Introduction

The past thirty years, a large literature has been developed to explain how we can represent trends in macroeconomic time series models, such as GNP. Until the late 70's it was common to simply fit a linear trend to the logarithm of the series, and then define the stochastic part of the time series as deviations from this trend. Working in this sense, macroeconomists routinely detrended data, and regarded business cycles as the stationery deviation about that trend or, as it was wisely accepted, short-run deviations from the trend. However, the permanence of the shocks, led them to question this time-honored assumption and start wonder whether these shocks resemble the permanent shocks of a random walk. This new approach motivated Nelson and Plosser to test macroeconomic series for unit roots and in their seminal paper in 1982 they found that they could not reject the random walk hypothesis in most of them.

Throughout these years, financial economists got interested in the question of whether stock price and exchange rates movements are less than perfect random walks. It turns out that the same techniques that are good for quantifying how much macroeconomic series behave like random walks are useful for quantifying the extent to which stock price movements and exchange rates follow a random walk or not. This approach motivated some authors to consider these techniques as convincing evidence of "efficient markets", an approach most of them recognize now that it is not the case.

On the other hand, all these new issues motivated econometricians to try to develop methods and tests in order to capture and explain the behavior of these series. Since a well established result is that autoregressive models are those which describe these series, and particularly those of order one (AR(1) models), the random walk hypothesis became of great interest. All of these years, lot of different tests have arisen having as forefront the unit root tests by Dickey and Fuller (1976, 1979 and 1981). Related work was done by Evans and Savin (1981, 1984) and by Sargan and Bhargava (1983) and Bhargava (1986). Although their work was influential, the assumptions they used on the innovations that drives the model (either $iid(0,\sigma^2)$ or $iidN(0,\sigma^2)$) were rather strong, leading to rather restrictive results.

These strong assumptions were first relaxed by P. C. B. Phillips in 1987. In his paper Phillips studied the random walk in a general time series setting that allows for weakly dependent and heterogeneously distributed innovations. Phillips's work, and the framework for testing the unit root hypothesis he developed in collaboration with Perron, has proven to be very influential to econometricians and it is very important for the purpose of this paper.

In this paper we will attempt to apply a new test, developed by Kourogenis and Pittis, on several time series, such as stock indexes and exchange rates, a test that is characterized by an even general setting. Moreover, it extends the theory developed by Phillips to include cases where the variance grows without limit in a polynomial fashion. In particular they relax the restrictive assumption that u_t are all bounded, that is $\sup_t E|u_t|^{\beta} < \infty$ for some β>2, thus precluding trending moments. What is interesting about this test is that it embodies the Phillips-Perron traditional test as a special case.

The discussion that follows is organized in four parts: In the first part, we analyze the probabilistic structure of autoregressive models, and particular those of order one, which host the random walk models as a special case. In the second part, we examine some of the important properties of random walk models and we display the difference between them and the stable autoregressive ones. The third part is divided into two parts: In the first part we give an explicit citation of unit root tests; especially the Phillips-Perron framework is given a special treat and in the second part the new test is presented in detail. Finally, in the fourth path we present the methodology that was used in the implementation of the test and the results from it. The details about the data and the rundown of the results can be found in the appendix.

PART 1

1.Introduction to the Probabilistic Theory of AR(p) Models

The main objective of this chapter is to define and explain the theory that underlies autoregressive models; particularly the discussion will refer to the restrictions of dependence and heterogeneity needed to specify such statistical models. Moreover the probabilistic structure will be discussed in relation to the three basic categories of probabilistic assumptions:

1. Distribution
2. Dependence
3. Homogeneity

1.1 Distribution

Consider a stochastic process $\{Y_t, t \in T\}$ whose joint distribution $f(y_1, y_2, ..., y_n; \varphi)$ for any finite collection $(Y_1, Y_2, ..., Y_n)$ is Normal, i.e.

$$\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{n1} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \right),$$

is said to be a Normal (or Gaussian) process. That is, the only definitional characteristic is the distribution assumption of Normality. At this point it is essential to add that the Normality assumption is being done to simplify our discussion. Later this assumption will be waved off.

1.2 Dependence

From basic probability theory we know that without any restrictions on the dependence and heterogeneity of this process no operational model is possible. The only possible reduction of the joint distribution is the one based on sequential conditioning:

$$f(y_1, y_2, ... y_n; f) \overset{non-IID}{=} f_1(y_1; y_1) \prod_{k=2}^{n} f_k(y_k | y_{k-1}, ..., y_1; y_k), \text{ for all } y \in R^n. \; y \in R^n.$$

with the conditional distributions being Normal. The autoregressive and autoskedastic functions take the form:

$$E(Y_k | s\,(Y_k, ..., Y_1)) = b_0(k) + \sum_{i=1}^{k-1} b_i(k) Y_{k-i} \text{ , k=2,3,…,n}$$

$$Var(Y_k | s\,(Y_k, ..., Y_1)) = s_0^2(k) \text{, k=2,3,…,n}$$

This however, does not give rise to an operational model because the overparameterization problem remains: the number of unknown parameters in $\{y_1, y_2, ..., y_n\}$ is the same as those in **φ** (and increasing with n).

As it is well known, the way to deal with both problems, the increasing conditioning information set and the overparametrization, is to impose some restrictions on the dependence and heterogeneity of the set of random variables $(Y_1, Y_2, ..., Y_n)$.

Firstly we will pursue this line of argument by imposing Markov dependence without any restrictions on heterogeneity in order to bring out the role of each set of restrictions and then proceed to impose Markovness and stationarity to derive the family of the models of interest, that is autoregressive models.

The Markov dependence (first order dependence) when applied to the only possible reduction of the joint distribution, the sequential conditioning, yields:

$$f(y_1, y_2, \ldots y_n; \varphi) \overset{Markov}{=} f_1(y_1; \psi_1) \prod_{k=2}^{n} f_k(y_k | y_{k-1}; \psi_k), \text{ for all } y \in R^n.\ y \in R^n.$$

That is the dependence structure between Y_k and $(Y_{k-1}, \ldots, Y_1)$ is fully captured by its conditional distribution given its most recent past Y_{k-1}. It is very important to emphasize that Markovness does not involve any heterogeneity restrictions. Therefore, under the Normality assumptions the first two stochastic conditional moments take the form:

$$E(Y_k | \sigma(Y_{k-1}, \ldots, Y_1)) = \alpha_0(k) + \alpha_1(k) Y_{k-1}, \text{ k=2,3,…,n}$$

$$Var(Y_k | \sigma(Y_{k-1}, \ldots, Y_1)) = \sigma_0^2(k), \text{ k=2,3,…,n}$$

If we compare these moments with the unrestricted ones we can see that the Markov dependence assumptions deals with the problem of increasing conditioning information set but the parameters still remain index dependent. In order to deal with the last problem we need to impose some restrictions on the heterogeneity of the process.

1.3 Heterogeneity – Three Different Assumptions

This category is the most important of all three, because the models which derive under certain assumptions on the heterogeneity of the process are those of interest to the purpose of this paper. Univariate autoregressive models with characteristics like stationarity, first order non-stationarity and unit root non-stationarity are nothing but models with different assumptions on the heterogeneity of the process.

The first restriction on heterogeneity that we are going to deal with is that of second-order stationarity (Normality and Markovness are still on). The joint distribution under this assumption takes the following form:

$$f(y_1, y_2, \ldots y_n; f) \overset{Markov}{=} f_1(y_1; y_1) \prod_{k=2}^{n} f_k(y_k | y_{k-1}; y_k) \overset{Stationary}{=} f_1(y_1; y_1) \prod_{k=2}^{n} f_k(y_k | y_{k-1}; y)$$

Roughly speaking, stationarity deals with the overparametrization problem and Markovness with the increasing information set. It is easy to see that by supplementing these assumptions with some distribution assumption, such as Normality, the above decomposition gives rise to operational models.

Under these assumptions we can concentrate only on a bivariate joint distribution. Therefore we have:

$$\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} \sim \mathrm{N}\left(\begin{bmatrix} m \\ m \end{bmatrix}, \begin{bmatrix} s(0) & s(1) \\ s(1) & s(0) \end{bmatrix} \right)$$

where $E(Y_t) = m, \forall t$ $\quad Var(Y_t) = s(0), \forall t$ $\quad Cov(Y_t, Y_{t-1}) = s(1)$ free of t.

By the properties of the binomial Normal distribution it is easy to calculate the conditional moments which are:

$$E(Y_t | s\,(Y_{t-1},...,Y_1)) = a_0 + a_1 Y_{t-1}$$

$$Var(Y_t | s\,(Y_{t-1},...,Y_1)) = s^{\,2}$$

where $a_0 = m - a_1 m = m(1-a_1)$ and $a_1 = {}^{s\,(1)}\!\big/_{s\,(0)}\ \mathbf{p}\ 1$

Thus, when we have a process with the structure defined above, the model that we construct is an AR(1) model which is stationary and homoskedastic:

AR(1): $y_t = a_0 + a_1 y_{t-1} + u_t$ with $Var(Y_t | s\,(Y_{t-1},...,Y_1)) = s^{\,2}$.

The next step that we are going to make is to change the assumption on heterogeneity. We relax the assumption of stationarity but not arbitrarily. We are going to assume first order non-stationarity which means that the unconditional mean depends on the index in the following form:

$$E(Y_t) = mt, \forall t$$

Therefore the process becomes:

$$\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} \square\ \mathrm{N}\left(\begin{bmatrix} mt \\ m(t-1) \end{bmatrix}, \begin{bmatrix} s\,(0) & s\,(1) \\ s\,(1) & s\,(0) \end{bmatrix} \right)$$

where $E(Y_t) = mt, \forall t$, $Var(Y_t) = s\,(0), \forall t$, $Cov(Y_t, Y_{t-1}) = s\,(1)$ free of t.

As before the conditional moments are:

$$E(Y_t | s\,(Y_{t-1},...,Y_1)) = d_0 + d_1 t + a_1 Y_{t-1}$$

$$Var(Y_t | s\,(Y_{t-1},...,Y_1)) = s^{\,2}$$

where $d_0 = a_1 m$, $d_1 = m(1-a_1)$ and $a_1 = {}^{s\,(1)}\!\big/_{s\,(0)}\ \mathbf{p}\ 1$

Thus, when we have a process with the structure defined above, the model that we construct is an AR(1) model which is first order non-stationary and homoskedastic:

$$\text{AR(1):}\quad y_t = d_0 + d_1 t + a_1 y_{t-1} + u_t \quad \text{with} \quad Var(Y_t \,|\, s\,(Y_{t-1},\ldots,Y_1)) = s^{2}$$

The last heterogeneity assumption that we are going to analyze is that of separable non-stationarity. This is the most important assumption of all, because the unit root non-stationarity is a special case of this kind of heterogeneity. Thus the process under this assumption becomes:

$$\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} \sim \mathrm{N}\left(\begin{bmatrix} mt \\ m(t-1) \end{bmatrix}, \begin{bmatrix} s\,(0)t & s\,(1)(t-1) \\ s\,(1)(t-1) & s\,(0)(t-1) \end{bmatrix} \right)$$

where $E(Y_t) = mt, \forall t$, $Var(Y_t) = s\,(0)t, \forall t$, $Cov(Y_t, Y_{t-1}) = s\,(1)(t-1), \forall t$.

As before the conditional moments are:

$$E(Y_t \,|\, s\,(Y_{t-1},\ldots,Y_1)) = d_0 + d_1 t + a_1 Y_{t-1}$$

$$Var(Y_t \,|\, s\,(Y_{t-1},\ldots,Y_1)) = k_0 + k_1 t$$

where $d_0 = a_1 m$, $d_1 = m(1-a_1)$, $a_1 = {s\,(1)(t-1)}/{s\,(0)(t-1)} = {s\,(1)}/{s\,(0)}$,

$k_0 = {s^{2}(1)}/{s\,(0)}$ and $k_1 = {(s^{2}(0) - s^{2}(1))t}/{s\,(0)}$.

The above model under the unit root assumption, that is $a_1 = 1 \Rightarrow s(0) = s(1)$, takes the following form regarding its conditional moments:

$$E(Y_t | s(Y_{t-1},...,Y_1)) = d_0 + Y_{t-1}$$

$$Var(Y_t | s(Y_{t-1},...,Y_1)) = k_0$$

$$\text{where } d_0 = m, k_0 = s(1)$$

Thus, when we have a process with the structure defined above, the model that we construct is an AR(1) model which is unit root non-stationary:

$$\text{AR(1):} \quad y_t = d_0 + y_{t-1} + u_t \quad \text{with} \quad Var(Y_t | s(Y_{t-1},...,Y_1)) = k_0$$

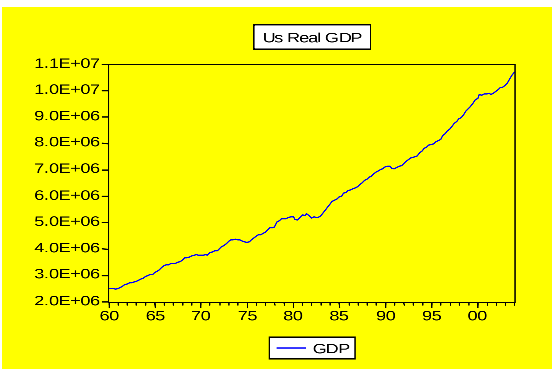
3.4 The Unit Root Case

What it is important from this analysis is that the two last models:

$$\text{AR(1):} \quad y_t = d_0 + d_1 t + a_1 y_{t-1} + u_t \quad \text{with} \quad Var(Y_t | s(Y_{t-1},...,Y_1)) = s^2$$

$$\text{AR(1):} \quad y_t = d_0 + y_{t-1} + u_t \quad \text{with} \quad Var(Y_t | s(Y_{t-1},...,Y_1)) = k_0$$

are both efficient to describe stochastic processes like the movement of the GDP:



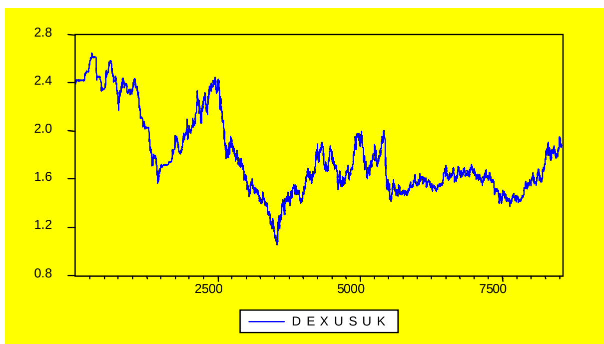
In the case where $m = 0$, the driftless case, the models become:

$$\text{AR(1):}\quad y_t = a_1 y_{t-1} + u_t \quad \text{with} \quad Var(Y_t \mid s\,(Y_{t-1},...,Y_1)) = s^{2}$$

$$\text{AR(1):}\quad y_t = y_{t-1} + u_t \quad \text{with} \quad Var(Y_t \mid s\,(Y_{t-1},...,Y_1)) = k_0$$

Once again, the above models are efficient to describe stochastic processes like the movement of exchange rates:

The US/UK exchange rate



Which of these models qualify to describe better the process will be decided by unit root tests which we are going to analyze later.

<u>Remark:</u> The results derived above can be easily extended to the case where we replace Markov dependence with pth order Markov dependence giving rise to autoregressive models of order p, that is AR(p). These models though are out of the scope of this paper and therefore will not be analyzed.

PART 2

Random Walks

2. A General Discussion on Random Walks

In the previous chapter we were introduced in the probabilistic structure of univariate autoregressive models. We saw that a stochastic process, like the GDP time series, can be described by two models, one of which is:

$$\text{AR(1)}: \quad y_t = d_0 + y_{t-1} + u_t \quad \text{with} \quad Var(Y_t \,|\, s\,(Y_{t-1},...,Y_1)) = k_0$$

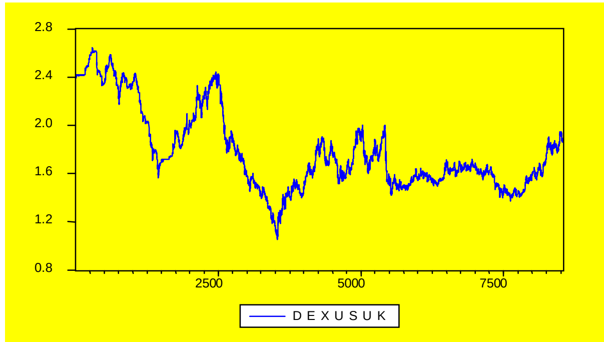
The above AR(1) model is called a random walk with a drift when $d_0 \neq 0$ and a simple random walk when $d_0 = 0$. The later case, that is the AR(1) model:

$$\text{AR(1)}: \quad y_t = y_{t-1} + u_t \quad \text{with} \quad Var(Y_t \,|\, s\,(Y_{t-1},...,Y_1)) = k_0$$

is the one of interest for this paper.

The simple random walk, as we saw earlier, is a model that describes very common phenomena in economics such as stock prices movements and foreign exchange rates. For example:

The US/UK exchange rate



Therefore, it is important to analyze the properties of these processes in order to establish a point of the importance of unit root tests.

2.1 Properties of Random Walks

Random walks have a number of interesting properties:

1. The impulse response function of a random walk is one at all horizons, while the impulse response function of a stationary process dies out eventually.

The impulse response function is a path that y_t follows if it is kicked by a single unit shock, i.e. $u_{t-j} = 0$, $u_t = 1$, $u_{t+j} = 0$. This function is interesting for several reasons. First, is another characterization of our models and second, and more importantly, it allows us to start thinking about "causes and effects". For example, you might compute the response of GNP to a shock in money in a GNP-M1 VAR and interpret the result as the "effect" on GNP of monetary policy. To illustrate this consider the following table which depicts the impulse response function of a AR(1):

The model is:

AR(1): $y_t = r\, y_{t-1} + u_t$ or $y_t = \sum_{j=0}^{\infty} r^j u_{t-j}$ (the $MA(\infty)$ representation of y_t)

The impulse response function is:

T	t-2	t-1	t	t+1	t+2	t+3	t+4
u_t	0	0	1	0	0	0	0
y_t	0	0	1	r	r^2	r^3	…

It is obvious that when the true model is $y_t = r\, y_{t-1} + u_t$ with $|r| \mathbf{p} 1$ a single unit shock would have transitory effects on the process.

On the other hand, when the true model becomes $y_t = y_{t-1} + u_t$, that is $r = 1$, the impulse response function becomes:

T	t-2	t-1	t	t+1	t+2	t+3	t+4
u_t	0	0	1	0	0	0	0
y_t	0	0	1	1	1	1	…

and it obvious that a single unit shock would have permanent effects on the process.

Therefore we can understand how important is to distinguish between the random walk hypothesis and its stationary alternative returning to the previous example, that is a change in monetary policy could have a transitory or permanent effect on real output.

2. The forecast variance of a random walk grows linearly with the forecast horizon $\mathrm{var}(y_{t+k} | y_t) = \mathrm{var}(y_{t+k} - y_t) = ks_u^2$ to infinity, while the forecast variance of a stationary process converges to the unconditional variance.

One of the most interesting things to do with an AR is form predictions of the variable given its past, i.e. we want to know what is the conditional expectation of y_{t+k} given the past values of y_t and u_t:

$$E_t(y_{t+k}) \equiv E(y_{t+k} | y_t, y_{t-1}, y_{t-2}, ...u_t, u_{t-1}, ...)$$

We also want to know how certain we are about the predictions, which we can quantify with

$$\mathrm{var}_t(y_{t+k}) \equiv \mathrm{var}(y_{t+k} | y_t, y_{t-1}, y_{t-2}, ...u_t, u_{t-1}, ...)$$

For the AR(1) model $y_t = r\, y_{t-1} + u_t$, we have:

$$E_t(y_{t+k}) = r^k y_t$$

$$\mathrm{var}_t(y_{t+k}) = (1 + r^2 + r^4 + ... + r^{2(k+1)}) s_u^2$$

Under stationarity, that is $|r| \mathbf{p} 1$, as $k \uparrow \infty$ we have:

$$E_t(y_{t+k}) \to 0 = E(y_t) \text{ and } \mathrm{var}_t(y_{t+k}) = \sum_{j=0}^{\infty} r^{2j} s_u^2 = \frac{1}{1 - r^2} s_u^2 = \mathrm{var}(y_t),$$

while under the unit root hypothesis, $\rho = 1$, as $k \uparrow \infty$ we have:

$$E_t(y_{t+k}) = y_t \text{ and } \mathrm{var}_t(y_{t+k}) \to \infty$$

Once again, the tradeoff between stationary and unit root non-stationary processes is obvious, a result that points out the importance of unit root tests.

<u>Remark:</u> The conditional moments are forecast functions that have the minimum mean square error from any other forecast function.

3. The autocovariances of a random walk aren't defined, strictly speaking.

We can see the above result in our AR(1) model $y_t = \rho y_{t-1} + u_t$, if we consider it to be stable, that is $|\rho| < 1$. The autocovariance and autocorrelation function of our model given the fact that $|\rho| < 1$ are:

$$\gamma_k = \frac{\rho^k}{1-\rho^2}\sigma_u^2, \quad corr_k = \rho^k, \; k = 0,1,2,...$$

It is obvious that when $\rho = 1$ the autocovariance function cannot be defined. However, we can think of the limit of an AR(1) model as the autoregression parameter ρ goes to unity. Thus, a sign of a random walk is that all the estimated autocorrelations are near one, or die out "too slowly".

4. The variance of a random walk is primarily due to low-frequency components; therefore the signature of a random walk is its tendency to wander around low frequencies.

The spectral density function of the AR(1) process is:

$$h_Y(w) = \frac{s_u^2}{2p}[1 - r\exp(-iw)][1 - r\exp(iw)]^{-1} = \frac{s_u^2}{2p}[1 + r^2 - 2r\cos(w)]^{-1}$$

In the limit $r \to 1$ we get:

$$h_Y(w) = \frac{s_u^2}{p}[1 - \cos(w)]^{-1}$$

It is obvious that as $w \to 0$, $h_Y(w) \to \infty$

PART 3

Tests for Unit Roots

3.1 General Discussion

The implications of unit roots in macroeconomic data are, at least potentially, profound. If a structural variable as real output, is truly I(1), then as we saw in the discussion before, shocks to it will have permanent effects. If confirmed, then this observation would mandate some rather serious reconsideration of the analysis of macroeconomic policy. For example, the argument that a change in monetary policy could have a transitory effect on real output would vanish.

Therefore it is understandable that we have to be in a position where we can infer whether the series that is under examination is characterized by a unit root or not. For the purpose of the following discussion we will restrict ourselves to the zero mean AR(1) model with white noise innovations:

$$\text{AR(1)}: \quad y_t = \rho y_{t-1} + u_t \text{ where } u_t \sim iid(0, \sigma_u^2)$$

What we will focus on how alternative values of ρ affect not only the behavior of $\{y_t\}$ but also the OLS estimator of ρ.

The OLS estimator of ρ is: $\hat{\rho} = \dfrac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=1}^{T} y_t^2}$

The properties of the OLS estimator depend to whether the real value of ρ is less or equal to unity.

If the real value is less than unity then the estimator of ρ, $\hat{r}_{OLS}$ is consistent and asymptotically Normal:

$$\sqrt{T}(\hat{r}_{OLS} - r) \xrightarrow{a} N(0, 1 - r^2)$$

The rate with which $\hat{r}_{OLS} \xrightarrow{p} r$ is of order $\sqrt{T}$ (which is referred as the standard asymptotic)

If the real value is equal to unity then things are completely different.

In that case the random quantity $\sqrt{T}(\hat{r}_{OLS} - 1) \xrightarrow{a} N(0,0)$, a result that is useless for statistical inference. The reason of this bad result is the sequence $\sqrt{T}$ which in the unit root case doesn't work. The solution to this problem was given by Fuller (1976). Fuller replaced the standard asymptotic with the sequence T and concluded that:

$$T(\hat{r}_{OLS} - 1) \xrightarrow{a} \frac{\frac{1}{2}\left\{w(1)^2 - 1\right\}}{\int_0^1 w(z)^2 dz} \text{ and } t_T = \frac{\hat{r}_T - 1}{\hat{s}_{r_T}} \xrightarrow{a} \frac{\frac{1}{2}\left\{w(1)^2 - 1\right\}}{\left\{\int_0^1 w(z)^2 dz\right\}^{1/2}}$$

where w(z) is a Brownian Motion

For the purpose of statistical inference the above distributions where calculated using the Monte Carlo simulation method by Fuller and Dickey and the critical values for it can be found in several textbooks. These results can be extended to include cases where we have non-zero mean and trend or both. The asymptotic distributions for these cases are reasonably different from the above distributions but we are not going to refer to them.

3.2 The Philips-Perron framework

The past few decades there have been considerable research in the random walk hypothesis (that is autoregressive models with a unit root) which resulted in the development of the distribution theory that is necessary to construct tests for checking such hypothesis.

Investigations by Dickey and Fuller (1976, 1979 and 1981), Evans and Savin (1981, 1984) have been at the forefront of this research (a small part of this research we saw earlier). Related work on regression residuals has been done by Sargan and Bhargava (1983) and Bhargava (1986). **All of this research has been confined to the case where the sequence of innovations driving the model is either** $iid(0, s^2)$ **or** $iidN(0, s^2)$ **(independent and homoskedastic) which are rather strong assumptions in most empirical econometric work.**

These strong assumptions were first relaxed by P. C. B. Phillips in 1987. In his paper Phillips studied the random walk in a general time series setting that allows for weakly dependent and heterogeneously distributed innovations. It was shown that simple least squares regression consistently estimates a unit root under very general conditions in spite of the presence of autocorrelated errors. The limiting distribution of the standardized estimator and the associated regression **t-**statistic are found using functional central limit theory. New tests are developed which permit a wide class of dependent and heterogeneous innovation sequences. A new limiting distribution theory is constructed based on the concept of continuous data recording.

Phillips's work has proven to be very influential to econometricians and it is very important for the purpose of this paper, therefore we will give an extensive citation of his work.

Let $\{y_t\}_{t=1}^{\infty}$ be a stochastic process generated in discrete time according to:

1. $y_t = ry_{t-1} + u_t$ (t=1,2,…)
2. r =1

Under the unit root assumption (2) our autoregressive model has the following representation:

$y_t = S_t + y_0$ where $S_t = \sum_{j=1}^{\infty} u_j$, $\{u_t\}$ is the innovations sequence and y_0 is the initial condition having three alternative forms as proposed by White (1958):

3a. $y_0 = c$ a constant, with probability one

3b. y_0 has a certain specified distribution

3c. $y_0 = y_T$ where T = the sample size

Equation (3c) is a circularity condition, due to Hotelling, that is used mainly as a mathematical device to simplify distribution theory. (3b) is a random initial condition that is frequently used to achieve stationarity in stable models. In this paper the condition that is used is (3b), which permits the greatest flexibility in the specification of the model. It allows for nonstationary series and includes (3a) as a special case, usually $y_0 = 0$.

At this point it is essential to display the limiting distribution of the standardized sums which will play an important role to the following discussion. Therefore we have:

4a. $X_T(r) = \frac{1}{\sqrt{T\sigma}} S_{[Tr]} = \frac{1}{\sqrt{T\sigma}} S_{j-1}, \quad (j-1)/T \leq r \leq j/T, (j = 1,2,\ldots,T)$

4b. $X_T(1) = \frac{1}{\sqrt{T\sigma}} S_T$

where [] denotes the integer part and σ is a certain constant defined later.

Under certain conditions $X_T(r)$, which is a random element in the function space $D[0,1]$, can be shown to <u>converge weekly</u> to a limit process known as the standard Brownian motion or the Wiener process.

That is:

$$X_T(r) \xrightarrow{L} W(r)$$

This property is known as the functional central limit theorem (FCLT) Moreover, $W(r) \in C[0,1]$ is a Gaussian process (for fixed r, W(r) is $N(0,r)$) and has independent increments (W(s) is independent of W(r)-W(s) for all $0 < s < r \leq 1$). What is important to add at this point is that the conditions that $X_T(r)$ converges to $W(r)$ are very general and extend to a wide class of nonstationary, weakly dependent and heterogeneously distributed innovations sequences $\{u_t\}_1^\infty$.

Returning back to the discussion about the unit root case we must be precise about the innovations sequence that drives the model. Therefore the following assumptions are made:

Assumptions:

a) $E(u_t) = 0$ for all t

b) $\sup_t E|u_t|^{\beta} < \infty$ for some β>2

c) $\sigma^2 = \lim_{T\to\infty} E(T^{-1}S_T^2)$ exists and $\sigma^2 > 0$

d) $\{u_t\}_1^{\infty}$ is strong mixing with mixing coefficients α_m that satisfy

$$\sum_1^{\infty} \alpha_m^{1-2/\beta} < \infty \qquad (d')$$

These conditions allow for both temporal dependence and heteroskedasticity in the process $\{u_t\}_1^{\infty}$. Especially condition (d) controls the extent of the temporal dependence in the process $\{u_t\}_1^{\infty}$, so that, although there may be substantial dependence amongst recent events, events which are separated by long intervals of time are almost independent. In particular, the summability requirement (d′) on the mixing coefficients is satisfied when the mixing decay rate is $\alpha_m = O(m^{-\lambda})$ for some $\lambda > \beta/(\beta-2)$. The same condition also controls the mixing decay rate in relation to the probability of outliers as determined by the moment existence condition (b). Thus, as β approaches 2 and the probability of outliers rises (under weakening moment condition (b)) the mixing decay rate increases and the effect of outliers is required under (d′) to wear off more quickly. This tradeoff between moment and mixing conditions was first developed by McLeish (1975b) in the context of strong laws for dependent sequences. Condition (c) also controls the allowable heterogeneity in the process by ruling out unlimited growth in the βht absolute moments of $\{u_t\}$.

Condition (c) is a convergence condition on the average variance of the partial sum S_T. It is a common requirement in much central limit theory although it is not strictly a necessary condition. However if $\{u_t\}$ is weakly stationary, then

$$s^2 = E(u_1^2) + 2\sum_{k=2}^{\infty} E(u_1 u_k)$$

And the convergence of the series is implied by the mixing conditions (d′)

These assumptions allow for a wide variety of possible generating mechanisms for the sequence of innovations $\{u_t\}_1^\infty$. These include all Gaussian and many stationary finite order ARMA models under very general conditions on the underlying errors.

For following discussion we shall make extensive use of the following two results for the purpose of theoretical development. The first is a functional limit theorem that is due to Herrndorf, and the second is the continuous mapping theorem:

- Lemma1: If $\{u_t\}_1^\infty$ satisfies the above assumptions, then as $T \uparrow \infty$ $X_T(r) \xrightarrow{L} W(r)$, a standard Wiener process on C
- Lemma2: If $X_T(r) \xrightarrow{L} W(r)$ as $T \uparrow \infty$ and h is any continuous functional on D (continuous, that is, except for at most a set of points $D_h \subset D$ for which $P(W \in D_h) = 0$, then $h(X_T) \xrightarrow{L} h(W)$ as $T \uparrow \infty$.

Under these assumptions, Phillips developed the asymptotic distributions of the OLS estimators for both r and the t-statistic.

Furthermore the ordinary least squares (OLS) estimator of r in (1) is:

$$\hat{r} = \sum_{1}^{T} y_t y_{t-1} \Big/ \sum_{1}^{T} y_{t-1}^2$$

Which appropriately centered and standardized is:

$$T(\hat{r}-1) = T^{-1}\sum_{1}^{T} y_t(y_t - y_{t-1}) \Big/ T^{-2}\sum_{1}^{T} y_{t-1}^2$$

And the subsequent t-statistic of the regression is:

$$t_r = \left(\sum_{1}^{T} y_{t-1}^2\right)^{1/2} (\hat{r}-1) \Big/ s \quad \text{where} \quad s^2 = T^{-1}\sum_{1}^{T}(y_t - \hat{r}y_{t-1})^2$$

Both the above statistics have been suggested as test statistics for detecting the presence of a unit root in our AR model. The distributions of these statistics under both the null hypothesis $r = 1$ and certain alternatives $r \neq 1$ have been studied recently by Dickey and Fuller (1979, 1981), as we saw before, Evans and Savin (1981, 1984) and Nankervis and Savin (1985). The work of these authors concentrates altogether on the special case in which the innovations sequence that drives the model $\{u_t\}_{t=1}^{\infty}$ is $iid(0, s^2)$.

Phillips's approach relies on the theory of weak convergence on D. It leads to rather simple characterizations of the limiting of the above statistics in terms of functionals of a Wiener process. The main advantage of the approach is that the results hold for a very wide class of error processes in the AR model.

The limiting distributions are given in the following theorem:

<u>Theorem1:</u> If $\{u_t\}_1^{\infty}$ satisfies the former assumptions and if $\sup_t E|u_t|^{b+h}$ **p** ∞ for some n>0 (b >2), then as $T \to \infty$:

a) $T^{-2}\sum_1^T y_{t-1}^2 \xrightarrow{L} s^{2}\int_0^1 W(r)^2\,dr$

b) $T^{-1}\sum_1^T y_{t-1}(y_t - y_{t-1}) \xrightarrow{L} (s^{2}/2)(W(1)^2 - s_u^{2}/s^{2})$

c) $T(\hat{r}-1) \xrightarrow{L} (1/2)(W(1)^2 - s_u^{2}/s^{2}) \Big/ \int_0^1 W(r)^2\,dr$

d) $\hat{r} \xrightarrow{p} 1$

e) $t_r \xrightarrow{L} (s\ /2s_u)(W(1)^2 - s_u^{2}/s^{2}) \Big/ \{\int_0^1 W(r)^2\,dr\ \}^{1/2}$

where $s_u^{2} = \lim_{T\to\infty} T^{-1}\sum_1^T E(u_u^2)$, $s^{2} = \lim_{T\to\infty} E(T^{-1}S_T^2)$ and W(r) is a standard Wiener process.

It is obvious that when the sequence $\{u_t\}_{t=1}^{\infty}$ is $iid(0, s^{2})$ we have $s_u^{2} = s^{2}$, leading to the following simplification of part (c) of the Theorem1:

$$T(\hat{r}-1) \xrightarrow{L} (1/2)(W(1)^2 - 1) \Big/ \int_0^1 W(r)^2\,dr$$

a result we have seen before and was first given by White (1958), although his expression is incorrect in terms of his standardization of $\hat{r}$, that is he proposed $g(T) = T/2$ instead of $g(T) = T$.

Theorem1 extends the distribution theory to include very general cases of weakly dependent and heterogeneously distributed data. The differences between (c) of theorem1 and the trivial case were the sequence that drives the model is $iid(0, s^2)$ can be demonstrated with a simple example:

Suppose that the generating process of $\{u_t\}_{t=1}^{\infty}$ is a moving average 1 process:

$u_t = e_t + qe_{t-1}$ where e_t is an $iid(0, s_e^2)$ process. Then:

$$s_u^2 = p\lim_{T\to\infty} T^{-1}\sum_1^T u_t^2 = (1+q^2)s_e^2 \quad \text{and} \quad s^2 = p\lim_{T\to\infty} T^{-1}E(S_T^2) = (1+q)^2 s_e^2$$

and we have $T^{-1}\sum_1^T y_{t-1}u_t \xrightarrow{L} (s_e^2/2)[(1+q)^2 W(1)^2 - (1+q^2)]$

which can also be verified by direct calculation. In this case:

$$T(\hat{r}-1) \xrightarrow{L} (1/2)[W(1)^2 - (1+q^2)/(1+q)^2] \Big/ \int_0^1 W(r)^2 dr$$

which of course generalizes the trivial case and reducing to it when $q = 0$.

Part (d) of Theorem1 shows that, unlike the stable AR(1) with $|r|\,\mathbf{p}\,1$, OLS retains the property of consistency when there is a unit root even in the presence of substantial serial correlation. The robustness of the consistency of $\hat{r}$ in this case is rather extraordinary, allowing for a wide variety of error processes that permit serious misspecifications in the usual random walk formulation of our AR model with white noise errors. Intuitively, when the model has a unit root, the strength of the signal (as measured by the sample variation of the regressor y_{t-1}) dominates the noise by a factor of O(T), so that the effects of any regressor-error correlation are annihilated as $T \uparrow \infty$.

Finally part (e) of Theorem1 gives the limiting distribution of t_r. This distribution, like that of the coefficient estimator, depends on the variance ratio s_u^2 / s^2.

At this point it is important to discuss how the unknown parameters s_u^2 and s^2 are estimated, since they appear in the limiting distributions in Theorem1, and how we deal with them with finite samples. Therefore it is understandable that the distributions Phillips derived are not directly useable for statistical testing and in order to modify these statistics and make them useful (that is the distributions have to be independent of s_u^2 and s^2), the unknown parameters have to be consistently estimated.

In the proof of Theorem1 (the proof can not be found in this paper because it is out of its scope) Phillips showed that:

$$T^{-1}\sum_{1}^{T} u_t^2 \xrightarrow{a.s.} s_u^2 \text{ as } T \uparrow \infty$$

This provides us with the simple estimator:

$$s_u^2 = T^{-1}\sum_{1}^{T} y_{t-1}(y_t - y_{t-1}) = T^{-1}\sum_{1}^{T} u_t^2$$

which is consistent for s_u^2 under the null hypothesis r =1. Since $\hat{r} \xrightarrow{p} 1$ by Thoerem1 we may also use $T^{-1}\sum_{1}^{T} y_{t-1}(y_t - \hat{r}\, y_{t-1})$ as a consistent estimator of s_u^2.

Consistent estimation of $s^{2} = \lim_{T\to\infty} E(T^{-1}S_T^2)$ is more difficult. The problem is essentially equivalent to the consistent estimation of an asymptotic covariance matrix of weakly dependent and heterogeneously distributed observations. Therefore we have:

$$s_{\mathrm{T}}^{2} = \mathrm{var}(T^{-1/2}S_T^2) = T^{-1}\sum_{1}^{T} E(u_1^2) + 2T^{-1}\sum_{t=1}^{T-1}\sum_{t=t+1}^{T} E(u_1 u_{t-t})$$

and introducing the approximant

$$s_{\mathrm{T}l}^{2} = T^{-1}\sum_{1}^{T} E(u_1^2) + 2T^{-1}\sum_{t=1}^{l}\sum_{t=t+1}^{T} E(u_1 u_{t-t})$$

where l is the lag truncation number.

For large T and large $l\,\mathbf{p}\,T$, $s_{\mathrm{T}l}^{2}$ may be expected to be very close to s_{T}^{2} if the total contribution in s_{T}^{2} of covariances such as $E(u_t u_{t-t})$ with long lags $T\,\mathbf{f}\,l$ is small. This will be true if $\{u_t\}_{t=1}^{\infty}$ satisfies the assumptions we previously stated. All this can be stated in the following lemma:

- Lemma3: If the sequence $\{u_t\}_{t=1}^{\infty}$ the assumptions and if $l \uparrow \infty$ as $T \uparrow \infty$ then $s_{\mathrm{T}}^{2} - s_{\mathrm{T}l}^{2} \to 0$ as $T \uparrow \infty$.

This lemma suggests that under suitable conditions on the rate at which $l \uparrow \infty$ as $T \uparrow \infty$ we may proceed to estimate s^{2} from finite samples of data by sequentially estimating $s_{\mathrm{T}l}^{2}$. Therefore we define:

$$s_{\mathrm{T}l}^{2} = T^{-1}\sum_{1}^{T} u_1^2 + 2T^{-1}\sum_{t=1}^{l}\sum_{t=t+1}^{T} u_t u_{t-t}$$

which establishes that $s_{\mathrm{T}l}^{2}$ is a consistent estimator of s^{2}

The previous result establishes the following theorem:

<u>Theorem2:</u>

1. If $\{u_t\}_{t=1}^{\infty}$ satisfies (a), (c), and (d), and part (b) of the assumptions is replaced by the stronger moment condition: $\sup_t E|u_t|^{2\beta} < \infty$, for some $\beta > 2$,
2. If $l \uparrow \infty$ **as** $T \uparrow \infty$ such that $l = o(T^{1/4})$, then $\sigma^2_{Tl} \xrightarrow{p} \sigma^2$ as $T \uparrow \infty$.

According to this result, if we allow the number of estimated autocovariances to increase as $T \uparrow \infty$ but control the rate of increase so that $l = o(T^{1/4})$ then σ^2_{Tl} yields a consistent estimator of σ^2. White and Domowitz (1984) provide some guidelines for the selection of l. Inevitably the choice of l will be an empirical matter. In our own case, a preliminary investigations of the sample autocorrelations of $u_t = y_t - y_{t-1}$ will help selecting an appropriate choice of l. Since the sample autocorrelations of first differenced economic time series usually decay quickly it is likely that in moderate sample sizes quite a small value of l will be chosen.

Rather than using the first differences $u_t = y_t - y_{t-1}$ in the construction of σ^2_{Tl}, we could have used the residuals $\hat{u}_t = y_t - \hat{\rho}\, y_{t-1}$ from the least squares regression. Since $\hat{\rho} \xrightarrow{p} 1$ as $T \uparrow \infty$ this estimator is also consistent for σ^2 under the null hypothesis $\rho=1$. Moreover this estimator is consistent for σ^2 under explosive alternatives to $\rho=1$ (i.e. when $\rho > 1$) and may, therefore, be preferred to σ^2_{Tl} when such cases seem likely.

We remark that σ^2_{Tl} is not constrained to be nonnegative as it is defined in $s^2_{Tl} = T^{-1}\sum_{1}^{T} u_1^2 + 2T^{-1}\sum_{\tau=1}^{l}\sum_{t=\tau+1}^{T} u_t u_{t-\tau}$. When there are large negative sample serial covariances, σ^2_{Tl} can take on negative values. In a related context, Newey and West (1985) have suggested a modification to variance estimators such as s_{Tl}^{2} which ensures that they are nonnegative. In the presence case the modification yields:

$$s_{Tl}^{2} = T^{-1}\sum_{t=1}^{T} u_t^2 + 2T^{-1}\sum_{t=1}^{l} w_{Tl} \sum_{t=t+1}^{T} u_t u_{t-t}$$

$$\text{where } w_{Tl} = 1 - t/(l+1)$$

The above expression represents the weighted variance estimator. When $\{u_t\}_{t=1}^{\infty}$ is weakly stationary, $s^{2} = 2p\, f_u(0)$ where $f_u(l)$ is the spectral density of u_t. In this case, $(1/2p)s_{Tl}^{2}$ is the value of the origin $l = 0$ of the Bartlett estimate:

$$\hat{f}_u(l) = (1/2p)\sum_{t=-l-1}^{l+1} [1-|t|/(l+1)]C(t)e^{-ilt}$$

$$\text{where } C(t) = T^{-1}\sum_{t=|t|+1}^{T} u_t u_{t-|t|}$$

of $f_u(l)$. Since the Bartlett estimate is nonnegative everywhere, we deduced that $s_{Tl}^{2} \geq 0$ also. Of course, weights other than the ones we proposed are possible and may be inspired by other choices of lag window in the density estimate.

3.3 Relaxing further the Assumptions – A New Test

The following discussion concerns the asymptotic theory for first-order autoregressions with asymptotically unbounded error variance. This theory is the product of the work made by N. Kourogenis and N. Pittis and it extends the theory developed by Phillips to include cases where the variance grows without limit in a polynomial fashion. In particular they relax the restrictive assumption that u_t are all bounded, that is $\sup_t E|u_t|^b \ \mathbf{p} \ \infty$ for some β>2, thus precluding trending moments. They consider both stable and unit root processes assuming martingale difference and weakly dependent innovations respectively. Moreover for both these cases the asymptotic distributions of the OLS estimator of the autoregressive parameter along with that of the corresponding t-statistic are derived. For the purpose of this paper we will restrict the discussion in the unit root case.

As before, we need to begin the discussion with the assumptions imposed on the innovations process that drives the model assuming the same autoregressive process as before:

Let $\{y_t\}_{t=1}^{\infty}$ be a stochastic process generated in discrete time according to:

1. $y_t = ry_{t-1} + u_t$ (t=1,2,…)
2. $r = 1$

The initial conditions about y_0 are that y_0 is either an arbitrary constant or a random variable.

Assumptions on u_t:

a) $u_t = \sqrt{f(t)}u_t$ where $f(t) = b_k t^k + g(t)$ $\mathbf{f}$ 0 with $g(t) = \mathrm{O}(t^{k-1})$ if $k \geq 1$

and $g(t) = 0$ if $0 \leq k \leq 1$

b) $T^{-1}\sum_{1}^{T} E(u_t^2) \xrightarrow{p} s_u^2$ $\mathbf{p}$ ∞ (the variance of u_t)

c) $T^{-1}E(S_T^2) \rightarrow s^2$ $\mathbf{p}$ ∞

d) $X_T(r) = \frac{1}{\sqrt{T}s} S_{[Tr]} \xrightarrow{L} W(r)$

Assumption (a) allows the second unconditional moment of the error term u_t to grow in a polynomial-like fashion. Assumptions (b)-(d) allow for quite general weekly dependent and heterogeneously distributed u_t, similarly to those Phillips imposed.

<u>Remark:</u> What is very important to add at this point is that assumption (a) can be simplified as $f_1(t) = t^k + g_1(t)$ where $g_1(t) = \frac{1}{b_k} g(t)$ without any loss of generality, therefore for the rest of the discussion we assume we have $b_k = 1$.

The asymptotic behavior of u_t under the assumptions (a) and (b) is:

$$\frac{1}{T^{k+1}} \sum_{1}^{T} E(u_t^2) \xrightarrow{p} \frac{s_u^2}{k+1} \ \mathbf{p} \ \infty,$$

The previous results are necessary to obtain the asymptotic behavior of the OLS estimator of r and the t-statistic t_r.

This is shown in the following theorem:

<u>Theorem2:</u> If $\{u_t\}_1^\infty$ satisfies the former assumptions, then as $T \to \infty$:

$$T(\hat{r}-1) \xrightarrow{L} \frac{\frac{1}{2}\left(Q(1)^2 - \frac{s_u^2}{s^2(k+1)}\right)}{\int_0^1 Q(r)^2\,dr} \quad \text{and}$$

$$t_r \xrightarrow{L} \frac{s}{s_u} \frac{\frac{1}{2}\sqrt{k+1}\left(Q(1)^2 - \frac{s_u^2}{s^2(k+1)}\right)}{\left(\int_0^1 Q(r)^2\,dr\right)^{1/2}}$$

Where $Q(r) = r^{k/2}W(r) - \frac{k}{2}\int_0^r s^{k/2-1}W(s)ds$

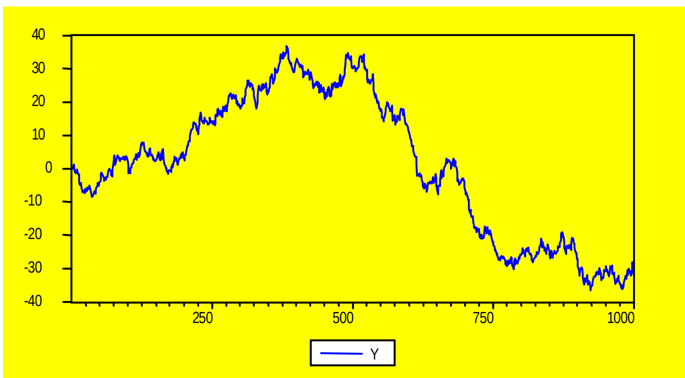
It is obvious that when $k = 0, b_0 = 1$ the asymptotic distributions are the same with the ones Phillips proposed.

3.4 From Theory to Real Data

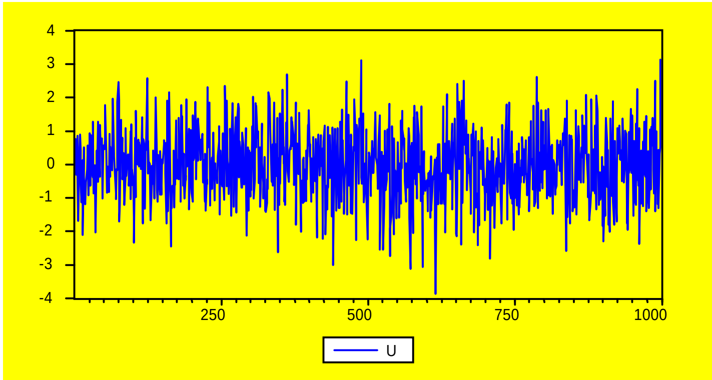
In time series analysis, what has been observed is that most of the economic and financial data tend to satisfy the unit root hypothesis. This result though, in several cases is rather misleading in away that these time series are proven to be, as econometricians say, “almost unit roots”. This means that sometimes the value the t-statistic very close to the critical value which in this case means that there is a relevant misconception whether the series under consideration has a unit root or is stationary.

As we said in the beginning, this paper is going to do is to test macroeconomic and financial time series under the assumption of polynomial trend in their conditional variance and compare the results to those of traditional tests, like the Phillips-Peron test.

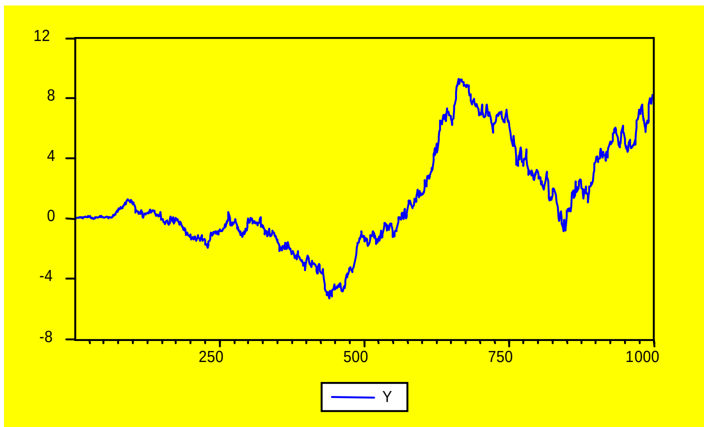
To demonstrate all these we will compare a case of a random walk with $iidN(0,1)$ disturbances and a case with a random walk with $u_t = \sqrt{t}u_t$ disturbances:



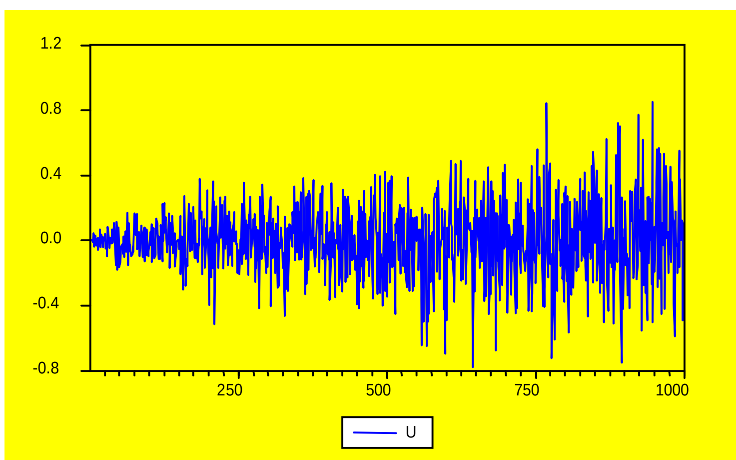
Random walk with $iidN(0,1)$



Disturbances $iidN(0,1)$



Random walk with $u_t = \sqrt{t}u_t$ (k=1)



Disturbances $u_t = \sqrt{t}u_t$ (k=1)

What is apparent from these diagrams is that both of them can describe a process:

1. $y_t = ry_{t-1} + u_t$ (t=1,2,…)
2. r =1

Which one describes better the process will be decided in unit root tests under different assumption on the sequence that drives the model.

PART 4

4. Methodology and Empirical Application of the new test

In order to test the new test we decided to use several time series which exhibit the statistical behavior of interest, that is pure random walks. This behavior is very common in foreign exchange rates, equity indexes, simple stocks, commodities and some macroeconomic variables.

Therefore we used daily data of several foreign exchange rates and equity indexes, few intraday data on equity indexes and some commodities. Information about the data can be found in the appendix (there someone can find the name of the series, its code (we used on the program) , the number of observations that are included for the test and the source from which each series was taken).

4.1 Methodology – Monte Carlo Simulation:

The new test is exclusively programmed in EViews

Firstly, for each series (the natural logarithm of them) we calculate the value of kappa (the exponent of the power of the polynomial $f(t) = b_k t^k + g(t) \mathbf{f}\ 0$ with $g(t) = O(t^{k-1})$ if $k \geq 1$ and $g(t) = 0$ if $0 \leq k \leq 1$) with its sample estimator:

$$k = \frac{1}{\log 2} \log \frac{\sum_{t=1}^{T} \hat{u}_t^2}{\sum_{t=1}^{T/2} \hat{u}_t^2} - 1$$

Then the values of s_u^2 and s^2 are estimated by their sample estimators s_u^2 and s_{Tl}^2 respectively:

$$s_u^2 = \frac{k+1}{T^{k+1}} \sum_{t=1}^{T} \hat{u}_t^2 \xrightarrow{p} s_u^2 \ \mathbf{p} \ \infty$$

$$s_{\mathrm{T}l}^2 = \frac{(k+1)}{T^{(k+1)}} \sum_{t=1}^{T} \hat{u}_t^2 + 2T^{-1} \sum_{t=1}^{l} w_{Tl} \sum_{t=t+1}^{T} \frac{\hat{u}_t \hat{u}_{t-t}}{(t(t-t))^{k/2}} \xrightarrow{p} s^2$$

What is important to add at this point is that we estimate s^2 using the methodology Phillips used in his framework. That is we weigh the autocovariance sample estimators (the second part of the above estimator s_{Tl}^2) with $w_{Tl} = 1 - t/(l+1)$ where l is the lag truncation number (the Newley-West bandwidth), which is taken by the Phillips-Perron test.

Then we calculate the values of the OLS estimator of r and the subsequent t-statistic t_r with their sample estimators $Z_{k,r}$ and $Z_{k,t}$ respectively:

$$Z_{k,r} = T(\hat{r}_T - 1) - \frac{1}{2(k+1)} \frac{s^2 - s_v^2}{T^{-(k+2)} \sum_{t=1}^{T} y_{t-1}^2}$$

$$Z_{k,t} = \frac{s_v}{s} t_r - \frac{1}{2\sqrt{k+1}} \frac{s^2 - s_v^2}{s\left(T^{-(k+2)} \sum_{t=1}^{T} y_{t-1}^2\right)^{1/2}}$$

Finally for all series we calculate the Phillips-Perron t-statistic without intercept or tend, using the Bartlett spectral estimation method (Bartlett kernel) and the Newley-West bandwidth.

The critical values for the new test are calculated through Monte Carlo analysis for various sample sizes (30, 50, 100, 250, 500, 1000 and 5000$\approx\infty$), for kappa = 0,1,2,3 and with three different assumptions on u_t (IID, AR(1) with r=0.5 and MA(1) with θ=0.7). The number of replications we chose for this analysis is 1000 and the results can be found in the Appendix II. For the case of infinite sample and IID secondary innovations the critical values are shown in the following tables:

Table 1: Critical Values for the $Z_{k,p}$ Statistic with IID secondary innovations (∞)

Kappa	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0	-12.476	-9.897	-7.349	-5.197	0.938	1.258	1.600	2.097
1	-15.629	-11.746	-9.242	-7.083	1.485	2.193	2.762	3.466
2	-21.677	-16.291	-12.589	-9.079	2.017	2.882	3.646	5.165
3	-23.722	-19.702	-15.575	-10.912	2.605	3.488	4.564	6.244

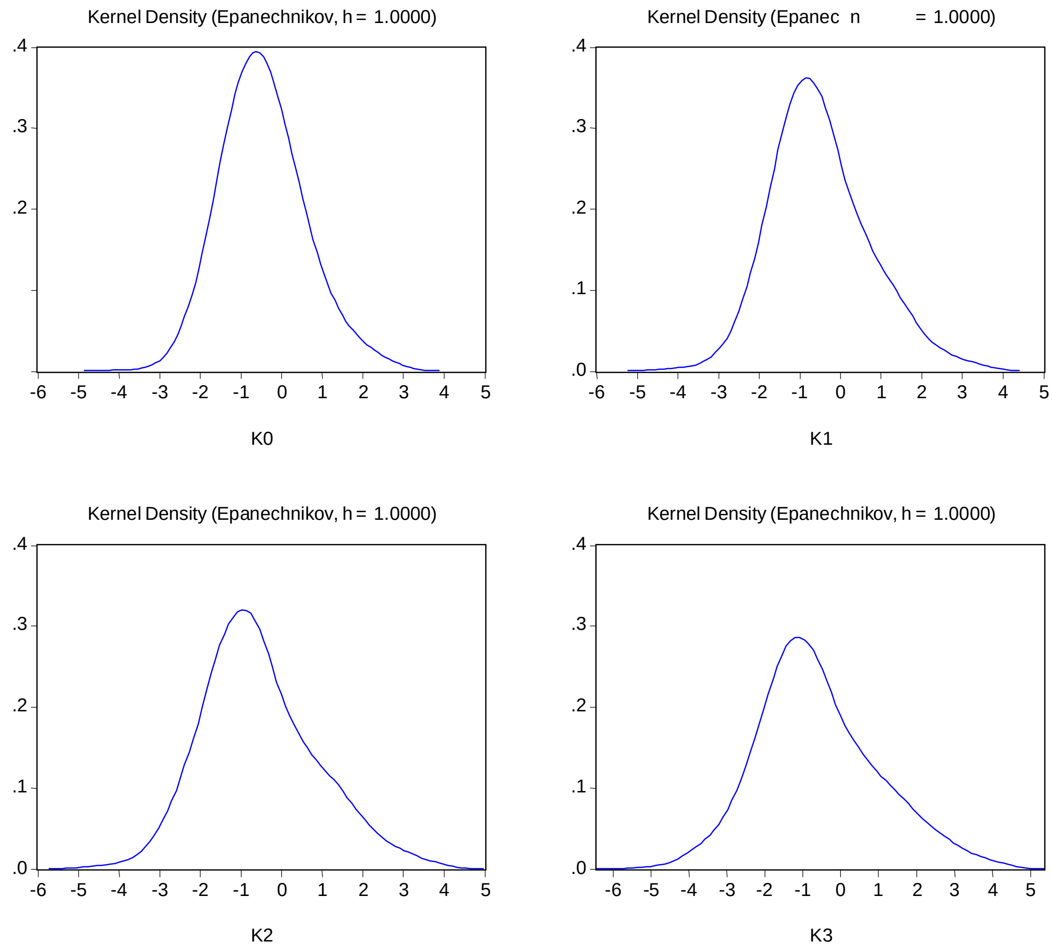
Table 2: Critical Values for the $Z_{k,t}$ Statistic with IID secondary innovations (∞)

Kappa	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0	-2.462	-2.143	-1.839	-1.556	0.897	1.312	1.586	1.897
1	-2.719	-2.366	-2.098	-1.822	1.150	1.590	2.079	2.646
2	-3.283	-2.826	-2.456	-2.034	1.237	1.938	2.594	3.094
3	-3.437	-3.065	-2.650	-2.226	1.348	2.163	2.829	3.459

These values will be used to compare the new test with the traditional Phillips-Perron test.

<u>Remark</u>**:** Secondary innovations are u_t.

It is apparent from the above tables that the distribution of $Z_{k,t}$ opens up. This can be seen at the following kernel densities of $Z_{k,t}$ for k=0,1,2,3:



4.2 Empirical Results

The following tables depict all the series that have significant value of kappa, the exponent of interest of polynomial. As it obvious, all values of kappa are smaller than unity; therefore the first table has the critical values for k=1 and IID secondary innovations. The rest of them can be found in Appendix I

Critical values

Levels	KP Critical Values K=1	PP Critical Values
1%	-2.719	-2.565
5%	-2.098	-1.940
10%	-1.822	-1.616

Results of daily data

Variable	Y9	Ro estimate	1.000028
Description	FX Thailand / U.S.	Kappa estimate	0.873441
Observations	5997	KP t-statistic	1.313605
Source	DataStream	PP t-statistic	1.141234

Variable	Y1	Ro estimate	0.99997
Description	FX Canada / U.S.	Kappa estimate	0.83381
Observations	8551	KP t-statistic	-0.2536
Source	DataStream	PP t-statistic	-0.2563

Variable	Y20	Ro estimate	1.00003
Description	FX Malaysia / U.K.	Kappa estimate	0.83023
Observations	6501	KP t-statistic	0.42752
Source	DataStream	PP t-statistic	0.44213

Variable	Y53	Ro estimate	1.00002
Description	FX H. Kong / Austr	Kappa estimate	0.77954
Observations	3025	KP t-statistic	0.4085
Source	DataStream	PP t-statistic	0.33096

Variable	Y99	Ro estimate	1.00003
Description	Total Mkt: Portugal	Kappa estimate	0.65951
Observations	3915	KP t-statistic	0.60306
Source	DataStream	PP t-statistic	0.69406

Variable	Y19	Ro estimate	1.00001
Description	FX Korea / U.K.	Kappa estimate	0.58943
Observations	6501	KP t-statistic	0.46833
Source	DataStream	PP t-statistic	0.46256

Variable	Y54	Ro estimate	1.00001
Description	FX H Kong / Canad	Kappa estimate	0.53572
Observations	3025	KP t-statistic	0.27859
Source	DataStream	PP t-statistic	0.27664

Variable	Y81	Ro estimate	1.00001
Description	Nikkei (Japan)	Kappa estimate	0.45745
Observations	6523	KP t-statistic	0.48408
Source	DataStream	PP t-statistic	0.49006

Variable	Y72	Ro estimate	1.00001
Description	Nikkei Average	Kappa estimate	0.42946
Observations	6565	KP t-statistic	0.50608
Source	DataStream	PP t-statistic	0.51154

Variable	Y66	Ro estimate	1.00004
Description	DAX (Germany)	Kappa estimate	0.42304
Observations	6565	KP t-statistic	1.88896
Source	DataStream	PP t-statistic	1.88505

Variable	Y32	Ro estimate	1.00001
Description	FX Denmark / Japan	Kappa estimate	0.4194
Observations	4696	KP t-statistic	0.12164
Source	DataStream	PP t-statistic	0.12733

Variable	Y79	Ro estimate	1.00003
Description	Dow Jones Utilities	Kappa estimate	0.40709
Observations	6524	KP t-statistic	1.33601
Source	DataStream	PP t-statistic	1.37376

Variable	Y64	Ro estimate	1.00006
Description	AEX INDEX (AEX)	Kappa estimate	0.32716
Observations	5781	KP t-statistic	1.73979
Source	DataStream	PP t-statistic	1.80589

Variable	Y15	Ro estimate	1.00011
Description	FX China / U.S.	Kappa estimate	0.29248
Observations	6017	KP t-statistic	2.18117
Source	DataStream	PP t-statistic	2.18656

Variable	Y7	Ro estimate	0.99991
Description	FX Singapore / U.S.	Kappa estimate	0.29138
Observations	6077	KP t-statistic	-1.2438
Source	DataStream	PP t-statistic	-1.2383

Variable	Y3	Ro estimate	0.99997
Description	FX Japan / U.S.	Kappa estimate	0.24959
Observations	8551	KP t-statistic	-1.8939
Source	DataStream	PP t-statistic	-1.9058

Variable	Y4	Ro estimate	0.99999
Description	FX Norway / U.S.	Kappa estimate	0.23117
Observations	8551	KP t-statistic	-0.359
Source	DataStream	PP t-statistic	-0.359

Variable	Y75	Ro estimate	1.00004
Description	BEL 20 (Belgium)	Kappa estimate	0.22877
Observations	6524	KP t-statistic	2.20486
Source	DataStream	PP t-statistic	2.25605

Variable	Y5	Ro estimate	1.00001
Description	FX Sweden / U.S.	Kappa estimate	0.20831
Observations	8551	KP t-statistic	0.32697
Source	DataStream	PP t-statistic	0.32798

Variable	Y48	Ro estimate	0.99956
Description	FX Rate Korea/US.	Kappa estimate	0.20716
Observations	2670	KP t-statistic	-0.9322
Source	DataStream	PP t-statistic	-0.9346

Variable	Y48	Ro estimate	0.99956
Description	FX Rate Korea/US.	Kappa estimate	0.20716
Observations	2670	KP t-statistic	-0.9322
Source	DataStream	PP t-statistic	-0.9346

Variable	Y78	Ro estimate	1.00005
Description	Dow Jones Transp.	Kappa estimate	0.18247
Observations	6524	KP t-statistic	2.17664
Source	DataStream	PP t-statistic	2.21488

Variable	Y25	Ro estimate	1
Description	FX Norway / U.K.	Kappa estimate	0.16337
Observations	6524	KP t-statistic	0.07633
Source	DataStream	PP t-statistic	0.07495

Variable	Y29	Ro estimate	0.99932
Description	FX Switzer. / Japan	Kappa estimate	0.16269
Observations	8550	KP t-statistic	-1.7804
Source	DataStream	PP t-statistic	-1.796

Variable	Y50	Ro estimate	1.00017
Description	FX Brazil / Switzer.	Kappa estimate	0.15444
Observations	2670	KP t-statistic	0.14469
Source	DataStream	PP t-statistic	0.14745

Variable	Y28	Ro estimate	0.9999
Description	FX Switzer. / U.K.	Kappa estimate	0.14631
Observations	6524	KP t-statistic	-1.3548
Source	DataStream	PP t-statistic	-1.3664

Variable	Y40	Ro estimate	1.00011
Description	FX Brazil / Canada	Kappa estimate	0.1358
Observations	2670	KP t-statistic	0.11193
Source	DataStream	PP t-statistic	0.10242

Variable	Y80	Ro estimate	1.00006
Description	MILAN MIB 30	Kappa estimate	0.0479
Observations	6524	KP t-statistic	2.1828
Source	DataStream	PP t-statistic	2.19417

Variable	Y10	Ro estimate	0.99984
Description	FX U.S. / Australia	Kappa estimate	0.04566
Observations	8551	KP t-statistic	-0.7599
Source	DataStream	PP t-statistic	-0.7598

Variable	Y67	Ro estimate	1.00004
Description	FTSE100 (England)	Kappa estimate	0.02948
Observations	6565	KP t-statistic	2.46396
Source	DataStream	PP t-statistic	2.47287

Variable	Y93	Ro estimate	1.00005
Description	Total Mkt: France	Kappa estimate	0.02882
Observations	8351	KP t-statistic	2.15179
Source	DataStream	PP t-statistic	2.1573

Variable	Y84	Ro estimate	1.00003
Description	Singap Straits Times	Kappa estimate	0.01525
Observations	5216	KP t-statistic	0.95856
Source	DataStream	PP t-statistic	0.95724

Variable	Y12	Ro estimate	0.99989
Description	FX U.S. / U.K.	Kappa estimate	0.01177
Observations	8551	KP t-statistic	-1.0188
Source	DataStream	PP t-statistic	-1.0232

Variable	Y22	Ro estimate	0.99999
Description	FX Denmark / U.K.	Kappa estimate	0.0021
Observations	6524	KP t-statistic	-0.3854
Source	DataStream	PP t-statistic	-0.3862

Results of intraday data with kappa>0.4

Dow Jones Industrial

Variable	Day Y89	Ro estimate	1
Description	Dow Jones Indust.	Kappa estimate	0.994163
Observations	2340	KP t-statistic	0.046627
Source	Dukascopy.net	PP t-statistic	0.029331

Variable	Day Y83	Ro estimate	0.9999998
Description	Dow Jones Indust.	Kappa estimate	0.6476177
Observations	2340	KP t-statistic	-0.4487252
Source	Dukascopy.net	PP t-statistic	-0.4846127

Variable	Day Y67	Ro estimate	1.0000003
Description	Dow Jones Indust.	Kappa estimate	0.5920893
Observations	2340	KP t-statistic	0.969043
Source	Dukascopy.net	PP t-statistic	0.5402505

Variable	Day Y52	Ro estimate	0.9999996
Description	Dow Jones Indust.	Kappa estimate	0.5244324
Observations	2340	KP t-statistic	-1.3556701
Source	Dukascopy.net	PP t-statistic	-1.3506685

FTSE100

Variable	Day Y13	Ro estimate	0.9999997
Description	FTSE 100	Kappa estimate	1.3505868
Observations	3000	KP t-statistic	-0.1316636
Source	Dukascopy.net	PP t-statistic	-1.1002111

Variable	Day Y14	Ro estimate	0.9999999
Description	FTSE 100	Kappa estimate	1.030446
Observations	3000	KP t-statistic	-0.0807578
Source	Dukascopy.net	PP t-statistic	-0.377579

Variable	Day Y12	Ro estimate	1.0000002
Description	FTSE 100	Kappa estimate	0.5929725
Observations	3000	KP t-statistic	0.3656941
Source	Dukascopy.net	PP t-statistic	0.9928287

S&P 500

Variable	Day Y21	Ro estimate	1
Description	S&P 500	Kappa estimate	0.41133
Observations	2340	KP t-statistic	-0.0494
Source	Dukascopy.net	PP t-statistic	-0.0491

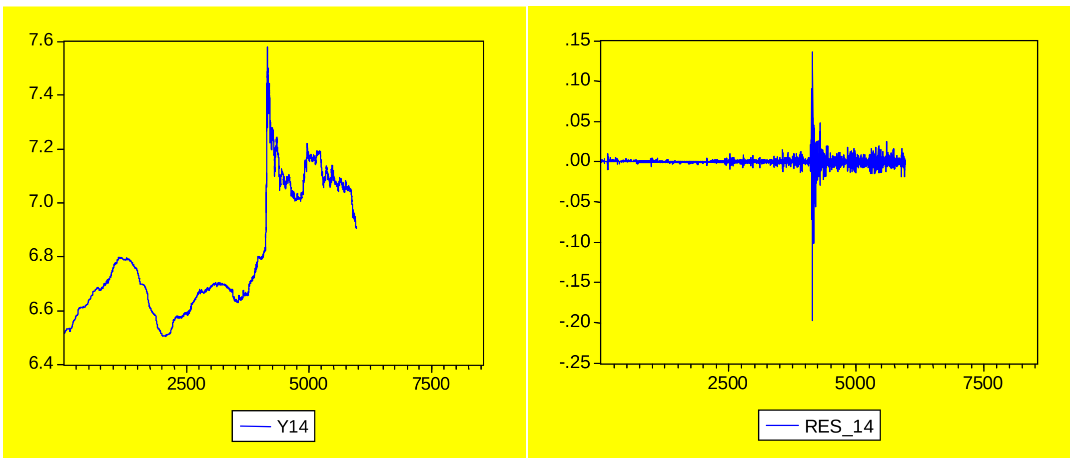
4.3 Comments on the Results

From the tables above, someone can infer that we cannot reject the null hypothesis; the variable Y has a unit root. The values that our t-statistic takes don't differ significantly from the Phillips-Perron values, especially when kappa takes values around zero. As kappa grows, all we can infer is that our t-statistic values are smaller than the values of Phillips-Perron t-statistic, but not significantly enough, in a way that we could reject the null.

4.4 The New Test when the Model is Misspecified

Many of the key macroeconomic and financial variables are characterized by permanent volatility shifts. It is known that conventional unit root tests are unreliable in the presence of such behavior. Similar cases were examined in this paper in which the implementation of the new test led to rather misleading results when the model is not a pure random walk. The case of Argentina's exchange rates, where we have an exogenous shock, a devaluation of Argentina's currency (Variables: Y16, Y33 Y34 Y35 Y38), the model was inappropriate to describe these unusual movements and the similar but extreme case of the FX Rate between Korea and U.S.:

Variable	Y14	Ro estimate	1.0000090
Description	FX Rate Korea/US.	Kappa estimate	5.1953457
Observations	5964	KP t-statistic	0
Source	DataStream	PP t-statistic	0.5581636



What we observe is that the exogenous shock is responsible for the extreme value kappa takes (Kappa=5.1953). This estimated value is incorporated in the calculation of all the other estimators' values ($s_u^2, s_{Tl}^2, Z_{k,r}$ and $Z_{k,t}$) and literally cancels out the test. To be precise, the extreme kappa is responsible the negative value of the estimator s_{Tl}^2, even in context of the Newey-West modification, which makes impossible to take a value for the t-statistic $Z_{k,t}$.

PART 5

5 Conclusions

In this paper we have examined a new test for unit roots, both theoretically and empirically. The new test is built upon more general assumptions than the traditional tests, like the Phillips-Perron test, and contain the latter as a special case. The old tests have a common assumption; the variances of the innovations that drives the model are bounded, thus precluding trending moments. For the development of the test we have used the asymptotic results obtained by the work of Pittis and Kourogenis for first-order autoregressive models with a unit root, when the innovations that drives the model grows in a polynomial fashion.

Theoretically, using Monte-Carlo simulation, we examined the behavior of the test for four different cases of error terms; we used polynomials of order zero to three. This approach showed that as the value of the exponent of the polynomial grows, the critical values of the test becomes significantly smaller than those of the traditional Phillips-Perron test, a result someone can observe watching the distribution of the t-statistic; the distribution opens up.

On the other hand, empirically the test didn’t provide us with any interesting result. The new test, that we used on several time series, failed to reject the null, that is unit root under the polynomial trend, due to small estimates of kappa while it misbehaved when our series were characterized by structural brakes.

APPENDIX I

Daily Data

Foreign Exchange Rates

Variable Y	Observations	Code	Description
1	8551	DEXCAUS	Canada / U.S.
2	8071	DEXINUS	India / U.S.
3	8551	DEXJPUS	Japan / U.S.
4	8551	DEXNOUS	Norway / U.S.
5	8551	DEXSDUS	Sweden / U.S.
6	8551	DEXSFUS	South Africa / U.S.
7	6077	DEXSIUS	Singapore / U.S.
8	8551	DEXSZUS	Switzerland / U.S.
9	5997	DEXTHUS	Thailand / U.S.
10	8551	DEXUSAL	U.S. / Australia
11	8551	DEXUSNZ	U.S. / New Zealand
12	8551	DEXUSUK	U.S. / U.K.
13	5094	DEXTAUS	Taiwan / U.S.
14	5964	DEXKOUS	South Korea / U.S.
15	6017	DEXCHUS	China / U.S.
16	5403	(ARG_UK	Argentina / U.K.
17	6501	CHILE_UK	Chile / U.K.
18	6501	HK_UK	Hong Kong / U.K.
19	6501	KOR_UK	Korea / U.K.
20	6501	MAL_UK	Malaysia / U.K.
21	6501	SING_UK	Singapore / U.K.
22	6524	DEN_UK	Denmark / U.K.
23	6524	IND_UK	India / U.K.
24	6524	IRL_UK	Ireland / U.K.
25	6524	NOR_UK	Norway / U.K.
26	6524	PHIL_UK	Philippines / U.K.
27	6524	CAN_UK	Canada / U.K.
28	6524	SWISS_UK	Switzerland / U.K.
29	8550	SWISS_JAP	Switzerland / Japan
30	8550	SWISS_CAN	Switzerland / Canada
31	4696	DEN_CAN	Denmark / Canada
32	4696	DEN_JAP	Denmark / Japan
33	3681	ARCADSP	Canada / Argentine
34	3681	ARHKDSP	Hong Kong / Argentine
35	3681	ARJPYSP	Japan / Argentine
36	3681	ARNZDSP	Argentine / New Zealand
37	3681	ARSEKSP	Sweden / Argentine
38	3681	ARZARSP	South Africa / Argentine
39	2670	BRAUDSP	Brazil / Australia
40	2670	BRCADSP	Brazil / Canada
41	2670	BRDKKSP	Brazil / Denmark

Variable Y	Observations	Code	Description
42	2670	BRHKDSP	Brazil / Hong Kong
43	2670	BRINRSP	Brazil / India
44	2670	BRMYRSP	Brazil / Malaysia
45	2670	BRNZDSP	Brazil / New Zealand
46	2670	BRSARSP	Brazil / Saudi Arabia
47	2670	BRSEKSP	Brazil / Sweden
48	2670	BRSGBSP	Brazil / Singapore
49	2670	BRSURSP	Brazil / Russia
50	2670	BRSWFSP	Brazil / Switzerland
51	1106	BRXEUSP	Brazil / ECU
52	2670	BRZARSP	Brazil / South Africa
53	3025	HKAUDSP	Hong Kong / Australia
54	3025	HKCADSP	Hong Kong / Canada
55	3025	HKCGFSP	Hong Kong / Switzerland
56	3025	HKJPYSP	Hong Kong / Japan
57	3025	HKMYRSP	Hong Kong / Malaysia
58	3025	HKSGDSP	Hong Kong / Singapore
59	3025	HKTHBSP	Hong Kong / Thailand
60	2669	MXBECSP	Mexico / Belgium
61	2669	MXCADSP	Mexico / Canada
62	2669	MXDEMSP	Mexico / Germany
63	2669	MXFRFSP	Mexico / France

Stock Market Indexes

Variable Y	Observations	Code	Description
64	5781	AMSTEOE	AEX INDEX (AEX)
65	6565	AUSTOLD	ASX ALL ORDINARIES 1971
66	6565	DAXINDX	DAX (Germany)
67	6565	FTSE100	FTSE100 (England)
68	6565	HNGKNGI	Hang Seng NGI
69	6565	IFGMAR$	S&P/IFCG M ARGENTINA
70	6565	IFGWJO$	S&P/IFCG W JORDAN
71	5779	ISEQUIT	RELAND SE OVERALL (ISEQ)
72	6565	JAPDOWA	NIKKEI 225 STOCK AVERAGE
73	4998	PSECOMP	PHILIPPINES SE COMPOSITE
74	6281	WIEIREL	FTSE W IRELAND
75	6524	BEL	BEL 20 (Belgium)
76	6524	US_S_P50001	S&P 500 (U.S.)
77	6524	DJ_INDUS	Dow Jones Industrial (U.S.)
78	6524	DJ_TRSPT	Dow Jones Transportation (U.S.)
79	6524	DJ_UTILS	Dow Jones Utilities (U.S.)
80	6524	IT_30	MILAN MIB 30
81	6523	JP_NIKKEI	Nikkei (Japan)
82	6524	NASCOMP	NASDAQ COMPOSITE
83	6524	NYSE_ALL	New York Stock Exchange All
84	5216	SNGPORI	SINGAPORE STRAITS TIMES
85	4436	TOTMKAR	Total Market: Argentina

Variable Y	Observations	Code	Description
86	8351	TOTMKAU	Total Market: Australia
87	2741	TOTMKBR	Total Market: Brazil
88	3486	TOTMKCA	Total Market: China A
89	2986	TOTMKCH	Total Market: China
90	8350	TOTMKCN	Total Market: Canada
91	3139	TOTMKCP	Total Market: Cyprus
92	4377	TOTMKFN	Total Market: Finland
93	8351	TOTMKFR	Total Market: France
94	3914	TOTMKIN	Total Market: India
95	8351	TOTMKIR	Total Market: Ireland
96	3132	TOTMKIS	Total Market: Israel
97	8351	TOTMKIT	Total Market: Italy
98	4519	TOTMKPH	Total Market: Philippines
99	3915	TOTMKPT	Total Market: Portugal

Commodities

Variable Y	Observations	Code	Description
100	5571	OILBREND	Crude Oil Brent

All the above series are taken from DATASTREAM

Results of daily data

Variable Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
1	0.9999707	0.8338131	-0.2536414	-0.2562589
2	1.0000677	0	4.1715508	4.1733739
3	0.9999709	0.2495874	-1.8938849	-1.9057779
4	0.9999876	0.2311723	-0.3590067	-0.3589686
5	1.0000125	0.2083144	0.3269656	0.3279819
6	1.0001046	0	1.2815273	1.2806069
7	0.9999141	0.2913789	-1.243778	-1.2382703
8	0.9996725	0	-2.7126547	-2.7138421
9	1.0000281	0.8734408	1.3136054	1.141234
10	0.9998404	0.0456553	-0.7599131	-0.7598407
11	0.9999147	0	-0.4986292	-0.4998042
12	0.9998895	0.011766	-1.0187524	-1.0231831
13	0.9999848	0.9445832	-0.9738193	-1.0750342
14	1.000009	5.1953457	0	0.5581636
15	1.0001128	0.2924766	2.181174	2.1865608
16	0.9993461	0	-6.1586826	-6.1557286
17	1.0000592	0	2.9675866	2.9698059
18	1.0000158	0	0.4637754	0.4637308
19	1.0000081	0.5894274	0.4683254	0.462557
20	1.0000303	0.8302329	0.4275193	0.442128
21	0.999913	0	-1.3116266	-1.3118303
22	0.9999903	0.0020966	-0.3854415	-0.386195
23	1.0000631	0	2.877065	2.8725584
24	0.9995947	0	-1.1112485	-1.1108701
25	1.0000016	0.1633665	0.0763273	0.0749472

Variable Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
26	1.0000746	0	2.3760169	2.3776463
27	0.9999398	0	-0.5996107	-0.6011298
28	0.9999028	0.1463132	-1.3547501	-1.3664101
29	0.9993173	0.1626944	-1.7804224	-1.7959747
30	0.9994835	0	-3.1753988	-3.1727641
31	0.999994	0	-0.3728774	-0.3721629
32	1.0000079	0.4193964	0.1216353	0.1273335
33	0.999597	1.4305138	-4.8218928	-0.7939126
34	0.999724	1.4172179	-1.5041179	-2.1886985
35	0.9998769	1.0468564	-1.1663394	-2.1476681
36	0.9994937	1.3309166	-3.0136533	-1.3761911
37	0.9997694	0.9054598	-1.2559599	-1.6665566
38	0.9997978	1.5568904	-2.1632116	-1.0512582
39	0.9992414	0	-0.2402401	-0.240541
40	1.0001118	0.1358012	0.1119296	0.1024209
41	0.9988382	0	-1.4710767	-1.4687993
42	0.9994115	0	-1.9240541	-1.9241605
43	0.9997221	0	-1.1982429	-1.196479
44	0.9990076	0	-1.4583044	-1.4583729
45	0.928559	0	-10.210154	-10.213923
46	0.9994085	0	-2.2459075	-2.2469373
47	0.9995156	0	-1.8586985	-1.8601199
48	0.9995601	0.2071611	-0.9322145	-0.9345966
49	0.9990013	0	-0.9674308	-0.9679724
50	1.0001678	0.1544358	0.1446855	0.1474507
51	0.9962369	3.5399614	0	0.051151
52	0.9975255	0	-1.0364866	-1.0366921
53	1.0000217	0.7795396	0.4085028	0.330959
54	1.0000105	0.5357223	0.2785861	0.2766428
55	1.0000399	0	0.5781722	0.578127
56	0.9999898	0	-0.2062466	-0.2060771
57	0.9998226	0	-1.3170731	-1.3139719
58	0.9999954	0	-0.1083743	-0.1084983
59	0.9995869	3.8953817	0	0.227894
60	0.9997107	0	-1.7409684	-1.7377726
61	1.0002084	0	1.8350284	1.8336286
62	1.0001197	0	0.737739	0.739301
63	0.9992999	0	-0.9904285	-0.9897958
64	1.0000594	0.3271574	1.739786	1.8058918
65	1.0000419	0	2.0599089	2.0539434
66	1.0000406	0.4230436	1.8889587	1.8850465
67	1.0000409	0.0294801	2.463962	2.4728683
68	1.0000461	0	1.5993488	1.5979701
69	1.0000033	0	0.0414952	0.041503
70	1.0000589	0	2.3113589	2.3119897
71	1.0000631	0	2.6000714	2.5965983
72	1.0000078	0.4294596	0.5060807	0.5115449
73	1.0000632	0	1.3772162	1.3753175
74	1.0000688	0	1.9078881	1.9059861
75	1.0000387	0.2287684	2.204859	2.2560538

Variable Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
76	1.0000572	0	2.875528	2.879621
77	1.0000462	0	3.0256141	3.0340286
78	1.0000538	0.182465	2.1766389	2.2148838
79	1.000031	0.407091	1.3360102	1.3737551
80	1.0000633	0.0478995	2.1827956	2.1941718
81	1.0000075	0.4574524	0.4840795	0.4900639
82	1.0000581	1.2048984	2.5237051	2.1463637
83	1.0000455	0	3.0594352	3.056678
84	1.0000292	0.0152542	0.9585598	0.9572358
85	1.0001251	0	2.2783119	2.2811516
86	1.000052	0	2.0802548	2.0775625
87	1.0001081	0	1.6993828	1.6982825
88	1.0000819	0	1.1621464	1.1616611
89	1.0000461	0	0.4927743	0.4926476
90	1.00005	0	2.6188102	2.6171039
91	1.0000035	1.4290038	-0.1480898	-0.0372473
92	1.0000587	1.3385485	0	1.0317691
93	1.0000525	0.0288218	2.1517923	2.1572968
94	1.0000899	0	1.4610483	1.4593207
95	1.0000636	0	2.3353134	2.3412533
96	1.0000635	0	1.2866299	1.2873796
97	1.0000538	0	1.9303757	1.9320312
98	1.0000596	0	1.2601102	1.2616222
99	1.0000273	0.6595127	0.6030598	0.694058
100	0.9999873	0	-0.1039725	-0.1038991

Intraday Data

Description	Dow Jones Industial	FTSE100	S&P 500
Days Included	99	39	39
Obsrevations per day	2340	3000	2340

Dow Jones Industrial

Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
1	0.9999997	0	-0.74635	-0.7463219
2	0.9999996	0	-1.2306	-1.2326473
3	0.9999999	0	-0.4904	-0.4905861
4	1.0000001	0	0.356722	0.356634
5	0.9999999	0	-0.35669	-0.3566132
6	1.0000001	0	0.292782	0.2921831
7	0.9999998	0	-0.88798	-0.8874949
8	1.0000003	0	0.97478	0.9739179
9	0.9999995	0	-1.89704	-1.8976871
10	1.0000002	0	0.930554	0.9286539
11	0.9999999	0	-0.32038	-0.3200663
12	0.9999997	0.112752	-0.90325	-0.9160382
13	0.9999996	0	-1.24944	-1.2474792
14	0.9999999	0	-0.29144	-0.2915111
15	1.0000004	0	1.267315	1.2680576
16	1.0000002	0	0.634536	0.6346197
17	0.9999999	0	-0.4642	-0.463917
18	1.0000002	0	0.71292	0.7129301
19	1.0000003	0	1.472171	1.4762488
20	1.0000002	0.3379745	1.004307	0.9432111
21	1	0	-0.01791	-0.0179289
22	1	0	-0.00543	-0.0054265
23	1.0000001	0	0.396926	0.3968255
24	0.9999997	0	-1.34717	-1.3477095
25	1.0000004	0.0451988	1.467121	1.5110625
26	1.0000002	0	0.885641	0.8836909
27	1	0	-0.126	-0.1256318
28	1.0000002	0.0295961	1.046034	1.0451242
29	1	0	-0.04941	-0.0493626
30	0.9999997	0	-1.60343	-1.6004732
31	1.0000001	0	0.667484	0.6661017
32	0.9999993	0	-2.40836	-2.4014226
33	1.0000003	0	1.018557	1.020969
34	1.0000003	0	1.114688	1.1125094
35	1.0000004	0	2.041538	2.0468145
36	0.9999997	0	-1.39986	-1.3941108
37	1.0000003	0	1.204947	1.2021396
38	1	0	-0.14699	-0.1470633
39	1	0	0.140532	0.1402408
40	1.0000005	0	1.609797	1.6065288
41	1	0.1100235	-0.00459	-0.0045964

Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
42	0.9999999	0	-0.52945	-0.5281415
43	0.9999996	0	-1.48778	-1.4870612
44	1.0000002	0	0.690863	0.6902216
45	0.9999997	0	-1.24372	-1.2420475
46	1.0000001	0	0.383565	0.3836917
47	0.9999998	0	-0.95149	-0.9524743
48	0.9999995	0	-1.74557	-1.7454725
49	1	0	-0.10188	-0.102002
50	1	0	-0.00032	-0.0003172
51	0.9999997	0	-0.99646	-0.9954187
52	0.9999996	0.5244324	-1.35067	-1.3556701
53	0.9999999	0	-0.293	-0.2933386
54	0.9999999	0	-0.29328	-0.2927449
55	1	0	1.003445	1.0001971
56	1.0000002	0	0.861188	0.8592039
57	0.9999997	0.0845513	-1.15493	-1.1600806
58	1.0000006	0.03226	2.075914	2.1070663
59	0.9999998	0	-0.76413	-0.7656735
60	0.9999995	0	-1.12222	-1.1210455
61	1.0000001	0.0087057	0.244353	0.2449898
62	1.0000002	0.0666542	0.707773	0.70976
63	1.0000001	0	0.49511	0.4956345
64	1.0000003	0	1.26958	1.2708859
65	0.9999996	0	-1.92525	-1.9209998
66	1	0	-0.17107	-0.1710934
67	1.0000003	0.5920893	0.540251	0.969043
68	0.9999995	0.0837395	-1.77653	-1.7878442
69	0.9999995	0.2100317	-1.61729	-1.6286761
70	0.9999991	0	-1.981	-1.9828129
71	0.9999999	0	-0.20738	-0.2076723
72	1.0000003	0	0.811244	0.810155
73	0.9999995	0	-1.29625	-1.2949706
74	1.0000009	0	1.809303	1.8047464
75	0.9999997	0.1582096	-0.6734	-0.6805607
76	1.0000003	0	1.037981	1.0387046
77	0.9999996	0.1029103	-1.17915	-1.2149342
78	1.0000002	0	0.607472	0.6070301
79	0.9999994	0	-1.57994	-1.5790744
80	1.0000005	0	1.080878	1.0795005
81	1	0.9941632	0.029331	0.0466267
82	1.0000005	0	1.645433	1.6443097
83	0.9999998	0.6476177	-0.48461	-0.4487252
84	1	0	0.078162	0.0781511
85	1.0000002	0	0.785666	0.7864734
86	0.9999995	0	-1.34303	-1.3404792
87	1.0000001	0	0.259024	0.2592452
88	0.9999995	0	-1.41917	-1.4170477
89	0.9999998	0.1473979	-0.57757	-0.5758451
90	1.0000005	0	2.089144	2.0864832
91	1.0000003	0	1.187509	1.1888702

Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
92	1.0000006	0	1.869516	1.872302
93	1.0000001	0.0011425	0.467541	0.467701
94	0.9999999	0.1132398	-0.34533	-0.3512107
95	1.0000002	0.0441087	1.056443	1.0569606
96	0.9999999	0.0926769	-0.25658	-0.2634659
97	0.9999998	0	-0.68718	-0.6873847
98	1.0000003	0	1.445787	1.4437251
99	1	0	0.292615	0.2924833

FTSE100

Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
1	1.0000002	0.1769204	1.3781449	1.3345659
2	0.9999999	0.0303881	-0.5461821	-0.5399847
3	1.0000003	0	1.5864642	1.5879972
4	1.0000001	0.0454218	0.4216716	0.4266769
5	1.0000001	0	0.3463147	0.3460903
6	0.9999999	0	-0.4780599	-0.4768736
7	0.9999998	0	-1.7077357	-1.705959
8	1.0000001	0	0.3512806	0.3505513
9	0.9999999	0	-0.4163291	-0.4157222
10	0.9999997	0	-1.2137493	-1.2136475
11	0.9999994	0	-0.921111	-0.919905
12	1.0000002	0.5929725	0.3656941	0.9928287
13	0.9999997	1.3505868	-0.1316636	-1.1002111
14	0.9999999	1.030446	-0.0807578	-0.377579
15	1.0000003	0	0.8076676	0.8091399
16	1.0000001	0	0.7827722	0.7814985
17	0.9999998	0	-0.9043655	-0.9064747
18	0.9999995	0.0371362	-2.4484276	-2.474518
19	1	0	-0.0577079	-0.0577812
20	1.0000001	0	0.4998888	0.5005384
21	1.0000005	0	1.6985702	1.6952759
22	1.0000001	0.0966392	0.7082944	0.7185353
23	1.0000002	0	0.978991	0.9782307
24	1.0000001	0.4320143	0.868033	0.8524921
25	0.9999999	0.107974	-0.6229485	-0.6506796
26	0.9999998	0.0130946	-0.8462739	-0.8570892
27	0.9999998	0	-0.7059849	-0.7049329
28	1.0000001	0	0.5390581	0.5380466
29	0.9999998	0	-0.6918686	-0.6931598
30	1	0.0937272	-0.1139593	-0.1232681
31	1.0000001	0.0628699	0.6830179	0.7002388
32	1.0000004	0	2.2219257	2.2271285
33	1.0000002	0	0.9265202	0.9288115
34	1.0000001	0	0.9005873	0.9006045
35	1.0000001	0	1.1163364	1.1164712
36	0.9999999	0	-0.6195189	-0.6183376
37	0.9999999	0	-0.9750424	-0.9757035

Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
38	1.0000002	0	1.0405482	1.0429308
39	0.9999999	0.0530023	-0.8165225	-0.8220193

S&P 500

Y	Ro estimate	Value of kappa	PK t-statistic	PP t-statistic
1	0.9999996	0	-0.7931226	-0.7941018
2	1.0000001	0	0.2897283	0.2897957
3	1.0000002	0	0.6207726	0.62171
4	1.0000002	0	0.4308946	0.4300748
5	0.9999995	0	-1.8453526	-1.8377208
6	1	0	0.0330782	0.0330766
7	1.0000004	0.3310292	0.733865	0.8343056
8	0.9999993	0	-1.8242714	-1.8255177
9	0.9999994	0.0332517	-1.3339044	-1.3333692
10	0.999999	0	-1.7998345	-1.8009953
11	1.0000002	0	0.4822901	0.4828529
12	1.0000003	0	0.7921867	0.7922196
13	0.9999992	0	-1.7043788	-1.7052683
14	1.0000011	0	1.7500755	1.7512465
15	0.9999997	0.0445938	-0.5720423	-0.5715161
16	1.0000004	0	0.9694359	0.9702058
17	0.9999995	0.0208233	-1.1492595	-1.1469534
18	1.0000002	0	0.5461368	0.5462838
19	0.9999993	0	-1.485686	-1.4870747
20	1.0000006	0	1.1216295	1.1208482
21	1	0.4113282	-0.0493657	-0.0490828
22	1.0000007	0	1.8664525	1.8649272
23	0.9999999	0.2555814	-0.2292944	-0.2218176
24	0.9999998	0	-0.3956649	-0.3953176
25	1.0000004	0	1.40145	1.4002716
26	0.9999994	0	-1.2922201	-1.2912806
27	1.0000002	0	0.3746721	0.3751307
28	0.9999994	0	-1.3794625	-1.3789554
29	0.9999997	0.1325458	-0.5564943	-0.5533239
30	1.0000006	0	1.7698606	1.7713568
31	1.0000004	0.0358128	0.950508	0.9531353
32	1.0000005	0	1.419085	1.4186148
33	1.0000002	0	0.6988074	0.699304
34	0.9999999	0.0474209	-0.3622195	-0.3663404
35	1.0000002	0	0.7501349	0.7519226
36	1.0000001	0.0604867	0.3417231	0.3426619
37	1	0	-0.0934764	-0.0940071
38	1.0000002	0	0.7093226	0.7097283
39	1.0000003	0	0.8133465	0.812594

All the above series are taken from Dukascopy.net

Appendix II

Table 1: Critical Values for the $Z_{k,p}$ Statistic with IID secondary innovations

T	1%	2.5%	5%	10%	90%	95%	97.5%	99%
K=0								
30	-11.236	-8.811	-7.392	-5.508	1.064	1.512	1.960	2.434
50	-12.144	-10.467	-8.211	-5.980	1.064	1.466	1.926	2.244
100	-14.151	-10.846	-8.643	-6.690	0.978	1.372	1.824	2.264
250	-13.871	-10.750	-8.385	-6.095	0.946	1.278	1.701	2.084
500	-14.691	-11.534	-8.081	-5.638	0.914	1.261	1.515	1.725
1000	-12.669	-9.721	-7.723	-5.718	0.897	1.281	1.653	2.060
∞	-12.476	-9.897	-7.349	-5.197	0.938	1.258	1.600	2.097
K=1								
30	-14.495	-12.009	-9.686	-7.255	1.780	2.576	3.198	4.107
50	-15.550	-12.504	-9.990	-7.531	1.863	2.468	3.216	3.817
100	-17.507	-13.426	-10.138	-7.669	1.542	2.354	3.073	4.284
250	-17.686	-13.738	-10.509	-8.042	1.517	2.223	2.856	3.531
500	-19.510	-14.297	-11.221	-7.270	1.511	2.042	2.452	3.093
1000	-15.180	-11.696	-8.653	-6.454	1.694	2.312	3.029	3.429
∞	-15.629	-11.746	-9.242	-7.083	1.485	2.193	2.762	3.466
K=2								
30	-23.060	-18.094	-13.654	-9.695	2.465	3.431	4.604	6.338
50	-22.340	-17.845	-13.043	-9.492	2.512	3.639	4.389	5.095
100	-23.457	-18.474	-14.288	-9.828	2.331	3.287	4.255	5.631
250	-24.810	-18.529	-13.319	-10.037	2.160	3.039	3.881	4.792
500	-23.757	-18.415	-14.178	-9.820	2.062	2.725	3.328	4.433
1000	-23.396	-17.537	-13.701	-9.828	2.288	3.104	3.994	4.912
∞	-21.677	-16.291	-12.589	-9.079	2.017	2.882	3.646	5.165
K=3								
30	-30.864	-24.073	-17.678	-12.208	3.179	4.505	5.771	8.166
50	-36.127	-23.675	-18.796	-12.092	3.125	4.390	5.384	6.659
100	-36.570	-25.133	-18.841	-13.394	2.971	4.424	5.981	7.298
250	-30.032	-22.474	-18.229	-13.117	2.947	3.917	5.078	6.202
500	-33.539	-22.180	-18.670	-12.816	2.601	3.453	4.362	6.001
1000	-32.128	-23.946	-18.419	-12.572	2.903	3.961	4.958	6.499
∞	-23.722	-19.702	-15.575	-10.912	2.605	3.488	4.564	6.244

Table 2: Critical Values for the $Z_{k,t}$ Statistic with IID secondary innovations

				K=0				
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-2.561	-2.194	-1.889	-1.626	1.103	1.563	2.226	3.176
50	-2.500	-2.309	-2.004	-1.703	1.009	1.566	2.003	2.408
100	-2.724	-2.311	-2.086	-1.788	0.927	1.417	1.869	2.378
250	-2.645	-2.317	-2.000	-1.697	0.907	1.310	1.659	1.981
500	-2.640	-2.321	-1.959	-1.608	0.864	1.305	1.701	2.104
1000	-2.476	-2.158	-1.909	-1.639	0.829	1.302	1.794	2.275
∞	-2.462	-2.143	-1.839	-1.556	0.897	1.312	1.586	1.897
				K=1				
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-2.881	-2.495	-2.168	-1.861	1.381	2.026	2.768	4.577
50	-2.872	-2.547	-2.211	-1.870	1.335	2.024	2.522	3.391
100	-2.973	-2.568	-2.218	-1.882	1.238	1.906	2.418	3.066
250	-2.943	-2.578	-2.258	-1.942	1.177	1.673	2.136	2.803
500	-2.989	-2.576	-2.335	-1.825	1.175	1.675	2.108	2.506
1000	-2.824	-2.527	-2.182	-1.819	1.114	1.618	2.127	2.825
∞	-2.719	-2.366	-2.098	-1.822	1.150	1.590	2.079	2.646
				K=2				
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-3.674	-3.160	-2.704	-2.170	1.568	2.274	3.326	5.094
50	-3.411	-3.046	-2.570	-2.110	1.744	2.448	2.929	4.303
100	-3.425	-2.965	-2.601	-2.137	1.481	2.164	2.950	3.930
250	-3.485	-2.988	-2.521	-2.196	1.352	1.963	2.592	3.550
500	-3.456	-2.956	-2.585	-2.142	1.378	1.941	2.513	2.800
1000	-3.380	-2.881	-2.528	-2.139	1.348	1.979	2.503	3.254
∞	-3.283	-2.826	-2.456	-2.034	1.237	1.938	2.594	3.094
				K=3				
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-4.114	-3.619	-3.027	-2.479	1.786	2.656	3.647	6.452
50	-4.248	-3.511	-3.053	-2.417	1.912	2.726	3.452	4.187
100	-4.174	-3.486	-3.031	-2.500	1.751	2.657	3.524	4.416
250	-3.836	-3.292	-2.956	-2.510	1.537	2.226	2.926	4.046
500	-4.038	-3.342	-2.964	-2.457	1.495	2.233	2.727	3.234
1000	-3.901	-3.382	-2.967	-2.373	1.577	2.328	2.881	3.596
∞	-3.437	-3.065	-2.650	-2.226	1.348	2.163	2.829	3.459

Table 3: Critical Values for the $Z_{k,p}$ Statistic with MA=0.7 secondary innovations

K=0								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-7.125	-6.399	-5.072	-3.847	1.319	1.828	2.244	2.739
50	-8.641	-6.991	-5.869	-4.313	1.257	1.697	2.003	2.807
100	-10.823	-8.672	-6.769	-5.017	1.096	1.553	2.006	2.398
250	-12.033	-8.727	-7.008	-5.366	1.032	1.487	1.804	2.280
500	-12.853	-9.649	-6.996	-4.896	0.975	1.345	1.572	1.784
1000	-11.957	-8.499	-6.957	-5.181	0.950	1.367	1.718	2.053
∞	-11.455	-9.205	-6.786	-4.971	0.956	1.246	1.642	2.162

K=1								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-9.118	-7.360	-5.862	-4.501	2.125	2.868	3.609	4.496
50	-10.912	-8.555	-7.038	-5.061	2.073	2.850	3.456	4.201
100	-12.459	-10.126	-8.147	-6.043	1.744	2.475	3.200	4.381
250	-13.754	-10.776	-8.593	-6.403	1.732	2.459	3.101	3.678
500	-15.486	-11.952	-9.856	-6.434	1.632	2.135	2.701	3.176
1000	-15.180	-11.696	-8.653	-6.454	1.694	2.312	3.029	3.429
∞	-15.295	-11.344	-8.748	-6.773	1.532	2.283	2.811	3.515

K=2								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-14.679	-10.257	-8.207	-5.967	2.945	3.917	4.989	6.335
50	-14.511	-11.777	-8.867	-6.724	2.706	3.786	4.629	5.612
100	-15.745	-13.311	-10.703	-7.874	2.559	3.476	4.474	5.340
250	-19.339	-13.668	-10.737	-8.462	2.357	3.183	4.228	5.096
500	-18.715	-14.182	-12.136	-8.364	2.210	2.929	3.558	4.605
1000	-21.547	-15.006	-12.129	-8.760	2.432	3.293	4.075	5.008
∞	-20.235	-15.025	-11.870	-8.488	2.055	2.933	3.718	5.407

K=3								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-25.152	-14.741	-11.083	-7.671	3.695	4.902	6.618	8.721
50	-21.875	-16.091	-12.146	-8.321	3.558	4.853	5.676	7.072
100	-23.562	-18.066	-13.259	-9.477	3.298	4.496	6.040	7.011
250	-22.830	-18.140	-13.575	-10.130	3.052	4.089	5.207	6.846
500	-25.141	-17.584	-14.796	-10.823	2.771	3.658	4.578	6.355
1000	-25.902	-20.297	-16.027	-10.897	3.046	4.201	5.206	6.442
∞	-22.894	-18.623	-14.500	-10.140	2.796	3.648	4.624	6.473

Table 4: Critical Values for the $Z_{k,t}$ Statistic with MA=0.7 secondary innovations

T	1%	2.5%	5%	10%	90%	95%	97.5%	99%
K=0								
30	-1.866	-1.710	-1.531	-1.332	1.362	2.014	2.839	3.634
50	-2.054	-1.817	-1.650	-1.402	1.215	1.804	2.261	2.763
100	-2.316	-2.057	-1.801	-1.516	1.081	1.575	2.194	2.897
250	-2.434	-2.044	-1.819	-1.576	1.020	1.479	1.767	2.222
500	-2.454	-2.112	-1.823	-1.501	0.959	1.417	1.809	2.265
1000	-2.372	-2.045	-1.829	-1.552	0.918	1.405	1.892	2.393
∞	-2.340	-2.075	-1.798	-1.519	0.953	1.363	1.618	1.991
K=1								
T	1%	2.5%	5%	10%	90%	95%	97.5%	99%
30	-2.086	-1.906	-1.667	-1.432	1.849	2.701	3.499	5.158
50	-2.295	-2.038	-1.807	-1.493	1.666	2.399	3.072	3.638
100	-2.413	-2.187	-1.951	-1.623	1.461	2.227	2.849	3.707
250	-2.590	-2.298	-2.047	-1.742	1.303	1.944	2.438	3.059
500	-2.726	-2.396	-2.115	-1.709	1.307	1.837	2.225	2.608
1000	-2.655	-2.356	-2.053	-1.711	1.188	1.788	2.248	3.001
∞	-2.668	-2.317	-2.029	-1.766	1.211	1.632	2.210	2.706
K=2								
T	1%	2.5%	5%	10%	90%	95%	97.5%	99%
30	-2.637	-2.210	-1.964	-1.645	2.149	2.895	4.155	5.817
50	-2.690	-2.328	-2.037	-1.709	2.043	2.798	3.571	4.359
100	-2.774	-2.434	-2.217	-1.834	1.694	2.495	3.476	4.126
250	-3.087	-2.590	-2.265	-1.971	1.557	2.123	2.724	3.980
500	-2.950	-2.647	-2.397	-1.961	1.517	2.103	2.484	2.975
1000	-3.175	-2.676	-2.372	-2.012	1.480	2.211	2.746	3.484
∞	-3.147	-2.681	-2.385	-1.974	1.320	2.006	2.678	3.195
K=3								
T	1%	2.5%	5%	10%	90%	95%	97.5%	99%
30	-3.547	-2.652	-2.324	-1.843	2.415	3.554	4.794	7.341
50	-3.308	-2.743	-2.366	-1.943	2.383	3.240	3.967	5.000
100	-3.332	-2.871	-2.522	-2.050	2.064	2.863	3.788	4.638
250	-3.300	-2.895	-2.522	-2.143	1.767	2.524	3.140	4.526
500	-3.486	-2.931	-2.621	-2.241	1.670	2.343	2.889	3.485
1000	-3.569	-3.129	-2.691	-2.189	1.714	2.494	2.984	3.708
∞	-3.357	-2.935	-2.579	-2.146	1.467	2.226	2.865	3.630

Table 5: Critical Values for the $Z_{k,p}$ Statistic with AR(1)(r=0.5) secondary innovations

K=0								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-6.870	-5.184	-4.343	-3.228	1.448	1.962	2.401	2.930
50	-7.541	-5.850	-4.861	-3.557	1.355	1.854	2.202	2.965
100	-9.487	-7.724	-6.052	-4.552	1.156	1.595	2.069	2.531
250	-10.799	-8.097	-6.470	-5.048	1.100	1.504	1.847	2.336
500	-11.801	-8.657	-6.561	-4.609	1.040	1.371	1.633	1.856
1000	-10.678	-8.091	-6.696	-4.998	0.975	1.368	1.799	2.086
∞	-11.359	-9.180	-6.649	-4.926	0.971	1.272	1.643	2.192

K=1								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-7.651	-6.172	-5.011	-3.746	2.311	3.059	3.789	4.975
50	-8.542	-6.986	-5.362	-4.082	2.259	2.879	3.511	4.377
100	-10.965	-9.059	-7.047	-5.353	1.905	2.614	3.340	4.408
250	-12.132	-10.009	-7.770	-5.925	1.875	2.519	3.333	4.025
500	-14.837	-11.348	-8.591	-5.972	1.714	2.181	2.757	3.327
1000	-14.818	-10.934	-8.303	-5.998	1.761	2.457	3.047	3.482
∞	-14.626	-10.950	-8.454	-6.654	1.560	2.275	2.779	3.610

K=2								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-11.694	-8.397	-6.443	-4.762	3.151	4.210	5.146	6.197
50	-10.411	-9.012	-6.960	-5.038	3.032	3.969	4.832	5.568
100	-13.690	-11.205	-9.074	-7.187	2.788	3.710	4.716	5.410
250	-16.839	-12.550	-8.927	-7.414	2.575	3.339	4.364	5.420
500	-16.721	-12.883	-10.965	-7.788	2.367	3.072	3.808	5.231
1000	-20.715	-14.375	-11.233	-8.348	2.493	3.329	4.131	5.509
∞	-20.310	-14.907	-11.707	-8.264	2.078	2.939	3.742	5.421

K=3								
T	**1%**	**2.5%**	**5%**	**10%**	**90%**	**95%**	**97.5%**	**99%**
30	-16.764	-11.769	-8.735	-5.997	4.004	5.279	6.796	8.387
50	-15.059	-11.588	-9.001	-6.449	3.877	4.989	5.744	7.091
100	-19.276	-14.544	-11.361	-7.994	3.552	4.828	6.176	7.286
250	-19.702	-15.481	-11.471	-8.727	3.284	4.332	5.471	6.823
500	-21.338	-16.369	-12.961	-9.537	2.982	3.908	4.800	6.419
1000	-23.748	-19.634	-14.597	-10.579	3.128	4.235	5.287	6.844
∞	-21.899	-17.646	-14.553	-9.988	2.790	3.712	4.802	6.783

Table 6: Critical Values for the $Z_{k,t}$ Statistic with AR(1)(r=0.5) secondary innovations

T	1%	2.5%	5%	10%	90%	95%	97.5%	99%
K=0								
30	-1.823	-1.568	-1.394	-1.194	1.706	2.330	3.065	3.921
50	-1.929	-1.668	-1.511	-1.264	1.454	2.097	2.564	3.155
100	-2.125	-1.918	-1.691	-1.436	1.201	1.831	2.328	3.085
250	-2.300	-1.954	-1.742	-1.536	1.112	1.587	1.874	2.361
500	-2.315	-2.024	-1.773	-1.457	1.037	1.489	1.853	2.311
1000	-2.245	-1.973	-1.791	-1.514	0.949	1.473	1.996	2.407
∞	-2.338	-2.062	-1.782	-1.499	0.939	1.386	1.635	1.985
K=1								
30	-1.902	-1.664	-1.513	-1.294	2.161	3.019	3.988	5.421
50	-2.043	-1.788	-1.571	-1.356	1.950	2.720	3.386	4.072
100	-2.283	-2.021	-1.809	-1.540	1.630	2.452	2.949	3.808
250	-2.405	-2.175	-1.925	-1.677	1.487	2.079	2.542	3.290
500	-2.705	-2.288	-2.006	-1.652	1.347	1.924	2.397	2.790
1000	-2.575	-2.271	-1.988	-1.678	1.283	1.816	2.422	3.106
∞	-2.621	-2.293	-2.028	-1.737	1.239	1.649	2.192	2.772
K=2								
30	-2.360	-1.944	-1.702	-1.428	2.535	3.545	4.723	6.190
50	-2.236	-2.024	-1.785	-1.490	2.389	3.215	4.315	5.058
100	-2.535	-2.263	-2.019	-1.752	2.021	2.766	3.544	4.701
250	-2.858	-2.389	-2.065	-1.834	1.759	2.456	3.038	4.093
500	-2.811	-2.492	-2.228	-1.884	1.664	2.216	2.696	3.249
1000	-3.087	-2.626	-2.287	-1.949	1.576	2.291	2.902	3.549
∞	-3.131	-2.666	-2.358	-1.942	1.365	2.054	2.717	3.215
K=3								
30	-2.826	-2.345	-1.943	-1.613	2.852	4.065	5.450	7.178
50	-2.678	-2.351	-1.991	-1.680	2.653	3.731	4.810	5.842
100	-3.034	-2.600	-2.295	-1.888	2.339	3.184	3.950	5.051
250	-3.085	-2.707	-2.300	-2.011	2.015	2.782	3.621	4.917
500	-3.244	-2.815	-2.462	-2.080	1.831	2.606	3.023	3.624
1000	-3.409	-3.038	-2.594	-2.128	1.862	2.595	3.249	4.011
∞	-3.291	-2.884	-2.633	-2.140	1.504	2.328	2.892	3.636

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