CHAPTER ONE

INDEX

1.1 Scope of the research	2
1.2 Aim of the research	2
1.3 Outline	3

1.1 Scope of the research

The present work deals with the investigation of the behavioural aspects of bidders in electronic auctions. The subject is of interest as it has been noticed that behavioural patterns exist in auctions of sibling items. This has triggered many research attempts, which employ different approaches in modeling these behaviours.

Modeling human behaviour is of interest as the decision making process of a bidder of whether and when to place a bid, may not be subject to rationality; bounded rationality asserts that decision-makers are intendedly rational; that is, they are goal-oriented and adaptive, but because of human cognitive and emotional architecture, oftentimes, rationality fails and as a consequence, there is a mismatch between the decision-making environment and the choices of the decision-maker [4].

This research approaches the problem at hand using splines (cubic polynomials) to fit curves between knot points that have been collected from numerous auctions. As a testbed the electronic platform of eBay is used, while as family of auctioned items, without loss of generality, the family of antique maps and engravings has been selected; then, these are classified per auction duration, that is in 5, 7 and 10 days. The results extracted show that: (i) auctions with the same duration exhibit uniform characteristics, and (ii) no significant differences are noticed when duration changes. Also our approach is evaluated in other types of items and the results are encouraging and justify our decision to model behaviour using splines.

1.2 Aim of the research

The aim of this work is to explore whether behaviour can be modeled and whether the polynomial approach is adequate. In the first place a second order polynomial is used, which proves to suffer from insufficiencies in modeling behaviour, which justifies the proceeding to use cubic polynomials (splines). Splines, in general, are very commonly used in curve fitting, due to their excellent properties in smooth curves (continuity in 1st and 2nd derivative); in statistics, to smooth a data set is to create an approximating function that attempts to capture important patterns in the data, while leaving out noise or other fine-scale structures/rapid phenomena. Curve fitting concentrates on achieving as close a match as possible between any two points that the order of equation involves; curve fitting implies that regression analysis is used, so that it may be attempted to explain how the typical value of the dependent variable changes when any one of the independent variables is varied, while the other independent variables are held fixed.

The above are explained in detail in the respective chapters.

1.3 Outline of the thesis

In order to better present the above, this thesis is structured as follows. In Chapter 2, we overview auctions and we present their basic properties, characteristics and mechanisms. Yet it is presented why e-auctions show such popularity. In Chapter 3, an introduction in the research takes place, while the advantages of this approach are listed. Also, a review of the relevant literature studied is presented, whilst the parameters that affect behaviour and their interrelations are gathered. Chapter 4 deals with the methodology that was used for the research, having at first introduced the challenges that come along with data that have to surpass. Finally, in Chapter 5, we present the findings of our research and in Chapter 6, we highlight the conclusions of our work and we suggest guidelines for future research.

CHAPTER TWO

-	
NI	cv
III N	 $\Gamma_{i}\Lambda$

2.1 Introduction	
2.2 Origins of auctions	
2.3 e-Auctions	8
2.4 Attractiveness and popularity of e-auctions	I III

Auctions have proven to be an excellent trading mechanism to allocate goods, services, resources, etc., to individuals and firms; they have known considerable and continuous growth during the last decades due to their interesting properties in price formation when the value of goods traded is either not known or it varies.

"Auction", as a word, is derived from the Latin <u>augeō</u>, which means "I increase" or "I augment". A widely-accepted definition of an auction has been proposed by Wolfstetter (1996). It is "a bidding mechanism, described by a set of auction rules that specify how the winner is determined and how much he has to pay. In addition, auction rules may restrict participation and feasible bids and impose certain rules of behaviour". This definition may extend to the purchasing case too, also known as reverse auction, where a buyer uses an auction for procurement. In reverse auctions – in contrary to forward auctions- the price of the item decreases and the winner is the one who submits the lowest bid (offer). **Figure 2-1** depicts a typical auction process which constitutes of bid submission, bid evaluation, price computation, closing, and the (optional) feedback to bidders.

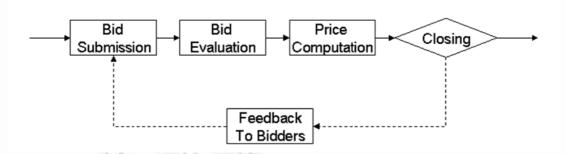


Figure 2-1: simplified auction process (adapted from Bichler et al. 2002).

The aim of all trading mechanisms (e.g., auctions, negotiations, bargaining, posted pricing, etc.) is to reach a final price which is acceptable to the involved parties (that is, sellers and buyers) and which serves as the dominant evaluation criterion for a bid.

Auctions may be distinguished between single-good (*auction* if the object of the trade is of the same type) and combinatorial auctions (*combinatorial* if the object of the trade concerns, positively or negatively, inter-related goods of different types, characteristics or functionalities), depending on the nature of the object of trade. In both cases, the number of units for the good(s) under auction helps in characterizing the auction as either *single-unit* or *multi-unit*. Yet another division between auctions, depending on the number of attributes (or criteria) used for the evaluation of a bid is

that in single-attribute and multi-attribute auctions. In addition to the above, auctions may be distinguished between sealed-bid and outcry (or open-cry) auctions, depending on the visibility to information that bidders have on rivals' bids. In *sealed-bid* auctions, bidders normally submit their sealed offers in order to be evaluated and they have no information on the contents of the opponents' offers. In *outcry* auctions, as the name implies, the contents of each bid are open to all bidders, usually at the time when the auction takes place. Typically, outcry auctions generate multiple rounds of auction, while sealed-bid auctions may terminate in a single round.

Finally, depending on how the auction progresses and on the resulting awarding rule, auctions may be classified into English or Dutch auctions in the outcry case and k-th price sealed-bid (where k=1,2,...) in the sealed-bid case, where the most common values of k are 1 and 2. These types are regarded as fundamental and are further elaborated below:

- English auctions are also encountered in the literature as "oral", "open", "ascending-bid", "ascending-price", or "ascending open-cry". In this type of auctions, the price offered is raised successively until only one bidder remains; this bidder is the winner and is awarded the object of the auction at the final price. Prices may be announced by buyers or sellers during the course of the auction. It is by far the most common type.
- Dutch auctions are also encountered in the literature as "descending-bid",
 "descending-price", or "descending-clock". In this type of auctions, the
 auctioneer announces the highest desirable price; then, the price decreases
 continuously or in discrete intervals until a bidder announces his willingness
 to pay the current price. This bidder is the winner and is awarded the object of
 the auction at the final price.

In *k-th* price sealed bid auctions each bidder submits independently a single sealed bid. The bidder that submitted the highest bid is awarded the object of the auction at the price corresponding to the k-th ranked bid. The case where k=1, is most commonly referred as *first-price sealed-bid* auction (FPSB) and is widely used in business procurement. The case where k=2, is most commonly referred as *second-price sealed-bid* auction (SPSB) or *Vickrey* auction [2].

For most of history, auctions have been a relatively uncommon way to negotiate the exchange of goods and commodities. In practice, both haggling and sale by set-price have been significantly more common. Indeed, prior to the seventeenth century the few auctions that were held were sporadic and infrequent.

Nonetheless, auctions have a long history, having been recorded as early as 500 B.C. According to Herodotus, in Babylon, auctions of women for marriage were held annually. The auctions began with the woman the auctioneer considered to be the most beautiful and progressed to the least. It was considered illegal to allow a daughter to be sold outside of the auction method.

During the Roman Empire, following military victory, Roman soldiers would often drive a spear into the ground around which the spoils of war were left, to be auctioned off. Later slaves, often captured as the "spoils of war", were auctioned in the forum under the sign of the spear, with the proceeds of sale going towards the war effort. The Romans also used auctions to liquidate assets of debtors whose property had been confiscated. For example, Marcus Aurelius sold household furniture to pay off debts, the sales lasting for months. One of the most significant historical auctions occurred in the year 193 A.D., when the entire Roman Empire was put on the auction block by the Praetorian Guard. On March 23, the Praetorian Guard first killed emperor Pertinax, and then offered the empire to the highest bidder: Didius Julianus outbid everyone else for the price of 6,250 drachmas per Guard, an act that initiated a brief civil war. Didius was then beheaded two months later when Septimius Severus conquered Rome.

From the end of the Roman Empire to the eighteenth century auctions lost favour in Europe, while they had never been widespread in Asia. In some parts of England during the seventeenth and eighteenth centuries, auction by candle was used for the sale of goods and leaseholds. This type of auction was named after the way the auction was launched which involved the lighting of a candle; then the bids were offered in ascending order until the candle spluttered out. The highest bid at the time the candle was melt down and its flame was put out won the auction.

The earliest modern era records of auctions appeared in the Oxford English Dictionary in 1595. Therefore, the presence of auctions in England preceded this date yet but how much is not known. During the end of the 18th century, soon after the French Revolution, auctions came to be held in taverns and coffeehouses to sell art. The auctions were held daily, and catalogues were printed to announce available items. Such Auction catalogues are frequently printed and distributed before auctions of rare or collectible items. In some cases, these catalogues were elaborate works of

art themselves, containing considerable detail about the items being auctioned. In America, early accounts of the use of auctions occurred in the South, when slaves were often sold at auction. Moreover, auctions were often used to liquidate estates. It is worth mentioning that oftentimes, owners of the goods were not disclosed because the current social norms did not look favourably upon auctions.

Aside from the early modern records of the use of auction mechanisms in England and America, auctions were used in the Netherlands and Germany as well in the later part of the 19th century. Auctioning in the Netherlands dates back to 1887 when it was used for the sale of fruits and vegetables. Reportedly, a grower named Jongerling arrived at the inland harbour, Broek op Langendijk in North Holland. Upon arrival he discovered a strong demand for his produce and instead of liquidating his produce in the usually customary fashion of selling to a specific dealer, he decided to allow the buyers to compete with each other by using an auction. In the same year as did Jongerling in North Holland, fishermen in Germany began to use auctions to sell their catch when arriving in port. These fish auctions allowed the fishermen to rapidly liquidate their catch and spend more time fishing to satisfy consumer demand [3].

Typically, the conduction of an auction is a responsibility for an intermediary auctioneer: an independent agent, experienced in conducting and securing the auction process, possibly receiving commission from the other participants [2]. During the American civil war, goods seized by armies were sold at auction by the Colonel of the division. Thus, some of today's auctioneers in the U.S. carry the unofficial title of "colonel" [3].

The oldest auction house in the world is Stockholm Auction House (*Stockholms Auktionsverk*) which was established in Sweden in 1674. Nowadays, the two widest known auction houses are Sotheby's, which held its first auction in 1744; and Christie's that was established around 1766. Other early auction houses that are still in operation include Dorotheum (1707), Bonhams (1793), Phillips de Pury & Company (1796), Freeman's (1805) and Lyon & Turnbull (1826) [3].

2.3 e-Auctions

Participating in an auction do not always demand for the physical presence of a bidder; there are cases, where the auctions are hosted by online platforms that play the part of the auctioneer organizing and conducting the auction; in online or e-auctions, a bidder can place their bids, in the auction they are interested in, remotely. The benefits of participating in an e-auction include:

- Transparent process
- It will be clear to participants why they won/lost the e-Auction.
- Participants will receive real-time market information.
- Contracts can be awarded faster.
- Time saving compared to face-to-face negotiations [5].

On the other hand, risks in relation to the online auctions emerge; internet transactions include weaknesses that deal with the open environment (bids may be exposed to attacks), anonymity (real identities cannot be determined), yet non secure payment transactions and difficulty in verifying reliability and rule keeping. These risks are further sharpened by threats that deteriorate the bidders' position; such are payment methods (credit card "phantom" transactions), shill bidding (where sellers themselves bid), yet late bidding by sniping with the use of proxy agents and threats related to protocol security, privacy and to encryption.

Table 2-1 summarizes the advantages and the risks involving in online auctions.

Advantages	Disadvantages/Risks
Market enlargement	Increase of competition
Global coverage – no time zone restrictions	No face-to-face contact
Inventory reduction	Possibility of shortage
Reduced transaction costs	Intermediary fees
Cost savings	Unknown supplier
Market for 2 nd -hand products – liquidation	No middleman
Efficient pricing	Lower profits
Lower risk of collusion	Requirement for prequalification
Real-time transaction	New forms of vulnerabilities
Quick transactions	Transaction security
Easiness	Hard to physically inspect items
Price testing	
No need for physical presence	
Disintermediation	
Complement to traditional store	
24-hour operation	
Reduced search costs	
Development of communities	_

Table 2-1: advantages and risks of e-auctions [9][12][13][14].

Such issues demand for preventive treatment, so that so no disagreeable effect would menace to blow up the auction: participant certification and use of evaluation

service; item rating service; participants' feedback services; possible use of escrow services. In cases of withdrawal, a penalty could be applied.

The auction workflow entails that several steps happen; at first, registration is a requisite; then the setup of the auction is followed by its publicity, which is a responsibility of the auctioneer; in parallel, bidding is taking place; the closing of the auction concurs with the bid evaluation and the announcement of the winner; the final step of this process is the settlement between the auctioneer and the winner of the auction. **Figure 2-2** demonstrates thoroughly the process described.

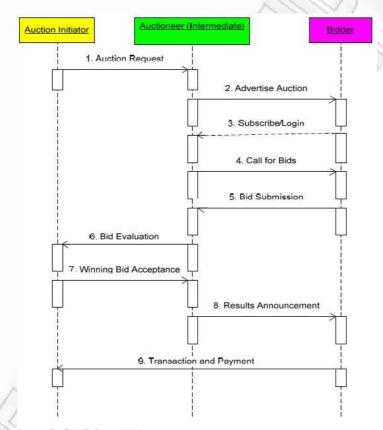


Figure 2-2: the auction workflow [10].

The selection of a marketplace involves several factors: to start with, decisions parameters are inserted: personal factors (ease of use, ease of learning, convenience), social factors (community, market, customer types), economic factors (market, customer types, bid limits, listing cost, traffic) and last but not least, reputation issues (name recognition, trust, reputation, security, integrity); then there are some core decisions for participants to be made: for the bidder that would be where to search, for the initiator, where to sell, whilst for the auctioneer that would be how to promote the item; Yet two important factors include experience and cognitive factors.

Nowadays, online auctions have become a popular way for both businesses and consumers to exchange goods. One of the biggest online marketplaces and currently the biggest C2C online auction place is eBay (www.ebay.com). The

dominant auction format on eBay is a variant of the second price sealed-bid auction [Krishna, 2002] with "proxy bidding". This means that individuals submit a "proxy bid", which is the maximum value they are willing to pay for the item. The auction mechanism automates the bidding process to ensure that the person with the highest proxy bid is in the lead of the auction. The winner is the highest bidder and pays the second highest bid [1]. Hence, information revelation in eBay entails that the leader of the auction is displayed (that is, the highest bidder) but the proxy bid revealed concerns the amount they will have to pay if they are the winners of the auction, which is the second highest bid. In addition, any bidder can submit a new proxy bid at any time until the auction ends. Unlike other auctions, eBay has strict ending times, ranging between 1 and 10 days from the opening of the auction, as determined by the seller [1]. Access to complete bid histories of closed auctions in eBay is possible for a depth of time of at least 15 days on its web site. Such data is available in abundance, notwithstanding in HTML format. Figure 2-3 shows an example of bid histories found on eBay's web site.

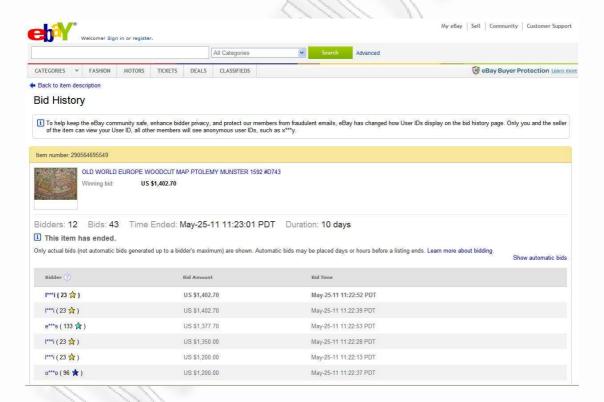


Figure 2-3: bid history in eBay.

At the top of **Figure 2-3** a summary of the information relevant to the auction is displayed: a description of the item and the item number, the current bid (which is identical to the winning bid since the auction is closed), the quantity of items auctioned (equal to one in the "proxy bid" auction type), and the number of bids received during the auction. Also, it is showed the starting price required for the first

bid to be, which is not shown to the picture above since there were too many bids in this particular auction to be cited here. Furthermore, information about the start and the end times of the auction as well as the seller's username and their rating (in parentheses) are apposed. The bottom of the page includes detailed information on the history of bids. Specifically, starting with the highest bid, the page displays the bidder's user name, their rating and the time and date when the bid was placed and continues with the other bidders in descending order according to their bid amount.

Although the transition of the marketplaces to the web field entailed the disintermediation of the market, it also introduced certain new types of intermediaries, the so called "infomediaries". These infomediaries are classified below, in **Table 2-2**, according to the services they provide.

Service / Tool	Usage	Provider	User	Example
Search tools	Locate auction web sites and items	ASP	Bidders	Online Auctions Network – (online-auction.net) Internet Auction List (internetauctionlist.com) Yahoo-auctions.yahoo.com Bidder's Edge (biddersedge.com) Turbobid (etusa.com)
Alerting and notification services	Alert bidder about auction evolution	ASP, S/W	Bidder	Auction Watch, auctionservices.com
Certification	Certify and check eligibility	ASP	Bidder, Initiator	BidSafe-auctions.com
Advertisement	Auction site advertisement	ASP	Auctioneer	illumix.com, auctionwizard.com
Bid evaluation and comparison tools	Bid evaluation, winner determination for complex auction mechanisms, comparison with bid in other auction sites	S/W	Auctioneer, Initiator	BidFind.com, freemerchand, AuctionWatch.com
Monitoring	Simultaneous monitoring of many auction sites	S/W, ASP	Bidder	BidMonitor (brucelay.com), EasyScreenLayout (auctionbroker.com)
Sniping	Automated bidding in soft-closing auctions	S/W, ASP	Bidder	
Proxy bidding	Automated bidding on behalf of bidder	S/W, ASP	Bidder	
Communication tools	Mailing lists, message boards, chat groups etc.	S/W, ASP	Bidder	
Billing and payment collection services		ASP	Bidder, Initiator, Auctioneer	PayBWeb.com, PAyPal, BidPay.com
Escrow services		ASP, Physical service provider	Bidder, Initiator	tradeenable.com, guzooescrow.com, escrow.com
Credit card payment services		ASP	All participants	billpoint.com, CcNow.com
Carrier and postal services		Physical service provider	Bidder, Initiator	iship.com, auctionship.com, stamps.com (USPS)

Table 2-2: infomediaries [11].

The most usual infomediaries encountered are intelligent agents. They are programs involving benefits from artificial intelligence that consist clearly identifiable entities with well-defined (and limited) resources and interfaces, designed to fulfill specific roles; they are situated or embedded in a particular environment; not only are they autonomous in the sense that they have control over their behavior, but also they are capable of exhibiting flexible behavior which can be either reactive, or proactive,

or even sociable. Their applications include proxy bidding, complex bidding (demanding either zero or zero-plus intelligence), auction exploring and pooling.

In online auctions, certain issues have emerged, in terms of quality of service provided to anyone interested to participate in such an auction. These issues include, for example, the quick response to users, the capability of providing 24-hour services, 7 days a week, the fairness among participants and security guaranty; certainly, these issues that represent requirements on behalf of users are connected to performance indicators, so that their efficiency is measured.

Table 2-3 summarizes the issues existent, in terms of quality of service, as much as the communication performance metrics of online auctions.

Quality of Service to online auction	Communication Performance
users	Metrics of Online Auctions
Fast response	Response time
No site failures	Traffic reliability
Predictable response	Predictability
24 X 7 services	Availability
Low transaction fee	Costs
Secure	Security
Fairness	Information sharing

Table 2-3: quality of service requirements and the respective performance indicators [15].

Obviously, the issues cited above can be further analyzed to requirements of technical, operational and quality nature. **Table 2-4** refers to such further disaggregation:

Bid facilitation (checking, storage)	Technical & operational
Real-time	requirements
Multi-casting	
Expected workload	
Usability (navigation, personalization, friendly	
interface)	() ()
Item display (content, guides, interactivity, discussion	
boards)	
Accessibility (direct access, minimal effort for	Operational requirements
installation and operation)	
S/W and H/W Costs (minimal or even zero)	
Additional services (notification, identification)	
Site-related quality	Quality requirements
Information quality	\rightarrow User perceptions
Interaction quality	
Domain-specific quality	
Auction model	Quality requirements
Motives and results	→ Business model
Exchange processes	
Stakeholder relationships	2)

Table 2-4: elements of technical, operational and quality requirements [2].

To sum up, participating in an auction do not always demand for the physical presence of a bidder; there are cases, where the auctions are hosted by online platforms that play the part of the auctioneer organizing and conducting the auction and where placing a bid can be done remotely, too; these auctions are so-called as "online auctions" or as "e-auctions". One of the biggest online marketplaces and currently the biggest C2C online auction place is eBay; the dominant auction format on eBay is a variant of the second price sealed-bid auction with "proxy bidding".

Although the transition of the marketplaces to the web field entailed the disintermediation of the market, it also introduced certain new types of intermediaries, the so called "infomediaries". The most usual infomediaries are the intelligent agents. Finally, in online auctions, certain issues have emerged, in terms of quality of service provided to anyone interested to participate; among these are cited the quick response to users, the capability of providing 24/7 services, the fairness among participants and security guaranty; these issues represent requirements on behalf of users; certainly, they are connected to performance indicators, so that their efficiency is measured.

Online auction hold a lot in store if you are looking forward to get the best of them. Without a doubt, they make it possible for the participants to win big. Online auction is other options that take a small investment but make it worthwhile for a lot of customers.

Online auctions can help in getting some of the best deals right off the market. Not only you may get the same product for a much better price but to make it better still, coupons that may come along with online shopping can help you get some major discounts on your next purchase. Moreover there is a facility of increased selection between same or similar merchandise that gives you the peace of mind that you made the right choice while purchasing the product at auction sites. Not to mention product or seller reviews that are provided to assist in deciding whether the product worth the money you are planning to spend. Yet, guarantees and warrantees come alongside with the product, so that it is insured while being shipped, so you are pretty much covered with every purchase you make [23].

Online auction is an easy process regardless of whether a bidder or a seller; for the latter, there are numerous stories about business ideas generated from simply looking around the house and been creative. Consider what items or electronics can be turned into profit, particularly collector's items that have been in your family for a long time. Look around your neighbourhood for stores or businesses which might be struggling and are selling items for pennies. Be creative finding your inventory, do your diligent research online, and find out the estimated prices of these items. You will want to negotiate accordingly and remember to consider shipping costs. There is plenty of money to make for the smart and savvy exploring online auctions. The tendency of turning to auctions for income is growing at a rapid rate partly due to the effects of the worldwide recession. Nevertheless, you must proceed cautiously due to the presence of smart scam artists who often take advantage of unsuspicious consumers [24]. But such jeopardies can be rather diminished if the potential bidder addresses experts: established players in the industry, such as eBay, eBid or Online Auction; then not only can they feel safe in reliable and secure hands but also they will be experiencing a higher level of customer service; this is due to the intense competition existent, that they rely on this pillar to maintain themselves on top. Besides, when it comes to auctions for income, customer satisfaction is critical [24].

Still, one should bear in mind that online auctions do not offer services of collecting payments and distributing goods over against the traditional auction houses that serve as third-party mediators; that is because online auction sites do not actually house the merchandise, so that buyers must deal directly with the individual seller to

complete the sale, since you have placed the highest bid over an item [25]. Despite that fact, bidders can be protected by taking a few precaution actions that are suggested by Illinois Attorney General:

- 1. Pay by credit card instead of using pay by check or money order; in some cases, if you use a credit card and don't get the merchandise, you may be able to challenge the charges with your credit card company. Check with your credit card company before making a bid.
- 2. Use an escrow agent, or pay by COD; a third-party escrow agent will collect both your payment and the product you are buying, and process the delivery of each. Be aware, however, that most escrow services charge a fee.
- 3. Verify the seller's identity; to complete a sale, you will need the seller's name, address, and phone number. If the seller is unwilling to provide this information and you are unable to verify their identity, do not do business with him or her. Some sellers may also use a forged e-mail header, making it impossible to contact them.
- 4. Get follow-up service if necessary; many sellers don't have the necessary expertise or facilities to provide services for the merchandise they put up for auction.
- 5. Make sure all the terms of the deal are known: shipping costs, returnability of the merchandise;
- 6. Report any problems you have to the auction site. They may be able to use your report to keep the seller from marketing goods and services on the site again. If you feel you've been a victim of Internet fraud, you can file a complaint with the Attorney General's Office, the Federal Trade Commission (FTC) Consumer Response Center or the Better Business Bureau [25].

Such actions that nowadays with the penetration of the internet are considered as self-evident ones enhance even more the attractiveness of auctions hosted online and wax them as the main scenery used for hosting auctions.

CHAPTER THREE

	-		
- 1	M	U.	$\mathbf{F}\mathbf{X}$

3.1	Introduction to research	18
3.2	Benefits	20
3.3	Literature review	22
3.4	Parameters affecting behaviour	23

Auctions have proven to be an excellent trading mechanism to allocate goods, services, resources, etc., to individuals and firms; The decision making process of a bidder of whether and when to place a bid it may be supported using price forecasts that can be done either in a static or in a dynamic way. The methods used widely (also referred as "traditional" methods) so far for price forecasting have been collapsed due to the fact that one of their main hypotheses is that the bidders act rationally. Yet findings from behavioural organization theory, behavioural decision theory, survey research and experimental economics leave no doubt about the failure of rational choice as a descriptive model of human behaviour [4]. That does not mean that people are irrational. Bounded rationality asserts that decision-makers are intendedly rational; that is, they are goal-oriented and adaptive, but because of human cognitive and emotional architecture, oftentimes, rationality fails and as a consequence, there is a mismatch between the decision-making environment and the choices of the decisionmaker [4]. That is why, in recent years, a vigorous experimental movement in economics has emerged that involves a direct methodology; derive a result from theoretical economics, set up a laboratory situation that is analogous to the real-world economic situation, and compare the behavior of subjects to the predicted [4].

By using regression analysis with a curve fitting of 3rd class polynomial equations it is intended to analyse the bidding behaviour, as is expressed in the evolution of the price. The sample studied concerned for antique maps and engravings which is a category involving high collectibility and perhaps perishability as features of a product. Such features may lead to an aggressive behaviour on behalf of bidders and therefore they may lead to a segmentation of the sample and the clustering of auctions studied; that comprises the goal of this study: the segmentation of a certain category of items, through the analysis of a series of auctions of this category, according to the bidding behaviour as it is expressed in the evolution of the price; this is due to the observation that the existent price forecasting models consider that the bidding behaviour be rational, that is they prejudice the plenary conformity to the classic expected utility model; in reality, however, "non-rational" as is to say non-expected behaviours are reported from the participants in an auction.

Therefore, in our experiment, a set of auctions was observed during the training and learning phase, in order for the bidding behavior to be recorded of a certain type of item; that is because different features drive different behavior; what was observed was that as bids are expressed as the auction goes, the bidding behavior follows a certain route, regardless of the duration of the auction; this route is of type $y = at^3 + bt^2 + ct + d$; what may differ in each auction examined in terms of price

evolution is the magnitude of this route. These different magnitudes of auctions that concern products that belong to the same family, altogether, they consist an area of behavior that **we** call "span of confidence". In this span of confidence the most-likely functional expression arises that reflects the most-expected-to-happen behavior. The **Figure 3-1,** in the following summarizes the above mentioned.

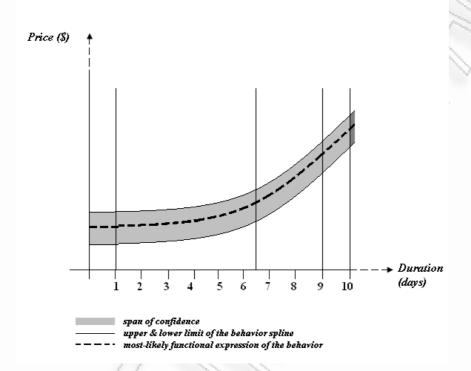


Figure 3-1: span of confidence arises from the different magnitudes of auctions concerning products that belong to the same family.

The training phase deals with determining the parameters a, b, c and d of the spline, so that the relations w.r.t. d would be determined with greater precision. It is noted that d corresponds to the initial bid y_I ; therefore, it is possible, by providing the system (**Figure 3-2**) with various bids and w.r.t. the duration of the auction, to predict and forecast the price at which the item auctioned will be valued.

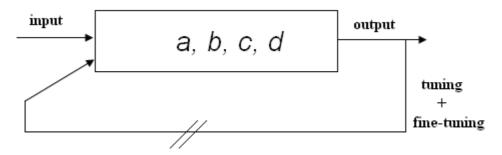


Figure 3-2: this approach circumvents the analytical econometric models that exhibit limited flexibility by neglecting the particular drivers of such models and focuses on the behavior of the bidders.

Therefore, in the application phase, the results extracted during training phase will be we employed in order to be able to, given only the starting bid and the time span of the auction, predict and forecast the behavior of the bidders for every item of that family.

3.2 Benefits

The benefits of this approach are manifold; to start with, traditional forecasting methods assume that data arrive in evenly-spaced time intervals. In such a setting a model may be trained up to the current period t which then is used to predict at time t+1. Implied in this process is the important assumption that the distance between two adjacent time periods is the same. In online auctions, on the contrary, bids arrive in very unevenly-spaced time intervals, determined by the bidders and their bidding strategies and the number of bids within a short period time can be sometimes very sparse and othertimes be very dense. Secondly, traditional forecasting methods assume also that the time-series continue, at least in theory, for an infinite amount of time. This is not the case for online auctions in platforms, such as eBay that hosted this study; eBay has strict ending times, ranging between 1 and 10 days from the opening of the auction, as determined by the seller. The implication of this is a discrepancy in the estimated forecasting uncertainty. Furthermore, traditional models do not account for instantaneous change and its effect on price forecast; in online auctions, however, even the same products can experience price paths with very heterogeneous price dynamics (Jank and Shmueli, 2005). By price dynamics is denoted the speed at which price travels during the auction and the rate at which this speed changes [1]. Moreover, the analytical econometric models take into account various specific drivers that differ according to the perspective of the author of each

model; nevertheless, it is not obligatory for the participants in an auction to indeed take into account all these particular drivers that limit the flexibility of the model. This approach creates a shortcut on the analytical econometric models, as was shown in Figure 2, by neglecting the particular drivers of such models and focuses on the behavior of the bidders. Last but not least, in cases of auctions with "buy now" possibility, when opening bid d is known and therefore, the price span expected for the item to be placed is revealed, if a bidder, they can judge whether to hit the auction; if a seller, they can predict at and forecast the price at which the item auctioned will be valued and perhaps involve a strategy so that that the item will be valued at the high predicted price C, as represented in **Figure 3-3**. It is obvious that the closer we get to the closing of the auction, the narrower the span of confidence will become, so that it will be restrained to the line concurring with the most-likely functional expression; hence, by the time that t is equal to the time of the closing, it will be unveiled whether the item will end up in the predicted price A, B or C.

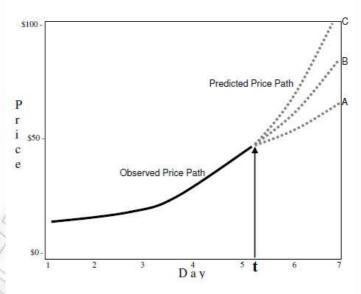


Figure 3-3: schematic of the dynamic forecasting model of an ongoing auction.

In conclusion, such an approach allows for dynamic forecasts in the sense that it incorporates information from the ongoing auction; it overcomes the unevenly spacing of data; it takes the finite-time horizon into account; it incorporates change in the price dynamics; it circumvents the analytical econometric models that exhibit limited flexibility by neglecting the particular drivers of such models and focuses on the behavior of the bidders; in auctions with "buy now" possibility, when opening bid d is known, if a bidder, they can judge whether to hit the auction; if a seller, they can predict at and forecast the price at which the item auctioned will be valued and

perhaps involve a strategy so that that the item will be valued at the highest possible price.

3.3 Literature Review

Auctions have been of interest for many researchers during the latter years. Emiris & Marentakis, having presented an overview of recent literature in auction theory that focuses on contemporary auction techniques, they propose an *Auctions Classification Ecosystem (ACE)* that encompasses auction features as well as mechanism design parameters in a single scheme; this scheme facilitates the understanding of auction characteristics and supports auction practitioners in designing the appropriate format depending on the application requirements [2]. Marentakis conducts a thorough study of advanced auctions in dynamic marketplaces that includes classification, technologies, applications and behavioural study of such auctions [17].

Price forecasting has been a field of pondering for many researchers; Koulouriotis, Emiris, Diakoulakis & Zoupounidis suggest an integrated computational short-term stock price forecasting system that encompasses representative techniques (multiple regression, exponential smoothing, neural networks and Adaptive-Networkbased Fuzzy Inference System - ANFIS); furthermore, a practically exhaustive comparative study of the performance is conducted that takes into account the behaviour of these techniques, as well as the role of their core parameters [19]. Jank, Shmueli & Wang propose a dynamic model for forecasting in ongoing auctions in the sense that it produces price forecasts for future time periods up to the auction-end. In this process the concepts of price velocity and price acceleration are incorporated [1]. In online auctions, on bids arrive in very unevenly-spaced time intervals, determined by the bidders and their bidding strategies; therefore, the number of bids within a short period time can be sometimes very sparse and othertimes be very dense [1]. Ovchinnikov and Milner deal with wait or buy decisions w.r.t. revenue management policies. That is, he is occupied with decisions of nature of whether to place an offer (or bid) or wait for potential last-minute offers on behalf of sellers [20].

In such decisions, the issue of bounded rationality is involved. Kauffman suggests that extremes in emotional arousal contribute to bounded rationality. He observes that emotions are largely absent from economics discourse, despite the prominent role they are given in human affairs by authors and playwrights from the time of the Greeks to the present both in the modern social and behavioral sciences

[21]. For the same topic, Jones asserts that rationality sometimes fail, due to human cognitive and emotional architecture and as a consequence, there is a mismatch between the decision-making environment and the choices of the decision-maker; he argues that in recent years, a vigorous experimental movement in economics has emerged that involves a direct methodology; derive a result from theoretical economics, set up a laboratory situation that is analogous to the real-world economic situation, and compare the behavior of subjects to the predicted [4].

3.4 Parameters affecting behaviour

The key parameter in auctions is the number of bidders; an increased participation has a positive effect from the point of view that an increased interest over an item auctioned may push its value up to a higher price. The number of bidders, on their behalf, is found to be positively affected by parameters like the number of units, the length of the auction and certainly, the value of the item; on the other hand, a high minimum bid could repel potential bidders.

Furthermore, parameters like experience, reserve price and buy-it-now feature are attractive to bidders. Also, usually bidders head for auctions where an increased number of participants are observed rather than look into for auctions based on their attributes; that is what's entailed by herding bias. However, a low starting price, the quality of information provided and perhaps an image of the item auctioned are parameters positively affecting the expression of interest. Participation fee, in reverse, is not explicit whether it influences positively or negatively bidders into placing their offer.

Other factors affecting bidding behaviour include opponents' bid (who, when, how), cultural differences, word-of-web (which is analogue to word-of mouth) spreading out reputation and bidirectional rating; the latter increases the trust level but is rather vulnerable as a feature due to possible changes in ID, shilling and the lack of feedback. Yet another parameter concerns on the reselling possibility (extra purchase, temporary ownership, unintentional, disposition); last but not least, channel duality may affect behaviour in terms of bidding or posting price.

Table 3-1 presents the main findings from Section 2.8.3[17]; it summarizes the influence over the main parameters mentioned.

Parameters	Influence (+/-)
# units	+
Length	+
Value	+ (
high minimum bid	- 11
Experience	1 111
reserve price	
"buy-it-now" feature	/ F
"herding bias"	7/14)/ /
starting price	+/-/
quality of information	-/4
image of the item	
participation fee	\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
opponents' bid (who, when, how)	-7+
cultural differences	
word-of-web	+/-
Bidirectional rating	1-1-1-

Table 3-1: main parameters that affect behaviour and their influence.

Figure 3-4 represents the interrelations amongst the main factors of an auction in terms of the parameters cited above.

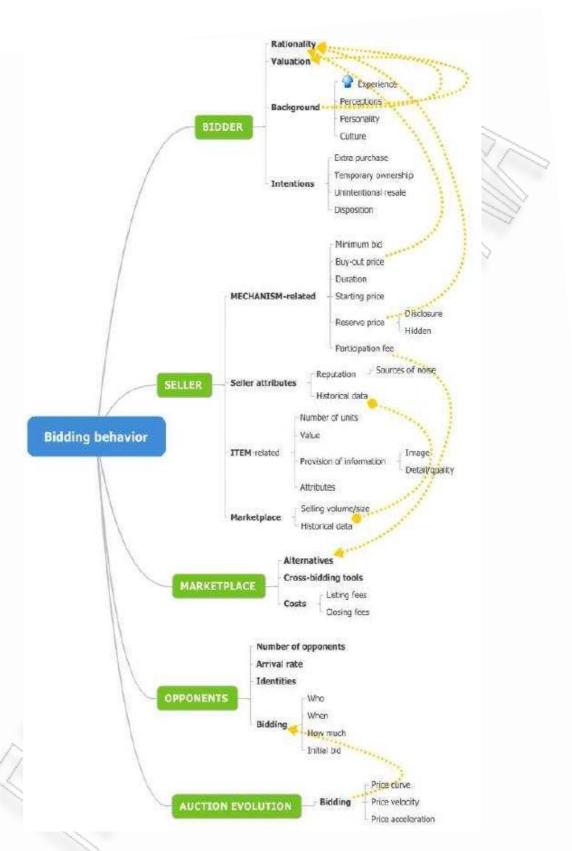


Figure 3-4: the interrelations amongst the main factors of an auction in terms of the parameters affecting behaviour [18].

CHAPTER FOUR

INDEX

4.1 Data Cha	allenges		27
4.2 Methodol	logy of research		28
4.3 Descripti	on of research		28
4.3.1.	Set target		28
4.3.2.	Data mining		29
4.3.3.	Data filtering	and sample determination	30
4.3.4.	Data analysis		33
4.3.4.	1 Mathematical	modelling	35
	4.3.4.1.1	Regression analysis	35
	4.3.4.1.2	Curve fitting	37
	4.3.4.1.3	Frequency distribution	37
4.3.4.	2 Excel modelli	ng	38

As it was mentioned on § 3.2 (benefits) above, online auction data arrive with many challenges that are not well-handled by traditional forecasting methods. The first such challenge is handling the extremely unevenly-distributed measurements. That is, the number and location of bids varies drastically from one auction to another. Traditional forecasting methods assume that the data be measured at equally spaced time intervals and are therefore not directly applicable. Another limitation of traditional forecasting approaches lies in their assumption about the process duration: traditional approaches assume that the process continues infinitely long into the future, and that the further into the future we attempt to forecast, the more uncertainty we face. This is quite different in the auction setting. Online auctions last for only a very short period of time (maximally 10 days on eBay). Moreover, the forecasting uncertainty is typically bounded. The reason for this is that for most auctions, the product being sold has a certain market value. Thus, while at the beginning of the auction the uncertainty about the next bid may be high, it decreases towards the auction end. An appropriate forecasting model should be able to account for this decreasing uncertainty as forecasts move further into the future. And lastly, yet another challenge of fast moving markets on the internet is change and the incorporation thereof into the forecasting model. Auctions on eBay operate in the online world, free of the barriers of geography and time. Online markets can react to stimulants in real time and therefore often experience fast change. If, for instance, demand for a particular product increases unexpectedly in one part of the world, it can affect the price of an auction that is hosted by a seller in a completely different part of the world. Even on a much smaller scale, auctions for the same product within eBay affect each other, and so do bidders within the same auction [1].

Therefore, all these interrelations are better filtered within the parties interested in participating in an auction. And since that existent price forecasting models have collapsed in front of the bounded rationality theory, the aim is, through the analysis of a series of auctions of a certain category of items (because the placing of a bid is item-sensitive; that is, different features drive different behavior), the segmentation of this category according to the bidding behaviour as it is expressed in the evolution of the price.

The following summarizes the methodology that was used during research:

- Step 1. Set target: selection of item to observe. Hypothesis of what is expected to be observed.
- Step 2. Data mining: set data to be collected and time span of collection. Gathering of data.
- Step 3. Data filtering: set rules to screen out the raw data and as regards to the conversion of data in order for the information hidden to be revealed; sample determination: the auctions remaining after the filtering has taken place in the previous step, consist the sample that will be processed.
- Step 4. Data analysis: how the sample will be processed.
- Step 5. Results and feedback: what is the derivation of the processing; comparison to the hypothesis that has been made.
- Step 6. Conclusion

4.3 Description of research

4.3.1 Set target

It was cited above that the decision making process of a bidder of whether and when to place a bid it may be supported using price forecasts; the methods that were used widely (also referred as "traditional" methods) so far for price forecasting have been collapsed due to the fact that one of their main hypotheses is that the bidders act rationally. That is why, in recent years, a vigorous experimental movement in economics has emerged that involves a direct methodology; derive a result from theoretical economics, set up a laboratory situation that is analogous to the real-world economic situation, and compare the behavior of subjects to the predicted [4].

Therefore, in our experiment, a set of auctions was observed during the training and learning phase, in order for the bidding behavior to be recorded of a certain type of item; that is because different features drive different behavior; This certain type of item that was chosen to be observed was antique maps & engravings.

This item prevailed among others because it involves collectibility and perishability - features that are expected to be accounted by potential bidders; that is due to the fact that in most cases, antique maps are "hunted down" by collectors that are cognizant not only of features of artistry like antiquity and virtuosity but of others like uniqueness and possible evaluation in a market as well. Moreover, they can be considered that they are aware of exactly what they are looking for, because usually they search for an impression of certain parts of the world; consequently, they are not expected to be easily drifted to place a bid for something that is not truly of their interest.

What is expected to be observed is that, as bids are expressed as the auction goes, the bidding behavior is likely to follow a certain route, regardless of the duration of the auction; this route is of type $y = at^3 + bt^2 + ct + d$; by determining the a, b, c and d of the spline in a numerous set of auctions (let it be n in number), the relations w.r.t. d in a n+1 auction of the same type of item would be determined with greater precision; this phase consists the training phase.

4.3.2 Data mining

The observation took place from late march to late may on eBay.com; The data that was collected for each auction during the second step of the research involved first and foremost the duration of the auction that was the primary factor of categorization of the auctions; also, the electronic address of the auction (URL), a description for the item(s) auctioned and the quantity of them, the name and rating of the seller, the number of bids and the number of bidders; therewithal, the complete bid history including date and time of bid, bid amount, bidder ID and rating of bidder, and starting price of the auction. Also, a picture of the item(s) auctioned was retained.

However, in the type of item observed (i.e. antique maps & engravings) due to its collectibility feature, usually only one item was auctioned at the time, which brings up the case of single item type of auction, or at least those were the cases of auctions examined, because of the minor number of multi-item auctions. What also is worth mentioning is the fact that the description of the item observed is rather indicative (i.e. it summarizes the main features of the item) than a decoy for bidders to be attracted.

According to what the third step of the methodology suggests, the data filtering concerns to the setting of rules to discard and convert the raw data in order for the information hidden beneath to be revealed. Firstly, bearing in mind that the analysis in a following of the research will incur by using polynomial equations of 3^{rd} class; that entails a prerequisite of 4 points minimum for every auction used in the analysis; if not, such an auction is filtered out of the experiment. It is noted that by the word "point" is implied a (y, t) combination, were y represents the bid amount, while t declares the time of bid of that bid amount.

These points should be a remainder after the clearing of the "noise"; that is, sudden high bids that blur the smooth monotonical augmentation of the price that is usually observed during an ongoing auction; those peaks are generally considered either as "noise" or as "noisy bids". Such cases usually incur when the first bid is expressed only a few seconds after the auction is launched. Beyond that, as it was mentioned above, one should watch out over bids that are suddenly increasing price during an ongoing auction.

Yet another factor of rejection of an auction is the lack of information provided as regards to the bidders. Particularly, information for the winner of the auction is mandatory to be provided, in order for an auction to be included in the samples examined. That is, auctions occurring private listing of bidders, or whether no information for the winner of the auction is provided, then such an auction is excluded.

It may be worth mentioning that, after the initial discard of the auctions in terms of the filtering rules that was cited above, the results indicated that:

- in the 10-day-duration auctions, the 17,97% of them were rejected,
- in the 7-day-duration auctions, the 33,89% of them were rejected and
- in the 5-day-duration auctions, the 25,95% of them were rejected.

Videlicet, approximately the 1/5 of the 10-day-duration auctions, the 1/3 of the 7-day-duration auctions and the ½ of the 5-day-duration auctions did not provide enough information in order to be examined.

Regarding to the data processing, in eBay.com, data is provided in .HTML format, as was shown in **Figure 2-3**, in Chapter 2. Due to the fact that the data would be processed using .XLS sheets, there had to be some kind of processing that would result first and foremost in the capability of the data to be processed and secondly but

equally important, in the uniformity of data. Hence, there was a currency conversion so that all auctions were assessed in US dollars, using the parity given by eBay.com at the time the auction was out. Moreover, it is obvious that day & time of bid info cannot be processed in the form provided by ebay.com (that is, for instance, Apr-06-11 11:48:54 PDT). The resolution proceeded was for the time to be represented as the following total:

$$t =$$
Serial # of the day of the auction + Decimal # of time passed in that day (4-1).

For example, if the day & time of bid above was expressed in a 10 day auction, that was out in March 27^{th} (that is, of serial number 0), then April 6^{th} is the 10^{th} day of the auction. As for the time of bid, the 11:48:54 hour represents the 49,23% of that day, or, as a decimal is number 0,49229 with 5-digit precision that is very accurate in practice, since in online auctions, as are those held in ebay.com, bidders can submit their at any time of day and no evenly-spaced time intervals of acquiring bids are contained. On the contrary, in online auctions bids arrive in very unevenly –spaced time intervals, determined by their bidders and their bidding strategies, so that the number of bids within a short period of time can sometimes be very sparse and othertimes can be very dense. In such a setting, the distance between t and t+1 can be more than a day, or only seconds [1]. Therefore, in such a case, the Apr-06-11 11:48:54 PDT will be represented as t=10,49229.

Furthermore, in order to study upon whether and in what extent the experience of the bidder affects the possibility of who is going to win the auction, elements over the bidders are kept, and mainly over the winner of the auction. However, nowadays, eBay.com has about 135 million registered users, which is a mass of people that is unlikely to be studied; not to mention that not all of those people are of interest, since in the set of auctions observed it is obvious that only a tiny minority of those 135 million users was engaged. What is important here is the relative ranking in between the participants of each auction studied. In order to achieve that, a fraction representing the ratio between the winner of the auction and the other participants of this auction was introduced; a descending sorting over the ratings of the bidders is necessary in order for that ratio to be made; then the first of this sorting with the highest rating is ranked as 1 (one), that is first over the other participants, the second is ranked as 2 etc., until the winner of the auction is reached to be ranked. Then ratio is given as below:

$$ratio = \frac{ranking \quad of \quad winner}{\# of \quad participants} \tag{4-2}$$

For example, in the 10 day auction shown below, the winner of the auction is user $r^{***}b$ with rating 113, which is rated 6^{th} in the relative rank in a total of 8 bidders participating in this auction. Thereupon, the ration is 6:8 or 0,75 decimal, as shown in **Figure 4-1**.

	A	В	C	D	E	F	G	H	1	J	K	L	M	N
1	URL:	http://cgi.ebar	y.com/EMPIRE-CH	INA-COREA-TA	IWAN-COL-MA	AP-BONNE-17	73-D740-/2905	49223791?pt=A	ntiquarian C	ollectible	&hash=item43	3a615516fi	#ht 3784w	vt 1139
2		V 6						The same of the sa		_			NI NI	81
3	Type of item:	1						personal linear	京西子文 》	T. 188	1225 5	THE R. P. LEWIS CO., Land		
	Description:	EMPIRE OF	CHINA COREA TAI	WAN COL MAR	BONNE 1773	#D740		1000 ×		1	57			
5	Quantity of items:	1						100 mg 100	2	建工	1			
6	Rating of the seller:	28064	antiqua.rare.books							是以是	7.			
7	Duration:	10 days						1 - 3 - 3			10			
	Opening bid:	\$2,00						4743			-			
9	**									3 5				
10	Humber of bids:	11								SELVE Y	1			
	Humber of bidders:	8								THE	70000	the second		
12	Rating of the winner:	113						1	10000000000000000000000000000000000000		Course Cours			
	Classification of the rating of the winner:	6/8	0,75	>				1900	1100	1				
14		-						- SARA	160	THE REAL PROPERTY.	MATERIAL PROPERTY.	CONTRACT OF THE PARTY.		
15	BID HISTORY							Control Control		-				
16										ets area	19			
	Serial Number	Date of Bid	Day (#)	Time of Bid	t	Bid Amount	User ID	Rating of User	Ranking					
18	1	Mar-27-11	0		0,73535	\$2,00	r***p	1720	- 1					
19		Apr-03-11	7		7,47534			1496						
20 21		Apr-06-11	10		10,10498			752						
21		Apr-01-11	5		5,60120			655						
22		Apr-05-11	9		9,47777			655						
23		Apr-04-11	8		8,27635			137	5					
24		Apr-06-11	10		10,49229	\$61,01		113	6)				
25 26		Apr-06-11	10		10,28959			77						
26		Apr-06-11	10		10,28385			77						
27 28 29		Apr-04-11	8		8,27390	\$7,00		77						
28	11	Apr-06-11	10		10,17715			9						
29		Mar-27-11	0	0,49237	0,49237	\$1,00	Starting price							

Figure 4-1: presentation of the relative ranking of the winner against the other participants in an auction.

At this point, a note has to be made; since only the non existence of information over the winner of the auction constitutes a reason for this auction to be excluded from examination, if a bidder of the auction that is NOT the winner gives no information of their rating so that to be ranked, then those bidders are treated as having an average ranking; that is, the ranking of the winner (and therefore, the ratio in the following) is estimated as if all the ratings where known and when the ranking of the winner is of the first half of the total of the bidders (excluding the bidders with private rating) then is supposed that the ranking of the bidders with private rating is lower than the winner's ranking; correspondingly, when the ranking of the winner is of the second half of the total of the bidders (excluding the bidders with private rating) then is supposed that the ranking of the bidders with private rating is higher than that of the winner. What is implied in such an assumption is that, in a set of 210 auctions that were finally examined, after all filtering was made, the distribution among the categories that will be formed (top, high, medium and low rating) is balanced, so that it lies near in the case where information about all bidders, with no exceptions made, was available. In any case, when an auction includes private ratings for non-winning bids, the ratio is followed by a question mark "?", nevertheless the decimal number representing that ratio, that is the one to be used in the analysis, is regularly used as suggested from the assumption mentioned above.

The final form of bid history table data involves a serial number of bid, t as sum of day & time of bid expression, bid amount, user ID and user rating; an example is shown to **Figure 4-2.**

							3		State State of the last of the
BID HISTORY									
Serial Number		Date of Bio	Day (#)	Time of Bio	<u>t</u>	Bid Amour	<u>User ID</u>	Rating of U	<u>lser</u>
	1	May-26-11	5	0,51481	5,51481	\$32,02	I***2	13	
	2	May-26-11	5	0,51478	5,51478	\$30,02	I***2	13	
	3	May-26-11	5	0,51476	5,51476	\$33,02	o***u	704	
	4	May-26-11	5	0,41973	5,41973	\$14,00	l***u	243	
	5	May-26-11	5	0,24667	5,24667	\$11,00	l***u	243	
	6	May-23-11	2	0,90800	2,90800	\$7,33	I***2	13	
	7	May-21-11	0	0,91000	0,91000	\$5,00	d***t	27	
	8	May-21-11	0	0,55865	0,55865	\$6,00	o***u	704	
		May-21-11	0	0,51483	0,51483	\$0,99	Starting P	rice	
		<u> </u>							
					1. 1	75, 75,	74. 74.		

Figure 4-2: the final form of bid history table data that is retained; it involves a serial number of bid, *t* as sum of day & time of bid expression, bid amount, user ID and user rating.

4.3.4 Data analysis

The first step in every functional analysis is to recover from the observed data the underlying continuous functional object. This is typically done with the help of smoothing techniques [1]. Therefore, the following step in the process involves curve fitting, so that a mathematical function depicting the evolution of price over time would be constructed. Known as it is, curve fitting is the process of constructing a curve, or mathematical function that has the best fit to a series of data points, possibly subject to constraints. Curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing, in which a "smooth" function is constructed that approximately fits the data. By using a third degree polynomial equation (i.e. $y = ax^3 + bx^2 + cx + d$), it is possible to construct a smoothing spline representing the price evolution over time for each auction studied. For such, exactly fit four points are mandatory.

As it was mentioned earlier on, by the word "point" is implied a (y, t) combination, were y represents the bid amount, while t declares the time of bid of that bid amount. In order to retrieve these (y, t) combinations (from now on they will be referred as "points"), first data is sorted by t column (see Figure 4-2); then "peaks" are

revealed, so that they can be screened out: sudden high bids that blur the smooth monotonical augmentation of the price that is usually observed during an ongoing auction; those peaks are generally considered either as "noise" or as "noisy bids". Such cases usually incur when the first bid is expressed only a few seconds after the auction is launched. Beyond that, as it was cited above, one should watch out over bids that are suddenly increasing price during an ongoing auction.

Bearing this in mind, the first point to be picked is the first bid, then the winning bid, and for the distance in between, there were picked two of the points suggesting *greater increase*, after the clearing of the noise. In other words, one should capture the inflection points by computing the second derivative of the price function. Wolfgang Jank, Galit Shmueli and Shanshan Wang refer to these curve derivatives as price dynamics [1]. They suggest that the smoothing spline, while it describes the magnitude of the current price, nevertheless, it does not reveal how fast the price is changing or moving. Attributes typically associated with a moving object are its velocity (1st derivative of the smoothing spline) and its acceleration (2nd derivative).

The price velocity has several interesting features; its starts out at a relative high mark which is due to the starting price that the first bid has to overcome. After the initial high speed, the price increase may slow down over the next several days, perhaps reaching a close-to-zero price velocity: an extremely low price increase. This is in stark contrast with the price increase on the last day where the price velocity picks up pace and the price is sprung.

Yet an important indicator of dynamics is acceleration, since a change in velocity is preceded by a change in acceleration. In other words, a positive acceleration today will result in an increase of velocity tomorrow. Conversely, a decrease in velocity must be preceded by a deceleration. Concluding, when an auction experiences forces during its entire duration that change price velocity that results in an increase of price acceleration; a flat figure of price acceleration is observed when no bids are placed; with every new bid, the auction experiences new forces; the magnitude of the force depends on the size of the price-increment: smaller price-increments will result in a smaller force, while on the other hand, a large number of small consecutive price-increments will result in a large force [1].

Yet another way of picking the four points for each auction is a grading of the bid-time combinations w.r.t. to time; having sorted the bids in descending order over time, the points can be picked according to the following:

• 1st point: 0%-10% of time,

• 2nd point: 10%-70% of time,

• 3rd point: 70%-90% of time and

• 4th point: 90-100% of time

However, the way of picking that was actually used involved both ways of picking: with reference to the grading mentioned above, the point that prevailed in each fraction was the one that involved the greater increase. Moreover, for uniformity reasons, the first point was the opening bid, having the noisy bids filtered, while the fourth point was the winning bid. Uniformity also suggested that the auctions be starting at $t_1 = 0$ and be closing at $t_4 = 1$. That lies in two causes: firstly, the data that was collected during the data-mining stage start at a different time of a day, even among auctions of the same duration; secondly, as regards to the winning bids and their corresponding time, inference is not possible when, for instance, the winning bid of an auction is at time $t_4 = 10$, 90015, while the one of another is at time $t_4 = 5$, 55784. Note that these two cases are both referring to 10-day-duration auctions. Hence, a formula was used, that suggests that all auctions be converted, so that they are starting at time $t_1 = 0$ (time referring to the opening bid after the clearing of the noise; that is, the 1st point) and they are closing at time $t_4 = 1$ (time referring to the winning bid; that is, the 4th point). This formula states that:

$$t_i = \frac{t_{0i} - t_{01}}{t_{04} - t_{01}} \tag{4-3},$$

where: t_i , the time referring to the i point,

 t_{0i} , the initial time of the *i* point, before the conversion has taken place, t_{01} , the initial time of the first point, before the conversion has taken place and t_{04} , the initial time of the fourth point, the conversion has taken place.

4.3.4.1 Mathematical Modeling

4.3.4.1.1 Regression Analysis

Regression analysis, in statistics, provides the tools needed in order to help one understand how the typical value of the dependent variable changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables — that is, the average value of the dependent variable when the independent variables are held fixed. Consequently, regression analysis is widely used for prediction and forecasting, where its use has

substantial overlap with the field of machine learning. Regression analysis is also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships. In restricted circumstances, regression analysis can be used to infer causal relationships between the independent and dependent variables. The performance of regression analysis methods in practice depends on the form of the data generating process, and how it relates to the regression approach being used [6].

Regression models involve the following variables:

- The unknown parameters denoted as β ; this may be a scalar or a vector.
- The independent variables *X*.
- The dependent variable, *Y*.

A regression model relates *Y* to a function of *X* and β : $Y \approx f(X, \beta)$ (4-4).

Assume now that the vector of unknown parameters β is of length k. In order to perform a regression analysis, providing information about the dependent variable Y is a prerequisite:

- If N data points of the form (Y, X) are observed, where N < k, most classical approaches to regression analysis cannot be performed: since the system of equations defining the regression model is underdetermined, there is not enough data to recover β .
- If exactly N = k data points are observed, and the function f is linear, the equations $Y = f(X, \beta)$ can be solved exactly rather than approximately. This reduces to solving a set of N equations with N unknowns (the elements of β), which has a unique solution as long as the X are linearly independent. If f is nonlinear, a solution may not exist, or many solutions may exist.
- The most common situation is where N > k data points are observed. In this case, there is enough information in the data to estimate a unique value for β that best fits the data in some sense, and the regression model when applied to the data can be viewed as an overdetermined system in β .

In the last case, under certain statistical assumptions, the regression analysis uses the surplus of information to provide statistical information about the unknown parameters β and predicted values of the dependent variable Y[6].

Curve fitting is the process of constructing a curve, or mathematical function that has the best fit to a series of data points, possibly subject to constraints. Curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing, in which a "smooth" function is constructed that approximately fits the data. Extrapolation refers to the use of a fitted curve beyond the range of the observed data, and is subject to a greater degree of uncertainty since it may reflect the method used to construct the curve as much as it reflects the observed data.

In order to construct the polynomial curve representing the bid expression, a cubic function will be used; then this fits in exactly four points [7]:

$$[y^*] = a[t^{*3}] + b[t^{*2}] + c[t] + d$$
 (4-5), where: $y^* = (y_1, y_2, y_3, y_4)$ and $t^* = (t_1, t_2, t_3, t_4)$.

$$\Rightarrow [y^*] = \begin{bmatrix} t_1^3 & t_1^2 & t_1 & 1 \\ t_2^3 & t_2^2 & t_2 & 1 \\ t_3^3 & t_3^2 & t_3 & 1 \\ t_4^3 & t_4^2 & t_4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Leftrightarrow \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} t_1^3 & t_1^2 & t_1 & 1 \\ t_2^3 & t_2^2 & t_2 & 1 \\ t_3^3 & t_3^2 & t_3 & 1 \\ t_4^3 & t_4^2 & t_4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$
(4-6)

It is noted that t is described by the relation (4-3), cited above.

The reason why polynomial curve of third degree is used lies in the smoothness between the connection of any two points that this order of equation involves; in statistics, to smooth a data set is to create an approximating function that attempts to capture important patterns in the data, while leaving out noise or other fine-scale structures/rapid phenomena. The aim of smoothing is to give a general idea of relatively slow changes of value with little attention paid to the close matching of data values, while curve fitting concentrates on achieving as close a match as possible [8].

4.3.4.1.3 Frequency Distribution

The step that follows suggests that it should be investigated whether the results of the process described above are subject to any statistic rules and whether there is any uniformity among these data.

In statistics, a frequency distribution is an arrangement of the values that one or more variables take in a sample. Each entry in the table contains the frequency or count of the occurrences of values within a particular group or interval, and in this way, the table summarizes the distribution of values in the sample. A frequency distribution shows us a summarized grouping of data divided into mutually exclusive classes and the number of occurrences in a class. It is a way of showing unorganized data; Managing and operating on frequency tabulated data is much simpler than operation on raw data. Statistical hypothesis testing is founded on the assessment of differences and similarities between frequency distributions. This assessment involves measures of central tendency or averages, such as the mean and median, and measures of variability or statistical dispersion, such as the standard deviation or variance.

A frequency distribution is said to be skewed when its mean and median are different. The kurtosis of a frequency distribution is the concentration of scores at the mean, or how peaked the distribution appears, if depicted graphically; if the distribution is more peaked than the normal distribution it is said to be leptokurtic; if less peaked it is said to be platykurtic. [22].

4.3.4.2 Excel Modeling

-____

As it was mentioned in § 4.3.3 (data filtering & sample determination), data will be processed using .XLS sheets. In Microsoft Excel, in order to be possible for the expression (6) to be solved, for each element of the set of auctions, two of the excel functions are involved: the MINVERSE() function and the MMULT () function.

The process is as follows: in order to solve the expression (6), for every element of the set of auctions, the formation of the matrix t is mandatory; then this matrix t will be inversed by using the excel function MINVERSE(); that could be represented as: $t^{-1} = \text{MINVERSE}(t)$; afterwards, the formation of the matrix Y is needed; then this matrix Y will be multiplied with matrix t^{-1} by using the excel function MMULT(); that would be represented as $(a, b, c, d) = \text{MMULT}(t^{-1}; Y)$; finally, set prices for a, b, c, d in the respective aggregated matrix, w.r.t. the auction duration; to wit, the result is the formation of three aggregated matrices, w.r.t. the auction duration, that is 5-day-duration, 7-day-duration and 10-day duration, respectively.

The step that follows suggests that it should be investigated whether the results of the process described above that ensued the formation of the three aggregated

matrices lied in the Appendix, are subject to any statistic rules and whether there is any uniformity among these data. Therefore, for each category per duration, figures of the frequency distribution of the features –b/a, c/a and d/a are plotted, using the excel function FREQUENCY().

FREQUENCY () function uses as arguments two arrays: the data array and the bin array; the data consisting the data array are allocated to the intervals (or "bins") they are associated with, depending on how many times these intervals are encountered; these intervals consist the bin array. Due to the fact that, for each one of the figures –b/a, c/a and d/a, a joint graph will be plotted for all auction categories depending on the duration, consequently all three distributions for each one of the figures will be subject to the same range. Thereupon:

For each one of the figures (-b/a, c/a and d/a):

{three data arrays are consisted depending on the duration; each one of these data arrays are then sorted in ascending order; for the creation of the bin array: find the minimum value of all three data arrays; the first value of the bin array is consisted by this minimum value minus ε , where $\varepsilon \in R$, $\varepsilon <<< 1$; the second value of the bin array is consisted by this minimum value minus ε , plus an increment; the third value is consisted by this minimum value minus ε , plus two times the increment and so on, till overlapping the maximum value of all data arrays; function FREQUENCY () results into counting, for each interval, how the data is allocated; then, plot the frequency-bin distribution}.

Following this, it is investigated whether there is any connection between bidding experience and the price height of the winning bid, using the corresponding data obtained as referred in 4.3.3 (data filtering & sample determination).

Relation (7) presents the formula used to calculate the average ratio of the winning bid to the opening bid:

average
$$(win.bid/open.bid) = \sum_{n=1}^{k} \frac{winning\ bid}{opening\ bid}$$
 (4-7),

where k = 64, for 5-day-duration auctions, k = 53, for 7-day-duration auctions and k = 74, for 10-day-duration auctions.

CHAPTER 5

INDEX

		Care Care	
5.1 Results		41	
5.1.1	Figure –b/a	41	, 9
5.1.2	Figure c/a	42	,
5.1.3	Figure d/a	43	•
5.2 Remarks		44	ĺ
5.3 Bidding e.	xperience to winning bid height association	44	!
		1 ((// //	
	/	1 11 11 11 11 11	

5.1 Results

5.1.1 Figure –b/a

According to the preceding, **Figure 5-1** shows the distribution of figure -b/a, for 5-day-duration auctions, 7-day-duration auctions and 10-day duration auctions, respectively.

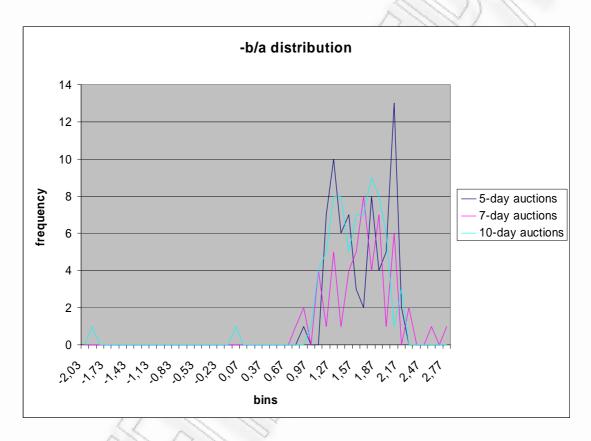


Figure 5-1: (-b/a) distribution for 5-day-duration auctions, 7-day-duration auctions and 10-day duration auctions.

Table 5-1 presents the main characteristics of figure -b/a, w.r.t. the duration of the auctions:

	Duration		
Figure: -b/a	5 days	7 days	10 days
Mean	1,58	1,59	1,46
Std. Deviation	0,34	0,40	0,54
Deviation/Average	0,22	0,25	0,37

Table 5-1: main characteristics of figure (-b/a).

Figure 5-2 shows the distribution of figure c/a, for 5-day-duration auctions, 7-day-duration auctions and 10-day duration auctions, respectively.

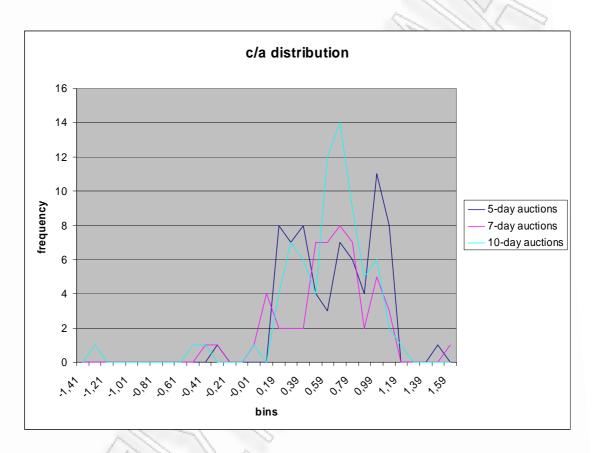


Figure 5-2: (c/a) distribution for 5-day-duration auctions, 7-day-duration auctions and 10-day duration auctions.

Table 5-2 presents the main characteristics of figure c/a, w.r.t. the duration of the auctions:

		Duration	
Figure: c/a	5 days	7 days	10 days
Mean	0,60	0,56	0,53
Std. Deviation	0,34	0,37	0,38
Deviation/Average	0,56	0,66	0,71

Table 5-2: main characteristics of figure (c/a).

Figure 5-3 shows the distribution of figure d/a, for 5-day-duration auctions, 7-day-duration auctions and 10-day duration auctions, respectively.

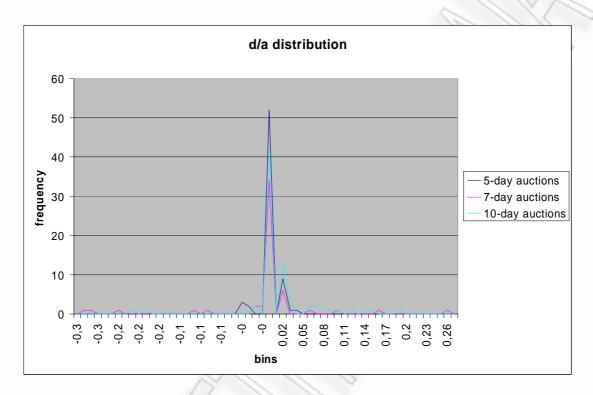


Figure 5-3: (d/a) distribution for 5-day-duration auctions, 7-day-duration auctions and 10-day duration auctions.

Table 5-3 presents the main characteristics of figure d/a, w.r.t. the duration of the auctions:

		Duration	
Figure: d/a	5 days	7 days	10 days
Mean	0,00	-0,01	0,01
Std. Deviation	0,01	0,08	0,05
Deviation/Average	26,90	-13,78	3,67

Table 5-3: main characteristics of figure (d/a).

Figure d/a seems to follow a gaussian but in order for safe results to be drawn, this should be combined with the other figures which, unfortunately, don't seem to be subject to any familiar uniformity. Perhaps, under these other two distributions (those are, -b/a and c/a) more information is hidden, remained to be revealed only after more profound research which entails further disaggregation. Such a consideration is not unlikely to occur, since engravings could be considered as either usual, fabricated items or "artifacts", depending on each item itself; in this process, items where examined altogether regardless of their perishable (or not) nature.

What is of interest though is that, in spite of the unfamiliarity in terms of usual distribution, each one of the three figures graphed present rather similar "behaviour" among the evolution for each duration. That alone amplifies the decision of using splines with 3rd order polynomials, suggesting further disaggregation for better examination in depth; such a further disaggregation is explained in detail in Chapter 6.

5.3 Bidding experience to winning bid height association

Table 5-4 associates the bidding experience and the average ratio of winning bid to opening bid in order to study upon whether and in what extent the experience of the bidder affects the possibility of who is going to win the auction; it is noted that the rating of the bidder is calculated as described in § 4.3.3 (data filtering & sample determination); therefore, high rating is associated with low bidding experience; for example, when rating p>0.9, this usually means that the winners of these auctions had the lowest relative score (rating) among the other participants of the auction; e.g. they were the fifth among five participants, so:

5/5=1>0.9, thereupon, they are classified with the others of this rating and the respective duration of auctions.

The average ratio of winning bid to average bid is calculated in two columns, the average (win.bid/open.bid) and average (win.bid/open.bid)*. The difference between them is that on average (win.bid/open.bid)* ratings are divided on quarters, or, in other words, in the average (win.bid/open.bid)* column, the first two fractions are incorporated in one and so are the two last fractions. Hence, the four fractions that are formed, are of rating: $p \le 0.25$, 0.25 , <math>0.5 and <math>p > 0.75. The reason

why both divisions had to take place, was to enhance the two special fractions, $0,1 \le p$ and $p \ge 0,90$, that are presented in the *average* (*win.bid/open.bid*) column.

		average		average
%rating	5-days	(win.bid/open.bid)		(win.bid/open.bid)*
0.1≤p	1	21,28		(Williams of Stringia)
0.10 <p≤0.25< td=""><td>12</td><td>13,34</td><td>→ \(\sigma\)</td><td>13,95</td></p≤0.25<>	12	13,34	→ \(\sigma\)	13,95
0.25 <p≤0.50< td=""><td>16</td><td>15,27</td><td>\rightarrow</td><td>15,27</td></p≤0.50<>	16	15,27	\rightarrow	15,27
0.50 <p≤0.75< td=""><td>14</td><td>34,93</td><td>→/ <u></u></td><td>34,93</td></p≤0.75<>	14	34,93	→/ <u></u>	34,93
0.75 <p≤0.90< td=""><td>9</td><td>8,53</td><td>4.</td><td>8,85</td></p≤0.90<>	9	8,53	4.	8,85
p≥0.90	12	9,09)) //
SUM	64			3// //
	7-days		//	
				All III
0.1≤p	0	0		1111
0.10 <p≤0.25< td=""><td>9</td><td>16,67</td><td></td><td>16,67</td></p≤0.25<>	9	16,67		16,67
0.25 <p≤0.50< td=""><td>13</td><td>23,68</td><td></td><td>23,68</td></p≤0.50<>	13	23,68		23,68
0.50 <p≤0.75< td=""><td>15</td><td>16,94</td><td>\longrightarrow</td><td>16,94</td></p≤0.75<>	15	16,94	\longrightarrow	16,94
0.75 <p≤0.90< td=""><td>3</td><td>7,79</td><td>7</td><td>42,01</td></p≤0.90<>	3	7,79	7	42,01
p≥0.90	13	49,90		
SUM	53		7	
			11	
	10-days			
0.1≤p	1	84,59		
0.10 <p≤0.25< td=""><td>17</td><td>24,62</td><td>$a_{i_1 i_2 i_3 i_4 i_5 i_5 i_5 i_5 i_5 i_5 i_5 i_5 i_5 i_5$</td><td>27,95</td></p≤0.25<>	17	24,62	$a_{i_1 i_2 i_3 i_4 i_5 i_5 i_5 i_5 i_5 i_5 i_5 i_5 i_5 i_5$	27,95
0.25 <p≤0.50< td=""><td>12</td><td>26,69</td><td>\longrightarrow</td><td>26,69</td></p≤0.50<>	12	26,69	\longrightarrow	26,69
0.50 <p≤0.75< td=""><td>17</td><td>20,03</td><td>→ ·</td><td>20,03</td></p≤0.75<>	17	20,03	→ ·	20,03
0.75 <p≤0.90< td=""><td>12</td><td>13,50</td><td>→ —</td><td>14,04</td></p≤0.90<>	12	13,50	→ —	14,04
p≥0.90	15	14,47	3	
SUM	74	11 11		

Table 5-4: association of the bidding experience and the average ratio of winning bid to opening bid that was noted.



Auctions have proven to be an excellent trading mechanism to allocate goods, services, resources, etc., to individuals and firms; In recent years, a vigorous experimental movement in economics has emerged that involves a direct methodology in relation to the decision making process; derive a result from theoretical economics, set up a laboratory situation that is analogous to the real-world economic situation, and compare the behavior of subjects to the predicted [4].

By using regression analysis with a curve fitting of 3rd class polynomial equations it is intended to analyse the bidding behaviour, as is expressed in the evolution of the price. The sample studied concerned for antique maps and engravings which is a category involving high collectibility and perhaps perishability as features of a product. Such features may lead to an aggressive behaviour on behalf of bidders and therefore they may lead to a segmentation of the sample and the clustering of auctions studied; that comprises the goal of this study: the segmentation of a certain category of items, through the analysis of a series of auctions of this category, according to the bidding behaviour as it is expressed in the evolution of the price; this is due to the observation that the existent price forecasting models consider that the bidding behaviour be rational, that is they prejudice the plenary conformity to the classic expected utility model; in reality, however, "non-rational" as is to say non-expected behaviours are reported from the participants in an auction.

This experiment concerned, thereupon, for the setting of the training phase, in order for the bidding behavior to be recorded of a certain type of item; that is because different features drive different behavior. Videlicet, the training phase deals with determining the parameters a, b, c and d of the spline, so that the relations w.r.t. d would be determined with greater precision; that is, the system presented in Figure 2 would be at first tuned and then fine-tuned, so that in the application phase, the results extracted will be we employed in order to be able to, given only the starting bid and the time span of the auction, predict and forecast the behavior of the bidders for every item of that family.

What was found is that, due to the particularity of antique items raised upon the collectibility matter, no common behavior is revealed with certainty; what is of interest though is that, in spite of the unfamiliarity in terms of usual distribution, each one of the three figures graphed present rather similar "behaviour" among the evolution for each duration. That alone amplifies the decision of using splines with 3rd order polynomials, suggesting further disaggregation that it may occur upon the one of the duration that was induced, for better examination in depth; thus, such further disaggregation could involve:

- Regional division (for instance, division of large scale such as Europe, Africa etc., or of small scale such as Greece, France etc.);
- Division per virtuoso;
- Research over common characteristics among works of art of the same seller; besides, it was noted, among the auctions examined, that each one of the sellers follows the same policy in terms of the duration of the items they place to be auctioned;
- Set periods of antiquity; for example, before 1500 B.C., from 1500 to 1700 B.C., from 1700 to 1900 B.C. or else wise;
- Ubiquity (as it is expressed on price acceleration);

Considering to the finite time horizon of the research that expanded to approximately two months long, it seemed unlikely, eventually, to discrete the truly ubiquitous items, because in such a distance of time, it is not possible to come across to many auctions of exactly the same item, especially in a category professing rarity. However, it appears that fabrication has settled for good in the field of artifacts, where virtuosity has reined for centuries. Despite that, a disaggregation, as it was proposed above, perhaps could uncloud the landscape; yet, it may reveal the individual distributions hidden beneath the blurry –b/a and c/a, as **Figure 14** shows:

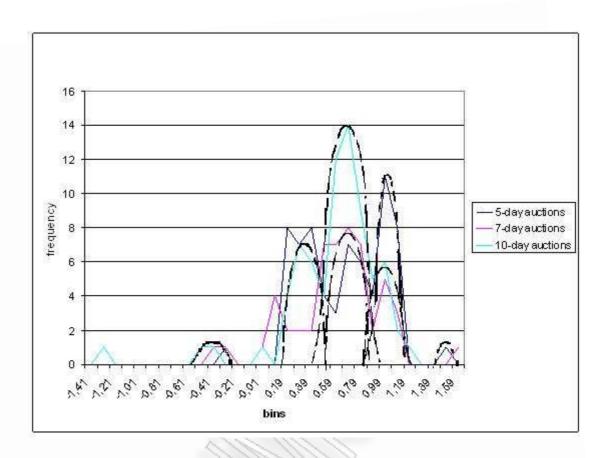


Figure 6-1: futher disaggregation, w.r.t. to region, virtuoso, period of antiquity, or else wise could uncloud the landscape of the bidding behavior in antique maps and engravings.

It may be then possible to unveil the true connection between bidding experience and the relative price height.

BIBLIOGRAPHY

- (1) Dynamic Price Forecasting in Online Auction using Functional Models, Wolfgang Jank, Galit Shmueli & Shanshan Wang.
- (2) A Unified Classification Ecosystem for Auctions, Dimitrios M. Emiris, Charis A. Marentakis.
- (3) Auctions and Auctioneering, Ralph Cassidy, Jr., University of California Press, 1967.
- (4) Bounded Rationality, Bryan D. Jones, Annual Review of Political Science, Nelson Polsby, Editor, November 1998.
- (5) http://www.maersk.com/Procurement/ProcurementForSuppliers/e-Sourcing/Pages/What%20is%20an%20e-Auction.aspx (11/07/2011)
- (6) http://en.wikipedia.org/wiki/Regression_analysis (18/07/2011)
- (7) http://en.wikipedia.org/wiki/Curve_fitting (18/07/2011)
- (8) http://en.wikipedia.org/wiki/Smoothing (18/07/2011)
- (9) Comparative Evaluation of the Unique Elements of e- and m- Auctions, Marentakis C., Emiris D., 24th European Conference on Operational Research (EURO XXIV), Lisbon, Portugal, July 11-14, 2010.
- (10) Advanced Auctions in Dynamic Marketplaces: Classification, Technologies, Applications and Behavioral Study, Marentakis (2011), p.57
- (11) Advanced Auctions in Dynamic Marketplaces: Classification, Technologies, Applications and Behavioral Study, Marentakis (2011), p.55
- (12) The On-line Auction Phenomenon: Growth, Strategies, Promise, and Problems, Ku G., Malhotra D. (2001), Negotiation Journal, October, 2001.
- (13) The Role of Internet Auctions in the Expansion of B2B Markets, Sashi C., O'Leary B. (2002), Industrial Marketing Management, Vol. 31, pp. 103-110.
- (14) Internet Auction Sellers: Does Size Really Matters?, Halstead D., Becherer R.C. (2003), Internet Research: Electronic Networking Applications and Policy, Vol. 13, pp. 183-194.
- (15) Multicast-based Online Auctions: a Performance Perspective, Liu H., Wang S., Fei T. (2003), Benchmarking: An International Journal, Vol. 10, pp. 54-64.
- (16) Advanced Auctions in Dynamic Marketplaces: Classification, Technologies, Applications and Behavioral Study, Marentakis (2011), chapter 2.8.2.
- (17) Advanced Auctions in Dynamic Marketplaces: Classification, Technologies, Applications and Behavioral Study, Marentakis (2011).
- (18) Auctions in Dynamic Marketplaces: Classification, Technologies, Applications and Behavioral Study, Marentakis (2011), p.60.
- (19) Analysis and Comparative Evaluation of Intelligent Methodologies for Short-term Stock Price Forecasting, D.E. Koulouriotis, D.M. Emiris, I.E. Diakoulakis, C. Zopounidis (2002), Fuzzy Economic Review, Vol. III, No. 2, November 2002, p. 23-57.
- (20) Strategic Response to Wait-or-Buy: Revenue Management through Last Minute Deals in the Presence of Customer Learning, Anton Ovchinnikov, Joseph M. Milner, September 2005.

- (21) Emotional Arousal as a Source of Bounded Rationality, Bruce E. Kaufman, Journal of Economic Behavior & Organization, Vol. 38 (1999), p. 135-144.
- (22) http://en.wikipedia.org/wiki/Frequency_distribution (06/09/2011)
- (23) http://www.w3journal.org/online-auction/modern-way-of-shopping-attractive-benefits-of-online-auction/ (20/09/2011)
- (24) http://www.helpful.com/online-auctions-attractive-option-extra-income 381 (20/09/2011)
- (25) http://illinoisattorneygeneral.gov/consumers/onlineauctions.html (20/09/2011)

