University of Piraeus
Department of Banking and Financial Management

Should investors invest in volatility? Evidence from the volatility derivatives market.

DIMITRIOS KRANAS
(mxrh 1010)

George Skiadopoulos : Associate Professor (supervisor)
Dimitrios Malliaropoulos : Professor
Dimitrios Voliotis : Lecturer

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Dimitris Kranas

Supervisor : Associate Professor George Skiadopoulos

Abstract

This paper investigates whether investors can improve their investment opportunity set that consists of traditional asset classes through the addition of VIX-related assets (Spot VIX and futures on VIX). First, we revisit the posed question within an in-sample setting by employing mean-variance and non-mean-variance spanning tests. To the best of our knowledge no previously published study has ever examined the results of non mean-variance spanning regarding VIX-related assets. Then, we form optimal portfolios by taking into account the higher order moments of the portfolio returns distribution and evaluate their out-of-sample performance. Under the in-sample setting, we find that VIX-related assets are beneficial both to mean-variance and to non mean-variance investors. Furthermore, these benefits are preserved out-of-sample. Our findings confirm the diversification benefits of VIX-related assets and are robust across a number of performance evaluation measures ,utility functions and datasets. The results hold even when transaction costs are considered .

Key words:

Volatility

VIX

VIX FUTURES

Spanning

Generalized method of moments (GMM)

Direct utility maximization

In-Sample analysis

Out-of-sample analysis

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1. Introduction

VIX is the ticker symbol for the Chicago Board Options Exchange Market Volatility Index, a popular measure of the implied volatility of S&P 500 index options .Volatility analysis shows that it hits its highest levels during periods of market turbulence .VIX is often referred as the fear index or the fear gauge, and it represents one measure of the market's expectation of stock market volatility . VIX represents the 30-calendar-day volatility, which is calculated from a model-free formula using the prices of a portfolio of out-of-the-money S&P 500 index (SPX) calls and puts whose weights are inversely proportional to the squared strike price. Moreover, the VIX formula also implies that the VIX-squared is replicable with a static portfolio of SPX options and thus allows the implementation of a tradable strategy for volatility speculation and hedging. Therefore, VIX futures and VIXsquared portfolios offer investors opportunities to expose their investment positions to the volatility risk directly. It is also possible to trade volatility by pure exposure to volatility alone without being affected by directional movements of the underlying asset. Classic methods for trading volatility, such as buying at-the-money (ATM) straddles, do not meet the demand of pure volatility exposure. They require frequent rebalancing to keep the options portfolio delta-neutral, which imposes high transaction costs. At first sight, investing in volatility appears attractive since volatility movements are known to be negatively correlated with stock index returns. Thus, adding volatility exposure to a portfolio of common stocks promises to improve risk diversification. In addition, past experience indicates that negative correlation is particularly pronounced in stock market downturns, offering protection against stock market losses when it is needed most.

For example, Dumas et al.(1998) find that the correlation between S&P 500 index returns and changes in the Black–Scholes implied volatility of S&P 500 index options is _0.57 from June 1, 1988 through December 31, 1993. Moreover, Whaley (2000) find not only a negative correlation but an asymmetric relation between stock market returns and changes in the VIX.2 He shows that if the VIX falls by 1%, the S&P 100 index will rise by 0.47%, whereas when the VIX rises by 1%, the S&P 100 index will fall by _0.71%. The evidence indicates that volatility derivatives may offer different risk return characteristics in comparison to the existing assets in the financial markets. This paper examines empirically the common perception on the diversification role of VIX-related assets by investigating the benefits of investing in VIX-related assets in a more general setting than the one that the previous literature has adopted so far .

In fact there exist only a few papers that examine whether the incorporation of VIX-related assets in a benchmark portfolio (that consists of stocks and bonds) is beneficial to investors . Chen , Chung and Ho (2010) employ mean-variance spanning and intersection tests to examine whether the addition of a VIX-related asset (spot VIX , VIX futures , VIX-squared portfolio) can significantly expand the investment opportunity set for investors relative to different groupings of benchmark portfolios . They prove that occurs a shift in the mean-variance frontier indicating that adding VIX-related assets may provide diversification benefits . Also they perform out-of-sample tests to support the robustness of their in-sample results. They find out that Sharpe ratios for the augmented portfolios that include VIX-related assets are much larger than for the benchmark ones not including VIX-related assets .Optimal portfolio weights are positive for spot VIX and negative for VIX futures . Alexander and Korovilas (2011) investigate whether is has ever been optimal to add a long VIX futures position to a long position on the S&P 500 within the Markowitz and Black-Litterman framework . Their analysis is limited to long equity investors and to VIX futures. They find that S&P achieves higher Sharpe ratios than VIX futures during the bullish period (April 2004-May 2007) while VIX futures achieve higher Sharpe ratios than S&P during the bearish period (June 2007- June 2010). Hafner and Wallmeier (2008) use an ex-post analysis to demonstrate the benefits of adding variance swaps to European equity portfolios . Egloff , Leippold , and Wu (2010) also focus on variance swaps , modeling their dynamics in a two-factor model and promoting their diversification benefits as well as those of volatility futures .

Szado (2009) considers the diversification of S&P 500 exposure using VIX futures and options and SPX put options, claiming that a long volatility exposure is beneficial for diversification and that VIX derivatives are more efficient than an exposure through SPX options. They consider the crash period and they find that the spot VIX performs better that VIX futures. Several studies, e.g. Daigler and Rossi (2006), Dash and Moran (2005) and Pezier and White (2008) use the spot VIX, which is not tradable. Moran and Dash (2007) show that the desirable qualities of VIX do not always carry over to VIX futures and options. Delisle Doran Krieger (2010) find that due to the asymmetric and negative relation between VIX and S&P 500 returns, the VIX index provides a particularly effective hedge against market declines without proportionally penalizing performance when there are market gains. They find that the returns to VIX futures contracts are more negative when the S&P 500 is increasing, while the opposite is true of the VIX index itself. This contrast between the VIX futures and the index suggests the futures contracts did not offer the same downside protection investors would expect given the asymmetric relationship between volatility and returns. They state as well that it is not clear if this is due to low liquidity or mispricing of futures contracts.

Briere, Burgues and Signora (2010) advocate a sliding approach when hedging , in which more (fewer) VIX futures contracts are held when VIX levels are notably lower (higher) due to the mean-reverting nature of the index. Jacob Rasiel (2008) prove that the VIX Index performs well as a hedge to a long equity portfolio . Inclusion of the VIX Index in a long-only diversified portfolio of equities and bonds substantially improves the efficient frontier of risk/return tradeoffs. Moreover they show that VIX futures perform better in bearish markets. The convexity of VIX futures returns, when plotted against the S&P 500 Index, implies a decreasing marginal hedge (i.e., the more severe the equity correction, the fewer incremental VX Futures are required to hedge). Thus, the portfolio allocation to VX Futures remains low, and relatively stable, across a broad range of negative return scenarios for the S&P 500 Index.

Therefore the above mentioned literature has provided unanimous evidence that the investor is better off by including spot VIX in their portfolio. Unfortunately spot VIX is not tradable , however we use it a proxy of VIX ETF's. Also the existing literature claim that VIX futures perform better during periods of market turmoil . Nevertheless this conclusion has been reached in most of the cases in a MV setting. This approach is subject to shortcomings though. The Markowitz setting may not reflect accurately the gains from investing in VIX-related assets since it is founded on two assumptions, i.e. that either the distribution of the asset returns is normal or investor's preferences are described by a quadratic utility function.

Neither of these two conditions is expected to hold. In particular, there is ample empirical evidence that asset returns are not distributed normally, especially for relatively short horizons .In the case where the non-normality of returns is not taken into account in the optimal portfolio formation process, then there is a utility loss (Jondeau and Rockinger, 2006). This is because a risk averse investor has a preference for positive skewness and dislikes high kurtosis, and therefore one should consider these moments in the portfolio choice process. In fact spot VIX (1990-2011) demonstrates positive skewness (1.24) as futures on VIX (2004-2011) do so (2.63). Furthermore, a quadratic utility function exhibits negative marginal utility after a certain finite wealth level and increasing absolute risk aversion with respect to wealth; both these features are not consistent with rational behavior. All the previously published papers have studied the in-sample benefits of diversifying in VIX-related assets within a mean-variance framework.

In light of the previously mentioned shortcomings, we take a more general approach to examining whether VIX-related assets should be included in an investors' portfolio. In particular, we consider an investor who allocates funds between equities, bonds, a risk-free asset and VIX-related assets in a standard static asset allocation context and make the following contributions to the existing literature. First, we revisit the posed question within an in-sample setting that has also been employed by the previous literature in order to draw direct comparison with previous findings.

The novelty though is that we employ rigorous tests that take into account the higher moments of the asset returns distributions instead of eyeballing the relative position of the efficient frontiers based on the traditional and the traditional augmented with VIX-related asset universes, respectively. To this end, we apply the regression-based spanning techniques to test for spanning when investor preferences are described by utility functions that are consistent with the MV setting, as well as, a more general non-MV one (see e.g., Huberman and Kandel, 1987, and DeRoon and Nijman, 2001, for MV spanning, and DeRoon et al., 1996, for generalized non-MV spanning tests). Chen Chung Ho (2010) propose in their paper that since traditional mean-variance spanning test ignore higher order moments , that someone can extend their analysis to mean-variance spanning tests on the diversification benefits of VIX-related assets in future research . To the best of our knowledge it is the first time in literature that a non-mean variance spanning test regarding VIX-related assets .

Second, we examine the question under scrutiny by employing an *out-of-sample* setting. In line with DeMiguel et al. (2009) and Kostakis et al. (2010), we form static one-period optimal portfolios at any point in time, calculate their corresponding realised returns and evaluate their performance under a number of performance measures. Third, we construct optimal portfolios by taking into account the higher order moments of the returns distributions of the involved assets. To this end, direct utility maximization is performed (e.g., Cremers et al., 2005, Adler Kritzman, 2007). The appeal of this approach compared to the MV optimization applied by previous studies is that it yields optimal portfolios by maximizing the expected utility of the investor for any assumed type of returns distribution and description of her preferences. Alexander and Korovilas (2011) have also employ Black-Litterman analysis in order to include higher order moments in their out of sample analysis regarding VIX futures.

Forth we study the posed question by considering alternative ways of investing in VIX . We use the spot VIX as a proxy for VIX ETF's , because the historical data for VIX ETF's is very small , as they were introduced in 2009. Also in our analysis we use historical data for VIX futures starting from 2004. Finally we employ a robustness test using sub-samples. We examine the performance of spot VIX and VIX futures during (2004-2007) a bullish period and during (2007-2011) a bearish period .

We conduct a number of tests in order to check the robustness of the obtained results. First, we employ various utility/value functions and degrees of risk aversion that describe the preferences of the individual investor. This is because the formation of optimal portfolios is investor specific. In particular, exponential and power utility functions, as well as, the disappointment aversion setting introduced by Gul (1991) are adopted. The latter takes into account behavioral characteristics in investor's preferences.

Second, we use a number of performance measures (Sharpe ratio, opportunity cost, portfolio turnover and risk-adjusted returns net of transaction costs) to compare the performance of the optimal portfolio based on traditional and augmented with VIX-related opportunity sets, respectively. This enables us to take into account the impact of the higher order moments as well as that of transaction costs on performance evaluation.

The rest of the paper is structured as follows. Section 2 describes the dataset. Section 3 outlines the tests for spanning and discusses the results. Section 4 sets the asset allocation framework and then compares the out-of-sample performance of optimal portfolios that contain VIX-related assets with that of those that do not contain VIX-related assets and Section 5 conducts a number of further robustness tests. We summarize results in the last section.

2. The dataset

The dataset consists of monthly closing prices of a number of indexes , spot VIX and VIX futures provided by Bloomberg. We employ the S&P 500 total return index, Spartan U.S. Bond Index Fund and the Libor one-month rate to proxy the equity market, bond market and the risk-free rate, respectively. Spartan U.S. Bond Index Fund (FBIDX) replicate the performance of the Barclays Capital Aggregate Bond Index , investing at least 80% of the fund's assets in bonds included in the Barclays Capital U.S. Aggregate Bond Index. Using statistical sampling techniques based on duration, maturity, interest rate sensitivity, security structure, and credit quality to attempt to replicate the returns of the Index using a smaller number of securities. To get exposure to the VIX-related asset class, we employ the spot VIX and VIX futures returns **.** The dataset for all assets spans the period from April 1990 to August 2011 with the exception of VIX futures that covers the period from April 2004 to August 2011 due to data availability constraints.

VIX is an index, like the Dow Jones Industrial Average (DJIA), computed on a real-time basis throughout each trading day. The only meaningful difference is that it measures volatility and not price. VIX was introduced in 1993 with two purposes in mind. First, it was intended to provide a benchmark of expected short-term market volatility. To facilitate comparisons of the then-current VIX level with historical levels, minute-by-minute values were computed using index option prices dating back to the beginning of January 1986. This was particularly important since documenting the level of market anxiety during the worst stock market crash since the Great Depression—the October 1987 Crash—would provide useful benchmark information in assessing the degree of market turbulence experienced subsequently.

Second, VIX was intended to provide an index upon which futures and options contracts on volatility could be written. The social benefits of trading volatility have long been recognized. The Chicago Board Options Exchange (CBOE) launched trading of VIX futures contracts in May 2004 and VIX option contracts in February 2006. In attempting to understand VIX, it is important to emphasize that it is forward-looking, measuring volatility that the investors expect to see. It is not backward looking, measuring volatility that has been recently realized, as some commentators sometimes suggest. Conceptually, VIX is like a bond's yield to maturity. Yield to maturity is the discount rate that equates a bond's price to the present value of its promised payments. As such, a bond's yield is implied by its current price and represents the expected future return of the bond over its remaining life. In the same manner, VIX is implied by the current prices of S&P 500 index options and represents expected future market volatility over the next 30 calendar days.

Since April 2004, it has been possible for a portfolio to buy or sell cash-settled VIX futures. Entering into VIX futures may be a close substitute to theoretical investment in the VIX index. A portfolio that invests in VIX futures, starting with the availability of futures data in April 2004, through the end of August 2009, returns a statistically insignificant -3.12% a month. By comparison, the monthly return of the VIX index over the same period was 1.8% per month. When separated into periods of positive and negative S&P 500 returns, the returns to VIX futures contracts are more negative when the S&P 500 is increasing, while the opposite is true of the VIX index itself.5 The difference between mean returns of the VIX futures and the index were -4.73% when S&P 500 futures fell and -6.84% when S&P 500 futures rose. Thus, the holders of futures contracts overpaid relative to the index regardless of the direction of the S&P 500 futures returns. This contrast between the VIX futures and the index suggests the futures contracts did not offer the same downside protection investors would expect given the asymmetric relationship between volatility and returns.

3. In-sample benefits of VIX-related assets: Testing for spanning

Huberman and Kandel (1987) were the first to introduce the concept of spanning that was initially restricted to a mean– variance framework. This method statistically tests whether adding a group of new assets can improve the investment opportunity set of an existing group of basis assets used as a benchmark, by analyzing the effects of the added assets on the mean–variance frontier.

For ease of illustration, we call the combined group of new assets and benchmark assets ‘‘augmented assets.” If the mean–variance frontier of the benchmark assets coincides with that of the augmented assets, we refer to this result as ‘‘spanning,” which indicates that investors will gain no benefit from the addition of the new assets, regardless of their level of risk aversion . If, however, the mean–variance frontier of the benchmark assets is smaller than that of the augmented assets, this indicates an expanded opportunity set, showing that investors would gain diversification benefits from adding the new assets. In this section, we investigate the economic benefits from investing in VIX-related assets by means of tests for spanning, without restricting ourselves in an MV framework though. To this end, we follow DeRoon et al., (1996, 2003) and analyze the concept of spanning by means of the stochastic discount factor (SDF) that sets the ground for the ensuing discussion of spanning tests within a non-MV framework.

3.1. Definition of spanning : The stochastic discount factor approach

Let an investor who considers a set of *K* benchmark assets (stocks, bonds, and the risk-free asset) with R_{t+1} be the (K×1) vector of the respective gross returns. Asset pricing theory dictates that there exists a SDF M_{t+1} such that

$$E[M_{t+1}R_{t+1} \mid I_t] = \iota_\kappa \tag{1}$$

where I_t denotes the information available at time *t* and ι_κ a *K*-dimensional unit vector. In fact equation (1) is the fundamental equation of asset pricing .The SDF is derived from the first order conditions of a portfolio choice problem where the investor maximizes the expected utility of her terminal wealth (DeRoon and Nijman, 2001). In this case, the SDF is proportional to the first derivative of the assumed utility function of wealth, given the investor's optimal portfolio choice *w** :

$$M_{t+1} = \lambda U'(w^* R_{t+1}) \tag{2}$$

Where λ is a constant and *w** the (K×1) vector of optimal portfolio weights (see also DeRoon, et al., 2003). Equation (2) shows that the SDF varies across investors who have different utility functions or the same utility function with different risk aversion coefficients. Here M_{t+1} stands for the intertemporal marginal rate of substitution .

The investor has to decide whether she will incorporate a set of test assets (in our case one commodity asset), with gross return R_{t+1}^{test}, in the initial *K*-asset universe. Let M be a set of SDF's that price the *K* benchmark assets , i.e. for each M_{t+1} that belongs to *M* equation (1) holds.

DeRoon et al. (1996, Proposition 1, page 6) show that the returns R_{t+1}^{test} of the test asset are *M*-spanned by the returns R_{t+1} of the benchmark assets if and only if

$$\hat{R}_{t+1}^{test} = proj\left(R_{t+1}^{test} \,\middle\|\, \left[M \cup \{w' R_{t+1} : w \in W\}\right]\right) \text{ for some } w \in W \tag{3}$$

Where $w \in W = \{w \in \mathbb{R}^k : w'\iota_\kappa = 1\}$. Proposition 1 yields the following testable hypothesis : the new asset is *M*-spanned by the benchmark assets if and only if the return of the new asset can be written as the return of a portfolio of the benchmark assets , and a zero-mean error term ε_{t+1} i.e.

$$H_0 : R_{t+1}^{test} = w' R_{t+1} + \varepsilon_{t+1} \tag{4}$$

where ε_{t+1} is orthogonal to the set M of the pricing kernels under consideration .

3.2. Mean-variance spanning tests

First, we test for MV spanning. Hansen and Jagannathan (1991) show that the SDFs associated with MV optimizing behavior have the lowest variance among all admissible ones (that price correctly a set of asset returns) and are linear in asset returns. Hence, equation (3) can be estimated by the following linear regression

$$R_{t+1}^{test} = \alpha + \beta R_{t+1} + \varepsilon_{t+1} \tag{5}$$

From a financial theory perspective, it is legitimate to use the risk-free rate as a regressor in equation (5). However, from an econometric perspective, this is an unattractive regressor given its persistency and therefore the stated reformulation is preferred. Notice also that in the case that the risk-free asset is included in the set of benchmark assets, testing for spanning is equivalent to testing for intersection. This can be easily perceived by means of the MV efficient frontier. In the case where there is a risk-free asset, two mutual fund separation theorem holds, i.e. the efficient frontier is linear and constructed by combining the risk-free asset with the tangency portfolio. Hence, testing for spanning amounts to testing whether the two linear frontiers, that of the test and benchmark assets and the one that includes only benchmark assets, are the same. This is equivalent to testing whether the tangency portfolios are the same, i.e. testing for intersection.

The null hypothesis for spanning is (see also Huberman and Kandel , 1987)

$$H_0 : \alpha = 0 \text{ and } \beta \iota_{\kappa} = 1 \tag{6}$$

Since in our case the *K*-benchmark asset universe includes also the risk-free asset , the test for MV spanning is reformulated in excess return terms. To fix ideas, define α_J to be the intercept in the regression of the test asset's excess returns of the *K* benchmark assets , i.e.

$$R_{t+1}^{test} - R_t^f = \alpha_J + \beta(R_{t+1} - R_t^f i_K) + \varepsilon_{t+1} \tag{7}$$

with R^f being the risk-free rate of return and $E(\varepsilon_{t+1}) = E(\varepsilon_{t+1} R_{t+1}) = 0$. In Appendix A, we derive the equivalence between the intercepts of equations (5) and (7) , i.e .

$$\alpha_J = \alpha - R_t^f (1 - \beta i_K) = 0 \tag{8}$$

Given the regression model in equation (7), imposing the spanning constraints of equation (6) yields $\alpha_J = 0$, i.e.

$$H_0 : \alpha_J = \alpha - R_t^f (1 - \beta i_K) = 0 \tag{9}$$

Notice that in the case of the excess returns formulation, the hypothesis of spanning amounts to testing only the intercept term. The slope coefficients of the risky assets do not need to add up to one since they multiply only the excess returns of the *(K-1)* risky assets; the missing allocation is filled by the investment in the risk-free asset (see also Huberman and Kandel , 1987 , Scherer and He , 2008) .

It is important to underline that in the mean-variance case the α intercept term of the regression can be interpreted as the Jensen's alpha . Whereas the Sharpe ratio is defined in terms of the characteristics of one portfolio (the expected excess portfolio return and its standard deviation), Jensen's alpha is defined in terms of one asset or portfolio relative to another. Sharpe ratios answer the question whether one portfolio is to be preferred over another, whereas Jensen's alpha answers the question whether investors can improve the efficiency of their portfolio by investing in the new asset. So it becomes obvious that an α statistically significant different from zero means that the addition of VIX related assets to a traditional portfolio leads to improvement of efficiency .

3.3. Non mean-variance spanning tests

Next, we outline the test for spanning in the non–MV case. Let investors' preferences be described by a non-MV utility function $U(\cdot)$, i.e. not a quadratic one. Consequently, the set *M* of pricing kernels under consideration includes the MV linear SDFs as well as the SDFs of the assumed non-MV utility function that correspond to different risk aversion coefficients. Equation (2) implies that any given value for the risk aversion coefficient imposes a different SDF that should be included in the set *M*. Therefore, in the case where a non-MV utility function is considered, the test for spanning should be carried out by examining whether the relative restrictions hold for *any* value of risk aversion. For the purposes of our study, we employ a wide range of risk aversion coefficients for each non-MV utility function $U_i(\cdot)$ of interest with *i=1,2,…,n* corresponding to the *i*th risk aversion value. Following the approach suggested by DeRoon et al. (1996, 2003), we estimate equation (3) by projecting the excess returns of the test assets on the set *M* of SDFs, i.e.:

$$R_{t+1}^{test} = \alpha + \beta R_{t+1} + \sum_{i=1}^{n} \gamma_i U_i'(w_i^{*'} R_{t+1}) + \varepsilon_{t+1} \tag{10}$$

where $U_i'(w_i^{*'}R_{t+1})$, i=1,2,…n are the derivatives of the (non mean-variance) utility functions of interest, i.e., for all utility functions that are in M. In fact this regression plots the relation between the returns of the test asset and the increase in the wealth of the potential investor .The increase of wealth of the potential investor is expressed by the marginal utility of their respective utility function .

and test jointly for spanning in the mean-variance and non mean-variance case by evaluating the restrictions

$$H_0 : \beta \iota_k = 1 \text{ and } \alpha = \gamma_i = 0 \forall i \tag{11}$$

Again, the test for non-MV spanning is reformulated in excess returns terms and the following linear regression equation is estimated (see Appendix B):

$$R_{t+1}^{test} - R_t^f = \alpha_J + \beta(R_{t+1} - R_t^f i_K) + \sum_{i=1}^{n} \gamma_i U_i'(w_i^{*'} R_{t+1}) + \varepsilon_{t+1} \tag{12}$$

So the restrictions that need to hold for the joint existence of mean-variance and non mean-variance spanning become

$$H_0 : \alpha_J = 0 \quad \text{and} \quad \gamma_i = 0 \forall i \tag{13}$$

a_J can be interpreted as Jensen's alpha only in the mean-variance framework.

From an implementation point of view, the restrictions in (9) and (13) are tested by Wald test (see e.g., DeRoon and Nijman, 2001). We correct the standard errors of the estimators by the Newey and West (1987) method to account for the presence of autocorrelation and heteroskedasticity in the residual term. Moreover, to perform the regression in equation (12), we need to estimate the unobserved regressors (i.e. the marginal utilities).

To this end, we make an assumption about the utility function and estimate the optimal portfolio weights. In particular, we consider an investor whose preferences are described by either an exponential utility function or a power utility function, for different levels of risk aversion. The negative exponential utility function is defined as:

$$U(W) = -exp\{ -nW \} / n \ , \ n>0 \tag{14}$$

where n is the coefficient of absolute risk aversion (ARA) . The power utility function is defined as

$$U(W) = \frac{W^{1-\gamma} - 1}{1-\gamma}, \gamma \neq 1 \tag{15}$$

where γ is the coefficient of relative risk aversion (RRA).

We estimate the optimal portfolio weights by applying the Generalized Method of Moments (GMM, see e.g., Cochrane, 2005). The moment conditions generated by the SDFs of interest need to be defined. Given the assumed non mean-variance utility function, equations (1) and (2) imply that the returns on the *K* benchmark assets should satisfy the following conditions:

$$E\left[\lambda_i U_i'(w_i^{*'} R_{t+1}) R_{t+1} \mid I_t\right] = \iota_K \quad \forall i \tag{16}$$

Let the parameter vector $\Theta_i = [c_i w_i^*]$ that corresponds to the *ith* value of risk aversion, i = 1,2,..,n . Define the errors , $u_{t+1}(\Theta_i)$:

$$u_{t+1}(\Theta_i) = \lambda_i U_i'(w_i^{*'} R_{t+1}) R_{t+1} - \iota_K \tag{17}$$

Then , for a sample of size T , the moment conditions $g_T(\theta_i)$ are defined as the sample mean of the errors $u_{t+1}(\Theta_i)$ i.e.

$$g_T(\Theta_i) \equiv \frac{1}{T}\sum_{t=1}^{T} u_t(\Theta_i) = E_T[u_t(\Theta_i)] = E_T\left[\lambda_i U_i'\left(w_i^{*'} R_t\right) R_t - \iota_K\right] \tag{18}$$

By definition, the SDF (for each i) should price each one of the three benchmark assets. This provides us with three moment conditions in order to estimate Θ_i . We obtain the GMM estimate of Θ_i by minimizing the quadratic function

$$J_T(\Theta_i) = g_T(\Theta_i)' W g_T(\Theta_i) \tag{19}$$

where W is a positive definite weighting matrix . We set W equal to the identity matrix I since the number of unknowns equals the number of moment conditions . We performed the GMM estimation using EVIEWS .

3.4. Results and discussion

This section tests the spanning hypothesis when a VIX-related asset is included in a traditional asset universe , consisting of stocks , bonds and the risk-free rate .

We conduct the analysis using as test asset either the spot VIX , which we also use it as a proxy for the VIX ETF's or VIX futures .Table 1 reports the Wald test statistics and the respective *p*-values for testing the null hypothesis that there is spanning . We test the following hypotheses separately : only mean-variance spanning , mean-variance and non mean-variance spanning jointly (MV & exponential , MV & power) as well as only non mean-variance spanning (exponential , power). We use risk aversion coefficients for a range of values (ARA ,RRA = 2,4,6,8,10) to conduct the non mean-variance spanning tests (equation (12)) . We can see that the null hypothesis of MV spanning can be rejected at a 5% significance level. This holds for the spot VIX as well as for the VIX futures .Therefore the results suggest that under a MV framework , the performance of traditional portfolios , consisting of stocks, bonds and cash can be significantly improved by in investing VIX-related assets .

These findings are in line with those reported by Chen,Chung,Ho (2010) , who find that there is not spanning when the test asset is either spot VIX ,or VIX futures , or VIX squared portfolios .This is the only published paper that conducts spanning test regarding VIX-related assets .

It is common knowledge that the returns are characterized by non zero skewness and kurtosis . The S&P 500 total index returns demonstrate negative skewness and excess positive kurtosis , while the VIX-related asset returns demonstrate high positive skewness and positive excess kurtosis .So except for the mean-variance spanning test we conducted also non mean-variance spanning tests in order to take into account these higher moments .To the best of our knowledge no previously published study has ever examined the results of non mean-variance spanning regarding VIX-related assets . In the non mean-variance framework we can see that the spanning hypothesis is rejected for spot VIX and for VIX futures as well .Results hold regardless of whether testing is carried out for joint MV and non-MV or for only non-MV spanning and are in agreement with the mean-variance results. Therefore in our in-sample analysis we show that VIX-related assets offer added value to investors in a mean-variance (Markowitz) framework as well as in a non mean-variance framework that takes into account higher order moments .

4. Out-of-sample benefits of VIX-related assets

Next, we investigate whether the (MV as well as the non-MV) in-sample diversification benefits provided by VIX-related assets are preserved in an out-of-sample setting, too. To this end, we calculate optimal portfolios separately for an asset universe that includes "traditional" asset classes (stock, bond, risk-free asset) and an "augmented" one that also includes VIX-related assets. Next, we evaluate their relative performance in an out-of-sample setting which is the ultimate test given that at any given point in time, the investor decides on the portfolio weights; the portfolio returns to be realized over the investment horizon are uncertain.

4.1. The asset allocation setting

In about the 4th century, Rabbi Issac bar Aha proposed the following simple rule for asset allocation: 'One should always divide his wealth into three parts: a third in land, a third in merchandise, and a third ready to hand.' Since then many things has changed so we have to conduct a different method. Let a myopic investor with fixed initial wealth W_t who faces an asset universe of *N* assets that pay off at time *t*+1. Their utility function U(.) is assumed to be continuous, increasing, concave and differentiable. Let be the weight of wealth invested in the risky asset *i* over the next period. We construct the optimal portfolio at time *t* by maximizing the investor's expected utility of wealth at time *t+1* with respect to the portfolio weights, i.e.

$$\max_{w_i} E[U(W_{t+1})] \quad \text{, subject to } \sum_{i=1}^{N} w_i = 1 \tag{20}$$

Let also $r_{i,t+1}$ be the simple rate of return n the individual asset i and $r_{p,t+1}$ the portfolio return . Without loss of generality , we assume that the initial wealth is normalized to one i.e . $W_t = 1$.

The end-of-period wealth is given by :

$$W_{t+1} = W_t(1+\sum_{i=1}^{N} w_i r_{i,t+1}) = 1+\sum_{i=1}^{N} w_i r_{i,t+1} = 1+r_{p,t+1} \tag{21}$$

To solve the expected utility maximization problem, an assumption about the utility function of the investor needs to be made. First, we assume that the preferences of the investors are described either by the negative exponential or the power utility function (equations (14) and (15), respectively) that are commonly used in the finance literature. To ensure the robustness of our results, we use various levels of absolute and relative risk aversion (ARA, RRA=2, 4, 6, 8, 10). In addition, we consider the disappointment aversion (DA) setting introduced by Gul (1991) to capture behavioral characteristics in investors preferences. In particular, this framework has been employed in recent asset allocation studies so as to capture the presence of loss aversion (see e.g., Driessen and Maenhout, 2007, Kostakis et al., 2010), i.e. the fact that investors are more sensitive to reductions in their financial wealth than to increases relative to a reference point. The advantage of Gul's (1991) DA setting over other behavioral models is that it is founded on formal decision theory that retains all assumptions and axioms underlying expected utility theory but the independence axiom that is replaced by a weaker one to accommodate the Allais paradox. In line with Driessen and Maenhout (2007) and Kostakis et al. (2010), we employ a DA value function based on a power utility function, i.e.

$$U(W) = \frac{W_T^{1-\gamma}-1}{1-\gamma} \qquad \text{if} \quad W_T > \mu_W$$

$$U(W) = \frac{W_T^{1-\gamma}-1}{1-\gamma} - \left(\frac{1}{A}-1\right)\left[\frac{\mu_W^{1-\gamma}-1}{1-\gamma} - \frac{W_T^{1-\gamma}-1}{1-\gamma}\right] \qquad \text{if} \quad W_T < \mu_W \tag{22}$$

where γ denotes the RRA coefficient that controls the loss function in each region, A≤1 is the coefficient of DA that controls the relative steepness of the value function in the region of gains versus the region of losses and is the reference point relative to which gains or losses are measured; the investor gets disappointed in the case where her wealth drops below the reference point. Notice that the loss aversion decreases as *A* increases; *A*=1 corresponds to the case of the standard power utility function where there is no loss aversion. In accordance with Driessen and Maenhout (2007), we employ two values for *A*=0.6, 0.8. Furthermore, in line with Kostakis et al. (2010) and references therein, we set equal to the initial wealth invested at the risk-free rate, i.e. . This choice of the reference point implies that the investor uses the risk-free rate as a benchmark to distinguish gains from losses. The DA function is not globally differentiable and hence it cannot be employed in the spanning tests described in Section 3 .

4.2. Calculating the optimal portfolio

We implement the optimization problem in equation (20) by performing direct utility maximization defined as the following non-linear optimization problem:

$$\max_{w_i} E\left[U(W_{t+1})\right] = \int \ldots \int U\left[W_t(1+\sum_{i=1}^{N} w_i r_i)\right] dF(r_1 \ldots r_N) \quad \text{, subject to } \sum_{i=1}^{N} w_i = 1 \tag{23}$$

where *F* ($r_{1..}r_N$)is the joint cumulative distribution function (CDF) of the *N* returns at time *t*+1. Direct utility maximization provides a more general asset allocation setting compared with the Markowitz MV one since it takes into account the higher order moments of the joint CDF as well.

On the other hand, the joint CDF needs to be estimated; this requires assuming either a specific estimator or a parametric form for the CDF leading to an estimation error. To circumvent this, we estimate optimal portfolios by applying the full scale optimization method proposed by Cremers et al. (2005) and Adler and Kritzman (2007). This is a non-parametric technique based on a numerical grid search procedure that uses as many asset mixes as necessary to identify the weights that yield the highest expected utility. The method requires no assumptions about the joint CDF of returns or potential estimators. On the other hand, the absence of simplifying assumptions comes at the cost of computational burden.

We performed directed utility maximization using MATLAB . We used fmincon to minimize the negative expected utility function.

4.3. Out-of-sample performance measures

To ensure the out-of-sample nature of our study, a "rolling-sample" approach is employed. Let the dataset consist of T monthly observations for each asset and K be the size of the rolling window to be used for the calculation of the portfolio weights, where $K \leq T$. Standing at any given point in time (month) t, we use the previous K observations to estimate the asset allocation weights that maximize expected utility. The estimated weights at time t are then used to compute the out-of-sample realised return over the period $[t,t+1]$. This process is repeated by incorporating the return for the next period and ignoring the earliest one, until the end of the sample is reached. To ensure the robustness of the obtained results, we use alternative rolling windows sizes of K=36, 48, 60, 72 monthly observations. This rolling-window approach allows deriving a series of T-K monthly out-of-sample optimal portfolio returns, given the preferences of the investor and length of the estimation window. The time series of realised portfolio returns is then used to evaluate the out-of-sample performance of the formed optimal portfolios.

Following DeMiguel et al. (2009) and Kostakis et al. (2010), we employ a number of performance measures, namely the Sharpe ratio (SR), opportunity cost, portfolio turnover and a measure of the portfolio risk-adjusted returns net of transaction costs.

To fix ideas, let a specific strategy *v*. The estimate of the strategy's *SRv* is defined as the fraction of the sample mean of out-of-sample excess returns $\hat{\mu}_v$, divided by their sample standard deviation $\hat{\sigma}_v$.

$$\hat{SR}_v = \frac{\hat{\mu}_v}{\hat{\sigma}_v} \tag{24}$$

The sharpe ratio , also known as the ' reward to variability' ratio , measures the slope of the line from the risk-free rate to any portfolio in the mean-standard deviation plane.

To test whether the SRs of the two optimal portfolio strategies are statistically different, we use the statistic proposed by Jobson and Korkie (1981) and corrected by Memmel (2003).

However, the SR is suitable to assess the performance of a strategy only in the case where the strategy's returns are normally distributed .We have already mentioned that the returns of stocks , bonds and VIX aren't normally distributed , as they demonstrate positive skewness and excess kurtosis . Hence, we use next the concept of opportunity cost (Simaan, 1993) to assess the economic significance of the difference in performance of the two optimal portfolios based on the traditional and augmented with VIX-related asset universes, respectively. Let r_{wv}, r_{nv}denote the optimal portfolio realized returns obtained by an investor with the expanded investment opportunity set that includes VIX-related assets and the investment opportunity set restricted to the traditional asset classes, respectively.

The opportunity cost θ is defined as the return that needs to be added to the portfolio return r_{nv} so that the investor becomes indifferent (in utility terms) between the two strategies imposed by the different investment opportunity sets, i.e.

$$E\left[U\left(1+r_{nv}+\theta\right)\right]=\mathrm{E}\left[U\left(1+r_{wv}\right)\right] \tag{25}$$

So, a positive opportunity cost implies that the investor is better off in case of an investment opportunity set that allows VIX-related assets investing.

Notice that the opportunity cost takes into account all the characteristics of the utility function and hence it is suitable to evaluate strategies even when the return distribution is not the normal one , as it is in our case .

Moreover, we use the portfolio turnover metric so as to quantify the amount of trading required to implement each one of the two strategies. The portfolio turnover PT_v for a strategy *v* is defined as the average absolute change in the weights over the *T-K* rebalancing points in time and across the *N* available assets i.e.

$$PT_v = \frac{1}{T-K}\sum_{t=1}^{T-K}\sum_{j=1}^{N}\left(| w_{v,j,t+1} - w_{v,j,t+} |\right) \tag{26}$$

where $w_{v,j,t}, w_{v,j,t+1}$ are the derived optimal weights of asset *j* under strategy *c* at time *t* and *t+1*, respectively; $w_{v,j,t+}$ is the portfolio weight before the rebalancing at time *t+1*; the quantity $| w_{v,j,t+1}$ - $w_{v,j,t+} |$ shows the magnitude of trade needed for asset *j* at the rebalancing point *t+1*. The *PT* quantity can be interpreted as the average fraction (in percentage terms) of the portfolio value that has to be reallocated over the whole period. In simple words the turnover quantity defined above can be interpreted as the average percentage of wealth traded in each period .

Finally, we also evaluate the two investment strategies under the risk-adjusted, net of transaction costs, returns measure proposed by DeMiguel et al. (2009). To fix ideas, let *pc* be the proportional transaction cost and $r_{v,p,t+1}$ the realized portfolio return at *t+1* (before rebalancing). The evolution of the net of transaction costs wealth NW_v for strategy *v*, is given by:

$$NW_{v,t+1} = NW_{v,t}\left(1 + r_{v,p,t+1}\right)\left[1 - pc \times \sum_{j=1}^{N}\left(| w_{v,j,t+1} - w_{v,j,t+} |\right)\right] \tag{27}$$

So , the return net of transction costs is defined as

$$RNTC_{v,t+1} = \frac{NW_{v,t+1}}{NW_{v,t}} - 1 \qquad (28)$$

The return-loss measure is calculated as the additional return needed for the strategy with the restricted opportunity set to perform as well as the strategy with the expanded opportunity set that includes VIX-related assets. Let μ_{wv} ,μ_{nv}be the monthly out-of-sample mean of *RNTC* from the strategy with the expanded and the restricted opportunity set, respectively, and σ_{wv} ,σ_{nv} be the corresponding standard deviations. Then, the return-loss measure is given by:

$$return - loss = \frac{\mu_{wv}}{\sigma_{wv}} \times \sigma_{nv} - \mu_{wv} \qquad (29)$$

To calculate $NW_{v,t+1}$, we set the proportional transaction cost pc equal to 50 basis points per transaction for stocks and bonds (see DeMiguel et al., 2009, for a similar choice) , 50 basis points for VIX-related assets for the period 2004-2011 and 27 basis points for the period 2007-2011 (based on discussion with practitioners in the volatility market) .Nowadays the VIX transaction costs have been significantly reduced reaching 5 basis points per transaction for VIX futures. We set the proportional transaction cost for the risk-free rate equal to zero .

4.4. Direct Maximization: Results and discussion

This section discusses the results on the out-of-sample performance of the traditional and augmented with VIX-related assets portfolios formed by direct maximization of expected utility . Tables 2 , 3 and 4 show the results for the cases where the preferences of the investor are described by a power utility , DA value function and exponential utility , respectively .Investors access investment in volatility market via the spot VIX (sample from 1990 to 2011) , which although is not tradable , is used as a proxy for the VIX ETF's .Results are reported for the four performance measures and various levels of (absolute/ relative) risk and disappointment aversion , as well as different sample sizes of the estimation window .

To assess the statistical significance of the superiority in SRs , the *p*-values of Memmel's (2003) test are reported within parentheses . The null hypothesis is that the SRs obtained from the traditional investment opportunity set and the augmented investment opportunity set that also includes VIX-related assets are equal . We can see that the augmented portfolios with volatility yield definitely greater SRs than the corresponding traditional portfolios that don't invest in volatility. This happens throughout tables 2,3,4 without an exception. However , the *p*-values of Memmel's (2003) test indicate that the differences in SRs are not statistically significant. Interestingly , we can see that for any given level of risk aversion , the SRs decrease as the size of the rolling window increases . This implies that the recently arrived information should be weighted more heavily (see also Kostakis et al., 2010 for a similar finding) .

As far as the opportunity cost is concerned , we can see that it is positive in all cases .The positive sign indicates that the investor is not willing to pay a premium / or is willing to accept a premium in order to replace the optimal strategy that includes investment in spot VIX with the optimal one that invests only in the traditional assets. This implies that the investor is better off when the augmented investment opportunity set is considered . These results are in accordance with the ones obtained under the SR despite the fact the distribution of the optimal portfolio returns deviates from normality . Interestingly , in most cases , the opportunity cost decreases (in absolute terms) as the risk aversion increases . This implies that the investor becomes indifferent in utility terms between including and excluding spot VIX in their asset portfolio as they become more risk averse .

In contrast the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that also include spot VIX in almost all cases . Interestingly we can see that in most cases the difference in the portfolio turnovers of the two strategies decreases as the risk aversion increases . This suggest that as the investor becomes more risk averse , they decrease their rebalancing activity since they are willing less to undertake an active bet . Finally , we can see that the return-loss measure that takes into account transaction costs is also positive . The positive sign simply confirms the out-of-sample superiority of the expanded portfolios that include spot VIX , even after deducting the incurred transaction costs. We can see that the return-loss measure decreases (in absolute terms) as the risk aversion increases, just as was the case with the opportunity cost. These findings hold regardless of the assumed utility/value function, degree of the investor's relative/absolute risk and disappointment aversion, and the employed size of the estimation window.

The results reported in the tables mentioned above are in agreement with the ones reported in Chen Chung Ho (2010) , who mention that portfolios that consist of spot VIX or VIX squared achieve higher SRs than the traditional ones .

Tables 4 , 5 , 6 show the results when investors access investment in volatility markets via VIX futures and their preferences are described by a power utility , DA value function , power utility respectively . Results are reported for all values of risk aversion .

However due to historical data limitations (VIX futures were introduced in April 2004) we employ rolling windows of the sizes : 36 months , 48 months , 60 months . Results are similar to the ones obtained in the case where spot VIX is considered . i.e. in almost all cases augmented portfolios that invest in VIX futures outperform the traditional ones . In particular , we can see that the expanded portfolios yield higher SRs than the traditional ones . However the *p*-values of Memmel's (2003) test indicate that the differences in SRs are not statistically significant . These findings hold regardless of the assumed utility/value function , degree of investor's relative /absolute risk and disappointment aversion , and the employed size of the estimation window .

Regarding the opportunity cost we can see that it is positive in all cases .In most cases , the opportunity cost decreases as the (in absolute terms) as the risk aversion increases . In contrast the portfolios that include traditional assets induce less
Portfolio turnover compared to the ones that also include VIX futures . This was the case with the spot VIX as well . Interestingly we can also see that in most cases the difference in the portfolio turnovers of the two strategies decreases as the risk aversion increases . Finally regarding the return-loss measure , it is always positive . This implies that even though the portfolios based on an investment opportunity set that includes VIX futures / or spot VIX have greater turnover than the ones based on the traditional opportunity set , investors can still earn positive risk-adjusted return by investing in volatility .

After all , the reported out-of-sample results are in total agreement with the with the in-sample results (whether the framework employed is mean-variance or non mean-variance) .

5. Further robustness tests

In this section we perform further tests to assess the robustness of the results reported in sections 3.4 and 4.4.In fact , we divide the sample to two sub-periods and repeat the previous analysis. We divide the sample into two sub-periods in order to examine the effect of the recent subprime crisis . Like Carr , wu (2008) we split our sample in a bullish and a bearish period , to investigate how VIX-related assets behave in these two extremely different circumstances . We take the same sub-sample as Alexander and Korovilas do i.e. the bullish period outspreads from April 2004 to May 2007 , while the bearish period outspreads from June 2007 to August 2011.

5.1. sub-sample analysis : The bullish period (2004-2007)

During these years S&P 500 total index has exhibited a significant growth . In April 2004 the index was at 1624 points and in May 2007 reached 2377 points . In April 2004 VIX futures were introduced , that's why our sub-sample starts from this date .In contrast during this quite tranquil and booming period spot VIX was at low levels . In April 2004 spot VIX was at 17.19 points and in May 2007 was at 13,05 points .During this bullish period next to none had predicted that in the upcoming years things will take an different route. This specific sub-sample analysis attempts to examine whether an investor should have included VIX-related assets in their portfolio during a period characterized by a bullish market and low volatility. We will examine spot VIX and VIX futures separately to investigate the potential differences between them.

Table 8 show the results for the cases where the preferences of the investors are described by a power utility function , and they access the volatility market via spot VIX during the two different sub-sample periods . Table 9 show the respective results when an exponential utility function is assumed . Table 10 show the results for the cases where the preferences of the investors are described by a power utility function and they access the volatility market via VIX futures during the two different sub-sample periods. Table 11 show the respective results when an exponential utility function is assumed .Due to historical data limitations and to facilitate the calculation of performance measures we employ rolling window of 24 months for each period , nevertheless we experimented also with a rolling window of 36 months that produced the same results .

During the bullish period we can see that the inclusion of spot VIX significantly benefit the potential investor (whether a power or an exponential function is assumed) as shown in panel A of tables 8 and 9 respectively . We can see that the augmented with spot VIX portfolios yield greater SRs than the traditional ones . However the differences in SRs are not statistically significant . Regarding the opportunity cost we can see that it is positive in all cases. This implies that the investor is better off when spot VIX investment is allowed . Finally , the positive reported return-loss measure also confirms the out-of-sample superiority of the augmented with spot VIX even after deducting the incurred transactional costs.

The results for spot regarding to this period are similar to those of Chen , Chung ,Ho that show that the augmented portfolios with spot VIX perform better than the traditional ones . The results are also in line with Delisle , Doran , Krieger (2010) who find that due to the asymmetric and negative relation between VIX and S&P 500 returns, the VIX index provides a particularly effective hedge against market declines without penalizing performance when there are market gains.

As stated before spot VIX is not tradable but it is a proxy for VIX ETF's , for that reason we also investigate the performance of VIX futures during this tranquil period . Panel A of tables 10 and 11 show the performance of VIX futures during the bullish period under a power and an exponential utility respectively .

Interestingly we can see that the traditional portfolios that don't invest in VIX futures perform better than the augmented ones .The augmented portfolios demonstrate lower SRs than the traditional ones and negative return-loss measure . However for risk aversion 2 and 4 the augmented have positive opportunity cost while for greater level of risk aversion they have negative . The positive sign of the opportunity cost shows that SRs cannot capture the higher order moments characteristics of the augmented with VIX futures , because SRs are based in the mean-variance framework (1^{st} and 2^{nd} moments) , so this measure may underrate the augmented portfolios as far as the 2^{nd} and 4^{th} level of risk aversion is considered. The results mentioned above hold for both power and exponential utility function . Although when the whole sample is considered portfolios that invest in VIX futures outperform the traditional ones , when the question comes to bullish period the superiority of VIX futures is at stake .

These results are in line with the ones obtained by Alexander Korovilas (2011) who find that during (April 2004- May 2007) portfolios that refrain from investing in VIX futures outperform the augmented ones , while the opposite is true in the bearish period (June 2007 – August 2010) . Szado (2009) reports also that VIX futures perform better during 2008 rather than the bullish period .He also notes that VIX futures does not directly mimic holdings in the spot levels of VIX given that the mean-reverting nature of derivative instruments are priced into their values.

Jacob Rasiel (2009) also report that VIX futures perform better during periods of market turmoil . They claim that the reason for this, resides in the following matter. The convexity of VIX Futures returns, when plotted against the S&P 500 Index, implies a decreasing marginal hedge (i.e., the more severe the equity correction, the fewer incremental VX Futures are required to hedge). Thus, the portfolio allocation to VX Futures remains low, and relatively stable, across a broad range of negative return scenarios for the S&P 500 Index. Delisle , Doran , Krieger (2010) show that when separated into periods of positive and negative S&P 500 returns, the returns to VIX futures contracts are more negative when the S&P 500 is increasing, while the opposite is true of the VIX index itself. This contrast between the VIX futures and the index suggests the futures contracts did not offer the same downside protection investors would expect given the asymmetric relationship between volatility and returns. This is maybe due to their term structure , where sellers of the futures incorporate a premium for the upside risk in the index futures(i.e VIX futures are in contango) , since on average, VIX futures have an upward sloping term-structure. As a matter of fact we can see that the moderate performance of VIX futures during periods of bullish markets has been examined quite thoroughly by the bibliography existing already.

5.2. sub-sample analysis : The bearish period (2007-2011)

During these years S&P 500 total index has exhibited an unprecedented decline. In June 2007 the index was at 2338 points and in February 2009 was only at 1188 points. In contrast during this highly volatile and bearish period spot VIX spiked at a high . In June 2007 spot VIX was at 13.05 points and in October 2008 reached the historical high level of 59.89 points .The large negative correlation between daily returns on the S&P 500 and those on VIX , averaging about -0.7 before the banking crisis , became even more negative -0.85 during the crisis. During these volatile years VIX-related assets became beyond any doubt the new effective diversifier for traditional asset classes .The motivation for undertaking the analysis under this period is to examine the diversification benefits of VIX-related assets during periods of turbulence markets . The already existing bibliography unanimously reports the diversification benefits of VIX-related assets during bearish periods.

During the bearish period we can see that the inclusion of spot VIX significantly benefit the potential investor (whether a power or an exponential function is assumed) as shown in panel A of tables 8 and 9 respectively . We can see that the augmented with spot VIX portfolios yield greater SRs than the traditional ones . However the differences in SRs are not statistically significant .We have to underline that during these turbulence years the traditional portfolios demonstrate negative SRs for 2 level of risk aversion (whether a power or an exponential utility function is considered). Therefore investing in volatility during these years appears to be a protection again substantial loses . Regarding the opportunity cost we can see that it is positive in all cases. This implies that the investor is better off when spot VIX investment is allowed . Finally , the positive reported return-loss measure also confirms the out-of-sample superiority of the augmented with spot VIX even after deducting the incurred transactional costs. Moreover we can see that during the bearish period the turnover for both portfolios is greater compared to the bullish period , a fact that shows the magnitude of volatility that prevailed the specific period.

It comes to no surprise that the superiority of volatility augmented portfolios is preserved when investors access the volatility market during this bearish period via VIX futures .The expanded portfolios with VIX futures perform better than the traditional ones in terms of higher SRs , significantly positive opportunity cost , and positive return-loss measure. This holds under power utility as well as under exponential utility . Alexander , Korovilas (2011) find similar result for this bearish period (as mentioned above) , so does Szado (2009) and Jacob ,Rasiel (2009).

Investment in VIX-related assets (either VIX futures or spot VIX) during this period delivers the potential investor from substantial and unprecedented loses.

6. Conclusions

This Msc Thesis investigates whether an investor can improve his performance by including VIX-related assets in a portfolio that consists of traditional asset classes , namely stocks , bonds and cash. To this end we take a general approach conducting in-sample and out-of-sample analysis.

In particular we depart from the previous literature in the following aspects. First we revisit the posed question within an in-sample setting by employing rigorous spanning tests that are consistent with mean-variance as well as non mean-variance preferences. No previously published paper has ever examined the results of non mean-variance spanning tests regarding VIX-related assets. Then we study the diversification benefits of VIX-related assets within an out-of-sample non mean-variance framework, in order to take into account higher order moments like skewness and kurtosis. To this end, we form optimal portfolios under the traditional and augmented with VIX-related asset universes, separately, by taking into account the higher order moments of returns distribution. Next, we evaluate their comparative performance. To check the robustness of the obtained results, we consider alternative ways of investing in volatility (spot VIX and VIX futures) and various utility/value functions that describe the preferences of the individual investor. Furthermore, we employ a number of performance measures and take into account the presence of transaction costs. Finally we conduct further robustness tests considering sub-samples.

We find out that within the in-sample setting , VIX-related assets (either spot VIX or VIX futures) do yield added value whether a mean-variance framework is considered or higher order moments are taken into account (non mean-variance framework). These benefits are also preserved in the out-of-sample framework. In the vast majority of the cases , the augmented portfolios with volatility have superior performance than the traditional ones.

Given that the out-of-sample setting is the ultimate test for addressing the primary question of this Msc Thesis, our results confirm that VIX-related assets should be included in investor’s portfolio , especially during periods of bearing markets. Most importantly, the findings are robust given that they hold regardless of the performance measure, and specification of utility function . Furthermore, the superiority of the augmented portfolios is confirmed even under the presence of transaction costs. The only exception appears when the optimal portfolio invests in VIX futures over the 2004-2007 bullish period. Previously published literature has already discussed the moderate performance of VIX futures during bullish periods. Therefore, VIX-related assets do provide diversification benefits , but should be used with caution and by experienced investors. Our finding come to light in a period that the VIX-related assets are considered as the definite diversification instrument .

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Appendix A : Mean-Variance Spanning Tests in Excess Returns

In case where the initial *K*-benchmark asset universe includes also the risk-free asset , we modify the test for MV spanning to formulate it in excess returns terms. In particular , subtracting the risk-free rate from both sides of (5) , yields

$$R_{t+1}^{test}-R_t^f=\alpha+\beta R_{t+1}-R_t^f+\varepsilon_{t+1}\Rightarrow R_{t+1}^{test}-R_t^f=\alpha+\beta R_{t+1}-(R_t^f(1-\beta\iota_K)+R_t^f\beta\iota_K)+\varepsilon_{t+1}\Rightarrow$$

$$R_{t+1}^{test}-R_t^f=\left[\alpha-R_t^f(1-\beta\iota_K)\right]+\beta(R_{t+1}-R_t^f\iota_K)+\varepsilon_{t+1} \qquad (30)$$

Let α_J denote the intercept term in the regression of the test asset's excess returns on the excess returns of the *K*-benchmark assets [see equation (7)]. Equation (30) establishes the equivalence between the intercepts of equations (5) and (7) , i.e. $\alpha_J=\alpha-R_t^f(1-\beta i_K)$. Given that the restrictions in the case where the test is formulated in gross returns are $\alpha=0$ and $\beta i_K=1$, the equivalent restriction in excess returns is that $\alpha_J=0$.

Appendix B : Non Mean-Variance Spanning Tests in Excess Returns

In the case where the initial *K*-benchmark asset universe includes also the risk-free asset , we formulate the test for non-MV spanning in terms of excess returns . In particular , subtracting the risk-free rate from both sides of (10) yields

$$R_{t+1}^{test}-R_f=\alpha+\beta R_{t+1}-R_f+\sum_{i=1}^{n}\gamma_i U_i'(w_i^{*'}R_{t+1})+\varepsilon_{t+1}\Rightarrow$$

$$R_{t+1}^{test}-R_f=\alpha+\beta R_{t+1}-(R_f(1-\beta\iota_K)+R_f\beta\iota_K)+\sum_{i=1}^{n}\gamma_i U_i'(w_i^{*'}R_{t+1})+\varepsilon_{t+1}\Rightarrow$$

$$R_{t+1}^{test}-R_f=\left[\alpha-R_f(1-\beta\iota_K)\right]+\beta(R_{t+1}-R_f\iota_K)+\sum_{i=1}^{n}\gamma_i U_i'(w_i^{*'}R_{t+1})+\varepsilon_{t+1} \qquad (31)$$

Let α_J again denote the intercept term in the regression (31) , i.e. $\alpha_J=\alpha-R_f(1-\beta\iota_K)$. In the case where test is formulated in gross returns the constraints are $\alpha=\gamma_i=0\forall i$ and $\beta i_K=1$.

So , the equivalent restriction in excess returns is that $\alpha_J=\gamma_i=0\forall i$.

TABLE 1
Testing for spanning: Results

Entries report the Wald test statistics and respective *p*-values for the null hypothesis that a set of benchmark assets consisting of stocks, bonds and the risk-free asset spans a given test asset from the volatility market. The first column reports results for the null hypothesis that there is mean-variance spanning. The next column reports results for the null hypothesis that there is both mean-variance and exponential utility spanning with risk aversion coefficient ranging from 2 to 10. The third column reports results for the null hypothesis that there is spanning only for investors with exponential utility function. The forth column reports results for the null hypothesis that there is both mean-variance and power utility spanning with risk aversion coefficient ranging from 2 to 10. The last column presents the respective results when only power utility function is considered. The initial set of assets is the S&P 500 Total Return Index, Spartan Fidelity Bond Index and Libor 1-month. Results are based on monthly observations from April 1990 –August 2011 for spot VIX and April 2004-August 2011 for VIX futures. All test statistics are based on a Newey-West covariance matrix with five lags.

Test Asset	Mean - Variance (MV)	MV & Exponential	Exponential	MV & Power	Power
VIX	8.47 (0.003)	6.11 (0.000)	3.97 (0.003)	6.13 (0.000)	5.25 (0.001)
FUTURES VIX	5.44 (0.021)	7.16 (0.000)	7.34 (0.000)	7.08 (0.000)	7.23 (0.000)

*Notice that the Disappointment Aversion function is not globally differentiable and hence it cannot be employed in the spanning test.

TABLE 2
Direct Utility Maximization :Volatility Index (VIX) and Power Utility Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a power utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes spot VIX. Results are reported for different sizes of the rolling window (*K*=36,48,60,72 observations) and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via spot VIX. Results are based on monthly observations from April 1990 to August 2011.

		Volatility Index (VIX) (1990-2011)									
		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	Traditional	Expanded	Traditional	Expanded	Traditional	Expanded	Traditional	Expanded	Traditional
K=36	Sharpe ratio	0.33	0.22	0.37	0.30	0.39	0.34	0.40	0.37	0.42	0.38
	(p-value)	(0.267)		(0.212)		(0.225)		(0.273)		(0.186)	
	Opp. Cost	1.26%		0.65%		0.49%		0.41%		0.36%	
	Port.Turnover	56.15%	36.51%	42.08%	40.49%	45.53%	36.42%	46.33%	37,61%	37.25%	33.50%
	Return-Loss	0.65%		0.41%		0.30%		0.25%		0.19%	
K=48	Sharpe ratio	0.31	0.21	0.36	0.26	0.37	0.29	0.38	0.32	0.39	0.34
	(p-value)	(0.218)		(0.248)		(0.281)		(0.368)		(0.439)	
	Opp. Cost	1.10%		0.62%		0.45%		0.34%		0.25%	
	Port.Turnover	44.74%	34.22%	39.93%	31.35%	28.73%	26.97%	29.76%	26.65%	24.41%	27.33%
	Return-Loss	0.59%		0,45%		0.32%		0.21%		0.13%	
K=60	Sharpe ratio	0.30	0.17	0.31	0.21	0.31	0.23	0.31	0.25	0.32	0.27
	(p-value)	(0.114)		(0.209)		(0.245)		(0.323)		(0.371)	
	Opp. Cost	1.00%		0.61%		0.41%		0.29%		0.20%	
	Port.Turnover	34.70%	26.71%	27.07%	22.73%	20.88%	21.37%	16.25%	16.81%	16.63%	16.26%
	Return-Loss	0.87%		0.54%		0.33%		0.22%		0.13%	
K=72	Sharpe ratio	0.28	0.13	0.26	0.16	0.30	0.19	0.27	0.22	0.28	0.23
	(p-value)	(0.133)		(0.185)		(0.289)		(0.403)		(0.401)	
	Opp. Cost	1.13%		0.63%		0.37%		0.22%		0.16%	
	Port.Turnover	37.36%	23.13%	19.00%	13.70%	17.08%	11.67%	12.99%	10.52%	11.67%	10.79%
	Return-Loss	0.90%		0.52%		0.30%		0.16%		0.09%	

TABLE 3 / PANEL A (A=0.6)

Direct Utility Maximization: Volatility Index (VIX) and Disappointment Aversion Value Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a power utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes spot VIX. Results are reported for different sizes of the rolling window (*K*=36,48,60,72 observations) and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via spot VIX. Results are based on monthly observations from April 1990 to August 2011.

Panel A: Volatility Index (VIX) (1990-2011) (A=0.6)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,42	0,38	0,44	0,39	0,45	0,40	0,46	0,42	0,46	0,43
	(p-value)	(0,550)		(0,511)		(0,410)		(0,496)		(0,569)	
	Opp. Cost	0,47%		0,38%		0,40%		0,37%		0,33%	
	Port.Turnover	41,82%	33,79%	35,21%	26,00%	30,92%	24,97%	30,85%	24,67%	28,84%	19,80%
	Return-Loss	0,15%		0,12%		0,10%		0,04%		0,00%	
K=48	*Sharpe ratio*	0,36	0,30	0,38	0,33	0,38	0,34	0,39	0,37	0,40	0,37
	(p-value)	(0,389)		(0,498)		(0,583)		(0,645)		(0,565)	
	Opp. Cost	0,34%		0,31%		0,29%		0,28%		0,29%	
	Port.Turnover	27,52%	19,26%	25,76%	19,70%	24,37%	19,46%	19,32%	17,89%	20,19%	16,54%
	Return-Loss	0,22%		0,14%		0,05%		0,00%		0,00%	
K=60	*Sharpe ratio*	0,30	0,23	0,29	0,24	0,29	0,28	0,30	0,29	0,32	0,30
	(p-value)	(0,325)		(0,543)		(0,853)		(0,822)		(0,728)	
	Opp. Cost	0,34%		0,24%		0,15%		0,14%		0,14%	
	Port.Turnover	27,90%	20,10%	19,18%	18,14%	21,19%	15,18%	23,33%	14,49%	22,89%	13,28%
	Return-Loss	0,28%		0,10%		-0,03%		-0,05%		-0,04%	
K=72	*Sharpe ratio*	0,24	0,20	0,25	0,23	0,27	0,25	0,28	0,26	0,29	0,28
	(p-value)	(0,525)		(0,717)		(0,708)		(0,696)		(0,702)	
	Opp. Cost	0,23%		0,16%		0,17%		0,15%		0,12%	
	Port.Turnover	24,60%	17,15%	19,48%	17,35%	19,54%	17,57%	23,15%	18,33%	20,35%	16,37%
	Return-Loss	0,19%		0,03%		0,00%		-0,01%		-0,02%	

TABLE 3 / PANEL B (A=0.8)

Direct Utility Maximization: Volatility Index (VIX) and Disappointment Aversion Value Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a power utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes spot VIX. Results are reported for different sizes of the rolling window (*K*=36,48,60,72 observations) and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via spot VIX. Results are based on monthly observations from April 1990 to August 2011.

Panel B : VIX (1990-2011) (A=0,8)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,36	0,27	0,41	0,34	0,44	0,38	0,45	0,39	0,46	0,40
	(p-value)	(0,337)		(0,385)		(0,405)		(0,312)		(0,327)	
	Opp. Cost	0,79%		0,49%		0,39%		0,38%		0,35%	
	Port.Turnover	52,90%	38,29%	40,68%	31,80%	36,54%	34,40%	32,42%	26,88%	28,82%	23,75%
	Return-Loss	0,39%		0,25%		0,20%		0,17%		0,10%	
K=48	*Sharpe ratio*	0,34	0,25	0,37	0,30	0,38	0,32	0,38	0,34	0,40	0,36
	(p-value)	(0,320)		(0,395)		(0,372)		(0,562)		(0,443)	
	Opp. Cost	0,66%		0,40%		0,37%		0,27%		0,27%	
	Port.Turnover	37,41%	20,76%	30,63%	20,17%	29,43%	20,89%	29,12%	21,05%	25,16%	17,52%
	Return-Loss	0,43%		0,24%		0,21%		0,06%		0,05%	
K=60	*Sharpe ratio*	0,32	0,19	0,31	0,23	0,31	0,25	0,31	0,27	0,32	0,28
	(p-value)	(0,180)		(0,256)		(0,373)		(0,490)		(0,528)	
	Opp. Cost	0,75%		0,42%		0,29%		0,21%		0,15%	
	Port.Turnover	24,00%	16,00%	19,75%	17,91%	19,12%	17,48%	20,03%	15,81%	20,45%	15,17%
	Return-Loss	0,70%		0,33%		0,38%		0,08%		0,02%	
K=72	*Sharpe ratio*	0,26	0,14	0,25	0,19	0,26	0,23	0,27	0,24	0,29	0,26
	(p-value)	(0,171)		(0,403)		(0,561)		(0,587)		(0,502)	
	Opp. Cost	0,70%		0,32%		0,18%		0,15%		0,15%	
	Port.Turnover	24,64%	10,15%	17,64%	10,96%	12,65%	10,73%	11,53%	9,59%	15,50%	9,71%
	Return-Loss	0,68%		0,24%		0,09%		0,04%		0,02%	

TABLE 4

Direct Utility Maximization: Volatility Index(VIX) and Exponential Utility Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under An exponential utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes spot VIX. Results are reported for different sizes of the rolling window (*K*=36,48,60,72 observations) and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via spot VIX. Results are based on monthly observations from April 1990 to August 2011.

		VIX (1990 -2011)									
		ARA=2		ARA=4		ARA=6		ARA=8		ARA=10	
		Expanded	Traditional	Expanded	Traditional	Expanded	Traditional	Expanded	Traditional	Expanded	Traditional
K=36	*Sharpe ratio*	0,33	0,23	0,37	0,30	0,39	0,33	0,41	0,24	0,41	0,29
	(p-value)	(0,167)		(0,123)		(0,227)		(0,067)		(0,152)	
	Opp. Cost	1,02%		0,57%		0,38%		0,74%		0,52%	
	Port.Turnover	51,34%	36,89%	44,27%	34,97%	41,95%	38,95%	39,31%	38,51%	37,51%	36,81%
	Return-Loss	0,63%		0,37%		0,24%		0,58%		0,49%	
K=48	*Sharpe ratio*	0,32	0,21	0,36	0,27	0,37	0,30	0,34	0,21	0,37	0,26
	(p-value)	(0,219)		(0,301)		(0,301)		(0,078)		(0,090)	
	Opp. Cost	1,00%		0,56%		0,43%		0,58%		0,47%	
	Port.Turnover	46,01%	36,77%	31,67%	27,65%	27,12%	24,41%	39,46%	33,56%	37,86%	36,41%
	Return-Loss	0,61%		0,44%		0,31%		0,43%		0,37%	
K=60	*Sharpe ratio*	0,31	0,16	0,32	0,21	0,31	0,23	0,31	0,24	0,30	0,23
	(p-value)	(0,134)		(0,171)		(0,210)		(0,326)		(0,296)	
	Opp. Cost	1,10%		0,67%		0,43%		0,26%		0,24%	
	Port.Turnover	31,52%	26,99%	20,45%	16,00%	17,75%	15,10%	19,49%	18,27%	18,38%	17,81%
	Return-Loss	0,95%		0,64%		0,35%		0,18%		0,23%	
K=72	*Sharpe ratio*	0,27	0,12	0,26	0,16	0,27	0,19	0,24	0,21	0,28	0,23
	(p-value)	(0,154)		(0,189)		(0,281)		(0,641)		(0,489)	
	Opp. Cost	1,06%		0,61%		0,35%		0,00%		0,14%	
	Port.Turnover	30,07%	20,01%	16,58%	12,91%	15,29%	11,36%	17,62%	13,49%	17,75%	15,04%
	Return-Loss	1,00%		0,59%		0,29%		0,00%		0,00%	

TABLE 5

Direct Utility Maximization: VIX Futures and Power Utility Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under A power utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes VIX futures. Results are reported for different sizes of the rolling window (*K*=36,48,60 observations) .We don't employ rolling window 72 due to historical data limitations and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via VIX FUTURES. Results are based on monthly observations from April 2004 to August 2011.

		VIX FUTURES (2004-2011)									
		RRA=2		**RRA=4**		**RRA=6**		**RRA=8**		**RRA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,19	0,09	0,25	0,13	0,29	0,16	0,30	0,18	0,31	0,18
	(p-value)	(0,414)		(0,409)		(0,389)		(0,391)		(0,409)	
	Opp. Cost	1,41%		1,37%		1,31%		1,24%		0,97%	
	Port.Turnover	73,51%	67,66%	74,83%	53,74%	61,55%	49,19%	51,29%	47,44%	45,66%	39,85%
	Return-Loss	0,18%		0,17%		0,11%		0,08%		0,05%	
K=48	*Sharpe ratio*	0,20	0,11	0,24	0,09	0,25	0,11	0,25	0,12	0,26	0,12
	(p-value)	(0,362)		(0,375)		(0,389)		(0,396)		(0,363)	
	Opp. Cost	1,40%		1,39%		1,27%		1,12%		0,73%	
	Port.Turnover	84,29%	62,54%	58,42%	40,11%	40,82%	36,37%	31,89%	31,62%	28,15%	26,42%
	Return-Loss	0,45%		0,42%		0,35%		0,27%		0,23%	
K=60	*Sharpe ratio*	0,10	0,03	0,06	-0,02	0,05	-0,02	0,08	0,00	0,07	0,00
	(p-value)	(0,402)		(0,398)		(0,394)		(0,405)		(0,331)	
	Opp. Cost	1,02%		0,81%		0,63%		0,35%		0,16%	
	Port.Turnover	51,93%	29,75%	27,59%	17,79%	18,53%	14,57%	14,44%	12,91%	13,03%	11,93%
	Return-Loss	0,38%		0,31%		0,21%		0,19%		0,21%	

TABLE 6 / PANEL A (A=0.6)

Direct Utility Maximization : VIX Futures and Disappointment Aversion Value Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a Disappointment Aversion utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes VIX futures . Results are reported for different sizes of the rolling window (*K*=36,48,60 observations) .We don't employ rolling window 72 due to historical data limitations and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via VIX FUTURES. Results are based on monthly observations from April 2004 to August 2011.

Panel A : VIX FUTURES (2004-2011) / (A=0.6)											
		RRA=2		**RRA=4**		**RRA=6**		**RRA=8**		**RRA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,33	0,18	0,35	0,24	0,36	0,26	0,37	0,26	0,37	0,28
	(p-value)	(0,460)		(0,471)		(0,514)		(0,482)		(0,520)	
	Opp. Cost	1,07%		1,03%		0,86%		0,74%		0,69%	
	Port.Turnover	67,75%	55,97%	64,81%	51,99%	57,18%	42,73%	49,03%	38,41%	45,70%	36,12%
	Return-Loss	0,07%		0,05%		0,02%		0,02%		0,00%	
K=48	*Sharpe ratio*	0,27	0,15	0,28	0,17	0,29	0,16	0,29	0,17	0,27	0,17
	(p-value)	(0,641)		(0,453)		(0,486)		(0,590)		(0,418)	
	Opp. Cost	1,05%		0,96%		0,80%		0,72%		0,66%	
	Port.Turnover	65,49%	53,00%	57,74%	43,61%	43,87%	35,20%	30,55%	29,72%	27,90%	25,41%
	Return-Loss	0,10%		0,08%		0,07%		0,05%		0,00%	
K=60	*Sharpe ratio*	0,13	0,04	0,11	0,05	0,10	0,04	0,09	0,00	0,09	-0,01
	(p-value)	(0,538)		(0,617)		(0,842)		(0,787)		(0,450)	
	Opp. Cost	0,95%		0,79%		0,64%		0,48%		0,43%	
	Port.Turnover	48,33%	31,56%	33,12%	21,59%	19,19%	13,87%	13,70%	12,66%	12,38%	11,71%
	Return-Loss	0,09%		0,07%		0,05%		0,02%		0,00%	

TABLE 6 / PANEL B (A=0.8)

Direct Utility Maximization : VIX Futures and Disappointment Aversion Value Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a Disappointment Aversion utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes VIX futures. Results are reported for different sizes of the rolling window (*K*=36,48,60 observations) .We don't employ rolling window 72 due to historical data limitations and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via VIX FUTURES. Results are based on monthly observations from April 2004 to August 2011.

Panel B : VIX FUTURES (2004-2011) / (A=0.8)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,31	0,17	0,33	0,23	0,34	0,23	0,35	0,25	0,35	0,26
	(p-value)	(0,425)		(0,317)		(0,361)		(0,430)		(0,419)	
	Opp. Cost	1,36%		1,25%		1,11%		0,89%		0,81%	
	Port.Turnover	69,54%	58,30%	66,73%	54,27%	59,91%	45,73%	51,23%	40,20%	46,81%	38,12%
	Return-Loss	0,12%		0,10%		0,07%		0,05%		0,05%	
K=48	*Sharpe ratio*	0,25	0,16	0,26	0,17	0,30	0,19	0,30	0,18	0,29	0,19
	(p-value)	(0,561)		(0,519)		(0,473)		(0,550)		(0,389)	
	Opp. Cost	1,14%		1,01%		0,96%		0,77%		0,68%	
	Port.Turnover	67,19%	55,80%	60,45%	48,94%	48,81%	37,10%	37,69%	32,43%	34,83%	29,51%
	Return-Loss	0,11%		0,11%		0,08%		0,07%			
K=60	*Sharpe ratio*	0,15	0,03	0,12	0,03	0,12	0,02	0,11	0,00	0,10	0,00
	(p-value)	(0,149)		(0,234)		(0,265)		(0,381)		(0,464)	
	Opp. Cost	1,04%		0,86%		0,63%		0,51%		0,46%	
	Port.Turnover	47,92%	31,89%	39,27%	24,65%	26,46%	18,03%	15,30%	12,71%	13,82%	11,99%
	Return-Loss	0,12%		0,08%		0,05%		0,03%		0,00%	

TABLE 7
Direct Utility Maximization: VIX Futures and Exponential Utility Function

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under an exponential utility function. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes VIX futures. Results are reported for different sizes of the rolling window (*K*=36,48,60 observations) .We don't employ rolling window 72 due to historical data limitations and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via VIX FUTURES. Results are based on monthly observations from April 2004 to August 2011.

		VIX FUTURES (2004 - 2011)									
		ARA=2		**ARA=4**		**ARA=6**		**ARA=8**		**ARA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,20	0,09	0,26	0,12	0,29	0,14	0,29	0,15	0,31	0,18
	(p-value)	(0,401)		(0,365)		(0,393)		(0,317)		(0,152)	
	Opp. Cost	1,39%		1,35%		1,31%		1,21%		0,88%	
	Port.Turnover	73,52%	64,02%	77,70%	49,99%	60,67%	48,78%	49,88%	45,72%	46,71%	40,50%
	Return-Loss	0,10%		0,19%		0,08%		0,02%		0,02%	
K=48	*Sharpe ratio*	0,21	0,11	0,24	0,10	0,26	0,12	0,27	0,13	0,27	0,13
	(p-value)	(0,541)		(0,492)		(0,411)		(0,278)		(0,090)	
	Opp. Cost	1,27%		1,13%		1,02%		0,97%		0,64%	
	Port.Turnover	82,18%	62,43%	57,32%	58,58%	40,96%	35,78%	33,26%	31,24%	27,41%	26,20%
	Return-Loss	0,47%		0,48%		0,33%		0,27%		0,19%	
K=60	*Sharpe ratio*	0,12	0,04	0,06	-0,02	0,05	-0,03	0,05	-0,02	0,06	0,00
	(p-value)	(0,330)		(0,371)		(0,293)		(0,326)		(0,294)	
	Opp. Cost	1,00%		0,88%		0,59%		0,47%		0,23%	
	Port.Turnover	59,14%	40,99%	28,91%	25,86%	18,56%	15,11%	15,06%	13,61%	13,34%	11,86%
	Return-Loss	0,34%		0,32%		0,22%		0,19%		0,18%	

TABLE 8

Direct Utility Maximization :Volatility Index(VIX) and Power Utility Function

further robustness sub sample (April 2004-May 2007 , June 2007-August 2011)

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a power utility function and the sample is divided to two sub-samples , a bullish period (2004-2007) and a bearish period (2007-2011). The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes spot VIX. Results are reported for different sizes of the rolling window (*K*=36 observations) .We don't employ rolling window 72 due to historical data limitations and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via spot VIX . Results are based on monthly observations from April 2004 to August 2011.

Panel A : VIX (2004-2007)

		RRA=2		**RRA=4**		**RRA=6**		**RRA=8**		**RRA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,81	0,68	1,05	0,68	1,14	0,76	1,19	0,82	1,27	0,79
	(p-value)	(0,195)		(0,201)		(0,263)		(0,062)		(0,058)	
	Opp. Cost	2,22%		1,41%		1,06%		1,02%		1,15%	
	Port.Turnover	54,92%	51,52%	48,91%	46,06%	43,70%	36,58%	56,38%	48,81%	52,17%	48,20%
	Return-Loss	1,12%		2,28%		2,29%		2,13%		2,12%	

Panel B : VIX (2007-2011)

		RRA=2		**RRA=4**		**RRA=6**		**RRA=8**		**RRA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,25	-0,02	0,30	0,06	0,30	0,11	0,29	0,13	0,29	0,14
	(p-value)	(0,520)		(0,514)		(0,524)		(0,468)		(0,471)	
	Opp. Cost	1,90%		1,14%		0,77%		0,58%		0,50%	
	Port.Turnover	83,76%	74,79%	74,13%	74,92%	74,93%	67,01%	66,04%	56,53%	65,58%	50,10%
	Return-Loss	2,39%		1,38%		0,92%		0,67%		0,55%	

TABLE 9

Direct Utility Maximization: Volatility Index(VIX) and Exponential Utility Function

further robustness sub sample (April 2004-May 2007 , June 2007-August 2011)

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under an exponential utility function and the sample is divided to two sub-samples , a bullish period (2004-2007) and a bearish period (2007-2011). The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes spot VIX. Results are reported for different sizes of the rolling window (*K*=36 observations) and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via spot VIX . Results are based on monthly observations from April 2004 to August 2011.

Panel A : VIX (2004-2007)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,83	0,69	1,06	0,69	1,14	0,76	1,20	0,86	1,28	0,81
	(p-value)	(0,530)		(0,207)		(0,330)		(0,133)		(0,121)	
	Opp. Cost	2,16%		1,40%		1,06%		1,02%		0,97%	
	Port.Turnover	53,21%	50,64%	46,76%	43,92%	41,54%	35,58%	52,09%	48,21%	50,55%	47,20%
	Return-Loss	1,25%		2,34%		2,22%		2,11%		2,04%	

Panel B : VIX (2007-2011)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,25	-0,01	0,30	0,07	0,29	0,10	0,29	0,12	0,29	0,11
	(p-value)	(0,514)		(0,500)		(0,521)		(0,562)		(0,655)	
	Opp. Cost	1,83%		1,08%		0,73%		0,55%		0,51%	
	Port.Turnover	84,43%	74,72%	73,56%	76,46%	68,41%	60,41%	60,64%	41,64%	58,65%	39,77%
	Return-Loss	2,34%		1,36%		0,89%		0,64%		0,49%	

TABLE 10

Direct Utility Maximization :Volatility Index (VIX FUTURES) and Power Utility Function

further robustness sub sample (April 2004-May 2007 , June 2007-August 2011)

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under a power utility function and the sample is divided to two sub-samples , a bullish period (2004-2007) and a bearish period (2007-2011). The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes VIX futures. Results are reported for different sizes of the rolling window (*K*=36 observations) . and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via VIX FUTURES . Results are based on monthly observations from April 2004 to August 2011.

Panel A : VIX FUTURES (2004-2007)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=24	*Sharpe ratio*	0,42	0,68	0,51	0,68	0,59	0,76	0,64	0,82	0,63	0,79
	(p-value)	(0,395)		(0,567)		(0,438)		(0,611)		(0,280)	
	Opp. Cost	0,81%		0,25%		-0,16%		-0,34%		-0,50%	
	Port.Turnover	60,43%	51,52%	57,81%	46,06%	68,23%	36,58%	71,09%	48,81%	56,17%	48,20%
	Return-Loss	-1,17%		-0,81%		-0,90%		-0,81%		-0,60%	

Panel B : VIX FUTURES (2007-2011)

		RRA=2		RRA=4		RRA=6		RRA=8		RRA=10	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,28	-0,02	0,34	0,06	0,36	0,11	0.37	0,13	0,38	0,14
	(p-value)	(0,553)		(0,585)		(0,602)		(0,519)		(0,576)	
	Opp. Cost	2,33%		1,54%		1,03%		0,81%		0,69%	
	Port.Turnover	84,30%	74,79%	75,12%	74,92%	72,86%	67,01%	67,49%	56,53%	65,23%	50,10%
	Return-Loss	2,36%		1,46%		1,02%		0,82%		0,72%	

TABLE 11

Direct Utility Maximization :Volatility Index(VIX FUTURES) and Exponential Utility Function

further robustness sub sample (April 2004-May 2007 , June 2007-August 2011)

Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return- Loss) for the case where the expected utility is maximized under an exponential utility function and the sample is divided to two sub-samples , a bullish period (2004-2007) and a bearish period (2007-2011). The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes VIX futures. Results are reported for different sizes of the rolling window (*K*=36 observations) . and different degrees of absolute risk aversion (*ARA*=2,4,6,8,10). Investors access investment in Volatility market either via VIX FUTURES . Results are based on monthly observations from April 2004 to August 2011.

		Panel A : VIX FUTURES (2004-2007)									
		RRA=2		**RRA=4**		**RRA=6**		**RRA=8**		**RRA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=24	*Sharpe ratio*	0,43	0,69	0,54	0,68	0,57	0,76	0,61	0,82	0,60	0,80
	(p-value)	(0,170)		(0,265)		(0,399)		(0,463)		(0,280)	
	Opp. Cost	0,63%		0,39%		-0,36%		-0,60%		0,00%	
	Port.Turnover	57,42%	50,62%	54,09%	44,75%	57,34%	35,58%	65,46%	48,21%	53,90%	47,20%
	Return-Loss	-1,20%		-0,80%		-0,90%		-0,87%		0,00%	

		Panel B : VIX FUTURES (2007-2011)									
		RRA=2		**RRA=4**		**RRA=6**		**RRA=8**		**RRA=10**	
		Expanded	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**	**Expanded**	**Traditional**
K=36	*Sharpe ratio*	0,28	-0,01	0,34	0,07	0,36	0,10	0,37	0,12	0,36	0,11
	(p-value)	(0,543)		(0,499)		(0,525)		(0,416)		(0,465)	
	Opp. Cost	2,29%		1,53%		1,09%		0,84%		0,68%	
	Port.Turnover	83,81%	74,72%	78,39%	76,46%	69,17%	60,41%	50,57%	41,64%	44,61%	39,77%
	Return-Loss	2,30%		1,44%		1,04%		0,81%		0,63%	