*	***Observations***	*148*	***Frequency***	*Years*
Estimated K	*1.297*	***Value of ρ***	*0.0312244*	***t-stat***	*0.3853316*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.158,-0.118] U [0.191,0.231]*	

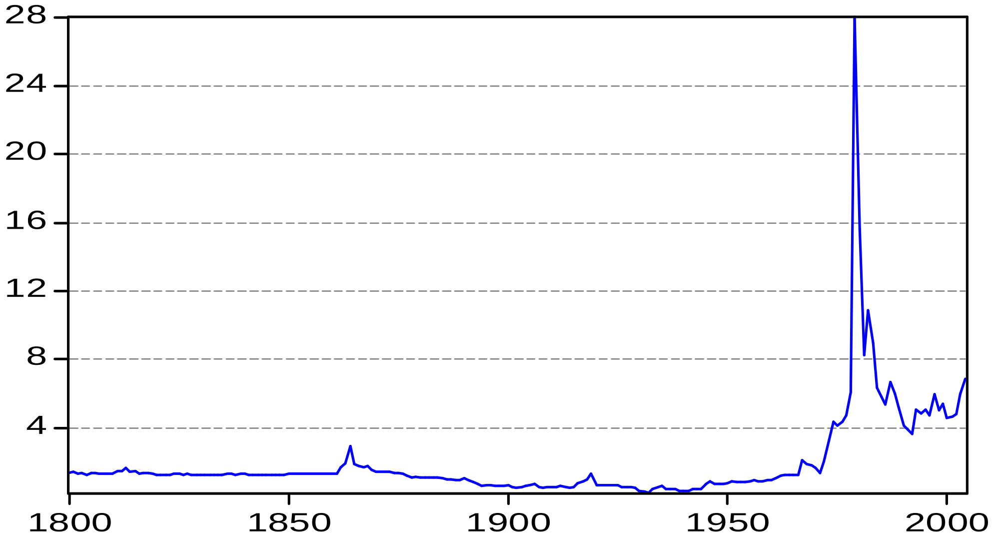


France SBF-250 Index (w/GFD extension)

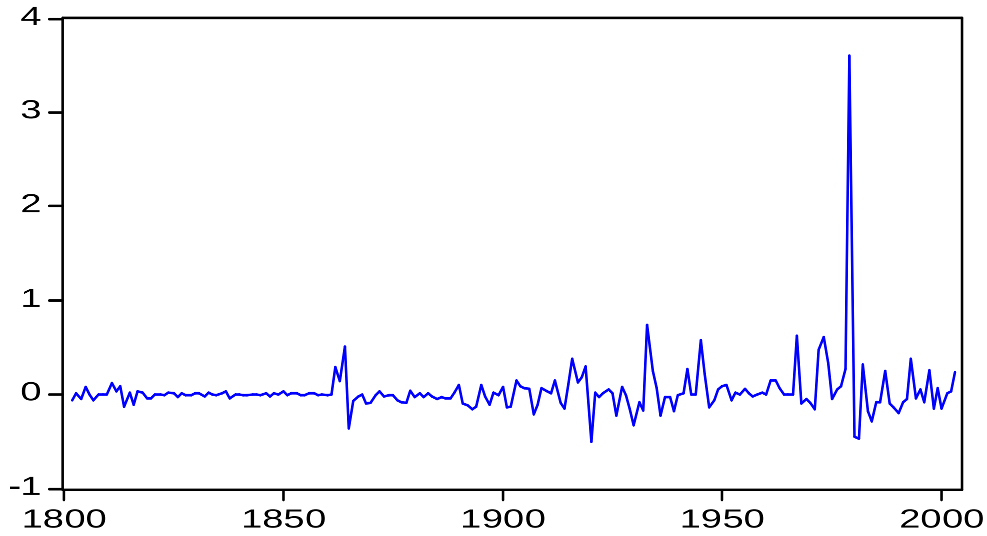


Fitted Residuals of AR(1) model over the returns

Description	*France SBF-250 Index (w/GFD extension)*				
Source	*Global Financial Data*			***Mnemonic***	*sbf250d*
Starting Date	*1856*	***Observations***	*149*	***Frequency***	*Years*
Estimated K	*2.294*	***Value of ρ***	*0.19151*	***t-stat***	*2.346084*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.028,0.041] U [0.351,0.421]*	

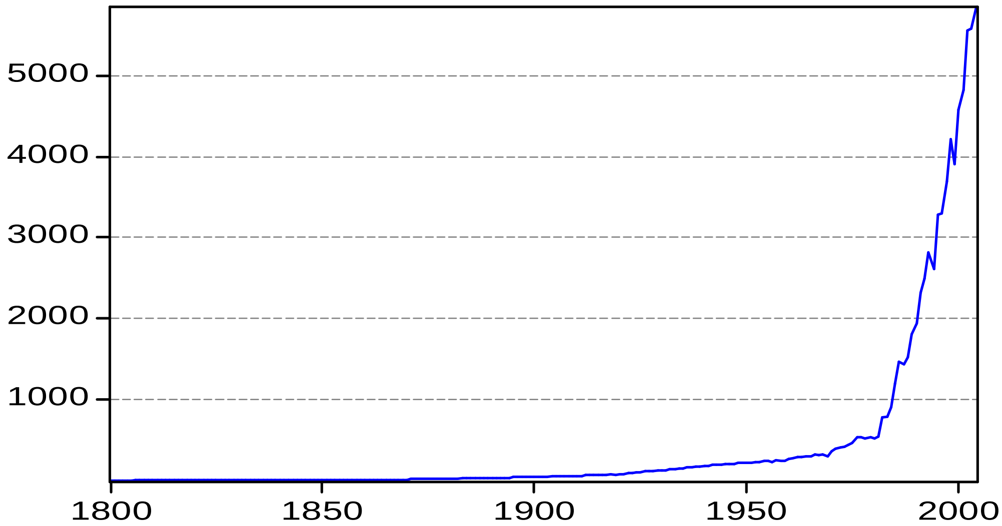


Silver Cash Price (US$/Ounce)

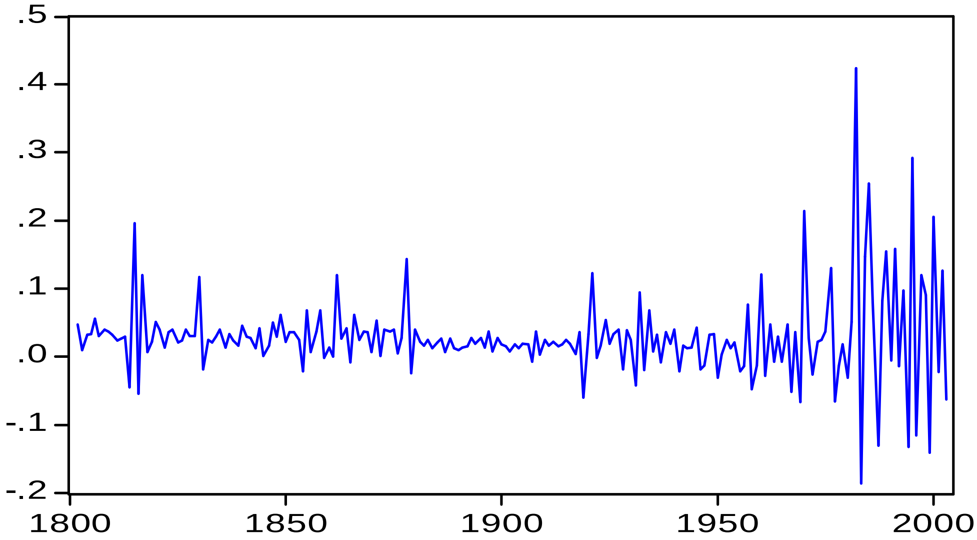


Fitted Residuals of AR(1) model over the returns

Description	*Silver Cash Price (US$/Ounce)*				
Source	*Global Financial Data*			***Mnemonic***	*silver*
Starting Date	*1800*	***Observations***	*205*	***Frequency***	*Years*
Estimated K	*3.594*	***Value of ρ***	*0.002542*	***t-stat***	*0.035986*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.217,-0.127] U [0.142,0.232]*	

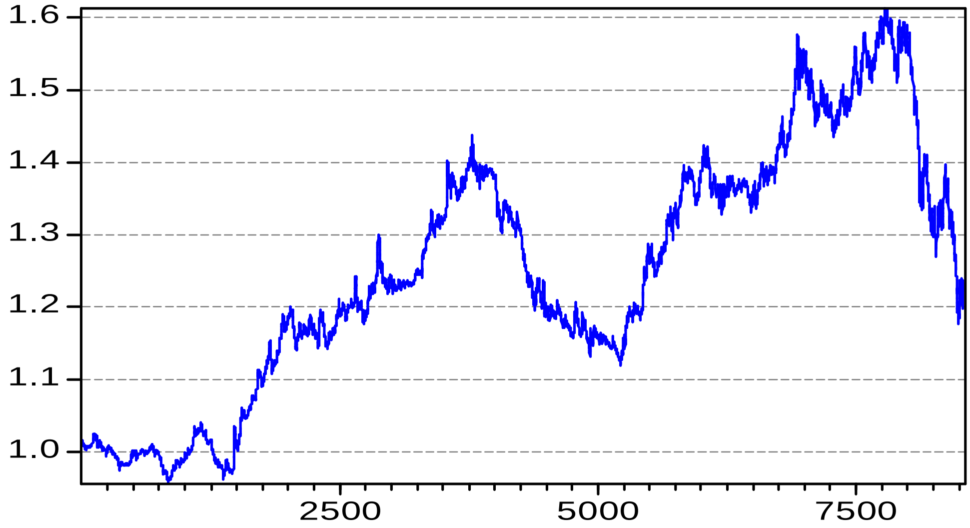


USA 10-year Government Bond Total Return Index

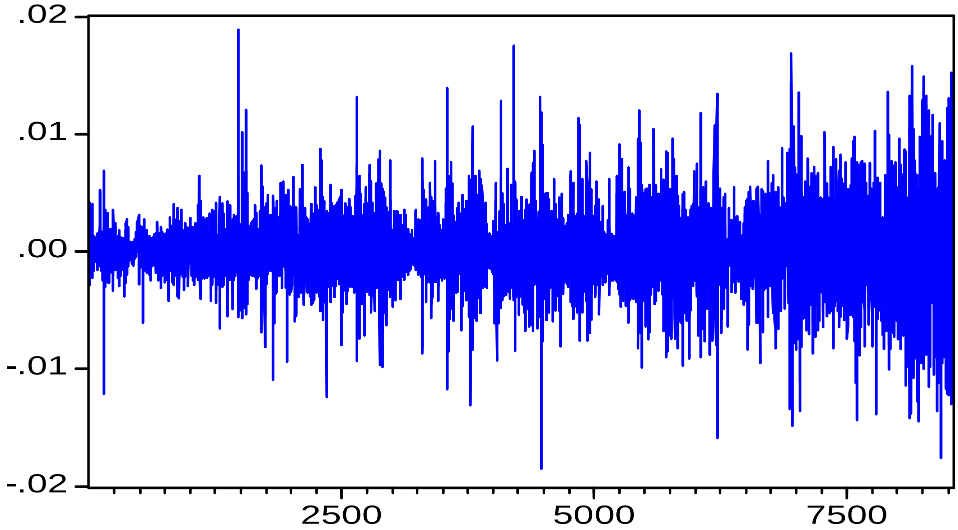


Fitted Residuals of AR(1) model over the returns

Description	*USA 10-year Government Bond Total Return Index*				
Source	*Global Financial Data*			***Mnemonic***	*trusg10m*
Starting Date	*1800*	***Observations***	*205*	***Frequency***	*Years*
Estimated K	*1.332*	***Value of ρ***	*0.450979*	***t-stat***	*7.204486*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.310,0.330] U [0.580,0.600]*	

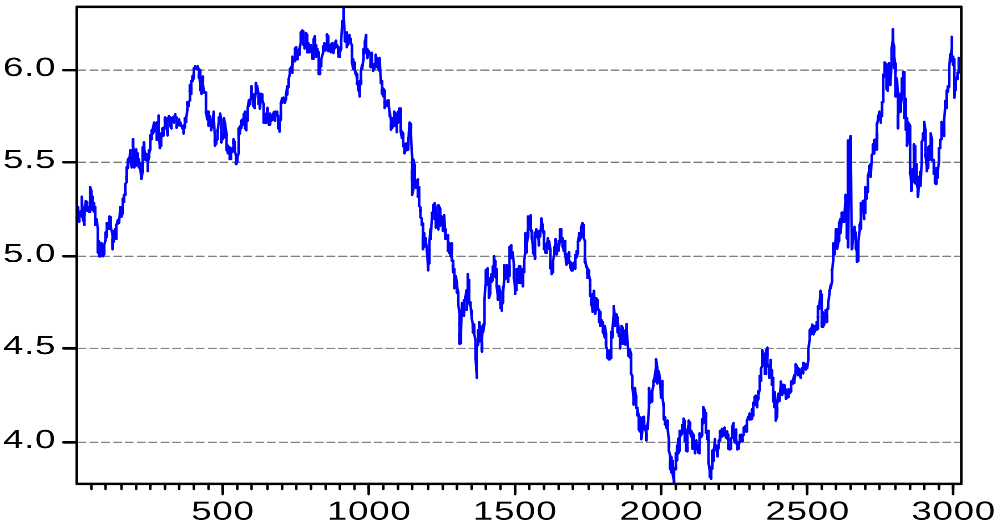


Foreign Exchange Rates Canada/U.S.

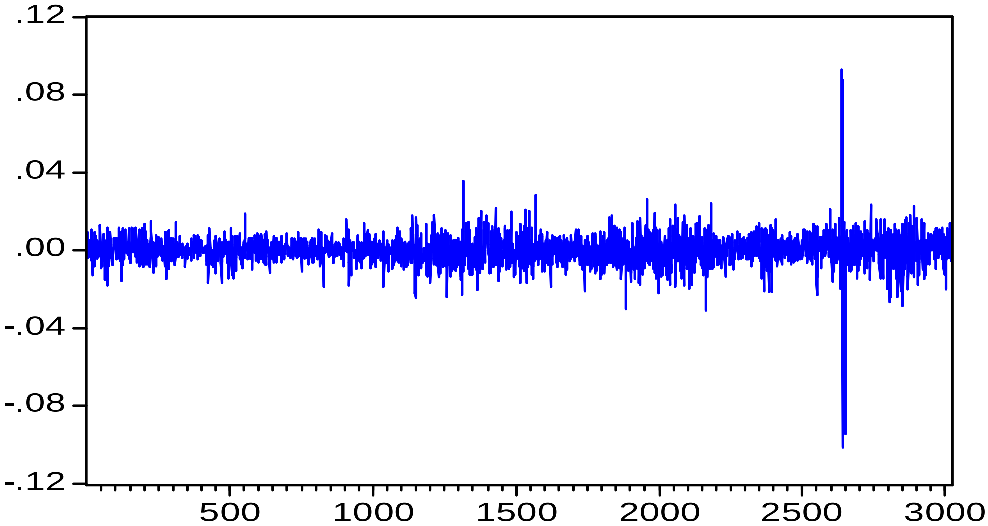


Fitted Residuals of AR(1) model over the returns

Description	*Foreign Exchange Rates Canada/U.S.*				
Source	*Federal Reserve Bank of St. Louis*			***Mnemonic***	*dexcaus*
Starting Date	*4/1/1971*	***Observations***	*8551*	***Frequency***	*Daily*
Estimated K	*0.836*	***Value of ρ***	*0.032209*	***t-stat***	*2.977272*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.009,0.011] U [0.054,0.056]*	

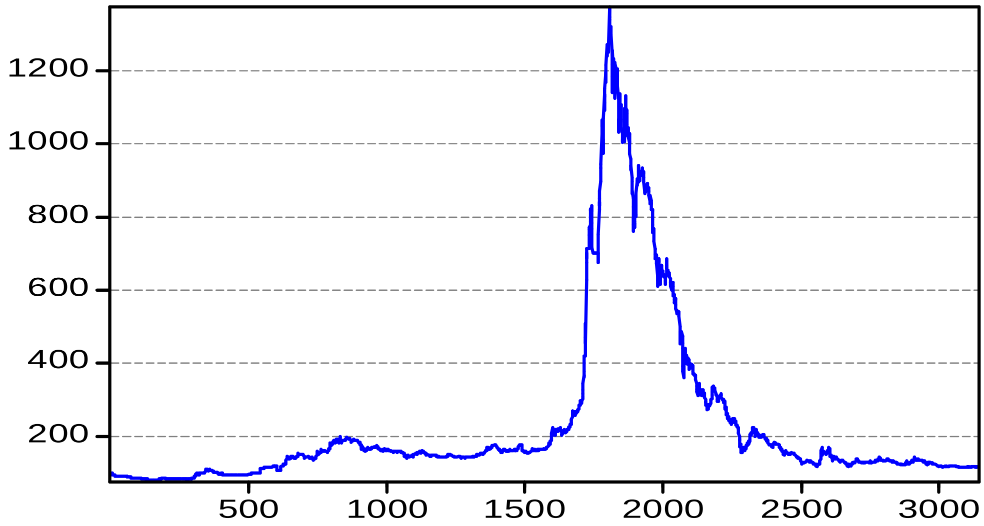


Foreign Exchange Rates Hong Kong $ To Australian $

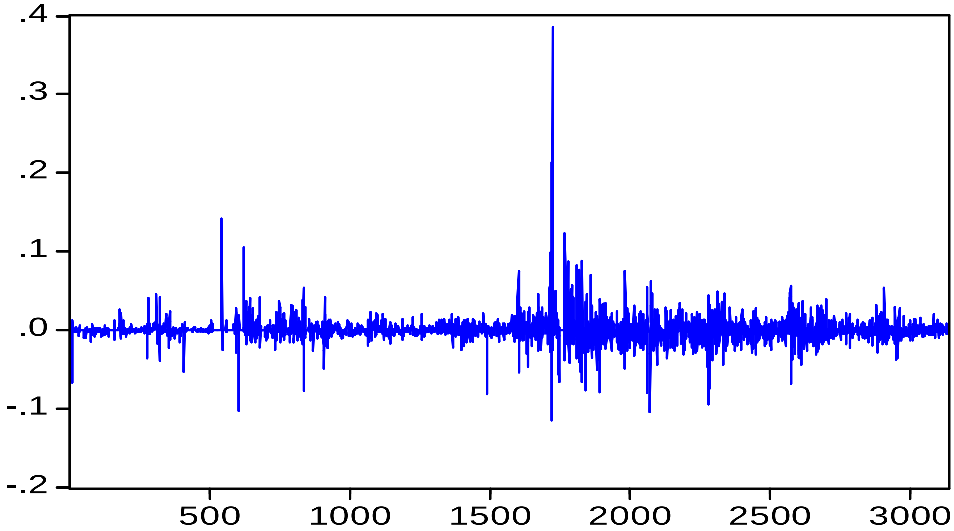


Fitted Residuals of AR(1) model over the returns

Description	*Foreign Exchange Rates Hong Kong $ To Australian $*				
Source	*Datastream*			***Mnemonic***	*hkaudsp*
Starting Date	*31/5/1993*	***Observations***	*3025*	***Frequency***	*Daily*
Estimated K	*0.749*	***Value of ρ***	*-0.090858*	***t-stat***	*-5.014283*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.129,-0.125] U [-0.054,-0.050]*	

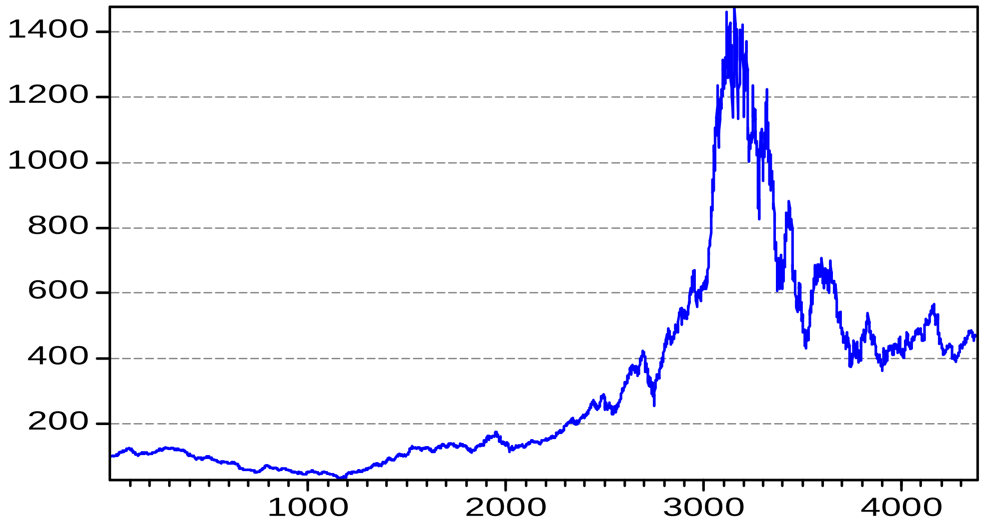


Stock Market: Cyprus Datastream Index

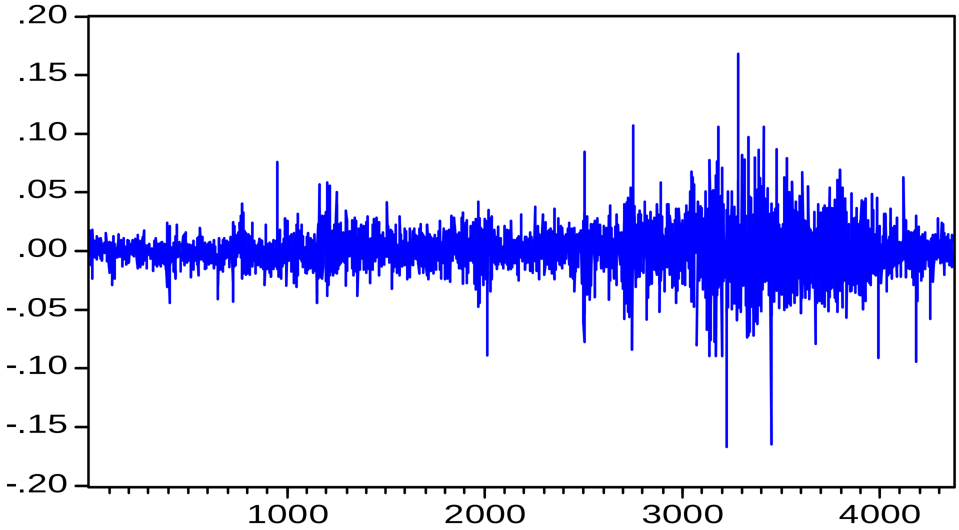


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: Cyprus Datastream Index*				
Source	*Datastream*			**Mnemonic**	*totmkcp*
Starting Date	*23/12/1992*	**Observations**	*3141*	**Frequency**	*Daily*
Estimated K	*1.480*	**Value of ρ**	*0.182317*	**t-stat**	*10.3738*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.142,0.152] U [0.222,0.232]*	

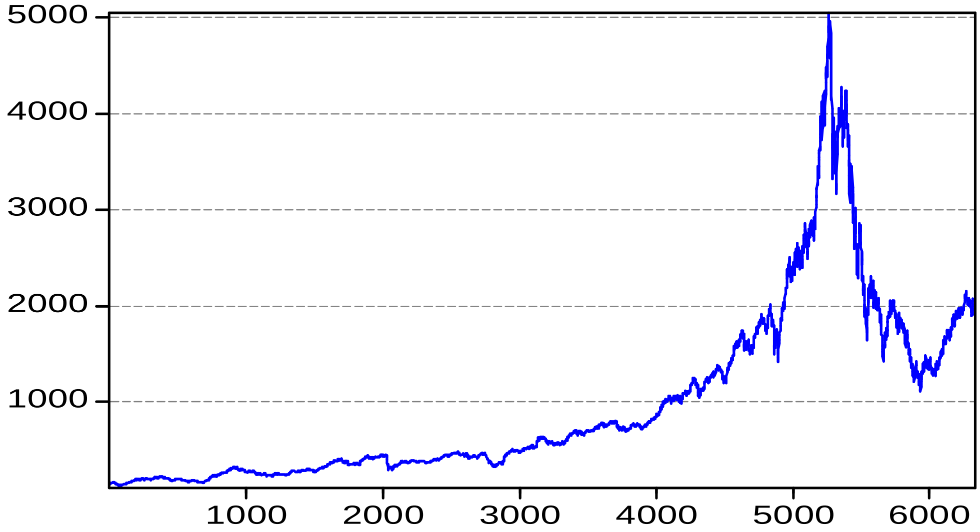


Stock Market: Finland Datastream Index

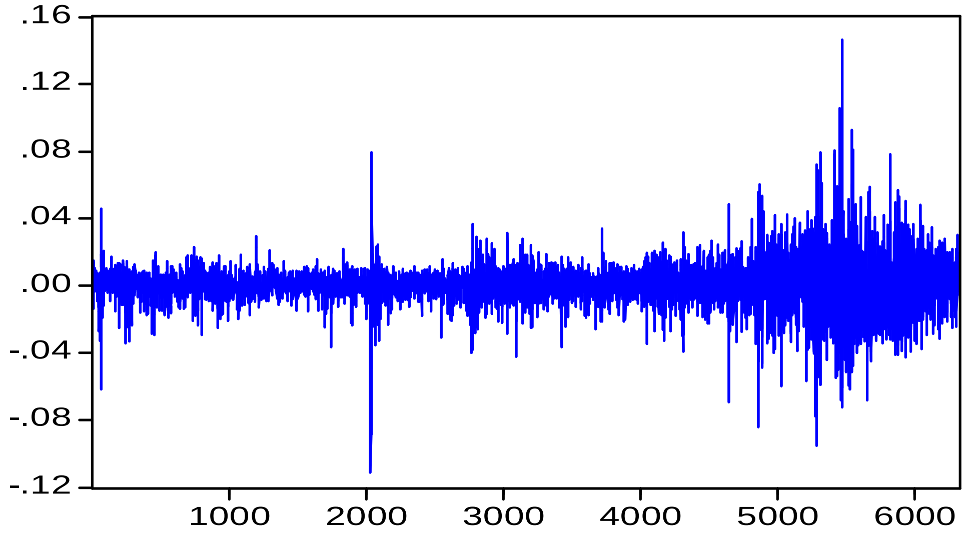


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: Finland Datastream Index*				
Source	*Datastream*			***Mnemonic***	*totmkfn*
Starting Date	*25/3/1988*	***Observations***	*4381*	***Frequency***	*Daily*
Estimated K	*1.355*	***Value of ρ***	*0.046651*	***t-stat***	*3.087529*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.016,0.026] U [0.076,0.086]*	

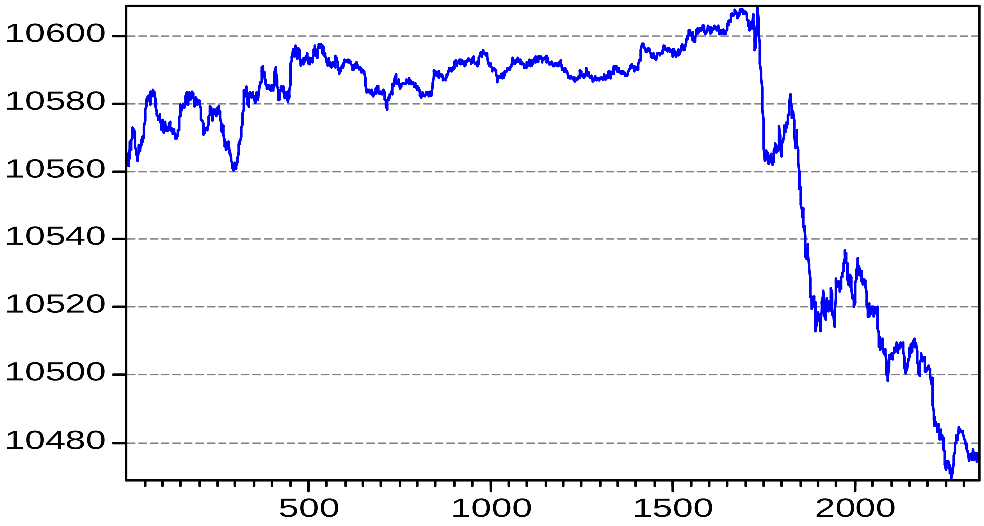


Stock Market: Dow Jones Industrial Index

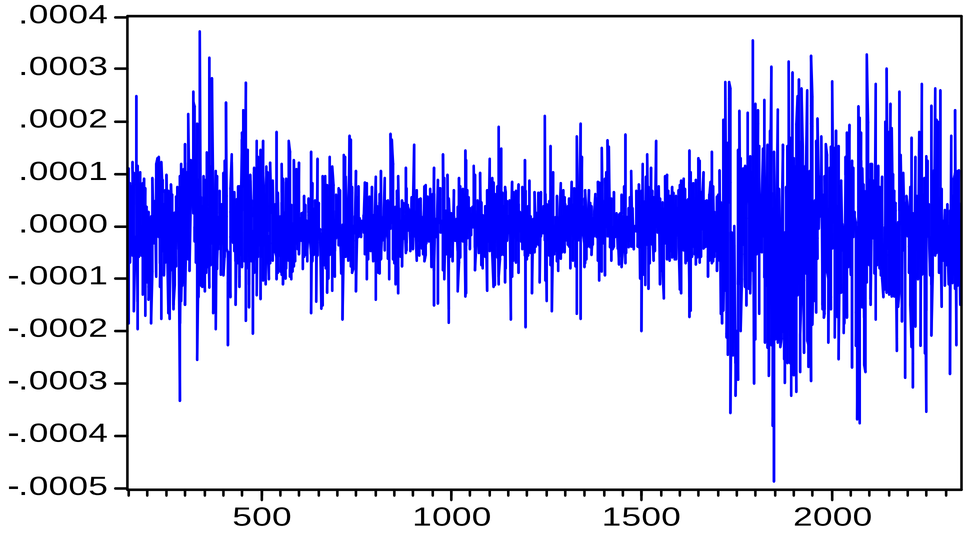


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: Dow Jones Industrial Index*				
Source	*Datastream*			***Mnemonic***	*djindus*
Starting Date	*1/1/1980*	***Observations***	*6331*	***Frequency***	*Daily*
Estimated K	*1.286*	***Value of ρ***	*0.063776*	***t-stat***	*5.083883*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.034,0.039] U [0.088,0.093]*	

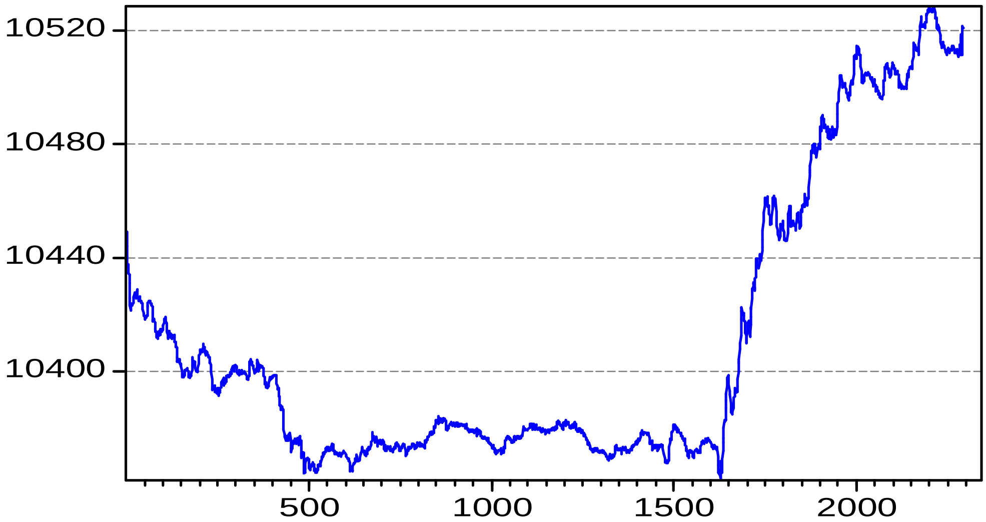


Stock Market: Dow Jones Industrial (Intraday Data)

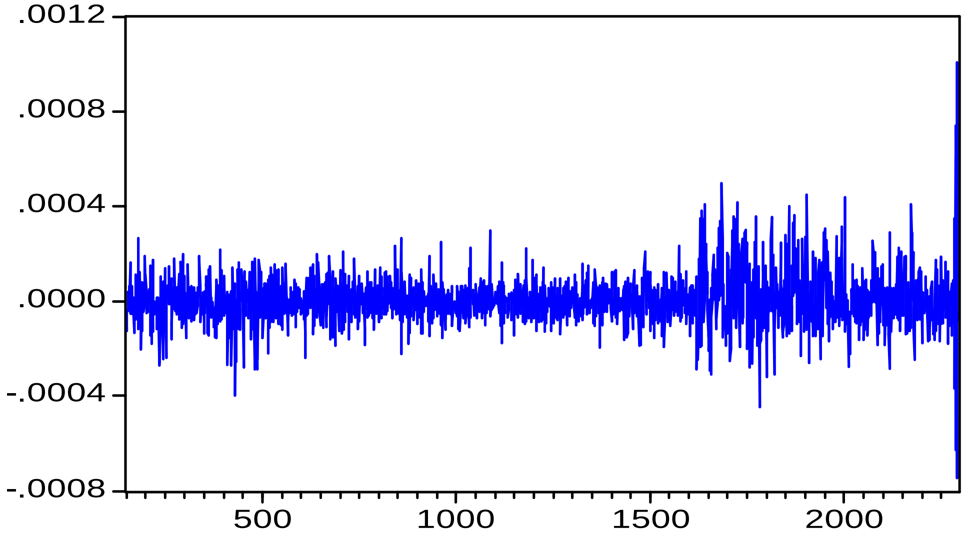


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: Dow Jones Industrial (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*d&j-ind*
Starting Date	*22 March 2005*	***Observations***	*2340*	***Frequency***	*10 sec*
Estimated K	*0.718*	***Value of ρ***	*0.178438*	***t-stat***	*8.487213*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.133,0.137] U [0.220,0.224]*	

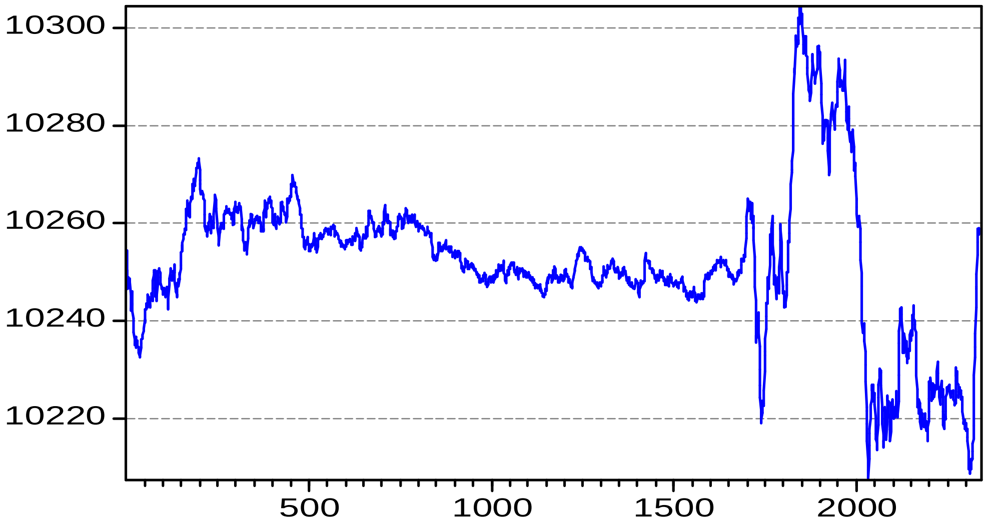


Stock Market: Dow Jones Industrial (Intraday Data)

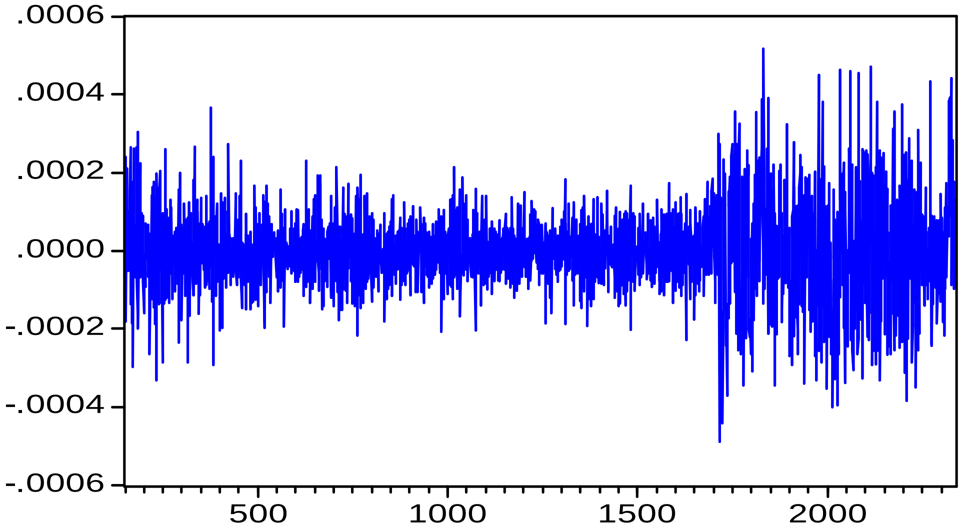


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: Dow Jones Industrial (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*d&j-ind*
Starting Date	*12 Aprli 2005*	***Observations***	*2295*	***Frequency***	*10 sec*
Estimated K	*0.846*	***Value of ρ***	***0.077444***	***t-stat***	*3.597466*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.030,0.035] U [0.120,0.125]*	

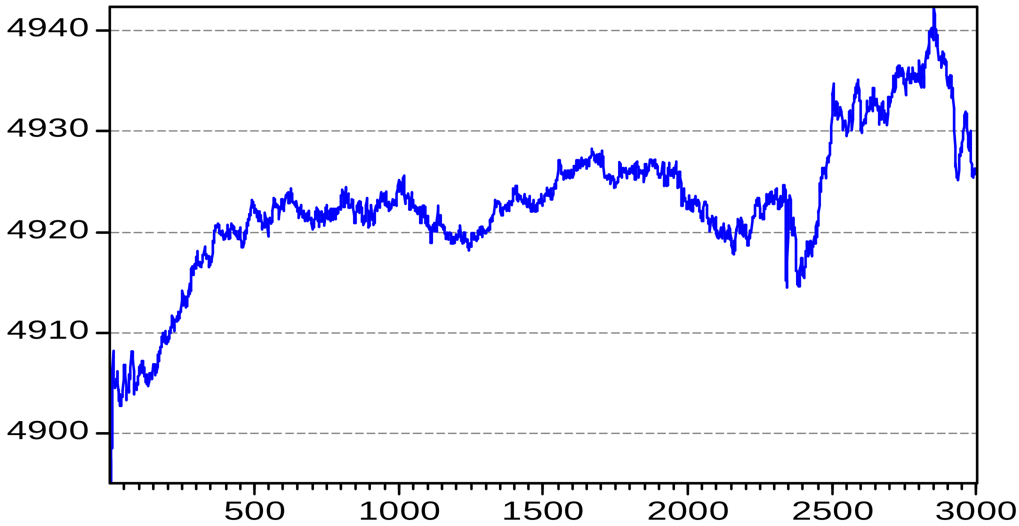


Stock Market: Dow Jones Industrial (Intraday Data)

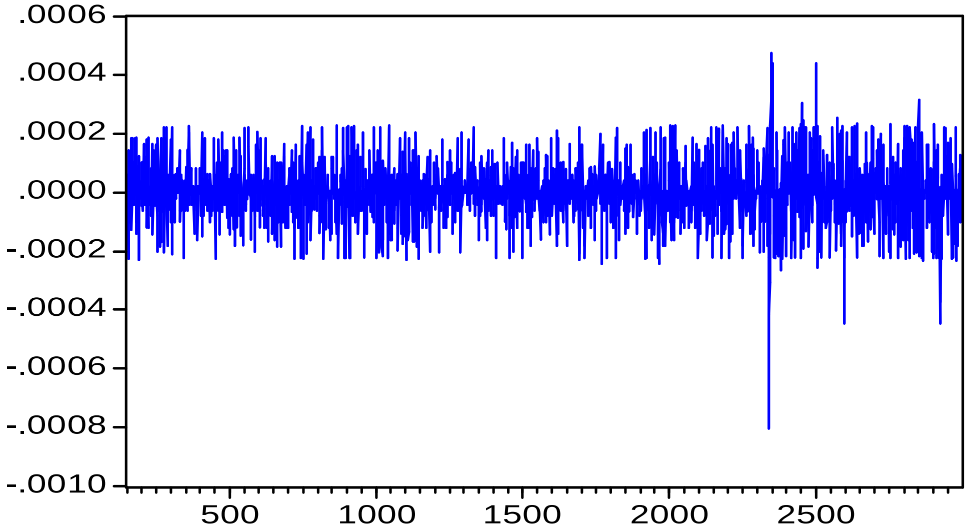


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: Dow Jones Industrial (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*d&j-ind*
Starting Date	*3 May 2005*	***Observations***	*2340*	***Frequency***	*10 sec*
Estimated K	*0.970*	***Value of ρ***	*0.329191*	***t-stat***	*16.30952*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.284,0.290] U [0.369,0.375]*	

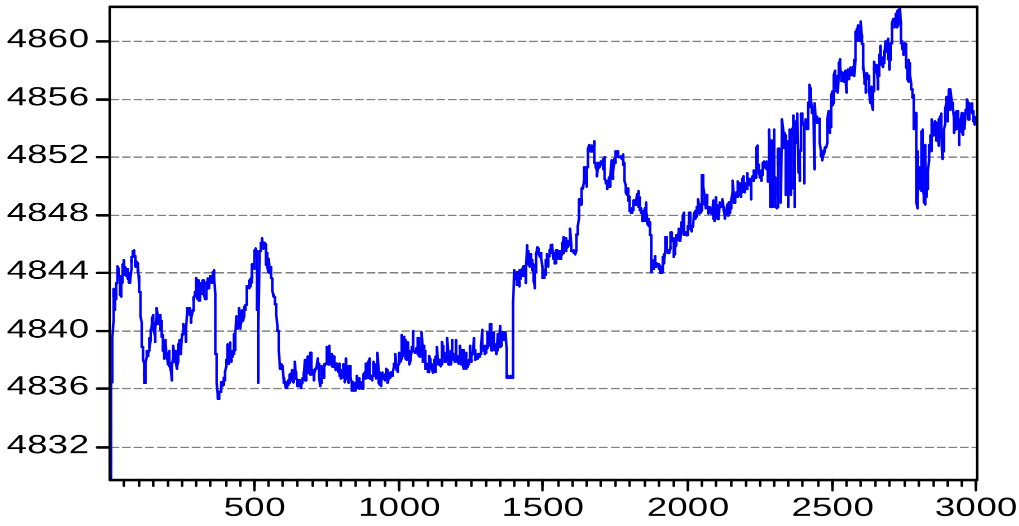


Stock Market: UK FTSE-100 (Intraday Data)

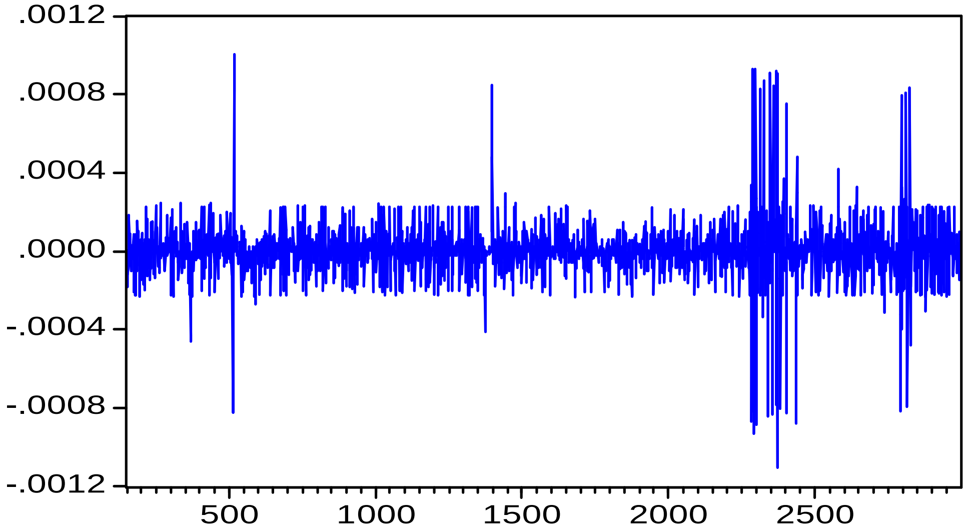


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: UK FTSE-100 (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*futsee-100*
Starting Date	*1 Aprli 2005*	***Observations***	*3000*	***Frequency***	*10 sec*
Estimated K	*0.441*	***Value of ρ***	*0.038181*	***t-stat***	*2.040977*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[0.000,0.002] U [0.075,0.077]*	

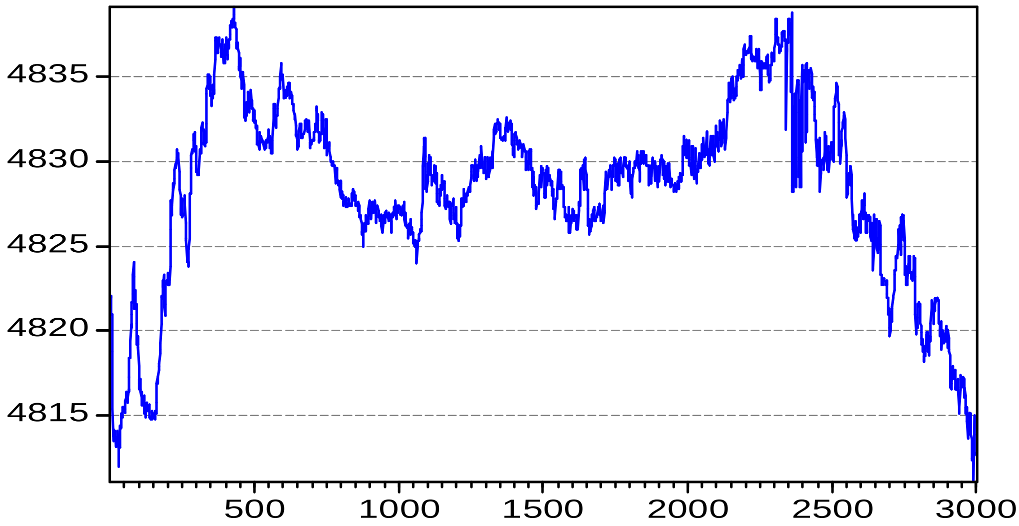


Stock Market: UK FTSE-100 (Intraday Data)

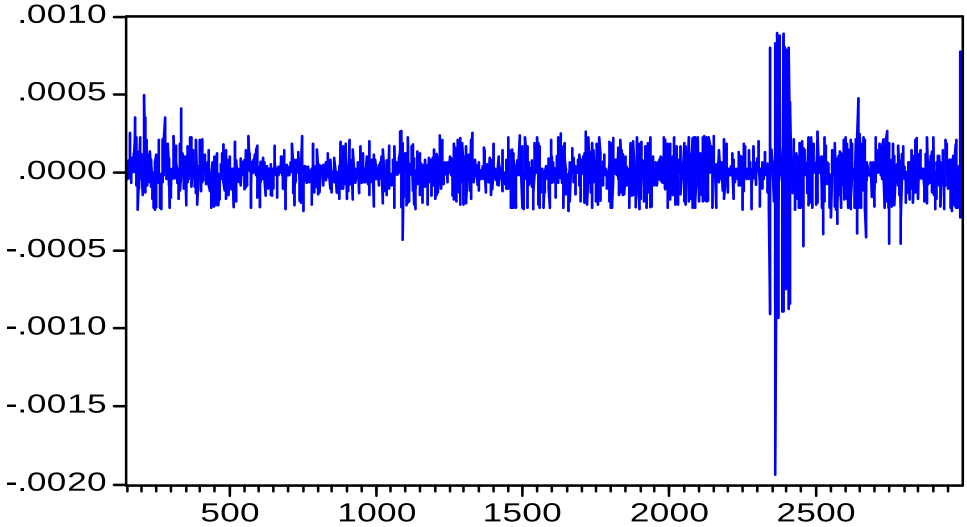


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: UK FTSE-100 (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*futsee-100*
Starting Date	*19 Aprli 2005*	***Observations***	*3000*	***Frequency***	*10 sec*
Estimated K	*0.681*	***Value of ρ***	*-0.114768*	***t-stat***	*-6.166664*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.153,-0.150]* ***U*** *[-0.077,-0.074]*	

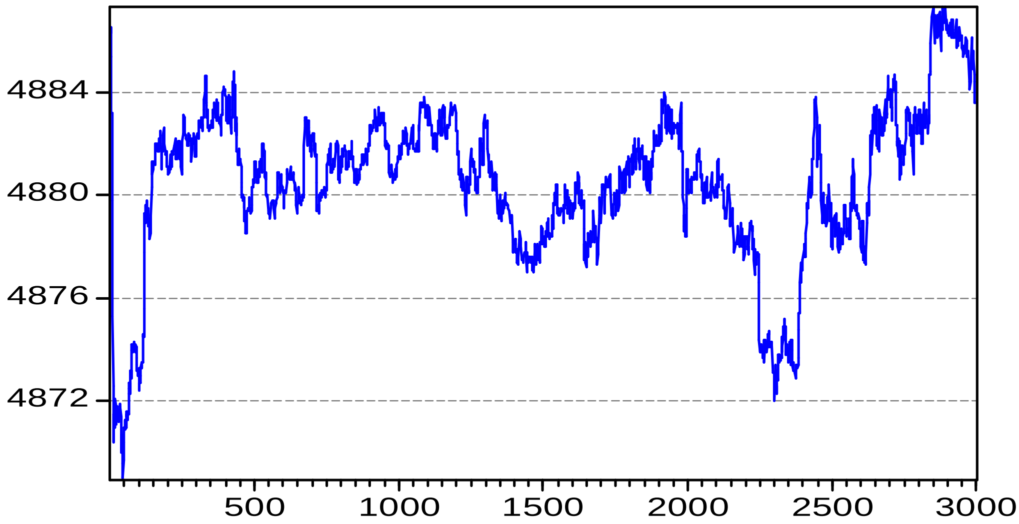


Stock Market: UK FTSE-100 (Intraday Data)

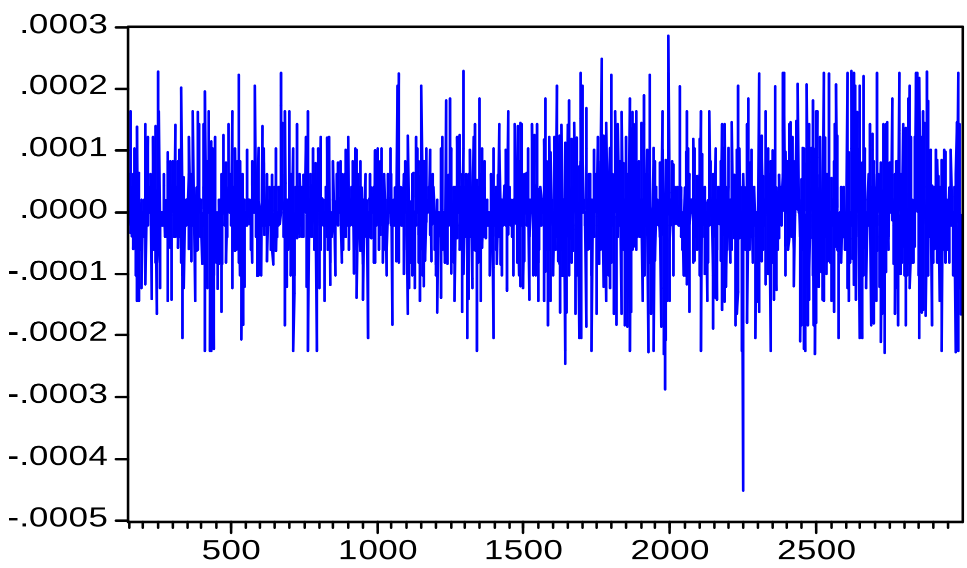


Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: UK FTSE-100 (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*futsee-100*
Starting Date	*21 Aprli 2005*	***Observations***	*3000*	***Frequency***	*10 sec*
Estimated K	*1.103*	***Value of ρ***	*-0.175646*	***t-stat***	*-9.523356*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.217,-0.211] U [-0.138,-0.132]*	



Stock Market: UK FTSE-100 (Intraday Data)



Fitted Residuals of AR(1) model over the returns

Description	*Stock Market: UK FTSE-100 (Intraday Data)*				
Source	*Dukascopy Trading Technologies*			***Mnemonic***	*futsee-100*
Starting Date	*16 May 2005*	***Observations***	*3000*	***Frequency***	*10 sec*
Estimated K	*0.484*	***Value of ρ***	*-0.038613*	***t-stat***	*-2.060674*
Values of ρ where exist conflicting results between common t-stat and t*-stat adjusted to estimated k				*[-0.076,-0.074] U [-0.001,0.000]*	