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‘ TESTING INVESTMENT STRATEGIES FOR STATISTICAL ARBITRAGE’

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ABSTRACT

In the following paper we are going to penetrate Market's equilibrium by introducing the concept of Statistical Arbitrage. Persistent anomalies that exist in the economy and cannot be fully explained by equilibrium models, will be put under the test of investment strategies that were designed to exploit circumstances like these. We will give the essence of Statistical Arbitrage, the theoretical base that constitutes it as well as the methodology that will be used for testing while the joint hypothesis dilemma is being bypassed without being invoked. Finally we will compare the abovementioned strategies concerning the statistical arbitrary level they produce.

Key words: market efficiency, stochastic processes, persistent anomalies, value - momentum strategies, zero initial cost, incremental trading profit, time-average variance, parameters' estimation, likelihood ratios.

SECTION 1

Introduction

Would it be possible for someone to make a fortune by fearlessly investing in securities, some money he had one day found on the street?

The first part of the sentence (would it be possible for someone) refers to a situation in which one or more incidents can or cannot occur, at the same or in distinctive time. The second part (to make a fortune) is modeling positive accumulated feedbacks. The third part (by fearlessly investing in securities) expresses the certainty that negative feedbacks are being eliminated. Finally the last part (some money he had one day found on the street) interestingly implies two things, the first would be the profound zero initial cost of the investment and the second and most important would be that the money that were found, were not missed enough by anyone. The rhetorical questions generated then would be: ‘’Can anyone make a fortune like this and how? Are there more money on the street waiting to be found and what does this imply for economy’s existence?’’

According to Steve Hogan, Robert Jarrow, Melvyn Teo and Mitch Warachka (2004), a long horizon trading strategy that generates riskless profit, with zero initial cost and is designed to exploit persistent anomalies constitutes a so called Statistical Arbitrage. Therefore we can state that Statistical Arbitrage tries to capitalize the statistical mispricing of one or more assets based on the expected value of the assets generated by a statistical model. Without any interference by a market efficiency or an equilibrium model, statistical arbitrage is a ‘phenomenon’ that has been empirically observed through testing value and momentum strategies. Most important, all conclusions were extracted after adjusting for transaction costs, influence of small stocks, margin requirements, liquidity buffers on short sales and higher borrowing rates.

SECTION 2

2.1 Value strategies

2.1.1 Common Risk factors on the returns of stocks

In our effort to capture the movements on stocks and bonds' returns, lot of studies were made. These studies' main objective was to determine the main variables that determine the returns' movements and to measure the scale of these movements in a more specific way. One of these studies and the first of such impact (after the CAPM theory) in financial history was the work by Eugene F. Fama and Kenneth R. French (1992).

Cross-section average returns on U. S. common stocks, after some filtering under the common asset-pricing model showed little relation either to the Sharpe's market βs or the consumption βs via the intertemporal of Breeden's asset-pricing model. Variables that were never at that time suggested such as Market Equity (number of company's shares times stock's price), leverage ratio, Earnings to Stock price ratio and Book to Market ratio (the book value BV of a company's shares to its' market value MV) showed that they were able to explain, the cross-section of average returns. In other words, these additional variables have significant reliable power.

Used in isolation the above new factors seemed to have explanatory power on stock returns. Used in combination to each other the explanatory power of E/P and the leverage ratio was absorbed by the ME and BV/MV factors (study on average returns of NYSE ,NASDAQ and AMEX stocks for almost a thirty year period confirmed that).

Provided that markets are integrated there should be a formula that would have enough explanatory power for stocks' returns as well as for bonds' returns. The new factors that were suggested above could not have a significant role on bonds' returns by all means. Bonds could only be interpreted through a maturity factor and a risk default factor. Indeed studies showed that these two factors on bonds' returns capture most of the variation.

Concerning the differences between the average returns and the one-month bills, by generating regressions that include the size and book to market value factors upon selected portfolios, slopes on the market factor were produced that were close to 1. This meant that risk premium for the market factor could explain the returns on stocks and bills. As for bonds the average returns were explained by the two new bond term factors, producing a slope in the regression used on the excess bond returns close to 0. Even though stocks' new factors could not explain bonds' returns, they seemed to have a reliable power on interpreting low-grated firms' bonds and on the other hand bonds' term-structure factors used alone, could capture a strong variation on stocks' returns. In that way a five risk factor model that could have enough explanatory power on average returns was born.

2.1.2 Stocks' three-factor model

Most investors are comfortable with the notion that taking higher levels of risk is necessary to expect to earn higher returns. But why should riskier companies have higher returns? Intuitively an investor would require a higher expected return in exchange for accepting greater risk. It is observable that this relationship exists when we look back at historical long-run stocks' returns.

In our effort to capture the picture, we can imagine an investment that is expected to generate $1 million per year in the long-run. How much is someone likely to pay for this asset? The answer is far from obvious. It depends on the riskiness of the expected cash flows. With complete certainty that the cash flows will all be paid as promised, an investor would discount the asset at the risk-free rate. As the degree of uncertainty increases, the return required to justify the risk to be overtaken will be much higher, resulting in a much lower price the investor would be willing to pay, simply because of the higher required discount rate.

After relative documentation on stocks' returns, it was shown that firms with a low stock price in relation to book value (high BE/ME) were having a low earnings on assets performance. On the other hand, firms with low BE/ME were showing a high earnings performance that persisted for a long period. Considering the 1980-'82 recession, where the earnings of small sized firms were devastated, the earnings of small firms before were slightly less than the earnings of big firms.

This leads us to a conclusion that the size factor is able to explain the negative relation between average returns and size and the BE/ME factor can stand up for explaining the positive relation between average returns and book-to-market equity values. Common variation on stocks' returns in relation to fundamentally analyzed Size and BE/ME elements can proxy as common risk factors.

2.1.3 Portfolio building and analysis

One widely accepted measure of risk is volatility, the amount that an asset's returns varies through successive time periods and is most commonly quoted in terms of the standard deviation of returns. An asset whose return fluctuates dramatically is perceived to have a wider range of potential future values. Volatility can be effectively reduced without significant cost by diversifying our portfolio with assets adding procedures or by changing the weights on the existing ones. Having enough assets in our portfolio, its volatility will ultimately match Market's volatility. In that case investors can only expect to be compensated for the risk that cannot be diversified, the systematic risk.

An asset exhibits both systematic and unsystematic risk. The portion of its volatility which is considered systematic is measured by the degree to which its returns vary relative to those of the overall market. To quantify this relative volatility, a parameter called beta was conceived as a measure of the risk contribution of an individual asset to a well diversified portfolio.

The equation that describes the asset's returns and that is used for the regression is:

R_a-R_f= α+β(R_a-R_f)+ε, (2.1)

where α stands for potential value addition beyond theory's prediction and ε stands for regression's residuals.

As we examine the more complex three factor model the estimated equation is:

R_a-R_f= α+ β(R_a-R_f)+dSMB+zHML+ε, (2.2)

where similarly β measures the exposure of the asset to market factor, d measures its exposure to size factor and z its exposure to value factor.

Economic data should be organized and be studied under a specified method. This method should also be able to mimic the underlying risk factors in returns related to size and BE/ME in order for the procedure to have a fundamental meaning.

The importance of the size factor could be established through the split of the index (NYSE) into two groups of stocks. In the first group, stocks with ME smaller than the

NYSE's median ME were participating and the second group included the remaining stocks that they had ME bigger than NYSE's median ME. Therefore the SMB factor accounts for the size premium, which is the additional return investors have historically received by investing in stocks of companies with relatively small market capitalization.

The book-to-market factor was established upon the categorization of the index (NYSE) into three equity groups. The first group involved firms with BE/ME that represented the low 30% ranked values, the second group firms that represented the medium 40% ranked ones and the third group firms that were on the top 30%. Therefore the HML factor accounts for the value premium, which is the additional return investors have historically received for investing in companies with high BE/ME values.

The constructed portfolios were the S/L, S/M, S/H, B/L, B/M, and B/H. The S/L portfolio for example holds stocks that participate in the small group which participate in the low BE/ME group also. The explanation for the rest is similar.

The SMB (small minus big) portfolio was set to capture the risk factor in returns related to size by presenting the monthly difference of the average returns between the small portfolios (S/L, S/M, S/H) and the big ones (B/L, B/M, B/H). The HML (high minus low) portfolio were set to capture the risk factor of average returns in relation to BE/ME by presenting similarly the monthly difference of average returns between high BE/ME portfolios (S/H and B/H) and low BE/ME ones (S/L and B/L) . As for the market risk factor the excess market return is used (Rm-Rf). The RM stands for the return of the value weighted portfolio of the six size BE/ME portfolios designed, including the portfolios that were rejected in the beginning for having negative BE, and the RF represents the one-month bill rate.

2.2 Contrarian Investments, Extrapolation and Risk

Value investment strategies that outperform the market based on the formation of portfolios that buy stocks with low price relative to earnings, dividends, historical prices, book assets, cash flow to price ratio and other measures, are under argue with portfolios that are formed in the basis of past losers can be today's winners, over subsequent several years.

Graham and Dodd(1934), Dreman(1977), Chan, Hamao and Lakonishok(1991) and Fama and French(1992) are the main supporters of the value strategies concerning price over book measures mentioned above. De Bondt and Thaler (1985, 1987) are some of the major expressers of the past losers-today's winners theory. Despite all criticism upon the value strategies(Chan, Ball, Kothari), their offered analysis managed to show that stocks with high book relative to market values of equity outperform the market (Chopra, Lakonishok, Rosenberg, Reid and Lanstein).

Analysts start feeling uncomfortable when they try to confront the reasons of value strategies' excess returns. Value strategies might produce higher returns cause of their contrarian character against the naïve strategies followed by other investors. Naïve strategies might range from extrapolating past earnings growth too far into the future to assume a trend in a stock price, to produce an overreaction to bad or good news or to equate a good investment with a well-run company regardless its price. Finally major arguments are taking place concerning the higher fundamental risk bearing of a value strategy that simply enjoys a compensation of higher average returns at the end.

2.2.1 Methodology

The methodology used to define all the above mentioned quotes, measures in the first case the past growth and the expected future growth. Contrarian models in their majority actually are consistent with the predictions over the differences between expected future growth rates and their link to past growth rates. Overestimation of actual future growth by the naïve strategies is resulted.

Regarding to the second set of argument, the methodology describes the necessity of existence of a certain frequency of underperformance of value stocks in comparison to glamour ones. Then the risk factor would be revealed and it would take its full meaning in the countries with high marginal utility of wealth. The result in this case also favors the non-existence of any view that value strategies are fundamentally riskier than glamour ones, especially in the long-run.

Overpriced-glamour stocks are those which first have performed well in the past and second are expected from the market to perform well in the future. Similarly the underpriced ones or value stocks are the ones that have performed poorly in the past and are expected to continue to perform poorly. Value strategies that bet against those investors who extrapolate past performance too far into the future, produce superior returns.

2.2.2 Portfolio formation

Within each portfolio formed by returns' decile, stocks were equally weighted and returns were computed by using annual buy and hold strategy for 1 to 5 years relative to the time of formation. If a stock disappeared, its return would be replaced by the return of a corresponding deciled size portfolio until the end of the year. At the end of each year the portfolio was rebalanced and each surviving stock got the same weight. To adjust portfolio returns for size, the market capitalization decile at the end of previous year was obtained for each stock in the sample. A size benchmark return for each portfolio was needed and was constructed by replacing each stock's return in the portfolio with an annual buy and hold return on an equally weighted portfolio of all stocks in its size decile for that year. Then the returns were equally weighted across all original portfolio's stocks and that constituted the original portfolio's benchmark. The annual size-adjusted return on the original portfolio is then computed as the difference of its return to its year's benchmark return.

Considering growth rates the case of computing earnings' growth from year to year was used. The investment rule was 1$ per stock at the end of each year. The proportion of each firm owned was then 1/firm's market capitalization (yearly measured). Afterwards the generated earnings per dollar were calculated for each stock at the end of each year. Total firm's earnings would be multiplied by the proportion of the firm owned, all stocks' results would sum up and be divided by the number of the stocks in the portfolio. In order to avoid negative or close to zero yearly growth rates, a new calculation was needed. Average rates for each period were used and the difference between year t and year t-1 as a proportion of the average rate of year *t-1* was resulted. The calculation of the analogous rates in cash flows and sales was conducted. Finally accounting ratios were produced such as earnings to price and cash flow to price ratios. For the classifications under the conditions of these ratios, only the stocks with positive ones were considered due to the impossible interpretation of the negative ones. But for other purposes these ratios were computed for the entire equally weighted portfolios and then averaged across all formation periods in order not to eliminate individual stocks in the portfolio that have negative values for the variable. This resulted into a ratio of a cash flow per invested 1$ in the portfolio with each stock receiving the same dollar investment.

2.2.3 Possible explanations on value strategies

Two alternative theories have been proposed to explain why value strategies have produced higher returns in the past. The first theory says that they have done so because they exploit the mistakes of naive investors. The previous section showed that investors appear to be extrapolating the past too far into the future, even though the future does not warrant such extrapolation. The second explanation of the superior returns to value strategies is that they expose investors to greater systematic risk. In the section, they examine this explanation directly.

Value stocks would be fundamentally riskier than glamour stocks if, first, they underperform glamour stocks in some states of the world, and second, those are on average "bad" states, in which the marginal utility of wealth is high, making value stocks unattractive to risk-averse investors. This simple theory motivates their empirical approach.

To begin, they look at the consistency of performance of the value and glamour strategies over time and ask how often value underperforms glamour. They then check whether the times when value underperforms are recessions, times of severe market declines, or otherwise "bad" states of the world in which the marginal utility of consumption is high. These test o not provide much support for the view that value strategies are fundamentally riskier. Finally, they look at some traditional measures of risk, such as beta and the standard deviation of returns, to compare value and glamour strategies.

While one can never reject the "metaphysical" version of the risk story, in which securities that earn higher returns must by definition be fundamentally riskier, the weight of evidence suggests a more straightforward model. In this model , out-of-favor (or value) stocks have been underpriced relative to their risk and return characteristics, and investing in them has been indeed earned abnormal returns.

This conclusion raises the obvious question: how can the 10 to 11 percent per year in extra returns on value stocks over glamour stocks have been persisted for so long? One possible explanation is that investors simply did not know about them. This explanation has some plausibility in that quantitive portfolio selection and evaluation are relatively activities. Of course, advocacy of value strategies is decades old, going back at least to Graham and Dodd (1934). But such advocacy is usually not accompanied by defensive statistical work and hence might not be entirely persuasive, especially since many other, strategies are advocated as well.

Another possible explanation is that they have engaged in data snooping (Lo and MacKinlay (1990)) and have merely identified an ex post pattern in the data. Clearly, these data have been mined in the sense that others have looked at much of these same data before them. On the other hand, they think there is good reason to believe that the cross-sectional return differences that they reported, reflect an important economic regularity rather than sampling error. First, similar findings on the superior returns of value strategies have been obtained for several different time series. Davis (1994) finds similar results on a subsample of large U.S. firms over the period 1931 to 1960. Chan, Hamao an Lakonishok (1991) find similar results for Japan. Capaul, Rowley and Sharpe (1993) find similar results for France, Germany, Switzerland and the United Kingdom, as well as for the United States and Japan.

Second, they have documented more than just a cross-sectional pattern of returns. The evidence suggests a systematic pattern of expectational errors on the part of investors that is capable of explaining the differential stock returns across value and glamour stocks. Investors expectations of future growth appear to have been excessively tied to past growth despite the fact that future growth rates are highly mean reverting. In particular, investors expected glamour firms to continue growing faster than value firms, but they are systematically disappointed. La Porta (1993) shows that a similar pattern of expectational errors and returns on value strategies obtains when growth expectations are measured by analysts' 5-year earnings growth forecasts rather than by financial ratios such as E/P or C/P. The evidence on expectational errors supports the view that the cross-sectional differences in returns reflect a genuine economic phenomenon.

Individual investors might focus on glamour strategies for a variety of reasons. First, they may make judgment errors and extrapolate past growth rates of glamour stocks, such as Walmart or Microsoft, even when such growth rates are highly unlikely to persist in the future. Putting excessive weight on recent past history, as opposed to a rational prior, is a common judgment error in psychological experiments and not just in the stock market. Alternatively, individuals might just equate well-run firms with good investments, regardless of price. After all, how can you lose money on Microsoft or Walmart? Indeed, brokers typically recommend "good" companies with "steady" earnings and dividend growth.

Presumably, institutional investors should be somewhat more free from judgment biases and excitement about "good companies" than individuals, and so should flock to value strategies. But institutional investors may have reasons of their own for gravitating toward glamour stocks. Lakonishok Shleifer and Vishny (1992b) focus on the agency context of institutional money management. Institutions might prefer glamour stocks because they appear to be "prudent" investments, and hence are easy to justify to sponsors. Glamour stocks have done well in the past and are unlikely to become financially distressed in the near future, as opposed to value stocks, from the universe of stocks they pick. Indeed, sponsors may mistakenly believe glamour stocks to be safer than value stocks, even though, as they have seen, a portfolio of value stocks is no more risky. The strategy of investing in glamour stocks, while appearing "prudent", is not prudent at all in that it earns a lower expected return and is not fundamentally less risky. Nonetheless, the career concerns of money managers and employees of their institutional clients may cause money managers to tilt towards "glamour" stocks.

Another important factor is that most investors have shorter time horizons than are required for value strategies to consistently pay off (De Long et al. (1990) and Shleifer and Vishny(1990)). Many individuals look for stocks that will earn them high abnormal returns within a few months, rather than 4 percent per year over the next 5 years.

Institutional money managers often have even shorter time horizons. The often cannot afford to underperform the index or their peers for any nontrivial period of time, for if they do, their sponsors will withdraw the funds. A value strategy that takes 3 to 5 years to pay off but nay underperform the market in the meantime (i.e., have a large tracking error) might simply be too risky for money managers from the viewpoint of career concerns, especially if the strategy itself is more difficult to justify to sponsors. If a money manager fears getting getting fired before a value strategy pays off, he will avoid using such a strategy. Important, while tracking error can explain why a money manager would not want too strong a toward value given its apparently superior risk/return profile. Hence, these horizon and tracking error issues can explain why money managers do not more aggressively "arbitrage" the differences in returns across value and glamour stocks, but they cannot explain why such differences are there in the first place. In their view, such return differences are ultimately explained by the tendency of investors to make judgmental errors and perhaps also by a tendency for institutional investors to actively tilt toward glamour to make their lives easier. Are the anomalous excess returns on value stocks likely to persist?

It is possible that over time more investors will become convinced of the value of being a contrarian with a long horizon and the returns to value strategies will fall. Perhaps the recent move into disciplined quantitative investment strategies, evaluated based only on performance and not o individual stock picks, will increase the demand for value stocks and reduce the agency problems that result in picking glamour stocks. Such sea changes rarely occur overnight, however. The time-series and cross-country evidence support the idea that the behavioral and institutional factors underlying the higher returns to value stocks have been pervasive and enduring features of equity markets.

2.3 Momentum strategies

Is Fama-French three factor model capable to explain drifts in future stocks' returns that arise when we examine past returns and surprise earnings, under the scope of Market risk, book-to-market and size effect? According to Louis K. C. Chan, Narasimhan Jegadeesh and Josef Lakonishok, markets miss-forecast future earnings of stocks with the worst past performance and only gradually respond to new information. Is there a connection between markets' under reaction and predictability of returns, when controlling for past returns and earnings in surprise?

Long term past losers outperform long term past winners over a period of three to five years, according to DeBondt and Thaler. On the other hand Jegagadeesh and Titman document that past winners continue to outperform past losers in an intermediate horizon of three to twelve months. These momentum strategies are well-implemented by professional analysts-investors and are established as an investment tool of world wide popularity.

These long term reversals have competing explanations such as microstructure biases that have major impact on low-priced stocks or time-variation in expected returns. Short term reversals can be tested for lead-lag effects between stocks or a bid-ask spread explanation power that derives from the tendency of an investor to over react.

Differences across stocks' past price performance tend to appear as differences in their book-to-market value of equity. In contradiction stock price momentum is not the same case, considering the difficulty in finding logical roots with enough explanatory power to support its existence. Fama and French (1993) tried to rationalize Jegagadeesh and Titman's strategies by related empirical regularities but failed to account for their profitability, facing at the same time lack of a credible factor with enough explanatory power. The absence of a logical explanation might be suggesting that momentum strategies cannot work out-of-samples and that a momentum strategy can be only considered as a statistical fluke.

Is it natural to investigate earnings in order to reveal the source of predictability of future stock returns? By examining the correlation between momentum in stock prices and market's under-reaction to earnings-related information, Bernard and Thomas, Thomas and Wahlen among others found that firms which reported unexpectedly high earnings outperformed, over a period of about six months, firms that announced unexpectedly poor ones. According to Lakonishok, profitability of momentum strategies is explained by the component of medium-horizon returns that are related to earnings news. This of course gives us the issue of how profitable a momentum strategy can be if past innovations in earnings are finally accounted.

Another approach by Affleck-Graves is the Value Line timeless ranking system. This engages past earnings and price momentum to account for the predictability of future returns. The possibility that arises from market's over-reaction over the fact of positive feedbacks, allows momentum strategies, by trend-chasers, to reinforce movements in stock prices even in the absence of fundamental information.

The general idea of the aforementioned paper consists on how momentum strategies can benefit from an under or over-reaction to information. An earnings momentum strategy can benefit in short-term by an under-reaction of the market to current information and on the other hand a price momentum strategy can benefit by a market's slow reaction to a wider range of information such as long-term profitability. Measuring the economic earnings of a firm does not necessarily mean that we can account the firm's future prospects. A high stock price against low earnings, may stand for capitalized information of a future profitability which leads by all means to a momentum success.

Furthermore in finance literature the most common way of measuring earnings surprises is by standardizing the unexpected earnings. The model used for such cases often enables the risk of specification errors. Against this idea, analysts' forecasts provide a more direct measure of expectations even though they are not widely used. Tracking changes in analysts' forecasts is also considered to be a popular technique among investment managers. Abnormal returns around earnings announcements is another way of unraveling market's thoughts upon news even though Foster, Olsen and Shevlin via their work found that residual returns (asset's excess return minus beta times benchmark's excess return) around announcement date have no explanatory power.

2.3.1 Methodology and variable analysis

Primary stocks that were listed on NYSE, AMEX and Nasdaq were used. Information about earnings news and past returns were taken by CRSP (Center of Research on Security Prices) and COMPUSTAT. For an almost twenty year period, stocks were ranked according to their earning news and past returns and finally after all variables formulated, each stock was assigned to one of ten portfolios formed.

The earnings momentum strategy involved reported earnings within the prior three months and this was used as a breakpoint. The three month period accomplished to take into account all earning announcements and furthermore the stocks were all equally-weighted within created portfolios. For price momentum strategy, compound returns of a six month period prior to portfolios' formation were used.

The variables that were used for the EMS were of three kinds. The first one was the SUE (standardized unexpected earnings):

SUE_{it}= (e_{iq}-e_{iq-4})/σ_{it}, (2.3)

were t is time, i is the stock, e_{iq} is quarterly earnings most recently announced, e_{iq-4} is earnings per share four quarters ago and σ_{it} is the standard deviation of unexpected earnings' difference in brackets for the last eight months.

The second was:

ABR_{it}= $\sum_{j=-2}^{+1}(r_{ij} - r_{mj})$ (2.4)

were r_{ij} is stock's i return on day j (with earnings announced on day 0) and r_{mj} the return on an equally-weighted market index. Cumulated returns up to one day after the earnings announcement in order to capture any market's delayed reaction upon earnings news. And the third one was:

$REV6_{it}$= $\sum_{J=0}^{6}\frac{(Fit-j)-(Fit-j-1)}{Pit-j-1}$ (2.5)

were it stands for a six month moving average of past changes in analysts' earning forecasts. When the forecasts are not revised for a month, they take the value of 0 for this month. The F_{it} is the mean's estimation in month t on stock's i earnings for the current fiscal year, which finally is being divided by prior month's stock price. Nevertheless as long as we consider analysts' estimations, we may enjoy the luxury of not needing an expected return's model but we suffer the possibility of enhancing colored incentives such as brokerage commissions.

To control for the spuriously related returns over contiguous intervals due to bid-ask bounce, the portfolio's performance starts being measured after the first five days of formation. During holding period delisted stocks are being replaced by a value-weighted index and at the end of each period portfolio's stocks are being rebalanced to equal weights in order to calculate returns in subsequent periods. Finally the attribution of book-to-market value of equity and the ratio of cash flow (earnings plus depreciation) to price, is reported at the time of portfolios' formation.

2.4 Returns on buying winners and selling losers- Implications for stock market efficiency

Over/underreaction to a specific situation, positive or negative, tends to be a 'normal' reaction of a neural system, that tries by all its means to be ensured against the uncertainty of its next moment. Coming to the world of financial instruments and especially the stock market's world, the over/underreaction tendency could not be absent.

The common logic in the financial world subscribes that in order to profit, we must buy a stock that performs and sell the one that underperforms. The questions that arises and cannot be explained by common logic is how performing ones will behave tomorrow, what does today's underperformance mean for the future and finally, is this 'gap' compatible to market efficiency?

Narasimhan Jegadeesh and Sheridan Titman (1993) by referring to De Bondt and Thaler's works (1985, 1987) support that overreaction/under reaction of stock prices to information is a direct extension of individuals' common logic. Nevertheless this same reaction is responsible for abnormal returns achieved by contrarian strategies. Stocks that performed poorly the previous three to five years, outperform the same period's good performed ones over a holding period of three to five years. In other words long-term losers outperform long-term winners. Furthermore Jegadeesh and Lehmann (1990) provide evidence that contrarian strategies with stocks' data of previous week or month generated significant abnormal returns. The argue on that was whether these transaction sensitive returns reflect overreaction or price pressures and lack of liquidity.

On the other hand Levy's work (1967) on strength strategies-of buying past winners and selling past losers- presented abnormal returns following a simple trading rule of buying stocks with current prices substantially higher than their average prices over the last 27 weeks. Additionally Grinblatt and Titman (1989,1991) showed that the majority of mutual funds has the tendency of buying stocks with an increased price over the previous quarter. Finally Value Line rankings are known to be based in large part on past relative strength, providing suggestive evidence of generating abnormal returns. A price momentum factor is being created by dividing the stock's latest ten-week average relative price by its fifty two-week average relative one designing in such a way a discriminative price performance rank among stocks.

More recent papers by Jegadeesh (1990) and Lehmann (1990) provide evidence of shorter term return reversals. These papers show that contrarian strategies that select stocks based on their returns of the previous week or month generate significant abnormal returns. However, since these strategies are transaction intensive and are based on short term price movements their apparent success may reflect the presence of short term price pressure or a lack of liquidity in the market rather than over reaction.

Jegadeesh and Titman (1991) provide evidence on the relation between short term return reversals and bid-ask spreads that supports this interpretation. In addition Lo and MacKinlay (1990) argue that a large part of the abnormal returns documented by Jegadeesh and Lehmann is attributable to a delayed stock price reaction to common factors rather than to overreaction.

2.4.1 Trading strategies

The trading strategies that were followed and examined in relation to over/underreaction to information, suggested a stock selection on the basis of their returns in previous 1, 2, 3, 4 quarters in row and holding periods that vary from one to four quarters. That gave in total 16 strategies for examination. In order to avoid price pressure and lagged reactions, 16 more portfolios were formed on the previous basis but with one substantial difference, a week was skipped between portfolio formation period and holding period.

Therefore when refer to j-month/k-month portfolio it means the portfolio started at time t, by selecting stocks according to their returns in previous j months and with a holding period of k months. At the beginning of each month t the securities were ranked in ascending order on the basis of their returns in the past j months. After ranking, ten decile portfolios were formed that equally weighted the stocks contained in the top decile, the second decile etc. The top decile portfolio was called 'losers' decile and the bottom one 'winners' decile. In each month t the strategy buys the winner portfolio and sells the loser, holding this position for k months. Also the strategy closes out its position initiated in month t-k and by that revise the weights-for equal weights' maintenance- on 1/k of the securities in the entire portfolio in any given month and carry over the rest from previous month. The profits' calculation was made not only for the buy and hold portfolios but also for the monthly rebalanced ones.

2.5 Industries and momentum

Both investment theory and its applications to investment management critically depend on our field's understanding of stock return persistence anomalies. Determining whether these anomalies are rooted in behavior that can be exploited by more rational investors at low risk has profound implications for our view of market efficiency and optimal investment policy.

The ability to outperform buy and hold strategies by acquiring past winning stocks and selling past losing ones, commonly referred to as 'individual stock momentum', remains one of the most puzzling of these anomalies, both because of its magnitude (almost 12 percent abnormal returns per dollar long on self financing strategy per year) as well as its peculiar horizon pattern that it seems to follow.

Trading based on individual stock momentum appears to be a poor strategy when using a short historical horizon for portfolio formation (especially less than a month) it is highly profitable at intermediate horizons (up to twenty four months and most strongest in the six to twelve months) and it is once again a poor strategy at long horizons.

According to Tobias J. Moskowitz and Mark Grinblatt (1999) there are strong evidence that persistence in industry return components generates significant profits that may account for much of the profitability of individual stock momentum strategies.

Industry portfolios exhibit significant momentum, even after controlling for size, book to market value, individual stock momentum , the cross sectional dispersion in mean returns and potential microstructure influences.

Once returns are adjusted for industry effects, momentum profits from individual equities are significantly weaker and for the most part are statistically insignificant. Industry momentum strategies are more profitable than individual stock momentum strategies while by robusting to various specifications and methodologies, they appear to be profitable even among the largest and most liquid stocks.

Profitability of industry strategies over intermediate horizons is predominately driven by the long positions. By contrast, profitability of individual stock momentum strategies is largely driven by selling past losers, particularly among the less liquid stocks.

Unlike individual stock momentum industry momentum is strongest in the short-term (one to three months) and then like individual stock momentum tends to dissipate after twelve months. Thus the signs of the short-term (less than one month) performances of the industry and individual stock momentum strategies are completely the opposite, yet the signs of their intermediate and long-term performances are identical.

The existence of industries as a key source of momentum profits may support the viability of behavioral models that have been offered for the individual stock momentum anomaly. Among these behavioral explanations is Jegadeesh and Titman's (1993) initial conjecture that individual stock momentum is driven by investor underreaction to information. Additionally recent behavioral theories rooted in investor cognitive biases have attempted to explain this phenomenon. Among them are Daniel, Hirshleifer and Subrahmanyam (1998), Barberis, Shleifer and Vishny (1998).

Behaviorally driven momentum profits should at least be constrained by the fact that some rational investors exist who may perceive momentum as an arbitrage opportunity. Rational investors can profit from their irrational counterparts at low risk with positions in large numbers of stocks if the bulk of investors persistently and irrationally under react to information that is sufficiently uncorrelated across firms. There are virtually no limits to this arbitrage if stock returns are generated by a factor model. A self-financing momentum portfolio that is long the high past return stocks and short the low past return stocks, could be created with zero factor risk.

Such a portfolio would have firm specific risk that was almost perfectly diversified away and because of momentum would enjoy a positive expected return. It seems unlikely that rational investors would not exploit such a low risk near arbitrage.

2.6 Possible explanations of momentum

There are several possible explanations for momentum. One is that momentum's higher returns are compensation for some unique risk associated with investments that have recently outperformed.

As of yet, no such risk factor has been convincingly identified. If it is not compensation for risk, the existence of momentum seems to challenge the efficient market hypothesis that past price behavior provides no information about future behavior.6 In other words, momentum is associated with some inefficiency in markets, perhaps due to investor behavior. Several possible behavioral explanations have been put forth

First, investors may be slow to react to new information. Efficient market theory assumes that once new information is released, it is instantly available to all investors and that prices immediately adjust to reflect the news. In practice, however, different investors (for example, a trader versus a casual investor) receive news from different sources, and react to news over different time horizons and in different ways.

Also, anchoring and adjustment is a behavioral phenomenon in which individuals update their views only partially when faced with new information ,slowly accepting its full impact. There is ample evidence supporting slow-reaction-to-information theories, ranging from market response to earnings and dividend announcements to analysts' reluctantance to update their forecasts.

Second, investors (as human beings) are prone to what behavioral economists and experimental psychologists call the disposition effect. Investors tend to sell winning investments prematurely to lock in gains, and hold on to losing investments too long in the hope of breaking even.

The disposition effect creates an artificial head wind when good news is announced, the price of an asset does not immediately rise to its value because of premature selling. Similarly, when bad news is announced, the price falls less because investors are reluctant to sell.

Third, investors are susceptible to the bandwagon effect(also called over-reaction). Short-term traders may use recent performance as a signal to buy or sell. Longer-term investors look to recent performance to confirm their convictions. The interaction between these investors can create price run-ups or -downs that can persist for many months until an eventual correction.9 Notable extreme examples include the technology bubble of the late 1990s and the energy rally of 2007-2008.

There continues to be a lively debate about the root causes of momentum. (A similar debate is ongoing for value investing as well). What is clear is that the overwhelming evidence from a range of markets, asset classes, and time periods supports the argument that momentum is neither a random occurrence nor an effect that disappears once the impact of transaction costs is incorporated

2.7 International Returns and value, momentum and size factors

In their paper, Fama and French examined North America, Europe, Japan and Asia Pacific's value premiums that in average stock returns, except from Japan, decrease with size. Except for Japan, there is return momentum everywhere and spreads in average momentum returns also decrease from smaller to bigger stocks. They tested whether empirical asset pricing models capture the value and momentum patterns in international average returns and whether asset pricing seems to be integrated across the four regions. Integrated pricing across regions, local models that use local explanatory returns provide passable descriptions of local average returns for portfolios formed on size and value versus growth. Even local models are less successful in tests on portfolios formed on size and momentum.

Banz (1981) found that stocks with lower market capitalization (small ones) tend to have higher average return. There were also evidence that value stocks, meaning stocks with high ratios of a fundamental like book value or cash flow to price, have higher returns than growth stocks, which have low ratios of fundamental to price. It is shown that the stocks that have done well over the past year tend to continue to do well. The value premium (higher average returns of value stocks relative to growth stocks) and momentum are also observed in international returns.

SECTION 3

3.1 Statistical Arbitrage’s philosophy

A large number of empirical studies have shown that stock prices tend to escape the market efficiency ‘picture’ and be consistent with continuous anomalies. Jegadeesh and Titman (1993) investigated a trading strategy that generated 12% excess return, under the CAPM, by selling poor-performing stocks and buying well-performing ones. Lakonishok (1994) reached into a similar conclusion by buying value stocks and selling glamour ones, using variables such as price to earnings ratios, book-to-market ratios, growth sales etc. Furthermore Chan (1996) confirmed excess returns of portfolios formed using past returns and earning announcements. The ‘phenomenon’ is out there, we just need to focus on it.

However Fama’s (1998) joint hypothesis problem and cautions against rejecting market efficiency is being bypassed with the absence of necessity for it to exist so the abnormal returns can appear on the scene. The only conjecture we need to make is to accept a misspecified equilibrium model. Finally the sensitivity of long-term (i.e. over five years) anomalies to the statistical methodology utilized is being treated with measurement of inferences during testing period.

Generating riskless profit in long-term horizon is a natural extension of already known strategies that are utilized in exploiting persistent anomalies. Therefore statistical arbitrage is the time-series analog of limiting arbitrage opportunity contained in Ross (1976) and its existence is inconsistent to any market equilibrium model. It rejects without invoking joint hypothesis. Similarly to arbitrage pricing theory on estimated covariances, the statistical arbitrage methodology utilizes historical data to detect its opportunities in the economy and furthermore it computes strategies’ probability of a loss.

3.2 Methodology used by Steve Hogan, Robert Jarrow, Melvyn Teo and Mitch Warachka (2004)

Momentum strategies that were tested in Jegadeesh and Titman (1993) and value strategies by Lakonishok (1994)-all original papers subsumed-were examined for a period of thirty five years. Six out of sixteen momentum strategies produced statistical arbitrage at the 5% level and three more at the 10% level, enclosing a probability of incurring a loss for longing the highest return and shorting the lowest return based on six months of past returns with a twelve month portfolio holding period, fell below 1% after just 89 months of trading. On the other hand five out of twelve value strategies that were examined produced statistical arbitrage at the 5% level and incurred a probability of a loss less than 1% after just 79 months of trading using past three years of sales growth with one year holding period, making market efficiency hypothesis not ‘feeling’ very comfortable.

All trading strategies tested above, incorporated the assumption that expected trading profits are constant over time (constrained mean version of stat. arbitrage). This assumption was chosen over the unconstrained mean version-variation of expected profits over time, whose complexity does not make things better-after figuring that rates of expected profits are often not statistically different from zero, the in sample as measured by the root mean squared errors for both (CM, UM) are indistinguishable, the sum of squared normalized residuals are almost identical and finally that likelihood ratio cannot reject the null hypothesis that expected returns are constant over time. Unavoidably to eliminate possible concerns of transaction costs that reset market efficiency, each portfolio’s turnover was computed and the results were combined with the estimated round-trip transaction costs.

3.3 Theoretical approach of Statistical Arbitrage

Self-financing trading strategies with zero initial cost, under the standard option pricing theory’s stochastic process [x(t),y(t): t≥0] were tested against market efficiency. These strategies involved x(t) units of a stock and y(t) units of a money market account at time *t*, formulated by using only available information on past returns, firm sizes, earnings announcements, market to book values and growth rates, without any equilibrium model required. By definition these trading strategies should have a zero initial cost [x(0)St+y(0)=0]. There is also the necessity of working with the discounted value of the cumulative trading profits, u(t)=V(t)/B*t*, were B*t* would be the money market account.

3.4 Black-Scholes-Merton world

Considering a non-dividend paying stock price S*t* that evolves in a standard BSM world as follows:

$St = S0e^{at - \frac{\sigma^2}{2} + \sigma Wt}$, (3.1) having a money market account of $Bt = e^{\mathrm{rt}}$.

The W*t* expresses a standard Brownian motion and α, σ^2 and r are non-negative constants with α>r. Considering a standard Brownian motion we mean a stochastic process that tries to model the change of a normal random variable from time t to time $t+1$. A fundamental property of SBM process is that increments on non-overlapping time intervals (touching each other only at their endpoints) are independent of one another meaning that knowing something about a change of a value over one interval, gives us no information about the change of that value over a non-overlapping next interval.

If we buy and hold a stock using the money market account then the value of our mini portfolio at time t would be :

V(t)= St- Soe^{rt} => V(t)= So[$e^{\left(a-\frac{\sigma^2}{2}\right)t+\sigma Wt} - e^{rt}$] (3.2) and u(t)= V(t)/ e^{rt} (3.3)

Thus the discounted expected trading profit of this evolving value as t approaches infinity would be:

E[u(t)]= So[$e^{(a-r)t}$-1]→∞ (3.4)

and its variance:

Var[u(t)]= $So^2 e^{2(a-r)t}(e^{\sigma^2\tau} - 1)$→∞ (3.5)

Time average variance would then be: Var[u(t)]/t →∞ as the numerator reaches infinity quicker than denominator. Any expected return would be shaded by the enormous risk taken.

3.5 Arithmetic Brownian Motion

Arithmetic Brownian Motion or Brownian motion with drifts, is a stochastic process that allows linearly growing means and variances to be proportional to time. Considering a discounted cumulative value of a strategy that evolves according to: u(t)= at+σWt with u(0)= 0, the expectation and variation of these profits would be:

E[u(t)]= at→∞ and Var[u(t)]= σ^2t→∞ (3.6)

The interesting part is the limitation of time-averaged variance which is now equal to : Var[u(t)]/t= σ^2.

Even though expected discounted trading profits increase over time, the aforementioned strategies ‘suffer’ from an increasing variance which is only limiting time-average one.

3.6 Capturing Statistical Arbitrage

Considering a trading interval (t_{k-1}, t_k) within which an arbitrage opportunity exists, the discounted trading profits would be:

u(t_k)-u(t_{k-1}) = μ+σZ_k (3.7)

where μ, σ>0 and Z_k to be i. i. d. random variables under zero mean and a variance of 1/k. The behavioral of this trading strategy is characterized by positive discounted trading profits (μ) in every interval and a random noise (σZ_k) which is diversifiable. As expected the variance of the noise is decreased over time.

For u(t)=0 the cumulative discounted trading profits at time t_n would be:

u(t_n)=

$$\sum_{k=1}^{n}[\mathrm{u}(t_k) - \mathrm{u}(t_{k-1})] = \mu n - \sigma \sum_{k=1}^{n} Z_k$$

(3.8)

The expected discounted trading profits and variance would be equal with:

E[u(t_n)]= μn and (3.9)

Var[u(t_n)]= $\sigma^2 \sum_{k=1}^{n}(\frac{1}{k})$ (3.10)

As *n* converges to infinity the expected profits and variance would converge also but when it comes to time-average variance we realize that:

Var[u(t_n)]/*n* = $\sigma^2 \sum_{k=1}^{n}(\frac{1}{k})$/*n*→0 as *n*→∞ . (3.11)

Positive discounted trading profits and a time-average variance that approaches zero is the idea that from the beginning gave the essence of a Statistical Arbitrage. Attempting to summarize all conditions that need to be fulfilled in order for a Stat. Arb. to exist, we end up with the following:

- u(0)= 0, which demonstrates the zero initial cost of our portfolio.
- $\lim_{t\to\infty}$ E[u(t)]> 0, that accounts for positive expected cumulative discounted trading profits.

- ☞ $\lim_{t\to\infty} \mathrm{P}[\mathrm{u(t)} < 0] = 0$, that stands for the probability of a loss that converges to zero.
- ☞ $\lim_{t\to\infty} \left(\frac{\mathrm{Var[u(t)]}}{t}\right) = 0$ if $\mathrm{P}[\mathrm{u(t)} < 0] > 0 \quad \forall \quad t < \infty$, representing the convergence of time-average variance to zero if the probability of a loss does not become zero in finite time.

If the probability of a loss for a specific transaction is zero ($\mathrm{P}[\mathrm{u(t)} < 0 = 0]$) then the specific transaction would apply in the standard arbitrage opportunity area where limitation exists in specific time. On the other hand if there is always a probability of encountering a loss and the time-average variance converges to zero in infinite time as the Sharpe's ratio $\left(\frac{\mathrm{E(R}p)-\mathrm{Rf)}}{\sqrt{var(Rp)}}\right)$ increases monotonically then a riskless incremental profit can be produced.

3.7 Critical observations

Ross's (1976) limiting arbitrage opportunity involves a limiting probability of a loss in a cross-sectional specific time while Statistical Arbitrage derives by limiting this probability across time. The necessity which becomes obvious is the currency denomination by the risk-free rate and the normalization by time, given the money market account in which all trading profits will be invested in order to serve to the reduction of each strategy's time-average variance. Furthermore at the end of a finite time *t* of the Stat. Arb. period, as the variance (per unit of time) will eventually become arbitrarily small, the only difference between the statistical arbitrage and a standard arbitrage opportunity will be an $\varepsilon = \mathrm{P}[\mathrm{u(t')} < 0]$.

As it was stated before statistical arbitrage methodology rejects without invoking an equilibrium model that sets market to efficiency. An efficient market that would be able to be explained by an equilibrium model would never highlighted investors that were willing to pursue arbitrage opportunities. Cochrane and Saa-Requejo (2000) invoke Sharpe's ratios to find asset prices that respond to an arbitrage opportunity while Bernardo and Ledoit (2000) exclude investments whose maximum gain-loss ratios are too attractive. Both of these approaches investigate acceptable trading opportunities that are too good for a reasonable investor to ignore, implying the market's hidden incompletion.

3.8 Jensen's Alfa and linear functions

Considering a mini-portfolio of one stock, its return based on the capital asset pricing model (E. Elton-M. Gruber-S. Brown-W. Goetzmann 2011) would be: R_p= α +$\beta_p R_m$+ε, where ε is the systematic risk which folds all market's risk that cannot be explained otherwise (error term). It is diversifiable to the extent possible but never totally and especially not in one market only (unlike unsystematic risk which is fully diversifiable as portfolio gets bigger). The level of its correlation to R_m gives us the accuracy of the equation. The more uncorrelated they are to each other, the more accurate the equation becomes. The R_m factor stands for market's return (benchmark in relation to its difference to the risk free rate) at the same time that β is our security's beta. Beta is a risk measure that arises from the relationship between the return on a stock and the return on the market. It is a 'constant' that measures the expected change in R_p (portfolio's return) given a change in R_m. For example a beta of 0.5 indicates that our stock's return will increase (decrease) by ½ of 1% if market's return increase (decrease) by 1%. The beta of the stock can be given as:

$$\beta = \frac{Cov(Rp-Rm)}{VarRm} \quad and \quad \alpha = E(Rp)\text{-}\ Rf\ \text{-}\ \beta_p(Rm - Rf), \quad given \ \ E(\varepsilon) = 0 \quad (3.12)$$

When it comes to α (Jensen's Alfa or ex-post alpha) we can support that it is the component of our stock's return that is independent of the market's performance-variable. We could also say that it can be divided into two parts (αi and εi) and that is the way that our abovementioned stock's return equation is formed. While beta represents the volatility of a security when compared to overall market, alfa represents a measure of excess return generated over what is expected by CAPM and it can take negative values also (when stock's return does not compensate enough over the average market's return given the security's beta). Maximizing the above mentioned equation in order to get the security's market line (optimal portfolio) we just need to solve for the alfa variable and beta coefficient that minimizes the $\sum \varepsilon^2$ (least squares linear regression). Accepting the analysis above we would expect every reasonable investor to behave in such a way.

According to Jensen (1976) economic trading profits reject market efficiency and by that Statistical Arbitrage satisfies this definition. Any portfolio bearing the same risk but stands above or under the SML is an arbitrary portfolio and by market's efficiency definition it should never exist. *Perhaps equilibrium models reflect efficient bubbles in arbitrary environments…*

3.9 Validating Statistical Arbitrage's methodology

It would be appropriate before examining the test for a Stat. Arb. to mention some key points which distinguish Stat. Arb. tests and the market's efficiency ones using a risk-adjusted α.

- As return's measurement requires an underlying model that sets market to an equilibrium, alpha becomes a test of market efficiency subject to joint hypothesis issue. Stat. Arbitrage test does not require any equilibrium to generate risk-adjustment or excess returns but only a currency denomination of incremental profits produced by a self-financing trading strategy.
- Risk-adjustment process implies a linear factor model (Fama & French 1993, Carhart 1997) unlike to Stat. Arb. test that can be applied to any asset even the ones that cannot be priced using linear factor models such as derivatives.
- Stat. Arbitrage test cannot be reduced to a t-ratio test- like alpha's test on the mean- and by that it is unable to detect the presence of a stat. arbitrage. This reduction would be implicitly assuming that the rate of change in profit's volatility would be zero, something that would be violating the critical fourth condition of Stat. Arbitrage.
- Stat. Arbitrage's theory incorporates a declining time-average variance, a condition that in alpha's test is not enacted.

Statistical Arbitrage's test starts by analyzing time series of currency denominated discounted cumulative trading profits produced by a trading strategy. Let u(t_1), u(t_2), ,… u(t_n) be these values and $\Delta\ u_i$ = u(t_i) - u(t_{i-1}) denote the increments of these cumulated trading profits at equidistant time points were Δ = t_i- $t_{(i-1)}$ and t_i = i × Δ.

Assuming that the discounted incremental trading profits satisfy:

Δu_i= μi^{θ}+ $\sigma i^{\lambda} z_i$ where i= 1,2,….,n and z_i are i.i.d of N(0, 1) random variables having a z_0= 0 with u(t_0) and Δu_0 are both zero.

As we try to generate undiscounted trading profits V(t) that will be invested in the risk free rate-asset, a constant amount of 1 € will be invested in the risky asset (long-short portfolio) and the value of that in statistical arbitrage opportunity would be 1 divided by 1 plus the trading profits (1/1+V(t)). The property of a declining marginal impact of risk free assets is consistent with a convex decreasing function when trading profits increase over time. This feature parallels the above mentioned i^{λ}. The λ<0 is an important condition of statistical arbitrage as the marginal decrease in the portfolio's volatility itself declines over time.

On the other hand σ is not needed to be a constant, being valid evolving as a Generalized Autoregressive Heteroskedasticity process (GARCH). The expectation and the variation of the above model would be : E[Δu_i]= μi^{θ} and Var[Δu_i]= $\sigma^2 i^{2\lambda}$. As by having λ<0 the fourth condition of the Stat. Arb. is being fulfilled, the μ>0 drives the dynamics of the discounted incremental trading profits' strategy to immediately begin trading and not wait for portfolio's volatility to decline. All lead to positive expected profit and a decreasing time-average variance.

The discounted cumulative trading profits generated by this strategy would be:

u(t_n)= $\sum_{i=1}^{n} \Delta u_i$~dN(μ$\sum_{i=1}^{n} i^{\theta}$, $\sigma^2 \sum_{i=1}^{n} i^{2\lambda}$) (3.13)

while log likelihood function for the abovementioned increments would be:

logL(μ, σ^2, λ, θ| Δu)= -1/2$\sum_{i=1}^{n} log\,(\sigma^2 i^{2\lambda})$ – 1/2σ^2 $\sum_{i=1}^{n} 1/\, i^{2\lambda}\, (\Delta u_i - \mu i^{\theta})^2$ (3.14)
allowing maximum likelihood estimation to generate the four required parameters.

Maximum likelihood estimation (MLE), given a fixed set of data and an underlying statistical model, selects the set of values of the model's parameters that maximizes the likelihood function.

This leads the score equations required to solve for the above to be:

$\frac{\partial \log L(\mu,\sigma^2,\lambda,\theta|\,\Delta u)}{\partial \mu}$: $\widehat{\widetilde{\mu}} = \frac{\sum_{i=1}^{n} \Delta u_i i^{\widehat{\widetilde{\theta}} - 2\widehat{\widetilde{\lambda}}}}{\sum_{i=1}^{n} i^{2(\widehat{\widetilde{\theta}} - \widehat{\widetilde{\lambda}})}}$ (3.15)

$\frac{\partial \log L(\mu,\sigma^2,\lambda,\theta|\,\Delta u)}{\partial \sigma^2}$: $\widehat{\widetilde{\sigma^2}} = 1/n \sum_{i=1}^{n} (\Delta u_i - \widehat{\widetilde{\mu}}\, i^{\widehat{\widetilde{\theta}}})^2 \frac{1}{i^{2\widehat{\widetilde{\lambda}}}}$ (3.16)

$\frac{\partial \log L(\mu,\sigma^2,\lambda,\theta|\,\Delta u)}{\partial \theta}$: $\sum_{i=1}^{n} \Delta u_i \log(i) i^{\widehat{\widetilde{\theta}} - 2\widehat{\widetilde{\lambda}}} = \widehat{\widetilde{\mu}} \sum_{i=1}^{n} \log(i) i^{2(\widehat{\widetilde{\theta}} - \widehat{\widetilde{\lambda}})}$ (3.17)

$\frac{\partial \log L(\mu,\sigma^2,\lambda,\theta|\,\Delta u)}{\partial \theta}$: $\widehat{\widetilde{\sigma^2}} \sum_{i=1}^{n} \log(i) = \sum_{i=1}^{n} \frac{\log(i)}{i^{2\widehat{\widetilde{\lambda}}}} (\Delta u_i - \widehat{\widetilde{\mu}}\, i^{\widehat{\widetilde{\theta}}})^2$ (3.18)

For θ=0 and λ=0 the standard normal MLE estimators for μ and σ would be:

$\widehat{\widetilde{\mu}} = 1/n \sum_{i=1}^{n} \Delta u_i$ and $\widehat{\widetilde{\sigma^2}} = 1/n \sum_{i=1}^{n} (\Delta u_i - \widehat{\widetilde{\mu}})^2$

Caution is needed for possible misspecification of the incremental trading profits' stochastic process which will only increase the likelihood of accepting the null hypothesis of no statistical arbitrage. Therefore a trading strategy is generating a statistical arbitrage by 1-*a* percent of confidence if the following conditions are satisfied:

H1: $\widehat{\widetilde{\mu}} > 0$

H2: $\widehat{\widetilde{\lambda}} < 0$

H3: $\widehat{\widetilde{\theta}} > \max\{\widehat{\widetilde{\lambda}} - 1/2, -1\}$

The sum of all individual p-values associated with the hypotheses above constitute an upper bound which must be below or equal to *a* (Type I error) to conclude that a trading strategy generates a Statistical Arbitrage. An λ< 0 satisfies the fourth condition as ensures that :

$\sigma^2 \sum_{i=1}^{n} i^{2\lambda} / n \to \infty$

while second condition requires the parameter μ> 0. As for θ, any value ensures that $\mu \sum_{i=1}^{n} i^{\theta} > 0$ provided μ >0. Finally the convergence of P(u(t)<0) to zero requires :

$$\frac{\sum_{i=1}^{n} i^{\theta}}{\sqrt{\sum_{i=1}^{n} i^{2\lambda}}} \to \infty \quad (3.19)$$

The tail probability converging to zero is whether $n^{\theta-\lambda+1/2}$ converges to infinity (using Bonferroni inequalities).

Summarizing we are now in position to declare that if:

- ✓ μ> 0 and θ> λ-1/2 → second and third conditions are satisfied ensuring that the probability of a loss converges to zero even though a θ= 0 does not reject the existence of stat. arb. as the situation when either μ or λ is zero
- ✓ λ< 0 → fourth condition is satisfied

then a Statistical Arbitrage opportunity arises.

3.10 Portfolios' formation

Portfolios are formed on the basis of momentum and value strategies and behave as follows:

MOM 3/3 denotes a momentum portfolio with a formation period of three months and a holding period of three months. Every three months the stocks are sorted based on the past three months of stocks' returns. Groups are being formed and each period we select the fifteen top stocks based on their three past months' returns against the fifteen bottom ones.

MOM 3/6 denotes a momentum portfolio with a formation period of three months and a holding period of six months. Every six months the stocks are sorted based on the past three months of stocks' returns. Groups are being formed and each period we select the twenty top stocks based on their past three months' returns against the twenty bottom ones.

MOM 3/9 denotes a momentum portfolio with a formation period of three months and a holding period of nine months. Every nine months the stocks are sorted based on the past three months of stocks' returns. Groups are being formed and each period we select the fifteen top stocks based on their past three months' returns against the fifteen bottom ones.

MOM 3/12 denotes a momentum portfolio with a formation period of three months and a holding period of twelve months. Every twelve months the stocks are sorted based on the past three months of stocks' returns. Groups are being formed and each period we select the fifteen top stocks based on their past three months' returns against the fifteen bottom ones.

The other portfolios are value portfolios and their characteristic variables are past one year book to market value (BM), cash flow to price value (CP), earnings to price value (EP) as well as past three year sales' growth (SALES).

Portfolio BM1 denotes the value portfolio that longs the top fifteen stocks in ranking sorted by their book to market value of the previous year, shorts the bottom fifteen ones and holds that spread for one year.

CP1 portfolio is the one that sorts the stocks by their past year's cash flow to price index and based on that ranking it longs the top fifteen stocks, shorts the fifteen bottom ones and holds that spread for one year.

Portfolio EP1 is the one that sorts the stocks by their past year's performance on earnings to price and based on that ranking it longs the top fifteen stocks, shorts the bottom fifteen and keeps that spread for one year.

Finally the SALES portfolio, sorts the stocks according to their past year's performance on sales' growth and based on that ranking it shorts the top fifteen ones and longs the bottom fifteen ones while it holds this spread for one year.

Each of the portfolios is managed as follows:

MOM 3/3

- The portfolio of the first quarter is being formed as we mentioned above based on the past three months of stocks' returns. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We move on a holding period of three months, were at the end of it and by not overlapping, we again sort our stocks based on the same characteristic and

place again a thousand euro on the top stocks, financed by shorting the bottom ones.

- Every three months along with sorting the stocks by their past three months returns, we calculate their return within the given period by the difference of each stock's price on the last quarter's day to its first . The top stocks' returns are responsible for our long position status and the bottom ones are responsible for our short position status.
- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first quarter. For the second quarter and beyond, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous quarter's portfolio multiplied by one plus the previous quarter's risk free rate (accumulated trading profit).
- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

MOM 3/6

- The portfolio of the first semester is being formed based on the past three months of stocks' returns. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We move on a holding period of six months, were at the end of it and by not overlapping, we again sort our stocks based on the past three months' returns and place again a thousand euro on the top stocks, financed by shorting the bottom ones.
- Every six months along with sorting the stocks by their past three months returns, we calculate their return within the given period by the difference of each stock's price on the last semester's day to its first . The top stocks'

returns are responsible for our long position status and the bottom ones are responsible for our short position status.

- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first period. For the second period and beyond, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous period's portfolio multiplied by one plus the previous period's risk free rate (accumulated trading profit).
- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

MOM 3/9

- The portfolio of the first period is being formed based on the past three months of stocks' returns. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We move on a holding period of nine months, were at the end of it and by not overlapping, we again sort our stocks based on the past three months' returns and place again a thousand euro on the top stocks, financed by shorting the bottom ones.
- Every nine months along with sorting the stocks by their past three months returns, we calculate their return within the given period by the difference of each stock's price on the last period's day to its first . The top stocks' returns are responsible for our long position status and the bottom ones are responsible for our short position status.
- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first period. For the second period and beyond, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous period's portfolio

multiplied by one plus the previous period's risk free rate (accumulated trading profit).

- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

MOM 3/12

- The portfolio of the first period is being formed based on the past three months of stocks' returns. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We proceed on a holding period of twelve months, were at the end of it and by not overlapping, we again sort our stocks based on the past three months' returns and place again a thousand euro on the top stocks, financed by shorting the bottom ones.
- Every twelve months along with sorting the stocks by their past three months returns, we calculate their return within the given period by the difference of each stock's price on the last period's day to its first . The top stocks' returns are responsible for our long position status and the bottom ones are responsible for our short position status.
- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first period. For the second period and beyond, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous period's portfolio multiplied by one plus the previous period's risk free rate (accumulated trading profit).

- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

BM1

- The portfolio of the first year is being formed based on the past year's stocks' performance. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We move on a holding period of one year, were at the end of it and by not overlapping, we again sort our stocks based on the past year's performance and place again a thousand euro on the top stocks, financed by shorting the bottom ones.
- Every one year along with sorting the stocks by their past year's BV/MV, we calculate their return within the given period by the difference of each stock's price on the last year's day to its first . The top stocks' returns are responsible for our long position status and the bottom ones are responsible for our short position status.
- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first year. For the second year and after, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous year's portfolio multiplied by one plus the previous year's risk free rate (accumulated trading profit).
- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.

- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

CP1

- The portfolio of the first year is being formed based on the past year's stocks' performance on cash flow to price. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We move on a holding period of one year, were at the end of it and by not overlapping, we again sort our stocks based on the past year's performance and place again a thousand euro on the top stocks, financed by shorting the bottom ones.
- Every one year along with sorting the stocks by their past year's CF/P, we calculate their return within the given period by the difference of each stock's price on the last year's day to its first . The top stocks' returns are responsible for our long position status and the bottom ones are responsible for our short position status.
- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first year. For the second year and after, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous year's portfolio multiplied by one plus the previous year's risk free rate (accumulated trading profit).
- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

EP1

- The portfolio of the first year is being formed based on the past year's stocks' performance on earnings to price. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to invest thousand euro to long the top stocks, financed by our short thousand euro position in the bottom twenty ones.
- We move on a holding period of one year, were at the end of it and by not overlapping, we again sort our stocks based on the past year's performance and place again a thousand euro on the top stocks, financed by shorting the bottom ones.
- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first year. For the second year and after, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous year's portfolio multiplied by one plus the previous year's risk free rate (accumulated trading profit).
- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

SALES1

- The portfolio of the first year is being formed based on the past year's stocks' performance on sales' growth. After the selection of the top fifteen stocks and the fifteen bottom ones, we select to short the top stocks and finance our thousand euro position in the bottom twenty ones.
- We move on a holding period of one year, were at the end of it and by not overlapping, we again sort our stocks based on the past year's performance and place again a thousand euro on the bottom stocks, financed by shorting the top ones.

- The current value of our portfolio is equal to the difference between our long and short positions but that only stands for the first year. For the second year and after, the current value of our portfolio is equal to the difference of our long and short position, plus the value of our previous year's portfolio multiplied by one plus the previous year's risk free rate (accumulated trading profit).
- For each period the current portfolio's value is being discounted by the rate that has been selected to represent the money market account on the same period.
- Each period except from the first one were no results can be produced, we control for the difference between the current discounted value of our portfolio and the previous one, to finally produce Δu which is our key material for the statistical arbitrage investigation.

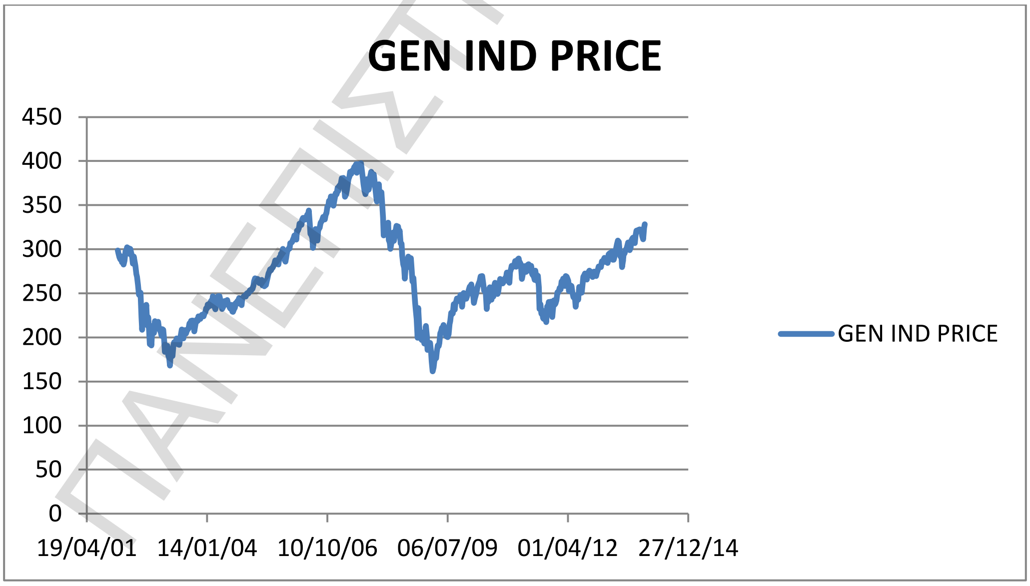
SECTION 4

4.1 Data selection

Data were selected with the optimal criterion of internationality. There was an effort to close the gap between 'failing' countries and 'dominant' states in order to capture the strength of a real remaining economy. That is why the Eurostoxx 600 was selected and more specifically the industrial and banking sector that participate in it. Unfortunately the Euro Stoxx 600 started only in 1998 and that is an obstacle in our effort to use the most representative index in Europe with the way that it should by going backwards many years.

The STOXX Europe 600 Index is derived from the STOXX Europe Total Market Index (TMI) and is a subset of the STOXX Global 1800 Index. With a fixed number of 600 components, the STOXX Europe 600 Index represents large, mid and small capitalization companies across 18 countries of the European region: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom.

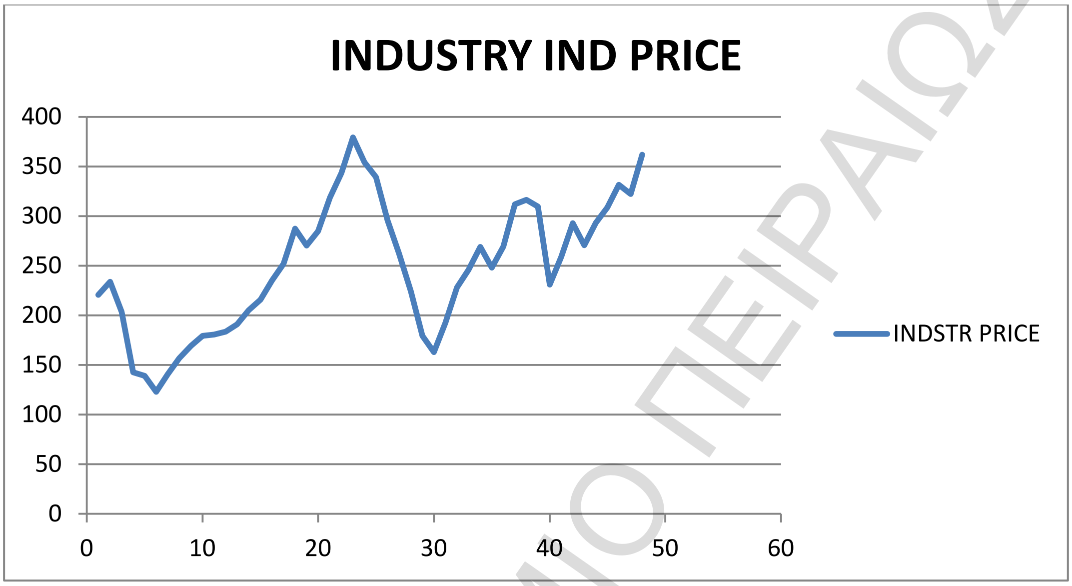
Chart 1. Eurostoxx 600 performance 19.04.2001 to 31.12.2013



There are ten industries, and derived from these – in increasingly finer classifications – there are19 super sectors, 41 sectors and 114 subsectors.

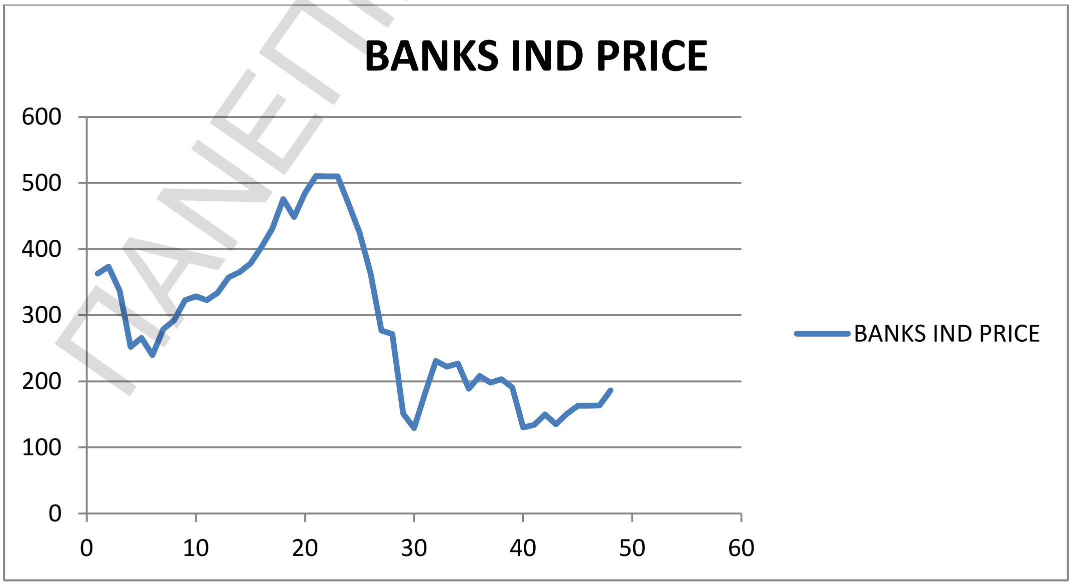
Each stock in the investable stock universe is uniquely classified, based on the company's primary revenue source, in one of the 114 subsectors. Consequently, it is automatically and uniquely classified into one of the 41 sectors, one of the 19 super sectors and one of the ten industries.

Chart 2. Industrial sector's performance from 01.01.2000 to 31.12.2013



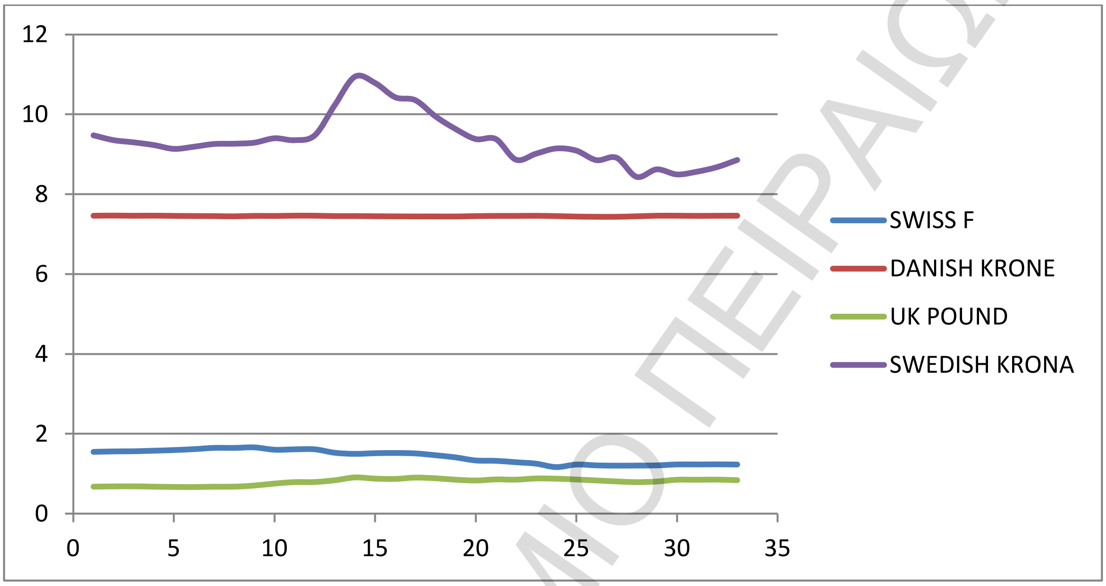
The industrial sector is consists of 126 components from which were selected the top hundred ranked. The banking sector consists of 47 institutions and the performance of it in the past seven years is seen as below:

Chart 3. Banking sector's performance from 01.01.2000 to 31.12.2013



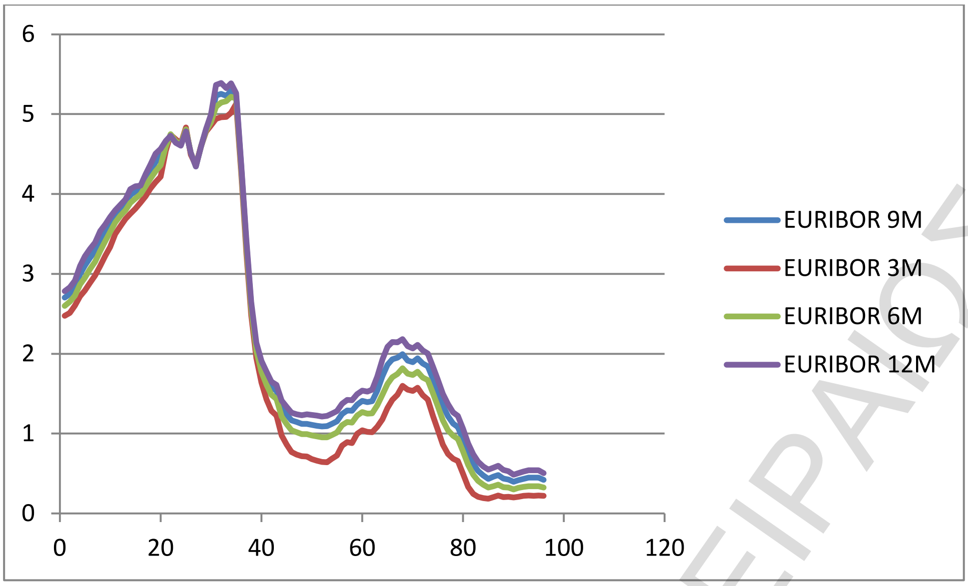
The problem of different currencies was solved by bringing the value of each stock to its euro equality by its currency's on date exchange rate.. The period that was finally selected was from first of January 1998 to thirty first of December 2013.

Chart 4. Fx rates from 01.01.2000 to 31.12.2013



Concerning the borrowing and lending rates that were used, there were: the Euribor 1 month rate for discounting our V(t) in each period so that the U(t) to be produced and the Euribor of three, six, nine and twelve months for accumulating our strategy's V(t) during each period by using its previous period's value. The rates' fluctuation during our sample period are presented in the following chart.

Chart 5. Borrowing and lending rates from 01.01.1998 to 31.12.2013



SECTION 5

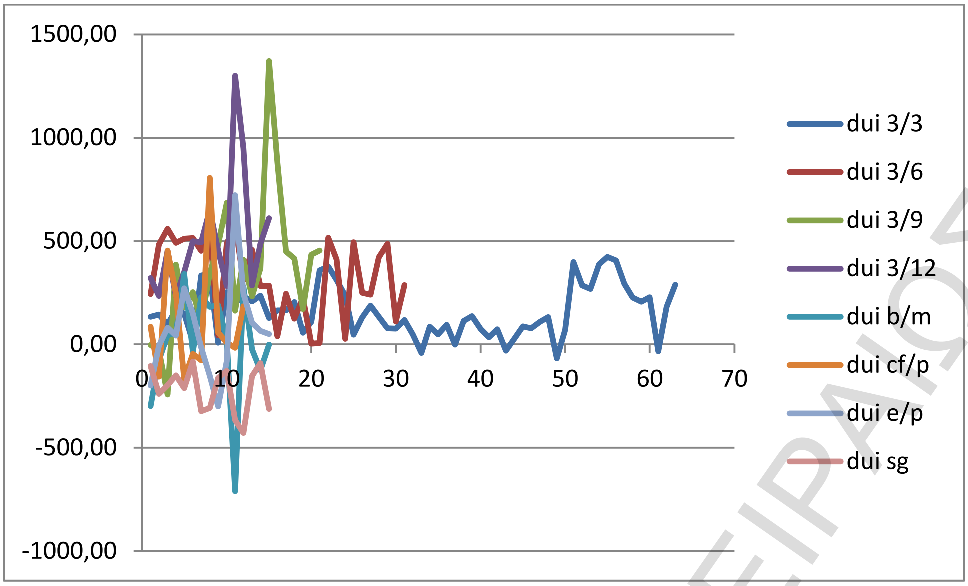
Data analysis

Our data were analyzed by all their characteristics as described in section three, with our target being to extract from all portfolios formed their Δu, the behavior of which will be analyzed under the statistical arbitrage's methodology. The table that follows shows our analysis' results for our portfolios.

Table 1. Δui results on Industrial portfolios

	Mean	Stdev	Max	Min
Δu 3/3i	159,44	121,23	423,19	-67,71
Δu 3/6i	327,56	186,86	581,62	3,88
Δu 3/9i	358,07	337,84	1370,21	-242,70
Δu 3/12i	507,19	288,76	1298,94	232,65
Δu b/mi	4,63	258,66	341,20	-710,17
Δu cf/pi	115,46	280,16	806,30	-167,55
Δu e/pi	65,44	238,85	722,13	-299,22
Δu sgi	-216,82	107,68	-82,84	-427,84

Chart 6.Industry sector's Δu

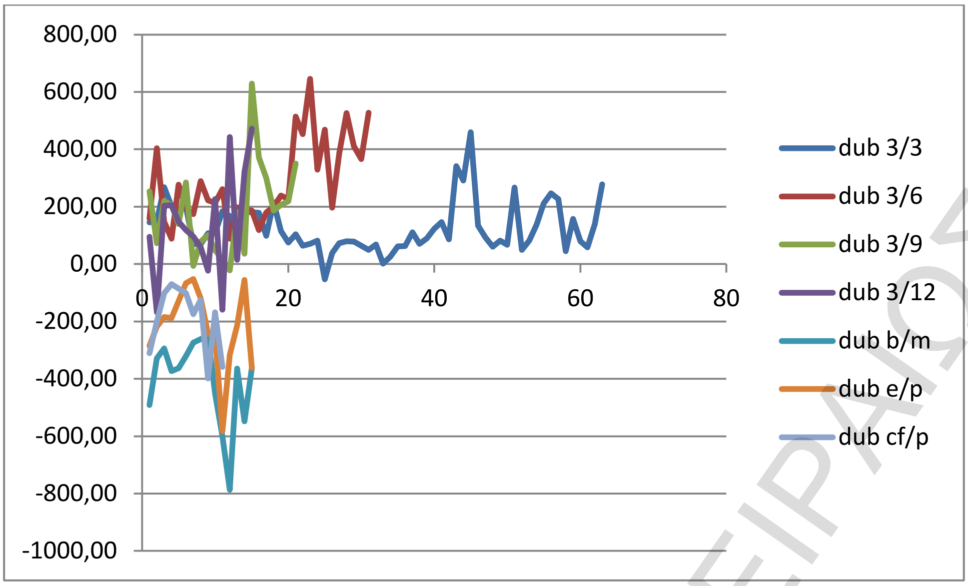


While the table that follows shows our analysis' results on our banking institutions' portfolios.

Table 2. Δui results on Banking institutions' portfolios

	Mean	Stdev	Max	Min
Δu 3/3b	127,34	87,20	459,23	-53,55
Δu 3/6b	286,35	145,87	645,57	88,12
Δu 3/9b	184,26	153,58	628,74	-22,37
Δu 3/12b	136,72	186,68	471,38	-168,02
Δu b/mb	-404,66	148,19	-249,89	-787,11
Δu e/pb	-217,39	137,67	-52,20	-582,18
Δu cf/pb	-190,17	115,13	-70,16	-398,22

Chart 7. Banking sector's Δu



The Δu of each strategy and of every sector will be taken and investigated for a statistical arbitrage opportunity.

Our methodology for statistical arbitrage, as described in section three, involves the minimization of the Maximum Likelihood function in order for our four parameters to be produced, the production of the individual p-values and a robustness check of root mean square errors which will support the strength of our model by simulating it one hundred thousand times.

While for the portfolios' formation and calculation excel was used, Matlab was used for the analysis of our portfolios' results under our statistical arbitrage's methodology. Our Banking portfolios' results are as follows:

Table 3. Results on Banking portfolios

	μ	λ	H1	H2	RMSE	Stat.Arb.
P- 3/3b	160,3073	0	0,004306	0,007947	0,002974	No
P- 3/6b	61,44035	-0,07351	0,003799	0,007245	0,004361	No
P- 3/9b	83,39969	0	0,005676	0,00788	0,003243	No
P- 3/12b	0,005531	0	0,007802	0,007838	0,003566	No
P- b/mb	0	0	0,007838	0,007838	0,005184	No
P- e/pb	0	0	0,007838	0,007838	0,003797	No
P- cf/pb	0	0	0,007782	0,007782	0,003439	No

While our Industrial sector's results are:

Table 4. Results on Industrial portfolios

	μ	λ	H1	H2	RMSE	Stat.Arb.
P-3/3i	153,7651	0	0,000198	0,007947	0,003137	No
P- 3/6i	511,2	-0,00275	0,000006	0,007911	0,005058	Yes*
P- 3/9i	73,32712	0	0,003985	0,00788	0,005117	No
P-3/12i	239,988	0	0,001186	0,007838	0,005149	No
P- b/mi	4,159084	0	0,004057	0,007838	0,003109	No
P- cf/pi	230,6397	0	0,002091	0,007827	0,004333	No
P- e/pi	6,212588	0	0,007651	0,007838	0,003058	No
P- sgi	0	0	0,007838	0,007838	0,004066	No

Fifteen portfolios formed that contained almost 200 international stocks, were bought and sold through a sixteen years period and by checking their performances only one was found (P-3/6 momentum industrial sector) that could hold against the statistical arbitrage's methodology at a significant 10% level and another one at the same level that almost touched methodology's barriers (P-3/6 momentum banking sector).

Furthermore our robustness check confirms that our model's incremental profits compared with those from 100.000 simulations using our estimated parameters converge significantly well.

SECTION 6

Conclusions

Momentum strategies seem to produce a more significant arbitrary level than Value strategies. The examined period is a period of great and global conflicts among governments and nations. Different policies are being followed almost by every nation but the goal is the same for everyone and that is profit. This diversity is the main reason for mispricing effects in global market. These effects are very hard not to be seen by investors that search for profits round the globe. And when profit gets riskless then new opportunities arise and investments are getting bigger .Soon enough the mispriced gap closes and push people-many times not very pleasantly- to start pricing things if not better at least in a more stringent way.

A common critique of financial anomalies is that the trading profits from such anomalies tend to disappear after adjusting for transaction costs, higher borrowing rates or other market frictions such as liquidity buffers and margin required for both long and short positions. In response to concerns that statistical arbitrage could be an artifact of the observed sequence of returns , a further robustness check can be made on the mean-μ and the growth rate of standard deviation-λ by bootstrap procedures that compute residuals and alter the sequence of trading profits.

This study's evidence on statistical arbitrage's existence in the financial world even in its smallest portion among the portfolios formed, is not to establish a new economical model but to define markets' inefficiencies that are being revealed through persistent anomalies. Tools have been developed to exploit these kinds of anomalies and by looking at the bigger picture, these tools come to complete markets.

Furthermore statistical arbitrage's methodology seems to be a mathematically efficient one and powerful enough to proceed to the comparison of different investment strategies and establish an arbitrary level of success that they produce.

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