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STRATEGIC MARKET GAMES: DO MARKET INSTITUTIONS MATTER FOR EFFICIENCY?

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SUMMARY

This paper is a reference to certain economic models, which are already published by distinguished economists. These models refer to game theoretic approaches and they describe the different market supply and bidding strategies. Herewith, we want to reveal if and how the institutional intervention may offer another dimension in trading, trying to improve the general equilibrium in an economy with symmetric information.

Each model imposes certain rules and regulations that must be followed, which define the behavior and course of action of traders. We shall denote the relation between commodities and money, as well as between price and quantities and how these relations alter, as trade evolves. Therefore, we also refer to the different price formation mechanisms, in a single-period model in contrast with a multi-period model.

Furthermore, we shall refer to the role of money, the different kinds of it and we shall also describe how a gearing ratio allows agents to deal with trading opportunities and their resulting obligations respectively. All in all, we reveal the importance of the different strategic equilibria and how these can lead, whenever possible, to Pareto optimality.

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INTRODUCTION

It is said that in terms of perfect competition the market offers the best choice in terms of balancing the aggregate demand and supply. Everyone is fully aware of the market conditions and satisfies his needs, buying goods at given prices. Because of the nature of a perfectly competitive market, no one's trading decisions and actions can have an effect on prices. This very remark make him a price taker. But how perfect competition fits in a market in a real world anyway?

Many people claim that there is no perfect information and equal opportunities for everyone. Asymmetry of information and other exogenous or endogenous variables may make the trading game "unfair". What we are interested in though is, if under perfect foresight and full information one as an individual or as a part of a team can influence the prices of the goods and set a different equilibrium point.

This essay is consisted of five basic chapters. The first one discriminates and defines the three different types of modeling trade, using either commodity money or fiat money. We refer to the (a) "sell-all" model, (b) the "offer-for-sale" (bid-offer) model and (c) the "price-quantity strategy" model.

The second chapter refers to a diversified model where we import financial assets. We use a two-period model in order to show, how a trader can protect his property against uncertainty, and which strategy set allows him to do so. This model defines a very useful roadmap, if someone wants to confront with endogenous market danger and abrupt variations on prices etc.

The third chapter reveals the importance of having an infinite number of traders and how equilibrium is set when we first use one model with fiat money and credit, while in the second model we use only commodity money. The fourth chapter handles the existence of a "mutual bank" trying to ease trading, and how fiat money is being imported in a model with a continuum of traders. Moreover, we juxtapose another model, only that this time the banking sector operates with different criteria, trying to maximize its profit.

Finally, the fifth chapter addresses whether market institutions matter for efficiency and protect the most of the traders against disutility and economic imbalance.

CHAPTER 1

Three Basic Strategic Models

How can we achieve a market equilibrium regarding the demand and supply, or under which mechanism can the market secure this balancing relationship, if the market is capable of such a thing in first place? Is it possible for an individual to define and control the prices or the quantities of a commodity under specific circumstances and limitations? We shall examine below, how the trading relationship between goods evolves, where market functions under strict terms, that is to say, taking under strong consideration certain rules and regulations, which are known in advance, as well as they are unbreakable. Furthermore, we shall define "money" and it' s role as a means of payment during the trading procedure.

The classical theory suggests that the decisions being made by individuals are based on fixed prices of goods. Shapley and Shubik based on a strategically closed model though, showed that individuals can set different prices according to their trading decisions, and set a new strategic equilibrium, the so-called "Cournot-Nash equilibrium" (CNE).**[1],[14]**

In the following model we are using money as means of payment, the role of which, will play a specific commodity. This commodity may have an intrinsic value i.e. gold or silver. Money therefore, is a strategic element, which on the one hand can be traded like a common commodity, or it can be used as a price measuring tool on the other. The usual textbook definitions of money are functional – the three main functions being to serve as a medium of exchange, as a store of purchasing power, and as a standard of value.**[1]** Finally, money serves also an important economic purpose in lowering transaction costs. We quote from Shubik, *"Money in game* *theoretic models has three important properties: (1) money as strategic decoupling device; (2) monetary control as a criterion for identifying and distinguishing players; and (3) money as a vehicle for side payments".* **[1]**

It is also worth mentioning, that when the economy is modeled as a game in strategic form, the presence of money gives individuals a strategic freedom they would not have otherwise. In a no-money, no-credit society, an individual cannot commit himself to buy until he knows what he will be able to sell.

In order to describe a well defined-game of exchange, in which a commodity is used as a means of payment, we define how prices are formed. The classic general-equilibrium model is used to establish the existence of prices (often not unique) at equilibrium. Several price formation mechanisms might be introduced, each setting different restrictions on the strategic possibilities. In the model presented hereto, the strategic variables are quantities, not prices.

1.1 Sell-All Model (Lloyd Shapley and Martin Shubik)

In our first basic model we set the following terms. We trade m commodities, using the (m+1)st as money, but no credit is granted. All payments are required in advance on all purchases. There are n traders participating, using m trading posts. Our general model is simple and can be depicted as a one-shot game. Traders use goods, which are ready for trade, in other words we exclude the producing procedure in order to simplify our trading game.

Each trader has an initial group of goods which is symbolized with $a^i = \left(a_1^i, a_2^i, \dots, a_{m+1}^i\right)$, with a concave utility function $U = u^i\left(x_1^i, x_2^i, \dots, x_{m+1}^i\right)$.**[1],[4]** Traders have to use quantities of the (m+1)st good to buy quantities of the first m goods. Traders have to put all of their goods for sale no matter what the quantities are, but if they want some quantities of one or more goods to hold back, they have to go through the market and buy them back. It is important to discriminate, that traders are not obliged to use all of their (m+1)st commodity. So each trader can offer all of his endowments for sale and make bids, using different amounts of his (m+1)st commodity, but in no case he can surpass the specific amount of money at his disposal. Therefore, he can bid and finally spend b_1 money for the 1^{st} good, b_2 money for the second good and so on, but the relationship which defines his buying potentials is $b_1^i + b_2^i + \ldots + b_m^i < a_{m+1}^i, b_j^i > 0, j = 1, 2, \ldots, m$. We use a_{m+1}^i, when we want to refer to the specific good, that we chose to use as money.

At each trading post there are two standards, the amount of m goods to be supplied, and the amount of (m+1)st good to be used for the bids. So the prices are determined through this simple mechanism, as defined by the ratio of the amount of bids to the amount of the items to be supplied. Traders have decided in advance which amount of the (m+1)st good want to sacrifice, without previously knowing what the final price of each good will be.

At the first trading post the price is defined $P_1 = \frac{\sum_{i=1}^{n} b1}{\sum_{i=1}^{n} a1}$, $\Rightarrow P_1 = \frac{\bar{b}1}{\bar{a}1}$, while at the second is defined $P_2 = \frac{\sum_{i=1}^{n} b2}{\sum_{i=1}^{n} a2}$, $\Rightarrow P_2 = \frac{\bar{b}2}{\bar{a}2}$, until we reach the final one, where price is given by the type $P_m = \frac{\bar{b}m}{\bar{a}m}$. That leads us to the conclusion, that the final prices are defined with $P_j = \frac{\bar{b}j}{\bar{a}j}$, j =1,2,…m, meaning more or less, that the bids, which are made by every single trader, and which reflect his own behavior, tend to strongly influence the final price of one good, without prior knowing of what exactly the price will be.

If the model extends over several periods, we would see that traders already having known the previous prices, if they expected a slight change in them, they would not change their behavior. But if they expected radical changes, they would rather alter the quoted quantities than the amount of the (m+1)st good.

Each trader receives $x_j^i = \frac{b_j^i}{p_j}$, if $p_j > 0$, and $x_j^i = 0$, if $p_j = 0$. A trader receives nothing, if he bids nothing. In other words, each trader spends $\sum_{j=1}^{m} b_j^i$ and receives $\sum_{j=1}^{m} a_j^i p_j$. His final amount of the (m+1)st good is resulting, if we add his sales and deduct his purchases from his initial amount of "money".

$$x_{m+1}^i = a_{m+1}^i + \sum_{j=1}^{m} a_j^i p_j - \sum_{j=1}^{m} b_j^i$$

The payoff function is as follows:

$$\Pi^i = v^i(b^1, b^2, \ldots, b^n) = u^i\left(x_1^i(b), x_2^i(b), \ldots, x_{m+1}^i(b)\right)$$

Theorem 1.1.1 (Shubik M. and Shapley L.)

For each trader i, i = 1, 2, ..., n, let u^i be continuous, concave and non-decreasing. For each good j, j = 1, 2, ..., m, let there be at least two traders with positive initial endowments of m+1 good, whose utility for good j is strictly increasing. Then a strategic equilibrium exists.

Note: When there are only two commodities being exchanged, either can be regarded as the means of exchange. This is violated, when it comes to three or more goods to be exchanged.

It is quite important to mention that equilibrium exists no matter what the trade will be, that is to say that even if no match is accomplished between the two parties (the two traders), regarding bid and offer, although we have no trade, we have an equilibrium point, which reflects their up to that moment decisions. In other words, we can make any assumptions of equilibrium, which is determined by any "rational" or "irrational" behavior individually. Once traders make their decisions, prices are formed. What strategy is to be followed, depends on the initial rules and restrictions apart from individual courses of action. For example, different strategies will be set by traders, depending on whether coalitions are allowed or not. However, there will always be a certain point of CNE.**[4]**

It is also worth mentioning to explain the meaning of replication in an economic model, when a large number of traders participates. There is quite a high possibility that some traders belong to a specific type of a trader, that is to say with identical characteristics or behavior. They even have identical endowments or tastes, but they are not forced or theoretically constrained to act alike, which means that they retain their ability to make their own decisions, without forming a specific coalition or a cartel. This replication number, the number of traders that form a specific type, could alter the result of a trading game. In other words, replication may change the strategic equilibrium point.

In a replicated model let us consider n number of n traders, T types of traders and k of each type; n= k T. The initial amount of the *j* good regarding the trader *s* of type *t* will be denoted a_j^{ts}, also a_s^t since it does not depend on s. The whole amount of good j will be denoted $\bar{a}_j$. Regarding the bids made the strategy will be again denoted $b^{ts} = (b_1^{ts}, b_2^{ts}, \ldots, b_m^{ts})$. Again we have the following set:

$p_j = \bar{b}_j / \bar{a}_j$, j = 1,…, m;

$x_j^{ts} = b_j^{ts} / p_j$, j = 1,…, m and $p_j \neq 0$

$x_j^{ts} = 0$, j = 1,…, m and $p_j \neq 0$

$x_{m+1}^{ts} = a_{m+1}^{ts} - \sum_{j=1}^{m} b_j^{ts} + \sum_{j=1}^{m} a_j^{ts} \cdot p_j$;

$\Pi^{ts} v^i(b^{11}, \ldots, b^{ik}, b^{21}, \ldots, b^{nk}) = u^t(x_1^{ts}(b), \ldots, x_{m+1}^{ts}(b))$, where u is the utility function for the t traders. A NE (Nash Equilibrium) is established where all the “t” traders make the same bids. Always $\sum_{j=1}^{m} b_j^t < a_{m+1}^t$, that means that in no way can the amount of bids made, supersede the amount of u-money being at traders' disposal.**[4]**

As already mentioned above, each trader's total bids depend on his spending limit as previously defined, credit excluded. When all bids are made, prices are formed and a new CNE emerges. Every CNE point is suboptimal, compared to the Competitive Equilibrium (CE) point, for finite replications of the economy. Every CE point is also a CNE point but not vice versa. Our aim is to examine, how we can reach under specific assumptions the CE point, when traders make their decisions separately.

As a first step, we can relax the "sell-all" model by letting traders hold back some of the initial endowments, so the restriction of "sell-all" is violated. Again prices are denoted $p_j = \bar{b}_j$ / $\bar{q}_j$, ($q_j > 0$). b_j^i stands for bids, while q_j^i represents the quantities of goods sent to the trading post. $0 \leq q_j^i \leq a_j^i$, j$\epsilon[0, m]$. This means that the tradable quantities cannot surpass the amount of the ones at trader's disposal. The quantities held back are indicated by a_j^i - q_j^i. Again if $b_j^i = 0 \Rightarrow x_j^i = 0$, as well as if $q_j^i = 0 \Rightarrow x_j^i = 0$.

We do not want the trader to both buy and sell at the same trading post, therefore we impose the restriction $b_j^i . q_j^i = 0$, j = 1, …, m. In any other case one single trader could change and finally destroy the equilibrium by diversifying the marginal cost of the good j, if he decided for example to reduce both b_j^i and q_j^i . But it is worth mentioning to say, that in a model where coalitions are allowed, this restriction might not be active, because one trader on the one hand and a second one on the other, could buy and sell simultaneously in favor of this coalition, trying to keep the price unchanged. These general remarks are generalized in the following section.

1.2 Bid-Offer Model (Pradeep Dubey and Martin Shubik)

In the example above we saw how trade takes place, where individuals put all of their goods into market. This is a rather extreme case where a trader is required to sell all of his goods, making bids to purchase back, using one special commodity as money. Below we have another example with one difference. Traders are no longer forced to sell all of their commodities, so they can either be sellers or buyers. Additionally, fiat money can also be used. We define I_n the set of i traders, while I_{m+1} the set of commodities. Again each trader has a limited amount of (m+1)st commodity, in order to use it for trade, which at the end is the difference between the amount he spent to buy goods and the one he gained from his sales. So the strategy set is as follows:

Each i trader can sell q_j quantities of good j but in any case $q_j^i < a_j^i$, where a_j^i is the initial amount of good at his disposal. Furthermore, the range of his bids is subject to limitation prior set by his initial amount of (m+1)st commodity. That is to say $\bar{b}^i \leq \alpha^i_{m+1}$. The initial amount of good j is symbolized again $a^i, a^i \in \Omega^{m+1}$ and the initial money held by a trader i is symbolized a_{m+1}^i.

We denote utility function $u^i : \Omega^{m+1} \rightarrow \Omega^1$, $i \in I_n$.**[5]**

Discrimination: When $a_{m+1}^i > 0$, then the trader is called moneyed, while when $a_j^i > 0$, the trader is then called "j-furnished". We also symbolize the sum Σ with a bar (-), therefore by $\bar{b}$ we mean the total amount of bids for example.**[5]**

The strategy that a trader will follow is:

$$S^i = \{(q^i, b^i) : q^i \in \Omega^m, b^i \in \Omega^m, q_j^i \leq \alpha^i, b^i \leq a_{m+1}^i\}.$$

It means that traders should both buy and sell, according to some restrictions. The quantity they are able to offer cannot supersede the amount they already possess.

Regarding the bids they can make, these cannot be bigger than the amount of money, fiat or not, at their disposal.

The next step is to denote price $p_j = \bar{b}_j / \bar{q}_j$, j ∈ I_m, p ∈ Ω^m. Again if $\bar{q}_j = 0$ then $p_j = 0$, $s^i = (q^i, b^i)$. Each trader has a set of strategy regarding bids and quantities to offer as already mentioned. In this model trader i has the ability to influence the prices, either through the amount of quantities he wants to buy or sell, or through the amount of "money" he wants to offer.

In the end, when all the bids are executed:

- $p_j > 0 : \xi^i_j(s) = b^i_j / p_j + \alpha^i_j - q^i_j$
- $p_j = 0 : \xi^i_j(s) = \alpha^i_j - q^i_j$
- $J = m+1 : \xi^i_j(s) = \alpha^i_j - \sum_j b^i_j + \sum_j q^i_j p_j$.

The final payoff to the traders is $p^i(s) = u^i(\xi^i(s))$. So the CNE point is where a trader maximizes his payoff. We can define this n–person game "Γ" and also define the CNE of this Γ game $\hat{s}$ for each i ∈ I_n.**[5]**

Theorem 1.2.1 (Dubey P. and Shubik M.)

Given that all traders desire money, then for any good j there are at least two moneyed traders who desire j, and at least two j-furnished traders. Then an E.P. exists.

This means exactly that if there are two traders who have money and they are willing to buy j product, and there are also two traders who possess this product, then a specific strategic equilibrium is defined. This equilibrium point is set by the specific quantities to offer and specific bids to be made.

Apart from this, we must also point out that even if a trading post is inactive, that is, even if it were open for business, but it would attract no offer and money bid, again there should be a equilibrium point rather "pathogenic". This is called a trivial CNE.

All in all, comparing the two models we have to focus on the following great differences and similarities. In both models we can use a commodity as "u-money", that is as a means of payment, while in the second one we can also use fiat money. In both models inactive trading posts may exist, if demand does not meet supply, defined by the decisions of each individual. In our first model we can only set the quantities, which will finally define the prices and an equilibrium point. Each equilibrium point is a strategic one, which "desires" to reach the competitive equilibrium point, because only then the economy satisfies its needs as a whole. In the second one, we can change either the quantities, or the prices, or even both and set a CNE point. Furthermore, we are not forced to put all of our products for trade, but we can set aside those quantities we want to keep for our own purposes, or internal use.

What we shall try to do next, is to take the strategic trading relation one step further, only that this time the strategic variables are prices not quantities. So we convert the Cournot-Nash Equilibrium into Bertrand-Nash Equilibrium (BNE). The first one refers to a strategic equilibrium based on quantities, while the second one is based on prices. The following model handles this very strategic theory.

1.3 Price-Quantity Strategy Model (Pradeep Dubey, Pradeep Dubey and Martin Shubik)

In the third model similarly to the second one, the strategies again refer to price and quantity strategies, yet we shall focus on the prices. We will see how these strategies set by agents, affect the equilibria. Again we refer to a market with n traders and a finite number of commodities. At each trading post traders state how much they want to bid and offer, without prior knowledge of each other's bidding and offering preferences.

The market E consists of a set of n traders. Each trader i has a_j^i amount of commodities, which defines the amount he is able to bid and offer. He also has the freedom to buy and sell simultaneously the same commodity. It is rather unlikely a trader to both buy and sell at the same time, but the model gives this freedom as well, because this behavior also constitutes a specific strategy.

Given that there are at least two traders, which are positively endowed with j commodity, as well as two traders who sufficiently desire j commodity, then for any p $\in R^k_{++,}$ let

$B^i(p)$ = {x ∈ R^k_+ : $p\,.\,x\ \leq p\,.\,a^i$}, and

$\tilde{B}\,^i(p)$ = {x ∈ $B^i(p)$: $u^i\,(x)$ = max $u^i(y)$}.

A competitive equilibrium of the market E is a pair (p; x^i, …, x^n) of prices and an allocation such, that each x^i is optimal in i's budget set.**[6]**

Given a competitive equilibrium (p; x^1, …, x^n), we can associate with it *shadow prices* of income $\lambda^1, \ldots, \lambda^n$, where λ is a nonnegative number such that x^i is a solution of max $u^i(y) +\ \lambda^\iota[p.a^i -\ p.y]$.**[6]**

Regarding the market game $\Gamma^1(E,\lambda)$, one restriction is that a trader cannot bid more money than his own. Additionally, he cannot offer more goods than he possesses. The price to buy j commodity is symbolized with p^i, while the price to sell is symbolized with $\tilde{p}^i$. Regarding the quantities that one trader is willing to buy or sell are symbolized with q^i and $\tilde{q}^i$ respectively. If the price of a commodity is less than p_j^i, in other words the price to buy, then trader i is rather likely to purchase this specific j commodity. On the other hand, if the price of a commodity is higher than $\tilde{p}_j^i$ then trader i is likely to sell this j commodity. The aggregate demand and supply form the equilibrium price, if this is sufficient.

There may be occasions, where supply does not meet demand, so no such equilibrium price comes out. The demand curve is formed by a specific strategy set given at that time, including all quoted quantities. The defined strategy sets give also a specific shape, regarding the supply curve on the Cartesian chart. If suppliers require higher price, they sell nothing, while buyers who wish for lower price, purchase nothing as well. Then no trade takes place and each one retrieves his commodities. Another occasion is, when the supply or the demand is excessive. Then the marginal buyers and sellers trade proportionally to their demands and offers.

Under this occasion, without violating our initial restrictions and regulations, the highest priced seller could match with the highest buyer to a specific amount of quantities. Alternatively, the highest buyer purchases from the lowest seller, or even sellers could also keep the whole surplus for themselves. Finally, there could be a third party introduced as middlemen or agents, who could buy from sellers and sell to buyers at a price, which could benefit from, or they could even been paid a commission, for doing so.

This model gives individuals apart from the ability to satisfy their personal needs, also the ability to have an income and make a profit. This results from the fact that they quote at different prices, when they want to buy or sell. So let h^i denote the profit. We define $h^i = (\tilde{p}^i \cdot \tilde{q}^i) - (p^i . q^i)$ which can be either positive or negative, or even equal to zero. The money gained from sales are deducted from the money used for purchases.

Money: traders are allowed to resort to a mutual bank and ask for credit or a loan at a zero rate of interest without any limit. By doing so, a trader can finance his purchases, with sole obligation to give back the initial amount of money borrowed. This gives him on the one hand the possibility to make profit, if he ends up with a positive amount of money after the trade has ended, yet a potential inability to repay the loan to the bank may lead to a severe penalty with real dimensions. That is to say, he can either have some of his goods confiscated, or pay a toll / fine or he can even get excluded from trade. We shall not explain how such a mechanism could operate regarding the necessary cost, clearance procedures and documents written down. Neither we shall explain how fiat money enters our closed economy. What we should highlight, is that through the mechanism of borrowing, a trader could further improve his payoff, but perhaps a possible bankruptcy would certainly harm any extra utility gained.

So, in this model certain strategic equilibria exist, which depend on the decisions of the individuals, whether some trading posts are fully active or inactive, but the question is how we establish a competitive equilibria. But as mentioned many times before, a Nash Equilibrium exist regardless the activity or not of a trading post. In addition, we call "tight" N.E. when at each trading post all active traders quote the same price.**[6],[7]**

Theorem 1.3.1 (Dubey P.)

Consider any E and any $\lambda>0$. Then (a) the active N.E. of $\Gamma^1(E, \lambda)$ coincide with the C.E. of E; (b) the tight, active N.E. of $\Gamma^1(E, \lambda)$ also coincide with the C.E. of E; (c) every tight, active N.E. of $\Gamma^1(E, \lambda)$ is strong.

This means that if a certain strategic equilibrium obtains and all active traders are fully satisfied given the specific circumstances, and no other subtotal or subset of traders can achieve better equilibrium, then this equilibrium is a competitive one and it is strong and tight. There are times where certain coalitions or set of traders may bargain / trade in such a way that they could benefit the most from it. So they do not bother about what third parties or the rest individuals would do. Their priority of course is to maintain what is best for them, and how this can be "strategically" secured. The general equilibrium in other words does not fit them, and it can be put into doubt. These traders separate from the whole and set their own strategic equilibrium.

CHAPTER 2

2.1 Equilibria With Financial Markets – An Introduction of Securities

So far we have seen what happens, when trade takes place under specific game circumstances, what kind of equilibria are established, when we use fiat money or commodity money as a means of payment. Moreover, we examined how traders change or support their preferences and behaviors when it comes to alteration of game rules, such as money borrowing. We finally showed that the Nash Equilibrium tends to become Walrasian Equilibrium as more and more traders participate in this specific trade and how we can replicate a specific model to achieve competitive equilibrium.

In the following model we shall examine a two period financial market, based on the previous models. The basic difference is, that here a new element is introduced, the financial assets. Both assets and commodities are tradable in a specific way though. Again, our model consists of a finite number of n traders, a finite number of j commodities, one of which plays the role of money.

In an incomplete information environment traders attempt to maximize their utility payoff function by intertemporally allocating their resources, by using security markets. They are willing to pay a specific price in order to buy some financial security, especially when they want to hedge a potential loss. This is the primary road of financial markets. It turns out that financial markets provide a shield against uncertainty, especially when we refer to different periods of trade.

First we have to note that it is rather impossible that a strategic equilibrium in a commodity-money model is the same with the equilibrium established in a fiat money model. The only way for this to happen, is when no trade takes place in both models, then such an equilibrium is called a trivial one. What we expect from the model below, is a "nice" equilibrium where all traders are active.**[8]**

We impose two basic restrictions in order to avoid short-selling and arbitrage opportunities during the two periods. First of all, we require that all payments are in advance and because we also want to exclude default, we refer only to secure lending. These basic constraints help us avoid manipulation of the market, if for example some agents wanted to gain power for themselves by leading others to bankruptcy. But on the contrary, some may argue that these restrictions induce less liquidity in the market, as well as less efficiency.

In the first period our model works like a standard one. When the first period ends, we find ourselves in an "s-situation" or "s-statement". It is a critical situation, where traders should decide which set of assets to choose in their attempt to predict the future trading opportunities. They have to pay in advance and possess the sets of assets prior to the second opening of the market. So this time another factor is yet to affect the allocation of the commodities along with that of the assets; the combination of the assets chosen by all the traders. Because they pay in advance, they can reallocate their consumption drift.

2.2 The Model (Giraud G. and Stahn H.)

We present a two period model, in which there are N traders, $n \geq 2$, also L commodities, while the L+1st commodity is considered as money, therefore it also serves as a numeraire. Each trader is given an initial amount of commodities defined by the vector (ω^i, μ^i), each trader also chooses a consumption bundle (x, m), which specifies an amount of commodities (x_s) as well as numeraires (m_s) The preferences of the traders are defined by the common utility function U, as we have already seen in previous models. U^i is continuous, concave and increasing which satisfies the condition: ∀ (x, m) from the consumption set, $U^i(\omega^i, \mu^i) > U^i(x, m)$.**[8]**

We also define j number of securities or financial assets that can be traded at t=0, that is to say after the first period has ended, while the second one has not begun yet. Each asset should offer positive returns at some s state. This means that the matrix, which refers to a specific set of assets cannot be either of full rank, or zero. Or in other words, in each state we demand that at least one asset pays positive returns.

For example, a random set of assets could be this kind of a matrix: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the horizontal lines refer to the assets, while the vertical lines refer to certain states of economy. In detail, it means that a trader may buy a portfolio which can protect him in any likely situation. In first s_1 situation the first asset provides him no payback but the second set of assets does (second line), while in another situation s_2 the first set of assets is the winner. Zero means that unfortunately one trader gains nothing apart from paying for this bundle of assets, but 1 means that he can have his commodities back.

Each player's action set is defined by $A^i = \prod_{s=0}^{S} A_s^i$ where,

A_0^i : $\{(b_j^i, q_j^i) \in (R_+^J)^2$: ∀ j = 1, …, J $\sum_{j=1}^{J} b_j^i \leq \mu_0^i$ and R · $(q_j^i)_{j=1}^{J} \leq \mu_1^i$ and

A_s^i: = $\{(b_{\ell s}^i, q_{\ell s}^i)$ ℓ=1,…, L ∈ $(R_+^L)^2$: ∀ ℓ = 1,…, L, $q_{\ell s}^i \leq \omega_{\ell s}^\iota$ }.

Again no trader can bid more money than he possesses. We refer to securities which should be financed endogenously. Traders should be able to refund their assets individually, as they have to pay in advance by the net income of their portfolio. Nevertheless, each trader can defend his initial holding by neither bidding nor offering any assets. His strategy herewith is differentiated from all the rest strategies.

In other words he can play "safe". Now if all traders play safe, then no trade of assets takes place, and what we have now is the common trivial equilibrium. Each one keeps his position after the first period, which is the trade with commodities, but no one can improve that position.

Provided that a bundle of action set is chosen as follows a = $(a^i, a^{-i}) \in$ A : = $A^i \times \prod_{j\in I\setminus\{i\}} A^j$, price vectors for assets and commodities are also set as follows:

$$\begin{cases} \forall\, j = 1, \dots J \quad \pi_j = \dfrac{\sum_{i\in I} b_j^i}{\sum_{i\in I} q_j^i} := \dfrac{B_j}{Q_j} \\ \forall\, s = 1, \dots S, \forall \ell = 1, \dots L \;\; \pi_{\ell s} = \dfrac{\sum_{i\in I} b_{\ell s}^i}{\sum_{i\in I} q_{\ell s}^i} = \dfrac{B_{\ell s}}{Q_{\ell s}} \end{cases}$$

While the final holding of player i in security j is denoted

$$\theta_j^i(\alpha) := \begin{cases} \frac{b_j^i}{\pi_j} - q_j^i \; if\; \pi_j \neq 0 \\ \quad -q_j^i \; else \end{cases},$$

and his final allocation in numeraire is:

$$m_0^i(\alpha) = \mu_0^i - \sum_{j=1}^J b_j^i + \sum_{j=1}^J \pi_j q_j^i.$$

In the second period where we set s state s ≥ 1, each trader has to finance his bids for commodities, always under the reminder, that all payments are made in advance. Additionally, all bids are linked to the strategies followed in the first period, therefore we consider A^{-i} strictly bonded to A^i .

So the new set of action is denoted as follows:

$$a^i(a^{-i}) := \{a^i \in A^i : \sum_{\ell=1}^L b_{\ell s}^i \leq \mu_s^i + \sum_{j=1}^J r_{sj} \cdot \theta_j^i(\alpha_i, \alpha_{-i}) \text{ for all s} = 1, \dots, S\}$$

Our next step is to denote the final allocation of trader i regarding commodity $\ell = 1, \dots, L$ in state s = 1, …, S

$$x^{i}_{\ell s}\,(\alpha) := \begin{cases} \omega^{i}_{\ell s} - q^{i}_{\ell s} + \frac{b^{i}_{\ell s}}{\pi^{i}_{\ell s}}, if\ \pi_{\ell s} \neq 0 \\ \omega^{i}_{\ell s} - q^{i}_{\ell s},\ else \end{cases}$$

while the final allocation in numeraire goods in state s = 1, …, S is set by

$m^{i}_{s}\,(\alpha) := \mu^{i}_{s} + \sum_{j=1}^{J} r_{sj} \cdot \theta^{i}_{j}(\alpha) - \sum_{\ell=1}^{L} b^{i}_{\ell s} + \sum_{\ell=1}^{L} \pi_{\ell s}\, q^{i}_{\ell s}$.

This model is quite helpful and explanatory when we refer to stocks for example. Assume that a player has a quite well differentiated portfolio with money accounts, ready for liquidation, bonds, stocks etc, in order to minimize the market danger. He can buy a stock, while at the same time take a short position using a derivative for this stock, if he wants to hedge a potential decline on the stock's price. It indicates how a single player can built a protective shell against pessimistic predictions on prices or generally the market, or yet he can even follow an aggressive strategy to improve his trading position.

CHAPTER 3

3.1 Noncooperative General Exchange with a Continuum of Traders: Two Models

We have seen so far what strategies are selected by a specific number of traders (finite) with strategic power, given certain market constraints. Next there is no oligopolistic power in terms of a non-atomic continuum of traders. But as we are going to see, as our model evolves, there are some other difficulties we are called to deal with, such as illiquidity of the market, etc. We divide the model in two forms, regarding money and how it is used. In the first one we do not have the “cash in advance” constraint, we can use either paper money or IOU’s (I owe you), which are notes like checks or prescripts, while in the second one the above constraint is valid, and only a commodity is used as money.**[9]**

In detail, in the first model traders are free to create as much money as they please, but heavy spending means that overdraft penalties will be imposed to those who are caught short when accounts are settled. In the second, a specific commodity is treated as money, which has intrinsic value, therefore we do not bother of any penalties or payments in advance. Additionally, there is no need of a clearing house and the market is fully decentralized. The question which arises in both models is, if or how strategic and competitive equilibria can be established under a continuum of traders acting individually.

Let $\{T, \mathcal{B}, \mu\}$ *be a non-atomic measure space where T denotes the set of traders,* $\mathcal{B}$ *is the σ-algebra of measurable subsets of T, and μ is a non-atomic, non-negative, finite population measure on* $\{T, \mathcal{B}\}$.**[9]** Again we use m tradable commodities and we denote Ω^m the non-negative orthant of R^m. Vectors in Ω^m may stand for either commodity bundles or price vectors. A set $S \in \mathcal{B}$ is null if $\mu(S) = 0$, otherwise non-null. Finally, when we use the phrase “almost all” we refer to all i.e. traders except for a null set. The endowments are symbolized with α^t , $t \in T$. and every commodity j, j = 1,…, m is present in the economy. **[9]**

<u>Remark:</u> *If $u^t(x)$ is strictly increasing as a function of the component x_j, we shall say that t desires j, and if x is such that $u^t(x)$ maximizes $u^t(\cdot)$ over Ω^m we shall say that x satiates t.*

An allocation is a measurable function $z : T \rightarrow \Omega^m$ with $\int_T z^t \, d\mu \leq \int_T \alpha^t d\mu$, and indicates the output of a potential redistribution of goods, with possibly some waste. A competitive equilibrium is an ordered pair $\langle p, z \rangle$ where $p \in \Omega^m$ is a price vector and z is such an allocation that z^t is optimal in t’s budget set.

$$\begin{cases} z^t \in B^t(p), and \\ u^t(z^t) = \max\{u^t(x) : x \in B^t(p)\} \end{cases},$$

where

$$B^t(p) = \{x \in \Omega^m : px \leq p\}.$$

Furthermore, a shadow price for t is a number $\lambda^t \geq 0$ such that the bundle z^t maximizes the following $u^t(x^t) + \lambda^t(p\alpha^t - px^t)$.

3.2 Model 1: Trade with fiat money and credit (Shapley L. and Dubey P.)

In this exchange economy we use fiat money and credit in order to purchase and sell our commodities. We do that at a different and decentralized trading post for each commodity. It is quite obvious, that since we use just paper notes without any intrinsic value, such money has no other value outside the specific market or after the trade has ended. The existence of a clearing house or a bank is essential, but regarding credit or other means of payment like IOU's, there is no interest charge. They are interest-free, but as already pointed out, anyone who cannot meet his obligation to redeem loans, is penalized. Therefore, he should be very cautious and thoughtful about his spending money strategy; his purchases should be financed by his sales, if he wants to avoid such a bankruptcy, as well as its consequences.

A potential default could also influence others connected with this "waster" trader. This could lead to a domino effect, but we can overcome this by paying off the creditors exogenously. It is not in our purpose to model a default, or explain all the measures one should take to confront with, but the aim here is to study the action set of traders, who know that they are not free to act recklessly. In some other model perhaps such redemption measures could be like the further borrowing for someone to cover his previous dept, or the liquidation of some of his assets and so on.

All in all the strategy set is as follows:

$\Sigma^t = \{s^t = (q^t, r^{t,}) : q^t \in \Omega^m , r^t \in \Omega^m, q^t \leq \alpha_j^t, j = 1,\dots, m\}$

r^t symbolizes the money used for bid for good j and q^t symbolizes the quantity of good j, which is sent to be sold. Let there be a set of strategies $s \in \Sigma = X_{t \in T}\Sigma^t$, we then have to make clear that each trading post works independently. This means that in every single trading post we have a certain amount of money and a certain quantity of goods received, which determines the final price of good j and sets the respective ratio between price and quantity.

Each trader makes a certain selection $s = (q, r) \in \Sigma$, and each trading post provides the following:

$$p_j = \begin{cases} \int r_j / \int q_j \,, & if \ \int q_j > 0 \\ \infty, & if \ \int q_j = 0 \ and \ \int r_j > 0 \\ undefined, & if \ \int q_j = \int r_j = 0 \end{cases}$$

In words, it indicates the settled price, given the decisions of traders on the amount of their bids, and of the quantities to be sold. The next step is to see which amount of money or goods is allocated to each trader.

Regarding commodities we give the outcome in the following bracket:

$$\begin{cases} \dfrac{r_j^t}{p_j}, & if \ 0 < p_j < \infty \\ 1, & if \ p_j = 0 \ and \ r_j^t > 0 \\ 0, & else \end{cases}$$

Regarding money the outcome is presented as follows:

$$\begin{cases} p_j q_j^t, & if \ 0 < p_j < \infty \\ 1, & if \ p_j = 0 \ and \ q_j^t > 0 \\ 0, & else \end{cases}$$

The aim for each trader is to maximize his payoff function which is $\Pi^t(s) = U^t(\tilde{z}^t)$, $t \in T$. In other words, each trader wants to maximize his utility function, where $\tilde{z}$ describes the final allocation of z and $\tilde{z}_{m+1}$, which stands for goods and money respectively. But $U^t(\tilde{z}^t)$ is $u^t(z^t) + \zeta^t \min\{0, \tilde{z}_{m+1}^t\}$, where ζ describes the penalties imposed on money that each trader has, in case he is caught short.**[9]** In other words, his utility function is also determined by his spending behavior and it' s consequences. All in all, the final holdings of money and goods are shown in the bracket below:

$$\begin{cases} z_j^t = a_j^t - q_j^t + \dfrac{r_j^t}{p_j}, \ j = 1, \dots, m \\ \tilde{z}_{m+1}^t = -\displaystyle\sum_{j=1}^{m} r_j^t + \sum_{j=1}^{m} p_j \ q_j^t \end{cases}$$

The aforementioned strategies and outcome sets refer to a strategically formed game $\Gamma(\mathcal{E})$ of an exchange economy $\mathcal{E}$. What we are interested in, is mainly the achievement of a competitive equilibrium, or how a strategic equilibrium can lead to a competitive one. The following three theorems reveal how these two types of equilibrium are related.

First of all, it should be made clear again, that if some trading posts are inactive, we refer to a trivial equilibrium, which is something unlikely to happen, i.e. if all traders are reluctant to buy goods at a high price. If all trading posts are active, then all quoted quantities and final holdings in goods are positive, so in this case we refer to an active S.E.

Theorem 3.2.1 (Dubey P. and Shapley L.)

Every active SE allocation of $\Gamma_\zeta(\mathcal{E})$ that leaves almost every trader unsatiated is competitive for $(\mathcal{E})$.

This theorem indicates the relation between the SE of the Γ game and the CE of the economy we refer to.

Theorem 3.2.2 (Dubey P. and Shapley L.)

If each good is desired by a non-null set of traders, then each competitive allocation z of $(\mathcal{E})$ is an active SE allocation of $\Gamma_\zeta(\mathcal{E})$, provided that $K\zeta^t \geq \lambda^t \langle p, z\rangle$, a.e. for some $K > 0$ and some p such that $\langle p, z\rangle$ is a CE of $(\mathcal{E})$.

Regarding this theorem, we note that at a certain SE $\langle p, \tilde{z} \rangle$ traders have an amount of goods strategically distributed. We require that $\tilde{z}_{m+1} = 0$, meaning that upon the end of trade, there should be no money left in everyone's pocket. But if traders are not fully satisfied at z, then all traders of a set S are not satisfied. We remind that $\tilde{z} = z + \tilde{z}_{m+1}$. If we assume that traders still have some money to spend, in order to improve their utility individually, this contradicts to the first assumption that $\tilde{z}_{m+1} = 0$. Since this is not possible, then we conclude that this specific S set of traders cannot improve its position in a further strategic form game $\Gamma(\mathcal{E})$, so back in the economy $\mathcal{E}$, $\langle p, z \rangle$ is a CE.

Theorem 3.2.3 (Dubey P. and Shapley L.)

Let each good be desired by a non-null set of traders, and let each allocation satiate at most a null set of traders. Then the set of CE allocations of $\mathcal{E}$ and the set of active SE allocations of $\Gamma_\zeta(\mathcal{E})$ coincide, provided that $K\zeta^t \geq \lambda^t[\mathcal{E}]$, a.e. for some positive constant K.

Regarding the rate of penalty, it reflects the overdraft and the lack of paying it back, which has nothing to do with the amount of money. Multiplying by K, as in theorem 2, we emphatically disapprove of overspending and we therefore multiply this disutility. If the K factor is large then the SE is more likely to drop out such overdrafts, which make the final allocations more competitive. The complete bracket showing the utility after the trade has ended, is presented below:

$$\begin{cases} U^t(\tilde{z}^t) = U^t(z^t), & if\ \tilde{z}_{m+1} \geq 0 \\ U^t(\tilde{z}^t) \leq U^t(z^t) + \zeta^t \tilde{z}_{m+1}, & if\ \tilde{z}_{m+1} < 0 \end{cases}$$

It simply means that at the end of trade, if the amount of money left is larger or equal to zero, no penalty is imposed. Optimality is restored, if the "u-money" were equal to zero, meaning that i trader has spent his money in such way, that fully satisfies him. If there is a surplus in each trader's pocket, it means that they did not quite exhaust all of their buying opportunities. But if one trader needs more money to cover the financial gap ,which is created by the overdraft, he has to face a penalty. How severe the penalty can be, it depends on the initial assumptions.

3.3 Model 2: Trade using a commodity money (Shapley L. and Dubey P.)

The following model excludes paper money, so this time all traders use commodity money to make their payments. This means that no clearing house is needed, while the market is decentralized. Since there is no paper money, there is no credit, no overdraft, no bankruptcy, so there is no penalty imposed in the end. In addition, traders are now obliged to make all payments in advance.

The previous non-atomic measure space remains the same and we denote the economy $\mathcal{E}$ = {(T, $\mathcal{b}$, μ) α, u}. In this model all vectors α^t, z^t refer to Ω^{m+1}. The utility functions are quasi-concave. We denote three different games $\Gamma_1(\mathcal{E})$, $\Gamma_2(\mathcal{E})$ and $\Gamma_3(\mathcal{E})$, whisch are distinguished by the strategy spaces Σ_i^t, i = 1, 2, 3. $\Gamma_1(\mathcal{E})$ is a buy-and-sell game, is a sell-all game, and finally $\Gamma_3(\mathcal{E})$ is a buy-or-sell game.**[9]**

Regarding the first game the strategy set is given below:

$\Sigma_1^t = \{s^t = (q^t, r^t) \in \Omega^{2m} : q_j^t \leq a_j^t$, j = 1,..., m, and $\sum_{j=1}^{m} r_t^j \leq \alpha_{m+1}^t\}$.

It simply says that each trader has a strict set of goods and money, such as no amount of commodity to offer can supersede the initial amount at his disposal. In other words, no trader can offer more goods than he initially possesses and no trader can bid more money than what he has on hand. Below we give the strategy set for the second sell-all game.

Σ_2^t = {s^t ∈ Σ_1^t : $q_j^t = \alpha_j^t$, j = 1, …, m}.

This set is in connection with the first one, only that this time the offered quantity should be equal to the total amount of goods, according to the restriction that this game imposes. Finally the third strategy set is as follows:

Σ_3^t = {s^t ∈ Σ_1^t : $q_j^t r_j^t$ = 0, j = 1, …, m}.

In this game traders should decide between buying and selling.

The mechanism of trade is similar to the first model, as we earlier described. Traders should make a measurable selection s = (q, r) ∈ Σ, regarding the prices formed at every trading post, the goods purchased and the amount of money earned from the selling of the goods. The only difference comes when we refer to the final holdings of money. Since we use commodity as means of payment, we have to make the following amendment:

$z_{m+1}^t = a_{m+1}^t - \sum_{j=1}^{m} r_j^t + \sum_{j=1}^{m} p_j\, q_j^t$.

The commodity money has intrinsic value and can be either exchanged or consumed, hence its final allocation is subject to the individual preferences of each trader. The payoff function is Π^t(s) = u^t(z^t), t ∈ T. Each trader wants to maximize his payoff function.

The existence of inactive trading posts has great influence on the SE's. We know that if a trading post is inactive, then no price is formed, no trade takes place therefore suboptimal allocation obtains. This is certainly an equilibrium, but a trivial one. In general, every SE of the Γ games is a SE of the economy $\mathcal{E}$. If there is a SE, given that all trading posts are active, then this SE is called open SE. So basically the following remark refers to games Γ_1 and Γ_3.

We denote s = (q, r) an SE of $\Gamma_\iota(\mathcal{E})$ *and I(s) the set of all inactive trading posts at s.* **[9]** This means, that at these trading posts traders' money and goods disappear. We shall call s an *open* SE of $\Gamma_\iota(\mathcal{E})$, when given a price p_j, $j \in I(s)$, the set (q^t, r^r) maximizes the $u^t(z)$ for all traders. The final set of $z = (z^t, z^t_{m+1})$ is as follows:

$$\begin{cases} z_j^t = a_j^t - q_j^t + \dfrac{r_j^t}{p_j}, \; j = 1, \dots, m \\ z_{m+1}^t = a_{m+1}^t - \displaystyle\sum_{j=1}^{m} r_j^t + \sum_{j=1}^{m} p_j \; q_j^t \end{cases}$$

If we want to compare this set to the previous one, regarding the economy with fiat money and credit, we shall notice that the only difference refers to the final holdings of money, where one should take into account the initial amount of the m+1st commodity.

If one trader fails to spend all his money, then we shall call such a trader t interior. The mathematical relationship is $\sum_{j=1}^{m} r_j^t < a_{m+1}^t$. We shall call a trader competitive if he prefers z^t to z^t_{m+1}, since money is desirable and all prices are finite. This means that he is rather indifferent about what he really prefers.

CHAPTER 4

4.1 A Strategic Market Game with a Mutual Bank with Fractional Reserves and Redemption in Gold (a Continuum of Traders)

The following model reveals the importance and the role of a mutual bank. It also indicates how paper money should be issued and distributed. How bankruptcy or no bankruptcy affects the equilibria is yet to be examined, along with the relationship between the gearing ratio and the bankruptcy penalty.

Theoretically, whoever issues paper money, should do so, according to the reserves he has in gold, or any other metal (bullion). However, nowadays this relationship is rather violated. So again theoretically this kind of money issued tends to be rather inflationary. In our example we shall see that paper money is issued under specific check and control, in order to secure the former relationship.

The money which needs to be issued in one period strategic market game, is for the purpose of making the trade possible. In other words with this amount of money we finance the short term trade. Apart from that, it is also crucial to observe how money is distributed. The results differ, if the money is enough or inadequate, as well as if the money is rightfully distributed or not. Let us define mathematically what *money well distributed* means.

Suppose that i trader has an initial amount of α^i endowment. A W.E. (x, p) is the Walrasian Equilibrium of a finite number of potential equilibria of the Γ game of an economy $\mathcal{E}$. If money is well distributed then we have the following:

$$\sum_{j=1}^{m} p_j \max[0, (x_j^i - a_j^i)] \leq a_{m+1}^i \quad \textbf{(1)}$$

We remind that p_j refers to all prices at the W.E., while we also know that price is given by the ratio of the sum of bids offered to the sum of quantities quoted ($\frac{\sum b_j^i}{\sum q_j^i}$).

The previous relationship means that at a competitive equilibrium all prices that maximize the traders' payoff, if multiplied with the final holding of good j minus the initial holding of the same good j, should be less or equal to the initial amount of money, or the m+1st commodity, which also stands for money, if no paper money is used.

On the other hand if money is not well distributed, then again with respect to the Walrasian Equilibrium we have:

$\sum_{i=1}^{n} \sum_{j=1}^{m} p_j \max [0, (x_j^i - a_j^i)] \leq \sum_{i=1}^{n} a_{m+1}^i$ **(2i)**

and in some cases

$\sum_{j=1}^{m} p_j \max[0, (x_j^i - a_j^i)] > a_{m+1}^i$ **(2ii)**

If there is not enough money in the first place at a W.E. then,

$\sum_{i=1}^{n} \sum_{j=1}^{m} p_j \max [0, (x_j^i - a_j^i)] > \sum_{i=1}^{n} a_{m+1}^i$ **(3)**

In the previous models we used one commodity as a means of payment. This commodity could be any good j, that is fully agreed in advance to play the role of money. It is more likely that this m+1st commodity to be a certain kind of metal like gold or silver. In case of paper money, it should be issued with respect to the gold being held. As far as fiat money is concerned, it is basically divided to coin money (has some intrinsic value) and pure paper money. The basic contrast between coin and paper in terms of game theory is, that coin requires no trust or enforcement by the state or the banks, but paper money requires such a trust and enforcement.**[10]**

However the trend is that even more and more central banks hold smaller reserves in gold. So it is up to the faith in the state and its laws that support the whole mechanism of paper money issuing. Regarding gold, if the reserve ratio between a gold certificate and the unit of gold is near to one, then there is less need for trust in the system.**[10]** If this ratio requires less than one gold unit per gold certificate, we need more trust on the one hand, on the other hand we must issue harsher default penalties, in order to avoid deliberate bankruptcy.

The kind of the default penalties may vary, depending on the initial model structure, as long as there is no “cash in advance” constraint, which would make such penalties of no use at all. However, whether we speak of paper money or gold coins, specific rules and default redemptions and penalties are needed.

A mutual bank has a certain role to play in terms of an exchange game within an economy. This kind of bank has to ease the trade between the individuals, being an intermediary sort of speaking. Its main purpose should not be to make or maximize profit, but to grant credit, as well as centralize the size of debt. From this point of view, the mutual bank can also deal with everyone’s dept and default penalty, excluding any direct relationship between traders. In other words any financial difference is dealt “institutionally”. A more visualized example is when traders deposit an amount of gold at the bank, which makes them owners. From that point the mutual bank can manage every ownership, dept, or claim.

The bank can also set a gearing ratio, according to which, the bank can issue certificates of gold, which can outreach the total amount of gold deposited. In other words the gearing ratio multiplies the gold, allowing more paper money to be issued, and this is the mechanism, through which the bank can offer credit to someone.

The following model refers to a continuum of traders symbolized with T, consisting of n different types symbolized with n. Each trader has a set of endowments a^i= (a_1^i, …, a_m^i, a_{m+1}^i), while $a^i \in R_+^{m+1}$, and $\varphi^i = R_+^{m+1} \rightarrow R$. We note the following conditions:

1. $\sum_{i=1}^{n} a^i >> 0$;
2. $a^i \neq 0, i = 1, \ldots, n$;
3. φ^1 is continuous, concave, and strongly monotonic for all $i = 1, \ldots n$.

The following set refers to the strategy of each trader:

$S^i = \{u^i, v^i, b_1^i, q_1^i, \ldots, b_m^i, q_m^i, b_{m+1}^i, q_{m+1}^i\}$,

where u^irefers to the amount of gold deposited in the bank, v^i refers to the amount of I.O.U. notes bid by traders for banknotes, q_j^i, j = 1, 2, …, m, refers to the amount of good j offered for sale, b_j^i, j = 1, 2, …, m, refers to the percentage of banknotes bid on good j. We also present the following restrictions:

1. $0 \leq u^i \leq a_{m+1}^i$;
2. $0 \leq v^i \leq$ k $\sum_{i=1}^{n} a_{m+1}^i$, k≥ 1, where k refers to the gearing ratio for banknotes to be issued;
3. $\sum_{j=1}^{m+1} b_j^i \leq 1$, for all j = 1, …, m;
4. $q_j^i \leq a_j^i$, again for all j = 1, …, m;
5. $q_{m+1}^i + u^i \leq a_{m+1}^i$.

The bank is a strategic dummy in a three stage game. At the first stage traders deposit their gold in the bank. Their share of ownership is defined by the proportion of the amount of gold deposited, given by the following type:

$$\theta^i = \frac{u^i}{\int_T u^t dt}$$

At the second stage we have the bid for bank money. The bank therefore multiplies the amount of gold deposited by k (= the gearing ratio), and issues banknotes up to M, which defines the previous relationship (M = k . $\sum_{i=1}^{n} u^i$, or M = k. $\bar{u}$). The interest rate is given by the following type:

$$1+\rho = \frac{\int_T u^t dt}{M}$$

The third stage is all about the trade as we have previously referred to, that is to say the exchange of goods for bank money. In it' s simplest version we rule out asymmetric information, which would force us to deal with complex strategies, and we only admit that there is perfect foresight, at each stage. The amount of banknotes to be issued is given by the following type:

$$w^i = \frac{v^i M}{\int_T u^t dt}$$

Every strategy set is subject to the previous five restrictions. The price formation mechanism is described as follows:

$$\boldsymbol{p^j} = \begin{cases} \frac{\int_T b_j^t w^t dt}{\int_T q_j^t dt}, & \text{if } \int_T b_j^t w^t dt > 0 \text{ and } \int_T q_j^t dt > 0 \\ 0, & else \end{cases}$$

The below type gives us the final allocation of the first m goods.

$$x_j^i = a_j^i - q_j^i + \frac{b_j^i w^i}{p_j}, \text{ for all j = 1, …, m}$$

At the end of each trade also comes the final amount of gold given by the type below:

$$\boldsymbol{G_i} = \{a^i_{m+1} - u^i\} + \{\frac{v^i}{1+\rho} - \hat{v}^i - p_{m+1}g^i_L\} + \{\sum_{j=1}^{m+1} p_j q^i_j - \sum_{j=1}^{m+1} b^i_j\, w^i\} \; +$$

$$+\ \theta^i\{\mathrm{M} - \frac{\sum_{i=1}^{n} v^i}{i+\rho^*} + \sum_{i=1}^{n} \hat{v}^i + p_{m+1} \sum_{i=1}^{n} g^i_L\}$$

This type also connotes the potential debt of a trader at the end of the trading procedure.

<u>Symbol explanation</u>

$\hat{v}^i$: the part of repayment of his loan in banknotes

g^i_L : the part of repayment of his loan in gold

$p_{m+1}g^i_L$: par value of gold used for repayment

ρ : ex ante interest rate

ρ^* : ex post interest rate

There are four brackets in this type, and each one indicates something different. The first bracket identifies the amount of gold held up. The next bracket indicates the size of the loan and its repayment. The third one defines the final outcome of the buying/selling-goods activity. The final one just refers to all the financial elements that a bank needs to operate, as well as to organize the rules of our game. By this of course we mean liquidation, profit payments, repayment ratios and returns of the initial deposits of gold.

Each trader, unless otherwise imposed, has the option to repay the loan either in banknotes or in gold. He might either pay back the whole amount of the loan or partially. Furthermore, he can use only banknotes or gold, or even mixed. This means that gold may be used as a means of payment, so it can be also used in the trade. Or perhaps gold is used only at the first stage, when we want to issue paper money via the mutual bank.

As far as the interest rate is concerned, we have two types of rates, the ex ante rate and the ex post rate. In the beginning these two are equal. In case there is no bankruptcy, they are again equal. The ex ante rate is the one the bank uses, when a trader goes into depositing his gold and this rate defines the amount of extra money the bank has to pay, when a trader wants to take back his gold. It is symbolized with ρ. But if a borrower is unable to fulfill this obligation, then this ex ante rate after a certain calculation transforms into the ex post rate symbolized with ρ^* with $\rho^* < \rho$.**[10]** So the rest gold depositors are affected and they take back less than what was initially agreed.

Let us give an example for the ex post and ex ante rates, in order to visualize the previous reference. Suppose traders deposit their amounts of gold, silver etc and the mutual bank rewards them with a 10 percent. The traders may collect their deposits along with the interest not until the trade ends. But if one or more traders fail to meet his / their obligations – we refer to the money borrowed – then the bank will not grant the rest with a 10 percent but with a smaller one, let us say 7 percent for example. This procedure is crucial, because for the bank this is a way of protection against bankruptcy, and it distributes the losses of a potential default, between the traders.

In our model we assume that no deliberate bankruptcy is intended. This makes ex ante and ex post interest rate equal at the beginning. But in the end potential defaults and failures may cause a reduction on the ex post rate, so all these variables matter. The final payoff is not only a function of utility, but it also includes a function of dept, when we refer to the possibility of bankruptcy. Therefore we have to conduct a calculated value of dept from the utility payoff. The type below denotes this very thing.

Π^i(s) = $\varphi^i(\chi^i$(s)) – μD^i_+(s),

μ : the default penalty

D^i_+(s) : max[0, D^i(s)]

What about optimality when there is absence of bankruptcy? We assume therefore D^i < 0. This could be feasible if we chose for example a severe penalty rate. We also denote an exchange economy E(n, φ^i, α^i), where an allocation (x, p) is a W.E. if the following conditions are met:

1. $\sum_{i=1}^{n} x^i = \sum_{i=1}^{n} a^i$
2. x^i = arg max {φ^i(y) : y ∈ R_+^{m+1}, py = pα^i } ∀ i = 1,…, n.

*We denote a game Γ (n , φ^i, α^i, s^i) and if **s** such as for every t ∈ T and $\alpha^t \in \sum = X_{i=1}^{n} \sum^i$, then we consider a N.E. where $\Pi^t(s / a^t) \leq \Pi^t(s)$, where (s / a^t) is s with s^t replaced by a^t.*

A Type-Symmetric Nash Equilibrium (T.S.N.E.) is established when for every t, t´ ∈ T_i, i = 1,…, n → $s^t = s^{t'}$. An active T.S.N.E. is when ρ > -1 and p_j > 0, for all j = 1,…, m+1.

Theorem 4.1.1 (Shubik M. and Tsomocos D.)

The strategic market game Γ (n , φ^i, α^i, s^i), with a continuum of traders of n types and a mutual bank for any non-zero amount gold and any gearing ratio k ≥ 1 and a sufficiently harsh bankruptcy penalty μ, which results into no bankruptcy, has an active T.S.N.E.

Theorem 4.1.2 (Shubik M. and Tsomocos D.)

For a sufficiently high gearing ratio k and a sufficiently harsh penalty μ, which entails no bankruptcy, the strategic market game $\Gamma\ (n\ ,\ \varphi^i,\ \alpha^i,\ s^i)$ will produce $\rho = 0$ and give the same relative prices and distribution as the W.E. of the associated exchange economy $E(\ n,\ \varphi^i,\ \alpha^i)$.

Theorem 4.1.3 (Shubik M. and Tsomocos D.)

The strategic market game $\Gamma\ (n\ ,\ \varphi^i,\ \alpha^i,\ s^i)$ with a continuum of traders of n types and a mutual bank for any non-zero amount of gold and any gearing ratio $k \geq 1$ and bankruptcy penalty μ has an active T.S.N.E.

Theorem 4.1.4 (Shubik M. and Tsomocos D.)

For a sufficiently high gearing ratio k and a sufficiently harsh default penalty μ, the strategic market game $\Gamma\ (n\ ,\ \varphi^i,\ \alpha^i,\ s^i)$ will have T.S.N.E.'s, which do not involve bankruptcy, produce an endogenous interest rate $\rho = 0$, and give the same relative prices and distribution as the W.E. of the associated exchange economy $E(\ n,\ \varphi^i,\ \alpha^i)$.

Let us highlight the significance of the gearing ratio and the confrontation of a default by adopting a bankruptcy penalty. As already mentioned in the beginning of this section, the gearing ratio is this very mechanism, through which the bank may control the money supply and justify the rate and extent of the loans given. This k factor multiplies the deposited gold in reserves and as a result it defines the total amount of money in currency.

We also know that the level of circulated money directly affects the level of prices. If the money supply is characterized as inflationary, then we would expect the commodity prices to increase. If there is less money supplied, the prices tend to decrease. In other words it affects the pricing mechanism, by setting a higher and a lower bound on prices respectively.

It is very important to make the following notice. The positive supply of money should not change the relative prices, but only the actual ones, so the relationship between two goods will remain intact. For example if someone affords two gold coins to buy one apple, while he also spends 4 gold coins to buy one pencil, this sets an exchange ratio $\frac{2}{4}$, which is one pencil for two apples. Having more money now, a trader should give 3 coins for one apple and 6 coins for one pencil. Both prices of the goods have risen up, but none of it lost its value, since the exchange ratio remained unaffected. This phenomenon is called “money neutrality”.

We consider that there is no external intervention. Therefore all prices and interest rates are set endogenously by our initial rules and by the behavior of the traders. We exclude any exogenous uncertainty, as well as we keep the players fully informed, in order to deal with information asymmetry.

Concerning the bankruptcy, we rule out the intentional one. But no matter how hard someone tries to avoid bankruptcy, it is not always feasible. How hard these efforts will be, depends also on the penalty. The penalty can be a fine or confiscation etc., or it can be blockage from the game, or prison etc. Regardless the nature of the penalty, the size of it controls in a way the frequency and the size of defaults, which are likely to happen anyway. By this we mean that with the grant of credit comes greater responsibility in a trader’s decision making, if he wants to pay back his creditor. We should also notice that the penalty sets a lower bound on prices as well.

4.2 The Value Of Money In A Finite-Horizon Economy. A Role For Banks

We have said so far that the traders' preferences for commodities are strictly monotonic. But what about their preferences for money? What is the value of money in an one-period model as well as in a finite-horizon economy? Let us admit that there is no point in keeping fiat money after the trade has ended, when we refer to the one-period model of economy. Simply because money is used only as a medium in the trade. No one can consume it, since it is only paper when we refer to banknotes. Therefore the value of it equals to zero.

The same problem we are about to confront with, when we insert more periods in our model. In the last period no trader wants to keep a positive amount of money. Its price equals to zero according to the demands of the equilibrium. Going back one period we can also admit that its price is again zero since there is no reward in carrying money forward in the last period. If we do this comparison from period to period, going backwards from end to beginning, we shall realize that the value of money is zero, provided that the money supply is positive and that we refer to a finite-horizon economy.

There are different ways of arguing about the positive value of money. Some theories have proven that money has value in any period because it is a store of value, therefore it carries this value forward as our model evolves. Some others enact an agent, who has the task to exchange commodities for money at certain prices.**[11]** Let us see what happens if we insert a bank in our model. This bank wants to maximize its profit, by gaining more money. The bank's function has nothing to do with the reserve of a mutual, and the reward of its owners. This time the bank is competitive and works as a single unit, trying to increase its money without offering anything in exchange. However, the bank still may lend money to someone for some reason.

Agents again are linked with the store of value of money, but they also participate actively in the trade. We impose the following rules in our model. First of all, all payments are made in advance. The bank is provided with money from the outside, meaning that the bank has an initial stock of money, which is set exogenously. Traders have to exchange goods using fiat money. They can spend all of their money they already possess, but they can also go through borrowing, if they want to spend more money. If they do so, then they are obliged to repay the bank, taking into account an interest rate, which is set by the bank.

The main difference between this and the previous model is, that the bank here uses the gearing ratio, in order to increase its own amount of money and therefore its profit. In case of a default, the bank will not change the interest rate once used, in order to service the social good. Traders in debt will face the consequences, without affecting others as a sole unit. We saw earlier that, if someone fails to return the money borrowed, then the bank will change the rate promised to the depositors for using the bank.

Another main difference is, that traders also have an initial amount of money, which is set exogenously. So the general idea here is that traders can use the bank for their own purposes, for example to optimize their behavior, or to deal better with their debts at some point of the game, or to improve their position as single players.

Traders make their transactions by using only fiat money. The bank's profit is defined by the level of the interest rate. At this point, a remark is necessary. The rate of interest is subject among others to the level of money supplied in the economy. The more money is supplied, the less the interest rate becomes, when we refer to bank money. If we simply supply the market directly with more fiat money, it causes the increase of the interest rates.**[11]** In any case, as we introduce more fiat money, especially more bank money in the economy, we reach Pareto optimality. Because of its transaction role as well as its role of a store of value, we treat money as a durable good.

Finally, as far as default penalties are concerned, we have mentioned in the previous model, where we imposed the existence of a mutual bank, that they play a significant role for the equilibrium. Because we want to avoid "easy" defaults, we set rather high penalties, but always in proportion with the money supplied and the volume of trade. We do not want the penalties to be unreasonably high and affect the lending and the ambitious behavior of the traders, because this could cause the suffocation of the economy. We let defaults happen, in a sense that someone is unable to overcome his miscalculations or his misjudgments. Therefore, the size of the penalty is not always set a priori, but it depends on the size of the debt, that cannot be paid.

When we use a model to describe a finite economy, we refer to different types of traders, where each type can strongly support its preferences. We do not treat traders as units, but as sets, which indicate the different types. In the next model we denote the sets of commodities with L = $\{1,\ldots,\ \ell\}$, the sets of traders with H = $\{1,\ldots,h\}$. We also denote with M the initial bank money supply (M > 0), and with m^i the initial amount of money held by each trader. We symbolize with c^i the amount of money, that a trader i can borrow from the bank. In the end he has to pay back to the bank an amount equal to μ^i = (1+θ)c^i, where θ is the rate of interest for borrowing money.**[11]**

In simple words we may say, that a trader given the common constraints, i.e. he cannot sell more goods than those he has at his disposal as well as he cannot bid more than his pocket allows, he has at the end of each period to deal with his potential debt. He can either choose to pay his debt partially or as a whole. If he goes for the partial deal, he expects, that with the money that he kept on hand, he may gain more benefits in the next period, as well as pay back his loan under better circumstances. The choices he will make, depend also on the type of penalties and their relationship with the prices.

For example nominal penalties do not directly depend on prices, but the real penalties, as well as the endogenous ones depend on price levels, while they can also relate to other variables in terms of the economy.

4.3 Monetary Equilibrium

Speaking of fiat money, we introduce another type of equilibrium, the monetary one. This equilibrium is strongly related to the optimality of the goods. Upon Pareto optimality such an equilibrium is void and the money has no positive value. Money is needed, in order to make transactions / exchanges happen. On the other hand, if we have a positive initial endowment of money, then there exists a positive interest rate at monetary equilibrium.**[11]** This condition restrains, suspends trade. Since fiat money has no direct utility, given that money rate is large, no trader would do trade, knowing the outcome of a potential default.

The above conclusion indicates that if one trader borrows money with a money rate of 10%, while at the end of the trade he has to pay back an amount calculated with 12%, it is obvious that he will suffer a loss. This would weaken his trading position, as well as deteriorate his utility. This is the reason why positive money rates make traders reluctant to go further with trade, at least at that time.

It is very crucial to notice, that the most important thing with fiat money, especially when its supply is quite sufficient, are the so called “ gains from trade”. This theory explains that the more money we have, the more motivated we are to do trade. Such a relationship results in a better allocation of goods, improving everyone’s needs and wants, so finally this better commodity distribution enhances the equilibrium, as well as improves the utility function.

CHAPTER 5

Conclusions

We have examined so far how trading relationships and economic equilibria change due to different price formation mechanisms, as well as different institutional assumptions. The general idea is to observe how the basic three different strategically constructed models apply to complete or incomplete markets. Additionally, we quote further extensions on these models, in order to enhance our analysis on strategic equilibria and its significance to the social welfare.

In the first “sell-all” model, we introduced a finite number of traders and commodities, as well as a limited amount of money for each trader. We use only commodity money with intrinsic value and we demand that all payments are made in advance. Furthermore, each trader is obliged to put all of his endowments for sale. We also note that this model refers to a single-period trade. We conclude that as contrasted to general economic equilibrium theory a la Arrow – Debreu proves, that the final allocation of commodities is not optimal due to the strategic character of the equilibrium concept. We call this strategic equilibrium Cournot – Nash.

The next “bid-offer” model is an extension of the first one. Again we keep a finite number of traders and commodities. Quantities remain the strategic variable. There are two basic differences compared to the previous model. We relax the model by allowing traders to put for sale as many commodities as they prefer, therefore the sell-all constraint is violated. Fiat money can also be used as a means of payment. Regarding the equilibrium, still remains suboptimal. However, when we accommodate a continuum of traders, we reach optimality. In fact, the continuum case obtains in the limit, as we sequentially replicate the economy while the resulted equilibria are approximate.

The third "price – quantity strategy" model shows the way out of sub-optimality with no continuum of traders. Quantities are no longer the strategic variable, but prices. Traders both sell and buy, so they quote different bid and ask prices as well as quantities. The final outcome results from the total goods sold minus total goods bought. The described equilibrium is competitive and we call such an equilibrium Bertrand – Nash.

We also described the role of money, in particular. We recall that money is divided into commodity and fiat money. Regarding fiat money, we have coins as well as paper money. In some occasions we may also use promissory notes. In addition, we may allow credit and money borrowing or not. Traders' behavior and decision making depend on these varying discriminations. Therefore, different type of money has a direct impact on the allocation of goods.

Money is a strong institutional factor , because it eases trade. It helps us deal with transaction costs and other obstacles of the barter economy. Therefore, we say that money decouples a market. It can also be used to resolve problems of trust, uncertainty. It is a tool of measuring one's purchasing power, it is a store of value and in general it gives a strategic freedom to traders. All in all, a monetary market acts differently than a barter one. Monetized trade has a special effect on both the final allocation of commodities and the strategic equilibrium.

Taking monetized trade a step further we introduced credit, money borrowing and the effect of a potential default on the strategic actions and its social impact. We introduced the role of another, quite important institutional factor, the mutual bank.

Yet, we used in a different model a competitive banking sector, which strongly influences the price formation mechanism and the final drift of money and goods.

Finally, we introduced a market with financial assets. The institutional settings of the following market meet the so – called "securities". It is a two-period market model, where each trader may buy a bundle of securities in order either to protect his property against uncertainty and hedge his up to that moment portfolio, or curry forward a part of his income. By extending the Shapley – Shubik model, at the end of the second period a Nash Equilibrium is established. This model allows traders to reallocate their resources and come up with defensive or aggressive strategies.

This essay attempts to highlight the significance of the institutions and how they influence the price formation mechanisms. In fact, it indicates the relation between the institutional activity and the market efficiency. We realize that in real world institutions often do not work or they do not work properly. They are designed to service the social welfare, still appear quite incomplete and manipulable markets. Should these institutions get more enhanced or altered? Should they be in some cases aborted and replaced? We conclude that sometimes it is wise to do so.

For example, the aforesaid models illustrated the characteristics of an incomplete market, whether there is an oligopolistic competition or a collusion between two large groups of interests, which perverse the market, i.e. by keeping unreasonably high prices, while they both have low productivity and marginal costs. We may intervene and change the rules of trade, aborting for example the quantity – model and replacing it with a price – model. Specifically, we may set low / high price limits. So, first we can decode a market and its mechanisms, secondly we can explain why such inefficiency happens. Then we can suggest ways of solving such issues and rationalizing the market. Under these assumptions we may improve the general welfare of people and score out any disutility and indeterminacy.

The economic theory suggests that people find adequacy and fully satisfy their needs in a pure competitive economy, in Pareto terms. It also notices that strategic behavior is not preclude even under in complete markets. When strategic power is present, it is always the case of arbitrage opportunities,**[12]** and information plays no role. Some theories also refer that under incomplete markets with strategic behavior are better balanced than complete markets with strategic behavior. In other words strategic behavior tends to correct inadequacies and improve the utility function of people, while in a complete market tend to vitiate optimality. As a consequence, imperfect competition may Pareto-dominate perfect competition when markets are incomplete.**[13]**

Taking the previous assumptions under scrutiny, we support the following argument. Since pure competition in real world economies is rather unfeasible, if someone wants to talk about optimality, why to reach perfect competition and not try to strengthen and improve the institutional regulations within the imperfectly competitive economy. We argue that the market itself may offer corrected allocation of goods favorably of the social welfare.

Finally, a reverse–logic theory, known as Implementation Theory, is supported by a great part of literature, that is, we design mechanisms to implement Pareto outcomes. Strategic market games constitute a solid ground to develop new institutions within the framework of mechanism design towards this direction. In other words, I am aware of the final outcome and I try to design the game, which will bring in this result.

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