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Computer Aided Diagnosis: SVM-Based Breast Cancer Data Classification

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1 Intro: Computer Aided Diagnosis

Nowadays cancer is a leading cause of death worldwide. There were an estimated 14.1 million cases in 2012, of these 7.4 million cases were in men and 6.7 in women with mortality rate of 8.2 million. Cancer has more than 100 types and named for the organs or tissues the cancers form. They can start almost anywhere in the human body. Daily, human cells grow and divide to form new cells. When cells grow old or become damaged they die and new cells take their place. When cancer develops the old cells don't die, cell division continues, and may form growths called tumors. There are two different kind of tumors: Malignant tumors, which means they can spread into, or invade nearby tissues and benign tumors which unlike malignant they don't spread but they can be quite large however. As cancerous tumors grow, some cancer cells can travel through the blood to other places in the body and form new tumors. This phenomenon is called metastasis.

Not every change in the body's tissues is cancerous although some of them may develop into cancer if they are not well treated and monitored:

Hyperplasia occurs when cell's division within a tissue is faster than normal cells and extra cells build up or proliferate. Is considered to be a physiological response to a specific stimulus however can also occur as a pathological response, if an excess of hormone or growth is responsible for the stimuli. The way tissue is organized make cells to look normal under a microscope.

Dysplasia is way more dangerous than hyperplasia. The cells in this case look abnormal and there are changes in how the tissue is organized. The abnormality in cells and tissue defines the chance that cancer will form.

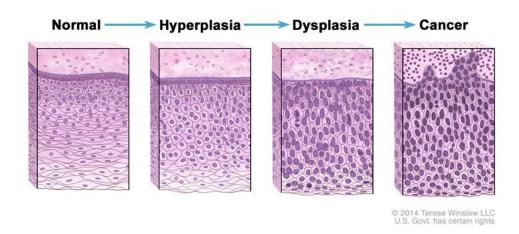


Figure 1.1: Normal cells may become cancer cells

Researches have shown that certain risk factors may increase a person's chance of developing cancer among five of them are responsible for about one third of cancer deaths: high body mass index, low fruit and vegetable intake, lack of physical activity, tobacco and alcohol use. The most common sites of cancer diagnosed in 2012 were lung, prostate, colorectum, stomach, and liver in men and breast, colorectum, lung, cervix and stomach for women. Especially in women the breast cancer was the most common one with nearly 1.7 million incidents in 2012 and the second most common overall which represents about 12 percent of all new cancer cases and 25 percent of all cancer in women. In the U.S.A only, an estimated 246.660 new cases of invasive breast cancer are expected to be diagnosed in 2016.

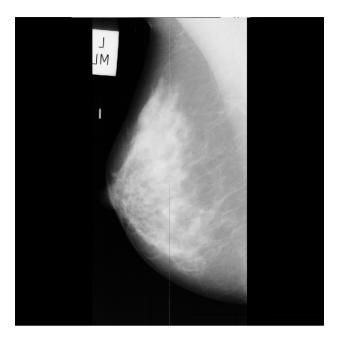


Figure 1.2: Mammogram of a patient

Over the past few years cancer research studies that involve the collection of mammograms from the patients participation have been funded. Each of these mammograms needs to be analyzed by radiologists which task can be extremely time consuming, expensive and some times hard to accomplish. As an example of how hard this task can be, in a study 115 doctors made a total of 6900 diagnoses. The tissue samples included 23 cases of invasive breast cancer (where the cancer already has spread into nearby breast tissue), 73 cases of ductal carcinoma, 72 cases of atypical hyperplasia and 72 benign cases. Three experts worked together to establish the correct diagnosis for each case. 96% of the time correctly diagnosed the cases of inancive carcinoma, 84% and 87%, respectively, of all cases of DCIS and benign lesions, but in samples of atypical hyperplasia, only a 48% correct opinion rate had been achieved.

Cancer is considered to be more of a developed world issue, but in fact it is around 43%. The 57% of all cancers, non-melanoma and skin cancer excluded, occur in third world countries. The lack of experts in these less developed countries provokes diagnosis to be slow and unreliable.

For the reasons mentioned above a Computer Aided Diagnosis system (CAD) would be very useful. It could be able to batch process mammograms more efficient, with the least amount of time and effort as well as hopefully help to objectively and accurately quantify cancer, with enabling reliable tracking of progression or regression of the disease.

2 Problem Definition: Breast-cancer Data Classification

Computer vision has been feasible over the past few decades by the rapid technology development, increasing the studies have been done in an effort to automate the analysis of mammograms. To extract features studies use texture analysis methods such as Gray level Co Occurence Matrix (GLCM) [3, 10, 19], edge-enhancing[19], local binary pattern (LBP)[19], fractal dimension [19], Spatial Gray-Level Dependence Method (SGLDM) [5]. One of the most famous and most used feature extraction method is gray level co occurrence matrix (GLCM) and in [3, 10] authors used five co occurrence matrices extracted with with

 $\theta \in \{0, \pi/4, \pi/2, 3\pi/4\}$ and pixel distance (d=1) to detect masses in patients mammograms. As mentioned in [5] the extracted features from these matrices won't be able to be discriminative with cases of cancers due to their non-uniform shape and margins. Therefore suggested a different approach, where increment of the number of spatial orientations and of range of pixel distances have been done to achieve better results. For texture classifications various methods have been introduced in this area, like SVM [7, 15], KNN [9, 12] and in some cases, as in an approach introduced by 3-D ultrasound (US) using run difference matrix (RDM) with neural networks[6],accuracy reaching 91.9%. For our study we will use [1] database where 300 benign 299 malignant and 100 normal mammograms of patients have been collected and classified. We will use the feature extraction algorithms from [5] and we will collect these datasets and try to find the best classification method from various SVM kernels by testing their accuracies.

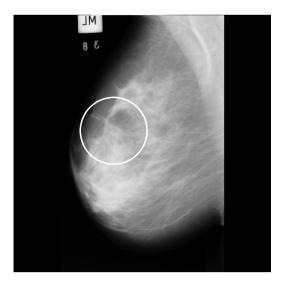


Figure 2.1: Mammogram with dense-glandular background tissue and spiculated masses - benign

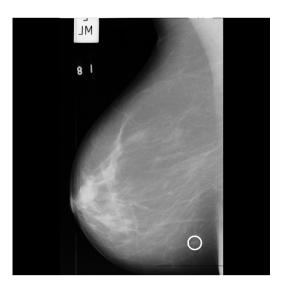


Figure 2.2: Mammogram with fatty background tissue and spiculated masses - benign



Figure 2.3: Mammogram with fatty background tissue and spiculated masses - benign

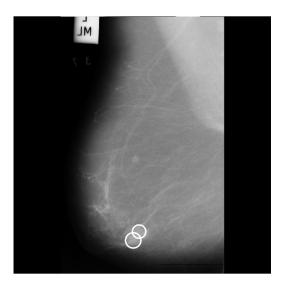


Figure 2.4: Mammogram with fatty background tissue and well-defined/circumscribed masses - benign

3 Feature Estimation Algorithms

Two of the most known pre-existing methods will be used for feature extraction which are Spatial Gray Level Dependence method (SGLDM) and Run Difference Method (RDM) in order to describe textual characteristics of the mammogram masses.

3.1 Spatial Gray Level Dependence Method: SGLDM

The spatial gray level dependence method (SGLDM) describes the spatial distribution and spatial dependence among gray levels on extraction areas throughout the image. Its a human texture discrimination in terms of texture statistical properties investigated by Julesz in 1975 who has introduced that texture pairs in gray level images with identical second order

statistics cannot be discriminated and hence he used the estimation of second order joint conditional probability density function (pdf) for texture feature extraction.

The SGLDM matrix is formed by computing the number of occurrences of each pixel's gray level (G) i that are away from any pixel with gray level j by distance d, in a direction defined by angle θ . The choice of distance, angle combination and quantization level is arbitrary. Before an image is processed using the SGLDM, it's gray levels are binned so that the resulting image contains only a few gray levels. The typical number of gray levels G typically used is 8, as in our case, or 16. Figure 3.1 shows an example for a pixel (middle) for d = 3 and $\theta = \{0, \pi/4, \pi/2, 3\pi/4\}$.

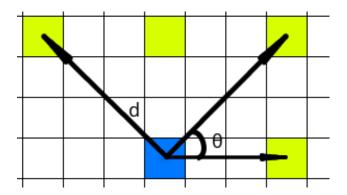


Figure 3.1: Co occurrence for d =3 and $\theta = \{0, \pi/4, \pi/2, 3\pi/4\}$

If image size is quite large would significantly increase the time of the algorithm processing so we have extract a region of interest (ROI) of every image.

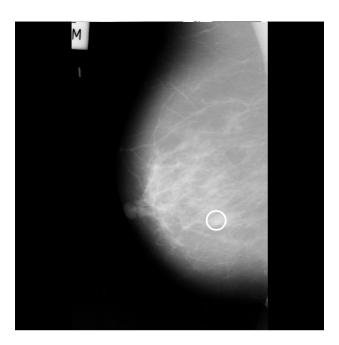


Figure 3.2: Mammogram with fatty-glandular background tissue and other, ill-defined masses - benign

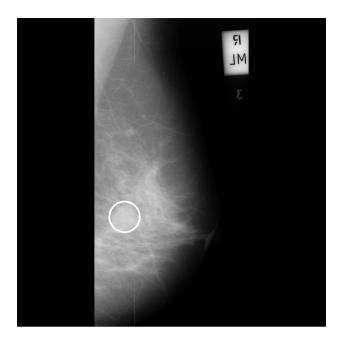


Figure 3.3: Mammogram with dense-glandular background tissue and asymmetry - benign

The purpose of ROI extraction and number of gray levels reduction is to enhance computational efficiency and the power of feature extraction. After the gray level reduction SGLDM estimates the second order conditional bivariate pdf $f(i,j|d,\theta)$, where $\theta \in \{0,\pi/4,\pi/2,3\pi/4\}$. $d \in \{1,2,\ldots,L-1\}$ defines how large the area of interest would be with L being the size of ROI. Usually SGLDM matrices are computed for a distance d=1 and for $\theta=\{0,\pi/4,\pi/2,3\pi/4\}$. In our case in order to gain more information we compute the co-occurrence matrix at eight different angles: $\Theta \in \{0,\frac{\pi}{8},\frac{\pi}{4},\frac{3\pi}{8},\frac{\pi}{2},\frac{5\pi}{8},\frac{3\pi}{4},\frac{7\pi}{8}\}$ and at all distances $d\in \{1,2,3,4,\ldots,L/2\}$. When ROI have been extracted and rotated at every angle we compute the SGLDM matrix in the horizontal direction $(\theta=0)$ for each one of them producing $8 \times \lfloor L/2 \rfloor$ matrices and obtain a rotation invariant matrix Md from the summation of these matrices.

$$f_{ij}^d = \sum_{\theta \in \Theta} f_{ij}^{\theta, d} \tag{3.1}$$

Then we extract the following eight features from these $8 \times |L/2|$ SGLDMs:

• Contrast:

Contrast is the difference between maximum and minimum intensity in an image.

$$\gamma_1^d = \sum_{i=1}^G \sum_{j=1}^G |i - j|^2 f_{ij}^d$$
(3.2)

• Correlation:

Correlation is an optical method that employs tracking and image registration techniques for accurate 2D and 3D measurements of changes in images. This is often used to measure deformation, displacement, optical flow and strain and also is widely applied in many areas of science and engineering.

$$\gamma_2^d = \sum_{i=1}^G \sum_{j=1}^G \frac{(i - \mu_i^d) (j - \mu_j^d) f_{ij}^d}{\sigma_i^d \sigma_j^d}$$
 (3.3)

Where:

$$\mu_i^d = \frac{1}{G} \sum_{i=1}^G f_{ij}^d \tag{3.4}$$

is the mean of the sum of co occurrence matrix at distance d of each row,

$$\mu_j^d = \frac{1}{G} \sum_{i=1}^G f_{ij}^d \tag{3.5}$$

is the mean of the sum of co occurrence matrix at distance d of each column,

$$\left(\sigma_i^d\right)^2 = \frac{1}{G} \sum_{j=1}^G \left(f_{ij}^d - \mu_i^d\right)^2$$
 (3.6)

is the standard deviation of each row,

and

$$\left(\sigma_j^d\right)^2 = \frac{1}{G} \sum_{i=1}^G \left(f_{ij}^d - \mu_j^d\right)^2$$
 (3.7)

is the standard deviation of each column.

• Energy:

Energy measures the occurrence of repeated pairs in an image:

$$\gamma_3^d = \sum_{i=1}^G \sum_{j=1}^G (f_{ij}^d)^2 \tag{3.8}$$

• Homogeneity:

Homogeneity measures an image's smoothness:

$$\gamma_4^d = \sum_{i=1}^G \sum_{j=1}^G \frac{f_{ij}^d}{1 + |i - j|} \tag{3.9}$$

• Variance:

Variance, measures variation of the gray level distribution:

$$\gamma_5^d = \sum_{i=1}^G \sum_{j=1}^G (i - \mu^d)^2 f_{ij}^d$$
 (3.10)

• Sum average:

Sum average, measures the average gray level in an image:

$$\gamma_6^d = \frac{1}{2} \sum_{i=1}^G \sum_{j=1}^G (i+j) f_{ij}^d$$
 (3.11)

• Maximum probability:

Maximum probability, determines predominance of the most predominant pixel pair:

$$\gamma_7^d = \max_{i,j} f_{ij}^d \tag{3.12}$$

• Cluster prominence:

Cluster prominence, measures grouping of pixels with similar gray levels:

$$\gamma_8^d = \sum_{i=1}^G \sum_{j=1}^G \left(i - \mu_i^d + j - \mu_j^d \right)^4 f_{ij}^d$$
 (3.13)

To define characteristics of the SGLDM matrices we use equations 3.2 - 3.13,and the following characteristic for $n=1,\ldots,8$ to compute all 48 SGLDM features as shown in tables 1,2 and 3:

$$\bar{\gamma_n} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \gamma_n^d \tag{3.14}$$

Table 1: SGLDM 1-16 Features

	1
Contrast	$x_1 = \gamma_1^1$
Correlation	$x_2 = \gamma_2^1$
Energy	$x_3 = \gamma_3^1$
Homogeneity	$x_4 = \gamma_4^1$
Variance	$x_5 = \gamma_5^1$
Sum Average	$x_6 = \gamma_6^1$
Maximum Probability	$x_7 = \gamma_7^1$
Cluster Prominence	$x_8 = \gamma_8^1$
Mean Contrast	$x_9 = \bar{\gamma_1}$
Mean Correlation	$x_{10} = \bar{\gamma_2}$
Mean Energy	$x_{11} = \bar{\gamma_3}$
Mean Homogeneity	$x_{12} = \bar{\gamma_4}$
Mean Variance	$x_{13} = \bar{\gamma_5}$
Mean Sum Average	$x_{14} = \bar{\gamma_6}$
Mean Maximum Probability	$x_{15} = \bar{\gamma_7}$
Mean Cluster Prominence	$x_{16} = \bar{\gamma_8}$

Table 2: SGLDM 17-32 Features

Minimum Contrast	$x_{17} = \min_d \gamma_1^d$
Minimum Correlation	$x_{18} = \min_d \gamma_2^d$
Minimum Energy	$x_{19} = \min_d \gamma_3^d$
Minimum Homogeneity	$x_{20} = \min_d \gamma_4^d$
Minimum Variance	$x_{21} = \min_d \gamma_5^d$
Minimum Sum Average	$x_{22} = \min_d \gamma_6^d$
Minimum Maximum Probability	$x_{23} = \min_d \gamma_7^d$
Minimum Cluster Prominence	$x_{24} = \min_d \gamma_8^d$
Maximum Contrast	$x_{25} = \max_d \gamma_1^d$
Maximum Correlation	$x_{26} = \max_d \gamma_2^d$
Maximum Energy	$x_{27} = \max_d \gamma_3^d$
Maximum Homogeneity	$x_{28} = \max_d \gamma_4^d$
Maximum Variance	$x_{29} = \max_d \gamma_5^d$
Maximum Sum Average	$x_{30} = \max_d \gamma_6^d$
Maximum Maximum Probability	$x_{31} = \max_d \gamma_7^d$
Maximum Cluster Prominence	$x_{32} = \max_d \gamma_8^d$

Table 3: SGLDM 33-48 Features

Variance of Contrast	$x_{33} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_1^d - \bar{\gamma_1} \right)^2$
Variance of Correlation	$x_{34} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_2^d - \bar{\gamma_2} \right)^2$
Variance of Energy	$x_{35} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_3^d - \bar{\gamma_3} \right)^2$
Variance of Homogeneity	$x_{36} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_4^d - \bar{\gamma_4} \right)^2$
Variance of Variance	$x_{37} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_5^d - \bar{\gamma_5} \right)^2$
Variance of Sum Average	$x_{38} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_6^d - \bar{\gamma_6} \right)^2$
Variance of Maximum Probability	$x_{39} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_7^d - \bar{\gamma_7} \right)^2$
Variance of Cluster Prominence	$x_{40} = \frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} \left(\gamma_8^d - \bar{\gamma_8} \right)^2$
Skewness of Contrast	$x_{41} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_1^d - \bar{\gamma}_1)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_1^d - \bar{\gamma}_1)^2\right)^{3/2}}$
Skewness of Correlation	$x_{42} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_2^d - \bar{\gamma}_2)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_2^d - \bar{\gamma}_2)^2\right)^{3/2}}$
Skewness of Energy	$x_{43} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_3^d - \bar{\gamma}_3)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_3^d - \bar{\gamma}_3)^2\right)^{3/2}}$
Skewness of Homogeneity	$x_{44} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_4^d - \bar{\gamma_4})^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_4^d - \bar{\gamma_4})^2\right)^{3/2}}$
Skewness of Variance	$x_{45} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_5^d - \bar{\gamma}_5)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_5^d - \bar{\gamma}_5)^2\right)^{3/2}}$
Skewness of Sum Average	$x_{46} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_6^d - \bar{\gamma}_6)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_6^d - \bar{\gamma}_6)^2\right)^{3/2}}$
Skewness of Maximum Probability	$x_{47} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_7^d - \bar{\gamma}_7)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_7^d - \bar{\gamma}_7)^2\right)^{3/2}}$
Skewness of Cluster Prominence	$x_{48} = \frac{\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_8^d - \bar{\gamma}_8)^3}{\left(\frac{1}{\lfloor L/2 \rfloor} \sum_{d=1}^{\lfloor L/2 \rfloor} (\gamma_8^d - \bar{\gamma}_8)^2\right)^{3/2}}$

At table 1 we use only d=1 to get the first eight features because in reality the first distance (d=1) contains the most useful information whereas the other SGLDM features contains progressively less information as d increases. However, as the remaining SGLCM matrices contains important information about the distribution of the features when different values of d are considered, we also keep the mean, minimum, maximum, variance, mean absolute deviation and skewness of these co occurrence features over all values of d.

Algorithm 1 shows the pseudo code of SGLDM algorithm we implemented.

Algorithm 1 SGLDM algorithm

```
1: procedure SGLDM(image)
        for \theta in \Theta do // \Theta \epsilon \left\{0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}\right\} create image(\theta) // Rotated image by \theta
 2:
 3:
             image(\theta) = crop image(\theta) // crop image to get the ROI for angle \theta
 4:
        end for
 5:
 6:
        L = length(image(\theta))
        features_to_compute = Array( 'correlation', 'homogeneity', 'sum average', 'cluster
 7:
    prominence', 'maximum probability', 'variance', 'energy', 'contrast')
        for every d do //d \in \{1, 2, 3, 4, \dots, L/2\}
 8:
             co\_occurrence\_matrix = 0
 9:
             for \theta in \Theta do // Sum all co occurrence matrices for this d
10:
                 co_occurrence_matrix += co occurrence matrix of I(\theta)
11:
             end for
12:
             for i=1 to length(features_to_compute) do
13:
                 S_i = computeFeature(co\_occurrence\_matrix, features\_to\_compute[i]) // For
14:
    every feature return the calculated feature based on equations 3.2 - 3.13
             end for
15:
        end for
16:
        for i=1 dolength(features_to_compute)
17:
18:
             S_{i+n} = mean(S_i)
             S_{i+2n} = mad(S_i)
19:
             S_{i+3n} = min(S_i)
20:
             S_{i+4n} = max(S_i)
21:
22:
             S_{i+5n} = variance(S_i)
             S_{i+6n} = skewness(S_i)
23:
        end for
24:
25:
        features = Array(S)
        return features
26:
27: end procedure
```

3.2 Run Difference Method: RDM

The Run Difference Method (RDM), as SGLDM seeks to extract texture features in an image. RDM is based on the estimation of the probability density function of the gray level difference along with a distance between the pixels,p, when the displacement vector between two pixels is given.

Before image is processed, the gray levels are binned so that the resulting image contains only eight gray levels. Then the conditional biviate pdf RDM $(r, g_{dif}|\theta)$ is estimated where $\theta \in \{0, \pi/4, \pi/2, 3\pi/4\}$, $r = 1, \dots, R$ and g_{dif} is the gray level difference.

As in cancer the shape and margins are non-uniform we extend RDM matrices for eight different angles Θ $\epsilon \left\{0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}\right\}$ by rotating an inscribe square inside the ROI and using bilinear interpolation to interpolate the values.

As a result eight RDM matrices are formed and summed to obtain the rotation invariance matrix M_{rdm} as shown in eq. 3.15:

$$M_{RDM} = \sum_{\theta \in \Theta} M_{RDM}^{\theta} \tag{3.15}$$

Rather than extracting features directly from matrix M_{rdm} , three characteristic vectors defined:

• Distribution of Gray level Difference (DGD):

$$DGD_{gdif} = \sum_{r=1}^{L/2} M_{RDM}$$
 (3.16)

• Distribution of Average Distance (DAD):

$$DAD_{gdif} = \sum_{r=1}^{L/2} M_{RDM} \cdot r \tag{3.17}$$

• Distribution Of average Difference (DOD):

$$DOD_r = \sum_{g_{dif}=0}^{G-1} M_{RDM} \cdot g_{dif}$$
 (3.18)

to help us extract the following features:

- Large difference emphasis (LDE): which measures the predominance of large gray level differences
- Long Distance Emphasis for Large difference (LDEL): which measures the prominence of large differences a long distance from each other.
- Sharpness: which measures the contrast and definition in an image
- Second Moment of DGD (SMG): which measures the variation of gray level differences.
- Second Moment of DOD (SMO): which measures the variation of average gray level differences.

as shown in Table 4:

Table 4: RDM Features

Large difference emphasis (LDE)	$x_{49} = LDE = \sum_{gdif=0}^{G-1} DGD(g_{dif}) \ln(2/g_{dif})$
Long Distance Emphasis for large difference (LDEL)	$x_{50} = LDEL = \sum_{gdif=0^{G-1}DAD(g_{dif})(g_{dif})^2}$
Sharpness	$x_{51} = Sharpness = \sum_{gdif=0}^{G-1} DGD(g_{dif}) g_{dif}^{3}$
Second Moment of DGD (SMG)	$x_{52} = SMG = \sum_{gdif=0}^{G-1} (DGD(g_{dif}))^2$
Second Moment of DOD (SMO)	$x_{53} = SMO = \sum_{r=1}^{L/2} (DOD(r))^2$

As before we present the pseudo code for RDM algorithm as shown in algorithm 2. As in SGLDM algorithm, we created eight images of the ROI in order to extract features from each of which at one direction ($\theta = 0$). Then RDM matrix has formed, and vectors DGD, DOD and DAD have computed. From the computed vectors, we have finally extracted the five features.

Algorithm 2 RDM algorithm

```
1: procedure RDM(image)
        for \theta in \Theta do // \Theta \epsilon \left\{0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}\right\} create image(\theta) // Rotated image by \theta
 2:
 3:
            image(\theta) = crop image(\theta) // crop image to get the ROI for angle \theta
 4:
        end for
 5:
        L = length(image(\theta))
 6:
        for each \theta do
 7:
            for d = 1 to L/2 do
 8:
 9:
                 for pixel p = 1 to (L-d) do
                     G_{dif} = |image_{\theta}(p, h) - image_{\theta}(p + d, h)| + 1
10:
                     RDM_{\theta}(d, G_{dif}) = RDM_{\theta}(d, G_{dif}) + 1
11:
                 end for
12:
            end for
13:
        end for
14:
         // Compute vectors
15:
16:
        DGD = computeDGD // See eq.
        DOD = computeDOD // See eq.
17:
        DAD = computeDAD // See eq.
18:
19:
         // Compute features
        Sharpness = computeSharpness // See eq.
20:
        LDE = computeLDE // See eq.
21:
        SMG = computeSMG // See eq.
22:
        SMO = computeSMO // See eq.
23:
        LDEL = computeLDEL // See eq.
24:
        RDMfeatures = Array(LDE, Sharpness, SMG, SMO, LDEL)
25:
        return RDMfeatures
26:
27: end procedure
```

4 SVM Classification

Support Vector machines are supervised learning models with associated algorithms that analyze data used for classification and regression analysis. They have been discovered by Vladimir Naumovich Vapnik in 1963 who in 1992 suggested a way to create a nonlinear classifiers by applying the kernel trick to maximum margin hyperplanes. Vapnik moved to the USA at the end of 1990 and joined the Bell Labs. At the time they were interested in hand written character recognition and in neural nets and he convinced them that support vector machines would do better than neural nets and this was the first time support vector machines and kernel trick were actually used.

To illustrate the concept of the Support Vectors we will first suppose that we have two linearly separable classes which we would like to separate with an optimal separating hyperplane. To get a solution we can use linear regression, but then we see that we can have many different lines that solve a particular problem. To get the best line instead we use the notion of **margin** (Figure 4.1) which is the region expanded from a particular selected line until it crossovers. The objective of the SVM is to find the optimal separating hyperplane which maximizes the margin of the training data.

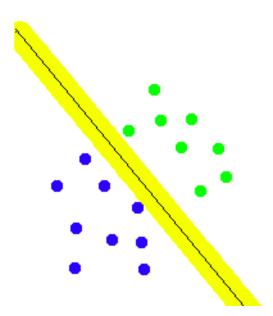


Figure 4.1: Example of svm margin in linearly separable data

As we know the line equation is y = ax + b. This can be represent as $w^T x$ as shown:

$$y = ax + b \implies y - ax - b = 0$$

Given two vectors:

$$\begin{pmatrix}
-b \\
-a \\
1
\end{pmatrix}$$
 and $x \begin{pmatrix} 1 \\
x \\
y \end{pmatrix}$

we have:

$$w^{T}x = -b \times (1) + (-a) \times x + 1 \times y$$

$$w^{T}x = y - ax - b$$

which is the same as the line equation describes a hyperplane.

A dataset D will be composed of n vectors $x_i \in \mathbb{R}^{p+1}$ and each x_i is associated with a value $y_i \in \{-1, 1\}$.

$$D = \{(x_i, y_i) | x_i \in \mathbb{R}^{p+1}, y_i \in \{-1, 1\}\}$$

Let x_i a point be the nearest data to the plane $w^T x = 0$.

For all the points in the dataset when we get $w^T x$ we get a number different than 0.

$$|w^T x_i| > 0$$

We would like to relate w to the margin but we find that any formula that takes w and produces the margin must have scale invariance. To do that we have to make this quantity $|w^T x_i| > 0$ to be normalized. Therefore:

$$|w^T x_i| = 1 (4.1)$$

We also pull out w_0 :

$$w = w1, \cdots, w_d \tag{4.2}$$

and the plane equation now becomes:

$$w^T x + b = 0 (4.3)$$

As shown to Figure 4.2 the vector w is perpendicular to the plane in the x space. To show that lets take two points x' and x" on the plane. These two points satisfy 4.3 so that

$$w^T x' + b = 0$$
 and $w^T x'' + b = 0$

If we get the difference of these two equations we get:

$$w^T \left(x' - x'' \right) = 0$$

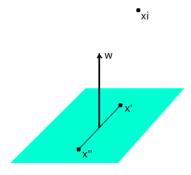


Figure 4.2: The geometric representation of vector w and x_i

w is orthogonal to x'-x" as a vector. As we did not take any restrictions for x' and x" w is orthogonal to every vector, therefore is perpendicular to the plane.

If we now take a random point x, and get the projection of $x_n - x$ on w we can find the distance between our point x_i and the plane.

We compute the unit vector $\hat{w} = \frac{w}{||w||}$:

$$\hat{w} = \frac{w}{||w||} \Longrightarrow$$

$$r = \hat{w} (x_n - x) \Longrightarrow$$

$$r = \frac{1}{||w||} |w^T x_i - w^T x| \Longrightarrow$$

$$r = \frac{1}{||w||} |w^T x_i + b - w^T x - b| \Longrightarrow$$

$$r = \frac{1}{||w||}$$

$$r = \frac{1}{||w||}$$

$$(4.4)$$

To find the biggest margin we have to maximize that distance.

$$\max \frac{1}{||w||}$$
subject to $\min_{i=1,\dots,N} |w^T x_i| = 1 \implies$

$$\min ||w|| \implies \min \frac{1}{2} w^T w$$
subject to $y_i (w^T x_i + b)$ for $i = 1, \dots, N$

where $w \in \mathbb{R}^d$. To find the extremum of a function with constraints we have to use Lagrange multipliers to get an expression which we can minimize without these constrains:

$$L = \frac{1}{2}w^{T}w\sum_{i=1}^{N} a_{i} \left[y_{i} \left(w \cdot x_{i} + b \right) - 1 \right]$$
(4.5)

We find the derivatives and set them to zero:

$$\frac{\partial L}{\partial w} = w - \sum_{i=1}^{N} a_i y_i x_i = 0 \implies$$

$$w = \sum_{i=1}^{N} a_i y_i x_i \qquad (4.6)$$

$$\frac{\partial L}{\partial b} = -\sum_{i=1}^{N} a_i y_i = 0 \implies$$

$$\sum_{i=1}^{N} a_i y_i = 0 \qquad (4.7)$$

And we discover that decision vector w is a linear sum of the samples of the dataset D. Replacing w from 4.6 to 4.5:

$$L = \frac{1}{2} \left(\sum_{i=1}^{N} a_i y_i x_i \right) \cdot \left(\sum_{j=1}^{N} a_j y_j x_j \right) - \sum_{i=1}^{N} a_i y_i x_i \cdot \left(\sum_{j=1}^{N} a_j y_j x_j \right) - \sum_{i=1}^{N} a_i y_i b + \sum_{i=1}^{N} a_i \quad (4.8)$$

From 4.7:

$$L = \frac{1}{2} \left(\sum_{i=1}^{N} a_i y_i x_i \right) \cdot \left(\sum_{j=1}^{N} a_j y_j x_j \right) - \sum_{i=1}^{N} a_i y_i x_i \cdot \left(\sum_{j=1}^{N} a_j y_j x_j \right) - b \sum_{i=1}^{N} a_i y_i + \sum_{i=1}^{N} a_i \right)$$

$$L = \frac{1}{2} \left(\sum_{i=1}^{N} a_i y_i x_i \right) \cdot \left(\sum_{j=1}^{N} a_j y_j x_j \right) - \sum_{i=1}^{N} a_i y_i x_i \cdot \left(\sum_{j=1}^{N} a_j y_j x_j \right) + \sum a_i \right)$$

$$L = \sum_{i=1}^{N} a_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j y_i y_j x_i \cdot x_j \Longrightarrow$$

$$\max_{a} \sum_{i=1}^{N} a_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j y_i y_j x_i \cdot x_j \Longrightarrow$$

$$(4.9)$$

This equation manifests that the maximization depends only on the dot product of pair of samples $x_i \cdot x_j$.

To defining support vector machines the following KKT condition holds:

for
$$i = 1, \dots, N$$

 $a_i (y_n (w^T x_i + b) - 1) = 0$ (4.11)

This equation implies that either a_i or slack is zero. If the slack is positive which means that we refer to external points - the points not touching the margin - a_i is definitely zero. That is the reason that many alphas have zero value and therefore every point which has $a_i > 0$ is a **support vector** x_i (**SV**).

Finally we compute the alphas and from 4.6 hence get the w.

$$w = \sum_{i=1}^{N} a_i y_i x_i \tag{4.12}$$

and then b from

$$y_i \left(w^T x_i + b \right) = 1 \tag{4.13}$$

using any SV.

As stated before, unfortunately, these support vectors are for linear separable data. When a dataset is not linear separable we have to move to a higher dimensional space, call it z. As we see from 4.9 that working in a higher dimensional space will only affect $x_i \cdot x_j$. So Lagrangian becomes:

$$\max_{a} \sum a_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j y_i y_j z_i \cdot z_j$$
 (4.14)

When we solve that for z space we observe that the alphas we get are the same as before. They relate only to the dataset we have, so it does not affect from the dimensionality of the problem we are trying to solve. Support vectors in x space in this case are just "pre-images" of support vectors in z. The distance between the svs and the surface we get in the x space is no longer the margin we had before as this margin now is maintained in Z space.

4.1 Polynomial kernel

Although we need to compute the dot product of $z_i \cdot z_j$ in z space. So given two points x and $x' \in X$ we need $z^T z'$. Let:

$$z^T z' = K(x, x') \tag{4.15}$$

This function K is the **kernel** and is correspond to some space z.As an example to illustrate this notion, let have x being two dimensional: $x = (x_1, x_2)$ and we use a non linear transformation, a second order polynomial Φ

$$z = \Phi(x) = (1, x_1, x_2, x_1^2, x_2^2, x_1 x_2)$$

and therefore to get the kernel K:

$$K(x,x') = z^T z' = 1 + x_1 x_1' + x_2 x_2' + x_1^2 x_1'^2 + x_2^2 x_2'^2 + x_1 x_1' x_2 x_2'$$

which shows that kernel K is just a function of only x and x'. Although what we want is to compute this function K without transforming the x and x'. Improvising the kernel K(x,x') to be:

$$K(x,x') = (1+x^Tx')^2 = (1+x_1x_1'+x_2x_2')^2 = 1+x_1^2x_1'^2+x_2^2x_2'^2+2x_1x_1'+2x_2x_2'+2x_1x_1'x_2x_2'$$

$$(4.16)$$

which looks like an inner product not considering the twos. The transformation to the space that make this an inner product is:

$$\left(1, x_1^2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1x_2\right)$$

$$\left(1, x_1'^2, x_2'^2, \sqrt{2}x_1', \sqrt{2}x_2', \sqrt{2}x_1'x_2'\right)$$

which is the space z. If instead of two order we had for example a 200 order polynomial, this is a tremendous accomplishment as we can solve this in our input space and never do the actual transformation. This called the **polynomial kernel** and its general form given $X = \mathbb{R}^d$ and $\Phi: X \to \mathbb{Z}$, a polynomial of order Q is:

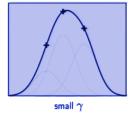
$$K(x, x') = (ax^{T}x' + b)^{Q}$$
(4.17)

4.2 Radial Basis Function Kernel (RBF)

In the polynomial kernel we manage to solve a problem to our input space knowing that a different dimension z existed and have shown that that solution exists if K(x,x') is an inner product in some space z. Lets now consider a different kernel:

$$K(x, x') = \exp(-\gamma ||x - x'||^2)$$
 (4.18)

The choice of gamma affects the sensitivity as shown in figure 4.3



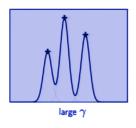


Figure 4.3: Gamma selection

We apply this kernel to one dimensional space so x and x' are scalars:

$$K(x, x') = \exp(-(x - x')^2)$$

where $\gamma = 1$

Expressing this using Taylor series we have:

$$K(x, x') = \exp(-x^{2}) \exp(-x'^{2}) \exp(2xx') =$$

$$\exp(-x^{2}) \exp(-x'^{2}) \sum_{k=0}^{\infty} \frac{2k (x)^{k} (x')^{k}}{k!} =$$

$$\exp(-\frac{1}{2}||x^{2}||) \exp(-\frac{1}{2}||x'^{2}||) \sum_{k=0}^{\infty} \frac{(x^{T}x')^{k}}{k!}$$
(4.19)

which indeed is an inner product of x and x' in infinite space and is named Radial Basis Function Kernel (RBF).

4.3 Other kernels

There are many equations that can serve as a kernel function which are commonly used in machine learning. A function considered to be a valid kernel iff:

- It is symmetric (inner product is commutative) and
- Must be compliant with Mercers Law condition Mercers law dictates that the matrix:

$$\begin{bmatrix} K(x_1, x_1) & K(x_1, x_2) & \dots & K(x_1, x_i) \\ K(x_2, x_1) & K(x_2, x_2) & \dots & K(x_2, x_i) \\ \vdots & \vdots & \vdots & \vdots \\ K(x_i, x_1) & K(x_i, x_2) & \dots & K(x_i, x_i) \end{bmatrix}$$

is semi-positive for any x_1, \dots, x_i .

The most widely used is the one described in the previous kernel, the RBF kernel. Other known kernels are:

- No Kernel or "Linear Kernel": $x \cdot x'$
- Sigmoid: $tanh(\gamma x \cdot x' + b)$
- Polynomial: $(\gamma * x \cdots x' + b)^Q$

Some other not so widely used esoteric kernels are:

- String Kernel
- Chi-square kernel
- Histogram Intersection Kernel

5 Results

The mammogram computed aided diagnosis is implemented in Arch Linux operating system using MATLAB R2014a with matlab statistical and image tools, on a desktop computer of Intel(R) Core(TM)2 CPU 6600 @ 2.40GHz with 6Gb of RAM.

For our purposes the MIAS[1] dataset have been used for feature extraction. The dataset is parted by 322 mammograms 1024 x 1024 each, and contained information of the exact location of the mass given x,y and mass radius as shown in table 5.

FFatty Character of background tissue Fatty-glandular GDense-glandular DCalcification CALCWell-defined/circumscribed masses CIRCSPICSpiculated masses Class of abnormality present MISCOther, ill-defined masses ARCHArchitectural distortion ASYMAsymmetry Normal NORMBBenign Severity of abnormality Malignant Mx,y image-coordinates of centre of abnormality. Approximate radius (in pixels) of a circle enclosing the abnormality.

Table 5: MIAS dataset information

The list of mammograms is arranged in pairs of films where each pair represents the left (even filename numbers) and right mammograms (odd filename numbers) of a single patient. x, y and radius coordinates have been given for mammograms with abnormalities. When calcifications are present, center locations and radius applied to clusters rather than individual calcifications.

Coordinate system origin is the bottom-left corner which is not the same matlab use. Hence for every processed image we did a transpose in y dimension so we can take the correct region of interest based on x,y and radius. In normal images and in some images where calcifications are widely distributed throughout the image, the mass center and radius have been omitted. Therefore we extracted fake regions of interests and seek for feature extraction on them as processing the whole image would be time consuming.

To accomplish that for every image we have extracted the number from the name and based on if was odd or even we find the mean x y and radius for left and right mammograms explicitly (Algorithm 3).

We have extracted 61 features in total, using the two algorithms described. The data stored in six different mat files. In the first three files we deliberately ignored the normal mammograms and stored only the benign and malignant using labels 1 and -1 accordingly

Algorithm 3 Mean x,y,radius algorithm

```
1: procedure MEANXYRADIUS(imagesInfoFromDataset)
      xeven = Array()
2:
      xodd = Array()
3:
      yeven = Array()
4:
5:
      yodd = Array()
      radiuseven = Array()
6:
      radiusodd = Array()
7:
      for each info in imagesInfoFromDataset do
8:
          get x info
9:
          get y info
10:
          get radius info
11:
12:
          if x and y and radius then
             num = get number from image name // HINT: \d^* regular expr.
13:
             if num % 2 then
14:
                Add x to xodd
15:
                Add y to yodd
16:
                Add radius to radiusodd
17:
             else
18:
                Add x to xeven
19:
                Add y to yeven
20:
                Add radius to radiuseven
21:
             end if
22:
          end if
23:
      end for
24:
      meanxeven = sum(xeven) / length(xeven)
25:
26:
      meanxodd = sum(xodd) / length(xodd)
      meanyeven = sum(yeven) / length(yeven)
27:
      meanyodd = sum(yodd) / length(yodd)
28:
      mean radius even = sum(radius even) / length(radius even)
29:
      meanradiusodd = sum(radiusodd) / length(radiusodd)
30:
      means = Array(meanxeven,meanxodd,meanyeven,meanyodd,meanradiuseven,
31:
32: meanradiusodd)
33:
      return means
34: end procedure
```

2

0

1

2

0

1

2

1

-1

1

-1

1

-1

benign

normal

malignant

benign

normal

malignant

benign

malignant

benign

malignant

benign

malignant

benign

for RDM, SGLDM and both of them. In the next three we stored normal mammograms too using labels 0,1 and 2 as shown in Table 6.

Table 6: Datasets

Labels Filename normal SGLDM.matmalignant

RDM.mat

all Features. mat

SGLDMmb.mat

RDMmb.mat

all Features mb. mat

0 1

Therefore x, y and radius of each image is used for either left or right mammogram, by using them to create a rectangle area around the circle formed by these coordinates with the following properties:

- side = 2 * radius
- $Upper\ x\ coordinate = x$ -radius
- $Upper\ y\ coordinate = y$ -radius

Each image has been rotated and cropped to form the eight squares each of the algorithm needs. A matter of importance here is, that the rotation has been made based on the x,y and not the center of the image as this gave us wrong roi results (Figure 5.1).

Instead of matlab's built in sym library we used libsym[4] a sym library developed by Chih-Chung Chang and Chih-Jen Lin. In our experiments, we use 10 fold cross-validation, by which the data set is divided into 10 subsets. One of them is used as a test and the remaining subsets are used for training. For each fold, each train set divided to 10 other folds to get the best parameters for every used kernel.

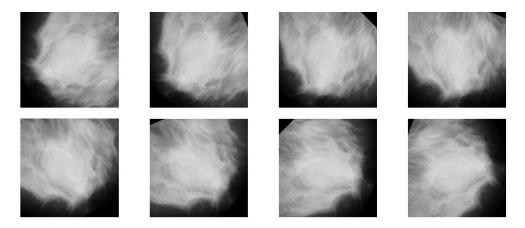


Figure 5.1: Example of extracted and rotated roi for Θ $\epsilon \left\{0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}\right\}$

5.1 Complete Set

Firstly we train our SVM using all the three labels. Due to the fact, that the number of classes are more than one we are using two different ways to classify our data. The one vs one or all pairs SVM, which is the standard way libsym classify multi class datasets, and one vs all.

In one vs all approach we break the k-class problem into k binary problems and solve separately for each one of them. We create k classifiers one per class, creating k hypothesis. For every new data we decide based on the output of these classifiers, where if more than one classify it as positive we take the highest confidence.

At all pairs, a method discovered by Friedman and Hastie & Tibshirani we create one binary problem for each pairs of classes therefore we have $n \times k^2$ pairs of them and in testing we take a vote for the best classified label. At table 7 we have the big o comparison for OVA and all-pairs during the training and testing assuming training time is $O(m^a)$ and testing time is $O(c_t)$.

Table 7: All-pairs and OVA comparison

5.1.1 RBF Kernel

One vs one: First we implement RBF kernel classification. Data normalized between -1 and 1 as this produced better results. We split the dataset in 10 folds and for each one we search for optimum parameters C and gamma using the -v parameter of the libsym library. The dataset contained 207 normal, 54 malignant and 69 benign left and right mammograms. C parameter search space was from 2⁻⁵ and 2¹⁵ and gamma's was between 2⁻¹⁵ and 2³. We selected the best one of them by selecting this pair maximizing the training accuracy. Because our data were not balanced and this could cause many false positives during the training we tried to solve this issue by specifying different weights for each class getting the proper ratio for each one.

An example of these weights is shown in table 8

Table 8: Weights for avoiding misclassification

Normal weight	0.62626
Benign weight	0.20875
Malignant weight	0.16498

We use the parameters to train the model with the train data and validate with the test data we excluded for each fold.

To plot the test data and the labels assigned from the validation stage we used the pairwise euclidean distance for each observation and the classical (metric) multidimensional scaling to reduce the dimensionality of the dataset. Some examples are in Figures 5.2 and 5.3 The filled markers represent data instance from the test set, and filled color represents the class label assigned by SVM whereas the edge color represents the true (ground-truth) label. The marker size of the test set represents the probability that the sample instance is assigned with its corresponding class label; the bigger, the more confidence.

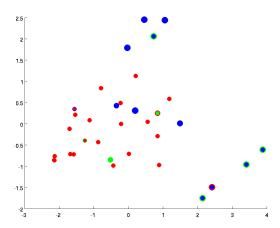


Figure 5.2: All features labels for fold n = 1 - rbf kernel

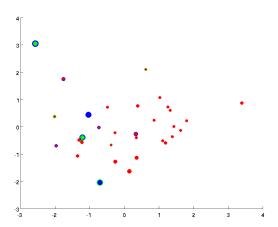


Figure 5.3: All features labels for fold n = 8 - rbf kernel

Also for each fold we plot the predicted and true classes for a visual comparison of the two.(Figures 5.4,5.5)

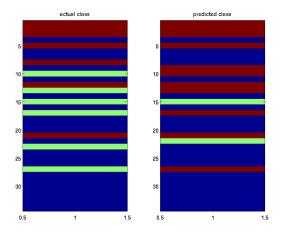


Figure 5.4: all features label comparison for fold n = 1 - rbf kernel

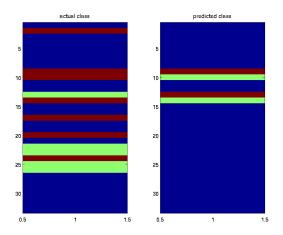


Figure 5.5: all features label comparison for fold n = 8 - rbf kernel

Finally we get the confusion matrices for each fold (Tables 9, 10)

Table 9: Confusion matrix for fold n = 3 - rbf kernel

	Normal	Malignant	Benign
Normal	21	0	0
Malignant	1	5	4
Benign	4	0	3

Table 10: Confusion matrix for fold n=10 - rbf kernel

	Normal	Malignant	Benign
Normal	21	0	0
Malignant	0	1	3
Benign	2	1	4

and we have extracted the total mean accuracy and the mean accuracies for each class (Table 11). As we discovered the classification accuracy for malignant class (cancerous mammograms) is not good at all.

Table 11: Mean accuracies for all features - rbf kernel

Normal	Malignant	Benign	Total
97.55%	18.66%	59.76%	76.65%

Afterwards we run 10 fold classification with *one vs all* approach. For each fold we search for the best c and gamma as before and we plot the test set and the assigned labels, after reducing dataset dimensionality (Figure 5.6). Also a visual comparison for predicted labels versus actual labels for fold 2 is shown in Figure 5.7. Finally the mean accuracy table is shown below in Table 12.

Table 12: Mean accuracies for all features - rbf kernel OVA

Normal	Malignant	Benign	Total
99.05%	20%	59.29%	77.88%

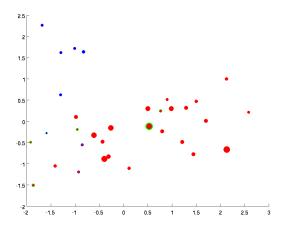


Figure 5.6: All features for fold n = 10 - rbf kernel OVA

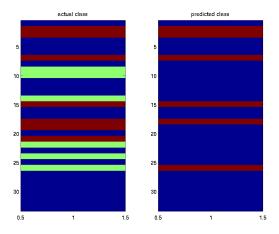


Figure 5.7: All features label comparison for fold n = 2 - rbf kernel OVA

In comparison with one vs one, we had some improvement. The malignant accuracy increased about 2 percent, and overall accuracy increased about 1 percent.

As one vs all was not much helpful, in an effort to eliminate the misclassification in malignant mammograms we repeat the training, but this time we keep only the malignant and benign mammograms.

Dataset included 54 cancer (malignant) and 69 non cancer cases (benign).

Once again the dataset was divided in 10 random created folds and some training and testing dataset were created. We calculated the best gamma and c for each fold and trained the SVM. With the model we obtained and plotted the testing data and assigned labels as before as shown in Figures 5.8, 5.9:

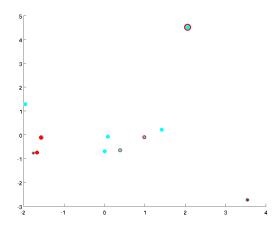


Figure 5.8: All features with only benign and malignant labels for fold n = 6 - rbf kernel

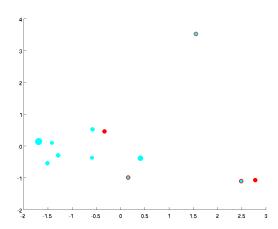


Figure 5.9: All features with only benign and malignant labels for fold n = 9 - rbf kernel

Below is a visual comparison of the true and predicted labels for fold 9. (Figure 5.10)

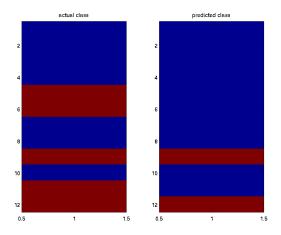


Figure 5.10: Label comparison for fold n = 9 - malignant & benign - rbf kernel

Finally we get the confusion matrices and calculated the mean accuracy for benign and malignant labels.(Table 13)

Table 13: Mean accuracies for all features - only benign and malignant - rbf kernel

Malignant	Benign	Total
84.05%	25%	57.88%

The overall classification accuracy was 57.88%, a drop around 20% than before. Although the malignant accuracy increased significantly from 20% to 84%.

5.1.2 Polynomial Kernel

Another widely used kernel is the polynomial. Polynomial kernel unlike rbf has a parameter Q which represents the order of the polynomial. Before training we searched for the best order Q, from values 1 to 11. An example of best found parameters are the following.

Table 14: Best values for fold n = 3 - polynomial kernel

Best Q	2
Best gamma	0.313

As with rbf we did a 10 fold classification. Some examples from the assigned labels shown in Figures 5.11 and 5.12.

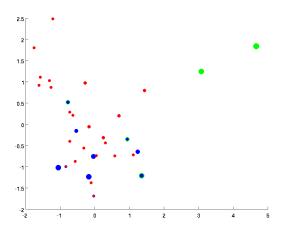


Figure 5.11: All features for fold n=1 - polynomial kernel

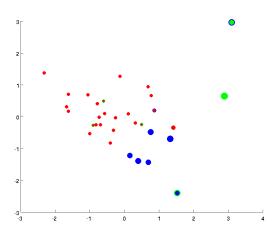


Figure 5.12: All features for fold n=3 - polynomial kernel

and the predicted and true classes for a visual comparison in Figures 5.13 and 5.14.

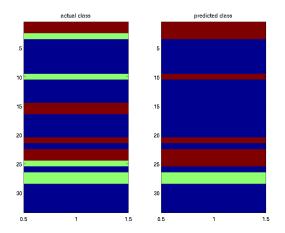


Figure 5.13: all features label comparison for fold n = 1 - polynomial kernel

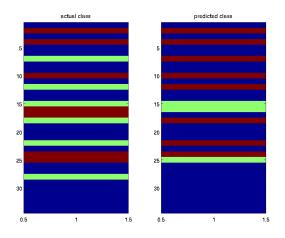


Figure 5.14: all features label comparison for fold n = 3 - polynomial kernel

Total mean accuracy and mean accuracies for the classes shown in Table 15.Once again the classification accuracy for malignant class (cancerous mammograms) is not good but it's a little better than rbf.

Table 15: Mean accuracies for all features - polynomial kernel

Normal	Malignant	Benign	Total
98.55%	20.33%	62.14%	78.16%

As seen the mean accuracy is about the same as the rbf kernel.

We also run 10 fold classification using *one vs all*. For each fold we search for the best order q and gamma and we plotted testing labels, after reducing dataset dimensionality (Figure 5.15). Also a visual comparison for predicted labels versus actual labels for fold 4 is shown in Figure 5.16. Finally the mean accuracy table is shown below in Table 16.

Table 16: Mean accuracies for all features - polynomial kernel OVA

Normal	Malignant	Benign	Total
99.02%	19.66%	53.80%	76.63%

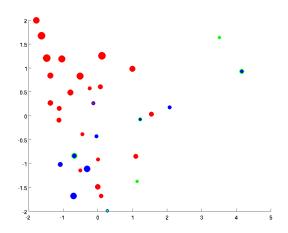


Figure 5.15: All features for fold n = 4 - polynomial kernel OVA

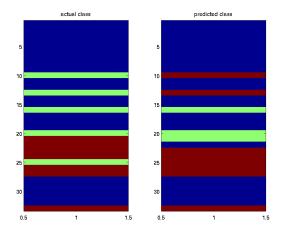


Figure 5.16: Label comparison all features for fold n = 4 - polynomial kernel OVA

The total accuracy is about the same with rbf here too. We had a drop around 1 percent which may be a statistical error. Once again we did the training and testing with only malignant and benign mammograms. Examples of assigned labels are shown in Figures 5.17 and 5.18

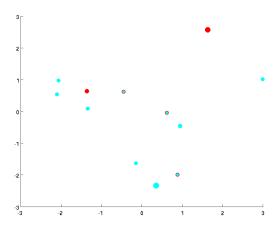


Figure 5.17: All features with only benign and malignant labels for fold n=8 - polynomial kernel

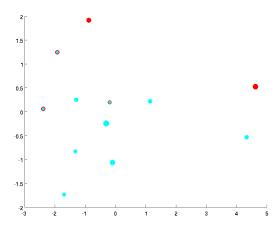


Figure 5.18: All features with only benign and malignant labels for fold n=10 - polynomial kernel

and a visual comparison for true and predicted labels for fold 8 is shown in Figure 5.19)

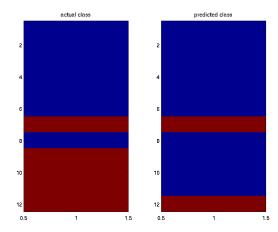


Figure 5.19: Label comparison for fold n = 8 - malignant & benign - polynomial kernel

Finally the mean accuracy for benign and malignant labels is shown in Table 17)

Table 17: Mean accuracies for all features - only benign and malignant - polynomial kernel

Malignant	Benign	Total
85.23%	31.66%	61.79%

In comparison with all features with normal mammograms included, we have loss in total accuracy by around 10%. Therefore in compare with rbf we gain an increment in total accuracy by 10% and we did better for benign mammograms about 5%.

5.1.3 Sigmoid Kernel

Sigmoid kernel is also a well known kernel, used a lot in neural networks. The steps we followed are the same as in the previous kernels. We first tested with one vs one and one vs all and then with only malignants and benign. For convinience we will skip repeating those steps and just show the results.

One vs one Testing dataset with assigned labels in Figures 5.20 and 5.21:

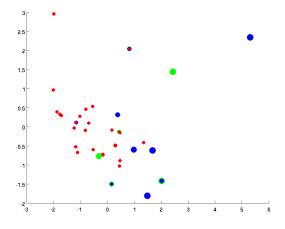


Figure 5.20: All features for fold n = 1 - sigmoid kernel

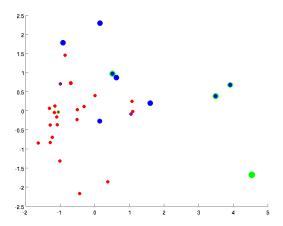


Figure 5.21: All features for fold n = 6 - sigmoid kernel

Predicted and true classes for a visual comparison in Figures 5.22 and 5.23.

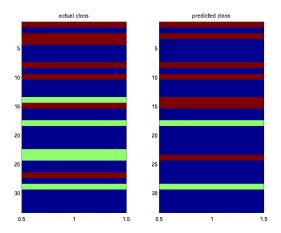


Figure 5.22: all features label comparison for fold n = 1 - sigmoid kernel

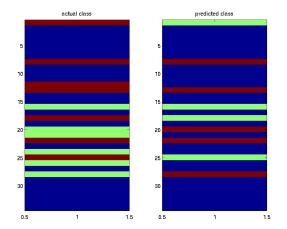


Figure 5.23: all features label comparison for fold n=6 - sigmoid kernel

mean accuracies in Table 18

Table 18: Mean accuracies for all features - sigmoid kernel

Normal	Malignant	Benign	Total
97.59%	16.66%	65.23%	77.60%

Once again no significant loss or gain was observed.

One vs all Testing dataset with assigned labels in Figures 5.24 and 5.25:

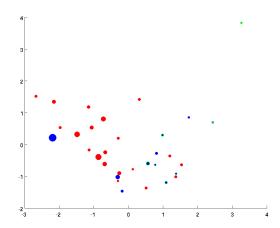


Figure 5.24: All features for fold n=2 - sigmoid kernel OVA

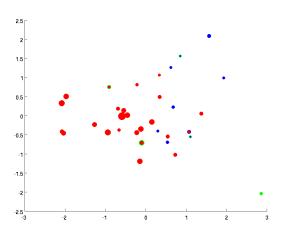


Figure 5.25: All features for fold n=3 -sigmoid kernel OVA

Predicted and true classes for a visual comparison in Figures 5.26 and 5.27.

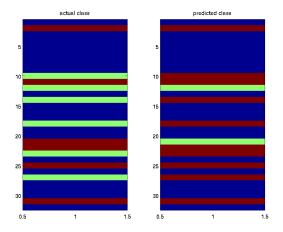


Figure 5.26: all features label comparison for fold n = 2 - sigmoid kernel OVA

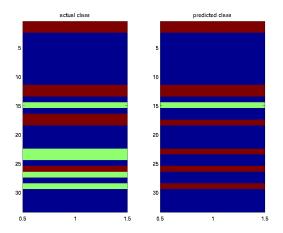


Figure 5.27: all features label comparison for fold n = 3 - sigmoid kernel OVA

Mean accuracies in Table 19

Table 19: Mean accuracies for all features - sigmoid kernel OVA

Normal	Malignant	Benign	Total
98.54%	16.66%	61.19%	77.30%

Comparing to one vs one, we had about the same results. Therefore comparing to polynomial kernel we had an increment about 1%.

All features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.28 and 5.29:

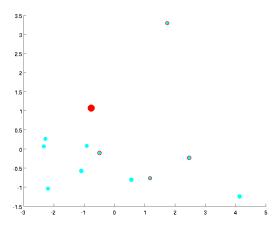


Figure 5.28: All features for fold n=3 - only benign and malignant - sigmoid kernel

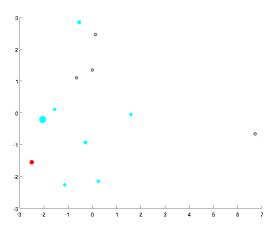


Figure 5.29: All features for fold n = 5 -sigmoid kernel

Predicted and true classes for a visual comparison in Figures 5.30 and 5.31.

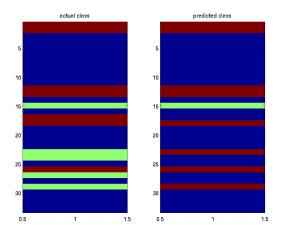


Figure 5.30: all features label comparison for fold n=3 - only benign and malignant - sigmoid kernel

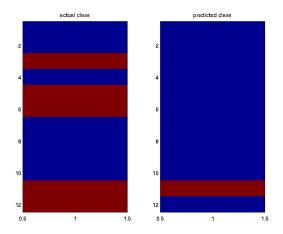


Figure 5.31: all features label comparison for fold n=5 - only benign and malignant - sigmoid kernel

and mean accuracies in Table 20

Table 20: Mean accuracies for all features - only benign and malignant - sigmoid kernel

Malignant	Benign	Total
85.71%	24.66%	58.73%

The total accuracy in sigmoid kernel is the lowest we seen so far, around 59%.

5.1.4 Linear Kernel

We hoped to get better results with the linear kernel as in some cases where dimensionality is high and no much data are given, the linear kernel approach may actually perform better than various other kernels. The procedure we follow is the same as before so we will just shown the produced results.

One vs one Testing dataset with assigned labels in Figures 5.32 and 5.33:

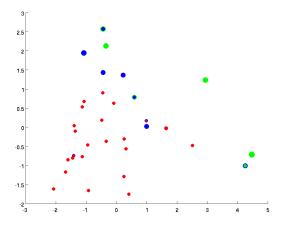


Figure 5.32: All features for fold n = 1 - linear kernel

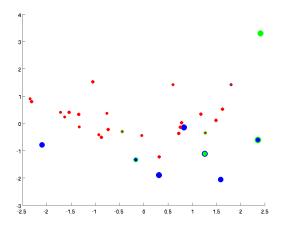


Figure 5.33: All features for fold n = 8 - linear kernel

Predicted and true classes for a visual comparison in Figures 5.34 and 5.35.

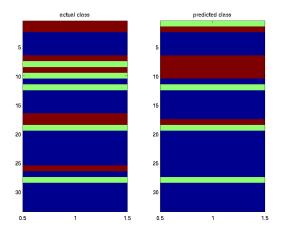


Figure 5.34: all features label comparison for fold n=1 - linear kernel

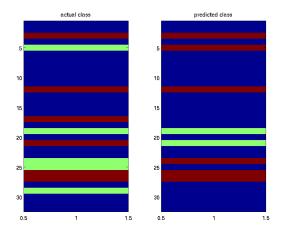


Figure 5.35: all features label comparison for fold n=8 - linear kernel

mean accuracies in Table 21

Table 21: Mean accuracies for all features - linear kernel

Normal	Malignant	Benign	Total
98.54%	24.66%	62.38%	78.81%

Indeed we got a very good accuracy, around 79%. Malignant percent is still pretty low, around 25%.

One vs all Testing dataset with assigned labels in Figures 5.36 and 5.37:

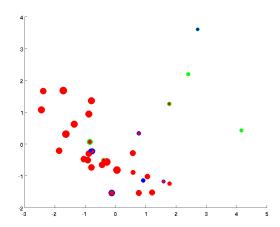


Figure 5.36: All features for fold n = 1 - linear kernel OVA

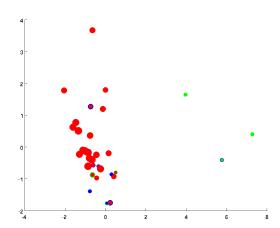


Figure 5.37: All features for fold n = 2-linear kernel OVA

Predicted and true classes for a visual comparison in Figures 5.38 and 5.39.

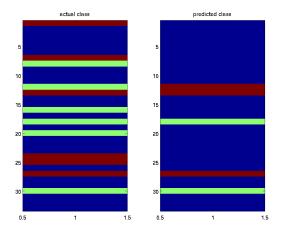


Figure 5.38: all features label comparison for fold n = 1 - linear kernel OVA

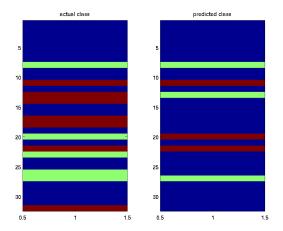


Figure 5.39: all features label comparison for fold n=2 - linear kernel OVA

and mean accuracies in Table 22

Table 22: Mean accuracies for all features - linear kernel OVA

Normal	Malignant	Benign	Total
100%	10.66%	29.04%	70.62%

We did worst than one vs one but we get 100% accuracy in normal images. Malignant mean accuracy dropped.

All features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.40 and 5.41:

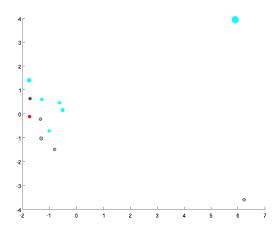


Figure 5.40: All features for fold n=4 - only benign and malignant - linear kernel

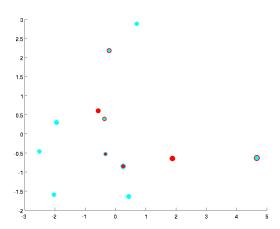


Figure 5.41: All features for fold n = 6-linear kernel

Predicted and true classes for a visual comparison in Figures 5.42 and 5.43.

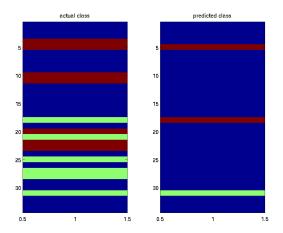


Figure 5.42: all features label comparison for fold n=4 - only benign and malignant - linear kernel

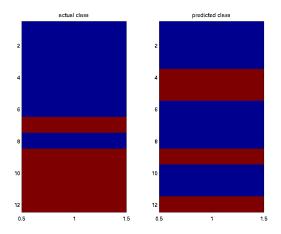


Figure 5.43: all features label comparison for fold n=6 - only benign and malignant - linear kernel

mean accuracies in Table 23

Table 23: Mean accuracies for all features - only benign and malignant - linear kernel

Malignant	Benign	Total
81.42%	22%	55.59%

Keeping only malignant and benign mammograms did worst than keeping all features. Mean accuracy dropped at 55%.

5.2 SGLDM Data Set

SGLCM dataset contained only the 61 features from the SGLDM algorithm. Because we had a non well distributed dataset and the number of benign mammograms was low, we tested keeping fewer features to see if that increased our accuracy. The process was the same as before, hence we will just provide the results.

5.2.1 RBF Kernel

One vs one Testing dataset with assigned labels in Figures 5.44 and 5.45:

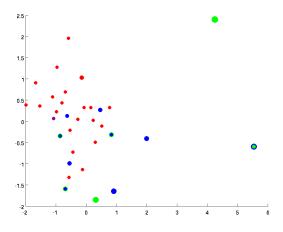


Figure 5.44: SGLDM labels for fold n = 3 - rbf kernel

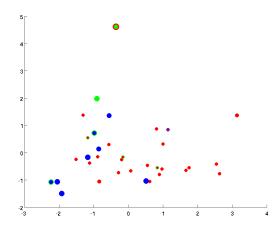


Figure 5.45: SGLDM labels for fold n = 4 - rbf kernel

Predicted and true classes for a visual comparison in Figures 5.46 and 5.47.

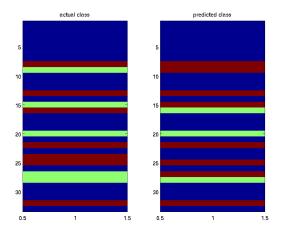


Figure 5.46: SGLDM features label comparison for fold n=3 - rbf kernel

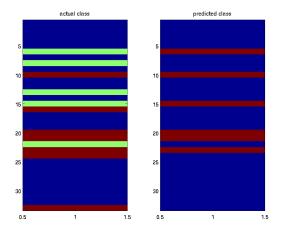


Figure 5.47: SGLDM features label comparison for fold n=4 - linear kernel mean accuracies in Table 24

Table 24: Mean accuracies for SGLDM dataset - rbf kernel

Normal	Malignant	Benign	Total
98.54%	17.66%	59.52%	76.94%

The mean accuracy is same as rbf. We haven't see any significant improvements. **One vs all** Testing dataset with assigned labels in Figures 5.48 and 5.49:

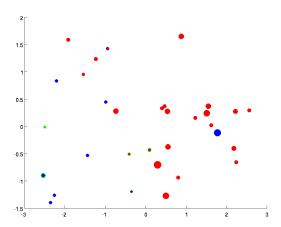


Figure 5.48: SGLDM for fold n=5 - rbf kernel OVA

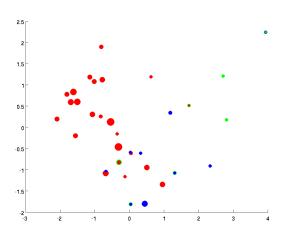


Figure 5.49: SGLDM for fold n=7 - rbf kernel OVA

Predicted and true classes for a visual comparison in Figures 5.50 and 5.51.

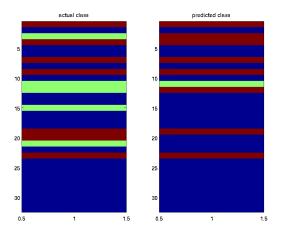


Figure 5.50: SGLDM label comparison for fold n=5 - rbf kernel OVA

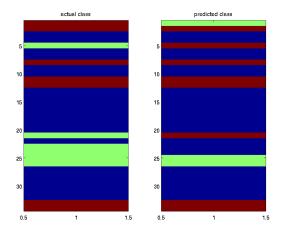


Figure 5.51: SGLDM features label comparison for fold n = 7 - rbf kernel OVA

and mean accuracies in Table 25

Table 25: Mean accuracies for SGLDM dataset - rbf kernel OVA

Normal	Malignant	Benign	Total
98.54%	14.66%	59.52%	76.71%

SGLDM features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.52 and 5.53:

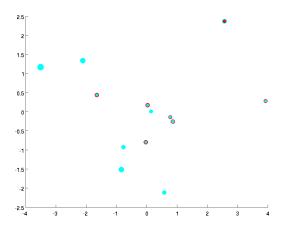


Figure 5.52: SGLDM features for fold n = 4 - only benign and malignant - rbf kernel

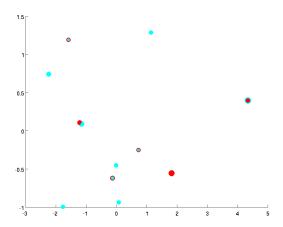


Figure 5.53: SGLDM features for fold n=8 - only benign and malignant -rbf kernel

Predicted and true classes for a visual comparison in Figures 5.54 and 5.55.

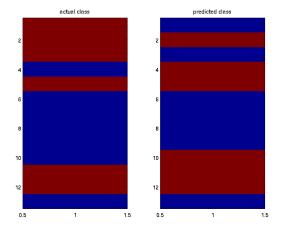


Figure 5.54: SGLDM features label comparison for fold n=4 - only benign and malignant - rbf kernel

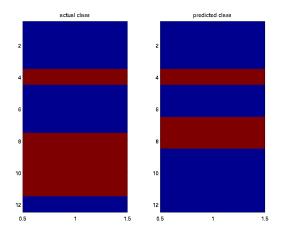


Figure 5.55: SGLDM features label comparison for fold n=8 - only benign and malignant - rbf kernel

mean accuracies in Table 26

Table 26: Mean accuracies for all features - only benign and malignant - rbf kernel

Malignant	Benign	Total
76.66%	42.66%	61.86%

5.2.2 Polynomial Kernel

One vs one Examples from the assigned labels shown in Figures 5.56 and 5.57.

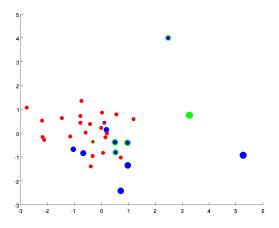


Figure 5.56: SGLDM features for fold n=2 - polynomial kernel

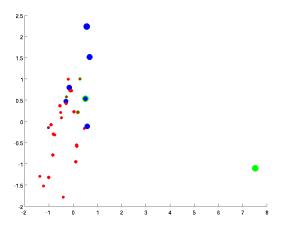


Figure 5.57: SGLDM features for fold n=4 - polynomial kernel

and the predicted and true classes for a visual comparison in Figures 5.58 and 5.59.

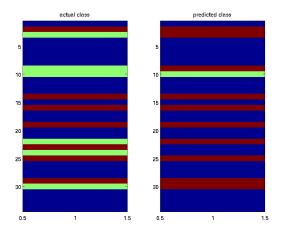


Figure 5.58: SGLDM features label comparison for fold n=2 - polynomial kernel

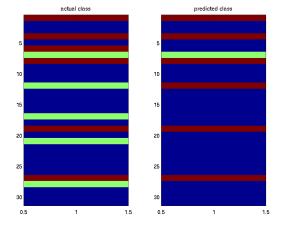


Figure 5.59: SGLDM features label comparison for fold n=4 - polynomial kernel

Total mean accuracy and mean accuracies for the classes in Table 27.

Table 27: Mean accuracies for SGLDM - polynomial kernel

Normal	Malignant	Benign	Total
98.54%	16.66%	64.04%	77.94%

One vs all Predicted labels shown in Figure 5.60 and a visual comparison for predicted labels versus actual labels in Figure 5.61. The mean accuracy table is shown below in Table 28.

Table 28: Mean accuracies for SGLDM - polynomial kernel OVA

Normal	Malignant	Benign	Total
98.54%	15.33%	63.57%	77.60%

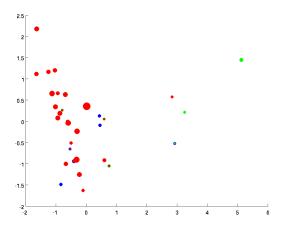


Figure 5.60: SGLDM for fold n = 1 - polynomial kernel OVA

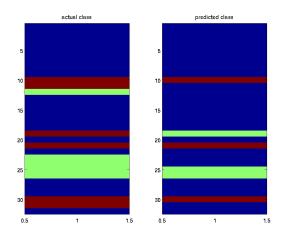


Figure 5.61: Label comparison all Features for fold n = 1 - polynomial kernel OVA

All features keeping only malignant and benign mammograms Examples of assigned labels are shown in Figures 5.62 and 5.63

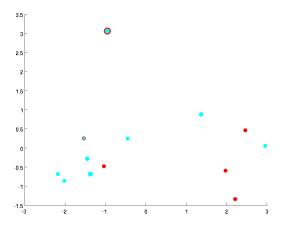


Figure 5.62: SGLDM features with only benign and malignant labels for fold n=2 -polynomial kernel

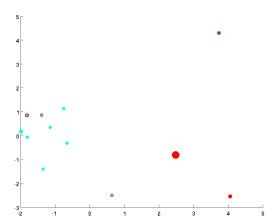


Figure 5.63: SGLDM features with only benign and malignant labels for fold n=6 -polynomial kernel

and a visual comparison for true and predicted labels for fold 2 is shown in Figure 5.64

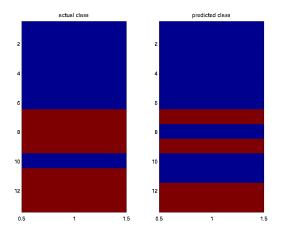


Figure 5.64: SGLDM features label comparison for fold n=2 - malignant & benign - polynomial kernel

Mean accuracy for benign and malignant labels is shown in Table 29)

Table 29: Mean accuracies for SGLDM features - only benign and malignant - polynomial kernel

Malignant	Benign	Total
81.19%	33.33%	60.19%

5.2.3 Sigmoid Kernel

One vs one Testing dataset with assigned labels in Figures 5.65 and 5.66:

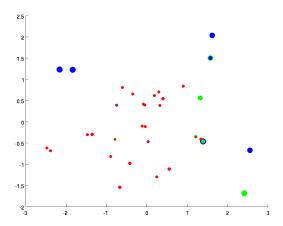


Figure 5.65: sgldm features for fold n=5 - sigmoid kernel

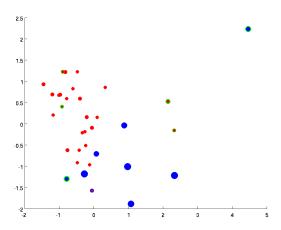


Figure 5.66: sgldm features for fold n=9 - sigmoid kernel

Predicted and true classes for a visual comparison in Figures 5.67 and 5.68.

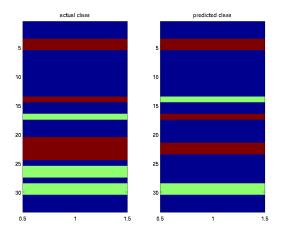


Figure 5.67: sgldm features label comparison for fold n=5 - sigmoid kernel

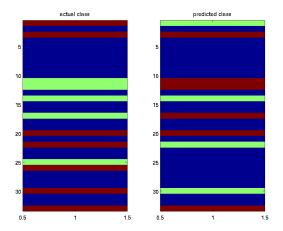


Figure 5.68: sgldm features label comparison for fold n=9 - sigmoid kernel

Table 30: Mean accuracies for sgldm features - sigmoid kernel

Normal	Malignant	Benign	Total
98.04%	16.66%	61.90%	77.25%

One vs all Testing dataset with assigned labels in Figures 5.69 and 5.70:

mean accuracies in Table 30

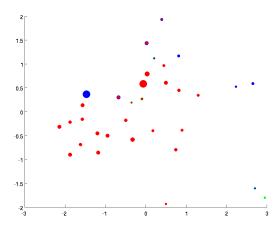


Figure 5.69: sgldm features for fold n=2 - sigmoid kernel OVA

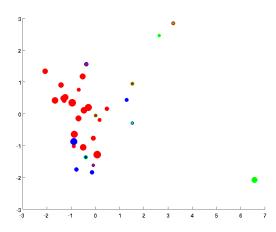


Figure 5.70: sgldm features for fold n=4-sigmoid kernel OVA

Predicted and true classes for a visual comparison in Figures 5.71 and 5.72.

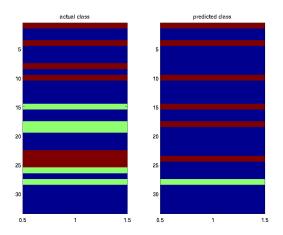


Figure 5.71: sgldm features label comparison for fold n=2 - sigmoid kernel OVA

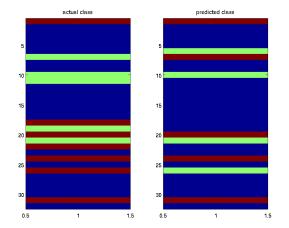


Figure 5.72: sgldm features label comparison for fold n=4 - sigmoid kernel OVA

Mean accuracies in Table 31

Table 31: Mean accuracies for sgldm features - sigmoid kernel OVA

Normal	Malignant	Benign	Total
98.07%	11.33%	60.71%	76.12%

sgldm features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.73 and 5.74:

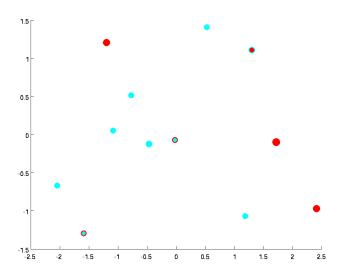


Figure 5.73: sgldm features for fold n=1 - only benign and malignant - sigmoid kernel

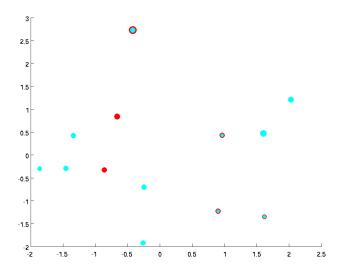


Figure 5.74: sgldm features for fold n = 6-sigmoid kernel

Predicted and true classes for a visual comparison in Figures 5.75 and 5.76.

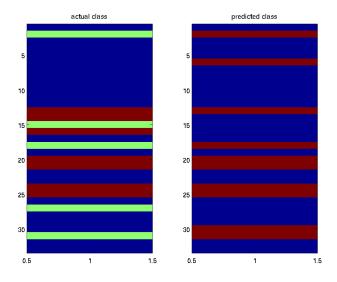


Figure 5.75: sgldm features label comparison for fold n=1 - only benign and malignant - sigmoid kernel

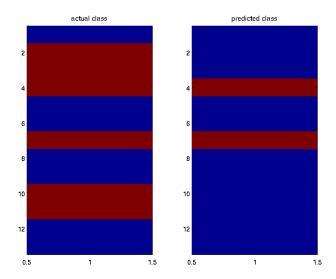


Figure 5.76: sgldm features label comparison for fold n=6 - only benign and malignant - sigmoid kernel

and mean accuracies in Table 32

Table 32: Mean accuracies for sgldm features - only benign and malignant - sigmoid kernel

Malignant	Benign	Total
81.42%	35.66%	60.97%

5.2.4 Linear Kernel

One vs one Testing dataset with assigned labels in Figures 5.77 and 5.78:

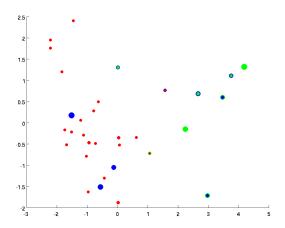


Figure 5.77: Sgldm features for fold n=2 - linear kernel

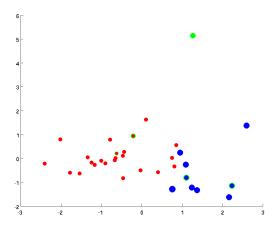


Figure 5.78: Sgldm features for fold n = 6 - linear kernel

Predicted and true classes for a visual comparison in Figures 5.79 and 5.80.

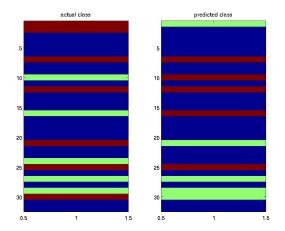


Figure 5.79: SGLDM features label comparison for fold n=2 - linear kernel

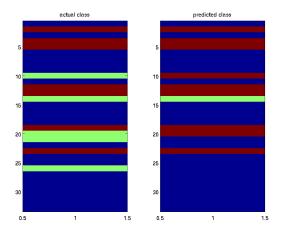


Figure 5.80: SGLDM features label comparison for fold n=6 - linear kernel mean accuracies in Table 33

Table 33: Mean accuracies for SGLDM features - linear kernel

Normal	Malignant	Benign	Total
98.09%	21.00%	57.85%	77.02%

One vs all Testing dataset with assigned labels in Figures 5.81 and 5.82:

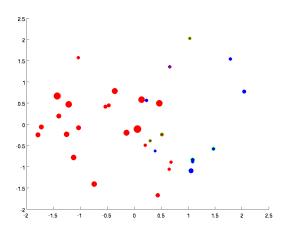


Figure 5.81: Sgldm features for fold n=3 - linear kernel OVA

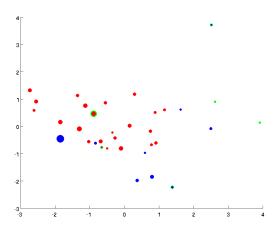


Figure 5.82: Sgldm features for fold n=10 -linear kernel OVA

Predicted and true classes for a visual comparison in Figures 5.83 and 5.84.

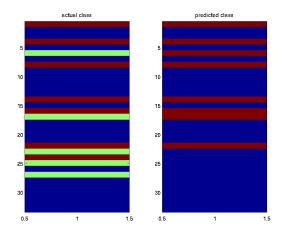


Figure 5.83: SGLDM features label comparison for fold n=3 - linear kernel OVA

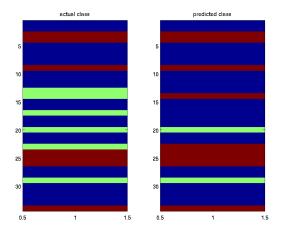


Figure 5.84: SGLDM features label comparison for fold n=10 - linear kernel OVA

and mean accuracies in Table 34

Table 34: Mean accuracies for SGLDM features - linear kernel OVA

Normal	Malignant	Benign	Total
99.02%	17%	54.76%	76.33%

Sgldm features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.85 and 5.86:

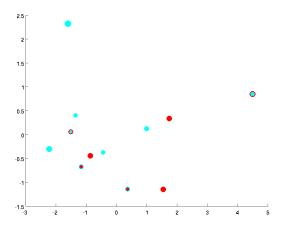


Figure 5.85: Sgldm features for fold n = 1 - only benign and malignant - linear kernel

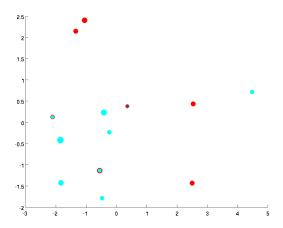


Figure 5.86: Sgldm features for fold n=3 -linear kernel

Predicted and true classes for a visual comparison in Figures 5.87 and 5.88.

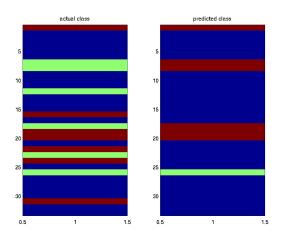


Figure 5.87: SGLDM features label comparison for fold n=1 - only benign and malignant - linear kernel

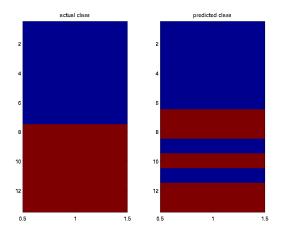


Figure 5.88: SGLDM features label comparison for fold n=3 - only benign and malignant - linear kernel

mean accuracies in Table 35

Table 35: Mean accuracies for SGLDM features - only benign and malignant - linear kernel

Malignant	Benign	Total
78.09%	31.33%	57.69%

5.3 RDM Data Set

At last we try to get better results by keeping only the features from RDM algorithm. As the number of features is only 6 we could get better results than the previous cases.

5.3.1 RBF Kernel

One vs one Testing dataset with assigned labels in Figures 5.89 and 5.90:

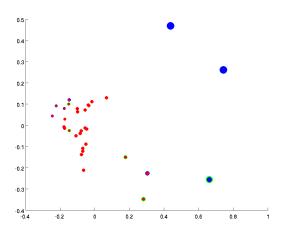


Figure 5.89: RDM labels for fold n = 5 - rbf kernel

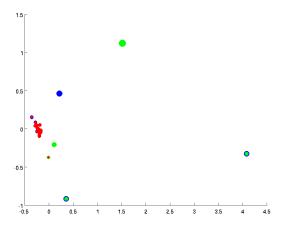


Figure 5.90: RDM labels for fold n = 8 - rbf kernel

Predicted and true classes for a visual comparison in Figures 5.91 and 5.92.

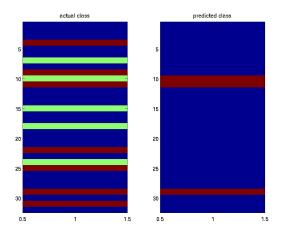


Figure 5.91: RDM label comparison for fold n=5 - rbf kernel

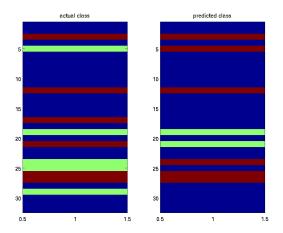


Figure 5.92: RDM label comparison for fold n = 8 - linear kernel

mean accuracies in Table 36

Table 36: Mean accuracies for RDM dataset - rbf kernel

Normal	Malignant	Benign	Total
99.04%	7.66%	10.00%	65.47%

One vs all Testing dataset with assigned labels in Figures 5.93 and 5.94:

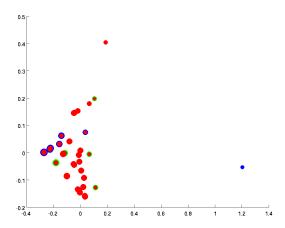


Figure 5.93: RDM for fold n=6 - rbf kernel OVA

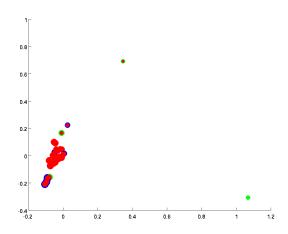


Figure 5.94: RDM for fold n=9 - rbf kernel OVA

Predicted and true classes for a visual comparison in Figures 5.95 and 5.96.

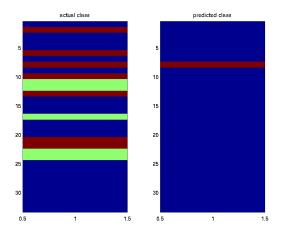


Figure 5.95: RDM label comparison for fold n=6 - rbf kernel OVA

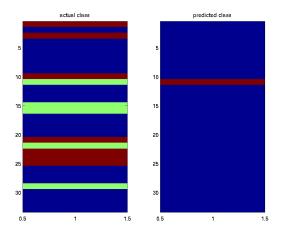


Figure 5.96: RDM label comparison for fold n = 9 - rbf kernel OVA

and mean accuracies in Table 37

Table 37: Mean accuracies for RDM dataset - rbf kernel OVA

Normal	Malignant	Benign	Total
100%	2%	4.52%	63.94%

All features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.97 and 5.98:

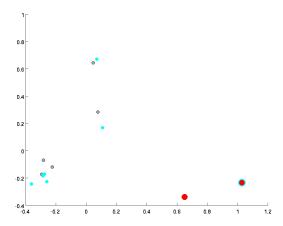


Figure 5.97: RDM features for fold n=4 - only benign and malignant - rbf kernel

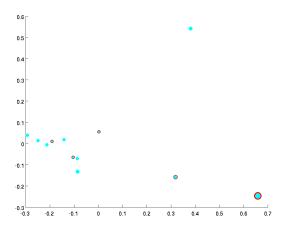


Figure 5.98: RDM features for fold n=5 -rbf kernel

Predicted and true classes for a visual comparison in Figures 5.99 and 5.100.

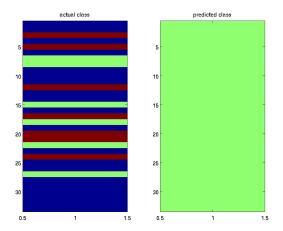


Figure 5.99: RDM features label comparison for fold $\mathbf{n}=4$ - only benign and malignant - rbf kernel

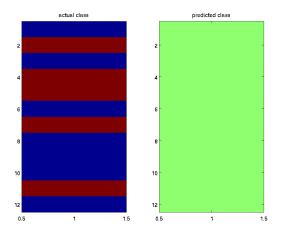


Figure 5.100: RDM features label comparison for fold n=5 - only benign and malignant - rbf kernel

mean accuracies in Table 38

Table 38: Mean accuracies for RDM features - only benign and malignant - rbf kernel

Malignant	Benign	Total
98.57%	2%	56.15%

5.3.2 Polynomial Kernel

One vs one Examples from the assigned labels shown in Figures 5.101 and 5.102.

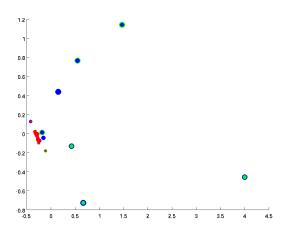


Figure 5.101: RDM for fold n = 6 - polynomial kernel

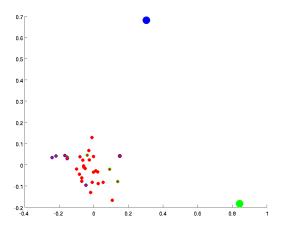


Figure 5.102: RDM for fold n = 8 - polynomial kernel

and the predicted and true classes for a visual comparison in Figures 5.103 and 5.104.

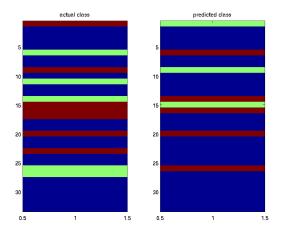


Figure 5.103: RDM label comparison for fold n = 6 - polynomial kernel

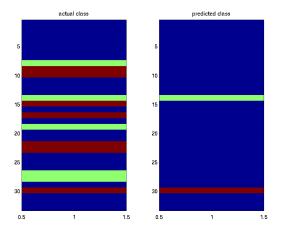


Figure 5.104: RDM label comparison for fold n = 8 - polynomial kernel

Total mean accuracy and mean accuracies for the classes in Table 39.

Table 39: Mean accuracies for RDM - polynomial kernel

Normal	Malignant	Benign	Total
99.02%	9%	8.80%	65.46%

One vs all Predicted labels shown in Figure 5.105 and a visual comparison for predicted labels versus actual labels in Figure 5.106. The mean accuracy table is shown below in Table 40.

Table 40: Mean accuracies for RDM - polynomial kernel OVA

Normal	Malignant	Benign	Total
100%	5.66%	1.42%	63.96%

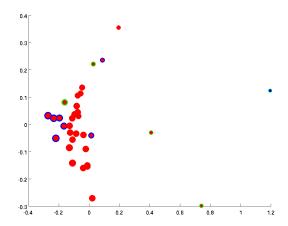


Figure 5.105: RDM for fold n=2 - polynomial kernel OVA

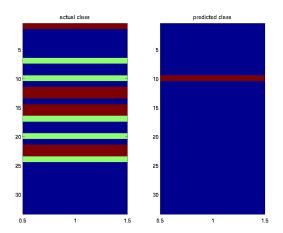


Figure 5.106: Label comparison all Features for fold n = 2 - polynomial kernel OVA

All features keeping only malignant and benign mammograms Examples of assigned labels are shown in Figures 5.107 and 5.108

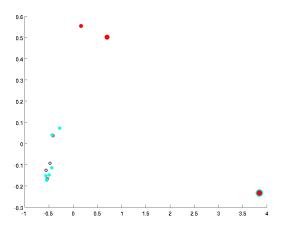


Figure 5.107: RDM with only benign and malignant labels for fold n=2 - polynomial kernel

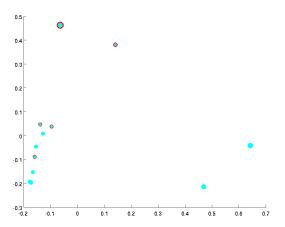


Figure 5.108: RDM with only benign and malignant labels for fold n=3 - polynomial kernel and a visual comparison for true and predicted labels for fold 2 is shown in Figure 5.109

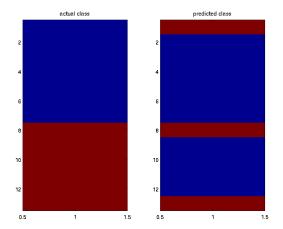


Figure 5.109: Label comparison for fold n=2 - malignant & benign - polynomial kernel Mean accuracy for benign and malignant labels is shown in Table 41)

Table 41: Mean accuracies for RDM - only benign and malignant - polynomial kernel

Malignant	Benign	Total
95.71%	3.33%	55.25%

5.3.3 Sigmoid Kernel

One vs one Testing dataset with assigned labels in Figures 5.110 and 5.111:

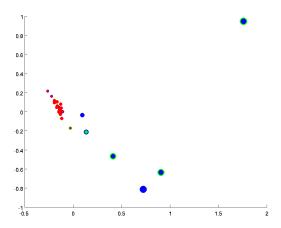


Figure 5.110: rdm features for fold n=8 - sigmoid kernel

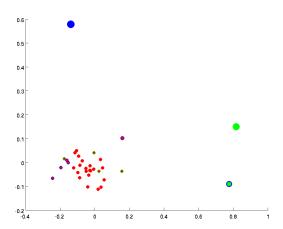


Figure 5.111: rdm features for fold n = 10 - sigmoid kernel

Predicted and true classes for a visual comparison in Figures 5.112 and 5.113.

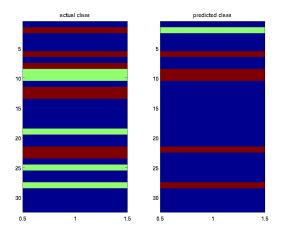


Figure 5.112: rdm features label comparison for fold n = 8 - sigmoid kernel

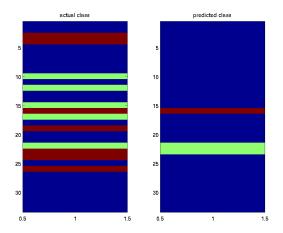


Figure 5.113: rdm features label comparison for fold n=10 - sigmoid kernel mean accuracies in Table 42

Table 42: Mean accuracies for rdm features - sigmoid kernel

Normal	Malignant	Benign	Total
99.04%	5.66%	10%	65.17%

One vs all Testing dataset with assigned labels in Figures 5.114 and 5.115:

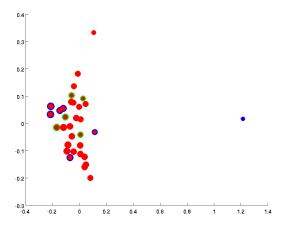


Figure 5.114: rdm features for fold n=5 - sigmoid kernel OVA

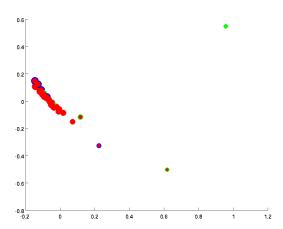


Figure 5.115: rdm features for fold n=10 -sigmoid kernel OVA

Predicted and true classes for a visual comparison in Figures 5.116 and 5.117.

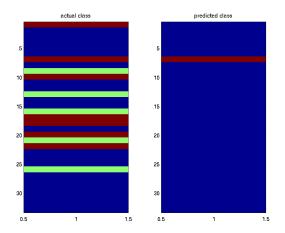


Figure 5.116: rdm features label comparison for fold n=5 - sigmoid kernel OVA

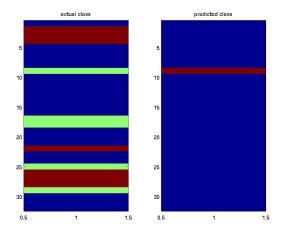


Figure 5.117: rdm features label comparison for fold n = 10 - sigmoid kernel OVA

Mean accuracies in Table 43

Table 43: Mean accuracies for rdm features - sigmoid kernel OVA

Normal	Malignant	Benign	Total
100%	2%	1.42%	63.36%

rdm features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.118 and 5.119:

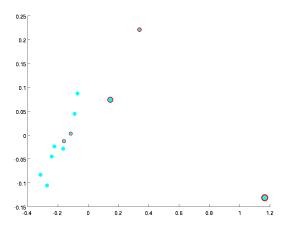


Figure 5.118: rdm features for fold n = 2 - only benign and malignant - sigmoid kernel

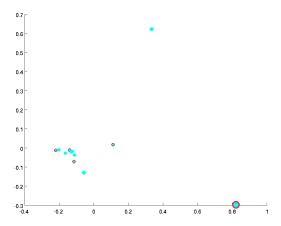


Figure 5.119: rdm features for fold n = 3-sigmoid kernel

Predicted and true classes for a visual comparison in Figures 5.120 and 5.121.

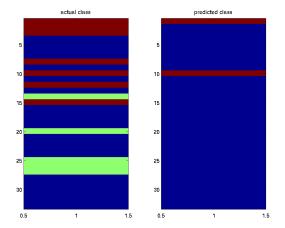


Figure 5.120: rdm features label comparison for fold n=2 - only benign and malignant - sigmoid kernel

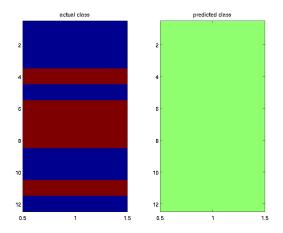


Figure 5.121: rdm features label comparison for fold n=3 - only benign and malignant - sigmoid kernel

and mean accuracies in Table 44

Table 44: Mean accuracies for rdm features - only benign and malignant - sigmoid kernel

Malignant	Benign	Total
94.28%	0%	52.88%

5.3.4 Linear Kernel

One vs one Testing dataset with assigned labels in Figures 5.122 and 5.123:

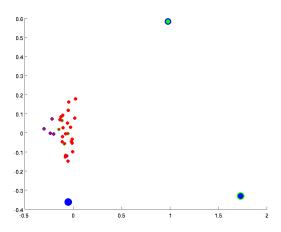


Figure 5.122: RDM features for fold n=1 - linear kernel

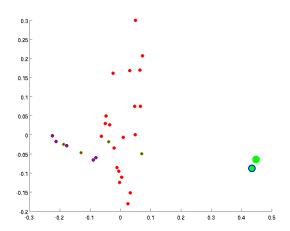


Figure 5.123: RDM features for fold n = 10 - linear kernel

Predicted and true classes for a visual comparison in Figures 5.124 and 5.125.

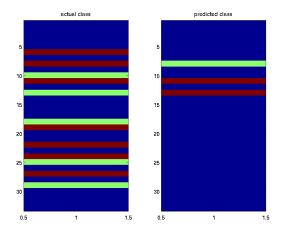


Figure 5.124: RDM features label comparison for fold n=1 - linear kernel

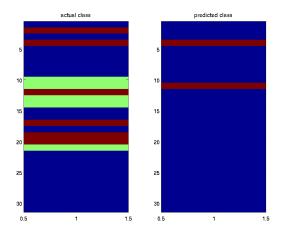


Figure 5.125: RDM features label comparison for fold n=10 - linear kernel mean accuracies in Table 45

Table 45: Mean accuracies for RDM features - linear kernel

Normal	Malignant	Benign	Total
99.52%	7.33%	7.14%	65.19%

One vs all Testing dataset with assigned labels in Figures 5.126 and 5.127:

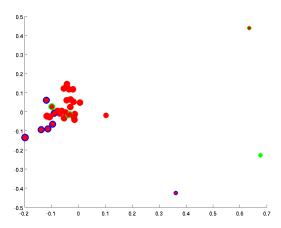


Figure 5.126: RDM features for fold n=4 - linear kernel OVA

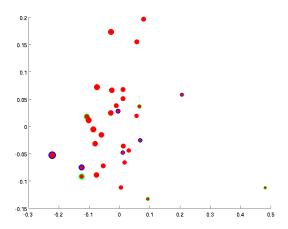


Figure 5.127: RDM features for fold n=5 -linear kernel OVA

Predicted and true classes for a visual comparison in Figures 5.128 and 5.129.

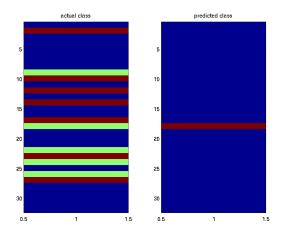


Figure 5.128: RDM features label comparison for fold n=4 - linear kernel OVA

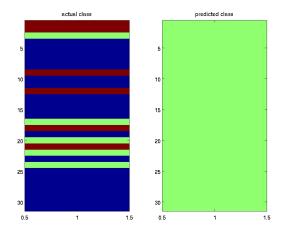


Figure 5.129: RDM features label comparison for fold n=5 - linear kernel OVA

and mean accuracies in Table 46

Table 46: Mean accuracies for RDM features - linear kernel OVA

Normal	Malignant	Benign	Total
100%	4%	1.42%	63.65%

RDM features keeping only malignant and benign mammograms Testing dataset with assigned labels in Figures 5.130 and 5.131:

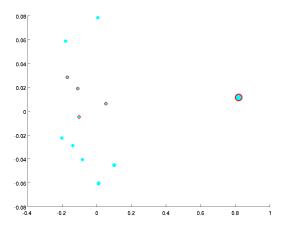


Figure 5.130: RDM features for fold n = 2 - only benign and malignant - linear kernel

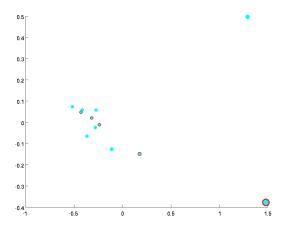


Figure 5.131: RDM features for fold n=3 -linear kernel

Predicted and true classes for a visual comparison in Figures 5.132 and 5.133.

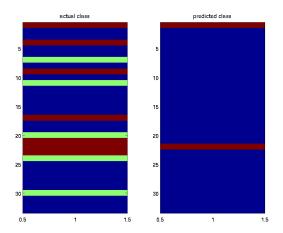


Figure 5.132: RDM features label comparison for fold n=2 - only benign and malignant - linear kernel

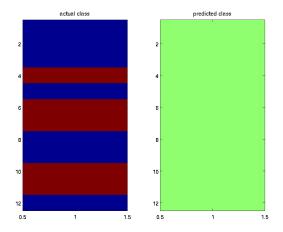


Figure 5.133: RDM features label comparison for fold n=3 - only benign and malignant - linear kernel

mean accuracies in Table 47

Table 47: Mean accuracies for RDM features - only benign and malignant - linear kernel

Malignant	Benign	Total
94.28%	1.66%	53.65%

6 Conclutions

The SGLDM and RDM algorithms are used in many studies and are two well studied algorithms. In our case we could not get the expected results. Our best results were accomplished with linear kernel with the full dataset and total mean accuracy 78.81% and the worst with sigmoid kernel keeping only malignant and benign with RDM algorithm, 52.88%. Although 78.81% is a pretty good accuracy we discovered that this was not completely accurate as was false positive. The number of normal and benign (non cancerous mammograms) much larger than malignant mammograms and that gives us pretty low accuracy in benign images which is not the desired result. Furthermore, high dimensionality of the dataset worsens the case. For deeper investigation we can use an algorithm like forward feature selection to select the best contributed features as an efort to increase the overall accuracy. Additionally more mammograms are needed to be added in order to be tested with other classification algorithms, a gesture which will produce us a better classification model. All the above are both very interesting and worth to be further investigated.

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