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THESIS

ON THE EFFECT OF MONETARY POLICY OF FED AND ECB ON MARKET EXPECTATIONS OF FUTURE EURO EXCHANGE RATES. EVENT STUDY ON PERIOD 2000-2001.

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1. Introduction

Market participants and policy makers require up to date information on market sentiment and market beliefs about the future. This information can be compactly expressed in terms of the probability distribution of financial asset prices. The price of assets themselves and the prices of derivative contracts on the assets contain much of that information. A classic example is the use of the forward exchange rate as an indicator of the market consensus on the first moment of the future exchange rate; another is the use of term structure of interest rates as an indicator of the expected future inflation rate. The payoffs on derivatives such as options are conditional on the future prices of the underlying assets, and therefore reflect market beliefs about the future prices of the underlying assets.

Implied probability density functions (PDFs) estimated from option prices are gaining increasing attention. They are used to price complex derivatives. A number of authors have used implied PDFs as indicators of market sentiment to examine whether options markets anticipated major economic events. Central banks, in particular, have been interested in using implied PDFs to assess market participants' expectations of future changes in interest rates, stock prices and exchange rates.

One of the factors that contribute most on the formation of future exchange rates is monetary policy. When central banks change the key interest rates, they cause excess or low demand for the corresponding currency.

This project investigates how monetary policy of the Fed and the ECB affected market expectations for future US $-Euro exchange rates. We examine the effect of monetary policy of Fed and ECB not only on traders' average expectation of the spot rate at a future date (i.e on $\mathbf{E(s_{\tau})}$), but also:

- on the uncertainty of the market about the exchange rate that will prevail over the near future(i.e on the implied **variance** of the exchange rate),
- on the weight the market participants put on a much higher and a much lower exchange rate in the near future with respect to the forward rate (i.e. on the implied **skewness** of the future exchange rate) and
- on the likelihood the market attributes to very large exchange rate movements in either direction in the near future. (i.e. on the implied **kurtosis** of the future exchange rate).

This is achieved by examining the risk-neutral first four moments of the implied distribution function of U.S.$/Euro. We make our analysis by using implied probability density functions (pdfs) derived from currency option prices.

In corresponding surveys, Galati and Melick (1999) looking at the effect of market perceptions of foreign currency intervention operations by the Federal Reserve and the Bank of Japan on the distribution of the dollar-yen exchange rate, found that while there was no statistically significant effect on the skewness of the PDFs, the market perception of intervention was associated with a higher variance of future spot rates.

Also, Malz (1997) examined over-the-counter foreign exchange rates, using one-month options from April 1992 through June 1996. Malz found that the explanatory value of the option-implied risk-neutral first four moments of the exchange rate contributes to explaining exchange rate excess returns (in a CAPM sense). More specifically, the explanatory value of the 2nd, 3rd and 4th option- implied moments for the most major currency pairs was quite high (R^2 between 26% and 40%). The first moment (forward premium), however, was not significant and contributed little to $\bar{R}^2$.

Though, the risk neutral exchange-rate moments were highly correlated with one another and with the forward premium, so inferences concerning the influence of individual moments cannot be made accurately and including subsets of risk neutral moments in the regression significantly changes some coefficients. Malz also found that investors can earn excess returns (in a CAPM sense) by holding currencies whose option prices indicate positive skewness.

Moreover, Melick and Thomas (1996) found that the option prices were consistent with the market commentary at the time, in that they reflected a significant probability of a major disruption in oil prices. Examination of particular days confirmed the large shift in market expectations that occurred when significant crisis-related news reached the oil market.

Furthermore, Campa, Chang and Reider (1998) looked at implied skewness in one and three-month over-the counter options on a number of different exchange rates between April 1996 and March 1997. Campa, Chang and Reider found that the direction of skewness was positively correlated with returns over the remaining length of the option.

In our analysis we investigate how the first four option- implied moments of the euro-dollar exchange rate change, due to the monetary policy of the ECB and the Fed .In other words, we see the effect of the monetary policy of the ECB and the Fed on market's expectation for future euro-dollar exchange rates.

We look at 12 events in total, in the period 2000-2001. During this period, 24 events took place. Data deficiency and other statistical reasons, explained in detail in proceeding chapters, made it impossible to examine the total of the events. We estimate implied probability density functions of future dollar/euro rates using risk reversals, strangles and at-the money currency option prices of one-month, three-month and one-year OTC options.

The tests we use are and two sample tests for the mean, the variance and the skewness of the implied pdfs and one-sample test for the kurtosis. We first test each of the events separately (using implied pdfs for one-month, three-months and one-year ahead) and we then make some total tests.

The results show that we have no significant effect on the reaction of the market in each event separately. In other words none of the first four moments of the implied pdfs changes significantly. On doing the total tests, we find that monetary policy significantly affects the first and the forth moment of pdfs, while it does not significantly affect the variance and the skewness.

Since this is the first time an event study on the effect of monetary policy on future exchange takes place, we cannot compare the results directly with previous surveys. Surely, an important finding of this survey is the significant effect of the monetary policy on the forth moment of the implied pdfs.

The remainder of the paper is organized as follows. Section 2 the methods that have been developed in the literature for estimating implied distribution density functions. Section 3 represents the data we use for estimating implied pdfs and section 4 describes the event study. In section 5 we make the statistical analysis of the project and in section 6 we represent the results. Section 7 concludes.

2. Implied pdf estimation

2.1. Literature review

A number of methods have been developed in the literature for estimating implied PDFs. To date, however, little attention has been paid to the robustness of these estimates or to the confidence that users can place in the summary statistics (for example the skewness or the 99^{th} percentile) derived from these fitted PDFs.Like all statistics estimated from finite data samples, implied PDFs and their summary statistics are point estimates subject to estimation error. However, while many papers have estimated and interpreted implied PDFs, surprisingly few have considered the reliability of estimated implied PDFs and their associated summary statistics. Methods for estimating implied PDFs fall into five groups:

-Stochastic process methods

Stochastic process methods for estimating PDFs begin by assuming a model for the stochastic process driving the prices of the underlying security, usually one for which it is possible to obtain an analytical solution to rhe implied PDF for a given horizon. After estimation, the parameters of the stochastic process are plugged into the analytical formula for the PDF. For instance, Malz (1996) fits a lognormal-jump diffusion process to OTC foreign exchange derivative prices and then analytically computes risk-neutral realignment probabilities around the time of the 1992 E.R.M. crises. The stochastic process approach can be used in the absence of option prices (the other approaches cannot).

-Implied binomial trees

The method was developed in Rubinstein (1994) and seeks to build a binomial tree for the value of the underlying asset. The tree is conducted so as to minimize deviations from a lognormal process subject to the tree fitting the observed option prices. The method is thus a non-parametric Bayesian technique related to stochastic process methods in that its focus is on modeling the evolution of the underlying asset's price.

-PDF approximating function methods

Approximating function methods begin with the option-pricing relation in Cox and Ross (1976), who show that the price of an option is the discounted risk-neutral expected values of the payoffs of the European calls or puts at time t before maturity. Parametric approximating function methods assume that the risk-neutral probability density function has a particular form chosen to allow a variety of possible shapes. Parameter values are found by minimizing some functions that have been used include: mixtures of lognormals, developed by Melick and Thomas,(1997), Hermite polynomials, developed by Madan and Milne (1994),and a Burr III disribution, used by Sherrick, Garcia and Tirupattur (1996). Alternatively, non-parametric methods can be used. Examples include the kernel estimator of Ait-Sahalia, and Lo (1998), and the maximum entropy methods developed by Buchen and Kelly (1996).

-Finite -difference methods

These methods begin with the observation, made by Breeden and Lintzenberger (1978) that differentiating twice the price C of a call with respect to K (strike price) gives the probability density function:

$$\frac{\theta^2 c(t, X, T)}{\partial X^2} = e^{-r\tau}\pi(\mathrm{X})$$

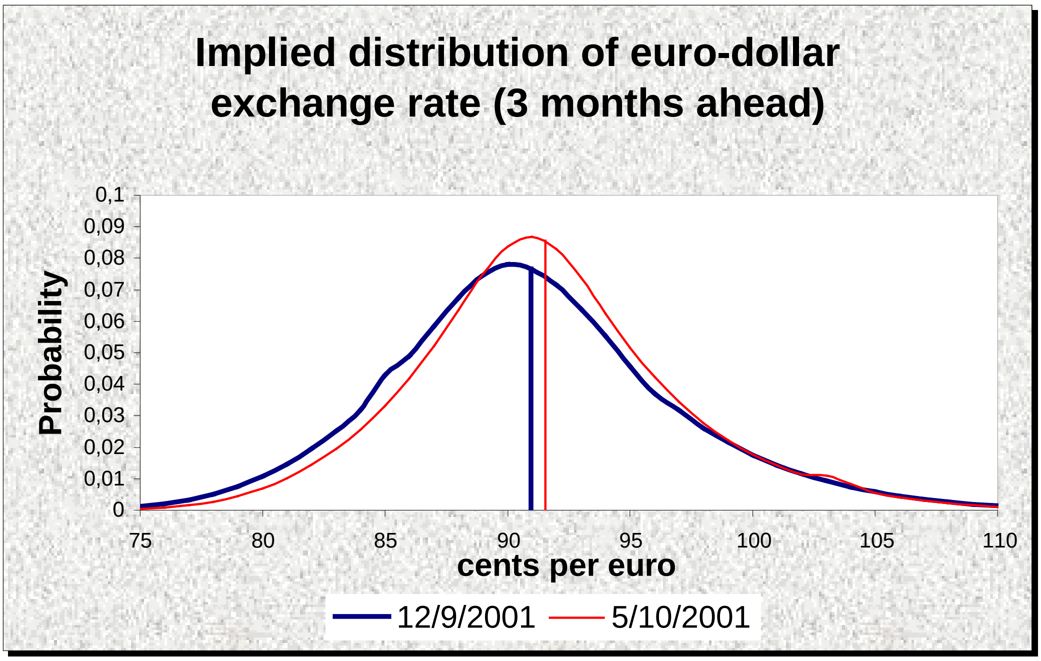
Breeden and Lintzenberger (1978) show that one can use finite difference methods to approximate the equation above, using strikes where bond prices are observed.

-Implied volatility smoothing methods.

_The method was originally developed by Shimko (1993). The method is an approximating function method applied to the volatility smile rather than to the PDF.

A number of papers have compared different implied PDF estimation methods. Campa, Chang and Reider (1997) compared binomial tree, smoothed implied volatility smile and mixtures of lognormal methods. Comparing various moments of the implied distributions they concluded that all methods produced similar results. Coutant, Jondeau and Rockinger (1999) compared single lognormal, mixtures of lognormals Hermite polynomials and maximum entropy methods. Again results were broadly the similar, although they noted that the maximum entropy method run into convergence problems.

AN EXAMPLE OF ESTIMATED PDFS.



·obMetrics

	Mean	Variance	Skewness	Kurtosis	25%	50%	75%	Pr(x<85)	Pr(85<x<95)	Pr(x>95)
/9/2001(1M)	91,1509	12,19509	0,3538679	0,5477893	88,86309	90,9513	93,22	3,430%	83,332%	13,238%
.0/2001(1M)	91,648	10,65396	0,339242	0,6093111	89,66659	91,466	93,456	1,327%	85,407%	13,266%
/9/2001(3M)	90,9353	11,58236	0,3200793	0,2646034	87,12219	90,6044	94,345	14,326%	63,880%	21,794%
.0/2001(3M)	91,5343	10,82194	0,3151462	0,3853225	88,06679	91,2373	94,66	10,337%	66,473%	23,189%
/9/2001(1Y)	89,7214	10,0725	0,132624	-0,27705	82,83709	89,3694	96,228	32,463%	38,797%	28,740%
10/2001(1Y)	90,1327	9,928935	0,1129455	-0,247024	83,42348	89,8302	96,509	30,400%	39,842%	29,757%

3. The data

We will estimate the 1M, 3M, 1Y implied PDFs for some specific days (around the date of each intervention in the key interest rates by the ECB or the FED).
We use (for that specific dates):
–The risk reversal (for 1M, 3M, 1Y).
–The strangle (for 1M, 3M, 1Y).
–The option price(for 1M, 3M, 1Y).
–The domestic (U.S.) interest rate.
–The foreign (Euro) interest rate.
–The spot rate (U.S.$/Euro).

The data we use for the risk reversal, the strangle and the option price are over-the-counter data. Using over-the-counter data has several advantages over exchange traded- data. For example, the exchange-traded data options are not exactly at-the money. Moreover, variations in the level of implied volatility calculated from them are commingled with variations in the curvature of the volatility smile. At-the-money forward over-the-counter options avoid this distortion.

4. The event study

This event study is about the effect of the F.E.D. and the E.C.B. monetary policy on market expectations for future euro exchange rates. More specifically, we examine how markets react each time when the F.E.D or the E.C.B. set up new interest rate levels.

In our survey, we are looking into the effect 7 events of the F.E.D. and 5 events of the E.C.B. had on market expectations for future euro exchange rates, as reflected in the implied probability density functions. These events took place during 2000-2001, at the following dates:

FED

Date	**Change**
02-02-2000	25 b.p. increase
21-03-2000	25 b.p. increase
16-05-2000	50 b.p. increase
21-08-2001	25 b.p. decrease
02-10-2001	50 b.p. decrease
06-11-2001	50 b.p. decrease
11-12-2001	25 b.p. decrease

ECB

Date	**Change**
17-03-2000	25 b.p. increase
28-04-2000	25 b.p. increase
09-06-2000	50 b.p. increase
31-08-2001	25 b.p. decrease
09-11-2001	50 b.p. decrease

These are not all but some of the events that took place at 2000 and 2001, as we can see in the table below:

Years\ Events	Total # of events for F.E.D.	# of events of F.E.D we examine	Total # of events for E.C.B.	# of events of E.C.B. we examine
2000	3	3	6	3
2001	11	4	4	2
TOTAL	**14**	**7**	**10**	**5**

It was impossible for us to examine all the events: First, data was available only for certain days (more specifically, from January up to July 2000 and from August up to December 2001). Second, some events (for which data was available) happened in successive days, so any inferences from the implied pdfs constructed for that dates would be useless, since the information of that pdfs would be on the effect of two events and not one.

For each one of the events we estimate the implied probability density functions for **1 month,** for **3months** and for **1 year** ahead. More specifically, we estimate the implied pdfs two working days before and two working days after the day each event took place. For these two specific dates (before and after the event) we compare the first three moments of the distributions (the mean, the variance and the skewness). We at first examine each event separately and we then make some total tests. Indeed, the "total" tests tell us whether monetary policy affects market expectations when:

- The Fed increases the key interest rates.
- The Fed cuts the key interest rates.
- The ECB increases the key interest rates.
- The ECB cuts the key interest rates.
- The Fed cuts or the ECB increases the key interest rates.
- The Fed increases or the ECB cuts the key interest rates.
- The Fed changes the key interest rates.
- The ECB changes the key interest rates.
- The Fed or the ECB changed the key interest rates.
- The Fed or the ECB changed the key interest rates in 2000.
- The Fed or the ECB changed the key interest rates in 2001.

5. Statistical analysis

5.1 The methodology

In order to test whether monetary policy changes market expectations for future euro exchange rates, we test the hypothesis that the moments of the distributions are the same vs. the alternative. **Our random variable is the exchange rate U.S.$ to Euro**, which we consider that takes discrete values. For 1M pdfs, we usually estimate 35 values of the variable (81-115), for 3M pdfs 35 values (81-115) and for 1Y pdfs 55 values (66-120). First we show by using the Colmogorov-Smirnov test for Goodness of fit for one sample that all the implied probability density functions can be considered that come from normal distributions with mean and variance their realized first and second moments respectively.

Then we go on by doing the hypothesis testing.

We start with the variance. We examine whether the pdfs under examination have the same variance or not, under normality. We then move on to the means of the distributions. If the variance turned out to be the same, we test whether the means of the pdfs are equal under the hypothesis that the variances of the pdfs are unknown, but equal. If the variances were not equal, we do the test for the means under inequality of the variances. Finally, we test whether the skewness of the pdfs changes or not, without any normality assumptions of the pdfs that time.

We estimate the implied pdfs two days before and two days after each event.

5.2 The tests we use

1) In order to prove that implied probability density functions come from normal distributions, we use the Colmogorov-Smirnov test for Goodness of fit for one sample. We show that our “samples” (implied pdfs) come from normal distributions with mean and variance the mean and the variance of the sample.

<u>Colmogorov-Smirnov test for Goodness of fit</u>

We want to support the null hypothesis
Ho: F=Fo vs.
Hα: F$\neq$Fo.

For $D_N = \sup_x \left| F_N(x) - F_O(x) \right|$, where F_N is our sample (empirical) distribution and F_O is the hypothesized distribution,
we reject Ho if and only if $\mathbf{D}_N \succ \mathbf{K\alpha}$, for critical value Kα given from:
$P\left[D_N \succ Ka / Ho\right] \prec \alpha$,
(level of confidence **a** given).

For an ordered sample $\{Yi, i = 1,2,\ldots,N\}$, with $Y_1 \prec Y_2 \prec \ldots \prec Y_N$, and
Zi= Fo(Yi)=G($\frac{Yi-\mu}{\sigma}$) (for Ho:Y~N(μ,σ^2)), we have the value D_N to be
$D_N = \max\left\{ \max_{0\le i\le N}\left\{ \frac{i}{N} - Z_i \right\}, \max_{0\le i\le N}\left\{ Z_{i+1} - \frac{i}{N} \right\} \right\}$.

Example: Random discrete variable that takes values from 80 to 115, with :

Mean	Variance	Skewness	Kurtosis
91,37645	10,43003	0,485962	0,754892

Kolmogorov-Sminov test for Goodness of fit

i	Yi	(Yi-ì)/ó	Zi=Fo(Yi)	i/N	(i/N)-Zi	$Z_{i+1} - i/N$
0	$-\infty$	0	0	0	0	0,137694
1	80	-1,09074	0,137694	0,02777	-0,10992	0,132131
2	81	-0,99486	0,159901	0,05554	-0,10436	0,12879
3	82	-0,89899	0,18433	0,08331	-0,10102	0,127646
4	83	-0,80311	0,210956	0,11108	-0,09988	0,128631
5	84	-0,70723	0,239711	0,13885	-0,10086	0,131632
6	85	-0,61135	0,270482	0,16662	-0,10386	0,13649
7	86	-0,51548	0,30311	0,19439	-0,10872	0,142999
8	87	-0,4196	0,337389	0,22216	-0,11523	0,150914
9	88	-0,32372	0,373074	0,24993	-0,12314	0,159953
10	89	-0,22785	0,409883	0,2777	-0,13218	0,169804
11	90	-0,13197	0,447504	0,30547	-0,14203	0,180134
12	91	-0,03609	0,485604	0,33324	-0,15236	0,190596
13	92	0,059784	0,523836	0,36101	-0,16283	0,20084
14	93	0,155661	0,56185	0,38878	-0,17307	0,210521
15	94	0,251538	0,599301	0,41655	-0,18275	0,21931
16	95	0,347415	0,63586	0,44432	-0,19154	0,226903
17	96	0,443292	0,671223	0,47209	-0,19913	0,233025
18	97	0,539169	0,705115	0,49986	-0,20525	0,237441
19	98	0,635046	0,737301	0,52763	-0,20967	0,239957
20	99	0,730923	0,767587	0,5554	-0,21219	**0,240425**
21	100	0,8268	0,795825	0,58317	-0,21265	0,238742
22	101	0,922677	0,821912	0,61094	-0,21097	0,234853
23	102	1,018554	0,845793	0,63871	-0,20708	0,228743
24	103	1,114431	0,867453	0,66648	-0,20097	0,22044
25	104	1,210308	0,88692	0,69425	-0,19267	0,210005
26	105	1,306185	0,904255	0,72202	-0,18224	0,197532
27	106	1,402062	0,919552	0,74979	-0,16976	0,183135
28	107	1,497939	0,932925	0,77756	-0,15537	0,166951
29	108	1,593816	0,944511	0,80533	-0,13918	0,149127
30	109	1,689693	0,954457	0,8331	-0,12136	0,129816
31	110	1,78557	0,962916	0,86087	-0,10205	0,109175
32	111	1,881447	0,970045	0,88864	-0,0814	0,087358
33	112	1,977324	0,975998	0,91641	-0,05959	0,064513
34	113	2,073201	0,980923	0,94418	-0,03674	0,040782
35	114	2,169078	0,984962	0,97195	-0,01301	0,016292
36	115	2,264955	0,988242	1	0,011758	0
37	$+\infty$	$+\infty$	1	_	_	_

As we can see here, **D_N = 0,24025 $\prec$ 0,2716 =Kα**, for **α=0,01**.(True $\alpha \approx 5\%$).

So, we can assume that Y~N(μ=91,37, σ^2 =108,785).
All implied pdfs we examine can be assumed normal due to this criterion.

2) We then go on with testing the variance of the implied pdfs. We assume as mentioned above that both 'samples' (the implied distribution of the exchange rate ***before*** the event and the implied distribution of the exchange rate ***after*** the event) come from normal distributions, $N_1(\mu_1, \sigma_1^2)$ the first and $N_2(\mu_2, \sigma_2^2)$ the second sample. We also assume that both samples have unknown means. We want to test the hypothesis of equality of the two variances. Our null hypothesis is

Ho: $\frac{\sigma_1^2}{\sigma_2^2}$ =1 (equality of variances) vs.

Hα: $\frac{\sigma_1^2}{\sigma_2^2} \succ 1$

The statistic we use is

F=$\frac{S_1^2}{S_2^2}$**, (S_1^2, S_2^2 are the sample variances)**

which under $\sigma_1^2 = \sigma_2^2$ follows Snedecor distribution $F_{n_1-1,\, n_2-1}$,
where n_1-1, n_2-1 are the degrees of freedom of the distribution.
If $F^* \succ F\alpha$, we reject Ho for Hα.

(In cases $S_2^2 \succ S_1^2$, our statistic F=$\frac{S_2^2}{S_1^2}$).

For the total tests (page 7), F=$\frac{\overline{S}^2_{before}}{\overline{S}^2_{after}}$,

($\overline{S}^2_{before} = \frac{\overline{S}^2_{1,before} + \overline{S}^2_{2,before} + ... + \overline{S}^2_{k,before}}{n_1 + n_2 + ... + n_k - k}$, k the number of events,

$\overline{S}^2_{after} = \frac{\overline{S}^2_{1,after} + \overline{S}^2_{2,after} + ... + \overline{S}^2_{k,after}}{n_1 + n_2 + ... + n_k - k}$, k the number of events,

before: before the event, *after*: after the event).

3) After having done the test for the two variances, we continue with the test of the two means, μ_1 and μ_2. The null hypothesis here is
Ho: $\mu_1 = \mu_2$ v s.
Hα: $\mu_1 \neq \mu_2$
In case the unknown variances in the previous hypothesis testing turn out to be equal, and they **always** are in our survey, the test we use is:

t= $\frac{\bar{X}_{after}-\bar{X}_{before}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ **~ t** $_{\mathbf{n_1-1,\, n_2-1}}$, with t to be the **Student** distribution,

$\bar{X}_{after}, \bar{X}_{before}$ the sample means ($\bar{X}=\frac{\sum_{i=1}^{n} X_i}{n}$) and

S²= $\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$ the common sample variance. (S_1 : before, S_2 : after).

$S_1^2=\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$.

If $|t^*| \succ t_{\frac{\alpha}{2}}$, we reject Ho for Hα.

In the total tests,

$t=\frac{\sum_{i=1}^{k}(|\bar{X}_{after}-\bar{X}_{before}|)_i}{\bar{S}\sqrt{\frac{1}{n_1+n_2+...+n_k}+\frac{1}{n_1+n_2+...+n_k}}}$, $\bar{S}$ the common variance, k the number of events
(one side t-test).

4) We then have the test for the skewness of the pdfs. The null hypothesis is:
Ho: $\text{skew}_1=\text{skew}_2$ v.s.
Hα: $\text{skew}_1 \neq \text{skew}_2$

or

Ho: $\text{skew}_1-\text{skew}_2=0$ v.s.
Hα: $\text{skew}_1-\text{skew}_2 \neq 0$

or

Ho: $\text{skew}_{1\text{-}2}=0$ v.s.
Hα: $\text{skew}_{1\text{-}2} \neq 0$, because the test we will use is a single-sample test.
The statistic we use for this test is
z=E ln(F+$\sqrt{F^2+1}$), (1), z~Z(0,1) (the standardized normal distribution), where:

A=$\sqrt{b_1}\sqrt{\frac{(n+1)(n+3)}{6(n+2)}}$, (2),

B= $\frac{3(n^2+27n-70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}$, (3),

C=$\sqrt{2(B-1)}-1$, (4),

D= $\sqrt{C}$, (5),

E=$\frac{1}{\sqrt{\ln D}}$, (6),

F=$\frac{A}{\sqrt{\frac{2}{C-1}}}$, (7), and

$$\sqrt{b_1} = \frac{(n-2)g_1}{\sqrt{n(n-1)}}.$$

$$g_1 = \frac{m_3}{\tilde{s}^3}$$

$$m_3 = \frac{n\sum X^3 - 3\sum X\sum X^2 + \frac{2(\sum X)^3}{n}}{(n-1)(n-2)}$$

$$\tilde{s} = \sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}}$$

Equations (1)-(7) summarize the steps that are involved in computing the test statistic for the single sample test for evaluating population skewness. Zar (1999) states that equation (1) provides a good approximation of the exact probabilities for the sampling distribution of g_1(which is employed to compute the value of $\sqrt{b_1}$ that is used in equation (2) , when n$\geq$9. In our survey n always exceeds 35.

Again, if $|z^*| \succ z_{\frac{\alpha}{2}}$, we reject Ho for Hα.

In the total tests, we test skew=$\sum_{i=1}^{k} |skew|_i$ in an one side z-test.

5) Finally, we test for the kurtosis of the pdfs.

Ho: kurtosis=0 v.s.

Hα: kurtosis$\neq$0,

The statistic we use for this test is

z=$\frac{1-\frac{2}{9K}-\sqrt[3]{L}}{\sqrt{\frac{2}{9K}}}$, (8), z~Z(0,1) (the standardized normal distribution).

G= $\dfrac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}$, (9),

H= $\dfrac{(n-2)(n-3)|g_2|}{(n+1)(n-1)\sqrt{G}}$, (10),

J= $\dfrac{6(n^2-5n+2)}{(n+7)(n+9)}\sqrt{\dfrac{6(n+3)(n+5)}{n(n-2)(n-3)}}$, (11),

K= $6+\dfrac{8}{J}\left[\dfrac{2}{J}+\sqrt{1+\sqrt{\dfrac{4}{J^2}}}\right]$, (12),

L= $\dfrac{1-\dfrac{2}{K}}{1+H\sqrt{\dfrac{2}{K-4}}}$, (13), and

$$g_2=\frac{m_4}{\tilde{s}^4},$$

$$m_4=\frac{\left[\left[\sum(X-\overline{X})^4 n(n+1)\right]/(n-1)\right]-3\left[\sum(X-\overline{X})^2\right]^2}{(n-2)(n-3)}$$

Equations (8)-(13) summarize the steps that are involved in computing the test statistic (which, as noted above, is a z-value) for the single sample test for evaluating population kurtosis. Zar (1999) states that equation (8) provides a good approximation of the exact probabilities for the sampling distribution of g_2, when $n \geq 20$. In our survey $n \geq 35$ always.

The **power** of our tests is very satisfactory, since in the most of the tests our random variable takes more than 35 values (the power of the test is a positive function of n), followed by relatively small standard deviation. In the total tests especially, our random variable takes from 70 to 720 values, followed also by relatively small standard deviation, so the power of that tests is very strong.

6. The results

6.1 Results on each event separately

PDFS 1-MONTH AHEAD

FED

DATES	EVENT	MEAN	ST.DEV.	SKEW	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
31/01/00	25 b.p. increase	97,205	12,597	0,142			
03/02/00		99,129	12,559	0,007			
RESULTS					0,63	1,006	-0,374
19/03/00	25 b.p. increase	97,333	12,959	0,17			
23/03/00		96,171	11,872	0,16			
RESULTS					-0,386	1,191	-0,027
14/05/00	50 b.p. increase	92,099	13,288	0,05			
18/05/00		89,589	12,232	0,19			
RESULTS					-0,825	1,18	0,388
19/08/01	25 b.p. decrease	91,537	11,014	0,445			
23/08/01		91,376	10,43	0,465			
RESULTS					-0,063	1,115	0,056
30/09/01	50 b.p. decrease	90,881	11,035	0,31			
04/10/01		91,309	11,017	0,30			
RESULTS					0,159	1,003	-0,027
02/11/01	50 b.p. decrease	90,098	9,629	0,24			
08/11/01		89,606	9,469	0,38			
RESULTS					-0,215	1,034	0,388
09/12/01	25 b.p. decrease	88,916	8,023	0,29			
13/12/01		89,683	8,129	0,33			
RESULTS					0,398	1,027	0,111
CRITICAL VAL.(5%)					**1,69**	**1,55**	**1,65**

ECB

DATES	EVENT	MEAN	ST.DEV.	SKEW	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
15/03/00	25 b.p. increase	96,877	13,66	0,17			
21/03/00		97,471	12,667	0,25			
RESULTS					0,191	1,164	0,222
26/04/00	25 b.p. increase	92,216	12,764	0,18			
02/05/00		91,704	12,67	0,05			
RESULTS					-0,168	1,014	-0,36
07/06/00	50 b.p. increase	95,539	13,965	0,27			
13/06/00		95,399	13,433	0,26			
RESULTS					-0,04	1,081	-0,027
29/08/01	25 b.p. decrease	91,027	10,322	0,32			
04/09/01		90,707	10,824	0,34			
RESULTS					-0,11	1,1	0,05
8/11/01	50 b.p. decrease	89,606	9,469	0,38			
13/11/01		89,305	9,62	0,14			
RESULTS					-0,132	1,016	0,66
CRITICAL VAL.(5%)					**1,69**	**1,55**	**1,65**

PDFS 3-MONTHS AHEAD

FED

DATES	EVENT	MEAN	ST.DEV.	SKEW	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
31/01/00	25 b.p. increase	97,421	11,576	0,056			
03/02/00		99,475	11,633	0,094			
RESULTS					0,704	1,009	0,105
19/03/00	25 b.p. increase	97,760	12,194	0,213			
23/03/00		96,567	11,508	0,186			
RESULTS					-0,425	1,122	-0,075
14/05/00	50 b.p. increase	92,368	12,337	0,15			
18/05/00		89,957	11,979	0,25			
RESULTS					-0,830	1,029	0,277
19/08/01	25 b.p. decrease	91,371	11,101	0,352			
23/08/01		91,258	10,947	0,362			
RESULTS					0,045	1,028	0,027
30/09/01	50 b.p. decrease	90,817	10,979	0,351			
04/10/01		91,105	10,774	0,343			
RESULTS					0,11	1,038	-0,022
02/11/01	50 b.p. decrease	89,895	9,994	0,319			
08/11/01		89,374	9,712	0,298			
RESULTS					-0,221	1,059	0,058
09/12/01	25 b.p. decrease	88,686	9,486	0,340			
13/12/01		89,429	9,531	0,326			
RESULTS					0,327	1,009	-0,038
CRITICAL VAL.(5%)					**1,69**	**1,55**	**1,65**

ECB

DATES	EVENT	MEAN	ST.DEV.	SKEW	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
15/03/00	25 b.p. increase	97,253	12,488	0,104			
21/03/00		97,856	11,798	0,17			
RESULTS					0,207	1,12	0,183
26/04/00	25 b.p. increase	92,525	12,307	0,179			
02/05/00		92,052	12,23	0,205			
RESULTS					-0,164	1,012	0,072
07/06/00	50 b.p. increase	95,633	12,979	0,12			
13/06/00		95,651	13,03	0,23			
RESULTS					0,006	1,007	0,305
29/08/01	25 b.p. decrease	90,885	10,687	0,334			
04/09/01		90,571	10,735	0,356			
RESULTS					-0,117	1,009	0,061
8/11/01	50 b.p. decrease	89,374	9,712	0,298			
13/11/01		89,088	9,804	0,279			
RESULTS					-0,129	1,019	-0,052
CRITICAL VAL.(5%)					**1,69**	**1,55**	**1,65**

PDFS 1-YEAR AHEAD

FED

DATES	EVENT	MEAN	ST.DEV.	SKEW	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
31/01/00	25 b.p. increase	98,199	10,03	0,016			
03/02/00		100,89	10,984	0,219			
RESULTS					**1,46 (α=8%)**	1,199	0,724
19/03/00	25 b.p. increase	98,539	10,549	-0,002			
23/03/00		97,615	10,324	0,04			
RESULTS					-0,505	1,044	0,116
14/05/00	50 b.p. increase	93,38	9,66	0,07			
18/05/00		90,94	10,004	0,124			
RESULTS					**-1,41 (α=8%)**	1,072	0,15
19/08/01	25 b.p. decrease	90,485	9,818	0,199			
23/08/01		90,34	9,74	0,215			
RESULTS					-0,081	1,016	0,044
30/09/01	50 b.p. decrease	89,336	10,098	0,152			
04/10/01		89,778	9,973	0,131			
RESULTS					0,252	1,025	-0,058
02/11/01	50 b.p. decrease	89,05	9,68	0,26			
08/11/01		88,39	9,547	0,25			
RESULTS					0,209	1,028	-0,027
09/12/01	25 b.p. decrease	87,767	10,611	0,241			
13/12/01		88,43	9,55	0,24			
RESULTS					0,378	1,234	-0,002
CRITICAL VAL.(5%)					**1,68**	**1,42**	**1,65**

ECB

DATES	EVENT	MEAN	ST.DEV.	SKEW	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
15/03/00	25 b.p. increase	96,801	9,46	-0,297			
21/03/00		97,54	9,28	-0,34			
RESULTS					0,45	1,039	-0,154
26/04/00	25 b.p. increase	92,775	9,776	0,10			
02/05/00		93,031	9,78	0,06			
RESULTS					0,149	1,001	-0,111
07/06/00	50 b.p. increase	96,034	9,305	0,009			
13/06/00		95,87	9,17	0,029			
RESULTS					-0,102	1,029	0,055
29/08/01	25 b.p. decrease	89,986	9,835	0,171			
04/09/01		89,798	9,755	0,22			
RESULTS					-0,109	1,016	0,136
8/11/01	50 b.p. decrease	88,39	9,547	0,25			
13/11/01		88,166	9,635	0,225			
RESULTS					-0,133	1,018	-0,069
CRITICAL VAL.(5%)					**1,68**	**1,42**	**1,65**

6.2 Analysis

In testing each event separately, we found no statistically important changes in any of the four moments of the implied pdfs (with the exception of a significant change (a=8%) in the first moment at the 1-Y ahead pdf of a Fed interest rate increase in 2000).

This means that the market did not react significantly each time the Fed or the ECB changed the interest rate in the period 2000-2001. In other words, the traders' average expectation of the spot rate at a future date (i.e the E(st)) did not change in each event separately.

Moreover, the uncertainty of the market about the exchange rate that would prevail over the near future(i.e the implied variance of the exchange rate) we see that was not significantly reduced. It is surprising that in some cases the uncertainty of the market concerning the future exchange rate was increased after interest rate changes.

Changes in the skewness of the implied pdfs were not significant, either. This means that the market did not significantly change the weight on a much higher or much lower dollar to euro, after dates of interest rate changes. Changes in the skewness did not follow a systematic pattern in the same categories of events, showing that the changes in the skewness were unpredictable.

Changes in the kurtosis were also statistically insignificant, that is why we do not represent any tests. However, we observe that kurtosis in almost each single event is reduced at the 1-Y ahead implied pdfs. This means that the probability of large interest rate movements of 1-Y ahead implied probability density functions was reduced after dates of interest rate changes during 2000-2001.

We also see that the bigger changes in the first moment of the pdfs are observed on actions by the Fed rather than on actions by the ECB. However, changes are not always in the direction we would expect as the effect of monetary policy. This happens because in many cases the market had anticipated earlier these interest rate changes, so either it did not actually react after the announcement of interest rate changes, or in cases it had anticipated a larger than finally realized interest rate change, it reacted in the opposite direction immediately after the announcement of the interest rates change.

6.3 Results on the total tests

PDFS 1-MONTH AHEAD

CASE	#OF EVENTS	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
Fed increases i. r.	3	1,074	1,12	0,421
Fed cuts i.r.	4	0,393	1,034	0,266
ECB increases i.r.	3	0,239	1,085	0,324
ECB cuts i.r.	2	0,258	0,968	0,481
Fed cuts / ECB increases i.r.	7	0,441	1,059	0,401
Fed increases / ECB cuts i.r.	5	1,005	1,054	0,61
Fed changes i.r.	7	1,068	1,076	0,466
ECB changes i.r.	5	0,312	1,032	0,536
Fed or ECB change i.r in 2000	6	0,939	1,103	0,513
Fed or ECB change i.r. in 2001	6	0,425	1	0,476
Fed or ECB change i.r. in 2000-2001	12	1,006	1,056	0,688

PDFS 3-MONTHS AHEAD

CASE	#OF EVENTS	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
Fed increases i. r.	3	1,16	1,057	0,244
Fed cuts i.r.	4	0,337	1,029	0,067
ECBincreases i.r.	3	0,211	1,039	0,299
ECB cuts i.r.	2	0,247	0,986	0,076
Fed cuts / ECB increases i.r.	7	0,387	1,034	0,237
Fed increases / ECB cuts i.r.	5	1,046	1,031	0,231
Fed changes i.r.	7	1,058	1,042	0,204
ECB changes i.r.	5	0,274	1,020	0,273
Fed or ECB change i.r. in 2000	6	0,948	1,048	0,372
Fed or ECB change i.r. in 2001	3	1,16	1,057	0,244
Fed or ECB change i.r. in 2000-2001	4	0,337	1,029	0,067

PDFS 1-YEAR AHEAD

CASE	#OF EVENTS	t-stat. (mean)	F-stat. (var)	z-stat. (skew)
Fed increases i. r.	**3**	**1,937***	0,933	0,583
Fed cuts i.r.	4	0,551	1,073	0,081
ECB increases i.r.	3	0,303	1,022	0,201
ECB cuts i.r.	2	0,171	0,999	0,181
Fed cuts / ECB increases i.r.	7	0,617	1,052	0,195
Fed increases / ECB cuts i.r.	**5**	**1,658***	0,958	0,557
Fed changes i.r.	**7**	**1,701***	1,009	0,433
ECB changes i.r.	5	0,341	1,012	0,266
Fed or ECB change i.r. in 2000	**6**	**1,401***	0,974	0,55
Fed or ECB change i.r. in 2001	6	0,552	1,048	0,167
Fed or ECB change i.r. in 2000-2001	**12**	**1,329***	1,011	0,5

*** statistically important for α= 5%**
*** statistically important for α= 10%**

We now represent the events in which we had a significant change on the kurtosis of the pdfs:

Ho: The pdf platykurtic,
Hα: The pdf non- platykurtic.

CASE	#OF EVENTS	Kurtosis (before)	Kurtosis (after)	z-stat. (before)	z-stat. (after)
Fed or ECB change i.r. in 2000-2001	**12**	-0,37	-0,33	**1,785***	**1,625***
Fed changes i.r. in 2000-2001	**7**	-0,36	-0,31	**1,427***	1,265

*** statistically important for α= 5%**
*** statistically important for α= 10%**

6.4 Analysis

We will focus our analysis on the results of 1-Y ahead pdfs, since in 1-M ahead and in 3-M ahead pdfs the results are on the same direction, but are not statistically important. This probably happens because monetary policy affects most long-term than short-term economic evolutions.

As far as for the monetary policy of the Fed, we see that in 2000, when we had 3 actions (interest rate increases), the monetary policy significantly affected the market's expectation for the level of future euro-dollar exchange rates. The second and the third moment of the implied pdfs did not significantly change.

On the other hand, the monetary policy of the Fed in 2001 does not seem to have affected the market expectations for forthcoming euro-dollar exchange rates. Neither the first moment, nor the second, the third or the fourth significantly changed.

As a whole, monetary policy of the Fed in 2000-2001 significantly affected the market's expectations for future euro-dollar exchange rates and significantly reduced the kurtosis of the pdfs. In that way, monetary policy of Fed in 2000-2001 has been effective.

As far as for the monetary policy of the ECB, neither in 2000 nor in 2001 significantly affected the market. We did not have any statistically important change in any of the first four moments of the option- implied pdfs. In that way, monetary policy of the ECB in 2000-2001 has been ineffective.

Moreover, actions from the two central banks that were in favor of the dollar (increase in the interest rates by the Fed or decrease in the interest rates by the ECB) significantly affected the market. This is probably more due to the strong effect of the Fed interest rate increases in 2000 rather than due to the interest rate cuts by the ECB in 2001.

Furthermore, monetary policy of the Fed and the ECB in 2000 significantly affected market's expectations for the level of future euro-dollar exchange rates, in contrast with monetary policy of the two central banks in 2001,that did not significantly affect the market view for future exchange rates . This probably happened because in 2000 we had very fewer events in number than in 2001, so the market had a stronger reaction in each event.

Finally, the monetary policy of the Fed and the ECB in 2000-2001 we could say that significantly changed the market views for the level of future exchange rates, did not significantly reduce the uncertainty of the market nor the weight the market put on a much higher or lower level of the future exchange rate, but it did significantly reduce the likelihood the market attributed to very large exchange rate movements 1 year ahead (mainly due to the monetary policy of Fed).

Since it is the first time such an event study takes place, we can't compare our results with previous surveys.

Comparing the results with corresponding surveys, our findings show a significant change in the first and in the fourth moment of the pdfs. Changes in the kurtosis is a very important finding, since very few papers up to now have tested for kurtosis changes in event studies, and those who have did not find significant changes in the kurtosis. As far as for the variance and the skewness of the pdfs, our results do not show significant changes, in contrast with other surveys, that have found in corresponding event studies significant changes in the second and the third moment of the implied pdfs.

The power of the tests we use would probably be higher if instead of assuming the implied pdfs to follow a normal distribution we had assumed them to be lognormals. This could probably be the case if we had a parametric method (i.e. a mixture of two lognormals) in estimating the implied pdfs and this is a suggestion for corresponding surveys.

7. Conclusions

The first result of our survey, and that of the greatest importance, is that the expected level of the future exchange rate, $E(s_t)$ (the forward rate),changes significantly each time the Fed changes its key interest rates.

More specifically, $E(s_t)$ changed significantly each time the Fed changed the key interest rates in the year 2000 and totally for the period 2000-2001. This means that the monetary policy of Fed affected significantly the market view for the level of future (1 year ahead) euro-dollar exchange rates, especially in 2000.

Specifically in 2000, when we had only 3 actions by the Fed –3 interest rate increases- each event was accompanied by a strong reaction of the market. Indeed, in February 2002, the forward rate moved to the opposite than expected direction (the Euro was strengthened!) in an interest rate increase, and that happened because the market had anticipated a 50 b.p. increase in the interest rates by the Fed, which eventually announced a 25 b.p. increase.

On the other hand, the ECB did not seem to affect the markets so much as far as for the expected level of the future exchange rate U.S. $ to euro. Both in 2000 and in 2001 the markets did not react significantly in the monetary policy of the ECB. This probably happened because the ECB is a recently established bank, so it does not affect the market so much. Moreover, the target of the bank is mainly the maintenance of price stability in the Eurozone and not the level of Euro towards other currencies.

Fed, on the other hand, is a more decisive bank. The market seems to trust Alan Greenspan and to believe that each decision he takes helps the American economy perform better and the dollar to strengthen towards foreign currencies.

As far as for the changes in the variance of the implied pdfs, i.e in the level of un certainty of the market about the exchange rate that will prevail over the near future, these were in general insignificant, with the exceptions of some (very few in number) specific events. That means that monetary policy of the Fed and the ECB did not at general affect the level of uncertainty of the market about the exchange rate that will prevail over the near future.

The same conclusion comes for the skewness, too. Neither at each single event separately neither at the whole the monetary policy of the Fed and the ECB affected the weight the market participants put on a much higher and a much lower exchange rate in the near future with respect to the forward rate.

Finally, as far as for the kurtosis, we could say that it was significantly reduced the dates after the Fed or the ECB had changed the interest rates. That means that monetary policy of the Fed and the ECB as a whole reduced the likelihood the market attributed to very large exchange rate movements in either direction on the near future. This specific conclusion could stand for the monetary policy of the Fed specifically, but not for the monetary policy of the ECB.

The later result shows that the Fed was more effective in reducing market uncertainty for a large movement in the exchange rate in the near future than the E.C.B. This result is on the same direction of what we had stated before, i.e. that the market tends to trust more monetary policy of the Fed rather than that of the ECB.

The former analysis was for based on the findings on the total tests. In testing each event separately, as we mentioned before, we found no statistically important changes in any of the four moments with an exception of a change in the first moment at the 1-Y ahead pdf of a Fed interest rate increase in 2000. This is only due only to statistical reasons: Our sample in the total tests is much larger than the sample of each single event.

APPENDIX 1

OPTION-IMPLIED PROBABILITY DISTRIBUTIONS- DISCRIPTION OF METHODS

The payoffs of derivatives such as options are conditional on the future prices of the underlying assets and their valuation is based heavily on the perception the traders have about them. Thus, they reflect market beliefs about the potential value that the underlying asset will experience at the maturity of the option contract. We can consider a European call option at time t with exercise price X and maturity at T to have a market value:

$$c(t, X, T) = e^{-r\cdot\tau}\int_{\chi}^{\infty}(S_T - X)\pi(S_T)dS_T, \quad (A)$$

S_t : time-t asset price,
τ =T-t : time to maturity
X: strike price
r :domestic risk-free continuously compounded discount rate
π(X): risk-neutral pdf of the terminal asset price S_T conditional on S_t

We use the equation above and the observed market prices c (t,X,T) in order to draw inferences on $\pi(\chi)$, the risk-neutral probability density function. There are many ways to succeed in it employing different categories of methods .We present two specific methods of different categories.

Non-Parametric Methods. Malz ´s quadratic approximation.
(The method we follow).

This group of methods is using the following result.The risk-neutral probability density function is the second derivative of the market call price with respect to the exercise price(Breeden D., Litzemberger R, 1978):

$$\frac{\theta^2 c(t, X, T)}{\theta X^2} = e^{-r\cdot\tau}\pi(X) \text{ , (B)}$$

Therefore if we can provide a continuous function of the price of the call and the strike price (c(X)) we can calculate the pdf. It is difficult, in general, to provide such a function. Hence the efforts are concentrated in providing functions of volatility with respect to X (σ(X)) or of volatility with respect to δ ($\sigma(\delta)$) .We can transform these function to C(x) by using the Black &Scholes formula (see Appendix for details) and calculate the derivative afterwards.

The method of Malz that we employ is approximating the $\sigma(\delta)$ function. In order to do so we must define certain things. We observe that out of the money options on currencies with flexible exchange rates often have higher implied volatilities than at-the-money options, and out-of-the money call options most of the times have implied volatilities which differ from those of equally out–of-money puts. The latter two phenomena are known as the **volatility smile**.

Exercise prices of over-the-counter currency options are often set equal to the forward exchange rate of like maturity .In such a case the option is called at-the-money forward and options for which the exercise price equals the spot exchange rate are called at-the-money outright.

We define a strangle to be the action that the dealer sells or buys both out-of-the money options (call & put with the same delta) from the counterpart. The risk-reversal is the dealer ´s sell of the one of the options and the buy of the other (call & put with the same delta) with the counterpart. Because the put and the call are generally of not equal value, the dealer pays or receives a premium for exchanging the options, expressed as the difference between the implied volatility of the puts and the call. The dealer quotes the implied volatility differential at which he is prepared to exchange a 25-delta call for a 25-delta put. The midpoint of the time-t one month-strangle price, can be expressed as

$str_t=0,5\cdot(\sigma^{(0,75)}-\sigma^{(0,25)})-atm_t$

and the risk reversal price as
$rr_t= \sigma^{(0,25)}-\sigma^{(0,75)}$

where str_t, rr_t and atm_t denote the one month strangle price, risk reversal price, and at the money volatility, in vols, $\sigma_t^{(0,25)}$ and $\sigma_t^{(0,75)}$ refer to the implied volatilities of the one month 25-delta call and 25-delta put.

The method

Malz uses a quadratic approximation of the volatily smile. He considers the volatility smile to have three components ($\sigma\ (\delta)=\alpha\chi^2+\beta\chi+\gamma$). More specifically, it is a linear function of at-the-money volatility, a linear function of the risk reversal price and the deviation of delta from 0,5:

$\sigma(\delta) = b_0\, atm_t+b_1\, rr_t(\delta-0,5)+b_2\, str_t(\delta-0,5)^2$

The at-the-money (straddle) volatility gives the general level of implied volatility, the risk reversal indicates the skew in the volatility smile and the strangle price indicates the degree of curvature of the volatility smile.
Solving for specific values $(b_0, b_1, b_2)=(1,-2,16)$ we have :

$\boldsymbol{\sigma(\delta)=atm_t-2rr_t(\delta-0,5)+16str_t(\delta-0.5)^2}$

Combining the equation above with the definition of the delta of an option ,

$$\delta_U(S_{t,}\tau,\mathrm{X},\sigma,r,r^*)\equiv\frac{\theta u(S_{t,}\tau,\mathrm{X},\sigma,r,r^*)}{\theta S_t}=e^{-r\tau}\Phi\left[\frac{\ln\left(\frac{S_t}{X}\right)+(r-r^*+\frac{\sigma^2}{2})}{\sigma\sqrt{\tau}}\right]$$

we get the implied volatility as a function of exercise prices:

$$\sigma = atm_t - 2rr_t\left(e^{-r^*\tau}\cdot\Phi\left[\frac{\ln(\frac{S}{X})+(r-r^*+\frac{\sigma^2}{2})\cdot\tau}{\sigma\cdot\sqrt{\tau}}\right]-0{,}50\right)+16str_t\left(e^{-r\tau}\cdot\Phi\left[\frac{\ln(\frac{S}{X})+(r-r^*-\frac{\sigma^2}{2})\cdot\tau}{\sigma\cdot\sqrt{\tau}}\right]-0{,}5\right)^2$$

The last equation shows great difficulties to be solved analytically and provide a continuous function of σ(X). However we are able to solve it numerically and to obtain pairs of values (X,σ).The range of X and the values that we use to calculate the corresponding σ´s is up to the computational power that we use.

Using the B&S formula, we transform the (X,σ) pairs to (X,C) pairs. Then we calculate the second derivative of the C(X) function as suggested in equation (B).It is clear that we will perform this calculation numerically since we have pairs of (X,C(X)) and not a continuous, analytically described function. In the end we take pairs of (X,p(X)) ,where p(X) is the estimated probability density function.

Appendix

<u>Transforming Option prices to volatilities</u>

Even though neither traders nor academics believe in its literal truth, the language and convention of the currency option trading are drawn from the Black-Scholes model. The specific model results in only for European call and put values.

Due to one-to-one relationship between the parameter σ and the Black-Scholes call pricing function for given values of the remaining arguments, market prices can be expressed either in units of volatility or in currency units, called the **Black-Scholes implied volatility**, or in currency units. For a given a market price c (t,X,T) in currency units, the corresponding implied volαtility σ_t can be found by substituting the maturity, exercise price, the observed spot rate, and the τ-period domestic and foreign interest rates and solving the equation:

$$c(t,X,T) = u(S_{t,}\tau, \mathrm{X}, \sigma_t, r_t^{\tau}, r_t^{*\tau})$$

Therefore market participants quote option prices in terms of implied volatility (vols).

The rate of change of the Black-Scholes call pricing function with respect to the spot exchange rate, a fraction called **delta,** is the optimal hedge of an option position under ideal conditions and is referred to by option dealers in managing option price risk:

$$\delta_U(S_{t,}\tau, X, \sigma, r, r^*) = \frac{\partial U(S_{t,}\tau, X, \sigma, r, r^*)}{\partial S_t} = e^{-r\tau}\Phi\left[\frac{\ln\left(\frac{S_t}{X}\right)+(r-r^*+\frac{\sigma^2}{2})}{\sigma\sqrt{\tau}}\right]$$

(The price of a *put* option: $\delta(w) = 1-\delta(u)$).

Parametric Methods. Mixture of two Lognormals

The double log-normal approximation method is based on the theoretical pricing relations for European calls and puts:

$$C_t(K) = e^{-rt}\int_K^{\infty}(S_T - K)df(S_T)$$

$$P_t(K) = e^{-rt}\int_K^{\infty}(S_t - K)df(S_T)$$

where C and P are the call and the put prices observed at time t ; r is the riskless rate ; τ is the time to expiry; K is the exercise price and $df(S_T)$ the risk neutral probability function for the value of the underlying asset, S , at expiry, $T=t+\tau$.

The double-lognormal method approximates the density function with a mixture of two lognormal density functions:

$$dfS_T = \theta \cdot L(S_T | \mu_{1,} \sigma_1, S_t) + (1-\theta)L(S_T | \mu_{2,} \sigma_{2,} St)$$

with

$$L(S_T) = \frac{1}{S_T \sigma \sqrt{2\pi\tau}} \exp\left\{ \frac{-\left[\log S_T - \log S_t - (\mu - \frac{1}{2}\sigma^2)\tau\right]^2}{2\sigma^2\tau} \right\}$$

where S_t is the current value of the underlying asset $\{\mu_1,\sigma_{1,}\mu_2,\sigma_2,\theta\}$are the unknown parameters that define the double lognormal density function ;
$\theta \; \varepsilon \; [\, 0,1]$.

Thus the fitted value for a call price, given parameters $\{\mu_1,\sigma_1,\mu_2,\sigma_2,\theta\}$, is given by

$$\widehat{C}_t(KI\mu_{1,}\sigma_{1,}\mu_{2,}\sigma_2,\theta) = e^{-r\tau}\{\theta\int_K^{\infty}(S_T - K)L(S_T l\mu_1,\sigma_1,S_t)dS_T + (1-\theta)\int_K^{\infty}(S_T - K)L(S_T I\mu_{2,}\sigma_2,S_t)dS_T\}$$

with an equivalent expression for the value put option.

Given observations of calls and put prices, the parameters $\{\mu_1,\sigma_{1,}\mu_2,\sigma_2,\theta\}$, of the implied double-lognormal PDF can be estimated using non-linear optimization methods to minimize the weighted sum of fitted price errors:

$$\min_{\{\mu_1,\sigma_1,\mu_2,\sigma_2,\theta\}} \sum_{i=1}^{N_c} w_i \cdot \left[C_t(K_i) - \hat{C}\left(K_i \middle| \mu_1, \sigma_1, \mu_2, \sigma_2, \theta\right)\right]^2 + \sum_{j=1}^{N_p} w_j \cdot \left[P_t(K_i) - \hat{P}(K_i \middle| \mu_1, \sigma_1, \mu_2, \sigma_2, \theta)\right]^2$$

subject to : $\sigma_1, \sigma_2 \geq 0$ and $0 \leq \theta \leq 1$

and where N_c and N_p are the number of call and put contracts in the estimation sample for a given pair of observations and expiry dates {t, T} and the w_i , w_j are the weights placed on each option (the weights are depended on volume or open interest).

APPENDIX 2

INTEREST RATE PARITY

From **interest rate parity** we know that the spot exchange rate S_t, the forward rate $F_{t,T}$ and the interest rates i and i* of two currencies are linked by the relationship:

$$\frac{F_{t,T} - S_t}{S_t} = \frac{i - i^*}{1 + i^*}\text{,or}$$

% forward = %interest

premium differential

Thus, when interest rates change, forward rates are change. So, when central banks change the key interest rates, the forward rate is affected.

We also know that the forward rate is a biased estimator of future spot exchange rate. Analysis of forward bias is framed in terms of the relationship:

$$E_t(s_T) = f_{t,T} + p_t\,,$$

where s_T and $f_{t,T}$ equal the logarithms of the spot and forward exchange rates and $p_t \equiv E_t(s_T) - f_{t,T}$ is the predictable logarithmic return on an open currency position (Fama 1984). So, expectation of future exchange rate changes, when central banks change interest rates.

Under rational expectations, the market's subjective expectation and the conditional mathematical expectation of the future exchange rate are equal. The deviation of the conditional forecast from the realized exchange rate should average zero over a long period and exhibit no correlation with elements of the conditioning set. Under the additional hypothesis of risk-neutrality, the market's subjective expectation of the future exchange rate is equal to the risk-neutral expectation, the forward exchange rate.

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