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Διπλώματος Ειδίκευσης στην Ναυτιλία

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Υπογραφή

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Τα μέλη της Επιτροπής ήταν:

- **Θαλασσινός Λευτέρης (Επιβλέπων)**
- **Γκιζιάκης Κωνσταντίνος**
- **Πελαγίδης Θεόδωρος**

Η έγκριση της Διπλωματικής Εργασίας απο το Τμήμα Ναυτιλιακών Σπουδών του Πανεπιστημίου Πειραιώς δεν υποδηλώνει αποδοχή των γνωμών του συγγραφέα.

ABSTRACT - 3

Since the exploration of the first liter of oil, this has been ruling the world. All the industries depend on oil even with direct or indirect way. Producers and consumers with daily contracts, which worth billions of dollars, define the price of oil per barrel after strong and long negotiations. The price of oil defines inflations in countries and as a result the growth rates of them. As a conclusion, oil means power even politically or financial to the people have it. So the prediction of its price is very essential for oil's counterparties. This thesis aims to predict the price of the Brent Oil's future contract price.

ΠΕΡΙΛΗΨΗ - 4

Από το πρώτο λίτρο πετρελαίου που αντλήθηκε για πρώτη φορά στον κόσμο μέχρι και σήμερα, φαίνεται πως ορίζει τον κόσμο όποιος το κατέχει. Όλες οι βιομηχανίες εξαρτώνται απο αυτό είτε με άμεσο είτε με έμμεσο τρόπο. Παραγωγοί και καταναλωτές καθημερινά ορίζουν την τιμή του πετρελαίου ανα βαρέλι, σε συμβόλαια δισεκατομμυρίων μετά απο χρονοβόρες και σκληρές διαπραγματεύσεις. Η τιμή του πετρελαίου μπορεί να καθορίσει ακόμα και τον πληθωρισμό σε μια χώρα αλλά και τον ρυθμό ανάπτυξής της. Εν κατακλείδι το πετρέλαιο σημαίνει δύναμη είτε πολιτικά είτε οικονομικά σε όποιον το κατέχει, άρα η πρόβλεψη της τιμής του ειναι υψίστης σημασίας για τα εμπλεκόμενα μέρη αλλά και για όλο το κόσμο. Η διπλωματική εργασία αυτή, έχει στόχο την πρόβλεψη της τιμής των μελλοντικών συμβολαίων του πετρελαίου Brent.

Brent has served as a leading global benchmark for Atlantic Basin crude oils in general, and low-sulfur ("sweet") crude oils in particular, since the commercialization of the U.K. and Norwegian sectors of the North Sea in the 1970s. These crude oils include most grades produced from Nigeria and Angola, as well as U.S. Gulf Coast (USGC) sweet crude oils such as Louisiana Light Sweet (LLS) and U.S. benchmark West Texas Intermediate (WTI). This degree of substitutability for refiners in the USGC, U.S. East Coast (USEC) and Northwest Europe explains why Brent is useful as a pricing basis.

The Brent field, located in the U.K. sector of the North Sea and delivered by pipeline to the terminal at Sullom Voe, is the namesake of the Brent futures and options market. However, the name has lapsed into shorthand for BFOE, or Brent-Forties-Oseberg-Ekofisk family of North Sea crude oils, each of which has a separate delivery point. Many of the crude oils traded as a basis to Brent actually are traded as a basis to Dated Brent, a cargo loading within the next 10-21 days (23 days on a Friday).

In a circular turn, the active cash swap market for the differentials (contracts for differences, or CFDs) between Dated Brent and various crude oils traded on a BFOE basis in the so-called 21-day Brent market determine where Dated Brent is assessed. If the forward curve of the Brent market is in backwardation, the condition wherein each successive futures contract is priced lower than its predecessor, the CFD should be a positive value. If the forward curve of the Brent market is in contango, the condition wherein each successive futures contract is priced higher than its predecessor, the CFD should be a negative value.
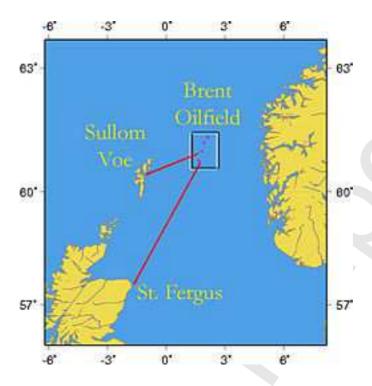
Figure 1: (Location: North Sea, Brent oil field)

Even though Dated Brent itself is not an actual spot market, but rather a short-term forward market affected by CFDs derived from the forward curve of Brent futures and short-dated cash market options, it is the basis used to price approximately 65% of the world's trade in crude oil, including deals done for immediate delivery.

A second forward market, the 21-day BFOE market, involves the actual cash market trade in the cheapest-to-deliver crude from the BFOE market. This historically was Brent itself, but that has changed with time to make Forties the cheapest-to-deliver crude oil more often than not. The 21-day BFOE index is used to compile the Brent Index on a daily basis and then used to cash-settle the Brent futures contract.

THE WTI MARKET – 5.1

While Brent is a waterborne cargo market where crude oil arrives in discrete quantities over a short period of time, WTI is a mid-continent pipeline market where crude oil flows continuously at near-constant rates.

The crude oil industry in the U.S. began in western Pennsylvania and eastern Ohio; in Canada it started in southern Ontario. However, the respective industries soon discovered much larger sources of crude oil elsewhere. In Canada, the industry soon centered in Alberta, which is a long way by pipeline or railcar from major

refining centers. In the U.S., the industry first boomed in Southern California, followed in quick succession by discoveries along the U.S. Gulf Coast, Oklahoma, and then both West and East Texas.

Oklahoma's early prominence, and the need to build long-distance pipelines to refining centers in the Midwest, gave rise to a pipeline terminus at Cushing. When crude oil was discovered in the Permian basin of West Texas and New Mexico in the 1920s, pipelines were laid to Cushing and refining centers along the U.S. Gulf Coast. Gulf Coast crude oil shipped north could connect to this pipeline system, along with Canadian crude oil moving south.

The network of pipelines and storage tanks at Cushing made WTI at Cushing a natural marker price for U.S. pipeline crude oil. The U.S. pipeline market revolves around pipeline scheduling considerations. The window after the 25th day of the previous month and before the start of the next month is the scheduling period. Crude oil priced for the next month's delivery flows is delivered ratably at that price in the following month. That fixed price serves as the basis for swaps against crude oil priced in the daily posting market. The posting, or posting-plus market, involves daily prices set by crude oil resellers and constitutes the floating leg of the pipeline market.

FACTORS AFFECTING PETROLEUM ECONOMICS – 5.2

Energy markets are highly volatile, and natural gas and electricity tend to be more so than crude oil, yet neither affects the world's economic psyche as much as crude oil. The introduction of petroleum-based fuels for purposes of lighting, space heating, and for transportation in the 19th century, ushered in an acceleration of economic growth the likes of which had been unseen in the history of man.

The growing dependence on what has been recognized from the start as a finite resource base of naturally occurring conventional petroleum has led to a fear of depletion. Unlike agricultural commodities, which can be replaced each season, or metals, which can be recycled indefinitely, fossil fuels such as crude oil, natural gas and coal are consumed with little possibility of replacement or recycling.

Moreover, the law of diminishing returns applies on the supply side: Producers spend ever-greater amounts of money to discover and bring to market ever-smaller

quantities of petroleum. This fear and the strategic importance of crude oil to the global economy assure the permanent interest of governments in the crude oil market.

This is true for producers, who formed the Organization of Petroleum Exporting Countries (OPEC) at the behest of Venezuela in 1960, and who have attempted to maintain some measure of control over production ever since as non-OPEC producers in the North Sea, Mexico and Russia have sought increased market share, as well as for consumers interested in secure supply and stable prices.

The inelastic nature of crude oil prices assures price volatility in both the short- and long-term. Income elasticity, or the change in total demand as a function of global growth and recession enters into the picture as well; economic downturns in the early 1980s, in 1998 and in 2008 led to sharp decreases in price.

The interplay of the resource base, demand growth, politics and random events leads to an inescapable and highly demonstrable conclusion: Despite more than 150 years of effort, the next person to forecast crude oil prices successfully for any sustained period of time will be the first. Due to these market characteristics, price risk is always present and must be managed.

For decades, prices and production levels were controlled by the international oil firms, the so-called Seven Sisters. After the introduction of OPEC and successful attempts by new firms to offer preferential terms to producing nations, pricing and production control began to shift to the producing nations by the early 1970s. This led to the first oil shock of 1973-1974. A second oil shock came about from the Iranian Revolution of 1979 and the Iran-Iraq War beginning in 1980. This was followed by a price collapse in the mid-1980s as new supplies emerged and as energy consumption habits changed.

All of these events took place prior to the introduction of Brent futures in 1988. A new cycle began shortly thereafter with the Persian Gulf War in 1990-1991; realized historic volatility in Brent jumped to its all-time high during this disruption. However, nothing compared to the bull market beginning in 1999 and extending into 2008. Prices surged along with demand from China, India and other newly industrializing countries - and then collapsed as a global financial crisis slashed demand growth.

With the profile of Brent steadily rising, traders have increasingly turned to Brent for managing price risk in the global oil market. The best measure of any contract's success is whether volume is independent of events and price trends.
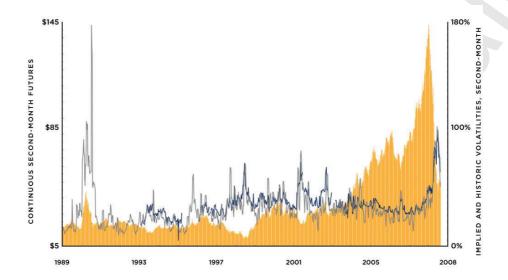

Figure 2: (Brent volatilities rose as market spiked, Source:Bloomberg)

In February 2006, a cash-settled WTI futures contract began trading at ICE Futures Europe. The contract was an immediate success and soon reached a strong level of volume and open interest.

BRENT TRADES AND ISSUES – 5.3

Traders quickly learn to focus on the spread between WTI and Brent, usually expressed as the easier-to-say "Brent-TI spread" even though the number is WTI minus Brent. That this number is the focus of trade is a tribute to the importance of the ICE Brent and WTI contracts — the spread between a waterborne cargo in the North Sea and ratable pipeline delivery in mid-continent Cushing, Oklahoma always requires explaining.

The pipelines running into Cushing flow in a northerly direction from Texas and points along the USGC, although they obviously flow in a southerly direction for crude oil coming in from Canada. This means WTI at Cushing cannot be delivered back out to the USGC when inventories at Cushing rise and depress the price of WTI. Those storage conditions will be addressed later. A better comparison for the incentive to bring Brent-basis waterborne cargoes into the USGC refining markets is the LLS-Brent spread.

A second consideration rises, and that is voyage time. It takes a cargo moving across the Atlantic approximately two weeks to get to the USGC, during which time its price should either increase or "ride up" the forward curve in the case of a backwardated market or decrease or "ride down" the forward curve in the case of a contango market. Accordingly, the price of Dated Brent should be adjusted by one-half of the spread between first- and second-month Brent futures to afford a proper comparison for refinery economics.

The LLS-Brent spread has exhibited mean-reverting tendencies for much of recent history. The only major exception here was a delayed expansion of this spread during the final rally in 2008 and another delayed reaction to the downside once prices of LLS turned lower.

Contrast this spread to the one between WTI at Midland, Texas, a point with pipelines to both Cushing and the USGC. The spread has put in some rather large moves, particularly to the downside, as storage conditions at the Cushing market pushed WTI prices there higher and lower.

The Brent-WTI spread tends to be seasonal, albeit not as much as it was before the markets witnesses in the spring of 2007 and the winter of 2008-2009. The divisors for this spread are greater than 1.00 for June, September, October and November, and just slightly so for all other months. Market participants should be aware of this seasonality.
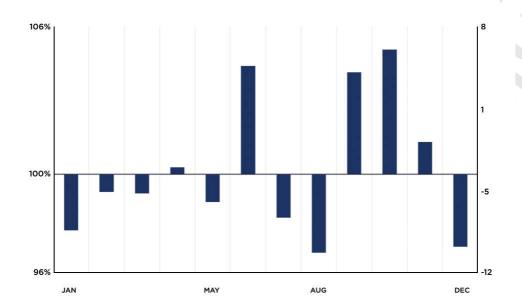
Figure 3: (Seasonal Adjustment Divisors For The BRENT-WTI Spread, source:Bloomberg)

Another important relationship, and one that has a more direct effect on the price trend of Dated Brent is the refining margin, or crack spread, between it and second-month New York Harbor heating oil and gasoline prices. Dated Brent prices tend to track the "2/1/1" crack spread, or two barrels of Brent refined into one barrel each of heating oil and gasoline. This close relationship suggests marginal changes in the U.S. refined products market have a profound, and tradable, impact on Dated Brent prices.

Another critical spread is the one between Dated Brent and Dubai crude oil. Dubai is a high-sulfur or "sour" crude oil and serves, either by itself or averaged in with Omani crude oils, as the marker grade for many of the Persian Gulf crudes exported eastward into Asian markets.

Westbound crude exports from the Persian Gulf to Europe are priced against an average of trades in the ICE Brent crude oil futures contract; this is called the Bwave (Brent Weighted Average). U.S.-bound crudes from the Persian Gulf are priced against Platts WTI.

The Brent-Dubai spread, as is the case with all so-called "sweet-sour" spreads, tends to spike in favor of the more expensive sweet crude oil during times of maximum refinery demand. The value of sour crude oils will also be affected by the

relative value of fuel oil. Higher sulfur crude oils typically yield a relatively higher volume of fuel oil.

Sweet-sour spreads, here illustrated by the spread between WTI and West Texas Sour (WTS) at Midland, Texas, are in turn a function of crack spreads. As a spread such as the second-month 2/1/1 rises, refiners find it profitable to bring on incremental processing units only capable of processing the more expensive sweet crude oil. This takes time, as there is a lead of 96 trading days on average between this crack spread and this sweet-sour spread. A similar, less-easy-to-illustrate dynamic takes place in the global crude oil market and drives spreads such as the Brent-Dubai spread.

TRADING BRENT FUTURES AND OPTIONS – 5.4

Futures markets exist for the purposes of price discovery and risk transfer. Price discovery is the more straightforward. Buyers and sellers meet in a competitive market place, and the prices resulting from each transaction signal to other traders what a given commodity might be worth. This process is vastly different from the fundamental analysis approach to a market, in which a theoretical market clearing price is deduced from supply and demand data. There is no theory involved in price discovery: It is what it is.

Once accepted by a clearing firm or other licensed futures brokerage, it is possible to participate in the markets. For regulatory and reporting purposes, a market participant not in the petroleum business will be classified as non-commercial, and a market participant in the petroleum business will be classified as a commercial or hedging trader.

Hedgers tend to utilize Brent crude oil options. Producers can put a floor underneath their selling price with long put options, and buyers can put a ceiling over their costs with long call options, among other strategies. In a futures trade, the trader and the counterparty to the trade will post initial or original margin a futures commission merchant or clearing member.

Risk transfer is the second purpose of a futures market. Any producer of Brent-basis crude oil, any holder of Brent-basis inventories or any party at risk if the price of Brent-basis crude oil declines is long the market. These participants can offset

risk by going short a futures contract. A refinery or any user at risk; if the price of Brent-basis crude oil increases is short the market and can offset risk by going long a futures contract.

For example a Brent-basis crude oil producer at risk to prices falling can acquire a financial asset, the short futures position, which will rise in value as the market declines. The opposite is true for a refinery at risk to prices rising; there a long futures position will rise in value as the market rises.

The Brent oil's future contract price is very sensitive, due to all risks that are applied on it (political, economical etc), also the price of Brent is very important because in nowadays defines even the growth rates of countries. So, it is obvious that the prediction of the price of such a valuable future contract is very important achievement for producers and consumers.

DESCRIPTIVE STATISTICS - 6

As a first step, before forecasting values for Brent Oil's future contract price, it is important Brent's oil's time series to be identified. So, in order to be achieved that, some descriptive statistics should be calculated. For this purpose the descriptive statistics that have been chosen, are shown below:

-**Arithmetic Mean**

The arithmetic mean is the standard average, which is often called the "mean". More specifically, for a data set, the mean is the sum of the values divided by the number of values. The mean of a set of numbers $x_1, x_2,\ldots, x_n$ is typically denoted by $\bar{x}$, pronounced "x bar". The mathematical formula is:

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$$

-**Standard deviation**

Standard deviation is a widely used measure of variability or diversity used in statistics and probability theory. It shows how much variation or "dispersion" exists

from the average (mean, or expected value). A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data points are spread out over a large range of values. In the case we have a data set of $x_1, x_2, x_3, \ldots, x_N$, the standard deviation is calculated by the formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}, \quad \text{where} \quad \mu = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

-**Variance**

The variance is a measure of how far a set of numbers is spread out and lie from their arithmetic mean. In particular, the variance is one of the moments of probability distribution. The formula is used in order to calculate the variance is:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 \qquad \mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$
, where

-**Skewness**

Skewness is a measure of the asymmetry of the data around the sample mean. If skewness is negative, the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right. The skewness of the normal distribution (or any perfectly symmetric distribution) is zero. The skewness of a distribution is defined as:

$$s_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)^3}$$

-**Kurtosis**

Kurtosis is a measure of how outlier-prone a distribution is. The kurtosis of the normal distribution is 3. Distributions that are more outlier-prone than the normal distribution have kurtosis greater than 3; distributions that are less outlier-prone have kurtosis less than 3. The kurtosis of a distribution is calculated by the formula:

$$k_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2}$$

-**Minimum & maximum**

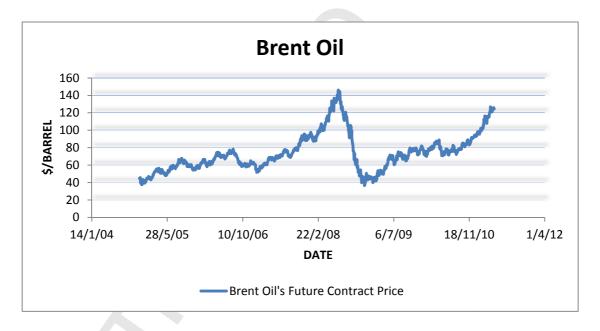Figure 4: (Brent's Oil Future Contract Price)

After our calculations on the Brent Oil's future contract price, for the descriptive statistics, we concluded to the following tables which present the results:

	N	Mean	Std. Deviation	Variance
Brent Oil	1672	73.8967	21.44846	460.036
Valid N (listwise)	1672			

Figure 5: (Main Descriptive Statistics Table)

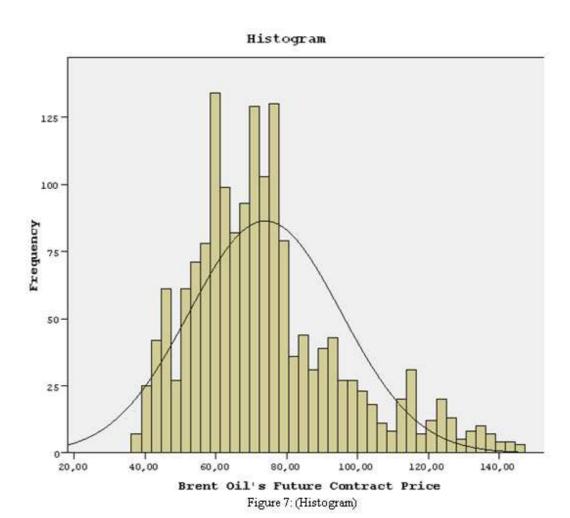|	N	Minimum	Maximum	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Brent Oil	1672	36.49	146.19	.966	.060	.744	.120
Valid N (listwise)	1672						

Figure 6: (Additional Descriptive Statistics Table)

As it is stated on the Table 1, the **arithmetic mean** has a value of 73.9. As a value, the **arithmetic mean** doesn't have many things to show us for the time series but is valuable for the phase of identification of the time series.

On the Table 2, is stated the **minimum** and **maximum** values of the Brent Oil's future contract price. The **minimum** is equal to 36.49 and the **maximum** is equal to 146.19. So, that means for the period of 24th of November 2004 until 5th of February 2011 the future contract's prices were between those limits.

On the Table 1, is stated the value for the **standard deviation** which is equal to 21.45. This value presents the historical volatility of the time series. So if it is considered the value which is stated on the Table 1, it is obvious that the Brent's Oil future contract price in the examined period was high volatile. So, a buyer of Brent's Oil future contract should expect a big amount of volatility on his investment, from the date he buys until the time he sells the contract.

As a last value on the Table 1 is stated the **variance**, which is equal to 460.04. This value is the dispersion of Brent's Oil future contract prices around their **arithmetic mean** value of 73.9. Also the **variance** presents the volatility of Brent's Oil, so the risk an investor might take if he considers buying this particular future contract. As it seems from the high value of **variance** the risk is quite big.

Figure 7: (Histogram)

On the Table 2, is stated the **Skewness** statistic value which is equal to 0.966. The **Skewness** for the Brent's Oil future contract prices is positive, which means that the most of the prices are laying at the right of their **arithmetic mean**. By knowing which side data is skewed, we are able to estimate if a future value will be more or less than the **arithmetic mean**. In this time series a future value will be more than the calculated **arithmetic mean** of 73.9.

On the other hand, **Kurtosis** statistic value is presented on the Table 2 and has a value equal to 0.744. As it is observed, the number is positive, that means that the distribution is relatively peaked. If the value was negative then the distribution would be relatively flat. In addition, the value of **Kurtosis** is useful, to be used as indicator for the chart's trend of the Brent's oil future contract price. The value of **Kurtosis**

statistic (equal to 0.744) indicates that, in Brent's oil time series chart the distribution tends to be concentrated toward the **arithmetic mean** and tends to have skinny tails.

<div align="right">**STATIONARITY - 7**</div>

Econometric models used in finance and commodity markets typically require the assumption that the data are stationary. In this thesis, for using the chaos theory and getting predictions for Brent Oil's future contract price, is essential the time series to be non stationary. For this purpose in early stages the stationarity of the time series is calculated.

The stationarity is an essential property to define a time series process. A process is said to be covariance stationary or weak stationary, if its first and second moments are time invariant. A stationary process has the property to be mean reverting, i.e., it will fluctuate around its mean. Stationarity implies that no trend is observed in the time series.

Stationarity of time series is a necessary condition for many applications of time series econometrics, especially in the empirical analysis of macro-economic variables. The results derived through classical econometric theory presume the stationarity of time series as the non-stationarity of data largely invalidates standard econometric techniques. And thus, testing for stationarity should necessarily be the first step in any time series data analysis. Stationarity of a time series matters because it provides a framework in which averaging make sense. Unless properties like the mean and covariance are either fixed or evolve in a known manner, observed data cannot be averaged.

Stationarity is essentially a restriction on the data generating process over time. Stationarity is a longitudinal measure in which the process generating return is identical over time. More specifically, stationarity means that the fundamental form of the data generating process remains the same over time. This is indicated in the moments of the process. It means a stationary time series has constant mean, time invariant variance, and covariance between lag values depends only on the length of the lag.

In other words:

i) Mean stationarity means that the expected value of the process is constant over time, i.e.,

$$E(Y_t) = \mu$$

ii) Variance stationarity means that the variance is temporally stable, i.e.,

$$VAR(Y_t) = \sigma^2$$

iii) Covariance stationarity is the same, i.e.s

$$cov(Y_t, Y_{t-s}) = \gamma_s$$

In this last case, this means that the serial correlation of two observations (Y_t, Y_{t-s}), depends only on the lag 's' and not on 'where' in the series they fall.

Thus, a stationary time series is identified when mean, variance, and autocovariances are finite and constant over time. What is described, here, is called weak stationarity (stationarity in the moments). The time series that is not stationary in the way just described is called non- stationary. A non-stationary time series is characterized by time varying mean or variance or both. The nonstationarity of data facilitates the empirical and behavioural study of the underlying time series only for the time period under consideration which delimits the scope of generalization of the result vis-à-vis other time periods. And, thus non-stationarity of time series should be identified first, and then the series should be made stationary, if found non-stationary, before they are applied to empirical works.

This concept of weak stationarity of time series has got wide applications in the empirical literature of financial economics. Recently the studies based on time series pertaining to stock markets of developed and emerging capital markets have attracted the attention of good many researchers, market analysts, academicians and others who keep interest in market studies.

The empirical studies concerning stock markets such as testing efficient market hypothesis, performing cointegration and

causality tests between leading macro-economic variables, volatility modelling, predicting stock returns and similar other studies calls for the stationarity of underlying time series.

Yule (1926) showed that spurious correlation could persist in non-stationary time series even if the sample is very large. Granger (1974) also showed that non-stationarity lead to estimation of spurious regression. Pagan and Schwert (1990) proposes several non-parametric tests for covariance stationarity and applies them to common stock return data from 1834-1987. Dehay and Kow (1995) observes that in recent research on the volatility of asset returns, the time series data frequently show a lack of covariance stationarity. Ho and Wan (2002) suggests that neither the Asian crisis nor the 1998 currency crisis of Russia and Latin America has any significant impact on the stock return series of Australia and the US, which are found to be covariance stationary and covariance non-stationary, respectively.

In order to calculate the autocorrelation and partial autocorrelation the SPSS software is used. Below are presented all the results from the calculations.

Case Processing Summary

		BRENT
Series Length		1672
Number of Missing Values	User-Missing	0
	System-Missing	0
Number of Valid Values		1672
Number of Computable First Lags		1671

Figure 8: (Case Processing Summary)

As it is shown on the table above for calculating the autocorrelation is used the whole time series of Brent Oil's future contract price.

Autocorrelations

Series: BRENT

Lag	Autocorrelation	Std. Error[a]	Box-Ljung Statistic		
			Value	df	Sig.[b]
1	.995	.024	1657.906	1	.000
2	.990	.024	3300.695	2	.000
3	.985	.024	4928.748	3	.000
4	.980	.024	6541.607	4	.000
5	.975	.024	8138.651	5	.000
6	.970	.024	9720.431	6	.000
7	.965	.024	11286.038	7	.000
8	.960	.024	12835.707	8	.000
9	.955	.024	14369.964	9	.000
10	.950	.024	15888.504	10	.000
11	.944	.024	17390.711	11	.000
12	.939	.024	18877.128	12	.000
13	.933	.024	20346.480	13	.000
14	.928	.024	21798.823	14	.000
15	.922	.024	23233.427	15	.000
16	.915	.024	24649.630	16	.000

a. The underlying process assumed is independence (white noise).

b. Based on the asymptotic chi-square approximation.

Figure 9: (Autocorrelations Table)

The Box–Ljung test (named for Greta M. Ljung and George E. P. Box) is a type of statistical test of whether any of a group of autocorrelations of a time series are different from zero. Instead of testing randomness at each distinct lag, it tests the "overall" randomness based on a number of lags, and is therefore a portmanteau test. This test is sometimes known as the Box–Ljung Q test, and it is closely connected to the Box–Pierce test (which is named after George E. P. Box and David A. Pierce). In fact, the Box–Ljung test statistic was described explicitly in the paper that lead to the use of the Box-Pierce statistic, and from which that statistic takes its name. The Box-Pierce test statistic is a simplified version of the Box–Ljung statistic for which subsequent simulation studies have shown poor performance. The Box–Ljung test is widely applied in econometrics and other applications of time series analysis.

BRENT
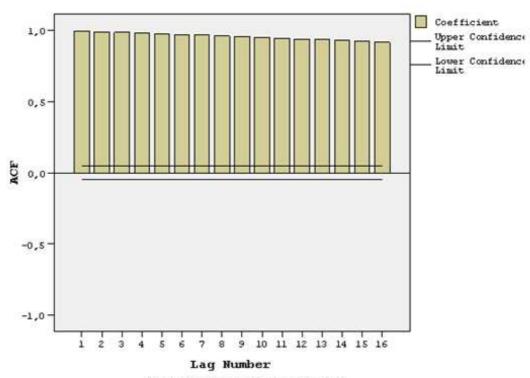
Figure 10: (Autocorrelation Diagram)

As it is shown above in the table from the Box–Ljung test, it is concluded that Brent Oil's future contract price time series is not independently distributed. This conclusion is supported also from the fact that the correlation is not 0, as it is shown on the table in combination with the autocorrelation plot.

Partial Autocorrelations

Series: BRENT

Lag	Partial Autocorrelation	Std. Error
1	.995	.024
2	.024	.024
3	.008	.024
4	-.017	.024
5	-.024	.024
6	.011	.024
7	-.031	.024
8	.002	.024
9	.011	.024
10	-.013	.024
11	-.023	.024
12	.009	.024
13	-.044	.024
14	-.005	.024
15	-.030	.024
16	-.028	.024

Figure 11: (Partial Autocorrelations Table)

On the table above are presented all the calculated results for the partial autocorrelation. Also it is shown that the partial autocorrelation results are not 0.

BRENT
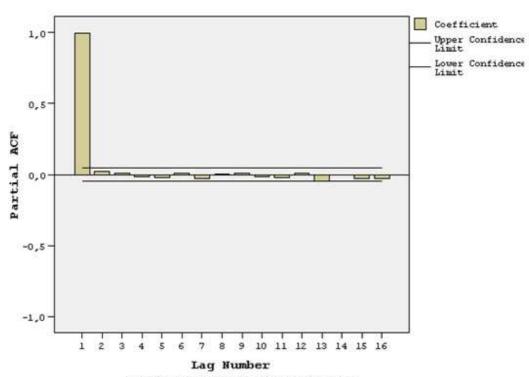
Figure 12: (Partial Autocorrelations Diagram)

As a conclusion from both the autocorrelation and partial autocorrelation calculated results, the Brent Oil's future contract price time series is not stationary. This conclusion defines that other method than linear methods for forecasting values must be used. The non-linear method will be used in this thesis is Chaos Theory.

On the other hand if the goal was to be used a linear method for forecasting, the data should be transformed with '1' degree of difference. As it is shown below the time series after the transformation is made stationary.

Autocorrelations

Series: BRENT

Lag	Autocorrelation	Std. Error[a]	Box-Ljung Statistic Value	df	Sig.[b]
1	-.066	.024	7.274	1	.007
2	-.004	.024	7.305	2	.026
3	.026	.024	8.424	3	.038
4	.053	.024	13.175	4	.010
5	-.044	.024	16.495	5	.006
6	.031	.024	18.131	6	.006
7	-.014	.024	18.444	7	.010
8	.008	.024	18.555	8	.017
9	.002	.024	18.561	9	.029
10	.018	.024	19.135	10	.039
11	-.004	.024	19.156	11	.058
12	.050	.024	23.325	12	.025
13	.026	.024	24.484	13	.027
14	.030	.024	25.956	14	.026
15	.019	.024	26.553	15	.033
16	.028	.024	27.858	16	.033

a. The underlying process assumed is independence (white noise).

b. Based on the asymptotic chi-square approximation.

Figure 13: (Autocorrelations Table)

BRENT
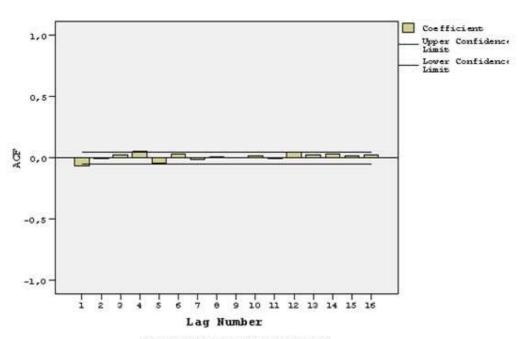
Figure 14: (Autocorrelations Diagram)

Partial Autocorrelations

Series: BRENT

Lag	Partial Autocorrelation	Std. Error
1	-.066	.024
2	-.009	.024
3	.025	.024
4	.057	.024
5	-.037	.024
6	.026	.024
7	-.013	.024
8	.006	.024
9	.006	.024
10	.015	.024
11	.002	.024
12	.048	.024
13	.033	.024
14	.032	.024
15	.022	.024
16	.024	.024

Figure 15: (Partial Autocorrelations Table)

BRENT
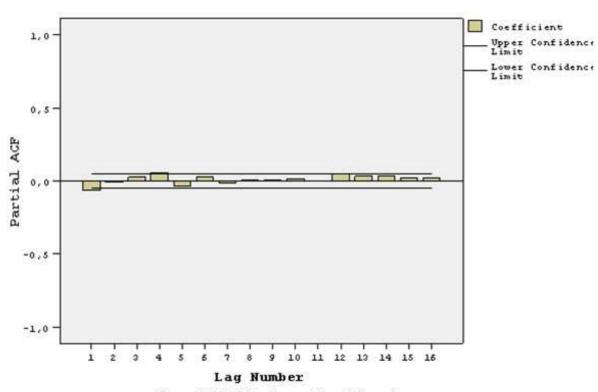
Figure 16: (Partial Autocorrelations Diagram)

In this thesis the Brent Oil's future contract price time series is going to be used is the first one, the time series which is not transformed in order to become stationary. Because, as it is mentioned before in Chaos Theory method, are not considered stationary time series. Time series in which joint probability distribution does not change when shifted in time or space.

CHAOS THEORY - 8

The term `chaos theory' suggests disorder and unpredictability, which is partially right, but does not represent the other very important aspect of chaotic systems—that they are deterministic, with changes occurring only within prescribed borders. Their unpredictability arises because minuscule changes in the starting conditions of a chaotic system can produce widely different outcomes.

A good example of this is a simplified vertical pinball machine, where a ball is dropped and its path is interrupted by two rows of equally spaced pins, finally landing in one of 16 different pockets. The interaction with the first row of pins greatly amplifies any minor difference between one try and the next, and this is again amplified by the interaction with the next row. In practical terms, it is impossible to predict in which pocket the ball will come to rest, even if the greatest care is taken to reproduce the previous result.
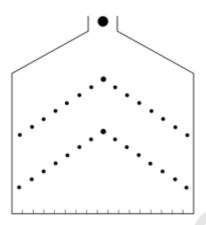
Figure 17: (Cartoon of apparatus to demonstrate the unpredictability of a dropped ball from the same site and height and ending up in a specific hole, its route being altered at random by the first and/or second row of pins.)

A more familiar example of this principle is the problem of weather forecasting. Very small changes in initial conditions can produce large changes in the weather, even in the short to medium term, making forecasting extremely difficult. Lorenz tried to solve this problem during WWII so that the weather could be more accurately predicted for air-force sorties.

Working with a simple computer, he realized that repeated equations in which several decimal places were rounded up (e.g. 2.978658 becomes 2.97866) gave different results, leading to the conclusion that small changes in the initial conditions can lead to highly diverse outcomes. Lorenz predicted that if we had supercomputers (to handle all the decimal places) and means of obtaining accurate information, such as satellites, then we would be able to forecast the day-to-day weather much more accurately.

The advent of supercomputers and satellites has enabled this prediction to come true, at least for short- to medium-term forecasting. Lorenz himself devised a computer and a program to perform some of these calculations, which form the basis of what is loosely known as chaos theory.
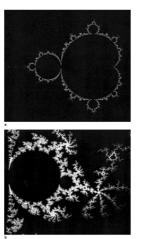
Figure 18: (The famous Mandelbrot set (**a**) derived from iteration of an equation plotted in 2D. Ever-increasing magnification gives rise to complex patterns which have their analogies in nature (**b**,**c**). Note the recurrent appearance of the fractal of (**a)** within these 'sets'.)

It is important to appreciate that a lot of `chaos' results from simple re-iteration of simple equations. Indeed, re-iterating some of these equations can lead to wonderful complex structures, such as the Mandelbrot set which, while ever diversifying at higher magnification, always appears to come back to its original shape, with many repetitions of recognizable forms or fractals visible at different levels of magnification.

It is the ability of chaos to form repeatable, recognizable structures on different scales, such as fractals, and also to return to previous starting points of forms, encompassing what Lorenz called the strange attractor, which provides the excitement in applying this approach to financial time series.

The limitation of variability in that it takes place within a certain wedge of possibilities, was first noted by the French mathematician Poincaré, who noted that the expression of a variable mathematical problem was always confined to a certain area or box.

That chaotic variability has an inherent predictability can be seen in the weather. Although it is extremely difficult to predict the weather in 3 to 4 weeks, it is likely that in 12 months the weather will be roughly similar to now, and that the seasons will follow in a recognizable pattern of change.

Historically, economists have, whenever possible, used linear equations to model economic phenomena, because they are easy to manipulate and usually yield

unique solutions. However, as the mathematical and statistical tools available to economists have become more sophisticated, it has become impossible to ignore the fact that many important and interesting phenomena are not amenable to such treatment.

There is a strong support in economics for both the significance of linear models, and the advantages of nonlinear models. But nonlinear models clearly outperform linear models. Clearly the economic world is nonlinear, so it would appear that focusing on linear dynamics is of limited interest. However economists have typically found nonlinear models to be so difficult and intractable that they have adopted the technique of linearization to deal with them.

Important phenomena for which linear models are not appropriate include depressions and recessionary periods, stock market price bubbles and corresponding crashes, persistent exchange rate movements and the occurrence of regular and irregular business cycles. Therefore, economic

theorists are turning to the study of non-linear dynamics and chaos theory as possible tools to model these and other phenomena.
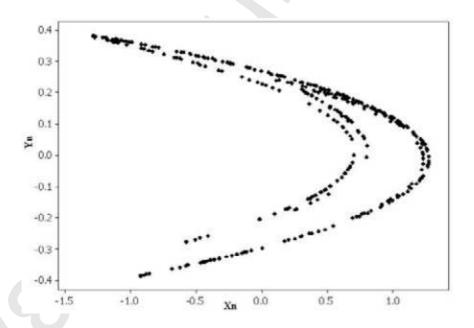

Figure 19: (One dimensional Henon map)

The most exciting feature of nonlinear systems is their ability to display chaotic dynamics. Much economic data has this random-like behavior, but it comes from agents and markets that are presumably rational and deterministic. Random-like

data that economists often encounter might not be coming from a random system. The generating system could be deterministic and perhaps the economy can be explained by a relatively simple nonlinear system.
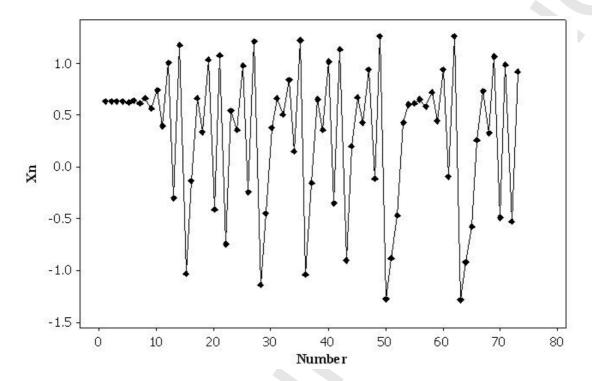
Figure 20: (Two dimensional Henon map)

Chaos is widely found in the fields of physics and other natural sciences. However, the existence of chaos in economic data is still an open question. Since the mid eighties several economists have tried to test for nonlinearity and in particular for chaos in economic and financial time series. One route toward finding a nonlinear underlying system in the economy would be to show that the data itself demonstrates nonlinear or chaotic properties.

In this thesis the non-linearity of the data is tested by calculating the stationarity. As it was found from the autocorrelation and partial autocorrelation plots, the Brent Oil's future contract price time series is a non-linear time series. So for the prediction of the future prices of the time series, the chaos theory will be used.

BRENT OIL'S FUTURE CONTRACT PRICE TIME SERIES – 8.1

According to the theory of observed chaotic data (Abardanel, 1996; Sprott, 2003; Hanias et al., 2007a, 2007b) any non-linear time series can be presented as a set of signals $x = x(t)$. A daily time series of Brent's oil future contract price covering the

period from 24th of November 2004 to 2nd of May 2011 totaling 1672 observations is used for testing the forecasting ability of chaos theory.
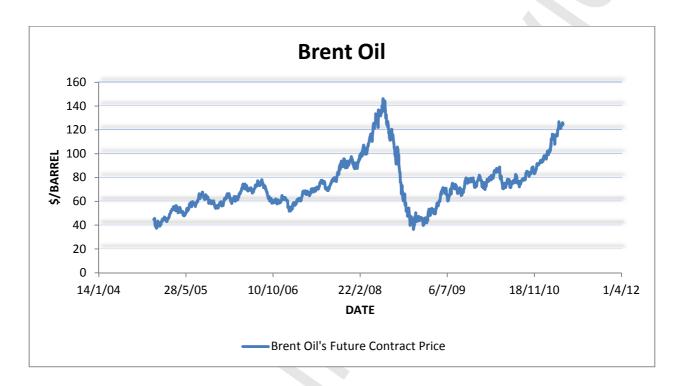
Figure 21: (Brent's Oil Future Contract Price)

As it is presented above in the figure, the Brent's oil future contract price has a maximum of $ 146.19 at 3rd of June 2008 (942nd observation) which was exactly before the financial crisis and a minimum of $ 36.49 at 24th of December 2008 (1066th observation) which was after the financial crisis.

STATE SPACE RECONSTRUCTION – 8.2

TIME DELAY t – 8.2.1

Using the time series which was described above, a vector $_i$, i = 1,2,3….,n, in the m order dimensional phase space given by the following relation (Kantz and Schreiber, 1997; Takens, 1981; Hanias et al., 2007a, 2007b) is constructed as shown in following equation:

$$_i = \{ x_i, x_{i-t}, x_{i-2t}, \ldots, x_{i+(m-1)t} \}$$

where $\vec{x}_i$ represents a point in the m order dimensional phase space in which the attractor is embedded each time. The time delay $\tau = i\Delta t$ is τ and the element x_i represents a value of the examined time series in time, corresponding to the i-th component of the Brent's oil future contract price time series.

The next step is to be found the time delay (τ) and the embedding dimension (m) without considering any other information except of the past values of the corresponding time series.

Fraser and Swinney (1986) and Abarbanel (1996) proposed an equation in order to calculate the time delay by using the average mutual information, the mentioned equation is presented below:

$$I(\tau) = \sum_{xi\,,xi+\tau} P(x_i, x_{i+\tau})log_2\left(\frac{P(x_i, x_{i+\tau})}{P(x_i)P(x_{i+\tau})}\right)$$

Where $I(\tau)$ shows the information in bits being extracted from the value in time x_i to the value in time $x_{i+\tau}$, $P(x_i)$ is the probability of value x_i and $P(x_i, x_{i+\tau})$ denotes joint probability.
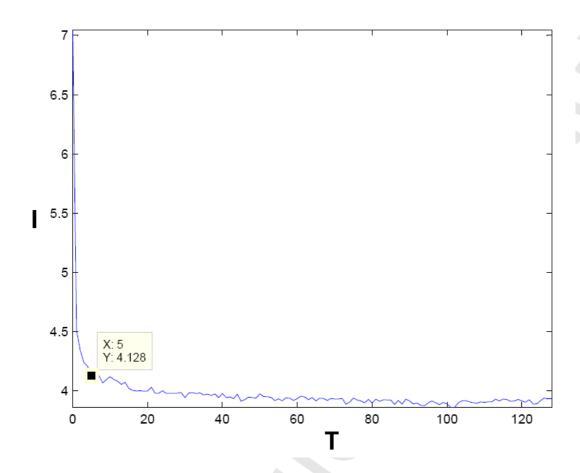
Figure 22: (Mutual information I versus time delay τ)

The time delay is calculated as the first minimum of the mutual information as it is stated in Kantz and Schreiber (1997). For the Brent's oil future contract price time series, the mutual information I versus time delay τ is calculated by using the previous equation for I(τ) and presented on the above table. The mutual information I versus the time delay τ shows a steep diminishing pattern from time delay 0 to time 17 and then the pattern is considerable stable from 17 to time delay 120.

EMBEDDING DIMENSION – 8.2.2

By obtaining the satisfactory value of τ, the embedding dimension m is to be determined in order to finalize the phase space reconstruction. For this purpose a method which was presented by Kantz and Schreiber (1997), Kennel et al. (1992), known as the FNN is used. In more details, the method is based on a fact that when the embedding dimension is too low, the trajectory in the phase space will cross itself. In order to detect these crossings, a decision must be made whether the used m is large enough for correcting the reconstruction of the original phase space in fact when intersections occur or not.

If for a given m, intersections are present it means that the embedding dimension is too low so it is necessary to increase it at least by one. In order to verify it, one tests the eventual presence of self-crossings again as proposed by Kennel et al. (1992) and Abarbanel (1996).

The practical realization of the described method is based on testing the neighboring points in the m-dimensional phase space. Typically, a certain number of points in the phase space are considered in an attempt to find the nearest neighbor to each them.

In the following step the method calculates the distances for all these pairs and also their distances in the (m+1)-dimensional phase space. The rate of these distances is given by the following equation:

$$P = \frac{||y_i(m+1) - y_{n(i)}(m+1)||}{||y_i(m) - y_{n(i)}(m)||}$$

Where P is the rate, $y_i(m)$ represents the reconstructed vector which belongs to i-th point in the m-dimensional phase space, $y_i(m+1)$ represents the reconstructed vector which belongs to i-th point in the m+1 dimensional phase space and index n(i) denotes the nearest neighbor to the i-th point.

If P is greater than a certain max value (P_{max}), the corresponding pair of points is known as the FNN, in fact neighbors which arise from trajectory self-intersections and not from the closeness in the original phase space. In the ideal case, when the number of FNN falls to zero, then the value of m is precisely determined. For this purpose the rate of FNN in the reconstructed phase space is computed by using the following equation:

$$\left| x_{i+m\tau} - x_{n(i)+m\tau} \right| \geq R_A$$

Where R_A is the radius of the attractor as it is given in the following equation:

$$R_A = \frac{1}{N} \sum_{i=1}^{N} |x_i - \bar{x}|$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Where $\bar{x}$ is the average value of Brent's oil future contract price time series, which is calculated for finding the R_A.

If $P \geq P_{max}$, then $P = \dfrac{||y_i(m+1) - y_{n(i)}(m+1)||}{||y_i(m) - y_{n(i)}(m)||}$ can be

used to distinguish between true and FNN as it is stated in Kennel et al. (1992). The dimension m is set when the percentage of the FNN decreases below some limit, typically set to 1%, as it is stated at Kugiumtzis et al. (1994).

By choosing $P_{max} = 10$ the matlab code is able to calculate the mutual information I and the corresponding quantity P. The rate of the FNN that is under the above limit is achieved for m= 7, thus this value should be suitable for the purpose of phase space reconstruction as it shown in the figure below for the Brent's oil future contract price time series:
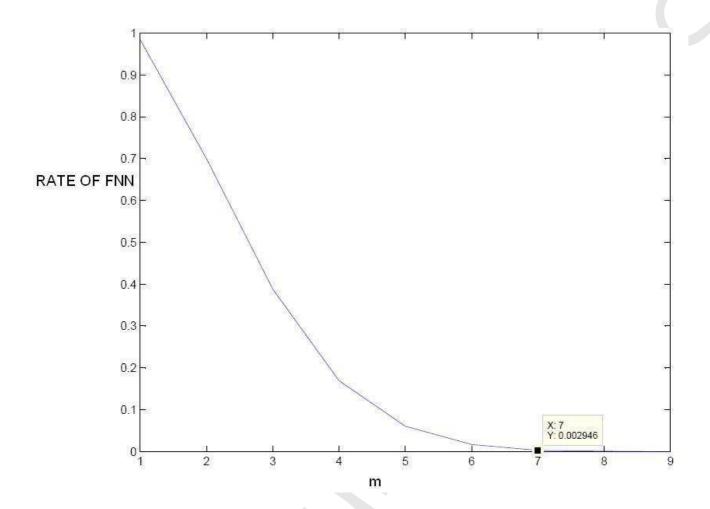
Figure 23: (Percent rate of the FNN versus m)

THE EMPIRICAL EVIDENCE FOR BRENT'S OIL PRICE – 8.3

The reconstructed m-dimensional signal projected into the state space can exhibit a range of trajectories, some of which have structures or patterns that can be used for system prediction and modeling. Essentially, in order to be predicted k steps ahed the future from the last m-dimensional vector point $\{x_N^m\}$, all the nearest neighbors $\{x_{NN}^m\}$ are calculated in the ε-neighborhood of this point.

For getting this more clear, let $B_\varepsilon(\chi_N^m)$ be the set of point within ε of $\{x_N^m\}$, in other words the ε-ball. Thus any point in $B_\varepsilon(\chi_N^m)$ is closer to the $\{x_N^m\}$ within the ε. All these points $\{x_{NN}^m\}$ come from the previous trajectories of the system, therefore it is possible to follow their evolution k-steps ahead the future $\{x_{NN+k}^m\}$. The final prediction for the point $\{x_N^m\}$ is obtained by averaging over all neighbors' projections k-steps ahead the future as it is shown in the following equation:

$$\{x_{N+k}^m\} = \frac{1}{|B_\epsilon(\chi_{NN}^m)|} \sum_{\chi_{NN}^m \in B_\epsilon(\chi_{NN}^m)} (x_{NN+k}^m)$$

Where $|B_\epsilon(\chi_{NN}^m)|$ denotes the number of the nearest neighbors in the neighborhood of the point $\{x_N^m\}$ (Kantz and Schreiber, 1997).

As an example let us suppose that we want to predict k = 2 steps ahead. The $\{x_N^m\}$ represents the last known sample from which we want to predict one and two steps into the future. The next step in the algorithm by using the equation for the $\{x_{NN+k}^m\}$ is to be checked that the projections, one and two steps into the past, of the points in $\{x_{NN}^m\}$ are also nearest neighbors of the two previous readings $\{x_{N-1}^m\}$ and $\{x_{N-2}^m\}$ respectively. This criterion excludes unrelated trajectories that enter and leave the ε-neighborhood of $\{x_N^m\}$ but do not track back to ε-neighborhoods of $\{x_{N-1}^m\}$ and $\{x_{N-2}^m\}$, thus making them unsuitable for prediction.

By assuming that any nearest neighbors have been found and checked using the appropriate criterion as it is stated above the projection of their trajectories into the future is possible. Therefore, by taking their average, projections for $\{x_{N+1}^m\}$ and $\{x_{N+2}^m\}$ are obtained.

By using the values of τ and m from the previous step the appropriate time delay τ is chosen. In the present case the time delay τ is chosen to be τ = 5, as the first minimum in the plot (I vs τ) and the best number of the nearest neighbors of embedding dimension m = 7, because it is below the limit 1% (percentage of the FNN) as it was stated by Kugiumtzis et al. 1994 for selecting the embedding dimension.

The above procedure was followed for every k=10 time steps ahead prediction in this essay. The prediction error for establishing the quality of the fit was chosen to be the classical normalized root mean square error (NRMSE) as it is presented in the equation below:

$$NRMSE = \frac{\sqrt{\frac{\sum_{t=1}^{N}(x - \hat{x})^2}{N}}}{x_{max} - x_{min}}$$

Where x denotes the original data, $\hat{x}$ denotes the predicted data, and x_{max} is the maximum of time series while x_{min} is the minimum value of time series.

As it is calculated the NRMSE = 2.47%. Below are presented, the plot of the predictions with k=10 steps ahead vs the actual values of Brent's oil future contract price time series for the tested period of 7th of June 2010 to 2nd of May 2011 together with the forecasted values of the time series ,and the table with all the prices and errors.
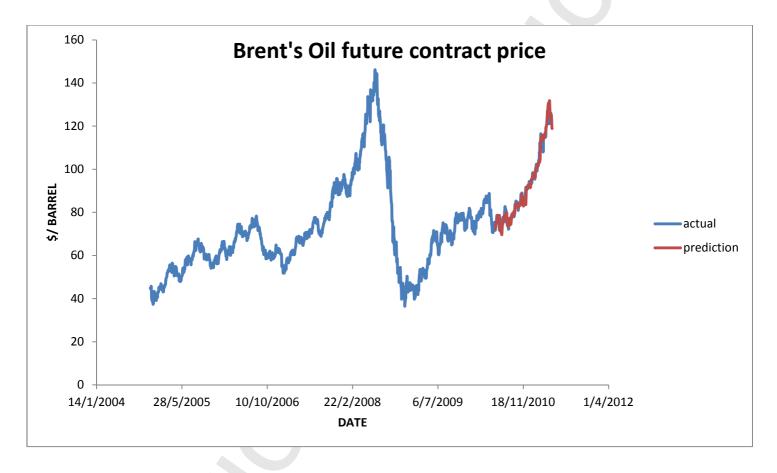
Figure 24: (Brent's oil future contract price actual and predicted.)

In addition below is presented a more close look on the predictions and the actual close prices of the Brent's Oil future contract price.
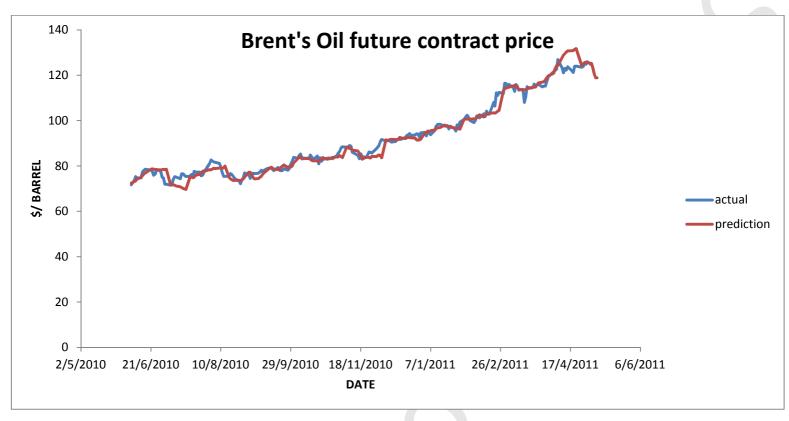
Figure 25: (Brent's oil future contract price actual and predicted.)

Below is presented a table with all the values together, are presented the predicted close prices of Brent's Oil future contract price and the actual close prices. In addition are presented the APE (absolute percentage error) for each time step ahead and the NRMSE.

DATE	ACTUAL PRICE	PREDICTED PRICE	APE	NRMSE = 2.47%
7/6/2010	71.69	72.45	1.05%	
8/6/2010	72.51	72.77	0.36%	
9/6/2010	73.6	73.18	0.57%	
10/6/2010	75.31	73.4	2.60%	
11/6/2010	74.79	74.17	0.84%	
14/6/2010	74.73	75.01	0.37%	
15/6/2010	77.32	75.77	2.05%	
16/6/2010	78.12	76.37	2.29%	
17/6/2010	78.59	76.99	2.08%	
18/6/2010	78.42	77.23	1.54%	
21/6/2010	78.22	78.59	0.47%	
22/6/2010	77.75	78.73	1.24%	
23/6/2010	75.92	78.59	3.40%	
24/6/2010	76.41	78.5	2.66%	
25/6/2010	78.41	78.35	0.08%	
28/6/2010	77.65	78.32	0.86%	
29/6/2010	75	78.36	4.29%	
30/6/2010	74.73	78.55	4.86%	
1/7/2010	71.96	78.35	8.16%	

2/7/2010	71.93	78.58	**8.46%**
5/7/2010	71.5	71.86	**0.50%**
6/7/2010	71.66	71.6	**0.08%**
7/7/2010	74.1	71.69	**3.36%**
8/7/2010	75.25	71.56	**5.16%**
9/7/2010	75.16	71.2	**5.56%**
12/7/2010	74.35	70.84	**4.95%**
13/7/2010	76.55	70.48	**8.61%**
14/7/2010	76.56	70.08	**9.25%**
15/7/2010	75.92	69.9	**8.61%**
16/7/2010	75.35	69.62	**8.23%**
19/7/2010	75.5	75.11	**0.52%**
20/7/2010	76.25	75.1	**1.53%**
21/7/2010	75.45	74.78	**0.90%**
22/7/2010	77.7	75.03	**3.56%**
23/7/2010	77.34	75.87	**1.94%**
26/7/2010	77.3	76.32	**1.28%**
27/7/2010	75.6	76.71	**1.45%**
28/7/2010	76	77.65	**2.12%**
29/7/2010	77.35	77.9	**0.71%**
30/7/2010	78.26	77.74	**0.67%**
2/8/2010	81.05	78.4	**3.38%**
3/8/2010	82.7	78.22	**5.73%**
4/8/2010	82.1	78.66	**4.37%**
5/8/2010	81.73	78.98	**3.48%**
6/8/2010	81.61	78.86	**3.49%**
9/8/2010	81.14	79.1	**2.58%**
10/8/2010	79.61	79.14	**0.59%**
11/8/2010	77.12	78.97	**2.34%**
12/8/2010	75.4	79.23	**4.83%**
13/8/2010	75.4	79.98	**5.73%**
16/8/2010	75.49	74.84	**0.87%**
17/8/2010	76.65	74.28	**3.19%**
18/8/2010	76.18	73.88	**3.11%**
19/8/2010	75.35	73.57	**2.42%**
20/8/2010	74.33	73.68	**0.88%**
23/8/2010	73.44	73.65	**0.29%**
24/8/2010	72.15	73.5	**1.84%**
25/8/2010	73.83	73.92	**0.12%**
26/8/2010	74.72	74.23	**0.66%**
27/8/2010	76.93	75.07	**2.48%**
30/8/2010	76.07	77.25	**1.53%**
31/8/2010	74.48	77.26	**3.60%**
1/9/2010	76.26	76.39	**0.17%**
2/9/2010	76.82	75.06	**2.34%**
3/9/2010	76.62	74.25	**3.19%**
6/9/2010	76.81	74.46	**3.16%**
7/9/2010	77.38	75.02	**3.15%**
8/9/2010	78.17	75.5	**3.54%**
9/9/2010	77.35	76.3	**1.38%**
10/9/2010	78.15	76.95	**1.56%**
13/9/2010	79.11	78.62	**0.62%**
14/9/2010	78.91	78.98	**0.09%**
15/9/2010	79.2	79.43	**0.29%**

16/9/2010	78.47	79.02	**0.70%**
17/9/2010	77.97	78.59	**0.79%**
20/9/2010	79.36	78.45	**1.16%**
21/9/2010	78.1	78.31	**0.27%**
22/9/2010	77.98	79.45	**1.85%**
23/9/2010	77.93	79.96	**2.54%**
24/9/2010	78.8	80.48	**2.09%**
27/9/2010	78.14	79.28	**1.44%**
28/9/2010	78.85	79.75	**1.13%**
29/9/2010	80.67	79.48	**1.50%**
30/9/2010	82.11	79.94	**2.71%**
1/10/2010	83.81	81.34	**3.04%**
4/10/2010	83.3	83.14	**0.19%**
5/10/2010	84.65	84.16	**0.58%**
6/10/2010	85.3	83.59	**2.05%**
7/10/2010	83.11	83.63	**0.62%**
11/10/2010	83.46	83.24	**0.26%**
12/10/2010	83.39	83.23	**0.19%**
13/10/2010	84.84	82.96	**2.27%**
14/10/2010	84.02	82.49	**1.85%**
15/10/2010	82.76	82.27	**0.60%**
18/10/2010	84.34	82.25	**2.54%**
19/10/2010	80.89	82.01	**1.37%**
20/10/2010	83.65	82.45	**1.46%**
21/10/2010	82.04	83.11	**1.29%**
22/10/2010	82.9	83.74	**1.00%**
25/10/2010	83.38	82.93	**0.54%**
26/10/2010	83.5	83.15	**0.42%**
27/10/2010	83.34	83.16	**0.22%**
28/10/2010	83.55	83.55	**0.00%**
29/10/2010	83.14	83.83	**0.82%**
1/11/2010	84.59	83.94	**0.77%**
2/11/2010	85.65	84.02	**1.94%**
3/11/2010	86.57	84.53	**2.41%**
4/11/2010	88.13	84.11	**4.78%**
5/11/2010	88.47	83.7	**5.70%**
8/11/2010	88.22	88.33	**0.12%**
9/11/2010	87.97	88.42	**0.51%**
10/11/2010	89	87.67	**1.52%**
11/11/2010	88.72	87.73	**1.13%**
12/11/2010	86.04	86.98	**1.08%**
16/11/2010	84.75	86.64	**2.18%**
17/11/2010	83.16	85.73	**3.00%**
18/11/2010	85.35	84.37	**1.16%**
19/11/2010	84.13	82.96	**1.41%**
22/11/2010	83.73	83.92	**0.23%**
24/11/2010	86.21	83.72	**2.97%**
25/11/2010	85.91	83.54	**2.84%**
26/11/2010	85.8	84.3	**1.78%**
29/11/2010	87.45	84.09	**4.00%**
1/12/2010	88.92	84.85	**4.80%**
2/12/2010	90.64	84.66	**7.06%**
3/12/2010	91.68	83.57	**9.70%**
6/12/2010	90.99	91.53	**0.59%**

8/12/2010	91.11	91.39	0.31%
9/12/2010	90.97	91.63	0.72%
10/12/2010	90.53	91.8	1.38%
13/12/2010	90.79	91.64	0.93%
15/12/2010	92.05	91.7	0.38%
16/12/2010	91.73	92.6	0.94%
17/12/2010	91.75	92.33	0.63%
20/12/2010	92.6	92.07	0.58%
21/12/2010	93.5	92.39	1.20%
22/12/2010	93.74	92.71	1.11%
23/12/2010	94.33	92.56	1.91%
24/12/2010	93.46	92.4	1.15%
27/12/2010	93.67	92.25	1.54%
28/12/2010	94.21	91.5	2.96%
29/12/2010	94.05	91.35	2.96%
30/12/2010	92.65	91.55	1.20%
31/12/2010	94.59	91.75	3.10%
3/1/2011	94.84	94.67	0.18%
4/1/2011	93.26	94.96	1.79%
5/1/2011	95.43	94.95	0.51%
6/1/2011	94.43	95.22	0.83%
7/1/2011	93.72	95.56	1.93%
10/1/2011	95.68	95.9	0.23%
11/1/2011	97.5	96.52	1.02%
12/1/2011	98.26	96.76	1.55%
13/1/2011	98.4	96.86	1.59%
14/1/2011	98.36	97	1.40%
17/1/2011	97.59	98.08	0.50%
18/1/2011	97.58	97.78	0.20%
19/1/2011	97.88	97.35	0.54%
20/1/2011	96.32	97.41	1.12%
21/1/2011	97.59	97.11	0.49%
24/1/2011	96.52	96.79	0.28%
25/1/2011	95.38	96.8	1.47%
26/1/2011	98.14	96.67	1.52%
27/1/2011	97.24	96.43	0.84%
28/1/2011	99.33	96.19	3.26%
31/1/2011	100.5	100.6	0.10%
1/2/2011	101.55	100.7	0.84%
2/2/2011	102.33	100.7	1.62%
3/2/2011	101.75	100.7	1.04%
4/2/2011	100.1	100.7	0.60%
7/2/2011	99.15	100.8	1.64%
8/2/2011	100.2	100.7	0.50%
9/2/2011	102.1	101.4	0.69%
10/2/2011	101.62	102	0.37%
11/2/2011	101.16	102.6	1.40%
14/2/2011	102.99	101.6	1.37%
15/2/2011	101.79	102.1	0.30%
16/2/2011	104.22	102.7	1.48%
17/2/2011	102.83	103.1	0.26%
18/2/2011	102.82	103.2	0.37%
21/2/2011	108	103.4	4.45%
22/2/2011	106.39	103.3	2.99%

23/2/2011	112.21	103.6	**8.31%**
24/2/2011	111.13	104.2	**6.65%**
25/2/2011	112.43	104.5	**7.59%**
28/2/2011	111.94	113.2	**1.11%**
1/3/2011	116.46	114.2	**1.98%**
2/3/2011	116.34	114.3	**1.78%**
3/3/2011	114.59	114.6	**0.01%**
4/3/2011	115.99	114.8	**1.04%**
7/3/2011	114.55	115.3	**0.65%**
8/3/2011	112.89	115.5	**2.26%**
9/3/2011	115.88	115.7	**0.16%**
10/3/2011	115.2	114.6	**0.52%**
11/3/2011	113.44	113.8	**0.32%**
14/3/2011	113.78	113.5	**0.25%**
15/3/2011	108	113.6	**4.93%**
16/3/2011	110.4	113.7	**2.90%**
17/3/2011	115	114.1	**0.79%**
18/3/2011	114.49	114.2	**0.25%**
21/3/2011	114.6	114.5	**0.09%**
22/3/2011	116.09	114.8	**1.12%**
23/3/2011	115.27	114.8	**0.41%**
24/3/2011	115.75	115.8	**0.04%**
25/3/2011	116.02	116.6	**0.50%**
28/3/2011	114.85	117	**1.84%**
29/3/2011	115.2	117.2	**1.71%**
30/3/2011	115.23	117.9	**2.26%**
31/3/2011	117.14	118.8	**1.40%**
1/4/2011	119.1	119.7	**0.50%**
4/4/2011	120.98	120.5	**0.40%**
5/4/2011	121.58	120.9	**0.56%**
6/4/2011	122.12	122.9	**0.63%**
7/4/2011	122.61	124.2	**1.28%**
8/4/2011	126.9	124.3	**2.09%**
11/4/2011	123.2	127.8	**3.60%**
12/4/2011	121.07	128.9	**6.07%**
13/4/2011	123.04	129.5	**4.99%**
14/4/2011	122.29	130.2	**6.08%**
15/4/2011	123.73	130.7	**5.33%**
18/4/2011	121.91	130.8	**6.80%**
19/4/2011	121.18	130.9	**7.43%**
20/4/2011	123.95	131.4	**5.67%**
21/4/2011	124.04	131.8	**5.89%**
25/4/2011	123.55	124.3	**0.60%**
26/4/2011	123.8	124.9	**0.88%**
27/4/2011	125.52	125.5	**0.02%**
28/4/2011	124.81	125.7	**0.71%**
29/4/2011	126.03	125.5	**0.42%**
2/5/2011	124.56	125.2	**0.51%**
3/5/2011		123.6	
4/5/2011		124.1	
5/5/2011		124	
6/5/2011		122.8	

9/5/2011	121.5
10/5/2011	121.3
11/5/2011	121.5
12/5/2011	121.6
13/5/2011	121.5
16/5/2011	121.3

Figure 26: (Predictions)

For these last 10 time steps ahead values of the Brent's Oil future contract price, will be calculated the APE (absolute percentage error) will be expected in 95% confidence interval. Below is presented the APE in 95% confidence interval, as calculated using the previous APE on the tested period.

	N	Mean	Std. Deviation	Std. Error Mean
APE	229	.0204784	.02077197	.00137265

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
	Lower	Upper	Lower	Upper	Lower	Upper
APE	14.919	228	.000	.02047843	.0177737	.0231831

Figure 27: (Tables showing the APE in 95% Confidence Interval.)

As it is shown above in the tables, in a 95% Confidence Interval the APE it will be expected for these predictions to be from 1.78% to 2.32%, which is an accepted APE.

In the following chapter it will be described in details the conclusion from the predictions according to the APE each of them has related to the actual close price of the Brent's Oil future contract price time series.

CONCLUSION - 9

As it is shown on the table above the APE (Absolute Percentage Error) that was calculated is below 5% in the majority of the predictions. The APE which was calculated above 5% was found because that specific date an unexpected event took place and shocked the financial markets even positive or negative.

That unexpected event could be a political or economical event. Financial markets are very sensitive to that kind of events because investors are humans and their decisions are influenced by news related to the financial instruments they decide to invest.

In this chapter will be presented the unexpected events which took place the dates when our predictions' APE were calculated and found above 5%. In order to be achieved that; the online newspaper REUTERS will be used.

At 1st of July 2010's prediction it is presented an APE of 8.16%. That day an unexpected event by the markets shocked the investors and made them sold aggressively the Brent's future contract and posed it to post losses. The unexpected event was that weak economic data from China and the United States added to worries over the global economic recovery and oil demand.

The economic data for United States of America were that the growth in U.S. manufacturing activity in the previous month had slowed to its lowest level since December 2009 and that the U.S. contracts for pending sales of previously owned homes had plunged a record 30 percent in May 2010.

On the other side an official survey showed the pace of Chinese manufacturing activity the previous month was the weakest since February 2010.

The next day, at 2nd of July 2010's prediction, again it is presented an APE of 8.46%, and it is appeared because of the previous day's unexpected event in combination with the consequences.

At 8th and 9th of July 2010's prediction it is shown an APE of 5.16% and 5.56% respectively. These two absolute percentage errors occurred because of an unexpected event which took place at 7th of July 2010 and caused the price of Brent's future contract to rise instead of being stable as it is predicted.

The unexpected event was the release of the EIA's report, which had been revised upwards, of its monthly world oil demand forecast. The agency had said that expects world demand to climb by 1.56 million barrels per day in 2010, to 85.82 million bpd, from a year earlier.

Traders also were closely monitoring a potential tropical depression, that United States of America's government storm trackers had said could hit the southern Gulf of Mexico. All the previous in combination with the positive mood of the investors that day, who made the Wall Street rallied, helped the Brent's future contract price to rise and differ from the prediction made by Chaos Theory.

Above 5 percent error, related to the actual close price of Brent's future contract, it is presented at $13^{th} - 16^{th}$ of July 2010. At 13th of July 2010 the Brent's future contract price rose after a Wall Street rally sparked by earnings optimism which had encouraged energy investors. That day the future contract's price gained nearly 3 percent.

Also, supportive for crude oil was the fact that EUR/USD exchange rate hit a two month high above \$1.27, after a smooth Greek Treasury bill auction had eased some concerns about Europe's debt crisis.

After 13th of July 2010's rise on Brent's oil future contract price which caused by the mentioned unexpected events, the close prices for the above mentioned dates were almost stable instead of getting lower and being closer to the predictions.

At 3rd of August 2010's prediction it is shown an APE 5.73 percent which actually caused by an unexpected event's huge upward trend that took place at 2nd of August 2010 and continued till 3rd of August 2010.

That date was given to publicity a statement from Enbridge Inc's chief executive who had said on Sunday that "the company would not restart its ruptured pipeline in Michigan this week as it continues to clean up 800000 gallons of oil spilled in and around the Kalamazoo River".

In addition Mr But Lutts had said to investors that "were focused on BP's planned "static kill" on Tuesday 3rd of August 2010, when it plans to inject drilling mud into the top of the well to push oil back where it came from -- a reservoir 13,000 feet/ (4,000 metres) beneath the seabed". Those both announcements caused the Brent's oil future contract price to draw an upside trend because of the disruption of the crude oil's production.

The price of Brent's oil future contract as all the products is a result of demand and supply. On the above paragraph it is stated a shortage in the oil market because the production was reduced, so the price of the future contract increased by considering that unexpected event.

On Friday 13th of August 2010's prediction it is presented an APE 5.73%. The contraction of Brent's oil future contract price caused by an event that took place at 12th of August 2010 and was not expected by financial markets.

The financial markets erased gains because of an unexpected rise in jobless claims in United States of America. Which was, according to analysts, the worst percentage decline in more than a month. The number of U.S. workers filing new claims for unemployment benefits had risen to nearly six-month high and had increased ongoing fears about the weak labor market.

So, that posed the Brent's oil future contract price to write losses in combination with the restart of gas operations in Gulf of Mexico. Several oil and gas companies had restarted operations and begun returning nonessential personnel to offshore infrastructure, as Tropical Depression Five, had fizzled out into a cluster of thundershowers after it had moved through the Gulf of Mexico and inland over southeastern Louisiana the previous day.

At 5th of November 2010's prediction it is shown an APE 5.7 percent. That APE was caused by a big rise in Brent's oil future contract prices which actually was happened on Wednesday 3nd of November 2010. After that the Brent's prices followed that upward trend and caused the APE 5.7%.

That date the Federal Reserve launched a controversial policy to buy more government debt. The Federal Reserve announced that would buy $ 600 billion more in Treasuries by the middle of the following year in an attempt to reinvigorate a flagging recovery.

With that decision automatically the Federal Reserve pumped cheap money to the financial markets and made the investors thinking riskier assets for their portfolio. So the investors attracted by the commodities, and most of them from the Brent's oil future contracts, as it was shown from the huge rise in the price of Brent that date. On Wednesday 3rd of November the Brent's oil future contract price gained almost 1$ which means around 1.1 percent increase.

The following days, investors continued investing in riskier assets and continued their strategy which was buying the Brent's oil future contract. In financial

markets, that is called "carry trade". They used the dollar as a funding currency, as it was cheap after the Federal Reserve's decision to print more money, to invest it into commodities, emerging markets and higher – yielding currencies.

At 2nd and 3rd of December 2010's predictions the APE , which are presented on the table, are above 5 percent. Again the absolute percentage errors caused by an unexpected event, which made investors buying Brent's oil future contract. The unexpected event was happened on Tuesday 23rd of November 2010, at the time the market for Brent's oil future contact was closed.

That date North Korea rained artillery shells on the island of Yeonpyeong, south of the disputed maritime boundary with the South. Two soldiers and two civilians were killed and was the first time civilians had died in an attack by the North since 1987. Tensions remained high on the peninsula as a U.S. aircraft carrier group set off Korean waters.

That unexpected event is a political event in the category of war. As always happens when a war appears or war's rumors appear, the crude oil's market experiences a huge boost. So, the following date at 24th of November 2010, when the market for Brent's oil future contract opened, many investors started buying Brent aggressively. That posed the Brent's oil future contract price to post gain $ 2.48, almost 3 percent rise from the previous date's close.

The following days the Brent's oil future contract price continued that upward trend and that situation causes the shown absolute percentage errors, above 5 percent to the dates 2nd and 3rd of December 2010.

On Wednesday 23rd of February 2011 till Friday 25th of February 2011's predictions are shown APE above 5 percent which actually caused from the unexpected event that was happened on Wednesday 23rd of February 2011. That event made the Brent's oil future contract price to gain $ 5.82, which was almost 5.5 percent gain.

The unexpected event that happened that date was a political event in Libya which disrupted country's oil production. In details, that date fears persisted that Libya's unrest could spread to other producers in the Middle East and North Africa, which would be raising oil's geopolitical risk premium by investors.

The fact that European Union's governments agreed to pave the way for possible sanctions against Libya after Muammar Gaddafi's violent suppression of anti – government protests (Muammar Gaddafi had killed as many as 1000 people in his

way to crush a revolt against his rule), in addition with the statement of Barack Obama, who said that the violent crackdown in Libya had violated international norms and that he had ordered his national security team to prepare the full range options dealing with the crisis, reinforced the assumption that this Libya's unrest wasn't a short time crisis.

Also, the Brent's oil future contract price ended up after the American Petroleum Institute said that United States of America's crude stockpiles rose 163000 barrels the previous week.

Both the above unexpected events caused the huge gains on the Brent's oil future contract prices and made it differ from the predictions which are presented on the table above.

At the dates 12th, 14th, 15th, 18th, 19th, 20th and 21st of April 2011's predictions are shown on the table absolute percentage errors above 5 percent. Instead of an upside trend as it was predicted by Chaos Theory, there is a downside trend presenting that APE above 5 percent. The trend of the Brent's oil future contract price, changed because some events which were not expected by the market. The date when everything started was on Monday 11th of April 2011.

First of all was the fact that, the African Union said that the Libyan leader Muammar Gaddafi had accepted a road map to end the civil war, including an immediate ceasefire. This statement created hopes that the Libyan issue would come to an end in the short term, giving to traders of Brent's oil future contract the excuse to start selling the future contract.

Another event was a statement from the investment bank Goldman Sachs, which was stated that there was a strong chance commodity prices to reverse and the recommendation to take profits. The Goldman Sachs' Jeffrey Currie said in a note to clients that "not only there are now nascent signs of oil demand destructions in the United States, but also record speculative length in the oil market, elections in Nigeria and a potential ceasefire in Libya that has begun to offset some of the upside risk owing to contagion".

As it seems, all the above unexpected events in addition with the investor's concerns that high prices could erode demand and could pull economic recovery back, made the Brent's oil future contract price that date to post loss around 3% and follow that downside trend the following days.

So, as a conclusion the APE (Absolute Percentage Error) that was calculated and found to be above 5 percent on the predictions made by Chaos Theory, seems to be justified because of the existence of all those unexpected events. Of course must be done further research in order to be achieved lower APE on the predictions.

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