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Thesis

Parameter instability in exchange rate models

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Abstract

This study examines the forecasting performance of time varying coefficient regression models under alternative structures of coefficient variation. We compare the forecasting performance of fixed and stochastic coefficient models when the true Data Generating Process includes either constant or varying coefficients with stationary and I(1) regressors. Furthermore, we investigate the cost of misspecification in the coefficients structure and we try to find which structure provides the smallest deterioration in forecast accuracy. Besides, we examine the ability of recursive coefficient plots, Cusum and Cusum Square test to correctly advise for parameter instability. Finally, we use the stochastic coefficient regression models in order to improve the out-of-sample forecasts of two structural macroeconomic models, the Frenkel-Bilson and Dornbusch-Frankel, for the case of the Dollar/Euro Exchange Rate.

CONTENTS

CHAPTER 1

INTRODUCTION

The purpose of our study is to examine the out-of-sample forecasting performance of a class of monetary exchange rate models employing a methodology that allows for time varying coefficients.

Relaxing the conventional assumption of fixed regression slopes we will attempt to estimate structural models of exchange rate determination with the hope that they perform better than the random walk model in predicting out-of-sample values of exchange rates.

Our motivation derives from the seminal work of Meese and Rogoff (1983a) that casts serious doubts on the ability of international macroeconomic theory to predict exchange rate movements. They found that the structural models failed to improve on the simple random walk forecasting rule, even though the models' forecasts were based on actual realized values of future explanatory variables.

As representatives of structural models we use two monetary models of exchange rate determination that have gained much popularity and have been subjected to extensive empirical work, the Frenkel-Bilson (1976) and the Dornbusch-Frankel (1979) model.

After an overview of the techniques that have been applied many researchers have concluded that structural models cannot explain the variability of exchange rate at all. Nevertheless, we find this conclusion unsatisfactory and we hold the view that one should test the stability of models[1] before reject them out of hand. There is a variety of reasons that may call for stochastic coefficients. We hope that imposing varying coefficients we may "capture" the lost dynamics of those variables that have been omitted and may play a predominant role in the variability of coefficients.

Before applying the time varying parameter models in our real data we conduct several Monte Carlo Experiments in order to investigate their forecasting performance under alternative coefficient structures. In the literature there have been analyzed three different forms of parameter variation, the Hildreth-Houck, the Random Walk and the Return to Normalcy Model. Using these structures we compare the forecasts of fixed and stochastic coefficient regression models when the true Data Generating Process includes either fixed or stochastic. Besides, we evaluate the cost of selecting a different structure for coefficients than the true and we look for cases where this misspecification leads to as small as possible deterioration of the forecast accuracy. Furthermore, in the Monte Carlo study we try to investigate the ability of the Cusum, Cusum Square test and recursive coefficients plots to correctly advise for parameter instability.

All the analysis is made both for a univariate and a bivariate regression model with either stationary or integrated of order 1 regressors.

Our final task is the implementation of stochastic coefficient models in the Dollar/Euro Exchange Rate. Using monthly data we compare the forecasting performance of Frenkel-Bilson and Dornbusch-Frankel model with and without

[1] "Stability" is used in the sense of time invariant.

stochastic coefficients with the forecasting performance of the simple Random Walk Model. Similar empirical studies have been performed by Wolff (1978 and 1989) and Swamy-Schinasy (1989) for the cases of the Dollar/Yen, Dollar/Mark and Dollar/Pound exchange rates.

The organization of chapters is as follows. Chapter 2 includes a brief description of the two structural macroeconomic models and an overview of the alternative techniques that have been applied in order to improve the forecast accuracy of these models. In Chapter 3 after an analytical discussion of the reasons for parameter instability we represent the Kalman filter, an algorithm that recursively estimates the hidden states of stochastic coefficients. Chapter 4 includes the Monte Carlo analysis on the forecasting performance of state space models. Finally, in Chapter 5 we apply the stochastic coefficient models for the case of the Dollar/Euro Exchange Rate.

CHAPTER 2

EMPIRICAL EVIDENCE ON EXCHANGE RATE MODELS

Introduction

Following the collapse of the Bretton Woods agreement in 1973 and the generalized floating of the major international currencies there have been many attempts to find a structural model capable of explaining the behavior of exchange rates. A number of structural models of the exchange rate have been proposed but none have proved to be entirely satisfactory and most have performed poorly.

Virtually all of these structural models have been based on the monetary approach of exchange rate determination. According to this theory foreign exchange should be viewed as a financial asset with its price determined by the demand and supply for the stock of foreign exchange. Therefore, monetary theory suggests that the spot exchange rate is the relative price of two national monies.

All these models of exchange rate determination assume a high degree of *capital mobility* between assets denominated in different currencies. In these models, *Perfect Capital Mobility* is implied by the Interest Rate Parity Condition.

The monetary approach can be characterized by the assumption that domestic and foreign currency denominated bonds display *perfect substitutability.*

Finally, the asset markets are *efficient* and that is affected by the use of *rational expectations.* This implies that asset approach to exchange rate is inherently "forward looking". Asset approach assumes that the current spot rate should reflect everything that is known or expected to happen in the macroeconomic environment (notion of *Market Efficiency).*

The chapter is organized as follows. First, we briefly discuss two widely used monetary models of exchange rate determination. Secondly, we represent an overview of the alternative techniques and approaches that have been used in order to improve the forecasting performance of structural macroeconomic models. Our overview is divided into two periods. The first covers the period since the Meese-Rogoff results and the second the period following Meese-Rogoff results.

2.1 EXCHANGE RATE MODELS:

We present two reduced form equations that are representative of the monetary approach to the exchange rate. We choose these models that have attained a considerable degree of popularity and have been subject of extensive empirical testing:

- Flexible Price Monetary Model (Frenkel – Bilson, 1976)
- Sticky Price Monetary Model (Dornbusch – Frankel, 1979)

Flexible Monetary Model

This approach assumes that domestic prices are fully flexible, the purchasing power parity holds continuously and that the real exchange rate never changes. In addition it is assumed that the money supply and real income are determined exogenously. The model involves the following equations:

$$m_t - p_t = a_1 y_t - a_2 i_t, \quad a_1, a_2 \succ 0 \quad (1)$$

$$m_t^* - p_t^* = a_1 y_t^* - a_2 i_t^*, \quad a_1, a_2 \succ 0 \quad (2)$$

$$s_t = p_t - p_t^* \quad (3)$$

where m is the natural logarithm of domestic money supply, p is the natural logarithm of price levels, y is the natural logarithm of the level of real national income and i is the interest rate in levels. The parameter a_1 is the income elasticity of the demand for money (the same for both countries) and a_2 is the interest rate semi-elasticity (the same for both countries). Asterisks denote the corresponding foreign variables.
(These equations are typical Cagan demand for money functions in logarithmic form). Solving the first two equations with respect to p_t *and* p_t^* and then substituting into (3) yields the final exchange rate equation:

$$s_t = (m_t - m_t^*) - a_1(y_t - y_t^*) + a_2(i_t - i_t^*) \quad (4)$$

Sticky Price (Overshooting) Monetary Model.

The Sticky Price Monetary Model was introduced by Rudiger Dornbusch (1976) to highlight the impact of assuming that the speed of adjustment of goods prices is slow relative to the speed of adjustment of asset prices. This model has the same properties as the FLMA in the long run but it differs fundamentally in its short run properties because prices are assumed to be sticky. Dornbusch showed that the gradual adjustment of goods prices following a monetary shock imparts a dynamic adjustment path to the exchange rate, so that while the real exchange rate changes in the short run (deviations from *PPP*), it reverts to its original level in the long run.

In the FLMA the expected change in the exchange rate was continuously equal to the expected inflation differential. Now, due to the failure of PPP, in the short run is assumed that the expected change in the exchange rate is equal to a constant proportion φ of the differential between the equilibrium values $\bar{s}_t$, and the current levels s_t:

$$\Delta s^e_{t+1} = \phi(\bar{s} - s)_t \quad 0 < \phi < 1 \quad (11)$$

i.e. expectations are assumed to be governed by the regressive expectations mechanism.
Substituting (11) into *UIP* equation and using the FLMA reduced form equation (4) as the equilibrium value we obtain:

$$s_t = (m_t - m_t^*) - a_1(y_t - y_t^*) - (1/\vartheta)(i_t - i_t^*) \quad (12)$$

where ϑ is the rate at which the exchange rate adjusts toward its long-run equilibrium and the other variables are as in our earlier FLMA equation. Note that in equation (12) the expected coefficient of interest rate is negative.
Frankel (1979a), suggested that expected change in the exchange rate should be governed by a regressive expectation component and, additionally, a term from the FLMA capturing the expected inflation differential:

$$\Delta s^e_{t+1} = \phi(\bar{s} - s)_t + (\Delta p^e - \Delta p^{e*})_{t+1} \quad (13)$$

Substituting (13) into *UIP* equation and using the FLMA reduced form equation (4) as the equilibrium value we obtain:

$$s_t = (m_t - m_t^*) - a_1(y_t - y_t^*) + a_2(\Delta p^e_{t+1} - \Delta p^{e*}_{t+1}) - (1/\phi)[(i_t - \Delta p^e_{t+1}) - (i_t^* - \Delta p^{e*}_{t1+})] \quad (14)$$

2.2 EMPIRICAL EVIDENCE ON EXCHANGE RATE MODELS

The empirical evidence of the monetary exchange rate models can be divided into two periods. The first-period evidence relates studies until about 1978 and is supportive of the monetary model. The second-period evidence, which covers the period of the recent float extending beyond the late 1970s, produces results that become at best mixed and often inconsistent with the theoretical predictions of the monetary approach. Besides, the review of empirical evidence is divided into two sections: a) *in sample results* designed to measure how well actual exchange rates conform to the predictions and specifications of an estimated model, and b) *out-of sample results* designed to measure how accurately a model can forecast exchange rates once the model's coefficients have been estimated.

One of the first tests of equation of the FLMA model was conducted by Frenkel in 1976 for the deutsche mark–U.S. dollar exchange rate over the period 1920-23. The results were supportive of the model. Coefficients were both correctly signed and significant and besides the model explained almost 99% of the exchange rate variability over the sample period.

A number of researchers have estimated the FLMA equation and found supportive results. Bilson (1978) tested the deutsche mark-pound sterling over the period January 1972 through April 1976 and found both correctly singed and statistically significant coefficients. The same highly supportive results were found by Putnam and Woodbury (1979) for the sterling-dollar exchange rate over the period July 1972 to June 1975, Dornbusch (1979) for the mark-dollar exchange rate over the period March 1973 to May 1978. Furthermore, Driskill (1981) presented an estimate of the Dornbusch overshooting model for the Swiss franc-U.S. dollar rate for the period 1973-77 and reported favorable results for the model. The same results found Hacche and Townend (1981) for the effective exchange rate of the pound sterling from May 1972 to February 1980.

Despite the above supportive evidence for the period up to 1978, the picture alters dramatically once the sample period is extended. Estimates of the Monetary Models of the exchange rate determination reported by Dornbusch (1980), Haynes and Stone (1981) for the mark-dollar exchange rate and other researchers for the period extending beyond 1978 reveal that these models have little if any power to explain exchange rate behavior. These results relate essentially to the *in-sample* fit of the equations in terms of their explanatory power (determined by the R^2) and the fact that model's coefficient values differ dramatically, in terms of sign and magnitude, from their prior values.

Confidence in the monetary approach of exchange rate determination was further shaken when *out-of sample* studies showed that these models could hardly provide accurate forecasts.

The first study of exchange rates in the modern flexible rate era to employ post sample methodology was conducted by *Richard Meese and Kenneth Rogoff* in 1983. The result of their study was the most damning indictment of the monetary class of models. They conducted such a survey for the dollar-pound sterling, dollar-mark, dollar-yen, and trade-weighted dollar exchange rates using monthly data running from March 1973 through June 1981. The exchange rate models they tested were the FLMA (Frenkel-Bilson), the sticky-price monetary model (Dornbusch-Frankel) and the sticky-price asset (Hooper-Morton) model. They compared the out-of-sample forecasting performance of these models to the forecasting performance of the random walk (without drift), the forward exchange rate, 6 univariate AR models, and a vector autoregression. The competing models were estimated using monthly data from March 1973 to November 1976, and four forecasts were made for 1, 3, 6 and 12 months ahead (so as to correspond to the available forward rate data). In order to avoid the difficulty of predicting the explanatory variables they used *actual realized values*. They estimated the models using ordinary leas squares, generalized least squares (correcting for serial correlation) and Fair's method. The parameters of each model were estimated on the basis of the most up-to-date information available at the time of a given forecast. This was accomplished by using *rolling regressions* to re-estimate the parameters of each model every forecast period. Out-of-sample accuracy was measured by three statistics: Mean Error (ME), Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). The Meese and Rogoff results are reported in the table below:

Root Mean Square Forecast Errors

		Pure Forecast Models				Macroeconomic Models		
	Model:	**Random walk**	**F. Rate**	**AR**	**VAR**	**Frenkel Bilson**	**Dornb. Frankel**	**Hooper Morton**
S	**Horizon**							
$/mark	1 month	3.72	3.20	3.51	5.40	3.17*	3.65	3.50
	6 months	8.71*	9.03	12.40	1.83	9.64	12.03	9.95
	12 months	12.98	12.60*	22.53	15.06	16.12	18.87	15.69
$/yen	1 month	3.68*	3.72	4.46	7.76	4.11	4.40	4.20
	6 months	11.58*	11.93	22.04	18.90	13.38	13.94	1.94
	12 months	18.31*	18.95	52.18	22.98	18.55	20.41	19.20
$/pound	1 month	2.56*	2.67	2.79	5.56	2.82	2.90	3.03
	6 months	6.45*	7.23	7.27	12.97	8.90	8.88	9.08
	12 months	9.96*	11.62	13.35	21.28	14.62	13.66	14.57
Trade Weighed $	1 month	1.99*	N.A	2.72	4.10	2.40	2.50	2.74
	6 months	6.09*	N.A	6.82	8.91	7.07	6.49	7.11
	12 months	8.65*	14.2	11.14	10.96	11.40	9.80	10.35

** Lowest RMSE forecast.*
The three structural models are estimated using Fair's instrumental variable technique to correct for first-order serial correlation.

These results show that a macroeconomic model achieved the lowest RMSE (3.17%) in the 1-month $/DM forecast horizon, and the forward rate achieved the lowest RMSE (12.60%) in the 12-month $/DM forecast horizon. In the other 10 cases, the random walk model (without drift) produced the lowest RMSE. Although RMSE at three-month horizon are not listed in the above table, they give the same result. Furthermore, the same ranking is taken by using the MAE as a measure of forecasting accuracy, while random walk is somewhat less dominant in ME than in RMSE or MAE, particularly for the $/DM.
In summarizing their results, Meese and Rogoff observed:

> *"We find that a random walk model would have predicted major-country exchange rates during the recent floating-rate period as well as any of our candidate models. Significantly, the structural models fail to improve on the random walk model in spite of the fact that we base their forecasts on actual realized values of future explanatory variables." (R. Meese and K. Rogoff, 1983, page 3).*

In an attempt to improve on the poor performance of these models, Meese and Rogoff tried a number of alternative approaches:

- They estimated the model in first differences
- They imposed theoretical coefficient constraints to account for simultaneous equation bias

- They allowed for separate coefficients on domestic and foreign explanatory variables
- They used different definitions of the money supply
- They replaced long-term interest rates with other proxies for inflationary expectations

But unfortunately, the modified reduced-form equations still failed to outperform the simple random walk.

They proposed other possible explanations without arriving at any definite conclusions. They stressed the importance of structural instability during the sample period and they used weighted least squares methods but without any improvement. They mentioned the misspecification of the money demand functions but substituting price levels for monetary variables did not yield better results. Finally, the failure of the models to incorporate real disturbances and the difficulties in modeling expectations of the explanatory variables were thought as another obvious source of trouble.

2.3 EMPIRICAL EVIDENCE FOLLOWING MEESE AND ROGOFF'S RESULTS.

Undoubtedly, these results were very disappointing to those who had spent a great deal of time and effort trying to model exchange rates. Others took these results as a challenge to understand what factors were behind the poor post-sample performance of these structural macroeconomic models.

An outstanding large segment of the literature has been devoted to determining whether Meese and Rogoff's specification of the equations, their estimation strategy, or the models themselves are at fault.

Many researchers have linked the failure of monetary models with the instability in the conventional *money demand functions*. Recent studies of the demand for money (Frankel (1982), Goldfeld (1976)) document for instabilities in the empirical money demand functions for the U.S. and for Germany. Woo W. (1985) based on Goldfeld's seminal paper on U.S. short-run money demand function relates the inability of models to give accurate forecasts to an inappropriate specification of the money demand function. Allowing for partial adjustment mechanism he finds that the structural model outperforms the random walk model and its unconstrained equivalent in out-of-sample forecasts for the $/DM rate (1974:3 ~ 1981:10).

Other researchers seek to address the question as to whether *future values* of the exchange rate can be predicted more accurately by a monetary model or a random walk model. Hoffman (1983) estimated a monetary model for the dollar/mark, dollar/ franc and dollar/pound rate (period: 1974:6 ~ 1979:12) with rational expectations and tested the parameter restrictions associated with this model. His results suggest that both the parameter constrains associated with the monetary model and those implied by the Rational Expectation hypothesis (REH) are consistent with the data. Finn M. (1986) reported that the simple FLMA for the dollar/pound rate is not supportive by the data (1974:5 ~ 1982:12) while its rational expectations counterpart is. Besides, he found that the REM model forecasts as well as the Random Walk model, but it fails to outperform it (forecasting period: 1980:1 ~ 1982:12). The same supportive results were found by Woo (1985) for the $/DM.

In 1986 Somanath considered that a *lagged adjustment* might contribute towards better performance. Using the DM/$ exchange rate for the period 1975:1 ~ 1983:12 estimated a variety of models (including those used by Meese-Rogoff (1983)). Out-of-sample results of the M-R period confirm the Meese-Rogoff conclusion. Interestingly, though, they indicate that lagged models dominate the random walk model. (In their seminal paper Meese-Rogoff (1983) report that lagged effects did not provide any further result). In the post M-R period (and subperiod) the results are much different. Some of the basic monetary models do better than the random walk but yet the lagged models remain the dominating models.

In order to explain the poor empirical performance of monetary approach of exchange rate determination, some authors have suggested that foreign exchange rates might have consistently deviated from their underlying "fundamental" levels due to the presence of rational bubbles (Meese (1986)). Other researchers have concentrated on the influence of foreign exchange analysts who base their forecasts not on economic theory (as "fundamentalists" do) but on the identification of supposedly recurring patterns in graphs of exchange rate movements (bandwagon expectations, "chartists"). For example, Frankel and Froot (1986, 1990) find that investors have heterogeneous expectations. Besides, they find that at short-horizons analysts tend to forecast by extrapolating recent trends while at long-horizons by relying on fundamental values.

In an attempt to discriminate between the short-run and long-run aspects of the monetary models researchers have used the *cointegration methodology* to estimate a long-run monetary model. MacDonald and Taylor (1991), using the multivariate cointegration technique proposed by Johansen, have demonstrated that the null of no cointegration may be rejected for the bilateral U.S. dollar rates of the UK pound, German mark and Japanese yen for the period from 1976:1 to 1990:12. Using the same methodology as MacDonald, Sarantis (1994) concludes the opposite over the period 1973-1990 for the pound-streling exchange rates of the United States, Germany, Japan and France. Furthermore, MacDonald and Taylor in 1993 found evidence for the long-run validity of monetary models for the deutsche mark/dollar exchange rate (1976-1990). Besides, imposing the long-run model restrictions in a dynamic error-correction framework they found that the model outperformed the simple random walk. Mark Nelson demonstrated the same evidence (1995) for the U.S. dollar prices of the Canadian dollar, the deutsche mark, the Swiss franc and the Yen from 1973 to 1991.

CHAPTER 3

THE LINEAR STATE SPACE MODEL AND THE KALMAN FILTER

Introduction

Considering the drawbacks of classical linear regression models econometricians developed models that account for time varying coefficients. Most of the models with time varying coefficients, that have been put forward, may be cast in State Space Form. State Space Models and the associated Kalman Filter were developed in the technical literature, namely Kalman (1960) and Kalman & Bucy (1961).

Regression models with time varying coefficients have found wide spread use in various fields of empirical economics. The state space model has been applied to estimate commodity demand functions, money demand functions, or capital asset pricing models.

Because these are a rather general class of models covering many different models it will not be presented in its full generality. Actually, the presentation will be restricted to the formulation, interpretation and estimation of those models, which may be interpreted as linear regression models with time varying coefficients.

The structure of the chapter is as follows. In section 3.1 we discuss the reasons that lead us to the use of time varying coefficients. Then, the state space model is setup in section 3.2. In section 3.3 different forms of parameter variation are discussed. In section 3.4 and 3.5, after representing the Kalman filter recursions the estimation problem is discussed. Finally, in section 3.6, we describe two tests for parameter instability that are commonly used (Cusum and Cusum Square Test).

3.1 WHY TIME VARYING COEFFICIENT REGRESSION MODELS?

Undoubtedly, the results for the forecasting performance of structural models are very disappointing especially to those who have spent a great deal of time trying to understand the factors for their poor forecasting performance.

One might be tempted to conclude from these studies that economic variables convey little or no useful information about exchange rate movement.

In our opinion, though, one crucial condition that should be tested (and not assumed to hold, as in the existing literature) is that the *model is stable over time*. This is because in several cases the type of *time heterogeneity* exhibited by the Data Generating Process (DGP) is more complex than that associated with unit roots, and therefore estimation of the system will result in parameter time dependence, misleading statistical inference and eventually poor forecasting performance.

The question that naturally arises is why should parameters change over time. The most important reason that made us to adopt such a methodology is the potential *misspecification* of these models due to the *exclusion* of some important explanatory variables (*omitted variables).* If some of these variables are excluded from the model the effects of the included variables on the dependent variables can be expected to vary across the sample and as a minimum produce variations in the coefficients. Allowing for stochastically varying coefficients we expect to *incorporate these lost dynamics* of the omitted variables into the model. In other words, we expect to incorporate the lost information of the omitted variables into the system through time varying coefficients.

In the literature, there have been analyzed several other reasons that explain the importance of parameter instability. Some of these are the following:

- Economists often use proxy variables that only partially reflect the economic effects they represent. If the relationship between the proxy and the true counterpart is not constant across observations, the coefficients of the proxy variables will not be constant.
- When aggregate data are used, the potential for coefficient variation is present. Coefficients in the aggregate equation will remain constant as long as the relationship between micro units remains constant.
- Coefficients of a linear model can vary across the sample if the true relationship is nonlinear and observations fall outside the narrow range where the linear approximation is acceptable.
- Many regression coefficients have a certain theoretical meaning. E.g. in a simple macroeconomic consumption model, $c_t = b_1 + b_2 y_t$, where c_t shall be the macroeconomic consumption and y_t the gross national product, b_2 is the marginal propensity to consume and there is no real reason why it should not change over time.
- Changes in government economic policies will systematically alter the structure of econometric models in general (Lucas 1976). In this context varying-parameter models can play a useful role in tracking parameter changes over time and can help improve models' forecasting performance.
- Instability in the conventional money demand functions. Since the simplest monetary models are derived from domestic and foreign transactions demand equations, it is not surprising that models emphasizing monetary phenomena

work poorly over a period characterized by money demand instability. Recent studies have documented instabilities in empirical money demand functions for the United States and for Germany (Frankel 1982, Goldfeld 1976).

- Factors leading to changes in the long-run real exchange rate (such as changes in oil prices, global trade patterns etc.) may lead to instability in the parameters of the class of structural exchange rate models.
- Finally, heterogeneous beliefs by traders, leading to a diversity of responses to macroeconomic developments over time, could impart parameter instabilities.

In summarizing, we believe that even though it is not possible (so far) to explain the poor forecasting performance of structural exchange rate models with sounds and rigorous economic principles, one should examine the type o stochastic coefficient models before rejecting existing exchange rate models out of hand. Before applying stochastic coefficients models in our real data we will proceed with the description of this methodology and an investigation of the forecasting performance of these models in a Monte Carlo Study. These are the subjects of this and the following chapter respectively.

3.2 THE MODEL SET-UP

A linear state space model is characterized by the measurement (or observation) equation, and a set of transition equations.

(A1) Observation equation:

$$y_t = x_{1,t}^{'} b_t + x_{2,t}^{'} \gamma + u_{2,t} \qquad t = 1,2,\dots,T$$

(A2) Transition equations:

$$b_t = \Phi b_{t-1} + x_{3,t}^{'} \zeta + u_{1,t} \qquad t = 1,2,\dots,T$$

The variables have the following dimensions and meanings:

y_t : (1 x 1) observable system output

$x_{1,t}$: (k_1 x 1) observable measurement vector

$x_{2,t}$: (k_2 x 1) observable input to the measurement equation

$x_{3,t}$: (k_3 x k_1) observable input to the transition equation

b_t : (k_1 x 1) unobservable system state or state of nature

$u_{1,t}$: (k_1 x 1) unobservable transition noise

$u_{2,t}$: (1 x 1) unobservable measurement or observation error.

The (k_1 x k_1) matrix Φ is the so-called transition matrix, γ and ζ are (k_2 x 1) and (k_3 x 1) vectors, respectively.

The stochastic properties of the error terms are:

(A3)
$$E\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} = 0 \;\forall t$$

$$E\begin{bmatrix} u_{1,s}u_{1,t}^{'} & u_{1,s}u_{2,t} \\ u_{2,s}u_{1,t}^{'} & u_{2,s}u_{2,t} \end{bmatrix} = \begin{cases} \begin{bmatrix} \Omega_1 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} & s = t \\ 0 & s \neq t \end{cases}$$

The (k_1 x k_1) matrix Ω_1 is the variance matrix of the transition noise and the scalar σ_2^2 is the measurement error variance.

To complete the model some assumptions about the initial state b_0 and the distribution of the error processes have to be made. The initial state b_0 is usually taken to be a random variable, which is independent of the error processes:

(A4) 1. The process is assumed to start with the initial state b_0, which is random with the following first two moments:

$$\mu_0 := E(b_0) \; and \; \Sigma_0 := Var(b_0)$$

2. b_0 is independent of $\{u_{1,t}\}$ and $\{u_{2,t}\}$.

(A5) The error processes $\{u_{1,t}\}$, $\{u_{2,t}\}$ and the initial state b_0 are normally distributed with the moments given above.

The input sequences to the system $\{x_{1,t}\}, \{x_{2,t}\} \; and \; \{x_{3,t}\}$ are taken to be predetermined. This means that they are either exogenous variables or at least known at time point t. This means that lagged output variables may be included in the inputs. The analysis is then carried out conditional on the observations at time t. In order to ease the presentation the following abbreviations will be used:

$$x_i(t) := \{x_{i,1}, \dots x_{i,t}\}, \; y(t) := \{y_1, \dots, y_t\} \text{ and } b(t) := \{b_0, \dots b_t\}$$

The analysis of a state space model of form (A1) and (A2) has mainly two aims:

1. If the hyperparameters of the system $(\gamma, \Phi, \zeta, \Omega_1, \sigma_2^2, \mu_0 \; and \; \Sigma_0)$ are known, then the analysis consists in the reconstruction of the time path of the system state $\{b_t\}$, i.e. the estimation of b_t, t=1,2,...T based upon the sample information up to and including time point T $(y(T))$. These estimates will be called:
   - Filtering solution, for t=T,
   - Smoothing solution, for t<T,
   - Prediction solution, for t>T
2. If some of the hyperparameters are unknown, we would like to derive their likelihood functions, so that these parameters can be estimated.

For both purposes the Kalman Filter and smoother recursions provide the analytical tool. The Kalman Filter is a means to recursively estimate the hidden state b_t based on the information up to time point t. If the entire information available up to time point T is used then Kalman smoothing recursions have to be applied.

3.3 INTERPRETATION OF THE STATE SPACE MODEL

Quite a lot of commonly used models in econometrics may be cast into a State Space form, such as:

- All kinds of VARMAX models (vector autoregressive moving average models with exogenous regressors),
- VARX models with randomly or systematically varying coefficients,
- Multiple regression models with unobservable regressors.

For most of the above-mentioned models it is rather difficult to evaluate the likelihood function even if normality in the random variables is assumed and, therefore, ML estimates are hard to derive. But much research has been spent on the likelihood theory for state space models, so that having transformed those models to state space forms the ML estimation task is often much easier to solve.

Since the state space form covers so many different models there is no unique interpretation at hand. In fact, it varies from model to model. Here the state space model will be interpreted as a linear regression model with stochastic coefficients, with different forms of parameter variation.

3.3.1 REGRESSION MODELS WITH TIME VARYING COEFFICIENTS

The observation equation:

$$y_t = x_{1,t}' b_t + x_{2,t}' \gamma + u_{2,t} \quad t = 1,\dots,T$$

may be viewed as a normal regression equation, where $x_{1,t}$ and $x_{2,t}$ contain the exogenous variables and possibly also lagged output variables. If regression coefficients were constant over time, $b_t = b_0 \ \forall t$, one might estimate the system by OLS:

$$\begin{bmatrix} \hat{b}_0 \\ \hat{\gamma} \end{bmatrix} = (X'X)^{-1} X'y, \text{ where } X := (X_1, X_2),\ X_i = (x_{i,1}, \dots x_{i,T})' \ y = (y'_1, \dots y'_T)'$$

However, in this linear regression models the regressors are divided into two groups, those with constant regression coefficients and those with time varying regression coefficients. The law of motion of the coefficients that assumed to be time varying is described by the transition equation:

$$b_t = \Phi b_{t-1} + x'_{3,t} \zeta + u_{1,t} \quad \text{t=1,...T}$$

In econometrics the emphasis rests mainly on three different linear regression models, namely the Hildreth-Houck, the Random Walk and the Return to Normalcy. A brief description of the above models follows.

3.3.1.1 HILDRETH-HOUCK MODEL

In this model proposed by Hildreth & Houck (1968) the coefficients are thought to follow a time invariant (normal) distribution:

$$b_t = \bar{b} + u_{1,t} \qquad u_{1,t} \overset{iid}{\sim} N(0,\Omega_1), \qquad t = 1,\dots,T$$

In this structure, knowledge of b_{t-1} is of no value in predicting b_t, and the stochastic innovation $u_{1,t}$ has no persistence effect on subsequent b's.

The idea behind this concept is that b_t describes the response of y_t to the inputs $x_{1,t}$. If some important input variables have been omitted, because e.g. they are unobservable, the system response to a certain input might not be constant. If the missing variables show no systematic behavior it seems reasonable to assume also a time invariant distribution of b_t. So, $\bar{b}$ represents the average response and $u_{1,t}$ the deviation from the average response due to missing variables.

3.3.1.2 RANDOM WALK COEFFICIENTS

In this model the time varying regression coefficients are assumed to follow a multivariate random walk:

$$b_t = b_{t-1} + u_{1,t} \quad u_{1,t} \overset{iid}{\sim} N(0,\Omega_1) \qquad t = 1,\dots,T$$

and with recursive substitutions we have:

$$b_t = \mu_0 + u_{1,0} + \sum_{i=0}^{t} u_{1,i} \quad u_{1,0} := (b_0 - \mu_0) \sim N(0,\Sigma_0)$$

This model has become very popular due to its simple interpretation: the new state is the old state plus some white noise. This model might be viewed as the other extreme to the Hildreth-Houck model. Whereas in the Hildreth-Houck model shocks in the coefficients were absorbed by the system within in the same time period, they never fade out, here. In other words: While the Hildreth-Houck model has no or a zero memory the random walk model has an infinite one. This assumption as such does not seem too plausible for two reasons. First, it involves the assumption that shocks, which occurred in the very remote past, have the same influence on b_t as recent ones. And second, that

$Var(b_t) = \Sigma_0 + t\Omega_1$, which implies that $Var(b_t) \to \infty \text{ as } t \to \infty$, and it seems hard to justify an infinitely growing variance of coefficients in regression models.

But, on the other hand the random walk assumption can be viewed as a method of modeling smooth transitions. "Smooth" is used in the sense that the system state does not erratically jump around. As b_t forms a martingale process the new system state is the previous system state with some white noise added and, hence, the expected value of the new state is just the last realization. Quite often there is no economic theory that directly leads to random walk coefficients, so that time varying parameters are regarded as a relaxation of too strong assumptions. If one gives up the assumption of constant regression coefficients, one might wish to have coefficients that evolve only slowly over time. The desired property of smooth and perhaps trending regression coefficients can be captured quite well by random walks. Also, one has to note that in econometrics one often deals with time series of only moderate length T so that the property of increasing variance will not become too serious.

Hence, the assumption that the time varying regression parameters follow a random walk will in general not be made for theoretically founded economic reasons, but it will be employed as a flexible tool for modeling very differently shaped, smooth, and perhaps trending, time paths of the regression coefficients by a very parsimoniously parameterized model set-up.

3.3.1.3 THE RETURN TO NORMALCY MODEL

The return to normalcy model, originally proposed by Rosenberg (1973), supposes that the time varying coefficients fluctuate around a constant mean $\bar{b}$:

$$(b_t - \bar{b}) = \Phi(b_{t-1} - \bar{b}) + u_{1,t} \qquad t = 1,...,T$$

where all eigenvalues of Φ are less than unity in absolute value. Hence, disregarding the white noise process $\{u_{1,t}\}$, the parameters b_t will slowly return to their normal values $\bar{b}$. Differences between the actual parameter value b_t and its mean value $\bar{b}$ will fade away eventually. Rearranging of the above equation yields:

$$b_t = (I - \Phi)\bar{b} + \Phi b_{t-1} + u_{1,t}$$

If we let $\Phi = I$ the model would represent a random walk and if we assume that Φ=0 the Hildreth-Houck model emerges. Therefore, the return to normalcy model is a mediator between the random walk and the Hildreth-Houck model in case that Φ is a diagonal matrix with only positive elements. Anyhow, the closer the eigenvalues move to unity the stronger will be the influence of past shocks on the actual system state and if the eigenvalues approach zero the influence dies out.

Also in analyzing the variance of b_t it can be seen that this model mediates between the two previous ones. Whereas the conditional variance of b_t given b_{t-1} is constant for all models $Var(b_t / b_{t-1}) = \Omega_1$, the unconditional variance changes from model to model:

$$\text{Hildreth-Houck: } Var(b_t) = \Omega_1$$

$$\text{Random Walk: } Var(b_t) = \Sigma_0 + t\Omega_1$$

$$\text{Return to Normalcy: } Var(b_t) = \Phi^t \Sigma_0 \Phi'^t + \sum_{i=0}^{t-1} \Phi^i \Omega_1 \Phi'^i$$

The variance of the return to normalcy model increases but remains bounded. It converges to:

$$vec(\lim_{t \to \infty} Var(b_t)) = vec(\sum_{i=0}^{\infty} \Phi^i \Omega_1 \Phi'^i) = (I_{k_1^2} - \Phi \otimes \Phi)^{-1} vec(\Omega_1)$$

3.4 THE KALMAN FILTER AND SMOOTHER

Given the observations $(y_1,...,y_T)$ the researcher is interested in estimating the trajectory of the b's coefficients. Note that b_t is unobservable and the aim of the "game" is to uncover these unobserved states and it is actually the information that $y(s)$ contains about $\{b_t\}$ that will be used by the Kalman filter and smoother to derive estimates of $\{b_t\}$.

The Kalman filter, smoother and prediction recursions can be derived as a method to recursively calculate the means and variances of the system state, b_t, and the predicted system output y_t. Some abbreviations have to be defined before the recursions can be given. The (px1) vector θ collects the unknown elements of the hyperparameters $\gamma,\Phi,\zeta,\Omega_1,\sigma_2^2,\mu_0,\Sigma_0$. The first two conditional moments of y_t and b_t are abbreviated as:

$$\hat{b}_{t|s} := E[b_t \mid y(s),\theta] \qquad \Sigma_{t|s} := Var[b_t \mid y(s),\theta]$$
$$\hat{y}_t := E[y_t \mid y(t-1),\theta] \qquad D_t := Var[y_t \mid y(t-1),\theta]$$

Therefore $\hat{b}_{t|s}$ is the estimate of b_t based on the observations $(y_1,...y_s)$. In the same manner are defined and the rest abbreviations.

Under the assumptions (A1)~(A5) that we made in 3.1 paragraph and given that inputs $x_{1,t},x_{2,t},x_{3,t}$ $t=1,...,T$ are known the conditional distribution of $(b_t \mid y(s))$ and $(y_t \mid y(t-1))$ are normal with the following moments:

1. ***Filtering Solution:***

   The algorithm consists of two steps conducted for periods 1,…,T- the prediction step and the correction step (or filter step).

   The prediction step includes the following:

   (Forward recursive for t=1,…T)

   Initialization

$$\hat{b}_{0|0} = \mu_0$$
$$\Sigma_{0|0} = \Sigma_0$$

   One step ahead forecast for the system state

$$\hat{b}_{t|t-1} = \Phi\hat{b}_{t-1|t-1} + x_{3,t}^{'}\zeta$$
$$\Sigma_{t|t-1} = \Phi\Sigma_{t-1|t-1}\Phi' + \Omega_1$$

   One step ahead forecast for the system output

$$\hat{y}_t = x_{1,t}^{'}\hat{b}_{t-1|t-1} + x_{2,t}^{'}\gamma$$
$$D_t = x_{1,t}^{'}\Sigma_{t|t-1}x_{1,t} + \sigma_2^2$$

Based on information in t-1 estimates for $b_{t|t-1}$, $\Sigma_{t|t-1}$, $\hat{y}_t$ *and* D_t are derived in the prediction step. The correction steps compares these estimates to the true realizations of y_t in t and updates the estimates for information obtained in t. The correction step includes the following recursions:

Correction Step:

$$\hat{b}_{t|t} = \hat{b}_{t|t-1} + K_t(y_t - \hat{y}_t)$$

$$\Sigma_{t|t} = (I - K_t x_{1,t}^{'})\Sigma_{t|t-1} = (\Sigma_{t|t-1}^{-1} + x_{1,t}\sigma_2^{-2}x_{1,t}^{'})^{-1}$$

with $K_t := \Sigma_{t|t-1}x_{1,t}D_t^{-1} = \Sigma_{t|t}x_{1,t}\sigma_2^{-2}$ (Kalman Filter Gain).

2. ***Smoothing solution, fixed interval:***
The Kalman filter recursions deliver estimates for b_t, t=1,…,T, based on information up to time point t. This is a drawback as we have a time series of length T at hand whose law of motion we want to determine. So we would like to incorporate the neglected sample information of $\{y_s\}_{s=t+1}^{T}$ into the estimates for b_t. This is done by the fixed interval smoother. This smoother may be interpreted as a backward filter that starts at time point T:

Initialization:

$\hat{b}_{T|T}$ *and* $\Sigma_{T|T}$ from *filtering solution*

Smoothing recursions:

$$\hat{b}_{t|T} = \hat{b}_{t|t} + S_t(\hat{b}_{t+1|T} - \hat{b}_{t+1|t})$$

$$\Sigma_{t|T} = \Sigma_{t|t} - S_t(\Sigma_{t+1|t} - \Sigma_{t+1|T})S_t^{'}$$

with $S_t := \Sigma_{t|t}\Phi'\Sigma_{t+1|t}^{'}$ *(Kalman smoother matrix)*

3. ***h-step ahead prediction solution***
(forward recursive for h=1,…)

Initialization

$\hat{b}_{T|T}$ *and* $\Sigma_{T|T}$ from filtering solution

Prediction recursions

$$\hat{b}_{T+h|T} = \Phi\hat{b}_{T+h-1|T} + x_{3,T+h}^{'}\zeta$$

$$\Sigma_{T+h|T} = \Phi\Sigma_{T+h-1|T}\Phi' + \Omega_1$$

$$E[y_{T+h} \mid y(T)] = x_{1,T+h}^{'}\hat{b}_{T+h|T} + x_{2,T+h}^{'}\gamma$$

$$Var[y_{T+h} \mid y(T)] = x_{1,T+h}^{'}\Sigma_{T+h|T}x_{1,T+h} + \sigma_2^2$$

3.5 ESTIMATION OF THE HYPERPARAMETERS

The first of the two tasks in analyzing a state space model has been solved: Given the hyperparameters $\gamma, \Phi, \zeta, \Omega_1, \sigma_2^2, \mu_0, \Sigma_0$ one can reconstruct the time path of the varying coefficients (either by the use of the filter or the smoother solution).

If some of the hyperparameters are unknown, we would like to replace them by ML estimates. It is the use of the Kalman Filter again that plays a predominant role also for the solution of this estimation problem.

3.5.1 SETTING UP THE LIKELIHOOD

Collecting the unknown elements $\gamma, \Phi, \zeta, \Omega_1, \sigma_2^2, \mu_0, \Sigma_0$. in the (px1) vector θ its likelihood can be calculated by the Kalman filter recursions. By recursive use of Bayes' theorem the density of $y(T)$ given θ can be written as:

$$f(y(Y) \mid \theta) = f(y_T \mid y(T-1) \mid \theta).f(y(T-1) \mid \theta)$$
$$= ... = f(y_1 \mid \theta) f(y_2 \mid y_1 \theta) \cdots f(y_T \mid y(T-1) \mid \theta).$$

Assuming the error processes and the initial state to be normally distributed all these densities are also normal and hence the log-likelihood is given by:

$$l(\theta \mid y(T)) := \ln L(\theta \mid y(T)) =$$
$$= cst - 1/2 \sum_{t=1}^{T} (\ln(D_t(\theta)) + D_t(\theta)^{-1}[e_t(\theta)]^2$$

(The *cst* denotes a constant term that is not depend on any parameter).

This is the *prediction error decomposition* of the likelihood. Note, for any given θ we can use the Kalman filter in order to derive the one step ahead forecast and to evaluate the above log-likelihood.

Maximization of the log-likelihood is started by making an initial guess as to the numerical values of the unknown parameters (θ). With some stationary models, steady-state conditions allow us to use the system matrices to solve for the values of $\hat{b}_{0|0}$ and $\Sigma_{0|0}$ (one way to do this is to take the OLS estimators). In other cases, we may have preliminary estimates of $\hat{b}_{0|0}$ and $\Sigma_{0|0}$, along with measures of uncertainty about those estimates. But in many cases, we may have no information, or diffuse priors, about the initial conditions. Having computed the initial values of the parameters the sequences $\{\hat{b}_{t|t-1}\}_{t=1}^{T}$ and $\{\Sigma_{t|t-1}\}_{t=1}^{T}$ are computed and used to evaluate the log likelihood function that results from these initial parameters values. The numerical optimization methods can then be employed to make better guesses as to the value of the unknown parameters until the log-likelihood is maximized. Two methods that have been extensively analyzed are the Scoring Algorithm and the Expectation Maximization (EM) algorithm of Dempster, Laird & Rubin (1977). In our study we will use the first derivative method of Marquardt that is provided by the Eviews 4.1.

3.6 CUSUM AND CUSUM SQUARE TEST FOR PARAMETER INSTABILITY

Brown, Durbin and Evans (1975) have suggested two tests of the stability of regression coefficients, both of which use the recursive residuals in order to determine for parameter instability in the regression.

In recursive least squares the equation is estimated repeatedly, using ever larger subsets of the sample data. If there are k coefficients to be estimated in the b vector, then the first k observations are used to form the first estimate of b. The next observation is then added to the data set and k+1 observations are used to compute the second estimate of b. This process is repeated until all the T sample points have been used, yielding T-k+1 estimates of the b vector. At each step the last estimate of b can be used to predict the next value of the dependent variable. The one-step ahead forecast error resulting from this prediction, suitably scaled, is defined to be a recursive residual.

More formally, let X_{t-1} denote the (t-1) x k matrix of the regressors from period 1 to period t-1, and y_{t-1} the corresponding vector of observations on the dependent variable. These data up to period t-1 give an estimated coefficient vector, denoted by b_{t-1}. This coefficient vector gives you a forecast of the dependent variable in period t. The forecast is $x_t^{'}b$, where $x_t^{'}$ is the row vector of observations on the regressors in period t. The forecast error is $y_t - x_t^{'}b$, and the forecast variance is given by:

$$\sigma^2(1+x_t^{'}(X_t^{'}X_t)^{-1}x_t)$$

The recursive residual is w_t is defined as:

$$w_t = \frac{(y_t - x'_t b)}{(1+x_t^{'}(X_t^{'}X_t)^{-1}x_t)^{1/2}}$$

These residuals can be computed for t=k+1,...,T. If the maintained model is valid, the recursive residuals will be independently and normally distributed with zero mean and constant variance σ^2.

The ***Cusum Test*** is based on the cumulative sum of the recursive residuals. The statistic is computed as:

$$W_t = \sum_{r=k+1}^{t} w_r / s \quad t = k+1,...,T$$

where w is the recursive residual defined above and s is the standard error of the regression fitted to all T sample points. If the vector b remains constant from period to period, $E(W_t)=0$, but if b changes, W_t will tend to diverge form the zero mean value line. The significance of any departure from the zero line is assessed by reference to a pair of 5% significance lines, the distance between which increases with t. The 5% significance lines are found by connecting the points

$$[k, \pm -0.948(T-k)^{1/2}] \quad and [T, \pm 3 \cdot 0.948(T-k)^{1/2}]$$

Movement of W_t outside the critical lines is suggestive of coefficient instability.

The ***Cusum Square*** test is based on the test statistic:

$$S_t = \left(\sum_{r=k+1}^{t} w_r^2 \right) / \left(\sum_{r=k+1}^{T} w_r^2 \right)$$

S_t is a monotonically increasing sequence of positive numbers with $S_T = 1$. Under the null hypothesis of stability $1-S_t$ has a beta distribution with parameters $\alpha = -1+(T-k)/2$ and $\beta = -1+(t-k)/2$ and therefore S_t has mean value $(t-k)/(t-k)$. Brown, Durbin and Evans suggest constructing a confidence interval for S_t as $[(t-k)/(T-k)] \pm c_0$, where c_0 is chosen from tables constructed by Durbin (1969). If $\left|S_t - ((t-k)/(T-k))\right| > c_0$ for any $t \in [k+1, T]$ the null hypothesis is rejected.

The ability of the above two tests to determine a structure of stochastically varying coefficients, under alternative structures, will be examined in the next chapter in a Monte Carlo Study.

CHAPTER 4

A MONTE CARLO STUDY ON THE FORECASTING PERFORMANCE OF STATE SPACE MODELS

INTRODUCTION

In this chapter we conduct several Monte Carlo Experiments in order to investigate the forecasting performance of stochastic coefficient regression models under alternative structures of coefficient variation. More precisely, we compare the forecasting performance of the classical fixed coefficient regression model (estimated with OLS) with the forecasting performance of the simple Random Walk Model and a State Space Model when the true Data Generating Process (DGP) includes fixed and stochastic coefficients.

The first question that we try to answer is what happens in the case where the true DGP contains fixed coefficients and we wrongly decide to use a stochastic coefficient model. Therefore, before experimenting in an environment with stochastic coefficients we generate data with constant coefficients and compare the forecasting performance of the fixed OLS model with that of state space models.

It is almost obvious that when the true DGP contains coefficients that change over time in a stochastic way the forecasting performance of the fixed coefficient model will be deteriorated. For this purpose, we want to compare this deterioration with respect to the forecasting performance of a simple Random Walk Model and a correct Stochastic Coefficient Model.

Nevertheless, the question that arises is that if we accept that the DGP contains stochastic coefficients then how should we model this variation. As we have mentioned there have been analyzed three types of parameter variation in the literature, the Hildreth-Houck Model, the Random Walk Model and the Return to Normalcy Model. But all these models represent a different behavior of parameter variation over time. From this point it is obvious that even if we agree that there is parameter variation we have to decide on the specific structure of this variation. What we try to see is how changes the forecast accuracy of the stochastic coefficient models when we choose a structure of parameter variation that is different from that we have in the true DGP. In other words, we want to see if the misspecification on the structure of the coefficients causes a deterioration of the forecasting accuracy even in the case we use a state space model. For this purpose, we generate data with a specific structure of parameter variation and compare the RMSE of the incorrectly specified State Space Model with the RMSE of the correctly specified State Space Model, the fixed coefficient regression model and the simple Random Walk Model. Finally, we try to see if there is any specific coefficient structure that should be used in order to avoid any deterioration of the forecasting performance due to the misspecification in the variation of coefficients.

Furthermore, as in practice a first step to investigate whether any equations are characterized by parameter instability is recursive estimation of coefficients, for this reason we find the "average" recursive coefficient plots. Our intention is double. First, we want to check whether this procedure is trustworthy in deciding on parameter variation.

Second, by plotting the average recursive coefficient series we want to see if there is any specific pattern which may help to find the correct structure of parameter variation that the true DGP represents.

Moreover, in our experiments we conduct the widely used Cusum and Cusum Square tests. We want to investigate their ability to reject a false null hypothesis of parameter constancy. More precisely, we want to see how their ability to correctly advise us for parameter variation is behaved under different structures of coefficient variation.

All the above analysis is conducted both for a univariate and a bivariate regression model with stochastic coefficients where regressors are either stationary or integrated of order 1 (I(1)). We generate N=500[1] observations for k=1000 replication in its experiment. We make h=1,2,3,5,10 steps ahead forecasts. The measure of the forecast accuracy will be the Root Mean Square Error. Forecasts are generated taking the actualized value of the explanatory variables. We carried out the forecasts with expected values of the explanatory variables but the results remain the same.

The structure of our Data Generating Process has the following form:

- *Univariate case:*

$$\begin{aligned} y_t &= a + b_t x_t + e_{1t} \\ b_t &= r_0 + r_1 b_{t-1} + e_{2t} \quad \text{with} \\ x_t &= k_1 x_{t-1} + e_{3t} \end{aligned} \qquad \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{bmatrix} \right) \qquad (1)$$

- *Bivariate case:*

$$\begin{aligned} y_t &= a + b_{1t} x_{1t} + b_{2t} x_{2t} + e_{1t} \\ b_{1t} &= p_0 + p_1 b_{1t-1} + e_{5t} \ \forall t \\ b_{2t} &= r_0 + r_1 b_{2t-1} + e_{2t} \\ x_{1t} &= k_1 x_{1t-1} + e_{3t} \\ x_{2t} &= l_1 x_{2t-1} + e_{4t} \end{aligned} \quad \text{with} \quad \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \sigma_{34} & \sigma_{35} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_4^2 & \sigma_{45} \\ \sigma_{15} & \sigma_{25} & \sigma_{35} & \sigma_{45} & \sigma_5^2 \end{bmatrix} \right) \qquad (2)$$

with $p_0 = 1,\ p_1 = 0,\ \sigma_5^2 = 0 \Rightarrow b_{1t} = 1\ \forall t$

Giving different values to the coefficients of the mechanism with which we generate the stochastic coefficients we take the three different structures that have been applied in the literature. More precisely, both for b_t *and* b_{2t} we have:

[1] We generate 550 observations but we drop the first 50 in order to avoid the effect of initial values.

Hildreth-Houck Model	$r_0 \neq 0, r_1 = 0$
Random Walk Model	$r_0 = 0,\ r_1 = 1$
Return to Normalcy Model	$r_0 \neq 0, r_1 < 1$

The chapter is organized as follows. First, we compare the RMSE of fixed and stochastic coefficient models when the DGP includes constant coefficients. Secondly, we represent the forecasting performance of univariate and bivariate regression models with stochastic coefficients in the case the regressors are stationary ($k_1, l_1 < 1$). Thirdly, we represent the same analysis in the case of I(1) explanatory variables ($k_1, l_1 = 1$). Finally, we investigate the ability of the Cusum and Cusum Square Test to detect for parameter instability.

4.1 True DGP: constant coefficients

Using the DGP (1) we generate data with fixed coefficients. Our intention is to compare the forecasting performance of the fixed OLS model with the state space models. The analysis is conducted both with X_t be a stationary process and a I(1) process. The results are represented in the tables below:

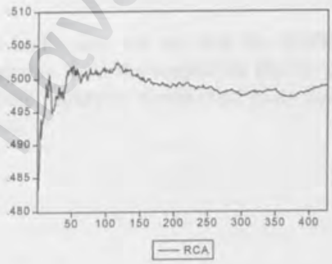
X_t Stationary process

Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	0.8033573	0.7920559	0.8018794	0.8158629	0.8120131
Random Walk	1.4410981	1.5719336	1.6928819	1.9665545	2.0451513
OLS*	1.1961677	1.2733917	1.2809643	1.4222815	1.4579162
OLS with Lag of Y	0.8031151	0.7935706	0.8011907	0.8166728	0.8122041
OLS Lag*	1.1938611	1.2760254	1.2811661	1.4276861	1.4648209
State Space Models					
AR(1)	0.7998224	0.7989409	0.8018967	0.8149911	0.8156078
AR(1)*	1.1947253	1.2746736	1.2806326	1.4226276	1.4579952
Random Walk	0.8014789	0.7970094	0.8017862	0.8170696	0.8142413
Random Walk*	1.1952331	1.2748264	1.2801257	1.4227517	1.4578345
Hildreth-Houck	0.8032034	0.7920661	0.8018236	0.8158910	0.8118072
Hildreth-Houck*	1.1960808	1.2734943	1.2810513	1.4221046	1.4578702
AR(1) with drift	0.8050082	0.7944509	0.8015604	0.8193289	0.8125185
AR(1) with drift*	1.1973429	1.2722863	1.2804715	1.4221990	1.4580041

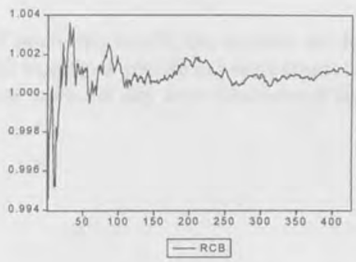
(* denotes forecasts without using actualized values for X_t)

The recursive coefficient plots for the intercept and the slope where:

RECURSIVE COEFFICIENT INTERCEPT



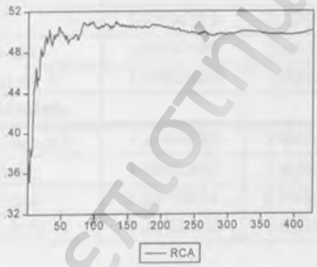
RECURSIVE COEFFICIENT SLOPE



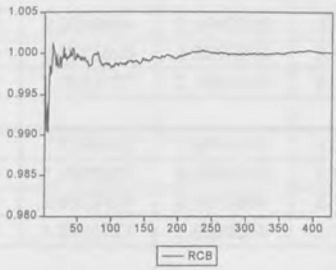
X_t I(1) process

Constant Coefficient Models	1	2	3	5	10
OLS	0.7631830	0.7733595	0.8153890	0.8187291	0.8064980
Random Walk	1.3546727	1.6167211	1.7681237	2.0334962	2.7271822
OLS*	1.1133254	1.3733633	1.5807643	1.9198065	2.7000474
OLS with Lag of Y	0.7654248	0.7755017	0.8129506	0.8166752	0.8069155
OLS Lag*	1.1157792	1.3742453	1.5817083	1.9236331	2.7036244
State Space Models					
AR(1)	0.7708007	0.7761614	0.8094086	0.8238440	0.8250485
AR(1)*	1.1183912	1.3816612	1.5852900	1.9337260	2.7113350
Random Walk	0.7665512	0.7884879	0.8230653	0.8416046	0.8200957
Random Walk*	1.1207408	1.3836173	1.5816511	1.9213378	2.6885014
Hildreth-Houck	0.7659112	0.7761475	0.8184180	0.8197597	0.8064766
Hildreth-Houck*	1.1171692	1.3771386	1.5842716	1.9221763	2.7008057
AR(1) with drift	0.797949	0.806861	0.807868	0.827741	0.830142
AR(1) with drift*	1.161465	1.387940	1.579919	1.967925	2.832755

RECURSIVE COEFFICIENT
INTERCEPT



RECURSIVE COEFFICIENT
SLOPE



In both cases we see that the RMSE of stochastic coefficient models are hardly distinguishable from the respective RMSE of the true fixed coefficient regression model. Finally, both recursive coefficient plots does not represent any time dependence for the coefficients.

4.2 UNIVARIATE CASE $X_t \sim I(0)$ PROCESS

In this section we use the Data Generating mechanism (1) assuming that the intercept of the regression is $a = 0.5$. X_t and the trivariete distribution of errors are:

$$X_t = 0.75X_{t-1} + e_{3t} \sim I(0) \qquad \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

In every case the forecast is generated using actualized values of the explanatory variable. Forecasts with X_t replaced by its expected values are reported in appendix A.

4.2.1 True DGP: b_t~AR(1)

In this case we generate the data assuming that the stochastic coefficient follows an AR(1) process of the following form: $b_t = 0.85b_{t-1} + e_{2t}$ with $v(e_{2t}) = 1$. Then we make forecasts for the series y that is generated by the mechanism (1). We used all available structures of b_t in order to investigate the cost of misspecification in the variation structure of b_t. The results are summarized in the following table:

Table 1

DGP $b_t \sim$ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	2.0834709	2.0704211	2.0836564	2.1840239	2.1729160
Random Walk	2.0670186	2.4432259	2.7515525	2.9252290	2.9862839
OLS with Lag of Y	1.7962989	1.9610320	2.0642707	2.1855148	2.1802664
State Space Models					
AR(1) (*correct model*)	1.4194885	1.6874647	1.7794691	1.9746511	2.0729880
Random Walk	1.4682330	1.8364989	1.9563911	2.2268750	2.5627150
Hildreth-Houck	2.0818974	2.0466801	2.0673151	2.1679584	2.1498400
AR(1) with drift	1.4192130	1.6929570	1.7924767	1.9935256	2.1129986

The first part of the table includes the RMSE of fixed coefficient models. The second part has the RMSE of state space models. In this case the correct state space model is the b_t~AR(1) and the others are wrong. It is clear that forecasting with a fixed coefficient regression model the forecast accuracy deteriorates in comparison with the RMSE of the correct model ($b_t \sim$ AR(1)). Nevertheless, the forecast of the simple OLS is not worse than the simple Random Walk model except for the case of 1 step ahead forecast. Besides, including a lag in the fixed coefficient model as a further explanatory variable it improves the RMSE but not so as to be lower than the RMSE of the correct stochastic coefficient model. (Table 1a in appendix A contains RMSEs without actualized values of X_t).

Another important point is that in the case we use a state space model but with a b_t's structure different from the true one we have a deterioration of the RMSE. For instance, assuming that b_t ~ R.W then we see that for h=5,10 steps ahead forecast the RMSE is even worse than the RMSE of the simple OLS model. The deterioration is even worse in the case we put r_1=0.6 instead of 0.85. In that case we see that as we decrease the influence of past shocks on the actual system state the cost of misspecification in the structure of b_t becomes worse. Assuming that b_t follows a Random Walk process the RMSE is worse than the simple OLS for h=2,3,5,10 steps ahead forecast. The table when b_t follows an AR(1) process with r_1=0.6 is in appendix A(Table1b).

Only in the case we assume an AR(1) with drift structure we have almost no loss on the forecast accuracy. In that case the AR(1) with drift estimates are very close to the true values of b_t's structure:

DGP	r_0=0	r_1=0.85	σ_2^2=1
AR(1) (correct)	-	0.845552	0.995925
	-	(0.032928)	(0.145336)
AR(1) with drift	-0.002461	0.840478	0.988731
	(0.050384)	(0.034102)	(0.147991)

(Standard errors in parenthesis)

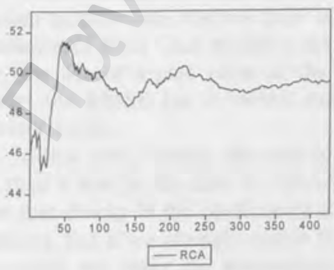
When we used OLS estimation we also computed the mean value and t-statistic of the estimator for the (wrongly supposed) fixed coefficient b.

	$\hat{b}_{OLS}$	*t-statistic*
Mean	0.0012326	-0.0005401
Standard Deviation	0.3700114	3.9688222

From these results we see that in case we wrongly decide to use a fixed coefficient model we will have totally misleading statistical inference. Though, the mean value of $\hat{b}_{OLS}$ is equal to the mean value of the stochastic coefficient.

The recursive plots of coefficients are:

RECURSIVE COEFFICIENT
INTERCEPT



RECURSIVE COEFFICIENT
SLOPE

From these plots it is evident that we should be concern with the time dependence of coefficient b. Nevertheless, the same happens for the intercept even though we have assumed to be fixed over time. It is worth to note that when we put r_1=0.6 this variation on the intercept was not clear. All the above analysis was conducted assuming that the error of b_t has a smaller variance. We put $var(e_{2t}) = 0.2$ and the results were almost the same. The table with forecasts (table1c) and the plots of recursive coefficients are represented in appendix A.

4.2.2 True DGP: b_t~Hildreth-Houck

We generate data assuming that the structure of b_t is defined by the Hildreth-Houck model. More precisely:

$$b_t = 0.8 + e_{2,t} \quad \text{with } var(e_{2t}) = 1$$

Again we make forecasts with all available structures of b_t in order evaluate the potential deterioration of the RMSE in the case of misspecification. The RMSE of its model is represented in the table below:

<u>**Table 2**</u>

DGP b_t~ Hildreth-Houck r0=0.8, r1=0					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.2995642	1.2682143	1.3205568	1.3620113	1.3519805
Random Walk	2.0155874	2.0776741	2.1872210	2.3616805	2.3235852
OLS with Lag of Y	1.2951865	1.2681951	1.3223114	1.3629275	1.3517205
State Space Models					
Hildreth-Houck (*correct*)	1.2920307	1.2594309	1.3170853	1.3558265	1.3509205
AR(1)	1.4657628	1.4251352	1.5375571	1.6330546	1.5834074
Random Walk	1.6275870	1.5833414	1.5869684	1.7031429	1.6447555
AR(1) with drift	1.2995436	1.2602616	1.3171701	1.3559104	1.3505647

From the above results it is evident the cost of assuming fixed coefficient regression models when the true DGP contains stochastic. Again, though, the forecasting performance of fixed OLS model is not worse than the simple random walk model. As before, the use of a past value of the dependent variable as an explanatory variable decreases the RMSE but it cannot outperform the forecasts of the correct stochastic coefficient model.

In this case, though, the cost of wrong selection of the structure of b_t is even worse than it was in the case 4.1.1(b_t~AR(1)). Choosing the Hildreth-Houck model we assume that shocks in the coefficients are "absorbed" by the system within in the same time period. But if we wrongly decide that the correct structure for b_t is an AR(1) model without drift, we make the assumption that shocks don't fade out immediately. As a consequence the RMSE of the mispecified state space model is worse than the simple fixed coefficient regression model for all steps. Things deteriorate further when we assume that there is fully persistence, i.e. when b_t~R.W. Taking these results into consideration we may conclude that the cost of choosing a wrong structure for b_t becomes greater when r_1 is close to zero, i.e. when the model of b_t has small or zero memory. (RMSE with expected values of X_t are reported in table 2a in appendix A).

Again, if we choose the AR(1) with drift model we produce similar forecast with the correct state space model since the estimated parameters of the model are very closed to the true DGP:

DGP	$r_0 = 0.8$	$r_1 = 0$	$\sigma_2^2 = 1$
Hildreth-Houck	0.801686	-	1.000065
	(0.071547)	-	(0.128415)
AR(1) with drift	0.804957	0.000373	0.990528
	(0.111942)	(0.107058)	(0.129542)

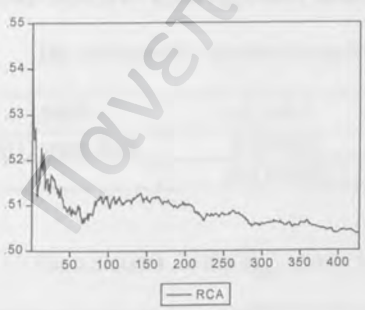
(Standard errors in parenthesis)

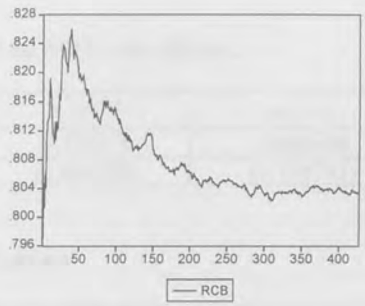
The estimates for the $\hat{b}_{OLS}$ and its t-statistic are:

	$\hat{b}_{OLS}$	*t-statistic*
Mean	0.8034800	0.0641789
Standard Deviation	0.0880944	1.539550

Again, it is evident the misleading statistical inference that appears in the case of using a model with fixed coefficients. Nevertheless, we see that the mean value of $\hat{b}_{OLS}$ is almost the same with the steady state value of b_t and with very small standard deviation. This may explain the small difference between the RMSEs of fixed models with the RMSEs of stochastic coefficient models.

Finally, the recursive plots of coefficients are:





The same analysis was carried out assuming that the error process of b_t has a variance $\text{var}(e_{2t}) = 0.2$ and we found the same results. The RMSE with the associated plots of recursive coefficients are represented in table 2b in appendix A.

4.2.3 True DGP: b_t ~ AR(1) with drift

Assuming that $r_0 = 0.7,\ r_1 = 0.3\ \textit{and}\ \text{var}(e_{2t}) = 1$ we allow b_t to follow an AR(1) with drift process. The RMSE of its model when the true DGP contains this structure of parameter variation are represented in the following table:

Table 3

DGP b_t~AR(1) with drift r0=0.7, r1=0.3					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.3357775	1.2343828	1.3728616	1.3086771	1.4595255
Random Walk	1.9952715	2.0938472	2.3367813	2.5294920	2.6273566
OLS with Lag of Y	1.3328765	1.2341178	1.3792583	1.3145584	1.4641537
TVP MODELS					
AR(1) with drift (*correct*)	1.2943835	1.2224082	1.3604499	1.3008546	1.4564665
AR(1)	1.4224689	1.3848218	1.5227358	1.5923051	1.7518272
Random Walk	2.014574	2.0121404	2.2415751	2.2241573	2.3898712
Hildreth-Houck	1.3154584	1.2296548	1.3720745	1.3158712	1.4687331

In this case we see that choosing any structure different from the correct one there is always a deterioration of the RMSE that in some cases (Random Walk) is even worse than the fixed coefficient regression model.

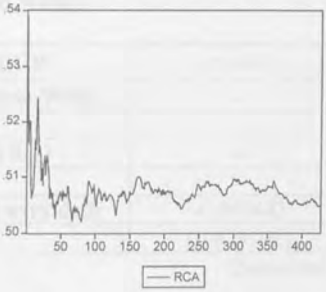
The estimates for the correct model and the AR(1) with drift are:

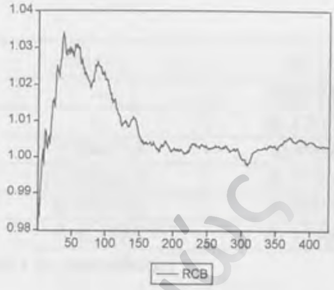
DGP	$r_0 = 0.7$	$r_1 = 0.3$	$\sigma_2^2 = 1$
AR(1) with drift	0.708027	0.292778	0.996466
	(0.110604)	(0.086139)	(0.134741)

The estimates for the $\hat{b}_{OLS}$ and its t-statistic are:

	$\hat{b}_{OLS}$	*t-statistic*
Mean	1.0030018	0.0556282
Standard Deviation	0.1125198	1.9087424

Finally, the recursive coefficient plots are:





The case where forecasted values for X_t are used is represented in appendix A table3a.

4.2.4 True DGP: $b_t \sim$ Random Walk

Assuming that the process of b_t is defined by the Random Walk Model with $r_0 = 0, r_1 = 1$ *and* $\text{var}(e_2 t)$ we have the following results:

Table 4

DGP $b_t \sim$ Random Walk r0=0, r1=1					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	11.609309	11.573643	11.786424	11.734197	12.224767
Random Walk	14.669134	19.788273	22.267452	25.440130	29.300482
OLS with Lag of Y	9.3470632	11.265964	11.814486	11.997287	12.927582
TVP MODELS					
Random Walk (*correct*)	1.4681018	1.8631705	2.0562546	2.5354073	3.3437753
AR(1)	1.4689436	1.8851306	2.1177453	2.7129386	3.6765501
AR(1) with drift	1.4654841	1.8895214	2.1324575	2.7725452	3.7025478
Hildreth-Houck	11.625006	11.614123	11.802773	11.791659	12.169561

Again the picture is the same. The fixed coefficient regression models have a worse forecasting performance in comparison with the correct state space model. Choosing a different structure for b_t than the true one leads to a deterioration in the RMSE, especially in the case of the Hildreth-Houck model. This may be clear since we impose b_t's structure to have no memory, when the true one has infinite (Random Walk Model). Using the AR(1) or the AR(1) with drift model for b_t causes a much smaller deterioration of the RMSE. We should note that in comparison with the previous cases the RMSE of the AR(1) with drift is closer but not so to the true state space model (Random Walk).

DGP	$r_0 = 0$	$r_1 = 1$	$\sigma_2^2 = 1$
Random Walk	-	-	0.994451
		-	(0.170657)
AR(1)	-	0.994733	0.993684
	-	(0.089011)	(0.177364)
AR(1) with drift	-0.000443	0.987602	0.989542
	(0.039192)	0.013385	(0.177854)

(*Standard errors in parenthesis*)

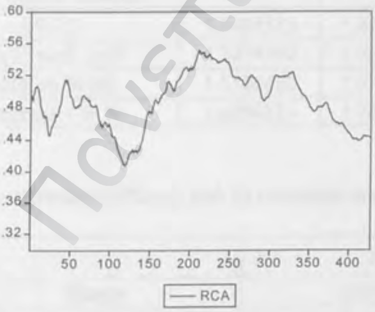
The estimated b (fixed) and its t-statistic were:

	$\hat{b}_{OLS}$	*t-statistic*
Mean	-0.9004699	0.7291830
Standard Deviation	14.164572	38.122423

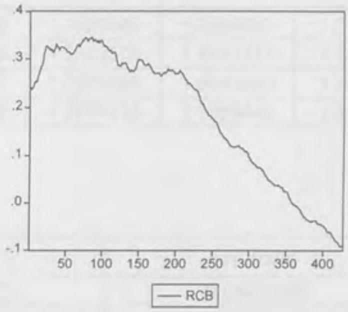
While in cases where b_t has a steady state the $\hat{b}_{OLS}$ could "catch" the average value of coefficient, now since b_t has no steady state the behavior of the OLS is worse and the discrepancy between the RMSE of fixed coefficient models with stochastic even larger.

The recursive plots of coefficients are represented below. It is interesting to note, that in this case that we have assume b_t ~ Random Walk the recursive plot of the intercept represents a time dependence even though we have assume to be fixed and equal to 0.5.

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE



In appendix A, table 4a and table 4b represent the RMSE in case we used expected values for X_t and when we generated data with $\text{var}(e_{2t}) = 0.2$ respectively. The only difference with the above results is that the recursive plot of intercept doesn't represent time dependence as it was in the case of $\text{var}(e_{2t}) = 1$.

In summarizing the above results we see that in case we do not know the correct structure for b_t it is safe to use an AR(1) with drift structure. In all cases it produces forecasts that are close to the forecasts of the correct state space models.

We conducted the same analysis, for the case where $b_t \sim$ AR(1), but assuming that there is, firstly, correlation between the dependent variable and the stochastic coefficient and, secondly, correlation between the dependent variable and the explanatory variable. The results were almost the same. The tables with the RMSE are represented in appendix A, table A and table B respectively.

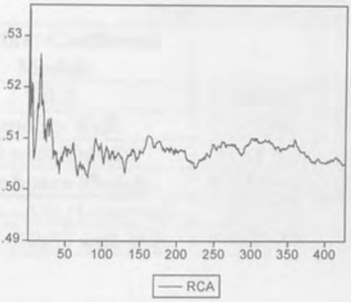
Having in mind that the AR(1) with drift may be a safe structure (in case we cannot distinguish the correct), we tried to see if this holds in the case where the structure of b_t is a Moving Average Process. As the AR(1) with drift may be regarded as a general structure that can accommodate as substructures our examined models we would like to see its performance when the true structure of b_t does not belongs to this "family". Unfortunately, in this case we cannot have the RMSE of the correct model since it is not available by the Eviews 4.1 to model b_t as a Moving Average. Our comparison was with respect with the other 3 models, the R.W, the Hildreth-Houck and the AR(1) without drift. Again the picture was the same. Modeling b_t as $b_t = e_{2t} + 0.5e_{2t-1}$ we took the following results:

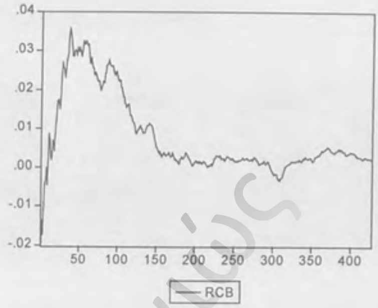
DGP b_t Moving Average					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.4119575	1.2916757	1.4085529	1.3566622	1.5155534
Random Walk	1.8476026	1.9343575	2.1275359	2.0883138	2.1961411
OLS with Lag of Y	1.3849229	1.2964003	1.4095650	1.3551992	1.5153035
State Space Models					
AR(1)	1.3425375	1.2829205	1.3920566	1.3564832	1.5106760
AR(1) with drift	1.3356160	1.2842603	1.3915575	1.3501213	1.5092354
Random Walk	1.5188460	1.5759927	1.6302858	1.6693800	1.7630826
Constant +W.N	1.4058721	1.2863470	1.3971216	1.3609446	1.5104498

The estimated b (fixed) and its t-statistic were:

	$\hat{b}_{OLS}$	*t-statistic*
Mean	0.0024522	0.0441058
Standard Deviation	0.1190723	1.9296474

Recursive coefficient plots:





RMSE results in the case where expected values of X_t were used are represented in table C in appendix A.

4.3 BIVARIATE CASE $X_t \sim I(0)$ PROCESS

We conducted the same analysis in the case of a bivariate regression model where the first coefficient was constant and equal to 1 and the second was stochastic. Again, X_t was supposed to be an I(0) process. The data was generated by the mechanism (2) with

$$\begin{array}{l} X_{1t} = 0.7X_{1t-1} + e_{3t} \\ X_{2t} = 0.8X_{2t-1} + e_{4t} \end{array} \quad \text{and} \quad \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)$$

In this section we computed the RMSE of the fixed coefficient models, the RMSE of the correct state space model and the RMSE of the state space model with $b_t \sim AR(1)$ with drift in order to see if again the AR(1) with drift can capture correctly the unknown time path of b_t.

4.3.1 True DGP: $b_{1t}=1$, $b_{2t} \sim AR(1)$

Letting $b_{1t} = 1\ \forall t,\ b_{2t} = 0.85b_{2t-1} + e_{3t}\ \textit{with}\ \text{var}(e_{2t}) = 1$ we found the following results:

Table 5

DGP $b_{1t}=1$ for all t $b_{2t}\sim AR(1)$ r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	2.2845507	2.2575106	2.2836071	2.3464585	2.3406614
Random Walk	2.2182892	2.8114443	3.1288191	3.4915022	3.6407552
OLS with Lag of Y	1.9120162	2.1877129	2.2860402	2.4349799	2.4042678
State Space Models	2.3965060	2.3005085	2.3103645	2.6266122	2.2614473
$b_{1t}=1, b_{2t}\sim AR(1)$ (*correct*)	1.5062798	1.7214782	1.8413855	2.0975423	2.2445512
b_{1t}, $b_{2t}\sim AR(1)$ with drift	1.5036054	1.7293236	1.8518660	2.0993967	2.2657953

Using the correct state space model yields the lowest RMSE, but we see that the same happens in case we suppose that both coefficients follow an AR(1) with drift process. This is clear if we see the estimates of the AR(1) with drift structure.

b_{2t}	$r_0=0$	$r_1=0.85$	$\text{var}(e_{2t})=1$
Correct State Space Model	-	0.845921	1.000969
	-	(0.031948)	(0.115007)
Both coefficients follow an AR(1) with drift process			
b_{1t}	$p_0=1$	$p_1=0$	$\text{var}(e_{5t})=0$
AR(1) with drift	0.342508	0.657402	0.00910
	(0128363)	(0.131392)	(0.01102)
b_{2t}	$r_0=0$	$r_1=0.85$	$\text{var}(e_{2t})=1$
AR(1) with drift	-0.000176	0.839992	0.983752
	(0.053925)	(0.069769)	(0.134605)

(*Standard errors in parenthesis)*

For b_{1t} we see that if we impose an AR(1) with drift process the estimated variance of the error process e_{5t} is almost zero. Therefore, we have a relationship of the form:

$$b_{1t}=0.342508+0.657402b_{1t-1}$$

and with recursive substitutions we get a geometric sequence which for 0.657402<1 it converges to 1:

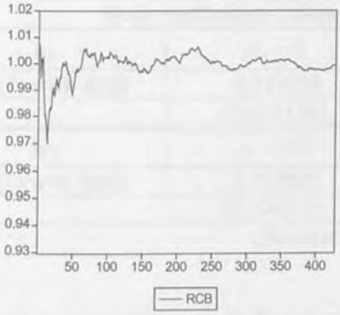
$$b_{1t}\underset{t\to\infty}{\to}\frac{0.342508}{1-0.657402}\approx 1.$$

The estimates for the OLS where:

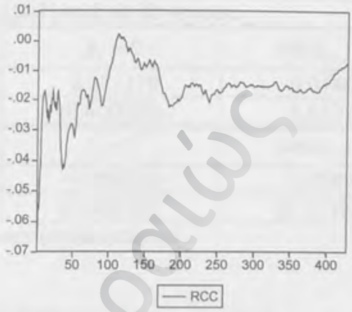
	$\hat{b}_{1,OLS}$	*t-statistic-b_1*	$\hat{b}_{2,OLS}$	*t-statistic-b_2*
Mean	0.9994371	-0.0062695	-0.0072446	-0.0830015
Standard Deviation	0.1739812	1.5411483	0.4007282	4.3227818

The recursive plots of coefficients are:

RECURSIVE COEFFICIENT b_{1t}



RECURSIVE COEFFICIENT b_{2t}



Again it is evident the misleading information of the recursive coefficient as far as the time dependence of the intercept is concerned.

In appendix A, table 5a contains the RMSE in case we use the forecasted values for X_t

4.3.2 True DGP: $b_{1t}=1$, $b_{2t} \sim$ Hildreth-Houck

Assuming that b_{2t} follows the Hildreth-Houck model we took the below results:

Table 6

DGP $b_{1t}=1$ for all t b_{2t}~Hildreth Houck r0=0.8, r1=0					
Constant Coefficient Models	1	2	3	5	10
OLS	1.4083313	1.4384124	1.3650980	1.3858138	1.4724581
Random Walk	2.1579081	2.4562104	2.5287784	2.8540823	2.8045767
OLS with Lag of Y	1.4113147	1.4396023	1.3645753	1.3868294	1.4797854
State Space Models					
$b_{1t}=1, b_{2t}$~H-H (*correct*)	1.3968588	1.4320868	1.3573203	1.3799223	1.4609702
b_{1t}, b_{2t}~AR(1) with drift	1.4009457	1.4335317	1.3598403	1.3816449	1.4637684

We see that imposing in both b_{1t} and b_{2t} an AR(1) with structure we take approximately the same results with the correct state space model. As above coefficients of the AR(1) with drift structure coincide with the coefficients of the true DGP.

	$r_0 = 0.8$	$r_1 = 0$	$\text{var}(e_{1t}) = 1$
Correct State Space Model	0.800671	-	0.996125
	(0.072252)	-	(0.133332)
Both coefficients follow an AR(1) with drift process			
b_{1t}	$p_0 = 1$	$p_1 = 0$	$\text{var}(e_{5t}) = 0$
AR(1) with drift	0.949788	0.053370	0.014621
	(0.613126)	(0.610000)	0.023713
b_{2t}	$r_0 = 0.8$	$r_1 = 0$	$\text{var}(e_{2t}) = 1$
AR(1) with drift	0.807820	-0.009074	0.988912
	(0.100533)	(0.086159)	(0.133978)

(*Standard errors in parenthesis*)

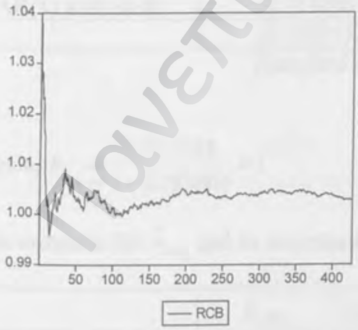
For b_{1t} we have the same result. By recursive substitution it converges to 1.

The estimates for $\hat{b}_{OLS}$ and its associated standard errors are:

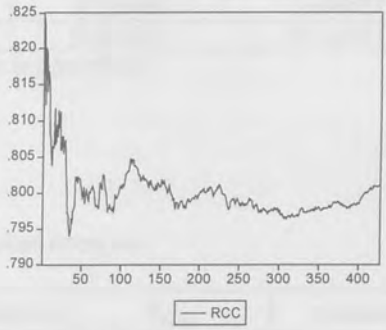
	$\hat{b}_{1,OLS}$	*t-statistic-b_1*	$\hat{b}_{2,OLS}$	*t-statistic-b_2*
Mean	1.0028353	0.0431759	0.8011061	0.0230744
Standard Deviation	0.0655000	0.9740418	0.0895863	1.5883009

Again, the mean value of $\hat{b}_{2,OLS}$ is equal to the mean value of the stochastic coefficient, but the t-statistic is totally misleading.

RECURSIVE COEFFICIENT b_{1t}



RECURSIVE COEFFICIENT b_{2t}



In table 6a of appendix A RMSE are reported without using actualized values for X_t.

4.3.3 True DGP: b_{1t}=1, b_{2t} ~ AR(1) with drift

The forecasting performance of the models where as follow:

Table 7

DGP b_{1t}=1 for all t b_{2t}~AR(1) with drift r0=0.7, r1=0.3					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.4010865	1.4431236	1.3633531	1.4251549	1.5534331
Random Walk	2.2293096	2.6801785	2.7771348	3.1508603	3.0583624
OLS with Lag of Y	1.3937067	1.4461395	1.3587349	1.4196468	1.5526991
State Space Models					
b_{1t}=1,b_{2t}~AR(1) with drift (*correct*)	1.3774982	1.4289584	1.3637951	1.4110462	1.5389854
b_{1t}, b_{2t}~AR(1) with drift	1.3781490	1.4299060	1.3681749	1.4119139	1.5396093

Estimates for the AR(1) with drift structure where:

	$r_0 = 0.7$	$r_1 = 0.3$	$\text{var}(e_{2t}) = 1$
Correct State Space Model	0.713617	0.285568	0.994037
	(0.106263)	(0.083306)	(0.138425)

Both coefficients follow an AR(1) with drift process

b_{1t}	$p_0 = 1$	$p_1 = 0$	$\text{var}(e_{5t}) = 0$
AR(1) with drift	0.919988	0.085600	0.015208
	(0.613558)	(0.606785)	(0.024306)
b_{2t}	$r_0 = 0.7$	$r_1 = 0.3$	$\text{var}(e_{2t}) = 1$
AR(1) with drift	0.713790	0.285486	0.994427
	(0.106915)	(0.083331)	(0.138941)

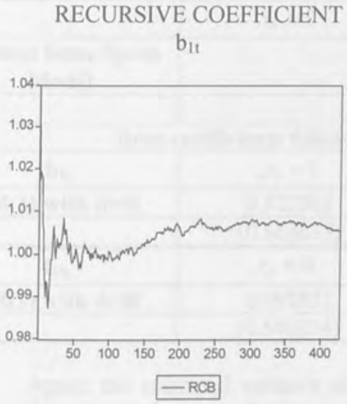
(Standard errors in parenthesis)

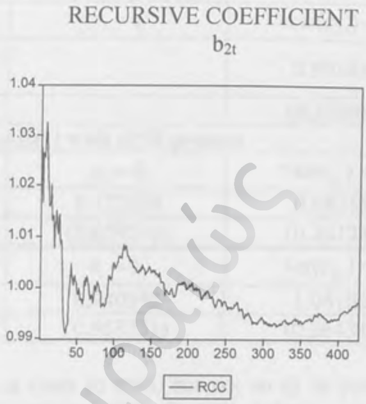
Again, $b_{1t} \underset{t\to\infty}{\rightarrow} \frac{0.919988}{1-0.085600} \approx 1$

The estimates for $\hat{b}_{OLS}$ and its associated standard errors are:

	$\hat{b}_{1,OLS}$	*t-statistic-b_1*	$\hat{b}_{2,OLS}$	*t-statistic-b_2*
Mean	1.0054755	0.0798508	0.9975474	-0.0417575
Standard Deviation	0.0788675	1.1359106	0.1178466	2.0346842

And the recursive coefficient plots for b_{1t} and b_{2t} are respectively:





(In appendix A, table 7a includes the RMSE without using actualized values of X_t)

4.3.4 True DGP: b_{1t}=1, b_{2t} ~ Random Walk

The RMSE of its model in this case is:

Table 8

DGP b_{1t}=1 for all t b_{2t}~Random Walk r0=0, r1=1					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	5.8877161	6.2188266	6.5657492	6.6910551	6.7145483
Random Walk	6.6067272	8.7671209	11.034701	12.396745	15.016038
OLS with Lag of Y	4.5841147	5.8089001	6.7030590	6.6573829	6.9972650
State Space Models					
b_{1t}=1,b_{2t}~Random Walk (*correct*)	1.5999083	2.1409731	2.3585758	2.8153893	3.8059302
b_{1t}, b_{2t}~AR(1) with drift	1.7200615	2.2211722	2.5945843	2.9252777	4.1294967

Looking at the estimates of the AR(1) with drift model we have:

	$r_0 = 0$	$r_1 = 1$	$\text{var}(e_{2t}) = 1$
Correct State Space Model	-	-	0.991454
	-	-	(0.352983)
Both coefficients follow an AR(1) with drift process			
b_{1t}	$p_0 = 1$	$p_1 = 0$	$\text{var}(e_{5t}) = 0$
AR(1) with drift	0.822563	0.172844	0.083586
	(0.664637)	(0.659533)	(0.261283)
b_{2t}	$r_0 = 0$	$r_1 = 1$	$\text{var}(e_{2t}) = 1$
AR(1) with drift	0.092332	0.940942	1.061866
	(0.540374)	(0.053351)	(0.364312)

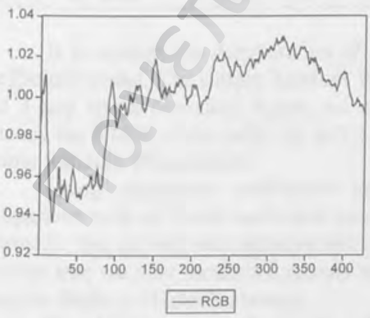
Again, the estimated variance of b_{1t} is close to zero, but not so as in previous cases. This may be the cause for the small discrepancy of the RMSE of the correct state space model with the AR(1) with drift model.

The estimates for $\hat{b}_{OLS}$ and its associated standard errors are:

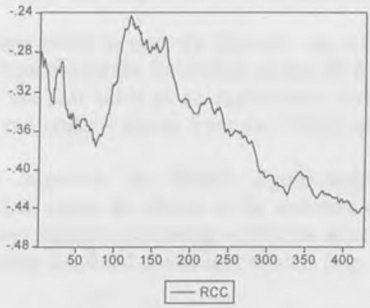
	$\hat{b}_{1,OLS}$	*t-statistic-b_1*	$\hat{b}_{2,OLS}$	*t-statistic-b_2*
Mean	0.9886993	0.0243203	1.4207447	4.3608719
Standard Deviation	0.8103209	1.7636139	7.4732029	22.353825

Finally from the recursive coefficient plots it is evident the wrong information we should retrieve for the time dependence of the intercept.

RECURSIVE COEFFICIENT
b_{1t}



RECURSIVE COEFFICIENT
b_{2t}



The case where we do not use actualized values for the explanatory variables is represented in appendix A table 8a.

4.4 UNIVARIATE CASE $X_t \sim I(1)$ PROCESS

In this section we carried out the same analysis as above but with the assumption that the regressor X_t is an I(1) process (the DGP is of the form (1)). Our intention is to check whether the Kalman filter can correctly "uncover" the time path of stochastic coefficient in an environment with integrated regressors and to measure again the cost of misspecification in the structure of b_t. The process of X_t and the trivariate distribution of errors are:

$$X_t = X_{t-1} + e_{3t} \sim I(1) \text{ and } \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

4.4.1 True DGP: $b_t \sim$ AR(1)

Assuming that $b_t = 0.85 b_{t-1} + e_{2t}$ with var(e_{2t}) we took the following results:

Table 9

DGP $b_t \sim$ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	25.654898	24.922812	25.464393	26.855880	27.101611
Random Walk	14.575316	18.985485	23.058544	30.143178	33.115341
OLS with Lag of Y	14.226902	17.498545	20.720804	25.596025	27.108223
State Space Models					
AR(1) (*correct model*)	13.843131	17.192413	20.2034970	25.532374	26.499894
Random Walk	13.967978	18.978744	22.608439	29.716108	32.301826
Hildreth-Houck	26.102457	25.075142	26.321454	27.001423	27.114201
AR(1) with drift	13.899050	17.215918	20.589910	25.738918	27.094357

It is evident the deterioration of the forecasting in case we wrongly use a fixed coefficient model. The simple Random Walk outperforms the fixed OLS model for h=1,2 and 3 step ahead forecasts. Again, the use of the past value as an explanatory variable reduces the RMSE (especially for h=1,2,3,5) and ranks it above from the RMSE of the simple Random Walk model.

Using stochastic coefficient models improves the RMSE significantly in comparison with all fixed coefficient models. But, again, the choice of the structure of b_t is crucial. The correct state space model is always superior, but using a different structure than the true we may produce forecasts worse than the fixed coefficient models (e.g. $b_t \sim$ Random Walk or Hildreth-Houck).

The AR(1) with drift structure for b_t remains a good approximation. This is evident looking at the estimates of its coefficients:

DGP	r_0=0	r_1=0.85	σ_2^2=1
AR(1)	-	0.846352	0.997911
	-	(0.024850)	0.075239
AR(1) with drift	0.003343	0.841114	0.995129
	(0.055371)	(0.025843)	(0.074919)

(*Standard errors in parenthesis)*

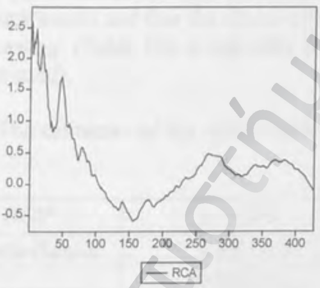
Estimates for b_{OLS} and its t-statistic where:

	$\hat{b}_{OLS}$	*t-statistic*
Mean	-0.0022107	0.0888210
Standard Deviation	0.6264857	4.0643743

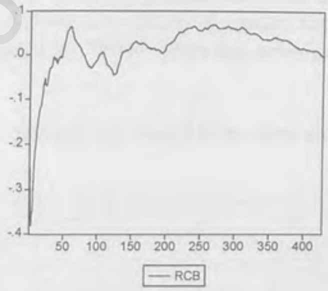
As in the case with stationary regressors, if coefficient has a steady state then the $\hat{b}_{OLS}$ mean value equals to the mean of the stochastic coefficient.

Finally, the recursive coefficients for the intercept and slope where:

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE



From the above plots it is evident the time dependence for b_t. But as we see the same indicates and the recursive coefficient plot for the intercept even though it has been imposed to be fixed and equal to 0.5.

(In appendix B, table 9a includes RMSEs without using actualized values of X_t).

4.4.2 True DGP: $b_t \sim$ Hildreth-Houck

Letting $b_t = 0.8 + e_{2,t}$ with $\text{var}(e_{2t}) = 1$, we had the following results:

Table 10

DGP $b_t \sim$ Hildreth-Houck r0=0.8, r1=0					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	13.642323	13.977513	13.322198	14.302548	14.555497
Random Walk	19.597235	20.706484	20.266892	21.197960	20.924806
OLS with Lag of Y	13.647103	13.969199	13.324837	14.303998	14.555613
State Space Models					
Hildreth-Houck(*correct*)	13.584099	13.954224	13.303987	14.292164	14.535497
AR(1)	16.165448	16.554812	16.485493	17.484086	17.639186
Random Walk	19.569647	20.660344	20.128019	20.984013	20.722154
AR(1) with drift	13.623562	13.967189	13.313984	14.352337	14.537792

In this case the correct state space model is the Hildreth-Houck. Again, we see that the performance of fixed coefficient models is worse in comparison to the correct state space model and that the choice of the structure of b_t is very important for purposes of forecasting. (Table 10a in appendix B includes the RMSE when not actualized values of X_t are used).

The estimates of the AR(1) with drift structure are found to be very close to the true:

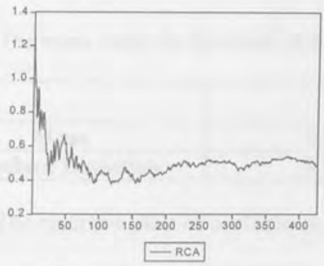
DGP	$r_0 = 0.8$	$r_1 = 0$	$\sigma_2^2 = 1$
Hildreth-Houck	0.801402	-	0.998953
	(0.067061)	-	(0.072906)
AR(1) with drift	0.807060	-0.008933	0.993813
	(0.081083)	(0.074131)	(0.073181)

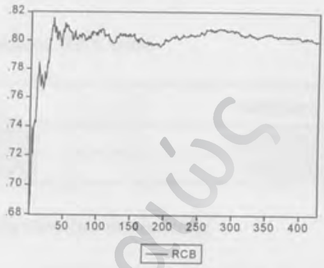
(*Standard errors in parenthesis*)

The estimates for the $\hat{b}_{OLS}$ and its t-statistic are:

	$\hat{b}_{OLS}$	*t-statistic*
Mean	0.7998855	0.0244544
Standard Deviation	0.1004540	1.1753163

And the recursive coefficient plots:





4.4.3 True DGP: $b_t \sim$ AR(1) with drift

The forecasting performance in the case where $r_0 = 0.7, r_1 = 0.3$ *and* $\text{var}(e_{2t}) = 1$ was:

Table 11

DGP b_t~AR(1) with drift r0=0.7, r1=0.3					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	14.831466	14.092687	13.937074	15.341365	16.073700
Random Walk	17.509618	19.845952	20.923788	22.100610	22.567203
OLS with Lag of Y	14.049675	14.036605	13.955784	15.343504	16.082759
State Space Models					
AR(1) with drift (*correct*)	13.963600	13.978040	13.865795	15.385462	16.074124
AR(1)	16.086209	17.031142	17.210129	18.998418	20.198192
Random Walk	17.045176	18.830579	19.954898	21.320060	21.669623
Hildreth-Houck	14.768898	14.039225	13.914185	15.380674	16.070427

The results are the same as in the case of X_t~I(0) process. The AR(1) with drift structure for b_t is the only structure that gives the lowest RMSEs. Its estimates for the structure of b_t are:

DGP	$r_0 = 0.7$	$r_1 = 0.3$	$\sigma_2^2 = 1$
AR(1) with drift	0.711776	0.291456	0.994122
	(0.083239)	(0.047363)	(0.072943)

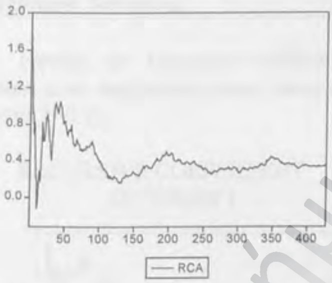
(*Standard errors in parenthesis)*

The mean value for the fixed OLS estimator for b was:

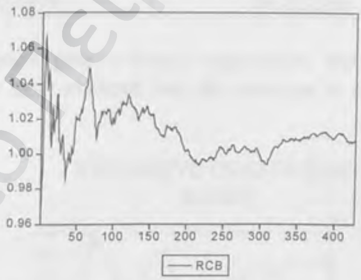
	$\hat{b}_{OLS}$	*t-statistic*
Mean	0.9976225	0.0062826
Standard Deviation	0.1456374	1.5892500

The recursive plots of coefficients in this case are misleading:

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE



(Table 11a in appendix B includes RMSE without actualized values of X_t)

4.4.4 True DGP: b_t ~ Random Walk

Finally, assuming that b_t ~ Random Walk, the RMSE of its model was:

Table 12

DGP b_t ~ R.W r0=0, r1=1					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	114.07371	115.64601	115.79923	117.01485	122.83102
Random Walk	21.205572	30.359542	36.529043	49.027810	64.871509
OLS with Lag of Y	20.985843	30.207110	37.468527	48.558382	63.481475
State Space Models					
Random Walk (*correct*)	13.496078	18.738312	23.271922	31.808357	42.179804
AR(1)	13.612541	18.866985	23.542466	32.698205	43.531180
AR(1) with drift	14.004172	19.235418	24.041579	33.041725	44.071352
Hildreth-Houck	144.58632	146.40871	146.70938	146.85223	151.07192

The fixed OLS model produces higher RMSE even than the simple Random Walk Model. Again, the selection of the structure for b_t is important since choosing a wrong one we may have RMSEs worse than the fixed OLS model (b_t~ Hildreth Houck).

The estimates for the AR(1) with drift model when the true is the Random Walk are:

DGP	$r_0 = 0$	$r_1 = 1$	$\sigma_2^2 = 1$
Random Walk	-	-	1.008435
		-	(0.137581)
AR(1) with drift	-0.009638	0.975282	0.981445
	(0.824242)	(0.014201)	(0.180217)

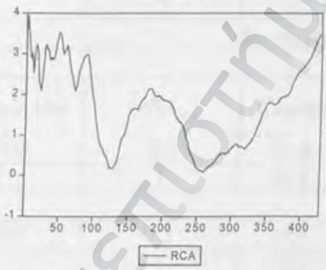
(*Standard errors in parenthesis*)

The estimated with OLS b and its t-statistic were:

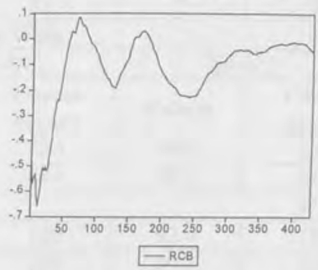
	$\hat{b}_{OLS}$	*t-statistic*
Mean	-0.0487559	-0.2380399
Standard Deviation	17.864979	40.233554

Finally, the recursive coefficient plots give a mixed impression. Both plots represent time dependence even though we have assumed that the intercept is constant over time (=0.5).

RECURSIVE COEFFICIENT
INTERCEPT



RECURSIVE COEFFICIENT
SLOPE



We noticed that the RMSE in the case of X_t~ I(1) is larger than in the case of X_t~I(0). This can be attributed to the large values of series. Nevertheless, if we compare the RMSE with the mean value of series in both cases the result of forecast accuracy is the same.

Using the Data Generating Mechanism (2) in order to produce a bivariate regression with partially stochastic coefficients we took the same results. The AR(1) with drift may be a good approximation in case we do not know the true process for b_t. The recursive coefficients in some cases may lead to misleading decisions regarding with the time dependence of coefficients. We assumed that the first coefficient is equal to one while the second has one of the alternative stochastic structures. The tables with the RMSEs and the estimates of the AR(1) with drift are reported in appendix B (tables 13~16).

4.5 CUSUM TEST AND CUSUM SQUARE TEST FOR PARAMETER INSTABILITY

Incidence of Rejection of Null Hypothesis of stability at 5 % level of confidence when the true model is Hildreth-Houck, Random Walk and AR(1)

		X_t~I(0)		X_t~I(1)	
Sample size	$var(e_{2t}) = 0$	**Cusum**	**Cusum square**	**Cusum**	**Cusum square**
100		4.3	7.2	4.33	8.8
500		4.2	6.5	4.8	7.9
		b_t ~ Hildreth-Houck			
		X_t~I(0)		X_t~I(1)	
Sample size	$var(e_{2t})$	**Cusum**	**Cusum square**	**Cusum**	**Cusum square**
100	0.2	3.8	39.4	3.7	40
	1	4.4	71.6	4.5	86.9
500	0.2	3.8	44.7	1.2	99.4
	1	3.7	84.3	2	99
		b_t ~ Random Walk			
		X_t~I(0)		X_t~I(1)	
Sample size	$var(e_{2t})$	**Cusum**	**Cusum square**	**Cusum**	**Cusum square**
100	0.2	51.9	93.4	88.9	96.8
	1	56.4	96.5	89.2	97.2
					97.2
500	0.2	77.4	100	92.4	100
	1	78.1	100	99.5	100
		b_t ~ AR(1) r_1=0.85			
		X_t~I(0)		X_t~I(1)	
Sample size	$var(e_{2t})$	**Cusum**	**Cusum square**	**Cusum**	**Cusum square**
100	0.2	31.4	67.8	57.5	92.8
	1	41	87	60	93.1
500	0.2	52.8	87	78.1	99.9
	1	62.3	97	78.4	99.9

$b_t \sim AR(1)\ r_1 0.6$					
		$X_t \sim I(0)$		$X_t \sim I(1)$	
Sample size	$\mathrm{var}(e_{2t})$	**Cusum**	**Cusum square**	**Cusum**	**Cusum square**
100	0.2	13.4	50.1	26.7	87.7
	1	19.1	79.4	26.9	91.1
500	0.2	18	64	34.3	99.9
	1	27.8	90.3	36.1	99.9

(Entries are in percent)

From inspection of the case $\mathrm{var}(e_{2t}) = 0$ we see that the confidence levels on the Cusum Square statistic overstates the actual probability of rejecting a true null hypothesis.

Furthermore, letting $\mathrm{var}(e_{2t})$=0.2 or 1 we always see that the Cusum test is dominated by the Cusum square test. The power of both tests changes with the sample size, the level of variation and the structure of coefficient. The power to reject a false null hypothesis of both tests is increased when changes in the coefficients are completely persistent (i.e. $b_t \sim$ Random Walk). Garbade (1977) was the first to examine these two tests and compare their performance with the Wald Coefficient Test. The results where almost the same, with some cases finding the Wald test to be superior than the Cusum Square Test. Nevertheless, Wald coefficient test can be used to test the null hypothesis that particular diagonal term in the covariance matrix of b_t is zero under the maintained hypothesis that the remaining are zero. Therefore, it cannot be used in a multivariate regression. For instance, let:

$$y_t = x_{1t} b_{1t} + x_{2t} b_{2t} + e_t$$

$$\mathrm{var}(b_{1t}) = 0,\ \mathrm{var}(b_{2t}) > 0$$

If we want to test whether b_{1t} varies over time it will be done with the assumption that b_{2t} is constant. But, since the null hypothesis (of constant b_{1t}) is false the test will probably also reject this one and could lead the investigator to the wrong conclusion that b_{1t} varies over time.

4.6 RESULTS FROM THE MONTE CARLO STUDY

Having concluded our Monte Carlo Study we may summarize our results as follow:

First, when the true DGP contains fixed coefficients the RMSE of fixed and stochastic coefficient regression models is hardly distinguishable.

Nevertheless, when the true DGP includes stochastic coefficients the forecasting performance of fixed coefficient models is worse than the correct stochastic coefficient model. In case where the stochastic coefficient has a steady state then the estimate of the simple OLS equals to the mean value of the stochastic coefficient. The discrepancy between the RMSE of fixed and stochastic coefficient models increases as we move from structures with no memory to structures with infinite one (i.e. from Hildreth-Houck to Random Walk). The use of a past value of the dependent variable improves the RMSE of fixed coefficient model but it cannot outperform the RMSE of the correct stochastic coefficient model.

What was impressing though is the fact that relaxing the assumption of fixed coefficients when the true DGP includes stochastic is not enough so as to produce accurate forecasts. From the Monte Carlo analysis it is evident that the choice of the structure for the stochastic coefficient plays an important role for purposes of forecasting. Even if we correctly choose an environment with stochastic coefficients we may produce forecasts that are even worse than the fixed coefficient regression model. This is the case when we choose a structure that has a different form from the true one. For instance, if the correct structure for b_t is AR(1), AR(1) with drift or Hildreth-Houck and we choose the Random Walk structure then the RMSE of the mispecified state space model is in some cases worse than the RMSE of the wrong fixed coefficient regression model. More generally, the RMSE deteriorates when the true process for b_t has short or no memory and we impose to have infinite or when there is infinite and we impose no merory.

Having measured the cost of misspecification in the structure of b_t we looked for a structure with which we will not have an extreme deterioration of the RMSE in comparison to the RMSE of the correct state space model. This structure is the AR(1) with drift. In all cases it can correctly capture the time path of the true stochastic coefficient. This structure is less restrictive since it can accommodate all other structures as sub cases. Having this in mind we model b_t with a structure that does not belong to this "family" and compare the RMSE of its state space model. We assumed that $b_t \sim MA(1)$[1]. Again, the AR(1) with drift gave the lowest RMSEs.

The interesting, though, is the fact that the Kalman filter can correctly capture the type of stochastic time dependence of coefficients in an environment with integrated regressors. Letting X_t be an I(1) process we saw that the Kalman filter can correctly uncover the time path of stochastic coefficients. Again, the cost of misspecification in the structure of b_t is quite important for purposes of forecast accuracy and in some cases more severe. The AR(1) with drift remains a structure which can accommodate all other structures as sub cases.

The above results are robust for different levels of coefficient variation, both for a univariate and a bivariate regression model with stochastic coefficients.

[1] Because Eviews4.1 cannot model b_t as a MA(1) we compare the RMSEs of the alternative structures that we have mentioned.

Furthermore, examining recursive coefficients plots we observe that in all cases they correctly advise for time dependence of the true stochastic coefficient. The time dependence is more clear when b_t follows Random Walk or an AR(1) with r_1 near to unity. Nevertheless, in these cases we saw that they represent time dependence and for the intercept, even though it was imposed to be fixed.

Finally, testing the ability of the Cusum and Cusum Square test to detect for parameter instability we saw that the Cusum Square test is always more powerful than the Cusum test. The ability of both tests to correctly advise for parameter instability changes with the sample size, the level of coefficient variation and the structure of coefficients. (i.e when b_t is Random Walk, AR(1) or Hildreth Houck).

CHAPTER 5

EXCHANGE RATE MODELS AND PARAMETER VARIATION: THE CASE OF THE DOLLAR-EURO EXCHANGE RATE

In this chapter we use time varying coefficient models in order to improve the forecasting performance of two structural monetary models for the case of the Dollar-Euro Exchange Rate.

Other researchers that have used the same methodology in order to evaluate the forecasting performance of monetary models are Wolff (1987 and 1989) and Schinasi-Swamy (1989).

Wolff (1987) using the Kalman Filter methodology reworked Meese and Rogoff results (using the same currencies and time period) for the reduced forms of the Frenkel-Bilson (1976) and Dornbusch-Frankel (1979) models. He assumed that coefficients follow a Random Walk. As he mentioned the choice of this structure was inspired by the empirical regularity that the natural logarithm of the spot exchange rate follows approximately a Random Walk process. The forecasting results, though, where not so satisfactory. The two models could outperform the simple Random Walk Model only for the case of the Dollar-Mark exchange rate. Taking the average of RMSE across all currencies and time forecast horizons it is obvious the dominance of the simple Random Walk Model. It is important to mention that even though Wolff relaxed the fixed coefficient assumption he did it in a rather restrictive way. More precisely, he used prior values for the covariance matrix of the multivariate Random Walk process of stochastic coefficients[1]. In a subsequent paper, Wolff (1989), performed a true *ex-ante* forecasting experiment in order to test whether the earlier findings for the Dollar-Mark exchange rate remains valid and he found that again the state space models outperform the simple Random Walk Model for horizons up to about one year.

Schinasi and Swamy used Meese –Rogoff data for estimation of variable and fixed coefficient versions of the structural models of Frenkel-Bilson, Dornbusch-Frankel and Hooper-Morton (which includes Current Account). They used a less restrictive time varying parameter model than Wolff, assuming that coefficients follow an AR(1) with drift process:

$$b_t - \overline{b} = \Phi(b_{t-1} - \overline{b}) + v_t$$

While their results on fixed coefficient models where consistent whith Meese-Rogof conclusions, they found that allowing for time varying coefficients structural models outperform the simple Random Walk Model. Referring to the results of Wolff (1987) they attributed the failure to the fact that he used prior values both for the covariance matrix of the multivariate process of b_t and for Φ (= I).

[1] He considered a range of matrices that were proportional to the prior covariance matrix: $Q=k^*\Sigma(0|0)$, with Q the covariance matrix of the multivariate Random Walk process and k=0, 0.01, 0.05, 0.1 and 0.25

In our application we use monthly data for the dollar-euro exchange rate. The data set covers the period from 1983:2 to 2000:12[2]. We examined the forecasting performance of the Frenkel-Bilson and Dornbusch-Frankel models. The reduced form of these models is:

$$s_t = b_0 + b_1(m_t - m_t^*) + b_2(y - y_t^*) + b_3(i - i_t^*) + b_4(\pi_t^e - \pi_t^{e*}) + u_t$$ [3]

- Frenkel-Bilson: $b_1=1,\ b_2<0,\ b_3>0,\ b_4=0$
- Dornbusch-Frankel: $b_1=1,\ b_2<0,\ b_3<0,\ b_4>0$

where:

s	=	log of spot exchange rate[4]
m	=	log of money supply
y	=	log of industrial production
i	=	short-term interest rate
π^e	=	long-term interest rate

and * indicates a foreign variable

In our analysis we used actualized values for the explanatory variables (ex-post) and we used the RMSE as a measure of forecast accuracy.

$$RMSE = \left[\frac{1}{k}\sum_{i=1}^{k}(\hat{y}_{T+i} - y_{T+i})^2\right]^{1/2}, \quad k = 1,2,3,6\ and\ 12$$

where $\hat{y}_{T+i}$ is an i-step-ahead forecast of y_{T+i} and T is the terminal period of the fitting period.

We used two periods in order to compare the forecasting performance of fixed and stochastic coefficient models. Firstly, we used data from 1983:2 to 1999:9 to estimate both fixed and stochastic coefficient models. Then we generate forecasts for h=1,2,3,6 and 12 months ahead. The same analysis was conducted for the period from 1986:5 to 1998:6. We assumed that coefficients follow all available structures (Hildreth-Houck, Random Walk, AR(1) with drift). Results were in most cases encouraging when we assumed that the vector of coefficients follow either a Random Walk or an AR(1) with drift process.

Tables with the RMSE of its case are represented below. Plots of the Cusum, Cusum square test and recursive coefficients are in the appendix C.

[2] Data description is in appendix C.

[3] In the estimation we allowed for separate coefficients on domestic and foreign variables. Letting domestic and foreign variables have the same coefficients yielded the same results.

[4] By comparing predictors on the basis of their ability to predict the logarithm of the spot exchange rate we circumvent any problems arising from Jensen's inequality.

ESTIMATION PERIOD: 1983:2 ~ 1999:9 FORECASTING PERIOD: 1999:10 ~ 2000:9

Dornbusch-Frankel Model

	1	2	3	6	12
OLS	9.8344742	9.2301920	9.6221441	10.390202	15.322959
Random Walk	0.2182232	1.3774262	3.2733270	5.2973752	9.9127450
State Space Models					
b_t~AR(1) with drift	1.3963327	1.1624547	0.9505469	1.9386335	7.3040158
b_t~Random Walk	1.0734575	0.8056282	2.1505704	2.9615892	5.1606993

Frenkel-Bilson Model

	1	2	3	6	12
OLS	12.213407	12.376299	12.495288	14.485457	16.810817
Random Walk	0.2182232	1.3774262	3.2733270	5.2973752	9.9127450
State Space Models					
b_t~AR(1) with drift	0.8638225	2.2853604	3.6834609	5.4418453	10.444702
b_t~Random Walk	1.5156263	1.0776422	2.3140490	2.9333730	5.0165400

ESTIMATION PERIOD: 1986:5 ~ 1998:6 FORECASTING PERIOD: 1998:7 ~ 1999:6

Dornbusch-Frankel

	1	2	3	6	12
OLS	1.8977203	1.8747759	5.4204765	8.1117529	6.7433396
Random Walk	0.3100093	0.8626096	2.3657813	3.7268016	3.5277177
State Space Models					
b_t~AR(1) with drift	0.2500894	0.7436486	2.4249816	3.0879835	3.6174741
b_t~Random Walk	0.2473651	0.2984526	3.3655327	5.3183691	4.5425110

Frenkel-Bilson

	1	2	3	6	12
OLS	3.1474084	2.8191603	2.6115271	2.8841874	5.1939313
Random Walk	0.3100093	0.8626096	2.3657813	3.7268016	3.5277177
State Space Models					
b_t~AR(1) with drift	0.0486756	0.9102525	2.0920868	3.2226100	4.9792182
b_t~Random Walk	0.0959193	0.4156305	2.4881898	3.8943296	3.8261695

The analysis was conducted taking the first difference of series. Tables with the RMSE of its model are represented below. Plots of the Cusum, Cusum square test and recursive coefficients are in the appendix C.

ESTIMATION PERIOD: 1983:2~1999:9 FORECASTING PERIOD: 1999:10~2000:9

Dornbusch-Frankel Model

	1	2	3	6	12
OLS	0.4043656	2.6247236	2.1525584	1.5646651	3.2935267
Random Walk	7.1730403	7.8148489	6.9460840	6.6896142	7.4762359
State Space Models					
b_t~AR(1) with drift	1.3846899	1.8546608	1.6014064	1.1881996	3.7577864
b_t~Random Walk	1.6720985	2.4654135	2.0964988	1.7652878	3.5170239

Frenkel-Bilson Model

	1	2	3	6	12
OLS	1.2498260	2.6828589	2.2386325	1.6278333	3.2574875
Random Walk	7.1730403	7.8148489	6.9460840	6.6896142	7.4762359
State Space Models					
b_t~AR(1) with drift	1.4422691	2.0556476	1.8107865	1.6640634	3.4122866
b_t~Random Walk	2.7703665	2.8016030	2.3222542	1.9225546	3.4886848

ESTIMATION PERIOD: 1986:5~1998:6 FORECASTING PERIOD: 1998:7~1999:6

Dornbusch-Frankel Model

	1	2	3	6	12
OLS	0.1004611	3.8414134	3.3042289	2.4749115	2.2464143
Random Walk	12.476689	9.7575593	9.5153080	10.827417	11.608245
State Space Models					
b_t~AR(1) with drift	0.8737286	3.4253326	2.8054687	2.0273435	1.7151575
b_t~Random Walk	0.2338107	0.2137602	3.1993060	2.4970139	2.1173195

Frenkel-Bilson Model

	1	2	3	6	12
OLS	0.1322867	0.4735148	2.8414000	2.3422551	2.0428550
Random Walk	12.476689	9.7575593	9.5153080	10.827417	11.608245
State Space Models					
b_t~AR(1) with drift	0.3770832	0.7615208	2.7943011	2.4658127	2.2373162
b_t~Random Walk	0.1661777	0.4692160	3.1599337	2.6872749	2.1525310

Looking at the plots of Cusum, Cusum Square and recursive coefficients (appendix C) there is an evident prompt, both in levels and first difference, for parameter variation in the examined models. The Cusum Square test always moves outside the critical lines while in most cases the same happens and for the Cusum test.

As far as forecasting performance is concerned in the case of levels we observe that always the fixed coefficient model is worse than the simple Random Walk. The forecasts improve significantly when we use state space models. In most cases the stochastic coefficient model with either an AR(1) with drift or a Random Walk structure outperforms the simple Random Walk model.

Finally, the picture in first difference is rather mixed. The simple OLS model is always better than the simple Random Walk process. The use of stochastic coefficients in some forecast steps improves the RMSE and in others is at the same rank or a little worse than the RMSE of the fixed coefficient model.

To sum up, we could say that using the estimated variation in coefficients in an efficient statistical forecasting procedure might improve the forecasting ability of the structural macroeconomic models when compared to fixed coefficient models and the simple Random Walk model.

CONCLUSION

From the Monte Carlo Study it is evident the deterioration of fixed coefficient models when the true DGP includes stochastic. Nevertheless, the use of state space models is not enough in order to produce accurate forecasts. We should select the correct structure of coefficients otherwise the deterioration may be even worse than the simple OLS model. In case we do not know the true structure we may use the AR(1) with drift with small loss in forecast accuracy. These results are robust both for a univariate and a bivariate model with either stationary or I(1) regressors. Furthermore, the Cusum Square test is proved to be helpful in advising for parameter instability. Finally, applying the time varying parameter methodology in Frenkel-Bilson and Dornbusch-Frankel models for the case of the Dollar/Euro rate we found an improvement of forecast accuracy in comparison to the simple Random Walk model.

APPENDIX A

Includes tables and graphs for the univariate and bivariate case with stationary regressors.

Table1a - b_t~AR(1) with r_1=0.85

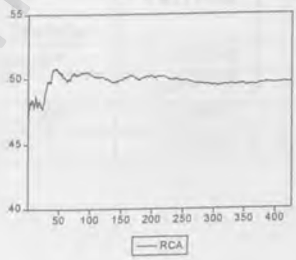
DGP $b_t \sim$ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	2.0719813	2.0488536	2.0590991	2.1455177	2.1118131
OLS Lag*	1.7914541	1.9526425	2.0510532	2.1564317	2.1395410
State Space Models					
AR(1)*	1.7644601	1.9216996	2.0244752	2.1221848	2.1003812
AR(1) with drift*	1.7649323	1.9246024	2.0251865	2.1260625	2.1000794
R.W*	1.8075038	1.9757978	2.0996492	2.1481669	2.0983289
Hildreth-Houck*	2.0537271	2.0383393	2.0494958	2.1335130	2.1010023

(* Denotes forecasts were X was replaced by its expected value)

Table 1b – b_t~AR(1) with r_1=0.6 and var(e_2)=1

DGP $b_t \sim$ AR(1) r0=0, r1=0.6 var(e_2)=1					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.5111935	1.5302735	1.5136056	1.5679362	1.5357668
Random Walk	1.8215517	2.0529900	2.1635775	2.1711013	2.1876082
OLS*	1.5088265	1.5223086	1.5052271	1.5537785	1.5180966
OLS with Lag of Y	1.4520337	1.5102596	1.5131255	1.5667869	1.5366900
OLS Lag*	1.4516825	1.5044964	1.5069729	1.5555739	1.5220764
State Space Models					
AR(1) (correct model)	1.3620267	1.4785718	1.4849078	1.5428863	1.5148752
AR(1)*	1.4345946	1.4927900	1.4948994	1.5453397	1.5145368
Random Walk	1.4676703	1.7477292	1.7628903	1.8494124	1.9376203
Random Walk*	1.5272597	1.5959142	1.5889682	1.5739528	1.5177032
Hildreth-Houck	1.5135202	1.5251732	1.5044185	1.5618461	1.5279853
Hildreth-Houck*	1.5030796	1.5198763	1.4987714	1.5488857	1.5147471
AR(1) with drift	1.3633021	1.4797561	1.4868542	1.5545784	1.5208454
AR(1) with drift*	1.4385214	1.4993548	1.4998715	1.5493365	1.5241571

INTERCEPT



SLOPE-b_t

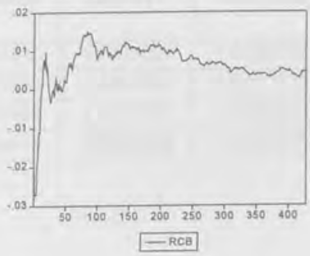
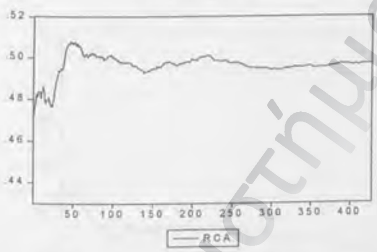


Table 1c – b_t~AR(1) with r_1=0.85 and var(e_2)=0.2

DGP b_t ~ AR(1) r0=0, r1=0.85 var(e_2)=0.2					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.1955501	1.1991883	1.1958830	1.2310947	1.2442642
Random Walk	1.3733806	1.5025313	1.6231637	1.6732845	1.6927351
OLS*	1.1919274	1.1896609	1.1876365	1.2175061	1.2176156
OLS with Lag of Y	1.1263978	1.1702104	1.1904429	1.2321040	1.2443311
OLS Lag*	1.1232298	1.1669856	1.1841706	1.2202703	1.2238886
State Space Models					
AR(1) (correct model)	1.0081949	1.1004302	1.1101562	1.1619191	1.2123248
AR(1)*	1.1004982	1.1458899	1.1851504	1.2085073	1.2169098
Random Walk	1.0408242	1.1568809	1.1883573	1.2604078	1.3926387
Random Walk*	1.1242313	1.1601825	1.2109195	1.2176091	1.2154729
Hildreth-Houck	1.1952497	1.1959743	1.1959263	1.2265757	1.2383006
Hildreth-Houck*	1.1851848	1.1904278	1.1887969	1.2140295	1.2155445
AR(1) with drift	1.0091234	1.1230458	1.1354824	1.1821548	1.2354871
AR(1) with drift*	1.1102154	1.1542158	1.192548	1.223548	1.235210

(* Denotes forecasts were X was replaced by its expected value)

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE

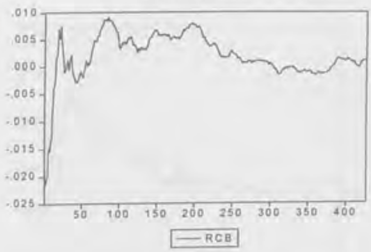


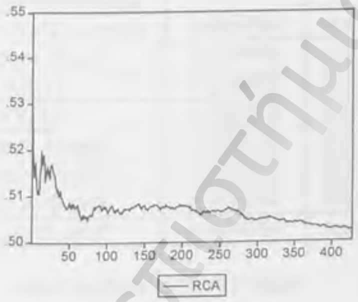
Table 2a – DGP b_t ~ Hildreth-Houck Model

DGP b_t ~ H.H r0=0.8, r1=0					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	1.4324481	1.4482355	1.5429083	1.6255052	1.5816215
OLS Lag*	1.4278486	1.4485408	1.5464117	1.6312260	1.5839621
State Space Models					
Hildreth-Houck*	1.4317918	1.4493342	1.5423337	1.6248358	1.5804947
AR(1)*	1.5068110	1.4736236	1.5694962	1.6451135	1.5830838
AR(1) with drift*	1.4249873	1.4501004	1.5425221	1.6246805	1.5807609
R.W*	1.6348196	1.5913074	1.6237868	1.6733582	1.5839402

Table 2b – DGP $b_t \sim$ Hildreth-Houck Model

DGP $b_t \sim b_o+e_t$ r0=0.8, r1=0 var(e_{2t})=0.2					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	0.9474321	0.9275019	0.9386462	0.9725935	0.9629725
Random Walk	1.4904576	1.6035478	1.6893757	1.8480907	1.8496589
OLS*	1.1176296	1.1763307	1.2465698	1.3332709	1.3170597
OLS with Lag of Y	0.9475378	0.9278219	0.9396350	0.9736354	0.9611741
OLS Lag*	1.1167767	1.1778705	1.2487683	1.3385781	1.3193786
State Space Models					
Hildreth-Houck(*correct*)	0.9465309	0.9280098	0.9388907	0.9716847	0.9621536
Hildreth-Houck*	1.1176694	1.1773181	1.2470532	1.3336704	1.3163397
AR(1)	0.9823960	0.9520002	0.9801987	1.0221592	1.0376814
AR(1)*	1.1370957	1.1959674	1.2558966	1.3371770	1.3169996
Random Walk	0.9793915	0.9474585	0.9612064	1.0131372	0.9822312
Random Walk*	1.1295765	1.1940596	1.2517181	1.3318569	1.3161898
AR(1) with drift	0.9447349	0.9263224	0.9394631	0.9715825	0.9622472
AR(1) with drift*	1.1157022	1.1779733	1.2480250	1.3336699	1.3162989

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE

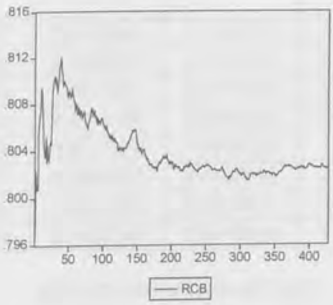


Table 3a DGP $b_t \sim$ AR(1) with drift

DGP $b_t \sim$ AR(1) with drift r0=0.7, r1=0.3					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	1.5715996	1.4990194	1.6510619	1.6845876	1.7879868
OLS Lag*	1.5569922	1.4857822	1.6243830	1.6456957	1.7371044
State Space Models					
AR(1) with drift*	1.5471210	1.4928735	1.6504869	1.6845132	1.7833185
AR(1)*	1.6027307	1.5366593	1.6467259	1.6915744	1.7910837
R.W*	1.7012454	1.7254101	1.7524036	1.8001478	1.8436512
Hildreth-Houck*	1.5571651	1.5154247	1.6510071	1.6894215	1.7896314

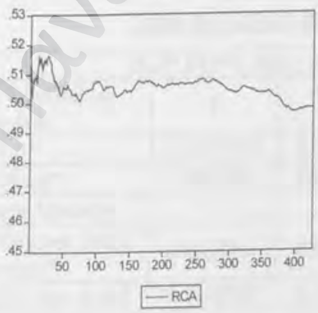
Table 4a DGP $b_t \sim$ Random Walk

DGP $b_t \sim$ Random Walk r0=0, r1=1					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	17.039691	19.145264	20.121159	20.385733	21.197035
OLS Lag*	14.408121	17.495631	18.262713	18.490911	19.484589
State Space Models					
R.W*	13.626322	17.371081	19.044884	20.066119	21.266809
AR(1)*	13.633434	17.400426	19.143594	20.171117	21.327967
AR(1) with drift*	13.624155	17.423254	19.165245	20.192514	21.352154
Hildreth-Houck*	17.135121	19.280008	20.260783	20.554286	21.209117

Table 4b DGP $b_t \sim$ Random Walk

DGP $b_t \sim$ Random Walk r0=0, r1=1 var(e_{2t})=0.2					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.5428190	1.5614329	1.4986551	1.5849671	1.6342918
Random Walk	2.0318527	2.4621521	2.6372517	3.0482137	3.3446573
OLS*	2.0182481	2.1462257	2.2045005	2.4304204	2.4917756
OLS with Lag of Y	1.4000421	1.5179608	1.5079485	1.6153633	1.6623376
OLS Lag*	1.8570561	2.0248208	2.0926030	2.2991072	2.3504160
State Space Models					
Random Walk(*correct*)	0.8637834	0.8768897	0.8722383	0.9067270	0.9477024
Random Walk*	1.7376164	2.0014960	2.1406035	2.3935865	2.4798275
AR(1)	0.8719602	0.8897626	0.8910325	0.9215592	0.9608794
AR(1)*	1.7356878	2.0006792	2.1402104	2.3961529	2.4796431
Hildreth-Houck	1.5261765	1.5510807	1.4821548	1.5995542	1.6375112
Hildreth-Houck*	1.9248797	2.3750177	2.5985334	2.4463932	2.4973564
AR(1) with drift	0.8719752	0.8863582	0.8900508	0.9293717	0.9727369
AR(1) with drift*	1.7426584	2.0016148	2.1330343	2.3943860	2.4869075

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE

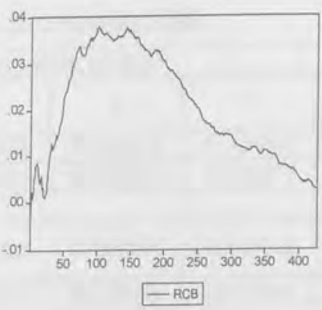


Table 5a – DGP: $b_{1t}=1$, $b_{2t} \sim$ AR(1)

DGP b1t=1, b2t ~ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	2.4395554	2.4799693	2.5132408	2.5814240	2.5757872
OLS Lag*	2.0416547	2.3356438	2.4232594	2.5139496	2.4729577
State Space Models					
$b_{1t}=1, b_{2t}\sim$AR(1) (*correct*)*	1.9873610	2.3199920	2.4288054	2.5282685	2.5689932
b_{1t}, $b_{2t}\sim$AR(1) with drift*	1.9885846	2.3216402	2.4332021	2.5314031	2.5692285

Table 6a – DGP: $b_{1t}=1$, $b_{2t} \sim$ Hildreth-Houck

DGP b1t=1, b2t ~ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	1.7290270	1.8504908	1.8858300	2.0638960	2.0027677
OLS Lag*	1.7268270	1.8586814	1.8933125	2.0654694	2.0127239
State Space Models					
$b_{1t}=1, b_{2t}\sim$H-H (*correct*)*	1.7246775	1.8479943	1.8860123	2.0627266	1.9986938
b_{1t}, $b_{2t}\sim$AR(1) with drift*	1.7258391	1.8501584	1.8858948	2.0629381	1.9989692

Table 7a – DGP: $b_{1t}=1$, $b_{2t} \sim$ AR(1) with drift

DGP b1t=1, b2t ~ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	1.8308107	1.9748857	2.0460685	2.2159929	2.1023034
OLS Lag*	1.8183000	1.9441568	1.9860447	2.1231959	2.0251017
State Space Models					
$b_{1t}=1, b_{2t}\sim$AR(1) with drift (*correct*)*	1.8094475	1.9700308	2.0475185	2.2164105	2.1007597
b_{1t}, $b_{2t}\sim$AR(1) with drift*	1.8104836	1.9702979	2.0475598	2.2167828	2.1011609

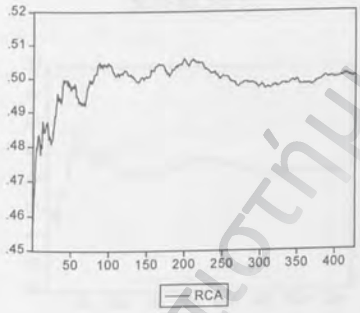
Table 8a – DGP: $b_{1t}=1$, $b_{2t} \sim$ Random Walk

DGP b1t=1, b2t ~ AR(1) r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	8.3388435	9.1323548	10.359519	11.130855	11.495168
OLS Lag*	7.0468784	8.3883523	9.8653248	10.222879	10.556088
State Space Models					
$b_{1t}=1, b_{2t}\sim$ Random Walk (*correct*)*	6.4334753	8.0595220	9.7960371	10.601048	11.372621
b_{1t}, $b_{2t}\sim$AR(1) with drift*	6.7296746	8.4556318	9.9187990	10.973440	12.118207

Table A – $cov(e_{1t}, e_{2t})=0.5$

DGP $b_t \sim AR(1)$ r0=0, r1=0.6 $cov(e_{1t}, e_{2t})=0.5$					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.5189543	1.4931600	1.4449696	1.5577145	1.5475145
Random Walk	1.8408560	2.0366662	2.0971469	2.1719336	2.1952894
OLS*	1.5176460	1.4893183	1.4337681	1.5521503	1.5974578
OLS with Lag of Y	1.4722073	1.4760182	1.4444166	1.5561500	1.6082020
OLS Lag*	1.4692801	1.4751732	1.4367719	1.5511509	1.5994586
State Space Models					
AR(1)	1.3616312	1.4569633	1.4128214	1.5479637	1.5358184
AR(1)*	1.4512800	1.4644911	1.4230308	1.5450150	1.5948391
Random Walk	1.4649139	1.6976330	1.7034450	1.8561417	1.9577450
Random Walk*	1.5273124	1.7612839	1.7364647	1.8955710	1.9501524
Hildreth-Houck	1.5091758	1.4913973	1.4312812	1.5566815	1.5538021
Hildreth-Houck*	1.5170125	1.4867959	1.4267206	1.5573031	1.5751037
AR(1) with drift	1.3695411	1.4631021	1.4201423	1.5543810	1.5452414
AR(1) with drift*	1.4601245	1.4702141	1.4324150	1.5502154	1.6100154

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE

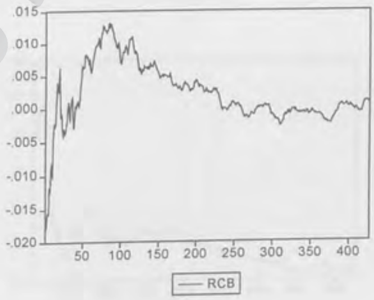
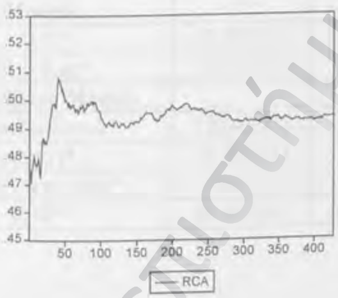


Table B - cov(e_{1t},e_{3t})=0.5

DGP $b_t \sim AR(1)$ r0=0, r1=0.6 cov(e_{1t},e_{3t})=0.5					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	1.4472044	1.5295960	1.5063892	1.5153752	1.4996334
Random Walk	1.8399113	2.1356620	2.2012770	2.2187169	2.2185496
OLS*	1.4791727	1.5539257	1.5092492	1.5203294	1.4625135
OLS with Lag of Y	1.4143638	1.5329223	1.5175263	1.5266956	1.4632509
OLS Lag*	1.4414882	1.5557435	1.5203720	1.5269780	1.4670552
State Space Models					
AR(1)	1.3254087	1.4836773	1.4632661	1.5022360	1.4786857
AR(1)*	1.4213352	1.5305951	1.4941944	1.5082482	1.4584010
Random Walk	1.4580115	1.7397548	1.7304367	1.7845704	1.8413236
Random Walk*	1.5319562	1.6622870	1.6079789	1.5521580	1.4622086
Hildreth-Houck	1.4477220	1.5215383	1.4929135	1.5189858	1.4906742
Hildreth-Houck*	1.4783966	1.5496438	1.5021690	1.5143733	1.4794507
AR(1) with drift	1.3302541	1.4901245	1.4612341	1.5110241	1.4924154
AR(1) with drift*	1.44251431	1.5210141	1.4994151	1.5105147	1.4621541

RECURSIVE COEFFICIENT INTERCEPT



RECURSIVE COEFFICIENT SLOPE

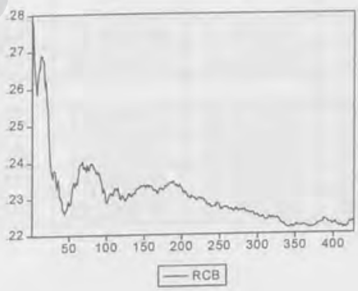


Table C – DGP $b_t \sim$ Moving Average

DGP $b_t = e_{2t} + 0.5e_{2t-1}$					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	1.4196375	1.2949394	1.3984905	1.3582812	1.5120018
OLS Lag*	1.3925059	1.2981134	1.4018594	1.3564349	1.5108989
State Space Models					
AR(1)*	1.3671189	1.2911444	1.3971538	1.3566428	1.5106924
AR(1) with drift*	1.3660849	1.2943143	1.3979590	1.3553221	1.5104568
R.W*	1.4904920	1.4258861	1.5082006	1.3858423	1.5175990
Hildreth-Houck*	1.4151107	1.2918822	1.3970379	1.3549576	1.5112724

APPENDIX B

Includes tables and graphs for the univariate and bivariate case with I(1) regressors.

Table 9a- $b_t \sim AR(1)$

DGP $b_t \sim AR(1)$ r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	25.655341	24.955574	25.460791	26.868195	27.017094
OLS Lag*	14.227603	17.505228	20.713636	25.595589	27.087322
State Space Models					
AR(1)*	14.131467	17.914668	20.700621	25.112832	25.850313
AR(1) with drift*	14.082386	17.370326	20.544849	25.732103	27.063884
R.W*	14.168881	19.202299	22.749151	29.774861	32.368305
Hildreth-Houck*	26.102142	25.235479	25.392147	26.103214	27.047142

(* denotes forecasts were X was replaced by its expected value)

Table 10a – DGP $b_t \sim$ Hildreth-Houck Model

DGP $b_t \sim$ H.H r0=0.8, r1=0					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	13.701076	14.008640	13.492706	14.453327	14.892575
OLS Lag*	13.711905	14.003232	13.490574	14.453178	14.886548
State Space Models					
Hildreth-Houck*	13.643671	13.995896	13.487751	14.515557	14.882325
AR(1)*	16.207971	16.545198	16.492581	17.484357	17.639204
AR(1) with drift*	13.691322	13.998372	13.488116	14.515418	14.884701
R.W*	19.565561	20.652593	20.229375	21.162284	20.875264

Table 11a – DGP $b_t \sim AR(1)$ with drift

DGP $b_t \sim AR(1)$ with drift r0=0.7, r1=0.3					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	14.922125	14.127643	14.199375	15.497242	16.476735
OLS Lag*	14.123104	14.079606	14.136131	15.449161	16.304854
State Space Models					
AR(1) with drift*	14.086794	14.039206	14.129828	15.545473	16.501845
AR(1)*	16.162621	17.003596	17.241287	18.994718	20.201400
R.W*	17.124413	18.887229	20.192034	21.467333	21.841976
Hildreth-Houck*	14.862909	14.082287	14.184718	15.540411	16.498613

Table 12a DGP $b_t \sim$ Random Walk

DGP $b_t \sim$ Random Walk r0=0, r1=1					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS*	114.75805	117.30444	118.66715	120.64813	130.39154
OLS Lag*	21.858809	31.436431	38.659663	50.255137	65.564035
State Space Models					
R.W*	21.284734	30.411146	36.804001	49.342197	65.895188
AR(1)*	21.254563	30.517231	36.653851	49.277710	66.128132
AR(1) with drift*	23.279824	32.441572	38.447214	52.447214	69.221471
Hildreth-Houck*	145.82917	148.27111	149.64389	151.00193	158.15392

Bivariate case $X_t \sim I(1)$ process

Table 13

DGP $b_{1t}=1$ for all t $b_{2t}\sim AR(1)$ r0=0, r1=0.85					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	20.840127	21.224769	21.975350	21.839975	21.881977
Random Walk	11.759640	15.917646	18.796223	22.369605	27.656201
OLS*	20.894753	21.376138	22.160019	22.082533	22.096771
OLS with Lag of Y	11.605705	15.244556	17.739385	20.342090	22.293561
OLS Lag*	11.633346	15.273319	17.808043	20.386309	22.248515
State Space Models					
$b_{1t}=1, b_{2t}\sim AR(1)$ (*correct*)	11.254634	14.621672	17.138129	19.079822	20.946293
$b_{1t}=1, b_{2t}\sim AR(1)$ (*correct*)*	11.473089	14.923739	17.557971	19.388618	21.123928
b_{1t}, $b_{2t}\sim AR(1)$ with drift	11.580624	15.520531	17.523343	19.281852	23.058251
b_{1t}, $b_{2t}\sim AR(1)$ with drift*	11.642259	15.717872	17.963384	19.722402	23.974360

Estimates of AR(1) with drift

b_{2t}	$r_0=0$	$r_1=0.85$	$var(e_{2t})=1$
Correct State Space Model	-	0.837402	0.988213
	-	(0.037825)	(0.104301)

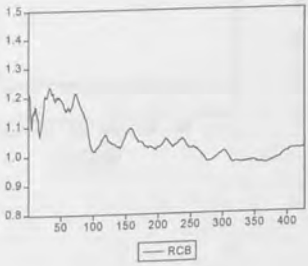
Both coefficients follow an AR(1) with drift process

b_{1t}	$p_0=1$	$p_1=0$	$var(e_{5t})=0$
AR(1) with drift	0.512147	0.486009	0.036961
	(0.663322)	(0.558759)	(0.310431)
b_{2t}	$r_0=0$	$r_1=0.85$	$var(e_{2t})=1$
AR(1) with drift	0.004275	0.837606	0.976918
	(0.094014)	(0.039086)	(0.121610)

(*Standard errors in parenthesis*)

For b_{1t} again since $var(b_{1t}) \approx 0$ it converges $b_{1t} \underset{t\to\infty}{\rightarrow} \frac{0.512147}{1-0.486009} \approx 1$

RECURSIVE COEFFICIENT
b_{1t}



RECURSIVE COEFFICIENT
b_{2t}

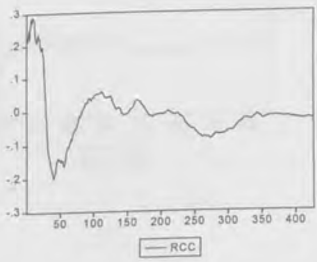


Table 14

DGP b_{1t}=1 for all t b_{2t}~Hildreth Houck r0=0.8, r1=0					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	12.123407	12.392979	12.176324	12.099919	12.091445
Random Walk	13.510377	15.803302	16.791840	17.184510	17.981089
OLS*	12.171142	12.638400	12.462161	12.656411	13.118211
OLS with Lag of Y	11.208430	12.265073	12.219739	12.051452	12.101353
OLS Lag*	11.260011	12.467511	12.421296	12.453013	12.774107
State Space Models					
b_{1t}=1,b_{2t}~H-H (*correct*)	11.067161	12.276140	12.043485	12.012362	11.985905
b_{1t}=1,b_{2t}~H-H (*correct*)*	11.146765	12.454484	12.337584	12.594158	13.075256
b_{1t}, b_{2t}~AR(1) with drift	11.095843	12.319609	12.023203	12.022948	11.962535
b_{1t}, b_{2t}~AR(1) with drift*	11.172596	12.485428	12.321980	12.577541	13.054304

Estimates of the AR(1) with drift

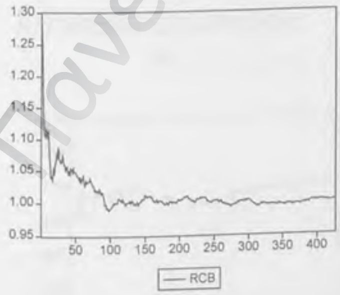
	$r_0 = 0.8$	$r_1 = 0$	$\text{var}(e_{1t}) = 1$
Correct State Space Model	0.804154	-	0.983695
	(0.0941520	-	(0.106293)

Both coefficients follow an AR(1) with drift process

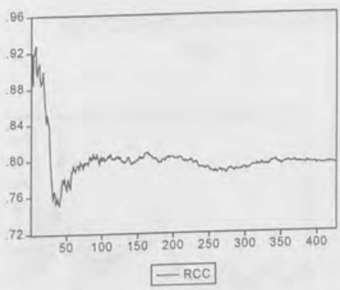
b_{1t}	$p_0 = 1$	$p_1 = 0$	$\text{var}(e_{5t}) = 0$
AR(1) with drift	0.912017	0.077435	*0.004801*
	(0.584082)	(0.555731)	(0.452448)
b_{2t}	$r_0 = 0.8$	$r_1 = 0$	$\text{var}(e_{2t}) = 1$
AR(1) with drift	0.808657	-0.003266	0.971419
	(0.101053)	(0.095181)	(0.116264)

(Standard errors in parenthesis)

RECURSIVE COEFFICIENT
b_{1t}



RECURSIVE COEFFICIENT
b_{2t}



<u>**Table 15**</u>

DGP $b_{1t}=1$ for all t $b_{2t}\sim AR(1)$ with drift $r_0=0.7$ $r_1=0.3$					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	12.164477	11.782572	12.299160	11.631280	11.438380
Random Walk	14.024937	15.616347	16.865913	16.759979	17.709296
OLS*	12.257928	11.940145	12.647464	11.941754	12.424173
OLS with Lag of Y	11.457447	11.669231	12.349689	11.608115	11.485555
OLS Lag*	11.590858	11.815508	12.621775	11.789219	12.128699
State Space Models					
$b_{1t}=1, b_{2t}\sim AR(1)$ with drift (*correct*)	11.326804	11.487455	12.323021	11.508403	11.300453
$b_{1t}=1, b_{2t}\sim AR(1)$ with drift (*correct*)*	11.497122	11.639052	12.706566	11.843956	12.316685
b_{1t}, $b_{2t}\sim AR(1)$ with drift	11.318224	11.470598	12.334307	11.486224	11.323888
b_{1t}, $b_{2t}\sim AR(1)$ with drift*	11.488779	11.623076	12.714413	11.810874	12.355222

Estimates for the AR(1) with drift

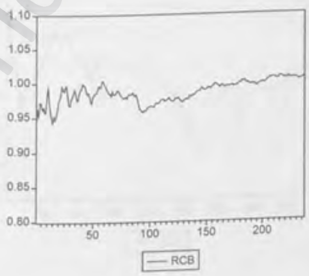
	$r_0=0.7$	$r_1=0.3$	$var(e_{2t})=1$
Correct State Space Model	0.711572	0.290392	0.985653
	(0.140775)	(0.067929)	(0.100543)

Both coefficients follow an AR(1) with drift process

b_{1t}	$p_0=1$	$p_1=0$	$var(e_{5t})=0$
AR(1) with drift	0.871255	0.122533	0.034554
	(0.574683)	(0.555315)	(0.166110)
b_{2t}	$r_0=0.7$	$r_1=0.3$	$var(e_{2t})=1$
AR(1) with drift	0.712585	0.288326	0.972440
	(0.144701)	(0.071370)	(0.106731)

(Standard errors in parenthesis)

RECURSIVE COEFFICIENT
b_{1t}



RECURSIVE COEFFICIENT
b_{2t}

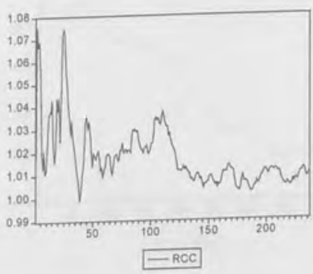


Table 16

DGP $b_{1t}=1$ for all t b_{2t}~Random Walk $r_0=0$ $r_1=1$					
Constant Coefficient Models	**1**	**2**	**3**	**5**	**10**
OLS	67.490333	69.750473	70.990030	73.099558	81.793320
Random Walk	17.241714	26.006153	30.656543	38.431960	55.880011
OLS*	68.503957	71.296475	71.977320	75.242158	85.831533
OLS with Lag of Y	17.139590	25.408617	29.524034	37.204674	65.121470
OLS Lag*	18.162280	26.918637	31.139064	38.789693	56.318232
State Space Models					
$b_{1t}=1$,b_{2t}~Random Walk (*correct*)	14.279264	19.703515	23.960282	35.202027	58.197487
$b_{1t}=1$,b_{2t}~Random Walk (*correct*)*	17.161465	26.097055	31.221128	49.962411	77.629788
b_{1t}, b_{2t}~AR(1) with drift	14.731810	22.707751	28.994950	40.531811	69.119836
b_{1t}, b_{2t}~AR(1) with drift*	20.160620	31.485678	39.132470	53.560583	85.676484

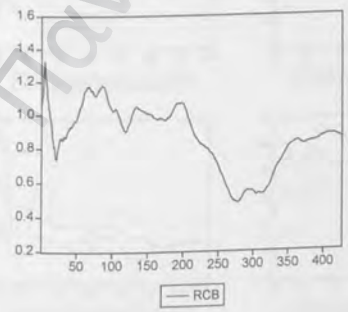
Estimates for the AR(1) with drift

	$r_0=0$	$r_1=1$	$\text{var}(e_{2t})=1$
Correct State Space Model	-	-	1.008955
	-	-	(0.135597)

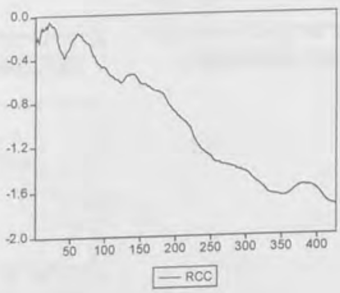
Both coefficients follow an AR(1) with drift process

b_{1t}	$p_0=1$	$p_1=0$	$\text{var}(e_{5t})=0$
AR(1) with drift	0.020201	0.998129	0.0302144
	(0.463441)	(0.445711)	(0.511725)
b_{2t}	$r_0=0$	$r_1=1$	$\text{var}(e_{2t})=1$
AR(1) with drift	-0.055219	0.981579	0.987437
	(0.592779)	(0.016726)	(0.180876)

RECURSIVE COEFFICIENT
b_{1t}



RECURSIVE COEFFICIENT
b_{2t}



APPENDIX C

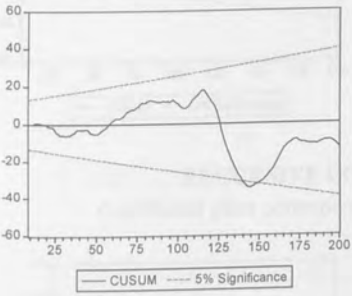
Includes plots of the recursive coefficients, the Cusum and Cusum Square test from chapter 5.

LEVELS

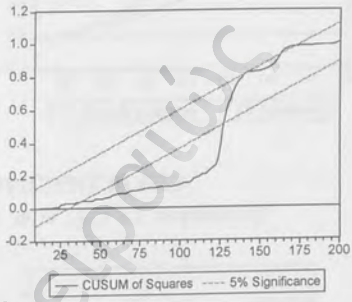
ESTIMATION PERIOD: 1983:2~1999:9 **FORECASTING PERIOD: 1999:10~2000:9**

Dornbusch-Frankel Model

CUSUM TEST

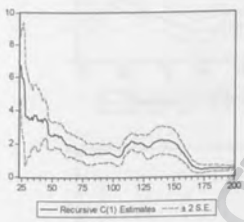


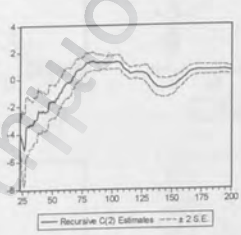
CUSUM SQUARE TEST

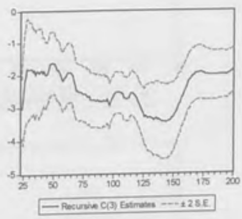


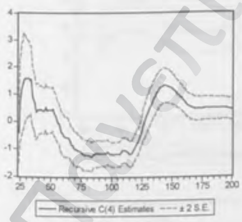
RECURSIVE COEFFICIENT PLOTS

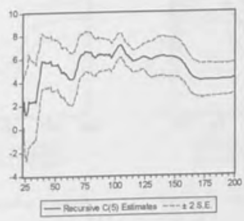
Coefficient plots correspond to $m, m^{*}, y, y^{*} i, i^{*}, \pi^{e}, \pi^{e*}$ respectively

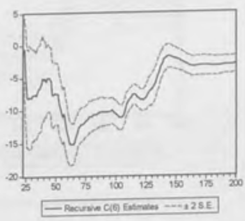


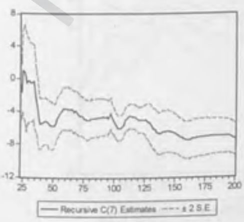


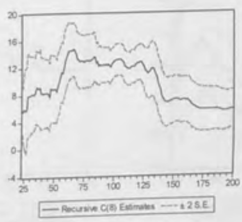






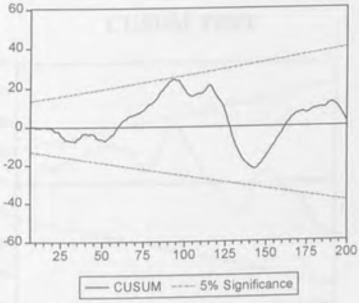




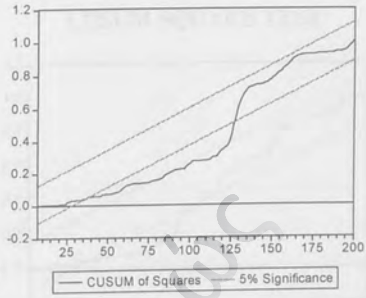


Frenkel-Bilson Model

CUSUM TEST

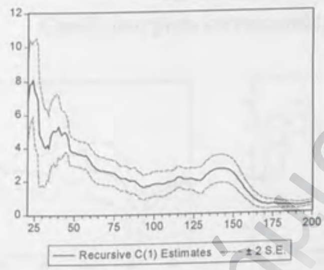


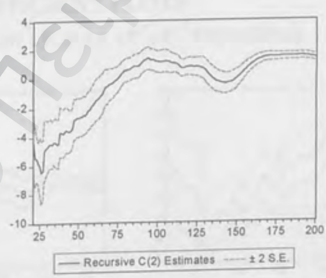
CUSUM SQUARE TEST

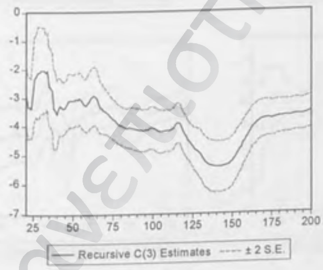


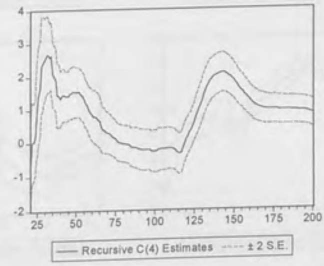
RECURSIVE COEFFICIENT PLOTS

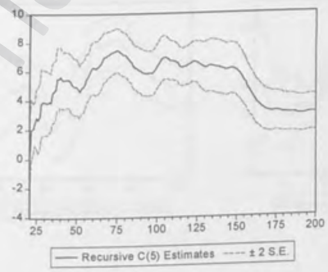
Coefficient plots correspond to $m, m^*, y, y^* i, i^*$ respectively

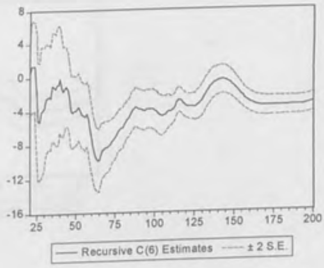








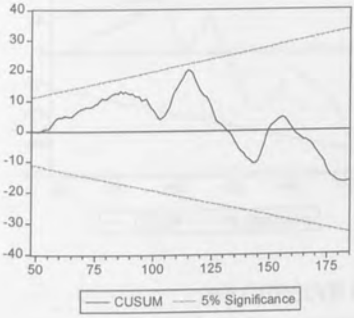




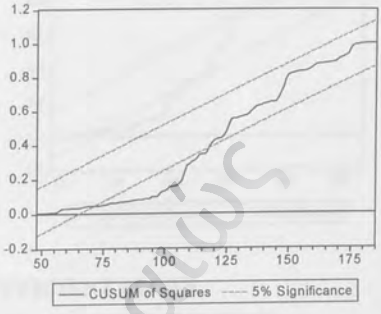
ESTIMATION PERIOD: 1986:5~1998:6 **FORECASTING PERIOD: 1998:7~1999:6**

Dornbusch-Frankel Model

CUSUM TEST

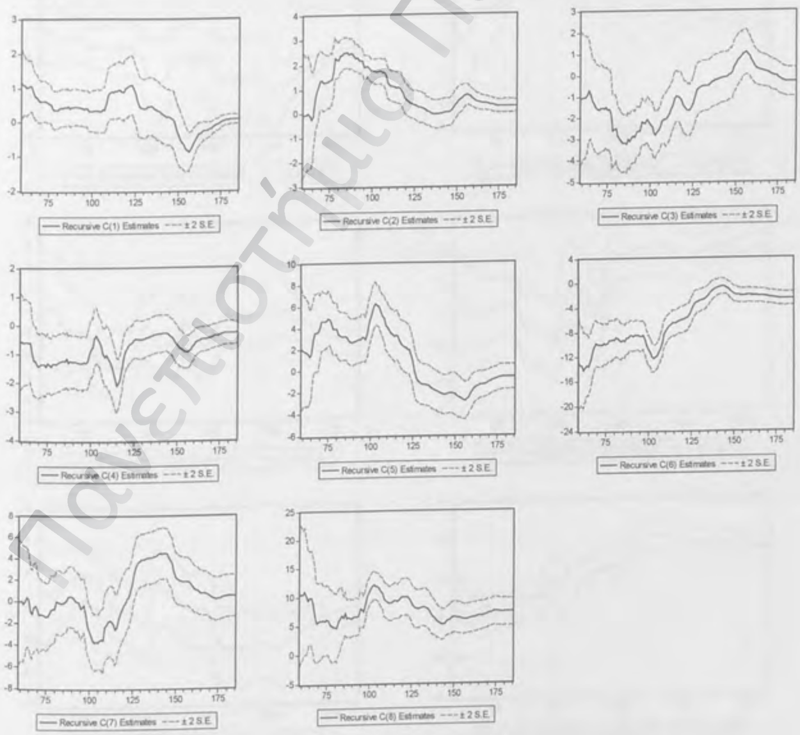


CUSUM SQUARE TEST



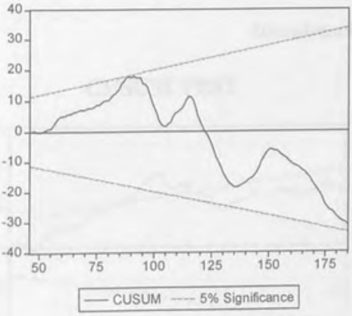
RECURSIVE COEFFICIENT PLOTS

Coefficient plots correspond to $m, m^*, y, y^* i, i^*, \pi^e, \pi^{e*}$ respectively

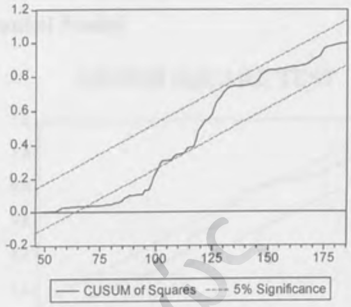


Frenkel-Bilson Model

CUSUM TEST

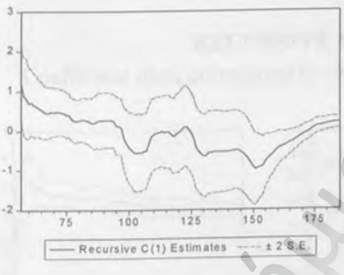


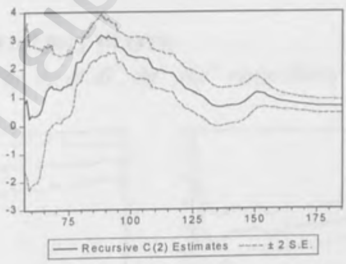
CUSUM SQUARE TEST

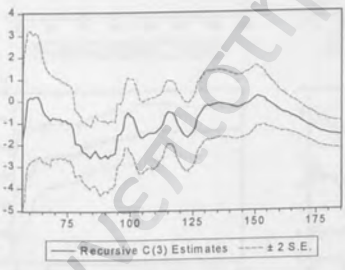


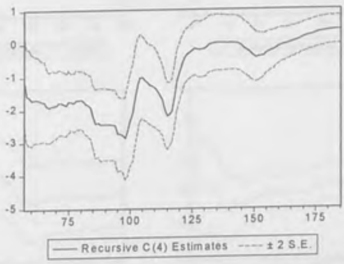
RECURSIVE COEFFICIENT PLOTS

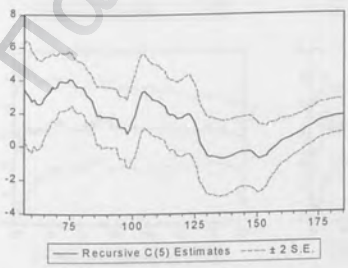
Coefficient plots correspond to $m, m^*, y, y^* i, i^*$ respectively

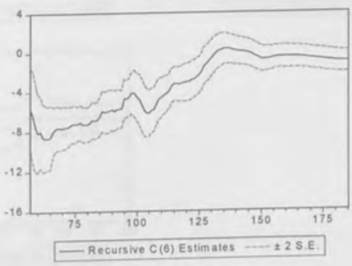










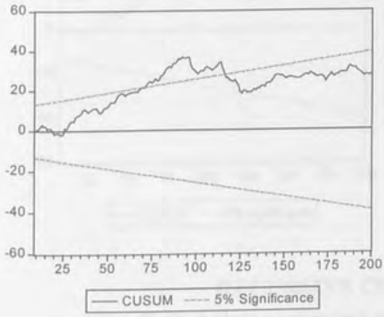


FIRST DIFFERENCE

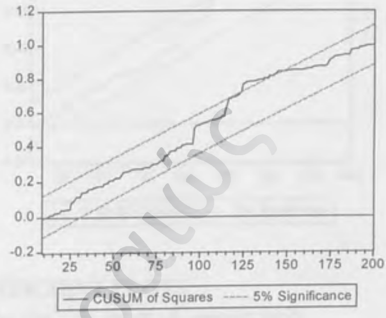
ESTIMATION PERIOD: 1983:2~1999:9 FORECASTING PERIOD: 1999:10~2000:9

Dornbusch-Frankel Model

CUSUM TEST

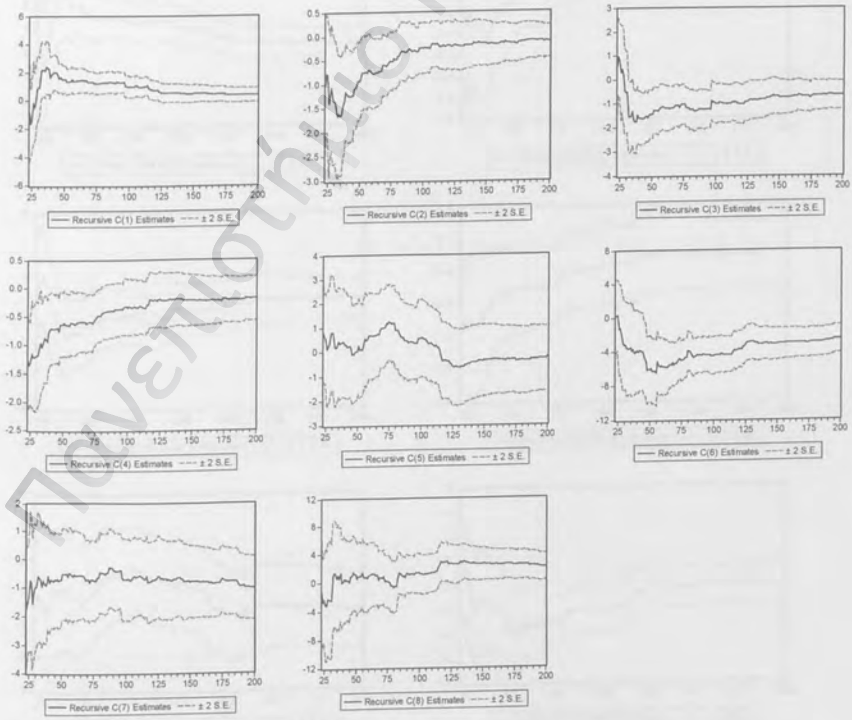


CUSUM SQUARE TEST



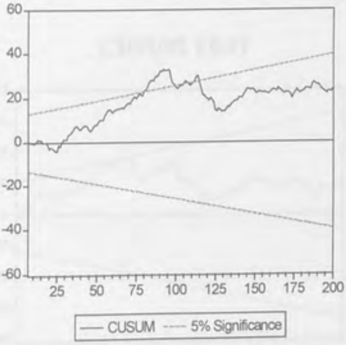
RECURSIVE COEFFICIENT PLOTS

Coefficient plots correspond to $dm, dm^*, dy, dy^* di, di^*, d\pi^e, d\pi^{e*}$ respectively

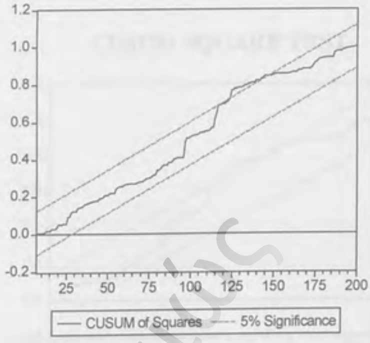


Frenkel-Bilson Model

CUSUM TEST

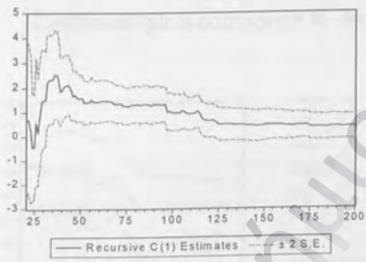


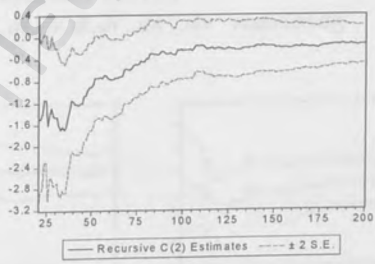
CUSUM SQUARE TEST

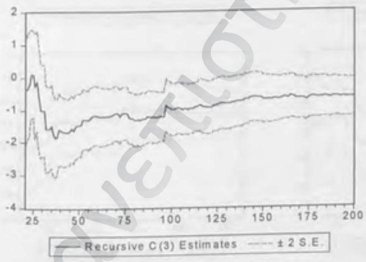


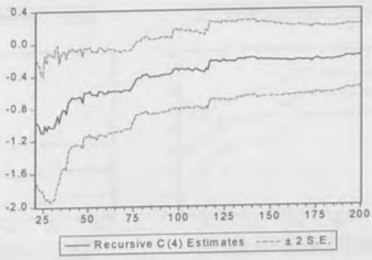
RECURSIVE COEFFICIENT PLOTS

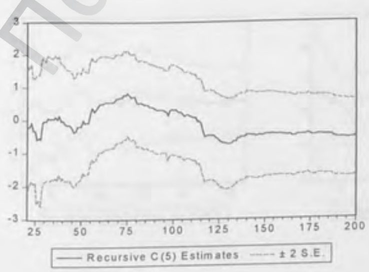
Coefficient plots correspond to $dm, dm^*, dy, dy^* di, di^*$ respectively

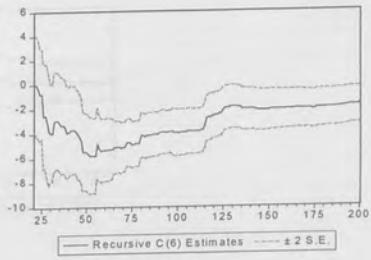








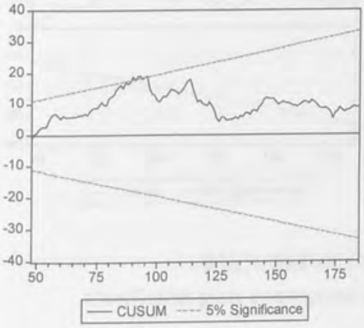




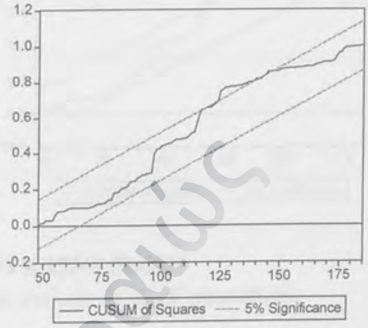
ESTIMATION PERIOD: 1986:5~1998:6   FORECASTING PERIOD: 1998:7~1999:6

Dornbusch-Frankel Model

CUSUM TEST

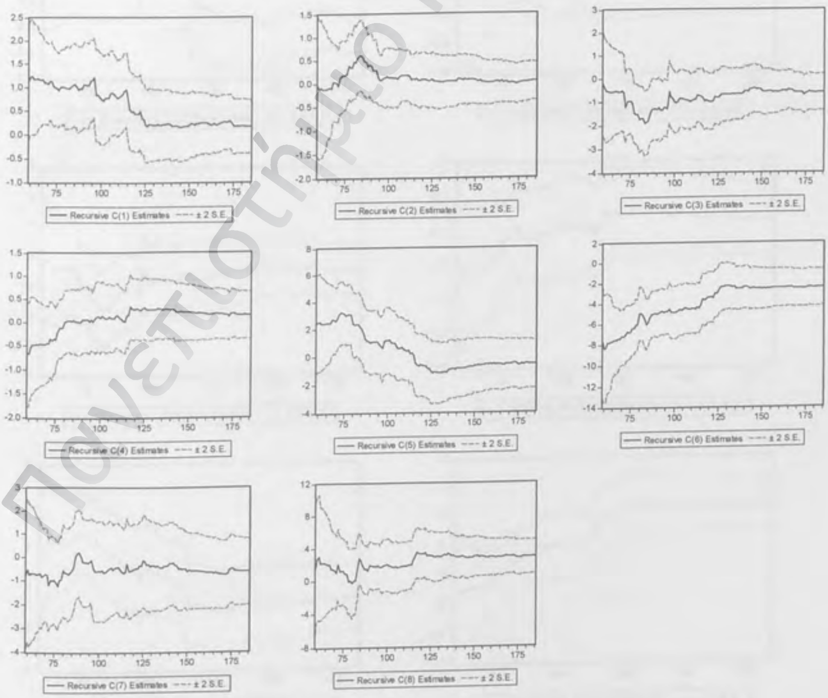


CUSUM SQUARE TEST



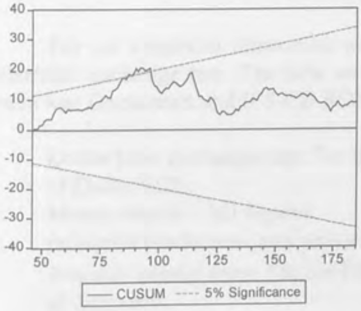
RECURSIVE COEFFICIENT PLOTS

Coefficient plots correspond to $dm, dm^{*}, dy, dy^{*} di, di^{*}, d\pi^{e}, d\pi^{e*}$ respectively

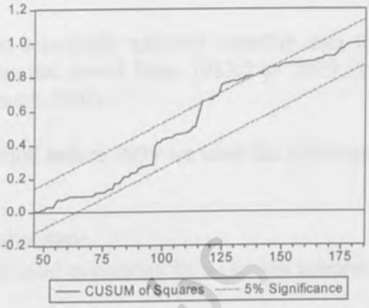


Frenkel-Bilson Model

CUSUM TEST

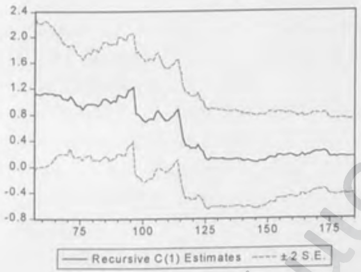


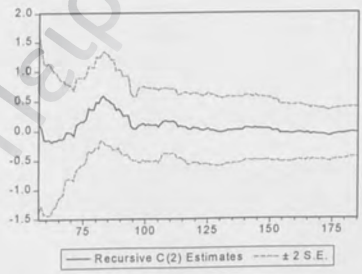
CUSUM SQUARE TEST

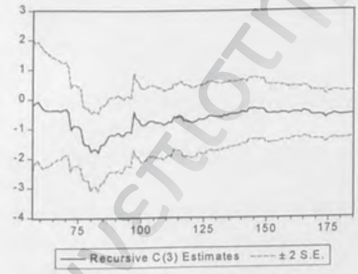


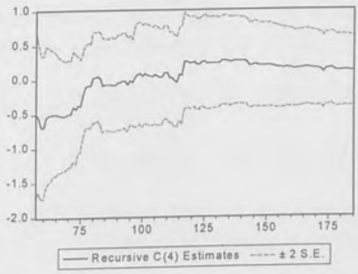
RECURSIVE COEFFICIENT PLOTS

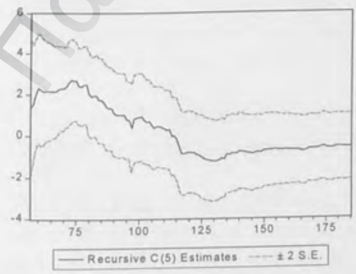
Coefficient plots correspond to $dm, dm^*, dy, dy^* di, di^*$ respectively

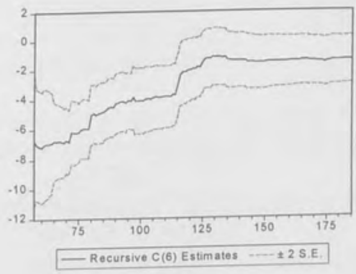












DATA DESCRIPTION

For our empirical illustration we used seasonally adjusted monthly data for the dollar/euro exchange rate. The data set covers the period from 1983:2 to 2000:12. Our source was Datastream and IFS-CD ROM (March 2002).

s : Dollar/Euro exchange rate. For the period before 1999 we used the exchange rate of Dollar/ECU.

m : Money supply – M1 figures.

y : Industrial production, as a proxy of real income.

i : 3-month interest rates. For the Euro we used as proxies the 3- month interest rates of Germany.

π^e : expected inflation rate. As a proxy we used the 10-year Government Bond yield for U.S and German for the case of Euro.

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