University of Piraeus
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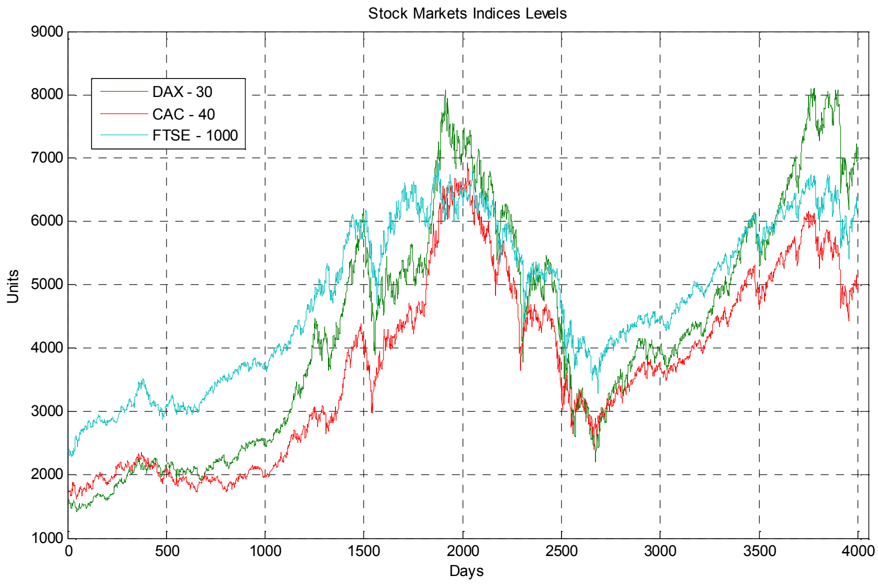
Thesis:

'Finite Sample Properties of Causality in Variance Tests: A Monte Carlo Study'



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INTRODUCTION

The globalisation of trade and financial transactions and the relatively recent deregulation of equity markets, combined with the rapid growth in telecommunications technology have strengthened the international interdependence among the various capital markets. Under these circumstances the need for a quantitative investigation of the causal interconnections between the individual markets, becomes more necessary than ever. This kind of knowledge is of particular importance not only for the individuals participating in these markets but also for the governmental supervisory and regulatory agencies. Inside this framework and during the last twenty years, an entire literature that seeks to study the volatility transmission mechanism has been developed. Simultaneously an increasing number of studies report the presence of a phenomenon that appears to characterize a large number of financial and economic time series. This is the so called long memory in the volatility of financial time series.

The principal aim of our study is to investigate the finite sample statistical properties for a group of methodologies which have been proposed in the econometric literature for the detection of causal relations in variance as well as in the mean. More specifically through extensive Monte Carlo simulations we attempt a thorough examination of the performance of the various methodologies under different states for the series studied as well as under the presence of long memory. For this purpose we use alternative Data Generating Processes such as the GARCH and FIGARCH models that will be described in detail in the following chapters. The methodologies we use belong in two categories. The first one is based on the estimation of the cross correlation function for the (squared) standardized residuals that are obtained from the estimation of univariate GARCH models, while the second one uses the residuals of the same models but takes the form of a Lagrange Multiplier test. The innovative part of this study lies on the fact that for the first time to our knowledge such an extensive in depth analysis of the competing causality tests is conducted. This feature makes our study particularly important for both theoretical and applied econometricians as well as for the professionals in the field of financial economics.

Our work has the following structure. In the first chapter we introduce the reader in the notion of volatility. In the second chapter we initially discuss the general concept of causality and then provide a thorough description of the various causality in variance / mean tests. In the third chapter we define the notion of long memory in both the first and second moments and then describe in detail the FIGARCH model. In the fourth chapter the Monte Carlo design of our study is discussed and we present the most important of our empirical results. The appendix is organized in four parts. In the first one we provide the MatLab Code that was used in this study, in the second part we present a large number of figures and graphs that were omitted from the main part of the dissertation for brevity reasons, in the third part we present the results of an empirical application using real time series data, and in the fourth part we provide the empirical results of three additional Monte Carlo experiments. Finally we would like to note that part of this study has been included in a working paper that we are preparing this period and in which a great part of our MatLab code has been used.

In this point I would like to express my thanks to Dr. Christina Christou for her important support and valuable guidance during the preparation of my Dissertation. Dr. Christou was always willing to answer any of my questions.

CHAPTER 1 : MODELING FINANCIAL VOLATILITY

1.1 INTRODUCTION TO VOLATILITY

The notion of volatility is of great importance in the field of finance. This is due to a number of reasons. Firstly, when asset prices fluctuate in an extreme way and in very short time intervals then it is difficult for the participants in the stock markets to accept that the reasons of these abrupt changes, are exclusively related with the arrival information that concerns the economic fundamentals. This fact leads the investors to distrust against the capital markets with a direct consequence the reduction of the capital inflows in the financial markets. Secondly for every listed corporation the volatility of its stock constitutes a factor that determines the probability of default of this company. The higher the volatility given the capital structure of a company, the higher also the probability of bankruptcy. Thirdly, volatility affects the liquidity of the markets and it is positively related with the bid ask spreads defined by market makers. Fourthly the hedging strategies applied from investment portfolios managers depend directly on the level of financial market volatility. Fifthly, economic and financial theory converge to the fact that consumers/investors are risk averse something implying that the higher the volatility in the stock markets, the higher the compensation of the investors should be. A high degree of risk will lead to decreased participation in the investment process through the financial mechanism. Finally a systematically high level of volatility can lead the regulatory and supervisory authorities to force the various financial corporations to allocate a higher percentage of their available capital in more liquid but less efficient investments.

Regarding the concept of volatility we indicatively mention two important studies, those of **Bollerslev, Chou & Kroner** (1992) and **Daly** (2008). The concept of volatility is directly linked with the notions of non forecasting ability, uncertainty and risk. It is also synonymous with the non efficient operation of financial markets in the sense that the stock titles are not priced always in their true intrinsic value but occasionally there are significant deviations between the latter value and the market price of a stock. Traditionally volatility was interpreted as the variance of financial returns and constituted the building block of the modern portfolio theory, the capital asset pricing models and the arbitrage pricing models. The oldest and most frequently used, measure of variation is the standard deviation of returns derived from the discipline of statistics. In this chapter however we will not describe this vital but very simple notion but we will directly present an important concept of measuring financial volatility that depends on a new class of heteroscedastic models.

1.2 QUANTIFYING VOLATILITY

1.2.1 ARCH MODELS

The most important step in the field of financial volatility forecasting was made with the development of the Autoregressive Conditionally Heteroscedastic class of models from **Engle** (1982). The most significant contribution of these models was the introduction of a strict mathematical formalization of the notion of volatility, facilitating in this way the in depth analysis of the dynamics of economic and financial time series. The idea behind this new class of volatility models was the observation that earlier researchers have made, about the phenomenon that big (small) changes in asset prices tend to be followed by big (small) changes. The ARCH specification as it is usually referred to, captures some of the most frequently observed properties of financial time series, such as the excess kurtosis of returns, the time varying volatility and the volatility clustering. Alternatively the ARCH effects are used to denote the presence of autocorrelation in the second order moments process. This serial correlation of volatility is the key element behind the predictability of this quantity.

We must say now that the presence of ARCH effects is not in contrast with the efficient market hypothesis according to which the past returns of financial assets cannot be used in order to achieve systematically abnormal returns in the future. The absence of autocorrelation in the returns does not necessarily means that the forecast errors are generally independent. This means that a possible existence of non linear relations governing the stochastic process of residuals can simultaneously be in accordance with the efficiency of the markets. Non linearities in the innovations process can simply denote the presence of linear relations in the conditional variance process of the errors. A possible source of these volatility clustering effects is the autocorrelation in the news arrival process meaning that information arrives in the market, in clusters and not in uniformly distributed time points. In other words the stochastic process that describes the flow of information in the market is characterized by linear dependence with the quantity of information that reaches in the market to be a function of time. An important research regarding the presence of time varying conditional heteroscedasticity is of **Lamourex & Lastrapes** (1990).

The ARCH models have been used in a vast number of empirical applications such as the statistical tests of the CAPM and APT pricing models, in the study of the information flow across countries and financial markets, in the design of efficient hedging strategies, in modelling the relation between time varying volatility and risk premium, in the analysis of the monetary policies effects in the economy etc. Returning now to the intuition of these models we have to say that an important characteristic of this new class is the fact that the variance of the forecast error is a function of the size of past errors. In a regression model a shock (error) is generally the deviation of the specific realized value of the dependent variable under study from the conditional mean of the latter variable. In an ARCH model the conditional variance of the forecast error is a positive function of the size of its own past values. This means that large errors (irrespectively of their sign) will tend to be followed by large errors and conversely. The order of the model will be determined from the time duration that a shock will influence the conditional volatility. An obvious advantage of this kind of models is that they consider the conditional volatility to be time varying instead of being constant through time. The traditionally assumed constancy of variance is a very simplified and not realistic hypothesis. However the unconditional or long term variance in the ARCH models is considered to be constant. It is now time to formalize our discussion.

From the field of econometrics we know that the best possible forecast for the value that a random variable y_t can take in a certain future time point, conditional on the information from the past realization of this variable, is given from the conditional mean $E(Y_t / Y_{t-1})$. The conditional mean can be calculated using either the conditional probability function or the conditional probability density function when using discrete or continuous variables respectively. The above mentioned forecast is also a random variable and thus follows a certain probability distribution and has a variance equal to $Var(Y_t / Y_{t-1})$. Once more this conditional variance can be assumed to be a random variable as it is dependent on past information. Let's consider now the classic autoregressive time series model of first order $y_t = \gamma y_{t-1} + \varepsilon_t$ where the error $\{\varepsilon_t\}$ constitutes a white noise process with variance $V(\varepsilon) = \sigma^2$. The conditional mean of y_t

is equal to γy_{t-1} while the unconditional mean is equal to zero. The most important contribution of time series models sources from the use of the conditional mean concept. The conditional variance of y is equal to σ^2 while the unconditional Variance is equal to $\sigma^2 / 1 - \gamma^2$. Obviously there is a direct decrease in the uncertainty of the future forecasts of y when using past information. This observation leads us to the parallel conclusion that we could also achieve more accurate variance forecasts if we incorporated into the corresponding models the information from past realizations of volatility. The classic approach to the concept of heteroscedasticity is with the incorporation in the regression models of an extra regressor, an exogenous variable χ_t. In the field of financial econometrics however a relatively different approach must be followed. This is due to the need to adequately describe the reasons behind the time varying nature of volatility. A first attempt was made by **Granger & Anderson** (1978) but in their case the unconditional variance could be equal either to zero or infinity and this characteristic meant that their model was inadequate.

An alternative and more preferable model is the following

$$y_t = \varepsilon_t h_t^{1/2}$$

$$h_t = a_0 + a_1 y_{t-1}^2$$

$$V(\varepsilon_t) = 1$$

This model is called ARCH. If we additionally impose the normality assumption and given the information set Ψ_t available at time t we can formulate the following relations using the conditional distribution.

$$y_t / \psi_{t-1} \sim N(0, h_t)$$

$$h_t = a_0 + a_1 y_{t-1}^2$$

The conditional variance or skedastic function can alternatively be written in the form $h_t = h(y_{t-1}, y_{t-2}, y_{t-3}, \ldots, y_{t-p}, a)$ Where p denotes the order of the ARCH stochastic process and α denotes a vector with unknown parameters. The regression ARCH models can be constructed assuming that the conditional mean of y_t is equal to $\chi_t \beta$ and is simply a linear combination of lagged endogenous and exogenous variables that are all contained in the information set. In a more general formulation we can have the following:

$$y_t / \psi_{t-1} \sim N(x_t \beta, h_t)$$

$$\varepsilon_t = y_t - x_t \beta$$

$h_t = h(\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-p}, a)$

or

$h_t = h(\varepsilon_{t-1}, ..., \varepsilon_{t-p}, x_t, x_{t-1}, ..., x_{t-p}, a)$

The ARCH regression model with its intrinsic property to allow the conditional variance to be time varying and predictable is very useful for studying a characteristic of forecasting procedures that has to do with the fact that the extent of uncertainty is a function of the time horizon that we use for conducting forecasts. This specification is also the mathematical representation of the empirical observation that the forecast errors tend to cluster through time and depending on their size. We can alternatively consider an ARCH as a more complicated regression model in which the disturbances are not the occasional white noise but instead follow an ARCH process. In this way we can also partially resolve the misspecification problems often encountered in the classic regression models. Let's assume now that the underlying generating mechanism behind the values of y_t is an ARCH process with the properties mentioned above. The mean of y_t as well all the autocovariances will be equal to zero. The joint probability density function (PDF) of this process is equal to the product of all the univariate PDF and taking the logarithm of this product we get the log likelihood function.

$$l_t = -\frac{1}{2}\log h_t - \frac{1}{2} y_t^2 / h_t$$
$$h_t = h(\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-p}, a)$$

In order now to estimate the unknown vector α we must maximize the log likelihood function. The first and second order conditions are the following:

First Order Conditions: $\frac{\partial l_t}{\partial a} = \frac{1}{2h_t}\frac{\partial h_t}{\partial a}(\frac{y_t^2}{h_t} - 1)$

Second Order Conditions: (Hessian) $\frac{\partial^2 l_t}{\partial a \partial a'} = -\frac{1}{2h_t^2}\frac{\partial h_t}{\partial a}\frac{\partial h_t}{\partial a'}(\frac{y_t^2}{h_t}) + [\frac{y_t^2}{h_t} - 1]\frac{\partial}{\partial a'}[\frac{1}{2h_t}\frac{\partial h_t}{\partial a}]$

The Information Matrix is equal to the negative value of the expectation of the Hessian Matrix and is given by:

$$\wp_{aa} = \sum_t \frac{1}{2T} E[\frac{1}{h_t^2} \frac{\partial h_t}{\partial a'} \frac{\partial h_t}{\partial a'}]$$

With its consistent estimator $\hat{\wp}_{aa} = \frac{1}{T} \sum_t [\frac{1}{2h_t^2} \frac{\partial h_t}{\partial a} \frac{\partial h_t}{\partial a'}]$

The most efficient estimation method of the unknown parameters is the Maximum Likelihood estimation that is the default method used in the previous lines but the least squares could be an alternative but less efficient equivalent. The function h of order p has the following specification $h_t = a_0 + a_1 + y_{t-1}^2 + ... + a_p y_{t-p}^2$

The ARCH model of order one is very simple but can also be particularly useful. According to this specification a high value of y in time t will lead in an increased forecast for the variance in the next period. However in the more distant future (e.g. in time t+2) the memory of the process will not remember this high value of y. An ARCH process generates realizations with fatter tails compared with the normal distribution. The conditions in order the ARCH process to have finite long term variance (to be second order stationary) are that all the roots of the characteristic equation lie outside the unit circle given that $a_0 > 0$ $a_1, ..., a_p \geq 0$. This long run unconditional Variance is equal to $E(y_t^2) = \frac{a_0}{(1 - \sum_{j=1}^{p} a_j)}$

1.2.2 GARCH MODELS

The innovation of this class of models that were introduced in financial econometrics by **Bollerslev** (1986) is that they allowed the past conditional variance to enter the conditional volatility ARCH model. They also helped to obtain a more parsimonious representation for the variance specification reducing significantly the number of unknown parameters. Thus in a GARCH model the forecast for the next period's variance depends both on the past realized volatility (as estimated from the squared errors) and on the past conditional volatility. This parameterization constitutes a generalization of ARCH representations and allows a more flexible structure in the lagged residuals that enter the model. GARCH models are able to capture more parsimoniously the long memory characteristics that govern the volatility stochastic process. Let's now assume that we have a discrete time stochastic process {ε_t} and that ψ_t is the information set (or σ – algebra) available on time t. Then the GARCH (p,q) process is given by the following formulations:

$$\varepsilon_t / \psi_{t-1} \sim N(0, h_t)$$

$$h_t = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \quad = a_0 + A(L)\varepsilon_t^2 + B(L)h_t,$$

$$a_0 > 0, \quad a_i \geq 0, \quad i = 1, ..., q$$

$$\beta_i \geq 0, \quad i = 1, ..., p$$

$$p \geq 0, \quad q > 0$$

When p equals zero then the GARCH process is equivalent to an ARCH(q) process. If both p and q equal zero then {ε_t} is simply a white noise process. In contrast to the simple ARCH model where the conditional variance is a function of past realized volatility, in the more advanced GARCH specification the conditional variance is also a function of past conditional volatility allowing to formulate a better adapting learning mechanism for the volatility modeling procedure. The GARCH model can be constructed by utilizing the residuals (innovations) of a linear regression model, $\varepsilon_t = y_t - x_t' b$, where y_t is the dependent variable, with χ_t being the vector containing all the independent variables and b being the vector with the unknown parameters. In the case that all the roots of the characteristic equation 1-B(z) = 0 lie outside the unit circle then the conditional variance model can also be equivalently expressed as

$$h_t = a_0(1-(B(1))^{-1} + A(L)(1-B(L))^{-1}\varepsilon_t^2 \quad = a_0(1-\sum_{i=1}^{p}\beta_i)^{-1} + \sum_{i=1}^{00}\delta_i \varepsilon_{t-i}^2$$

From the above formulation we can observe that a GARCH model is simply an infinite order ARCH representation with δ_i derived from $D(L) = A(L)(1-B(L))^{-1}$

$$= \sum_{j=1}^{n} \beta_j \delta_{i-j} \qquad i = q+1, ...$$

and calculated through $\delta_i = a_i + \sum_{j=1}^{n} \beta_j \delta_{i-j}, \quad i = 1, ..., q$

where n = min (p , t-1) If however B(1) < 1 then δ_i will decay for all i which are greater than m with m = max{p,q}. In regard with the stationarity conditions we have the following facts. The GARCH stochastic process will be stationary if given that E (ε_t) =0 , cov (ε_t, ε_s) = 0 & Var (e_t) = a_0 (1-A (1)-B (1))$^{-1}$ we have that A(1)+B(1) < 1 An equivalent representation for the GARCH model is the following:

$$\varepsilon_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \varepsilon_{t-j}^2 - \sum_{j=1}^{p} \beta_j v_{t-j} + v_t$$

$$v_t = \varepsilon_t^2 - h_t = (n_t^2 - 1) h_t,$$

$$n_t \sim NIID(0,1)$$

Where v_t is the shock to volatility. Thus we can also consider the GARCH process as an ARMA (m,p) process in the squared residuals with m=max{p,q}. The more simple but also occasionally used in empirical applications is the GARCH(1,1) model given by $\mathrm{h}_t = a_0 + a_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$, $a_0 \geq 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$

The GARCH (1,1) process will be stationary if and only if $a_1 + \beta_1 < 1$

The necessary and sufficient conditions in order to ensure the existence of the second order moments of a GARCH (1,1) process are the following:

$$\mu(\alpha_1, \beta_1, m) = \sum_{j=0}^{m} \binom{m}{j} a_j a_1^j \beta_1^{m-j} < 1,$$

$$a_j = \prod_{i=1}^{j} (2j-1), \quad \text{j=1...} \qquad a_0 = 1,$$

If β_1 = 0 then the condition mentioned above takes the same form as that of the ARCH models, $\alpha_m \alpha_1^m < 1$. If now in an ARCH (1) we have that $a_1 > (a_m)^{-1/m}$ then the second order moments will not exist. In contrast to that, if in a GARCH(1,1)

$$\sum_{i=1}^{00} \delta_i = \alpha_1 (1-\beta_1)^{-1} > (\alpha_m)^{-1/m}$$

this does not imply anything about the second order moments existence. This is due to the long memory feature underlying this kind of process. The kurtosis coefficient is by construction equal to

$$k = \left(E\left(\varepsilon_t^4\right) - 3E\left(\varepsilon_t^2\right)^2 \right) E\left(\varepsilon_t^2\right)^{-2} = 6\alpha_1^2 (1 - \beta_1^2 - 2\alpha_1\beta_1 - 3\alpha_1^2)^{-1}$$

as these processes are more leptokurtic (fatter tails) than what the normal distribution hypothesis defines.

In regard now with the estimation procedures for the GARCH models we will simply recall some important formulations without extending our discussion in a more depth analysis as it is not necessary for the purposes of this study. Thus we have the following:

$$\varepsilon_t = y_t - x_t^{'}b$$

$$\varepsilon_t / \psi_{t-1} \sim N(0, h_t) \qquad \omega^{'} = (a_0, a_1, ..., a_q, \beta_1, ..., \beta_p)$$

$$h_t = z_t^{'}\omega \qquad z_t^{'} = (1, \varepsilon_{t-1}^2, ..., \varepsilon_{t-q}^2, h_{t-1}, ..., h_{t-p})$$

$$\mathrm{E}(\varepsilon_t^2) < \infty \qquad \theta = (\mathrm{b}^{'}, \omega^{'})$$

$$\theta \in \Theta$$

$$\left.\begin{aligned} L_T(\theta) &= T^{-1}\sum_{t=1}^{T} l_t(\theta) \\ l_t(\theta) &= -\frac{1}{2}\log h_t - \frac{1}{2}\varepsilon_t^2 h_t^{-1} \end{aligned}\right\} \textit{Log Likelihood}$$

In order to estimate the unknown parameters of this model we just maximize the log likelihood function in respect with these parameters. Taking the partial derivatives of these functions we have for the conditional variance model:

$$\frac{\partial l_t}{\partial \omega} = \frac{1}{2} h_t^{-1} \frac{\partial h_t}{\partial \omega}\left(\frac{\varepsilon_t^2}{h_t} - 1\right)$$

$$\frac{\partial^2 l_t}{\partial \omega \partial \omega'} = \left(\frac{\varepsilon_t^2}{h_t} - 1\right)\frac{\partial}{\partial \omega^{'}}\left[\frac{1}{2} h_t^{-1} \frac{\partial h_t}{\partial \omega}\right] - \frac{1}{2} h_t^{-2} \frac{\partial h_t}{\partial \omega}\frac{\partial h_t}{\partial \omega^{'}}\frac{\varepsilon_t^2}{h_t}$$

with

$$\frac{\partial h_t}{\partial \omega} = z_t + \sum_{i=1}^{p} \beta_i \frac{\partial h_{t-j}}{\partial \omega}$$

and for the conditional mean model

$$\frac{\partial l_t}{\partial b} = \varepsilon_t x_t h_t^{-1} + \frac{1}{2} h_t \frac{\partial h_t}{\partial b}\left(\frac{\varepsilon_t^2}{h_t} - 1\right)$$

$$\frac{\partial^2 l_t}{\partial b \partial b^{'}} = -h_t^{-1} x_t x_t^{'} - \frac{1}{2} h_t^{-2} \frac{\partial h_t}{\partial b}\frac{\partial h_t}{\partial b^{'}}\left(\frac{\varepsilon_t^2}{h_t}\right)$$

$$= -2 h_t^{-2} \varepsilon_t x_t \frac{\partial h_t}{\partial b} + \left(\frac{\varepsilon_t^2}{h_t} - 1\right)\frac{\partial}{\partial b^{'}}\left[\frac{1}{2} h_t^{-1} \frac{\partial h_t}{\partial b}\right]$$

with

$$\frac{\partial h_t}{\partial b} = -2\sum_{j=1}^{q} a_j x_{t-j} \varepsilon_{t-j} + \sum_{j=1}^{p} \beta_j \frac{\partial h_{t-j}}{\partial b}$$

Concluding our presentation of the ARCH/GARCH family of models we would like to mention a weakness of these parameterizations. This drawback lies on the fact that they are not able to capture the asymmetries occasionally observed in the volatility process. More specifically **Nelson** (1991) were among the first researchers to discover the difference between the impact of good and bad news in the volatility of financial time series. This means that it is not only the size but also the sign of a shock that can determine and influence the conditional volatility. Unfortunately the GARCH model can capture only the size effects. Frequently however we observe these asymmetric effects occurring in the volatility processes. Through these considerations, a new alternative approach was born and used to model the time varying nature of volatility. This is the so called Exponential GARCH model.

In contrast to the conventional GARCH model it is not necessary anymore to impose restrictions in the ARCH and GARCH parameters to ensure that the conditional variance will receive positive values. This new model allows for a different impact of good/bad news in volatility. More specifically in this specification we have two important innovative features:

1. Negative shocks can have a stronger impact in the conditional volatility forecasts compared with positive shocks of the same magnitude.
2. Powerful shocks have a bigger effect in the volatility process compared with the effects of the same shocks when using the GARCH model.

The mathematical representation of this class of models is the following:

$$\log \sigma_t^2 = k + \sum_{i=1}^{P} G_i \log \sigma_{t-i}^2 + \sum_{j=1}^{Q} A_j \left[\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} \right] + \sum_{j=1}^{Q} L_j \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right)$$

$$E\{|z_{t-j}|\} = E\left(\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right) = \begin{cases} \sqrt{2/\pi} & \text{Gaussian} \\ \sqrt{\dfrac{v-2}{\pi}} \dfrac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} & \text{Student's t} \end{cases}$$

With degrees of freedom, v>2

CHAPTER 2 : THE CONCEPT OF CAUSALITY

2.1 INTRODUCTION TO GRANGER CAUSALITY

One of the most influential econometricians in the concept of causality is **Clive Granger** (1969, 1980). According to his theoretical formulations we say that the random variable Y_n causes X_{n+1} when the following relation is in effect.

$$\text{Prob}\left(X_{n+1} \in A \mid \Omega_n\right) \neq \text{Prob}\left(X_{n+1} \in A \mid \Omega_n - Y_n\right) \qquad (1)$$

with A being the set that contains all the possible values that X can take, and Ω being the information set (or σ-algebra) involving the maximum available information regarding the history of these two random variables. Thus a causal relation with direction from Y to X will exist when Y_n contains some kind of information regarding the values that X_{n+1} can take. The whole theory on causality depends on the fundamental axiom that the past or the present can influence the future. We may interpret the notion of causality from a number of different viewpoints. In order to be more precise we will confine now our discussion in the context of Granger causality between two random variables. In contrast to the field of Mathematics, in Statistics and Econometrics the existence of deterministic relations is a utopia and that's why we must be satisfied with the derivation of stochastic causal relations. So we will say that event A will possibly (but not certainly) happen when event B is realized.

In this point we must also refer to another important axiom according to which every true and existent causal relation must remain constant through time regarding the direction of movement but on the contrary the power of this relation as well as its time span can vary. In the next lines we will present some more practical and empirically testable definitions of Causality. In order to achieve this we must modify the definition (1). So let's assume that we want to examine whether a vector stochastic process Y_t influences another multidimensional random process X_t. We define as J_n the information set available at time n that contains all the terms of the vector process Z_t. Z_t does not contain any of the terms of Y_t while it includes all the terms of X_t. We thus have that $J_n : Z_{n-j}$, $j >= 0$. We also define the information set J'_n for which J'_n : Z_{n-j} , Y_{n-j}, $j >= 0$. Consequently we have the following definitions:

1. $Y_n \xrightarrow{\text{Not Cause}} X_{n+1}$, (J'_n)

when $F(X_{n+1}/J_n) = F(X_{n+1}/J'_n) \Rightarrow E(X_{n+1}/J_n) = E(X_{n+1}/J'_n)$

If $J'_n = \Omega_n$ then we will also have that

2. $Y_n \xrightarrow{\text{Cause}} X_{n+1}$, (J'_n)

when $F(X_{n+1}/\Omega_n) \neq F(X_{n+1}/\Omega'_n) \Rightarrow E(X_{n+1}/\Omega_n) \neq E(X_{n+1}/\Omega'_n)$

3. $Y_n \xrightarrow{\text{Not Cause in Mean}} X_{n+1}$, (J'_n)

when $E(X_{n+1}/J'_n) - E(X_{n+1}/J_n) = 0$

4. $Y_n \xrightarrow{\text{Cause in Mean}} X_{n+1}$, (J'_n)

when $E(X_{n+1}/J'_n) - E(X_{n+1}/J_n) \neq 0$

When we discuss about causality in mean we refer to a kind of causality that is substantially weaker than the general concept of causality as it does not involve the whole probability distribution of the random variable. Simultaneously this type of causality is much more useful from a practical perspective as it can be empirically tested using the point estimates from a least squares estimation methodology. In order to prove the validity of the above argument we can simply recall the fact that the conditional mean can be expressed as a regression function that may be specified as a linear model and for which we have to estimate some unknown parameters.

In the case that $\sigma^2(X \mid J_n, Y) < \sigma^2(X \mid J_n)$ then we will say that random variable Y is causally related with X with the former causing the latter. In other words the knowledge of the information in Y_n can help us to conduct more accurate forecasts regarding X_{n+1} and decreases the uncertainty of the predictions. We have to remind here that the causal relations we have discussed up to this point are linear. The introduction of non linear relations would require that the information contained in information set J_n can also be utilised in a non linear way. Another causal link that must also be mentioned is that of a feedback relation. When Y_n causes X_{n+1} and simultaneously X_n causes Y_{n+1} then there will exist a bidirectional feedback causality.

In the following lines it is necessary to refer to some of the problems that occur during the examination of causal relations. Firstly the frequency of the empirical data that sometimes cannot be exclusively determined by the researcher but depends on the nature and availability of this kind of data can in some occasions lead to misleading results regarding the type of causality. For example due to a non suitable frequency selection it is possible to derive the conclusion of the presence of instaneous causality when the true relationship is of some other type. In this point we must formalize the type of causality just mentioned. We will say that there exists an instaneous causal relation between X_{n+1} and Y_{n+1} when the following condition is fulfilled. $E(X_{n+1} / J_n, Y_{n+1}) \neq E(X_{n+1} / J_n)$.

A different but equally important problem often occurring in empirical research is the omission of important exogenous explanatory variables from the econometric models used during the causality tests. This misspecification of the models can lead once more to biased results. Characteristic of this problem is the following case. Let's assume that we ignore the existence of variable X that truly influences the values of Y. Then the forecast error that we conduct can be considered of being dependent on another variable Z which can erroneously be thought to influence Y when this is not true in practice. In finance it is not unusual to conduct forecast errors. There will always exist an independent variable that will have been mistakenly omitted from the estimated models. Another important consideration that must also be taken into account by the researchers in this field is that occasionally the time period in which we measure a certain variable may differ from the time period that an event that led this variable to take the measured value, has been realized. For example the unemployment rates of March, will become known on April the earliest.

Concluding our introductory remarks in the concept of causality we must illuminate the ultimate objective of the study of causal relations from an econometric perspective. This is not just the construction of models that can make a good fit in the sample data but of models that perform an equally good job when forecasting out of sample. In other words the increased forecasting accuracy of the econometric models that we use will possibly mean that the correct causal relation was detected.

2.2 CAUSALITY IN VARIANCE TESTS

2.2.1 THEORETICAL MOTIVATION

Ross (1989) formulated the opinion that the volatility of a stock market in a no arbitrage economy is directly related to the flow of information. Under this framework we can interpret the transmission of volatility as the result of information transmission among the markets or other variables. If two capital markets are informationaly efficient then it will not be expected to observe any volatility spillovers between them. It must be also noted that the observed interactions among the international markets are not of the intensity and power that one would expect given the low cost of information, the globalization of the financial markets and the simplicity of conducting financial transactions in various different places around the world. We must not however surge to decide that the capital markets are efficient. The absence of volatility spillovers may be in fact attributed to the varying construction methods of the stock indices used in empirical studies or to differences in industrial structures and foreign exchange policies across the different international markets.

There are two main channels of volatility transmission. The first is through the existing structural linkages between the financial markets and the real economies of the volatility interrelated countries. The second is due to the portfolio rebalancing applied by international investors when a crisis or an unfortunate event takes place in the country that they have accumulated a part of their investment capital. We must also note that there are different setups inside which we can study the existence of volatility transmission phenomena. More specifically we can have markets with non overlapping trading hours and markets with perfect synchronization in their trading hours. In the first case it is advisable to use only trading time returns when conducting causality in variance tests. This is essential because a possible existence of asynchronous trading or stale quotes would spuriously induce volatility spillovers effects. In the second case we can choose between Trading Time or Daily returns. Finally we would like to note the importance of the correct specification of the econometric models used through the tests in order to derive more reliable results.

2.2.2 THE IMPORTANCE OF VOLATILITY SPILLOVERS

The causality in variance tests are of particular importance for a number of reasons.

1. From the investors viewpoint the knowledge of possible interactions among the financial markets is useful in order to be able to effectively hedge the various types of undertaken risks and simultaneously efficiently diversify his investment portfolio. For example if it is known in advance that two specific stock markets exhibit significant volatility linkages then the rational investor would be expected to allocate a part of his capital in an another more isolated market and not just between the two above mentioned interdependent markets in order to reduce his non systematic risk.

2. From the viewpoint of the government and monetary authorities this kind of knowledge can be of particular importance. A foreign stock market crash could have a serious negative impact in the domestic financial system as well as in the welfare of this country. Thus in order to be able to achieve a better stabilizing and shielding policy, an essential step should be the study of the causality interrelations among the various countries.

3. The perception of the nature and direction of volatility spillovers can lead in important conclusions in regard with the extent of financial contagion events (e.g. volatility transmission mechanism being more powerful during a crisis). The absence for example of volatility transmissions from foreign economies towards the domestic one will mean that the volatility of the domestic stock market is generated mainly from domestic shocks and that this economy is shielded from foreign disturbances. These characteristics must be taken in account from the regulatory agencies as well as from other supervisory authorities in order to implement the more suitable policy for each country.

4. The study of causal relations in variance is also of particular importance from an applied econometrician's perspective as it will help in the construction of well specified models that can more accurately capture the volatility dynamics and will also lead to the design of more accurate asset pricing models. A huge leap can also be made in the design of more effective risk management tools and methodologies.

2.2.3 THE CHEUNG & NG TEST

This methodology was proposed by **Cheung and Ng** (1996) for testing the null hypothesis of non causality in variance between two time series and it is based on the estimation of the cross correlation function (CCF). We must however mention that the concept of CCF based testing was firstly introduced by **Haugh** (1976). This researcher has designed a two step methodology for testing the interdependence between two covariance stationary time series with homoscedastic errors. His main contribution is the proof that the asymptotic distribution of the CCF estimators for two white noise processes (that is already known from theory) is identical to the asymptotic distribution of the CCF estimators for two residuals series obtained from the estimation of univariate ARMA models. Haugh has also proved also that the presence of autocorrelation in a time series could inflate the variance of the CCF estimators and induce linear dependence among the different estimates.

The above considerations have led to the important conclusion that a robust causality in variance test could be constructed using as inputs the residuals of well specified conditional models. This conclusion was however not yet complete. The stylized feature of volatility clustering should also be removed from the residuals of the ARMA models. This step would finally transform the residuals in stochastic processes closely resembling the white noise random processes. This necessitated the use of GARCH models for filtering the persistence of volatility. It is necessary to note also that even after the use of GARCH models for standardizing the estimated residuals if there still remains some temporal dependence in these series this could be the source of spurious correlations in the conditional second moments falsely leading to the conclusion of the existence of lead/lag relationships between the volatility processes. We can now turn our discussion to the Cheung & Ng causality in variance test.

Cheung & Ng have developed a technique based on the two step CCF approach. In the first stage they estimate univariate ARMA/GARCH models and obtain the squared standardized innovations and in the second step they estimate the CCF and test for the significance of the various cross correlation coefficients being able in this way to detect any possible volatility spillovers between the two series studied. This method can also be used in order to check for causality in the mean by simply using the standardized residuals instead of their squares, as input in the CCF. But let’s now formalize our discussion. The basic building block is the existence of two stationary and ergodic time series and two information sets that are generated as following

$$I_t = \{X_{t-j}, j \geq 0\} \qquad J_t = \{X_{t-j}, Y_{t-j}, j \geq 0\} \qquad I_t \subseteq J_t$$

We will say that Y_t Granger causes X_{t+1} if the following relation is true.

$$E\{(X_{t+1} - \mu_{X,t+1})^2 / I_t\} \neq E\{(X_{t+1} - \mu_{X,t+1})^2 / J_t\}$$

Where $\mu_{x,t+1}$ is the conditional mean of X_{t+1} conditioned on the σ-algebra I_t.

There will exist feedback in variance when X causes Y and simultaneously Y causes X. Finally we will have instaneous causality in variance when

$$E\{(X_{t+1} - \mu_{X,t+1})^2 / J_t\} \neq E\{(X_{t+1} - \mu_{X,t+1})^2 / J_t + Y_{t+1}\}$$

In order to be able to empirically test these types of causal relations it is needed to impose some additional structure. This will be done in the framework of specific econometric models.

$$X_t = \mu_{X,t} + h_{x,t}^{0.5}\varepsilon_t \qquad \{\varepsilon_t\} \sim WN(0,1) \Rightarrow \text{Standardized Innovations}$$

$$Y_t = \mu_{Y,t} + h_{y,t}^{0.5}\zeta_t \qquad \{\zeta_t\} \sim WN(0,1) \Rightarrow \text{Standardized Innovations}$$

The conditional mean and conditional Variance models are given through the following relations.

$$\mu_{z,t} = \sum_{i=1}^{00} \phi_{z,i}(\theta_{z,\mu}) Z_{t-i} \quad (1) \rightarrow \text{ARMA MODEL}$$

$$h_{z,t} = \varphi_{z,0} + \sum_{i=1}^{00} \varphi_{z,i}(\theta_{z,h})\{(Z_{t-i} - \mu_{z,t-i})^2 - \varphi_{z,0}\} \quad (2) \rightarrow \text{GARCH MODEL}$$

$$\theta_{z,w} : p_{z,w} \times 1 \rightarrow \text{Parameters Vector}$$

Where $W = \mu, h$ $\quad \phi_{z,i}(\theta_{z,\mu}) \rightarrow$ Uniquely Defined Function of $\theta_{z,\mu}$

$Z = X, Y$ $\quad \varphi_{z,i}(\theta_{z,h}) \rightarrow$ Uniquely Defined Function of $\theta_{z,h}$

We can now obtain the squared standardized innovations by simply estimating the above models. These appropriately filtered residuals are given from the following relations.

$$U_t = ((X_t - \mu_{x,t})^2 / h_{x,t}) = \varepsilon_t^2 \;,\; V_t = ((Y_t - \mu_{y,t})^2 / h_{y,t}) = \zeta_t^2$$

These residuals will be used in the next step as inputs for the estimation of the Cross Correlation Function. The CCF is calculated through the sample cross correlation coefficients. For example the sample cross correlation at lag k, $r_{uv}(k)$ is computed from the following equation $r_{uv}(k) = C_{uv}(k)\{C_{uu}(0)C_{vv}(0)\}^{-1/2}$

With

$C_{uu}(0)$: sample variance of U

$C_{vv}(0)$: sample variance of V

$C_{uv}(k) : k_{th}$ Cross Covariance at Lag k

$$C_{uv}(k) = T^{-1}\sum(U_t - \bar{U})(V_{t-k} - \bar{V}), k = 0, \pm1, \pm2, ...$$

$$k = 1 \Rightarrow Y_{t-1} \rightarrow X_t$$
$$k = -1 \Rightarrow X_t \rightarrow Y_{t+1}$$

Given now the assurance that the second order moments of the squared standardized innovations are existent and finite and in combination with the hypothesis that these two residual series are independent we will have that

$$\begin{pmatrix} \sqrt{T r_{uv}(k)} \\ \sqrt{T r_{uv}(k')} \end{pmatrix} \sim \text{AN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right), \quad \kappa \neq \kappa' \rightarrow \text{Joint Limiting Distribution} \quad .$$

Consequently based on the discussion made above, a proper causality in variance test can be constructed. As it is widely known the squared standardized residuals are not observable and thus we should use their estimators. This means that we will finally use the sample estimators of the CCF $\hat{r}_{uv}(k)$ in order to test the null hypothesis of non causality in variance. Thus we will have:

$$\hat{\theta}_z \equiv \{\hat{\theta}_{z,\mu}, \hat{\theta}_{z,h}, \hat{\varphi}_{z,0}\} \xrightarrow{\text{CONSISTENT ESTIMATOR OF}} \theta_z^0 \equiv \{\theta_{z,\mu}^0, \theta_{z,h}^0, \varphi_{z,0}^0\}$$

$$\left.\begin{aligned} & Z = Y, X \\ & \theta^0 = (\theta_x^0, \theta_y^0) \\ & \hat{\theta} = (\hat{\theta}_x^0, \hat{\theta}_y^0) \\ & \theta = (\theta_x, \theta_y) \end{aligned}\right\} \hat{r}_{uv}(k) = r_{uv}(k)\,|_{\theta=\hat{\theta}}$$

Finally using all the above results in combination with the fact that the asymptotic distribution of the CCF estimator is already known, the following test statistics can be constructed that follow either the standard normal or the chi-square distribution.

1. $$S_N = \sqrt{T}\hat{r}_{uv}(k) \sim N(0,1)$$

The above test statistic is used to test the significance of a cross correlation coefficient at a specified lag. Another test statistic that we can use is the following:

2. $$S = T\sum_{i=j}^{k}\hat{r}_{uv}(i)^2 \sim X^2(k-j+1)$$

Using the above test function we can check for the joint existence of causality from lag j up to lag k, and this test function follows a chi-square distribution. We must note here that the choice of the values of indicators j and k will depend from the exact form of the alternative hypothesis. When we do not know in advance the direction of causality it is preferable to conduct a bidirectional test setting –j = k = m. In the opposite case that we want to test for a specific direction in causality for example whether y causes x then we should set j =1 και k = m. It is also necessary to note here the importance of the correct specification of the ARMA/GARCH models. In order to test whether a proper specification has been done a usual post estimation test is that based on the **Ljung-Box** Q-statistics (1978) on the standardized and squared standardized residuals. We can now continue the presentation of the available test statistics. The following test statistic is optimized for using with smaller samples.

3. $$S_M = T\sum_{i=j}^{k}\omega_i\hat{r}_{uv}(i)^2 \sim X^2(k-j+1)$$

where $\omega_i = (T+2)/(T-|i|)$

The advantage of this test statistic is that its distribution is closer the chi-square density when we make use of small samples.

The Cheung & Ng methodology has some certain advantages compared with alternative methodologies that make use of multivariate GARCH models. Firstly this technique does not demand the simultaneous modeling of the inter and intra series dynamics something that renders it much easier to use. This fact also implies an observable superiority of this method in the case of a multiple series study. Secondly there exists a greater degree of uncertainty in regard with the correct specification of a multi dimensional model as well as with the asymptotic properties of its Maximum Likelihood Estimators. The Cheung & Ng methodology is robust against asymmetric and leptokurtic errors and asymptotically robust in deviations from the normality assumption for the unconditional distribution. It can also be used to study causal relations extending for longer time spans in comparison with the multivariate methods. A draw back of the Cheung & Ng test is that it is not designed to detect the presence of non linear causal relations.

2.2.4 THE HONG TEST

The methodology that will be presented in this section is due to **Hong** (2001) and is an extensively modified version of the Cheung & Ng Test. The main contribution of Hong lies on using weighting functions inside the test functions. More specifically he had claimed that there is an inverse relation between the lag length and the weight that must be given to the corresponding cross correlation coefficient. We can now proceed with the description of the test. Let's assume that we have two strictly stationary stochastic processes.

$$\{Y_{1t}, t \in \Re^*\}$$
$$\{Y_{2t}, t \in \Re^*\} \qquad \Re^* = \Re \cup \{-00, 00\}$$

We also have the following information sets (σ – algebras)

$$I_{it}, i = 1,2 \xrightarrow{\text{Information Set on time t}} \{Y_{it}\}$$
$$I_t = (I_{1t}, I_{2t})$$

As we have already discussed in previous sections, according to **Granger** (1969)

$$\text{If } \Pr(Y_{1t} / I_{1t-1}) \neq \Pr(Y_{1t} / I_{t-1}) \Rightarrow Y_{2t} \xrightarrow{\text{Granger Causes (general)}} Y_{1t}$$

$$\text{If } f(Y_{2t} / I_{2t-1}) \neq f(Y_{2t} / I_{t-1}) \Rightarrow Y_{1t} \xrightarrow{\text{Granger Causes (general)}} Y_{2t}$$

We shall remind here that the null hypothesis that we want to test, is the non causality in variance between two time series. The null and the alternative hypotheses are:

$$H_0 : E\{(Y_{1t} - \mu_{1t}^0)^2 / I_{1t-1}\} = E\{(Y_{1t} - \mu_{1t}^0)^2 / I_{t-1}\} \equiv Var(Y_{1t} / I_{t-1})$$

$$H_A : E\{(Y_{1t} - \mu_{1t}^0)^2 / I_{1t-1}\} \neq Var(Y_{1t} / I_{t-1})$$

The three different possible types of causal relations in Variance are the following:

1. Unidirectional Volatility Spillover

If H_0 : accepted $\rightarrow Y_{2t} \xrightarrow{\text{Does not Granger Cause in Variance}} Y_{1t}$

If H_0 : rejected $\rightarrow Y_{2t} \xrightarrow{\text{Granger Cause in Variance}} Y_{1t}$

2. Feedback in Variance

$$\text{if } \begin{Bmatrix} Y_{1t} \xrightarrow{\text{Granger Causes in Variance}} Y_{2t} \\ Y_{2t} \xrightarrow{\text{Granger Causes in Variance}} Y_{1t} \end{Bmatrix}$$

3. Instaneous Causality in Variance

$$E\left\{\left(Y_{1t} - \mu_{1t}^0\right)^2 / I_{t-1}\right\} \neq E\left\{\left(Y_{1t} - \mu_{1t}^0\right)^2 / I_{1t-1}, I_{2,t}\right\}$$

The absence of causal relations in the first or second moments between two time series does not imply anything about the general causal link that may exist between them. It is possible that the two processes are causally interrelated in higher order moments. We can now continue with the Hong testing procedures. The cornerstone for the construction of this type of tests are the residuals.

$$\varepsilon_{it} = Y_{it} - \mu_{it}^0, \quad i = 1, 2$$

$$\mu_{it}^0 = E(Y_{it} / I_{t-1})$$

The underlying generating mechanism of residuals is $\varepsilon_{it} = \xi_{it}(h_{it}^0)^{1/2}$ where $\{\xi_{it}\}$ are the standardized residuals and $h_{it}^0 = f(I_{it-1})$ is the conditional volatility. In a more illustrative way we will have the following relations:

$$\left.\begin{matrix} \{\xi_{it}\}\begin{cases} \mathrm{E}(\xi_{it} / I_{it-1}) = 0 \text{ a.s} \\ \mathrm{E}(\xi_{it}^2 / I_{it-1}) = 1 \text{ a.s} \end{cases} \\ \{\varepsilon_{it}\}\begin{cases} \mathrm{E}(\varepsilon_{it} / I_{it-1}) = 0 \text{ a.s} \\ \mathrm{E}(\varepsilon_{it}^2 / I_{it-1}) = h_{it}^0 \text{ a.s} \end{cases} \end{matrix}\right\} \varepsilon_{it} = \xi_{it}(h_{it}^0)^{1/2} \leftarrow \left[\mathrm{h}_{it}^0 = \omega_i^0 + \sum_{j=1}^{q} a_{ij}^0 \varepsilon_{it-j}^2 + \sum_{j=1}^{p} \beta_{ij}^0 h_{it-j}^0 \right]$$

The null and the alternative hypotheses can be also rewritten in terms of the standardized residuals.

$$H_0: \ Var(\xi_{1t}/I_{it-1}) = Var(\xi_{1t}/I_{t-1}) \Rightarrow \xi_2 \xrightarrow{\text{Does Not Granger Cause in Variance}} \xi_1$$

$$H_A: \ Var(\xi_{1t}/I_{it-1}) \neq Var(\xi_{1t}/I_{t-1}) \Rightarrow \xi_2 \xrightarrow{\text{Granger Causes in Variance}} \xi_1$$

The model specifications that will be used through the tests are the following:

$\mu_{it}^0 = \mu_{it}(b_i^0)$ i=1,2 $\qquad b_i^0 \rightarrow$ Unknown Parameters Vector

$$h_{it}^0 = \omega_i^0 + \sum_{j=1}^{q} a_{ij}^0 \varepsilon_{it-j}^2 + \sum_{j=1}^{p} \beta_{ij}^0 h_{it-j}^0 \rightarrow GARCH(P,Q)$$

$\{Y_t\}_{t\geq 1} \rightarrow$ Vector Stochastic Process

$$Y_t = (Y_{1t}, Y_{2t})'$$

$$\beta_i^0 = (\beta_{1i}^0, \ldots, \beta_{pi}^0)'$$

$$\alpha_i^0 = (\alpha_{1t}^0, \ldots, \alpha_{qi}^0)'$$

$$\hat{\theta}_i = (\hat{b}_i', \hat{\omega}_i, \hat{a}_i', \hat{\beta}_i')' \xrightarrow{\text{CONSISTENT ESTIMATOR OF}} \theta_i^0 = (b_i^{0'}, \omega_i^0, a_i^{0'}, \beta_i^{0'})'$$

From the estimation of the above models we can now obtain the centralized squared standardized innovations.

$$\hat{u}_t \equiv u_t(\hat{\theta}_1) = \left(\hat{\varepsilon}_{1t}^2 / \hat{h}_{1t}\right) - 1, \qquad \hat{\varepsilon}_{it} \equiv \varepsilon_{it}(\hat{\theta}_i)$$

$$\hat{v}_t \equiv v_t(\hat{\theta}_2) = \left(\hat{\varepsilon}_{2t}^2 / \hat{h}_{2t}\right) - 1, \qquad \hat{h}_{it} \equiv h_{it}(\hat{\theta}_i)$$

$$\varepsilon_{it}(\hat{\theta}_i) \xrightarrow{\text{ESTIMATOR OF}} \varepsilon_{it}(\theta_i) = Y_{it} - \mu_{it}(b_i)$$

$$h_{it}(\theta_i) = \omega_i + \sum_{j=1}^{q} a_{ij} \varepsilon_{it-j}^2(\theta_i) + \sum_{j=1}^{p} \beta_{ij} h_{it-j}(\theta_j)$$

These residuals will be subsequently used as input for the estimation of CCF. As we have already discussed in the Cheung & Ng Test, the cross correlation function is calculated through the following procedure $\hat{\rho}_{uv}(j) = \{\hat{C}_{uu}(0)\hat{C}_{vv}(0)\}^{-1/2}\hat{C}_{uv}(j)$

$$\hat{C}_{uv}(j) = \begin{cases} T^{-1} \sum_{t=j+1}^{T} \hat{u}_t \hat{v}_{t-j}, \ j \geq 0 \Rightarrow Y_2 \rightarrow Y_1 \\ T^{-1} \sum_{t=-j+1}^{T} \hat{u}_{t+j} \hat{v}_t, \ j < 0 \Rightarrow Y_1 \rightarrow Y_2 \end{cases} \qquad \begin{matrix} \hat{C}_{vv}(0) = T^{-1} \sum_{t=1}^{T} \hat{v}_t^2 \\ \hat{C}_{uu}(0) = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2 \end{matrix}$$

Hong has claimed that the volatility transmission between two variables gradually weakens as the time distance between them increases. This theoretical suggestion according to him should also be implemented during the causality in variance tests.

The answer to these considerations came with the introduction of a new generation of test statistics that are designed to weight more heavily the low lag cross correlation coefficients. The weighting scheme depends upon specific kernel functions. The most important such functions, proposed in literature are the following:

1. Truncated $\rightarrow k(z) = \begin{cases} 1, & |z| \leq 1 \\ 0 \end{cases}$

2. Bartlett $\rightarrow k(z) = \begin{cases} 1-|z|, & |z| \leq 1 \\ 0 \end{cases}$

3. Daniell $\rightarrow k(z) = \sin(\pi z)/\pi z \ , \ -00 < z < 00$

4. Parzen $\rightarrow k(z) = \begin{cases} 1-6z^2+6|z|^3, |z| \leq 0.5 \\ 2(1-|z|)^3, 0.5 < |z| \leq 0.5 \\ 0 \text{ , otherwise} \end{cases}$

5. Quadratic Spectral $\rightarrow k(z) = \dfrac{3}{\sqrt{5}(\pi z)^2}\{\sin(\pi z)/\pi z - \cos(\pi z)\}, -00 < z < 00$

6. Tukey-Hanning $\rightarrow k(z) = \begin{cases} \frac{1}{2}(1+\cos(\pi z)), |z| \leq 1 \\ 0, \text{ otherwise} \end{cases}$

The proposed test statistic is the following:

$$Q_1 = \left\{ T \sum_{j=1}^{T-1} k^2(j/M)\hat{\rho}_{uv}^2(j) - C_{1T}(k) \right\} / \{2D_{1T}(k)\}^{1/2} \xrightarrow{\text{Regularity Conditions}} N(0,1)$$

$$C_{1T}(k) = \sum_{j=1}^{T-1} \underbrace{(1-j/T)}_{\text{Finite Sample Corrections}} k^2(j/M)$$

$$D_{1T}(k) = \sum_{j=1}^{T-1} \underbrace{(1-j/T)\{1-(j+1)/T\}}_{\text{Finite Sample Corrections}} k^4(j/M)$$

The values obtained through the above test must be compared with upper tailed standard normal distribution critical values. A slightly modified version of aforementioned test function that is asymptotically equivalent is the following:

$$Q_1^* = \{T \sum_{j=1}^{T-1} (1 - j/T)^{-1} k^2 (j/M) \hat{\rho}_{uv}(j) - C_{1T}^*(k)\} / \{2D^*_{1T}(k)\}^{1/2}$$

$$C_{1T}^*(k) = \sum_{j=1}^{T-1} k^2 (j/M)$$

$$D_{1T}^*(k) = \sum_{j=1}^{T-1} \{1 - (T-j)^{-1}\} k^4 (j/M)$$

Finally another test statistic that it is recommended in cases that we do not have any ex ante information in regard with the direction of the causal relation. Is the following:

$$Q_2 = \left\{ T \sum_{j=1-T}^{T-1} k^2 (j/M) \hat{\rho}_{uv}^2 (j) - C_{2T}(k) \right\} / \{2D_{2T}(k)\}^{1/2} \xrightarrow{d} N(0,1)$$

$$C_{2T}(k) = \sum_{j=1-T}^{T-1} (1 - |j|/T) k^2 (j/M)$$

$$D_{2T}(k) = \sum_{j=1-T}^{T-1} (1 - |j|/T)\{1 - (|j|+1)/T\} k^4 (j/M)$$

The values obtained using this test statistic must once more be compared with the critical values that correspond to the upper tail of a standard normal distribution.

2.2.5 THE LAGRANGE MULTIPLIER TEST

The causality in variance test that we will discuss in this section was proposed by **Hafner & Herwartz** (2006) and is in fact a Lagrange Multiplier test. We must also note that their study was based on an earlier work of **Lundbergh & Terasvirta** (2002) who used the LM tests for misspecification testing of GARCH models. But let's proceed now with the presentation of the methodology. The discussion that will be made in the following lines describes a unidirectional test for volatility spillover for a certain direction. Of course if we want to test for the existence of causality in variance in the opposite direction we will just use an exactly analogous procedure. Thus we assume that we have the stochastic process $\{\varepsilon_t \in \Re^n, t \in N\}$ that is defined in a probability space (Ω,F,P). We also make the assumption of stationarity for this process and that $E[\varepsilon_t \mid F_{t-1}] = 0$ where F denotes the σ-algebra (or more simply the information set).

The Null hypothesis that we want to test is that of non causality in variance and can be mathematically formulated as $H_0 : Var(\varepsilon_{it} / F_{t-1}^{(j)}) = Var(\varepsilon_{it} / F_{t-1})$

With $i, j = 1,...,N, i \neq j$ and $F_t^{(j)} = F_t \setminus \sigma(\varepsilon_{jt,} \tau \leq t)$. For testing the above hypothesis we should consider the use of a specific econometric specification. So we will have the following mathematical expressions:

$\sigma_{it}^2 = \omega_i + a_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \rightarrow$ GARCH(1,1) Model

$\varepsilon_{it} = \xi_{it} \sqrt{\sigma_{it}^2 g_t} \rightarrow$ Underlying Generating Mechanism of Residuals

$g_t = 1 + z_{jt}^{'} \pi \rightarrow$ Misspecification Indicator

$z_{jt} = (\varepsilon_{jt-1}^2, \sigma_{jt-1}^2)' \rightarrow$ Past Foreign Conditional & Realized Volatility

In order to be accurate we must clarify that the realized volatility is approximated through the use of the squared residuals. In the term z_{jt} we included the conditional and realized volatility of variable j that we want to examine whether it transmits its own volatility to variable i. It is clearly observable that in the way that we have specified our GARCH model, we have implanted inside the underlying process of residuals (shocks) of variable i, the dynamics of the residuals of another foreign variable j. In this way the proposed specification has by construction a causality in variance structure from variable j to variable i with the shocks in market/variable j being directly transmitted in market/variable i.

From a different viewpoint however we technically introduced a misspecification in the constructed GARCH model as the proper one would be that in which vector z_{jt} would be empty. Thus in the case of a misspecification test where the null hypothesis considers that the selected model is correctly specified this would also imply the absence of volatility spillovers. If the estimated model is found to be misspecified then this would also mean that there exists a causal relation in variance. The concluding remark and the key concept behind this test is to perceive the causality in variance test as a misspecification test. The true correctly specified GARCH model will be that where there are not any volatility spillovers while the misspecified would be that in which the underlying generating mechanism of residuals process will also include the influences from shocks in another variable. Under this framework the null and alternative hypotheses can be expressed as following:

$H_0 : \pi = 0$

$H_1 : \pi \neq 0$

It is now possible to construct a test function that will be based on the estimation of univariate GARCH models. The score function of the log likelihood function of ε_{it} that corresponds to the normal distribution is obtained from $x_{it}(\xi_{it}^2 - 1)/2$ where $x_{it} = \sigma_{it}^{-2}(\partial \sigma_{it}^2 / \partial \theta_i)$ and $\theta_i = (\omega_i, a_i, \beta_i)'$.

The proposed test statistic will have the following specification

$$\lambda_{LM} = \frac{1}{4T}\left(\sum_{t=1}^{T}(\xi_{it}^2 - 1)z_{jt}'\right)V(\theta_i)^{-1}\left(\sum_{t=1}^{T}(\xi_{it}^2 - 1)z_{it}\right) \xrightarrow{d} X^2(2)$$

$$V(\theta_i) = \frac{k}{4T}\left\{\sum_{t=1}^{T} z_{jt}z_{jt}' - \sum_{t=1}^{T} z_{jt}x_{it}'(\sum_{t=1}^{T} x_{it}x_{it}')^{-1}\sum_{t=1}^{T} x_{it}z_{jt}'\right\}$$

$$k = \frac{1}{T}\sum_{t=1}^{T}(\xi_{it}^2 - 1)^2$$

This test statistic converges in distribution in a random variable that is chi-square distributed with two degrees of freedom. An alternative method to estimate λ_{LM} is through a regression based procedure. The steps for this equivalent methodology are:

1. We estimate GARCH(1,1) models for ε_{it} and ε_{jt} and then obtain the standardized residuals ξ_{it} , the derivatives χ_{it} and the conditional Variances σ_{jt}^2

2. We then regress $\xi_{it}^2 - 1$ on the terms x_{it}' and $\varepsilon_{jt-1}^2, \sigma_{jt-1}^2$. The latter pair is contained inside the z_{jt}'

3. The λ_{LM} test statistic would be obtained by calculating the product T * R^2 , where T is the sample size and R^2 is the coefficient of determination obtained from the regression in step II. The asymptotic distribution of this alternative test statistic will be a chi square with two degrees of freedom (under the null hypothesis)

In the following section we provide an extensive literature review regarding the field of causality as well as the empirical findings of some important studies.

2.3 LITERATURE REVIEW ON CAUSALITY

Engle, Ito & Lin (1990) attribute the volatility clustering pattern observed in economic and financial time series in two factors. Firstly, behind these non linear dynamics there exists the autocorrelation in the news arrival process that describes the flow of new information in the market. They argue that occasionally, the news come in clusters (big volume of news becomes known in the same time point) something implying the existence of a non smooth flow of information. Secondly the existence of heterogeneous expectations as well as the use of inside information (violation of the efficient market hypothesis) among the participants in the markets create the sufficient conditions for a persistent turbulence in the market after a shock. An absence of volatility spillovers is a sign that the sources of disturbances in a certain market have local characteristics and influence only the domestic market. Conversely the existence of volatility transmission can be attributed to various factors such as synergies or competitive policies of the central banks, the distribution of expectations/fears of a market in others relating with the former and finally to changes in common fundamental factors that jointly affect the markets. In their study Engle, Ito & Lin use daily data on the exchange rate Dollar/Yen exchange rate for the period from October 1985 until September 1986. The innovative element of their research is the decomposition of the daily change in Exchange rate, in four individual changes that take place in the four most important international foreign exchange markets (Pacific , Tokyo , London , New York) that are opened in non overlapping time periods, and 24 hours per day. Their final conclusion is in favour of volatility transmission across the markets studied.

Hamao, Masulis & Ng (1990) investigate the existence of cross-correlation between the volatilities of different stock markets. They use daily opening and closing prices from Tokyo, London and New York equity markets with their sample spanning from April 1985 until March 1988. The fundamental objective of their research was to examine whether the volatility of a stock market is transmitted to the next market to open. An important procedure followed through their study was the decomposition of the total daily price changes of each of the indexes used, in the open to close and close to open components. This technique allowed the separate analysis of the effects of foreign market volatility in the opening prices of the domestic market and in the prices during trading hours. They also attempted to determine the reasons behind the cross market volatility transmission. According to these authors such phenomena can either be attributed in fundamental international factors that simultaneously influence the stock markets or alternatively constitute a causal relation that characterizes the markets that open and close sequentially during a trading day. The specification used in this study is the GARCH in Mean model as it can adequately describe the effects of volatility in the conditional mean. In order to overcome the spurious autocorrelation caused by the asynchronous trading of the markets they used a moving average model, filtering in this way the autocorrelation in the first moments. The necessity for the above action is dictated from the hypothesis of the ARCH models, that the conditional residuals are not serially correlated. The basic conclusion of this study is that there exists volatility transmission from the New York and London markets to Tokyo but no reverse relation is observed.

Baillie & Bollerslev (1991) use a GARCH specification with dummy variables for modelling the intradaily pattern in financial volatility using data from four exchange rates in an hourly basis and for a period of six months (January 1986 until July 1986) in order to examine the presence of volatility spillovers among the foreign exchange markets. The reason behind the usage of hourly data lies on the fact that in this way the researchers are able to discriminate between factors that are related exclusively to the exchange rates and factors that characterize the corresponding markets. For the conditional mean specification they choose a moving average parameterization that is compatible with the efficient market hypothesis while describing sufficiently accurate, the spurious autocorrelation caused by the asynchronous trading in the markets. According to these authors the existence of time varying conditional heteroscedasticity roots in the instability observed in the stock market prices after the arrival of new information which results in particularly extensive transactions until the complete incorporation of the new information in the prices. According to Baillie & Bollerslev and in contrast to the conclusions of other studies there are minimal signs of systematic volatility transmission among the financial markets investigated in their research. The observed persistence in volatility appears to be short lived. It lasts for a few hours and is eliminated after a few days.

Ito, Engle & Lin (1992) study the nature of volatility transmission across international stock markets. More specifically they analyze the importance of monetary policy coordination among the central banks of various countries and the effects of gradual private information dissemination, in the creation of volatility spillovers. Through their study three non overlapping time periods are used during which policy coordination of different intensity and nature among the monetary authorities is exhibited. They call as a 'Heat Wave' the case when the existence of autocorrelation in the volatility of a specific market has local/national characteristics and is not transmitted to other countries and as a 'Meteor Shower' the case when volatility is transmitted across the various markets. According to these authors the Meteor Shower phenomenon constitutes a failure of the market to completely make use of all the available information and is clearly a violation of the efficient market hypothesis. An important factor that is researched in this study is the role of the cooperative/competitive policies of central banks in the creation of Meteor Shower effects. Also using a variance decomposition methodology, the authors analyze the separate contribution of Heat Wave and Meteor Shower characteristics in the volatility of financial markets. The Meteor Shower hypothesis presupposes that the information concerning a specific market can be effectively used for foreign markets volatility forecasting. Regarding the econometric specification selected in this particular study a GARCH model is used while the dataset is consisted from daily observations for four non overlapping foreign exchange markets (Pacific , Tokyo , Europe And New York.). The sample covers the period from February 1979 until December 1988. The final conclusion of this study is that the volatility of the exchange rates has the characteristics of a Meteor Shower in the sense that it is transmitted across the various markets as the globe turns.

Karolyi (1995) attempts an inquiry in the dynamic characteristics of returns and volatility in the stock markets of New York and Toronto. The dataset used includes daily observations of the closing prices of the two basic indexes of these markets. The time span of the sample covers the period from April 1981 until December 1989. An important feature of the data used in this study is that the two markets are trading in overlapping trading hours a fact that helps to overcome the problems occurring due to the asynchronous trading in these markets. An important consideration according to the author is that the conclusions in regard with the nature and intensity of the causal relations between the two markets may also depend on the parameterization used to describe the intra market dynamics. He argues that when using Multivariate GARCH models the volatility transmission effects appear to be weaker than the case where VAR models are used. In order to prove his suggestions Karolyi uses various alternative specifications for the causality in variance test. These are a BEKK-

GARCH model, a VAR model consisted of a vector white noise stochastic process for the residuals and with a constant variance covariance matrix, a bivariate constant correlation GARCH and finally a pair of two univariate GARCH models that allow any interaction between the two time series to be demonstrated only in the first order moments. The empirical conclusion of this study is that a bivariate GARCH model describes adequately the cross market dynamics of the markets examined. It seems also that the spurious autocorrelation generated by the asynchronous trading of the markets cannot explain in this case the cross market dependencies. Finally through the comparison of Canadian stocks dually listed in both exchanges and of stocks listed only in the domestic market Karolyi illuminates the impact of restrictions and investment barriers on the dynamic behaviour and interdependence among the financial markets.

Koutmos & Booth (1995) examine the existence of causal relations in the first and second order moments of returns across the stock markets of New York, Tokyo and London. They also mention the importance of modeling the asymmetric impact of news in the volatility transmission mechanism. For capturing this effect they make use of an Exponential GARCH specification through their study. Their dataset consists of daily observations for the basic index of each market. The sample period spans from September 1986 until December 1993. The choice of the exact parameterization for the conditional variance models is made, seeking to obtain the best possible modeling capability of the volatility dynamics of the stock markets examined in this study and with the additional feature of capturing the asymmetric effects of domestic and foreign shocks. The authors also refer to the bias in results that can be induced when using data from markets with overlapping trading hours. The asynchronous trading often observed in the financial markets may be a source of spurious autocorrelations. That's why they use moving average representations for the conditional first moments modeling, in order to filter the serial correlation. In this direction it is also helpful the use of value weighted stock market indexes.

Finally the additional assumption of constant conditional correlation is made in order to reduce the number of unknown parameters. According to Koutmos & Booth there is an asymmetry in the transmission of shocks in the markets studied in their research. They also detect that after the stock markets crash in 1987, there is an increased interconnection between the markets. Regarding now the spillover effects across the markets they observe spillovers in the mean from New York to Tokyo and from Tokyo to New York and London. In the second moments there exist volatility spillovers from New York and London to Tokyo, from Tokyo and New York to London and from London and Tokyo to New York. The basic concluding remark is that the examined markets demonstrate a significant interconnection, with the impact of negative news being more powerfully transmitted across the countries compared with the news of positive sign.

Booth, Martikainen & Tse (1997) investigate the volatility linkages among the stock markets of Denmark, Norway, Sweden and Finland. They use daily observations for the main index of each market with their dataset spanning from May 1988 until June 1994. An important element that characterizes this study is the uniformity in the structural characteristics of the markets studied something that renders them ideal for an unbiased examination of the volatility transmission patterns across them. There is also an impressive convergence in the policies followed by the above mentioned countries. Regarding to the methodology used, they select a multivariate exponential GARCH model. The results of their research are as follows. There is evidence for volatility transmission from Sweden to Finland with a weaker pattern observed in the reverse direction. The causality in variance discovered in this research can be mainly attributed in the economic cooperation of these countries.

Hu, Chen, Fok & Huang (1997) study the presence of causal relations among the volatilities of different financial markets. More specifically they use the returns of basic indices from the markets of New York, Tokyo, Taiwan, Hong-Kong and Shanghai. The data are in a daily basis and cover the period from October 1992 until February 1996. The methodology used is based on the Cheung & Ng Test Statistic. The information received through the above causality in variance tests, is successively used for the construction of econometric models enriched with (empirically verified) augmented forecasting capabilities that allow the more accurate modelling of the second order moments processes. In regard with the existence of causal relations in volatility the researchers report firstly the existence of volatility transmission from Tokyo to New York, the existence of bidirectional causal relations in variances between Hong-Kong and New York and finally the existence of powerful simultaneous interactions between all the markets. An equally important finding is that when incorporating the effect of volatility spillovers in the models used then a decrease in volatility persistency is observed. In other words the persistence in volatility can some times be partially attributed to the effect of foreign volatility in the domestic one. It is also argued that geographic proximity and the economic ties between two countries do not essentially involve the existence of volatility transmission phenomena. Finally it is reported that the volatility of the Tokyo and New York markets is the result of a generating mechanism which is influenced from fundamental factors. These factors indirectly influence the other markets and this is the basic reason behind volatility spillovers from the aforementioned markets to the two other developing markets.

Tse (1998) examines the information transmission mechanism between the American and Japanese financial markets. More specifically the author uses data for the three month interest rate futures contracts in Eurodollar and Euro Yen covering a period from June 1990 until July 1996. The reason for the choice of these two specific markets is according to Tse, the fact that they do not suffer from the asynchronous trading/stale quotes problems as well as from the market segmentation, that are factors which often lead to spurious results concerning the volatility spillovers across the markets. According to Tse the volatility transmission is not the result of the impact that certain fundamental factors have on the markets but constitute signs of efficient markets hypothesis violation. He also argues that when the financial markets work efficiently then the opening prices of the domestic markets will incorporate in a fast pace, the new (if any) information concerning the foreign markets and consequently the closing prices will not carry any additional information in regard to the foreign markets. Thus when using opening and closing prices it is not expected to observe any volatility spillovers (given the efficiency of the specific market).

The author uses a two dimensional constant correlation EGARCH model in order to examine the volatility linkages between the markets and the contemporaneous correlations for the investigation of the information transmission mechanism. The reason for choosing the exponential model is in order to investigate a possible asymmetry in the volatility transmission, a feature implying that negative news in a market may cause more powerful increase in the volatility of the other market in comparison with the effect of good news. The final conclusion reached through this study is that the information is generally transmitted across the international markets. However there is an absence of any causal relations in variance as the opening prices in the various markets absorb sharply any foreign information and thus do not allow any volatility spillovers to become observable in the empirical data.

Comte & Lieberman (2000) in their research formulate two new definitions regarding the causality in the second moments from which the first one is of a Granger type while the second one is a linear version of the Granger non Causality and depends on the projection on Hilbert spaces. Their theoretical definitions acquire a more explicit mathematical structure through the use of multidimensional ARMA models enriched with GARCH disturbance terms. The above econometric structure allows the empirical testing of the causality definitions proposed by the two researchers through the imposition of alternative restrictions in the parameters of the models. Then, alternative types of tests such as the Likelihood Ratio Test , the Lagrange Multiplier Test and the Wald Tests can be conducted in order to detect the existence and the direction of any causal relations in the second moments. More specifically the authors use daily returns of dually listed stocks in the Tel Aviv stock market and New York stock exchange as well as the returns of the general index of the Tel Aviv stock market for the period between June 1988 and March 1998. Due to the non overlapping trading hours in the two equity markets, it is possible to investigate with a high degree of credibility, the effects of the volatility of the American stock market on the Israeli market. Finally it has been proved that there exist statistically important causal relations in the volatilities of the two markets despite a negligible ambiguity on the asymptotic properties of the tests conducted using the multivariate methodologies.

Brooks & Henry (2000) seek to model the transmission of shocks and the interdependence of volatility among the stock markets of America, Japan and Australia. They use weekly observations for the basic index of each market and the sample period spans from January 1980 until June 1988. The use of this particular frequency of observations is chosen in order to smooth the problem of spurious volatility spillovers observed in markets characterized from asynchronous trading something that is intensified in the case of daily data. They use both parametric and non parametric techniques through their research. In the first step the non linear Granger causality test proposed by Hiemstra & Jones (1994) is used. According to this test there is powerful evidence for the presence of causal relations from the U.S. and Japan to Australia and weaker signs for a spillover direction from Japan to the U.S. In the second step and having in mind the results from the previous method they make use of a VARMA – GARCH (BEKK) model modified to incorporate any potential asymmetries in the volatility transmission mechanism. The asymmetry can characterize the effect of the sign of the past own market's innovations on the conditional volatility of the foreign market as well as on the conditional covariance between the two markets. The main conclusions of this study are the existence of volatility transmission from the American to the Australian market that is also characterized from asymmetry. There is also a causal interconnection between the Japanese and the American markets.

Vilasuso (2001) investigated whether the presence of autocorrelation in the second order moments and the simultaneous existence of causal relations in the volatility of the financial time series, could distort the reliability of the causality in mean tests. His research concerns the regression based method for causality in mean detection that has been proposed from Granger (1969). This method is based on the use of VAR models as a specification for the conditional mean. The most important results that the author has attained are the following:

1. VAR models have two important drawbacks:

1.1 The O.L.S standard errors are not consistent because of the use of lagged dependent variables inside the VARs and this can lead to serious errors in the hypothesis testing procedures.

1.2. The corresponding techniques for the detection of causal relations that are based on the least squares approach cannot discriminate between the two kinds of causality (In the first and in the second moments)

2. When we have the presence of time varying conditional heteroscedasticity in the time series used in the empirical study then it is possible to have serious distortions in the reliability of the causality in variance methodologies.

3. If the above observable pattern of the financial time series is combined with the simultaneous existence of causal relations in the variance processes then the problem of discriminating between the two Causality types is even more intense. This argument is proved through Monte Carlo simulations.

Kanas and Kouretas (2002) studied the existence of causal relations in the means and variances of the exchange rates among four (main and parallel) Latin American markets (Argentina, Brazil, Mexico and Chile). Their sample includes data in a monthly basis and covers the period from 1976 until 1993. Their methodology is based on the two step Cheung & Ng test for causality in variance. In the first step the researchers use EGARCH models in order to capture the leverage effects of volatility shocks. The conditional distribution selected for the residuals is the Generalized Error Distribution in order to account for the severe kurtosis in the time series used in this particular study. The authors also investigate whether the presence of causality in variance can influence the existence of causal relations in the first order moments.

Their main empirical finding is that the existence of causality in volatility can have a significant impact in causality in mean tests in the case that a GARCH in mean or EGARCH in mean specification is used. More specifically they generate data characterized from causality in variance and then use two alternative specifications for the conditional mean model before studying for mean spillovers. In the first case the GARCH term is included in the mean model and in the second case it does not. They then test for causality in mean using both specifications and not surprisingly derive different results. Their basic conclusions regarding volatility spillovers are that there exist important signs of the presence of causal relations among the second order moments both between the official and parallel markets inside each country and across the different markets. Causality characterizes also the first order moments. Finally it is suggested that the preferable specification for modelling the volatility in the various markets, is the Exponential GARCH that has the added capability to model the asymmetric effects of shocks in volatility.

Sola, Spagnolo & Spagnolo (2002) introduce a new methodology for testing the causality in variance hypothesis. According to these authors, the existing GARCH-based causality in variance methodologies have a serious drawback which is the hypothesis of symmetric volatility transmission under periods of either high or low volatility. In other words stock market volatility is transmitted in the same way regardless of the state (turbulent / calm) of the financial market. The proposed technique makes use of a Markov Switching parameterization allowing for four different states of nature in the volatility process. Their methodology has the key advantage of considering a financial crisis as a non frequent event by not inducing it as a structural relationship. Thus financial crises are assumed to be sporadic, not systematic events. For their empirical research they use data for three emerging markets namely, Thailand, South Korea and Brazil. Two bivariate models are estimated, where the dependent variable (stock market returns) is assumed to be a four state Markov Process. The sample covers the period from January 1980 until January 2001. The authors denote as a volatility spillover from market i to market j, the case when a high volatility state in market i at time t-1, leads market j to a high volatility state in time t or later. The concluding result of their empirical study is in favor of volatility transmission from Thailand to South Korea. No other variance spillovers were observed during Likelihood Ratio tests of restricted (Contagion) versus unrestricted parameterizations (Independency) as the alternative hypothesis of independence was not rejected.

Caporale, Pittis & Spagnolo (2002) investigate the presence of causal relations between the stock market volatility and the exchange rates volatility in four East Asian countries. They use daily data from the financial markets of Japan Indonesia, South Korea and Thailand for the period from January 1987 up to January 2000. In order to check for the existence of an interconnection between the second order moments, they make use of a multivariate BEKK GARCH framework in which they impose specific alternative restrictions in the cross market volatility transmission parameters. They afterwards conduct Likelihood Ratio tests through which they compare the restricted GARCH models against the unrestricted Full BEKK parameterization. In this way they are able to detect the nature and the direction of volatility spillovers (if any). The authors also conduct a Monte Carlo simulation in order to asses the probability of type I errors when using the LR tests in a multivariate framework. They show that for a large number of observations (larger that 3000 observations) the frequency of rejection of the null hypothesis approaches satisfactorily its asymptotic equivalent. Regarding the results of their empirical study of the volatility transmission between the above mentioned markets the authors report that the direction of volatility is from the stock market to the exchange rates.

Alaganar & Bhar (2003) studied the transmission of information between pairs of economic variables using causality in variance detection methodologies. According to these researchers, the conclusions in regard to the presence or not of causal relations facilitate the construction of more objective multidimensional models and help the selection of a more realistic lag length in the models used. In this way there is no need to presuppose the volatility interconnection structure between two markets/variables something that is essential to be done when more complex models are directly used. The methodology followed by the authors is based on the two step causality in variance test proposed by Cheung & Ng and makes use of GARCH models. Their sample includes daily observations of the basic stock market indices of seven countries of OECD as well as weekly data for the prices of ten year government bonds and three month treasury bills. The sample covers the period from January 1990 until December 2000. The study concludes with the proposition that there exist significant signs of volatility transmission from the stock markets to the interest-rates and reversely with the intensity and the direction of transmission varying across countries. The important element here is the discovery of bidirectional causal relations instead of unidirectional causality from interest-rates to the stock markets as suggested from the economic theory.

Pantelidis & Pittis (2004) investigate the impact that the presence of neglected causality in mean can have on the finite sample properties of the various causality in variance tests. More specifically in their study they quantitatively describe the interdependence that exists between the two most frequently used notions of causality in the discipline of finance. Their critique is focused on the tests that make use of the cross correlation function. These are the Cheung & Ng and Hong methodologies that are designed to detect the presence of causal relations in variance but without however accounting by construction for any simultaneous existence of causality in the first moments. This weakness can be in some occasions materialized through a negative effect in the empirical size of the tests. Hence using directly these techniques without first, modeling any first order moments linkage can induce spurious results in the causality in variance tests. This misperception and confusion of the two concepts of causality when using the cross correlation based methods is demonstrated through a Monte Carlo simulation that the authors have conducted. In this way they demonstrate the importance of the correct conditional mean specification and the fact that this procedure must not only depend on the correct modeling of the temporal structure of the time series studied but also on the correct modeling of the inter series dynamics.

Malik (2005**)** investigates the volatility interdependence between the British Pound and Euro, Dollar denominated exchange rates. He uses data both in an hourly and daily basis. The sample with the hourly observations covers the period from December 2001 until March 2002 and contains 1892 trading hours. The alternative sample with the daily observations spans from January 1999 through July 2002. He initially observes that the two currencies demonstrate the highest volatility during the trading hours that the European markets are open. This increased volatility can be attributed either to the increased information flowing to these markets during these trading hours or to the use of private information from some investors. The significance of the impact of the American economy on the European currencies can be clearly seen from the fact that the volatility remains high even after the closing of the European markets when only the New York market is open. It can also be observed that the Euro currency is more volatile than the British Pound irrespectively of the frequency of the data used in the study.

Afterwards, Malik attempts to conduct a comparison between Standard GARCH models and Long Memory FIGARCH models, in order to choose the specification that can more accurately describe the exchange rates series dynamics. A separate comparison is made for the two frequencies used. He firstly uses the hourly data. In this case he also incorporates hourly dummy variables in order to capture the intradaily pattern in the exchange rates. The two currencies demonstrate an impressive homogeneity in their dynamic behavior through the trading day. The conclusion in this case is that the use of conventional GARCH models is preferable in order to capture the volatility dynamics in the exchange rate series. The conclusions are substantially different in the case that daily data are used. The highest volatility appears to be realized on Mondays and a possible reason for this anomaly is the accumulation of considerable volume of information during the weekends. The sum of the estimated coefficients is very close to unity this time, something that initially can lead to the conclusion of being in front of an infinite memory IGARCH type stochastic process but finally the FIGARCH model is found to work better.

Dijk, Osborn & Sensier (2005) investigate the effects of structural breaks in volatility, on the empirical size of various causality in variance tests when the former are not taken into account while performing the above mentioned tests. The critique focuses on the Cheung & Ng and Hong cross correlation based methodologies. The authors build up their discussion referring briefly to the empirical evidence of frequent structural changes in the conditional volatility processes of a large number of financial time series. This fact is the reason behind the necessity of testing for the existence of breaks in the variance process before attempting to test for causal relations in volatility. For this reason they conduct a Monte Carlo study in which they generate data enriched with the aforementioned characteristic. The empirical conclusions reached by these authors are the following:

1. When the structural break in volatility concerns only one of the time series studied then the consequences in the credibility of the causality in variance tests if this event is ignored will be of minimal importance.

2. In the case that there is a simultaneous structural break in both of the series then as is demonstrated through Monte Carlo simulation there will be an important increase in the empirical size of the tests that of course will dampen the reliability of the tests.

3. It is observed that the greatest distortion in the empirical size of the tests happens when the structural break is simultaneous for the two series. As the time span between the structural changes in the volatility processes of the two series increases the distorting effect of breaks, gradually weakens.

Inagaki (2007) examined the existence of systematic volatility transmission between two nominal exchange rates, the U.S. dollar denominated Euro currency and the U.S. dollar denominated British Pound currency. The sample used includes daily observations covering the period from January 1999 up to December 2004. The main reason that this study focuses in these particular exchange rates is in order to investigate the importance of Euro in comparison with other European currencies, in terms of informational efficiency. The volatility of exchange rates is a useful 'gauge' for the flow of information in a specific market. The volatility spillovers will take place when some new information regarding the corresponding markets arrives something that will eventually lead to a rebalancing in the portfolios investing in those markets. The methodology used here is that proposed by Cheung & Ng and depends on the cross correlation of the squared standardized innovations of suitably specified GARCH models. Through the empirical research it was found that the Euro, Granger causes in Variance the British Pound while the reverse relation is not verified in this study. Consequently there seems to be a unidirectional pattern in volatility transmission. An additional important ascertainment is that the Euro currency traders process efficiently the information received from the British Pound exchange rate something demonstrated through the absence of volatility transmission from the British Pound to Euro.

Francis In (2007) examines the existence or not of volatility transmission across three basic financial markets in which interest rate swaps are traded. These are the U.S., the Japanese and the U.K. markets. He uses daily observations for Swaps of three different maturities and for the period between January 1996 and June 2001. The researcher points out the necessity of using high frequency data instead of using lower frequency weekly data. In regard to the methodology used, a VAR - EGARCH specification is chosen with the additional hypothesis of conditional constant correlation in order to decrease the number of unknown parameters and make use of a more parsimonious representation. According to the author such a parameterization is preferable to the alternative methodology proposed by Cheung & Ng, as it makes an extensive use of the entire information contained in the variance covariance matrix. By using multivariate models it is also possible to investigate the volatility transmission mechanism in a more direct way. Through this research an important result that can be derived is that the American market has an important effect in the two other markets and that the Japanese market influences the British market. Bidirectional Causality can be observed between Japan and U.K., while the bidirectional relation between U.S. and U.K. is weaker.

Rodrigues & Rubia (2007) provide the probabilistic confirmation of the argument that the non diagnostic checking for volatility transmission can have a negative and significant impact on the reliability of the causality in variance tests. For their theoretic analysis they make use of a generalized class of non stationary volatility processes in which it is possible to have a very large number of structural breaks and not only one as in the case of other studies. They also prove that the problem of the distortion of the empirical size of the tests remains in effect even in an asymptotic level. The source of this distortion is the fact that in case of structural breaks it is impossible to consistently estimate the cross correlation coefficients regardless of the sample size used in the empirical study. This weakness exists when both series studied are non stationary as in this case the cross covariance function estimator cannot stochastically converge to the true covariance function. As we know the cross covariance is one of the two components of cross correlation. So the problem mentioned above is of course also transferred in the cross correlation. Finally with the use of the central limit theorem it is proved that when one of the series used in the causality in variance test is not stationary then the Cheung & Ng test statistic will not converge to the standard normal distribution but rather to a non standard normal distribution. In the even worse case that both series are non stationary then both Cheung & Ng and Hong Test functions will diverge to infinity.

CHAPTER 3 : LONG MEMORY PROCESSES

3.1 INTRODUCTION TO FRACTIONAL INTEGRATION

Nature's predilection towards long range dependence has been well documented in hydrology, geophysics and meteorology and to the extent that the ultimate sources of uncertainty in economics are also natural phenomena, it is possible that long term memory exists in many economic variables. The fact that economic time series may exhibit long range dependence has been a hypothesis of many early theories of the Trade and Business Cycles. Such theories were motivated by the distinct but non periodic cyclical patterns that typified plots of economic aggregates over time. The presence of long memory in economic time series can have important implications in the field of financial economics. Firstly the portfolio allocation decisions may become extremely sensitive to the investment horizon, if stock returns are long range dependent. Secondly, the pricing methods of financial securities such as options and futures that are based in continuous time martingale stochastic processes must be modified since these models are not consistent with the long term memory. Thirdly the Capital Asset Pricing Model, testing methodologies may stop being valid because the existing statistical inference procedures do not apply in the presence of such a strong persistence. For defining a suitable description of the phenomenon that we have just mentioned, we can simply say that the presence of long memory means that the financial market does not immediately respond to an amount of information flowing into it, but reacts gradually over a period.

From an empirical perspective long memory is related to a high degree of persistence in the realized data. It can be observed in the slow rate of decline (often defined as a hyperbolic decay) in the autocorrelation functions. This phenomenon was noted for the first time in non econometric literature **Hurst** (1951). The starting point of the literature on fractionally integrated processes is the fact that many financial and economic time series show evidence of being neither I(0) nor I(1). They demonstrate however significant autocorrelation up to very long lags. This feature is typical of long memory processes. The order of integration of such a series is most of the times denoted as 'd'. One key feature of the fractionally integrated processes is that they are more flexible than their extreme non stationary (unit root) counterparts. A common feature of the former is the very slow adjustment to equilibrium. From a different point of view and when using high frequency data it seems over restrictive to focus our attention only on integer values of integration.

Long memory processes are studied either in the time domain or in the frequency domain. In the time domain long memory is manifested through a hyperbolically decaying autocorrelation function. It is important to note at this point the importance of using large sample sizes in order for the long memory pattern to be observable in the graphs we use. In frequency domain the same information can be extracted from the spectrum of the variable under study. Returning at the time domain, a covariance stationary time series is said to exhibit long memory if it satisfies the following condition:

$\sum_{k=-n}^{n} \left|\rho(k)\right| \rightarrow \infty$ as $n \rightarrow \infty$, where ρ (κ) denotes the autocorrelation coefficient in lag k.

This means that correlations at long lags are not negligible. In fact such a series is a stochastic mixture of a non unit root series with a simultaneous impressive persistence feature. For historical consistency we must also briefly refer to the Hurst Exponents, which are a different measure of the long memory property of financial time series. Estimated values of this parameter between ½ and 1 are indicative of this kind of persistence.

There exists a considerable number of models that have been designed to capture the long memory in time series. Long memory can be observed in the first or/and in the second (non observable) moments of a financial variable. In our study we are exclusively interested in the volatility processes, however we simply mention the alternative specifications of Fractional White Noise and ARFIMA models used extensively to work with the long memory in the first moments. Concerning the ARFIMA processes, this class of models contains the fractional white noise as a particular case and was introduced by **Granger and Joyeux** (1980) and **Hosking** (1981). In the above mentioned models the propagation of shocks to the mean occurs at a hyperbolic rate of decay when $0 < d < 1$ and this is the main difference with the invertible and stationary ARMA models in which we observe an exponential rate of decay. This behavior differs also, from the infinite memory ARIMA models in which a shock persists for an infinite horizon.

3.2 MODELING LONG MEMORY IN VOLATILITY

Long memory in volatility can be attributed to various different factors such as the contemporaneous aggregation of stable GARCH(1,1) processes and weakly dependent information flow processes. It may also be the result of specific intradaily patterns in financial market volatility and thus related with some microstructure phenomena. More recently, research has shown that structural changes and/or regime switches can spuriously induce a long range dependence in the second order moments process. In our research we study among others, the consequences of the presence of long memory in volatility, in the finite sample properties of the various causality in variance Tests. For this reason we use a fractionally integrated GARCH model as our Data Generating Process in a Monte Carlo Simulation. That's why we provide in the following lines a detailed description of this model and discuss some relevant theoretical aspects of this parameterization.

Loosely speaking fractional integration is a more general & flexible way to describe the long range dependence than the unit root specification providing simultaneously an alternative perspective to examine the unit root hypothesis. The traditional GARCH models (see chapter 1) account for volatility persistence but have the feature that this persistence decays relatively fast. In practice however volatility often exhibits very long temporal dependence. This long run persistence may occur due to the aggregation of a large number of heterogeneous autocorrelated news arrival processes and constitute an intrinsic feature of the return generating process. From a different perspective volatility persistence may be attributed to the persistence in the trading volume which is also known to exhibit long memory properties. This explanation considers the information arrival process to influence the volatility through trading decisions. This means that long memory is transmitted to the volatility process and is not an internal feature of the latter.

It has been demonstrated through Monte Carlo experiments, that when the true underlying generating mechanism of the time series data is an FIGARCH process but erroneously a conventional GARCH model is used in the estimation procedures, this will possibly spuriously lead to the conclusion of the presence of an unrealistic extreme memory IGARCH process. This result is of special importance in the case of high frequency data because the infinite memory behavior occurs mainly with this kind of data. A frequent argument in support of the fractional integrated models is that the distinction between I(0) and I(1) can be too restrictive especially when we use high frequency data. There is also a continuously increasing number of studies that report the presence of autocorrelation in the squared or absolute returns. From all the aforementioned considerations and empirical findings a new class of models was born.

The new promising member of the Heteroscedastic family of models is called the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity Model or simply FIGARCH and was developed by **Baillie, Bollerslev & Mikkelsen**, (1996) The appealing feature of this parameterization is that it combines many of the features of the fractionally integrated processes for the mean, with the regular time varying conditional heteroscedasticity property of the GARCH models. It also has an important advantage compared with the ARFIMA models and this is the asymptotic consistency of its Maximum Likelihood Estimator.

But let's formalize now our discussion. The starting point as in the case of the conventional GARCH models is the discrete time real-valued stochastic GARCH process $\{\varepsilon_t\}$ with the underlying generating mechanism of the former being $\varepsilon_t \equiv z_t\sigma_t$ Two important properties of the z_t standardized innovations process are the following: $\begin{matrix} E_{t-1}(z_t) = 0 \\ Var_{t-1}(z_t) = 1 \end{matrix}$ For the rest of our discussion the operators $\left.\begin{matrix} E_{t-1}(.) \\ Var_{t-1}(.) \end{matrix}\right\}$ will depend on the same information set Ω_{t-1} . The commonly used GARCH (p,q) model is given from the equation $\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$ where $\begin{matrix} \alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + ... + a_q L^q \\ \beta(L) \equiv \beta_1 L + \beta_2 L^2 + ... + \beta_q L^q \end{matrix}$ are the Lag Polynomials. The GARCH process can be also written in an infinite order ARCH representation as it is shown immediately below:

$$\sigma_t^2 = \frac{\omega}{1-\beta(1)} + \frac{\alpha(L)}{[1-\beta(L)]}\varepsilon_t^2 = \frac{\omega}{1-\beta(1)} + \lambda(L)\varepsilon_t^2$$

We can alternatively consider the GARCH model as an ARMA(m,p) in the squared residuals. For this purpose we must first define the shock to volatility term v_t with $v_t = \varepsilon_t^2 - \sigma_t^2$. So we now have $[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)]v_t$, with $m \equiv \max(p,q)$. For Stability and second order stationarity all the roots of the polynomials $\left.\begin{matrix} [1-\alpha(L)-\beta(L)] \\ [1-\beta(L)] \end{matrix}\right\}$ must lie outside the unit circle. The stationarity in this framework means that the effect of the squared past innovations of the current conditional volatility decays with an exponential rate. In the case that the autoregressive polynomial $[1-\alpha(L)-\beta(L)]$ contains a unit root we say that the GARCH process in integrated in variance. The integrated GARCH model that we simply denote as IGARCH is given by the following equation: $\phi(L)(1-L)\varepsilon_t^2 = \omega + [1-\beta(L)]v_t$ where $\phi(L) \equiv \frac{[1-\alpha(L)-\beta(L)]}{(1-L)}$ and is of order m-1.

The only step needed in order to transfer our discussion to the FIGARCH representation is the replacement of the first difference operator with the fractional differencing operator. The fractional differencing operator $(1-L)^d$ has a binomial expansion which can be expressed in terms of the hypergeometric function:

$$(1-L)^d = F(-d,1,1;L) = \sum_{k=0}^{00}\Gamma(k-d)\Gamma(k+1)^{-1}\Gamma(-d)^{-1}L^k \equiv \sum_{k=0}^{00}\pi_k L^k$$

with Γ(.) denoting the Gamma function. So the FIGARCH(p,d,q) process is defined as $\phi(L)(1-L)^d\varepsilon_t^2 = \omega + [1-\beta(L)]v_t$ with 0 < d < 1. All the roots of the corresponding polynomials lie outside the unit circle. This model has also an alternative representation:

$$[1-\beta(L)]\sigma_t^2 = \omega + \left[1-\beta(L)-\phi(L)(1-L)^d\right]\varepsilon_t^2$$

Finally, the conditional volatility of ε_t is obtained from the following infinite order ARCH representation:

$$\sigma_t^2 = \frac{\omega}{[1-\beta(1)]} + \left\{1-\frac{\phi(L)(1-L)^d}{[1-\beta(L)]}\right\}\varepsilon_t^2 = \frac{\omega}{[1-\beta(1)]} + \lambda(L)\varepsilon_t^2$$

The FIGARCH process is not weakly stationary as is the case with the GARCH processes because the second moment of ε_t is not finite. However both processes are strictly stationary for $0 \le d \le 1$. The persistence of volatility can be expressed in terms of the impulse response coefficients of the optimal forecasts for the conditional variance. $\gamma_k \equiv \frac{\partial E_t(\varepsilon_{t+k}^2)}{\partial v_t} - \frac{\partial E_t(\varepsilon_{t+k-1}^2)}{\partial v_t}$. The impulse response coefficients can be found if we first modify the equation given for the FIGARCH Model in the equivalent form: $(1-L)\varepsilon_t^2 = \frac{\omega}{(1-L)^{d-1}\phi(L)} + \frac{[1-\beta(L)]}{(1-L)^{d-1}\phi(L)}v_t \equiv \zeta + \gamma(L)v_t$

To assess now the long term impact of shocks in the volatility process we compute the limit of the cumulative impulse response weights.

$$\gamma(1) = \lim_{k \to 00} \sum_{i=0}^{k} \gamma_i = \lim_{k \to 00} \lambda_k = F(d-1,1,1;1)\phi(1)^{-1}\left[1-\beta(1)\right]$$

$\gamma_i \to$ impulse response weight

$\lambda_k \to$ Cumulative impulse response weight

For $0 \leq d < 1$, we will have $F(d-1,1,1;1) = 0$ and so even in the long memory FIGARCH model, with 0 < d < 1 the limit of the cumulative impulse response weights will tend to zero. This means that eventually any volatility shock will die out in the long term horizon. This conclusion applies also in the case of the simple GARCH process. However the corresponding shocks will dissipate much faster in the latter. More specifically a certain shock in volatility will decay at an exponential rate in the standard GARCH model and at a hyperbolic rate in the FIGARCH model, despite the fact that the cumulative impulse response weights for both parameterizations will tend to become zero. For d = 1 the FIGARCH model becomes an IGARCH and so this means that any shock will persist forever. Finally the Maximum likelihood estimator of the parameters of the FIGARCH(p,d,q) can be obtained by the maximization of the log likelihood function given the specific realization: $\log L\left(\theta;\varepsilon_1,\varepsilon_2,...,\varepsilon_T\right) = -\frac{T}{2}\log\left(2\pi\right) - \frac{T}{2}\log\left(\sigma_t^2\right) - \frac{1}{2}\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\sigma_t^2}\right]$

An important point that we must note here is that when estimating GARCH models, it is needed to use some initial values in order to be able to start the recursions for the conditional variance function. Usually the unconditional variance is used to define the presample values of the squared innovations that will be used as initial values. Unfortunately the long term variance does not exist in the case of FIGARCH models. However, through asymptotic analysis it has been shown that conditioning on the same as in conventional GARCH, presample values will not distort the asymptotic distribution of the estimators. We must also note that because of the long memory in the volatility process as expressed from the slow decay of the cumulative impulse response weights, the presample values will affect our process for a much longer number of steps in the recursive estimation. That's why it is necessary to set a truncation lag at large number. Usually it is set to 1000. The above considerations will be of great importance as will be shown in the next chapter when we will use the FIGARCH as the data generating process in our Monte Carlo simulation study. In the rest of this chapter we provide a brief literature review.

3.3 LITERATURE REVIEW ON LONG MEMORY

Lo (1991) develops a test for long run memory that is robust to short range dependence. It is an extension of the range over standard deviation or R/S Statistic for which the relevant asymptotic sampling theory is derived via functional central limit theory. Lo also presents an interesting distinction between the short range and long range statistical dependence using the concepts of mixing conditions. He adopts strong mixing as an operational definition of short range dependence. He then conducts a Monte Carlo experiment in order to investigate the finite sample size and power properties of the test statistic that he has introduced. Finally using empirical data his research is focused in the detection of possible long memory in the first moments of specific time series. The dataset used in this study consists of monthly and daily observations of the CRSP value and equal weighted indexes. The daily data are available from July 1962 to December 1987 (6.409 Observations) and the monthly data are available from January 1926 to December 1987 (744 observations). According to this study, there is no evidence of long range dependence in any of the indexes over any sample period or sub period once short range dependence is taken into account. These findings mean that there is little support for long term memory in U.S. Stock returns.

Cheung (1993) uses the semi nonparametric Geweke-Porter-Hudak test for the detection of long memory in exchange rate data and the use of fractionally integrated autoregressive moving average models for the examination of the time series dynamics of exchange rates. The time series properties of five nominal dollar spot rates (British Pound, Deutsche Mark, Swiss Franc, French Franc and Japanese Yen) are examined. The dataset consists of weekly observations of the exchange rates mentioned above and spans from January 1974 to December 1989. It is important to note that the author mentions the serious weakness of using ARFIMA models in that they do not incorporate the conditional heteroscedasticity pattern frequently observed in the foreign exchange data. He implicitly refers to the need for the creation of long memory models that will incorporate the time varying nature of the conditional volatility. The results indicate the presence of long memory in the exchange rates. This implies that the empirical evidence of unit roots in exchange rate data is not robust to long memory alternatives. Finally the author defines two possible sources of long memory in the foreign exchange rates. A first explanation is the Purchasing

Power Parity. This means that the exchange rates are tied to the movements of relative national prices. As has been observed in other studies, there is evidence of long memory in Capital Price Indexes. This characteristic may be transmitted also in the exchange rates and gives them the observed long memory property. A second explanation is that the Macroeconomic Fundamentals (Money Supply, Output etc.) may be fractionally integrated. He denotes also the need for further research in this field.

Baillie, Bollerslev & Mikkelsen (1996) propose a new class of models, the so called fractionally integrated generalized autoregressive conditional heteroscedasticity processes. The shocks in these processes die out at hyperbolic rate that is determined from the value of the long memory/fractional differencing parameter. The Quasi Maximum Likelihood Estimator of these models is proven through Monte Carlo to have very good finite sample properties and is also argued to be asymptotically consistent. The authors apply their proposed model in the estimation of the long memory characteristic in the Deutschmark-US dollar exchange rate volatility process. Their sample contains daily data for the period starting in March 1979 through December 1992 (3.454 Observations). In the empirical research that they conduct, the FIGARCH model is compared with the alternative IGARCH specification as well as with the standard GARCH model. They conclude that the long run dynamics of the series used are better modeled by the fractional differencing parameter. A one sided test for $d = 1$ against $d < 1$ also rejects the null hypothesis of an IGARCH process. The superiority of the long memory specification is also demonstrated from the analysis of the cumulative impulse response weights for the influence of a shock (innovation) in the forecasts of the conditional variance.

Baillie (1996) presents a summary of the empirical findings of a vast number of research studies in the field of long memory processes. According to the author the presence of long memory can be defined from an empirical data-oriented approach in terms of the persistence of observed autocorrelations. The extent of the persistence is consistent with that of stationary processes but in contrast to them the autocorrelations decay far more slowly than the exponential rate associated with the stationary ARMA class of models. In his study Baillie also refers to the credibility of the semiparametric and parametric estimators proposed in the literature. The main advantage of the former is that they focus on the key parameter of interest while allowing for short memory effects to be neglected. Unfortunately however, simulation work has generally revealed a disappointing performance of these estimators. In the alternative parametric case that makes use of certain specifications, it is argued that the identifiability of high-order ARFIMA models often appears problematic as it is shown from the available empirical studies. In some cases the estimated value of d appears sensitive to the parameterization of the high frequency components of the series and in other cases, a unit root is included in the confidence interval of the fractional differencing parameter estimate. Finally the author briefly mentions the most common applications of long term memory models. Paradigms in the areas of geophysical sciences, macroeconomics and asset pricing are discussed.

Andersen and Bollerslev (1997) investigate the long memory characteristics of foreign exchange rates. The volatility now is interpreted as a mixture of numerous heterogeneous short run information arrivals. They estimate the degree of long term memory in the volatility process of the spot Deutsche Mark, U.S. dollar denominated exchange rate, using two frequency domain semi parametric estimators. Their sample is consisted of five minute returns over a period of one year amounting to the vast number of 74.880 Observations. Analyzing the frequency spectrum, it is found that at low intradaily frequencies the spectrum is approximately log-linear, indicating the presence of long memory in volatility. By using the log-periodogram GPH estimator and the frequency–domain semi parametric estimator of Robinson (1994) they estimate the long-memory parameter d. It is also shown that the corresponding fractional differencing operator constructed from the estimated d , is able to filter all the long run dependencies in the conditional second moments process. Consequently, a low pass filter in the frequency domain eliminates the powerful intradaily pattern of volatility and makes it possible to reveal a hyperbolically decaying autocorrelation function, a fact indicative of the presence of long memory effects in the exchange rates volatility process. It is finally argued that the long memory characteristics of a time series constitute an intrinsic feature of the return generating process rather than the manifestation of occasional structural shifts.

Teyssiere (1998) makes use of alternative multivariate long memory models, trying to capture the long term dependence and the volatility dynamics of foreign exchange rates. The reason for extending the long memory ARCH models to a multivariate framework is according to the author, the fact that some time series appear to share a common degree of long memory in their conditional variances. The author proposes two main multivariate specifications. The first is the Conditional

Constant Correlation FIGARCH and the second is an unrestricted multivariate long memory ARCH model. The drawback of the first parameterization is the oversimplifying hypothesis that the conditional covariances are proportional to the product of the two corresponding conditional standard deviations. From the other side the unrestricted alternative is much more complicated demanding the estimation of a large number of parameters. His data set consists of 30 min data for three series of foreign exchange returns, USD/DM, USD/GBP and USD/JPY. The total number of observations is 12528.

As a first step the author estimates univariate long memory models in order to investigate separately the long memory features in each of the series. The observation of a common degree of long memory in the conditional variance of the three exchange rates leads to the estimation of a trivariate FIGARCH model. The author also mentions the seasonality patterns in the series used and denotes the importance of correctly modeling these features before proceeding with the estimation of long memory models. From the estimations of alternative multivariate models the author concludes that the unrestricted FIGARCH is preferred. The three series demonstrate (as previously mentioned in the univariate case) the same degree of long memory. Finally it is argued that the FIGARCH model is inadequate to capture the seasonal component of volatility and this is exhibited from the significance of the autocorrelations of the squared standardized residuals from both FIGARCH models, implying a misspecification in the conditional volatility models. A possible solution of this problem according to the author is the use of a different time scale in which the time will be normalized with the level of market activity.

Bollerslev & Mikkelsen (1999) study the dynamics of stock market volatility. Their research supplements the existing literature regarding the long term dependence of stock market volatility by indirectly inferring the degree of fractional integration in an aggregate equity market, using a panel dataset of financial options transactions prices on the S&P 500 composite index. The dataset used in their analysis is consisted of daily closing prices for all the CBOE (Chicago Board of Options Exchange) traded S&P 500 long term equity anticipation stocks contracts (leaps) for a 141 week period spanning from January 1991 until September 1993. The begin their study with a brief description of the theoretical financial options pricing framework demonstrating the impact of the conditional volatility forecasts in the formulation of the theoretical options prices. The authors use as a benchmark model through their analysis, the

Black and Scholes (1973) option pricing formula that assumes a constant variance and a continuous time random walk process for the underlying asset price.

For the estimation of the theoretical leaps prices the authors use Monte Carlo simulation techniques through which they generate a large number of different paths for the simulated prices. They calculate the leaps prices with both a daily and a weekly sample frequency in order to ensure the robustness of their empirical findings. The authors compare the observed leaps prices with their simulated theoretical counterparts obtained using alternative specifications for the volatility process such as the EGARCH, IEGARCH, FIGARCH, and FIEGARCH parameterizations. The leaps valuations lead to the conclusion that the FIEGARCH model tends to produce the most accurate prices in both daily and weekly frequencies as demonstrated from the low average pricing errors. The above results are in favor of the presence of long memory in stock market volatility.

Baillie, Cecen and Han (2000) investigate whether the FIGARCH long memory volatility process can adequately model the time series dynamics of the Deutschemark / US dollar spot exchange rate returns in multiple different frequencies. Two distinct datasets are used. The first contains low frequency, daily observations spanning from March 1979 to December 1998 (4.989 Observations). The second sample consists of higher frequency data. More specifically the authors use a huge set of 30-minute exchange rates with a sample period from 00:30 GMT January 1, 1996 through 00:00 GMT, January 1, 1997 (12.576 Observations). The most important empirical findings are the almost similar values of the estimated long memory volatility parameters across the various frequencies. This fact is indicative of a common process underlying the generation mechanism of returns and supportive of the fact that long memory is an intrinsic feature of the system and is not caused by exogenous shocks or regime switches. The FIGARCH model used by the authors is also tested against the standard GARCH specification via the classic Wald tests. These tests just confirm the superiority and robustness of the long memory volatility model. An observable pattern in the autocorrelation function of the squared and absolute returns is the intradaily periodicity. This phenomenon is generally being attributed to the opening of the European, Asian and North American markets superimposed on each other and it is filtered from the series by using the Flexible Fourier Form method. The conclusive remark of this paper is that the FIGARCH models appear to be successful in accounting for the dynamics of the returns series studied irrespectively of the frequency of the data used in the estimations.

Brunetti & Gilbert (2000) use a bivariate FIGARCH framework in order to test for fractional cointegration. They provide an analytical presentation of the methodology proposed for cointegration testing in a multivariate FIGARCH framework along with a detailed discussion of Cointegration theory. There is an important point that we must note here. Unlike the Fractional Cointegration tests in the first moments processes, when we work with the unobservable skedastic processes we cannot presuppose stationarity since the volatility processes often encountered in finance are by nature stationary. The empirical testing takes place on the NYMEX and IPE crude oil markets which are known to be closely related using monthly data for the period from June 1988 till March 1999. It is investigated whether the volatility processes of these two markets are fractionally integrated, and also whether there is a common order of fractional cointegration (e.g. if there is a linear combination of the two series that exhibits a lower order of fractional integration).

A causality in variance test is also conducted using the squared standardized residuals obtained from the estimation of the univariate FIGARCH Models. There are no signs of any volatility spillovers between the two series. The results of the rest of the tests show the high degree of persistence in the volatility processes and a common degree of fractional integration. This implies that a linear combination of the two processes may be less persistent than the volatilities themselves even though it remains fractional. The most important outcome of this study is that the two volatility series are indeed fractionally cointegrated with the NYMEX appearing to be the dominant market as shown from the significance of the corresponding cross market volatility transmission parameters estimated in the bivariate FIGARCH model.

Szilard and Laszlo (2001) develop a multivariate diagonal FIGARCH model that builds upon the hypothesis of a common order of fractional integration. According to researcher's claims this parameterization is the most logical and intuitive of the various alternative multivariate extensions and it is the most suitable choice for adapting the concept of long memory and fractional differencing to a multivariate framework. In order to overcome some serious difficulties occurring during the estimation of this model even in this parsimonious representation, the fractional differencing operator (1-L) ^d is kept a scalar. This means that a common structure on the long memory components is imposed. A theoretical justification of this hypothesis is discussed. It is argued that the market efficiency implies a similar long range behavior in the volatility of these series and some indicative empirical studies are also mentioned. The QMLE estimator of this parameterization is extensively tested through a Monte Carlo simulation. The proposed estimator demonstrates a satisfactorily good performance and is robust to distributional assumptions.

In the rest of the paper the researchers use a trivariate specification for jointly modeling the daily volatility of the German mark, British pound and Japanese Yen against the U.S. dollar. The dataset contains daily spot rates of the aforementioned exchange rates and spans from July 1981 through January, 2001. As a first step an unrestricted specification of the trivariate diagonal FIGARCH is estimated. In this case there are six long memory parameters estimates. Then the alternative restricted specification is estimated and contrasted to the former. A likelihood ratio test seems to be in favor of the restricted model. Finally it is commented that the estimated value of the scalar long memory parameter is very close to estimates of previous empirical studies.

Vilasuso (2002) conducts a comparative investigation of the forecasting accuracy of alternative conditional volatility models. Three specifications are tested through his research. These are the conventional GARCH, the IGARCH and the FIGARCH models. The sample dataset consists of various daily US dollar denominated spot exchange rates and more specifically the Canadian dollar, the French franc, the German mark, the Italian lira, the Japanese yen and the British pound. The sample period extends from March 1979 until December 1997. From an initial estimation of the variance models it seems that the FIGARCH captures more adequately the volatility dynamics of the exchange rates as can be demonstrated from the low values of the Ljung-Box portmanteau statistics for the squared standardized residuals. In the next step the author using the previous estimates, evaluates the out-of-sample forecasts of the models for the period 1 January 1998 to 31 December 1999. The volatility forecasts are then compared with the squared daily exchange rate returns with the forecasting accuracy being determined from the calculation of the mean square error (MSE) and the mean absolute error (MAE). Three forecasting horizons are used, a 1-day, a 5-day and a 10-day horizon. For all the criteria the FIGARCH model, generates superior out-of-sample forecasts with these results being more denounced when longer forecasting horizons are considered.

Banerjee & Urga (2005) discuss some of the most important developments in the field of modeling the long memory in financial time series. They also explicitly refer to the link that exists between long memory and Structural Breaks. A useful discrimination between the estimation methods proposed to test for the long range dependence is also conducted. The tests can be divided into two classes, 'semi parametric' and 'parametric' estimation methods. Semi parametric methods (like the GPH Test) do not require the modeling of a complete set of autocovariances, but are interested just in the parameter d. In contrast to this approach, parametric techniques involve the complete estimation of models like the FIGARCH or ARFIMA. The drawback with the parametric case is that it is computationally demanding and prone to misspecification. From the other side the semi parametric methods are not as efficient as their parametric counterparts. The paper concludes with an important notification. The estimates of the long memory parameter depend on the number of regime switches and where they occur in the sample. Processes with infrequent regime switching may generate a long memory effect in the autocorrelation function. In such a case fractional models may lead the researchers in spurious results.

Caglayan and Jiang (2006) propose a new parameterization in order to investigate in a multivariate framework the dual long memory properties in the first and second moments of inflation and output growth as well the causal relations between them. The new class of models is the bivariate Constant Conditional Correlation ARFIMA-FIGARCH models. Through this specification it is possible among others to study whether the long memory in inflation is due to the output process. Their data set is consisted of monthly consumer price index (CPI) and industrial production index (IPI) series covering the period from February 1957 until May 2005. Proceeding with their work the authors estimate univariate models in order to gain a first outlook in regard with the presence or not of dual long memory in the series. In the next stage they estimate the bivariate ARFIMA-CCC-FIGARCH model assuming a constant correlation coefficient structure. The fractional differencing parameters in the mean and the variance of the series obtained from the bivariate model are very similar to those obtained from the estimation of the univariate models suggesting that both inflation and output growth exhibit long memory in the means and conditional variances.

Morana (2006) analyzes the volatility dynamics of the Deutsche mark-US dollar exchange rate. He discusses the importance of accounting for structural breaks and stochastic intradaily patterns in the volatility process. The aims of his research are firstly to estimate the FIGARCH model using a large number of observations, secondly to compare the evidence from high frequency data with that from daily data and finally to evaluate the robustness of the econometric estimates to various models of intraday volatility. The sample data set consists of both daily and high frequency observations for the DM / US exchange rate. The low frequency sample spans from January 1972 to December 1997 with a sub-sample from 1992 to 1997 being also used. The high frequency sample consists of 30-min data points for the period from 1992 until 1997 and as a sub-sample the year 1996 is used.

The most important empirical results reached by the author are the following. In the daily frequency the estimates for the long memory parameter are of smaller size when the short sample is considered than when the longer sample is used. This result may be indicative of the fact that structural changes may induce the long memory in the volatility process. When the high frequency sample is considered the estimation for the long memory parameter is once more lower for the sub-sample than the corresponding estimate for the longer sample. Also when a filter for the intraday seasonal features is applied in the raw data, an increase in the fractional differencing parameters is observed for both short and long sample of high frequency data. This result declares the importance of accounting the intradaily pattern in the time series. The conclusive suggestion by this author is that the FIGARCH models are sensitive to both the length of the data set and the presence of intradaily repetitive patterns.

Baillie and Morana (2007) introduce a new long memory volatility model, the so called Adaptive FIGARCH that is able to capture both the long memory and structural changes in the conditional variance process. More specifically the adaptive FIGARCH process is formed from two basic components. A long memory volatility process and a deterministic time varying intercept that allows for breaks, cycles and changes in drifts. The structural changes are modeled with the use of a flexible functional form allowing the intercept to be time varying. The estimation of this new model can be done with the usual Quasi Maximum Likelihood method. The authors argue that the QMLE is asymptotically normal and consistent, maintaining the optimal properties of the estimator used in the case of IGARCH processes.

In order to investigate the impact of estimating A-FIGARCH models under different data generating processes a Monte Carlo simulation is conducted. They use various alternative scenarios with the existence or absence of structural breaks as well as of different values for the fractional differencing parameters. The results of the simulations are the following. Under a no structural breaks environment the A-FIGARCH estimation has approximately the same degree of small sample bias as the corresponding estimate of the more conventional FIGARCH model. In the case that structural breaks are present however, the degree of bias in the estimates of the long memory parameters is smaller for the Adaptive FIGARCH relative to the FIGARCH specification for all the three different true values of the long memory parameter. The authors also conduct an empirical research estimating the above models for the S&P 500 returns. Their data set consists of 20863 observations spanning from January 1928 through February 2007. From their estimations it is found that neglecting the presence of structural breaks, results in important biases in the estimated conditional volatility processes denoting in this way the usefulness of the proposed variance model.

Krämer and Azamo (2007) study the nature of the excess persistence estimates often encountered in empirical research, when standard GARCH(1,1) models are used. Their theoretical motivation, deviates from the already discussed long memory in volatility that may be an intrinsic (but some times not properly modeled or even ignored) feature of a certain financial time series. The source of excess persistence according to these authors is not the misspecified use of conventional GARCH specifications (instead of their long memory alternatives), but the presence of regime switches during the sample period used. The incentive for research in this direction was born through the observation that the often strong, estimated persistence in financial volatility was not directly materialized in the construction of more accurate forecasts in applied working. Instead of that it seems that there exists an upward bias in the persistence estimates towards the maximum value of one (e.g. persistence measured from the sum of the coefficients in the GARCH parameterization) that is more intense in larger sample sizes. More specifically the estimated persistence depends mainly on the calendar time span of the sample and not only on the sample size. The explanation of this result is the fact that the probability of a regime switch (that is a mechanism which enforces the persistence in variance), increases with increasing calendar time. The authors also conduct Monte Carlo simulations in order to study the effects of structural breaks in the estimated persistence, demonstrating in practice the validity of their theoretical suggestions.

Kang and Yoon (2007) examine the presence of dual long memory in the return and volatility of the Korean Stock market. According to these authors long memory is often observed simultaneously in the mean and the variance of returns. They also argue that equity returns may exhibit a statistically significant asymmetric leptokurtosis. Through their research they make use of both the Gaussian distributional assumption and a skewed version of the Student's t distribution in order to be better able to capture any possible asymmetries and/or fat tails in the stock market returns distributions. The main focus of these authors is to investigate whether a combinatorial parameterization such as the ARFIMA-FIGARCH model, can more adequately describe the dynamics of the Korean Stock Market, compared with the single ARFIMA or FIGARCH specifications. Their dataset consists of daily closing prices of the KOSPI index covering the period from January 1980 to December 2005 (7290 Obs.) and of the KOSDAQ index spanning from July 1996 through April 2006 (2539 Obs.) The ARFIMA-FIGARCH model is found to provide the best fitting tool for capturing the dual long memory in both Korean Equity Indices confirming the introductory suggestions of the two authors. The skewed Student's t distribution is also found to characterize the returns in the Korean Stock Market.

Chitkushev, Wang et al. (2008) compare the volatility return intervals of the S&P 500 index with those generated from two frequently used time series models namely the Fractional Brownian Motion and ARMA-FIGARCH specifications. A return interval is defined as the time between successive volatilities above a certain threshold. In order to study the memory property, the authors analyze among others the conditional probability density functions of the intervals with the conditioning set containing the preceding interval. Their dataset spans from January 1984 until December 1996 and is consisted of 10 min. points for a total of 132.000 observations. In their analysis they filter the intraday patterns from the data in order to avoid any possible spurious conclusions. It is demonstrated that both in empirical and long memory simulated data the short (long) return intervals are more likely to be followed by short (long) return intervals. This is clearly a sign of clustering in volatility returns intervals. This observable pattern is absent in the case that the real data are randomly shuffled. They also study the cluster size probabilities in order to investigate whether there exists long memory in the return intervals. For this purpose short memory data are generated in order to test if this kind of memory can explain the cluster size distributions. It is shown that the cluster size probabilities for both high and low return intervals of the S&P 500 have a different behavior than those of the short memory generated data. This suggests the existence of long memory in volatility.

Gabjin, Seunghwan & Cheoljun (2008) investigate whether there exists a long memory property in high frequency data concerning diverse stock market indices and foreign exchange rates. In order to investigate the presence of long term memory in the volatility of financial time series the authors used 1-min data from two Korean Indexes from 1995 to 2002 and 1997 to 2004 respectively. They also used 5-minute foreign exchange data for Euro, British Pound, Japanese Yen and other currencies. The method for the quantification of the long memory property is the detrended fluctuation analysis. For all market data studied, no strong long memory property was found in the returns series as exhibited from the fast decay of the autocorrelation function and the values of the Hurst Exponents fluctuating around 0.5. In contrast to this result significant long term memory was found in the volatility process and this is clearly seen in the Power Law decay of the autocorrelation function. The main results of this study are that the long memory property of the volatility processes when existing can be attributed mainly to the volatility clustering observed in financial time series. This conclusion is reached from the observation that the values of the Hurst Exponents are reduced (although not in the extent as in the FIGARCH case) when the long memory data are filtered by the standard GARCH Models.

Bentes, Menezes & Mendes (2008) study stock market volatility and the reasons that lie beyond price movements. Volatility clustering and long memory are two well documented characteristics related to financial time series. In this paper, a new method for detecting the presence of long memory is introduced. This technique is based on the concept of entropy. More specifically three different measures are presented. The Shannon entropy, the Renyi entropy and the Tsallis entropy all of them being used for over a century in the discipline of Physics. These measures can be used to explain the tendency of a natural phenomenon to flatten out and gradually disappear over time. The authors use daily data from S&P 500, Nasdaq 100 and Stoxx 50 indexes, constituting a sample spanning over the period of June 2002 till January 2007. They also compare the empirical evidence obtained from the use of standard heteroscedastic models with that from using the entropy measures. The results from the estimation of FIGARCH models and the entropy measures lead to the conclusion that long memory characterizes the volatility processes.

CHAPTER 4 : MONTE CARLO STUDY

4.1 INTRODUCTION

The main purpose of our study is the extensive investigation of the finite sample properties of the causality in variance / mean, tests that have been presented in the previous sections. We must note that we have not used through our research the Multivariate BEKK GARCH based test. The main reason for this omission is that this methodology is extremely demanding from a computational perspective and thus it would be very difficult to perform an adequate number of replications in order for this method to be comparable with the others. Thus we will focus on three other techniques, namely the Cheung & Ng, the Hong and the Lagrange Multiplier tests. During our research we have conducted a large number of simulation experiments and in this way an important volume of artificial data has been accumulated. These data are available from the author upon request. Our empirical work can be categorized in four discrete Monte Carlo designs. In each of them we make use of three different sample sizes. The reason for this sampling variety is that we want to derive empirical results that are of practical importance for both the field of macroeconomics (where low frequency data are usually used) and the field of finance (where high frequency data are more easily available).

One common choice through our study (with the exception of the last design) is the number of observations that we discard in each replication. This number is set equal to 1000 observations. For example for a generation of a 500 observations sample we in fact generate 1500 observations but afterwards we use only the last 500. This approach was undertaken in order to minimize the simulation bias effects that are caused by the arbitrarily chosen initial values (necessary for the recursive simulation initiation). The initial value for the conditional volatility process was set equal to the unconditional long term variance in the first three designs (in the fourth one it was set equal to 1000), while the initial value for the real valued series was set equal to zero. The initial value for the GARCH residuals process was estimated using the square root of the initial value for the unconditional variance multiplied by a standardized normal random number. More details regarding the Data Generating Processes, the causality in variance methodologies and the Monte Carlo Designs are available in the

Appendix A of our work where we provide the MatLab code that was used through our research.

The Monte Carlo experiments that we have conducted can be summarized as follows:

1. Calculation of the Finite Sample Properties of causality in variance tests under different distributional assumptions for the underlying residuals Process.

2. Calculation of the Finite Sample Properties of causality in variance tests under filtered & unfiltered causality in mean.

3. Calculation of the Finite Sample Properties of causality in mean Tests under the presence of volatility Spillovers (with or without GARCH in Mean Effects) & under no volatility spillovers.

4. Calculation of the Finite Sample Properties of causality in variance tests under long memory in volatility and assuming either a Standard Normal or a Normal Inverse Gaussian distribution for the residuals (errors).

For all the designs we have chosen to generate 2000 Replications using the sample sizes of 200, 600 & 1000 observations. For brevity reasons however we present the results for the 200 and 1000 observations only. In the following lines a brief revisit in the various tests is attempted in order the reader to instaneously refer to the various formulas without having to go back in the theoretical part of the dissertation.

$$\mu_{z,t} = \sum_{i=1}^{00} \varphi_{z,i}(\theta_{z,\mu}) Z_{t-i} \quad \rightarrow \text{Conditional Mean Specification}$$

$$h_{z,t} = \varphi_{z,0} + \sum_{i=1}^{00} \varphi_{z,i}(\theta_{z,h}) \{(Z_{t-i} - \mu_{z,t-i})^2 - \varphi_{z,0}\} \quad \rightarrow \text{Conditional Variance Model}$$

$$Z = X, Y$$

$$U_t = ((X_t - \mu_{x,t})^2 / h_{x,t}) = \varepsilon_t^2 \rightarrow \text{ Squared Standardized Residuals for the 1st Series}$$

$$V_t = ((Y_t - \mu_{y,t})^2 / h_{y,t}) = \zeta_t^2 \rightarrow \text{Squared Standardized Residuals for the 2nd Series}$$

$$r_{uv}(k) = C_{uv}(k) \{C_{uu}(0) C_{vv}(0)\}^{-1/2} \rightarrow \text{ Cross Correlation Function}$$

1st Class of Tests: Cheung & Ng Cross Correlation Function based Tests.

1. 'Standard' version of the Test.

$$S = T\sum_{i=j}^{k} \hat{r}_{uv}(i)^2 \sim X^2(k-j+1)$$

2. 'Modified' version of the Test.

$$S_M = T\sum_{i=j}^{k} \omega_i \hat{r}_{uv}(i)^2 \sim X^2(k-j+1)$$

$$\omega_i = (T+2)/(T-|i|)$$

3. 'Normal' version of the Test.

$$S_N = \sqrt{T}\hat{r}_{uv}(k) \sim N(0,1)$$

4. 'Bidirectional' version of the Test.

$$S_B = T\sum_{i=j}^{k} \hat{r}_{uv}(i)^2 \sim X^2(k-j+1)$$

–j = k = m

2nd Class of Tests: Hong Tests with Kernel functions for the lag weights.

1. 'Hong' version

$$Q_1 = \left\{T\sum_{j=1}^{T-1} k^2(j/M)\hat{r}^2_{uv}(j) - C_{1T}(k)\right\} / \{2D_{1T}(k)\}^{1/2} \xrightarrow{\text{Regularity Conditions}} N(0,1)$$

$$C_{1T}(k) = \sum_{j=1}^{T-1} \underbrace{(1-j/T)}_{\text{Finite Sample Corrections}} k^2(j/M)$$

$$D_{1T}(k) = \sum_{j=1}^{T-1} \underbrace{(1-j/T)\{1-(j+1)/T\}}_{\text{Finite Sample Corrections}} k^4(j/M)$$

2. 'Modified Hong' version

$$Q_1^* = \{T\sum_{j=1}^{T-1}(1-j/T)^{-1}k^2(j/M)\hat{r}_{uv}(j) - C^*_{1T}(k)\} / \{2D^*_{1T}(k)\}^{1/2} \sim N(0,1)$$

$$C^*_{1T}(k) = \sum_{j=1}^{T-1} k^2(j/M)$$

$$D^*_{1T}(k) = \sum_{j=1}^{T-1} \{1-(T-j)^{-1}\}k^4(j/M)$$

3. 'Bidirectional Hong' version

$$Q_2 = \left\{ T \sum_{j=1-T}^{T-1} k^2(j/M)\hat{r}_{uv}^2(j) - C_{2T}(k) \right\} / \{2D_{2T}(k)\}^{1/2} \sim N(0,1)$$

$$C_{2T}(k) = \sum_{j=1-T}^{T-1} (1-|j|/T)k^2(j/M)$$

$$D_{2T}(k) = \sum_{j=1-T}^{T-1} (1-|j|/T)\{1-(|j|+1)/T\}k^4(j/M)$$

Kernel Functions used in the Hong class of tests.

1. Truncated $\rightarrow k(z) \;= \begin{cases} 1, & |z| \le 1 \\ 0 \end{cases}$

2. Bartlett $\rightarrow k(z) = \begin{cases} 1-|z|, & |z| \le 1 \\ 0 \end{cases}$

3. Daniell $\rightarrow k(z) = \sin(\pi z)/\pi z \;,\; -00 < z < 00$

4. Quadratic Spectral $\rightarrow k(z) = \dfrac{3}{\sqrt{5(\pi z)^2}}\{\sin(\pi z)/\pi z - \cos(\pi z)\}, -00 < z < 00$

5. Tukey-Hanning $\rightarrow k(z) = \begin{cases} \frac{1}{2}(1+\cos(\pi z)), |z| \le 1 \\ 0, \text{ otherwise} \end{cases}$

3rd Class of Tests: Lagrange Multiplier Test

1. We estimate GARCH(1,1) models for ε_{it} and ε_{jt} and then obtain the standardized residuals ξ_{it} , the derivatives χ_{it} and the conditional variances h_{jt}

2. We then regress $\xi_{it}^2 - 1$ on the terms $x_{it}^{'}, \varepsilon_{jt-1}^2, h_{jt-1}$. The latter pair is contained inside the $z_{jt}^{'}$ variable

3. The λ_{LM} test statistic would be obtained by calculating the product $T * R^2$, where T is the sample size and R^2 is the coefficient of determination obtained from the regression in step II.

This procedure is followed in order to perform a test for volatility transmission from variable j to variable i. A completely analogous procedure we would follow if we wanted to test for variance causality in the opposite direction.

4.2 MONTE CARLO SIMULATION DESIGN 1

We have calculated the size and power of causality in variance Tests under alternative distributional assumptions for the underlying residuals process.

1. NIID (0,1)
2. Student's t distribution with eight degrees of freedom
3. Skewed Student's t distribution with one degree of positive asymmetry & eight degrees of freedom, see **Hansen** (1994**)** for details of this distribution.
4. Skewed Student's t distribution with one degree of negative asymmetry & eight degrees of freedom, see **Hansen** (1994**)** for details of this distribution.

In the **Appendix D** (Design5) we also provide the results obtained using the Normal Inverse Gaussian Distribution of **Barndorff-Nielsen** (1997) that were omitted from the main body of our work for brevity reasons.

4.2.1 DATA GENERATING PROCESS

$$Y_t = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$w_1 = 0.10 \qquad w_2 = 0.10 \qquad a_{11} = 0.52 \quad a_{22} = 0.52$

$$\left.\begin{matrix} a_{12} = 0.0 \\ a_{21} = 0.0 \end{matrix}\right\} \text{Mean Spillover Parameters}$$

$$\varepsilon_{it} = \xi_{it} (h_{it}^0)^{1/2}$$

1. $\xi_{it} \sim NIID(0,1)$
2. $\xi_{it} \sim t_8(0,1)$
3. $\xi_{it} \sim Skewed\ t_8$, 1 degree of Positive Skewness
4. $\xi_{it} \sim Skewed\ t_8$, 1 degree of Negative Skewness

$$h_{it}^0 = \omega_i^0 + \alpha_i^0 \varepsilon_{it-1}^2 + \beta_i^0 h_{it-1}^0 + \delta_{ij} \varepsilon_{jt-g}^2 + \gamma_{ij} h_{jt-g}^0, \quad g > 0,\ i \neq j, \quad i,j = 1,2$$

$$\left.\begin{array}{l}\omega_i^0 = 0.03 \\ \alpha_i^0 = 0.15 \\ \beta_i^0 = 0.5\end{array}\right\} H_0 \ \& \ H_1$$

$$\left.\begin{array}{l}\delta_{ij(H_0)} = 0.0 \ , \quad \delta_{ij(H_1)} = 0.1 \\ \gamma_{ij(H_0)} = 0.0 \ , \quad \gamma_{ij(H_1)} = 0.2\end{array}\right\} \text{Volatility Transmission Parameters}$$

In the following tables we provide the Monte Carlo simulation estimations of the empirical size and empirical power of the various tests. Details regarding these outputs are provided in the upper and lower parts of each table. The numbers beside the names of the tests represent the lag lengths (or bandwidths when considering the Hong class of tests)

TABLE 1.1

Empirical Size, Causality in Variance Tests, N(0,1), Student's t8(0,1), Positively Skewed Student's t8, Negatively Skewed Student's t8, Sample Size: 200 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **N(0,1)**	4,75	4,60	5,35	5,35	4,75	3,70	2,75	2,00
Modified	4,90	4,70	5,90	6,60	7,40	7,65	8,25	9,00
Normal	6,20	5,30	6,15	4,70	6,10	4,85	4,30	3,70
Bidirectional	5,90	5,50	5,85	5,20	3,85	2,65	1,65	0,95
Standard, **t(0,1)**	4,10	4,95	6,70	6,85	7,80	7,10	5,70	5,20
Modified	4,30	5,25	7,35	7,85	10,35	11,20	11,35	12,45
Normal	6,15	6,10	5,70	5,40	5,80	4,55	4,70	4,75
Bidirectional	6,45	6,65	8,55	7,95	7,05	5,80	4,45	2,65
Standard, **(+) Sk.t**	4,90	6,30	10,35	12,85	14,35	14,50	13,20	12,15
Modified	4,95	6,60	10,55	13,90	16,45	18,45	19,85	20,45
Normal	6,35	5,70	6,70	5,95	5,50	5,35	4,55	4,40
Bidirectional	7,85	9,15	12,40	13,85	15,55	13,60	10,70	7,90
Standard, **(-) Sk. t**	4,90	7,00	8,60	12,30	14,30	14,70	13,55	12,45
Modified	4,95	7,15	9,10	12,90	16,50	18,70	20,15	20,75
Normal	6,55	7,00	5,55	5,65	4,60	5,55	4,75	3,85
Bidirectional	6,70	9,30	12,15	14,90	15,35	14,20	11,35	8,60

Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	6,70	6,55	7,30	7,65	8,05	8,25	8,85	9,05
Bartlett	-	6,70	6,60	6,75	7,00	7,15	7,50	7,75
Daniel	-	6,80	6,50	7,20	7,55	7,90	7,65	8,20
Quadratic	6,65	6,75	6,75	7,05	7,60	7,65	8,10	8,75
Tukey	-	6,70	6,60	7,10	7,20	7,45	7,80	7,45
Truncated, **t(0,1)**	5,95	6,30	8,60	8,90	11,10	12,05	11,95	12,90
Bartlett	-	5,95	6,50	8,55	8,85	9,15	10,00	10,20
Daniel	-	6,00	6,90	8,65	8,85	9,85	10,55	10,90
Quadratic	5,95	6,15	7,20	9,00	9,40	10,60	10,95	11,50
Tukey	-	5,95	6,50	8,45	8,85	9,35	10,00	10,60
Truncated, **(+) Sk. t**	5,45	7,65	11,65	15,10	17,40	18,80	20,70	20,55
Bartlett	-	5,45	8,50	11,05	13,20	15,45	16,50	17,10
Daniel	-	6,00	8,65	11,15	14,15	16,65	17,15	18,05
Quadratic	5,45	6,10	9,85	12,10	15,95	17,05	18,00	18,40
Tukey	-	5,45	7,55	10,45	13,75	15,50	16,80	17,30
Truncated, **(-) Sk. t**	5,60	8,20	10,15	13,80	17,20	19,50	21,00	21,10
Bartlett	-	5,60	8,00	10,05	13,30	14,80	15,95	17,10
Daniel	-	5,95	8,30	10,40	13,90	16,00	16,90	18,35
Quadratic	5,70	6,10	9,30	11,55	15,05	16,95	18,45	19,05
Tukey	-	5,60	7,80	9,60	13,10	14,45	16,25	17,35

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	6,70	6,55	7,35	7,80	8,25	8,40	8,85	9,40
Bartlett	-	6,70	6,60	6,75	7,00	7,15	7,40	7,90
Daniel	-	6,80	6,45	7,20	7,55	7,85	7,75	8,40
Quadratic	6,65	6,75	6,75	7,10	7,60	7,55	8,40	8,85
Tukey	-	6,70	6,60	6,95	7,20	7,50	7,80	7,70
Truncated, **t(0,1)**	5,95	6,30	8,65	8,90	11,20	12,30	12,10	12,95
Bartlett	-	5,95	6,55	8,45	8,85	9,15	10,00	10,45
Daniel	-	6,00	6,90	8,65	8,90	9,90	10,75	10,85
Quadratic	5,95	6,15	7,30	9,00	9,45	10,65	11,25	11,80
Tukey	-	5,95	6,50	8,50	8,80	9,30	10,40	10,80
Truncated, **(+) Sk. t**	5,45	7,65	11,70	15,10	17,10	18,95	20,40	21,05
Bartlett	-	5,45	8,50	11,15	13,35	15,75	16,45	17,50
Daniel	-	6,00	8,70	11,15	14,10	16,70	17,60	18,20
Quadratic	5,45	6,10	9,90	12,05	16,05	17,25	18,05	18,70
Tukey	-	5,45	7,55	10,55	13,85	15,65	16,75	17,55
Truncated, **(-) Sk. t**	5,60	8,20	10,15	13,80	17,60	19,35	20,95	21,20
Bartlett	-	5,60	8,00	10,10	13,50	15,05	16,20	17,25
Daniel	-	5,95	8,30	10,50	14,05	16,15	17,20	18,25
Quadratic	5,70	6,15	9,30	11,60	15,10	17,15	18,65	19,10
Tukey	-	5,60	7,80	9,60	13,15	14,80	16,45	17,45

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	7,25	7,55	7,55	7,60	7,40	7,50	7,80	7,55
Bartlett	-	7,25	7,70	8,00	7,45	7,05	7,25	7,35
Daniel	-	7,25	7,70	7,90	7,40	7,00	7,00	7,40
Quadratic	7,15	7,30	7,80	7,45	7,25	7,15	7,70	7,75
Tukey	-	7,25	7,70	8,10	7,65	7,35	7,00	7,15
Truncated, **t(0,1)**	7,35	8,20	10,30	10,65	11,15	11,65	11,95	12,50
Bartlett	-	7,35	8,50	9,60	9,80	10,55	10,45	10,75
Daniel	-	7,35	8,50	9,90	10,25	10,65	10,55	11,15
Quadratic	7,40	7,45	9,15	10,05	10,65	10,65	11,00	11,20
Tukey	-	7,35	7,90	9,75	10,45	10,45	11,05	10,70
Truncated, **(+) Sk. t**	7,60	10,30	13,15	16,75	19,05	19,80	20,10	19,45
Bartlett	-	7,60	10,00	13,10	16,00	17,05	16,95	17,00
Daniel	-	7,70	10,45	13,50	16,45	16,45	17,05	17,85
Quadratic	7,55	7,85	11,40	14,75	16,80	17,25	17,75	17,75
Tukey	-	7,60	9,35	12,75	15,55	16,65	17,35	17,30
Truncated, **(-) Sk. t**	7,15	8,70	14,00	17,40	19,65	21,60	20,95	21,30
Bartlett	-	7,15	9,65	12,85	16,35	17,40	18,00	18,85
Daniel	-	7,55	9,90	12,90	16,90	18,10	18,45	19,35
Quadratic	7,25	7,60	11,25	14,55	17,80	18,55	20,00	20,25
Tukey	-	7,15	9,65	12,45	16,20	17,65	18,05	18,85

Lagrange Mult.	
N(0,1)	6,45
t(0,1)	7,10
(+) Sk. t	7,35
(-) Sk. t	7,50

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, Skewness Parameters: +1 / -1, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag.

TABLE 1.2

Empirical Size, Causality in Variance Tests, N(0,1), Student's t8(0,1), Positively Skewed Student's t8, Negatively Skewed Student's t8, Sample Size: 1000 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **N(0,1)**	4,20	5,50	4,70	4,70	4,10	4,05	3,80	3,80
Modified	4,20	5,55	4,70	4,85	4,45	4,85	4,95	5,15
Normal	5,25	6,00	4,50	5,50	5,65	5,45	5,35	5,00
Bidirectional	4,80	4,70	4,35	4,15	4,40	4,20	3,85	3,70
Standard, **t(0,1)**	5,50	6,30	7,35	8,15	9,10	9,20	8,75	9,10
Modified	5,55	6,30	7,40	8,35	9,60	9,70	10,10	10,90
Normal	6,80	6,40	6,65	6,35	5,50	6,50	5,75	5,85
Bidirectional	6,90	7,20	8,65	9,20	9,75	9,00	8,60	8,55
Standard, **(+) Sk. t**	4,80	6,40	9,00	11,40	12,55	14,10	14,55	15,00
Modified	4,80	6,50	9,05	11,50	13,10	14,70	15,90	16,35
Normal	6,30	5,55	6,45	5,00	5,20	5,55	6,00	6,00
Bidirectional	6,85	8,60	12,00	13,80	14,15	15,50	16,45	16,20
Standard, **(-) Sk. t**	4,05	5,60	8,95	10,20	13,10	13,40	14,55	14,05
Modified	4,10	5,65	8,95	10,50	13,65	13,90	15,50	15,75
Normal	6,55	5,40	6,30	6,55	5,70	5,90	5,45	5,80
Bidirectional	6,30	8,40	11,30	13,00	14,65	15,40	15,55	15,15

Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	5,85	7,00	6,40	6,05	5,75	5,80	5,95	5,80
Bartlett	-	5,85	6,45	6,55	5,90	6,10	5,85	5,55
Daniel	-	5,65	6,85	6,45	6,20	6,40	5,75	5,90
Quadratic	5,85	5,75	7,15	6,75	6,10	5,70	6,00	5,90
Tukey	-	5,85	6,20	6,65	6,30	6,00	5,95	5,65
Truncated, **t(0,1)**	6,70	7,45	8,70	9,55	10,95	10,45	10,95	11,20
Bartlett	-	6,70	8,15	8,55	9,55	10,05	10,15	10,25
Daniel	-	6,75	7,75	9,05	9,40	9,75	10,40	10,40
Quadratic	6,85	7,00	8,35	9,05	9,70	10,30	10,70	11,05
Tukey	-	6,70	7,70	8,30	8,95	9,75	9,95	10,35
Truncated, **(+) Sk. t**	5,60	7,30	10,40	12,60	14,10	15,60	16,40	17,05
Bartlett	-	5,60	7,85	10,05	12,05	13,55	14,15	14,85
Daniel	-	5,85	7,90	10,65	12,70	13,80	14,00	15,30
Quadratic	5,55	5,75	8,80	11,25	13,60	13,95	15,50	16,05
Tukey	-	5,60	7,20	9,90	11,75	13,35	14,10	14,20
Truncated, **(-) Sk. t**	5,30	6,55	9,85	11,60	14,40	14,80	16,50	16,55
Bartlett	-	5,30	6,95	8,80	11,45	12,60	13,75	14,20
Daniel	-	5,55	6,90	9,20	11,75	13,20	13,80	14,95
Quadratic	5,30	5,80	8,20	10,80	12,75	14,10	14,95	14,85
Tukey	-	5,30	6,70	8,55	11,45	12,45	13,50	14,15

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	5,85	7,00	6,40	6,05	5,85	5,75	6,00	5,75
Bartlett	-	5,85	6,45	6,55	5,80	6,10	5,90	5,55
Daniel	-	5,65	6,90	6,45	6,20	6,30	5,80	5,85
Quadratic	5,85	5,75	7,10	6,75	6,10	5,70	5,95	5,85
Tukey	-	5,85	6,20	6,65	6,30	6,00	5,90	5,70
Truncated, **t(0,1)**	6,70	7,45	8,70	9,60	10,95	10,40	10,95	11,40
Bartlett	-	6,70	8,15	8,60	9,55	10,00	10,15	10,25
Daniel	-	6,75	7,75	9,05	9,45	9,75	10,40	10,45
Quadratic	6,85	7,00	8,30	9,05	9,70	10,30	10,65	11,05
Tukey	-	6,70	7,70	8,30	8,95	9,75	9,95	10,35
Truncated, **(+) Sk. t**	5,60	7,30	10,40	12,65	14,10	15,55	16,40	16,95
Bartlett	-	5,60	7,85	10,05	12,10	13,55	14,10	14,85
Daniel	-	5,85	7,90	10,60	12,80	13,75	14,05	15,40
Quadratic	5,55	5,75	8,80	11,30	13,55	13,95	15,55	16,15
Tukey	-	5,60	7,20	9,90	11,80	13,35	14,15	14,20
Truncated, **(-) Sk. t**	5,30	6,55	9,85	11,60	14,45	14,75	16,65	16,65
Bartlett	-	5,30	6,95	8,80	11,45	12,60	13,80	14,20
Daniel	-	5,55	6,95	9,20	11,80	13,20	13,85	14,85
Quadratic	5,30	5,80	8,20	10,80	12,70	14,15	14,85	14,95
Tukey	-	5,30	6,70	8,55	11,45	12,50	13,50	14,15

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	6,70	6,60	6,60	5,90	6,35	6,05	5,45	6,40
Bartlett	-	6,70	6,50	6,30	5,85	5,25	5,80	6,45
Daniel	-	6,55	6,95	6,25	5,90	5,30	6,00	6,00
Quadratic	6,45	6,25	7,00	6,10	5,05	6,00	5,80	5,85
Tukey	-	6,70	6,40	6,15	5,85	5,10	5,85	5,90
Truncated, **t(0,1)**	7,75	8,35	9,65	10,80	11,20	11,00	11,55	11,30
Bartlett	-	7,75	8,30	9,20	10,60	10,90	11,35	11,40
Daniel	-	7,90	8,25	9,55	11,05	11,25	11,50	11,65
Quadratic	7,85	7,95	8,85	10,30	11,25	11,50	11,70	11,65
Tukey	-	7,75	8,20	9,35	10,80	11,20	11,20	11,25
Truncated, **(+) Sk. t**	5,75	9,80	12,15	14,95	15,80	16,95	18,65	18,75
Bartlett	-	5,75	9,05	11,75	14,55	15,30	15,85	16,60
Daniel	-	6,15	9,65	12,30	15,25	15,85	16,05	17,65
Quadratic	5,95	6,45	11,10	13,70	15,60	16,25	17,65	18,25
Tukey	-	5,75	8,45	11,35	14,85	15,50	15,75	16,05
Truncated, **(-) Sk. t**	5,70	8,45	11,40	13,75	16,85	17,65	17,50	18,30
Bartlett	-	5,70	8,40	11,60	13,10	14,45	15,60	16,20
Daniel	-	5,95	8,80	12,05	13,65	15,15	16,25	16,60
Quadratic	5,75	6,10	10,35	12,50	14,70	16,25	16,55	16,60
Tukey	-	5,70	8,20	11,15	13,25	14,40	15,60	16,30

Table 1.2 (Continued)

Lagrange Mult.	
N(0,1)	4,45
t(0,1)	6,05
(+) Sk. T	6,30
(-) Sk. T	5,65

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, Skewness Parameters: +1 / -1, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag.

TABLE 1.3

Empirical Power, Causality in Variance Tests, N(0,1), Student's t8(0,1), Positively Skewed Student's t8, Negatively Skewed Student's t8, Sample Size: 200 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard , **N(0,1)**	22,65	29,30	34,30	32,00	21,25	14,35	10,30	6,90
Modified	22,80	30,00	35,65	34,35	26,50	21,85	18,00	17,45
Normal	29,80	25,20	13,75	8,50	4,10	4,00	3,25	3,75
Bidirectional	29,60	41,05	50,80	45,85	29,70	18,25	10,60	6,15
Standard, **t(0,1)**	23,15	32,25	41,35	41,20	32,85	26,45	21,75	16,80
Modified	23,35	32,75	42,40	43,15	36,90	33,60	31,35	28,15
Normal	30,40	26,55	14,80	8,20	5,15	4,95	4,65	3,85
Bidirectional	34,85	49,40	62,85	62,20	47,55	35,30	27,10	18,70
Standard, **(+) Sk. t**	20,45	28,60	34,20	35,10	31,10	25,80	23,10	19,60
Modified	20,65	28,80	34,90	36,60	34,30	30,65	29,35	28,55
Normal	24,80	20,85	11,00	6,80	4,70	4,65	4,45	3,55
Bidirectional	29,40	41,85	51,95	49,60	40,20	32,95	25,10	19,20
Standard, **(-) Sk. t**	20,75	27,20	34,55	34,40	29,50	26,60	22,40	20,05
Modified	20,85	27,55	35,60	35,20	32,30	31,00	30,30	28,70
Normal	25,95	19,30	11,00	6,85	5,10	4,65	4,25	3,65
Bidirectional	30,60	40,20	49,05	49,35	41,25	32,60	24,80	17,70

Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	26,10	33,85	39,10	37,55	29,30	24,25	20,80	19,85
Bartlett	-	26,10	34,65	39,85	41,55	39,35	37,05	35,35
Daniel	-	26,75	35,50	40,30	41,00	37,10	33,85	31,15
Quadratic	26,85	28,45	37,90	41,00	38,70	34,30	30,60	28,15
Tukey	-	26,10	33,15	39,10	41,90	39,10	36,05	33,60
Truncated, **t(0,1)**	26,55	35,95	45,45	46,50	39,65	36,40	34,30	31,70
Bartlett	-	26,55	39,10	46,50	49,20	47,85	46,70	45,75
Daniel	-	28,35	40,15	47,15	48,65	46,70	43,85	41,80
Quadratic	27,80	30,00	44,00	48,70	47,90	44,60	41,45	39,70
Tukey	-	26,55	36,90	45,75	49,10	48,20	46,20	43,70
Truncated, **(+) Sk. t**	22,40	31,50	37,65	38,55	35,70	32,85	31,20	30,15
Bartlett	-	22,40	34,05	38,85	41,60	40,55	39,75	39,05
Daniel	-	24,65	35,00	39,05	40,95	39,50	38,35	36,90
Quadratic	23,50	25,35	37,40	40,70	39,85	38,40	36,90	34,90
Tukey	-	22,40	32,65	38,30	41,25	40,25	39,20	37,90
Truncated, **(-) Sk. t**	23,05	30,20	38,00	37,25	34,30	32,20	32,35	31,05
Bartlett	-	23,05	33,45	39,70	40,40	40,25	39,30	38,70
Daniel	-	24,90	34,00	40,40	39,95	38,25	36,95	36,15
Quadratic	24,50	25,60	36,35	41,35	39,40	37,70	36,10	34,90
Tukey	-	23,05	31,35	38,80	40,85	39,50	38,05	37,40

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	26,10	33,85	39,10	37,40	28,50	22,95	19,30	18,20
Bartlett	-	26,10	34,65	39,90	41,60	39,20	36,45	34,30
Daniel	-	26,75	35,55	40,25	40,90	36,50	33,20	29,65
Quadratic	26,90	28,45	37,85	41,10	38,50	33,55	29,80	26,25
Tukey	-	26,10	33,15	39,30	41,95	38,70	35,60	32,85
Truncated, **t(0,1)**	26,55	35,95	45,50	46,45	38,60	35,00	32,05	28,85
Bartlett	-	26,55	39,10	46,45	49,30	48,10	46,65	45,00
Daniel	-	28,45	40,25	47,20	48,50	46,35	43,00	40,90
Quadratic	27,80	30,00	43,95	48,75	47,60	43,60	40,70	38,40
Tukey	-	26,55	36,90	45,85	48,95	48,00	45,65	43,30
Truncated, **(+) Sk. t**	22,40	31,50	37,60	38,25	35,35	32,00	29,95	28,85
Bartlett	-	22,40	34,00	38,85	41,40	40,30	39,75	38,55
Daniel	-	24,65	34,95	39,20	40,85	39,55	37,70	36,30
Quadratic	23,50	25,35	37,50	40,75	39,70	38,00	36,20	33,80
Tukey	-	22,40	32,70	38,30	41,10	40,15	38,85	37,40
Truncated, **(-) Sk. t**	23,05	30,15	38,00	37,05	33,90	31,70	31,10	29,40
Bartlett	-	23,05	33,50	39,70	40,40	40,25	38,90	38,25
Daniel	-	24,85	34,05	40,35	39,85	37,90	36,40	35,10
Quadratic	24,50	25,60	36,30	41,40	39,15	37,20	35,30	34,55
Tukey	-	23,05	31,35	38,75	40,90	39,40	37,95	36,95

Bidirect. Hong	**1**	**2**	**5**	**10**	**20**	**30**	**40**	**50**
Truncated, **N(0,1)**	22,85	41,15	55,20	54,05	41,50	33,20	27,65	25,50
Bartlett	-	22,85	43,30	56,05	59,95	56,85	53,10	49,35
Daniel	-	24,65	45,60	57,05	58,80	53,40	48,70	43,45
Quadratic	23,95	26,40	51,10	59,55	55,85	49,00	42,00	37,55
Tukey	-	22,85	40,15	54,75	59,85	56,65	52,10	48,15
Truncated, **t(0,1)**	26,55	46,75	66,80	69,15	56,95	49,25	44,90	41,55
Bartlett	-	26,55	50,35	66,15	72,00	70,40	67,20	64,55
Daniel	-	29,50	52,45	68,65	71,75	67,90	63,80	59,10
Quadratic	28,65	31,40	59,85	71,70	69,45	64,20	58,70	54,80
Tukey	-	26,55	46,70	65,45	72,25	70,05	67,10	63,25
Truncated, **(+) Sk. t**	20,45	38,45	54,40	54,30	47,90	43,60	40,05	36,60
Bartlett	-	20,45	41,95	55,70	58,75	57,05	55,25	53,15
Daniel	-	23,40	44,45	56,85	57,80	55,25	52,20	49,60
Quadratic	22,10	26,10	50,60	57,80	56,70	52,60	49,70	46,70
Tukey	-	20,45	38,00	55,55	58,35	57,15	54,95	52,25
Truncated, **(-) Sk. t**	21,70	39,30	51,60	54,25	48,85	44,05	39,70	37,35
Bartlett	-	21,70	42,30	53,00	56,70	56,40	54,70	52,75
Daniel	-	24,50	43,60	53,85	56,75	55,00	51,60	49,30
Quadratic	23,25	27,35	48,95	55,95	55,80	52,20	49,20	47,40
Tukey	-	21,70	39,40	52,10	57,05	56,15	54,60	51,85

Lagrange Mult.	
N(0,1)	44,70
t(0,1)	37,70
(+) Sk. T	34,85
(-) Sk. T	34,95

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, Skewness Parameters: +1 / -1, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag.

TABLE 1.4

Empirical Power, Causality in Variance Tests, N(0,1), Student's t8(0,1), Positively Skewed Student's t8, Negatively Skewed Student's t8, Sample Size: 1000 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **N(0,1)**	63,30	82,35	92,10	91,10	82,85	74,90	66,45	59,85
Modified	63,30	82,45	92,10	91,25	83,75	76,15	68,90	64,20
Normal	72,95	69,15	32,10	11,00	5,90	4,80	4,40	4,30
Bidirectional	80,60	95,85	99,80	99,70	98,65	95,60	90,05	84,45
Standard, **t(0,1)**	60,15	78,10	89,65	90,05	85,20	79,60	74,80	71,55
Modified	60,30	78,15	89,75	90,10	85,70	80,30	76,45	73,20
Normal	69,75	61,15	30,80	13,50	8,50	7,20	7,00	6,15
Bidirectional	78,15	94,00	98,70	98,90	98,40	96,60	94,40	91,70
Standard, **(+) Sk. t**	48,65	62,65	72,80	70,95	61,95	55,60	51,40	47,25
Modified	48,65	62,85	73,05	71,20	62,65	56,70	52,80	49,05
Normal	56,55	44,00	18,80	8,20	4,65	4,45	4,85	5,20
Bidirectional	65,65	86,40	95,25	94,95	86,50	80,10	73,20	67,80
Standard, **(-) Sk. t**	47,35	61,75	71,10	67,60	60,65	54,45	50,75	46,85
Modified	47,40	61,90	71,35	67,70	61,15	55,50	51,95	48,75
Normal	55,70	44,50	16,75	8,45	5,35	5,45	6,25	5,40
Bidirectional	66,15	85,55	93,95	93,05	85,35	78,90	72,85	67,00

Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	67,55	85,20	93,75	92,45	86,20	79,25	72,00	66,95
Bartlett	-	67,55	89,05	94,35	94,95	94,50	93,30	91,75
Daniel	-	71,95	90,50	94,65	94,50	92,70	90,60	88,50
Quadratic	71,00	75,70	93,15	94,95	93,55	91,15	87,80	84,70
Tukey	-	67,55	87,00	94,05	94,90	94,05	92,45	90,55
Truncated, **t(0,1)**	64,15	81,40	91,40	91,65	87,05	82,05	78,10	75,45
Bartlett	-	64,15	86,50	93,20	93,60	92,95	92,15	90,95
Daniel	-	70,00	88,05	93,25	93,40	92,15	90,30	88,60
Quadratic	69,20	72,75	90,90	93,35	92,55	90,65	88,10	86,45
Tukey	-	64,15	84,10	92,35	93,45	92,85	91,85	90,20
Truncated, **(+) Sk. t**	51,60	66,05	76,10	73,30	64,75	59,15	54,85	51,10
Bartlett	-	51,60	71,80	78,30	78,00	76,55	74,70	71,65
Daniel	-	56,10	72,70	78,05	76,55	74,30	70,30	67,85
Quadratic	55,55	59,80	75,75	78,45	75,55	70,50	67,20	63,50
Tukey	-	51,60	69,25	77,95	77,75	76,15	73,05	70,00
Truncated, **(-) Sk. t**	50,60	65,60	73,75	70,30	63,05	57,55	53,50	50,90
Bartlett	-	50,60	70,05	76,90	76,65	74,15	72,15	70,50
Daniel	-	54,80	72,00	76,80	74,50	71,85	68,70	66,05
Quadratic	54,85	59,00	75,30	76,70	72,65	69,15	65,55	62,45
Tukey	-	50,60	67,95	76,35	76,30	73,40	71,15	68,75

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	67,55	85,20	93,75	92,40	85,95	78,90	71,25	66,15
Bartlett	-	67,55	89,05	94,35	94,95	94,40	93,15	91,75
Daniel	-	72,00	90,50	94,60	94,45	92,70	90,55	88,40
Quadratic	71,00	75,70	93,15	94,95	93,55	91,00	87,70	84,45
Tukey	-	67,55	87,00	94,05	94,95	94,05	92,40	90,30
Truncated, **t(0,1)**	64,15	81,35	91,40	91,65	87,05	81,95	77,65	74,65
Bartlett	-	64,15	86,50	93,20	93,55	92,95	92,15	90,95
Daniel	-	70,00	88,05	93,25	93,40	92,10	90,20	88,50
Quadratic	69,20	72,75	90,90	93,35	92,55	90,60	88,05	86,15
Tukey	-	64,15	84,15	92,45	93,45	92,75	91,90	90,20
Truncated, **(+) Sk. t**	51,60	66,10	76,00	73,25	64,60	59,10	54,20	50,30
Bartlett	-	51,60	71,80	78,30	78,00	76,50	74,50	71,60
Daniel	-	56,10	72,70	78,05	76,50	74,20	69,95	67,65
Quadratic	55,55	59,80	75,75	78,40	75,50	70,40	67,05	63,25
Tukey	-	51,60	69,25	77,90	77,75	76,05	72,70	70,00
Truncated, **(-) Sk. t**	50,60	65,60	73,75	70,20	62,90	57,25	53,35	50,40
Bartlett	-	50,60	70,05	76,90	76,65	74,10	72,10	70,25
Daniel	-	54,80	72,05	76,80	74,45	71,80	68,55	65,65
Quadratic	54,85	59,00	75,30	76,60	72,50	69,10	65,25	62,30
Tukey	-	50,60	67,95	76,35	76,30	73,35	71,00	68,60

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **N(0,1)**	56,75	92,30	99,75	99,90	99,25	97,20	93,50	89,15
Bartlett	-	56,75	96,60	99,85	99,95	99,95	99,85	99,80
Daniel	-	67,50	97,90	99,90	99,95	99,95	99,80	99,45
Quadratic	64,35	74,20	99,55	99,95	99,95	99,80	99,40	98,90
Tukey	-	56,75	94,10	99,85	99,95	99,95	99,80	99,80
Truncated, **t(0,1)**	56,70	90,10	98,75	98,90	98,70	97,05	95,70	94,15
Bartlett	-	56,70	95,35	99,15	99,30	99,25	99,15	99,20
Daniel	-	69,35	96,60	99,40	99,20	99,25	99,15	98,80
Quadratic	66,85	75,90	98,55	99,40	99,25	99,05	98,80	98,40
Tukey	-	56,70	92,55	99,00	99,30	99,25	99,15	99,10
Truncated, **(+)Sk.T**	44,05	79,95	96,05	96,05	88,90	82,75	77,35	72,15
Bartlett	-	44,05	88,20	96,45	97,50	96,45	95,35	94,25
Daniel	-	53,40	90,25	97,10	97,30	95,50	93,80	91,25
Quadratic	52,25	61,45	94,40	97,70	96,40	93,80	90,60	87,55
Tukey	-	44,05	84,45	96,20	97,65	96,60	95,10	93,40
Truncated, **(-) Sk. T**	45,30	80,30	94,50	94,35	87,15	81,25	76,80	71,30
Bartlett	-	45,30	88,15	96,45	96,95	95,85	93,80	91,90
Daniel	-	54,30	90,55	96,80	96,20	94,20	91,30	88,55
Quadratic	53,70	63,05	94,55	97,20	95,45	91,75	88,25	85,70
Tukey	-	45,30	83,75	96,00	96,90	95,65	93,25	91,10

Table 1.4 (Continued)

Lagrange Mult.	
N(0,1)	95,00
t(0,1)	47,10
(+) Sk. T	71,35
(-) Sk. T	70,90

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, Skewness Parameters: +1 / -1, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag.

4.2.2 EMPIRICAL RESULTS OF MONTE CARLO DESIGN 1

A) SMALL SAMPLE

1. The presence of excess kurtosis (the case when considering the Student's t distribution) causes an upward distortion in the empirical size of the vast majority of tests irrespectively of the kernel used or the direction of the considered. This effect is more pronounced from the 5^{th} lag and ownwards. The only test that seems to be robust against leptokurtosis in the unconditional distribution of the residuals process is the 'Normal' version of Cheung & Ng tests.

2. The Standard and Modified versions of the Cheung & Ng class of tests exhibit a different pattern in their empirical size performance. More specifically the first one demonstrates a decaying empirical size as the lag length increases. The latter however displays an inverse behavior with the type I error probability being an increasing function of the lag length.

3. In regard with the Hong class of tests we observe that the Truncated and Quadratic kernels are more heavily influenced from excess kurtosis, than the rest of the kernel functions used.

4. The presence of positive skewness, also affects the majority of the tests. More specifically an inflation in the empirical size of the various methodologies is observed

when we consider the Skewed Student's t distribution in which it must be noted that the degrees of freedom parameter is set equal to eight degrees of freedom (the same as in the case of the symmetric Student's t distribution) The impact of skewness can be more clearly observable if we compare the empirical sizes of the various tests under the presence of symmetric leptokurtosis with those under positively / negatively skewed leptokurtosis. The Normal version of the Cheung & Ng tests remains robust to this additional deviation from the Gaussian distribution, while the Truncated and Quadratic kernels are the most sensitive among the alternative kernels, to the presence of asymmetry in the residuals process. The Lagrange Multiplier test seems to be weakly influenced from the presence of asymmetry.

5. In the case of negative skewness the effects in the empirical size of the tests are more or less the same with the case of positive asymmetry. This similarity may be attributed to the fact that all the causality in variance test functions make use of squared standardized innovations in which the sign effect is eliminated. Thus it is possible that we would observe a different response in alternative directions of skewness, if we considered causality in mean tests as in these types of tests we use as inputs the standardized residuals instead of their squares

6. In terms of empirical power and considering the NIID (0,1) distribution we have the following 'rankings':

1st Lag: The Lagrange Multiplier test exhibits the best performance

10th Lag: The bidirectional Quadratic Hong test exhibits the best performance.

30th Lag: The bidirectional Bartlett Hong test exhibits the best performance.

Under the Student's t distribution we have the following 'rankings':

1st Lag: Lagrange Multiplier and the Bidirectional Cheung & Ng test.

10th Lag: The bidirectional Quadratic Hong test exhibits the best performance.

30th Lag: The bidirectional Bartlett Hong test exhibits the best performance.

Under the Positively / Negatively Skewed Student's t distribution we have the following 'rankings':

1st Lag: The Lagrange Multiplier test exhibits the best performance.

10th Lag: The bidirectional Quadratic Hong test exhibits the best performance.
30th Lag: The bidirectional Bartlett Hong test exhibits the best performance.

As you may have already noticed there is a clear superiority of the bidirectional tests under all the alternative conditional distributions. The reason for this is that in the data generating process and under the alternative hypothesis (when computing the power of the tests) we have set a bidirectional volatility structure for the series we create. In other words the performance of the tests is investigated under a volatility feedback situation. For a more in depth analysis of this argument we have performed an additional Monte Carlo experiment, the results of which are placed in the **Appendix D** (Design 6) of the dissertation where we compare the performance of unidirectional and bidirectional tests under either unidirectional or bidirectional setups for the volatility transmission mechanism. In that point the reader will discover that in the case that a unidirectional volatility spillover pattern is considered, the unidirectional tests dominate their bidirectional counterparts in terms of empirical power.

7. The empirical power of the Normal Cheung & Ng test is a negative function of the lag length. This is not surprising however as this type of test is the only technique that searches for variance causality in a specific lag and not in a group of lags as happens with the other methodologies. In our case the volatility spillover takes place in the first lag. Thus it is logical that after a few periods it will not be detectable in the pattern of the volatility process and thus it will not be discovereable when using the aforementioned type of test.

8. Given the small sample size, the excess kurtosis in the underlying residuals process has a positive augmenting impact in the empirical power of the Cheung & Ng and Hong tests while a negative impact is observed for the Lagrange Multiplier tests.

9. The presence of skewness (positive or negative) has a negative impact in the power of all the tests that we have used. These effects are more powerful in the regime between the 10th and 30th lags. For the Cheung & Ng test however we observe an inversion of the direction of these effects for large lags such as the 50th. For example, the asymmetry in the distribution augments the power of the Standard version of the Cheung & Ng test when the 50th lag is considered.

10. In the Cheung & Ng class of tests the empirical power attains its maximum value somewhere between the 5th and 10th lags given of course the fact that the volatility spillover takes place in the first lag.

11. In the Hong class of tests and under the NIID (0,1) distribution we observe a slight heterogeneity in the behavior of the alternative kernels. More specifically the empirical power is maximized in the 5th lag for the Truncated kernel, in the 10th lag for the Quadratic kernel and in the 20th lag for the rest of the kernels. For the leptokurtic and asymmetric distributions the situation is almost the same with the empirical power being maximized in the region between the 10th and 20th lags.

12. We also observe that the Hong tests irrespectively of the kernel function used, have a tendency to over reject the null hypothesis compared with their more simple Cheung & Ng counterparts. This finding is in agreement with that of **Hong** (2001)

B) LARGE SAMPLE

1. The differentiation in the behavior of the empirical sizes of the Standard and Modified versions of the Cheung & Ng tests that was clearly observable in the case of the small sample size is now less obvious but still present.

2. The effects of excess kurtosis in the empirical size of the tests appear to be more pronounced for the large sample. As previously mentioned we observe an upward distortion in the type I error probability of the tests that is intensified when longer lags are considered.

3. The presence of positive skewness causes an upward distortion in the empirical size of all the tests for lags larger than the 5th one. The Lagrange Multiplier test seems not to be seriously affected from the positive skewness in the distribution of the residuals process.

4. In the case of negative skewness the effects in the empirical size of the various causality in variance tests are much about the same with those obtained under positive

asymmetry. We observe however a decrease in the empirical size of the Lagrange Multiplier test.

5. We observe a serious negative impact in the empirical power of the Lagrange Multiplier test under the presence of excess kurtosis.

6. In terms of empirical power and considering the NIID (0,1) distribution we have the following 'rankings':

1st Lag: The Lagrange Multiplier test exhibits the best performance

10th Lag: The bidirectional Quadratic Hong test exhibits the best performance

30th Lag: The bidirectional Bartlett and Quadratic Hong test exhibit the best performance.

Under the Student's t distribution we have the following 'rankings':

1st Lag: The bidirectional Cheung & Ng test exhibits the best performance

10th Lag: The bidirectional Quadratic Hong test exhibits the best performance

30th Lag: The bidirectional Bartlett Hong test exhibits the best performance

Under the Positively / Negatively Skewed Student's t distribution we have the following 'rankings':

1st Lag: The Lagrange Multiplier test exhibits the best performance

10th Lag: The bidirectional Quadratic Hong test exhibits the best performance

30th Lag: The bidirectional Bartlett Hong test exhibits the best performance

In general a considerable increase in the power of all the tests regardless of the lag or the distribution used, is observed when using large sample sizes. This practically means that the tests will work better in empirical applications where a large volume of data is available. In other words these methodologies are better suited for use in the field of finance than in the field of macroeconomics as in the former there is more frequently availability of high frequency data.

7. The empirical power of the Normal version of the Cheung & Ng class of tests is once more a monotonically decreasing function of the lag length. This technique offers us an insight in the memory of the volatility process. We observe that this test

has lost a significant portion of its empirical power when lags longer than the 20^{th} are considered. This means that the effects of a volatility spillover that takes place in a specific point in time decay and eventually die out after a number of periods. The exact time that will be needed in order for the volatility spillover effects to be completely wiped out depends on a number of factors such as the intensity of the volatility transmission etc.

8. The presence of excess kurtosis has a mixed impact in the empirical power of the Hong class of tests. For example in the case of unidirectional Hong tests the peakness of the distribution decreases the empirical power of the tests for lags smaller than the 10^{th} but increases the empirical power from the 10^{th} lag and afterwards. The same effects are true in the case of the Cheung & Ng class of tests. The excess kurtosis has a negative impact in the power of the Lagrange Multiplier tests.

9. The presence of skewness (positive or negative) has a negative impact on the tests with the exception of the Lagrange Multiplier test for which it seems that asymmetry increases its power.

10. In the Cheung & Ng class of tests and for all the distributional assumptions the empirical power is maximized in the region between the 5^{th} and 10^{th} lags. For the Hong tests the observed performance reveals that the empirical power of the tests attains its maximum value between the 5^{th} and 20^{th} lags, with the exact point depending on the kernel used and the distribution that is considered.

11. In the large sample the differences between the Hong and its Modified version are so small that it does not matter which of the two, one would choose in order to test for volatility Spillovers.

12. The power of all the Causality in Variance tests, as it was naturally expected, decreases with the lag length regardless of the underlying distribution.

13. We can also mention the fast decay in the power of the Normal Cheung & Ng test. This must not generate considerations and ambiguities however as we have already mentioned that this test is the only one that scans for causality in a specific lag and not

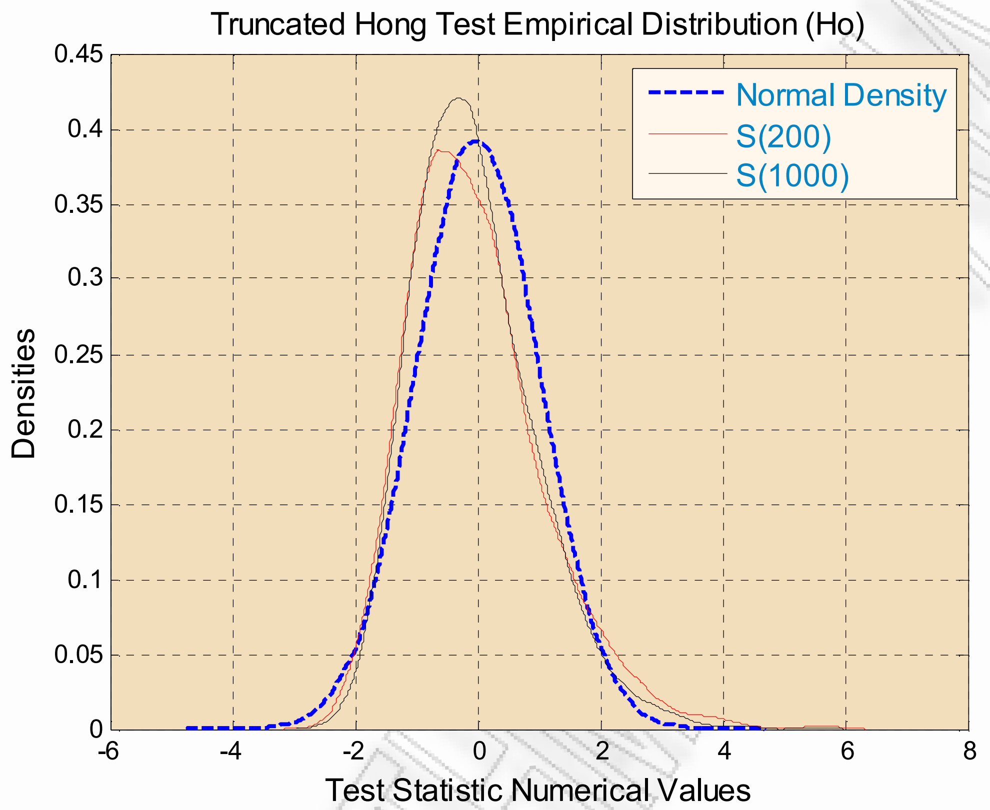
up to this lag. Thus it is natural to observe low values for the power of this test when considering large lag lengths as the data generating process of our Monte Carlo design involves a one lag volatility spillover. The memory of the causality with which we enrich the generated series is almost eliminated in lags such as the 30^{th} or 50^{th} and this is quantitatively depicted from the low power of the specific test when considering these lag lengths.

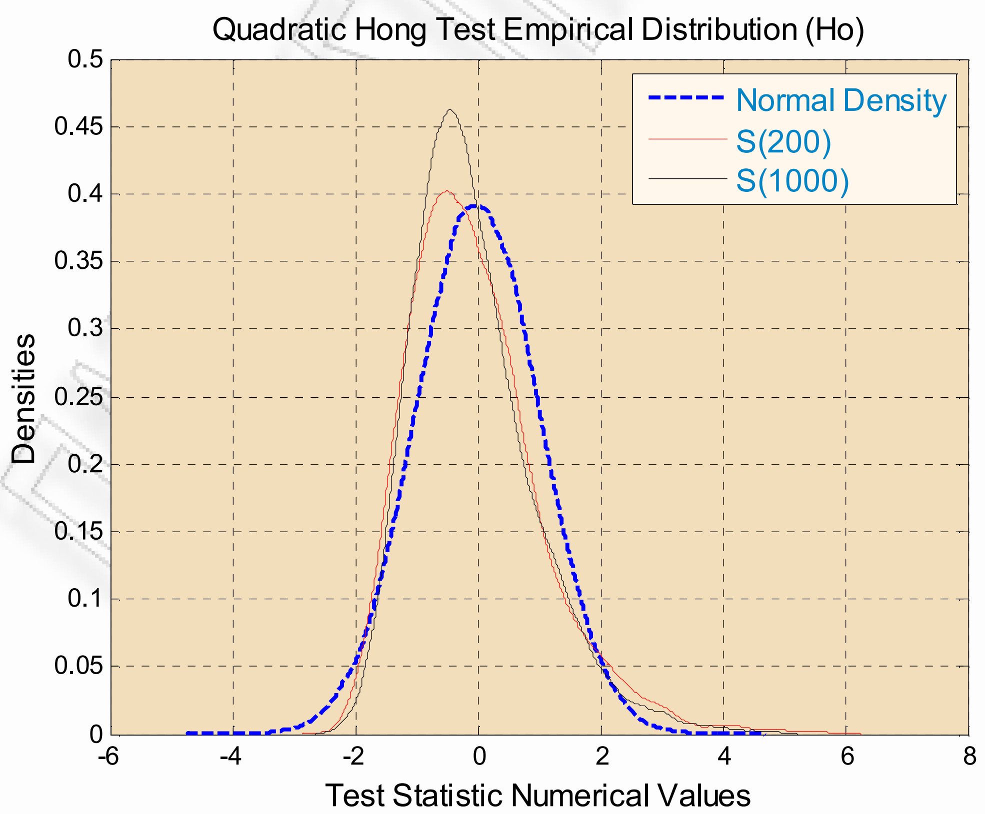
14. In the case of the large sample an important difference from the case of a small sample is observed that denotes the risk of spurious conclusions when studying only a specific sample size. In the occasion of a student's t distribution the impact of excess kurtosis on the power of both the Cheung & Ng and the Hong tests is negative. We can remind that in the case that a small sample was considered the impact was positive. The impact of skewness however remains negative and this ensures the validity of the claim that positive asymmetry in the distribution can dampen the causality detection capability of the cross correlation based causality in variance tests. The same conclusions apply also for the bidirectional version of these tests.

15. An interesting feature that we have observed is that when considering the Hong tests the Truncated kernel has a clear advantage over the alternative kernels when we consider lags up to the 5^{th} one. It is by far the most powerful in the 2^{nd} lag. This is true for both unidirectional and bidirectional tests. We have also discovered that the Tukey and Bartlett kernels are for the first initial lags considerably less powerful than their counterparts. In the 10^{th} lag and afterwards however this weakness disappears.

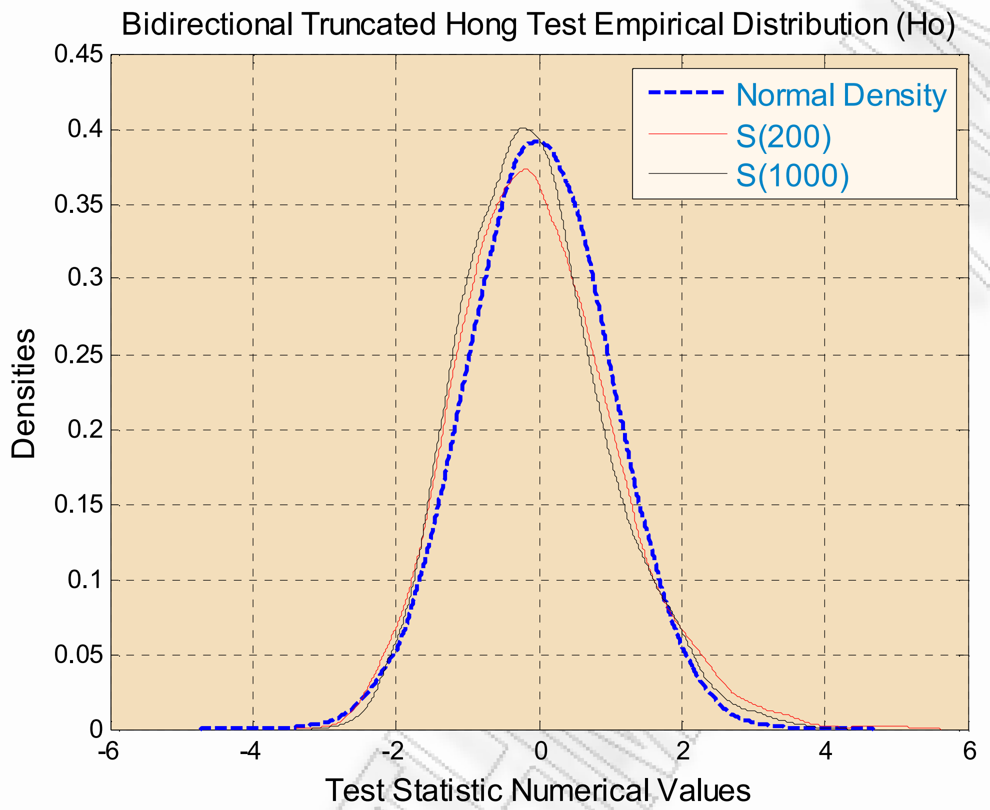
In the following figures we provide the smooth densities of the various causality in variance tests that were calculated by an Epanechnikov kernel using the test's realized values for the sample size of 600 obs. The results using this sample size were not included in the previous tables, as we wanted to depict the two limit cases of a small (200 obs.) and a large sample size (1000 obs.) This sample size however was chosen for the construction of the empirical distributions as it represents a mainstream situation between the two limits and allows us to briefly and concretely demonstrate our empirical results. In the **Appendix B** (Monte Carlo Design 1) you can find a number of additional figures that will help you to visualize the differences among the alternative tests as well as some other aspects of this Design.

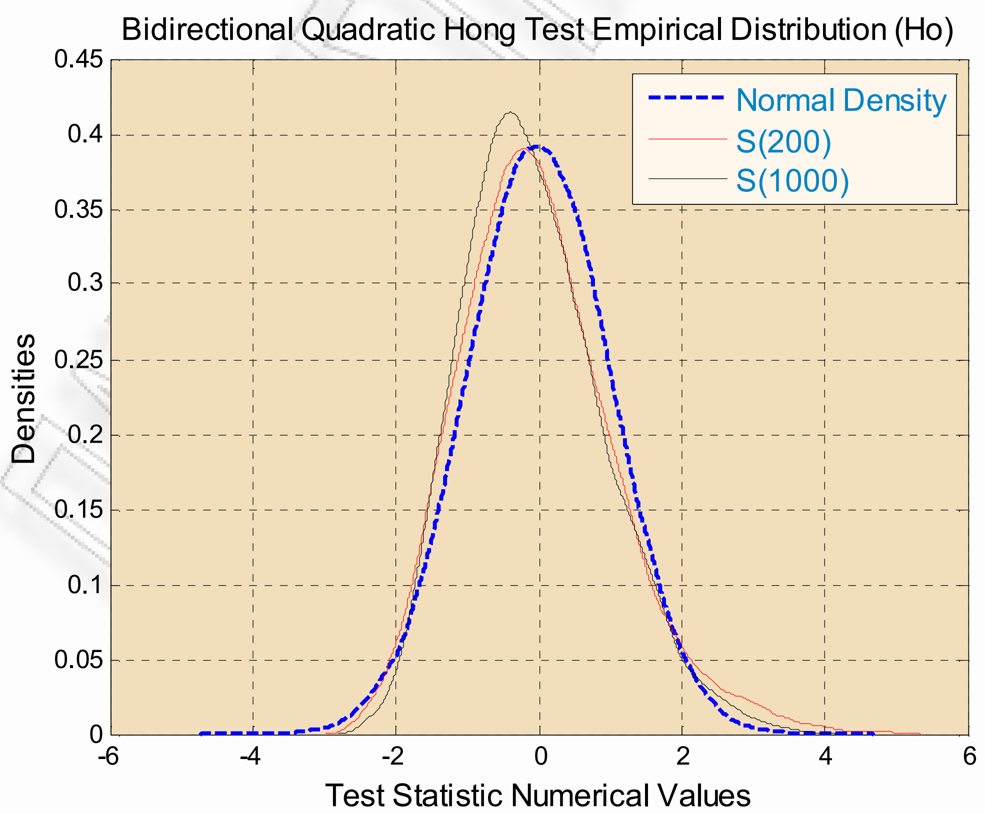
Figures 1.1 – 1.2 Unidirectional Hong Causality in Variance Tests using the Truncated & Quadratic Kernels (Bandwidth: 30), NIID(0,1) Residuals Process and two sample sizes.



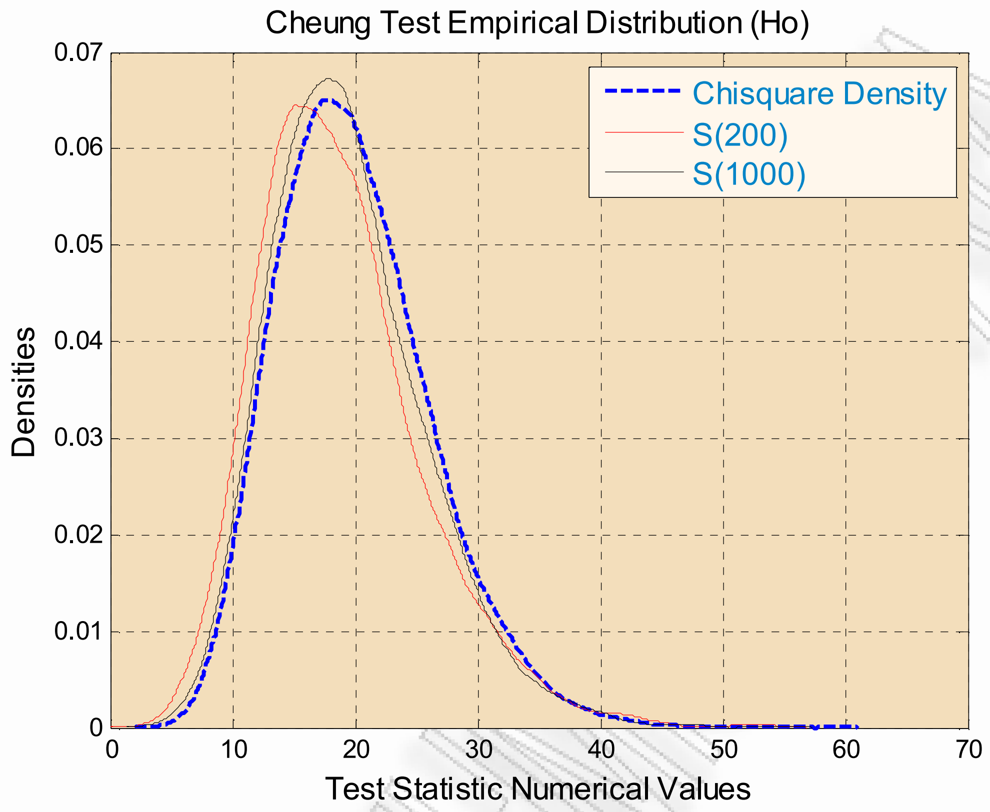


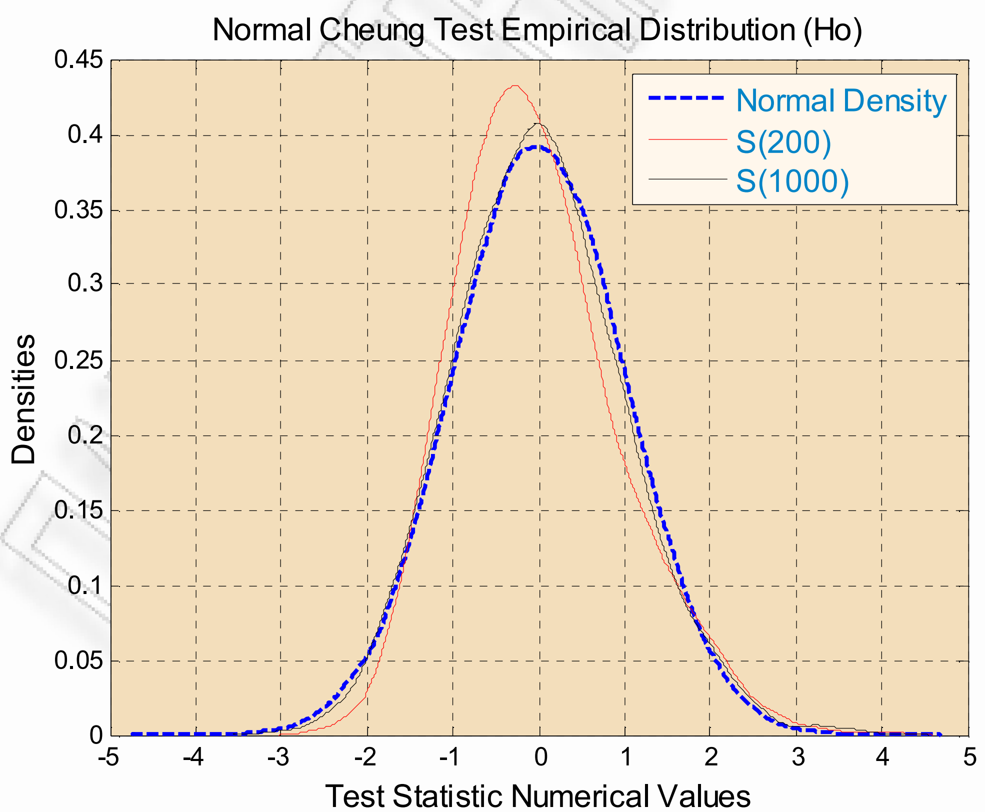
Figures 1.3 – 1.4 Bidirectional Hong Causality in Variance Tests using the Truncated & Quadratic Kernels (Bandwidth: 30), NIID(0,1) Residuals Process and two sample sizes.





Figures 1.5 – 1.7 Cheung & Ng Causality in Variance Tests (30th Lag), NIID(0,1) Residuals Process and two sample sizes.





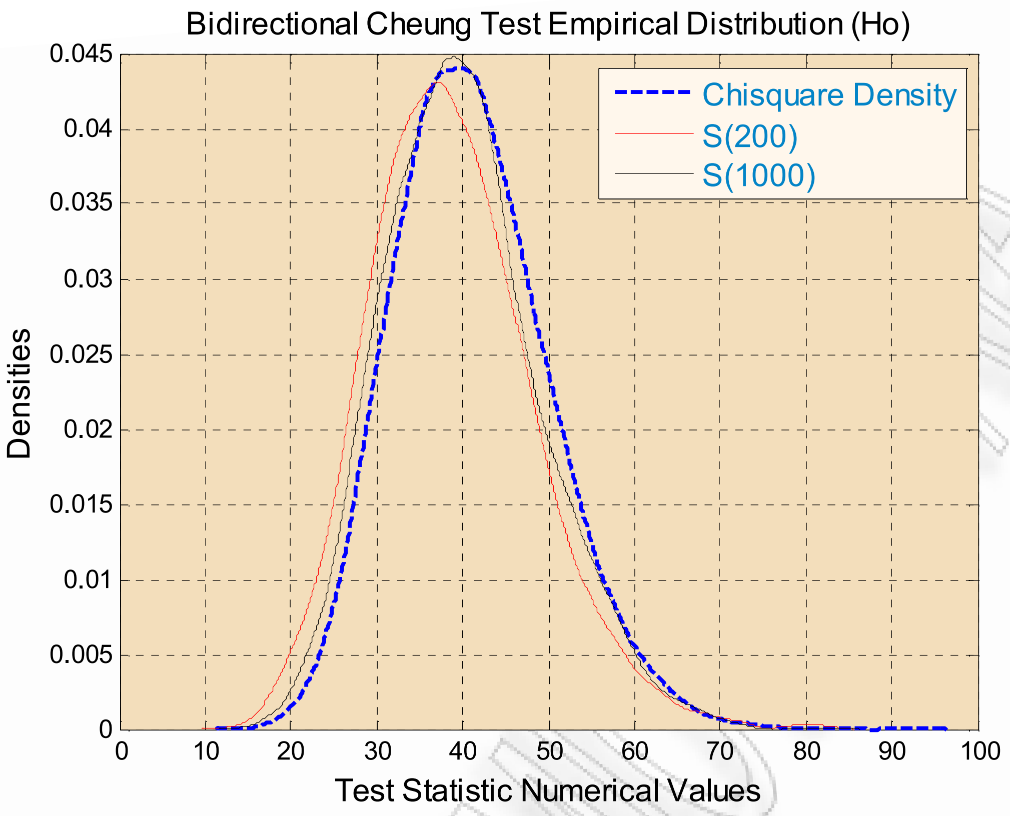
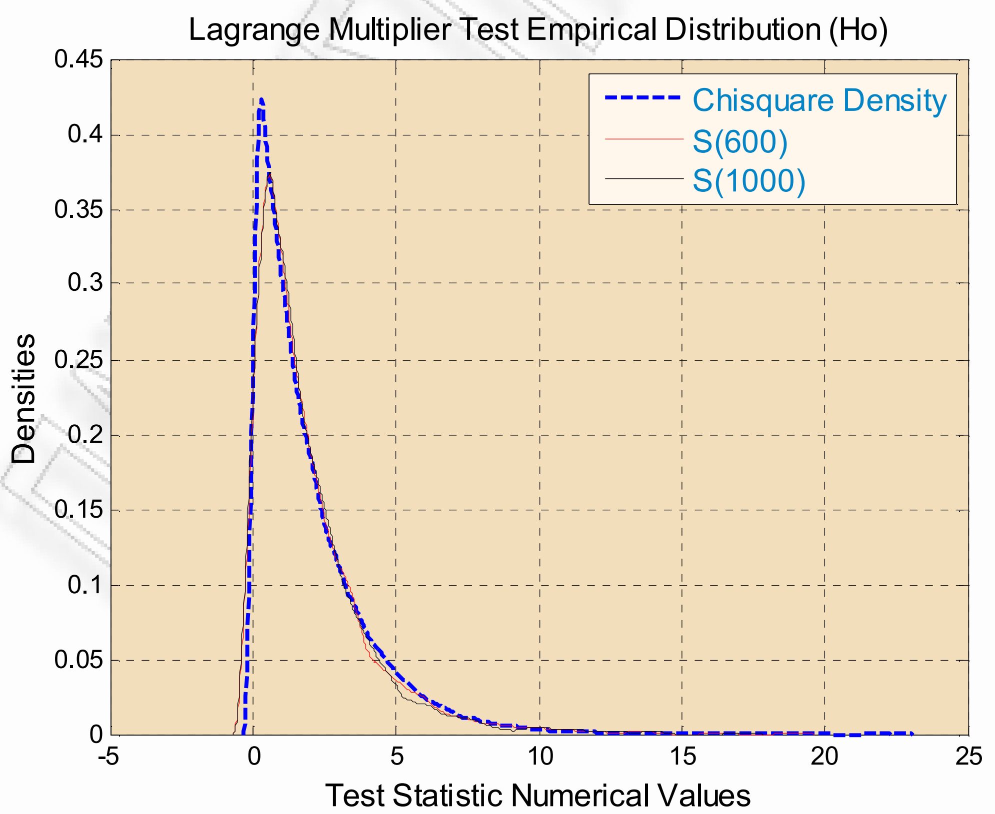
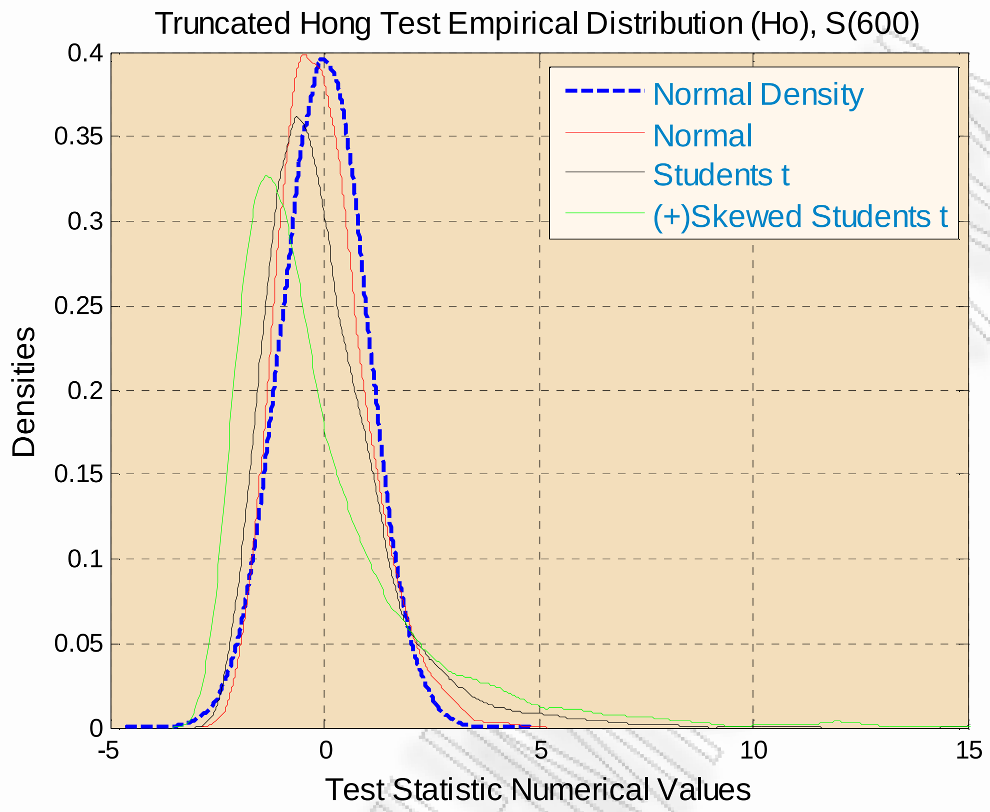
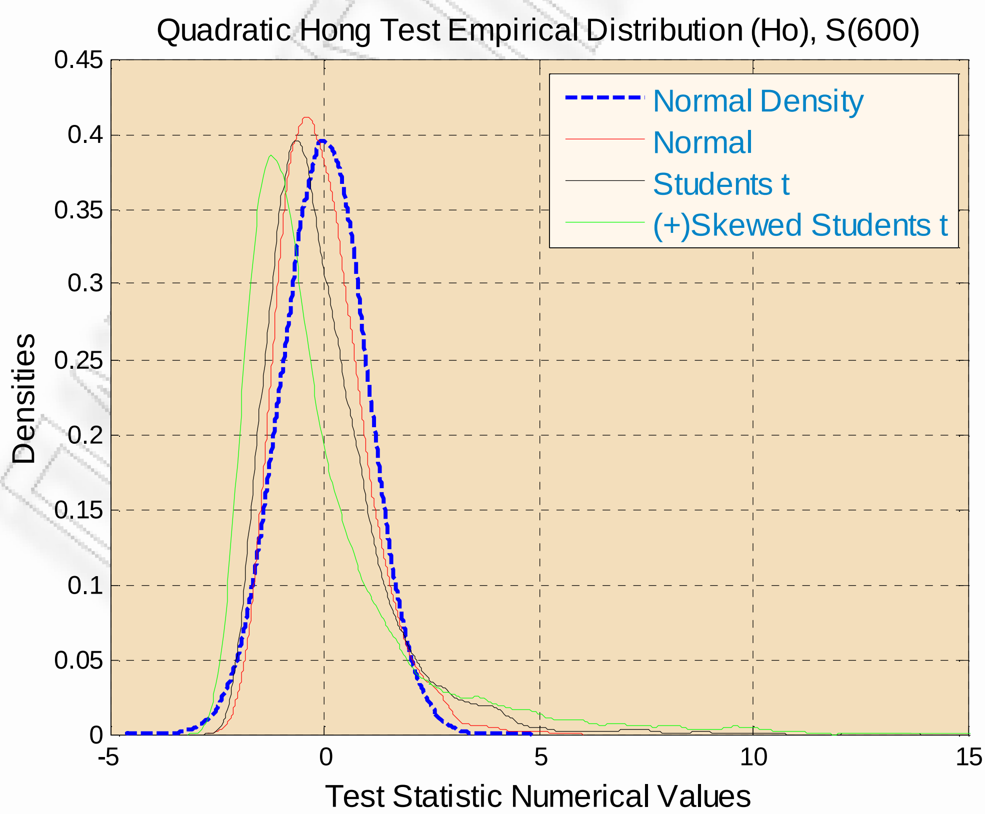


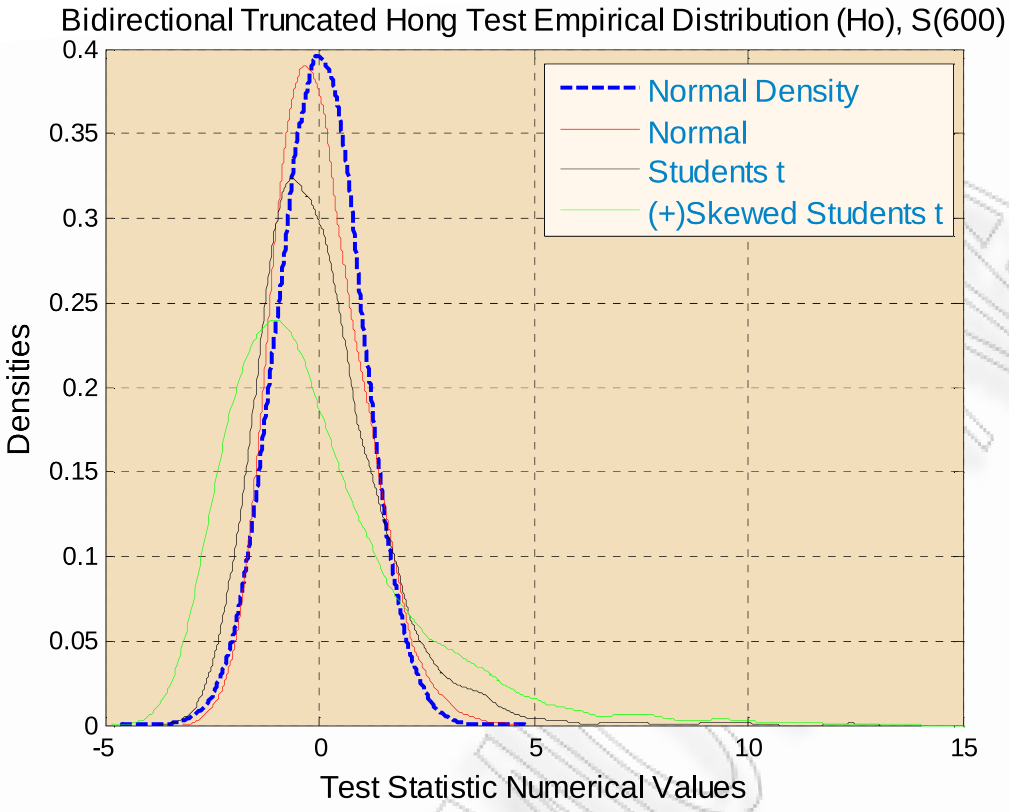
Figure 1.8 Lagrange Multiplier Causality in Variance Test (1st Lag), NIID(0,1) Residuals Process and two sample sizes.

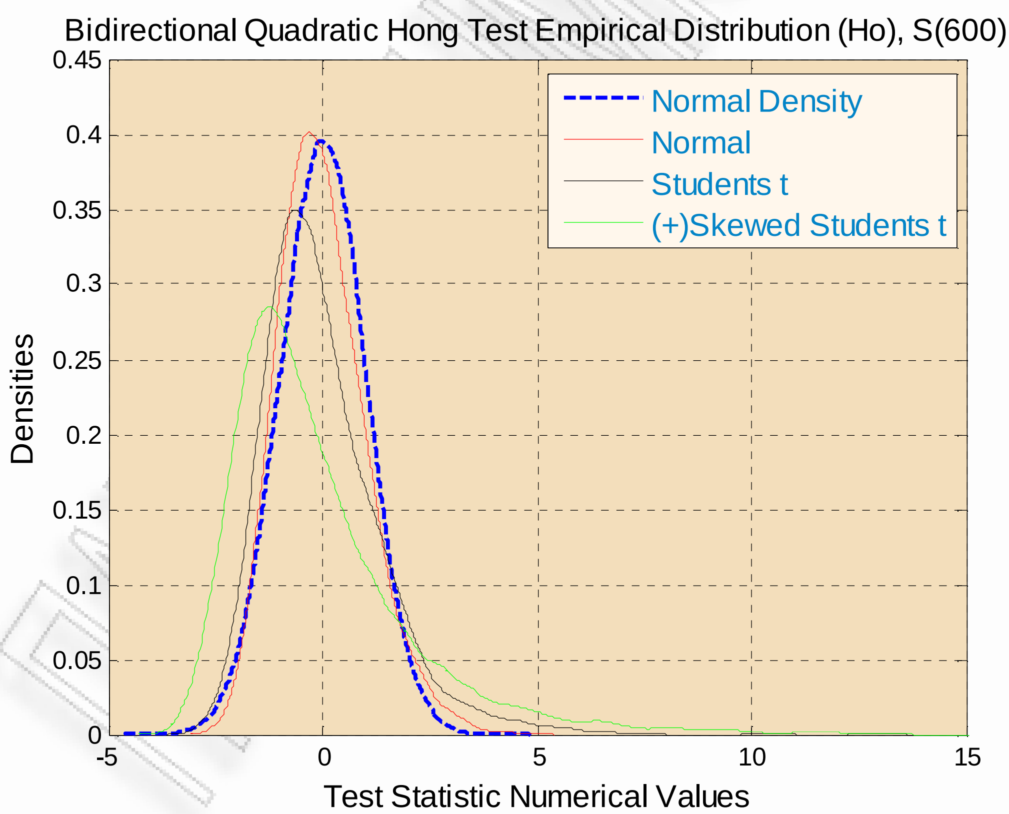


Figures 1.9 – 1.12 Unidirectional & Bidirectional Hong Causality in Variance Tests under alternative Hypotheses for the Residuals Processes (Bandwidth: 30)

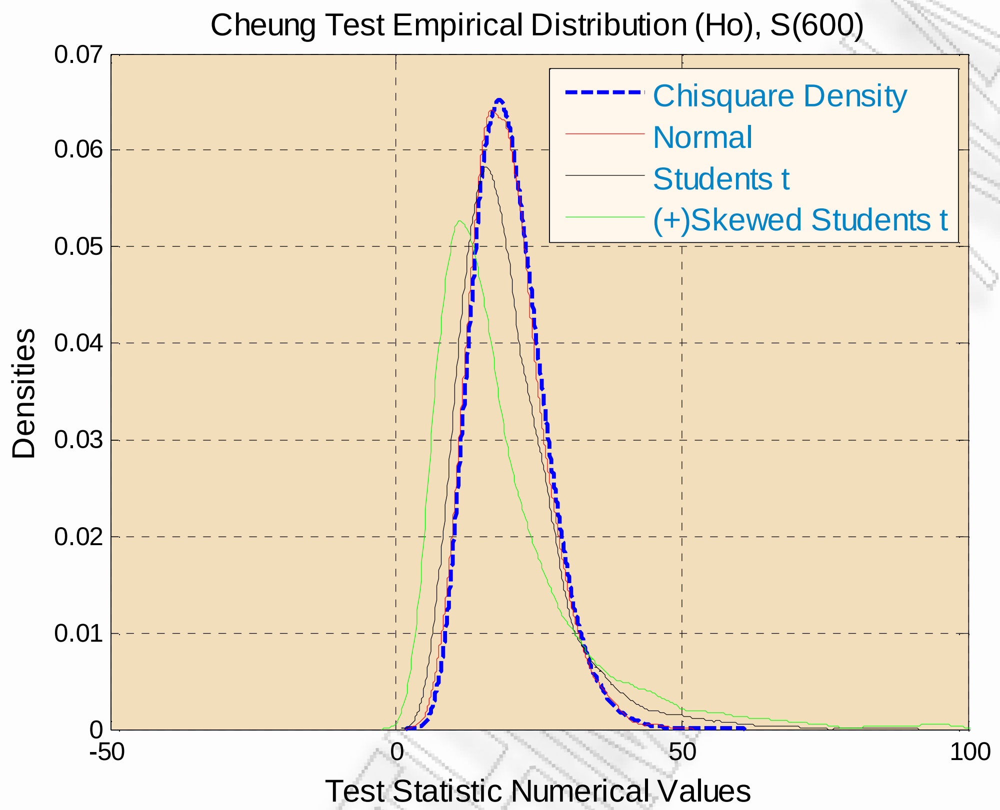


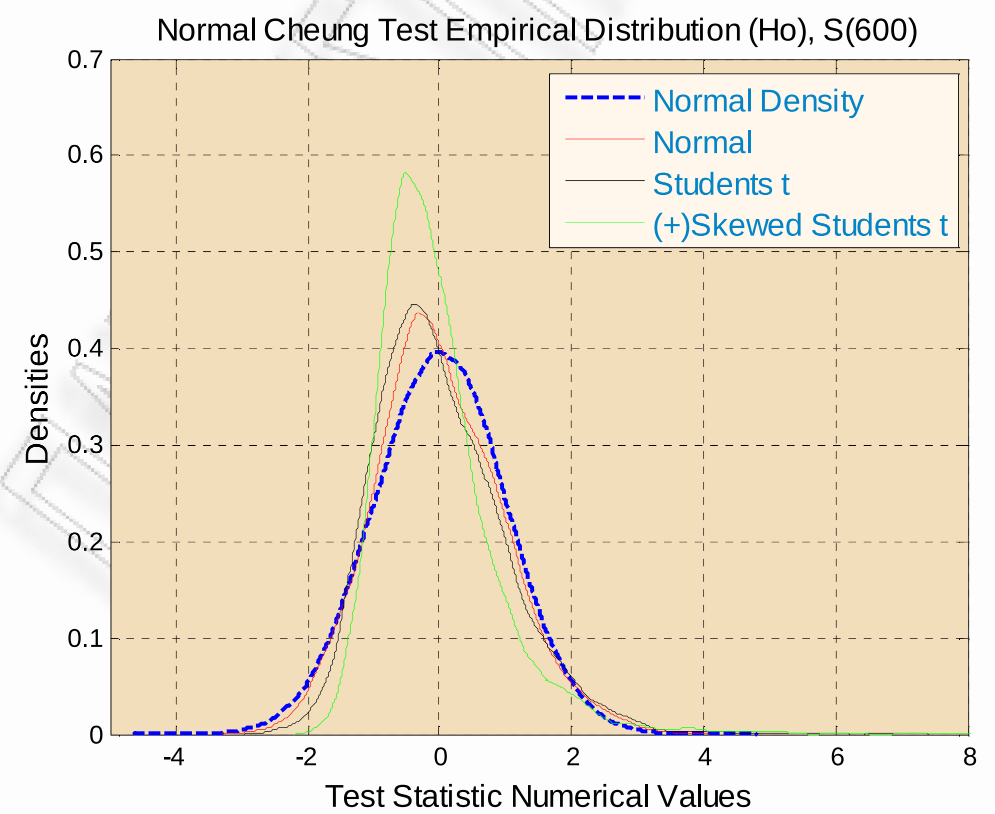



Bidirectional Truncated Hong Test Empirical Distribution (Ho), S(600)
Normal Density
Normal
Students t
(+)Skewed Students t
Densities
Test Statistic Numerical Values
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5
10
15


Bidirectional Quadratic Hong Test Empirical Distribution (Ho), S(600)
Normal Density
Normal
Students t
(+)Skewed Students t
Densities
Test Statistic Numerical Values
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5
10
15

Figures 1.13 – 1.15 Unidirectional & Bidirectional Cheung & Ng Causality in Variance Tests under alternative Hypotheses for the Residuals Processes (30th Lag)





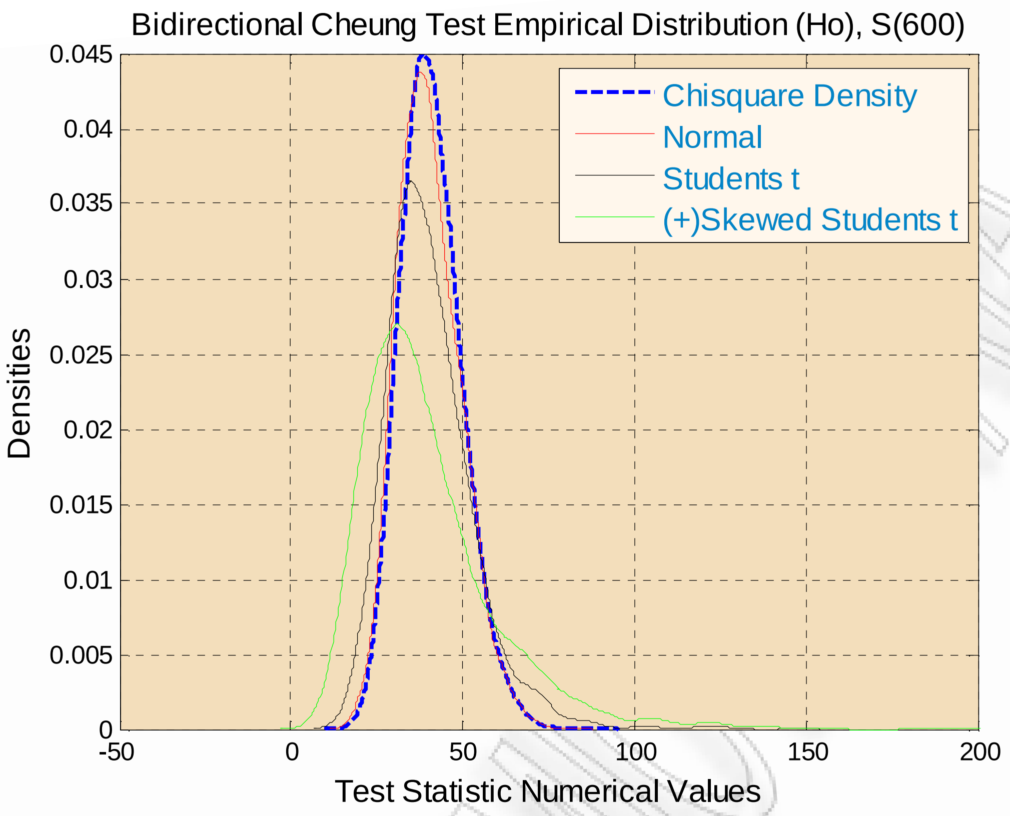
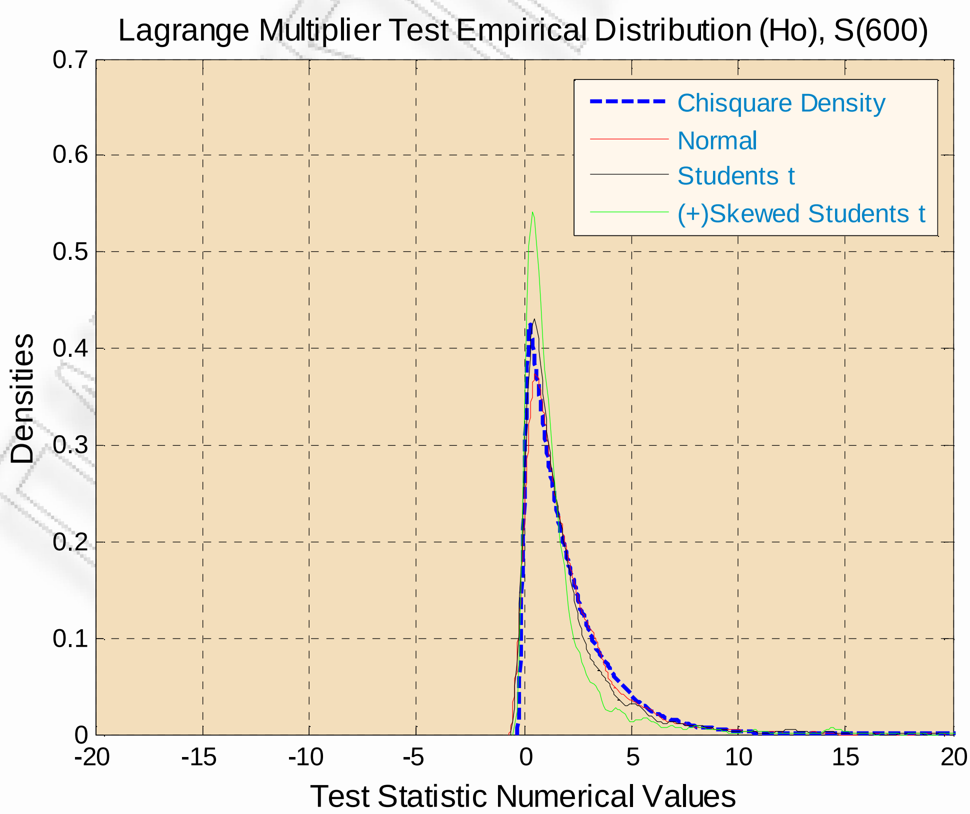


Figure 1.16 Lagrange Multiplier Causality in Variance Test under Alternative Hypotheses for the Residuals Process (1^{st} Lag)



4.3 MONTE CARLO SIMULATION DESIGN 2

In this study we perform an estimation of the empirical size of causality in variance tests under the alternative states of filtered causality in mean & not filtered causality in mean. The filter for removing mean spillovers is a first order VAR Model that coincides with the real specification of the conditional mean model of the Data Generating Process. The volatility and mean spillovers take place in the first lag.

4.3.1 DATA GENERATING PROCESS

$$Y_t = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$$w_1 = 0.1 \qquad w_2 = 0.1$$

$$a_{11} = 0.52 \qquad a_{22} = 0.52$$

$$\left.\begin{matrix} a_{12} = 0.3 \\ a_{21} = 0.3 \end{matrix}\right\} \text{Mean Spillover Parameters}$$

$$\varepsilon_{it} = \xi_{it} (h_{it}^0)^{1/2}$$
$$\xi_{it} \sim NIID(0,1)$$

$$h_{it}^0 = \omega_i^0 + \alpha_i^0 \varepsilon_{it-1}^2 + \beta_i^0 h_{it-1}^0 + \delta_{ij} \varepsilon_{jt-g}^2 + \gamma_{ij} h_{jt-g}^0, \quad g > 0,\ i \neq j, \quad i,j = 1,2$$

$$\left.\begin{matrix} \omega_i^0 = 0.03 \\ \alpha_i^0 = 0.15 \\ \beta_i^0 = 0.5 \end{matrix}\right\} H_0 \ \& \ H_1$$

$$\left.\begin{matrix} \delta_{ij(H_0)} = 0.0 \ , \quad \delta_{ij(H_1)} = 0.1 \\ \gamma_{ij(H_0)} = 0.0 \ , \quad \gamma_{ij(H_1)} = 0.2 \end{matrix}\right\} \text{Volatility Transmission Parameters}$$

In the following tables we provide the empirical results of this Monte Carlo design for both 5% and 10% nominal significance levels. More details can be found in the upper and lower parts of each table. The numbers beside the names of the tests represent the lag lengths (or bandwidths when considering the Hong class of tests)

TABLE 2.1

Empirical Size, Causality in Variance Tests, Filtering & Not Filtering Causality in Mean, Sample Size: 200 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **Filtered CM**	4,65	4,40	5,20	5,35	4,55	3,75	2,75	1,75
Modified	4,80	4,70	5,55	6,15	6,40	7,70	7,80	9,15
Normal	5,40	6,05	5,80	6,75	5,15	4,35	3,85	4,10
Bidirectional	4,50	4,55	5,05	5,40	3,95	2,30	1,35	0,55
Standard, **Not Filt. CM**	19,10	17,20	13,40	10,50	7,05	5,00	3,15	2,25
Modified	19,35	17,70	14,20	11,85	9,95	9,25	9,60	9,80
Normal	26,05	10,45	5,55	5,10	5,80	4,50	3,95	2,95
Bidirectional	23,85	22,50	18,20	11,70	6,70	4,60	2,65	1,60

Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	6,20	7,25	7,60	7,65	7,75	8,75	8,55	9,90
Bartlett	-	6,20	6,65	6,90	7,50	7,60	7,45	7,60
Daniel	-	6,20	6,90	7,10	7,60	7,55	7,85	7,75
Quadratic	6,10	6,00	7,05	7,05	8,00	7,80	7,80	8,20
Tukey	-	6,20	6,60	6,90	7,60	7,95	7,35	7,75
Truncated, **Not Filt. CM**	22,10	20,45	17,25	14,05	11,25	10,55	11,05	11,00
Bartlett	-	22,10	21,90	19,85	17,30	15,50	14,45	14,05
Daniel	-	22,20	21,50	19,05	15,35	14,00	12,90	12,10
Quadratic	22,20	22,20	21,00	17,70	14,70	12,95	12,45	12,05
Tukey	-	22,10	22,30	20,15	16,35	14,55	13,60	12,90

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	6,20	7,30	7,70	7,70	7,80	8,85	8,60	9,75
Bartlett	-	6,20	6,60	6,85	7,60	7,70	7,40	7,55
Daniel	-	6,15	6,95	7,30	7,55	7,55	7,75	7,95
Quadratic	6,10	6,00	7,10	7,00	7,80	7,75	7,70	8,35
Tukey	-	6,20	6,65	6,90	7,75	7,95	7,55	7,65
Truncated, **Not Filt. CM**	22,10	20,50	17,15	14,00	10,95	10,40	10,20	10,55
Bartlett	-	22,10	21,90	19,90	17,05	15,35	14,40	13,60
Daniel	-	22,20	21,40	19,00	15,30	13,85	12,75	12,20
Quadratic	22,20	22,25	21,00	17,65	14,50	12,60	12,40	11,45
Tukey	-	22,10	22,35	20,05	16,45	14,60	13,40	12,55

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	6,90	6,65	7,55	6,85	7,10	7,55	7,90	7,60
Bartlett	-	6,90	6,20	7,05	6,95	6,85	7,00	7,20
Daniel	-	6,60	6,10	7,25	6,90	7,05	7,05	7,05
Quadratic	6,95	6,70	6,15	7,00	7,10	7,05	7,05	7,25
Tukey	-	6,90	6,15	6,95	6,65	6,90	7,15	7,05
Truncated, **Not Filt. CM**	19,75	27,30	22,80	16,90	13,30	11,95	11,30	11,05
Bartlett	-	19,75	25,75	25,55	22,20	18,95	17,50	15,95
Daniel	-	19,90	26,30	25,00	19,60	16,95	14,45	13,55
Quadratic	20,55	21,70	26,35	23,15	17,50	14,65	13,60	12,80
Tukey	-	19,75	25,85	25,30	21,50	18,00	16,05	14,35

Lagrange Mult.	
Filtered CM	6,85
Not Filt. CM	20,75

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. Mean Spillover in the first Lag, The Filter is a VAR(1) Model

TABLE 2.2

Empirical Size, Causality in Variance Tests, Filtering & Not Filtering Causality in Mean, Sample Size: 1000 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **Filtered CM**	5,00	5,00	4,80	5,25	4,15	4,60	5,30	4,55
Modified	5,05	5,00	4,95	5,35	4,50	5,30	6,20	6,50
Normal	5,25	5,45	5,25	5,60	5,50	4,55	5,95	5,15
Bidirectional	4,65	4,60	4,50	4,45	4,25	4,25	4,40	3,95
Standard, **Not Filt. CM**	55,40	51,35	38,70	30,15	21,55	17,65	14,70	13,65
Modified	55,50	51,50	39,10	30,95	22,25	19,35	16,20	15,95
Normal	65,50	15,75	5,65	5,35	4,85	5,15	5,00	5,25
Bidirectional	73,35	70,25	57,80	46,25	32,45	23,50	20,10	16,80

Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	7,00	6,85	6,80	7,20	5,95	6,25	6,80	7,35
Bartlett	-	7,00	6,70	6,60	6,05	6,30	6,30	6,35
Daniel	-	6,65	6,65	6,70	6,45	5,95	6,40	6,25
Quadratic	6,85	6,90	6,70	6,70	6,35	6,30	6,35	6,25
Tukey	-	7,00	6,70	6,55	6,25	6,35	6,15	6,45
Truncated, **Not Filt. CM**	60,05	56,00	44,65	35,55	25,75	21,85	18,75	17,70
Bartlett	-	60,05	59,35	55,20	46,70	41,10	37,85	33,85
Daniel	-	59,85	58,90	51,60	41,20	35,60	31,75	28,50
Quadratic	60,35	60,30	56,70	47,10	37,55	32,15	28,05	24,65
Tukey	-	60,05	59,90	54,05	44,25	38,55	33,90	31,30

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	7,00	6,85	6,85	7,25	5,90	6,30	6,85	7,35
Bartlett	-	7,00	6,70	6,65	6,00	6,35	6,30	6,30
Daniel	-	6,65	6,65	6,75	6,35	6,05	6,40	6,25
Quadratic	6,85	6,90	6,70	6,65	6,30	6,35	6,25	6,20
Tukey	-	7,00	6,70	6,55	6,25	6,35	6,20	6,45
Truncated, **Not Filt. CM**	60,05	56,00	44,55	35,45	25,50	21,60	18,50	17,50
Bartlett	-	60,05	59,35	55,15	46,65	41,00	37,45	33,65
Daniel	-	59,85	58,90	51,60	41,05	35,35	31,60	28,30
Quadratic	60,35	60,30	56,70	47,00	37,45	32,05	27,90	24,35
Tukey	-	60,05	59,90	54,05	44,15	38,45	33,80	30,90

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	6,80	7,15	5,95	5,50	5,45	5,65	6,10	6,40
Bartlett	-	6,80	6,35	6,40	5,15	5,30	5,25	5,50
Daniel	-	6,70	6,30	6,25	5,45	5,15	5,55	5,55
Quadratic	6,60	6,35	6,60	5,50	5,05	5,40	5,75	5,75
Tukey	-	6,80	6,35	6,30	5,25	5,35	5,40	5,45
Truncated, **Not Filt. CM**	56,25	75,15	63,75	51,50	37,65	29,45	25,20	23,30
Bartlett	-	56,25	74,50	71,60	64,70	58,60	53,75	49,40
Daniel	-	56,65	74,65	70,10	59,30	52,60	46,25	41,25
Quadratic	58,70	62,70	73,75	66,55	54,50	46,65	40,80	36,70
Tukey	-	56,25	73,40	72,10	62,95	55,70	50,45	45,60

Lagrange Mult.	
Filtered CM	5,40
Not Filt. CM	51,30

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. Mean Spillover in the first Lag, The Filter is a VAR(1) Model

TABLE 2.3

Empirical Size, Causality in Variance Tests, Filtering & Not Filtering Causality in Mean, Sample Size: 200 obs., Replications: 2000, Nominal Size: 10%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **Filtered CM**	8,90	9,45	9,55	10,50	8,35	7,45	5,60	4,20
Modified	9,20	9,95	10,15	11,95	11,75	12,75	14,35	15,35
Normal	9,60	10,20	10,30	11,05	9,05	8,70	7,95	7,55
Bidirectional	9,10	9,05	10,40	9,00	7,25	5,20	3,55	1,60
Standard, **Not Filt. CM**	26,45	24,75	20,95	16,85	12,35	9,10	6,50	5,10
Modified	26,70	25,60	22,10	18,70	16,80	16,05	16,40	16,20
Normal	35,95	16,00	10,15	9,10	8,75	9,30	7,70	6,25
Bidirectional	32,50	32,30	26,25	19,30	12,80	8,35	4,75	2,65

Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	8,40	10,00	10,70	12,10	12,15	12,50	13,65	15,10
Bartlett	-	8,40	9,40	10,60	10,80	11,45	11,95	12,25
Daniel	-	8,50	9,70	11,20	11,95	12,00	12,35	12,50
Quadratic	8,35	8,25	10,10	11,20	11,95	12,00	12,35	13,20
Tukey	-	8,40	9,05	10,80	10,85	11,75	12,00	12,10
Truncated, **Not Filt. CM**	25,45	25,70	22,60	19,05	17,40	16,60	16,50	16,20
Bartlett	-	25,45	26,90	25,55	22,55	21,00	19,85	18,90
Daniel	-	25,50	27,00	24,15	20,35	19,10	18,15	17,40
Quadratic	25,60	25,85	26,85	22,95	19,75	18,05	17,30	17,25
Tukey	-	25,45	26,90	25,50	21,60	19,70	18,90	18,15

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	8,40	9,95	10,70	12,20	11,80	12,60	13,90	15,00
Bartlett	-	8,40	9,40	10,55	10,85	11,40	12,15	12,35
Daniel	-	8,50	9,70	11,10	11,80	12,10	12,35	13,05
Quadratic	8,35	8,25	10,15	11,30	12,00	12,15	12,85	13,25
Tukey	-	8,40	9,00	10,80	10,90	11,85	12,10	12,10
Truncated, **Not Filt. CM**	25,45	25,70	22,50	19,05	16,95	15,90	16,00	15,75
Bartlett	-	25,45	26,90	25,45	22,30	20,90	19,75	18,65
Daniel	-	25,60	27,10	24,10	20,50	18,80	18,05	17,35
Quadratic	25,60	25,90	26,85	22,80	19,30	17,85	17,30	17,35
Tukey	-	25,45	26,90	25,40	21,45	19,65	18,45	17,65

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	9,50	9,90	11,95	11,80	11,40	12,50	12,45	12,30
Bartlett	-	9,50	9,65	10,65	10,95	10,75	11,00	11,40
Daniel	-	9,75	9,85	11,10	10,95	11,20	11,15	11,75
Quadratic	9,70	9,65	9,70	11,25	11,15	11,25	11,55	11,55
Tukey	-	9,50	9,85	10,20	11,10	11,30	11,20	11,35
Truncated, **Not Filt. CM**	24,45	34,20	29,80	23,35	19,85	18,40	17,75	16,90
Bartlett	-	24,45	31,95	32,15	29,10	26,35	24,00	23,45
Daniel	-	24,70	32,50	31,85	26,85	22,95	21,85	20,25
Quadratic	24,80	26,30	33,50	30,90	24,80	22,10	20,25	19,50
Tukey	-	24,45	31,65	32,95	28,70	25,70	22,75	22,10

Lagrange Mult.	
Filtered CM	12,80
Not Filt. CM	28,00

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. Mean Spillover in the first Lag, The Filter is a VAR(1) Model

TABLE 2.4

Empirical Size, Causality in Variance Tests, Filtering & Not Filtering Causality in Mean, Sample Size: 1000 obs., Replications: 2000, Nominal Size: 10%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **Filtered CM**	10,00	9,10	10,20	10,65	9,25	9,00	9,25	9,45
Modified	10,00	9,20	10,25	10,85	9,95	10,35	11,05	11,70
Normal	9,95	10,40	10,00	10,75	9,70	9,00	10,60	10,00
Bidirectional	9,60	9,90	9,15	8,80	8,70	8,55	8,35	7,75
Standard, **Not Filt. CM**	65,50	61,60	50,50	41,95	31,20	26,50	23,00	20,60
Modified	65,50	61,60	50,95	42,45	32,85	28,15	25,10	24,20
Normal	77,20	25,00	11,80	10,35	10,10	9,40	9,50	9,10
Bidirectional	82,25	79,50	68,70	57,10	44,45	35,00	29,60	26,05

Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	9,35	9,35	10,75	11,40	10,55	10,90	11,40	12,15
Bartlett	-	9,35	10,00	10,20	10,70	10,50	10,35	10,45
Daniel	-	9,15	9,95	10,65	10,75	10,75	11,10	10,85
Quadratic	9,30	9,35	10,45	10,80	10,35	11,20	10,60	11,05
Tukey	-	9,35	9,40	10,25	10,85	10,55	10,85	10,85
Truncated, **Not Filt. CM**	64,30	62,00	52,10	43,70	34,25	29,60	26,40	25,10
Bartlett	-	64,30	65,10	60,95	53,45	49,15	45,35	43,20
Daniel	-	64,10	64,50	58,50	49,20	43,70	40,50	37,25
Quadratic	64,20	64,40	62,60	55,30	45,60	40,75	36,65	34,00
Tukey	-	64,30	65,35	60,15	52,10	46,35	42,60	39,90

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	9,35	9,35	10,75	11,45	10,60	10,85	11,50	12,15
Bartlett	-	9,35	10,00	10,15	10,80	10,50	10,45	10,50
Daniel	-	9,15	9,95	10,60	10,75	10,85	11,10	10,85
Quadratic	9,30	9,35	10,45	10,80	10,30	11,10	10,70	11,00
Tukey	-	9,35	9,40	10,30	10,85	10,60	10,95	10,90
Truncated, **Not Filt. CM**	64,30	61,95	52,05	43,50	34,00	28,90	25,95	24,75
Bartlett	-	64,30	65,05	60,85	53,45	49,10	45,30	43,05
Daniel	-	64,10	64,55	58,45	49,20	43,65	40,05	37,00
Quadratic	64,20	64,40	62,55	55,20	45,45	40,50	36,40	33,85
Tukey	-	64,30	65,35	60,10	51,90	46,40	42,50	39,65

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **Filtered CM**	9,60	10,60	10,55	9,65	10,15	10,55	10,40	11,50
Bartlett	-	9,60	9,55	9,75	9,85	9,05	9,55	9,80
Daniel	-	9,55	9,55	10,10	9,70	9,30	10,05	10,25
Quadratic	9,50	9,50	9,90	9,70	8,85	9,60	10,05	10,40
Tukey	-	9,60	9,35	10,10	9,95	8,70	9,50	9,85
Truncated, **Not Filt. CM**	61,75	81,25	71,15	59,95	48,35	39,65	35,80	34,05
Bartlett	-	61,75	79,50	77,70	71,65	66,10	61,80	58,30
Daniel	-	62,30	80,10	76,85	67,45	60,55	55,60	51,10
Quadratic	64,50	69,00	80,15	73,70	62,85	55,85	50,45	46,90
Tukey	-	61,75	79,35	78,00	70,20	64,05	58,40	54,85

Lagrange Mult.	
Filtered CM	11,35
Not Filt. CM	62,15

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. Mean Spillover in the first Lag, The Filter is a VAR(1) Model

4.3.2 EMPIRICAL RESULTS OF MONTE CARLO DESIGN 2

A) SMALL SAMPLE

1. The most important empirical discovery of this Monte Carlo design is the serious upward distortion in the empirical size of the various causality in variance tests that can be realized when we do not account for the presence of causal interdependencies in the first order moments. This finding is in agreement with that of **Pantelidis & Pittis** (2004). These authors were the first to detect the negative impact of mean spillovers in the credibility of causality in variance tests.

2. In regard with the Cheung & Ng class of tests, we observe that the distortion of empirical size is maximized in the 1^{st} lag while it persists until the 10^{th} lag. This pattern was naturally expected as the mean spillover takes place in the 1^{st} lag and continues to influence the series for a number of periods. The Normal version of Cheung & Ng tests is also affected from causality in mean but this effect decays more rapidly than in the rest of the tests. This happens because this type of test scans for causality only in a specific lag. Thus in this case, the causality in mean does not have a cumulative effect in the performance of this test as happens with the other methodologies. However we must note that when considering the first lag the Normal version is more heavily influenced compared with the other versions of the Cheung & Ng class of tests. For the 2^{nd} and 5^{th} lags the bidirectional version seems more sensitive to mean spillovers while for the 10^{th} lag and afterwards the differences in the distorting effects across the alternative versions of the Cheung & Ng class of tests are less obvious.

3. For the Hong tests the situation is slightly different. More specifically we observe that the effects of not accounting for mean spillovers are observable even at the 50^{th} lag. This characteristic can be simply attributed in the more complex lag weighting scheme that is adopted when using this type of tests. We have also found that the most robust kernel against causality in mean is the Truncated. From the other side, the Bartlett and Tukey are the most heavily influenced kernels. Finally it seems that the bidirectional version of the Hong tests suffers more heavily from size distortions compared with its unidirectional alternatives.

4. The Lagrange Multiplier test suffers from significant oversizing when mean spillovers are not accounted for. Thus none of the methodologies tested in this study can be regarded as trustful in an environment with causal linkages in the first moments.

B) LARGE SAMPLE

1. In the case of a large sample size we clearly observe that the distorting effects of not filtering causality in mean are even more pronounced. This means that in empirical applications in the field of finance where the researchers frequently use thousands of observations, it is very important to account for mean spillovers before proceeding with the causality in variance tests. We must also note that the empirical worker must also choose a correct specification for the conditional mean model in order to efficiently remove any causality in the first moments. In our case the spillover in mean takes place in the 1^{st} lag and we filter this feature using a first order VAR model. That's why causality in mean when filtered does not affect the empirical size of the tests. This would not however happen if we used the same VAR specification but the mean spillover was located in another lag.

2. We also observe that the bidirectional version of the Cheung & Ng class of tests is more negatively affected (increase in the empirical size) than the rest of the Cheung & Ng tests used in this study.

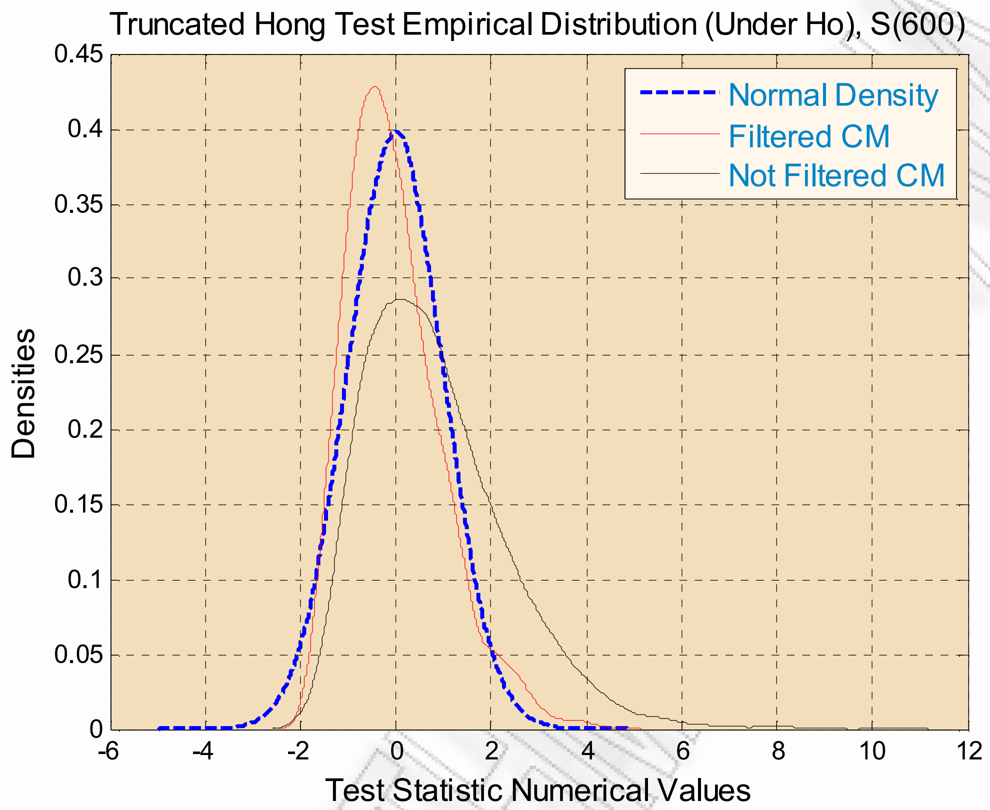
3. For the Cheung & Ng class of tests the distorting effects are maximized in the 1^{st} lag. For the unidirectional Hong tests, this happens between the 1^{st} and the 2^{nd} lags while for the bidirectional version of Hong tests the distortion is more pronounced in the 5^{th} lag.

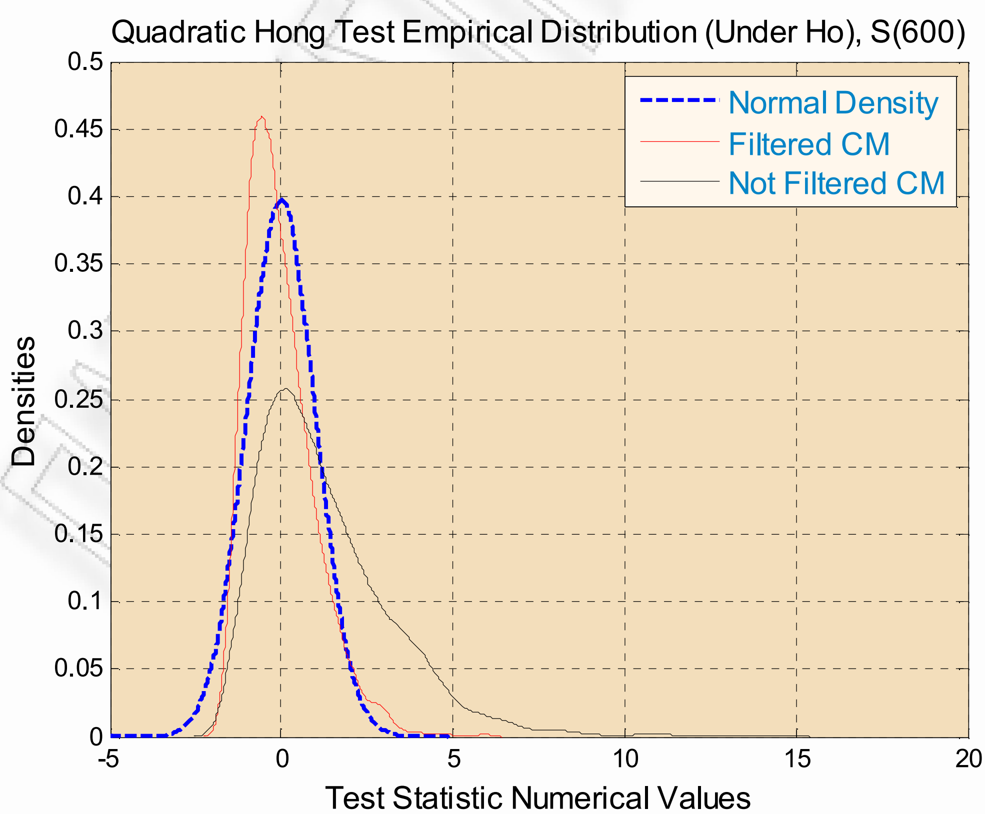
4. It is almost certain that the bidirectional version of the Hong class of tests is less robust to causality in mean compared with its unidirectional alternatives. This is not surprising however, as we have a bidirectional mean spillover structure in the generated data. Thus it is natural that this aggregate effect will have a more powerful impact on tests that search for volatility spillovers in both directions simultaneously.

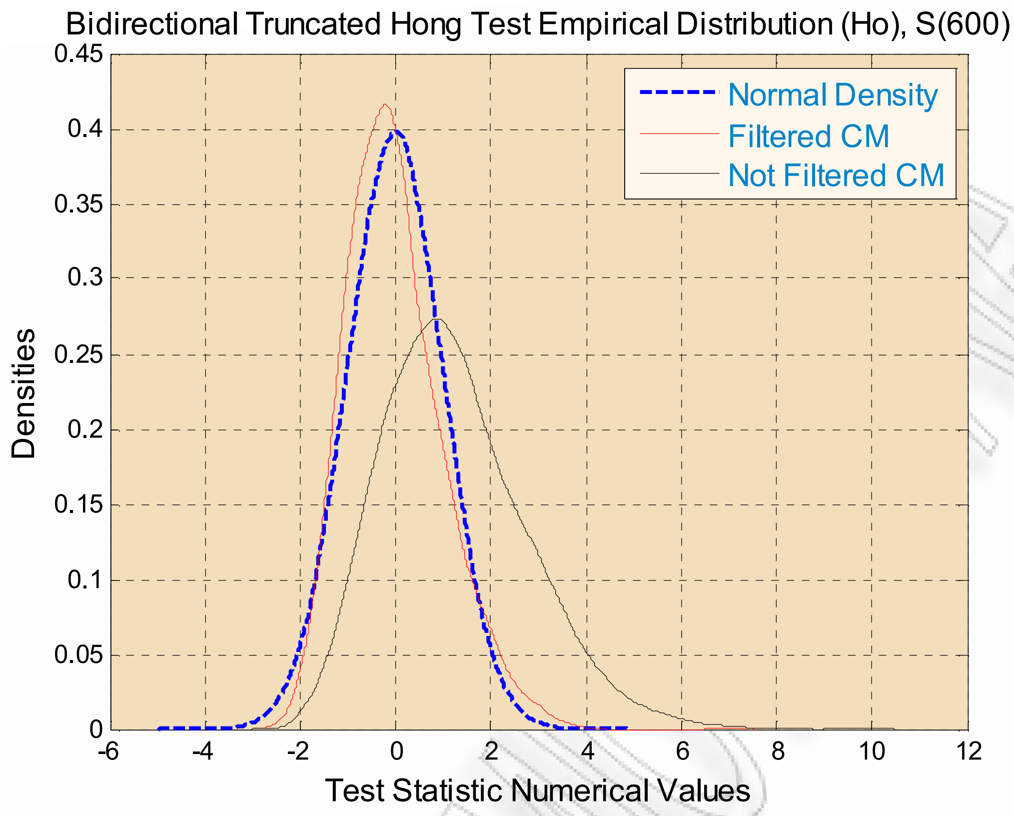
In this point we want to note that the reason for not providing outputs with the power of the tests under not filtering causality in mean is simply because the unfiltered mean spillovers have a positive impact in the power of the tests. This result is not however of any importance as it was naturally expected to happen. In the following figures we provide the smooth densities of the various causality in variance tests that were calculated by an Epanechnikov kernel using the test's realized values for the sample size of 600 obs. The results using this sample size were not included in the previous tables, as we wanted to depict the two limit cases of a small (200 obs.) and a large sample size (1000 obs.) This sample size however was chosen for the construction of the empirical distributions as it represents a mainstream situation between the two limits and allows us to briefly and concretely demonstrate our empirical results.

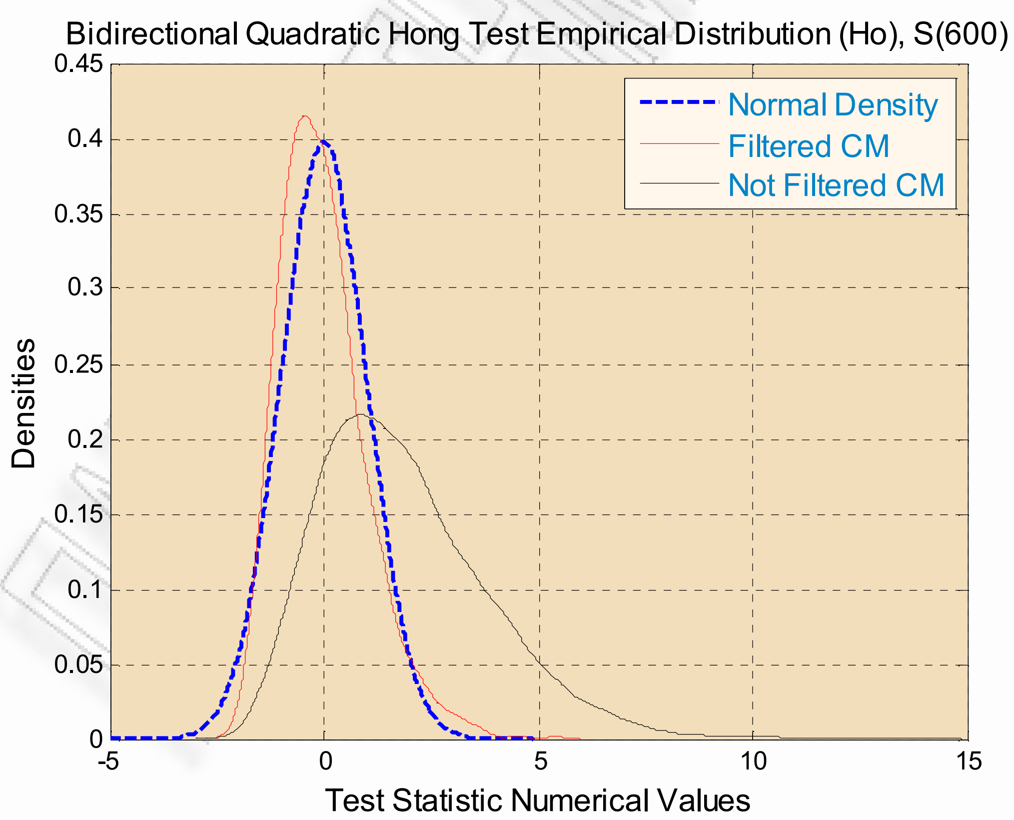
Previewing these figures we can safely conclude that when the presence of causality in mean is not correctly filtered (e.g. when using univariate ARMA models for the conditional mean specifications instead of VAR models) then a serious distortion in the empirical size of the various tests is observed. This means that the probability for type I error is increased, meaning that it is more possible to reject the null hypothesis of no causality in variance, when this is in fact true. Thus it is clear the underlying risk of finding causal interlinkages between series that are in fact independent or unrelated in their second moments when we do not account for any possible existent interdependence in the first moments. In the **Appendix B** (Monte Carlo Design 2) you will also find a number of additional figures that will help you to visualize the differences among the alternative tests as well as some other aspects of this Design.

Figures 2.1-2.4 Unidirectional & Bidirectional Hong Causality in Variance Tests under Filtered / Not Filtered Causality in Mean (Bandwidth: 10)

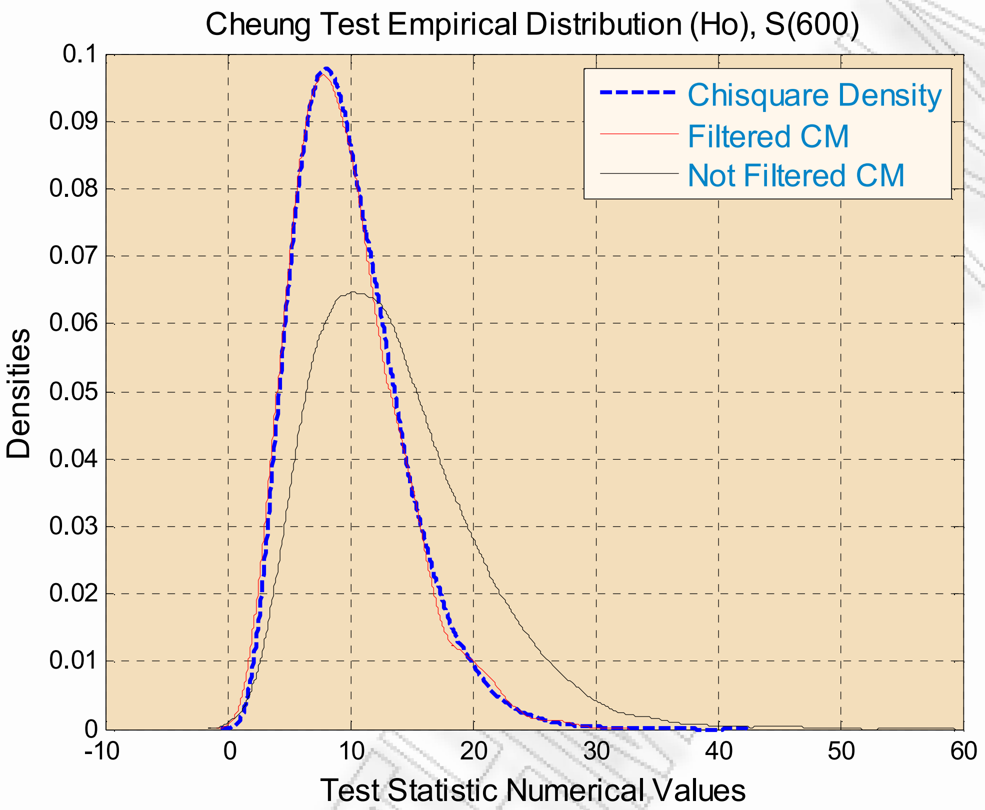


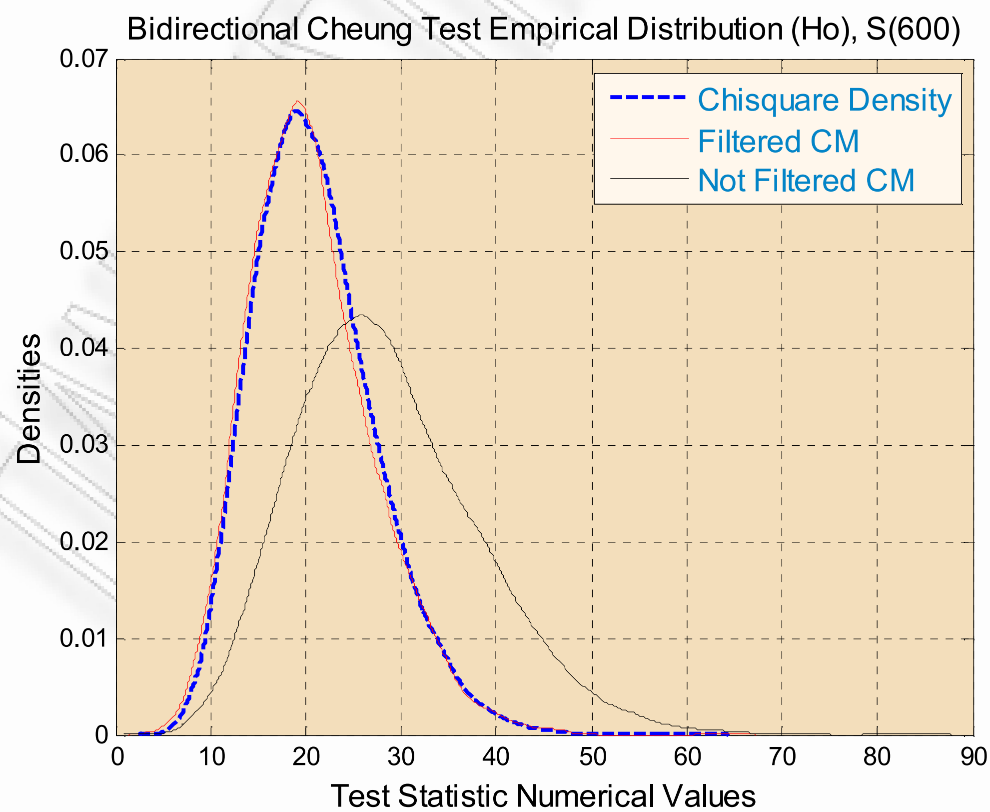



Bidirectional Truncated Hong Test Empirical Distribution (Ho), S(600)
Normal Density
Filtered CM
Not Filtered CM
Densities
Test Statistic Numerical Values
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-6
-4
-2
0
2
4
6
8
10
12


Bidirectional Quadratic Hong Test Empirical Distribution (Ho), S(600)
Normal Density
Filtered CM
Not Filtered CM
Densities
Test Statistic Numerical Values
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5
10
15

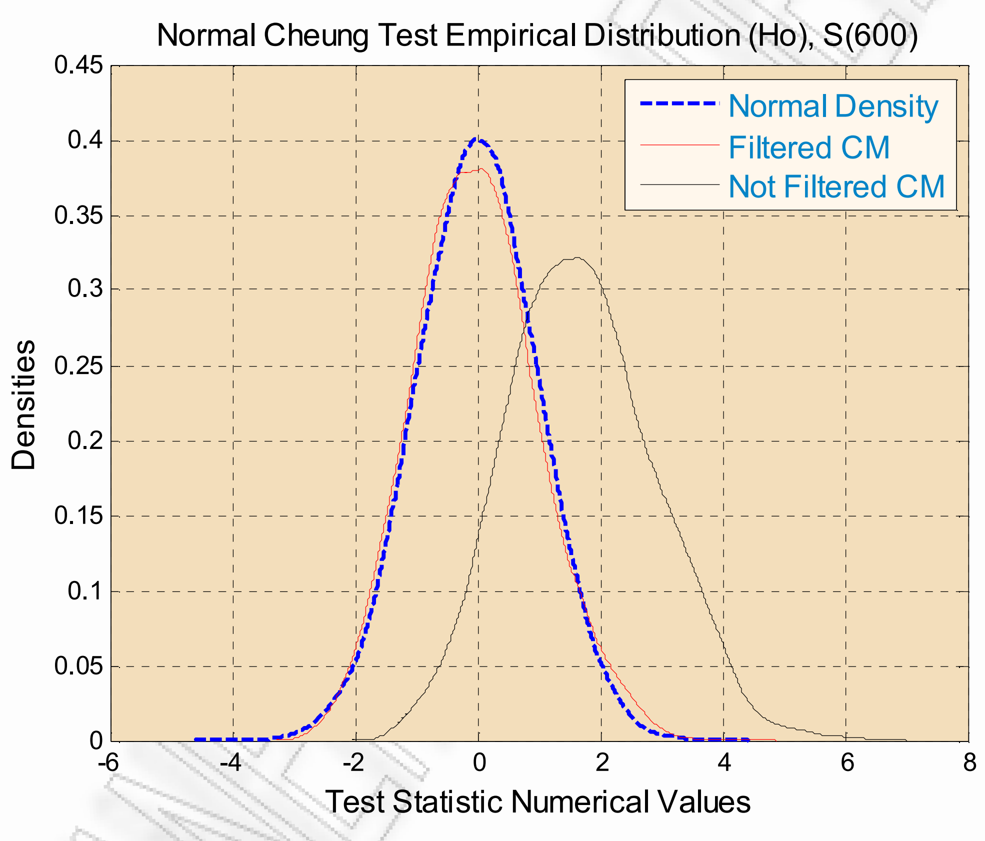
Figures 2.5-2.8 Unidirectional & Bidirectional Cheung & Ng Causality in Variance Tests under Filtered / Not Filtered Causality in Mean (10^{th} Lag)





In the following two graphs we plot the empirical distributions of the Normal Cheung & Ng Test in the 1st and 10th Lags. As we have already said this version of the Cheung & Ng tests, scans for causality only in a specific lag. In the 10th Lag the causality in mean is weak and thus the distorting effect in the empirical size of the test is not powerful and thus not observable. In the 1st Lag however the mean spillovers are strong and we can clearly observe their effects (when not filtered) in the empirical distribution of the Normal Cheung & Ng test.

- 1st Lag, Causality in Mean is Powerful



- 10th Lag, Causality in Mean has disappeared.

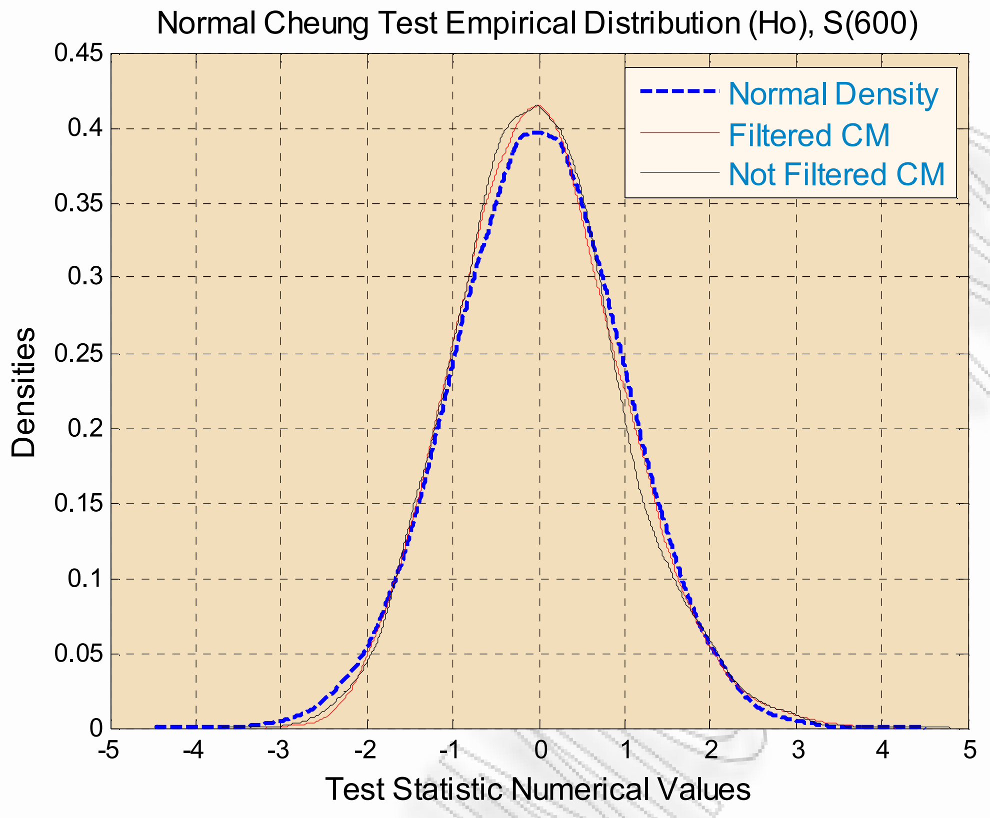
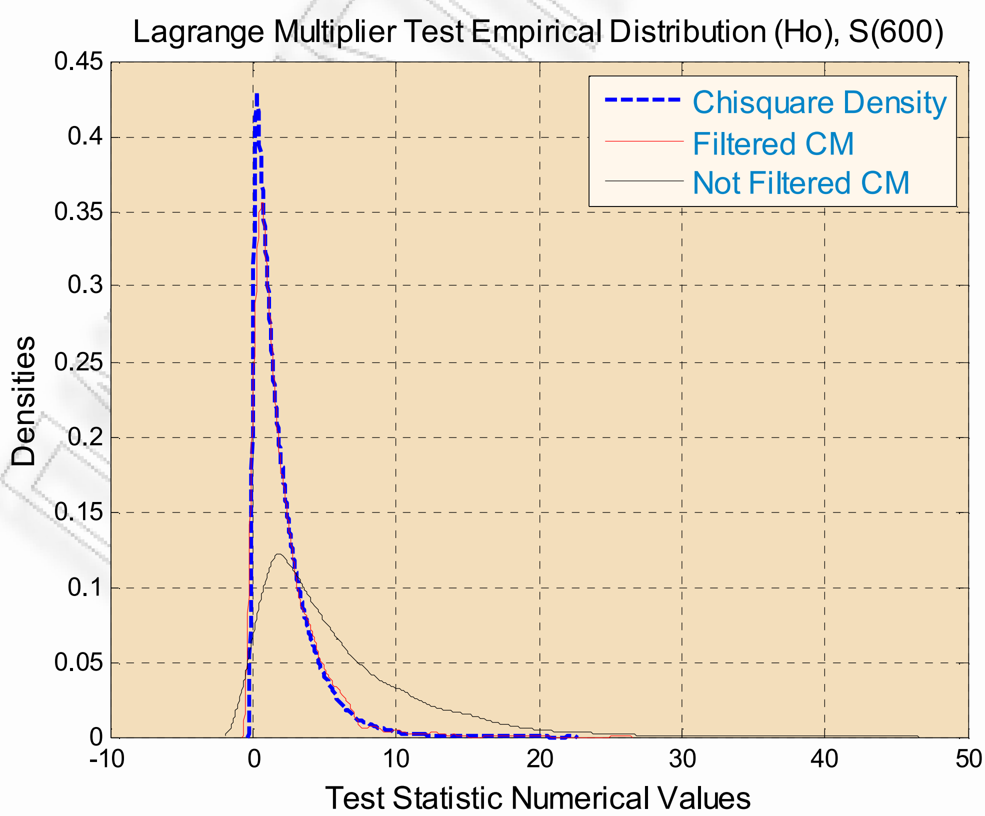


Figure 2.9 Lagrange Multiplier Causality in Variance Tests under Filtered / Not Filtered Causality in Mean (1st Lag)



4.4 MONTE CARLO SIMULATION DESIGN 3

In this experiment we have estimated the size and power of causality in mean tests under the alternative assumptions of no volatility spillovers, volatility spillovers and volatility spillovers with GARCH in Mean effects. The spillovers for both the first and second conditional moments are located in the first lag. We must note here that the LM test is not used in this experiment as it is designed to detect volatility spillovers only. It is also important to denote the versatility of the Cheung & Ng and Hong tests in that they can be used in both causality in mean and causality in variance applications. More specifically if we want to perform causality in mean tests, then the only thing that we have to do is to use the standardized residuals of the ARMA – GARCH models instead of their squares.

4.4.1 DATA GENERATING PROCESS

1. $Y_t = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$

2. $Y_t = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} + \begin{bmatrix} Gim_1 \times h_{1t} \\ Gim_2 \times h_{2t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$

$$\left.\begin{matrix} Gim_1 = 0.1 \\ Gim_2 = 0.1 \end{matrix}\right\} \text{GARCH in Mean Parameters}$$

$w_1 = 0.10 \quad w_2 = 0.10$

$a_{11} = 0.52 \quad a_{22} = 0.52$

$$\left.\begin{matrix} a_{12} = 0.0 \\ a_{21} = 0.0 \end{matrix}\right\} \text{Mean Spillover Parameters}(\text{H}_0)$$

$$\left.\begin{matrix} a_{12} = 0.3 \\ a_{21} = 0.3 \end{matrix}\right\} \text{Mean Spillover Parameters}(\text{H}_1)$$

$$\varepsilon_{it} = \xi_{it}(h_{it}^{0})^{1/2}$$

$$\xi_{it} \sim NIID(0,1)$$

$$h_{it}^{0} = \omega_i^{0} + \alpha_i^{0}\varepsilon_{it-1}^{2} + \beta_i^{0}h_{it-1}^{0} + \delta_{ij}\varepsilon_{jt-g}^{2} + \gamma_{ij}h_{jt-g}^{0}, \quad g > 0,\ i \neq j, \quad i, j = 1,2$$

$$\left.\begin{aligned} \omega_i^{0} &= 0.03 \\ \alpha_i^{0} &= 0.15 \\ \beta_i^{0} &= 0.5 \end{aligned}\right\}$$

1. No Volatility Spillovers

$$\left.\begin{aligned} \delta_{ij} &= 0.0 \\ \gamma_{ij} &= 0.0 \end{aligned}\right\}\text{Volatility Transmission Parameters}$$

$$g = 1$$

2. Volatility Spillovers in the 1st Lag

$$\left.\begin{aligned} \delta_{ij} &= 0.1 \\ \gamma_{ij} &= 0.2 \end{aligned}\right\}\text{Volatility Transmission Parameters}$$

$$g = 1$$

In the following tables we provide the most important of our Monte Carlo results in regard with the empirical size performance of the alternative causality in mean tests. We do not provide output for the power of these tests as this was out of our research interests. We have however computed the power of these tests and these results are available from the author upon request. The numbers beside the names of the tests represent the lag lengths (or bandwidths when considering the Hong class of tests)

TABLE 3.1

Empirical Size, Causality in Mean Tests, Presence of Volatility Spillovers and GARCH in Mean effects/ Volatility Spillovers / No Volatility Spillovers, Sample Size: 200 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	1	2	5	10	20	30	40	50
Standard, **No Vol Sp.**	4,90	5,05	4,70	4,35	3,50	2,55	1,60	0,85
Modified	5,10	5,30	5,40	5,15	5,65	5,80	6,40	6,40
Normal	5,10	4,90	5,35	4,25	4,25	3,60	2,55	3,35
Bidirectional	5,10	5,15	4,90	4,20	3,30	1,40	0,80	0,55
Standard, **Vol Sp.**	7,40	7,20	6,50	5,60	4,55	2,10	1,50	0,95
Modified	7,55	7,40	7,20	7,05	6,95	5,70	5,60	6,10
Normal	6,65	5,30	4,80	5,55	4,50	3,40	3,30	3,30
Bidirectional	7,45	7,65	7,85	6,20	3,90	2,10	1,25	0,45
Standard, **Vol Sp. & GIM**	7,85	7,85	8,30	6,15	4,55	3,05	2,05	1,05
Modified	8,00	8,45	9,05	7,65	7,05	7,00	6,65	7,05
Normal	6,90	5,95	4,60	4,30	4,45	3,90	3,35	2,10
Bidirectional	7,40	7,75	7,95	7,50	4,50	2,75	1,55	0,75

Hong	1	2	5	10	20	30	40	50
Truncated, **No Vol Sp**.	7,05	6,95	7,50	6,70	7,00	6,75	7,05	7,45
Bartlett	-	7,05	6,95	7,80	6,85	6,60	6,75	7,05
Daniel	-	6,90	7,10	7,45	6,85	6,90	7,25	7,30
Quadratic	6,95	6,95	7,30	7,05	6,75	7,10	7,20	7,50
Tukey	-	7,05	7,30	7,60	6,70	6,70	6,85	7,15
Truncated, **Vol Sp.**	9,50	9,60	10,40	9,30	8,35	6,45	6,15	6,55
Bartlett	-	9,50	10,15	10,15	9,80	9,40	8,95	8,35
Daniel	-	9,80	10,00	10,10	9,35	9,25	8,80	7,75
Quadratic	9,30	9,15	10,20	10,25	9,10	8,70	7,55	7,25
Tukey	-	9,50	9,95	10,25	9,85	9,00	8,75	8,75
Truncated, **Vol Sp. & GIM**	9,45	10,55	11,15	9,65	8,70	8,10	7,35	7,70
Bartlett	-	9,45	10,35	11,20	10,65	10,45	9,60	8,65
Daniel	-	9,55	10,85	11,45	10,60	9,60	8,90	8,15
Quadratic	9,50	9,55	11,00	10,90	9,60	8,95	8,10	8,00
Tukey	-	9,45	10,10	11,20	10,85	9,65	9,55	8,85

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **No Vol Sp.**	7,05	7,00	7,55	6,60	7,05	6,55	6,85	6,70
Bartlett	-	7,05	6,95	7,80	6,90	6,70	6,65	7,10
Daniel	-	6,90	7,10	7,45	6,95	6,90	7,25	7,30
Quadratic	6,95	6,95	7,35	7,00	6,80	7,15	7,45	7,25
Tukey	-	7,05	7,25	7,60	6,75	6,65	6,85	7,25
Truncated, **Vol Sp.**	9,50	9,60	10,25	9,30	8,25	6,55	6,00	6,60
Bartlett	-	9,50	10,10	10,15	9,80	9,45	8,90	8,45
Daniel	-	9,80	10,00	9,90	9,40	9,10	8,55	7,40
Quadratic	9,35	9,15	10,05	10,35	9,15	8,80	7,70	7,35
Tukey	-	9,50	9,95	10,20	9,75	9,15	8,80	8,55
Truncated, **Vol Sp. & GIM**	9,45	10,50	11,15	9,65	8,60	7,85	7,35	7,45
Bartlett	-	9,45	10,35	11,30	10,75	10,20	9,50	8,65
Daniel	-	9,55	10,80	11,35	10,75	9,55	8,65	7,90
Quadratic	9,50	9,55	11,05	10,70	9,50	8,90	7,80	7,80
Tukey	-	9,45	10,10	11,15	10,85	9,50	9,50	8,90

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **No Vol Sp.**	7,55	7,70	7,60	6,80	7,45	7,40	7,05	7,05
Bartlett	-	7,55	7,80	7,15	7,30	6,95	6,85	7,15
Daniel	-	7,30	7,60	7,70	6,95	6,70	6,65	7,25
Quadratic	7,40	7,20	7,35	7,05	7,05	6,55	7,25	7,80
Tukey	-	7,55	8,10	7,45	7,25	6,90	6,60	6,75
Truncated, **Vol Sp.**	10,10	11,20	10,95	11,05	9,00	8,75	7,95	8,30
Bartlett	-	10,10	10,95	12,05	10,95	10,60	10,10	10,05
Daniel	-	10,00	11,45	11,90	11,40	9,85	9,70	9,35
Quadratic	10,30	10,50	11,85	11,70	10,55	9,95	9,20	9,10
Tukey	-	10,10	10,70	12,30	11,15	10,65	9,85	9,50
Truncated, **Vol Sp. & GIM**	8,85	11,35	12,00	10,85	9,35	9,05	8,50	8,35
Bartlett	-	8,85	10,45	11,60	11,70	11,40	11,10	10,40
Daniel	-	8,80	10,75	11,35	11,70	10,90	10,35	10,10
Quadratic	9,35	9,55	11,20	11,75	11,30	10,45	9,95	10,05
Tukey	-	8,85	10,65	11,70	11,55	11,25	10,65	10,40

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Causality in Mean & H1: Bidirectional Symmetric Mean Spillover in the 1st Lag, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. Volatility Spillover (if existent) in the first Lag. The Unconditional Distribution of Residuals is N(0,1). GIM means Garch in Mean effects

TABLE 3.2

Empirical Size, Causality in Mean Tests, Presence of Volatility Spillovers and GARCH in Mean effects/ Volatility Spillovers / No Volatility Spillovers, Sample Size: 1000 obs., Replications: 2000, Nominal Size: 5%

Cheung & Ng	**1**	**2**	**5**	**10**	**20**	**30**	**40**	**50**
Standard, **No Vol Sp.**	4,70	4,95	4,65	4,60	4,00	4,75	4,15	3,90
Modified	4,70	5,05	4,75	4,90	4,65	5,20	5,15	5,40
Normal	5,20	4,85	4,85	5,10	5,20	4,40	4,65	3,75
Bidirectional	4,30	4,55	4,35	4,70	4,45	4,30	3,90	3,25
Standard, **Vol Sp.**	7,00	7,25	7,35	8,30	6,15	5,45	4,75	3,70
Modified	7,05	7,35	7,55	8,55	6,50	6,60	5,95	5,65
Normal	6,25	5,85	4,55	5,80	5,15	4,25	4,90	5,10
Bidirectional	5,75	6,00	8,55	7,95	8,00	6,90	5,70	4,65
Standard, **Vol Sp. & GIM**	8,55	8,25	8,50	6,80	5,65	5,25	4,80	4,65
Modified	8,60	8,25	8,50	7,10	6,45	6,25	5,90	6,60
Normal	8,75	6,80	5,70	5,50	4,60	6,05	6,00	4,45
Bidirectional	8,75	8,95	8,75	9,15	7,45	6,50	5,95	4,55

Hong	**1**	**2**	**5**	**10**	**20**	**30**	**40**	**50**
Truncated, **No Vol Sp**.	6,20	6,85	7,10	5,75	5,90	5,95	6,00	6,25
Bartlett	-	6,20	6,45	6,60	6,45	6,00	6,35	6,45
Daniel	-	6,05	6,50	6,60	6,15	6,05	6,35	6,25
Quadratic	6,15	6,10	6,45	6,10	6,30	6,55	6,55	6,75
Tukey	-	6,20	6,75	6,80	6,25	6,30	6,30	6,55
Truncated, **Vol Sp.**	8,80	10,10	10,40	10,80	8,15	8,40	7,20	7,10
Bartlett	-	8,80	10,10	10,85	10,15	10,40	10,00	9,30
Daniel	-	9,00	10,30	10,70	10,40	10,20	9,00	8,00
Quadratic	8,95	8,85	11,30	10,35	10,40	9,10	8,05	7,85
Tukey	-	8,80	9,75	10,95	10,10	10,30	10,10	9,20
Truncated, **Vol Sp. & GIM**	10,95	10,75	11,15	9,70	8,15	7,40	7,40	7,80
Bartlett	-	10,95	11,75	11,60	11,25	10,60	10,15	9,40
Daniel	-	11,25	11,95	11,30	11,15	9,95	9,30	8,40
Quadratic	11,25	11,10	11,45	11,45	10,45	9,35	8,55	7,90
Tukey	-	10,95	11,90	11,25	11,15	10,45	9,80	9,20

Modified Hong	1	2	5	10	20	30	40	50
Truncated, **No Vol Sp**.	6,20	6,85	7,10	5,75	5,85	5,90	5,95	6,30
Bartlett	-	6,20	6,45	6,55	6,45	6,00	6,35	6,45
Daniel	-	6,05	6,50	6,55	6,15	6,15	6,35	6,30
Quadratic	6,15	6,10	6,45	6,10	6,30	6,55	6,50	6,70
Tukey	-	6,20	6,75	6,80	6,25	6,35	6,15	6,60
Truncated, **Vol Sp.**	8,80	10,10	10,40	10,80	8,25	8,45	7,20	7,15
Bartlett	-	8,80	10,05	10,85	10,20	10,35	9,95	9,15
Daniel	-	9,00	10,30	10,75	10,30	10,15	8,90	8,05
Quadratic	8,95	8,85	11,30	10,35	10,35	9,10	8,10	7,85
Tukey	-	8,80	9,70	10,90	10,10	10,30	10,05	9,10
Truncated, **Vol Sp. & GIM**	10,95	10,75	11,15	9,65	8,05	7,45	7,45	7,70
Bartlett	-	10,95	11,75	11,60	11,25	10,60	10,15	9,40
Daniel	-	11,25	11,95	11,30	11,10	9,90	9,20	8,35
Quadratic	11,25	11,10	11,45	11,45	10,40	9,25	8,50	8,00
Tukey	-	10,95	11,85	11,25	11,15	10,45	9,75	9,15

Bidirect. Hong	1	2	5	10	20	30	40	50
Truncated, **No Vol Sp.**	6,50	6,55	5,15	6,50	6,05	5,80	6,20	6,30
Bartlett	-	6,50	6,30	5,90	5,65	5,75	5,80	5,30
Daniel	-	6,25	6,40	6,00	5,95	5,80	5,75	5,35
Quadratic	6,40	6,35	6,20	5,65	6,15	5,65	5,35	5,70
Tukey	-	6,50	5,95	6,00	6,10	6,00	6,05	5,45
Truncated, **Vol Sp.**	7,75	8,80	10,90	10,10	9,55	10,25	8,25	7,75
Bartlett	-	7,75	8,60	9,60	11,15	11,00	10,55	10,20
Daniel	-	7,65	8,65	10,55	11,10	10,60	10,45	9,75
Quadratic	7,65	7,85	9,20	11,35	10,90	10,25	9,70	9,45
Tukey	-	7,75	8,35	9,45	11,10	10,95	10,45	10,05
Truncated, **Vol Sp. & GIM**	9,50	12,40	11,35	11,75	9,10	8,90	8,70	8,55
Bartlett	-	9,50	11,70	11,95	11,75	11,85	11,10	10,65
Daniel	-	9,95	12,40	12,45	11,80	11,65	10,55	10,20
Quadratic	9,90	9,90	12,35	12,70	12,15	10,55	10,25	9,80
Tukey	-	9,50	11,20	12,25	11,75	12,10	10,85	10,45

Notes: DGP: VAR(1) - GARCH(1,1), Ho: No Causality in Mean & H1: Bidirectional Symmetric Mean Spillover in the 1st Lag, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. Volatility Spillover (if existent) in the first Lag. The Unconditional Distribution of the Residuals is N(0,1). GIM means Garch in Mean effects

4.4.2 EMPIRICAL RESULTS OF MONTE CARLO DESIGN 3

A) SMALL SAMPLE

1. The Cheung & Ng class of tests exhibit a pattern that is analogous to that observed in the case of causality in variance tests. The Standard and Modified versions of this class display a significantly different behavior. More specifically the empirical size of the first one decreases in a monotonic way with the lag length while the latter demonstrates exactly the opposite behavior. It is also impressive that in the 50th lag the Standard and Bidirectional tests exhibit a very small empirical size. This result however may be biased due to the small size of the sample used.

2. The Hong tests are slightly oversized for the entire regime of lag lengths irrespectively of the kernel we use.

3. In the case of volatility spillovers we observe an upward distorting effect in the empirical size of the Cheung & Ng class of tests that persists until the 20^{th} lag. The Normal version of the tests seems to be affected only in the 1^{st} lag as it was expected. In regard with the Hong tests the effects of variance causality, in the type I error probability of causality in mean tests, are slightly more powerful compared with those for the Cheung & Ng tests. The most heavily influenced kernels are the Bartlett and Tukey. We also observe that the bidirectional version of Hong tests is less robust against volatility spillover effects compared with its unidirectional alternatives. The effects of volatility transmission in the empirical size of Hong causality in mean tests persist for a long horizon. They seem to be strong even in the 50^{th} lag. Thus we can conclude that the existence of volatility spillovers has an inflationary impact on the empirical size of causality in mean tests. However this result is not as powerful as in the inverse case where the causality in mean caused a serious distortion in the empirical size of causality in variance tests.

4. An interesting pattern is exhibited from the Truncated kernel in the unidirectional Hong tests. More specifically until the 20^{th} lag the presence of volatility spillovers causes an upward distortion in the empirical size of this test. From the 30^{th} lag and afterwards the presence of these effects appears to push downwards the empirical size

of the Truncated kernel based tests. Interestingly this behavior is not exhibited from any of the other kernels or in the case of the bidirectional version of the Hong test.

5. When we enrich the generated process with GARCH in Mean effects, then the presence of volatility spillovers has a more powerful distorting impact in the empirical size of causality in mean tests. This effect is not however as strong as we would expect. A more concrete conclusion will be reached when we examine these effects for the large sample size.

B) LARGE SAMPLE

1. In the case of the large sample size we observe that the discrepancy in the behavior of the empirical size between the Standard and Modified versions of the Cheung & Ng class of tests is less intense.

2. Once more an upward distorting impact in the empirical size of the Cheung & Ng tests due to volatility spillovers is observed. These effects persist until the 40th lag. This however does not concern the Normal version of the tests that is affected only in the 1st and 2nd lags. We also observe an important upward distortion in the empirical size of the Hong class of tests that persists (as in the small sample case) for the entire lag length regime.

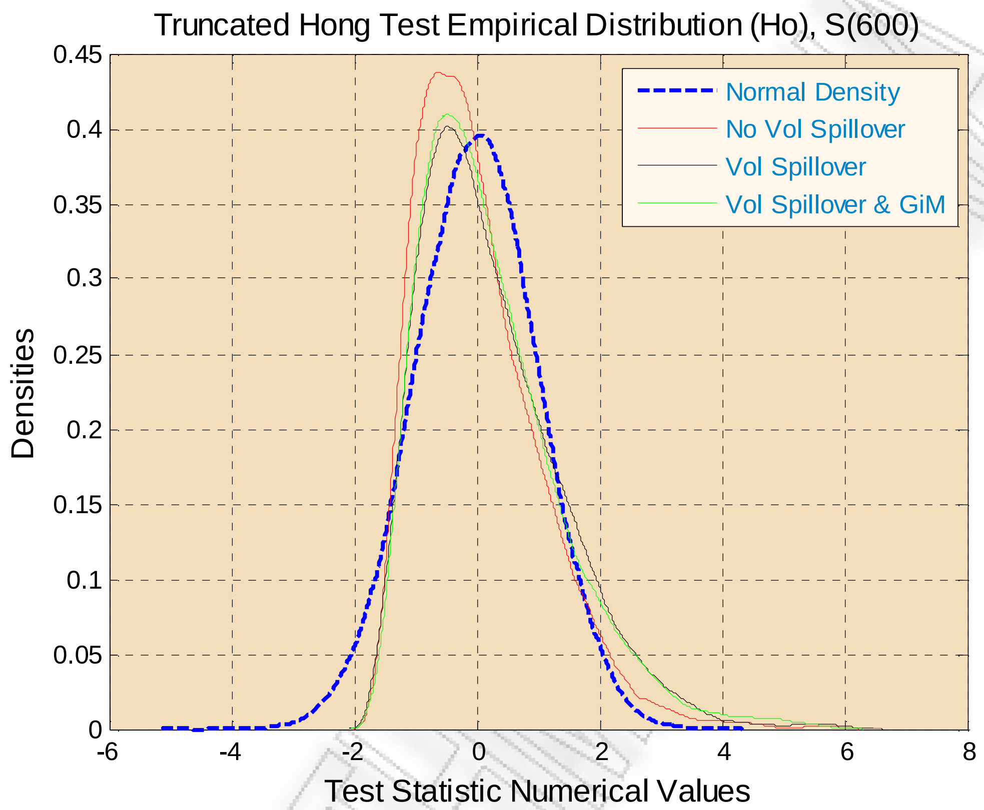
3. The response of the bidirectional Hong test in the influences of volatility spillovers, exhibits a small delay. More specifically the size distorting effects of variance causality are more pronounced in larger lag lengths compared with the unidirectional Hong tests, while in small lags they are substantially weaker.

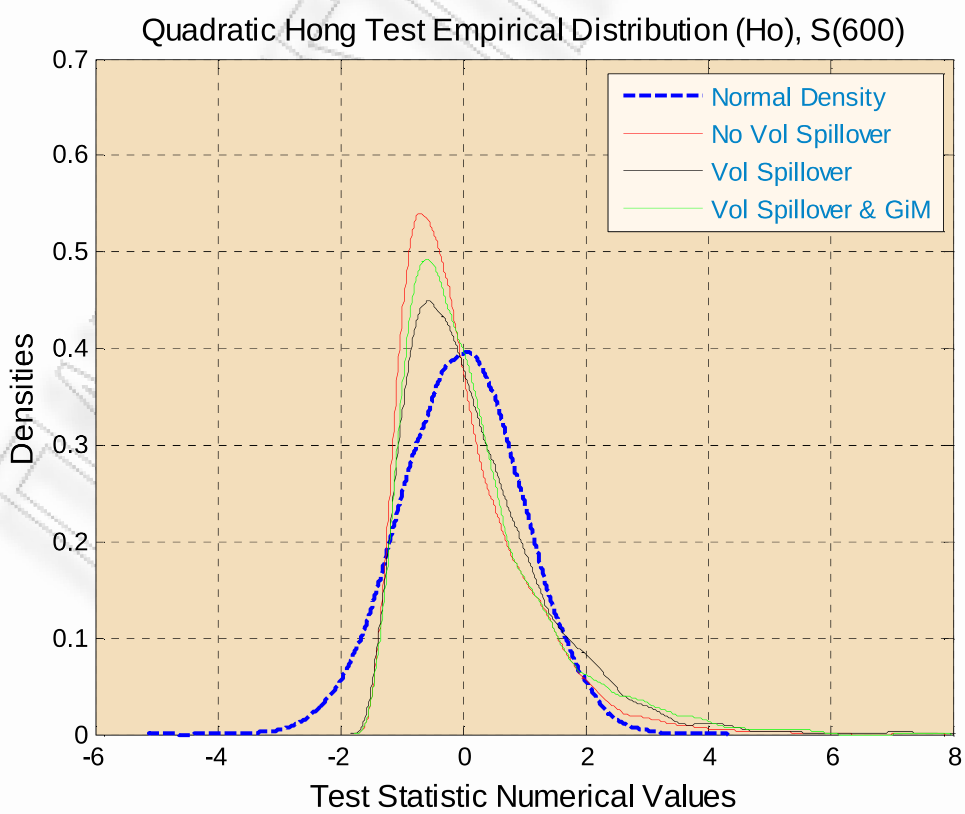
4. The GARCH in Mean effects have now a rather mixed impact in the empirical size of causality in mean tests. For the Cheung & Ng class of tests we observe that until the 5th lag the presence of GARCH in Mean effects causes an upward distortion in the probability for type I error while after that lag these effects are changing direction with the GARCH in Mean causing a decrease in the empirical size of these tests. An analogous behavior is exhibited from the Bidirectional and Normal Cheung & Ng tests with the latter not being influenced after the 10th lag.

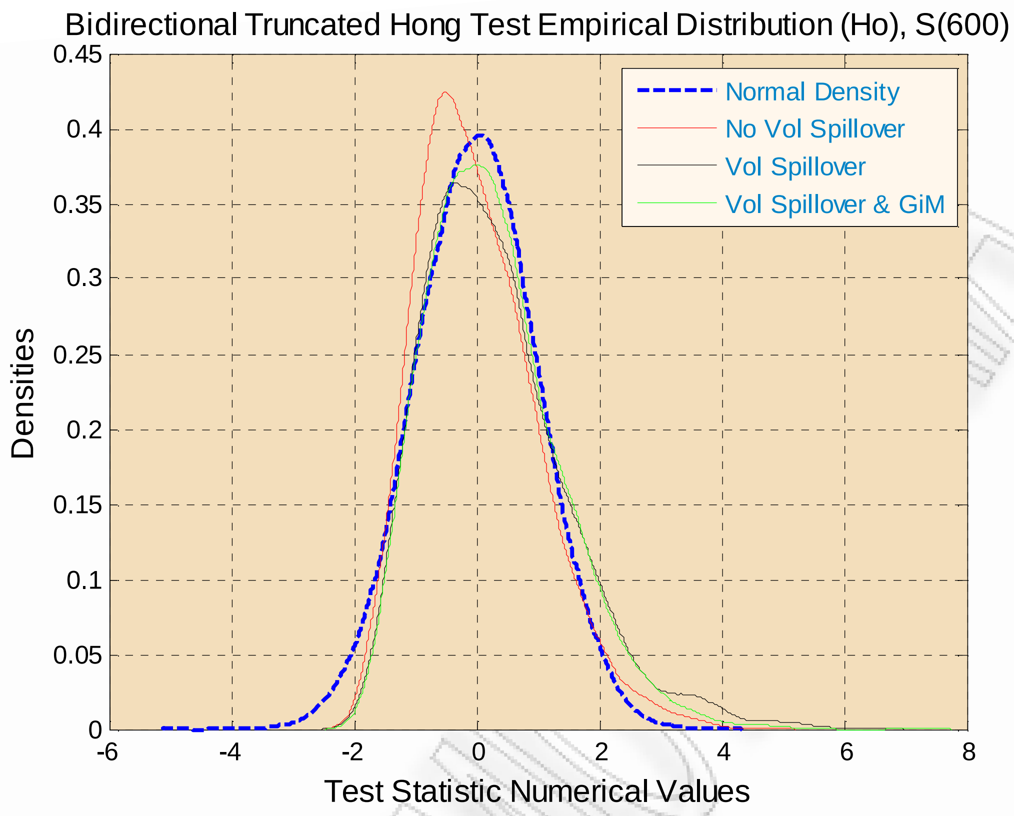
5. For the Hong tests the GARCH in Mean effects do not have a uniform impact for all the kernels. The empirical size of the Truncated kernel is upwardly distorted until the 5th lag, while after that lag the distorting effect has a downward direction. For the rest of the kernels used is this study the impact of GARCH in Mean effects, is an inflation in the empirical size of the tests irrespectively of the lag we considered. Finally for the bidirectional version of the Hong tests the distortion in the empirical size is more pronounced compared with the unidirectional version of these tests.

In the following figures we provide the smooth densities of the various causality in mean tests that were calculated by an Epanechnikov kernel using the test's realized values for the sample size of 600 obs. The results using this sample size were not included in the previous tables, as we wanted to depict the two limit cases of a small (200 obs.) and a large sample size (1000 obs.) This sample size however was chosen for the construction of the empirical distributions as it represents a mainstream situation between the two limits and allows us to briefly and concretely demonstrate our empirical results. In **Appendix B** (Monte Carlo Design 3) you can find a number of additional figures that will help you to visualize the differences among the alternative tests as well as some other aspects of this Design.

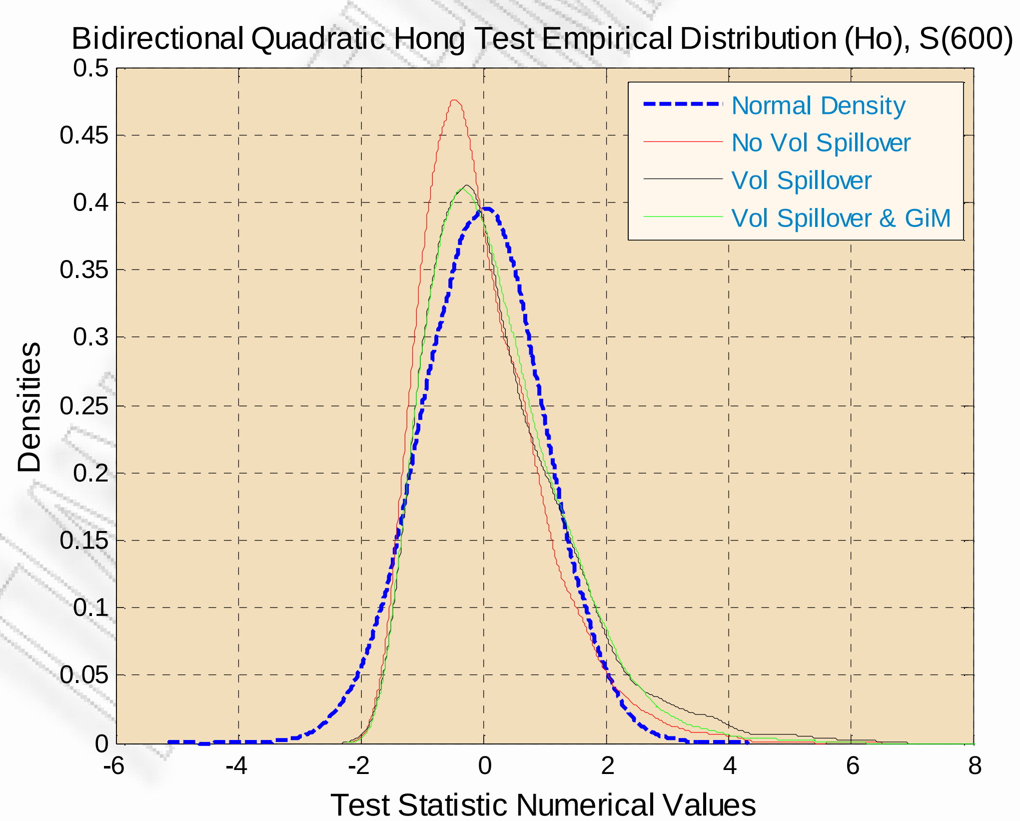
Figures 3.1-3.4 Unidirectional and Bidirectional Hong Causality in Mean tests under Volatility Spillovers / GARCH in Mean Effects (Bandwidth: 5)





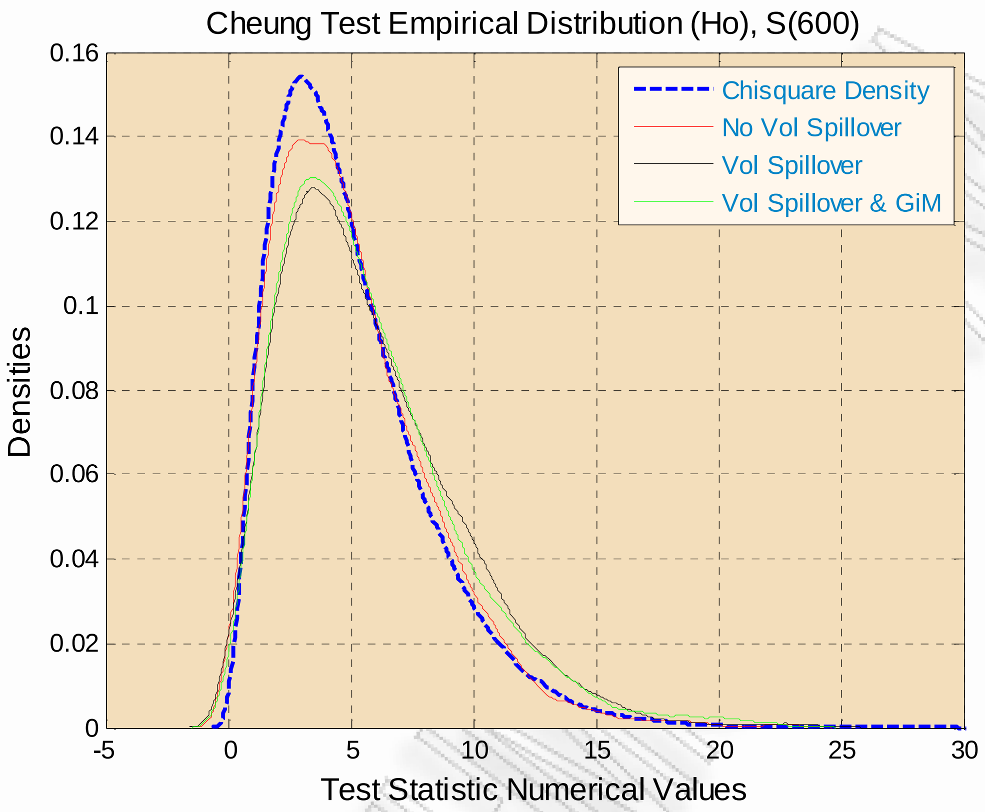


Bidirectional Truncated Hong Test Empirical Distribution (Ho), S(600)
Normal Density
No Vol Spillover
Vol Spillover
Vol Spillover & GiM
Densities
Test Statistic Numerical Values
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-6
-4
-2
0
2
4
6
8

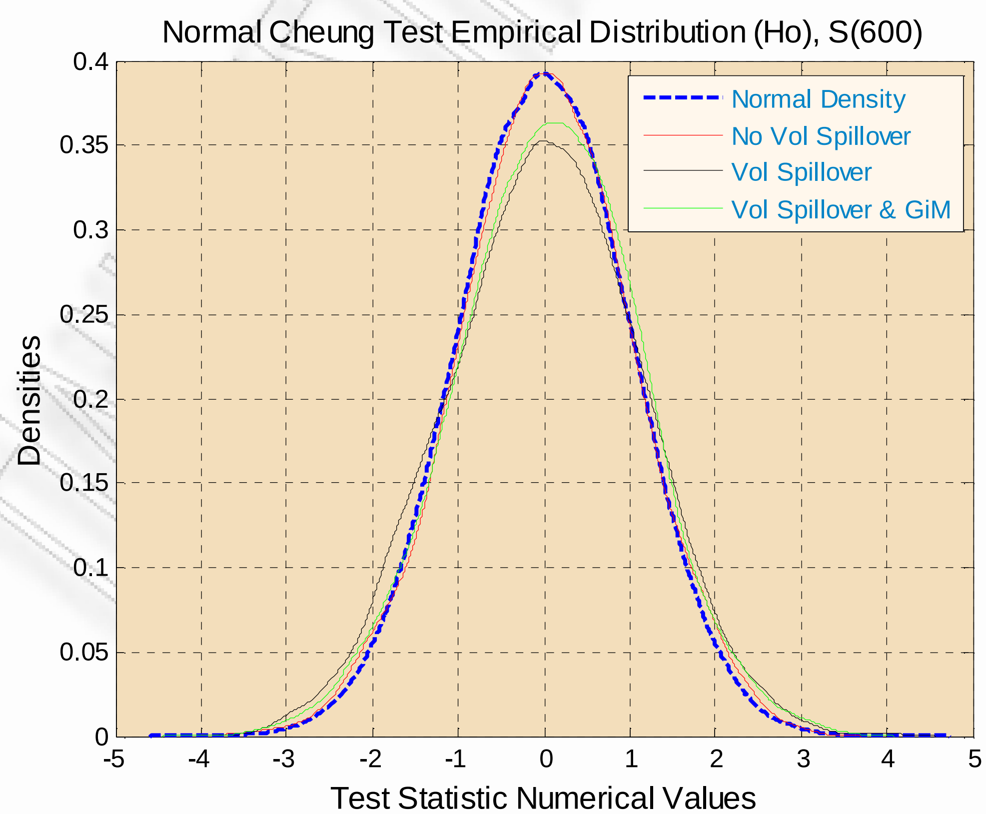


Bidirectional Quadratic Hong Test Empirical Distribution (Ho), S(600)
Normal Density
No Vol Spillover
Vol Spillover
Vol Spillover & GiM
Densities
Test Statistic Numerical Values
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-6
-4
-2
0
2
4
6
8

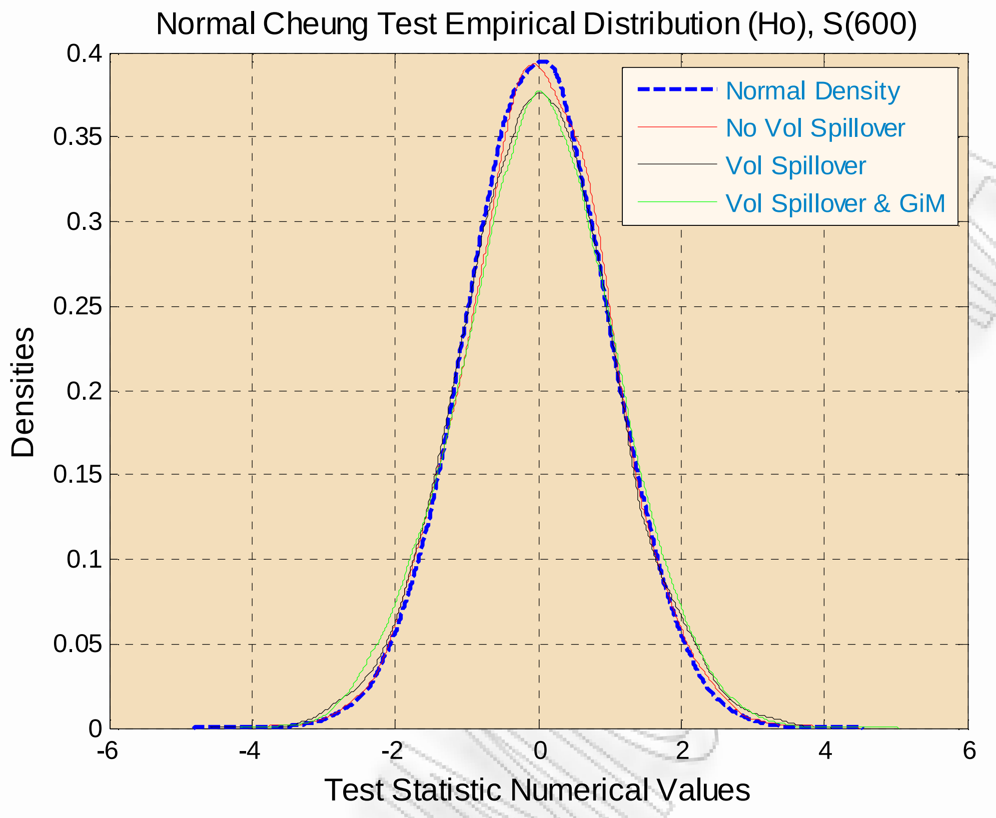
Figures 3.5-3.8 Unidirectional and Bidirectional Cheung & Ng Causality in Mean tests under Volatility Spillovers / GARCH in Mean Effects (5^{th} Lag)

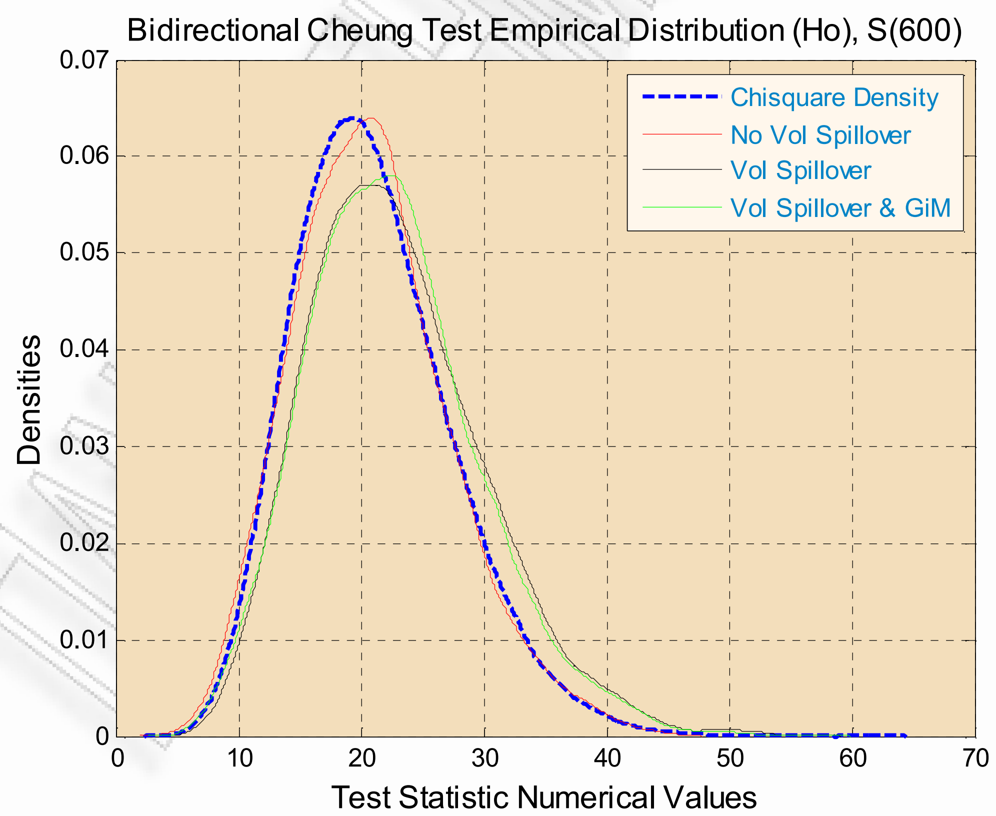


- In the following graph we used the 'Normal' test in the 1^{st} Lag.



- In the following graph we used the 'Normal' test in the 5th Lag.





4.5 MONTE CARLO SIMULATION DESIGN 4

In this experiment we perform an investigation of the size and power of causality in variance tests under the presence of fractional integration in the volatility process. We must note that the initial value for the conditional variance is now set equal to 1000 and not equal to the unconditional variance (as we did in the conventional GARCH process). The reason is that the FIGARCH process is not covariance stationary but instead is strictly stationary. We make use of three alternative distributional assumptions for the underlying residuals. In the two of them we use the Normal Inverse Gaussian distribution that has been found in the literature to frequently characterize the returns of financial variables as in **Andersson** (2001), **Jensen & Lunde** (2001), **Forsberg and Bollerslev** (2002), **Kilic** (2007) among others. The NIG distribution is a special case of the generalized hyperbolic distribution, which was introduced to the field of finance by **Barndorff Nielsen** (1997). The attractive features of the NIG distribution include the ability to fit leptokurtic and skewed data combined with nice analytical properties, such as being closed under convolution and having a closed form density.

4.5.1 DATA GENERATING PROCESS

$$Y_t = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$w_1 = 0.1 \qquad w_2 = 0.1$

$a_{11} = 0.52 \qquad a_{22} = 0.52$

$$\left.\begin{matrix} a_{12} = 0.0 \\ a_{21} = 0.0 \end{matrix}\right\} \text{Mean Spillover Parameters}$$

$$\varepsilon_{it} = \xi_{it} (h_{it}^0)^{1/2}$$

1. $\xi_{it} \sim NIID(0,1)$
2. $\xi_{it} \sim NIG(a,b,\mu,\delta), \quad a = 1.0, \quad b = 0.0$
3. $\xi_{it} \sim NIG(a,b,\mu,\delta), \quad a = 1.2, \quad b = 0.4060$

$$\gamma = \sqrt{\alpha^2 - b^2}, \quad \delta = \frac{\gamma^{3/2}}{a}, \quad \mu = \frac{-b\sqrt{\gamma}}{a}$$

$$h_{it}^0 = \omega_i^0 + \beta_i^0 h_{it-1}^0 + \left(1 - \beta_i^0 L - \left(1 - \phi_i^0\right)\left(1 - L\right)^{d_i}\right)\varepsilon_{it}^2 + \gamma_{ij} h_{jt-g}^0 \quad , g > 0, \; i \neq j, \quad i, j = 1,2$$

$$(1-L)^d = 1 - d\sum_{k=1}^{1000} \Gamma(k-d)\Gamma(1-d)^{-1}\Gamma(k+1)L^k$$

$$d_1 = 0.4$$

$$d_2 = 0.4$$

$$\left.\begin{matrix} \omega_i^0 = 0.03 \\ \beta_i^0 = 0.6 \\ \phi_i = 0.3 \end{matrix}\right\} H_0 \;\&\; H_1$$

$$\gamma_{ij(H_0)} = 0.0 \quad , \quad \gamma_{ij(H_1)} = 0.1 \rightarrow \text{Volatility Spillover in the 1st Lag}$$

Through a small scale simulation study we have calculated the sample moments of the Normal Inverse Gaussian distribution for six alternative pairs of values for the parameters **a** (steepness) and **b** (assymetry):

TABLE 4.1

	1	2	3	4	5	6
a	1,00	1,00	1,20	1,20	1,50	1,50
b	0,00	-0,40	0,4060	-0,4060	0,00	0,40
mean	0,00	0,00	0,00	0,00	0,00	0,00
var	1,00	1,14	0,99	0,99	0,82	0,82
skewness	0,00	-1,33	0,93	-0,95	0,01	0,29
kurtosis	5,97	9,00	6,69	6,87	4,63	4,78

We have used the parameter values of cases 1 and 3 for our Monte Carlo study of the finite sample properties of the causality in variance tests, along with the NIID (0,1) hypothesis and the results are provided in the following tables. Note also that in the **Appendix A** you can find the code for calculating the sample and theoretical moments of the NIG distribution for alternative parameter values if you want to study further the characteristics of this distribution. Finally we must mention that in the Data Generating process used in this Monte Carlo Design we discard 7000 observations in each replication instead of just 1000 as in the previous experiments in order to avoid startup problems and any possible distortion in the long memory features of the generated series due to the large truncation lag (in the fractional differencing operator) that was set equal to 1000. The numbers beside the names of the tests represent the lag lengths (or bandwidths when considering the Hong class of tests)

TABLE 4.2

Empirical Size, Causality in Variance Tests, Fractionally Integrated Volatility Process, N(0,1), NIG(1,0,m,delta), NIGb(1.2,0.4060,m,delta) , Sample Size: 200 obs., Replications: 2000, Nominal Size: 5%, d1 = d2 = 0.4,

Cheung & Ng	1	2	5	10	30	50	80	120
Standard, **N(0,1)**	5,55	5,30	5,70	5,25	4,25	2,30	1,20	0,10
Modified	5,75	5,70	6,40	6,20	7,80	9,90	13,20	19,45
Normal	6,85	5,25	6,00	5,35	4,75	3,75	2,75	0,85
Bidirectional	5,85	6,00	5,45	6,30	3,35	1,65	0,10	0,00
Standard, **NIG(1,0,m,delta)**	4,35	5,80	8,25	10,50	10,05	8,30	4,35	1,30
Modified	4,65	5,90	8,80	11,10	15,45	17,50	22,45	26,70
Normal	5,65	6,15	6,75	5,45	5,30	5,40	2,90	1,50
Bidirectional	7,00	8,00	9,50	11,50	8,35	4,45	0,30	0,00
Standard, **NIGb(1.2,0.4060,m,delta)**	5,30	6,50	9,10	10,50	11,00	9,65	6,15	2,25
Modified	5,50	6,70	9,40	11,05	15,50	16,65	22,30	28,60
Normal	6,70	6,35	5,45	6,00	4,45	4,70	3,60	2,50
Bidirectional	7,40	9,15	11,45	12,50	10,65	6,35	0,65	0,00

Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	7,25	7,70	8,55	7,50	8,65	9,85	12,35	17,50
Bartlett	-	7,25	7,65	7,50	8,05	8,05	8,25	9,35
Daniel	-	7,05	7,75	7,80	7,90	8,05	8,70	10,00
Quadratic	7,15	7,30	7,55	8,20	7,85	8,40	9,55	12,30
Tukey	-	7,25	7,50	7,30	7,90	7,95	8,35	9,40
Truncated, **NIG(1,0,m,delta)**	5,50	6,85	10,35	12,25	16,10	17,70	22,20	25,85
Bartlett	-	5,50	7,35	9,60	12,90	13,95	15,95	17,80
Daniel	-	5,60	7,50	9,80	13,60	15,15	16,90	18,95
Quadratic	5,75	5,90	8,20	10,90	14,05	15,65	18,30	20,75
Tukey	-	5,50	6,85	9,15	12,95	14,15	15,75	18,30
Truncated, **NIGb(1.2,0.4060,m,delta)**	6,10	8,00	10,70	12,35	16,25	17,10	22,15	27,20
Bartlett	-	6,10	8,35	10,20	13,15	14,60	16,60	17,55
Daniel	-	6,10	8,70	10,50	13,55	15,55	17,10	18,90
Quadratic	6,10	6,45	9,00	11,35	14,65	16,70	17,45	20,90
Tukey	-	6,10	8,05	10,10	13,50	15,25	16,65	17,10

Modified Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	7,25	7,60	8,50	7,60	8,60	10,10	13,45	19,30
Bartlett	-	7,25	7,65	7,50	8,05	8,15	8,40	9,90
Daniel	-	7,00	7,75	7,85	8,15	8,35	9,30	11,90
Quadratic	7,15	7,30	7,50	8,15	7,85	8,50	9,95	14,50
Tukey	-	7,25	7,50	7,40	7,95	8,00	8,55	9,60
Truncated, **NIG(1,0,m,delta)**	5,50	6,90	10,30	12,10	16,05	17,90	22,45	26,45
Bartlett	-	5,50	7,35	9,65	13,10	14,30	16,00	18,45
Daniel	-	5,60	7,55	9,85	13,65	15,75	18,50	21,20
Quadratic	5,75	5,85	8,20	11,05	14,25	15,90	18,75	23,90
Tukey	-	5,50	6,85	9,35	13,05	14,35	16,05	18,25
Truncated, **NIGb(1.2,0.4060,m,delta)**	6,10	8,00	10,70	12,30	16,25	17,05	22,35	28,40
Bartlett	-	6,10	8,35	10,20	13,05	14,70	16,85	18,70
Daniel	-	6,10	8,65	10,45	14,00	15,65	17,40	21,75
Quadratic	6,10	6,45	9,05	11,35	15,05	16,75	18,65	23,25
Tukey	-	6,10	8,05	10,10	13,55	15,45	16,75	17,80

Bidirect. Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	6,75	7,70	7,50	8,35	7,65	8,50	10,00	8,90
Bartlett	-	6,75	7,15	7,90	8,60	8,35	8,45	8,35
Daniel	-	6,90	7,45	7,65	8,50	8,65	8,45	8,45
Quadratic	6,55	6,60	7,80	7,85	8,20	8,70	8,10	8,70
Tukey	-	6,75	7,20	7,65	8,75	8,15	8,70	8,15
Truncated, **NIG(1,0,m,delta)**	7,60	8,75	11,35	13,95	15,90	14,75	16,65	12,25
Bartlett	-	7,60	8,65	10,70	13,50	13,75	13,95	13,70
Daniel	-	7,25	9,05	11,45	13,50	13,90	13,80	13,80
Quadratic	7,45	7,30	9,65	12,65	13,80	14,25	13,95	13,60
Tukey	-	7,60	8,35	10,70	13,40	13,90	14,25	14,25
Truncated, **NIGb(1.2,0.4060,m,delta)**	6,70	9,80	12,85	15,05	16,60	17,20	18,15	12,45
Bartlett	-	6,70	9,80	12,20	15,35	15,80	15,90	15,95
Daniel	-	7,00	10,25	12,55	15,35	15,65	15,90	15,50
Quadratic	6,65	7,00	11,70	13,20	15,90	15,85	16,45	15,25
Tukey	-	6,70	9,35	12,15	14,90	16,05	15,75	16,55

Lagrange Mult.	
N(0,1)	10,15
NIG(a,0,m,delta)	9,95
NIGb(a,b,m,delta)	9,00

Notes: DGP: VAR(1) - FIGARCH(1,d,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. NIG: Normal Inverse Gaussian Distribution

TABLE 4.3

Empirical Size, Causality in Variance Tests, Fractionally Integrated Volatility Process, N(0,1), NIG(1,0,m,delta), NIGb(1.2,0.4060,m,delta) , Sample Size: 1000 obs., Replications: 2000, Nominal Size: 5%, d1 = d2 = 0.4,

Cheung & Ng	**1**	**2**	**5**	**10**	**30**	**50**	**80**	**120**
Standard, **N(0,1)**	4,90	5,05	4,75	4,50	4,70	3,95	2,45	1,75
Modified	4,90	5,05	4,80	4,65	5,50	5,40	4,75	6,15
Normal	5,55	5,20	6,30	5,60	4,70	5,75	3,90	4,20
Bidirectional	5,05	5,70	5,75	5,55	3,95	3,65	3,10	1,70
Standard, **NIG(1,0,m,delta)**	4,60	5,35	7,50	9,05	10,10	10,70	9,95	8,65
Modified	4,65	5,35	7,60	9,20	11,00	12,50	13,30	14,70
Normal	6,55	5,70	5,00	6,00	5,65	5,50	6,05	5,30
Bidirectional	6,25	7,90	9,10	9,65	10,20	9,45	8,55	6,35
Standard, **NIGb(1.2,0.4060,m,delta)**	4,50	5,25	7,90	10,05	12,05	12,80	11,95	10,15
Modified	4,50	5,40	7,95	10,20	12,95	14,25	15,25	15,45
Normal	5,80	6,35	5,50	6,40	5,45	6,30	4,90	4,85
Bidirectional	6,60	7,70	9,50	11,70	13,20	13,40	12,40	9,80

Hong	**1**	**2**	**5**	**10**	**30**	**50**	**80**	**120**
Truncated, **N(0,1)**	6,50	7,25	6,25	5,95	6,45	6,25	5,80	6,90
Bartlett	-	6,50	6,55	6,50	5,85	5,95	6,35	6,60
Daniel	-	6,40	6,70	6,10	5,95	6,00	6,25	6,55
Quadratic	6,35	6,10	6,30	6,25	6,00	6,10	6,55	6,05
Tukey	-	6,50	6,80	6,30	5,90	5,85	6,05	6,65
Truncated, **NIG(1,0,m,delta)**	5,65	6,55	9,10	10,60	12,50	13,50	14,25	15,20
Bartlett	-	5,65	6,80	8,30	10,65	11,60	12,65	13,30
Daniel	-	5,55	6,85	8,95	10,95	12,15	12,55	13,20
Quadratic	5,75	5,70	7,80	9,25	11,60	12,85	13,05	14,40
Tukey	-	5,65	6,35	8,15	10,50	11,75	12,85	13,00
Truncated, **NIGb(1.2,0.4060,m,delta)**	5,15	7,20	9,30	11,55	14,00	15,35	16,40	16,25
Bartlett	-	5,15	7,25	8,60	12,20	13,40	14,25	15,15
Daniel	-	5,30	7,30	9,05	12,60	13,65	14,60	15,60
Quadratic	5,05	5,40	8,15	9,60	13,50	14,30	15,40	16,00
Tukey	-	5,15	6,65	8,50	11,75	13,65	14,35	15,30

Modified Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	6,50	7,25	6,20	5,95	6,45	6,20	5,85	7,00
Bartlett	-	6,50	6,55	6,50	5,85	5,95	6,45	6,60
Daniel	-	6,40	6,65	6,10	5,90	6,15	6,50	6,70
Quadratic	6,35	6,10	6,30	6,25	6,00	6,15	6,55	6,00
Tukey	-	6,50	6,80	6,30	5,90	5,85	6,05	6,65
Truncated, **NIG(1,0,m,delta)**	5,65	6,60	9,10	10,60	12,65	13,35	14,05	14,85
Bartlett	-	5,65	6,80	8,35	10,75	11,60	12,70	13,35
Daniel	-	5,55	6,85	8,95	11,00	12,25	12,45	13,20
Quadratic	5,75	5,70	7,80	9,30	11,55	12,85	13,15	14,55
Tukey	-	5,65	6,35	8,25	10,50	11,75	12,85	13,00
Truncated, **NIGb(1.2,0.4060,m,delta)**	5,15	7,20	9,30	11,55	13,85	15,50	16,55	16,00
Bartlett	-	5,15	7,25	8,60	12,25	13,35	14,35	15,35
Daniel	-	5,30	7,30	9,10	12,55	13,65	14,80	15,70
Quadratic	5,05	5,40	8,15	9,65	13,60	14,20	15,35	16,55
Tukey	-	5,15	6,70	8,50	11,80	13,70	14,25	15,20

Bidirect. Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	7,55	7,35	6,80	7,50	5,85	6,40	6,20	7,30
Bartlett	-	7,55	6,80	6,70	6,75	6,50	6,70	6,05
Daniel	-	7,25	6,50	7,10	6,90	7,05	6,20	6,20
Quadratic	7,45	7,65	7,00	7,25	6,80	6,65	6,30	6,15
Tukey	-	7,55	7,25	6,90	6,70	6,95	6,65	6,20
Truncated, **NIG(1,0,m,delta)**	6,45	9,10	10,50	10,75	12,30	12,10	13,25	14,45
Bartlett	-	6,45	8,50	10,30	12,10	12,30	12,60	13,00
Daniel	-	6,55	8,95	10,55	11,80	12,70	12,70	12,45
Quadratic	6,65	6,80	9,65	10,55	12,25	12,60	12,60	13,35
Tukey	-	6,45	8,25	10,10	11,65	12,35	12,60	12,80
Truncated, **NIGb(1.2,0.4060,m,delta)**	7,00	8,70	10,75	12,60	15,10	17,35	17,70	17,25
Bartlett	-	7,00	8,85	10,30	12,95	14,15	15,10	15,85
Daniel	-	6,80	9,10	10,65	13,60	14,45	15,40	17,10
Quadratic	7,10	7,20	9,35	11,50	14,30	15,10	16,40	17,20
Tukey	-	7,00	8,60	10,15	13,30	14,15	15,10	16,05

Lagrange Mult.	
N(0,1)	6,70
NIG(1,0,m,delta)	6,95
NIGb(1.2,0.4060,m,delta)	6,65

Notes: DGP: VAR(1) - FIGARCH(1,d,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. NIG: Normal Inverse Gaussian Distribution

TABLE 4.4

Empirical Power, Causality in Variance Tests, Fractionally Integrated Volatility Process, N(0,1), NIG(1,0,m,delta), NIGb(1.2,0.4060,m,delta) , Sample Size: 200 obs., Replications: 2000, Nominal Size: 5%, d1 = d2 = 0.4,

Cheung & Ng	1	2	5	10	30	50	80	120
Standard, **N(0,1)**	7,10	9,50	14,70	17,90	18,35	12,55	5,50	1,05
Modified	7,15	9,75	15,55	21,00	25,60	26,20	24,65	22,50
Normal	9,10	10,75	12,50	11,45	6,10	4,25	2,60	1,25
Bidirectional	8,75	9,90	17,40	22,55	21,45	13,65	3,35	0,15
Standard, **NIG(1,0,m,delta)**	5,25	8,55	13,15	16,70	19,75	15,00	8,20	2,25
Modified	5,30	8,80	13,65	18,00	26,30	25,50	25,10	26,15
Normal	7,05	10,20	9,60	8,85	6,00	5,15	2,00	1,70
Bidirectional	8,55	12,95	18,65	22,80	22,30	14,85	3,30	0,05
Standard, **NIGb(1.2,0.4060,m,delta)**	4,30	6,85	12,55	18,45	20,05	17,15	10,00	3,15
Modified	4,30	7,00	13,45	19,60	25,70	25,75	27,65	27,45
Normal	5,90	8,35	10,25	8,75	6,20	4,20	3,60	1,40
Bidirectional	7,00	9,20	15,75	23,45	23,40	15,50	3,60	0,05

Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	8,40	12,15	18,00	23,25	27,30	27,55	26,30	24,85
Bartlett	-	8,40	11,55	16,05	24,70	27,50	28,95	28,55
Daniel	-	9,00	12,35	16,75	26,05	28,60	29,15	28,50
Quadratic	8,70	9,00	13,85	18,90	27,00	29,10	28,75	27,25
Tukey	-	8,40	11,05	15,40	24,65	27,30	29,20	28,90
Truncated, **NIG(1,0,m,delta)**	6,15	10,30	15,45	19,90	27,25	26,45	26,70	27,35
Bartlett	-	6,15	11,00	14,85	21,85	24,45	26,75	27,85
Daniel	-	6,40	11,80	15,75	23,10	25,65	27,70	27,40
Quadratic	6,25	7,40	13,00	17,45	24,40	27,50	27,70	28,15
Tukey	-	6,15	10,10	14,50	21,85	24,60	27,50	27,55
Truncated, **NIGb(1.2,0.4060,m,delta)**	5,05	8,30	15,50	21,70	26,90	27,35	28,90	28,95
Bartlett	-	5,05	8,85	13,40	22,90	25,10	27,65	28,10
Daniel	-	5,95	9,50	14,75	23,80	26,05	28,20	28,15
Quadratic	5,05	5,70	10,90	17,40	25,00	27,95	28,20	28,60
Tukey	-	5,05	8,55	12,70	22,65	25,40	27,95	28,15

Modified Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	8,40	12,15	18,05	23,20	26,85	26,80	24,85	22,30
Bartlett	-	8,40	11,55	16,20	24,85	27,60	29,25	28,10
Daniel	-	9,00	12,40	16,85	26,30	28,25	28,50	27,70
Quadratic	8,70	9,00	13,85	18,90	27,05	29,40	28,55	26,40
Tukey	-	8,40	11,05	15,40	24,65	27,90	29,40	28,75
Truncated, **NIG(1,0,m,delta)**	6,15	10,30	15,45	20,05	27,10	25,95	25,15	25,95
Bartlett	-	6,15	11,05	14,95	22,15	24,95	27,05	27,55
Daniel	-	6,35	11,80	15,85	23,05	25,75	27,25	27,60
Quadratic	6,25	7,40	13,00	17,55	24,55	27,50	27,30	28,15
Tukey	-	6,15	10,15	14,60	22,10	24,90	27,50	27,50
Truncated, **NIGb(1.2,0.4060,m,delta)**	5,05	8,30	15,50	21,65	26,75	26,45	27,75	27,30
Bartlett	-	5,05	8,85	13,55	23,25	25,20	27,75	28,10
Daniel	-	5,95	9,55	14,85	23,95	26,35	28,15	28,20
Quadratic	5,05	5,70	10,95	17,55	25,25	28,00	28,00	29,30
Tukey	-	5,05	8,55	12,80	22,90	25,45	28,10	28,40

Bidirect. Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	9,70	12,60	19,65	27,80	34,85	33,30	29,30	21,25
Bartlett	-	9,70	12,25	18,30	30,85	34,40	35,50	34,60
Daniel	-	10,05	13,25	20,20	32,70	35,50	35,15	33,10
Quadratic	9,75	10,00	14,50	23,55	34,20	35,70	34,05	30,70
Tukey	-	9,70	11,35	16,75	30,85	34,65	35,90	34,70
Truncated, **NIG(1,0,m,delta)**	8,80	12,65	20,35	26,00	33,80	32,95	30,40	20,85
Bartlett	-	8,80	12,95	19,30	29,70	32,70	34,15	33,35
Daniel	-	9,15	13,90	21,30	30,60	33,65	34,15	32,70
Quadratic	9,00	9,40	16,65	23,15	33,05	34,60	33,65	30,30
Tukey	-	8,80	11,95	18,65	29,30	33,45	34,70	33,70
Truncated, **NIGb(1.2,0.4060,m,delta)**	7,10	9,50	16,40	26,00	32,25	31,75	28,80	20,30
Bartlett	-	7,10	10,25	15,70	26,60	30,10	31,95	31,50
Daniel	-	7,80	11,10	16,55	28,30	31,15	32,00	30,60
Quadratic	7,05	7,50	12,60	20,10	30,40	32,10	31,85	28,80
Tukey	-	7,10	9,75	15,20	26,90	30,40	32,10	32,05

Lagrange Mult.	
N(0,1)	39,65
NIG(1,0,m,delta)	27,00
NIGb(1.2,0.4060,m,delta)	24,30

Notes: DGP: VAR(1) - FIGARCH(1,d,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. NIG: Normal Inverse Gaussian Distribution

TABLE 4.5

Empirical Power, Causality in Variance Tests, Fractionally Integrated Volatility Process, N(0,1), NIG(1,0,m,delta), NIGb(1.2,0.4060,m,delta) , Sample Size: 1000 obs., Replications: 2000, Nominal Size: 5%, d1 = d2 = 0.4,

Cheung & Ng	1	2	5	10	30	50	80	120
Standard, **N(0,1)**	73,10	91,50	99,60	99,95	100,00	100,00	100,00	100,00
Modified	73,20	91,60	99,60	99,95	100,00	100,00	100,00	100,00
Normal	82,00	84,15	84,20	83,10	79,60	75,15	69,65	57,05
Bidirectional	94,85	98,60	99,95	100,00	100,00	100,00	100,00	100,00
Standard, **NIG(1,0,m,delta)**	26,10	42,45	68,20	84,05	93,55	94,70	94,95	93,75
Modified	26,10	42,60	68,30	84,05	93,60	95,15	95,65	95,25
Normal	34,10	39,05	40,70	39,30	34,45	30,90	26,40	19,75
Bidirectional	47,65	61,60	82,85	93,75	96,90	97,25	97,70	97,05
Standard, **NIGb(1.2,0.4060,m,delta)**	21,85	34,50	59,95	77,25	88,75	91,05	90,30	88,65
Modified	21,85	34,60	60,05	77,60	89,25	91,95	91,25	91,15
Normal	28,15	34,50	36,90	32,95	30,35	27,15	22,75	17,45
Bidirectional	39,00	53,05	76,80	89,05	95,00	95,60	94,95	94,40

Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	76,70	93,30	99,60	99,95	100,00	100,00	100,00	100,00
Bartlett	-	76,70	97,10	99,80	100,00	100,00	100,00	100,00
Daniel	-	89,05	98,70	99,90	100,00	100,00	100,00	100,00
Quadratic	83,35	87,30	99,10	99,90	100,00	100,00	100,00	100,00
Tukey	-	76,70	95,35	99,60	100,00	100,00	100,00	100,00
Truncated, **NIG(1,0,m,delta)**	29,40	45,60	70,90	85,45	94,25	95,25	95,80	95,65
Bartlett	-	29,40	52,50	73,20	91,65	94,10	95,20	95,75
Daniel	-	39,45	61,10	79,95	92,80	94,90	95,35	96,00
Quadratic	32,30	34,95	63,60	82,30	93,60	95,20	95,70	96,15
Tukey	-	29,40	48,15	70,70	90,75	93,75	95,20	95,55
Truncated, **NIGb(1.2,0.4060,m,delta)**	24,40	38,80	63,05	80,00	89,90	92,65	91,80	91,70
Bartlett	-	24,40	44,50	65,10	85,35	89,65	92,20	93,15
Daniel	-	31,45	51,45	69,80	88,30	90,95	92,65	93,10
Quadratic	26,65	28,90	55,65	74,35	89,35	91,95	93,05	93,10
Tukey	-	24,40	40,30	61,35	84,95	89,75	91,90	92,95

Modified Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	76,70	93,30	99,60	99,95	100,00	100,00	100,00	100,00
Bartlett	-	76,70	97,15	99,80	100,00	100,00	100,00	100,00
Daniel	-	89,05	98,70	99,90	100,00	100,00	100,00	100,00
Quadratic	83,35	87,30	99,10	99,90	100,00	100,00	100,00	100,00
Tukey	-	76,70	95,35	99,60	100,00	100,00	100,00	100,00
Truncated, **NIG(1,0,m,delta)**	29,40	45,55	70,90	85,45	94,20	95,25	95,75	95,50
Bartlett	-	29,40	52,60	73,25	91,70	94,20	95,20	95,75
Daniel	-	39,50	61,35	79,95	92,80	94,90	95,40	96,05
Quadratic	32,30	34,95	63,80	82,30	93,60	95,20	95,75	96,15
Tukey	-	29,40	48,15	70,70	90,80	93,75	95,20	95,60
Truncated, **NIGb(1.2,0.4060,m,delta)**	24,40	38,80	63,05	79,95	89,90	92,55	91,65	91,25
Bartlett	-	24,40	44,50	65,10	85,45	89,70	92,15	93,05
Daniel	-	31,70	51,50	69,90	88,40	91,00	92,70	93,20
Quadratic	26,65	28,90	55,70	74,35	89,35	92,10	93,00	93,10
Tukey	-	24,40	40,30	61,40	85,05	89,75	92,00	93,00

Bidirect. Hong	1	2	5	10	30	50	80	120
Truncated, **N(0,1)**	90,70	97,95	99,95	100,00	100,00	100,00	100,00	100,00
Bartlett	-	90,70	99,30	100,00	100,00	100,00	100,00	100,00
Daniel	-	96,75	99,85	100,00	100,00	100,00	100,00	100,00
Quadratic	93,65	95,30	99,90	100,00	100,00	100,00	100,00	100,00
Tukey	-	90,70	98,85	99,95	100,00	100,00	100,00	100,00
Truncated, **NIG(1,0,m,delta)**	41,10	59,30	82,50	93,65	97,20	97,80	98,25	98,00
Bartlett	-	41,10	66,95	84,55	96,25	97,45	97,85	98,40
Daniel	-	52,35	74,45	88,40	96,80	97,80	98,10	98,50
Quadratic	44,90	49,25	77,05	91,40	97,35	97,90	98,40	98,65
Tukey	-	41,10	62,40	82,10	95,90	97,40	97,85	98,30
Truncated, **NIGb(1.2,0.4060,m,delta)**	35,50	50,25	75,90	89,40	95,50	96,35	96,45	96,20
Bartlett	-	35,50	59,40	77,85	93,55	95,90	96,85	97,25
Daniel	-	45,60	66,05	82,80	94,65	96,45	97,00	97,20
Quadratic	38,65	41,55	69,40	86,10	95,55	96,60	97,05	97,20
Tukey	-	35,50	54,70	74,90	93,15	95,75	96,65	97,05

Lagrange Mult.	
N(0,1)	100,00
NIG(1,0,m,delta)	98,10
NIGb(1.2,0.4060,m,delta)	96,70

Notes: DGP: VAR(1) - FIGARCH(1,d,1), Ho: No Volatility Spillover & H1: Bidirectional Symmetric Volatility Spillover in the 1st Lag, The alternative distributional assumptions concern the underlying Residuals Process, The Direction of the Unidirectional Tests is from Series 1 to Series 2, The Normal Cheung & Ng, detects Causality in the Specific Lag. NIG: Normal Inverse Gaussian Distribution

4.5.2 EMPIRICAL RESULTS OF MONTE CARLO DESIGN 4

A) SMALL SAMPLE

1. We must firstly note that the NIG (1, 0, m, delta) is a leptokurtic and symmetric distribution and is used for the investigation of the effects of excess kurtosis in the finite sample properties of the alternative tests when the volatility of the generated series is fractionally integrated. The NIG (1.2, 0.4060, m, delta) is a leptokurtic and positively skewed distribution and is used for studying the combined effects of asymmetry and excess kurtosis in the empirical size and power of the tests.

2. Under the NIID (0,1) distribution the Standard version of Cheung & Ng tests exhibits a decreasing empirical size as the lag length increases. It is impressive that the Modified version of these tests exhibits a different behavior compared with the Standard version and more specifically it demonstrates an increasing distortion in its empirical size with the lag length. The discrepancy in the performance of these tests is more clearly observable for large lag lengths. We also observe that in the 120^{th} lag, the size of Standard version is almost zero while a significant distortion in the empirical size of the Modified test is exhibited. The Normal and Bidirectional versions of the test display also a decreasing empirical size as the lag length increases. Based on the above observations and comparing with the Table 1.1 values in Monte Carlo Design 1 we can conclude that the presence of long memory in the volatility process does not have a serious impact in the empirical size of the Cheung & Ng causality in variance tests.

3. Under the symmetric and leptokurtic NIG distribution we observe an upward distortion in the empirical size of the Cheung & Ng tests that is more pronounced between the 10^{th} and 30^{th} lags for the Standard and Bidirectional versions while the Modified version of the tests exhibits a more persistent distortion that lasts at least until the 120^{th} lag. Finally the Normal version of these tests seems to be robust against leptokurtosis. Under the asymmetric and leptokurtic NIG we do not observe important differentiations from the symmetric case. The Bidirectional version of the Cheung & Ng tests seems to be the most sensitive among the alternative versions to the combined presence of excess kurtosis and positive skewness.

4. In regard with the Hong tests and under the NIID (0,1) hypothesis we observe that the Truncated kernel is more seriously affected from the presence of long memory compared with the rest of the Kernels. This effect is more clearly observable for large lag lengths. The general picture however is that fractional integration in the second order moments does not have any serious impact in the empirical size of the Hong class of tests except from a slight but still negligible upward distortion.

5. This situation changes however when we consider the symmetric and leptokurtic NIG distribution. Once more the Truncated kernel is the least robust against excess kurtosis. We observe that irrespectively of the kernel used the unidirectional tests exhibit an increasing upward distortion in their empirical size as the lag length increases. It is impressive however that the effects of excess kurtosis are almost invisible before the 5^{th} lag. This means that a researcher who uses this type of tests, for studying causal relations in leptokurtic data will be able to get more robust conclusions if he considers only low lags. Thus when working with small samples and leptokurtic data that are characterized from fractional integrated volatility our advice is to consider small lag lengths for the Hong causality in variance tests. We must also note that in the case of the bidirectional version of Hong tests, we also observe an upward size distortion due to excess kurtosis, that is more powerful in the region between the 50^{th} and 80^{th} lag and not in larger lag lengths. This is not surprising however as this version searches for causality in both directions simultaneously.

6. In the case of the skewed and leptokurtic NIG distribution we observe a non uniform pattern in regard with the empirical size of Hong tests. The unidirectional versions of the tests appear to be not seriously influenced from the presence of asymmetry in the distribution. In the case of the bidirectional Hong tests, however we observe an upward distortion in the empirical size of the tests irrespectively of the kernel used. Thus we conclude that the bidirectional Hong test is more sensitive to asymmetries in the distribution.

7. In the case of Lagrange Multiplier tests we observe that the presence of long memory causes an upward distortion in the empirical size. These tests however appear to be robust against leptokurtic and asymmetric distributions.

8. In terms of empirical power we can observe that in the case of long memory and given the small sample size the majority of the tests display a low empirical power irrespectively of the distribution we consider.

9. We must also note that once more the Standard and Modified versions of the Cheung & Ng class of tests exhibit a different behavior under all the alternative distributional assumptions. For example under the NIID (0,1) distribution the empirical power of both versions is maximized around the 30^{th} lag but afterwards the Standard version loses in an exponential rate its empirical power while the Modified version maintains its power even at the 120^{th} lag. This pattern may however be spurious and caused by either the small size of the sample we use or / and by the long memory in the volatility process.

10. Under the symmetric and leptokurtic NIG distribution we observe a strange behavior. The presence of excess kurtosis seems to decrease the empirical power of the Standard and Modified versions of the Cheung & Ng tests until the 10^{th} lag, while the opposite is true when larger lags are considered. The picture is less clear in the case of the Normal and Bidirectional versions of the Cheung & Ng tests. Under the asymmetric and leptokurtic NIG distribution we do not observe any serious differentiations in the empirical power of the Cheung & Ng tests.

11. In regard with the Hong tests and under the symmetric and leptokurtic NIG distribution we observe that the presence of excess kurtosis leads to a slight decrease in the power of the unidirectional tests irrespectively of the kernel used. The bidirectional tests appear to be more robust against leptokurtosis. Under the skewed NIG hypothesis we do not observe any serious changes in the empirical power of the unidirectional tests. We would like however to mention that the presence of skewness appears to have a decreasing effect in the empirical power of the tests until the 30^{th} lag, while in larger lag lengths we observe an increase in the empirical power of the tests due to the asymmetry in the distribution. These effects however may be spurious and due to the use of a small sample size. For the bidirectional tests the picture is more clear leading us to the conclusion that the presence of asymmetry causes a decrease in the empirical power of bidirectional Hong tests. We must finally note that

more robust conclusions will be reached when we also analyze the results for the large sample size.

12. The Lagrange Multiplier test exhibits a clearer pattern. More specifically under the symmetric and leptokurtic NIG distribution we observe a decrease in its empirical power compared with the NIID (0,1) case. If we consider the asymmetric and leptokurtic NIG we will observe a slightly larger decrease in the power of the tests compared with the symmetric case. Thus we can conclude that when using small sample sizes the presence of excess kurtosis can cause a moderate decrease in the empirical power of the Lagrange Multiplier tests while the presence of skewness has a less powerful negative impact.

13. We can define the following 'rankings' for the tests used in this study and for the small sample size.

NIID (0,1) distribution:
1st Lag: The Lagrange Multiplier exhibits the highest empirical power.
30th Lag: The Truncated bidirectional Hong test is the dominant test.
120th Lag: The Bartlett bidirectional Hong test is the best choice.
Symmetric and leptokurtic NIG:
1st Lag: The Lagrange Multiplier exhibits the highest empirical power.
30th Lag: The Truncated bidirectional Hong test is the dominant test.
120th Lag: The Tukey bidirectional test is the best choice.
Asymmetric and leptokurtic NIG:
1st Lag: The Lagrange Multiplier exhibits the highest empirical power.
30th Lag: The Truncated bidirectional Hong test is the dominant test.
120th Lag: The Tukey bidirectional test is the best choice.

B) LARGE SAMPLE

1. In the case of the NIID (0,1) distribution we do not observe any serious distortions in the empirical size of the Cheung & Ng tests. The differentiation in the behavior of the Standard and Modified versions across the various lags still exists but it is less pronounced compared with the small sample case. Under the symmetric and

leptokurtic NIG distribution we observe a clear upward distortion in the empirical size of the Cheung & Ng tests with the exception of the Normal test that appears to be robust against leptokurtosis. This effect lasts at least until the 120th lag for the Standard and Modified versions and until the 80th lag for the bidirectional version of the test. Under the asymmetric and leptokurtic distribution we observe a more powerful upward distortion in the empirical size of the tests compared with the case that only leptokurtosis was present. The least robust version of the Cheung & Ng class of tests against skewness is the Bidirectional one and it is more seriously affected in the region between the 30th and 80th lags.

2. In regard with the Hong class of tests we must say that under the NIID (0,1) hypothesis we do not observe any oversizing under the presence of Long Memory in the volatility process. For comparison reasons the reader can refer to the Monte Carlo Design 1, Table 1.2 where we provide the empirical sizes of the Hong tests under a conventional GARCH process and a NIID (0,1) distribution. He will then possibly discover that under long memory in variance, we can even observe a slight decrease in the oversizing behavior of the Hong tests compared with the No long memory case. Finally we can also note that under the standard normal distribution and a fractionally integrated variance process, the bidirectional version of Hong tests appears to be slightly more oversized that its unidirectional alternative.

3. Under the symmetric and leptokurtic NIG law, we can observe a clear upward distorting effect in the empirical size of both unidirectional and bidirectional Hong tests. This effect is more pronounced after the 5th lag. We can also observe that the empirical size of Hong tests irrespectively of the kernel used is a positive monotonic function of the lag length. Thus we can safely conclude that excess kurtosis affects negatively the performance of Hong tests. The least robust kernels against leptokurtosis are the Truncated and Quadratic. Finally we observe that bidirectional Hong tests appear to be more negatively influenced from excess kurtosis in the region between the 1st and 30th lags.

4. Under the asymmetric and leptokurtic NIG hypothesis we observe an even more powerful upward impact in the empirical size of Hong tests compared with the symmetric NIG case. This empirical finding implies that skewness in the distribution

can have a negative impact in the type I error probability of Hong tests. Once more we can see that the Truncated and Quadratic kernels are the least robust kernels. Finally the bidirectional version of Hong tests appears to be more heavily influenced from asymmetries in the distribution compared with its unidirectional counterparts.

5. In regard with the Lagrange Multiplier test we can observe that despite the slight upward distortion due to the presence of long memory in the volatility Process,this method appears to be robust against leptokurtic and asymmetric data.

6. In terms of power, under the NIID (0,1) distribution the majority of tests exhibit a very high performance. This overpower is partially induced by the presence of long memory in the volatility process of the generated series. From this empirical observation it is obvious that the Cheung & Ng as well as the Hong tests may need to be adjusted in order to be better able to detect the true intensity and time pattern of volatility transmissions. If these tests are used with real data that are characterized with long memory in variance then it is possible that we will find very strong variance causalities even if the true volatility linkage is much weaker. One obvious solution to this problem is to make use of both GARCH and FIGARCH models in the first step of the cross correlation based techniques. Then we should compare the causality in variance results obtained from both types of volatility parameterizations in order to be able to reach more trustful conclusions.

7. In regard with the Cheung & Ng class of tests and under the symmetric and leptokurtic NIG distribution we can clearly observe a decrease in the empirical power of all the alternative versions. The negative impact in the empirical power is more pronounced in low lags while for large lag lengths the distorting effects gradually weaken. In other words the presence of excess kurtosis damages the performance of the Cheung & Ng tests and this effect is more powerful when considering the region between the 1^{st} and 10^{th} lags. When considering the asymmetric and leptokurtic NIG distribution we observe an even more powerful negative impact in the empirical power of the tests compared with the symmetric case. This means that the presence of skewness has also a negative impact in the performance of the Cheung & Ng tests.

8. In regard with the Hong tests we observe that under the symmetric and leptokurtic NIG hypothesis, the empirical power of all the different kernels is negatively affected. The distorting effects are more powerful for low lags and more specifically between the 1st and 10th lags. If we consider the asymmetric NIG distribution we will once more observe a downward movement in the empirical power values of the Hong tests irrespectively of the kernel used. Finally we would like to note that the most robust kernels against leptokurtosis and skewness in terms of power losses are the Truncated and Quadratic.

9. The Lagrange Multiplier tests appear once more to be robust against leptokurtic and asymmetric data. This is empirically demonstrated from the minor decrease in the power of these tests under both alternative NIG distributions.

10. In general we can conclude that in empirical applications where large samples of data (with fractionally integrated variances) are used the presence of excess kurtosis can have a relatively strong negative impact in the empirical power of causality in variance tests. The presence of skewness has also negative effects in the performance of the tests while these effects are not as powerful as in the case of the leptokurtic data.

11. We can define the following 'rankings' for the tests used in this study and for the small sample size.

NIID (0,1) distribution:

1st Lag: The Lagrange Multiplier exhibits the highest empirical power.

30th Lag: Very good performance from many tests

120th Lag: Very good performance from many tests.

Symmetric and leptokurtic NIG distribution:

1st Lag: The Lagrange Multiplier exhibits the highest empirical power.

30th Lag: The bidirectional Quadratic Hong test is the dominant test.

120th Lag: The bidirectional Quadratic Hong test is the best choice.

Asymmetric and leptokurtic NIG distribution:

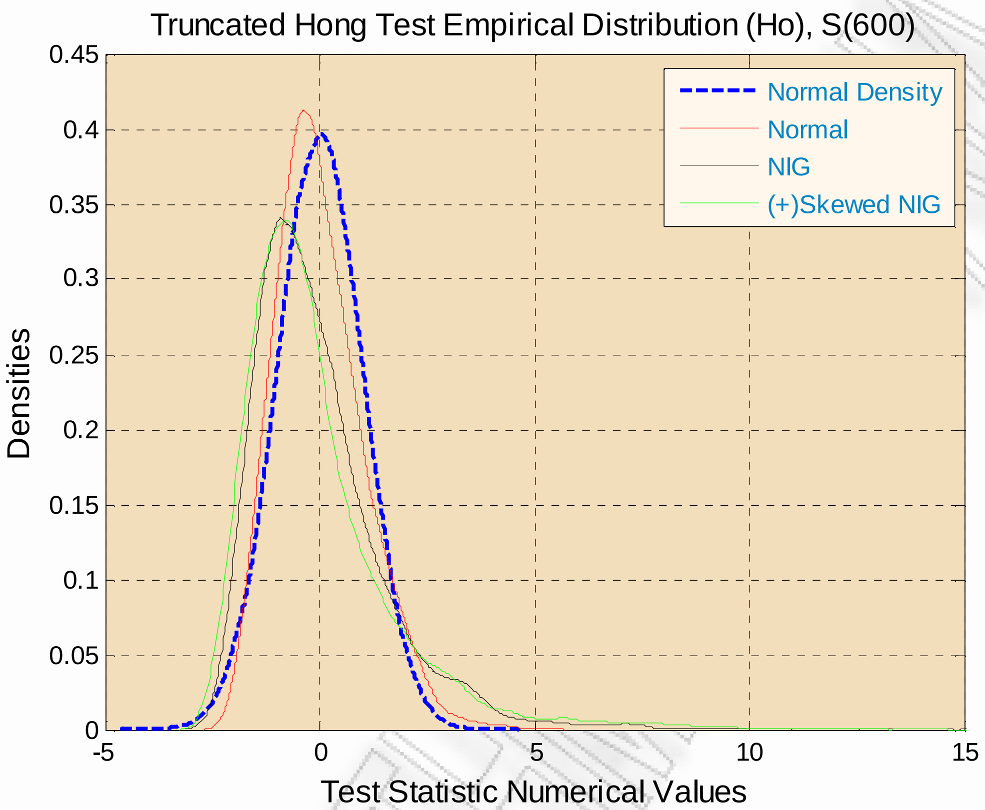
1st Lag: The Lagrange Multiplier exhibits the highest empirical power.

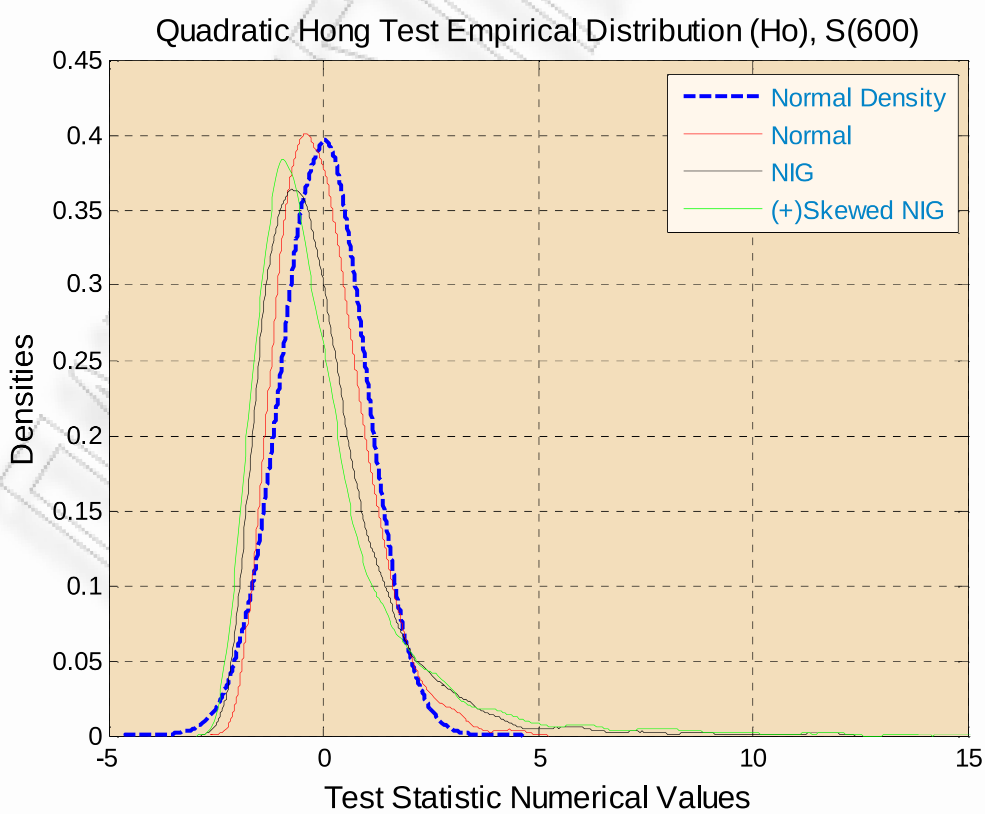
30th Lag: The bidirectional Quadratic Hong test is the dominant test.

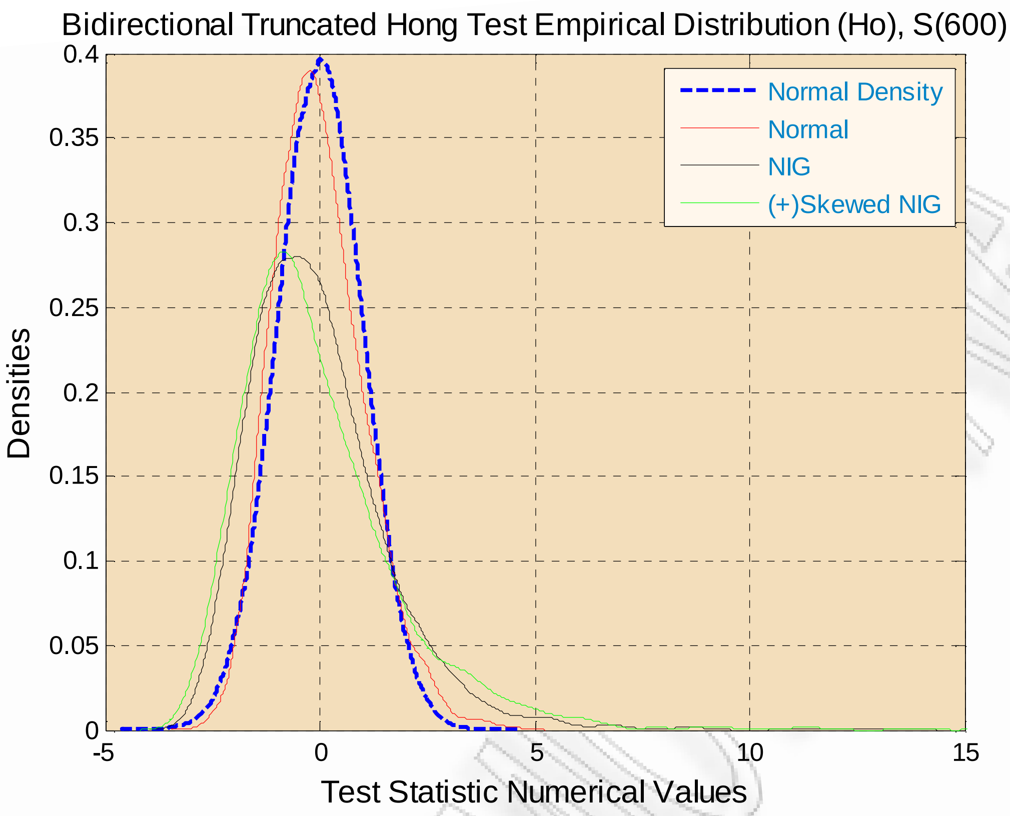
120th Lag: The bidirectional Bartlett Hong test is the best choice.

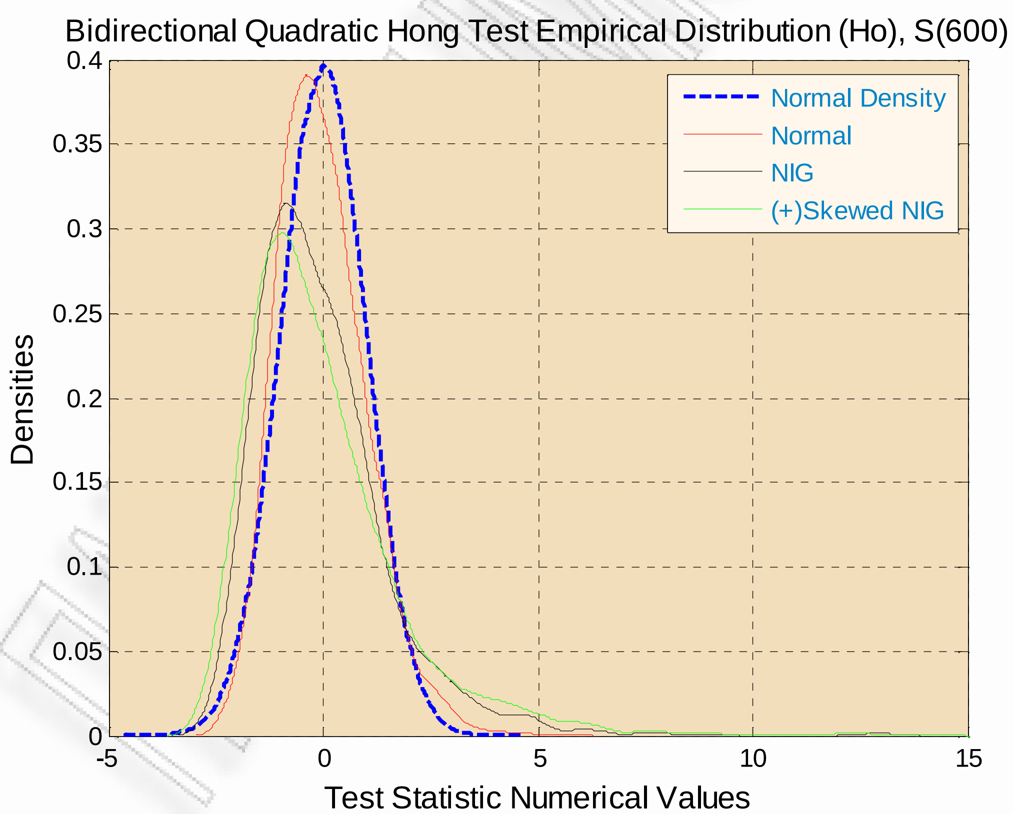
In the following figures we provide the smooth densities of the various causality in variance tests that were calculated by an Epanechnikov kernel using the test's realized values for the sample size of 600 obs. The results using this sample size were not included in the previous tables, as we wanted to depict the two limit cases of a small (200 obs.) and a large sample size (1000 obs.) This sample size however was chosen for the construction of the empirical distributions as it represents a mainstream situation between the two limits and allows us to briefly and concretely demonstrate our empirical results. In the **Appendix B** (Monte Carlo Design 4) you can find a number of additional figures that will help you to visualize the differences among the alternative tests as well as some other aspects of this Design.

Figures 4.1 – 4.4 Unidirectional and Bidirectional Hong Causality in Variance Tests under Long Memory in Volatility and alternative assumptions for the Residuals Process (Bandwidth: 30)

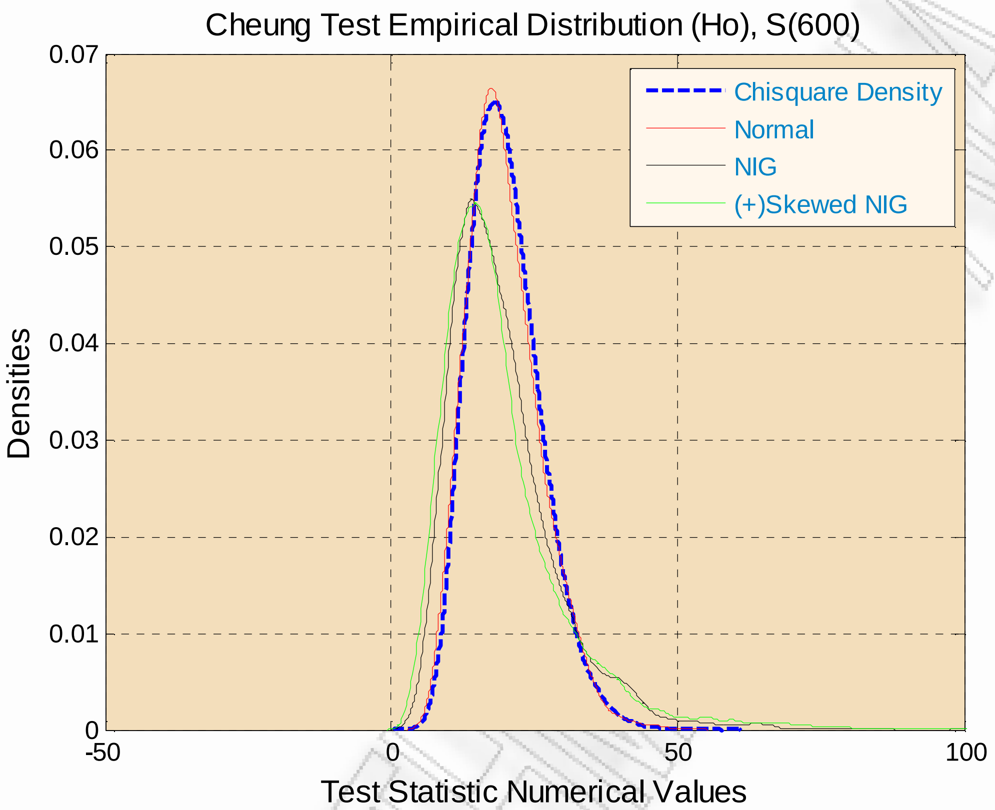


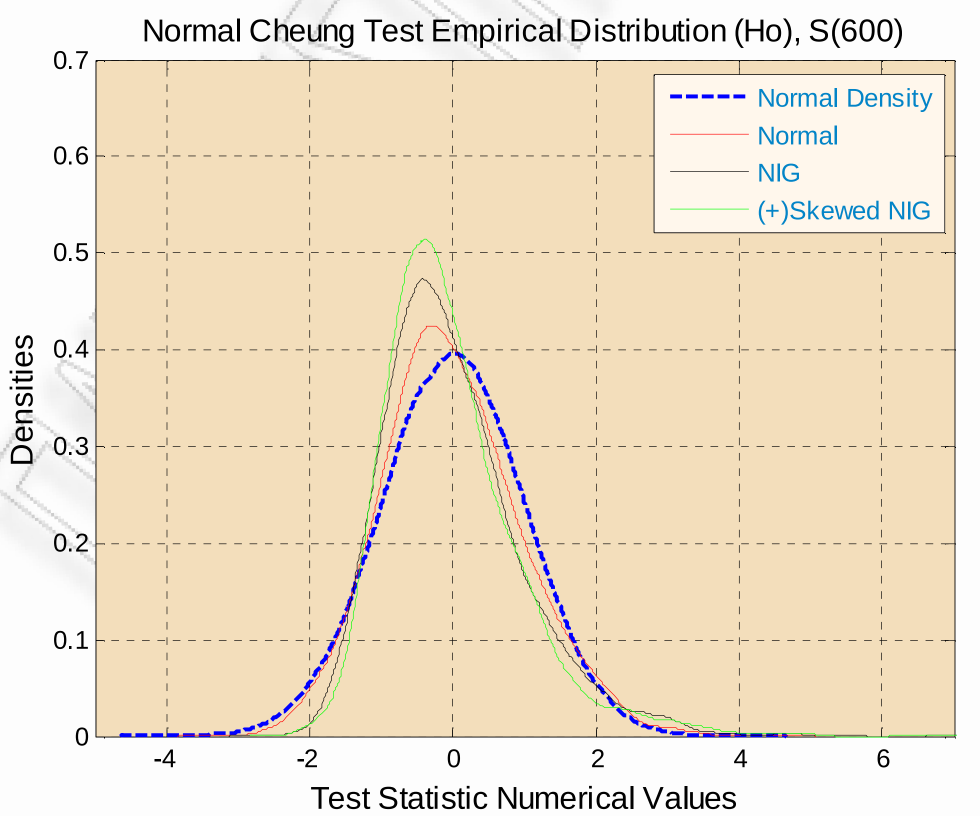



Bidirectional Truncated Hong Test Empirical Distribution (Ho), S(600)
Densities
Test Statistic Numerical Values
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5
10
15
Normal Density
Normal
NIG
(+)Skewed NIG


Bidirectional Quadratic Hong Test Empirical Distribution (Ho), S(600)
Densities
Test Statistic Numerical Values
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5
10
15
Normal Density
Normal
NIG
(+)Skewed NIG

Figures 4.5 – 4.7 Unidirectional and Bidirectional Cheung & Ng Causality in Variance Tests under Long Memory in Volatility and alternative assumptions for the Residuals Process (30^{th} Lag)





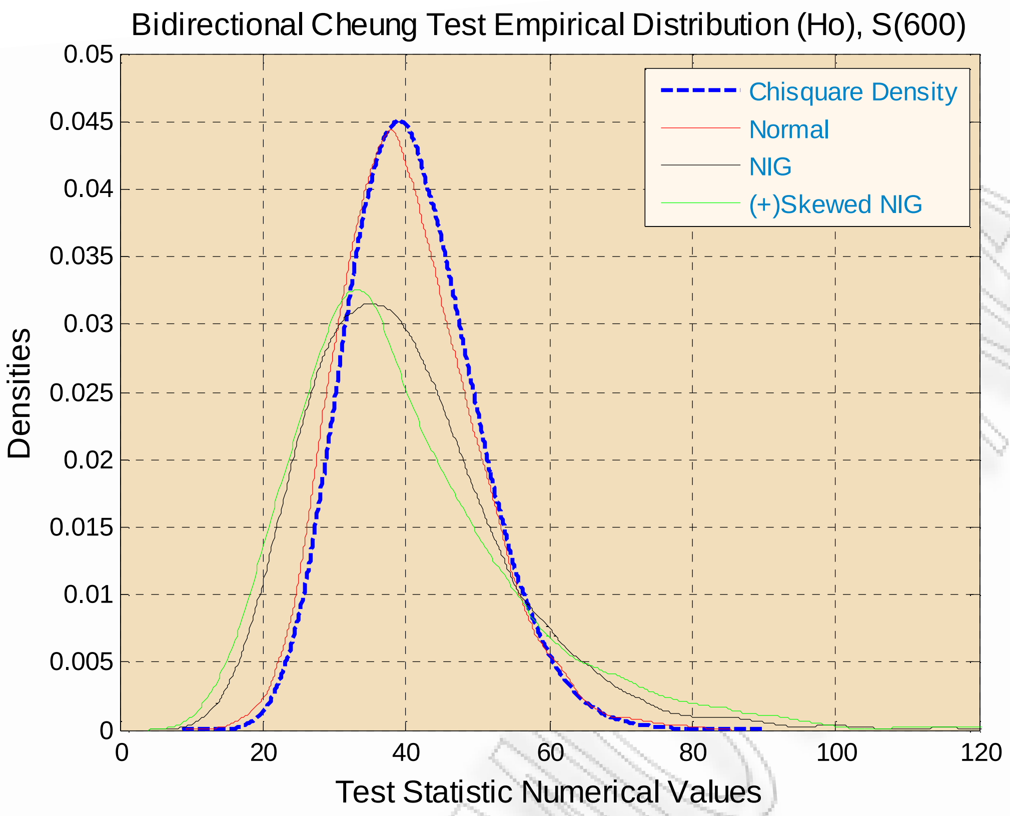
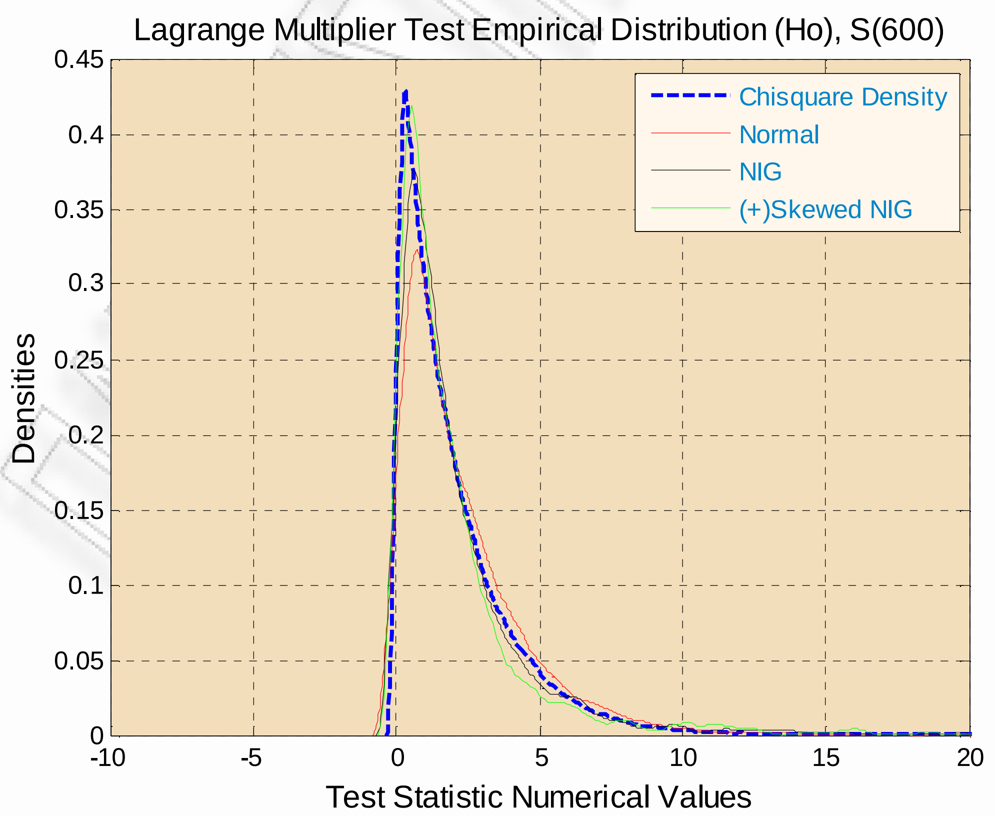
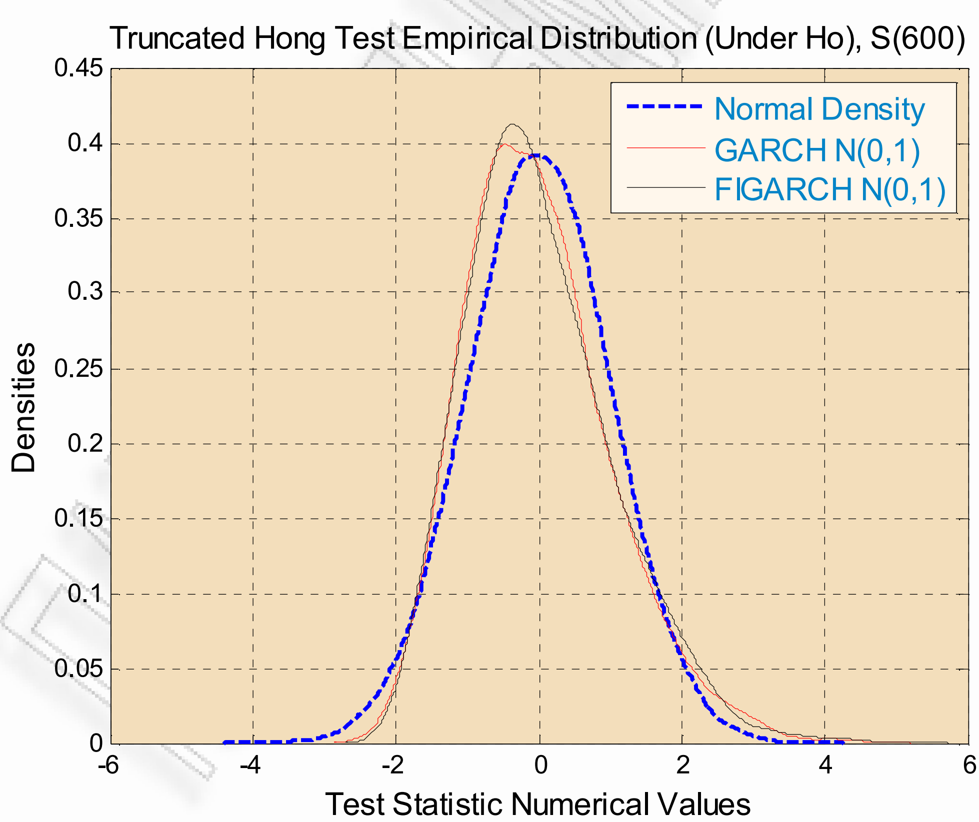


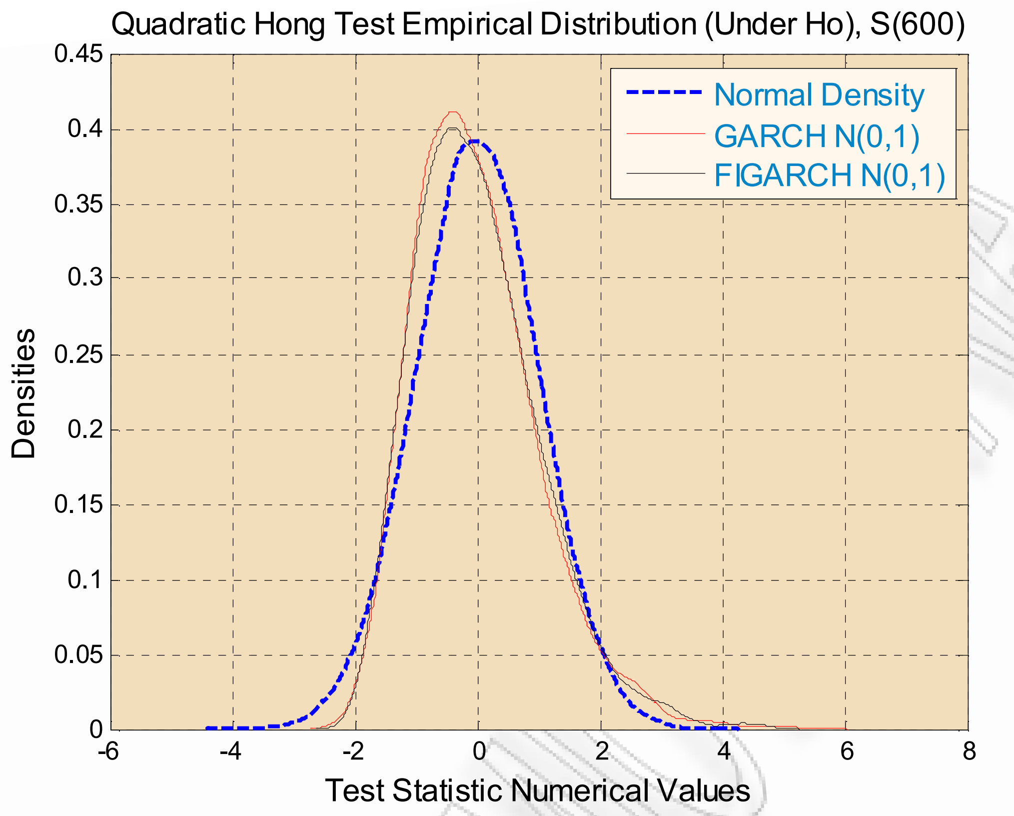
Figure 4.8 LM Causality in Variance Tests under Long Memory in Volatility and alternative assumptions for the Residuals Process (1^{st} Lag)

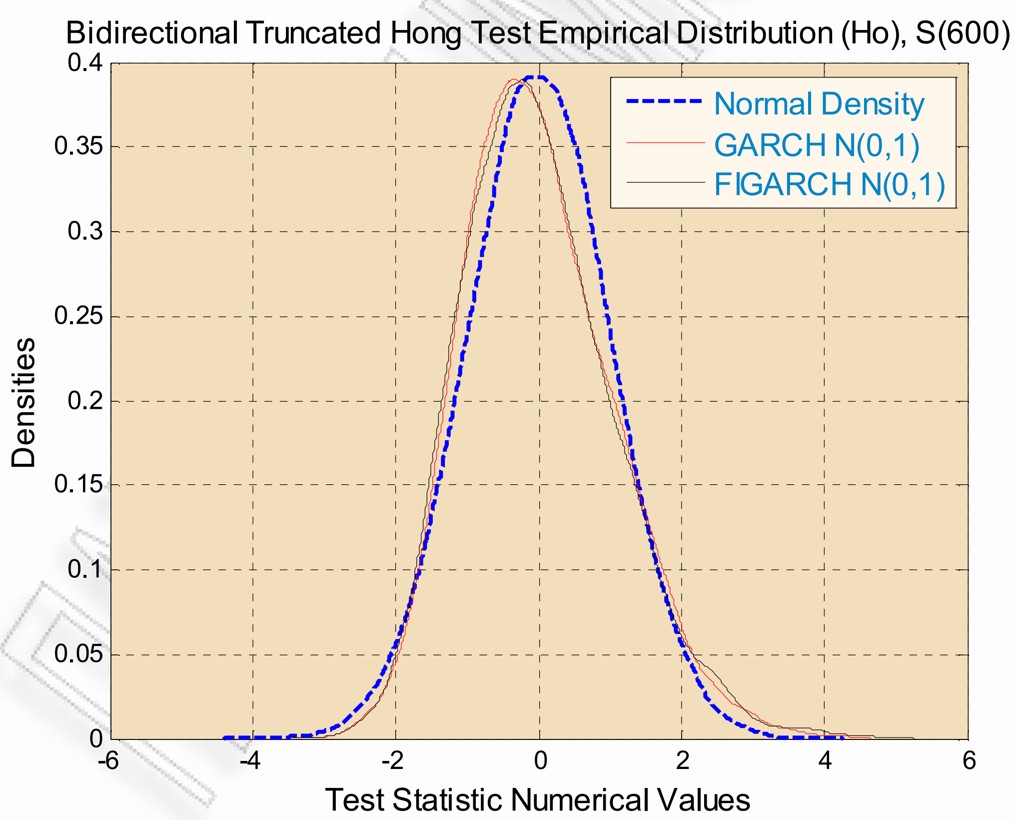


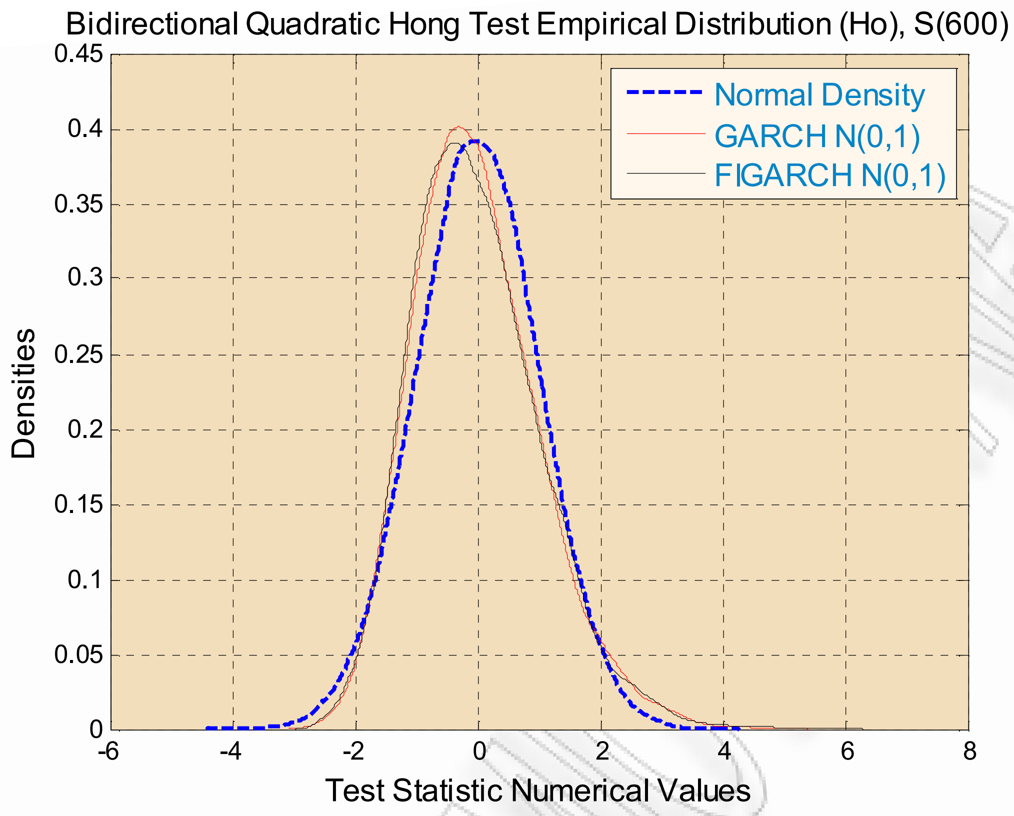
In the following figures we plot the empirical distributions of a number of selected causality in variance tests under the presence or absence of long memory in the volatility process. More specifically we have constructed these distributions using two different Data Generating Processes, a conventional GARCH(1,1) process and an FIGARCH(1,d,1) process. As you will notice for the specific lag we have used (30^{th}) there are not important differences in the two distributions. This means that the existence of fractional integration in the volatility process of the time series used in empirical applications will possibly not have a serious impact in the probability for type I error in the various tests used. We must also note that for these specific Data Generating Processes, a NIID (0,1) distribution was assumed for the underlying Residuals process.

Figures 4.9 – 4.12 Unidirectional and Bidirectional Hong Causality in Variance Tests under GARCH and FIGARCH data generating processes (Bandwidth: 30).

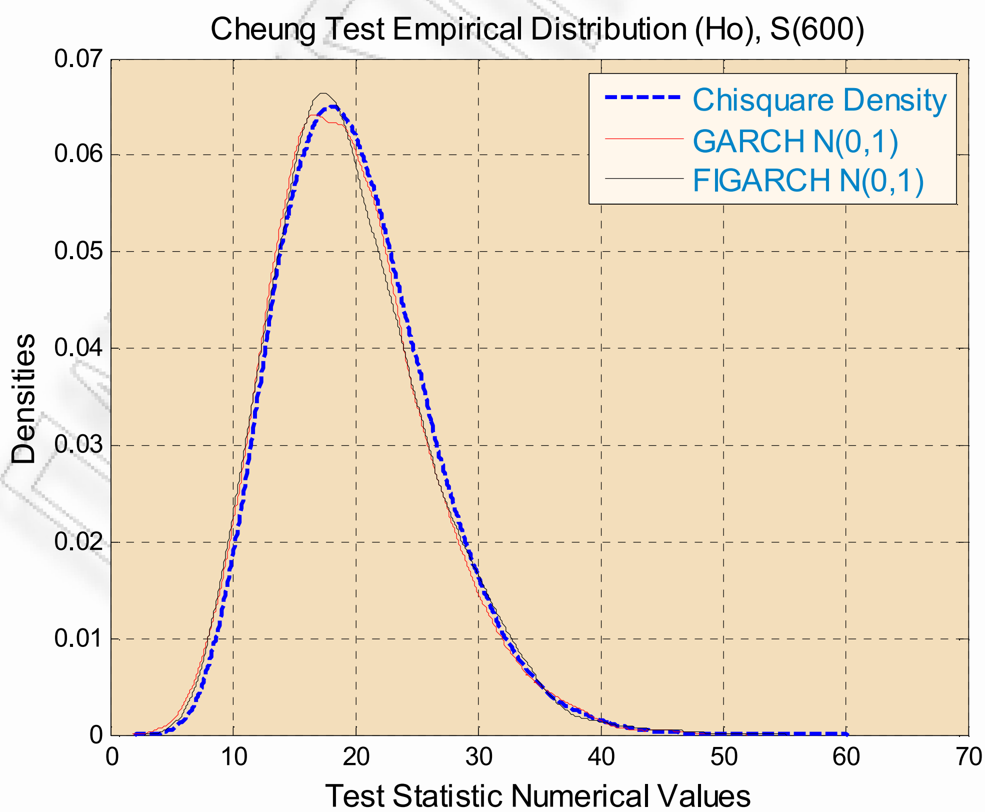


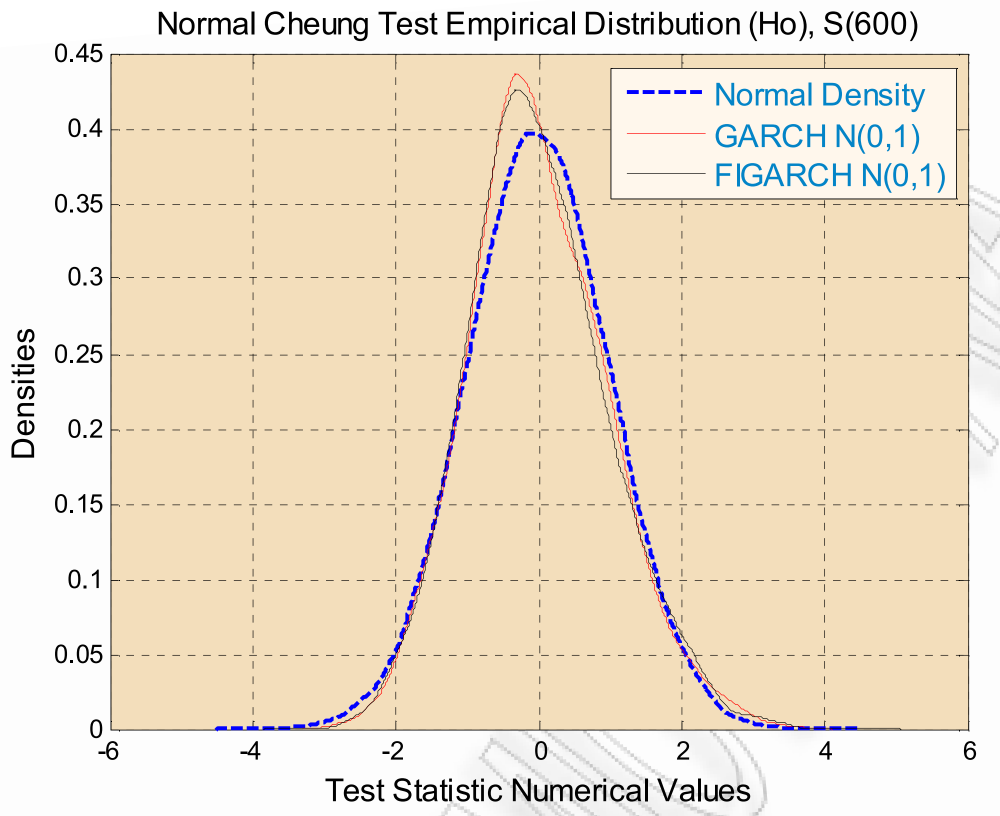

Quadratic Hong Test Empirical Distribution (Under Ho), S(600)
Normal Density
GARCH N(0,1)
FIGARCH N(0,1)
Densities
Test Statistic Numerical Values


Bidirectional Truncated Hong Test Empirical Distribution (Ho), S(600)
Normal Density
GARCH N(0,1)
FIGARCH N(0,1)
Densities
Test Statistic Numerical Values



Figures 4.13 – 4.15 Unidirectional and Bidirectional Cheung & Ng Causality in Variance Tests under GARCH and FIGARCH data generating processes (30th Lag)





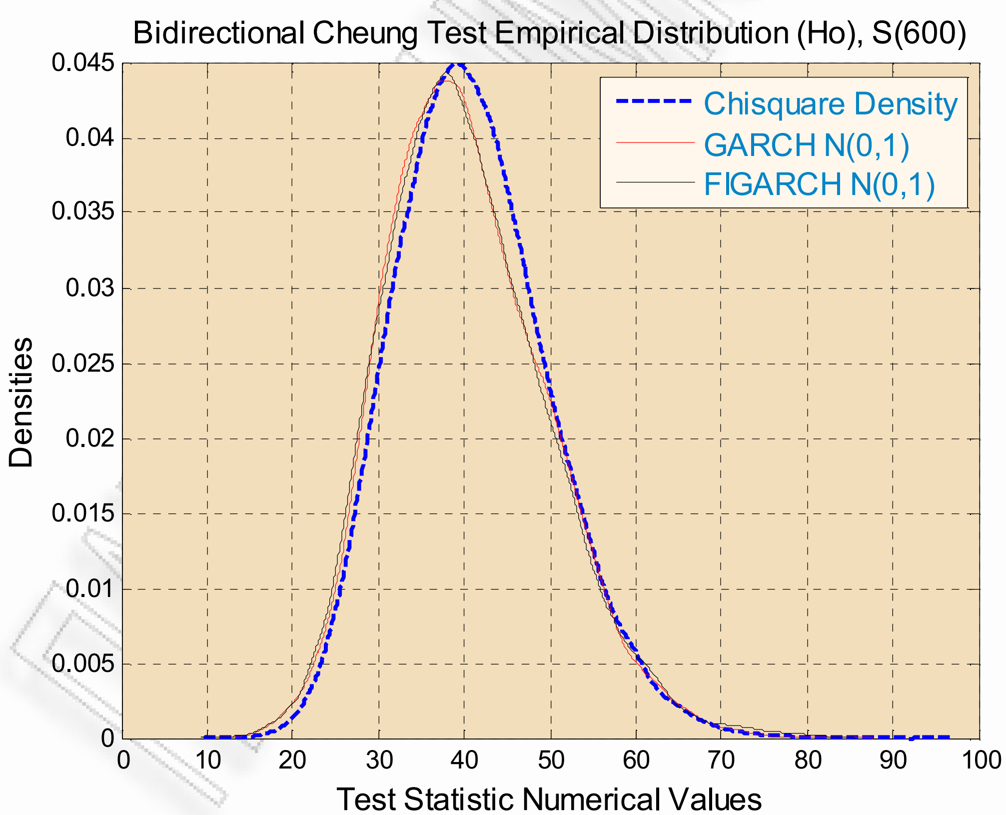


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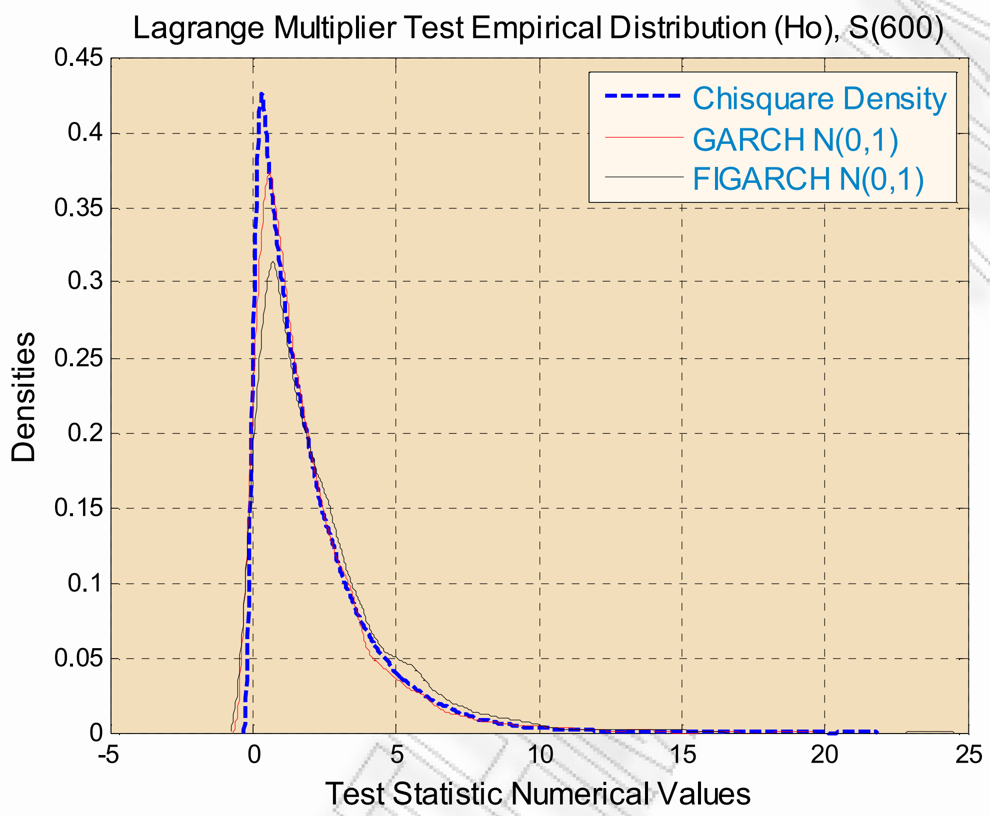
Figure 4.16 LM Causality in Variance Test under Tests under GARCH and FIGARCH data generating processes (1st Lag)



Concluding the presentation of the empirical results of our Monte Carlo simulations we would like to note that in **Appendix C** we provide the results of an empirical application using real stock market data. The purpose of this application was to investigate whether there exists long memory in the volatility process of the returns of five important stock market indices. Also in **Appendix D** we provide the results of three additional Monte Carlo experiments. In the first one we study the effects of a Normal Inverse Gaussian distribution in the performance of causality in variance tests when the Data Generating Process is the conventional VAR – GARCH process (No Long Memory). In the second one we perform a comparative study of the performance of unidirectional and bidirectional tests under unidirectional and bidirectional variance causality structures. Finally in the third design we investigate the effects of residual autocorrelation in the finite sample properties of the tests.

CONCLUSION

In the first part of this study we have analyzed the concepts of volatility, causality in variance and long memory. The empirical findings of a large number of studies have also been presented. In this way we have provided a solid theoretical foundation that can function as a reference for the rest of this study. As a next step we have conducted a large number of Monte Carlo simulations in order to perform an in depth inquiry in the empirical properties of some specific causality tests. Our simulation work was organized in four discrete parts. In the first one we have investigated whether the different distributional assumptions for the errors could have an impact in the causality in variance tests. In the next part we have analyzed the effects of neglected causality in mean in the empirical size of causality in variance tests. In the third part we have studied the effects of volatility spillovers and GARCH in Mean effects in the empirical size of causality in mean tests. In the last part of our study we have analyzed the effects of long memory in volatility and of Normal Inverse Gaussian distributed errors, in the finite sample properties of causality in variance tests. Summarizing our work we can report the following:

1. The presence of excess kurtosis (under a no long memory framework) causes a moderate upward distortion in the empirical size of the vast majority of causality in variance tests. The only test that seems to be robust against leptokurtosis is the Normal version of the Cheung & Ng class of tests.

2. The effects of skewness (positive or negative) have also size distorting effects. The similarity in the effects of positive and negative asymmetry can be attributed to the use of the squares of the standardized residuals in all the tests.

3. The Hong tests irrespectively of the kernel function used, have a tendency to over reject the null hypothesis compared with their more simple Cheung & Ng counterparts.

4. The empirical power of all the tests irrespectively of the distribution we consider, increases for larger sample sizes. This practically means that causality in variance / mean tests will work better for rich data sets that are usually available only in empirical applications in finance and not in macroeconomics.

5. Another important empirical discovery is the serious upward distortion in the empirical size of causality in variance tests when we do not account for the presence of causal interdependence in the first order moments and this effect is more pronounced in large sample sizes.

6. We have also observed that the presence of volatility spillovers has an inflationary but relatively weak impact on the empirical size of causality in mean tests. The impact of GARCH in Mean effects in the empirical size of causality in mean tests was not as powerful as expected and was more pronounced in small samples.

7. We have observed that the presence of long memory in the volatility process does not have a serious impact in the empirical size of causality in variance tests while it causes a slight increase in the empirical power of these tests. The latter effect may however be spuriously induced from the sample size choice.

8. We have also discovered that the presence of either excess kurtosis or asymmetry in the distribution of errors (as demonstrated from the use of a Normal Inverse Gaussian Distribution) and under fractionally integrated volatility processes has a negative (upward distorting) impact in the empirical size of the causality in variance tests with the leptokurtosis appearing to have a more powerful effect than asymmetry.

9. The presence of asymmetry and excess kurtosis under fractionally integrated volatility processes has a negative impact (decrease) in the empirical power of the various tests that is more pronounced when considering small lag lengths.

10. The Lagrange Multiplier test appears to be robust against leptokurtic and asymmetric data while it is more heavily influenced from the presence of long memory in the volatility processes compared with the rest of the tests.

Concluding our study we would like to note that an interesting field for future research would be the calculation of the finite sample properties of some more sophisticated causality in variance tests such as those that are based on a Multivariate GARCH framework. Methodologies that make use of such models are frequently used in literature. It would also be particularly useful to modify the parameterization used for the Data Generating Processes to a Multivariate setting for the long memory volatility process. This transformation would possibly give a better insight in the effects of fractional integration in the size and power of causality in variance tests. Finally among our aims is to investigate the empirical properties of the Vector Autoregression based **Granger** (1969) causality in mean test that has constituted the cornerstone for the development of the casaulity literature and we also seek to examine the performance of this class of tests under the presence of volatility spillovers.

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