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MSc in Banking and Financial Management

**THE RELATIONSHIP BETWEEN VOLATILITY OF ASSET
PRICES AND VOLATILITY OF OUTPUT GROWTH**

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1. Introduction

Nowadays, the world's major central banks have been largely successful at bringing inflation under control. Although it is premature to suggest that inflation is no longer an issue of great concern, it is quite conceivable that the next battles facing central bankers will lie on a different front. One development that has already concentrated the minds of policymakers is an apparent increase in financial instability, of which one important dimension is increased volatility of asset prices. With large movements in asset prices, in the US and Japan apparently coinciding with large swings in growth rates, many commentators have recently called for monetary policy makers to respond to asset price volatility. Moreover, the recent financial crises and associated output declines experienced by a number of emerging market economies have raised anew the issue of the links between financial and real variables, notably between stock market performance and economic activity. Furthermore, increasing capitalization in developed economies has attracted the attention of policymakers and economists with respect to the effects of unanticipated changes in stock returns on growth.

Hence, the main purpose of this paper is to investigate the relationship between asset price volatility and output growth volatility. We are interested in establishing a statistically significant relationship between asset returns volatility, especially stock returns, and the volatility of the economic activity in both developed and emerging economies. The examination of the conditional variance dynamics between stock markets and real economic activity and in particular the volatility transmission mechanism from stock markets to output growth is our ultimate goal. To this direction, two different causality-in-variance tests are used. First, we carry out a methodology widely used in the literature for detecting volatility spillovers that utilizes multivariate GARCH parameterizations. Then, the method developed by Cheung and Ng (1996) is utilized. We re-examine the causality in variance by this test, which provides insight into the dynamics of stock prices and can be used to construct better econometric models.

2. Bibliography

2.1) Asset prices and output growth

Firstly, we shed light to the relationship between asset prices and output growth, without taking volatilities in mind. As Paulo Mauro proved in his paper in 2002, the correlation between output growth and lagged stock returns is significant in emerging market economies as well as in advanced economies. Therefore, asset prices seem to contain valuable information to forecast output. The strength of the correlation is significantly related to a number of stock market characteristics, such as a high market capitalization to GDP ratio and, less robustly, English legal origin and the number of

listed domestic companies and initial public offerings. Supportive to Mauro's results is the paper by Peter Christoffersen and Torsten Slok. They argued that it is a well-known phenomenon that asset prices in developed economies contain information about future developments in the real economy. This is also the case in transition economies. It has been shown in their paper that lagged values of asset prices contain significant signals of changes in real economic activity, in particular industrial production.

2.2) Volatility definition and regularities

Uncertainty and risk are crucial issues in economic theory and finance. The measure of an asset's risk is its volatility, which is defined as the conditional variance of its return. Empirical studies as early as in Mandelbrot (1963) have demonstrated that the variance of stock returns is time varying and persistent. However, until two decades ago econometric models focused mainly on the modeling of the conditional first moments. The increasing importance of risk management and the need for accurate volatility forecasts led to the development of models for the time-varying second-order moments of financial time series in recent years.

Dimitrios Psychoyios, George Skiadopoulos, and Panayotis Alexakis carried out a review on the bibliography about stochastic volatility processes, their properties and implications in 2003. In that paper, they presented the main empirical properties of volatility. First, they mentioned that the probability that extreme events will occur is greater than the corresponding probability calculated under the normal distribution (fat tails). In addition to that, volatility oscillates around a constant value (Mean reversion in volatility) and there is a negative relationship between volatility and price changes (leverage effect). The clustering effect, dividend effect, overnight and weekend effect are some other empirical regularities of volatility. Moreover, there is a relationship between volatility and the information arrival or trading volume. Due to the globalization of equity markets news affecting equity prices in one market may also change the fundamentals in distant markets (volatility "spillovers").

There exist several methods for volatility modeling, most of which aim at capturing the above-mentioned characteristics. The key distinguishing features are the functional form for the conditional moments (mean and variance) and the variables of the information set (\mathfrak{I}_{t-h}), along with any additional distributional assumptions. The models included in this class are the discrete time models, most important of which are the ARCH (AutoRegressive Conditional Heteroscedasticity) models, the GARCH (Generalized ARCH) models and the stochastic volatility models. The main continuous time models are the continuous sample path diffusions and the jump diffusions & levy driven processes.

Financial market volatility is central to the theory and practice of asset pricing, asset allocation and risk management. Although most textbook models assume volatilities and correlations to be constant, it is widely recognized among both finance academics and practitioners that they vary importantly over time. The volatility of asset prices tends to change stochastically over time. This has important implications for option pricing and risk management.

One distinctive features of the stochastic process of stock returns that are often mentioned in the literature is time-varying volatility. It suggests that the unconditional distribution of an asset price exhibits “fat tails and high peak”, though the price is conditionally normal. The most well known models that characterize this feature are the Autoregressive Conditional Heteroscedasticity (ARCH) model by Engle and the more generalized version of it by Bollerslev.

The ARCH family is a sophisticated group of time series models. The ARCH (q) model relates time t volatility to q past squared returns, with no predetermined relationships between any of the q dependencies. GARCH models have been very popular and effective for modeling the volatility dynamics in many asset markets. In GARCH (p, q) additional dependencies are permitted on p lags of past estimated volatility.

The general structure of an ARCH model is the following:

Let r_t be the log return of an asset at time index t and let us consider the conditional mean and conditional variance of r_t given the information set \mathfrak{I}_{t-1} :

$$\mu_t = E(r_t | \mathfrak{I}_{t-1}) \text{ and } \sigma_t^2 = \text{Var}(r_t | \mathfrak{I}_{t-1}) = E[(r_t - \mu_t)^2 | \mathfrak{I}_{t-1}]$$

Assuming that r_t follows a time series model such as stationary ARMA (p, q) we get the model:

$$r_t = \mu_t + \varepsilon_t, \quad \mu_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} - \sum_{i=1}^q \theta_i \varepsilon_{t-i}, \text{ where } r_t, p, q \text{ are non-negative integers.}$$

Therefore, $\sigma_t^2 = \text{Var}(r_t | \mathfrak{I}_{t-1}) = \text{Var}(\varepsilon_t | \mathfrak{I}_{t-1})$. The manner in which σ_t^2 evolves over time distinguishes one volatility model from another.

ARCH supposes that the conditional variance, σ_t^2 , is a linear function of past squared values of the process ε_t , the mean-corrected asset return.

$$\varepsilon_t = \sigma_t \eta_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 = \alpha_0 + \alpha(L) \varepsilon_t^2$$

with $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$

Where $\{\eta_t\}$ is a sequence of IID random variables with mean zero and variance one. Under the ARCH model, large past shocks tend to be followed by other large shocks, allowing for the modeling of the so called "volatility clustering" in returns.

Bollerslev (1986) proposed an extension of this type of models, known as Generalized ARCH models. Using the same notation as before, a GARCH (p, q) is:

$$\varepsilon_t = \sigma_t \eta_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 = \alpha_0 + \alpha(L) \varepsilon_t^2 + \beta(L) \sigma_t^2$$

with $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$.

2.3) Asset price volatility

Financial market volatility is an important input for investment, option pricing and financial market regulation. Forecasting financial market volatility is an important

activity that has occupied the attention of academics and practitioners over the last decades since it does influence policy makers' behavior and it is the key concern in financial risk management. Rosa Rodriguez, Fernando Restoy and J Ignacio Pena proved in 1998 that asset prices are probably too volatile to be explained by its economic fundamentals through the standard present value relationships. It has been documented that the variability of price movements is too large to be justified in terms of simple statistical representations of the efficient market hypothesis, given the relatively low variability of output. Many scientific papers try to explain the sources of financial volatility. Warren Bailey in his paper in 1998 studied the impact of money supply releases on volatility by documenting the relationship between announced money surprises and implied standard deviations of stock, treasury bonds, gold and foreign currency prices and has confirmed that monetary policy and money supply releases affect the volatility of asset prices and, ultimately, the volatility of the entire economy.

Moreover, openness to international capital flows and the international integration of financial markets have been blamed for the excess volatility of financial assets. Helmut Wagner and Wolfram Berger in 2004 claimed that openness to international capital flows and the international integration of financial markets do not only have their benefits but also seem to make countries susceptible to rapid capital flow reversals. This may permit financial and 'balance of payments' crises to develop and impair real economic activity by requiring large output adjustments. Consequently, the increased volatility of international capital flows leads to a higher volatility of asset prices. Furthermore, Lucy F. Ackert, Bryan K. Church and Ann B Gillette in their paper published in 2001 proved that price volatility and profits dispersion are significantly higher and allocation efficiencies significantly lower when highly uncertain information is released, as compared to markets with more reliable information or even those in which information is withheld.

2.4) Output growth volatility and the Central's Bank behavior

As far as output growth volatility and economic activity is concerned, many academics have investigated in its potential sources and established statistically significant relationships. Olivier Blanchard and John Simon argued in their paper in 2001 that the decrease in volatility of US output can be traced to a number of proximate causes, from a decrease in the volatility of government spending early on (monetary and fiscal policy), to a decrease in consumption and investment volatility throughout the period (financial markets), to a change in the sign of the correlation between inventory investment and sales (logistics) in the last decade. Furthermore, they proved that there is a strong relationship between both output volatility and the level of inflation, and between output volatility and inflation volatility. Lower output volatility suggests lower risk, and thus changes in risk premiums, in precautionary saving, and so on. Interestingly, however, the decrease in output volatility has not been reflected in a parallel decrease in asset price volatility. This is not necessarily a puzzle. If we think of the better use of monetary policy as one of the factors behind the decrease in output volatility, stronger

stabilization efforts may require sharper movements in interest rates, and thus potentially stronger movements in asset prices.

An interesting approach was initiated in Christis Hassapis and Sarantis Kalyvitis' paper, which was published in *Journal of Policy Modeling* in 2001. They proved that in developed economies, output does not react significantly to unanticipated shocks in the domestic stock market. Domestic stock market fluctuations do not appear to have been a prominent source of output volatility. A notable exception is US, where these fluctuations appear to systematically account for a fraction of excess output volatility. Output volatility in major industrialized economies appears to be exposed to financial developments abroad. Empirical results in their paper from two-country VAR and a global (G-7) VAR suggest that part of excess variability in output growth can be attributed to disturbances in foreign (US) real returns.

It has been shown that there is a negative correlation between the mean and the variance of output growth irrespective of the source of fluctuations. The model by Keith Blackburn and Alessandra Pelloni predicts a negative correlation between short-run (cyclical) volatility and long-run (secular) growth. In addition to these results, Garey Ramey and Valery Ramey have examined thoroughly the relationship between volatility and output growth and showed using a panel of 92 countries as well as a subset of OECD countries, that countries with higher volatility have lower mean output growth even after controlling for other country specific growth correlates. There is evidence in their research that countries with higher year-to-year volatility in growth rates tend to have systematically lower growth rates.

Given the role that asset prices play on the transmission mechanism, central banks have been often tempted to use them as targets of monetary policy. Fluctuations of the stock market, which are influenced by monetary policy, have important impacts on the aggregate economy through effects on investment, firm balance-sheet effects, household wealth effects and household liquidity effects. Volatile monetary growth generates volatile interest rates, which raise the riskiness of bond holdings. This, in turn, raises the demand for money and the level of interest rates, which impedes corporate investment and lowers the level of real economic activity. There is strong evidence in the literature of significant linkage from either monetary or interest rate volatility to the level of real economic performance. Higher money growth volatility is transmitted directly to higher real output volatility and is associated with lower financial asset price and inflation volatility.

We conclude this brief review of the literature related to asset price and output growth volatility by raising the question of the Central Bank's appropriate response. While financial volatility in part reflects the nature of asset prices, driven primarily by revisions in expectations of future returns, large movements raise questions about the appropriate response of monetary policy. Frank Smets in 1997 claimed that the basic principle is simple: the central bank's response to unexplained changes in asset prices should depend on how these changes affect the inflation outlook, which in turn depends on two factors: the role of asset price in the transmission mechanism and the typical information content of innovations in asset prices. Finally, we should mention the paper "Monetary Policy and Asset Price Volatility" written by Ben Bernanke and Mark Gertler and published by *Economic Review* in 1999. They argued that fluctuations in asset prices should be a concern to policymakers if two conditions are met. The first is that 'non-

fundamental' factors sometimes underlie asset market volatility.(poor regulatory practice and imperfect rationality on the part of investors 'market psychology').The second is that changes in asset prices unrelated to fundamental factors have potentially significant impacts on the rest of the economy.(Via the effect on household wealth, 'balance-sheet channel-financial accelerator'). If these two conditions are satisfied, then asset price volatility becomes, to some degree, an independent source of economic instability, of which policymakers should take account.

2.5) Asset price volatility, output growth volatility and volatility spillovers

Volatility is often related to the rate of information flow (e.g., Ross, 1989). If information comes in clusters, asset returns or prices may exhibit volatility even if the market perfectly and instantaneously adjusts to the news. Therefore, study on volatility spillover can help understand how information is transmitted across assets and markets. Alternatively, the existence of volatility spillover may be consistent with the market dynamics which exhibits volatility persistence due to private information or heterogeneous beliefs (e.g., Admati and Pfleiderer, 1988; Kyle, 1985; Shalen, 1993). Here, whether volatilities are correlated across markets is important in examining the speed of market adjustment to new information. It is also hypothesized that the changes in market volatility are related to the volatilities of macroeconomic variables. In present value models such as those of Shiller (1981a, b), for example, changes in the volatility of either future cash flows or discount rates cause changes in the volatility of stock returns. Such a macroeconomic hypothesis can be checked by testing volatility spillover.

Hu John Wei-Shan, Mei-Yuan Chen b, Robert C.W. Fok, Bwo-Nung Huang in their paper published in 1997 ("Causality in volatility and volatility spillover effects between US, Japan and four equity markets in the South China Growth Triangular", Journal of International Financial Markets Institutions and Money 7) examined the spillover effects of volatility among two developed markets US and Japan and four emerging markets in the South China Growth Triangular using Chueng and Ng's causality-in-variance test. They arrived at the conclusion that Japanese stock market affects the US stock market and there is a feedback relationship between the Hong Kong and US stock market. Moreover, markets of the SCGT are contemporaneously correlated with the return volatility of the US market and econometric models constructed according to the results of causality-in variance tests have greater explanatory power than the conventional GARCH(1,1) model. Finally, they proved that using the return volatility of foreign exchange as a proxy for informational arrival can explain excess kurtosis of a stock return series, especially for the less open emerging market and that geographic proximity and economic ties do not necessarily lead to a strong relationship in volatility across markets.

Dušan Isakov and Christophe Pérignon in their paper "On the dynamic interdependence of international stock markets: A Swiss perspective" in April 2000 studied the links existing between the Swiss stock market and the five largest stock markets in the world (USA, Japan, United Kingdom, Germany and France) in terms of return and volatility. Firstly, they found that conditional heteroskedasticity was present in

every market and that conditional volatility responded asymmetrically to past shocks. By implementing a bivariate GARCH (1, 1) they proved that that the US market had the strongest influence on the Swiss market in terms of returns and volatility. Links with other markets in terms of returns were relatively weak. The German and British markets strongly influenced the volatility of the Swiss market. On the other hand, they found that the Swiss market had a statistically significant but economically weak influence on the foreign markets.

Another interesting study of volatility spillovers was carried out by Guglielmo Maria Caporale and Nicola Spagnolo. In their paper, published in June 2002, they investigated the real effects of financial crisis in 1997 in East Asian countries on the casual relationship between stock prices and output growth volatility. They used a bivariate GARCH (1, 1) model and tests for causality in variance were carried out for each model allowing for causality in one way direction at a time. They found positive and statistically significant volatility spillovers running from the stock markets to output growth in all six countries under examination. The East Asian crisis appears to have led to a sharp increase in the magnitude of the spillovers. They implemented their model in six countries Malaysia, Philippines, Thailand, Canada, U.S. and U.K. The result of a financial crisis is that the real economy becomes even more responsive to financial market turbulence, implying that under such circumstances policymakers should be even more concerned with the linkage between asset prices and output growth volatility.

2.6) Tests for Volatility Spillovers- Causality in Variance

Recently there has been increasing interest in the causation in conditional variance across various financial asset price movements. The study of causality in variance is of interest to both academics and practitioners because of its economic and statistical significance.

Changes in variance are said to reflect the arrival of information and the extent to which the market evaluates and assimilates new information. Ross in his paper published in 1989 proves that in a no-arbitrage economy the variance of price changes is directly related to the rate of information flow to the market. Engel, Ito and Lin in 1990 attributed movements in variance to the time required by the market participants in processing new information or in policy coordination. Thus, the relationship between information flow and volatility gives an interesting perspective to interpret the causation in variance between a pair of economic time series.

A methodology widely used in the literature for detecting volatility spillovers utilizes multivariate GARCH parameterizations. Theodossiou and Lee (1993), using a multivariate GARCH-M model, found that the US market was the major “exporter” of volatility.

The multivariate GARCH model is used as the unrestricted model. Then, we set some restrictions in the unrestricted models testing the hypothesis of no causality in variance. For the hypothesis testing, we use the Likelihood Ratio:

$$LR = -2 (l^{\text{restricted}} - l^{\text{unrestricted}}) \sim \chi^2 (\text{number of restrictions})$$

In 1996, Yin-Wong Cheung and Lillian K. Ng developed in their paper a new test for causality in variance. This test for causality-in-variance is an extension of Wiener-Granger causality in mean, based on the cross-correlation function (CCF). It is a two-stage procedure. The first stage involves the estimation of univariate time series models that allow for time variation in both conditional mean and conditional variance. In the second stage, the resulting series of squared residuals standardized by conditional variances are constructed. The cross-correlation function of these squared-standardized residuals is then used to test the null hypothesis of no causality in variance. This test is robust to distributional assumptions. This test will be implemented in this study as well and therefore we present the procedure followed by the writers in details.

They set off by considering two stationary and ergodic time series, X_t and Y_t . Let I_t, J_t be two information sets defined by $I_t = \{X_{t-j}, j \geq 0\}$ and $J_t = \{X_{t-j}, Y_{t-j}, j \geq 0\}$. Y_t is said to cause X_{t+1} in variance if:

$$E \{(X_{t+1} - \mu_{x,t+1})^2 | I_t\} \neq E \{(X_{t+1} - \mu_{x,t+1})^2 | J_t\}$$

Where $\mu_{x,t+1}$ is the mean of X_{t+1} conditioned on I_t . Feedback in variance occurs if X causes Y and Y causes X . There is instantaneous causality in variance if:

$$E \{(X_{t+1} - \mu_{x,t+1})^2 | J_t\} \neq E \{(X_{t+1} - \mu_{x,t+1})^2 | J_t + Y_{t+1}\}$$

In order to make the general causality concept applicable in practice they launched additional structure. They defined X_t and Y_t as:

$$X_t = \mu_{x,t} + \sqrt{h_{x,t}} \varepsilon_t$$

$$Y_t = \mu_{y,t} + \sqrt{h_{y,t}} \zeta_t$$

Where $\{\varepsilon_t\}$ and $\{\zeta_t\}$ are two independent white noise processes with zero mean and unit variance. Their conditional means and variances are given by

$$\mu_{z,t} = \sum_{i=1}^{\infty} \varphi_{z,i} (\theta_{z,\mu}) Z_{t-i}$$

$$h_{z,t} = \varphi_{z,0} + \sum_{i=1}^{\infty} \varphi_{z,i} \{(Z_{t-i} - \mu_{z,t-i})^2 - \varphi_{z,0}\}$$

where $\theta_{z,w}$ is a $p_{z,w} \times 1$ parameter vector; $W = \mu, h$; $\varphi_{z,i} (\theta_{z,\mu})$ and $\varphi_{z,i} (\theta_{z,h})$ are uniquely defined functions of $\theta_{z,\mu}$; and $Z = X, Y$.

Then, they defined U_t and V_t as the squares of the standardized innovations:

$$U_t = ((X_t - \mu_{x,t})^2 / h_{x,t}) = \varepsilon_t^2$$

$$V_t = ((Y_t - \mu_{y,t})^2 / h_{y,t}) = \zeta_t^2$$

And the sample cross-correlation at lag k as

$$r_{uv}(k) = c_{uv}(k) (c_{uu}(0) c_{vv}(0))^{-1/2},$$

Where $c_{uv}(k)$ is the k^{th} lag sample cross covariance given by

$$c_{uv}(k) = T^{-1} \sum (U_t - \bar{U})(V_{t-k} - \bar{V}), k=0, \pm 1, \pm 2, \dots$$

and $c_{uu}(0)$ and $c_{vv}(0)$ are the sample variances of U and V respectively.

They argued that since $\{U_t\}$ and $\{V_t\}$ are independent, the existence of their second moments implies

$$\begin{pmatrix} \sqrt{T} r_{uv}(k) \\ \sqrt{T} r_{uv}(k') \end{pmatrix} \rightarrow AN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), k \neq k'$$

As in the test for causality in mean, this expression suggests that the CCF of squared standardized residuals can be used to detect causal relations and identify patterns of causation in the second moment. The utility of the CCF has certain advantages over some possible alternative tests for causality in variance. For instance, compared with a multivariate method, the CCF approach does not involve simultaneous modeling of both intra- and inter-series dynamics, and hence it is relatively easy to implement. Further, the proposed test has a well-defined asymptotic distribution and is asymptotically robust to distributional assumptions. However, it is not designed to detect causation patterns that yield zero cross-correlations.

Since both U_t and V_t are unobservable, their estimators were used to test the hypothesis of no causality in variance. The sample correlation coefficient $\hat{r}_{uv}(k)$ computed from the consistent estimates of the conditional means and variances of X_t and Y_t in place of $r_{uv}(k)$. The property of $\hat{r}_{uv}(k)$ is given by:

(Theorem) $\sqrt{T} (\hat{r}_{uv}(k_1), \dots, \hat{r}_{uv}(k_m))$ converge to $N(0, I_m)$ as $T \rightarrow \infty$, where k_1, \dots, k_m are m different integers, if:

- (i) both $E(\varepsilon_t^8)$ and $E(\zeta_t^8)$ exist, and
- (ii) for all θ in an open convex neighborhood $N(\theta^0)$ of θ^0 and for all T , $\sqrt{T} \partial^2 c_{AB}(k) / \partial \theta_i \partial \theta_j$ exists and is bounded in probability for $\theta_i, \theta_j \in \theta$ and for $A, B = U, V$.

Given the asymptotic behavior of $\hat{r}_{uv}(k)$, a normal test statistic or a chi-square test statistic can be constructed to test the null hypothesis of noncausality. To test for a causal relationship at a specified lag k , we can compare $\sqrt{T} \hat{r}_{uv}(k)$ with the standard normal distribution. Alternatively, a chi-square test statistic defined by:

$$S = T \sum_{i=j}^k \hat{r}_{uv}(i)^2,$$

which has a chi-square distribution with $(k-j+1)$ degrees of freedom, can be used to test the hypothesis of no causality from lag j to lag k . The choice of j and k depends on the specification of alternative hypotheses. When there is no a priori information on the direction of causality, we may set $-j=k=m$. The parameter m should be large enough to include the largest nonzero lag that may appear in the causation pattern. When a uni-directional causality pattern, say, Y_t does not cause X_t , is considered, we set $j=1$ and $k=m$.

An interesting remark is that causality in the mean of X_t and Y_t can be tested as well by examining $\hat{r}_{\varepsilon\zeta}(k)$, the univariate standardized residual CCF, and using the test statistic that also converges to the standardized normal distribution.

While implementing the above-mentioned method, one should have in mind that the existence of serial correlation in ε_t and ζ_t or in U_t and V_t can affect the size of the proposed tests for causality in mean and variance. Therefore, the model specified in the first stage should “accurately” account for the serial autocorrelation in the data. In addition to that, the existence of causality in mean violates the independence assumption and hence may affect the CCF test. This, however, depends on the model specification.

Cheng and Ng apply the CCF test to a) daily index returns on Japan Nikkei 225 Index and US S&P index and b) 15-min returns on the S&P500 index futures and the corresponding returns on the underlying index. They find that the US stock index causes the Japanese stock index in variance, while a feedback appears in the variance of the 15-min stock index and futures returns.

Yongmiao Hong in his paper “A test for volatility spillover with application to exchange rates” published in October 2000 proposed a class of asymptotic $N(0, 1)$ tests for volatility spillover between two time series that exhibit conditional heteroskedasticity and may have infinite unconditional variances. The tests were based on a weighted sum of squared sample cross-correlations between two squared standardized residuals. We allow to use all the sample cross-correlations, and introduce a flexible weighting scheme for the sample cross-correlation at each lag. He tested Granger-causalities between two weekly nominal U.S. dollar exchange rates Deutschemark and Japanese yen. It was found that for causality in mean, there existed only simultaneous interaction between the two exchange rates. For causality in variance, there also existed strong simultaneous interaction between them. Moreover, a change in past Deutschemark volatility Granger-caused a change in current Japanese yen volatility, but a change in past Japanese yen volatility did not Granger-cause a change in current Deutschemark volatility.

3. The Data

We are interested in establishing a relationship between asset price volatility and output growth volatility for two groups of economies. The first group consists of six developed economies U.S., U.K., Canada, France, Germany and Japan, whereas three developing countries, Greece, Spain and Portugal, comprise our second group of interest. Output growth is measured in industrial production terms and asset prices in stock market returns. We define stock returns and industrial production growth rate as the logarithmic differences of stock indices and industrial production, respectively. The data comes in monthly terms from DATASTREAM and covers a time range of 15 years (January 1990-December 2004).

For U.S.A. we used the Industrial Production Index Total Industry (excluding Construction) from January 1990 till November 2004 in monthly terms. As far as asset prices, Standards & Poors 500 Composite Price Index for the same period was used.

U.K. Industrial Production Total Industry (excluding Construction) Index was the index chosen for U.K.’s Industrial production and FTSE 100 Price Index was the one for asset prices. The data came in monthly terms as well from January 1990 till December 2004.

For Canada, France and Germany Industrial Production SADI index measured the output growth and the changes in the DS ENG. General Price Index of each country represented the Stock Returns. Data came also in monthly terms from January 1990 till December 2004.

As far as Japan is concerned, the Industrial Production Total Industry (excluding Construction) Index composed our input data for Industrial Production and NIKKEI 225 Stock Average Price Index for asset prices. The period covered was from January 1990 until October 2004.

For Spain, we used the Industrial Production Total Industry (excluding Construction) Index from January 1990 till December 2004 and the IBEX 35 Price Index for the same period in monthly terms as well.

The relationship between volatility of asset prices and volatility of output growth

Industrial Production excluding Construction was the index used for Greece's Industrial Production. The stock price index was the Athens's SE General Price Index. The data for both indexes came in monthly terms and covered a 15 years period (January 1990- December 2004) .

Finally, for Portugal, we used the Industrial Production Industry (excluding Construction) Index and the PSI General Price Index for Industrial Production and asset prices respectively. The period was from January 1990 until December 2004.

Due to the fact that GARCH modeling assumes a return series, we need to convert the prices to returns. Stock returns are defined as percentage logarithmic differences of closing prices between two consecutive months, i.e.

$$\text{Stock Returns} = \ln \left(\frac{P_t}{P_{t-1}} \right) \quad (\text{continuously compounded stock returns})$$

Industrial Production Growth as percentage logarithmic differences of the index price between two consecutive months, i.e.

$$\text{Industrial Production Growth} = \ln \left(\frac{IP_t}{IP_{t-1}} \right) \quad (\text{continuously compounded Industrial Production Growth})$$

4. The Theory

The reason we decided to investigate the relationship between output growth volatility and asset price volatility is that a great deal of bibliography examines the causality patterns between asset prices and output growth. But only a few researches about the causality in variance have been carried out. So, we are interested in establishing a statistically significant relationship between the volatilities of these two macroeconomic factors.

Many theories have been put forward on the link between stock returns and output growth. The first one, called the "passive informant" hypothesis, states that the only mechanism underlying the correlation between stock returns and output growth is the following. Under the assumptions that stock prices reflect the present discounted value of all future dividends and that dividend growth is related to GDP growth, a correlation between this year's stock returns and next year's economic growth arises naturally: if next year's economic growth is buoyant, news revealed this year will typically be positive, resulting in large stock price increases this year. All theories reviewed below accept that the above mechanism plays a role, but leave room for additional mechanisms.

Under the "accurate active informant" hypothesis, stock price changes provide managers with information about the market's expectations of future economic developments. Managers base their investment decisions upon that information, thereby justifying the market's expectations. In this case, stock price changes turn out to be perfectly correlated with fundamentals.

The third theory, the "faulty active informant" hypothesis, suggests that managers' decisions about investment are influenced by stock price movements, but

managers cannot distinguish between movements reflecting fundamentals and those reflecting market “sentiment”. Stock market movements that are not motivated by fundamentals can therefore mislead managers into overinvesting or underinvesting compared with what later turns out to be warranted by fundamentals.

The “financing” hypothesis, based upon Tobin’s q theory (a formalization of Keynes’ reasoning in the quote reported above), argues that when stock prices are high compared to the replacement cost of capital, entrepreneurs will be more likely to expand their activities by investing in new physical capital (possibly financed by issuing new shares of their company) rather than by purchasing existing firms on the stock market.

Moreover, the “stock market pressure on managers” hypothesis suggests that stock price changes can affect investment even if they neither convey information nor change financing costs. If investors hold negative views on a firm’s prospects and drive down its stock price, managers may have to cut their investment projects to protect themselves from the possibility of being fired or taken over.

Fluctuations of the stock market, which are influenced by monetary policy, have important impacts on the aggregate economy. Transmission mechanisms involving the stock market are of three types:

- 1) Stock market effects on investment (Tobin’s q model)
- 2) Firm balance-sheet effects,
- 3) Household wealth effects: expansionary monetary policy which raises stock prices raises the value of household wealth, thereby increasing the lifetime resources of consumers, which causes consumption to rise.

An interesting question is whether fluctuations in asset prices should be of concern to policymakers? In the economist’s usual benchmark case, a world of efficient capital markets and without regulatory distortions, movements in asset prices simply reflect changes in underlying economic fundamentals. Under these circumstances, central bankers would have no reason to concern themselves with asset price volatility *per se*. Asset prices would be of interest only to the extent that they provide useful information about the state of the economy. Matters change, however if two conditions are met. The first is that “non-fundamental” factors sometime underlie asset market volatility. The second is that changes in asset prices unrelated to fundamental factors have potentially significant impacts on the rest of the economy. If these two conditions are satisfied, then asset price volatility becomes to some degree an independent source of economic instability, of which policy-makers should take account.

5. The 1st approach

5.1 Time series models for modelling financial volatility

A more sophisticated group of time series models is the ARCH family. The ARCH (q) model relates time t volatility to q past squared returns, with no predetermined

relationships between any of the q dependencies. In GARCH (p, q), additional dependencies are permitted on p lags of past estimated volatility. In general, empirical findings suggest that GARCH is a more parsimonious model than ARCH. Modeling volatility in logarithmic form resulting in the EGARCH model. In the EGARCH case, there is no need to impose estimation constraint in order to avoid negative variance because it is the logarithm of σ_t^2 that is formulated. With appropriate conditioning of the parameter, this specification captures the stylized fact that negative shocks lead to higher volatility in the subsequent period than that triggered by a positive shock. Other models that allow for nonsymmetrical dependencies are the GJR model (Glosten, Jagannathan and Runkle (1993)), QGARCH (Quadratic GARCH) and various non-linear GARCH, such as logistic smooth transition GARCH and exponential smooth transition GARCH, reviewed in Franses and van Dijk (2000). Both ARCH and GARCH models have also been implemented with a Hamilton (1989) type regime switching framework, where volatility persistence can take different values depending on whether it is in high or low volatility regimes. The most generalized form of regime switching model is the RS-GARCH (1, 1) model used in Gray (1996) and Klaassen (1998). The ARCH version of the regime switching model is very similar to the Threshold Autoregressive (TAR) model in Cao and Tsay (1992) except that TAR has independent noise processes in each volatility state and that the volatility and the return specifications are not estimated jointly. Finally, there is the Stochastic Volatility (SV) model, which involves a noise process in the variance equation that is independent from that in the return equation. The additional noise process in the variance equation makes the model a lot more flexible, but, as a result, the SV model has no closed form, making direct maximum likelihood estimation infeasible. Thus far, methods used to estimate the SV model are computationally more difficult to implement and, in some cases, the theoretical properties of the estimations are still unknown. For this reason, the SV model is not as popular as the ARCH model. One way to avoid this estimation problem is to abandon the structure of the mean and express the volatility simply as a function of its past values. This is known as the Simple Regression (SR) method. The SR method is principally autoregressive. If past volatility errors are included, one gets the ARMA model for volatility. Other nonparametric methods have been suggested. But in a forecasting exercise by Pagan and Schwert (1990), it was found that the nonparametric methods perform poorly.

5.2 Multivariate GARCH models

We may use a multivariate GARCH model in case we have a vector of asset returns whose conditional covariance matrix evolves over time. Suppose we have N assets with return innovations $\eta_{i,t+1}$, $i=1,2,\dots,N$. We form a vector with these innovations, $\eta_{t+1} = [\eta_{1,t+1} \dots \eta_{N,t+1}]'$ and define $\sigma_{ii,t} = \text{Var}(\eta_{i,t+1})$ and $\sigma_{ij,t} = \text{Cov}(\eta_{i,t+1}, \eta_{j,t+1})$; Hence $\Sigma = [\sigma_{ij,t}]$ is the conditional covariance matrix of all the returns.

We may now put all the no redundant elements of Σ_t (on and below the diagonal) into a vector. The operator which performs is known as the vech operator: $\text{vech}(\Sigma_t)$ with $N(N+1)/2$ elements.

The relationship between volatility of asset prices and volatility of output growth

Bollerslev et al (1988) in his VECH MODEL forms the covariance matrix as a set of univariate GARCH models. Each element of Σ_t follows a univariate GARCH model driven by the corresponding element of the cross-product matrix $\eta_t \eta_t'$.

$$\text{VECH}(\Sigma_t) = C + A \text{VECH}(\eta_t, \eta_{t-1}') + B \text{VECH}(\Sigma_{t-1}), \eta_t | \Psi_{t-1} \sim N(0, H_t),$$

where C is an $(N(N+1)/2)$ vector containing the intercepts in the conditional variance and covariance equations, A and B are $N(N+1)/2 * N(N+1)/2$ matrices containing the parameters on the lagged disturbance squares or cross-products and on the lagged variances or covariances respectively. The implied conditional covariance matrix is always positive definite if the matrices of the parameters C, A and B are all positive definite. The model has three parameters for each element of Σ_t thus $3N(N+1)/2$ in all. The weakness of the proposed model is its failure in capturing co-persistence in variance and asymmetries.

Another widely used multivariate GARCH model is the BEKK model proposed by Engle and Kroner(1995). That model guarantees the positive definiteness of Σ_t by working with quadratic forms rather than the individual elements of Σ_t .

$$\Sigma_t = C'C + B' \Sigma_{t-1} B + A' \eta_t \eta_t' A$$

where C is a lower triangular matrix with $N(N+1)/2$ parameters, B and A are square matrices with N^2 each, for a total parameter count $(5N^2+N)/2$. Weak restrictions on A and B guarantee that Σ_t is always positive definite.

Bollerslev (1990) has proposed a Constant Correlation model in which each asset return variance follows a univariate GARCH (1, 1) model and the covariance between any two assets is given by a constant-correlation coefficient multiplying the conditional standard deviation of returns:

$$\begin{aligned} \sigma_{ii,t} &= \omega_{ii} + \beta_{ii} \sigma_{ii,t-1} + \alpha_{ii} \eta_{it}^2 \\ \sigma_{ij,t} &= \rho_{ij} \sqrt{\sigma_{ii,t} \sigma_{jj,t}} \end{aligned}$$

$N(N+5)/2$ parameters

It gives a positive definite covariance matrix provided that the correlations ρ_{ij} make up a well-defined correlation matrix and the parameters ω_{ii} , β_{ii} and α_{ii} are positive.

A special case of the BEKK model is the single-factor GARCH (1, 1) model of Engle et al(1990). In this model we define N-vectors λ and w α and scalars α and β and then have:

$$\Sigma_t = C'C + \lambda \lambda' [\beta \omega' \Sigma_{t-1} w + \alpha (w' \eta_t)^2]$$

It is convenient to set $iw=1$, i is vector of ones. The vector w can be thought of as a vector of portfolio weights. We define: $\eta_{pt} = w' \eta_t$ and $\sigma_{ij,t} = \omega_{ij} + \lambda_i \lambda_j \sigma_{pp,t}$ and

$$\sigma_{pp,t} = \omega_{pp} + \beta \sigma_{pp,t-1} + \alpha \eta_{pt}^2$$

The covariance of two asset returns moves through time only with the variance of the portfolio return which follows a univariate GARCH (1, 1) model. The single-factor GARCH (1, 1) model is a special case of the BEKK where matrices A and B have rank one: $A = \sqrt{\alpha} \omega \lambda'$ and $B = \sqrt{\beta} \omega \lambda'$. It has $(N^2 + 5N + 2)/2$ parameters. The model can be extended forward to allow for multiple factors or a higher-order GARCH structure.

Finally, the orthogonal GARCH model is a generalization of the factor GARCH model introduced by Engle et al(1990) to a multi-factor model with orthogonal factors.

It allows $k \times k$ GARCH covariance matrices to be generated from just m univariate GARCH models. Normally, m , the number of principal components, will be much less than k , the number of variables in the system. This is so that extraneous “noise” is excluded from the data and the volatilities and correlations produced become more stable. In the orthogonal GARCH model the $m \times m$ diagonal matrix of variances of the principal components is a time-varying matrix denoted by D_t , and the time-varying covariance matrix V_t of the original system is approximated by

$$V_t = A D_t A'$$

where A is the $k \times m$ matrix of rescaled facto weights. This model is called orthogonal GARCH when the diagonal matrix D_t of variances of principal components is estimated using a GARCH model. This representation will give positive semi-definite matrix at every point in time, even when the number m of principal components is much less than the number k of variables of the system. Of course, the principal components are only unconditionally uncorrelated, but the assumption of zero conditional correlations has to be made, otherwise it misses the whole point of the model, which is to generate large GARCH covariance matrices from GARCH volatilities alone. The degree of accuracy that is lost by making this assumption is investigated by a thorough calibration of the model, comparing the variances and covariances produced with those from other models such as EWMA or, for small systems, with full multivariate GARCH.

5.3 The BEKK (1, 1) Model

In the first approach, we use a bivariate GARCH (1, 1) model to capture the joint processes that govern stock returns on one side and output growth on the other side. ρ_t represents output growth and r_t stands for stock returns. The mean equation process is a VAR (p) model:

(i) $X_t = A + B_1 X_{t-1} + \dots + B_p X_{t-p} + U_t$ where

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix},$$

$$X_{t-p} = \begin{bmatrix} \rho_{t-p} \\ r_{t-p} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

The relationship between volatility of asset prices and volatility of output growth

A is a constant vector (2*1) which is defined by two constants a_1 and a_2 and B is the matrix of coefficients b_{11} , b_{12} , b_{21} , b_{22} . X_{t-p} is a vector of explanatory variables that include p lagged values of X_t . Equation (i) for VAR (1) can be written as two different equations:

$$(1) \rho_t = a_1 + b_{11}\rho_{t-1} + b_{12}r_{t-1} + u_{1t}$$

$$(2) r_t = a_2 + b_{21}\rho_{t-1} + b_{22}r_{t-1} + u_{2t}$$

We assume that both stock returns and output growth follow a t-distribution and U_t is a vector of white noise residuals. The lag length of VAR mean equation model depends on specific criteria which are implemented in each country and varies from country to country. Two of them are Akaike (AIC) and Bayesian (BIC) information criteria. We can use them to compare alternative models. Since information criteria penalize models with additional parameters, the AIC and BIC model order selection criteria are based on parsimony.

The corresponding conditional variance covariance matrix of the residual vector U_t is H_t and is defined by the following equation (Variance Equation):

$$(ii) H_t = C_0' C_0 + G_1' H_{t-1} G_1 + A_1' U_{t-1} U_{t-1}' A_1 \text{ where}$$

$$H_t = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}, C_0 = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, G_1 = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

H_t is the conditional variance-covariance matrix of U_t . h_{11t} stands for the conditional variance of the output growth ρ_t and h_{22t} for the conditional variance of stock price returns. We model the dynamic process of H_t as a linear function of its own past values H_{t-1} and past values of squared residuals ($u_{1,t-1}^2$, $u_{2,t-1}^2$) in both cases allowing for own-market and cross-market influences in the conditional variance.

C_0 is restricted to be upper triangular ($c_{21}=0$) and G_1 , A_1 are two unrestricted matrices. Equation (ii) can be analyzed in the following equations:

$$(3) h_{11t} = c_{11}^2 + c_{21}^2 + g_{11}^2 h_{11,t-1} + g_{11}g_{21}h_{21,t-1} + g_{21}g_{11}h_{12,t-1} + g_{21}^2 h_{22,t-1} + a_{11}^2 u_{1,t-1}^2 + a_{11}a_{21}u_{2,t-1}u_{1,t-1} + a_{21}a_{11}e_{1,t-1}e_{2,t-1} + a_{21}^2 e_{2,t-1}^2$$

$$(4) h_{21t} = c_{12}c_{11} + c_{22}c_{21} + g_{11}g_{12}h_{11,t-1} + g_{11}g_{22}h_{21,t-1} + g_{21}g_{12}h_{12,t-1} + g_{21}g_{22}h_{22,t-1} + a_{11}a_{12}u_{1,t-1}^2 + a_{11}a_{22}u_{2,t-1}u_{1,t-1} + a_{21}a_{12}u_{1,t-1}u_{2,t-1} + a_{21}a_{22}u_{2,t-1}^2$$

$$(5) h_{12t} = c_{11}c_{12} + c_{21}c_{22} + g_{12}g_{11}h_{11,t-1} + g_{12}g_{21}h_{21,t-1} + g_{22}g_{11}h_{12,t-1} + g_{22}g_{21}h_{22,t-1} + a_{12}a_{11}u_{1,t-1}^2 + a_{12}a_{21}u_{2,t-1}u_{1,t-1} + a_{22}a_{11}u_{1,t-1}u_{2,t-1} + a_{22}a_{21}u_{2,t-1}^2$$

$$(6) h_{22t} = c_{12}^2 + c_{22}^2 + g_{12}^2 h_{11t-1} + g_{12}g_{22}h_{21t-1} + g_{22}g_{12}h_{12t-1} + g_{22}^2 h_{22t-1} + a_{12}^2 u_{1t-1}^2 + a_{12}a_{22}u_{2t-1}u_{1t-1} + a_{22}a_{12}u_{1t-1}u_{2t-1} + a_{22}^2 u_{2t-1}^2$$

5.4 Hypothesis Testing

The purpose of the paper is to investigate the relationship between volatilities in output growth and asset prices. Therefore, we perform three causality in variance tests on our data. Firstly, the maximum likelihood estimation method is used to estimate the parameters of the GARCH unrestricted model. Consequently we need assumptions about the distribution of the asset returns and the output growth. We assume they both follow t-student processes. For the hypothesis testing, we use the Likelihood Ratio:

$$LR = -2 (l^{\text{restricted}} - l^{\text{unrestricted}}) \sim X (\text{number of restrictions})$$

We perform three types of causality-in-variance tests. First, we test the hypothesis that there is causality from stock returns volatility to output growth volatility. The null hypothesis for that test is that the matrix G_1 is upper triangular ($g_{21}=0$) and A_1 is also upper triangular ($a_{21}=0$). Rejecting the null hypothesis means that the volatility of stock price returns Granger causes the volatility of output growth. In other words, the volatility of stock returns can be blamed for a part of output growth volatility.

Then, we test the hypothesis that output growth volatility affects the volatility of asset returns. In this case the null hypothesis is that A_1 and G_1 are both lower triangular ($a_{12}=0$ and $g_{12}=0$). If we reject this hypothesis, we arrive at the conclusion that there is causality from output growth volatility to stock returns volatility. Hence, a part of stock returns volatility could be attributed to output growth volatility.

Finally, we check if there is causality in variance in both directions. The null hypothesis is that all the parameters are simultaneously zero ($a_{21} = g_{21} = a_{12} = g_{12} = 0$), or A_1 and G_1 are both diagonal. Rejecting the null hypothesis means that there is causality in variance in both ways. Stock returns volatility Granger causes industrial production volatility and vice versa. In chapter 7, we implement the process described above in each country separately.

6. The 2nd approach

6.1 The procedure

In the second part of the paper, we will examine the spillover effects of volatility among stock returns and output growth in six countries using Chueng and Ng's causality-in-variance test. (*More details about the test can be reach in **section 2.6***)

Prior to the implementation of causality-in-variance test, we need to determine an appropriate model to describe return series. Most of the time series returns exhibit excess kurtosis. It is well known that a data series that is leptokurtic in nature can be specified by the ARCH (Engle, 1982) or the GARCH (Bollerslev, 1986). For each time series, we estimate three models based on the nature of autocorrelation. The model with the largest value of the Loglikelihood Function is selected to calculate standardized innovations.

Causality-in-variance between two data series can be evaluated by the following test statistic.

$t_k = \sqrt{T} \hat{\gamma}_{uv}(k)$ is to test the causal relationship at a specific lag k comparing with the standard normal distribution.

6.2 Remarks on the causality-in-variance test

The causality-in-variance test demands the estimation of a model whose conditional mean, μ_{it} , and conditional variance, h_{it} , are specified as ARMA (r, m) and GARCH (p, q) models, respectively for each of the two time series under investigation. In the case that causality in mean is present, the conditional mean should be modified accordingly to account for this additional dynamics. If not, the causality-in-variance tests are likely to suffer from size distortions.

Theologos Pantelidis and Nikitas Pittis proved in their paper (2004) "Testing for Granger causality in variance in the presence of causality in mean" that the tests for causality in variance suffer from severe size distortions when strong causality-in-mean effects are left unaccounted for. By means of Monte Carlo simulations they showed that the model used to filter out the conditional mean effects must account for possible causality in mean between the series. Otherwise, the causality-in-variance test statistics suffer from severe size distortions, especially when the neglected causality-in-mean effects are strong.

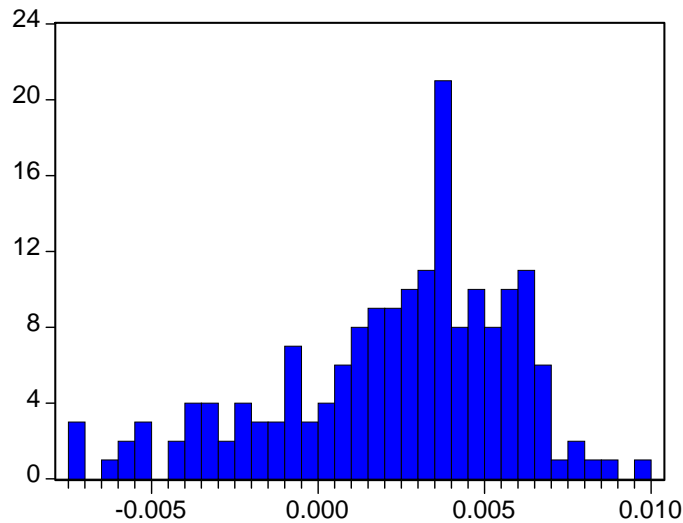
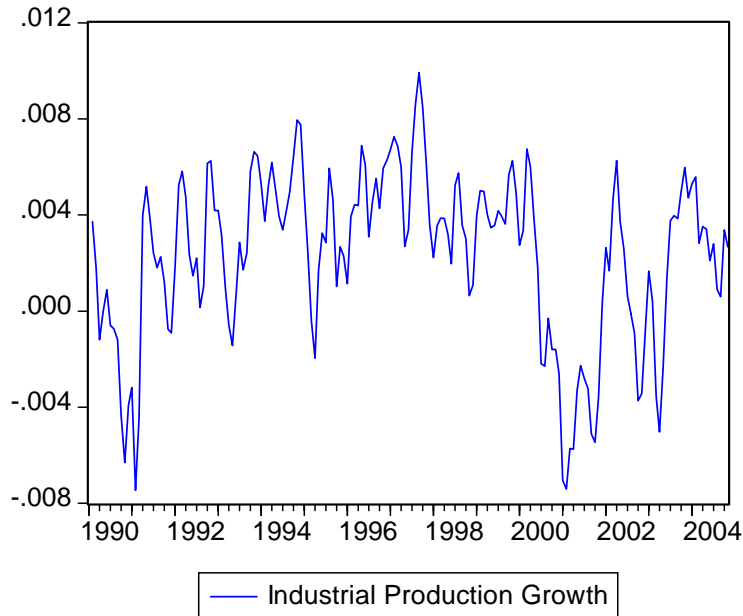
Hong (2001) put forward another interesting remark concerning Chueng and Ng's causality-in-variance test. He claimed that the S-test may not be fully efficient since it assigns equal weighting to each of the M sample cross-correlations. Instead, he proposed a weighting scheme, $k(\cdot)$, that gives a larger weight to a lower lag order j .

Having those remarks in mind, we proceed to the implementation of the test in the nine countries of interest.

7. Country Analysis

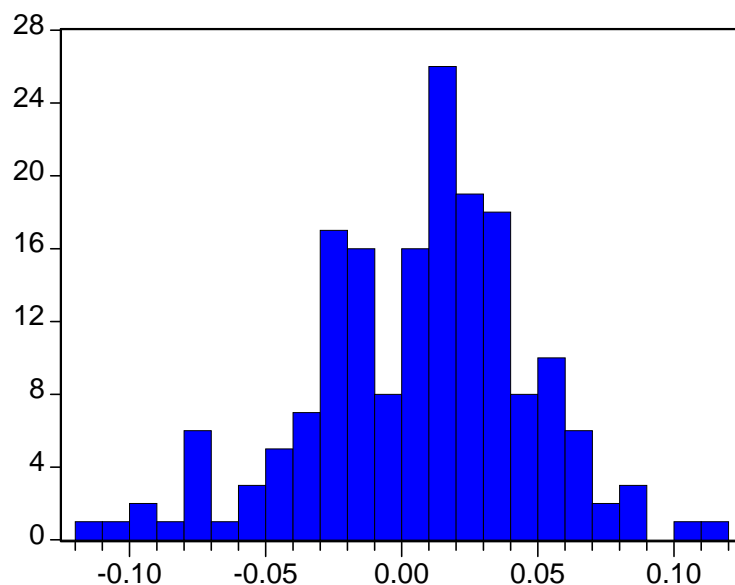
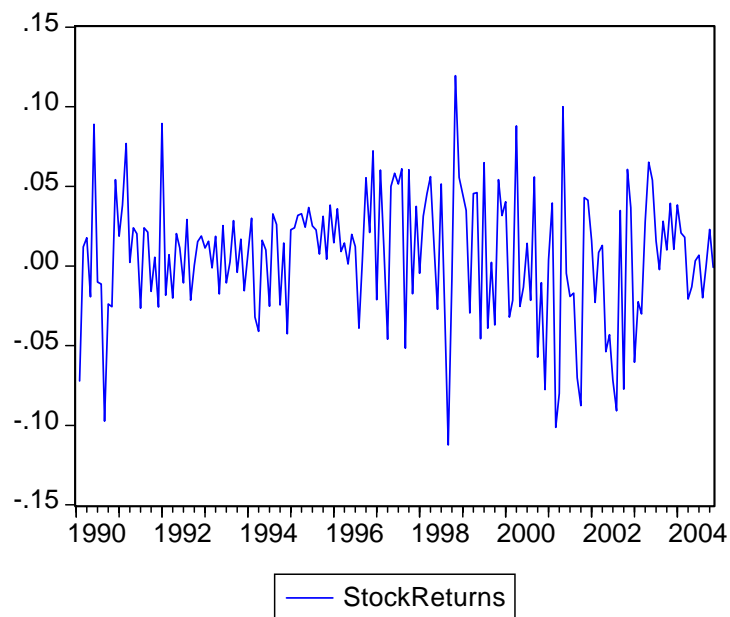
A) U.S.A.

It is necessary to start the analysis by presenting some interesting preliminary descriptive statistics for both Industrial Production Growth and Stock Returns.



Series: Industrial Production Growth	
Sample 1990:02 2004:11	
Observations 178	
Mean	0.002318
Median	0.003125
Maximum	0.009922
Minimum	-0.007455
Std. Dev.	0.003532
Skewness	-0.726606
Kurtosis	3.068125
Jarque-Bera	15.69711
Probability	0.000390

The relationship between volatility of asset prices and volatility of output growth



Series: StockReturns	
Sample 1990:02 2004:11	
Observations 178	
Mean	0.006533
Median	0.011426
Maximum	0.119504
Minimum	-0.112312
Std. Dev.	0.040203
Skewness	-0.357034
Kurtosis	3.437889
Jarque-Bera	5.203825
Probability	0.074132

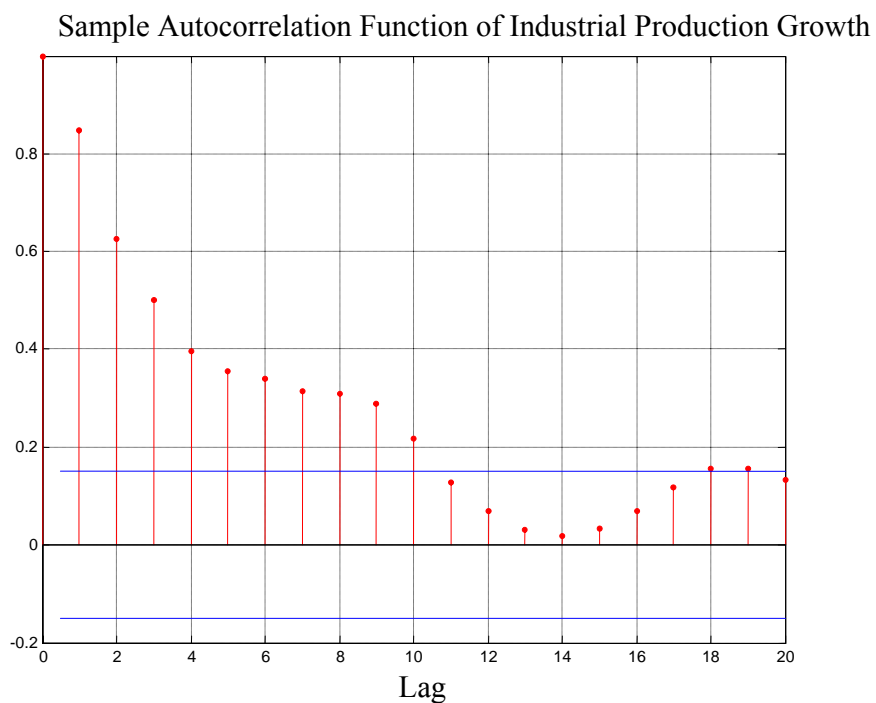
Jarque-Bera is a test statistic for testing whether the series is normally distributed. The test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. Under the null hypothesis of a normal distribution, the Jarque-Bera statistic is distributed as Chi-Square with 2 degrees of freedom. The reported Probability is the probability that a Jarque-Bera statistic exceeds (in absolute value) the observed value under the null hypothesis—a small probability value leads to the rejection of the null hypothesis of a normal distribution. In USA we should reject the Normality Hypothesis for Industrial Production Returns, but not for Stock Returns.

The relationship between volatility of asset prices and volatility of output growth

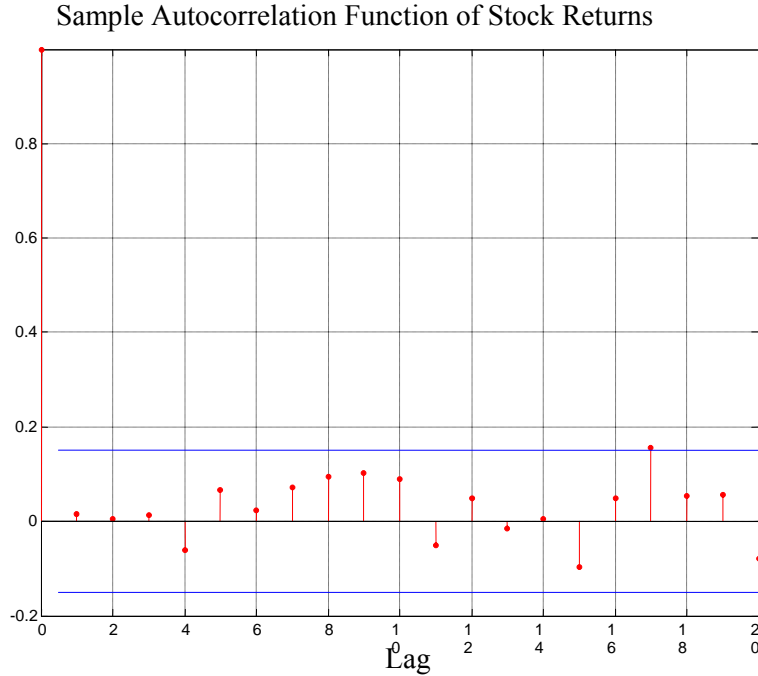
Skewness is a measure of asymmetry of the distribution of the series around its mean. The skewness of a symmetric distribution, such as the normal distribution, is zero. Positive skewness means that the distribution has a long right tail and negative skewness implies that the distribution has a long left tail. For both series the skewness coefficient is negative. The kurtosis coefficients are all larger than 3 indicating that the tails of the distribution are all fatter than those of the normal distribution.

When estimating the parameters of a composite conditional mean/variance model, we may occasionally encounter convergence problems. For example, the estimation may appear to stall, showing little or no progress. It may terminate prematurely prior to convergence. Or, it may converge to an unexpected, suboptimal solution. In order to avoid many of these difficulties we may perform a prefit analysis. We can plot the return series and examine the Autocorrelation Function or perform some preliminary tests, including Engle's ARCH test and the Q-test. The goal is to avoid convergence problems by selecting the simplest model that adequately describes your data.

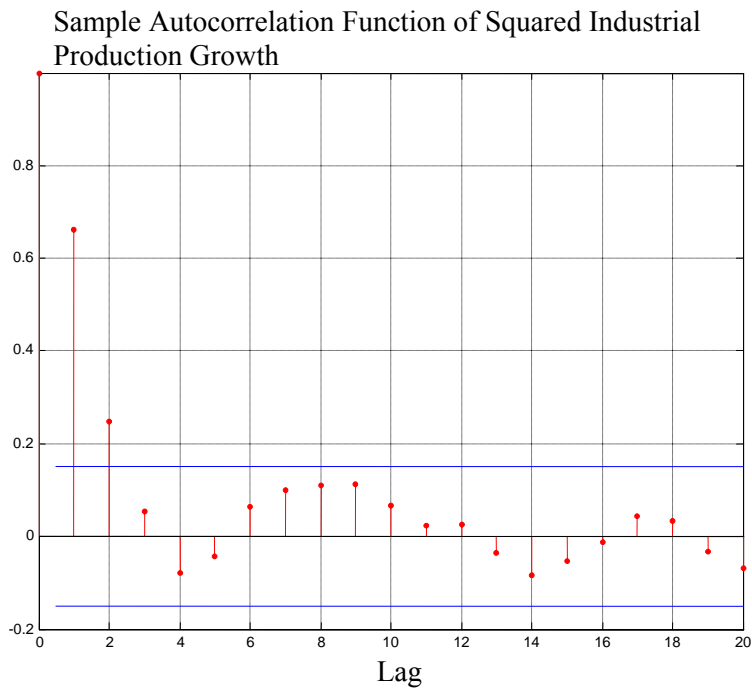
The autocorrelation function used by MATLAB 6.5p computes and plots the sample Autocorrelation Function of a univariate, stochastic time series with confidence bounds. In the diagrams below there are the plots for Industrial Production Growth and Stock Returns of U.S.A.. The blue line represents the bounds, which are computed with approximate 95% confidence level.



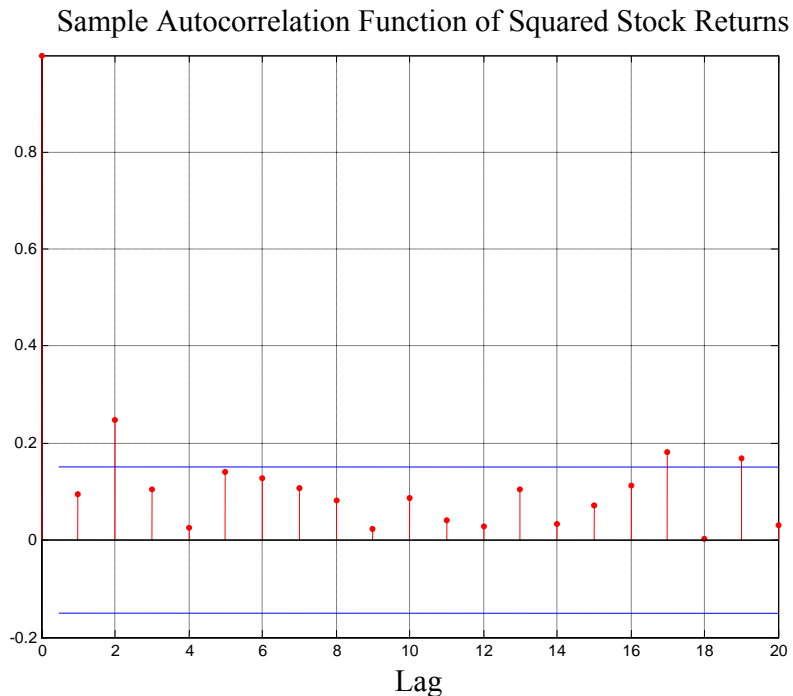
The relationship between volatility of asset prices and volatility of output growth



It is obvious that Stock returns series exhibit little autocorrelation, while Industrial Production Growth suffers from autocorrelation up to the 10th lag. Although the Autocorrelation function (ACF) of the observed returns exhibits little correlation, the ACF of the squared returns may still indicate significant correlation and persistence in the second-order moments. So it is appropriate to check for autocorrelation by plotting the ACF of the squared returns. As we can see in the diagrams below, both squared returns exhibit autocorrelation



The relationship between volatility of asset prices and volatility of output growth



We can now quantify the preceding qualitative checks for correlation using formal hypothesis tests, such as the Engle's ARCH test. ARCHTEST Hypothesis tests for the presence of ARCH/GARCH effects. The null hypothesis is that a time series of sample residuals is i.i.d. Gaussian disturbances (i.e. no ARCH effects exist). Given sample residuals obtained from a curve fit (e.g. a regression model), the presence of M^{th} order ARCH effects is tested by regressing the squared residuals on a constant and M lags, The asymptotic test statistic, $T \cdot R^2$, where T is the number of squared residuals included in the regression and R^2 is the sample multiple correlation coefficient, is asymptotically Chi-Square distributed with M degrees of freedom under the null hypothesis, When testing for ARCH effects, a GARCH(P,Q) process is locally equivalent to an ARCH(P+Q) process. Now, we impose two lags, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series. The results indicate persistent of ARCH effect in both series.

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0	0,0023
t-Statistic	88,2096	12,1227
Critical Value	5,9915	5,9915

** $H = 0$ indicate acceptance of the null hypothesis that no ARCH effects exist*

1st Approach

Mean Equation (VAR (10))

In order to choose the appropriate VAR model for our data we had to estimate many VAR models with various lags. We had to implement various lag length criteria such as Final Prediction Error (FPE), Akaike information criterion, Schwarz information criterion and Hannan-Quinn information criterion.

Furthermore, we performed in each model the residual LM autocorrelation test and we selected the model that had removed the autocorrelation from the residuals. This test is an alternative to the Q-statistics for testing serial correlation. The test belongs to the class of asymptotic (large sample) tests known as Lagrange multiplier (LM) tests. Unlike the Durbin-Watson statistic for AR(1) errors, the LM test may be used to test for higher order ARMA errors and is applicable whether or not there are lagged dependent variables. The null hypothesis of the LM test is that there is no serial correlation up to lag order, where is a pre-specified integer. The local alternative is ARMA() errors, where the number of lag terms = max(). The serial correlation LM test is available for residuals from either least squares or two-stage least squares estimation. The original regression may include AR and MA terms, in which case the test regression will be modified to take account of the ARMA terms. If the test indicates serial correlation in the residuals, LS standard errors are invalid and should not be used for inference.

In our case a VAR (10) model was selected as the one for the mean equation with the following parameters estimates:

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} \dots + b_{10} X_{t-10} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \quad A = \begin{bmatrix} 0.000374 \\ (0.00014) \\ 0.004673 \\ (0.00418) \end{bmatrix}$$

$$b_1 = \begin{bmatrix} 1.4819 & 0.0097 \\ (0.07940) & (0.00273) \\ 1.1536 & -0.0143 \\ (2.41170) & (0.08301) \end{bmatrix} \quad b_2 = \begin{bmatrix} -1.5281 & 0.0060 \\ (0.14043) & (0.00285) \\ -2.9987 & -0.0234 \\ (4.26519) & (0.08642) \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 1.6958 & 0.0069 \\ (0.17879) & (0.00282) \\ 1.4482 & -0.0223 \\ (5.43033) & (0.08560) \end{bmatrix} \quad b_4 = \begin{bmatrix} -1.7481 & 0.0017 \\ (0.20885) & (0.00284) \\ -0.0759 & -0.0320 \\ (6.34329) & (0.08616) \end{bmatrix}$$

The relationship between volatility of asset prices and volatility of output growth

$$b_5 = \begin{bmatrix} 1,7181 & 0,0018 \\ (0,22660) & (0,00284) \\ -0,5094 & 0,0785 \\ (6,88259) & (0,08619) \end{bmatrix} \quad b_6 = \begin{bmatrix} -1,4315 & -0,0007 \\ (0,22839) & (0,00277) \\ 2,2207 & 0,0346 \\ (6,93687) & (0,08427) \end{bmatrix}$$

$$b_7 = \begin{bmatrix} 1,0626 & -0,0019 \\ (0,21208) & (0,00277) \\ -5,8330 & 0,0637 \\ (6,44139) & (0,08423) \end{bmatrix} \quad b_8 = \begin{bmatrix} -0,7043 & 0,0012 \\ (0,18225) & (0,00278) \\ 6,5189 & 0,0994 \\ (5,53535) & (0,08433) \end{bmatrix}$$

$$b_9 = \begin{bmatrix} 0,5105 & 0,0004 \\ (0,14231) & (0,00278) \\ -1,3566 & 0,0869 \\ (4,32235) & (0,08433) \end{bmatrix} \quad b_{10} = \begin{bmatrix} -0,2768 & 0,0019 \\ (0,07539) & (0,00274) \\ -0,3236 & 0,0489 \\ (2,28993) & (0,08321) \end{bmatrix}$$

**standard errors in ()

The autocorrelation LM Test for the residuals of the VAR (10) is presented in the table above. We accept the null hypothesis of no autocorrelation in the first 12 lags:

Lags	LM-Statistic	P-Value
1	6,924142	0,1400
2	5,817972	0,2132
3	7,612491	0,1068
4	11,87737	0,0183
5	7,070343	0,1322
6	1,477447	0,8306
7	4,303489	0,3665
8	1,868395	0,7599
9	1,637853	0,8020
10	1,309843	0,8597
11	3,908815	0,4185
12	2,676241	0,6134

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

In this part of the country analysis we perform a causality-in-mean test to check whether industrial production growth and stock returns are related. We use the Pairwise Granger Causality Tests provided by EViews. We carry out pairwise Granger causality tests and test whether an endogenous variable can be treated as exogenous. For each equation in the VAR, the output displays (Wald) statistics for the joint significance of each of the other lagged endogenous variables in that equation. The results are presented in the table above:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	24,10719	0,0073

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	8,004	0,6284

So, there is causality in mean from stock returns to industrial production growth but, there is no causality in the opposite direction. In other words, Stock Returns Granger causes Industrial Production Growth.

Variance Equation (BEKK 1, 1)

We estimated a bivariate GARCH (1, 1) model with maximum likelihood and we assumed that output growth and stock returns follow a t-student distribution. The parameters estimates of the unrestricted model are presented in the table below:

The relationship between volatility of asset prices and volatility of output growth

Parameters (std.errors)	Unrestricted GARCH(1,1)
c₁₁	-0,0004 (0)
c₁₂	-0,0006 (0)
c₂₁	0
c₂₂	0,0070 (0)
GARCH g₁₁	0,1082 (0)
g₁₂	3,0331 (0,0001)
g₂₁	0,0020 (0)
g₂₂	0,3899 (0)
ARCH a₁₁	0,7582 (0)
a₁₂	-0,7453 (0,0002)
a₂₁	0,0040 (0)
a₂₂	0,8943 (0)

We performed three different LRatio Tests to check the causality in variance from output growth volatility to asset price volatility and vice versa. The results can be summarized in the table:

	Unrestricted	Restricted1 (a₂₁=g₂₁=0)	Restricted2 (a₁₂=g₁₂=0)	Restricted3 (a₁₂=g₁₂=a₂₁=g₂₁=0)
Loglikelihood	1215,9	1198,4	1214,9	1195,7
pValue		0	0,3625	0
LRatio		35,0816	2,0295	40,5344
Critical Value		5,9915	5,9915	9,4877

**H = 0 indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; H = 1 indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.*

The relationship between volatility of asset prices and volatility of output growth

In the first row, we present the value of the Loglikelihood Function for each model. As we expected, the unrestricted model has the greatest LLF value.

The restricted 1 model tests whether there is causality from stock returns volatility to output growth volatility. The LRatio test suggests that we should reject the null hypothesis that the restricted is better than the unrestricted. So there is causality in variance from stock returns volatility to output growth volatility.

The 2nd restricted model checks the causality from industrial production to stock returns volatility. According to the LRatio, we accept the null hypothesis that the restricted is preferable to the unrestricted and that there is no statistically significant causality from industrial production volatility to asset price volatility.

Finally, the restricted 3 model investigates the causality in both directions. The null hypothesis should be rejected, so the unrestricted model is better than the restricted.

We should now proceed to the post-estimation analysis, to check whether the unrestricted model selected for our data was sufficient. We used the Ljung-Box lack-of-fit hypothesis test. This model is based on the Q-Statistic:

$$Q = N*(N+2)* \sum_{k=1}^L \frac{r_k^2}{(N - k)}$$

where N = sample size, L = number of autocorrelation lags included in the statistic, and r_k^2 is the squared sample autocorrelation at lag k. Under the null hypothesis that the model fit is adequate, the test statistic is asymptotically Chi-Square distributed. We performed this test on the residuals of the unrestricted GARCH (1, 1) model. The lags used in the Q-Statistic are twenty and the significance level 5% and the results are presented below:

	Standardized Residuals1	Standardized Residuals2
pValue	0,8896	0,7644
Q-Statistic	12,7081	15,2081
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

2nd Approach

Firstly, we estimate univariate models for each series trying to take into account the various features of the data documented in the previous section. We model the mean equation as an autoregressive moving average process ARMA (p, q). The conditional variance equation is then modeled as a classical GARCH model. The error term ε_t is

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supposed to be conditionally normally distributed with mean 0 and conditional variance σ_t^2 .

We selected the lag structure of the models' equations according to the results of the equation estimates in EViews. For Industrial Production Growth the autoregressive coefficients were statistically significant up to eight lags. (The null hypothesis for the first eight lags coefficients being zero should be rejected). In the variance equation the ARCH (1) and GARCH (1) coefficients were also statistically significant. So, an Autoregressive Moving Average with eight lags model is selected for the mean equation of Industrial Production Growth and a univariate GARCH (1, 1) for the variance equation.

We also estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (8, 8), GARCH (1, 1) has the greatest LLF value.

LLF	GARCH(1,1)	ARMA(1, 1) GARCH(1,1)	ARMA(8,1) GARCH(1,1)	ARMA (8,8) GARCH(1,1)
Industrial Production Growth	799,5658	910,8923	916,6971	935,9357

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The Stock Returns series exhibits no autocorrelation, since the autoregressive coefficient of the first lag was not significant (the null hypothesis that the coefficient was zero should be accepted). So, the best model for the Stock Returns series is a GARCH (1, 1) model.

We estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The GARCH (1, 1) has the greatest LLF value.

LLF	AR (1) GARCH(0,1)	ARMA(1,1) GARCH(0,1)	ARMA (0, 0) GARCH(1,1)
Stock Returns	325,3864	325,4322	334,8276

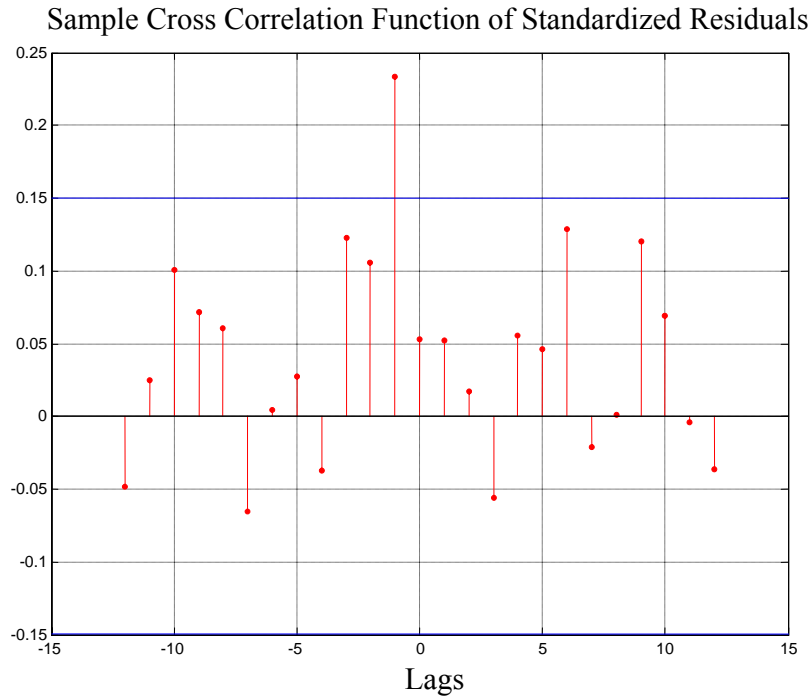
**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

After choosing the univariate model, the Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification is used. The innovations of the GARCH models are tested, in order to check whether the model fit is adequate. The lags used in the Q-Statistic are twenty and the significance level 5%. The null hypothesis should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,0918	0,5434
Q-statistic	28,8010	18,6696
Critical Value	31,4104	31,4104

* H_0 : the null hypothesis that the model fit is adequate (no serial correlation).

We plot now the Sample Cross Correlation Function of the standardized residuals of the series to check for causality in mean, and the squared standardized residuals to test for causality in variance. The blue line represents the confidence interval of 95 %:



The diagram exhibits causality in mean in the (-1) lag which means that $StockReturns_{t-1}$ Granger causes $IndustrialProduction_t$.

We proceed with the hypothesis testing using the t-statistics:

$$t = \sqrt{T} \hat{r}_{uv}(k) \rightarrow AN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

Where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

We apply this test on the standardized residuals. The table contains the t-statistic for each lag.

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Lags	t-statistic for st. residuals
-12	-0,6396
-11	0,3332
-10	1,3374
-9	0,959
-8	0,8045
-7	-0,8731
-6	0,0619
-5	0,3609
-4	-0,5004
-3	1,6386
-2	1,4074
-1	3,1133*
0	0,7106
1	0,6951
2	0,2329
3	-0,7504
4	0,7432
5	0,6121
6	1,7216
7	-0,2766
8	0,0097
9	1,5982
10	0,9191
11	-0,0525
12	-0,4895

H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

' indicates significance at the 1% level, '' indicates significance at the 5% level*

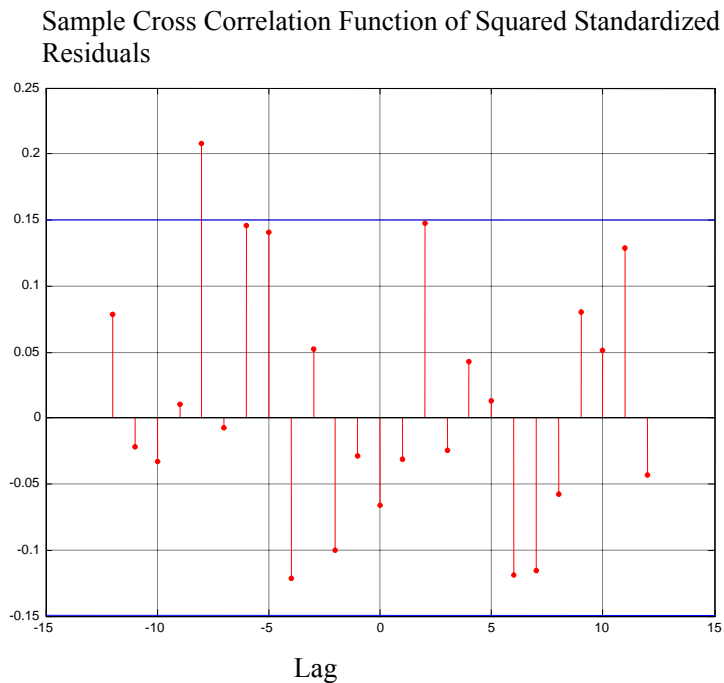
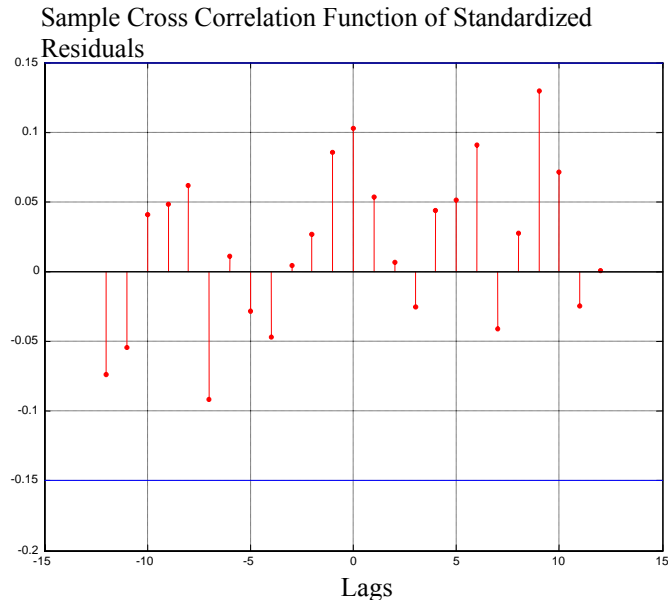
The hypothesis testing suggests that there is statistically significant causality in mean in (-1) lag. In other words, $StockReturns_{t-1}$ Granger causes $IndustrialProduction_t$.

In the case that causality in mean is present, the conditional mean should be modified accordingly to account for this additional dynamics. If not, the causality-in-variance tests are likely to suffer from size distortions. Theologos Pantelidis and Nikitas Pittis proved in their paper (2004) "Testing for Granger causality in variance in the presence of causality in mean" that the tests for causality in variance suffer from severe size distortions when strong causality-in-mean effects are left unaccounted for. By means of Monte Carlo simulations they showed that the model used to filter out the conditional

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mean effects must account for possible causality in mean between the series. Otherwise, the causality-in-variance test statistics suffer from severe size distortions, especially when the neglected causality-in-mean effects are strong.

Therefore, we add a time series regression vector of explanatory variable, a regression component, in the mean equation of Industrial Production Growth. The regression component is a lagged transform (by one lag) of the time series Stock Returns. We plot again the Sample Cross Correlation Function of the standardized residuals of the series and of the squared standardized residuals.



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As we can assume from the diagrams, the causality in mean has been successfully removed. There is volatility spillover from Stock Returns to Industrial Production Growth. The causality in variance from Stock Returns to Industrial Production Growth is statistically significant in the 8th lag, or the volatility of StockReturns_{t-8} Granger causes the volatility of IndustrialProductionGrowth_t.

We proceed with the hypothesis testing on the standardized residuals and on the squared standardized residuals. The table contains the t-statistic for each lag.

Lags	t-statistic for st. residuals	t-statistic for squared st. residuals
-12	-0,9846	1,0432
-11	-0,7254	-0,2879
-10	0,5453	-0,4447
-9	0,649	0,1423
-8	0,8284	2,7681*
-7	-1,2179	-0,1038
-6	0,1505	1,9436
-5	-0,3767	1,8755
-4	-0,6294	-1,621
-3	0,0544	0,6919
-2	0,3527	-1,3337
-1	1,1467	-0,3818
0	1,3684	-0,88
1	0,7103	-0,4209
2	0,0928	1,9495
3	-0,3421	-0,3317
4	0,5847	0,5727
5	0,6823	0,1769
6	1,2156	-1,588
7	-0,5464	-1,5426
8	0,3656	-0,7672
9	1,7274	1,0741
10	0,9491	0,6894
11	-0,3288	1,7221
12	0,005	-0,5709

H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

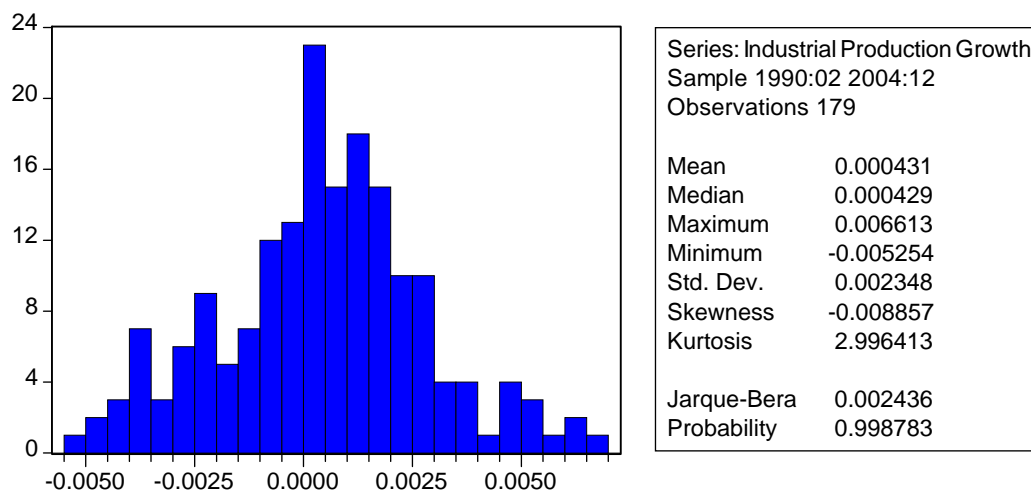
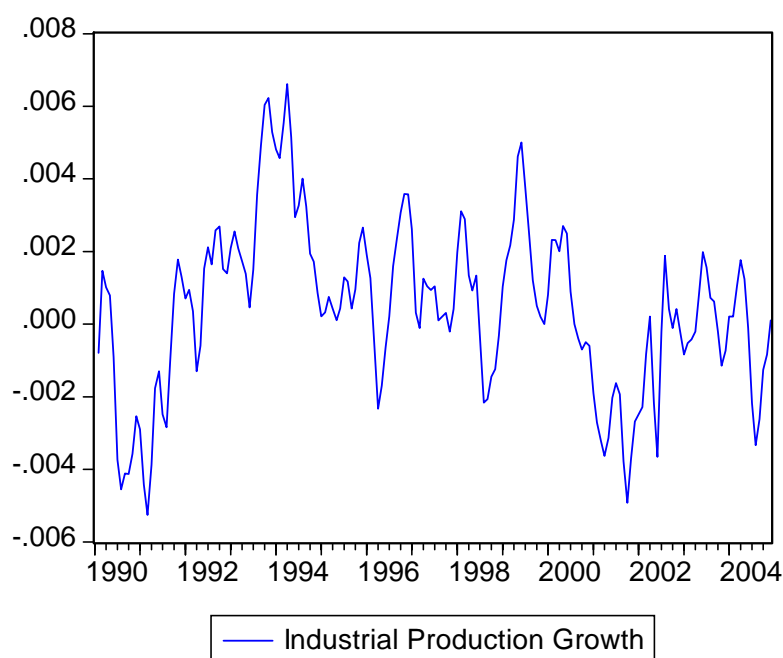
' indicates significance at the 1% level, '' indicates significance at the 5% level*

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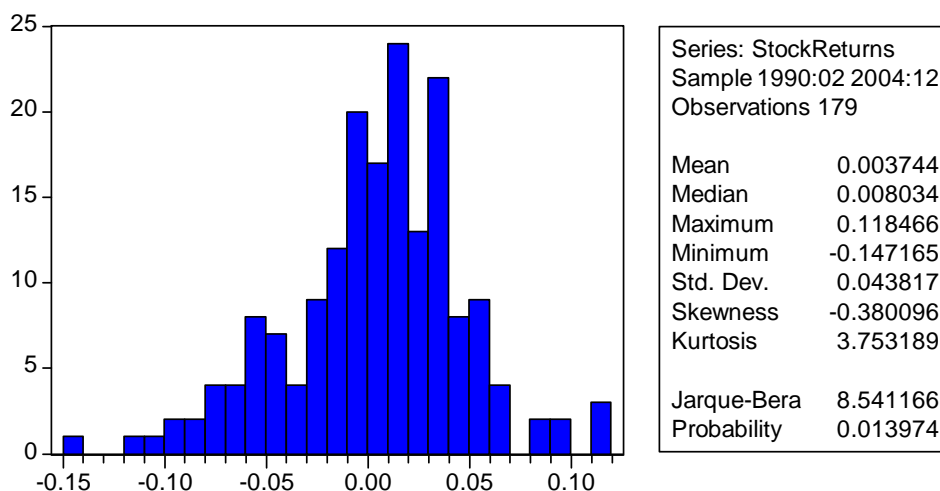
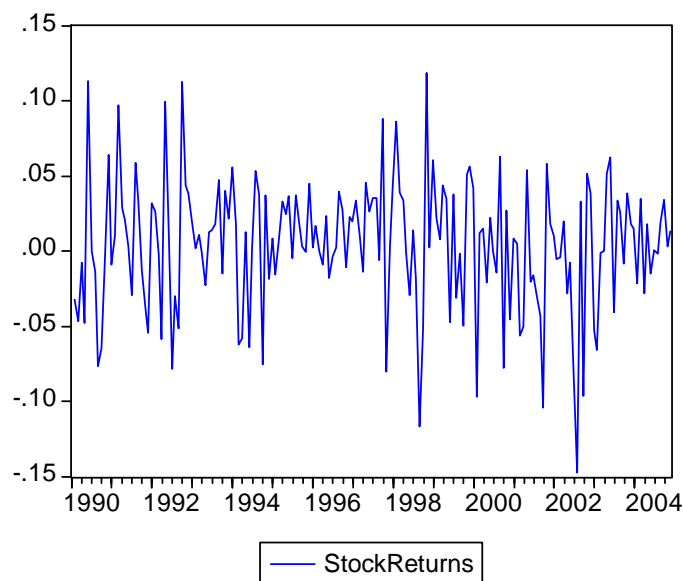
The Hypothesis Testing suggests that we should reject the null hypothesis of no causality in lag (-8) for the squared standardized residuals. Consequently, there is no causality in mean but there is statistically significant causality in variance from Stock Returns to Industrial Production Growth. The volatility of $StockReturns_{t-8}$ Granger causes the volatility of $IndustrialProductionGrowth_t$.

B) U.K.

In the beginning of our country analysis we shall present some preliminary statistics about our time series, industrial production growth and stock returns.



The relationship between volatility of asset prices and volatility of output growth

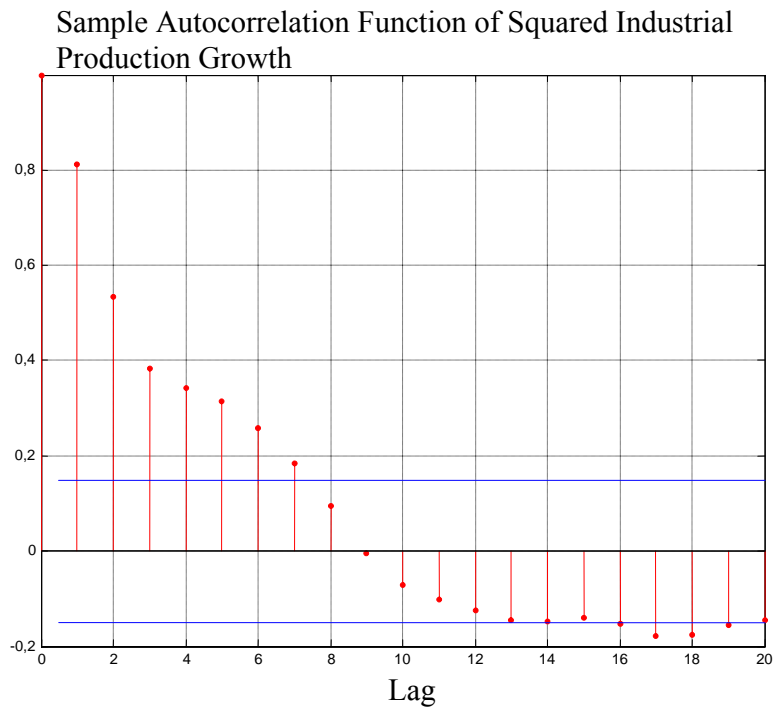
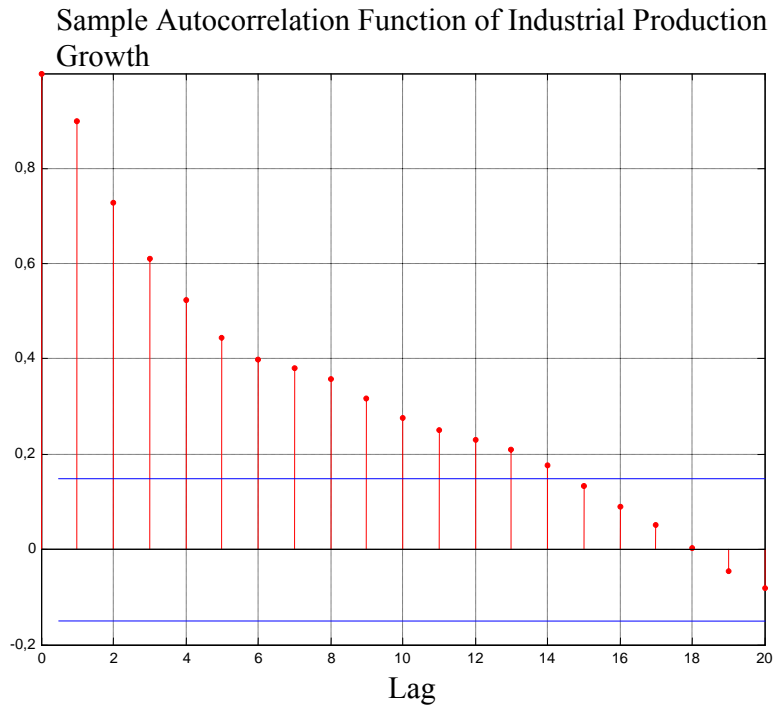


For stock returns series we should reject the hypothesis that it follows a Normal Distribution, according to the Jarque-Bera Test. Moreover, the skewness coefficient of the series is negative, indicating that the distribution has a long left tail. The kurtosis is greater than three (leptokurtic distribution).

Industrial Production Growth is normally distributed. The skewness and kurtosis coefficients approximate those of a normal distribution. Moreover, the Jarque-Bera test suggests that we should accept the normality assumption.

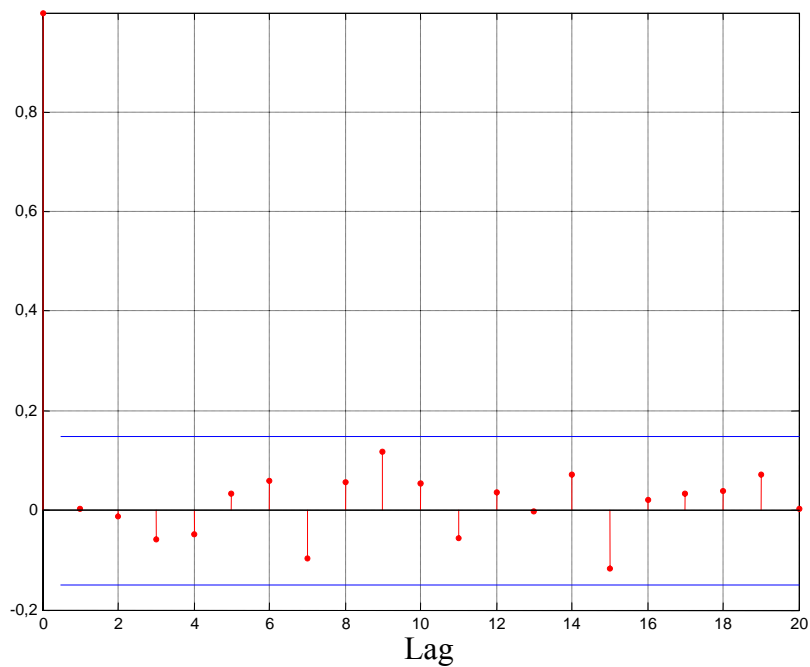
We shall now proceed to the autocorrelation tests. We check for autocorrelation in the Industrial Production Growth and Stock Returns as well as in the squared returns of Industrial Production and Stock Prices. The blue line represents the bounds, which are computed with approximate 95% confidence level.

The relationship between volatility of asset prices and volatility of output growth

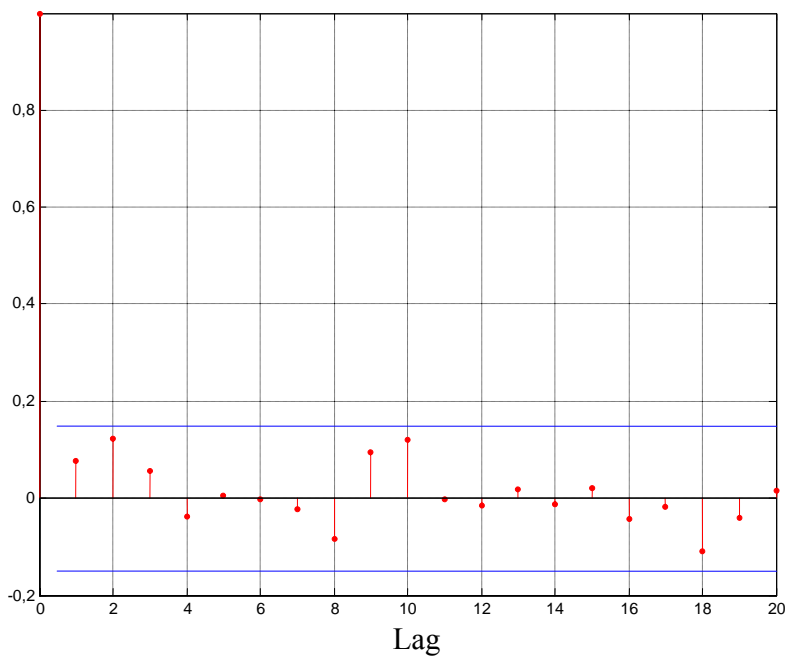


The relationship between volatility of asset prices and volatility of output growth

Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



The relationship between volatility of asset prices and volatility of output growth

The Stock Returns time series and the squared Stock Returns do not exhibit autocorrelation in any lag, in contrast to Industrial Production that suffers from autocorrelation in both raw series and squared series. We can perform Engle's ARCH test to check whether there are ARCH/GARCH effects on our series. The table below contains the results of this test. There is ARCH effect in Industrial Production Growth, while we can accept the null hypothesis of no ARCH/GARCH effect in Stock Returns. We impose one lag of squared sample residuals included in the ARCH test statistic and 5% significance level for each time series.

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0	0,2135
t-Statistic	113,9646	1,5477
Critical Value	3,8415	3,8415

**H = 0 indicate acceptance of the null hypothesis that no ARCH effects exist*

1st Approach
Mean Equation (VAR (6))

The Final Prediction Error (FPE), the Akaike information criterion, the Schwarz information criterion, the Hannan-Quinn information criterion and the residual LM autocorrelation test were used to help us choose the appropriate lag for our Vector Autoregressive model. Most of the criteria used, suggested that a VAR with five lags was appropriate for modeling the two time series. But, the VAR (5) model could not succeed in removing all the serial correlation from the residuals. So, we came down to a VAR (6) model that managed to remove all the serial correlation from our residuals. After implementing the serial LM autocorrelation test on the residuals of the VAR (6) we decided that it is the model that fits best to our data. The parameters estimates of the mean equation of the model are presented below (standard errors in “()”):

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + \dots + b_6 X_{t-6} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

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$$A = \begin{bmatrix} 0,00003 \\ (0,00005) \\ 0,004260 \\ (0,00351) \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1,8507 & 0,0019 \\ (0,07907) & (0,00124) \\ 1,5134 & -0,0032 \\ (5,07101) & (0,07945) \end{bmatrix},$$

$$b_2 = \begin{bmatrix} -1,9081 & 0,0005 \\ (0,15991) & (0,00122) \\ 2,8818 & -0,0253 \\ (10,2555) & (0,07817) \end{bmatrix}, \quad b_3 = \begin{bmatrix} 1,7160 & 0,0003 \\ (0,19647) & (0,00121) \\ -5,9730 & -0,0617 \\ (12,6005) & (0,07740) \end{bmatrix}$$

$$b_4 = \begin{bmatrix} -1,2008 & 0,0009 \\ (0,19538) & (0,00121) \\ 9,1160 & -0,0466 \\ (12,5304) & (0,07736) \end{bmatrix}, \quad b_5 = \begin{bmatrix} 0,5596 & -0,0001 \\ (0,15534) & (0,00121) \\ -8,1489 & 0,0213 \\ (9,96244) & (0,07732) \end{bmatrix}$$

$$b_6 = \begin{bmatrix} -0,1040 & 0,0010 \\ (0,07545) & (0,00120) \\ 1,2067 & 0,0414 \\ (4,83890) & (0,07725) \end{bmatrix}$$

According to the autocorrelation LM test, we can accept the null hypothesis of no autocorrelation in the residuals of our model (VAR (6)) up to 12 lags. The results are presented in the table below:

Lags	LM-Statistic	P-Value
1	6,701094	0,1526
2	8,730487	0,0682
3	8,214268	0,0840
4	6,400303	0,1712
5	1,325525	0,8570
6	1,345965	0,8535
7	10,70446	0,0301
8	4,972191	0,2902
9	2,168903	0,7047
10	2,779962	0,5953
11	4,613299	0,3293
12	3,055724	0,5485

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

Before proceeding to the causality in variance test, we investigate the relationship between Industrial Production Growth and Stock Returns. We try to find out the causality in mean patterns. We perform the pairwise Granger causality tests provided by EViews and we summarize the results in the tables below:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	3,582303	0,7330

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	3,731010	0,7130

In both cases, the null hypothesis that the variables are insignificant should be accepted. Hence, there is no causality in mean between Industrial Production Growth and Stock Returns in U.K. for this period.

Variance Equation (BEKK 1, 1)

A bivariate GARCH (1, 1) model has been estimated with maximum likelihood. The assumptions about output growth and stock returns are that they both follow a t-student distribution. The parameters estimates of the unrestricted model are presented in the table below: (standard errors in “()”)

Parameters	Unrestricted GARCH(1,1)
c₁₁	-0,0001 (0)
c₁₂	0 (0)
c₂₁	0
c₂₂	0,0003 (0)
GARCH g₁₁	0,0897 (0,0065)
g₁₂	-27,8410

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	(47,4109)
g ₂₁	0,0004 (0)
g ₂₂	0,0742 (0,0103)
ARCH	
a ₁₁	-0,9846 (0,0011)
a ₁₂	-1,6624 (2,2906)
a ₂₁	-0,0005 (0)
a ₂₂	-0,8961 (0,0018)

We checked for causality-in-variance from stock returns volatility to output growth volatility and vice versa. We checked also whether there is causality in both ways. The LRatio test was used for this purpose. We estimated three different restricted models. Each of the first two models checked for one way causality and the third one for causality in both ways. We also estimated the value of the Loglikelihood Function for each model. The significance level is 5% and the results are presented in the table below:

	Unrestricted	Restricted1 (a ₂₁ =g ₂₁ =0)	Restricted2 (a ₁₂ =g ₁₂ =0)	Restricted3 (a ₁₂ =g ₁₂ =a ₂₁ =g ₂₁ =0)
Loglikelihood	1336,8	1318,1	517,4650	534,1954
pValue		0	0	0
LRatio		37,4095	1638,7	1605,2
Critical Value		5,9915	5,9915	9,4877

H = 0 indicate acceptance of the restricted model (no **causality in variance) under the null hypothesis; H = 1 indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.*

In the first row, we present the value of the Loglikelihood Function for each model. As we expected, the unrestricted model has the greatest LLF value.

It is obvious that we should reject the null hypothesis in all cases, since p-Values are zero. The unrestricted model is better than all the three restricted models according to the LRatio tests. So, we arrive at the conclusion that there is statistically significant relationship between output stock volatility and stock returns volatility and vice versa.

Finally, we should proceed to the Ljung-Box lack-of-fit hypothesis test for model misspecification. We will test the residuals of our GARCH model. The lags used in the Q-Statistic are twenty and the significance level 5%. The null hypothesis is that the model fit is adequate. The table below contains the results:

	Standardized Residuals1	Standardized Residuals2
pValue	0,6646	0,6151
Q-Statistic	16,8203	17,5796
Critical Value	31,4104	31,4104

* H_0 : the null hypothesis that the model fit is adequate (no serial correlation).

2nd Approach

In this approach, we need to model the two time series separately. Two univariate models have to be estimated. The mean equation of each model is an Autoregressive Moving Average model and the variance equation a Generalized ARCH model. The lag structure of the models is selected according to the significance of each parameter. The estimation of the parameters is being performed by EViews, along with the determination of the statistical significance of each one.

The Industrial Production Growth series exhibits autocorrelation up to the fifth lag. Therefore, an ARMA (5, 5) process is selected. The parameters ARCH (1) and GARCH (1) are also statistically significant and therefore the variance equation was a simple GARCH (1, 1) model.

We also estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (5, 5), GARCH (1, 1) has the greatest LLF value.

LLF	GARCH(1,1)	ARMA(1, 1) GARCH(1,1)	AR(8) GARCH(1,1)	ARMA (5,5) GARCH(1,1)
Industrial Production Growth	882,2620	1028	1052,8	1057,3

*LLF - Optimized log-likelihood objective function value associated with the parameter estimates

The Stock Returns series does not suffer from any autocorrelation. So there was not any autoregressive or moving average term in the mean equation. But, there was autocorrelation in the second moment in the first lag. Therefore, the variance equation was a GARCH (1, 1) process.

Then, we estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The GARCH (1, 1) has the greatest LLF value.

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LLF	ARMA(0,0) GARCH(1,1)	ARMA(0, 0) GARCH(0,0)	ARMA(0,0) GARCH(0,1)
Stock Returns	311,7690	306,3739	308,9143

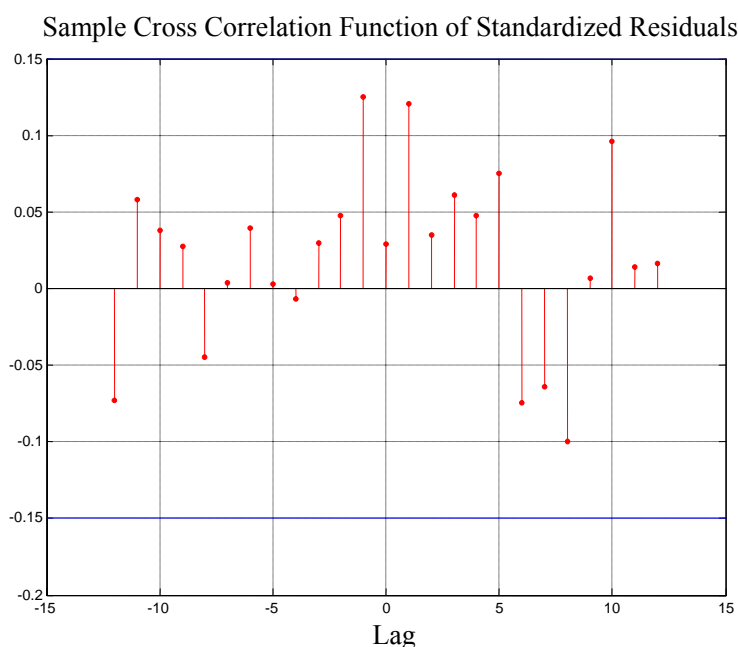
**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification is used to test each univariate model fit. The innovations of the models are tested, in order to check whether the model fit is adequate. The lags used in the Q-Statistic are twenty and the significance level 5%. The null hypothesis should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,9820	0,8456
Q-statistic	9,0728	13,6955
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

In this approach, the causality in mean is examined in two ways. Firstly, we plot the Sample Cross Correlation Function of the standardized residuals of the series. The null hypothesis is that the two residuals are uncorrelated (no causality in mean). The blue line represents the confidence interval of 95 %:



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The diagram suggests that we should accept the null hypothesis in every lag. The following t-statistic is used to test the hypothesis of no causality in mean:

$$t = \sqrt{T} \hat{r}_{uv}(\mathbf{k}) \rightarrow \text{AN} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

Where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

We apply this test on the standardized residuals. The table contains the t-statistic for each lag.

Lags	t-statistic for st. residuals
-12	-0,9783
-11	0,7772
-10	0,5105
-9	0,3718
-8	-0,5974
-7	0,0529
-6	0,5233
-5	0,0379
-4	-0,0916
-3	0,3942
-2	0,641
-1	1,6777
0	0,3906
1	1,6168
2	0,463
3	0,8123
4	0,6376
5	1,0083
6	-0,9932
7	-0,8574
8	-1,3402
9	0,0874
10	1,2867
11	0,1872
12	0,2197

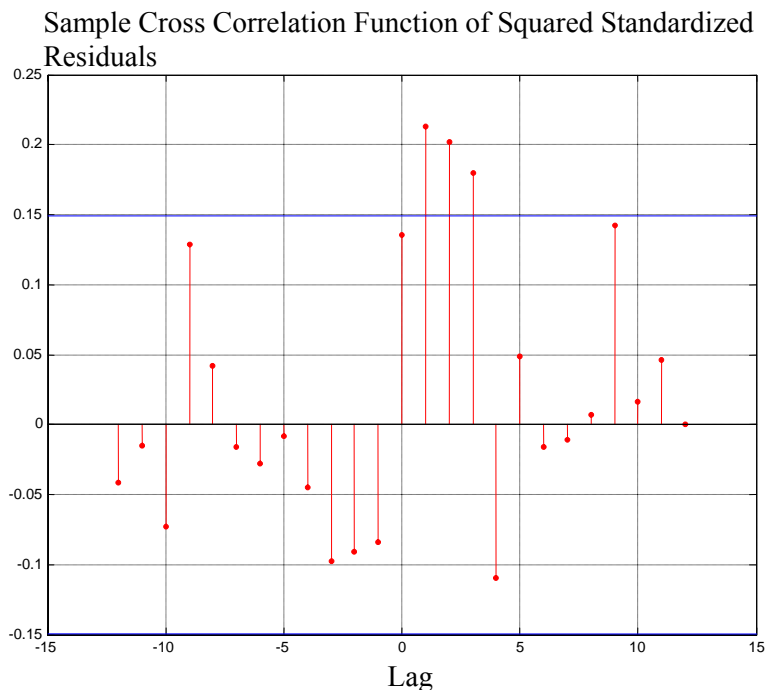
H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

‘’ indicates significance at the 1% level, ‘**’ indicates significance at the 5% level*

The relationship between volatility of asset prices and volatility of output growth

The null hypothesis is being accepted in every lag. There is no statistically significant causality in mean.

The causality in variance is tested by plotting the Sample Cross Correlation Function of the squared standardized residuals. The blue line represents the confidence interval of 95 %:



The causality in variance is present according to the diagram in the first, second and third lag. The hypothesis testing according to the t-statistic used before will be implemented. The table contains the t-statistic for each lag of the squared standardized residuals:

Lags	t-statistic for squared st. residuals
-12	-0,55
-11	-0,1975
-10	-0,9738
-9	1,7194***
-8	0,5581
-7	-0,2093
-6	-0,3767
-5	-0,1103
-4	-0,596
-3	-1,3043
-2	-1,2135
-1	-1,122

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0	1,8186
1	2,8513*
2	2,6999*
3	2,4109**
4	-1,465
5	0,6511
6	-0,2186
7	-0,1426
8	0,0884
9	1,9026
10	0,2243
11	0,615
12	0,0083

H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

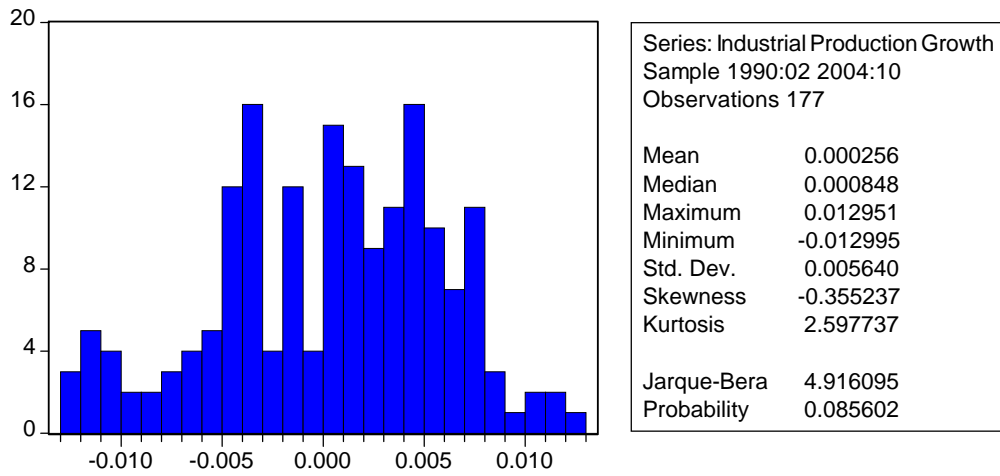
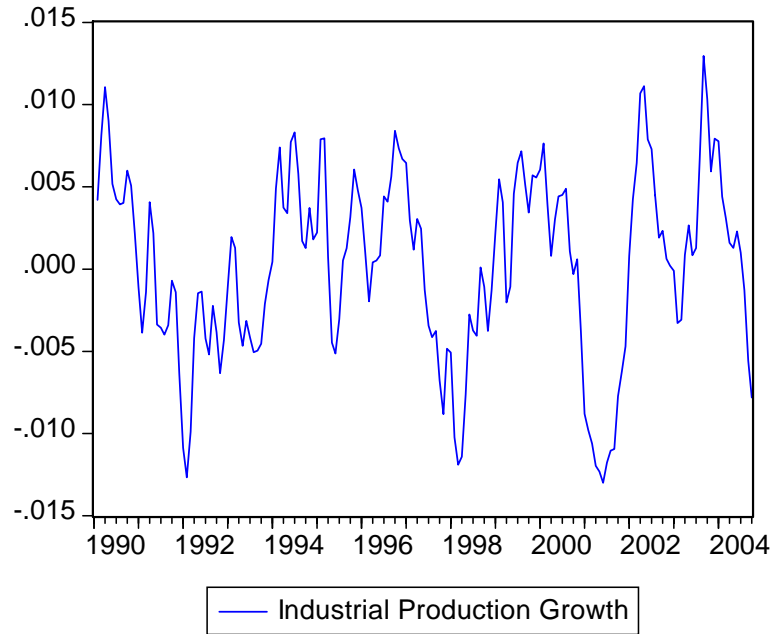
‘’ indicates significance at the 1% level, ‘**’ indicates significance at the 5% level ‘***’ indicates significance at the 10%*

There is volatility spillover from Industrial Production Growth to Stock Returns and vice versa. The volatility of IndustrialProduction_{t-1}, IndustrialProduction_{t-2} and IndustrialProduction_{t-3} Granger causes the volatility of StockReturns_t. In other words, the volatility of Stock Returns today has been influenced by the volatility of the Industrial Production Growth of the three previous months. But, there is also causality in volatility on the opposite direction. StockReturns_{t-9} volatility Granger causes IndustrialProductionGrowth_t volatility.

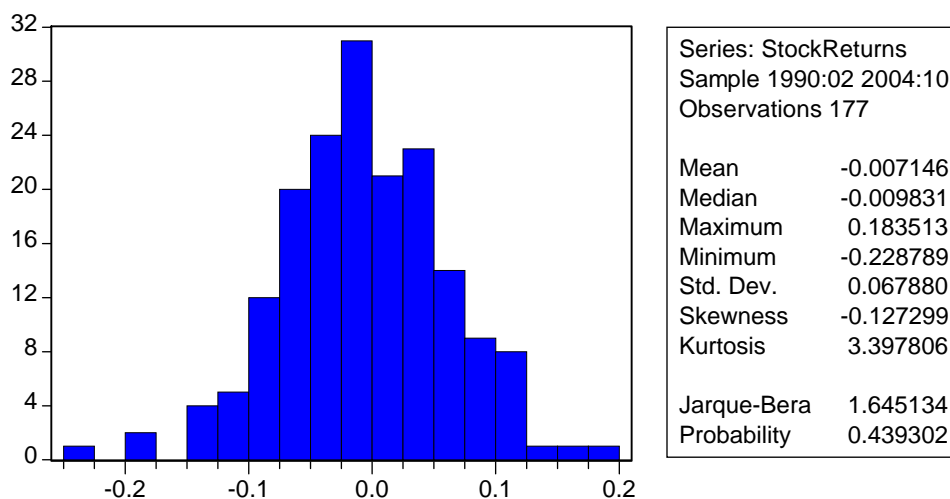
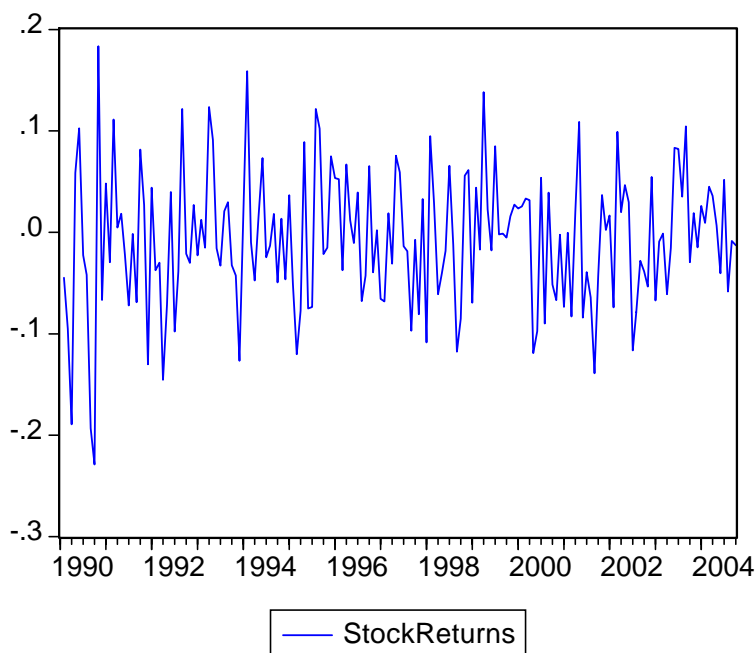
C) Japan

In this section, we will try to analyze the economic data regarding industrial production growth and stock returns of the last fifteen year for Japan. At the beginning, it is essential to present some preliminary, yet important for understanding the financial situation of the country, statistical information. The statistical analysis of the two times series for the past fifteen years could be summarized in the following diagrams and tables:

The relationship between volatility of asset prices and volatility of output growth



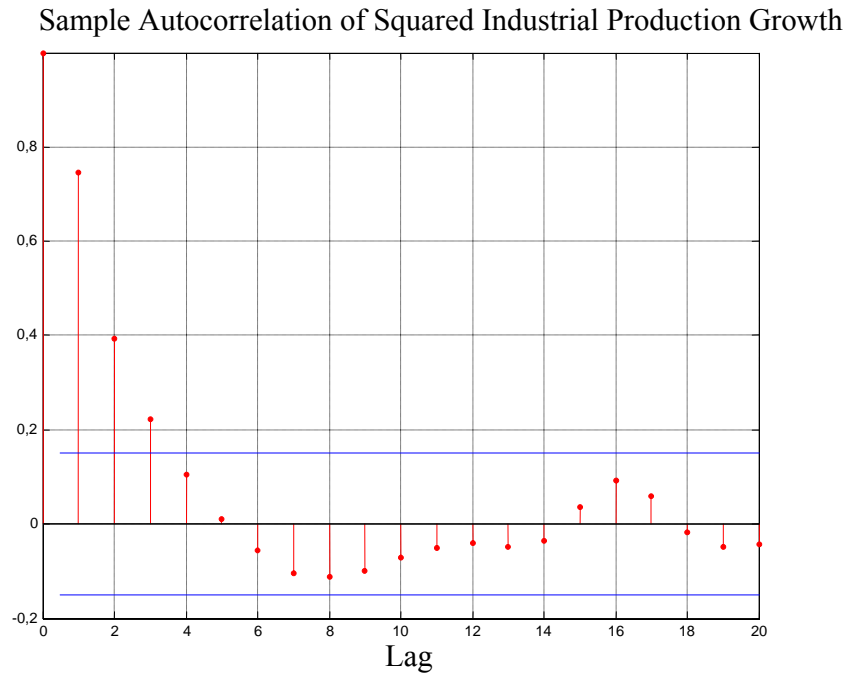
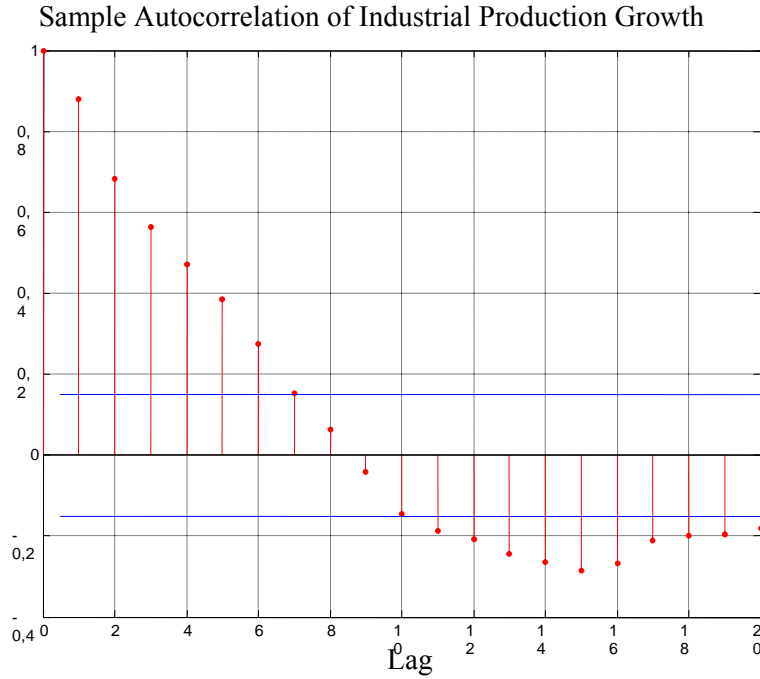
The relationship between volatility of asset prices and volatility of output growth



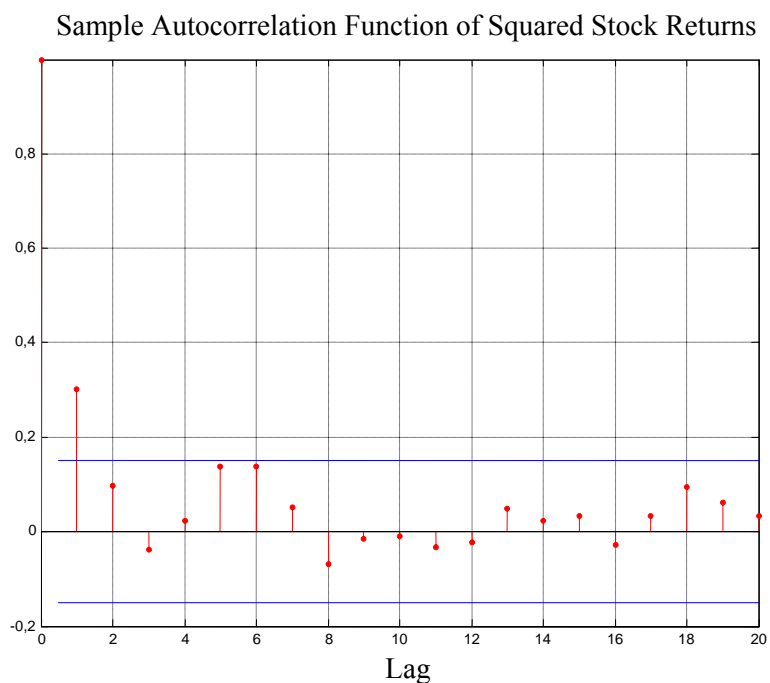
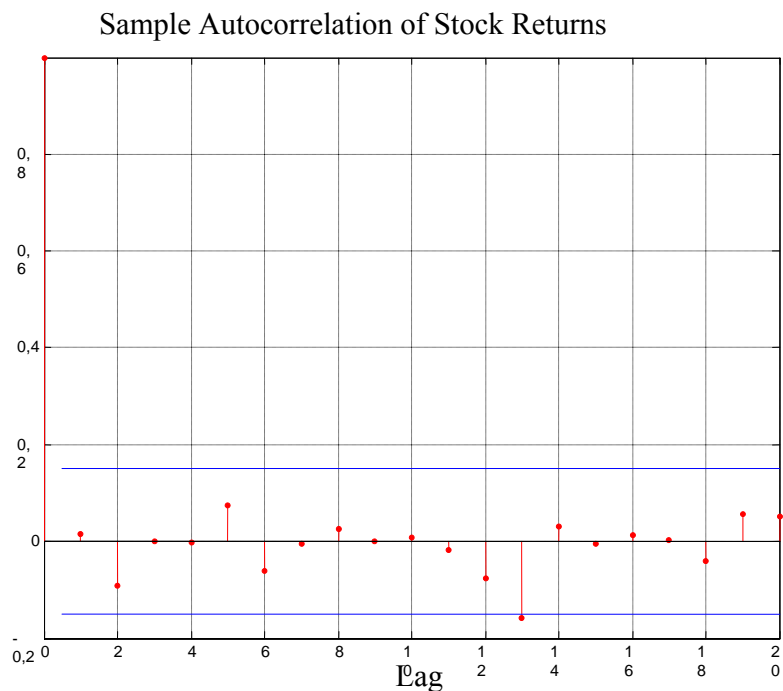
The statistical tables show that both series have asymmetric distributions as the skewness coefficient is different from zero. Mostly, they have more weight in the left part of the distribution, as the skewness coefficients are negative. The kurtosis coefficient for stock returns series is larger than 3 indicating that the tails of the distribution is fatter than those of the normal distribution. These two parameters are combined to test if the distribution is normal in the BERA-JARQUE (1980) test. The test indicates that the normality assumption can not be rejected for any of the time series.

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It is proper to check if there is statistically significant autocorrelation in the stock returns and industrial production series, as well as in the squared returns. The diagrams present the autocorrelation of the series in each lag. The blue line represents the bounds, which are computed with approximate 95% confidence level.



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The Industrial Production Growth series suffers from autocorrelation in many lags while the squared returns series only in the first three lags. As far as the Stock Returns is concerned, only the squared returns exhibit autocorrelation in the first lag. We will try to quantify the autocorrelation by performing the Engle's ARCH test. We impose one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and

The relationship between volatility of asset prices and volatility of output growth

5% significance level for each time series. The results indicate persistent of ARCH effect in both series. The null hypothesis of no ARCH effect should be rejected for both time series. The table contains the results of the test:

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0	0,00019713
t-Statistic	101,2731	13,8583
Critical Value	3,8415	3,8415

**H = 0 indicate acceptance of the null hypothesis that no ARCH effects exist*

Mean Equation (VAR (6))

The appropriate mean equation should be a model that could remove all the serial autocorrelation from the residuals. According to four criteria we have implemented in various models (Final Prediction Error (FPE), the Akaike information criterion, the Schwarz information criterion, and the Hannan-Quinn information criterion) and the residuals autocorrelation LM test a Vector Autoregressive model with 9 lags was the appropriate model. The parameters estimates are presented now (standard errors in “()”):

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + \dots + b_9 X_{t-9} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$A = \begin{bmatrix} 0,000018 \\ (0,00014) \\ -0,006048 \\ (0,00496) \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1,92 & 0,0036 \\ (0,08159) & (0,00212) \\ 4,7098 & -0,0976 \\ (2,93203) & (0,07603) \end{bmatrix}$$

$$b_2 = \begin{bmatrix} -2,2623 & -0,0001 \\ (0,17380) & (0,00205) \\ -8,1286 & -0,0896 \\ (6,24543) & (0,07374) \end{bmatrix}, \quad b_3 = \begin{bmatrix} 2,4733 & 0,0021 \\ (0,23758) & (0,00206) \\ 17,5653 & 0,0187 \\ (8,53757) & (0,07400) \end{bmatrix}$$

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$$b_4 = \begin{bmatrix} -2,4090 & 0,0008 \\ (0,27602) & (0,00206) \\ -25,282 & -0,0346 \\ (9,91877) & (0,07405) \end{bmatrix} \quad b_5 = \begin{bmatrix} 2,2621 & 0,0004 \\ (0,28695) & (0,00205) \\ 24,6939 & -0,0051 \\ (10,3115) & (0,07354) \end{bmatrix}$$

$$b_6 = \begin{bmatrix} -1,8559 & -0,0001 \\ (0,27857) & (0,00205) \\ -20,1079 & -0,1673 \\ (10,0106) & (0,07353) \end{bmatrix} \quad b_7 = \begin{bmatrix} 1,1737 & 0,0002 \\ (0,24171) & (0,00199) \\ 18,6492 & -0,0381 \\ (8,68569) & (0,07164) \end{bmatrix}$$

$$b_8 = \begin{bmatrix} -0,5033 & 0,0007 \\ (0,17850) & (0,00196) \\ -13,5360 & -0,0331 \\ (6,41435) & (0,07058) \end{bmatrix} \quad b_9 = \begin{bmatrix} 0,0472 & -0,0002 \\ (0,08396) & (0,00195) \\ 3,6468 & -0,0195 \\ (3,01697) & (0,07001) \end{bmatrix}$$

The autocorrelation LM test is now applied on the residuals of the VAR (9) model. The null hypothesis of no autocorrelation up to twelve lags is supported sufficiently and therefore is accepted. So the VAR (9) model is adequate for our data input. The table below contains the results:

Lags	LM-Statistic	P-Value
1	7,161002	0,1276
2	1,945619	0,7458
3	1,737387	0,7839
4	2,926252	0,5702
5	5,072457	0,2799
6	11,97032	0,0176
7	9,120800	0,0581
8	3,139550	0,5347
9	1,862193	0,7611
10	1,555087	0,8168
11	5,586140	0,2323
12	4,250731	0,3731

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

The causality in mean test will clarify the relationship between the Industrial Production Growth and Stock Returns in Japan. The pairwise Granger causality test provided by EViews is performed. The results are summarized in the tables:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	3,063536	0,6917

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	9,155117	0,1030

There is no causality in mean. Industrial Production Growth does not Granger causes Stock Returns and vice versa.

Variance Equation (BEKK 1, 1)

The appropriate model for the variance equation was a B.E.K.K. (1, 1) model. We estimated the parameters of this model assuming that both time series follow a normal distribution according to the results of the BERA-JARQUE (1980) test. The parameters estimates and standard errors (in “()”) are presented in the table above:

Parameters	Unrestricted GARCH(1,1)
c₁₁	0,0011 (0)
c₁₂	-0,0294 (0)
c₂₁	0
c₂₂	-0,0008 (0,0001)
GARCH g₁₁	-0,0964 (0,0061)
g₁₂	9,2051 (33,3377)
g₂₁	-0,0091 (0,0001)
g₂₂	-0,2671 (0,0079)
ARCH a₁₁	0,5156 (0,0156)
a₁₂	1,0256

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	(8,8846)
a₂₁	0,0109 (0)
a₂₂	0,7773 (0,0067)

Three L-Ratio tests are used to check whether there is causality-in-variance from industrial production growth to stock returns volatility and vice versa. The first one tests the causality from stock returns volatility to output growth volatility, the second one from output growth to stock returns and the last one the causality in both directions at the same time. The results could be summarized in the table below:

	Unrestricted	Restricted1 (a₂₁=g₂₁=0)	Restricted2 (a₁₂=g₁₂=0)	Restricted3 (a₁₂=g₁₂=a₂₁=g₂₁=0)
Loglikelihood	1088,7	734,0769	971,0255	691,8064
pValue		0	0,0065	0
LRatio		709,2777	1083,7	793,8187
Critical Value		5,9915	5,9915	9,4877

* $H = 0$ indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; $H = 1$ indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.

The first line contains the value of the Loglikelihood Function for each model we have estimated.

In all cases, the null hypothesis that the restricted model fits our data better than the unrestricted should be rejected. Thus, we arrive at the conclusion that there is strong evidence in our data of causality in variance in both directions. In other words, there is statistically significant relationship in Japan the last fifteen years between asset price volatility and output growth volatility.

At the end of this section, it is necessary to perform an after estimation analysis in order to check the validity of our results. The Ljung-Box lack-of-fit hypothesis test for model misspecification will be used. The GARCH residuals have been tested and the results are presented in the table. The lags used in the Q-Statistic are twenty and the significance level 5%.

	Standardized Residuals1	Standardized Residuals2
pValue	0,7308	0,8589
Q-Statistic	15,7693	13,4147
Critical Value	31,4104	31,4104

* H_0 : the null hypothesis that the model fit is adequate (no serial correlation).

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The results confirm that the model we selected for our time series was the appropriate one, since the null hypothesis that the model fit is adequate should be accepted in both series.

2nd Approach

The second approach we use to clarify the volatility spillovers between Stock Returns and Industrial Production Growth is based on the methodology proposed by Cheung and Ng in 1996. According to this, we have to estimate two univariate models for each time series. The models should be well specified and fit adequately our data. For the mean equation, an ARMA (R, M) process is usually used and for the variance equation a simple GARCH (p, q).

Industrial Production Growth series exhibits autocorrelation up to the eighth lag. Hence, we preferred an autoregressive model with eight lags for the mean equation. The ARCH/GARCH effect of the series was present in the first lag, so the variance equation was a GARCH (1, 1) model.

We also estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (8, 8), GARCH (1, 1) has the greatest LLF value.

LLF	AR(1) GARCH(1,1)	ARMA(1, 1) GARCH(1,1)	AR(8) GARCH(1,1)	ARMA (8,8) GARCH(1,1)
Industrial Production Growth	205,9284	245,2523	287,8358	400,8427

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The second time series, Stock Returns, has autocorrelation only in the second moments. So, we estimated a simple GARCH (1, 1) model to account for the autocorrelation of the series.

We also estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The GARCH (1, 1) has the greatest LLF value.

LLF	ARMA(0,0) GARCH(1,1)	ARMA(0, 0) GARCH(0,0)	ARMA(0,0) GARCH(0,1)
Stock Returns	228,2939	225,4820	228,1831

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

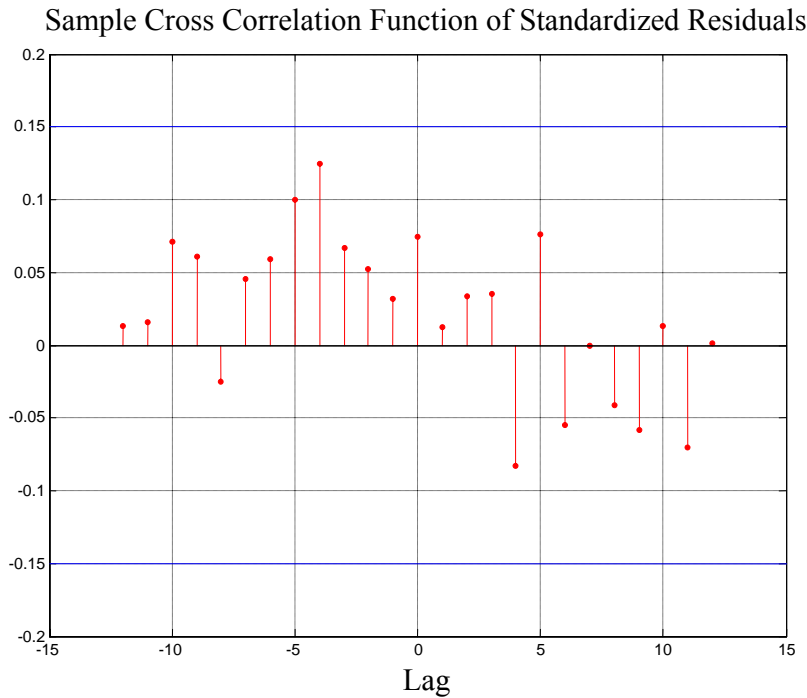
The relationship between volatility of asset prices and volatility of output growth

Then, the innovations of the two models were tested to check whether the model fits were adequate. The test that we used was the Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification. The lags used in the Q-Statistic were twenty and the significance level 5%. The null hypothesis should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,9633	0,9438
Q-statistic	10,1100	11,0922
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

We shall now plot the Sample Cross Correlation Function of the standardized residuals of the series to examine the causality in mean patterns. The null hypothesis is that the two residuals are uncorrelated (no causality in mean). The blue line represents the confidence interval of 95 %:



The null hypothesis of no autocorrelation (no causality) should be accepted in every lag. We may claim that IndustrialProductionGrowth does not Granger causes StockReturns and vice versa. The t-statistic for the hypothesis testing is:

$$t = \sqrt{T} \hat{r}_{uv}(\mathbf{k}) \rightarrow \text{AN} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

Where \mathbf{k} is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

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We apply this test on the standardized residuals. The table contains the t-statistic for each lag.

Lags	t-statistic for st. residuals
-12	0,1806
-11	0,2079
-10	0,9502
-9	0,8089
-8	-0,3326
-7	0,6063
-6	0,7837
-5	1,3272
-4	1,6561
-3	0,8907
-2	0,6984
-1	0,4214
0	0,9868
1	0,1674
2	0,4471
3	0,466
4	-1,1029
5	1,0119
6	-0,7277
7	-0,0108
8	-0,5448
9	-0,772
10	0,1734
11	-0,9356
12	0,0171

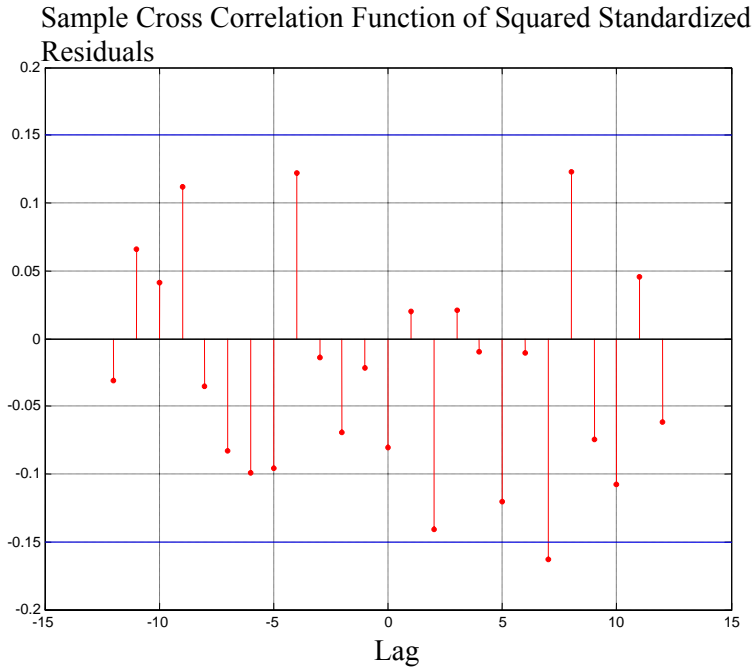
H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'' indicates significance at the 5% level, '**' indicates significance at the 1% level*

The null hypothesis should be accepted in every lag. There is no statistically significant causality in mean.

The causality in variance is tested by plotting the Sample Cross Correlation Function of the squared standardized residuals. The blue line represents the confidence interval of 95 %:

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The null hypothesis of no causality in variance should be rejected in lag (+7). The hypothesis testing according to the t-statistic used before is performed on the squared standardized residuals. The table contains the t-statistic for each lag of the squared standardized residuals:

Lags	t-statistic for squared st. residuals
-12	-0,4185
-11	0,8798
-10	0,5438
-9	1,4861
-8	-0,4716
-7	-1,1072
-6	-1,3238
-5	-1,2741
-4	1,6522***
-3	-0,1821
-2	-0,9258
-1	-0,2892
0	-1,0685
1	0,2694
2	-1,8713
3	0,2815
4	-0,1318
5	-1,5999
6	-0,1382

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7	-2,1665**
8	1,636
9	-0,9939
10	-1,4269
11	0,6041
12	-0,8192

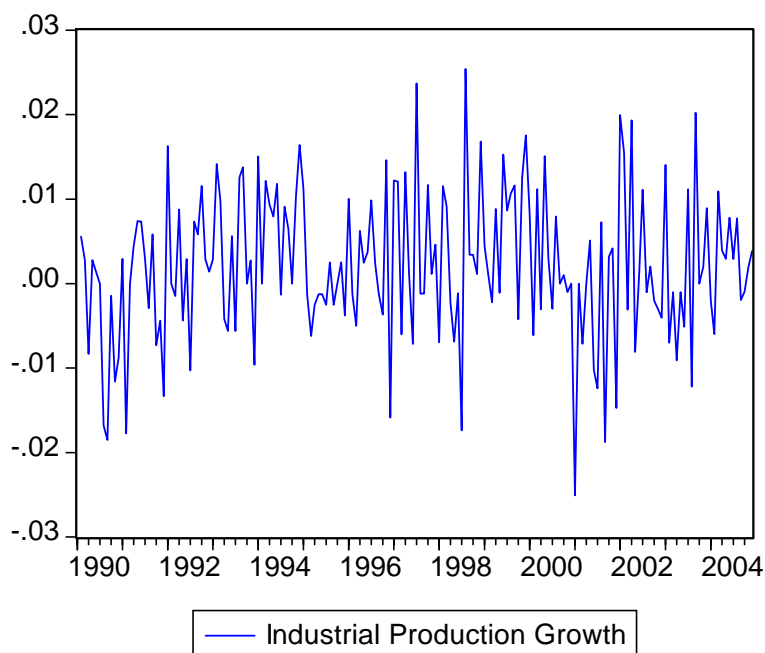
H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1 % level, '**' indicates significance at the 5% level '***' indicates significance at the 10% level

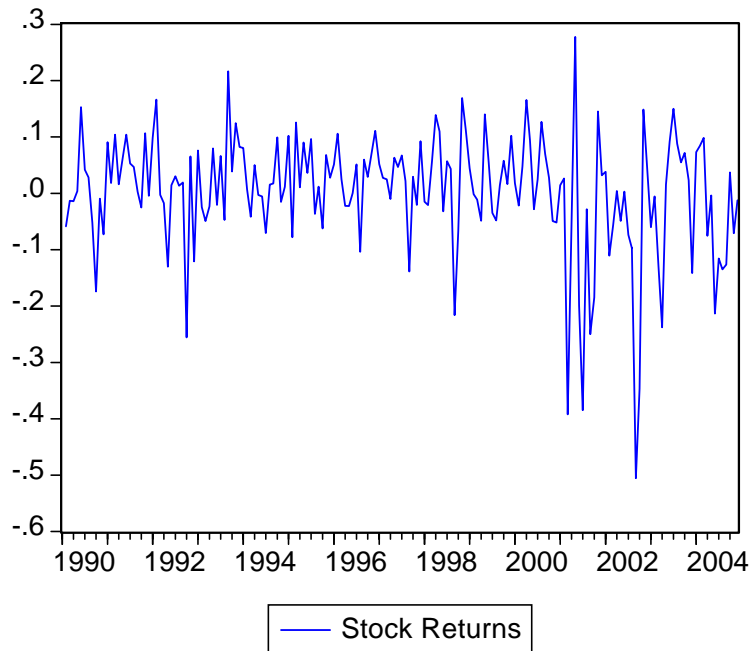
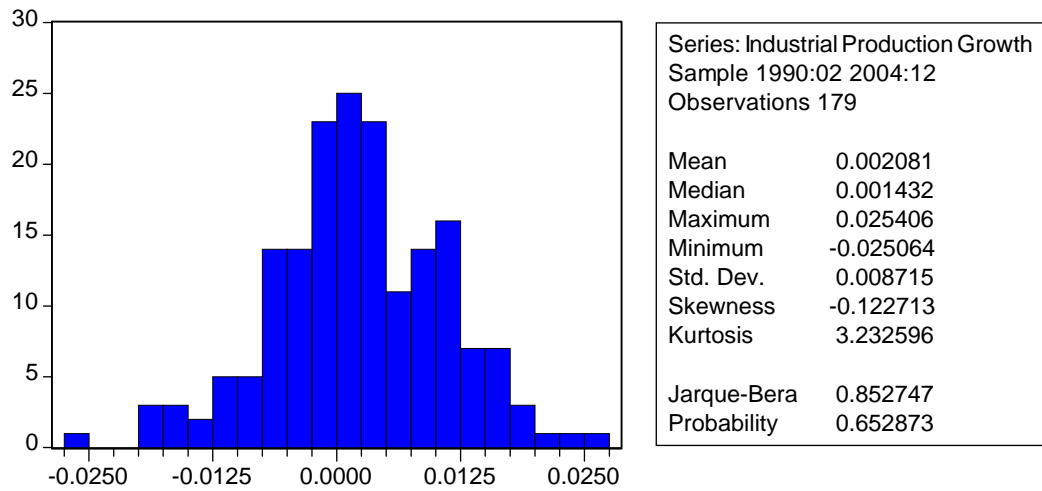
So, there is volatility spillover between Industrial Production Growth and Stock Returns in both directions. The volatility of IndustrialProductionGrowth_{t-7} Granger causes StockReturns_t. The volatility of Stock returns today is being influenced by the volatility of Industrial Production Growth seven months ago. Moreover, the volatility of StockReturns_{t-4} Granger causes the volatility of IndustrialProductionGrowth_t.

D) Canada

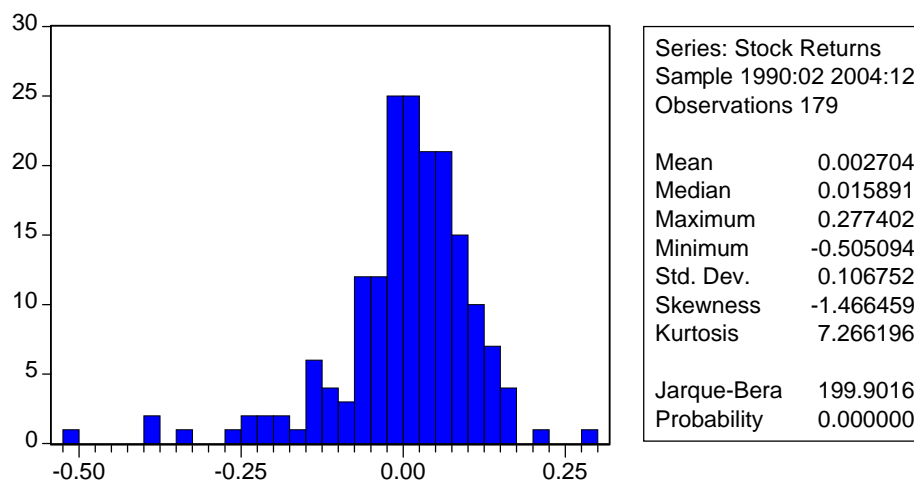
The causality in variance between Stock Returns and Industrial Production Growth in Canadian economy is under investigation in this section. Therefore, it is useful to present some preliminary and descriptive statistics about the two time series in interest. The period spans from January 1990 till December 2004.



The relationship between volatility of asset prices and volatility of output growth



The relationship between volatility of asset prices and volatility of output growth



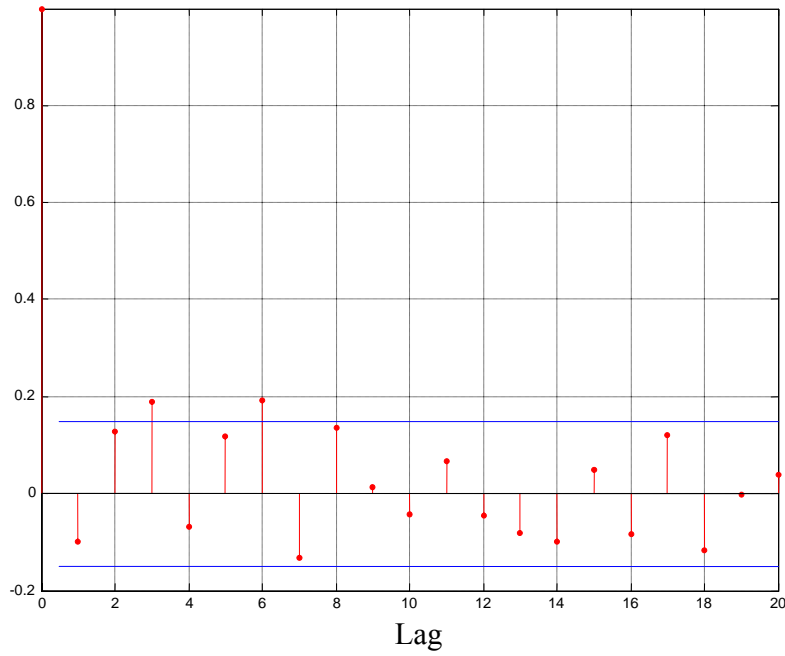
The distribution of Industrial Production Growth has similar features with the Normal Distribution. The skewness and kurtosis coefficients are close to zero and three respectively. Moreover, the Jarque-Bera test suggests that we should accept the null hypothesis that the time series is normally distributed.

On the other hand, stock returns series exhibits different features. The distribution is skewed to the left (negative skewness coefficient) and has fat tails (leptokurtic). The normality assumption should be rejected according to the Jarque-Bera test.

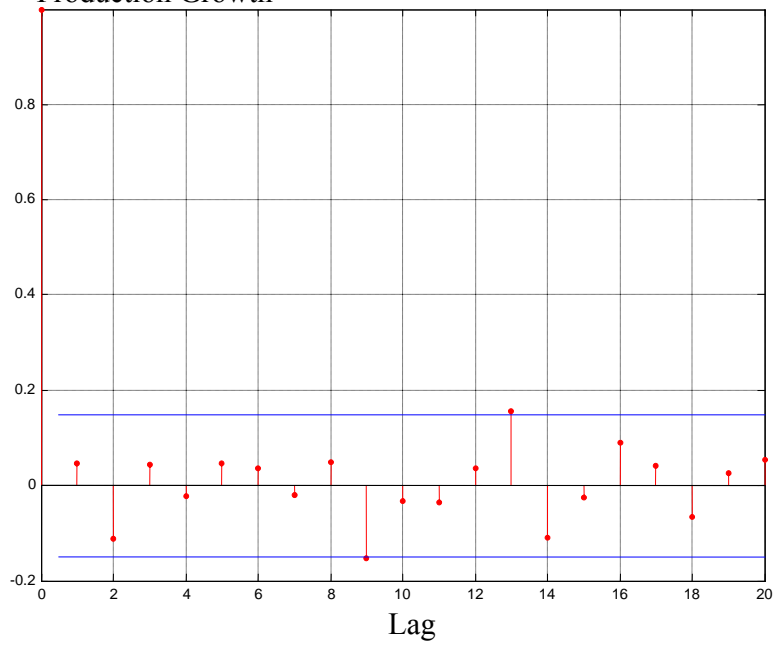
The autocorrelation of the series is examined by plotting the Sample Autocorrelation Function. The possibility of autocorrelation in the second moments is tested by the diagrams of the Sample Autocorrelation functions of the squared returns. (The blue line represents significance level of 5%)

The relationship between volatility of asset prices and volatility of output growth

Sample Autocorrelation Function of Industrial Production Growth

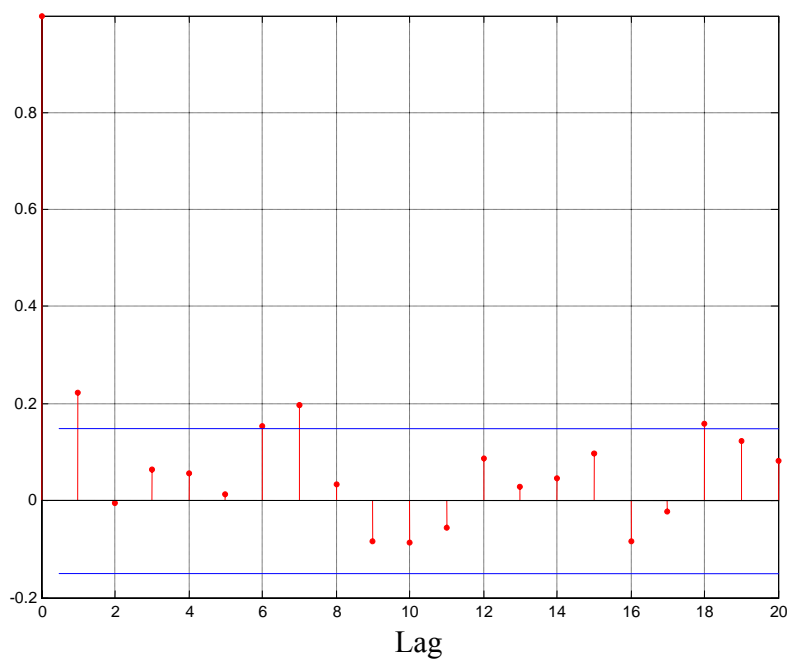


Sample Autocorrelation Function of Squared Industrial Production Growth

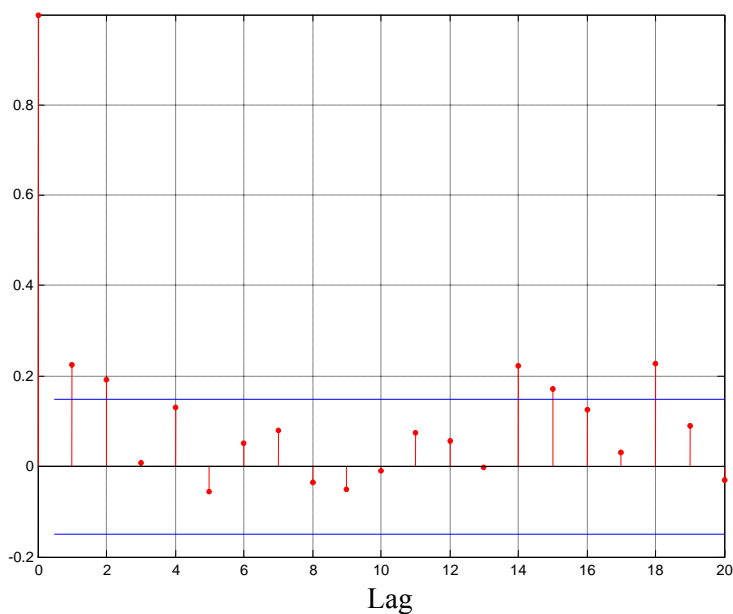


The relationship between volatility of asset prices and volatility of output growth

Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



The time series of Industrial Production Growth exhibits little autocorrelation in the third and sixth lag, but the squared series has no autocorrelation in any lag. On the other hand, Stock Returns suffers from autocorrelation in the first lag. The squared Stock returns exhibits also statistically significant autocorrelation in the first two lags.

The ARCH LM-test will help us figure out whether the two series exhibit ARCH effect, or serial autocorrelation. The null hypothesis of no ARCH effect should be

accepted for Industrial Production Growth and rejected for Stock Returns. We imposed one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series.

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0,5448	0,0025
t-Statistic	0,3668	9,1047
Critical Value	3,8415	3,8415

**H = 0 indicate acceptance of the null hypothesis that no ARCH effects exist*

1st Approach

Mean Equation (VAR (3))

The mean equation for our model is a Vector Autoregressive process. We chose the number of lags according to various criteria, namely the Final Prediction Error (FPE), Akaike information criterion, Schwarz information criterion and Hannan-Quinn information criterion. Moreover, we checked whether the model selected was capable of removing all the serial correlation from the residuals. We decided that a VAR (3) model was appropriate for our data. The parameters estimates are: (standard errors in ()):

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + b_3 X_{t-3} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

$$A = \begin{bmatrix} 0,001612 \\ (0,00069) \\ 0,001146 \\ (0,00867) \end{bmatrix} \quad b_1 = \begin{bmatrix} -0,119476 & 0,001741 \\ (0,07544) & (0,00617) \\ -0,256217 & 0,240992 \\ (0,94368) & (0,07717) \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 0,122447 & 0,012323 \\ (0,07428) & (0,00632) \\ 0,102120 & -0,077493 \\ (0,92924) & (0,07912) \end{bmatrix} \quad b_3 = \begin{bmatrix} 0,217552 & -0,002137 \\ (0,07425) & (0,00625) \\ 0,799336 & 0,078069 \\ (0,92888) & (0,07822) \end{bmatrix}$$

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The autocorrelation LM-test suggests that the VAR (3) residuals exhibit no autocorrelation for the first twelve lags. The LM-statistics and the corresponding PValues are presented in the table below:

Lags	LM-Statistic	P-Value
1	0,422254	0,9806
2	0,451708	0,9780
3	3,918614	0,4171
4	0,560459	0,9674
5	2,164619	0,7055
6	8,980518	0,0616
7	7,591766	0,1077
8	2,370500	0,6680
9	2,750917	0,6003
10	1,454214	0,8347
11	4,441664	0,3495
12	4,213091	0,3779

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

It is essential to investigate the relationship between Industrial Production Growth and Stock Returns before examining the volatility spillovers between these two series. Therefore, we perform the Pairwise Granger Causality test provided by EViews.

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	4,350595	0,2260

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	0,777756	0,8548

In both cases, the null hypothesis that the independent variable is statistically insignificant (zero) should be accepted. So, we arrive at the conclusion that there is no causality in mean in any direction.

Variance Equation (BEKK 1, 1)

The B.E.K.K. (1, 1) model is the variance equation. The parameters of the model are estimated with maximum likelihood estimation method. The parameters estimations are:

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Parameters (std.errors)	Unrestricted GARCH(1,1)
c ₁₁	0,0003 (0)
c ₁₂	0,0023 (0)
c ₂₁	0
c ₂₂	0,0193 (0)
GARCH g ₁₁	0,0641 (0,0070)
g ₁₂	2,6319 (0,9078)
g ₂₁	0,0051 (0)
g ₂₂	0,2318 (0,0050)
ARCH a ₁₁	0,9358 (0,0005)
a ₁₂	-2,2635 (0,0754)
a ₂₁	0,0113 (0)
a ₂₂	0,9112 (0,0014)

Before proceeding to the LRatio tests, it is necessary to perform the Ljung-Box lack-of-fit hypothesis test for model misspecification to check whether the unrestricted bivariate GARCH (1, 1) model is sufficient for our data. The lags used in the Q-Statistic are twenty and the significance level 5%.The residuals of the GARCH model have been tested and the results are in the table below:

	Standardized Residuals1	Standardized Residuals2
pValue	0,4391	0,2478
Q-Statistic	20,3033	20,4876
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

Three different LRatio test will help us figure out whether there is any causality in variance and the direction of the volatility spillovers.

	Unrestricted	Restricted1 ($a_{21}=g_{21}=0$)	Restricted2 ($a_{12}=g_{12}=0$)	Restricted3 ($a_{12}=g_{12}=a_{21}=g_{21}=0$)
Loglikelihood	763,0454	726,6618	583,3023	639,2621
pValue		0	0	0
LRatio		72,7672	359,4861	247,5655
Critical Value		5,9915	5,9915	9,4877

* $H = 0$ indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; $H = 1$ indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.

The first line of the table contains the value of the Loglikelihood Function for each model estimated. The unrestricted model has the greatest value of all.

In all cases, the unrestricted model seems to outperform the restricted ones. The LRatio test suggests that we should reject the restricted models. In other words, there is statistically significant causality in variance in both directions. The Industrial Production Growth volatility Granger causes the volatility of Stock Returns and vice versa.

2nd Approach

In the second part, we employ the causality in variance test proposed by Cheung and Ng in 1996. Firstly, we need to model our time series in a way that all the serial correlation of the residuals can be removed. The mean equation will be an ARMA(R, M) model and the variance equation a GARCH (p, q) model.

The significance of each parameter will determine the lag structure of each model. For Industrial Production Growth all the autoregressive parameters are insignificant. So we use for the variance equation the simple GARCH (1, 1) model and for the mean the raw series.

We also estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The GARCH (1, 1) has the greatest LLF value.

LLF	ARMA(0,0) GARCH(0,0)	AR(0,0) GARCH(0,1)	ARMA (0,0) GARCH(1,1)
Industrial Production Growth	595,4487	596,2236	596,2236

*LLF - Optimized log-likelihood objective function value associated with the parameter estimates

Stock Returns series exhibits autocorrelation in the first lag. Hence, the mean equation is an ARMA (1, 1). The ARCH (1), GARCH (1), GARCH (2) and GARCH (3)

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parameters are all statistically significant. Therefore, the variance equation is a GARCH (3, 1).

We also estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The ARMA (1, 1), GARCH (3, 1) has the greatest LLF value.

LLF	AR (1) GARCH(1,1)	AR (1) GARCH(3,1)	ARMA (1,1) GARCH(2,1)	ARMA (1,1) GARCH(3,1)
Stock Returns	159,4349	159,5077	159,5653	159,6378

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

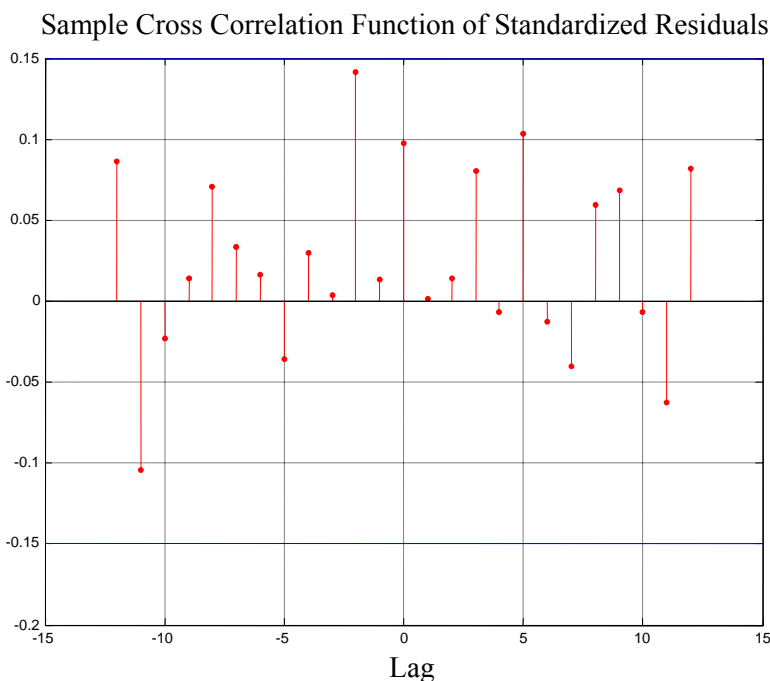
The Ljung-Box Q-statistic lack-of-fit hypothesis test will be implemented on the residuals of the series. This test is for model misspecification and the null hypothesis is that the model fit is adequate. The null hypothesis should be accepted for both series. The table summarizes the results of the test. The lags used in the Q-Statistic are twenty and the significance level 5%

	Industrial Production Growth	Stock Returns
P-Value	0,0908	0,3321
Q-statistic	4,7973	2,2045
Critical Value	5,9915	5,9915

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

The Sample Cross Correlation Function diagram of the standardized residuals of the two time series indicates the causality in mean patters. The blue line on the diagram represents the confidence interval of 95 %:

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According to the diagram, the null hypothesis of no correlation between the standardized residuals of the two series should be accepted in all lags. There is no causality in mean between Industrial Production Growth and Stock Returns in Canada.

The implementation of the following t-statistic confirms the above result.

$$t = \sqrt{T} \hat{r}_{uv}^{\wedge}(k) \rightarrow \text{AN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations). We apply this test on the standardized residuals. The table contains the t-statistic for each lag.

Lags	t-statistic for st. residuals
-12	1,1563
-11	-1,3984
-10	-0,3065
-9	0,1893
-8	0,9467
-7	0,4448
-6	0,2208
-5	-0,4777
-4	0,402
-3	0,0492
-2	1,8902
-1	0,1796
0	1,309
1	0,0187

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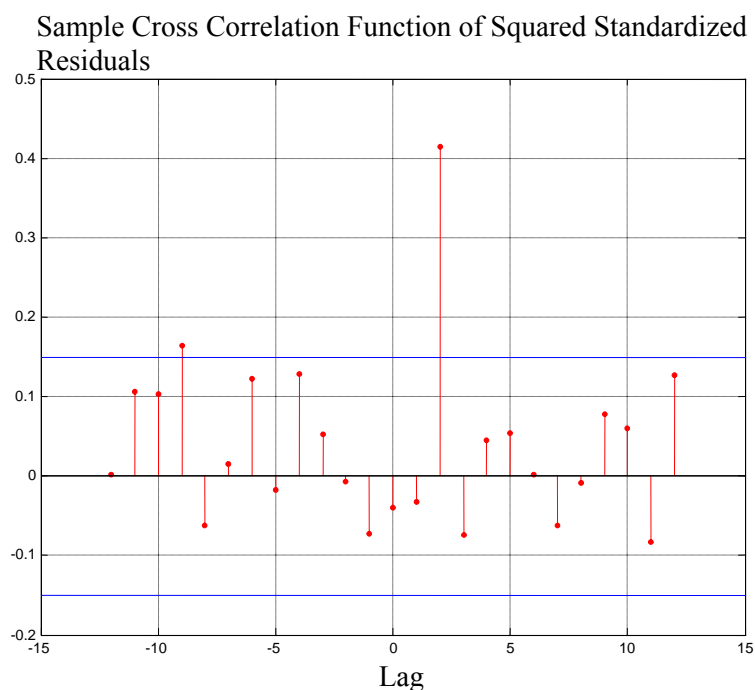
2	0,1905
3	1,0707
4	-0,0925
5	1,3794
6	-0,1701
7	-0,5418
8	0,7996
9	0,9158
10	-0,0923
11	-0,8423
12	1,0933

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level

The null hypothesis of no causality in mean should be accepted in all lags.

The plot of the Sample Cross Correlation Function of the squared standardized residuals of the two series is the causality in variance test. The blue line on the diagram represents the confidence interval of 95 %:



The null hypothesis of no causality should be rejected in two lags (-9, +2). There is causality in variance in both directions. $StockReturns_{t-9}$ volatility Granger causes the volatility of $IndustrialProductionGrowth_t$ and $IndustrialProductionGrowth_{t-2}$ volatility Granger causes the volatility of $StockReturns_t$.

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We will now proceed to the hypothesis testing using the previous t-statistic on the squared standardized residuals. The results are summarized on the table:

Lags	t-statistic for squared st. residuals
-12	0,0328
-11	1,4195
-10	1,385
-9	2,197**
-8	-0,8343
-7	0,1972
-6	1,6427
-5	-0,2308
-4	1,7186
-3	0,7008
-2	-0,097
-1	-0,9663
0	-0,5427
1	-0,4425
2	5,5453*
3	-0,9934
4	0,5981
5	0,7213
6	0,0151
7	-0,8363
8	-0,122
9	1,0361
10	0,7984
11	-1,1068
12	1,6932

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level

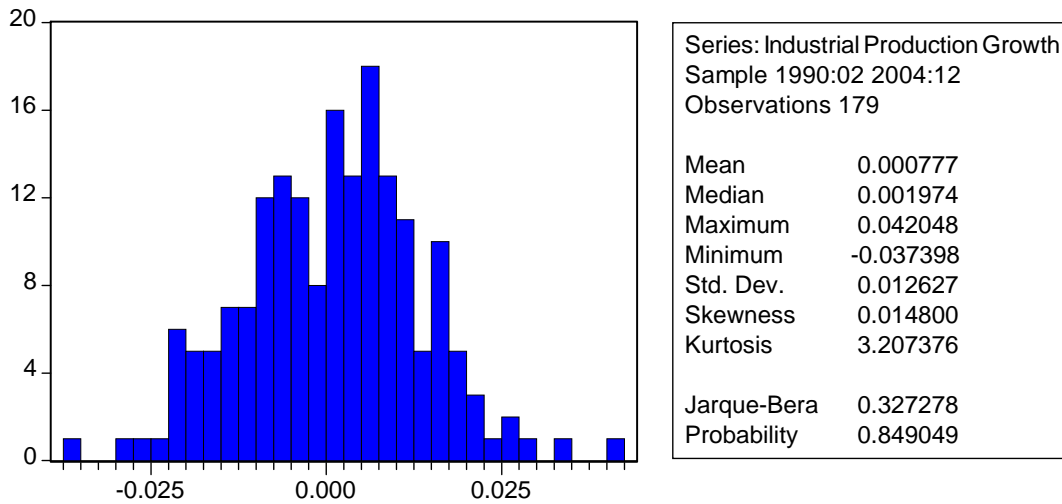
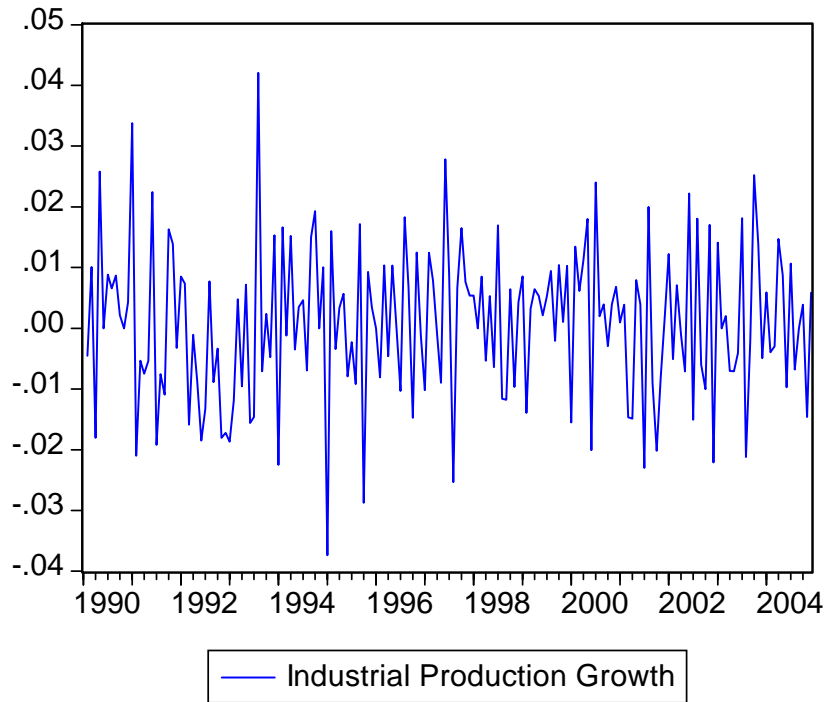
The volatility spillovers in Canada are present in both directions. StockReturns_{t-9} volatility Granger causes the volatility of IndustrialProductionGrowth_t and IndustrialProductionGrowth_{t-2} volatility Granger causes the volatility of StockReturns_t.

E) Germany

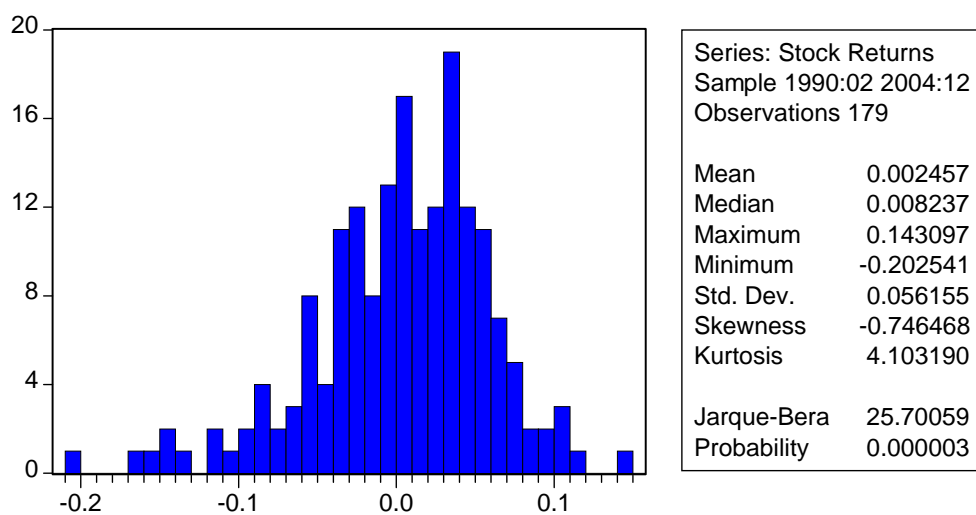
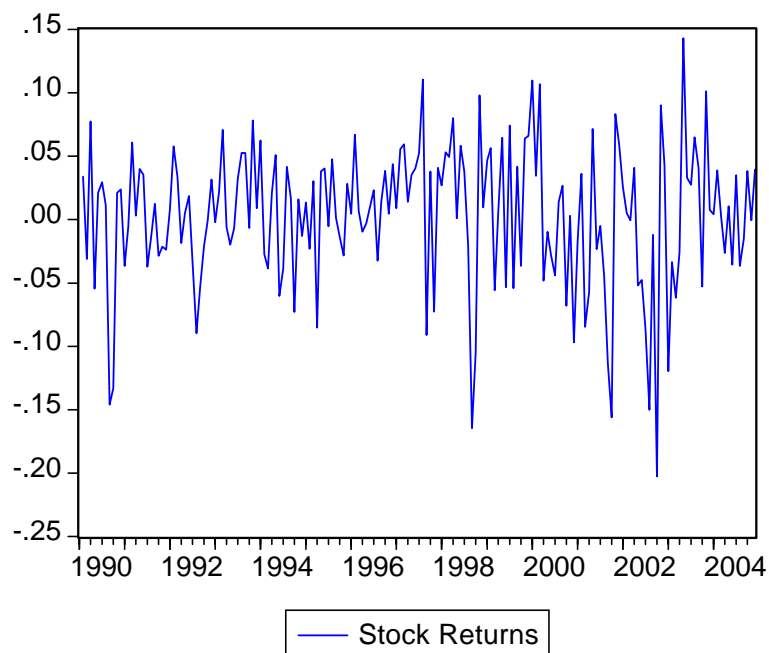
Firstly, it is useful to examine some preliminary statistics regarding the time series of Industrial Production Growth and Stock Returns in the last fifteen years. We

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present the diagrams of the evolution of the series during this period and two tables with the basic descriptive statistics.



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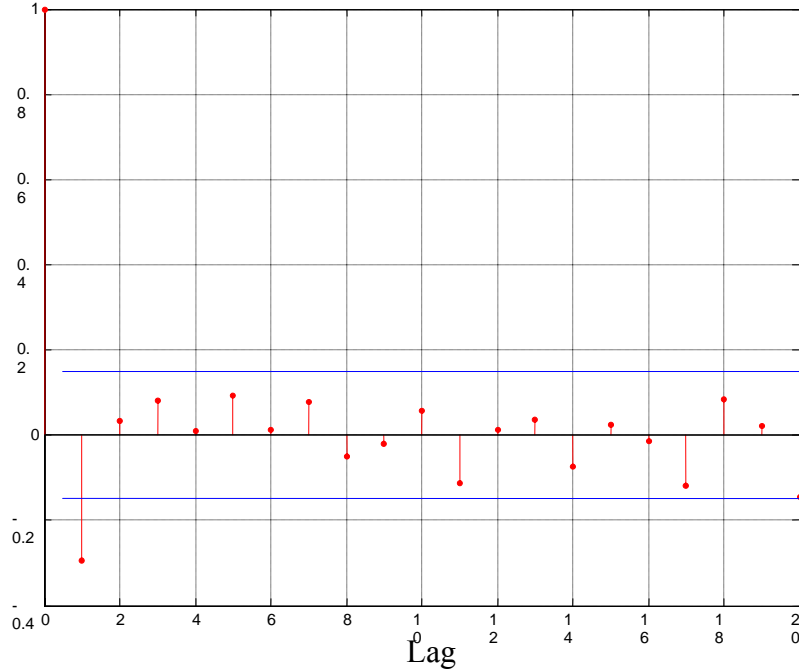
The Industrial Production Growth series exhibits strong evidence that follows a Normal distribution. The skewness and kurtosis coefficients are close to zero and three respectively. Moreover, according to the Jarque-Bera test we should accept the null hypothesis of Normality.

On the other hand, Stock Returns has a negative skewness coefficient which means that the distribution of the series is skewed to the left. Kurtosis is greater than three indicating leptokurtosis. Finally, the Normality assumption should be rejected as the Jarque-Bera test suggests.

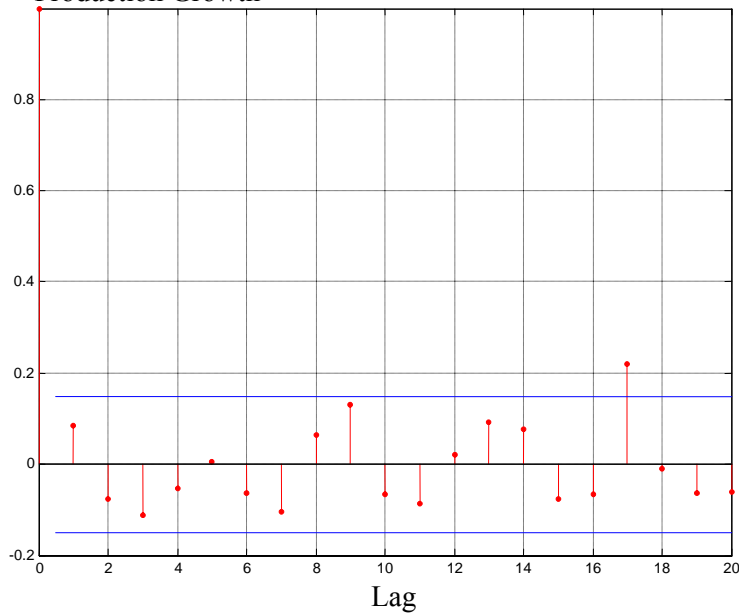
The relationship between volatility of asset prices and volatility of output growth

We should now plot the Sample Autocorrelation Function of the two series and the squared returns as well in order to check for autocorrelation in the series and in the second moments.

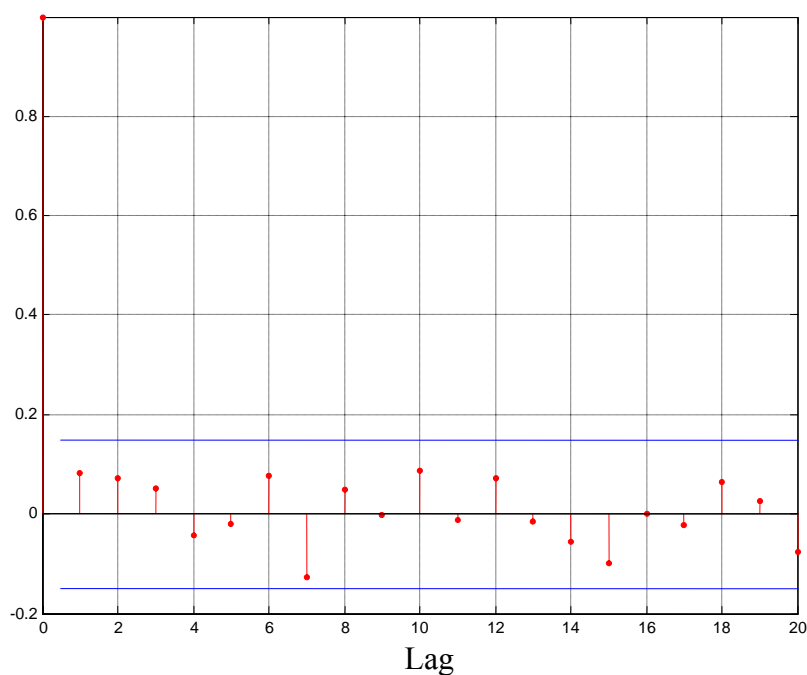
Sample Autocorrelation Function of Industrial Production Growth



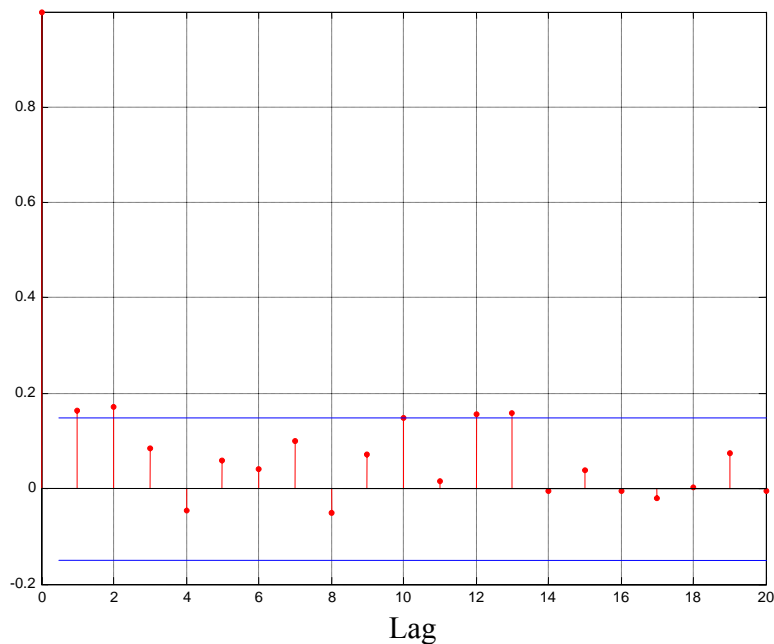
Sample Autocorrelation Function of Squared Industrial Production Growth



Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



Industrial Production Growth series exhibits autocorrelation in the first lag of the raw series, while no autocorrelation is observed on the squared returns series in any lag. We may also assume from the diagram that Stock Returns and the squared returns do not suffer from autocorrelation in any lag.

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The next test will help us examine whether there is an ARCH/GARCH effect on the series. The null hypothesis is that a time series is i.i.d. GAUSSIAN disturbances, hence no ARCH effect exists. We impose one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series.

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0,2518	0,0293
t-Statistic	1,3133	4,7484
Critical Value	3,8415	3,8415

* $H = 0$ indicate acceptance of the null hypothesis that no ARCH effects exist

1st Approach

Mean Equation (VAR (2))

We have to select a model for the mean equation. The model that fits best our data is a Vector Autoregressive model with two lags (VAR (2)). The model was selected according to various lag length criteria such as Final Prediction Error (FPE), Akaike information criterion, Schwarz information criterion and Hannan-Quinn information criterion. Moreover, the model succeeded in removing all the serial correlation from the residuals. The parameters estimates are: (the standard deviation in())

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

$$A = \begin{bmatrix} 0,000969 \\ (0,0092) \\ 0,002364 \\ (0,00419) \end{bmatrix} \quad b_1 = \begin{bmatrix} 0,313048 & 0,008912 \\ (0,07557) & (0,01653) \\ 0,482410 & 0,114971 \\ (0,34454) & (0,07537) \end{bmatrix}$$

$$b_2 = \begin{bmatrix} -0,055841 & 0,025156 \\ (0,07668) & (0,01638) \\ -0,721760 & 0,042138 \\ (0,34962) & (0,07467) \end{bmatrix}$$

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We perform the LM test for the residuals provided by EViews. The null hypothesis of no autocorrelation should be accepted for the first 12 lags. The results are presented in the table:

Lags	LM-Statistic	P-Value
1	3,636327	0,4575
2	3,486423	0,4799
3	3,489348	0,4795
4	1,135973	0,8885
5	5,748551	0,2187
6	5,841419	0,2113
7	12,81137	0,0122
8	1,350177	0,8528
9	0,126694	0,9981
10	3,538371	0,4721
11	3,755569	0,4401
12	2,924568	0,5705

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

We will now proceed to a test for causality in mean. We are interested in investigating the relationship between Industrial Production Growth and Stock Returns. The Pairwise Granger causality test provided by EViews will help us through.

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	2,842111	0,2415

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	8,492230	0,0143

We arrive at the conclusion that there is statistically significant causality from Industrial Production Growth to Stock Returns. Industrial Production Granger causes Stock Returns.

Variance Equation (BEKK 1, 1)

The variance equation is a B.E.K.K. (1, 1) model. Using the maximum likelihood estimation method, we estimated its parameters. The table contains the results:

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Parameters (std.errors)	Unrestricted GARCH(1,1)
c ₁₁	0,0094 (0)
c ₁₂	0,0034 (0)
c ₂₁	0
c ₂₂	0,0135 (0,0001)
GARCH g ₁₁	-0,4962 (0,0307)
g ₁₂	0,3939 (0,1895)
g ₂₁	0,0310 (0,0010)
g ₂₂	0,3688 (0,0104)
ARCH a ₁₁	0,1474 (0,0913)
a ₁₂	-0,4938 (1,6914)
a ₂₁	-0,0415 (0,0027)
a ₂₂	0,8764 (0,0112)

In order to check for causality in variance, we used the LRatio test. We estimated three different Restricted Models and performed LRatio tests. We compared each Restricted Model with the unrestricted one. The results can be summarized in the table:

	Unrestricted	Restricted1 (a₂₁=g₂₁=0)	Restricted2 (a₁₂=g₁₂=0)	Restricted3 (a₁₂=g₁₂=a₂₁=g₂₁=0)
Loglikelihood	1215,9	1198,4	1214,9	1195,7
pValue		0,0085	0,0447	0,0014
LRatio		9,5321	6,2139	17,6447
Critical Value		5,9915	5,9915	9,4877

**H = 0 indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; H = 1 indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.*

In the first row, we present the value of the Loglikelihood Function for each model. As we expected, the unrestricted model has the greatest LLF value.

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The null hypothesis that the restricted model is better than the unrestricted should be rejected for the first and the third case. The restrictions in the first and third restricted models should not have been imposed. Hence, there is statistically significant causality from Stock Returns volatility to Industrial Production Growth volatility. On the other hand, the pValue of the null hypothesis of acceptance the Restricted2 is close to 5% (4, 47%). Therefore, we may accept that the Restricted 2 model is marginally better than the Unrestricted, or there is no volatility spillover from Industrial Production Growth to Stock Returns.

The post estimation analysis with the use of Ljung-Box lack-of-fit hypothesis test on the residuals of the unrestricted GARCH model confirms that the model we selected was appropriate. The lags used in the Q-Statistic were twenty and the significance level 5%. The null hypothesis should be accepted.

	Standardized Residuals1	Standardized Residuals2
pValue	0,4206	0,6122
Q-Statistic	20,6063	17,6234
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

2nd Approach

We will now investigate the relationship between the volatility of Industrial Production Growth and Stock Returns with the method proposed by Chueng and Ng's. At first, we have to select two univariate models for the two time series separately. The equation for the mean will be an ARMA(R, M) process and for the variance a GARCH (p, q). The lag structure of each model will depend on the special features of each time series.

The mean equation for Industrial Production Growth is an ARMA (1, 1) and the variance equation a GARCH (1, 1). We estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (1, 1), GARCH (1, 1) has the greatest LLF value.

LLF	ARMA(0,0) GARCH(1,1)	AR(1) GARCH(0,1)	AR (1) GARCH(1,1)	ARMA (1, 1) GARCH(1,1)
Industrial Production Growth	530,6624	539,3985	539,3985	539,9441

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

Stock Returns do not exhibit any autocorrelation in the raw series, so the model for Stock Returns is a simple GARCH (1, 1). We estimated the Loglikelihood Function

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(LLF) of three different models. The results are presented in the table below. The GARCH (1, 1) has the greatest LLF value.

LLF	AR(0) GARCH(0,1)	ARMA(0,0) GARCH(0,2)	ARMA(0,0) GARCH(1,1)
Stock Returns	264,4063	268,5985	268,9800

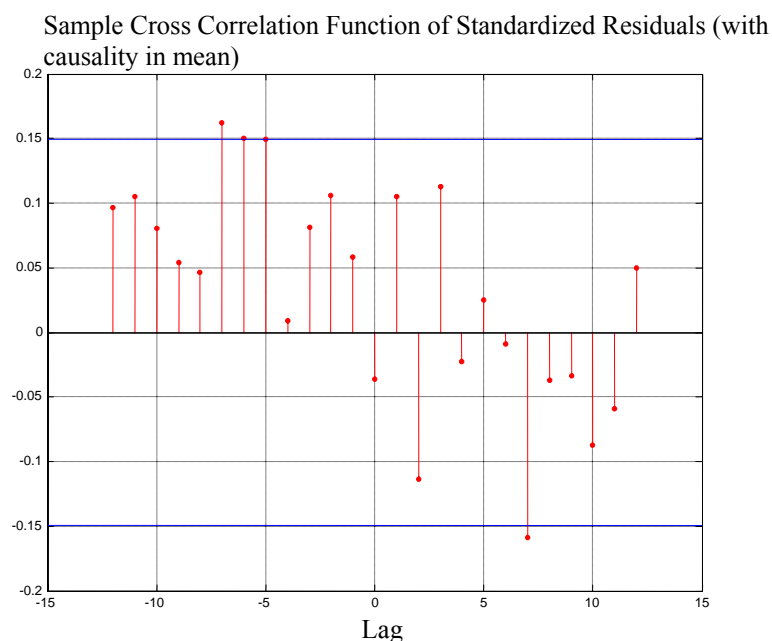
**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The innovations of the GARCH models are tested with the Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification. The lags used in the Q-Statistic were twenty and the significance level 5%. The null hypothesis that the univariate model we chose fit adequate should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,3326	0,7748
Q-statistic	22,1463	15,0286
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

We will now plot the Sample Cross Correlation Function of Standardized Residuals to check for causality in mean. The blue line represents the confidence interval of 95 %:



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From the diagram, we may conclude that there is causality in mean in both directions. StockReturns_{t-7} Granger causes IndustrialProductionGrowth_t and IndustrialProductionGrowth_{t-7} Granger causes StockReturns_t.

We proceed with the hypothesis testing using the t-statistics:

$$t = \sqrt{T} \hat{r}_{uv}(\mathbf{k}) \rightarrow \text{AN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

Where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

This t-statistic is being applied on the standardized residuals and the results are presented on the following table:

Lags	t-statistic for st. residuals
-12	1,2884
-11	1,4086
-10	1,0771
-9	0,7203
-8	0,6153
-7	2,1707**
-6	2,0118**
-5	2,0027**
-4	0,1169
-3	1,0904
-2	1,4224
-1	0,7788
0	-0,4879
1	1,4093
2	-1,5179
3	1,5093
4	-0,2994
5	0,3319
6	-0,1204
7	-2,1267**
8	-0,4993
9	-0,449
10	-1,1648
11	-0,7949
12	0,6713

H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

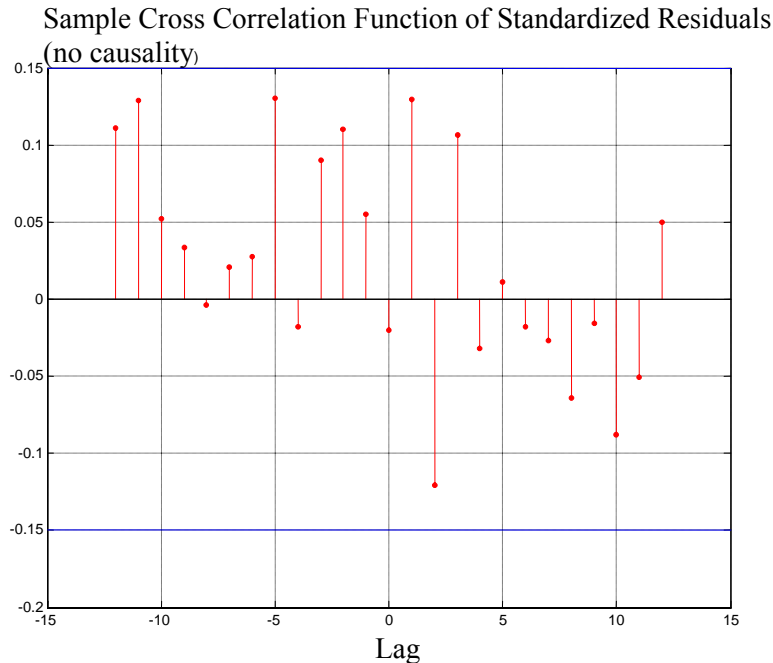
‘’ indicates significance at the 1% level, ‘**’ indicates significance at the 5% level*

The relationship between volatility of asset prices and volatility of output growth

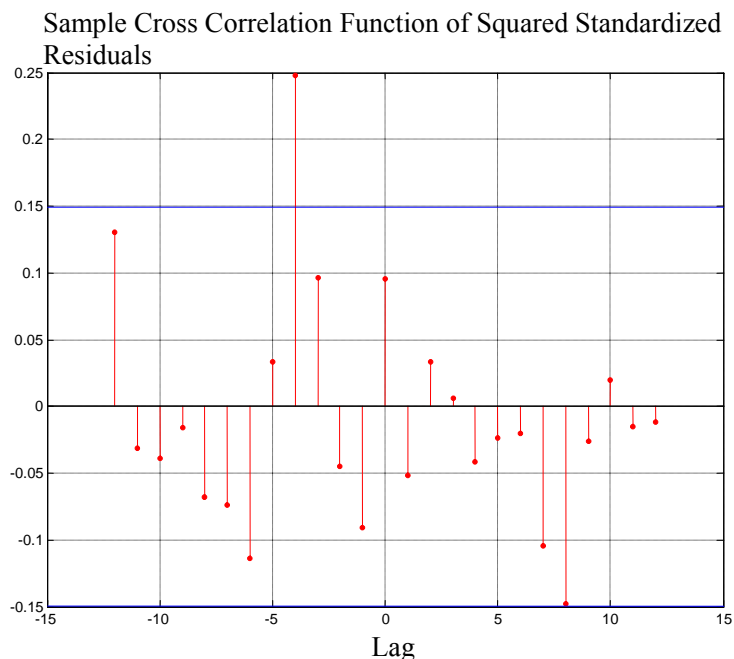
As we can see, there is causality in mean in lags (-5), (-6), (-7) and (+7). Stock Returns_{t-5}, Stock Returns_{t-6} and Stock Returns_{t-7} Granger cause Industrial Production Growth_t and simultaneously Industrial Production Growth_{t-7} Granger causes Stock Returns_t.

In the case that causality in mean is present, the conditional mean should be modified accordingly to account for this additional dynamics. If not, the causality-in-variance tests are likely to suffer from size distortions. Theologos Pantelidis and Nikitas Pittis proved in their paper (2004) “Testing for Granger causality in variance in the presence of causality in mean” that the tests for causality in variance suffer from severe size distortions when strong causality-in-mean effects are left unaccounted for. By means of Monte Carlo simulations they showed that the model used to filter out the conditional mean effects must account for possible causality in mean between the series. Otherwise, the causality-in-variance test statistics suffer from severe size distortions, especially when the neglected causality-in-mean effects are strong.

Therefore, we add a time series regression matrix of explanatory variables, a regression component, in the mean equation of Industrial Production Growth. The regression component is a lagged transform by 5, 6 and 7 lags of the time series Stock Returns (StockReturns_{t-5}, StockReturns_{t-6}, and StockReturns_{t-7}). Then, we follow the same procedure for the Stock Returns time series. The exogenous variable added in the mean equation of Stock Returns is Industrial Production Growth lagged by 7 lags (IndustrialProductionGrowth_{t-7}). We plot again the Sample Cross Correlation Function of the standardized residuals of the series and of the squared standardized residuals.



The relationship between volatility of asset prices and volatility of output growth



The diagrams reveal that there is no more causality in mean, but there is statistically significant causality in variance in lags (-4). StockReturns_{t-8} volatility Granger causes IndustrialProductionGrowth_t volatility.

We proceed to the hypothesis testing for standardized residuals and squared standardized residuals. The t-statistics for each lag are:

Lags	t-statistic for st. residuals	t-statistic for squared st. residuals
-12	1,4885	1,7409
-11	1,7247	-0,4158
-10	0,7001	-0,5243
-9	0,4433	-0,2164
-8	-0,0492	-0,9094
-7	0,2803	-0,9846
-6	0,3714	-1,5266
-5	1,7474	0,444
-4	-0,2416	3,311*
-3	1,2084	1,291
-2	1,4702	-0,5975
-1	0,7387	-1,2132
0	-0,2661	1,2816
1	1,7358	-0,688
2	-1,6148	0,4461

The relationship between volatility of asset prices and volatility of output growth

3	1,4215	0,0863
4	-0,4307	-0,5539
5	0,1448	-0,3121
6	-0,2382	-0,265
7	-0,3611	-1,394
8	-0,8549	-1,9764**
9	-0,2102	-0,3507
10	-1,1716	0,2601
11	-0,6768	-0,1979
12	0,6714	-0,152

H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

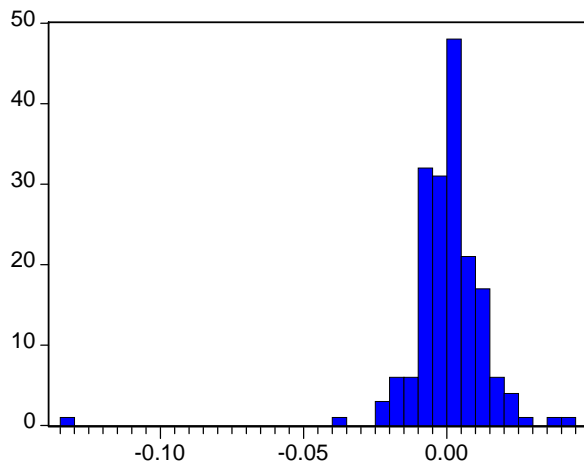
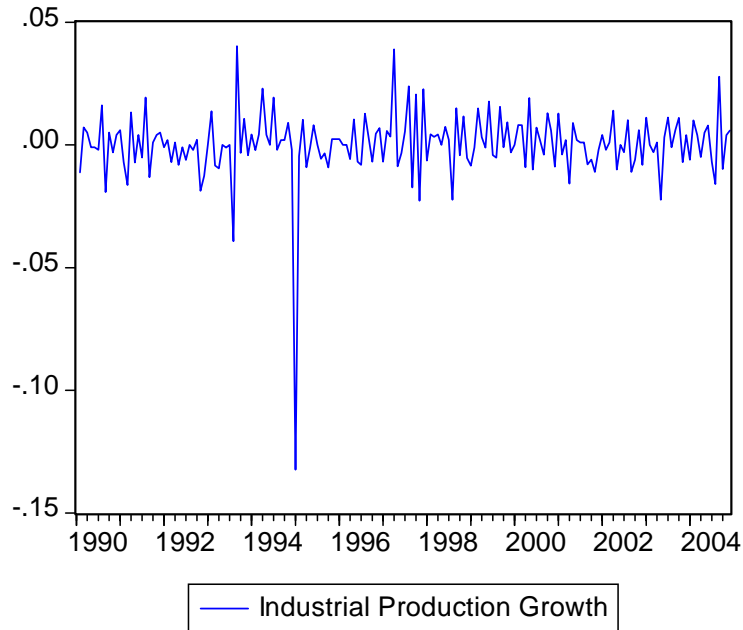
‘’ indicates significance at the 1% level, ‘**’ indicates significance at the 5% level*

The t-statistics suggest that there is no causality in mean but we should reject the null hypothesis for no causality in variance in two lags (-4, +8). So, we may assume that there is causality in variance in both ways. StockReturns_{t-4} volatility Granger causes IndustrialProductionGrowth_t volatility and simultaneously IndustrialProductionGrowth_{t-8} volatility Granger causes StockReturns_t volatility.

F) France

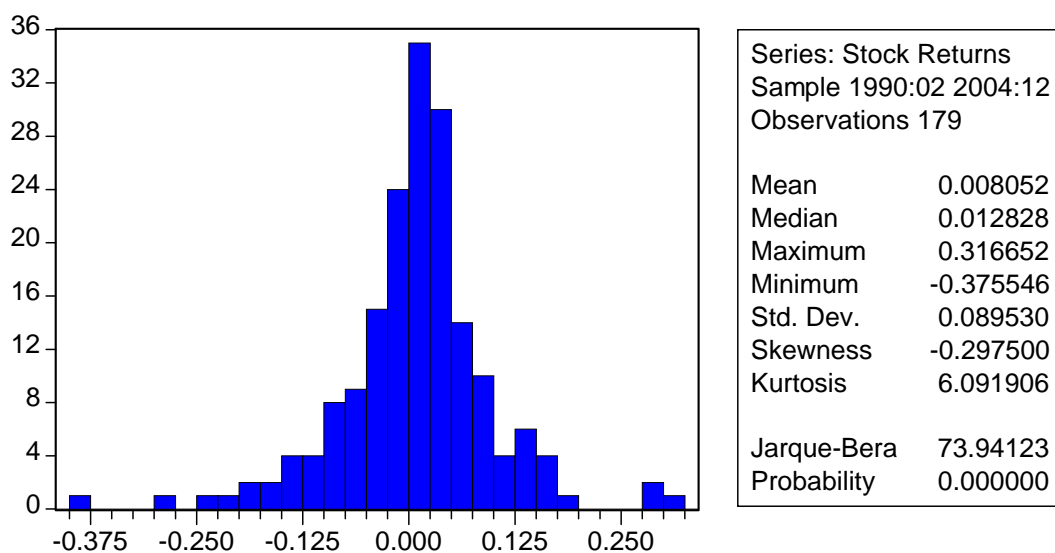
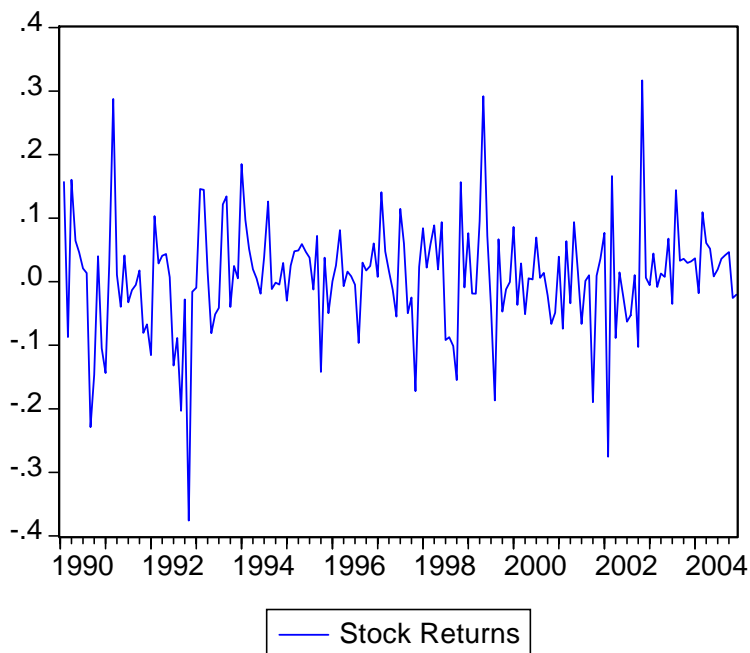
We start the analysis of the country with some descriptive statistics and diagrams representing the evolution of the time series under investigation the last fifteen years.

The relationship between volatility of asset prices and volatility of output growth



Series: Industrial Production Growth	
Sample 1990:02 2004:12	
Observations 179	
Mean	0.000184
Median	0.000000
Maximum	0.040245
Minimum	-0.132248
Std. Dev.	0.014411
Skewness	-4.171609
Kurtosis	41.79751
Jarque-Bera	11745.80
Probability	0.000000

The relationship between volatility of asset prices and volatility of output growth

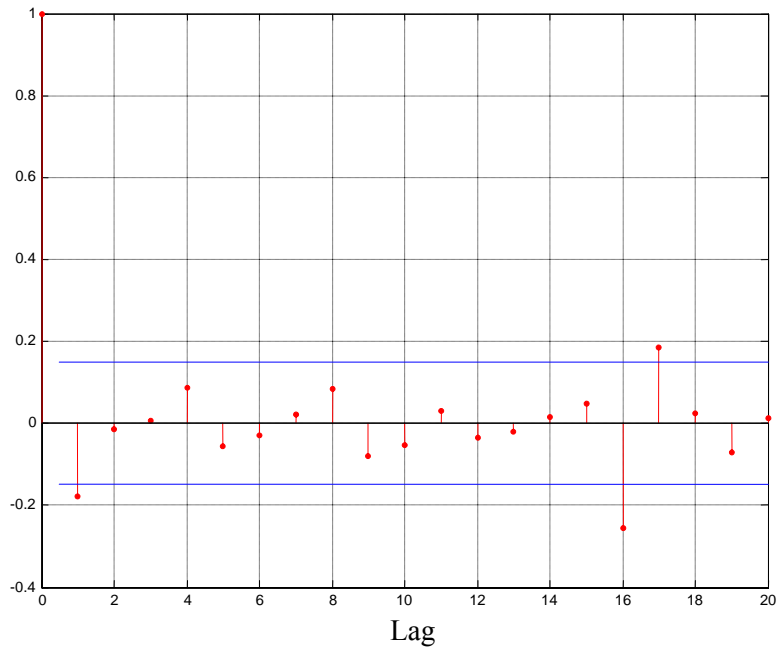


The skewness coefficients are negative for both series indicating that the distribution of the series is skewed to the left. The kurtosis coefficients are greater than three (fat tails). The Jarque-Bera test suggests that we should reject the Normality assumption for both series.

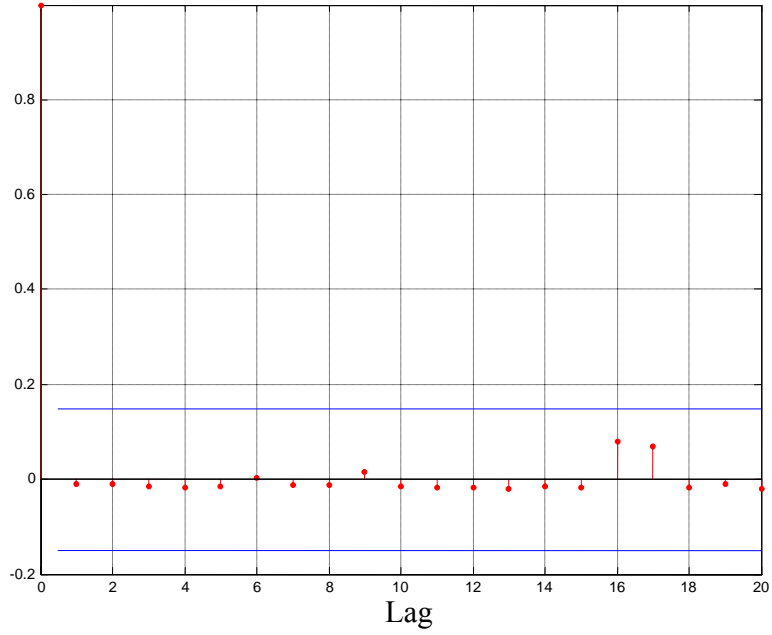
We shall now plot the sample autocorrelation function diagrams for both the returns of the series and the squared returns. The Autocorrelation Function (ACF) of the squared returns may still indicate significant correlation and persistence in the second-order moments. We may check this by plotting the ACF of the squared returns.

The relationship between volatility of asset prices and volatility of output growth

Sample Autocorrelation Function of Industrial Production Growth

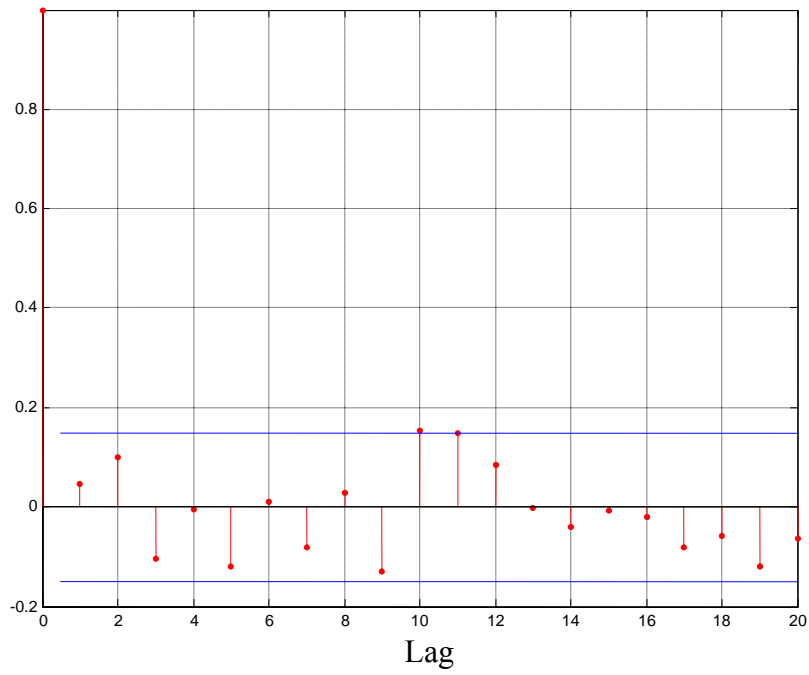


Sample Autocorrelation Function of Squared Industrial Production Growth

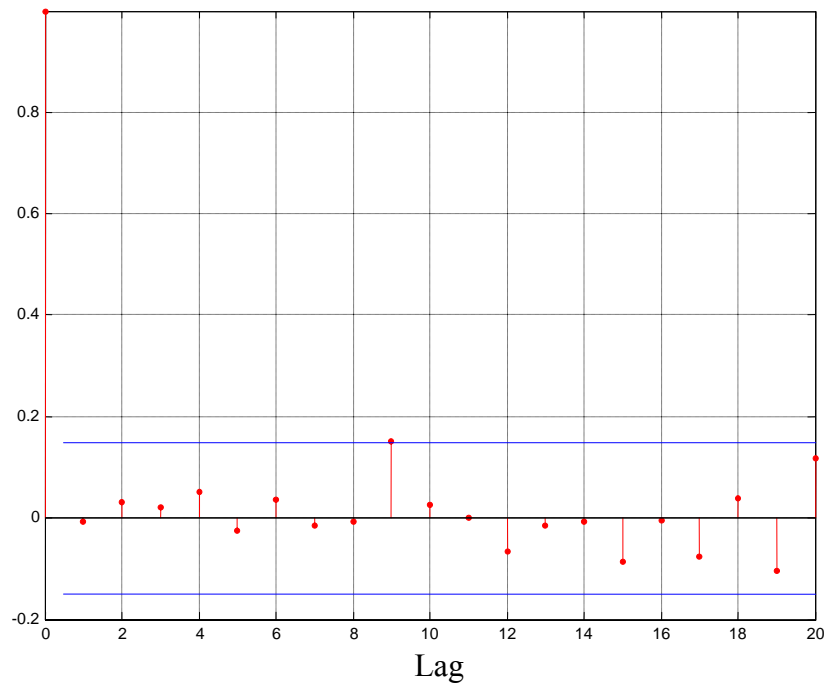


The relationship between volatility of asset prices and volatility of output growth

Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



The relationship between volatility of asset prices and volatility of output growth

We may assume that Industrial Production Growth series exhibits autocorrelation in the first lag while the Stock Returns show no autocorrelation. In the squared returns, there is no statistically significant autocorrelation in the first lags for both series.

We will now proceed to the ARCH test. The null hypothesis of this test is that no ARCH effect exists. Now, we impose one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series.

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0,9075	0,9253
t-Statistic	0,0135	0,0088
Critical Value	3,8415	3,8415

**H = 0 indicate acceptance of the null hypothesis that no ARCH effects exist*

So, that test suggests that no ARCH effect exists in any of our time series.

1st Approach

Mean Equation (VAR (1))

The mean equation selected for our data is a VAR (1) model. The criteria used to reach this decision are mentioned in the previous sections.

$$X_t = A + b_1 X_{t-1} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

$$A = \begin{bmatrix} 0,000211 \\ 0,00107 \\ 0,006815 \\ 0,00672 \end{bmatrix} \quad b_1 = \begin{bmatrix} -0,186448 & 0,007847 \\ 0,07491 & 0,01205 \\ -0,422783 & 0,056468 \\ 0,46915 & 0,07550 \end{bmatrix}$$

The autocorrelation LM Test for the residuals of the VAR (1) is presented in the table below. We accept in all lags the null hypothesis of no autocorrelation in the first 12 lags:

Lags	LM-Statistic	P-Value
1	2,399661	0,6627
2	3,518035	0,4751
3	2,499201	0,6448
4	1,628195	0,8037
5	3,670410	0,4524
6	0,519238	0,9716
7	0,936086	0,9193
8	2,276613	0,6850
9	5,660718	0,2260
10	6,582225	0,1597
11	5,250085	0,2626
12	5,297919	0,2581

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

Before studying the volatility spillovers, it is necessary to investigate the relationship between asset prices and Industrial Production Growth. We use the Pairwise Granger Causality Tests provided by EViews to check whether there is statistically significant causality in mean. We carry out pairwise Granger causality tests and test whether an endogenous variable can be treated as exogenous. For each equation in the VAR, the output displays (Wald) statistics for the joint significance of each of the other lagged endogenous variables in that equation. The results are presented in the tables below:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	0,423702	0,5151

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	0,812120	0,3675

In both cases, the null hypothesis that the independent variable is statistically insignificant (zero) should be accepted. So, there is no causality in mean in any direction. In other words, Stock Returns does not Granger causes Industrial Production Growth and vice versa.

Variance Equation (BEKK 1, 1)

The relationship between volatility of asset prices and volatility of output growth

We estimated a bivariate GARCH (1, 1) model with maximum likelihood and we assumed that output growth and stock returns follow a t-student distribution. The parameters estimates of the unrestricted model are presented in the table below:

Parameters (std.errors)	Unrestricted GARCH(1,1)
c ₁₁	0,0107 (0)
c ₁₂	0,0014 (0)
c ₂₁	0
c ₂₂	0,0897 (0,003)
GARCH g ₁₁	0,3714 (0,0165)
g ₁₂	0,5502 (0,0675)
g ₂₁	-0,0124 (0,0007)
g ₂₂	-0,4217 (0,1226)
ARCH a ₁₁	0,1597 (0,0223)
a ₁₂	0,4542 (0,3533)
a ₂₁	-0,0197 (0,0036)
a ₂₂	0,1576 (0,3092)

We will now perform the LRatio Tests to check the causality in variance from output growth volatility to asset price volatility and vice versa. The results can be summarized in the table:

	Unrestricted	Restricted1 (a₂₁=g₂₁=0)	Restricted2 (a₁₂=g₁₂=0)	Restricted3 (a₁₂=g₁₂=a₂₁=g₂₁=0)
Loglikelihood	761,5210	761,3519	760,9798	760,8630
pValue		0,8444	0,5814	0,5179
LRatio		35,0816	2,0295	40,5344
Critical Value		5,9915	5,9915	9,4877

**H = 0 indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; H = 1 indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.*

The relationship between volatility of asset prices and volatility of output growth

In the first row, we present the value of the Loglikelihood Function for each model. As we expected, the unrestricted model has the greatest LLF value.

The restricted 1 model tests whether there is causality from stock returns volatility to output growth volatility. The 2nd restricted model checks the causality from industrial production to stock returns volatility. The restricted 3 model investigates the causality in both directions. The null hypothesis should be accepted in all cases according to the LRatio test. Hence, there is no causality in variance at all, since the unrestricted model outperforms the three restricted models.

The post-estimation analysis reveals that the unrestricted model we chose fit adequate our data. The Ljung-Box lack-of-fit hypothesis test is implemented on the residuals of the unrestricted GARCH model. This model is based on the Q-Statistic. The lags used in the Q-Statistic were twenty and the significance level 5%. The results are presented in the table:

	Standardized Residuals1	Standardized Residuals2
pValue	0,2902	0,0894
Q-Statistic	22,9719	28,9174
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

2nd Approach

In the second method, we need to model each time series separately. So we construct two univariate models trying to take into account the various features of the data documented in the previous section. We model the mean equation as an autoregressive moving average process ARMA (p, q). The conditional variance equation is then modeled as a classical GARCH model. The error term ε_t is supposed to be conditionally normally distributed with mean 0 and conditional variance σ_t^2 .

The selection of the lag structure of each equation is made by the econometric package EViews. We estimate the parameters and calculate their significance. And we reject the insignificant parameters. For the Industrial Production Growth series the autoregressive first lag parameter is significant. Therefore, the mean equation is an ARMA (1, 1). The ARCH (1), GARCH (1) and GARCH (2) parameters are also statistically significant. Hence, the variance equation is a GARCH (2, 1) model.

We also estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The ARMA (1, 1), GARCH (2, 1) has the greatest LLF value.

The relationship between volatility of asset prices and volatility of output growth

LLF	ARMA(0,0) GARCH(1,1)	AR (1) GARCH(2,1)	ARMA (1, 1) GARCH(2,1)
Industrial Production Growth	505,4311	509,3706	509,6706

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The Stock Returns series exhibit no autocorrelation. The autoregressive coefficient of the first lag is not significant (the null hypothesis that the coefficient was zero should be accepted). The ARCH (1), GARCH (1) and GARCH (2) parameters are significant. Consequently, the variance equation of Stock Returns should be a GARCH (2, 1) model.

We also estimated the Loglikelihood Function (LLF) of three different models. The results are presented in the table below. The GARCH (2, 1) has the greatest LLF value.

LLF	AR(1) GARCH(0,1)	ARMA(0,0) GARCH(0,1)	ARMA(0,0) GARCH(2,1)
Stock Returns	179,5827	178,4713	181,4720

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

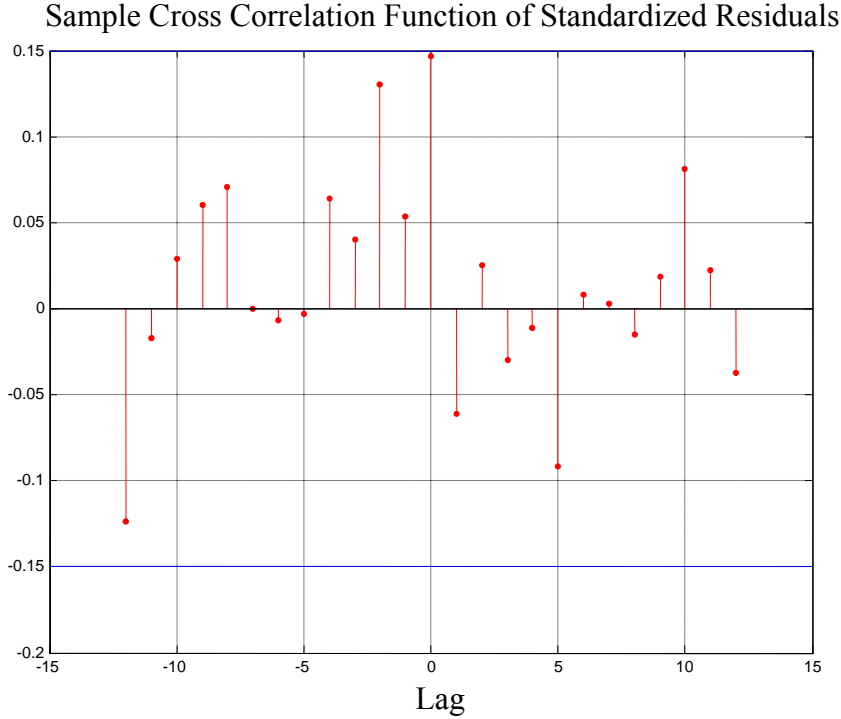
After choosing the univariate models, the Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification is used. The innovations of the GARCH models are tested, in order to check whether the model fit is adequate. The null hypothesis should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,3629	0,1161
Q-statistic	21,5951	27,7241
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

The causality in mean should be now tested. Therefore, we plot the Sample Cross Correlation Function of the standardized residuals of the series. The blue line represents the confidence interval of 95 %:

The relationship between volatility of asset prices and volatility of output growth



We may assume that no correlation exists between the standardized residuals and no causality in mean is present. The hypothesis testing using the following t-statistic confirms the above conclusion.

$$t = \sqrt{T} \hat{r}_{uv}^{\wedge}(k) \rightarrow AN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations). We apply this test on the standardized residuals. The table contains the t-statistic for each lag.

Lags	t-statistic for st. residuals
-12	-1,6504
-11	-0,2336
-10	0,383
-9	0,8025
-8	0,9426
-7	0,0025
-6	-0,0951
-5	-0,0385
-4	0,8529
-3	0,5385
-2	1,7439
-1	0,718

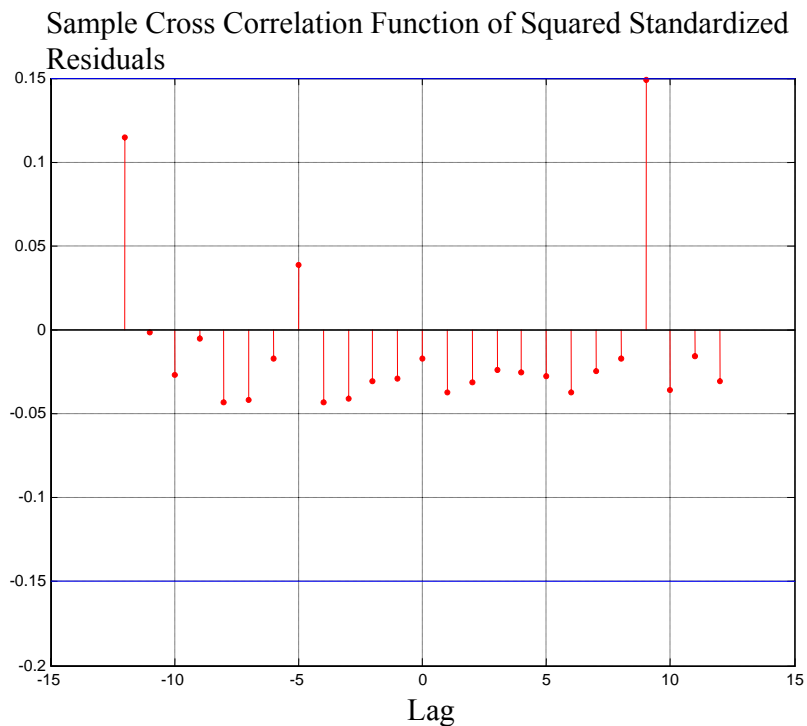
The relationship between volatility of asset prices and volatility of output growth

0	1,9597
1	-0,8128
2	0,3421
3	-0,3997
4	-0,1495
5	-1,2275
6	0,1081
7	0,0368
8	-0,1996
9	0,2503
10	1,0817
11	0,2955
12	-0,4951

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level

We proceed to the causality in variance test by plotting the Sample Cross Correlation function of the squared standardized residuals.



The relationship between volatility of asset prices and volatility of output growth

The diagram shows that there is no causality in variance in any lag. We proceed to the hypothesis testing using the same t-statistic as before. The null hypothesis of no causality should be accepted in every lag. The table contains the t-statistics for every lag:

Lags	t-statistic for squared st. residuals
-12	1,5299
-11	-0,0202
-10	-0,3599
-9	-0,0689
-8	-0,5811
-7	-0,5586
-6	-0,2337
-5	0,5198
-4	-0,5774
-3	-0,548
-2	-0,4063
-1	-0,391
0	-0,2311
1	-0,4963
2	-0,421
3	-0,3197
4	-0,3371
5	-0,37
6	-0,495
7	-0,3344
8	-0,2293
9	1,9523
10	-0,4821
11	-0,2086
12	-0,405

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

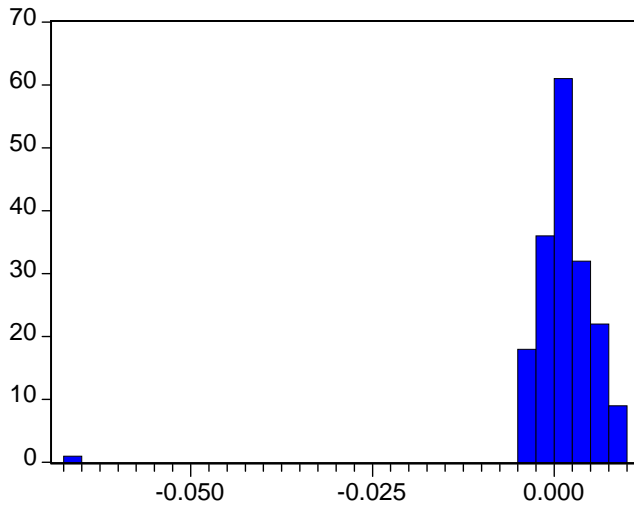
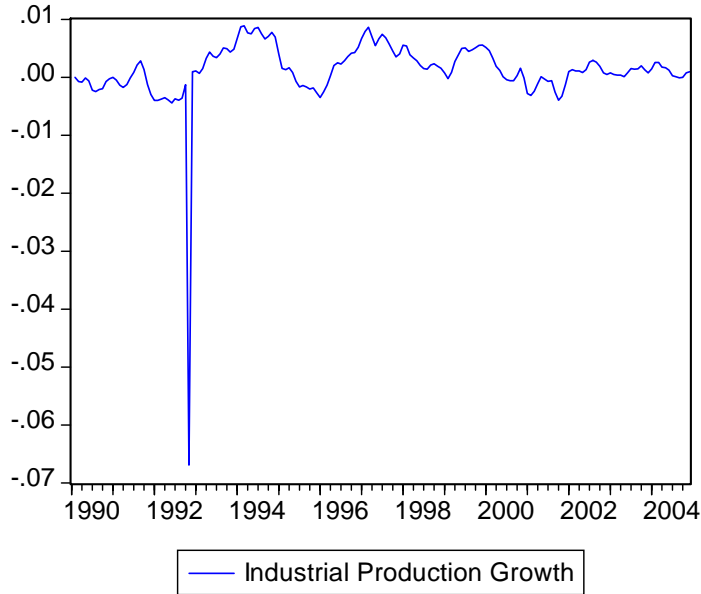
'' indicates significance at the 1% level, '**' indicates significance at the 5% level*

G) Spain

In the first part of the analysis, we present some preliminary statistical data about the Spanish Industrial Production Growth and the Spanish Stock Returns. The diagrams

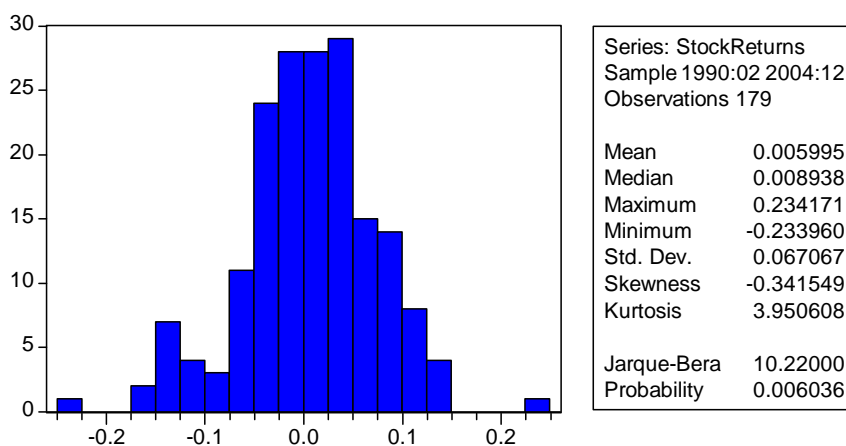
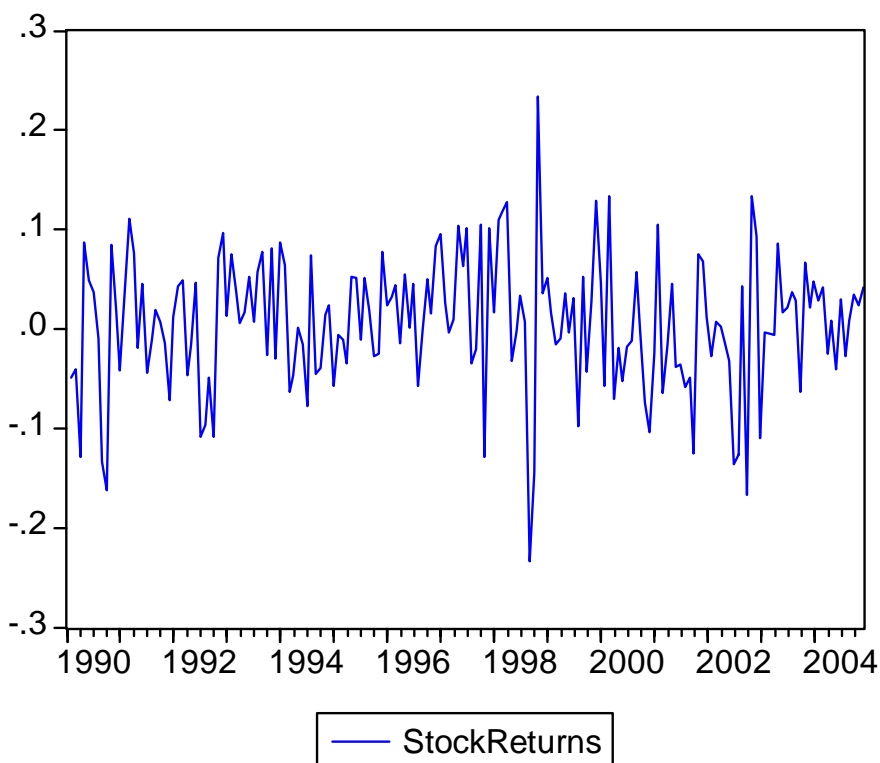
The relationship between volatility of asset prices and volatility of output growth

plot the evolution of the two series during the past fifteen year and the tables contains some descriptive statistics for both time series:



Series: Industrial Production Growth	
Sample 1990:02 2004:12	
Observations 179	
Mean	0.001252
Median	0.001309
Maximum	0.008884
Minimum	-0.066890
Std. Dev.	0.006019
Skewness	-8.077442
Kurtosis	93.04130
Jarque-Bera	62414.43
Probability	0.000000

The relationship between volatility of asset prices and volatility of output growth

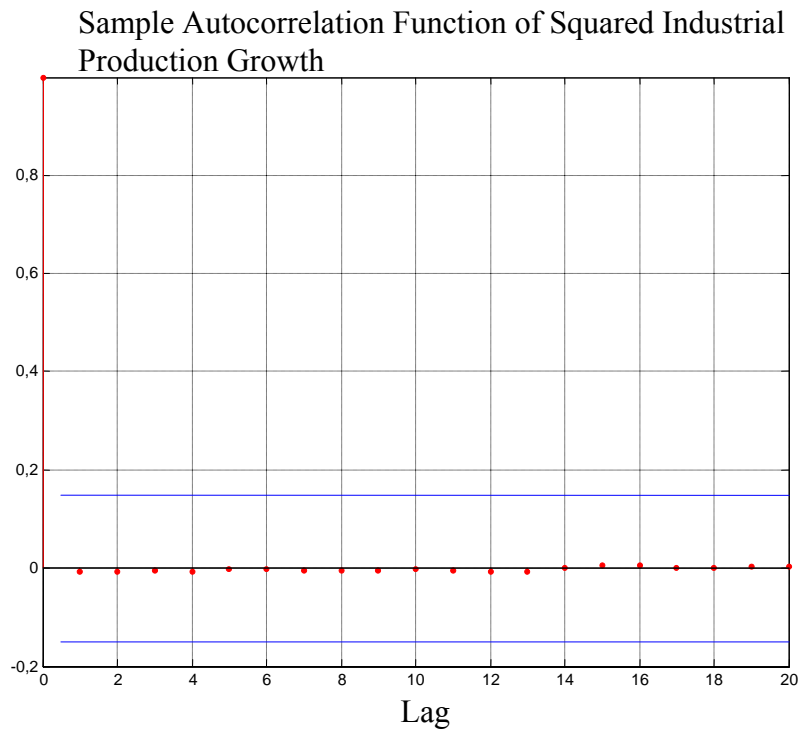
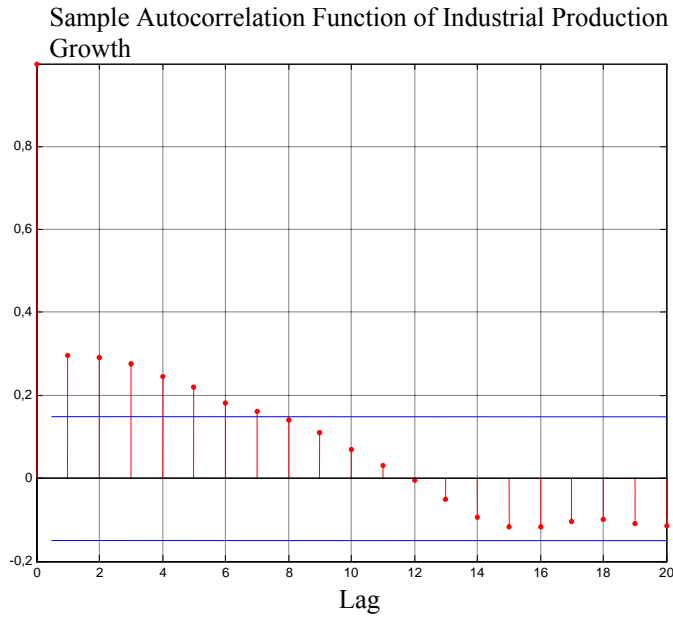


In November 1992 there was a huge crash in Spanish Industrial Production, resulting in a 6% monthly reduction. That was the minimum of the time series of industrial production growth. Some important characteristics of the time series are that both of them exhibit excess kurtosis and have negative skewness coefficients. So their distributions are skewed to the left and leptokurtic (fat tails). The normality assumption should be rejected for both of them according to the Jarque-Bera test.

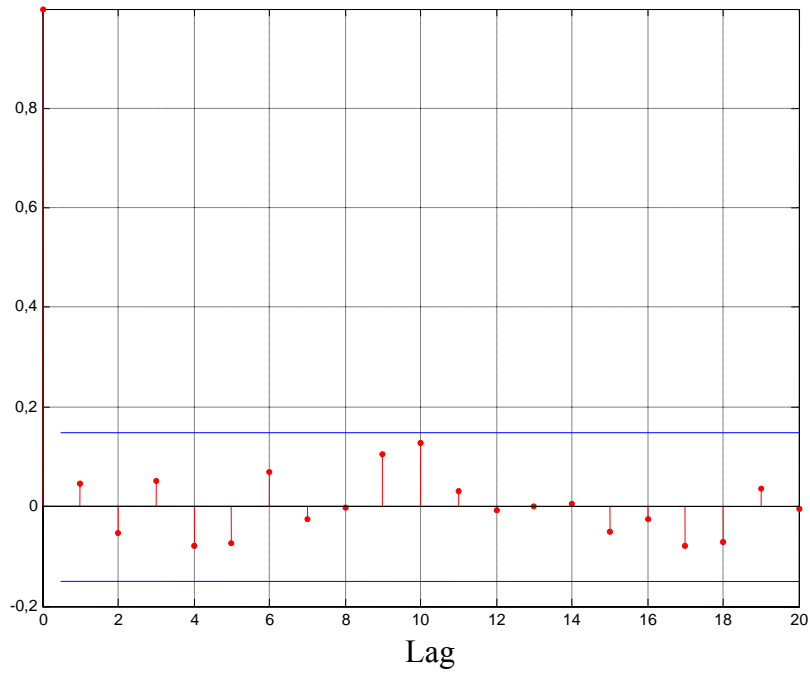
As far as autocorrelation is concerned, the diagrams below depict the autocorrelation in every lag for the return series and the squared returns as well. It is

The relationship between volatility of asset prices and volatility of output growth

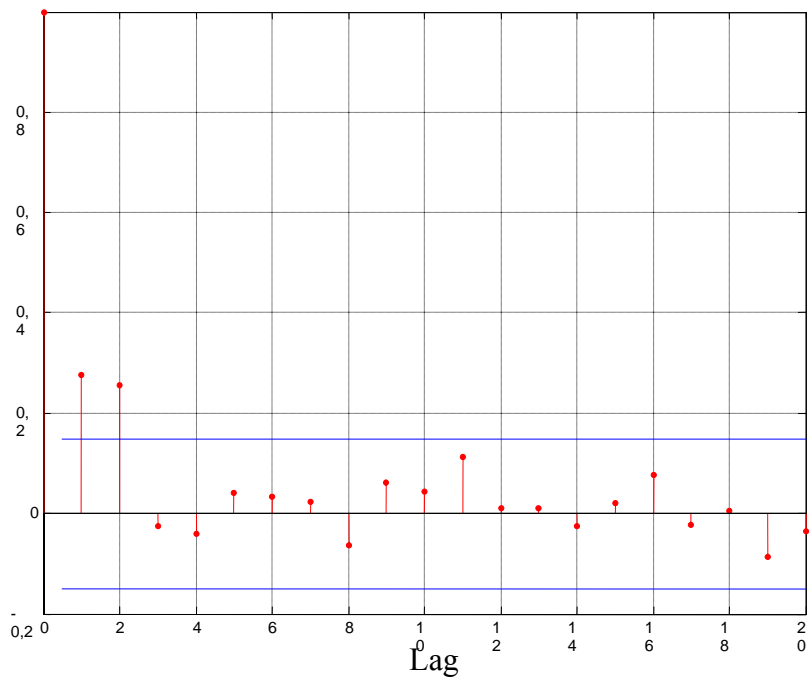
important to check the autocorrelation in the squared returns because sometimes even if the Autocorrelation Function of the observed returns exhibits little correlation, the autocorrelation function of the squared returns may still indicate significant correlation and persistence in the second-order moments. The blue line represents the bounds, which are computed with approximate 95% confidence level.



Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



The relationship between volatility of asset prices and volatility of output growth

The autocorrelation is statistically significant for both Industrial Production Growth and squared returns. The Industrial Production Growth series exhibits autocorrelation till the 7th lag and the squared returns series in the first lag. The stock returns series has no autocorrelation but the squared returns series suffers from autocorrelation till the 2nd lag.

In addition to the diagrams, we perform the Engle's ARCH test to check whether there is any ARCH/GARCH effect on our time series. The null hypothesis that a time series of sample residuals is i.i.d. Gaussian disturbances (i.e., no ARCH effects exist) can be accepted for the Industrial Production Growth but should be rejected for the Stock Returns. We have imposed one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series. The table contains the details for the test:

	Industrial Production Growth	Stock Returns
pValue	0,9214	0,000134
t-Statistic	0,0097	14,6387
Critical Value	3,8415	3,8415

**H = 0 indicate acceptance of the null hypothesis that no ARCH effects exist*

1st Approach

Mean Equation (VAR (3))

At this point we should choose the appropriate mean equation. The model that outperforms in all criteria and tests is the Vector Autoregressive model with three lags (VAR (3)). It manages to remove the autocorrelation from the residuals. The parameters estimates of the model are (standard errors in “()”):

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + b_3 X_{t-3} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$A = \begin{bmatrix} 0.000427 \\ (0.00043) \\ 0.007228 \\ (0.00532) \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0.1582 & 0.0166 \\ (0.07519) & (0.00617) \\ -0.3331 & 0.0433 \\ (0.92565) & (0.07593) \end{bmatrix}$$

The relationship between volatility of asset prices and volatility of output growth

$$b_2 = \begin{bmatrix} 0.1752 & 0.0086 \\ (0.07449) & (0.00628) \\ 0.6476 & -0.0624 \\ (0.91701) & (0.07729) \end{bmatrix}, \quad b_3 = \begin{bmatrix} 0.1703 & 0.0110 \\ (0.07375) & (0.00629) \\ -0.3757 & 0.0512 \\ (0.90786) & (0.07741) \end{bmatrix}$$

After having estimated the VAR (3) model we perform the autocorrelation LM test on the residuals. The autocorrelation has been removed up to the 10th lag.

Lags	LM-Statistic	P-Value
1	8.071794	0.0890
2	4.832431	0.3049
3	6.409572	0.1706
4	3.970393	0.4100
5	1.962505	0.7427
6	0.907225	0.9235
7	2.351051	0.6715
8	0.340981	0.9870
9	1.770188	0.7779
10	4.277953	0.3697

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

The relationship between Industrial Production Growth and Stock Returns is being investigated in this part of the country analysis. We try to check whether there is causality in mean in any direction. The results are presented in the following two tables:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	12,15085	0,0069

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	0,658137	0,8830

There is statistically significant causality from Stock Returns to Industrial Production Growth. In other words, Stock Returns Granger causes Industrial Production Growth.

Variance Equation (BEKK 1, 1)

According to the descriptive statistics of the two time series, we should reject the Normality Assumption for both of them (the Jarque-Bera test reject the normality for both series). But we can assume they follow a t-student distribution. So in this context we estimated the parameters of the BEKK (1, 1) model using the maximum likelihood estimation method. The parameters estimates are: (the standard errors in “()”)

Parameters	Unrestricted GARCH(1,1)
c₁₁	0,0000 (0)
c₁₂	0,0009 (0)
c₂₁	0
c₂₂	0,0262 (0,0004)
GARCH g₁₁	1,0250 (0,0261)
g₁₂	2,2592 (3,5917)
g₂₁	-0,0188 (0)
g₂₂	-0,2278 (0,0348)
ARCH a₁₁	0,2958 (0,0567)
a₁₂	2,9018 (6,0200)
a₂₁	0,0032 (0,0001)
a₂₂	-0,9107 (0,0201)

After estimating the parameters of the model, we will set some restrictions to test the causality in variance. Three different restricted models will be estimated and compared with the unrestricted model using the LRatio test. The results are presented in the following table:

	Unrestricted	Restricted1 ($a_{21}=g_{21}=0$)	Restricted2 ($a_{12}=g_{12}=0$)	Restricted3 ($a_{12}=g_{12}=a_{21}=g_{21}=0$)
Loglikelihood	1074,4	1047,6	1073,2	1047,3
pValue		0	0,2937	0
LRatio		53,4761	2,4504	54,2324
Critical Value		5,9915	5,9915	9,4877

* $H = 0$ indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; $H = 1$ indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.

The first line of the table contains the value of the Loglikelihood Function for each model estimated. The unrestricted model has the greatest value of all.

According to the LRatio test, the unrestricted model outperforms the Restricted1 model and the Restricted3 model. In other words, the hypothesis that there is causality in variance from stock returns to output growth should be accepted. On the other hand, the null hypothesis of acceptance the Restricted2 model should be also accepted. So there is no causality from output growth volatility to stock returns volatility. To sum up, Spain's stock returns volatility Granger causes output growth volatility but there is no statistically significant causality from output growth to stock returns volatility.

At the end of this section, we perform the Ljung-Box lack-of-fit hypothesis test for model misspecification to check whether the unrestricted bivariate GARCH (1, 1) model is sufficient for our data. The lags used in the Q-Statistic were twenty and the significance level 5%. The residuals of the GARCH model have been tested and the results are in the table below:

	Standardized Residuals1	Standardized Residuals2
pValue	0,9396	0,0428
Q-Statistic	11,2445	32,0490
Critical Value	31,4104	31,4104

* H_0 : the null hypothesis that the model fit is adequate (no serial correlation).

2nd Approach

At first, we should model the time series of Industrial Production Growth and Stock Returns. Two univariate models should be constructed according to the special features of the two series. The mean equation will be an autoregressive moving average process ARMA (R, M), whereas the conditional variance equation as a classical GARCH (p, q) model.

The relationship between volatility of asset prices and volatility of output growth

The mean equation for the Industrial Production Growth is an AR (3) model, while the variance equation an ARCH (1).

We estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The AR (3), GARCH (0, 1) has the greatest LLF value.

LLF	ARMA(0,0) GARCH(1,1)	AR(0,0) GARCH(0,1)	AR (3) GARCH(1,1)	AR (3) GARCH(0,1)
Industrial Production Growth	706,7654	661,7190	806,6669	806,6672

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

There is no statistically significant autocorrelation in the Stock Returns series neither in the squared returns in any lag. So the innovations of the model should be the series itself. The significance of each parameter is calculated by EViews, along with the estimation of the parameters.

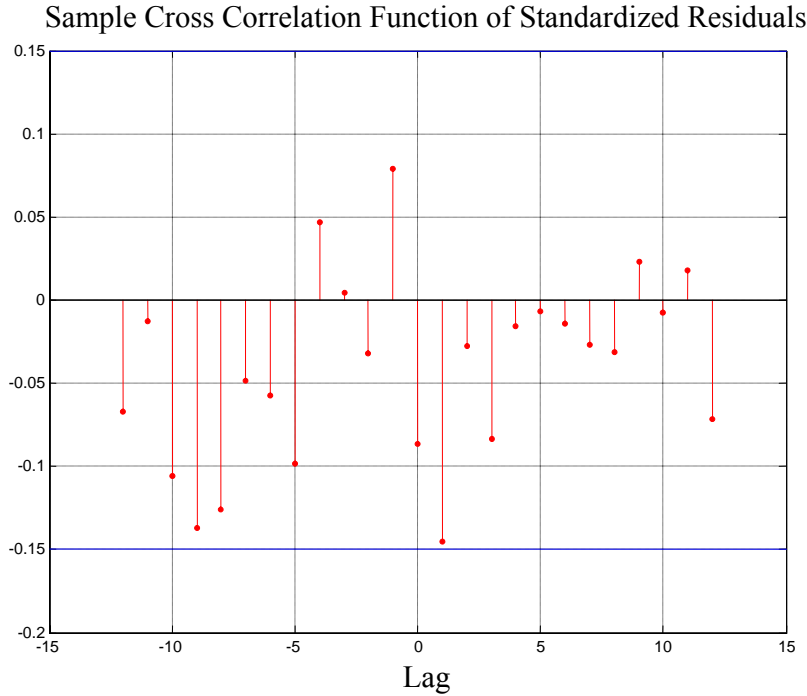
The Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification is performed on the innovations of the two models in order to check whether the model fit is adequate. The null hypothesis should be accepted in Stock Returns and rejected for Industrial Production.

	Industrial Production Growth	Stock Returns
P-Value	0	0,8745
Q-statistic	83,21	13,0670
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

We will now plot the Sample Cross Correlation Function of standardized residuals in order to check for causality in mean. The blue line represents the confidence interval of 95 %:

The relationship between volatility of asset prices and volatility of output growth



The null hypothesis of no causality in mean should be accepted for every lag, according to the diagram. The hypothesis testing using the following t-statistic is then performed.

$$t = \sqrt{T} \hat{r}_{uv}(k) \rightarrow AN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

We apply this test on the standardized residuals. The table contains the t-statistics for each lag.

Lags	t-statistic for st. residuals
-12	-0,8942
-11	-0,1688
-10	-1,413
-9	-1,835***
-8	-1,6879***
-7	-0,6524
-6	-0,7679
-5	-1,3193
-4	0,6233
-3	0,0609
-2	-0,4275
-1	1,0512
0	-1,1562

The relationship between volatility of asset prices and volatility of output growth

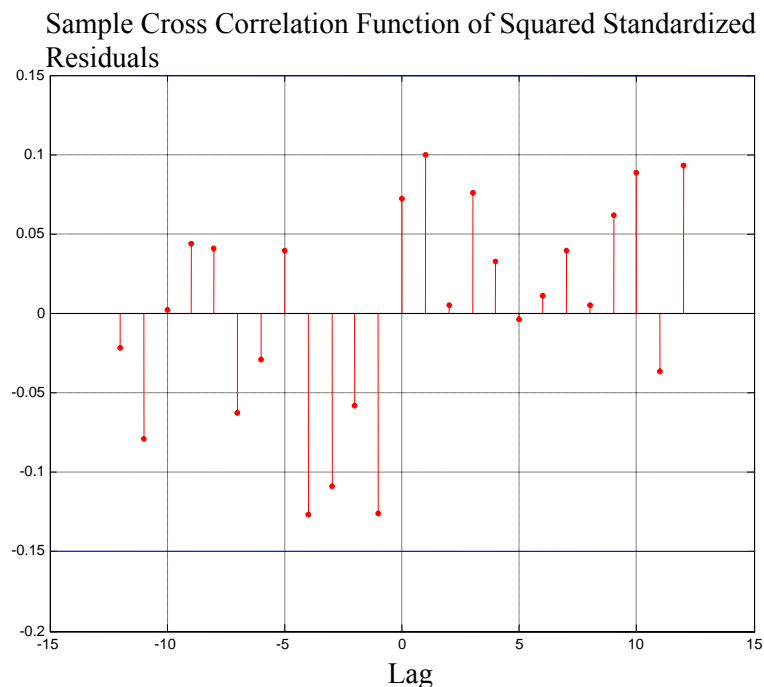
1	-1,5414
2	-0,3726
3	-1,119
4	-0,2087
5	-0,0932
6	-0,187
7	-0,3634
8	-0,4177
9	0,3117
10	-0,1014
11	0,2425
12	-0,9593

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level '***' indicates significance at the 10% level

The null hypothesis should be rejected in lags (-9,-8). There is causality in mean from Stock Returns to Industrial Production Growth. The $StockReturns_{t-9}$ and $StockReturns_{t-8}$ Granger causes $IndustrialProductionGrowth_t$

The diagram of the Sample Cross Correlation Function of the squared standardized residuals documents the causality in variance patterns. The blue line represents the confidence interval of 95 %:



The relationship between volatility of asset prices and volatility of output growth

We perform the hypothesis testing using the same t-statistic but on the squared standardized residuals of the two time series. The table contains the t-statistics for each lag.

Lags	t-statistic for squared st. residuals
-12	-0,2887
-11	-1,0569
-10	0,0328
-9	0,5914
-8	0,5468
-7	-0,8335
-6	-0,3897
-5	0,5284
-4	-1,6901***
-3	-1,4573
-2	-0,7742
-1	-1,6873***
0	0,9632
1	1,3375
2	0,0695
3	1,0189
4	0,4366
5	-0,0473
6	0,1462
7	0,5279
8	0,0663
9	0,8258
10	1,184
11	-0,487
12	1,2436

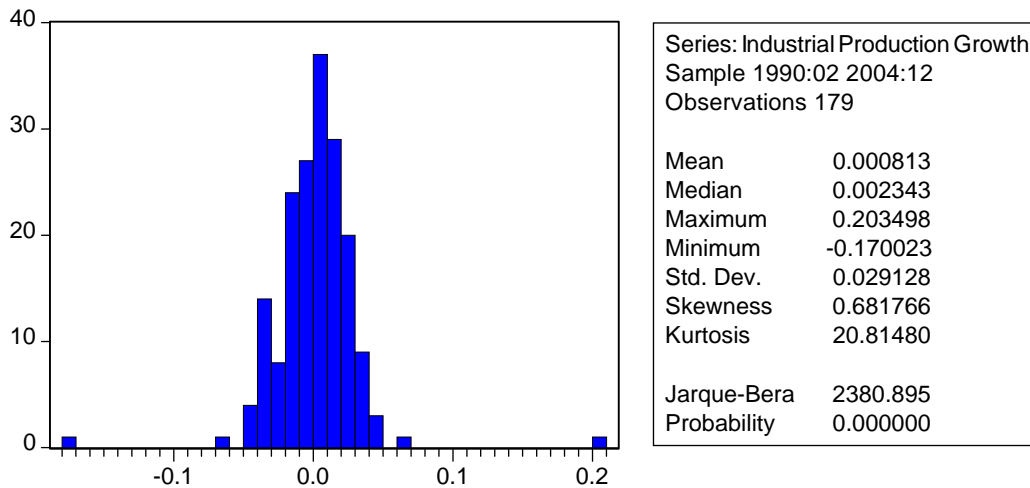
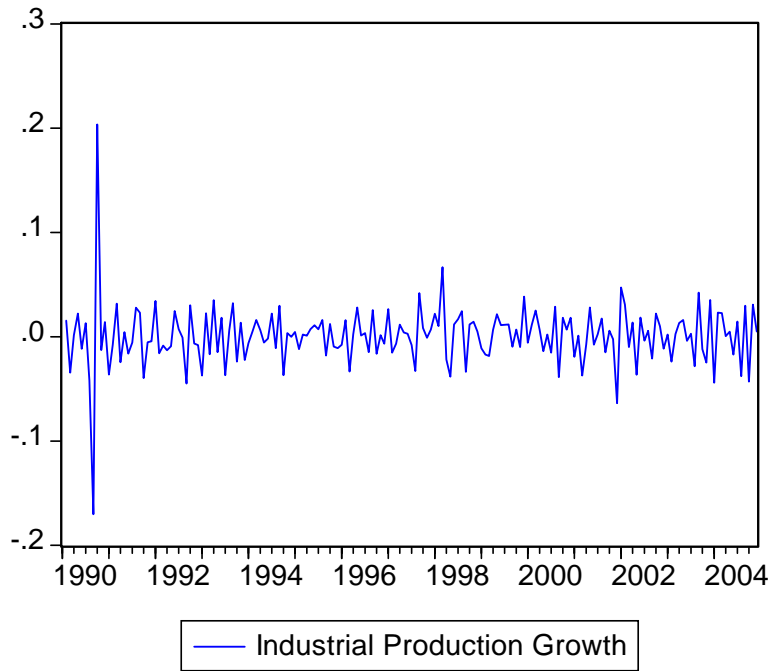
H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level, '***' indicates significance at the 10% level

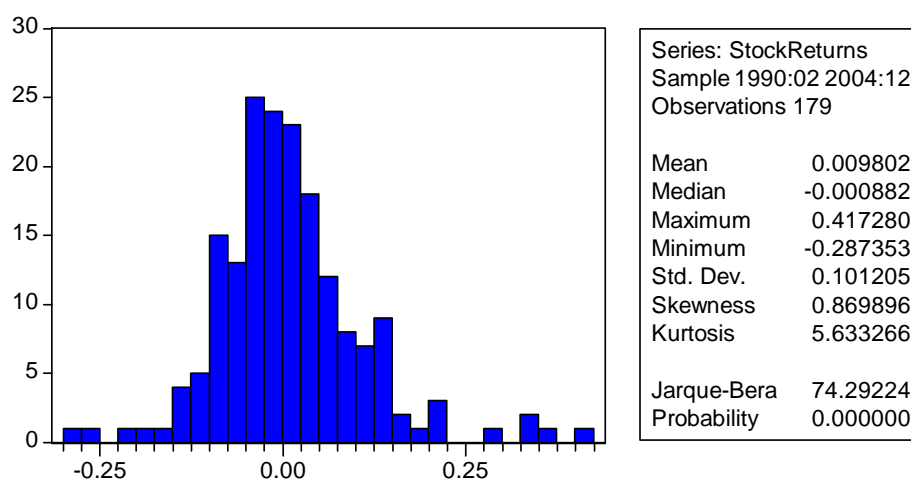
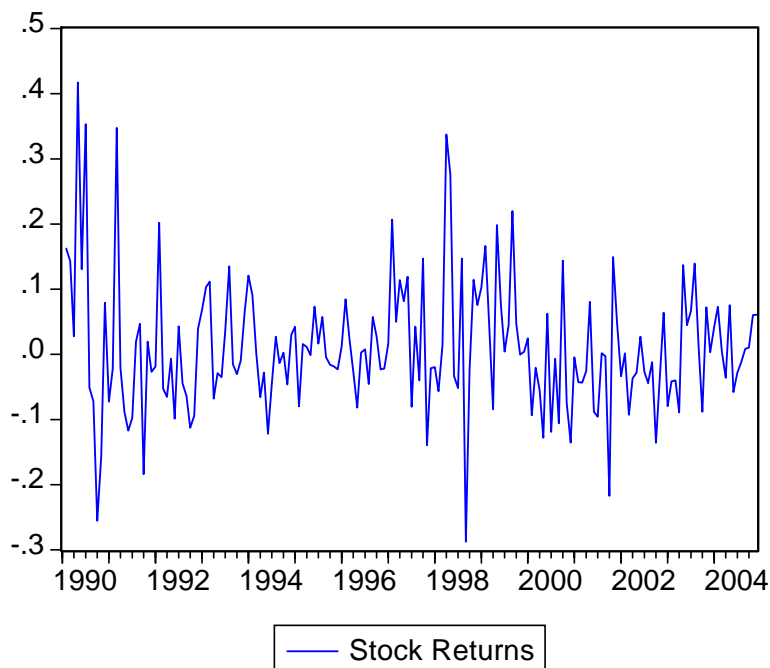
The hypothesis testing reveals causality in variance from Stock Returns to Industrial Production Growth. The null hypothesis of no causality should be rejected with 10% significance level in lag (-1) and lag (-4). The volatility of $StockReturns_{t-1}$ and $StockReturns_{t-4}$ Granger causes the volatility of $IndustrialProductionGrowth_t$. The volatility of today's $IndustrialProductionGrowth$ is influenced by the volatility of $StockReturns$ one and four months ago.

H) Greece

We will continue the country analysis with Greece. The data comes in monthly terms from January 1990 till December 2004. Some preliminary statistics for the time series and the diagrams of the evolution of the series in the last fifteen years are presented in the tables below:



The relationship between volatility of asset prices and volatility of output growth

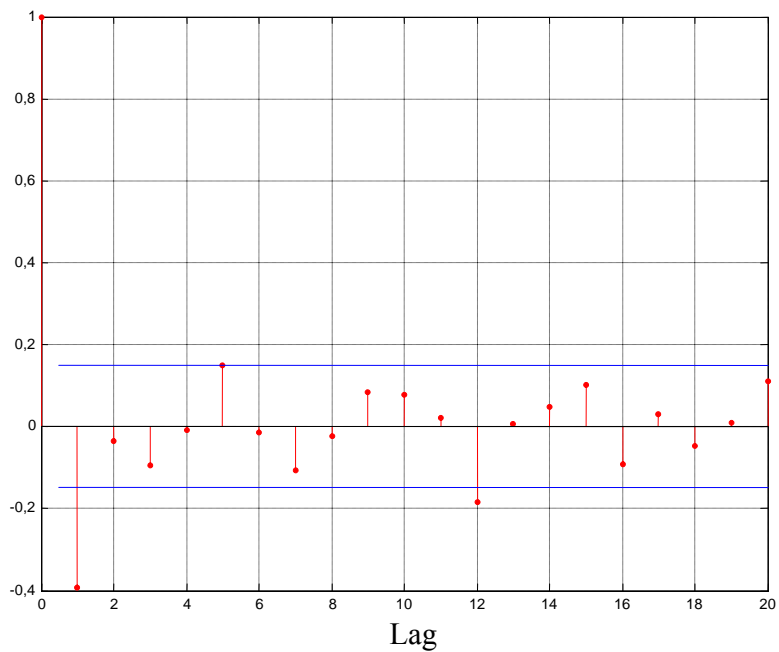


Industrial Production Growth and Stock Returns exhibit excess kurtosis (fat tails) and positive skewness coefficient (i.e., skewed to the right). The Jarque-Bera test suggests for both time series the rejection of the hypothesis that they follow a Normal Distribution.

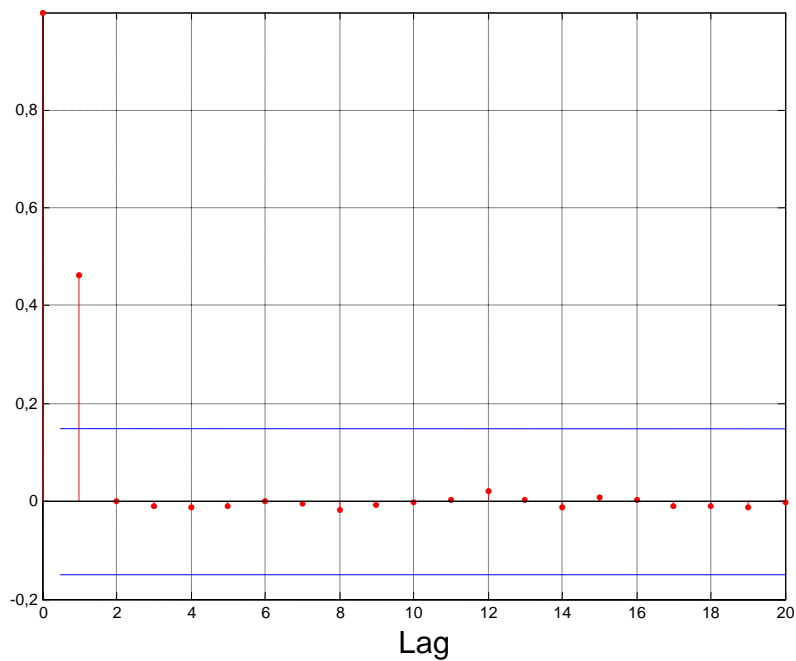
The autocorrelation of the two series is examined with two ways. At the beginning, we plot the autocorrelation diagrams for the series and the squared series. MERTON (1980) argues that one of the simplest ways to approximate the instantaneous volatility is to take the squared or absolute value of returns. This is one of the reasons why we also study the correlations of squared returns in order to detect if there is some non-linear (more precisely quadratic) dependence in returns and more specifically to check if there are some patterns in conditional volatility. The blue line represents the bounds, which are computed with approximate 95% confidence level.

The relationship between volatility of asset prices and volatility of output growth

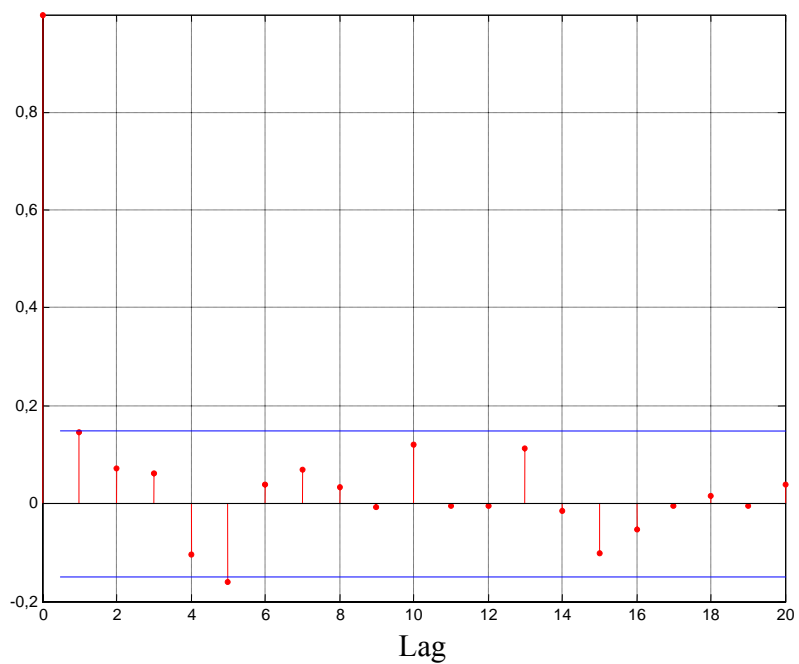
Sample Autocorrelation Function of Industrial Production Growth



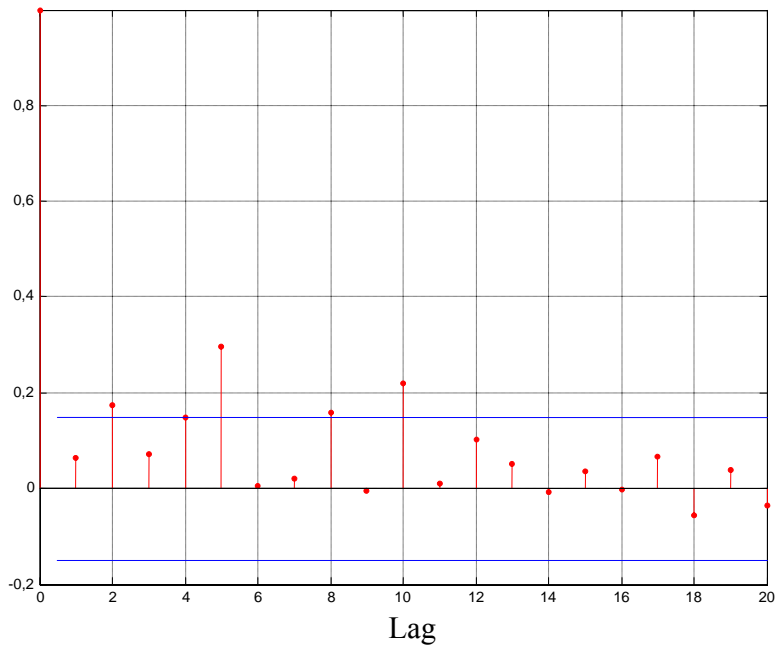
Sample Autocorrelation Function Of Squared Industrial Production Growth



Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



The relationship between volatility of asset prices and volatility of output growth

The results show that there is strong linear dependence in the first lag for Industrial Production Growth. In the squared returns the autocorrelation exceeds the bounds for the first lag as well. As far as the Stock Returns series is concerned, there is statistically significant autocorrelation in both returns and squared returns series in various lags.

We implement the ARCH test in order to check for the presence of ARCH/GARCH effects on the two series. We impose one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series. The results indicate persistent of ARCH effect in Industrial Production Growth series only.

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0	0,3874
t-Statistic	38,0064	0,7472
Critical Value	3,8415	3,8415

**H = 0 indicate acceptance of the null hypothesis that no ARCH effects*

1st Approach

Mean Equation (VAR (5))

The model which we chose for the mean equation is the Vector Autoregressive with five lags. The main criterion was its ability to remove all the serial correlation from the residuals. We also used the Final Prediction Error (FPE), Akaike information criterion, Schwarz information criterion and Hannan-Quinn information criterion. The parameters estimates are (the standard errors in “()”):

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + \dots + b_5 X_{t-5} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$A = \begin{bmatrix} 0.002194 \\ (0.00191) \\ 0.002792 \\ (0.00706) \end{bmatrix} \quad b_1 = \begin{bmatrix} -0.584883 & -0.020724 \\ (0.07723) & (0.02006) \\ 0.691599 & 0.080279 \\ (0.28494) & (0.07400) \end{bmatrix}$$

The relationship between volatility of asset prices and volatility of output growth

$$b_2 = \begin{bmatrix} -0.414905 & -0.037148 \\ (0.08892) & (0.01918) \\ 0.882043 & 0.060949 \\ (0.32809) & (0.07078) \end{bmatrix} \quad b_3 = \begin{bmatrix} -0.335638 & 0.007775 \\ (0.09118) & (0.01932) \\ 0.422255 & 0.053758 \\ (0.33642) & (0.07129) \end{bmatrix}$$

$$b_4 = \begin{bmatrix} -0.179212 & -0.026117 \\ (0.08871) & (0.01928) \\ 0.602310 & -0.112677 \\ (0.32731) & (0.07113) \end{bmatrix} \quad , \quad b_5 = \begin{bmatrix} 0.033483 & 0.042453 \\ (0.07691) & (0.01932) \\ 1.082122 & -0.117281 \\ (0.28376) & (0.07127) \end{bmatrix}$$

The autocorrelation LM test for the residuals of the VAR (5) confirms the autocorrelation has been removed from the residuals. The null hypothesis of no serial correlation should be accepted up to the second lag. The table contains the results:

Lags	LM-Statistic	PValue
1	5,031317	0,2841
2	3,513194	0,4759

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

Before the causality in variance test, we will perform a test for causality in mean. The pairwise Granger causality test is used. The results of the test are:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	12,16801	0,0326

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	19,62380	0,0015

In both cases the null hypothesis that the independent variable is statistically insignificant should be rejected. Thus, there is causality in mean in both directions. Industrial Production Growth Granger causes Stock Returns and vice versa.

Variance Equation (BEKK 1, 1)

The variance equation selected for our data is a B.E.K.K. (1, 1) model, with t-student errors, since the Normality assumption should be rejected for both time series. Using the maximum likelihood estimation method, we estimated the parameters of the unrestricted B.E.K.K. model: (standard errors in “()”):

Parameters (std. errors)	Unrestricted GARCH(1,1)
c₁₁	0,0000 (0,000)
c₁₂	0,0003 (0,000)
c₂₁	0
c₂₂	0,0737 (0,0013)
GARCH g₁₁	0,2298 (0,0706)
g₁₂	-0,4523 (1,0524)
g₂₁	-0,0936 (0,0012)
g₂₂	-0,2021 (0,0545)
ARCH a₁₁	0,7817 (0,0381)
a₁₂	-1,0770 (3,70220)
a₂₁	0,1026 (0,0050)
a₂₂	0,4269 (0,3117)

We will now estimate three restricted models, by setting some restrictions on the unrestricted models. The Restricted1 model will help us clear out the causality from Stock Returns volatility to Output Growth volatility. The Restricted2 model tests the causality from the opposite direction and the third the simultaneous causality in both directions. We will compare each restricted model with the unrestricted one. The choice will be made according to the LRatio test. We now present the results of the test:

	Unrestr icted	Restricted1 ($a_{21}=g_{21}=0$)	Restricted2 ($a_{12}=g_{12}=0$)	Restricted3 ($a_{12}=g_{12}=a_{21}=g_{21}=0$)
Loglikelihood	607,6657	304,8142	591,9625	312,3833
pValue		0	0	0
LRatio		605,7029	31,4063	590,5648
Critical Value		5,9915	5,9915	9,4877

* $H = 0$ indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; $H = 1$ indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.

At first, we evaluated the Loglikelihood Function for each model. The unrestricted model had the greatest value.

The LRatio tests indicate rejection of the null hypothesis (i.e. of the restricted model) for all cases. The unrestricted model outperforms all the restricted ones. So there is statistically significant causality in both directions. In other words, the volatility of the Greek output growth Granger causes the Greek stock returns volatility and vice versa.

At the end of the section, post estimation analysis has been conducted, with the use of the Ljung-Box test. This is a lack-of-fit hypothesis test for model misspecification. The lags used in the Q-Statistic were twenty and the significance level 5%. We checked the GARCH residuals to test the null hypothesis that the model fit is adequate. The null hypothesis should be accepted in both cases.

	Standardized Residuals1	Standardized Residuals2
pValue	0,4006	0,2084
Q-Statistic	20,9413	24,8198
Critical Value	31,4104	31,4104

* H_0 : the null hypothesis that the model fit is adequate (no serial correlation).

2nd Approach

The causality in variance between Industrial Production Growth and Stock Returns will be investigated in this part of the country analysis according to the methodology proposed by Chueng and Ng in 1996. The two time series have to be modeled in two separate univariate models. We choose for the mean equation an autoregressive moving average process and for the variance equation a generalized ARCH process. The two models should be well specified. The lag structure of each model depends on the special features of the time series.

The relationship between volatility of asset prices and volatility of output growth

For the Greek Industrial Production Growth series, an autoregressive moving average process with four lags was selected to account for the serial correlation that the series exhibits up to the fourth lag. For the variance equation, we chose a GARCH (3, 1). The GARCH parameters were statistically significant up to the third lag, while only the first ARCH parameter was different from zero (statistically significant).

We also estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (4, 4), GARCH (3, 1) has the greatest LLF value.

LLF	ARMA(0,0) GARCH(3,1)	ARMA(4,4) GARCH(0,1)	AR (4) GARCH(3,1)	ARMA(4,4) GARCH(3,1)
Industrial Production Growth	414,2916	440,7472	427,6997	441,4201

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The Stock Returns series was modeled as an ARMA (1, 1) process with variance equation a simple GARCH (1, 1).

We estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (1, 1), GARCH (1, 1) has the greatest LLF value.

LLF	AR(1) GARCH(1,1)	AR (2) GARCH(1,1)	ARMA(1,1) GARCH(0,1)	ARMA(1,1) GARCH(1,1)
Stock Returns	171,0358	170,9743	162,7072	171,2085

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

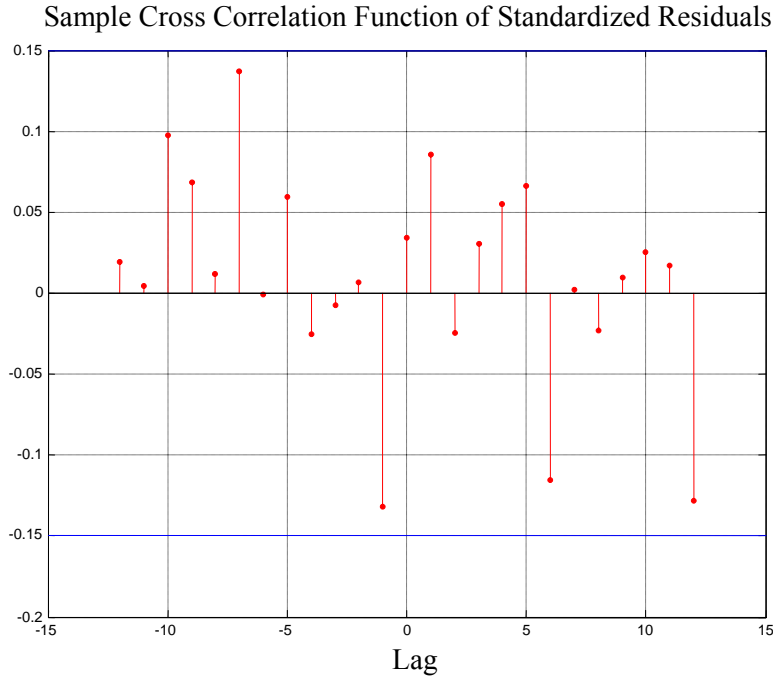
The Ljung-Box Q-statistic lack-of-fit hypothesis test for model misspecification is performed on the residuals of each univariate model. The lags used in the Q-Statistic were twenty and the significance level 5%. The null hypothesis that the model is well specified should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,1267	0,5703
Q-statistic	27,3111	28,2605
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

The relationship between volatility of asset prices and volatility of output growth

The causality in mean is tested using the Sample Cross Correlation Function of standardized residuals of the two time series. The diagram of the function is presented above. The blue line represents the confidence interval of 95 %:



According to the diagram, there is no causality in mean between the two time series. The null hypothesis of no autocorrelation should be accepted for every lag. The t-statistics is:

$$t = \sqrt{T} \hat{r}_{uv}(k) \rightarrow AN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

Where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

We apply this test on the standardized residuals. The table contains the t-statistics for each lag. The null hypothesis is that there is no causality, the standardized residuals are uncorrelated.

Lags	t-statistic for st. residuals
-12	0,2583
-11	0,0589
-10	1,2999
-9	0,9146
-8	0,1614
-7	1,832***

The relationship between volatility of asset prices and volatility of output growth

-6	-0,0142
-5	0,7928
-4	-0,3425
-3	-0,1025
-2	0,0845
-1	-1,763***
0	0,4593
1	1,1418
2	-0,3333
3	0,4108
4	0,7368
5	0,8906
6	-1,5467
7	0,0324
8	-0,3133
9	0,1271
10	0,3373
11	0,2318
12	-1,7125***

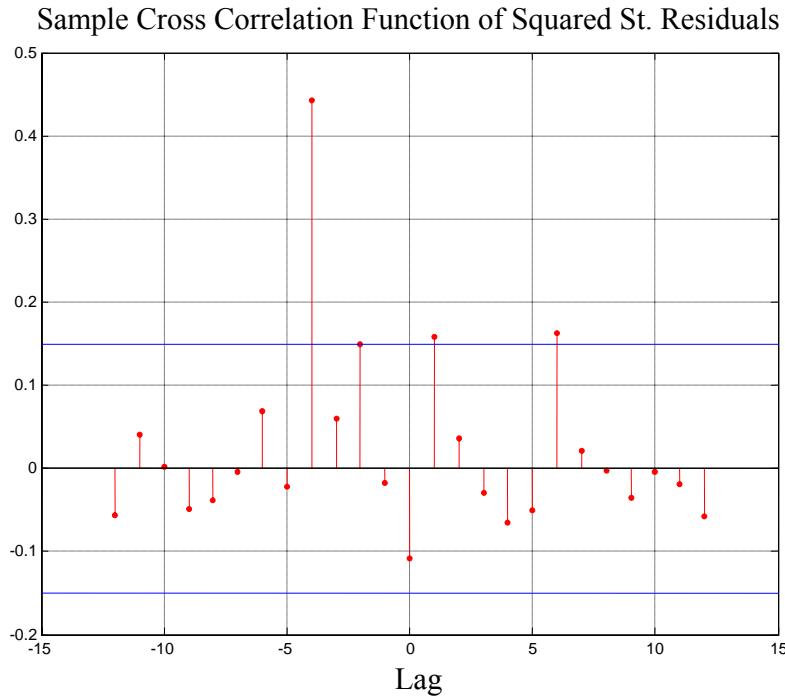
H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level, '***' indicates significance at the 10% level

There is causality in mean in both directions. $StockReturns_{t-1}$ and $StockReturns_{t-7}$ Granger causes $IndustrialProductionGrowth_t$ while $IndustrialProductionGrowth_{t-12}$ Granger causes $StockReturns_t$.

The diagram of the Sample Cross Correlation Function of the squared standardized residuals examines the causality in variance of the two time series. The blue line represents the confidence interval of 95 %:

The relationship between volatility of asset prices and volatility of output growth



The hypothesis testing is performed using the t-statistic mentioned before on the squared standardized residuals. The table contains the results:

Lags	t-statistic for squared st. residuals
-12	-0,7525
-11	0,5468
-10	0,0238
-9	-0,6445
-8	-0,5066
-7	-0,057
-6	0,9216
-5	-0,2847
-4	5,9135*
-3	0,8062
-2	2,0034**
-1	-0,236
0	-1,4485
1	2,113**
2	0,4754
3	-0,3875
4	-0,8816
5	-0,681

The relationship between volatility of asset prices and volatility of output growth

6	2,1701**
7	0,2882
8	-0,0424
9	-0,469
10	-0,0636
11	-0,2586
12	-0,7746

H₀: No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

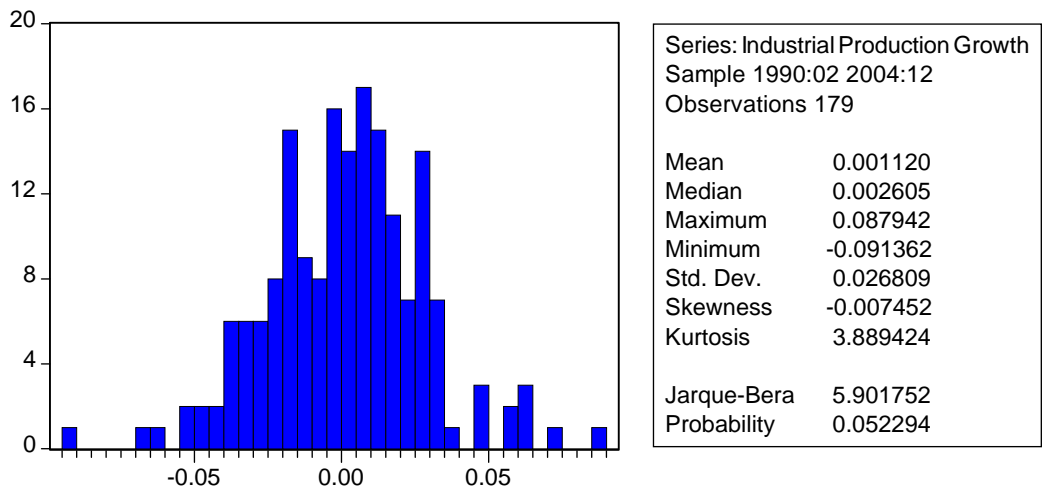
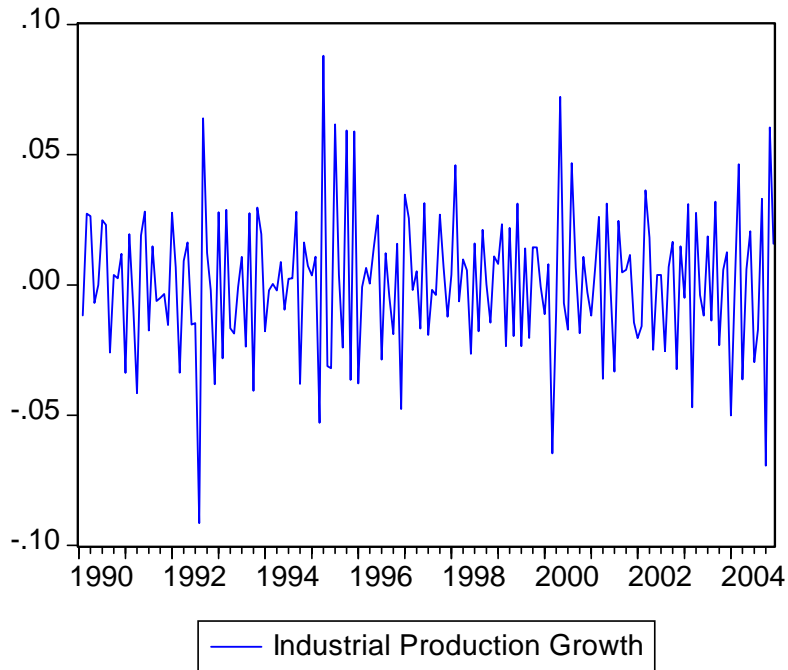
‘’ indicates significance at the 1% level, ‘**’ indicates significance at the 5% level*

The null hypothesis of no causality- the squared standardized residuals of the two series are uncorrelated- should be rejected in 4 lags (-4, -2, +1, +6). There are volatility spillovers between Industrial Production Growth and Stock returns, since there is statistically significant causality in variance in both directions. The volatility of $StockReturns_{t-2}$ and $StockReturns_{t-4}$ Granger causes the volatility of $IndustrialProductionGrowth_t$. On the other hand, $IndustrialProductionGrowth_{t-1}$ and $IndustrialProductionGrowth_{t-6}$ volatility Granger causes the volatility of $StockReturns_t$. To put it another way, the volatility of Stock Returns today is influenced by the volatility of Industrial production Growth one and six months ago, while the volatility of Industrial Production Growth today is influence by the Stock Returns volatility two and four months ago.

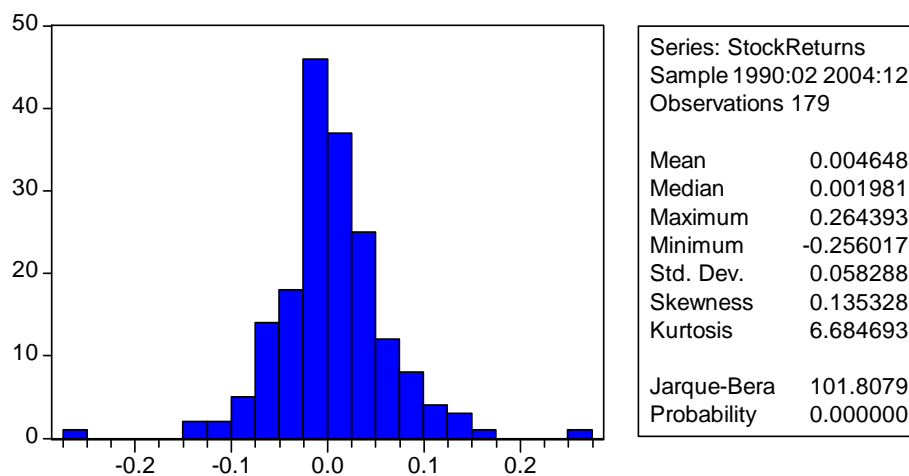
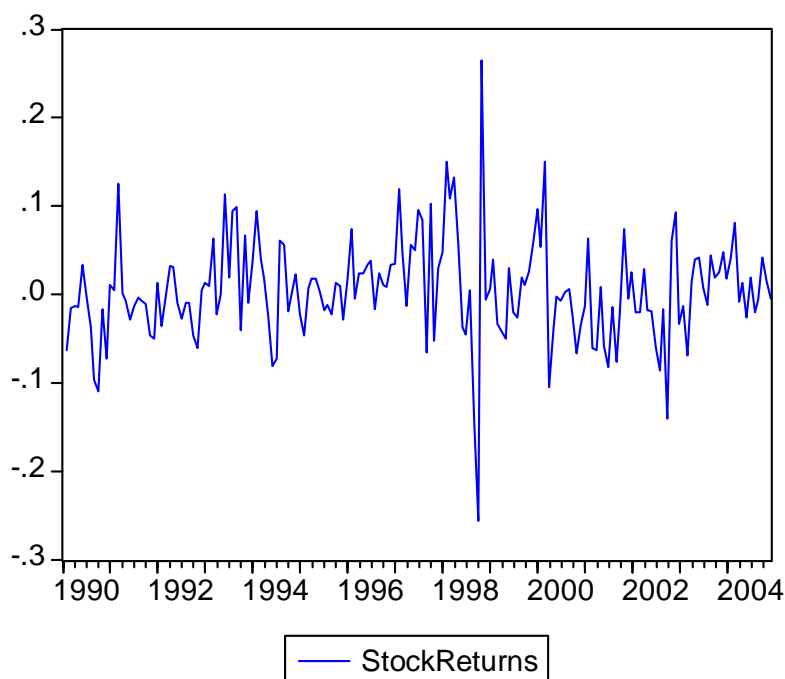
I) Portugal

The last country of our analysis is Portugal. At first, we analyze the time series of interest and plot the returns of the series for the fifteen years period. The descriptive statistics and the diagrams are presented below:

The relationship between volatility of asset prices and volatility of output growth



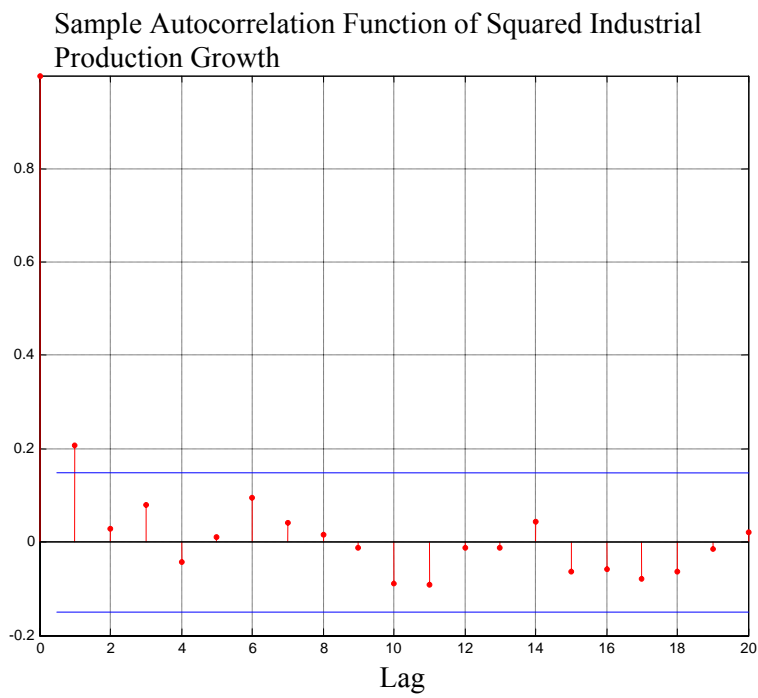
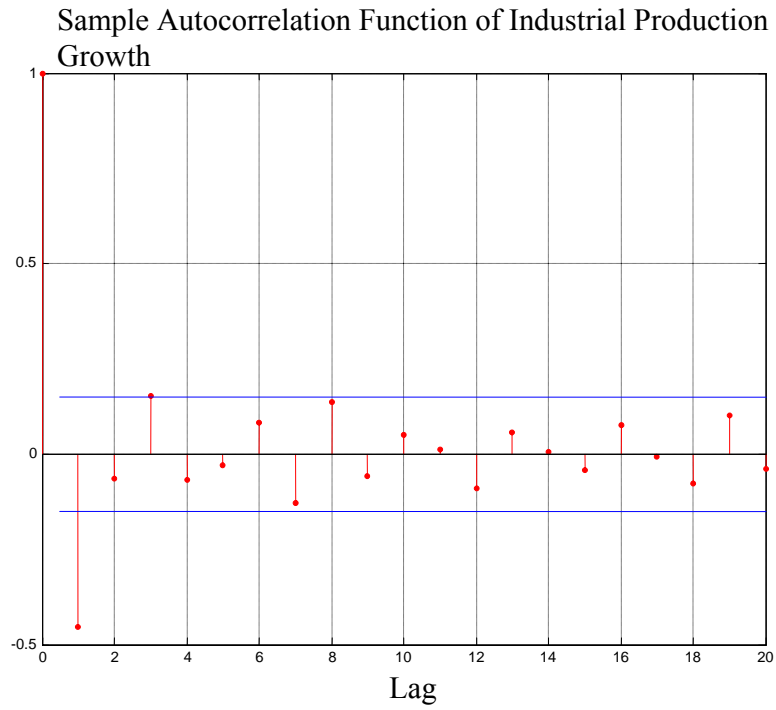
The relationship between volatility of asset prices and volatility of output growth



Both series exhibit excess kurtosis, since the kurtosis coefficient is greater than three (Fat tails). The skewness coefficient for Industrial Production Growth is negative; hence the distribution is skewed to the left, but close to zero. The Jarque-Bera test suggests that we should accept the Normality assumption for Industrial Production Growth. On the other hand, Stock returns distribution is skewed to the right and according to the Jarque-Bera test the normality hypothesis should be rejected.

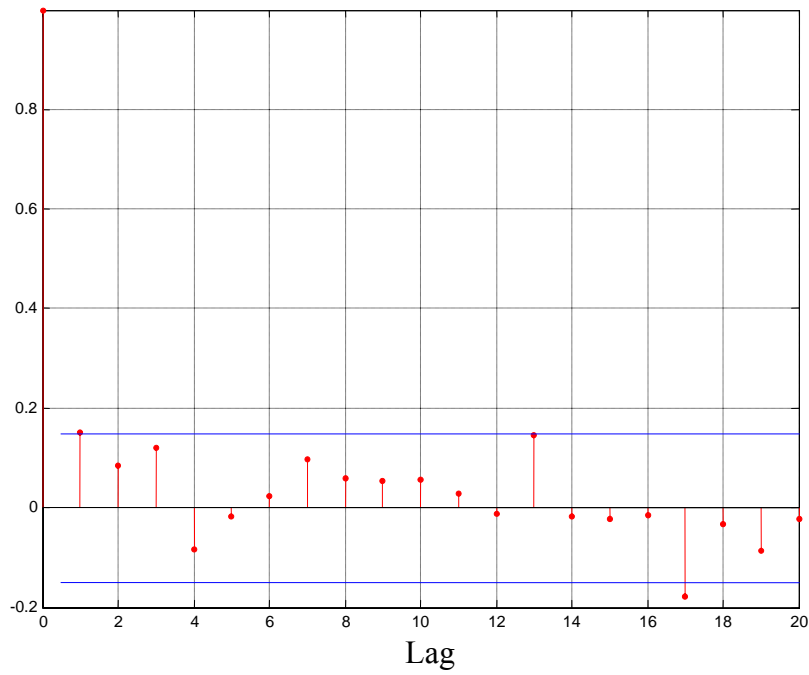
The following diagrams depict the Autocorrelation Functions for the two time series and the squared series as well. The blue line represents the bounds, which are computed with approximate 95% confidence level.

The relationship between volatility of asset prices and volatility of output growth

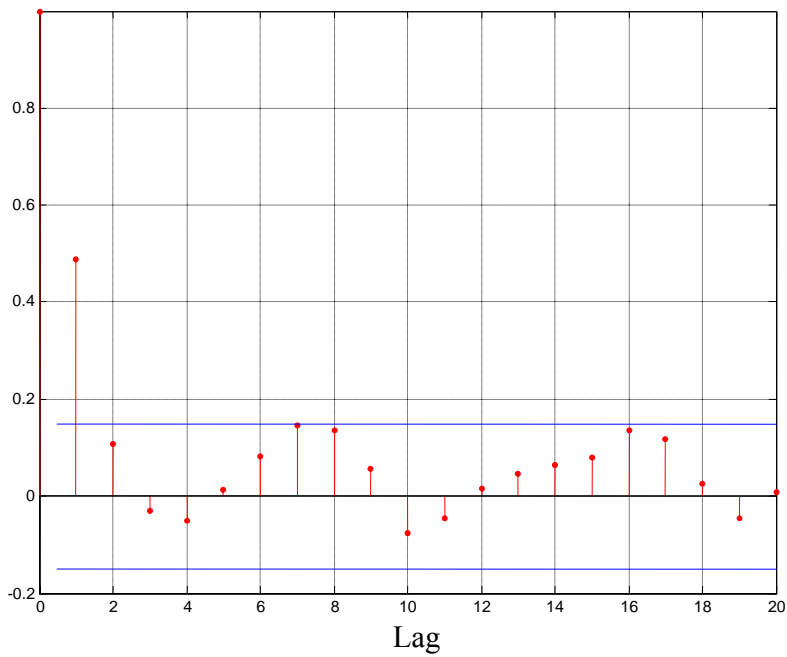


The relationship between volatility of asset prices and volatility of output growth

Sample Autocorrelation Function of Stock Returns



Sample Autocorrelation Function of Squared Stock Returns



The autocorrelation function for Industrial production growth and squared series exceeds the bounds on the first lag. In other words, the autocorrelation of the two series is statistically significant up to the first lag. The stock returns exhibits no

autocorrelation; on the contrary the squared stock returns exhibits autocorrelation in the first lag.

The ARCH/GARCH test is used to test the hypothesis that the time series have no ARCH effect (homoskedasticity at the corresponding Lag). We impose one lag, indicating the lags of the squared sample residuals included in the ARCH test statistic and 5% significance level for each time series. The results indicate persistent of ARCH effect in both series. The results of the test are shown in the table below:

	INDUSTRIAL PRODUCTION GROWTH	STOCK RETURNS
pValue	0,005	0
t-Statistic	7,8824	44,6821
Critical Value	3,8415	3,8415

* $H = 0$ indicate acceptance of the null hypothesis that no ARCH effects

1st Approach

Mean Equation (VAR (2))

The Vector Autoregressive model with two lags was the model selected for the mean equation. The autocorrelation LM test, the Final Prediction Error (FPE), the Akaike information criterion, Schwarz information criterion and Hannan-Quinn information criterion were the tests we used in order to choose the model. The parameters estimates and the standard errors in () are:

$$X_t = A + b_1 X_{t-1} + b_2 X_{t-2} + U_t \quad \text{where}$$

$$X_t = \begin{bmatrix} \rho_t \\ r_t \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} \rho_{t-1} \\ r_{t-1} \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

$$A = \begin{bmatrix} 0,002056 \\ (0,00172) \\ 0,004102 \\ (0,00441) \end{bmatrix}, \quad b_1 = \begin{bmatrix} -0,6397 & 0,0017 \\ (0,07177) & (0,02964) \\ -0,0052 & 0,1190 \\ (0,18409) & (0,07602) \end{bmatrix}, \quad b_2 = \begin{bmatrix} -0,3680 & -0,0179 \\ (0,07280) & (0,01918) \\ 0,1669 & 0,0559 \\ (0,02951) & (0,07570) \end{bmatrix}$$

The relationship between volatility of asset prices and volatility of output growth

The autocorrelation LM test on the residuals confirms that the autocorrelation has been successfully removed from the residuals up to the 12th lag. The null hypothesis that no serial correlation exists should be accepted at all lags. The table presents the results of the test:

Lags	LM-Statistic	P-Value
1	2,912179	0,5726
2	2,387478	0,6649
3	3,602742	0,4624
4	2,033484	0,7296
5	4,478129	0,3451
6	1,718682	0,7873
7	2,023880	0,7314
8	4,228247	0,3760
9	3,520079	0,4748
10	6,141671	0,1888
11	0,805396	0,9377
12	5,114506	0,2757

* H_0 : No serial correlation at lag order h

Causality-in-Mean Test

The relationship of Industrial Production Growth and Stock Return is under investigation in this part of the country analysis. The pairwise Granger causality test is carried out. The results are summarized in the tables below:

Dependent Variable: <i>Industrial Production Growth</i>		
	Chi-Square	PValue
Stock Returns	0,463272	0,7932

Dependent Variable: <i>Stock Returns</i>		
	Chi-Square	PValue
Industrial Production Growth	0,71890	0,6981

There is no statistically significant causality in mean between the two time series.

Variance Equation (BEKK 1, 1)

The BEKK (1, 1) model is our variance equation and the parameters of the model have been estimated with the maximum likelihood estimation method. The assumption we made is that the errors follow a t-student distribution.

The relationship between volatility of asset prices and volatility of output growth

Parameters	Unrestricted GARCH(1,1)
c₁₁	0,0048 (0)
c₁₂	0,0086 (0)
c₂₁	0
c₂₂	0,0156 (0)
GARCH g₁₁	-0,2335 (0,0071)
g₁₂	-0,1138 (0,0229)
g₂₁	0,0235 (0,0022)
g₂₂	0,5408 (0,0247)
ARCH a₁₁	0,8579 (0,0022)
a₁₂	-0,5255 (0,0180)
a₂₁	0,0084 (0,0007)
a₂₂	0,7759 (0,0141)

We estimated three models, by imposing different restrictions on the unrestricted GARCH model. Each model captures different direction of causality in variance. The Restricted1 checks the causality from stock returns volatility to the output growth volatility, the Restricted2 the causality from output growth to stock returns and the Restricted3 the causality in both directions. The LRatio test will be the rule for choosing among the restricted and the unrestricted model. The null hypothesis is that the restricted model fits our data better than the unrestricted. In other words, the acceptance of the null hypothesis indicates that there is no causality in this direction.

	Unrestricted	Restricted1 (a ₂₁ =g ₂₁ =0)	Restricted2 (a ₁₂ =g ₁₂ =0)	Restricted3 (a ₁₂ =g ₁₂ =a ₂₁ =g ₂₁ =0)
Loglikelihood	706,3987	704,1667	643,5877	659,9888
pValue		0,1073	0	0
LRatio		4,4640	125,6220	92,8198
Critical Value		5,9915	5,9915	9,4877

**H = 0 indicate acceptance of the restricted model (no causality in variance) under the null hypothesis; H = 1 indicate rejection of the restricted (causality-in-variance). The significance level of the hypothesis test is 5%.*

The relationship between volatility of asset prices and volatility of output growth

The first line contains the values of the Loglikelihood Functions for each model estimated. The unrestricted model has the greatest value of all.

The LRatio test suggests that the Restricted1 is better than the unrestricted. So, we conclude that there is no causality in variance from stock returns to output growth in Portugal. But, the unrestricted model outperforms the other restricted models. The LRatio test suggests in both cases the rejection of the restricted model. In other words, the causality from output growth volatility to stock returns volatility is statistically significant.

The post estimation analysis covers the last part of this section. The Ljung-Box lack-of-fit hypothesis test for model misspecification is used to test the GARCH residuals. The lags used in the Q-Statistic were twenty and the significance level 5%. The null hypothesis that the model fit is adequate (no serial correlation) should be accepted.

	Standardized Residuals1	Standardized Residuals2
pValue	0,9396	0,0910
Q-Statistic	11,2445	28,8403
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

2nd Approach

At first, we need two univariate models for our time series. We have to select the appropriate models to fit our data adequately. The mean equation for each time series will be an autoregressive moving average process and the variance equation a generalized ARCH process. The special features of the data and the autocorrelation of each time series will determine the lag structure of the models.

For Industrial Production Growth, we chose two autoregressive lags, since the first two autoregressive parameters of the ARMA process were statistically significant. The ARCH parameters were not significant, thus we did not use a variance equation for the residuals.

We also estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (2, 2) model has the greatest LLF value.

LLF	AR (1) GARCH(0,1)	ARMA(1,1) GARCH(0,1)	AR (2) GARCH(0,0)	ARMA (2, 2) GARCH(0,0)
Industrial Production Growth	415,5060	424,3884	426,1812	426,9126

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

The relationship between volatility of asset prices and volatility of output growth

Stock Returns series exhibits autocorrelation in the first lag and has an ARCH effect in the first lag as well. Hence, the mean equation for modeling this time series is an ARMA (1, 1) process and the variance equation a GARCH (1, 1) process.

We estimated the Loglikelihood Function (LLF) of four different models. The results are presented in the table below. The ARMA (1, 1), GARCH (1, 1) model has the greatest LLF value.

LLF	AR(1) GARCH(1,1)	AR (1) GARCH(0,1)	ARMA (1,1) GARCH(0,1)	ARMA(1,1) GARCH(1,1)
Stock Returns	279,5505	277,3135	278,7381	280,3612

**LLF - Optimized log-likelihood objective function value associated with the parameter estimates*

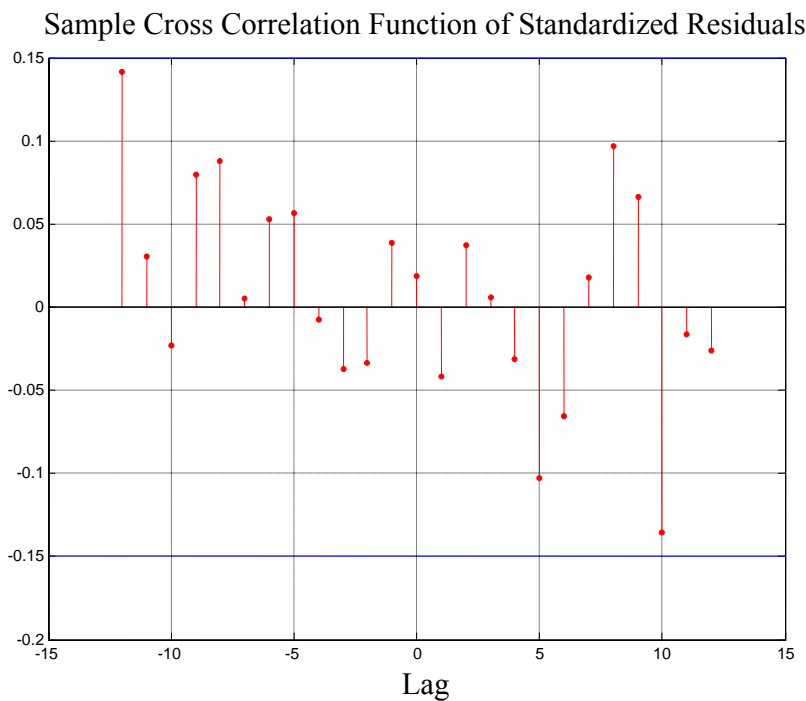
We shall now test our model structure whether the model fit is adequate. The Ljung-Box Q-statistic lack-of-fit hypothesis test will be implemented on the residuals of the series. This test is for model misspecification. The lags used in the Q-Statistic were twenty and the significance level 5%. The null hypothesis that no serial correlation has remained on the residuals should be accepted in both cases.

	Industrial Production Growth	Stock Returns
P-Value	0,9709	0,2498
Q-statistic	9,8442	23,8322
Critical Value	31,4104	31,4104

**H₀: the null hypothesis that the model fit is adequate (no serial correlation).*

The diagram of the Sample Cross Correlation Function of standardized residuals of the two time series will help us clarify the causality in mean patterns between Industrial Production Growth and Stock Returns. The null hypothesis is that the residuals are uncorrelated, or there is no causality in mean. The blue line represents the confidence interval of 95 %. The diagram of the Function is presented below:

The relationship between volatility of asset prices and volatility of output growth



The diagram suggests that we should accept the null hypothesis of no causality in mean in every lag. The t-statistic used for the hypothesis testing is defined as:

$$t = \sqrt{T} \hat{r}_{uv}(k) \rightarrow AN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

Where k is the number of periods the stock returns lag the industrial production and T the sample size (number of observations).

The t-statistic for the standardized residuals of the two time series for every lag is:

Lags	t-statistic for st. residuals
-12	1,8948
-11	0,4089
-10	-0,3141
-9	1,0656
-8	1,1736
-7	0,0728
-6	0,7034
-5	0,7527
-4	-0,1002
-3	-0,5012
-2	-0,4445
-1	0,5152

The relationship between volatility of asset prices and volatility of output growth

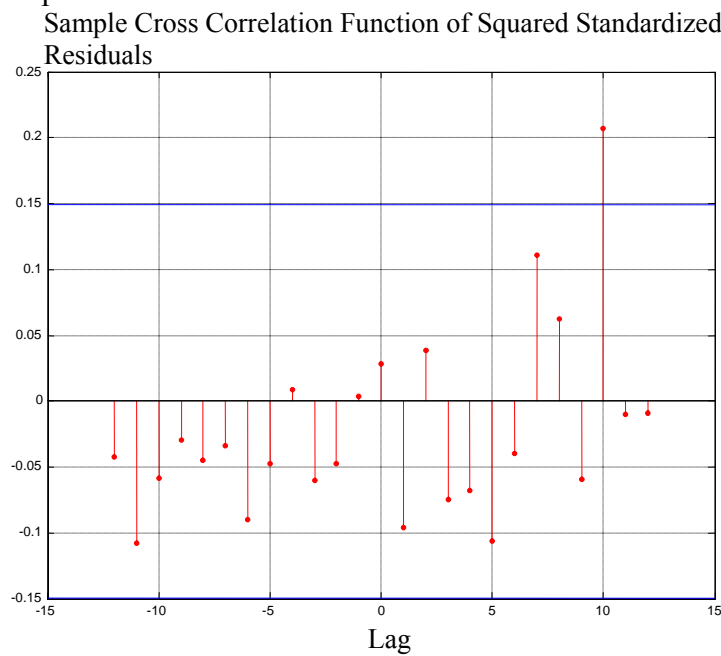
0	0,2516
1	-0,5597
2	0,4988
3	0,0828
4	-0,4227
5	-1,3758
6	-0,8795
7	0,2397
8	1,2988
9	0,8841
10	-1,8154
11	-0,2168
12	-0,3492

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level

The null hypothesis should be accepted for every lag. So there is no statistically significant causality in mean between Industrial Production Growth and Stock Returns.

As far as causality in variance is concerned, the diagram of the Sample Cross Correlation Function of the squared standardized residuals will be used. The null hypothesis is that the squared standardized residuals of the two series are uncorrelated. The blue line represents the confidence interval of 95 %:



The relationship between volatility of asset prices and volatility of output growth

The hypothesis testing uses the t-statistic mentioned before but performed on the squared standardized residuals. The table contains the results:

Lags	t-statistic for squared st. residuals
-12	-0,5709
-11	-1,4462
-10	-0,7885
-9	-0,3927
-8	-0,6057
-7	-0,4563
-6	-1,1995
-5	-0,639
-4	0,116
-3	-0,8113
-2	-0,6389
-1	0,0515
0	0,3791
1	-1,2824
2	0,5186
3	-0,9952
4	-0,9058
5	-1,424
6	-0,5307
7	1,4835
8	0,8332
9	-0,796
10	2,7665*
11	-0,1282
12	-0,1281

H_0 : No causality, the standardized residuals and standardized squared residuals of the two time series are uncorrelated.

'*' indicates significance at the 1% level, '**' indicates significance at the 5% level

The t-statistic suggests that the null hypothesis should be accepted in all but the 10th lag. In lag (+10) the null hypothesis of no causality in variance should be rejected. Hence, there is volatility spillover from Industrial Production Growth to Stock Returns. The volatility of IndustrialProductionGrowth_{t-10} Granger causes the volatility of StockReturns_t. The volatility of today's Stock Returns has been influenced by the volatility of Industrial Production Growth ten months ago.

8. Conclusions

The existence of causal links between volatility in the financial markets and in the real economy is a matter of great interest among policymakers, Central Banks and financial managers. At first, it is a necessary condition for asset price targeting to be effective as a means to stabilize the economy. Moreover, managers can obtain more insights in the management of their current assets and current liabilities from movements of these two economic variables.

In this paper, the short-term dynamic relationships between output growth and asset prices have been explored. We were interested in establishing statistically significant relationships about these two economic factors and their second moments (volatilities). The causality in mean and the causality in variance patterns were investigated in a group of nine economies. The economies varied in size, political power and economical situation and were located all around the world. We used two different approaches to reach our conclusions about volatility spillovers. In the first approach, we estimated a B.E.K.K. (1, 1) model with the two time series. Then, we imposed the appropriate restrictions and via LRatio tests we determined the direction of volatility spillovers. The second approach was the methodology proposed by Cheung and Ng in 1996 in their paper 'A causality in variance test and its application to financial market prices.

In general, Industrial Production Growth and Stock Returns are not closely related. Only in four countries out of nine, can man establish a strong relationship between them. In U.S.A. and in Spain asset prices Granger causes output growth, while in Germany and Greece the causality pattern is in both directions. In the rest countries of our sample, there is no statistically significant causality in mean. The table summarizes the results of both approaches:

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1st approach:

	Causality in mean
U.K.	No
U.S.A.	$r_{t-1} \rightarrow \rho_t^*$
Japan	No
Spain	$r_{t-9} \rightarrow \rho_t^{***}$ $r_{t-8} \rightarrow \rho_t^{***}$
Greece	$r_{t-1} \rightarrow \rho_t^{***}$ $r_{t-7} \rightarrow \rho_t^{***}$ $\rho_{t-12} \rightarrow r_t^{***}$
Portugal	No
France	No
Canada	No
Germany	$\rho_{t-7} \rightarrow r_t^{**}$ $r_{t-7} \rightarrow \rho_t^{**}$ $r_{t-6} \rightarrow \rho_t^{**}$ $r_{t-5} \rightarrow \rho_t^{**}$

r_t stands for Stock Returns

ρ_t stands for Industrial Production Growth

'*' indicates significance level of 1% '**' indicates significance level of 5% '***' indicates significance level of 10%

2nd approach:

	$r_t \rightarrow \rho_t$	$\rho_t \rightarrow r_t$
U.K.	No	No
U.S.A.	Yes	No
Japan	No	No
Spain	Yes	No
Greece	Yes	Yes
Portugal	No	No
France	No	No
Canada	No	No
Germany	No	Yes

* r_t stands for Stock Returns

** ρ_t stands for Industrial Production Growth

The empirical evidence of our study indicates that there exist volatility spillovers between output growth and asset prices, indicating that their second moments are related. In five countries, the causality in variance is statistically significant in both directions. In U.S.A. and Spain there is one way causality, since Stock returns volatility Granger causes Industrial Production Growth volatility. In Portugal, the causality in variance is on the

The relationship between volatility of asset prices and volatility of output growth

opposite direction, while in France there are no statistically significant volatility spillovers. The tables contain the results of the two approaches:

1st approach:

	Causality in variance
U.K.	$\text{Vol}(\rho_{t-1}) \rightarrow \text{Vol}(r_t)^*$ $\text{Vol}(\rho_{t-2}) \rightarrow \text{Vol}(r_t)^*$ $\text{Vol}(\rho_{t-3}) \rightarrow \text{Vol}(r_t)^{**}$ $\text{Vol}(r_{t-9}) \rightarrow \text{Vol}(\rho_t)^{***}$
U.S.A.	$\text{Vol}(r_{t-8}) \rightarrow \text{Vol}(\rho_t)^*$
Japan	$\text{Vol}(\rho_{t-7}) \rightarrow \text{Vol}(r_t)^{**}$ $\text{Vol}(r_{t-4}) \rightarrow \text{Vol}(\rho_t)^{***}$
Spain	$\text{Vol}(r_{t-1}) \rightarrow \text{Vol}(\rho_t)^{***}$ $\text{Vol}(r_{t-4}) \rightarrow \text{Vol}(\rho_t)^{***}$
Greece	$\text{Vol}(r_{t-2}) \rightarrow \text{Vol}(\rho_t)^{**}$ $\text{Vol}(r_{t-4}) \rightarrow \text{Vol}(\rho_t)^*$ $\text{Vol}(\rho_{t-1}) \rightarrow \text{Vol}(r_t)^{**}$ $\text{Vol}(\rho_{t-6}) \rightarrow \text{Vol}(r_t)^{**}$
Portugal	$\text{Vol}(\rho_{t-10}) \rightarrow \text{Vol}(r_t)^*$
France	No
Canada	$\text{Vol}(r_{t-9}) \rightarrow \text{Vol}(\rho_t)^{**}$ $\text{Vol}(\rho_{t-2}) \rightarrow \text{Vol}(r_t)^*$
Germany	$\text{Vol}(r_{t-4}) \rightarrow \text{Vol}(\rho_t)^*$ $\text{Vol}(\rho_{t-8}) \rightarrow \text{Vol}(r_t)^{**}$

*Vol (r_t) stands for Stock Returns Volatility

**Vol (ρ_t) stands for Industrial Production Growth Volatility

‘*’ indicates significance level of 1% ‘**’ indicates significance level of 5% ‘***’ indicates significance level of 10%

2nd approach:

	<u>Restricted 1</u>	<u>Restricted 2</u>	<u>Restricted 3</u>
	$\text{Vol}(r_t) \rightleftarrows \text{Vol}(\rho_t)$	$\text{Vol}(\rho_t) \rightleftarrows \text{Vol}(r_t)$	$\text{Vol}(\rho_t) \leftrightarrow \text{Vol}(r_t)$
U.K.	Yes	Yes	Yes
U.S.A.	Yes	No	Yes
Japan	Yes	Yes	Yes
Spain	Yes	No	Yes
Greece	Yes	Yes	Yes
Portugal	No	Yes	Yes
France	No	No	No
Canada	Yes	Yes	Yes
Germany	Yes	Yes	Yes

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