Instrumental Variable Estimation and Persistence of the Instruments

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ABSTRACT. In this paper, we investigate the behavior of the IV estimator under a specific regression model.Our purpose is to propose certain measures for selecting a good instrument, beyond the classical measure which is the degree of correlation between the regressor and the instrument.The survey is based in the theoretical analysis of mathematical types that describe the behavior of the IV estimator-such as the asymptotic bias-and it includes the proof of the theoretical results using Monte Carlo methods.

CONTENTS

1. INTRODUCTION.

Empirical researchers often wish to make causal inference about the effect of one variable on another. Based in Statistical theory, researchers developed regression models with purpose modelling this causality effect between the explanatory variable -or the regressor- and the dependent -or the outcome- variable. Very often the explanatory variable is endogenous; that is, it is influenced by some of the same forces that influence the outcome under study. When the explanatory variable is endogenous, the well known estimator the ordinary least squares (OLS) , gives biased and inconsistent estimates of the causal effect of the explanatory variable on the dependent variable. The most common strategy for dealing the specific problem is to use the instrumental variable (IV) estimator. The IV estimator uses one or more instruments variables that have : 1) no direct association with the dependent variable -or the instruments are orthogonal with the basic regression error- and 2) cause the explanatory variable -or the instruments are correlated with the endogenous explanatory variable.Under the previous assumptions, the IV estimates of the effect of the endogenous variable are consistent.

However, when searching for plausible instruments for a potentially endogenous variable, it is common to find that the candidates are only weak correlated with the endogenous variable in question. When the first assumption holds, the degree of correlation between the instrument and the explanatory variable is asymptotically irrelevant for the consistency of the IV estimator[1], provided that is not equal to zero. When the instrument does not fullfil the orthogonality condition, the finite sample performance of the IV estimator critically depends on the degree of correlation between the instrument and the regressor ; even a weak correlation between the instrument and the explanatory variable can lead to a large finite sample bias in IV estimates. It can be shown that the degree of misallocation is greater than that associated with the distribution of the OLS estimator.

When the IV estimator behaves in a such way, any statistical inference based on the IV estimates will lead to serious errors. The main idea of this survey is to examine whether or not the degree of correlation between the regressor and the instrument is the main suggestion for selecting a good instrument. The basic question that has not been answered yet, is what happens when the correlation coefficient is kept fixed or when we have to choose between two instruments that present the same degree of correlation. Our purpose is to investigate the properties of the IV estimator under a specific regression model, when the correlation coefficient is kept fixed. Additionally, knowing that the economic series are full of weak instruments, our motivation is to propose certain measures for selecting a good -if exists- instrument in a regression analysis.

[1](of course in finite samples is relevant)

2. MAIN STEPS OF THE SURVEY.

First of all, our purpose is to examine in a theoretical way the behavior of the IV estimator under a specific regression model. To be more specific, at first stage we will assume that the orthogonality condition does not hold and we will mainly focus on the Bias of the IV estimator and its mathematical formula; using Mathematical Analysis Theory, we will examine the behavior of the specific formula from a theoretical point of view. At second stage we will study the mathematical formula of the Asymptotic Variance of the IV estimator in the case where the IV estimator is consistent.

Afterwards, using Monte Carlo simulations, we will apply the theory in practice. More specifically, we will examine how the Bias and the Variance of the IV estimator behaves in finite samples -realizations- from specific data generating processes. Our survey will cover the study of some others measures that describe the behavior of the IV estimator, such as the Absolute bias,the T-statistic etc.

3. WHAT IS A MONTE CARLO SIMULATION.

A Monte Carlo simulation is an operation where we can generate random numbers in order to create series with specific characteristics. We can call each random number generation as a configuration and one advantage of the MC simulation is that we can create as many configurations as we want to. Afterwards, assuming specific regression models, we can create samples for time-series where we know from the beginning their characteristics (or more simply, we know their parameter of interest). The next step is to choose the estimators that we want to use for the estimation of the parameter of interest and to examine their results comparing them with the value of the true parameter. Some measures of comparison are the bias,the absolute bias,the MSE of the estimator, and the characteristics of the T-statistic : its mean, its standard error and its size.

Our survey will be based on the use of a specific program that runs Monte Carlo simulations. The program operates in the Gauss environment and it is programmed by Christina Christou. Each Monte Carlo simulation includes the generation of a n-size random number sample. The option to select the size is an another advantage of this method, because we can insure the appropriate application of the Central Limit Theorem. The program generates as much random samples as the defined configurations. Afterwards, assuming specific models, we are asked to define the true parameters in order to create the series. The program uses the IV estimator to give estimates for the parameter of interest. Each configuration leads to one estimate for the parameter of interest and one value for the bias,the absolute bias and the MSE of the estimator. For T-configurations, we have T-estimates, a well defined histogram for the estimator (mean value and standard error), the mean value for bias, absolute bias and MSE for the estimator and last but not least, a well defined histogram for the T-statistic.

4. DEFINING AN ESTIMATOR

4.1. Definition of an estimator. First of all, an estimator is a statistical function; that is a function that is not include the parameter of interest. More specifically, estimating the unknown parameter ϑ, amounts to defining the following function:

$$\widehat{\vartheta} = T(\widetilde{x}) : X \to \Theta$$

where X is the sample space (the set of all possible realizations), Θ denotes the parameter space (the set of all possible values of ϑ), and $\widetilde{x} = (X_1, X_2, ..., X_n)$ is the random variables's set. An estimator, is itself a random variable which takes different values depending on the sample realization. A particular value of this estimator, based on a particular sample realization $(x_1, x_2, ..., x_n)$, is called as an estimate of ϑ and denoted by :

$$\widehat{\vartheta} = T(x_1, x_2, ..., x_n)$$

An estimator has a density function. Its density function is denoted by:

$$f(\widehat{\vartheta}; x_1, ..., x_n) = f(\widehat{\vartheta}; x)$$

in order to emphasize its dependence on the sample $(X_1, X_2, ..., X_n)$.

4.2. Defining the 'ideal' estimator / properties of an estimator. We already know that it is easy to define estimators. This raises the problem of choosing the 'best' among these estimators. As we mentioned above, an estimator is a random variable which means it has a density function. Hence, any discussion of 'best' would be related to their distribution.

As it is shown in the previous paragraph, for each realization we get an estimate from the statistical function $\widehat{\vartheta}$. An 'ideal' estimator would be an estimator who would take only one value (ϑ, the true parameter), with probability one, irrespective of the sample realization. Its density function would have the following form:

$$P(\widehat{\vartheta} = \vartheta) = 1.$$

or using the moments of $\widehat{\vartheta}$, it is equivalent to say,

$$\begin{aligned} i)\ E(\widehat{\vartheta}) &= \vartheta \\ ii)\ Var(\widehat{\vartheta}) &= 0. \end{aligned}$$

The latter suggests that an 'optimal' estimator will be one whose mean is located at the true value of the parameter of interest and its variance is zero.

The definition of the previous properties is helping us to choose among a great number of estimators. Unfortunately, for a given sample size n, it is impossible to find estimators that would achieve the properties of an 'ideal' estimator (especially the second property). But as n goes to infinity, some estimators can emulate the specific properties. Because of this, we distinquish between finite sample properties (valid for any n) and asymptotic properties (valid as n goes to infinity). Some measures that describe the properties of an estimator are the following:

Unbiasedness. An estimator $\widehat{\vartheta}$ is said to be an unbiased estimator of ϑ if its sampling distribution has a mean equal to the parameter of interest ϑ :

$$E(\widehat{\vartheta}) = \vartheta$$

Otherwise, $\widehat{\vartheta}$ is said to be biased, and its bias is defined:

$$bias(\widehat{\vartheta};\vartheta) = E(\widehat{\vartheta}) - \vartheta$$

Minimum MSE estimators. The Mean Square Error measure compares biased and unbiased estimators and penalizes an estimator for its bias or the value of its variance:

$$\begin{aligned} MSE(\widehat{\vartheta};\vartheta) &= E\{(\widehat{\vartheta}-\vartheta)^2\} \\ or &= Var(\widehat{\vartheta}) + bias(\widehat{\vartheta};\vartheta) \end{aligned}$$

An estimator $\widehat{\theta}$ is said to be a minimum MSE estimator of ϑ if:

$$MSE(\widehat{\theta};\vartheta) \leq MSE(\widehat{\vartheta}_i;\vartheta) \ , i = 1,..,k$$

Consistency. An estimator $\widehat{\vartheta_n}$ is said to be consistent estimator of ϑ, if for any $\varepsilon \succ 0$:

$$\begin{aligned} \lim_{n\to\infty} P(|\widehat{\vartheta_n} - \vartheta| &< \varepsilon) = 1 \\ \text{or} \quad \widehat{\vartheta_n} &\Longrightarrow {}^{P}\vartheta \end{aligned}$$

This reads 'the limit of the probability of the event that $\widehat{\vartheta_n}$ differs from the true value of ϑ by any positive amount less than some constant $\varepsilon \succ 0$, goes to one as n goes to infinity. In the case where $\widehat{\vartheta_n}$ has a bounded variance we can use the following forms as a sufficient condition for the consistency of an estimator:

$$\begin{aligned} \lim_{n\to\infty} E(\widehat{\vartheta_n}) &= \vartheta \\ \lim_{n\to\infty} Var(\widehat{\vartheta_n}) &= 0 \end{aligned}$$

(Of course the previous moments of the sampling distribution of the estimator have to exist).

5. PRESENTATION OF THE EXISTING LITERATURE

5.1. A simple regression model. The general form of the multiple regression model is:

$$y_t = \vartheta_1 x_{1t} + \vartheta_2 x_{2t} + ... + \vartheta_k x_{kt} + u_t \quad t = 1, .., T.$$

where y_t is the dependent variable, x_{it} is the regressor and u_t is an error term. Using matrices the previous equation can be reformed :

$$\widetilde{y}_{Tx1} = x_{Txk}\widetilde{\vartheta}_{kx1} + \widetilde{u}_{Tx1}$$

For this model we assume the following assumptions,

$$\begin{cases} \text{i) } E(\widetilde{u}) = \widetilde{0} \\ \text{ii) } E\left(\widetilde{u}\widetilde{u'}\right) = \sigma^2 I \text{ , where I is a unitary } T \times T \text{ matrix} \\ \text{iii) } E(u_{t+i}x_t) = 0 \text{ , } \quad \forall i \succeq 0 \end{cases}$$

The last assumption is the predeterminess case which came to substitute the restrictive assumption that x is fixed in repeated samples. When the previous assumptions are fulfilled, it can be shown that the OLS estimator is the best linear unbiased estimator (B.L.U.E.) for the parameter vector $\widetilde{\vartheta}$. The OLS estimator in a form of vector is the following:

$$\widehat{\vartheta}_{ols} = (x'x)^{-1} x' \widetilde{y}$$

When we have only one explanatory variable x (that is k=1) , the number of parameters of $\widetilde{\vartheta}$ reduces to one and the OLS estimator takes the form :

$$\widehat{\vartheta}_{ols} = \left(\frac{\sum y_t x_t}{\sum x_{t-1}^2}\right)$$

Let us examine the case where the explanatory variable x is not orthogonal with the error term of the regression u.This means that the third assumption does no longer hold and x is not an exogenous variable. The OLS estimator is no longer the best estimator, in fact the OLS estimator is biased and inconsistent and it should not be used for estimation and for statistical inference. In this case, an instrument variable z will be used, which has the two characteristics that mentioned in the begging of this survey : orthogonality with the basic regression error and correlation with the endogenous explanatory variable.

5.2. Introduction of the IV estimator. In the case that predetermines does not hold, the existing literature introduces the IV estimator assuming the following regression models:

$$\begin{cases} \widetilde{y}_{Tx1} = \vartheta\, \widetilde{x}_{Tx1} + \widetilde{u}_{1,Tx1} \quad , \textit{basic regression} \qquad (1) \\ \widetilde{x}_{Tx1} = p\, \widetilde{z}_{Tx1} + u_{2,Tx1} \quad , \textit{first stage regression} \ (2) \\ \binom{u1,t}{u2,t}\sim NIID \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = N(0, \sum),\ i = 1,..,T. \end{cases}$$

The assumptions that hold for this regression analysis are the following :

$$\begin{cases} \text{i) } E\left(u_{1,t+i} x_t\right) \neq 0 \\ \text{ii) } Corr(z, x) \neq 0 \\ \text{iii) } Corr(z, u_1) = 0 \\ \text{iv) } Corr(z, u_{2)} = 0 \end{cases}$$

In the specific case the IV estimator is assigned. Because of the fact that z is a good instrument, the IV estimator of ϑ is consistent. The form of the specific estimator is :

$$\begin{aligned} \widehat{\vartheta}_{iv} &= (x'P_z x)^{-1} x' P_z y \\ \textit{where } P_z &= z(z'z)^{-1} z' \end{aligned}$$

This estimator is called also as the Two stages Least Squares estimator, because the whole procedure is numerically equivalent to estimating (1) and (2) by two stages least squares, where the first stage (equation 2) is estimated by OLS and the predicted values from this estimation, $\widehat{x} = z\widehat{p}$, are used in place of x in the second stage estimation of equation (1) by OLS. In the specific case, the regression coefficient is equal to the squared correlation coefficient between the regressor and the instrument; that is :

$$\rho^2_{Z,X} = \{corr(z, x)\}^2 = R^2$$

The asymptotic bias of the OLS relative to the asymptotic bias of the IV estimator, can be focalized to the following equation :

$$\frac{p\lim(\widehat{\vartheta}_{iv}-\vartheta)}{p\lim(\widehat{\vartheta}_{ols}-\vartheta)}=\frac{\rho_{z,u_1}}{\rho_{z,x}}\frac{1}{\rho_{x,u_1}}$$

From the previous equation we conclude that a weak correlation between the endogenous variable x and the instrument z , will exacerbate any problems associated with a correlation between the instrument and the regressor error u_1.These two effects can produce large inconsistency in the IV estimates of ϑ than in the OLS estimate.

As we mentioned before, in the case where the orthogonality condition holds, the selection of an instrument is asymptotically irrelevant. But when there is a correlation between the instrument and the basic regression error, the selection of a good instrument is critical and the existing literature suggests to choose the instrument that is the most correlated with the regressor, in order to achieve the least Asymptotic Bias. Is the correlation coefficient the only suggestion for selecting a good instrument?

6. REGRESSION MODEL PROPOSED BY N.PITTIS

We assume the following regression equation:

$$y_t=\vartheta x_t+u_{1t} \tag{1}$$

where the regressor is generated via an AR(1) process:

$$x_t=\rho_1 x_{t-1}+u_{2t} \tag{2}$$

In the case of an endogenous regressor, the estimation of the structural parameter, ϑ, requires the employment of an instrument, z_t, which is assumed to follow an AR(1) process as well:

$$z_t=\rho_2 z_{t-1}+u_{3t} \tag{3}$$

The error vector $\mathbf{u}_t=[u_{1t},u_{2t},u_{3t}]^\intercal$ is assumed to be normal, independent and identically distributed with zero mean and covariance matrix Σ. Specifically,

$$\begin{bmatrix}u_{1t}\\u_{2t}\\u_{3t}\end{bmatrix}\,\tilde{}\,NIID\begin{bmatrix}0\\0\\0\end{bmatrix}\begin{bmatrix}\sigma_{11}&\sigma_{12}&\sigma_{13}\\\sigma_{21}&\sigma_{22}&\sigma_{23}\\\sigma_{31}&\sigma_{32}&\sigma_{33}\end{bmatrix} \tag{4}$$

We further assume that $|\rho_1|$ and $|\rho_2|$ are less than one in order for the instrument and the regressor to be asymptotically stationary processes. When x_t and z_t are stationary processes, the equations (2) and (3) can be reformed as[2]:

[2]Most of the following equations are proved in the Appendix (A)

$$x_t = \sum_{j=0}^{\infty} \rho_1^j u_{2,t-j} \tag{5}$$

$$z_t = \sum_{j=0}^{\infty} \rho_2^j u_{3,t-j} \tag{6}$$

From equations (5) and (6) one can easily see that the characteristics of the x_t and z_t series can be described through the regression errors u_{2t} , u_{3t}. One of the advantages of the specific representation is that we can speak for the correlation of x_t and z_t through the correlation of the errors and more generally we can transfer all the assumptions about the y_t, x_t, z_t, in terms of the errors u_{1t}, u_{2t}, u_{3t}. Moreover, a study on the behavior of the OLS & IV estimators to regions near to unit-root, can be accomplished.

As we said at previous paragraphs, in the case that predeterminess does hold (that is - in terms of the errors u_{2t},u_{3t}– when $\sigma_{12} = 0$, because:

$$Cov(u_{1,t+i}x_t) = E\left(u_{1,t+i}x_t\right) = E(u_{1,t+i}u_{2t}) = \sigma_{12}\ , \forall i \geq 0 \tag{7}$$

) the parameter of interest, ϑ, can be consistently estimated by the OLS estimator $\widehat{\theta}_{LS} = (x^{\intercal}x)^{-1}x^{\intercal}y$, from the equation (1).

If $\sigma_{12} \neq 0$, the regressor, x_t, is no longer predetermined and the IV estimator has to be employed. Before we examine the properties of the IV estimator for various parameter configurations of the proposed system, let us derive the asymptotic bias of the OLS estimator for the case $\sigma_{12} \neq 0$. It is easy to show that:

$$p\lim \frac{x^{\intercal}x}{T} = \frac{\sigma_{22}}{1-\rho_1^2} \tag{8}$$

and

$$p\lim \frac{x^{\intercal}u_1}{T} = \sigma_{12} \tag{9}$$

Then, it immediately follows that,

$$p\lim \widehat{\vartheta}_{LS} = \vartheta + \frac{\sigma_{12}}{\sigma_{22}}(1-\rho_1^2) \tag{10}$$

It can be seen that the asymptotic bias of the OLS estimator is proportional to the degree of correlation between the regression error and the error that drives the regressor (σ_{12}) and inversely proportional to the degree of persistence of the regressor (ρ_1). If $\sigma_{12} = 0$, predeterminess assumption holds and OLS estimator is unbiased and consistent. In the case where $\sigma_{12} \neq 0$, the OLS estimator is biased and inconsistent, but as the regressor approaches the nonstationary region -that is when $\rho_1 \to \pm 1$- the asymptotic bias tends to zero, regardless of whether σ_{12} is zero. This is hardly surprising, given the (super) consistency properties of the OLS estimator in a cointegrating framework (see, among numerous others, Stock 1987, Phillips 1988).

Next, let us consider the asymptotic bias of the IV estimator.(we assumed that predeterminess assumption does not hold). In the framework (1)-(4), the IV estimator has the following form:

$$\widehat{\vartheta}_{IV} = (z^{\intercal}x)^{-1}z^{\intercal}y.$$

The predeterminess case for the instrument z_t can be written:

$$Cov(z_t, u_{1t-i}) = E(z_t\ u_{1t-i}) = \sigma_{13}\ , \forall i \geq 0 \tag{11}$$

We can speak for the asymptotic bias of the IV estimator in the case where $\sigma_{13} \neq 0$. In a fashion similar to the OLS case, we can show that:

$$p\lim \frac{z^{\intercal}u_1}{T} = \sigma_{13} \tag{12}$$

and

$$p\lim \frac{z^{\intercal}x}{T} = \frac{\sigma_{23}}{1 - \rho_1\rho_2} \tag{13}$$

Therefore,

$$p\lim \left(\widehat{\vartheta}_{IV} - \vartheta\right) = \frac{\sigma_{13}}{\sigma_{23}}(1 - \rho_1\rho_2) \tag{14}$$

As we mentioned from the beginning, our target is to find measures for selecting a good instrument beyond the classical measure which is the degree of correlation between the x_t and z_t. Specifically, our target is to keep the correlation coefficient stable and then examine the properties of the IV estimator.

Easily, using equations (2) and (3), it can be proved that the asymptotic variance of the regressor and the instrument can be given from the following equations:

$$\sigma_x = \frac{\sigma_{22}}{1 - \rho_1^2} \tag{15}$$

$$\sigma_z = \frac{\sigma_{33}}{1 - \rho_2^2} \tag{16}$$

The previous two equations suggest that as $\rho_1$and ρ_2 tend to unity, the x_t , z_t series become more volatile.

We know that the correlation coefficient is given from:

$$\rho_{x,z} = \frac{Cov(x,z)}{\sqrt{\sigma_x}\sqrt{\sigma_z}}$$

Then we can reform the previous equation to the following:

$$\rho_{x,z} = \frac{\sigma_{23}\sqrt{(1-\rho_1^2)(1-\rho_2^2)}}{\sqrt{\sigma_{22}\sigma_{33}}(1-\rho_1\rho_2)} \tag{17}$$

From the previous equation, solving for σ_{23} and substituting to equation (14), we have:

$$p\lim\left(\widehat{\theta}_{IV} - \theta\right) = \frac{\sigma_{13}}{\rho_{xz}}\sqrt{\frac{(1-\rho_1^2)(1-\rho_2^2)}{\sigma_{22}\sigma_{33}}} \tag{18}$$

or,

$$p\lim\left(\widehat{\theta}_{IV} - \theta\right) = \frac{\sigma_{13}}{\rho_{xz}}\frac{1}{\sqrt{\sigma_x\sigma_z}} \tag{19}$$

Equation (18) is the Asymptotic Bias of the IV estimator equation, which its properties we will study. We will assume the coefficients: $\sigma_{13}, \sigma_{22}, \sigma_{33}$ and $\rho_{x,z}$ as fixed and ρ_1, ρ_2 as variables. This means that we have to deal with a two-dimensional function $f(\rho_1, \rho_2)$. Afterwards, to generalize our conclusions, we will examine equation (19). Equation (19), describes the Asymptotic Bias as well, so it shares the same properties with equation (18), but it is voiced in terms of the asymptotic variances of the x_t and z_t series. From the equations we can easily see that when $\sigma_{13} = 0$, the asymptotic bias of $\widehat{\theta}_{IV}$ is zero, regardless of the 'weakness' of the instrument and of the degrees of persistence of the regressor and the instrument.

Next step is the expansion of our study for the behavior of IV estimator to its Asymptotic Variance. Asymptotic inference on θ requires a consistent estimate of the Asymptotic Variance of $\widehat{\theta}_{IV}$. Let us assume that the orthogonality condition holds, that is $\sigma_{13} = 0$. In general, we know that the covariance matrix for the IV estimator is:

$$Cov(\widehat{\vartheta}_{iv}) = \sigma_{11}(z^T x)^{-1}(z^T z)(z^T x)^{-1}$$

Asymptotically, it can be shown that:

$$\frac{z^\top u_1}{\sqrt{T}} \Rightarrow N\left(0, \frac{\sigma_{33}}{1-\rho_2^2}\right) \tag{20}$$

where '$\Rightarrow$' signifies convergence in distribution.

The last equation, in conduction with (13) and (16), gives rise to the following result:

$$\sqrt{T}\left(\widehat{\theta}_{IV}-\theta\right) \Rightarrow N\left(0, \frac{\sigma_{11}\sigma_{33}(1-\rho_1\rho_2)^2}{\sigma_{23}^2(1-\rho_2^2)}\right) \tag{21}$$

As in the Asymptotic Bias case, we will study the Asymptotic Variance of the IV estimator (when $\sigma_{13}=0$) in the case where the correlation coefficient is fixed. So substituting to the previous equation σ_{23}, we get:

$$\sqrt{T}\left(\widehat{\theta}_{IV}-\theta\right) \Rightarrow N\left(0, \frac{\sigma_{11}}{\sigma_{22}\rho_{x,z}^2}(1-\rho_1^2)\right) \tag{22}$$

7. THEORETICAL ANALYSIS

7.1. Is the correlation coefficient the only suggestion for selecting an instrument?. As we mentioned in a previous section, the function which describes the behavior of the IV estimator and we will study is the asymptotic bias of the IV, where the correlation coefficient is kept fixed:

$$p\lim\left(\widehat{\theta}_{IV}-\theta\right)=\frac{\sigma_{13}}{\sqrt{\sigma_{22}\sigma_{33}}}\frac{1}{\rho_{xz}}\sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)}=f(\rho_1,\rho_2) \tag{23}$$

We will assume that the coefficients $\sigma_{13},\sigma_{22},\sigma_{33}$, are also fixed, so the function (23) is two-dimensional with unknown variables ρ_1,ρ_2. More specifically, we have:

$$f: D \longrightarrow R$$

$$\begin{aligned} where\ D &= \{(\rho_1,\rho_2)\in R^2/\ (1-\rho_1^2)\geq 0,(1-\rho_2^2)\geq 0,\rho_1\neq 1,\rho_2\neq 1\} \\ &= \{(\rho_1,\rho_2)\in R^2/\ \rho_1^2<1,\rho_2^2<1\} \\ &= \{(\rho_1,\rho_2)\in R^2/-1<\rho_1,\rho_2<1\} \end{aligned}$$

We point out that ρ_1 and ρ_2 can not be equal to one, because x_t and z_t series are stationary. We can assume that function (23) could be rewritten as:

$$f(\rho_1,\rho_2)=\Pi\sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)} \tag{24}$$

$$\begin{aligned} where\ \Pi &= \frac{\sigma_{13}}{\sqrt{\sigma_{22}\sigma_{33}}}\frac{1}{\rho_{xz}} \\ and\ \Pi &\neq 0 \end{aligned}$$

In this section, the choice of value of the coefficients $\sigma_{13}, \sigma_{22}, \sigma_{33}$ and ρ_{xz}, does not matter from a theoretical point of view, because it does not misquote the behavior of our function. The choice of $\sigma_{13}, \sigma_{22}, \sigma_{33}$ and ρ_{xz}, only causes shifts in its field of values. To be more specific, the Asymptotic bias of IV is proportional to the degree of correlation between the basic regression error and the error that drives the instrument (σ_{13}) and inversely proportional to the variance of the regressor error-or instrument error; the latter means that the more volatile the regressor or the instrument the better for the researcher. Likewise, the Asymptotic bias is inversely proportional to the degree of correlation between the regressor and the instrument ; the smaller the ρ_{xz} the bigger the Asymtotic Bias.

Our theoretical study, will be focused in the case when $\Pi = 1$ (that is $\sigma_{22} = \sigma_{33} = 1$ and $\sigma_{13} = \rho_{xz}$), which has no difference with a case where Π takes another value-different from one.

Finding the extrema of the function f. To find the extrema points, we will use the second derivative criterion for multivariate functions. The first derivatives for $\rho_1$and ρ_2 are the following:

$$\begin{aligned} f_{\rho_1} &= -\frac{1}{\sqrt{(1-\rho_1^2)}}\sqrt{(1-\rho_2^2)}\rho_1 \\ f_{\rho_2} &= -\frac{1}{\sqrt{(1-\rho_2^2)}}\sqrt{(1-\rho_1^2)}\rho_2 \end{aligned}$$

Similar, the second derivatives for $\rho_1$and ρ_2 are:

$$\begin{aligned} f_{\rho_1\rho_1} &= -\frac{1}{\left(\sqrt{(1-\rho_1^2)}\right)^3}\sqrt{(1-\rho_2^2)}\rho_1^2 - \frac{1}{\sqrt{(1-\rho_1^2)}}\sqrt{(1-\rho_2^2)} \\ f_{\rho_2\rho_2} &= -\frac{1}{\left(\sqrt{(1-\rho_2^2)}\right)^3}\sqrt{(1-\rho_1^2)}\rho_2^2 - \frac{1}{\sqrt{(1-\rho_2^2)}}\sqrt{(1-\rho_1^2)} \end{aligned}$$

As we can see, $\rho_1$and ρ_2 are sharing the same type of derivative because of the fact that our function is symmetrical for $\rho_1$and ρ_2. The derivative for $\rho_1$and ρ_2 is:

$$f_{\rho_1\rho_2} = \frac{1}{\sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)}}\rho_2\rho_1$$

The stasis points of f are where its first derivatives are equal to zero:

$$\begin{cases} f_{\rho_1} = 0 \\ f_{\rho_2} = 0 \end{cases} \text{ where } \rho_1 \textit{ and } \rho_2 \in \Phi = (-1,1)$$
$$\textit{Solution is } (\rho_1, \rho_2) = (0,0) = P_o$$

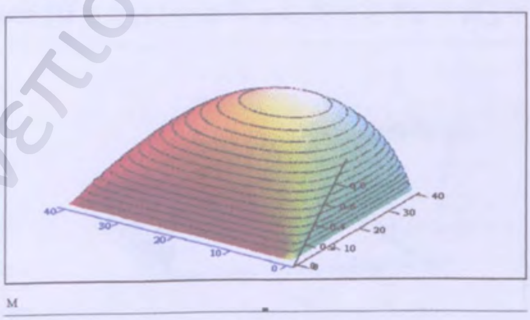
The next step is to find the value of second derivatives for the point of stasis $P_o = (0,0)$:

$$\begin{aligned} A &= f_{\rho_1\rho_1}(\rho_1, \rho_2)|_{P_o} = f_{\rho_1\rho_1}(0,0) = -1 < 0 \\ \Gamma &= f_{\rho_2\rho_2}(\rho_1, \rho_2)|_{P_o} = f_{\rho_2\rho_2}(0,0) = -1 \\ B &= f_{\rho_1\rho_2}(\rho_1, \rho_2)|_{P_o} = f_{\rho_1\rho_2}(0,0) = 0 \\ \Delta &= A\Gamma - B^2 = 1 > 0 \end{aligned}$$

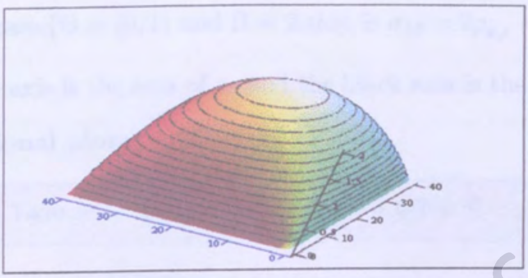
Because of the fact $\Delta > 0$ and $A < 0$, the stasis point $P_o = (0,0)$ is a local maximum point for the function $f(\rho_1, \rho_2)$. This means that the Asymptotic Bias of the IV estimator takes its maximum value when $(\rho_1, \rho_2) = (0,0)$, but it declines to zero when $(\rho_1, \rho_2) \to \pm 1$.

Three-dimensional and Two-Dimensional plots of the function f. In order to understand better the behavior of the function of Asymptotic Bias, we will plot some graphs in the three-dimensional and two-dimensional space. Our intention is to plot graphs of $f(\rho_1, \rho_2)$ for the space $\Phi = (-1,1)$ and $\Theta = (0,1)$ and for different values of Π. Moreover the diagonal elements in the variance-covariance matrix $\sum$, are equal to one.

Three-Dimensional plots .

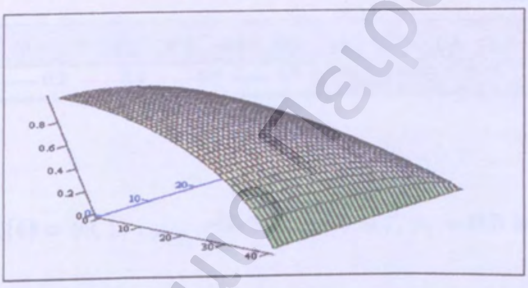


First case: $\{\Phi = (-1,1) \text{ and } \Pi = 1 \text{ that is } \sigma_{13} = \rho_{x,z} = \textit{fixed}\}$



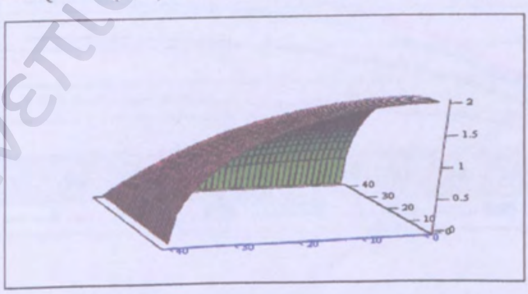
M

Second case:$\{\Phi = (-1,1)$ and $\Pi = 2$ (i.e. $\sigma_{13} = 2\ \rho_{x,z} = fixed\}$



M

Third case:$\{\Theta = (0,1)$ and $\Pi = 1$, that is $\sigma_{13} = \rho_{x,z} = fixed\}$

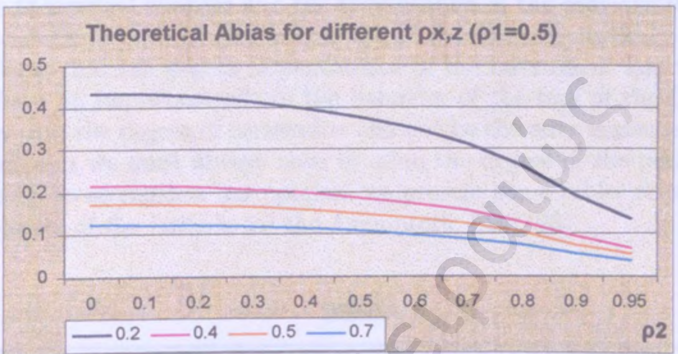


M

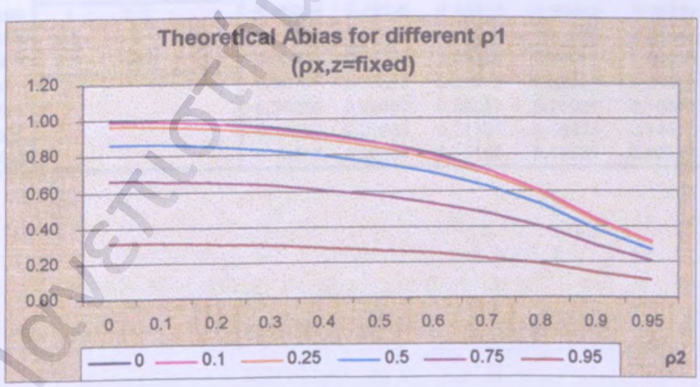
Fourth case:$\{\Theta = (0,1)$ and $\Pi = 2$,that is $\sigma_{13} = 2\rho_{x,z} = fixed\}$

(The blue axis is the axis of ρ_1 and the black axis is the axis of ρ_2)

Two-Dimensional plots.



Case where :$\{\Theta = (0,1)$, $\rho_{x,z} = 0.2, 0.4, 0.5, 0.7$, $\rho_1 = 0.5$ *and* $\sigma_{13} = 0.1\}$



Case where :$\{\Theta = (0,1)$ and $\Pi = 1$, that is $\sigma_{13} = \rho_{x,z} = 0.2$ *and* $\rho_1 = (0, 0.1, 0.25, 0.5, 0.75, 0.95)\}$

Tables of the Asymptotic Bias of the IV estimator. {In this part, we present some of the tables who contain the theoretical values of the asymptotic bias of the IV estimator. The tables contain values for specific data, as a specific variance covariance matrix $\sum$, specific degree of correlation $\rho_{x,z}$ and different values of ρ_1 (persistence of the regressor). The tables of the theoretical value of the Asymptotic Bias of IV are alike those who include the Monte Carlo results. The whole tables of results are presented in the appendix (B).}

From the theoretical analysis and the presentation of the previous plots, we confirm that the degree of correlation affects the Theoretical Asymptotic Bias, but we can also endorse that the degree of persistence of the instrument and the regressor as well, perform an important role in the behavior of the bias of the IV estimator. We could say that the degree of correlation can not be the only measure for selecting an instrument, but we must always have in mind the degree of the persistence of z_t and x_t. In order to strengthen our opinion, we present some tables that content the theoretical results of the formula for the Asymptotic Bias (23).

TABLE 1

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.2			0.50000	0.49749	0.48412	0.43301	0.33072	0.15612
0.1	1.010				0.49749	0.49500	0.48170	0.43084	0.32906	0.15534
0.2	1.042				0.48990	0.48744	0.47434	0.42426	0.32404	0.15297
0.3	1.099		Σ		0.47697	0.47458	0.46182	0.41307	0.31549	0.14893
0.4	1.190	1	0.5	0.1	0.45826	0.45596	0.44371	0.39686	0.30311	0.14309
0.5	1.333	0.5	1	σ23	0.43301	0.43084	0.41926	0.37500	0.28641	0.13521
0.6	1.563	0.1	σ23	1	0.40000	0.39799	0.38730	0.34641	0.26458	0.12490
0.7	1.961				0.35707	0.35528	0.34573	0.30923	0.23618	0.11150
0.8	2.778				0.30000	0.29850	0.29047	0.25981	0.19843	0.09367
0.9	5.263				0.21794	0.21685	0.21102	0.18875	0.14416	0.06805
0.95	10.256				0.15612	0.15534	0.15117	0.13521	0.10327	0.04875

TABLE 2

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			0.25000	0.24875	0.24206	0.21651	0.16536	0.07806
0.1	1.010				0.24875	0.24750	0.24085	0.21542	0.16453	0.07767
0.2	1.042				0.24495	0.24372	0.23717	0.21213	0.16202	0.07649
0.3	1.099		Σ		0.23848	0.23729	0.23091	0.20653	0.15774	0.07447
0.4	1.190	1	0.5	0.1	0.22913	0.22798	0.22185	0.19843	0.15155	0.07155
0.5	1.333	0.5	1	σ23	0.21651	0.21542	0.20963	0.18750	0.14321	0.06760
0.6	1.563	0.1	σ23	1	0.20000	0.19900	0.19365	0.17321	0.13229	0.06245
0.7	1.961				0.17854	0.17764	0.17287	0.15462	0.11809	0.05575
0.8	2.778				0.15000	0.14925	0.14524	0.12990	0.09922	0.04684
0.9	5.263				0.10897	0.10843	0.10551	0.09437	0.07208	0.03403
0.95	10.256				0.07806	0.07767	0.07558	0.06760	0.05163	0.02438

From the tables, we first observe that the Asymptotic Bias of the IV estimator is inversely proportional to the degree of correlation between the regressor and the instrument ; as $\rho_{x,z} \to 1$ the Asymptotic Bias decreases. Secondly, the Asymptotic Bias declines to zero also when ρ_1 and $\rho_2 \longrightarrow 1$. Moreover, we finally conclude that the degree of correlation is not the only measure for selecting an instrument. Comparing the tables 1 ($\rho_{x,z} = 0.2$) and 2 ($\rho_{x,z} = 0.4$) , we see that in some circumstances, we can achieve smaller bias in the case where $\rho_{x,z} = 0.2$, than that in the case where $\rho_{x,z} = 0.4$. For example, we report the following case : $\{\ \sigma_{13} = 0.1\ ,\ \rho_{x,z} = 0.2\ ,\ \rho_1 = 0.75\ ,\ \rho_2 = 0.9$ where the $Abias = 0.14416\}$ and $\{\sigma_{13} = 0.1\ ,\rho_{x,z} = 0.4\ ,\ \rho_1 = 0.5\ ,\ \rho_2 = 0.7$ where the $Abias = 0.15462\}$). This important result can be observed comparing and other tables for different degree of correlation.[3].

7.2. Selecting between two instruments. In the previous section, we have proved - in a theoretical way- that the correlation coefficient is not the only suggestion for selecting an instrument, but the degree of persistence has to be considered. So the next subject in discuss is what happens when we want to use an instrument to a regression analysis. Let us examine a general case where we have two instruments z_1 and z_2 that share the same degree of correlation and imagine the situation where we have to choose between these two series.

- In the case where the we know the innovation variances of the instruments and we know as well, that they are equal ($\sigma^1_{33} = \sigma^2_{33}$), the more persistent

[3]Just examine other cases for different $\rho_{x,z}$ from the tables who are presented in the appendix (B)

instrument will produce the smaller Theoretical Asymptotic Bias.

- In the case where the innovation variances are not equal, or more general are not known, the persistence criterion is not sufficient to select between the two instruments. For that reason we induct equation (19) :

$$\begin{aligned} p\lim\left(\widehat{\theta}_{IV}-\theta\right) &= \frac{\sigma_{13}}{\rho_{xz}}\frac{1}{\sqrt{\sigma_x\sigma_z}} \\ &= \Pi_1\frac{1}{\sqrt{\sigma_x\sigma_z}} \\ &= \varphi(\sigma_x,\sigma_z) \\ with\,\Pi_1 &= \frac{\sigma_{13}}{\rho_{xz}}\neq 0 \end{aligned} \tag{25}$$

As it was mentioned in a previous section, equation (19 or 25) arises when we substitute equations (15) and (16) to equation (18). The specific equation describes the Asymptotic Bias of the IV estimator as well, but in terms of σ_x and σ_z. With this transformation, we can examine the general case where we do not know the innovation variances σ_{33}^i with $i=1,2$, but we are able to estimate the variances of the instruments σ_{zi} with $i=1,2$.

The definition field of this function is:

$$\begin{aligned} \varphi &: V\longrightarrow R \\ where\,V &= \{(\sigma_x,\sigma_z)\in R^2/\ \sigma_x>0,\sigma_z>0\} \end{aligned}$$

In terms of (ρ_1,ρ_2) the definition field V can be transformed :

$$\begin{aligned} V &= \{(\sigma_x,\sigma_z)\in R^2/\ \sigma_x=\frac{\sigma_{22}}{1-\rho_1^2}>0,\sigma_z=\frac{\sigma_{33}}{1-\rho_2^2}>0\} \\ &= \{(\rho_1,\rho_2)\in R^2/\ (1-\rho_1^2)>0,(1-\rho_2^2)>0\} \\ &= \{(\rho_1,\rho_2)\in R^2/-1<\rho_1,\rho_2<1\} \end{aligned}$$

which is the definition field D of the function $f(\rho_1,\rho_2)$. The function of the Asymptotic Bias voiced in terms of (σ_x,σ_z) has the same behavior with the function $f(\rho_1,\rho_2)$, but in its definition field V does not present extrema points. In the case where $\Pi_1>0$, the first derivatives towards the (σ_x,σ_z) variables, are the following :

instrument will produce the smaller Theoretical Asymptotic Bias.

- In the case where the innovation variances are not equal, or more general are not known, the persistence criterion is not sufficient to select between the two instruments. For that reason we induct equation (19) :

$$\begin{aligned} p\lim\left(\widehat{\theta}_{IV}-\theta\right) &= \frac{\sigma_{13}}{\rho_{xz}}\frac{1}{\sqrt{\sigma_x\sigma_z}} \\ &= \Pi_1\frac{1}{\sqrt{\sigma_x\sigma_z}} \\ &= \varphi(\sigma_x,\sigma_z) \\ with\,\Pi_1 &= \frac{\sigma_{13}}{\rho_{xz}}\neq 0 \end{aligned} \tag{25}$$

As it was mentioned in a previous section, equation (19 or 25) arises when we substitute equations (15) and (16) to equation (18). The specific equation describes the Asymptotic Bias of the IV estimator as well, but in terms of σ_x and σ_z. With this transformation, we can examine the general case where we do not know the innovation variances σ_{33}^i with $i = 1, 2$, but we are able to estimate the variances of the instruments σ_{zi} with $i = 1, 2$.

The definition field of this function is:

$$\begin{aligned} \varphi &: V \longrightarrow R \\ where\,V &= \{(\sigma_x,\sigma_z)\in R^2/\ \sigma_x>0,\sigma_z>0\} \end{aligned}$$

In terms of (ρ_1, ρ_2) the definition field V can be transformed :

$$\begin{aligned} V &= \{(\sigma_x,\sigma_z)\in R^2/\ \sigma_x=\frac{\sigma_{22}}{1-\rho_1^2}>0,\sigma_z=\frac{\sigma_{33}}{1-\rho_2^2}>0\} \\ &= \{(\rho_1,\rho_2)\in R^2/\ (1-\rho_1^2)>0,(1-\rho_2^2)>0\} \\ &= \{(\rho_1,\rho_2)\in R^2/-1<\rho_1,\rho_2<1\} \end{aligned}$$

which is the definition field D of the function $f(\rho_1, \rho_2)$. The function of the Asymptotic Bias voiced in terms of (σ_x, σ_z) has the same behavior with the function $f(\rho_1, \rho_2)$, but in its definition field V does not present extrema points. In the case where $\Pi_1 > 0$, the first derivatives towards the (σ_x, σ_z) variables, are the following :

$$\begin{aligned}
\varphi_{\sigma x} &= -\frac{\Pi_1}{2\sqrt{\sigma_z}}\frac{1}{\sqrt{\sigma_x}^3} < 0 \\
\varphi_{\sigma z} &= -\frac{\Pi_1}{2\sqrt{\sigma_x}}\frac{1}{\sqrt{\sigma_z}^3} < 0
\end{aligned}$$

As we can see the first derivatives can not be equal to zero, so the function φ (σ_x, σ_z) does not present points of stasis in its definition field. Because of the fact that the first derivatives are negative for the whole definition field of φ, the function φ (σ_x, σ_z) is a decreasing function for the (σ_x, σ_z) variables. Moreover, it is easy to see that:

$$\lim_{(\sigma_x,\sigma_z)\to(0,0)} \varphi(\sigma_x, \sigma_z) = \lim_{(\sigma_x,\sigma_z)\to(0,0)} \frac{\Pi_1}{\sqrt{\sigma_x \sigma_z}} = +\infty$$

$$\lim_{(\sigma_x,\sigma_z)\to+\infty} \varphi(\sigma_x, \sigma_z) = \lim_{(\sigma_x,\sigma_z)\to+\infty} \frac{\Pi_1}{\sqrt{\sigma_x \sigma_z}} = 0$$

The specific analysis suggests that the Asymptotic Bias of the IV estimator defined in the space of (σ_x, σ_z), increases and goes to infinity as σ_x and σ_z decline and goes to zero as σ_x and σ_z increase. Because of the fact that σ_x and σ_z are not independent variables but they are well defined functions of ρ_1 and ρ_2 respectively, we know the limits of σ_x and σ_z a priory :

$$\begin{aligned}
\sigma_x &= \frac{\sigma_{22}}{1-\rho_1^2} \\
\textit{when } \rho_1 &\to 0 \text{ then } \sigma_x \to \sigma_{22} \\
\textit{when } \rho_1 &\to \pm 1 \text{ then } \sigma_x \to +\infty
\end{aligned}$$

$$\begin{aligned}
\sigma_z &= \frac{\sigma_{33}}{1-\rho_2^2} \\
\textit{when } \rho_2 &\to 0 \text{ then } \sigma_x \to \sigma_{33} \\
\textit{when } \rho_2 &\to \pm 1 \text{ then } \sigma_x \to +\infty
\end{aligned}$$

The previous relationships mean that the limits for the Asymptotic Bias in terms of (σ_x, σ_z) eventually are,

$$\lim_{(\sigma_x,\sigma_z)\to(\sigma_{22},\sigma_{33})} \varphi(\sigma_x, \sigma_z) = \Pi_1 \frac{1}{\sqrt{\sigma_{22}\sigma_{33}}} = \frac{\sigma_{13}}{\sqrt{\sigma_{22}\sigma_{33}}} \frac{1}{\rho_{x,z}}$$

$$\lim_{(\sigma_x,\sigma_z)\to+\infty} \varphi(\sigma_x,\sigma_z)=0$$

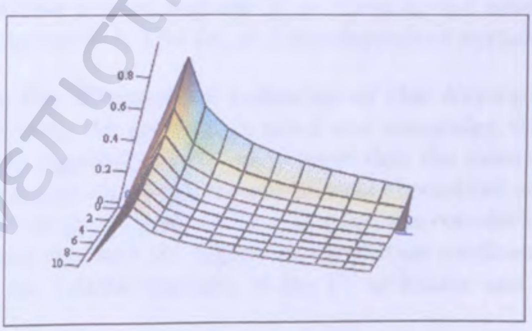
Keeping in mind the theoretical analysis of the Asymptotic Bias in terms of (ρ_1,ρ_2) in the general case, we conclude that,

$$\max AsBias(\widehat{\vartheta}_{iv}) = \max\varphi(\sigma_x,\sigma_z) = \max f(\rho_1,\rho_2) = \frac{\sigma_{13}}{\sqrt{\sigma_{22}\sigma_{33}}}\frac{1}{\rho_{x,z}} \qquad (26)$$

$$\begin{aligned} \textit{in the case } \sigma_{ii} &= 1, \textit{ where } i=1,2,3 \\ \textit{and } \rho_{x,z} &= \sigma_{13} \\ \textit{the } \max AsBias &= 1 \end{aligned}$$

Ultimately, our theoretical analysis suggests that the more volatile the instrument is, the smaller the Asymptotic Bias for the IV estimator. So in the general case where we have to choose between two instruments with unknown innovation variances and equal degrees of correlation, the more volatile instrument should be preferred. We will present some graphs in order to understand better these results.

Three-Dimensional and Two-Dimensional Plots. In this case, we forget the relationship of (σ_x,σ_z) with (ρ_1,ρ_2) and we plot a three-dimensional graph for the Asymptotic bias when is voiced in terms of (σ_x,σ_z). In other words, we consider the (σ_x,σ_z) as independent variables with only constrain $(\sigma_x,\sigma_z)>(0,0)$. The properties of the φ function when (σ_x,σ_z) are independent variables are summarized in the following plot :



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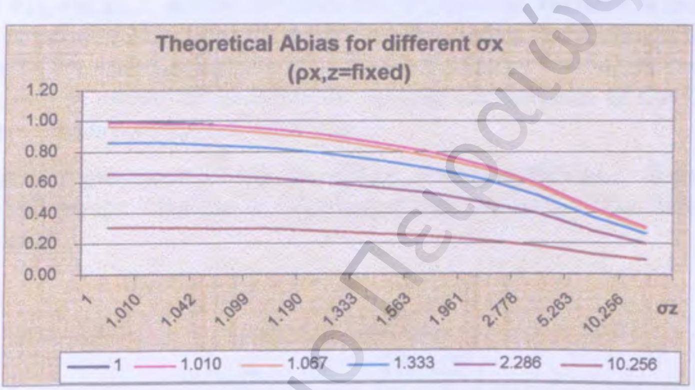
Case where :{$Asbias = \varphi(\sigma_x, \sigma_z)$ *and* $\Pi_1 = 1$ *and* $\sigma_{ii} = 1$ *where* $i = 1, 3$. *The* $\max Asbias = 1$. *The* (σ_x, σ_z) *are independent variables* }

(The blue axis is the σ_x axis, the black one is σ_z axis)

When we consider (σ_x, σ_z) as independent variables, the form of Asymptotic Bias looks like the previous plot. But actually, the (σ_x, σ_z) are dependent from (ρ_1, ρ_2) variables, so the true plots and tables of the Asymptotic Bias for (σ_x, σ_z) are the same as those in the case where Asymptotic Bias is voiced in terms of (ρ_1, ρ_2) :



Case where: { $Asbias = \varphi(\sigma_x, \sigma_z)$ *and* $\Pi_1 = 1$ *and* $\sigma_{ii} = 1$ *where* $i = 1, 3$. *The* $\max Asbias = 1$. *The* (σ_x, σ_z) *are dependent variables* }

Comments on the Theoretical behavior of the Asymptotic Bias of the IV estimator. Having the previous in mind and examining the tables 1 and 2 { or the other tables in Appendix (B) }, we observe that the more volatile instrument gives the smaller Asymptotic Bias. So, our general theoretical result is that in the selection of an instrument, we must to have in mind the correlation coefficient AND the variance of the instrument : the bigger the correlation coefficient and the variance of the instrument, the smaller the bias of the IV estimator and the better for the researcher.

7.3. Theoretical Analysis of the Asymptotic Variance of the IV estimator (in case $\sigma_{13} = 0$). As we mentioned in previous section, the function which describes the asymptotic variance of the IV estimator in the case where $\sigma_{13} = 0$, when the correlation coefficient is kept fixed, is the following:

$$\begin{aligned} AsVar(\widehat{\vartheta_{iv}}) &= \frac{1}{T}\frac{\sigma_{11}}{\sigma_{22}\rho_{x,z}^2}(1-\rho_1^2) = \Pi_2(1-\rho_1^2) = f(\rho_1) \\ with\ \Pi_2 &= \frac{1}{T}\frac{\sigma_{11}}{\sigma_{22}\rho_{x,z}^2} > 0 \end{aligned} \tag{27}$$

We can see that the Asymptotic Variance is an one-dimensional function and the only variable is the persistence of the regressor ρ_1. Again, we assume as fixed the coefficients $\sigma_{11}, \sigma_{22}, \rho_{x,z}$; we can see that the Asymptotic Variance is independent of the degree of persistence ρ_2 and inversely proportional to the square of the degree of the correlation between the regressor and the instrument. The bigger the correlation between x_t and z_t series, the smaller the Asymptotic Variance of the IV estimator and of course, the better for the researcher.

Finding the extrema of the function AsymptoticVar.. It is no difficult to see that the specific function is a decreasing function for ρ_1. The first and second derivatives are:

$$\begin{aligned} f_{\rho_1} &= -2\Pi_2\rho_1 \\ f_{\rho_1\rho_1} &= -2\Pi_2 < 0 \end{aligned}$$

The point of stasis Z_o is where $f_{\rho_1} = 0 \Longrightarrow \rho_1 = 0$. Because of the fact that $f_{\rho_1\rho_1}|_o < 0$, the point Z_o is a maximum point ; as $\rho_1 \to 1$ the Asymptotic Variance declines to zero.

Case where innovation variances are not known. In the case where the innovation variances are not known, equation (27) can not be used. Substituting equation (15) to the Asymptotic variance equation, we finally get:

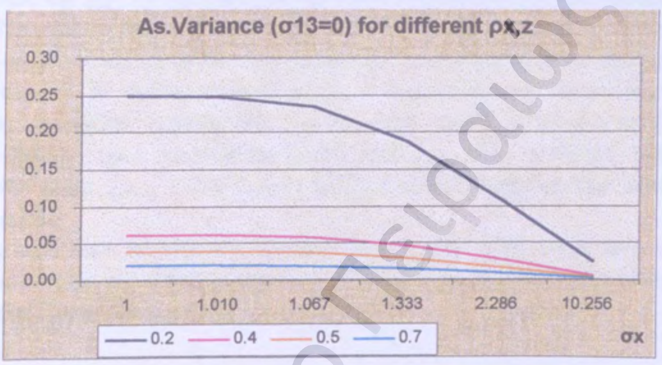
$$\begin{aligned} AsVar(\widehat{\vartheta_{iv}}) &= \frac{\sigma_{11}}{T}\frac{1}{\rho_{x,z}^2}\frac{1}{\sigma_x} = \Pi_3\frac{1}{\sigma_x} = \varphi(\sigma_x) \\ and\ \Pi_3 &= \frac{\sigma_{11}}{T}\frac{1}{\rho_{x,z}^2} > 0 \end{aligned} \tag{28}$$

From the previous equation we see that the Asymptotic Variance is independent of the variance of the instrument z. This means that in the case where we have to choose between two instruments, the Asymptotic Variance of the IV estimator is indifferent of their variances. Because of the fact that the $\rho_{x,z}$ is inversely proportional with the Asymptotic Variance, the most correlated instrument should be selected.

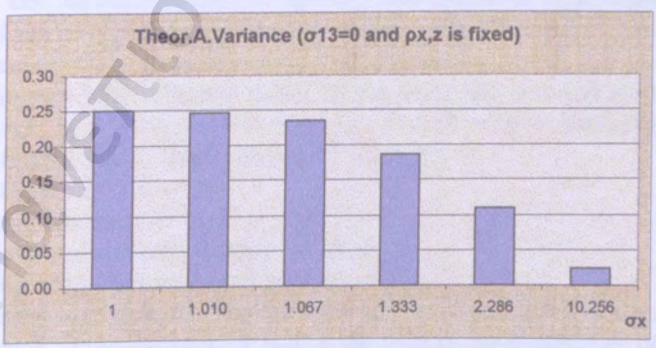
Furthermore, the Asymptotic Variance is inversely proportional to the variance of the regressor: the more volatile the regressor (or $\rho_1 \rightarrow \pm 1$) the smaller the Asymptotic Variance for the IV estimator. As in the case of the Asymptotic Bias of the IV estimator, we can prove that:

$$\max AsVar(\widehat{\vartheta}_{iv}) = \max \varphi(\sigma_x) = \max f(\rho_1) = \frac{1}{T}\frac{\sigma_{11}}{\sigma_{22}\rho_{x,z}^2} \tag{29}$$

Graphs of the Asymptotic Variance.



Case where: { $AsVar = \varphi(\sigma_x)$. $\rho_{x,z} = 0.2, 0.4, 0.5, 0.7$, $n = 100$ *and* $\sigma_{ii} = 1$ *where* $i = 1, 3$. *The* σ_x *is a dependent variable* }



Case where: $\{ AsVar = \varphi(\sigma_x)$. $\rho_{x,z} = 0.2$, $n = 100$ *and* $\sigma_{ii} = 1$ *where* $i = 1, 3$. *The* max $AsVar = 0.25$. *The* σ_x *is a dependent variable* $\}$

Comments on the Theoretical behavior of the Asymptotic Variance of the IV estimator. With a first look in the previous graphs and in the tables -who are located in the Appendix (B)- we observe the inversely proportional relationship of the Asymptotic Variance of the IV estimator with the degree of correlation between the regressor and the instrument. For example, comparing the tables for correlation : $\rho_{x,z} = 0.2$ and for $\rho_{x,z} = 0.4$, we see that the Asymptotic Variance for the $\rho_1 = 0.5$ case, is 0.18750 and 0.04688 respectively. Moreover, we conclude to an another result : the degree of persistence -or the variance- of the regressor, is an important measure for the behavior of the asymptotic variance of our estimator. In some circumstances, we can achieve a smaller Asymptotic Variance for the IV estimator in the case where we have a relative small correlation coefficient $\rho_{x,z}$. For example, the Asymptotic Variance in the case: $\{\rho_{x,z} = 0.4$, $\rho_1 = 0.75\}$ is 0.02734 and in the case $\{\rho_{x,z} = 0.5$, $\rho_1 = 0.5\}$ is 0.03.

Of course all these results are totally theoretical. The next step is to examine whether these results are valid in practice. We will try to apply our theory in practice through Monte Carlo simulations.

8. DESCRIPTION OF THE MONTE CARLO APPROACH

Before we proceed to the analysis of the Monte Carlo results, we believe that we have to describe first our Monte Carlo approach. As it was mentioned initially, when we run the program, we are asked to give some inputs; that are the size of the sample, the number of configurations, the definition of the persistence of the regressor, the definition of the variance-covariance matrix, the degree of correlation between the regressor and the instrument and finally the parameter of interest. The program gives estimates (and of course Bias estimates, MSE estimates, Absolute Bias estimates and Variance estimates) for different values of the persistence of the instrument, ρ_2.To be more specific, ρ_2 is taking values from zero to 0,95 with an increasing step of 0,1.

The inputs that where used, where the following:

1.Size of the sample, $n = 100$

2.Number of configurations, $T = 1000$

3.Specification of the parameters, $(\rho_1, \Sigma, \rho_{x,z})$.

4.Definition of the parameter of interest, $\vartheta = 1$.

Our intention is to examine the behavior of the IV estimator not only in the case of the persistence of the instrument, but in the case of the persistence of the regressor as well. For that reason we ran simulations for different values of ρ_1. More specifically, ρ_1 took values from the set: $(0, 0.1, 0.25, 0.5, 0.75, 0.95)$.

As for the variance covariance matrix, for simplicity we assumed that the elements $\sigma_{11}, \sigma_{22}, \sigma_{33}$, should be equal to one. Moreover, we thought that an IV estimator is applied when the OLS has problems; for that reason we defined as $\sigma_{12} = 0.5$. From the analysis that we presented initially, in the case where $\sigma_{13} = 0$, the IV estimator is unbiased and consistent. Examining the Bias of the IV estimator, the $\sigma_{13} = 0$ case, is not the subject that we want to study, so we ran simulations for different values of $\sigma_{13} \neq 0$, assuming the set $(0.1, 0.2, 0.4)$. Reversely, in order to study the behavior of the Variance, we ran simulations only for the $\sigma_{13} = 0$ case.

Finally, we defined the following set for the correlation coefficient : $(0.2, 0.4, 0.5, 0.7)$

Substantially, our simulations followed the following path:

$$\begin{aligned} & \textit{definition of } \sigma_{13} \\ \sigma_{13} &= (0.1, 0.2, 0.4) \text{ and } \sigma_{13} = 0 \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} & \textit{definition of } \rho_{x,z} \\ \rho_{x,z} &= (0.2, 0.4, 0.5, 0.7) \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} & \textit{definition of } \rho_1 \\ \rho_1 &= (0, 0.1, 0.25, 0.5, 0.75, 0.95) \end{aligned}$$

9. Monte Carlo Analysis

The tables who contain the results of our Monte Carlo simulations, are located in Appendix (B). The form of these tables is the same as in the form of the theoretical results. First, we will present the results of the bias of IV estimator which comprises the main body of this survey.

9.1. Monte Carlo results for the bias of the IV estimator. Analyzing the tables for the Bias of the IV estimator, we observe that the specific results confirm our theory. The Monte Carlo Bias of the IV estimator presents a behavior similar to the theoretical behavior which was presented in the Theoretical Analysis's section. More specifically, from the inspection of the presented tables we observe the inversely proportional relationship of the true Bias with the degree of correlation.. For example, the MC bias for the case { $\sigma_{13} = 0.1$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.5$ and $\rho_2 = 0.5$ } is 0.180567, while for the case { $\sigma_{13} = 0.1$, $\rho_{x.z} = 0.5$, $\rho_1 = 0.5$ and $\rho_2 = 0.5$ } is 0.114522 ; which is an already known result from previous research. Another result which was expected, is the direct proportional relationship of the bias of the IV estimator with the covariance coefficient (σ_{13}) : The MC bias for the case { $\sigma_{13} = 0.1$, $\rho_{x.z} = 0.5$, $\rho_1 = 0.75$ and $\rho_2 = 0.7$ } is 0.050689 while in the case { $\sigma_{13} = 0.4$, $\rho_{x.z} = 0.5$, $\rho_1 = 0.75$ and $\rho_2 = 0.7$ } is 0.247366, which is much bigger than the previous case, as it was expected.

As we mentioned in the last paragraph the reported results were a bit expected, the main subject to discuss is whether the MC behavior is similar with the Theoretical behavior of the Bias as concerned the degree of persistence -or the variance- of the regressor and the instrument. Examining the whole tables of the MC bias, we confirm the inversely proportional relationship of the Bias of our estimator with the degree of persistence -or the variance- of the x_t and z_t series. Indeed, the MC bias declines as ρ_1 or/and ρ_2 go to unity which is equivalent to say that the MC bias declines as σ_x or/and σ_z go to their maximum value. The MC results confirm the whole theoretical analysis for the bias of the IV estimator which was presented in previous sections[4]. In any case, the biggest values of the Bias are located when ρ_1 and ρ_2 are close to zero (or equivalent: when σ_x close to σ_{22} and σ_z close to σ_{33}), while the minimum value is located when $\rho_1 = \rho_2 = 0.95$ (or equivalent: when $\sigma_x = \sigma_z = 10.256$). For example, the MC bias for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.1$ and $\rho_2 = 0$ }is 1.05325, while for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.95$ and $\rho_2 = 0.95$ }is 0.017014. Conclusively, we confirm that the more volatile the instrument or/and the regressor, the smaller the bias for the IV estimator and of course the better for the researcher.

Furthermore, we can confirm another result: we can actually achieve smaller Bias for our estimator depending the degree of variance, even in a case where we have a small correlation.. For example in the case where we have { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.75$ and $\rho_2 = 0.95$ } the true bias is 0.1, while for the case where { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.4$, $\rho_1 = 0.5.$ and $\rho_2 = 0.5$ }the true bias is 0.27432 ; which proves the fact that the correlation coefficient can not be the only measure for selecting a good

[4] In any case the True Bias behaves exactly the same as the Theoretical one. Only in extremely few cases where $\rho_1 = 0$, we observe some problems in behavior of the Bias.

instrument.

Another important result is that the divergence percentage between the Theoretical and the MC bias, is greater in the region where ρ_1 is very close to one and ρ_2 is close to $(0.7 - 0.8)$. To be more specific, we calculated the degree of the divergence percentage between the Theoretical and the MC bias and we found that this degree is bigger in the region that was just reported. For example, the degree for the case{ $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.95$ and $\rho_2 = 0.95$ }is

$$\frac{MCbias - Theor.bias}{Theor.bias}100\% = \frac{0.017014 - 0.0975}{0.0975}100\% = -82.55\%,$$

while for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.95$ and $\rho_2 = 0.1$ } is

$$\frac{0.174516 - 0.31068}{0.31068}100\% = -43.83\%$$

but for the case $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.5$ and $\rho_2 = 0.95$ }is

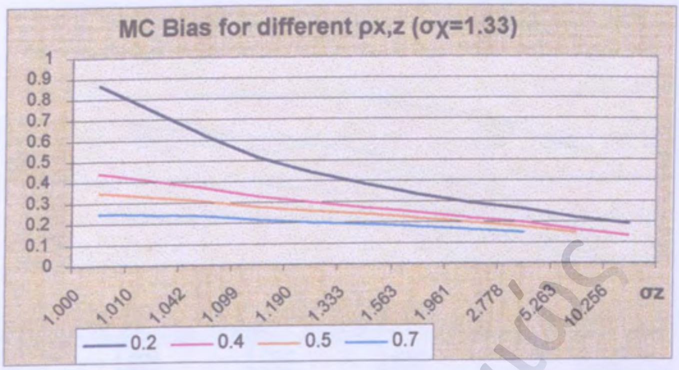
$$\frac{0.191097 - 0.270416}{0.270416} = -29.33\%$$

and finally for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.2$, $\rho_1 = 0.1$ and $\rho_2 = 0.1$ }is

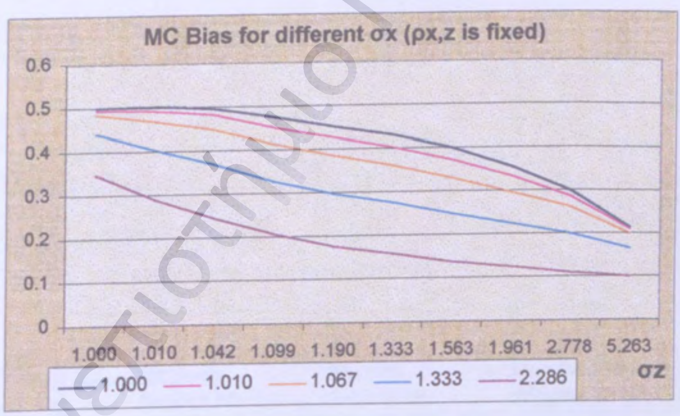
$$\frac{1.01146 - 0.99}{0.99}100\% = +2.17\%$$

When mainly the degree of persistence -or the variance- of the regressor , and secondly the degree of persistence -or the variance- of the instrument are close enough to the previous reported values, the MC Bias is much smaller from its Theoretical bias than in the case where the degree of persistence of these series takes small values from its definition field. In other words, the real Bias of the IV estimator in the case where we have to deal with a persistent instrument and a highly persistent regressor, is much smaller than the theoretical value that we suppose to have according to our theory.

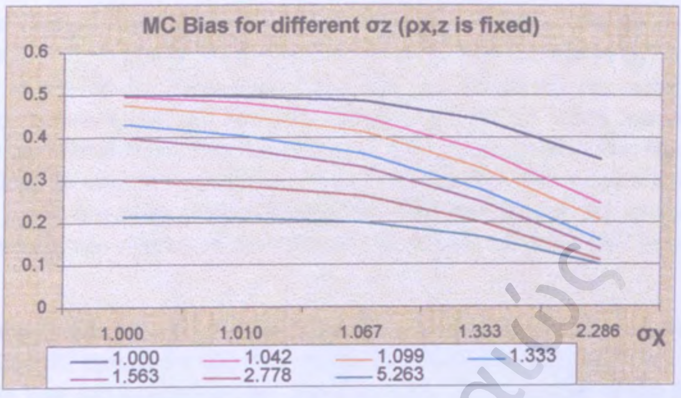
Graphs of the Monte Carlo results for the Bias of the IV estimator. In order to present the previous conclusions, we will apply some graphs who include some of the MC results. The first graph presents the inversely proportional relationship of the MC Bias with the degree of correlation.. The next two graphs confirm the inversely proportional relationship of the MC Bias with the variance of the instrument and also with the variance of the regressor. Finally, the last graph compares the Theoretical values with the respectively MC values of the Bias for a specific case.



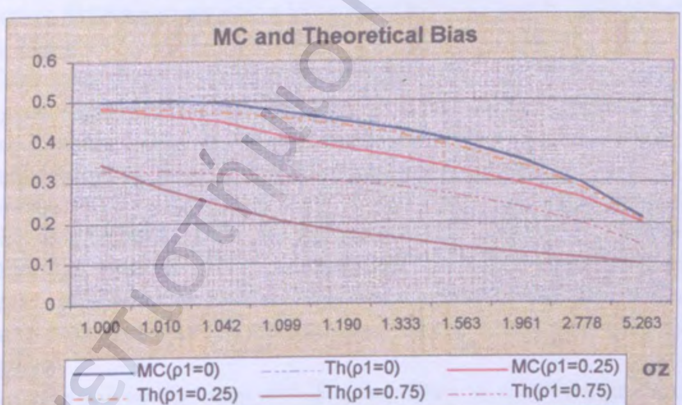
Case where: { $MCbias = function\ of\ \sigma_z$, $\rho_{x,z} = (0.2, 0.4, 0.5, 0.7)$, $\rho_1 = 0.5$ or $\sigma_x = 1.33$ $and\ \sigma_{13} = 0.1$}



Case where: { $MCbias = function\ of\ \sigma_z$, $\rho_{x,z} = 0.4$, $\sigma_{13} = 0.2$ $and\ \sigma_{ii} = 1\ where$ $i = 1, 3$. $The\ (\sigma_x, \sigma_z)\ are\ dependent\ variables$ }



Case where: { $MCbias = function\ of\ \sigma_x\ ,\ \rho_{x,z} = 0.4,\ \sigma_{13} = 0.2\ \ and\ \sigma_{ii} = 1\ where$ $i = 1, 3.\ The\ (\sigma_x, \sigma_z)\ are\ dependent\ variables$ }



Case where: { $MCbias = \varphi(\sigma_x, \sigma_z)\ ,\ \rho_{x,z} = 0.4,\ \sigma_{13} = 0.2\ \ and\ \sigma_{ii} = 1\ where$ $i = 1, 3.\ MC = Monte\ Carlo\ results,\ Th = Theoretical\ results$}

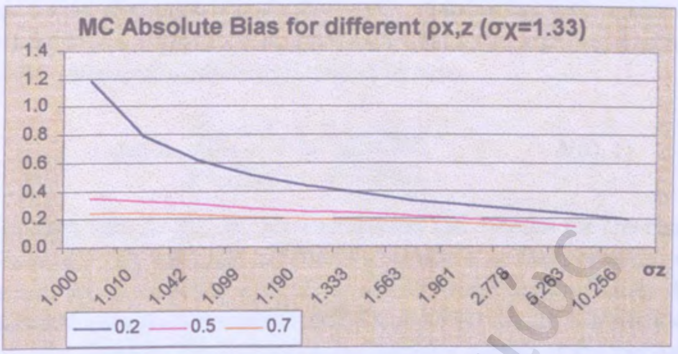
From the second and third graph, we can observe the decreasing properties of the Bias of the IV estimator. As the variance of the instrument or the the variance of the regressor increases, the true bias declines to zero. We can see that the maximum value of the bias is located in the region where $\rho_1 = \rho_2 = 0$ (or $\sigma_x = \sigma_{22} = 1$ and $\sigma_z = \sigma_{33} = 1$), while the minimum value is located where $\rho_1 = \rho_2 = 0.95$ (or $\sigma_x = \sigma_z = 10.256$) . Moreover, from the last graph we can notice the degree of divergence between the MC and the Theoretical results: when $\rho_1$and ρ_2 are close to zero, the MC results are very close to the Theoretical results for the bias. When ρ_1 is very close to one and ρ_2 is close to $(0.7 - 0.8)$, the MC results behave like the Theoretical ones, but they are much smaller. The last proves the previous reported result concerning the degree of divergence percentage between MC and Theoretical results.

9.2. Monte Carlo results for the Absolute bias of the IV estimator. As in the case of the MC results for the Bias of the IV estimator, the tables who present the MC Absolute bias are located in the Appendix (B). The behavior of the MC Absolute Bias is almost the same like the behavior of the MC bias. The true Absolute Bias is inversely proportional to the degree of correlation ; as $\rho_{x,z}$ tends to unity the MC Absolute Bias decreases. Comparing the values of the MC Absolute Bias for different values of correlation we can confirm the previous result.

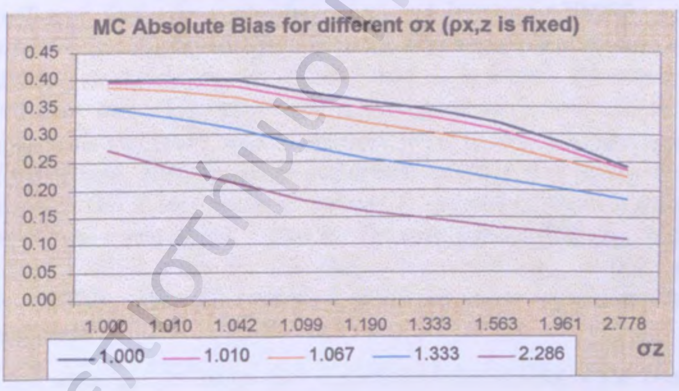
Moreover, the Absolute Bias declines to zero when ρ_1 or/and $\rho_2 \rightarrow 1$. The MC Absolute Bias is little more sensitive than the MC bias and it presents slightly bigger problems in behavior than the MC Bias in some cases where $\rho_1 = 0$; but as in the case of the MC bias, these problems are extremely few[5].As in the analysis of the MC Bias, the biggest values of the MC Absolute Bias are located when ρ_1 and ρ_2 are close to zero (or equivalent: when σ_x close to σ_{22} and σ_z close to σ_{33}), while the minimum value is located when $\rho_1 = \rho_2 = 0.95$ (or equivalent: when $\sigma_x = \sigma_z = 10.256$). For example, for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.5$, $\rho_1 = 0.1$ and $\rho_2 = 0.1$ }the MC Absolute Bias is 0.396165, while for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.5$, $\rho_1 = 0.95$ and $\rho_2 = 0.95$ }is 0.032780. Conclusively, the behavior of the MC Absolute Bias strengthen our opinion which endorses that the degree of variance of the instrument compared with the degree of correlation, can be the main measures for selecting a good instrument.

Before we proceed to the analysis of some other measures that describe the behavior of the IV estimator, we will present some graphs for the MC absolute Bias of our estimator.

[5]The specific problems could be small sample effects



Case where: { $MC\ Absbias = function\ of\ \sigma_z$, $\rho_{x,z} = (0.2, 0.5, 0.7), \sigma_x = 1.33,\ \sigma_{13} = 0.2\ \ and\ \sigma_{ii} = 1\ where\ i = 1,3$ }.



Case where: { $MC\ Absbias = function\ of\ \sigma_z$, $\rho_{x,z} = 0.5,\ \sigma_{13} = 0.2\ \ and\ \sigma_{ii} = 1$ $where\ i = 1,3.$ }.

9.3. Monte Carlo results for the T-statistic of the IV estimator. In this section we will examine the behavior of MC results for the T-statistic of the IV estimator. More specifically, we will investigate the behavior of the mean of T-stat and the behavior of the skewness and kurtosis coefficients. From Statistical Theory, a well-behaved T-stat is asymptotically Normally distributed with mean equal to zero and standard deviation equal to one; we remind that the kurtosis coefficient is equal to three and the skewness coefficient is equal to zero.

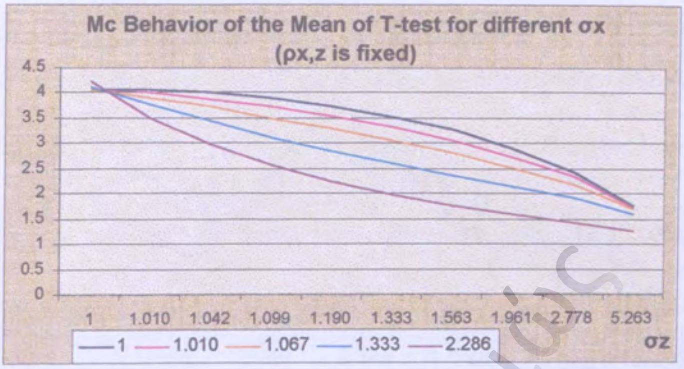
$$T - stat = \frac{\widehat{\vartheta}_{iv} - \vartheta}{\widehat{s.d.}(\widehat{\vartheta}_{iv})} = \frac{bias(\widehat{\vartheta}_{iv})}{\widehat{s.d.}(\widehat{\vartheta}_{iv})} \quad \sim \quad N(0,1)$$

We also remind that so far, we have examined MC results for the case of $\sigma_{13} \neq 0$, in order to examine the behavior of the IV estimator in the occasion where the IV is not consistent. Because of the fact that the orthogonality condition does not hold and the size of the sample is relative small ($n = 100$), the distribution of the T-stat can not be exactly Normal. We will examine how the degree of the persistence -or the variance- of the instrument and the regressor affects the behavior of the T-stat. In the Appendix (B) are presented some the MC tables with the results for the T-stat

MC results for the Mean of the T-statistic. Analyzing the tables who present the MC results for the Mean of the T-stat, we conclude that the degree of the persistence -or the variance- of the instrument and the regressor too, affect the behavior the T-stat. Indeed, we see that as $\rho_2 \to 1$ (or $\sigma_z \to 10.256$) and ρ_1 is fixed, the mean value of the T-stat declines to zero. The same phenomenon is observed in the case where $\rho_1 \to 1$ (or $\sigma_x \to 10.256$) and ρ_2 is fixed[6]. Moreover, we confirm another result that was a bit expected : the increase of the covariance coefficient of the instrument with the basic regression error (σ_{13}) causes a bigger misallocation in the mean of the T-test. But, the most important result is that in any case the degree of misallocation of the T-test tends to zero as ρ_1 or/and ρ_2 tend to unity.

For example, the mean of the T-stat for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.4$, $\rho_1 = 0.1$ and $\rho_2 = 0.1$ }is 1.9812, while for the case { $\sigma_{13} = 0.2$, $\rho_{x.z} = 0.4$, $\rho_1 = 0.5$ and $\rho_2 = 0.5$ }is 1.26702. We will present a graph in order to understand better the previous conclusion :

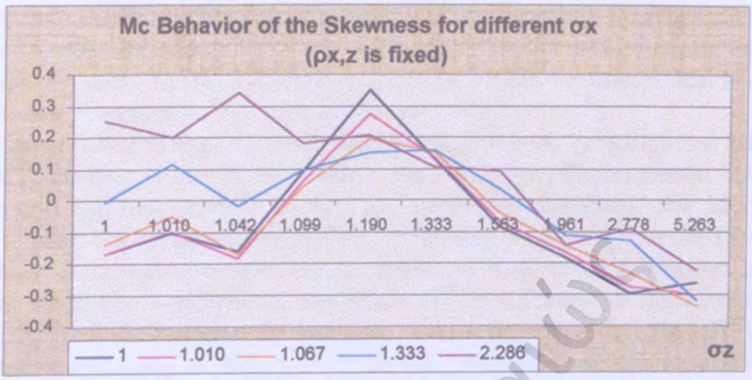
[6]exept the case where $\rho_2 = 0$, where have a problem in the behavior of the T-test probably by reason of the small sample.



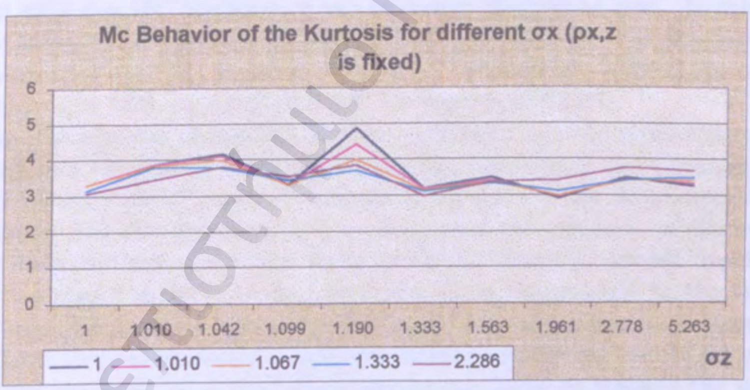
Case where: { $MC\ MeanT - test = function\ of\ \sigma_z$, $\rho_{x,z} = 0.4,\ \sigma_{13} = 0.4,$
$n = 100\ and\ \sigma_{ii} = 1\ where\ i = 1, 3$ }.

MC results for the Kurtosis and the Skewness coefficients. Examining the tables who present the MC results for the Kurtosis and the skewness coefficients, we can not observe any pattern in the behavior of these coefficients as we did in previous cases. In any case, the kurtosis coefficient is close to its appropriate value (the number three), but unfortunately it is not converging to this value as ρ_1 or/and ρ_2 tend to unity. Also, we observe the same behavior for the Skewness coefficient, which is close to zero but shows no convergency to this value. One reason for this phenomenon could be the small sample which we have used in our survey. Maybe a larger sample would weed out these problems and drive the reported coefficients to the convergency to their appropriate values.

Before we proceed to the next part of our study, which is the presentation of the MC results concerning the Variance of the IV estimator, we will present two graphs that describe the erratic behavior of the Skewness and Kurtosis coefficients.



Case where: { $MC\ a3T - test = function\ of\ \sigma_z\ ,\ \rho_{x,z} = 0.4,\ \sigma_{13} = 0.2,$
$n = 100\ and\ \sigma_{ii} = 1\ where\ i = 1, 3$ }.



Case where: { $MC\ a4T - test = function\ of\ \sigma_z\ ,\ \rho_{x,z} = 0.4,\ \sigma_{13} = 0.2,$
$n = 100\ and\ \sigma_{ii} = 1\ where\ i = 1, 3$ }.

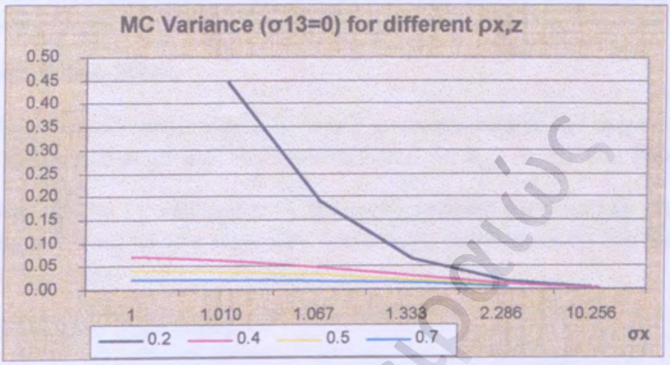
9.4. Monte Carlo results for the Variance in case where $\sigma_{13} = 0.$. The last part of our study is the analysis of the MC results for the Variance of our estimator. We remind to the reader that in this section we will analyze only the case where the covariance coefficient (σ_{13}) is equal to zero, because the forms of the variance-covariance matrix and the Asymptotic Variance of the IV estimator are known only for this specific case. In the Appendix (B) are presented the theoretical and the MC results for the Asymptotic Variance in the case where $\sigma_{13} = 0$.

Examining these tables, we first observe the inversely proportional relationship of the MC Variance of the IV estimator with the degree of correlation between the regressor and the instrument. As the $\rho_{x,z}$ coefficient increases, the MC Variance declines to zero. As we mentioned before, in the case where $\sigma_{13} = 0$ the IV estimator is consistent, but a strong correlation $\rho_{x,z}$ leads to better estimates for our parameter of interest since the it minimizes the value of the Variance of the estimator. For example, the MC Variance for the case { $\rho_{x,z} = 0.4$, $\rho_1 = \rho_2 = 0.5$ }is 0.029107, while in the case where { $\rho_{x,z} = 0.5$, $\rho_1 = \rho_2 = 0.5$ }the MC Variance is 0.0217305 and finally for the case { $\rho_{x,z} = 0.7$, $\rho_1 = \rho_2 = 0.5$ }is 0.013577.

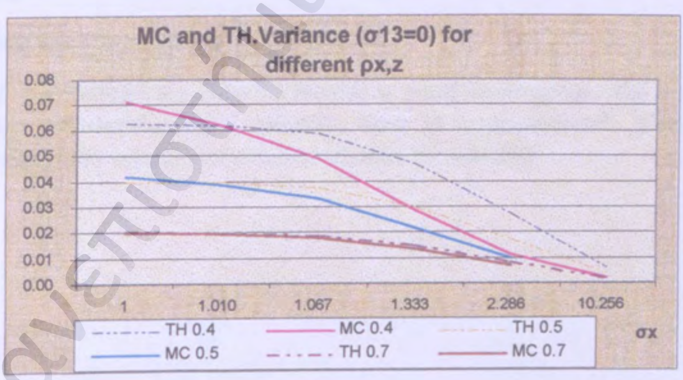
Another conclusion is the decreasing property that presents the true Variance towards the variance of the regressor.Analyzing the MC tables we can confirm the result which was presented in the Theoretical part of this study concerning the Variance of the IV estimator . As the variance of the regressor tends to its maximum value ($\sigma_x = 10.256$), the MC Variance of our estimator declines to zero. In other words, the more volatile the regressor the smaller the true Variance. For example, the MC Variance for the case { $\rho_{x,z} = 0.2$, $\rho_1 = \rho_2 = 0.5$ }is 0.065699, while in the case where { $\rho_{x,z} = 0.2$, $\rho_1 = 0.75$, $\rho_2 = 0.5$ }the MC Variance is 0.018511.

Ending the analysis of the MC Variance, we will report to a phenomenon which is contrary to a theoretical result concerning the behavior of the Variance of the IV estimator. During the Theoretical analysis of the form of the Asymptotic Variance, we concluded that the degree of the persistence -or the variance- of the instrument does not affect the behavior of the Variance. Unfortunatelly, our MC results do not confirm this result, in fact the MC Variance shows dependance to the variance of the instrument. A possible reason for this behavior is again the small sample which we have used for the estimation procedure. Maybe, an increase of the sample would result to the convergency of the MC results for the Variance towards the variance of the instrument to their respective Theoretical values.

Graphs of the Monte Carlo results for the Variance of the IV estimator. In order to understand better the previous conclusions, we will aply some graphs. The first graph confirms the inversely proportional relationship of the MC Variance of the IV estimator with the degree of correlation $\rho_{x,z}$; it also presents the inversely proportional relationship of the MC Variance with the variance of the regressor. The second graph compares the Theoretical values of the Variance (that are indifferent of ρ_2 or σ_x) for different $\rho_{x,z}$, with the respective MC results (for $\rho_2 = 0.5$ or $\sigma_z = 1.33$). Studying the second plot and the tables in Appendix (B), we can reach to a last conclusion : as the correlation coefficient increases, the MC results for the Variance are getting more closer to their respective Theoretical, even in the case where we use a small sample.



Case where: { $MCVar = \varphi(\sigma_x)$. $\rho_{x,z} = (0.2, 0.4, 0.5, 0.7)$, $n = 100$, $\rho_2 = 0.5$ *and* $\sigma_{ii} = 1$ *where* $i = 1, 3$. *The* σ_x *is a dependent variable* }



Case where: { $\rho_{x,z} = (0.4, 0.5, 0.7)$, $\rho_2 = 0.5$ *and* $\sigma_{ii} = 1$ *where* $i = 1, 3$.
$MC = Monte\ Carlo\ results,\ TH = Theoretical\ results$}

10. CONCLUSIONS

In this section, we will try to summarize the main conclusions concerning the behavior of the IV estimator. We will separate these results to the conclusions concerning the behavior of the Bias and to the conclusions concerning the behavior of the Variance (in the case $\sigma_{13} = 0$).

Behavior of the Bias.

- The Bias of the IV estimator is inversely proportional to the degree of correlation between the regressor and the instrument : As $\rho_{x,z} \to \pm 1$ the $Bias(\widehat{\vartheta}_{iv}) \to 0$.
- The Bias of the IV estimator is inversely proportional to the variance of the instrument (and to the regressor too). The more volatile the instrument the smaller the Bias.

Behavior of the Variance.

- The Variance of the IV estimator is inversely proportional to the degree of correlation between the regressor and the instrument : As $\rho_{x,z} \to \pm 1$ the $Variance(\widehat{\vartheta}_{iv}) \to 0$.
- The Variance of the IV estimator is inversely proportional to the variance of the regressor. The more volatile the regressor the smaller the Variance.

Selecting an Instrument.

- In the case where we have to choose a good instrument, the correlation coefficient and the degree of volatility have to be considered. The most highly correlated and volatile instrument should be selected.

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11. APPENDIX (A)

- Proof of equation (5): $x_t = \sum_0^\infty \rho_1^j u_{1,t-j}$

We know that,

$$\left.\begin{aligned} x_t &= \rho_1 x_{t-1} + u_{2t} \\ x_{t-1} &= \rho_1 x_{t-2} + u_{2t-1} \end{aligned}\right\} \Longrightarrow x_t = \rho_1^2 x_{t-2} + \rho_1 u_{2t-1} + u_{2t}$$

$$\left.\begin{aligned} x_t &= \rho_1^2 x_{t-2} + \rho_1 u_{2t-1} + u_{2t} \\ x_{t-2} &= \rho_1 x_{t-3} + u_{2t-2} \end{aligned}\right\} \Longrightarrow x_t = \rho_1^3 x_{t-3} + \rho_1^2 u_{2t-2} + \rho_1 u_{2t-1} + u_{2t}$$

after k-1 substitutions,we have,

$$x_t = \rho_1^k x_{t-k} + \sum_{j=0}^{k-1} \rho_1^j u_{2t-i}$$

For k=t , we have,

$$x_t = \rho_1^t x_0 + \sum_{j=0}^{t-1} \rho_1^j u_{2t-i}$$

and assuming that $t \to \infty$, we finally get:

$$x_t = \sum_{j=0}^{\infty} \rho_1^j u_{2t-i}$$

- Proof of equation (7): $Cov(u_{1,t+i}x_t) = \sigma_{12}$

$$\begin{aligned} Cov(u_{1,t+i}, x_t) &= E\left(u_{1,t+i}x_t\right) - E(u_{1,t+i})E(x_t) \\ &= E\left(u_{1,t+i}x_t\right) = E\left(u_{1,t+i}\sum_{j=0}^{\infty}\rho_1^j u_{2t-i}\right), \Upsilon i, j \geq 0. \end{aligned}$$

the errors $u_1 and\ u_2$ are only contemporaneous dependent, so the previous equation can be written as,

$$E\left(u_{1,t+i}\sum_{j=0}^{\infty}\rho_1^j u_{2t-i}\right) = E\left(u_{1,t}u_{2t}\right) = \sigma_{12}$$

- Proof of equation (8): $p\lim \frac{x^\intercal u_1}{T} = \sigma_{12}$

  Using the definition of the probabilistic limit we assume the following,

$$\begin{aligned}
\lim P\{|\frac{x^\intercal u_1}{T} - \sigma_{12}| &< \varepsilon\} = 1 \Longrightarrow \\
\lim P\{\sigma_{12} - \varepsilon &< \frac{x^\intercal u_1}{T} < \varepsilon + \sigma_{12}\} = 1 \Longrightarrow \\
E\{\lim P\{\sigma_{12} - \varepsilon &< \frac{x^\intercal u_1}{T} < \varepsilon + \sigma_{12}\}\} = E\{1\} \Longrightarrow \\
\lim E\{P\{\sigma_{12} - \varepsilon &< \frac{x^\intercal u_1}{T} < \varepsilon + \sigma_{12}\}\} = 1 \Longrightarrow \\
\lim P\{E\{\sigma_{12} - \varepsilon &< \frac{x^\intercal u_1}{T} < \varepsilon + \sigma_{12}\}\} = 1 \Longrightarrow \\
\lim P\{\sigma_{12} - \varepsilon &< E\{\frac{x^\intercal u_1}{T}\} < \varepsilon + \sigma_{12}\} = 1
\end{aligned}$$

in this point, we will prove that $E\{\frac{x^\intercal u_1}{T}\} = \sigma_{12}$:

$$E\{\frac{x^\intercal u_1}{T}\} = E\{\frac{(x_{1,..,}x_T)(u_{11,...,}u_{1T})^T}{T}\} = E\{\frac{(\sum_0^1 \rho_1^j u_{2t-i}, ..., \sum_0^T \rho_1^j u_{2t-i})(u_{11,...,}u_{1T})^T}{T}\}$$

again the $u_1 and\ u_2$ are only contemporaneous dependent, so we finally get,

$$\begin{aligned}
E\{\frac{(\sum_0^1 \rho_1^j u_{2t-i}, ..., \sum_0^T \rho_1^j u_{2t-i})(u_{11,...,}u_{1T})^T}{T}\} &= E\{\frac{u_{11}u_{21} + ... + u_{1T}u_{2T}}{T}\} \\
&= \frac{T\sigma_{12}}{T} = \sigma_{12}
\end{aligned}$$

so substituting the previous result to

$$\begin{aligned}
\lim P\{\sigma_{12} - \varepsilon &< E\{\frac{x^\intercal u_1}{T}\} < \varepsilon + \sigma_{12}\} = 1 \Longrightarrow \\
\lim P\{\sigma_{12} - \varepsilon &< \sigma_{12} < \varepsilon + \sigma_{12}\} = 1 \Longrightarrow \\
\lim P\{-\varepsilon &< 0 < \varepsilon\} = 1 \Longrightarrow \\
\lim 1 &= 1 \Longrightarrow 1 = 1 \\
&\textit{which is true}
\end{aligned}$$

- Proof of equation (10): $p\lim \widehat{\vartheta}_{LS} = \vartheta + \frac{\sigma_{12}}{\sigma_{22}}(1-\rho_1^2)$

  We know that,

$$\begin{aligned} p\lim \widehat{\vartheta}_{LS} &= p\lim\{(x^{\intercal}x)^{-1}x^{\intercal}y\} = p\lim\{(x^{\intercal}x)^{-1}x^{\intercal}(\vartheta x + u_1)\} \\ &= p\lim\{(x^{\intercal}x)^{-1}x^{\intercal}x\ \vartheta + (x^{\intercal}x)^{-1}x^{\intercal}u_1)\} \\ &= p\lim\{\vartheta + (x^{\intercal}x)^{-1}x^{\intercal}u_1\} \\ &= p\lim\{\vartheta + \frac{u_1}{x}\} \end{aligned}$$

  dividing equations (8) and (9) we get,

$$p\lim\{\vartheta + \frac{u_1}{x}\} = \frac{\sigma_{12}}{\sigma_{22}}(1-\rho_1^2)$$

  which is the result that we want.

- Proof of equation (11): $Cov(z_t, u_{1,t+i}) = \sigma_{13}$

  As in the OLS case,

$$Cov(z_t, u_{1,t+i}) = E(z_t u_{1,t+i}) = E(\sum_0^{\infty} \rho_2^j u_{3,t-j}\ u_{1,t+i})\ , \Upsilon i, j \geq 0.$$

  the errors $u_1 and$ u_3 are only contemporaneous dependent, so we can conclude that,

$$E(\sum^{\infty} \rho_2^j u_{3,t-j}\ u_{1,t+i}) = E(u_{3,t}u_{1,t}) = \sigma_{13}$$

- Proof of equation (12): $p\lim \frac{z^{\intercal}u_1}{T} = \sigma_{13}$

  It similar with the OLS case,

$$\begin{aligned} \lim P\{|\frac{z^{\intercal}u_1}{T} - \sigma_{13}| &< \varepsilon\} = 1 \Longrightarrow \\ \lim P\{\sigma_{13} - \varepsilon &< \frac{z^{\intercal}u_1}{T} < \varepsilon + \sigma_{13}\} = 1 \Longrightarrow \\ E\{\lim P\{\sigma_{13} - \varepsilon &< \frac{z^{\intercal}u_1}{T} < \varepsilon + \sigma_{13}\}\} = E\{1\} \Longrightarrow \end{aligned}$$

$$\begin{aligned}
\lim E\{P\{\sigma_{13}-\varepsilon &< \frac{z^{\intercal}u_1}{T} < \varepsilon+\sigma_{13}\}\} = 1 \Longrightarrow \\
\lim P\{E\{\sigma_{13}-\varepsilon &< \frac{z^{\intercal}u_1}{T} < \varepsilon+\sigma_{13}\}\} = 1 \Longrightarrow \\
\lim P\{\sigma_{13}-\varepsilon &< E\{\frac{z^{\intercal}u_1}{T}\} < \varepsilon+\sigma_{13}\} = 1 \Longrightarrow \\
\lim P\{\sigma_{13}-\varepsilon &< \sigma_{13} < \varepsilon+\sigma_{13}\} = 1 \Longrightarrow \\
\lim P\{-\varepsilon &< 0 < \varepsilon\} = 1 \Longrightarrow \\
\lim 1 &= 1 \Longrightarrow 1 = 1 \\
& \textit{which is true}
\end{aligned}$$

- Proof of equation (14): $p\lim\left(\widehat{\vartheta}_{IV}-\vartheta\right)=\frac{\sigma_{13}}{\sigma_{23}}(1-\rho_1\rho_2)$

  We know that,

$$\begin{aligned}
p\lim\widehat{\vartheta}_{IV} &= p\lim\{(z^{\intercal}x)^{-1}z^{\intercal}y\} = p\lim\{(z^{\intercal}x)^{-1}z^{\intercal}(\vartheta x+u_1)\} \\
&= p\lim\{(z^{\intercal}x)^{-1}z^{\intercal}x\,\vartheta+(z^{\intercal}x)^{-1}z^{\intercal}u_1)\} \\
&= p\lim\{\vartheta+(z^{\intercal}x)^{-1}z^{\intercal}u_1\} \\
&= p\lim\{\vartheta+\frac{u_1}{x}\}
\end{aligned}$$

  dividing equations (12) and (13) we will get,

$$p\lim\{\vartheta+\frac{u_1}{x}\}=\frac{\sigma_{13}}{\sigma_{23}}(1-\rho_1\rho_2)$$

- Proof of equation (15): $\sigma_x=\frac{\sigma_{22}}{1-\rho_1^2}$

  Using the equation (5),

$$x_t=\sum_0^{\infty}\rho_1^j u_{2,t-j}=u_{2t}+\rho_1 u_{2,t-1}+....+\rho_1^k u_{2,t-k}+....$$

  so taking second moment on the previous equation,

$$\begin{aligned}
Var(x_t) &= \sigma_x = \text{ remember that } u_2 \text{is NIID} \\
&= \sigma_{22}+\rho_1^2\sigma_{22}+....+\rho_1^{2k}\sigma_{22}+.... \\
&= \sigma_{22}(1+\rho_1^2+....+\rho_1^{2k}+...) \\
&= \sigma_{22}\frac{1}{1-\rho_1^2}
\end{aligned}$$

- Proof of equation (17): $\rho_{x,z} = \frac{\sigma_{23}\sqrt{(1-\rho_1^2)(1-\rho_2^2)}}{\sqrt{\sigma_{22}\sigma_{33}}(1-\rho_1\rho_2)}$

  We know that,

$$\rho_{x,z} = \frac{Cov(x,z)}{\sqrt{\sigma_x}\sqrt{\sigma_z}}$$

  where $\sqrt{\sigma_x}\sqrt{\sigma_z}$, can be given from equations (15) and (16). We also know that,

$$\begin{aligned}
Cov(x,z) &= E(xz) - E(x)E(z) \qquad (\text{ assuming that } x_o = z_o = 0) \\
&= E(xz) \\
&= E(\sum_0^\infty \rho_1^j u_{2,t-j} \sum_0^\infty \rho_2^i u_{3,t-i}) \ , \curlyvee i,j \geq 0 \\
&= E\{(u_{2t} + \rho_1 u_{2,t-1} + ... + \rho_1^k u_{2,t-k} + ...)(u_{3t} + \rho_2 u_{3,t-1} + ... + \rho_2^k u_{3,t-k} + ...)\} \\
&= E(u_{2t}u_{3t} + \rho_1\rho_2 u_{2,t-1}u_{3,t-1} + ... + (\rho_1\rho_2)^k u_{2,t-k}u_{3,t-k} + ...) \\
&= \sigma_{23}(1 + \rho_1\rho_2 + ... + (\rho_1\rho_2)^k + ...) \\
&= \sigma_{23}\frac{1}{1-\rho_1\rho_2}
\end{aligned}$$

  Using the previous equations we get the final equation.

- Proof of the equation (21):

  In the case where $\sigma_{13} = 0$, we already know that,

$$\begin{aligned}
Cov(\widehat{\vartheta}_{iv}) &= E\{(\widehat{\vartheta}_{iv} - \vartheta)(\widehat{\vartheta}_{iv} - \vartheta)^T\} \\
&= E\{(z^\intercal x)^{-1} z^\intercal u_1 u_1^T z (z^\intercal x)^{-1}\} \\
&= \sigma_{11}(z^T x)^{-1}(z^T z)(z^T x)^{-1}
\end{aligned}$$

  we know as well that,

$$\begin{aligned}
z^\intercal x &\longrightarrow {}^P \frac{\sigma_{23}}{1-\rho_1\rho_2} \\
z^T z &\longrightarrow {}^P \frac{\sigma_{33}}{1-\rho_2^2} \\
\widehat{\sigma}_{11} &\rightarrow {}^P \sigma_{11}
\end{aligned}$$

using these results we finally conclude to the following equation,

$$AsymptoticVar(\widehat{\vartheta}_{iv}) = \frac{\sigma_{11}\sigma_{33}(1-\rho_1\rho_2)^2}{\sigma_{23}^2(1-\rho_2^2)}\frac{1}{T}$$

- Proof of equation (22): $AVar(\widehat{\vartheta}_{iv}) = \frac{\sigma_{11}}{\sigma_{22}\rho_{x,z}^2}(1-\rho_1^2)\frac{1}{T}$

We already know that,

$$\sigma_{23} = \frac{\rho_{x,z}\sqrt{\sigma_{22}\sigma_{33}}(1-\rho_1\rho_2)}{\sqrt{(1-\rho_1^2)(1-\rho_2^2)}}$$

substituting the previous result to the AVar equation (21), we will have the object of research.

12. APPENDIX (B)

Presentation of the Tables with the Theoretical and Monte Carlo results.

DGPs		Theoretical As.bias (ρx,z is fixed)					
	ρ1	0	0.1	0.25	0.5	0.75	0.95
	σχ	1	1.010	1.067	1.333	2.286	10.256
ρ2	σz						
0	1	0.50000	0.49749	0.48412	0.43301	0.33072	0.15612
0.1	1.010	0.49749	0.49500	0.48170	0.43084	0.32906	0.15534
0.2	1.042	0.48990	0.48744	0.47434	0.42426	0.32404	0.15297
0.3	1.099	0.47697	0.47458	0.46182	0.41307	0.31549	0.14893
0.4	1.190	0.45826	0.45596	0.44371	0.39686	0.30311	0.14309
0.5	1.333	0.43301	0.43084	0.41926	0.37500	0.28641	0.13521
0.6	1.563	0.40000	0.39799	0.38730	0.34641	0.26458	0.12490
0.7	1.961	0.35707	0.35528	0.34573	0.30923	0.23618	0.11150
0.8	2.778	0.30000	0.29850	0.29047	0.25981	0.19843	0.09367
0.9	5.263	0.21794	0.21685	0.21102	0.18875	0.14416	0.06805
0.95	10.256	0.15612	0.15534	0.15117	0.13521	0.10327	0.04875

ρχ,z: 0.2

Σ

1	0.5	0.1
0.5	1	σ23
0.1	σ23	1

DGPs		Theoretical As.bias (ρx,z is fixed)					
	ρ1	0	0.1	0.25	0.5	0.75	0.95
	σχ	1	1.010	1.067	1.333	2.286	10.256
ρ2	σz						
0	1	1.00000	0.99499	0.96825	0.86603	0.66144	0.31225
0.1	1.010	0.99499	0.99000	0.96339	0.86168	0.65812	0.31068
0.2	1.042	0.97980	0.97488	0.94868	0.84853	0.64807	0.30594
0.3	1.099	0.95394	0.94916	0.92365	0.82614	0.63097	0.29787
0.4	1.190	0.91652	0.91192	0.88741	0.79373	0.60622	0.28618
0.5	1.333	0.86603	0.86168	0.83853	0.75000	0.57282	0.27042
0.6	1.563	0.80000	0.79599	0.77460	0.69282	0.52915	0.24980
0.7	1.961	0.71414	0.71056	0.69147	0.61847	0.47236	0.22299
0.8	2.778	0.60000	0.59699	0.58095	0.51962	0.39686	0.18735
0.9	5.263	0.43589	0.43370	0.42205	0.37749	0.28831	0.13611
0.95	10.256	0.31225	0.31068	0.30233	0.27042	0.20653	0.09750

ρχ,z: 0.2

Σ

1	0.5	0.2
0.5	1	σ23
0.2	σ23	1

DGPs		Theoretical As.bias (ρx,z is fixed)					
	ρ1	0	0.1	0.25	0.5	0.75	0.95
	σχ	1	1.010	1.067	1.333	2.286	10.256
ρ2	σz						
0	1	2.00000	1.98997	1.93649	1.73205	1.32288	0.62450
0.1	1.010	1.98997	1.98000	1.92678	1.72337	1.31624	0.62137
0.2	1.042	1.95959	1.94977	1.89737	1.69706	1.29615	0.61188
0.3	1.099	1.90788	1.89832	1.84730	1.65227	1.26194	0.59573
0.4	1.190	1.83303	1.82384	1.77482	1.58745	1.21244	0.57236
0.5	1.333	1.73205	1.72337	1.67705	1.50000	1.14564	0.54083
0.6	1.563	1.60000	1.59198	1.54919	1.38564	1.05830	0.49960
0.7	1.961	1.42829	1.42113	1.38293	1.23693	0.94472	0.44598
0.8	2.778	1.20000	1.19398	1.16190	1.03923	0.79373	0.37470
0.9	5.263	0.87178	0.86741	0.84410	0.75498	0.57663	0.27221
0.95	10.256	0.62450	0.62137	0.60467	0.54083	0.41307	0.19500

ρχ,z: 0.2

Σ

1	0.5	0.4
0.5	1	σ23
0.4	σ23	1

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			0.25000	0.24875	0.24206	0.21651	0.16536	0.07806
0.1	1.010				0.24875	0.24750	0.24085	0.21542	0.16453	0.07767
0.2	1.042				0.24495	0.24372	0.23717	0.21213	0.16202	0.07649
0.3	1.099		Σ		0.23848	0.23729	0.23091	0.20653	0.15774	0.07447
0.4	1.190	1	0.5	0.1	0.22913	0.22798	0.22185	0.19843	0.15155	0.07155
0.5	1.333	0.5	1	σ23	0.21651	0.21542	0.20963	0.18750	0.14321	0.06760
0.6	1.563	0.1	σ23	1	0.20000	0.19900	0.19365	0.17321	0.13229	0.06245
0.7	1.961				0.17854	0.17764	0.17287	0.15462	0.11809	0.05575
0.8	2.778				0.15000	0.14925	0.14524	0.12990	0.09922	0.04684
0.9	5.263				0.10897	0.10843	0.10551	0.09437	0.07208	0.03403
0.95	10.256				0.07806	0.07767	0.07558	0.06760	0.05163	0.02438

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			0.50000	0.49749	0.48412	0.43301	0.33072	0.15612
0.1	1.010				0.49749	0.49500	0.48170	0.43084	0.32906	0.15534
0.2	1.042				0.48990	0.48744	0.47434	0.42426	0.32404	0.15297
0.3	1.099		Σ		0.47697	0.47458	0.46182	0.41307	0.31549	0.14893
0.4	1.190	1	0.5	0.2	0.45826	0.45596	0.44371	0.39686	0.30311	0.14309
0.5	1.333	0.5	1	σ23	0.43301	0.43084	0.41926	0.37500	0.28641	0.13521
0.6	1.563	0.2	σ23	1	0.40000	0.39799	0.38730	0.34641	0.26458	0.12490
0.7	1.961				0.35707	0.35528	0.34573	0.30923	0.23618	0.11150
0.8	2.778				0.30000	0.29850	0.29047	0.25981	0.19843	0.09367
0.9	5.263				0.21794	0.21685	0.21102	0.18875	0.14416	0.06805
0.95	10.256				0.15612	0.15534	0.15117	0.13521	0.10327	0.04875

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			1.00000	0.99499	0.96825	0.86603	0.66144	0.31225
0.1	1.010				0.99499	0.99000	0.96339	0.86168	0.65812	0.31068
0.2	1.042				0.97980	0.97488	0.94868	0.84853	0.64807	0.30594
0.3	1.099		Σ		0.95394	0.94916	0.92365	0.82614	0.63097	0.29787
0.4	1.190	1	0.5	0.4	0.91652	0.91192	0.88741	0.79373	0.60622	0.28618
0.5	1.333	0.5	1	σ23	0.86603	0.86168	0.83853	0.75000	0.57282	0.27042
0.6	1.563	0.4	σ23	1	0.80000	0.79599	0.77460	0.69282	0.52915	0.24980
0.7	1.961				0.71414	0.71056	0.69147	0.61847	0.47236	0.22299
0.8	2.778				0.60000	0.59699	0.58095	0.51962	0.39686	0.18735
0.9	5.263				0.43589	0.43370	0.42205	0.37749	0.28831	0.13611
0.95	10.256				0.31225	0.31068	0.30233	0.27042	0.20653	0.09750

DGPs

$\rho_{\chi,z}$ = 0.5

Σ

1	0.5	0.1
0.5	1	σ23
0.1	σ23	1

Theoretical As.bias (ρx,z is fixed)

ρ2	σz \ ρ1	0	0.1	0.25	0.5	0.75	0.95
	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.20000	0.19900	0.19365	0.17321	0.13229	0.06245
0.1	1.010	0.19900	0.19800	0.19268	0.17234	0.13162	0.06214
0.2	1.042	0.19596	0.19498	0.18974	0.16971	0.12961	0.06119
0.3	1.099	0.19079	0.18983	0.18473	0.16523	0.12619	0.05957
0.4	1.190	0.18330	0.18238	0.17748	0.15875	0.12124	0.05724
0.5	1.333	0.17321	0.17234	0.16771	0.15000	0.11456	0.05408
0.6	1.563	0.16000	0.15920	0.15492	0.13856	0.10583	0.04996
0.7	1.961	0.14283	0.14211	0.13829	0.12369	0.09447	0.04460
0.8	2.778	0.12000	0.11940	0.11619	0.10392	0.07937	0.03747
0.9	5.263	0.08718	0.08674	0.08441	0.07550	0.05766	0.02722
0.95	10.256	0.06245	0.06214	0.06047	0.05408	0.04131	0.01950

DGPs

$\rho_{\chi,z}$ = 0.5

Σ

1	0.5	0.2
0.5	1	σ23
0.2	σ23	1

Theoretical As.bias (ρx,z is fixed)

ρ2	σz \ ρ1	0	0.1	0.25	0.5	0.75	0.95
	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.40000	0.39799	0.38730	0.34641	0.26458	0.12490
0.1	1.010	0.39799	0.39600	0.38536	0.34467	0.26325	0.12427
0.2	1.042	0.39192	0.38995	0.37947	0.33941	0.25923	0.12238
0.3	1.099	0.38158	0.37966	0.36946	0.33045	0.25239	0.11915
0.4	1.190	0.36661	0.36477	0.35496	0.31749	0.24249	0.11447
0.5	1.333	0.34641	0.34467	0.33541	0.30000	0.22913	0.10817
0.6	1.563	0.32000	0.31840	0.30984	0.27713	0.21166	0.09992
0.7	1.961	0.28566	0.28423	0.27659	0.24739	0.18894	0.08920
0.8	2.778	0.24000	0.23880	0.23238	0.20785	0.15875	0.07494
0.9	5.263	0.17436	0.17348	0.16882	0.15100	0.11533	0.05444
0.95	10.256	0.12490	0.12427	0.12093	0.10817	0.08261	0.03900

DGPs

$\rho_{\chi,z}$ = 0.5

Σ

1	0.5	0.4
0.5	1	σ23
0.4	σ23	1

Theoretical As.bias (ρx,z is fixed)

ρ2	σz \ ρ1	0	0.1	0.25	0.5	0.75	0.95
	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.80000	0.79599	0.77460	0.69282	0.52915	0.24980
0.1	1.010	0.79599	0.79200	0.77071	0.68935	0.52650	0.24855
0.2	1.042	0.78384	0.77991	0.75895	0.67882	0.51846	0.24475
0.3	1.099	0.76315	0.75933	0.73892	0.66091	0.50478	0.23829
0.4	1.190	0.73321	0.72954	0.70993	0.63498	0.48497	0.22895
0.5	1.333	0.69282	0.68935	0.67082	0.60000	0.45826	0.21633
0.6	1.563	0.64000	0.63679	0.61968	0.55426	0.42332	0.19984
0.7	1.961	0.57131	0.56845	0.55317	0.49477	0.37789	0.17839
0.8	2.778	0.48000	0.47759	0.46476	0.41569	0.31749	0.14988
0.9	5.263	0.34871	0.34696	0.33764	0.30199	0.23065	0.10889
0.95	10.256	0.24980	0.24855	0.24187	0.21633	0.16523	0.07800

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7			0.14286	0.14214	0.13832	0.12372	0.09449	0.04461
0.1	1.010				0.14214	0.14143	0.13763	0.12310	0.09402	0.04438
0.2	1.042				0.13997	0.13927	0.13553	0.12122	0.09258	0.04371
0.3	1.099		Σ		0.13628	0.13559	0.13195	0.11802	0.09014	0.04255
0.4	1.190	1	0.5	0.1	0.13093	0.13027	0.12677	0.11339	0.08660	0.04088
0.5	1.333	0.5	1	σ23	0.12372	0.12310	0.11979	0.10714	0.08183	0.03863
0.6	1.563	0.1	σ23	1	0.11429	0.11371	0.11066	0.09897	0.07559	0.03569
0.7	1.961				0.10202	0.10151	0.09878	0.08835	0.06748	0.03186
0.8	2.778				0.08571	0.08528	0.08299	0.07423	0.05669	0.02676
0.9	5.263				0.06227	0.06196	0.06029	0.05393	0.04119	0.01944
0.95	10.256				0.04461	0.04438	0.04319	0.03863	0.02950	0.01393

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7			0.28571	0.28428	0.27664	0.24744	0.18898	0.08921
0.1	1.010				0.28428	0.28286	0.27525	0.24620	0.18803	0.08877
0.2	1.042				0.27994	0.27854	0.27105	0.24244	0.18516	0.08741
0.3	1.099		Σ		0.27255	0.27119	0.26390	0.23604	0.18028	0.08510
0.4	1.190	1	0.5	0.2	0.26186	0.26055	0.25355	0.22678	0.17321	0.08177
0.5	1.333	0.5	1	σ23	0.24744	0.24620	0.23958	0.21429	0.16366	0.07726
0.6	1.563	0.2	σ23	1	0.22857	0.22743	0.22131	0.19795	0.15119	0.07137
0.7	1.961				0.20404	0.20302	0.19756	0.17670	0.13496	0.06371
0.8	2.778				0.17143	0.17057	0.16599	0.14846	0.11339	0.05353
0.9	5.263				0.12454	0.12392	0.12059	0.10785	0.08238	0.03889
0.95	10.256				0.08921	0.08877	0.08638	0.07726	0.05901	0.02786

DGPs					Theoretical As.bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7			0.57143	0.56856	0.55328	0.49487	0.37796	0.17843
0.1	1.010				0.56856	0.56571	0.55051	0.49239	0.37607	0.17753
0.2	1.042				0.55988	0.55708	0.54210	0.48487	0.37033	0.17482
0.3	1.099		Σ		0.54511	0.54238	0.52780	0.47208	0.36056	0.17021
0.4	1.190	1	0.5	0.4	0.52372	0.52110	0.50709	0.45356	0.34641	0.16353
0.5	1.333	0.5	1	σ23	0.49487	0.49239	0.47916	0.42857	0.32733	0.15452
0.6	1.563	0.4	σ23	1	0.45714	0.45485	0.44263	0.39590	0.30237	0.14274
0.7	1.961				0.40808	0.40604	0.39512	0.35341	0.26992	0.12742
0.8	2.778				0.34286	0.34114	0.33197	0.29692	0.22678	0.10706
0.9	5.263				0.24908	0.24783	0.24117	0.21571	0.16475	0.07778
0.95	10.256				0.17843	0.17753	0.17276	0.15452	0.11802	0.05571

DGPs						MC bias (ρx,z is fixed)					
					ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z			σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.2				0.49726423	-4.3862002	0.45837022	0.46193284	0.36904537	0.185388
0.1	1.0101					0.70722125	0.47675541	0.44127392	0.34184092	0.24057523	0.08412
0.2	1.0417					0.46854986	0.44916923	0.39678656	0.29518153	0.17580913	0.050816
0.3	1.0989		Σ			0.49988963	0.42916314	0.35342751	0.23761326	0.13121421	0.032855
0.4	1.1905	1	0.5	0.1		0.44523139	0.37522554	0.30155171	0.19966023	0.10129673	0.020348
0.5	1.3333	0.5	1	σ23		0.42500632	0.35552183	0.28066363	0.18056635	0.088658724	0.015207
0.6	1.5625	0.1	σ23	1		0.38503345	0.31532743	0.24417897	0.15186943	0.071479432	0.011113
0.7	1.9608					0.32076054	0.26142506	0.21518443	0.13417711	0.061281258	0.005779
0.8	2.7778					0.24726091	0.24032497	0.18723638	0.1199506	0.054551654	0.003785
0.9	5.2632					0.18885979	0.17175621	0.14565972	0.099502573	0.047633682	0.002665
0.95	10.256					0.14600282	0.13661769	0.12040277	0.085569331	0.04026101	-0.00031

DGPs						MC bias (ρx,z is fixed)					
					ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z			σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.2				1.1492921	1.0532529	0.93527453	0.86570867	0.73915545	0.401802
0.1	1.0101					1.19102	1.0114645	0.94279033	0.74581964	0.49863198	0.174516
0.2	1.0417					1.0807494	0.97560092	0.83819526	0.62065889	0.36288471	0.109671
0.3	1.0989		Σ			1.0086731	0.94277821	0.70563242	0.51325476	0.28169563	0.078853
0.4	1.1905	1	0.5	0.2		1.0494949	0.87587085	0.68262465	0.44069696	0.22824319	0.058177
0.5	1.3333	0.5	1	σ23		0.9558346	0.77899771	0.60397272	0.38507569	0.19345595	0.045101
0.6	1.5625	0.2	σ23	1		-0.50569634	0.70842395	0.53724513	0.33337763	0.16319945	0.037044
0.7	1.9608					0.75123803	0.62083505	0.47220913	0.29488989	0.14311095	0.029109
0.8	2.7778					0.6185367	0.52383613	0.41462955	0.26319935	0.12679474	0.023858
0.9	5.2632					0.43118452	0.38693491	0.3254218	0.22272058	0.11265253	0.021041
0.95	10.256					0.31028702	0.29189844	0.26009936	0.1910968	0.10001133	0.017014

DGPs						MC bias (ρx,z is fixed)					
					ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z			σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.2				0.97097118	2.4365965	2.2757471	1.6962461	1.3968982	1.047501
0.1	1.0101					2.272327	2.1155642	2.0552118	1.5944125	1.0054882	0.35523
0.2	1.0417					2.2202594	2.0781128	1.7475325	1.2669149	0.7355688	0.227432
0.3	1.0989		Σ			2.1426304	1.9712578	1.4868347	1.069395	0.58321425	0.170765
0.4	1.1905	1	0.5	0.4		-8.9031839	1.8656161	1.5286254	0.92436144	0.48314597	0.133981
0.5	1.3333	0.5	1	σ23		2.0335537	1.6294582	1.2514228	0.79439832	0.40332881	0.104908
0.6	1.5625	0.4	σ23	1		2.0047871	1.4844193	1.118202	0.69485454	0.34641044	0.088902
0.7	1.9608					1.3511641	1.310014	0.98656226	0.61663785	0.30709718	0.075824
0.8	2.7778					1.2943316	1.3264976	0.86514802	0.54867384	0.27112227	0.063996
0.9	5.2632					0.91079642	0.81374827	0.68262232	0.4681176	0.24243543	0.05778
0.95	10.256					0.63832308	0.60193772	0.53904985	0.40192495	0.21948849	0.051672

DGPs					MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			0.2417358	0.2407854	0.2353856	0.2142221	0.1700933	0
0.1	1.010101				0.2446992	0.2400676	0.2292866	0.1983822	0.14224	0
0.2	1.041667				0.2447261	0.2368096	0.2213385	0.1830534	0.1213609	0
0.3	1.098901		Σ		0.225141	0.2149157	0.1966184	0.1556442	0.0960942	0
0.4	1.190476	1	0.5	0.1	0.2097098	0.1984279	0.1789845	0.1371819	0.0793562	0.02020471
0.5	1.333333	0.5	1	σ23	0.2051171	0.1931208	0.1728782	0.1303714	0.0731872	0.01530918
0.6	1.5625	0.1	σ23	1	0.1888703	0.1766024	0.1562091	0.1146829	0.0617043	0.01076458
0.7	1.960784				0.1684169	0.1583857	0.1409528	0.103463	0.053821	0.00727855
0.8	2.777778				0.1435445	0.1369899	0.1244197	0.0939802	0.0490843	0.00244782
0.9	5.263158				0	0.0976561	0.093012	0.0760447	0.0426685	0.00206877
0.95	10.25641				0	0	0	0.0647686	0.0362971	0.00201485

DGPs					MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			0.4996693	0.4976215	0.4860711	0.4408473	0.3466326	0
0.1	1.010101				0.5027344	0.4928157	0.4697746	0.4045094	0.2880698	0
0.2	1.041667				0.4975847	0.4812032	0.4491549	0.3703838	0.2454315	0
0.3	1.098901		Σ		0.4749665	0.4530683	0.4142816	0.3285385	0.2055719	0
0.4	1.190476	1	0.5	0.2	0.4524976	0.4278055	0.3859369	0.2974675	0.1770956	0.05161037
0.5	1.333333	0.5	1	σ23	0.4306941	0.4049685	0.3622904	0.274319	0.1583442	0.04210629
0.6	1.5625	0.2	σ23	1	0.3987216	0.3733152	0.3314797	0.2466511	0.1386517	0.03402536
0.7	1.960784				0.3536737	0.3329702	0.2976605	0.2227871	0.1239737	0.02883087
0.8	2.777778				0.2998096	0.2865849	0.2614618	0.2008452	0.1119904	0.02240784
0.9	5.263158				0.2123493	0.2097231	0.2007586	0.1666021	0.0994262	0.01937881
0.95	10.25641				0	0	0	0.1392151	0.0874481	0.0187318

DGPs					MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4			1.0142478	1.0101991	0.9865413	0.8930478	0.6985294	0
0.1	1.010101				1.0151116	0.9947708	0.9474768	0.8141265	0.5783986	0
0.2	1.041667				0.9997955	0.9670284	0.9025333	0.7436301	0.4929402	0
0.3	1.098901		Σ		0.9735832	0.9286487	0.849356	0.674612	0.4247302	0.14025718
0.4	1.190476	1	0.5	0.4	0.9376489	0.8863511	0.7999132	0.6184816	0.3730797	0.11434432
0.5	1.333333	0.5	1	σ23	0.8813395	0.8282255	0.740762	0.5620451	0.3286966	0.09454917
0.6	1.5625	0.4	σ23	1	0.8168716	0.7652061	0.6807027	0.5099223	0.2924113	0.08296695
0.7	1.960784				0.72306	0.6812265	0.6104705	0.4612917	0.264397	0.07133222
0.8	2.777778				0.6109464	0.5845273	0.5345373	0.4140141	0.2376729	0.06118857
0.9	5.263158				0.4380611	0.4332426	0.4156384	0.3472223	0.2127418	0.05547395
0.95	10.25641				0	0	0	0.2880943	0.1897597	0.05111152

DGPs					MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.5			0.19456711	0.1937912	0.189312	0.1717763	0.1355878	0
0.1	1.0101				0.1976624	0.1948235	0.187238	0.1637282	0.1195558	0
0.2	1.0417				0.19772685	0.1930894	0.182803	0.1545084	0.1054095	0
0.3	1.0989		Σ		0.17997366	0.1741862	0.162392	0.132451	0.0846074	0
0.4	1.1905	1	0.5	0.1	0.16730837	0.1610928	0.148845	0.1185582	0.0715023	0
0.5	1.3333	0.5	1	σ23	0.16457873	0.158061	0.145412	0.1145219	0.0672858	0
0.6	1.5625	0.1	σ23	1	0.15282215	0.1459894	0.133078	0.1024866	0.0578951	0.010188875
0.7	1.9608				0.13592044	0.1307946	0.120189	0.0927934	0.0506891	0.007569651
0.8	2.7778				0.1164333	0.1135937	0.106513	0.0848357	0.0467365	0.004605508
0.9	5.2632				0	0	0	0.0680801	0.0405581	0.002562002
0.95	10.256				0	0	0	0	0.034708	-0.000576901

DGPs					MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.5			0.39846805	0.3968135	0.387333	0.3502691	0.2740158	0
0.1	1.0101				0.40159582	0.3957293	0.379926	0.3311047	0.2402897	0
0.2	1.0417				0.39733013	0.3880823	0.36734	0.3102092	0.2120043	0
0.3	1.0989		Σ		0.3769676	0.3649928	0.340648	0.2790692	0.1811618	0
0.4	1.1905	1	0.5	0.2	0.35832975	0.3451741	0.319453	0.2563386	0.1592827	0
0.5	1.3333	0.5	1	σ23	0.34275201	0.3292077	0.303179	0.2402053	0.1452218	0
0.6	1.5625	0.2	σ23	1	0.31815128	0.3049473	0.279655	0.2187705	0.1291334	0.033059271
0.7	1.9608				0.28267282	0.2726488	0.252042	0.1987491	0.1162291	0.027954957
0.8	2.7778				0.24024542	0.235023	0.221688	0.1798849	0.1058657	0.022890571
0.9	5.2632				0	0	0.168967	0.1481241	0.093943	0.019967547
0.95	10.256				0	0	0	0	0.0824216	0.016507755

DGPs					MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.5			0.80555565	0.8022311	0.782819	0.7065528	0.5500397	0
0.1	1.0101				0.80719045	0.795327	0.76323	0.664221	0.4809575	0
0.2	1.0417				0.79469498	0.7763933	0.734991	0.620587	0.4246805	0
0.3	1.0989		Σ		0.77053004	0.7462774	0.697026	0.5724708	0.3744065	0
0.4	1.1905	1	0.5	0.4	0.74007888	0.7131728	0.660694	0.532207	0.335236	0
0.5	1.3333	0.5	1	σ23	0.69869091	0.6711336	0.618401	0.4913834	0.3010829	0.091662876
0.6	1.5625	0.4	σ23	1	0.64803235	0.6220299	0.572016	0.4508768	0.2715124	0.07891403
0.7	1.9608				0.57532273	0.5556191	0.5152	0.4104597	0.2473661	0.067653051
0.8	2.7778				0.48687463	0.4769768	0.451269	0.3695142	0.2240014	0.06162277
0.9	5.2632				0	0	0.348518	0.3078545	0.2005357	0.053933122
0.95	10.256				0	0	0	0.2531435	0.1778831	0.049984897

		DGPs			MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7			0.14063505	0.14005256	0.13672075	0.12375043	0	0
0.1	1.0101				0.14383547	0.14254217	0.13804487	0.12240016	0	0
0.2	1.0417				0.1432957	0.14145474	0.13599774	0.11803772	0	0
0.3	1.0989		Σ		0.12822475	0.12606283	0.12018167	0.10175181	0.073576736	0
0.4	1.1905	1	0.5	0.1	0.11883385	0.11673861	0.11105237	0.09300117	0.067368818	0
0.5	1.3333	0.5	1	σ23	0.11813152	0.11607686	0.11040101	0.0920928	0.061110287	0
0.6	1.5625	0.1	σ23	1	0.1126748	0.10988859	0.1036758	0.08507537	0.049002639	0
0.7	1.9608				0	0	0.092511382	0.07675699	0.041392917	0
0.8	2.7778				0	0	0	0.07099456	0.041623023	0.004501726
0.9	5.2632				0	0	0	0	0.036083865	0.002827707
0.95	10.256				0	0	0	0	0	0.001525702

		DGPs			MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7			0.28492347	0.28371525	0.27677039	0.24970588	0	0
0.1	1.0101				0.28809235	0.28553261	0.27639567	0.24443211	0	0
0.2	1.0417				0.28452125	0.28099836	0.27029615	0.23484398	0.16449475	0
0.3	1.0989		Σ		0.26740716	0.26320074	0.25151944	0.21449375	0.15194083	0
0.4	1.1905	1	0.5	0	0.25341479	0.24923433	0.23770083	0.20079067	0.1394884	0
0.5	1.3333	0.5	1	σ23	0.24414494	0.24016136	0.22897542	0.19251369	0.12223681	0
0.6	1.5625	0	σ23	1	0.22884252	0.22491213	0.21422833	0.17908139	0.10998046	0
0.7	1.9608				0	0.20019407	0.19301099	0.16352111	0.1041815	0
0.8	2.7778				0	0	0	0.14898767	0.094202182	0.022342114
0.9	5.2632				0	0	0	0	0.081502938	0.019233659
0.95	10.256				0	0	0	0	0	0.017069962

		DGPs			MC bias (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z		σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7			0.5731746	0.57074143	0.55658201	0.50124085	0	0
0.1	1.0101				0.5755099	0.57043066	0.55207374	0.48771902	0.35910556	0
0.2	1.0417				0.56602978	0.55918451	0.53806092	0.46776847	0.33261813	0
0.3	1.0989		Σ		0.54556672	0.53729738	0.5140855	0.44001094	0.30665161	0
0.4	1.1905	1	0.5	0.4	0.52242665	0.51413656	0.49100929	0.41655457	0.27898788	0
0.5	1.3333	0.5	1	σ23	0.49588126	0.48806129	0.46588348	0.39317601	0.25668738	0
0.6	1.5625	0.4	σ23	1	0.46139931	0.45487618	0.43510563	0.36690424	0.23787292	0
0.7	1.9608				0.4097116	0.40699202	0.39344973	0.33677373	0.21890779	0
0.8	2.7778				0	0	0.34481568	0.30459598	0.19993558	0.058105069
0.9	5.2632				0	0	0	0	0.1778712	0.052039836
0.95	10.256				0	0	0	0	0.16039803	0.048118799

	DGPs				MC Absolute bias					
ρ2	ρχ,z			ρ1	0	0.1	0.25	0.5	0.75	0.95
0	0.2				0.568429	5.413974	0.550478	0.525344	0.454997	0.238371
0.1					0.825815	0.551691	0.486140	0.411045	0.256752	0.092218
0.2					0.596954	0.519939	0.446821	0.329715	0.192959	0.058793
0.3			Σ		0.610749	0.495475	0.390424	0.268935	0.148253	0.044748
0.4		1	0.5	0.1	0.556242	0.450625	0.355348	0.235575	0.125949	0.038076
0.5		0.5	1	σ23	0.495098	0.406741	0.318737	0.208232	0.110459	0.034080
0.6		0.1	σ23	1	0.496879	0.390371	0.297026	0.188807	0.099292	0.033277
0.7					0.429158	0.361617	0.270639	0.174211	0.093182	0.032435
0.8					0.398039	0.319337	0.248849	0.163206	0.087633	0.029425
0.9					0.270538	0.239846	0.201577	0.142634	0.081297	0.030595
0.95					0.201198	0.189974	0.171679	0.132079	0.080375	0.031502

	DGPs				MC Absolute bias					
ρ2	ρχ,z			ρ1	0	0.1	0.25	0.5	0.75	0.95
0	0.2				1.214060	1.157879	1.189411	1.189207	0.909252	0.454197
0.1					1.220436	1.056461	0.958604	0.791063	0.499292	0.174977
0.2					1.088089	0.983399	0.850329	0.623029	0.363770	0.110027
0.3			Σ		1.113747	0.948558	0.786657	0.514199	0.282875	0.079912
0.4		1	0.5	0.2	1.121840	0.883444	0.683579	0.441713	0.229861	0.061036
0.5		0.5	1	σ23	0.957912	0.780252	0.604857	0.385858	0.194806	0.049680
0.6		0.2	σ23	1	2.324898	0.714270	0.540942	0.336088	0.166290	0.043887
0.7					0.767037	0.626276	0.476406	0.299273	0.149100	0.039477
0.8					0.640572	0.546411	0.422367	0.269515	0.133881	0.034898
0.9					0.447571	0.399977	0.335951	0.230688	0.120576	0.033726
0.95					0.320649	0.301860	0.269525	0.201239	0.111840	0.033258

	DGPs				MC Absolute bias					
ρ2	ρχ,z			ρ1	0	0.1	0.25	0.5	0.75	0.95
0	0.2				3.601205	2.552402	2.332680	2.649366	2.021908	1.129386
0.1					2.354668	2.216278	2.097740	1.594413	1.005488	0.355230
0.2					2.277559	2.078113	1.747533	1.266915	0.735569	0.227432
0.3			Σ		2.368980	2.077396	1.627095	1.069395	0.583214	0.170765
0.4		1	0.5	0.4	13.673999	1.884736	1.528625	0.924361	0.483146	0.134005
0.5		0.5	1	σ23	2.033554	1.629458	1.251423	0.794398	0.403329	0.104953
0.6		0.4	σ23	1	2.004787	1.484419	1.118202	0.694855	0.346410	0.089110
0.7					1.789345	1.310014	0.986562	0.616638	0.307097	0.076492
0.8					1.377699	1.326498	0.865148	0.548674	0.271165	0.065515
0.9					0.910796	0.813748	0.682622	0.468118	0.242602	0.059089
0.95					0.638323	0.601938	0.539050	0.401925	0.219656	0.054494

	DGPs				MC Absolute bias					
ρ2	ρχ,z			ρ1	0	0.1	0.25	0.5	0.75	0.95
0	0.5				0.207772	0.206924	0.202084	0.183042	0.143557	0
0.1					0.211279	0.208189	0.199910	0.174463	0.127301	0
0.2					0.210318	0.205465	0.194574	0.164392	0.112126	0
0.3			Σ		0.194974	0.188848	0.176444	0.144797	0.094683	0
0.4		1	0.5	0.1	0.189647	0.182836	0.169647	0.137713	0.088512	0
0.5		0.5	1	σ23	0.183279	0.175948	0.162227	0.130305	0.082908	0
0.6		0.1	σ23	1	0.177331	0.169535	0.155454	0.123835	0.078162	0.027520
0.7					0.161513	0.155602	0.144276	0.116834	0.074742	0.029459
0.8					0.142955	0.139921	0.132841	0.111296	0.072177	0.029651
0.9					0	0	0	0.093506	0.067194	0.030140
0.95					0	0	0	0	0.064385	0.030600

	DGPs				MC Absolute bias					
ρ2	ρχ,z			ρ1	0	0.1	0.25	0.5	0.75	0.95
0	0.5				0.398512	0.396855	0.387360	0.350299	0.274096	0
0.1					0.402046	0.396165	0.380327	0.331427	0.240522	0
0.2					0.398376	0.389110	0.368310	0.310982	0.212429	0
0.3			Σ		0.377201	0.365280	0.341023	0.279658	0.181851	0
0.4		1	0.5	0.2	0.358722	0.345616	0.319968	0.256990	0.160281	0
0.5		0.5	1	σ23	0.343336	0.329745	0.303660	0.240751	0.146190	0
0.6		0.2	σ23	1	0.320067	0.306606	0.281026	0.220119	0.131341	0.038012
0.7					0.284766	0.274600	0.253895	0.201378	0.120671	0.036843
0.8					0.243558	0.238070	0.224531	0.183267	0.110967	0.034020
0.9					0	0	0.172315	0.152516	0.099839	0.033247
0.95					0	0	0	0	0.090981	0.032780

	DGPs				MC Absolute bias					
					ρ1	p1'	p1'	p1'	p1'	p1'
ρ2	ρχ,z				0	0.1	0.25	0.5	0.75	0.95
0	0.5				0.805556	0.802231	0.782819	0.706553	0.550040	0
0.1					0.807190	0.795327	0.763230	0.664221	0.480958	0
0.2					0.794695	0.776393	0.734991	0.620587	0.424681	0
0.3			Σ		0.770530	0.746277	0.697026	0.572471	0.374406	0
0.4		1	0.5	0.4	0.740079	0.713173	0.660694	0.532207	0.335236	0
0.5		0.5	1	σ23	0.698691	0.671134	0.618401	0.491383	0.301083	0.091694
0.6		0.4	σ23	1	0.648032	0.622030	0.572016	0.450877	0.271512	0.079062
0.7					0.575323	0.555619	0.515200	0.410460	0.247366	0.067981
0.8					0.486875	0.476977	0.451269	0.369514	0.224001	0.062537
0.9					0	0	0.348537	0.307911	0.200710	0.055888
0.95					0	0	0	0.253145	0.178048	0.052371

ρ2	DGPs: ρχ,z			ρ1	MC Absolute bias: 0	0.1	0.25	0.5	0.75	0.95
0	0.7				0.149016	0.148398	0.144803	0.130800	0	0
0.1					0.152844	0.151488	0.146634	0.129717	0	0
0.2					0.150167	0.148355	0.142787	0.123932	0	0
0.3		Σ			0.139390	0.137074	0.130833	0.111527	0.080851	0
0.4		1	0.5	0.1	0.134192	0.132145	0.126408	0.108039	0.077229	0
0.5		0.5	1	σ23	0.133219	0.130595	0.124171	0.105204	0.072306	0
0.6		0.1	σ23	1	0.127051	0.124771	0.119079	0.101130	0.066553	0
0.7					0	0	0.111235	0.096220	0.067787	0
0.8					0	0	0	0.091940	0.066208	0.026559
0.9					0	0	0	0	0.060570	0.029067
0.95					0	0	0	0	0	0.029210

ρ2	DGPs: ρχ,z			ρ1	MC Absolute bias: 0	0.1	0.25	0.5	0.75	0.95
0	0.7				0.284992	0.283785	0.276842	0.249765	0	0
0.1					0.288327	0.285766	0.276617	0.244631	0	0
0.2					0.285149	0.281631	0.270905	0.235315	0.164696	0
0.3		Σ			0.267564	0.263409	0.251797	0.214884	0.152337	0
0.4		1	0.5	0.2	0.253632	0.249477	0.238009	0.201212	0.139928	0
0.5		0.5	1	σ23	0.244820	0.240802	0.229570	0.193092	0.123633	0
0.6		0.2	σ23	1	0.229873	0.226012	0.215304	0.180029	0.113221	0
0.7					0	0.202193	0.194975	0.165691	0.107158	0
0.8					0	0	0	0.151468	0.098610	0.031421
0.9					0	0	0	0	0.088786	0.032597
0.95					0	0	0	0	0	0.031526

ρ2	DGPs: ρχ,z			ρ1	MC Absolute bias: 0	0.1	0.25	0.5	0.75	0.95
0	0.7				0.573175	0.570741	0.556582	0.501241	0.000000	0
0.1					0.575510	0.570431	0.552074	0.487719	0.359106	0
0.2					0.566030	0.559185	0.538061	0.467768	0.332618	0
0.3		Σ			0.545567	0.537297	0.514086	0.440011	0.306652	0
0.4		1	0.5	0.4	0.522427	0.514137	0.491009	0.416555	0.278988	0
0.5		0.5	1	σ23	0.495881	0.488061	0.465883	0.393176	0.256687	0
0.6		0.4	σ23	1	0.461399	0.454876	0.435106	0.366904	0.237873	0
0.7					0.409712	0.406992	0.393450	0.336774	0.218908	0
0.8					0	0	0.344816	0.304596	0.199957	0.058809
0.9					0	0	0	0	0.177876	0.053769
0.95					0	0	0	0	0.160491	0.049978

DGPs					T-test (mean of t)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		0.96694336	0.96799359	0.97242078	0.98944968	1.0286275	npd
0.1	1.010				0.97879666	0.96510794	0.94722476	0.91628833	0.8601868	npd
0.2	1.042				0.97890427	0.95201047	0.91438964	0.84548742	0.73392196	npd
0.3	1.099		Σ		0.90056392	0.86399364	0.81226635	0.71888986	0.58112331	npd
0.4	1.190	1	0.5	0.1	0.8388393	0.79771001	0.73941746	0.63361602	0.47990134	0.2588274
0.5	1.333	0.5	1	σ23	0.82046819	0.776375	0.7141914	0.60215949	0.44259463	0.19611444
0.6	1.563	0.1	σ23	1	0.75548113	0.70996825	0.6453281	0.5296975	0.37315259	0.13789693
0.7	1.961				0.67366771	0.63673457	0.58230188	0.47787509	0.32547855	0.093240017
0.8	2.778				0.57417792	0.5507199	0.51400058	0.43407617	0.29683418	0.031357143
0.9	5.263				npd	0.39259228	0.38424966	0.35123535	0.25803498	0.026501419
0.95	10.256				npd	npd	npd	0.29915343	0.21950405	0.025810678

DGPs					T-test (a3 of t)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		-0.4700449	-0.47026169	-0.4526513	-0.36814339	-0.13848863	npd
0.1	1.010				-0.50223017	-0.48167103	-0.41933368	-0.27017135	-0.12789777	npd
0.2	1.042				-0.64173706	-0.62768849	-0.57631302	-0.39904113	-0.06782989	npd
0.3	1.099		Σ		-0.40107533	-0.39712988	-0.37933069	-0.30493364	-0.20701931	npd
0.4	1.190	1	0.5	0.1	-0.09202139	-0.12555424	-0.165281	-0.19513185	-0.20918138	-0.0252513
0.5	1.333	0.5	1	σ23	-0.16365632	-0.13801652	-0.10765376	-0.0851892	-0.13910214	-0.31638767
0.6	1.563	0.2	σ23	1	-0.3890351	-0.33596575	-0.27066294	-0.18400545	-0.16017601	-0.21436548
0.7	1.961				-0.34469577	-0.32826176	-0.31349527	-0.32248936	-0.43493467	-0.46015613
0.8	2.778				-0.47354254	-0.44700803	-0.41253555	-0.37153921	-0.42734777	-0.67999478
0.9	5.263				npd	-0.33046013	-0.39791917	-0.47154308	-0.51587456	-0.62401345
0.95	10.256				npd	npd	npd	-0.3516977	-0.37046227	-0.66676194

DGPs					T-test (a4 of t)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		3.5914129	3.588726	3.5792306	3.5985672	3.7929197	npd
0.1	1.010				4.1647995	4.0685326	3.8868862	3.6408897	3.533686	npd
0.2	1.042				4.9676039	4.8581199	4.664428	4.2568872	3.969423	npd
0.3	1.099		Σ		4.1297154	4.1358678	4.1691787	4.1787651	3.9099289	npd
0.4	1.190	1	0.5	0.1	4.0975183	3.9184356	3.7272163	3.5989826	3.802279	3.981024
0.5	1.333	0.5	1	σ23	3.2368951	3.1570619	3.0948701	3.0899437	3.1594365	3.9880043
0.6	1.563	0.2	σ23	1	4.0160265	3.8366764	3.6371558	3.4059773	3.3847167	4.4896219
0.7	1.961				3.0005255	3.0106305	3.0699391	3.2939641	3.8553632	4.0204344
0.8	2.778				3.7275022	3.6502133	3.5487712	3.4790613	3.8984309	4.4158542
0.9	5.263				npd	3.3215318	3.4759441	3.6861107	4.0616872	4.6035796
0.95	10.256				npd	npd	npd	3.4162951	3.6139449	4.461268

DGPs				**T-test (mean of t)**						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		1.9986772	2.0005136	2.0080485	2.0361865	2.0962371	npd
0.1	1.010				2.0109376	1.9811936	1.9407244	1.8683488	1.7420825	npd
0.2	1.042				1.9903387	1.9345094	1.8555406	1.7107294	1.48423	npd
0.3	1.099		Σ		1.8998661	1.8214032	1.7114729	1.5174542	1.2431819	npd
0.4	1.190	1	0.5	0.2	1.8099905	1.7198429	1.5943756	1.3739437	1.0709733	0.66114183
0.5	1.333	0.5	1	σ23	1.7227765	1.6280346	1.4966878	1.2670252	0.95757585	0.53939215
0.6	1.563	0.2	σ23	1	1.5948862	1.5007837	1.3694031	1.1392324	0.83848646	0.4358734
0.7	1.961				1.4146949	1.3385905	1.2296899	1.0290096	0.74972258	0.36933069
0.8	2.778				1.1992383	1.1521146	1.0801464	0.9276643	0.67725441	0.28705007
0.9	5.263				0.84939715	0.84311841	0.8293704	0.76950205	0.60127287	0.24824744
0.95	10.256				npd	npd	npd	0.64300705	0.52883611	0.2399591

DGPs				**T-test (a3 of t)**						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		-0.17068124	-0.16987951	-0.13846765	-0.00442181	0.25279414	npd
0.1	1.010				-0.10123706	-0.09970207	-0.04770277	0.11745801	0.20047156	ndp
0.2	1.042				-0.16041772	-0.18118771	-0.16693767	-0.016524	0.34268379	ndp
0.3	1.099		Σ		0.093650503	0.060434287	0.043699459	0.098598949	0.18052532	npd
0.4	1.190	1	0.5	0.2	0.34947977	0.27354304	0.19650767	0.14993769	0.21076289	0.74204215
0.5	1.333	0.5	1	σ23	0.13133507	0.13995932	0.15610677	0.15840194	0.1054712	0.3931082
0.6	1.563	0.2	σ23	1	-0.08309601	-0.06669896	-0.03874495	0.033550532	0.094335366	0.46699708
0.7	1.961				-0.18406568	-0.16579988	-0.14107744	-0.10957327	-0.14091764	0.019469045
0.8	2.778				-0.2992169	-0.27518469	-0.22927503	-0.13109538	-0.08928399	-0.17773851
0.9	5.263				-0.26202959	-0.30305161	-0.33583889	-0.31930447	-0.22443029	-0.14824023
0.95	10.256				npd	npd	npd	-0.2140494	-0.19688906	-0.22796024

DGPs				**T-test (a4 of t)**						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		3.2997518	3.3246632	3.3063376	3.1610918	3.0676961	npd
0.1	1.010				3.906874	3.8853989	3.8478114	3.8052624	3.4704776	ndp
0.2	1.042				4.1540292	4.1336755	4.0315576	3.7874471	3.8178397	ndp
0.3	1.099		Σ		3.325376	3.3114042	3.3188089	3.4437321	3.5569162	npd
0.4	1.190	1	0.5	0.2	4.880738	4.4510103	4.0289865	3.7138849	3.8410953	4.4986168
0.5	1.333	0.5	1	σ23	3.1934353	3.1783218	3.1691784	3.1247507	3.0161755	4.4101233
0.6	1.563	0.2	σ23	1	3.5168256	3.4477587	3.3792751	3.3501365	3.407455	4.3408253
0.7	1.961				2.9356894	2.9431615	2.982381	3.1270629	3.4463129	3.7359308
0.8	2.778				3.5014272	3.4740771	3.4366229	3.4305903	3.766883	4.1341895
0.9	5.263				3.278152	3.3470525	3.4188551	3.5124382	3.6966146	4.1268757
0.95	10.256				npd	npd	npd	3.2877618	3.5392223	4.200927

DGPs					T-test (mean of t)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		4.0569912	4.0611531	4.0755818	4.1248113	4.2243089	npd
0.1	1.010				4.0604463	3.9991289	3.9141993	3.7602891	3.4978257	npd
0.2	1.042				3.9991818	3.8876004	3.7285296	3.4346802	2.9810221	npd
0.3	1.099		Σ		3.8943327	3.7333082	3.5088444	3.1158993	2.5685271	1.7967299
0.4	1.190	1	0.5	0.4	3.7505957	3.5632655	3.3045873	2.8566442	2.2561737	1.4647796
0.5	1.333	0.5	1	σ23	3.5253581	3.3295918	3.0602229	2.5959751	1.9877699	1.2111987
0.6	1.563	0.4	σ23	1	3.2674865	3.0762444	2.8121067	2.3552302	1.7683373	1.0628276
0.7	1.961				2.8922401	2.7386337	2.5219647	2.1306149	1.5989228	0.9137837
0.8	2.778				2.4437854	2.349888	2.2082709	1.9122492	1.4373104	0.783841
0.9	5.263				1.7522444	1.7417008	1.7170781	1.6037512	1.2865416	0.7106352
0.95	10.256				npd	npd	npd	1.3306506	1.1475586	0.6547515

DGPs					T-test (a3 of t)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		0.67788266	0.79063618	0.97257843	0.99939354	0.774643	npd
0.1	1.010				0.68245562	0.63821843	0.64086433	0.74425357	0.6540854	npd
0.2	1.042				0.58371428	0.49911948	0.44380419	0.455821	0.6537412	npd
0.3	1.099		Σ		0.90596149	0.80991995	0.68963612	0.62553751	0.6263811	1.2016284
0.4	1.190	1	0.5	0.4	0.98773393	0.87660114	0.78658979	0.68982809	0.6204585	1.1025611
0.5	1.333	0.5	1	σ23	0.73047395	0.69710471	0.65973157	0.56376969	0.4685191	1.1321178
0.6	1.563	0.4	σ23	1	0.39261356	0.33514465	0.28849186	0.32909023	0.4691379	0.7435987
0.7	1.961				0.2356488	0.23423703	0.25452493	0.34664432	0.483013	0.8102056
0.8	2.778				0.1044099	0.11270385	0.17884102	0.41465982	0.7339578	0.925384
0.9	5.263				-0.164765	-0.14139991	-0.07382915	0.10998402	0.3926067	0.7072234
0.95	10.256				npd	npd	npd	0.053860535	0.1686015	0.4222025

DGPs					T-test (a4 of t)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		4.9289514	6.0709701	7.7945346	6.7577461	4.2192472	npd
0.1	1.010				4.0056848	3.7987478	3.7556257	4.1017522	3.712844	npd
0.2	1.042				4.2954978	3.7787419	3.3936703	3.3292329	3.8722822	npd
0.3	1.099		Σ		5.1351268	4.7634057	4.2891414	4.0099096	3.8477933	5.5201379
0.4	1.190	1	0.5	0.4	6.0806242	5.5820236	5.4182028	5.1176403	4.0973141	5.4931429
0.5	1.333	0.5	1	σ23	4.3196604	4.29332	4.2498529	3.8092937	3.2861504	5.1315251
0.6	1.563	0.4	σ23	1	3.5678944	3.4579758	3.3452113	3.4130306	3.6451443	4.1792503
0.7	1.961				2.9793079	2.9637831	3.0127193	3.2968644	3.6518846	4.5352865
0.8	2.778				3.2951325	3.3280581	3.4815951	3.9921888	5.219171	4.8688205
0.9	5.263				3.1485282	3.1435499	3.1642096	3.3033385	3.7120704	4.9883704
0.95	10.256				npd	npd	npd	3.2570434	3.4724201	3.7765223

DGPs				Theor.A.Variance (σ13=0 and ρx,z is fixed)						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.2	100		0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.1	1.010				0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.2	1.042				0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.3	1.099		Σ		0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.4	1.190	1	0.5	σ13	0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.5	1.333	0.5	1	σ23	0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.6	1.563	σ13	σ23	1	0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.7	1.961				0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.8	2.778				0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.9	5.263				0.25000	0.24750	0.23438	0.18750	0.10938	0.02438
0.95	10.256				0.25000	0.24750	0.23438	0.18750	0.10938	0.02438

DGPs				Theor.A.Variance (σ13=0 and ρx,z is fixed)						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.1	1.010				0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.2	1.042				0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.3	1.099		Σ		0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.4	1.190	1	0.5	σ13	0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.5	1.333	0.5	1	σ23	0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.6	1.563	σ13	σ23	1	0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.7	1.961				0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.8	2.778				0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.9	5.263				0.06250	0.06188	0.05859	0.04688	0.02734	0.00609
0.95	10.256				0.06250	0.06188	0.05859	0.04688	0.02734	0.00609

DGPs				Theor.A.Variance (σ13=0 and ρx,z is fixed)						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.5	100		0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.1	1.010				0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.2	1.042				0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.3	1.099		Σ		0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.4	1.190	1	0.5	σ13	0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.5	1.333	0.5	1	σ23	0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.6	1.563	σ13	σ23	1	0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.7	1.961				0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.8	2.778				0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.9	5.263				0.04000	0.03960	0.03750	0.03000	0.01750	0.00390
0.95	10.256				0.04000	0.03960	0.03750	0.03000	0.01750	0.00390

DGPs				Theor.A.Variance (σ13=0 and ρx,z is fixed)						
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7	100		0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.1	1.010				0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.2	1.042				0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.3	1.099		Σ		0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.4	1.190	1	0.5	σ13	0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.5	1.333	0.5	1	σ23	0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.6	1.563	σ13	σ23	1	0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.7	1.961				0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.8	2.778				0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.9	5.263				0.02041	0.02020	0.01913	0.01531	0.00893	0.00199
0.95	10.256				0.02041	0.02020	0.01913	0.01531	0.00893	0.00199

DGPs					MC Variance (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.2	100		66492.781	1101.4618	11737.994	73.827481	6891.2754	4.770487
0.1	1.010				225825.76	338.91389	1516.6045	380538.79	23.878292	0.010736264
0.2	1.042				4471916.2	74.841618	1.7302786	0.97279381	0.051423791	0.005115914
0.3	1.099		Σ		798.31814	10032.825	23.415864	0.35319668	0.031686706	0.003758847
0.4	1.190	1	0.5	0	283.29118	6.3442391	0.95314625	0.091791999	0.024705901	0.003193268
0.5	1.333	0.5	1	σ23	1.6647343	0.4463629	0.18821251	0.0656987	0.018511446	0.002576779
0.6	1.563	0	σ23	1	4214992.8	0.61627489	0.18299142	0.058215141	0.016452886	0.002444963
0.7	1.961				9394.6568	677563.26	0.15033009	0.050127477	0.015077458	0.002423514
0.8	2.778				25.815298	7.9731683	0.52867877	0.045605375	0.013548624	0.002247961
0.9	5.263				0.37916633	0.13743016	0.075791504	0.034908929	0.012272989	0.002261793
0.95	10.256				0.058645277	0.051834822	0.042266865	0.026221187	0.011030835	0.002206227

DGPs					MC Variance (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.4	100		0.076279231	0.075444152	0.071767827	0.05884615	0.035377916	0
0.1	1.010				0.077472022	0.073748912	0.066019079	0.047717139	0.02355164	0
0.2	1.042				0.085397651	0.075385776	0.062128689	0.040527766	0.017813546	0
0.3	1.099		Σ		0.080869946	0.071891639	0.058659183	0.036379054	0.015150861	0
0.4	1.190	1	0.5	0	0.079348106	0.069512077	0.055649075	0.033465331	0.013483655	0
0.5	1.333	0.5	1	σ23	0.071228355	0.061879911	0.049076122	0.029107256	0.011608324	0.002003883
0.6	1.563	0	σ23	1	0.068382407	0.059292984	0.046879019	0.02760265	0.011044663	0.00195537
0.7	1.961				0.057090563	0.050238051	0.040612594	0.024988912	0.010572301	0.001909367
0.8	2.778				0.043236629	0.039478263	0.033536318	0.022100428	0.009799076	0.00201287
0.9	5.263				0	0.023361355	0.022086199	0.017289393	0.008993497	0.002105443
0.95	10.256				0	0	0	0.012214024	0.007781598	0.001980957

DGPs					MC Variance (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.5	100		0.04474573	0.044315822	0.04212233	0.034238356	0.020500073	0
0.1	1.010				0.04518881	0.043759948	0.04011186	0.0300939	0.015645417	0
0.2	1.042				0.04459807	0.042383695	0.03777137	0.026842957	0.012820451	0
0.3	1.099		Σ		0.0458412	0.042798574	0.03718977	0.025290682	0.011539768	0
0.4	1.190	1	0.5	0	0.04558231	0.042103124	0.03609934	0.023996063	0.01065161	0
0.5	1.333	0.5	1	σ23	0.04212434	0.038693983	0.03297409	0.021730525	0.009581391	0
0.6	1.563	0	σ23	1	0.03977532	0.036719885	0.03150713	0.020915032	0.009317631	0.00177913
0.7	1.961				0.03394831	0.031735748	0.02780177	0.019268026	0.009080785	0.00180562
0.8	2.778				0.02595081	0.025024296	0.02294882	0.017088406	0.008524487	0.00181559
0.9	5.263				0	0	0	0.013255287	0.007843863	0.00194437
0.95	10.256				0	0	0	0	0.006683338	0.00201194

DGPs					MC Variance (ρx,z is fixed)					
				ρ1	0	0.1	0.25	0.5	0.75	0.95
ρ2	σz	ρχ,z	n	σχ	1	1.010	1.067	1.333	2.286	10.256
0	1	0.7	100		0.02146328	0.021272799	0.02021623	0.016370715	0	0
0.1	1.010				0.02160094	0.021218689	0.01985866	0.015456961	0	0
0.2	1.042				0.02120689	0.020701534	0.01919764	0.014632581	0	0
0.3	1.099		Σ		0.02175067	0.021114754	0.01941343	0.014521738	0.007265657	0
0.4	1.190	1	0.5	0	0.0215455	0.020869788	0.01915088	0.014256103	0.006981215	0
0.5	1.333	0.5	1	σ23	0.02031374	0.019692977	0.01812425	0.013577241	0.006842422	0
0.6	1.563	0	σ23	1	0	0.018434666	0.01727396	0.013283518	0.006983763	0
0.7	1.961				0	0	0.01562025	0.012572911	0.006997425	0
0.8	2.778				0	0	0	0.011218421	0.00662831	0
0.9	5.263				0	0	0	0	0.006054326	0.00175168
0.95	10.256				0	0	0	0	0	0.00176436