Estimating Betas in Thinner Markets: The Case of the Athens Stock Exchange

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Abstract

This paper examines the impact of the return interval on the beta estimate known as the "interval effect" which causes securities that are thinly traded to give biased OLS beta estimates. The present study covers a 5-year period, from January 2002 through to December 2006, using three different return intervals: daily, weekly and monthly data for 60 continuously listed thinly-traded stocks on the main market of the Athens Stock Exchange. Results generally support findings from earlier studies [(Diacogiannis, Makri - 2008) & (Brailsford, Josev - 1997)] that beta estimates rise as the return interval is lengthened, yet the effect is not observed for the period chosen given the statistically insignificant differences between all different pairs of return intervals for the mean estimate.

Keywords: Beta, Return interval, Risk, Return

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A. Portfolio Analysis and Management

INTRODUCTION TO INVESTMENTS: What we mean by an *"investor"* can range from an individual to a pension fund, a regular company which purchases types of equity or financial securities to make a profit or hedge itself. Regardless of who the investor is or how simple or complex the investment needs are, he or she should develop a *policy statement* before making long-term investment decisions. The structure of this should be related to the age, financial status, future plans, risk aversion characteristics and needs whether we deal with an individual investor or an institutional. To build a framework for this process, we should take into consideration the investment's objectives and constraints.

A.1 Investment objectives: The investment's objectives are his or her investment goals expressed in terms of both risk and return. It's absolutely necessary that we express the goals not only in terms of returns but also in terms of investment risks, including the possibility of loss, so as to avoid unacceptable high-risk investment strategies. A person's return objective may be stated in terms of an absolute or a relative percentage return, but it may also be stated in terms of a general goal, such as "capital preservation" (earning a return on an investment that is at least equal to the inflation rate), "capital appreciation"(exceeding the inflation rate for a period of time), "current income" (as opposed to capital appreciation) and "total return" (both capital gains & reinvestment of current income).

A.2 Investment constraints: In addition to the investment objective that sets limits on risk and return, certain other constraints affect the investment plan including liquidity needs, the time horizon of the investment, tax factors and certain legal & regulatory constraints. Among these constraints, a close relationship exists. For example investors with long investment horizons generally require less liquidity and can bear greater risk. This is true if we consider that funds will probably not be needed for a long period of time and any losses in the process can be offset by potential earnings in the future.

A.3 The need for a Policy Statement

A policy statement, although it does not guarantee investment success, is a useful tool that guides the investment process. It helps the investor decide on the investment goals after learning about the financial market expectations and the risks of investments. It will prevent him from making inappropriate decisions that will not conform to specific, measurable financial goals. Secondly, it creates a standard by which to judge the performance of the investment. This last one is compared to guidelines specified in the policy statement.
A typical policy statement process includes 4 steps as presented below:

1. Policy Statement Introduction in relation to investment needs & expectations
2. Examination of economic, political & social conditions surrounding the investment
3. Construction of the Investment Plan taking into account the above 2 steps
4. Feedback: Evaluate investment performance

A.4 The Importance of Asset Allocation

A policy statement as presented briefly above, although it is of great value to the overall investment strategy, does not indicate either which specific assets to purchase or the relative proportions of the different asset classes. This is a process attributed to the *Asset Allocation Theory*. It is actually the process of deciding how to distribute an investor's wealth among different asset classes for investment purposes. As an "asset class", we regard a set of securities with similar characteristics & attributes. Asset allocation is not a theory implemented in practice alone. Much of this theory is part of the investor's policy statement and the 4-step procedure stated above.

Having mentioned the importance of developing an investment policy statement before implementing an investment plan, we go on to present the term of "*investment portfolio*" and the characteristics that accompany it. We must bear in mind though that an investor must consider the relationship among the investments to meet the optimum portfolio that will meet his or her investment objectives.

We need to clarify some **general assumptions** of this theory starting with the basic one that an investor wants to maximize the returns from the total set of investment for a given level of risk. The total set of investment includes all assets & liabilities varying from stocks and marketable securities to houses and furniture. The relationship among the returns for assets in the portfolio is important and an investor should not regard it just as a collection of marketable assets.

A.5 Definition of Risk: In everyday life we use the words "*risk*" and "*uncertain*ty" to mean the exact same things. In this paper and maybe for most investors, *risk* can be seen as the *uncertainty of future outcomes.* Portfolio theory assumes that investors are basically *risk-averse*, meaning that given a choice between two assets with equal rates of return, they will select the asset with the lower level of risk and the opposite: Given 2 assets with the same level of risk, a risk-averse investor will choose the one with the higher rate of return. The existence of risk means that the payoff the investor acquires in any investment must be described by a set of outcomes and each probability of occurrence, called "return distribution". We can describe such a distribution by two measures:

- a measure of central tendency, called the expected return
- a measure of risk or dispersion around the mean, called the standard deviation

Investors hold in reality a group or a *portfolio* of assets and not just a single asset, so a great concern arises with the estimation of the above 2 measures given the attributes of the individual assets.

B. MARKOWITZ PORTFOLIO THEORY

The basic portfolio model was developed by Harry Markowitz (1952, 1959) who derived the expected rate of return and a measure of risk for a portfolio of assets. Markowitz showed the meaning of variance to measure portfolio risk and used his theory not only to indicate the importance of diversifying investments to reduce the total risk of a portfolio but also the way to diversify effectively. The Markowitz model is based on several assumptions:

1. Investors consider each investment alternative as being represented by a probability distribution of expected returns over some holding period.
2. Investors maximize one-period expected utility and their utility curves demonstrate diminishing marginal utility of wealth.
3. Investors estimate the risk of the portfolio on the basis of the variability of expected returns.
4. Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and variance of returns only.
5. For a given level of risk, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk.

B.1 EXPECTED RATE OF RETURN

The meaning of the expected value of a random variable is the probability-weighted average of the possible outcomes. Knowing that when we are talking about portfolio return, we are actually talking about a set of securities or assets in general that appear at a certain percentage, we can easily interpret *portfolio return* as a weighted average of the returns on the securities in the portfolio with the weights being these percentages. Thus, the expected return on a portfolio is a weighted average of the expected returns on the securities using exactly the same weights. We can measure the expected returns in terms of future returns but, are alternatively calculated using historical data and then used as proxies for future returns.

The expected return for an individual investment:

The expected rate of return for a single risky asset can be calculated as follows:

$$E(R) = \sum P_i R_i = P_1R_1 + P_2R_2 + \ldots P_nR_n$$

where:

P_i : probability that *i* will occur
R_i : asset return if *i* state occurs

Properties of Expected Value:

Let w_i be any constant and R_i be a random variable. Then:

1. The expected value of a constant *"c"* times a return equals the constant times the expected return.

   $$E(cR_i) = c\,E(R_i)$$

2. The expected value of a weighted sum of random variables equals the weighted sum of the expected values, using the same weights.

   $$E(w_1R_1 + w_2R_2 + \ldots + w_nR_n) = w_1E(R_1) + \ldots + w_nE(R_n)$$

Implications of this second statement, helps us derive the expected return of a portfolio of assets. A portfolio with *n* securities is defined by its portfolio weights $w_1, w_2, \ldots, w_n$ which sum to 1. So we can calculate *portfolio return* R_p as

$$\longrightarrow R_p = w_1R_1 + w_2R_2 + \ldots + w_nR_n$$

And continue estimating the *expected return of a portfolio* as:

$$E(R_p) = E(w_1R_1 + w_2R_2 + \ldots + w_nR_n) = w_1E(R_1) + \ldots + w_nE(R_n)$$

B.2 DEFINITION OF VARIANCE:

Not only is it necessary to have a measure of the average return, it's also meaningful to have some measure of how much the outcomes differ from the average. A statistical definition of the variance states that it is the expected value (probability-weighted average) of squared deviations from the random variable's expected value.

$$\sigma^2 (X) = E\{[X-E(X)]^2\}$$

The two notations for variance are σ^2 (X) and Var(X).

Variance being the sum of squared terms means that we do not expect a number lower than zero. If we actually calculate variance to be 0, it means that there is no dispersion or risk. The general formula to calculate variance is:

$$\sigma^2 (X) = \sum P(X_i) [X_i - E(X)]^2$$

where: X_i is one of n possible outcomes of the random variable X.
$P(X_i)$ is the probability of state i occurring
and $E(X)$ is the expected return.

Variance is a standard statistical measure of spread and consequently risk. Measuring variability involves identifying the possible outcomes and assigning probabilities to them.

Standard deviation is the positive square root of variance. It's an alternative measure of dispersion denoted by σ_i.

$$\sigma_i = \sqrt{\sigma^2}$$

Standard deviation is easier to interpret than variance, as it is in the same units as the random variable. For example if the random variable return is expressed in percent, standard deviation of returns is also expressed in units of percent, whereas variance of return in units of percent squared.

Above all, we are interested in calculating *portfolio variance of returns* as a measure of investment risk. Letting R_p stand for the return on the portfolio, portfolio variance is:

$$\sigma^2 (R_p) = E\{[R_p-E(R_p)]^2\}$$

Before we proceed any further with portfolio theory, it's absolutely necessary to introduce two basic concepts in statistics, covariance and correlation.

Covariance of Returns:

Given two random variables R_i and R_j, the covariance between R_i and R_j is:

$$Cov(R_i,R_j) = E\{R_i-E(R_i)\}\{R_j-E(R_j)\}$$

Alternative notations are $\sigma(R_i,R_j)$ and σ_{ij}.
As the above equation states, it is the expected value of the product of two deviations: the deviations of the returns on the variable R_i from its mean and the deviations of the variable R_j from its mean. Covariance is a measure of the degree to which two variables move together relative to their individual mean values over time. A positive covariance ("positive relationship") means that the rates of return for two investments in general tend to move in the same direction relative to their individual means during the same time period whereas a negative covariance ("inverse relationship") means that they tend to move in different directions during specified time intervals over time. How great the number is depends on the variances of the individual return series, as well as on the relationship between the series. Covariance of returns is 0 if returns on the assets are unrelated. The reason why covariance is so important in our theory is its effect on portfolio variance. The covariance terms capture how the co-movements of returns affect portfolio variance. In practice, it is a positive number when the good or bad outcomes for each investment or asset occur together (product of two large positive or negative numbers is positive) and negative if good outcomes are associated with bad of the other. Covariance has to do with a major part of modern portfolio theory and that is ***diversification***. Holding a portfolio of assets and not just a single asset comes with a diversification benefit that is risk-reduction.

Above all, a portfolio strategy is designed to reduce exposure to risk by combining a variety of investments (stocks, bonds, real estate etc.). Its volatility is limited by the fact that not all assets move up and down in value at the same time and at the same rate. Diversification reduces both upside and downside potential and allows in general for a more consistent performance. This benefit increases with decreasing covariance.

Dividing the covariance between two assets by the product of the standard deviation of each asset produces a variable with the same properties as the covariance but with a range of -1 to +1. The measure is called ***correlation coefficient*** and is calculated as:

$$r_{ij} = Cov(R_i,R_j) \;/\; \sigma_i\sigma_j$$

where:

r_{ij} = correlation coefficient of returns

σ_i = standard deviation of R_{it}

σ_j = standard deviation of R_{jt}

Alternative notations are $Corr(R_i,R_j)$ and p_{ij}.

Correlation is a pure number, meaning one with no unit of measurement. In case we have uncorrelated variables (correlation = 0), this indicates an absence of any linear (straight-line) relationship between the variables. Increasingly strong positive correlation indicates an increasingly strong positive linear relationship (up to 1, which indicates a perfect linear relationship) whereas increasingly negative correlation indicates an increasingly strong negative (inverse) linear relationship (down to -1, which indicates a perfect linear inverse relationship).

B.3 INTRODUCING STANDARD DEVIATION TO A PORTFOLIO

Earlier in this paper, we showed that the expected rate of return of the portfolio was the weighted average of the expected returns for the individual assets in the portfolio, with the weights being the percentage of value of the portfolio. MARKOWITZ (1959) derived the general formula to calculate the variance of the portfolio as follows:

$$(1) \quad \sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{cov}(R_i R_j)}$$

where:

σ_p = standard deviation of the portfolio

w_i = the weight of each asset in the portfolio

σ_i^2 = the variance of rates of return for asset *i*

Cov(R_i,R_j) = the covariance between the rates of return for assets *i* and *j*, where

$$(2) \quad \boldsymbol{Cov(R_i,R_j) = r_{ij}\sigma_i\sigma_j}$$

The general formula indicated above shows that the standard deviation of a portfolio of assets is a function of the weighted average of the individual variances (weights squared) plus the weighted covariances between all assets in the portfolio. One of the things we should pay attention to, is the fact that the standard deviation for a portfolio of assets reflects not only the variances of the individual assets but also the covariances between all pairs of individual assets within. So we are not only interested in the risk of every asset alone, but also in the way they deal with each other.

One important aspect of portfolio theory has to do with the import of a new security in a portfolio and its properties that consequently change. What will happen to the standard deviation and expected rate of return when we add one more security to such a portfolio? The answer is derived from combining equation (1) and (2) from above.

Concerning its expected return, the contribution to the portfolio is the asset's own expected return multiplied by its importance (weight) within, while concerning its risk, not only do we care about the asset's own variability but more importantly, the asset's covariability (correlation) with other assets in the portfolio. In terms of portfolio standard deviation, an asset's contribution to portfolio risk is:

asset's own standard deviation of return	***X***	***asset's correlation with the portfolio return***	***X***	***asset's importance (weight) in the portfolio***

The relative weight of the numerous covariances between the assets already in the portfolio is substantially greater than the asset's unique variance, and tends to be even greater as the number of these assets grows. This means that the important factor to consider when adding an investment to a portfolio is not the new security's own variance but its average covariance with all other investments in the portfolio. So one can draw the conclusion that an asset's own variability (standard deviation) can be partitioned into two components:

non-diversifiable portion	asset's standard deviation of return	X	asset's correlation with the portfolio return
diversifiable portion	asset's standard deviation of return	X	(1 – asset's correlation with the portfolio)

The importance of the distinction between non-diversifiable risk and diversifiable risk is that only the non-diversifiable risk of the asset bears the investor while holding it. That is why he or she is compensated only for the non-diversifiable risk (via higher expected return) that he/she bares and not for the diversifiable risk which can be eliminated via diversification.

CALCULATING PORTFOLIO STANDARD DEVIATION

Based on the assumptions of the Markowitz portfolio model, any asset or portfolio of assets can be described by two characteristics: the expected rate of return and the expected standard deviation of returns. In the extreme case where the returns of two assets are perfectly correlated (p = 1), the standard deviation for the portfolio is in fact the weighted average of the individual standard deviations.

$$\sigma_p = w_1 \sigma(R_1) + w_2 \sigma(R_2)$$

Both risk and return of the portfolio are simply linear combinations of the risk and return of each security meaning that all combinations of two securities that are perfectly correlated will lie on a straight line in risk and return space.

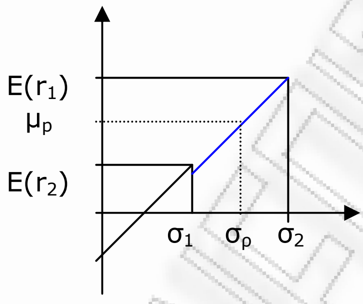


Exhibit (1)

where: $E(r_1)$ = expected rate of return for asset 1, or else "μ_1" stated
$E(r_2)$ = expected rate of return for asset 2, or else "μ_2" stated
σ_1 = standard deviation of returns for asset 1
σ_2 = standard deviation of returns for asset 2
μ_p = portfolio expected rate of return
σ_p = portfolio standard deviation of returns

In this case the important thing is that we get no real benefit from combing two assets that are perfectly correlated; they are more like one asset because their returns move together (blue straight line shown above). So the benefits of diversification as noted earlier in this paper are absent and there's no risk reduction from purchasing both assets.

Another situation worth mentioning is the extreme case when the correlation between two assets is perfectly negative (p = -1). In this case the negative covariance term exactly offsets the individual variance terms, leaving an overall portfolio standard deviation of zero. This is called a *risk-free portfolio.* One can remark that the returns in a two-asset case show no variability. Any returns above and below the mean for each asset are completely offset by the return for the other, leaving no variability for the overall portfolio.

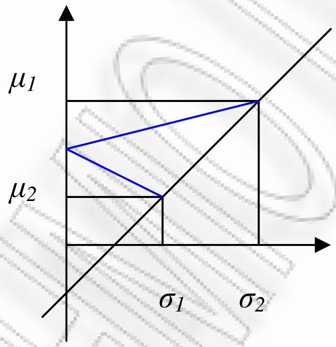


Exhibit (2)

The above scatter-gram indicates the ultimate benefits of diversification for the holder of this portfolio. These two assets move perfectly together but in exactly opposite directions. Note that these two blue lines that touch at the vertical line stated at Exhibit (2) above come from the equation:

$$\sigma_p^2 = (\mu_1\sigma_1 - \mu_2\sigma_2)^2$$

which gives us exactly 2 solutions: one positive and one negative since we took the square to obtain an expression for σ_p. These two straight lines, one for each solution of $\sigma_{p,}$ are actually derived if we examined the return on the portfolio as a function of the standard deviation.

The most common situation though of the correlation coefficient taking values within

$-1 < r_{ij} < +1$ but not the extreme ones (-1, +1), leaves us with a portfolio risk somewhere in-between (the red and blue line below inside the dashed lines with

$r_{red} > r_{blue}$). More specifically, combining assets that are not perfectly correlated does not affect the expected return on the portfolio, but it does reduce the risk of the portfolio as measured by its standard deviation.

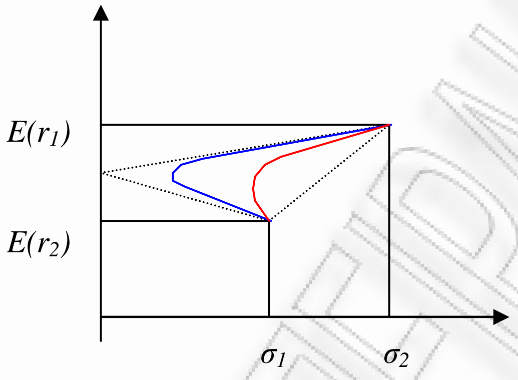
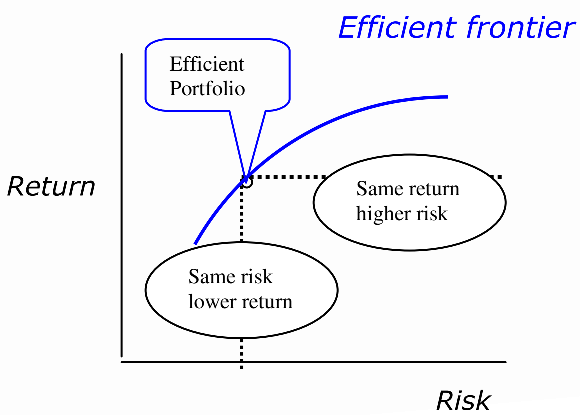


Exhibit (3): Two imperfectly correlated risky assets with $-1 < r_{ij} < +1$

B.4 <u>THE EFFICIENT FRONTIER</u>

"Mean–variance analysis" going back to Markowitz theory (1952), states that a marginal investor bases the portfolio decision solely on these two properties of the uncertain portfolio return. More specifically, it is postulated that a combination of higher means and lower variances is favored. Therefore the set of potentially <u>optimal</u> portfolios for the investor are those with the maximum rate of return for any given level of risk or the minimum risk for every level of return. Such portfolios are termed "*mean-variance efficient"* and the set of all these portfolios are called the **"efficient frontier"**. Every portfolio that lies on the efficient frontier has either a higher rate of return for equal risk or lower risk for an equal rate of return than some portfolio beneath the frontier (see exhibit below).

Exhibit (4): Efficient frontier for alternative portfolios



A distinction between the set of "minimum-variance portfolios", i.e., portfolios that have the smallest possible variance for an expected return and mean-variance portfolios as stated earlier must be made. All mean-variance efficient portfolios are also minimum-variance portfolios, but the converse is not true. Thus the efficient frontier is a subset of the minimum-variance set.

We must also note that all of the portfolios on the efficient frontier have different return and risk measures, with expected rates of return that increase with higher risk. Thus no portfolio on the frontier can dominate any other on it but as an investor, you reflect your attitude towards risk ("risk-lover", "risk-averse" etc.) by choosing the target point on the frontier. The optimal portfolio for each investor is the one that has the greatest utility for him/her. It's called the efficient portfolio and lies at the point of

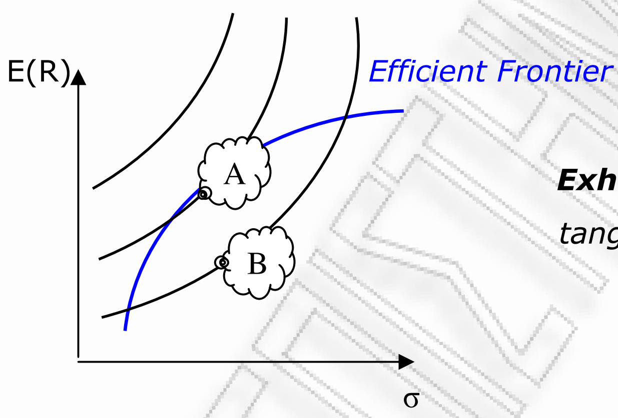


Exhibit (5): *The Efficient Frontier tangent to utility curve*

tangency (point A above) between the efficient frontier and the curve shaped "U" with the highest possible utility (as shown). Utility curves specify the trade-offs an investor is willing to make between expected return and risk. The investor is equally disposed towards any point along the same curve but can best achieve one at the point where the curve touches the efficient frontier. Although point B shown above is achievable, is not the optimal (lies on a lower utility curve) for the investor with these risk-tolerance characteristics.

B.5 INTRODUCTION TO THE ASSET PRICING MODEL OF CAPM

Risk-free asset: Following the development of portfolio theory by Markowitz, a model for the valuation of risky assets was introduced-that is, the *capital asset pricing model (CAPM)* independently by Sharpe (1964), Lintner (1965) & Mossin (1966). The major factor that allowed portfolio theory to develop into capital market theory is the concept of a *risk-free asset* that is an asset with zero variance. We have already defined a risky asset as one with uncertain future returns, whose uncertainty can measure by the variance or standard deviations of expected returns. On the other hand the risk-free asset has an expected return which is certain while the standard deviation of its expected return is zero.

$$\sigma_{RF} = 0$$

The covariance and the correlation of the risk-free asset with any risky asset or portfolio of assets will always equal zero. Like the expected return for a portfolio of two risky assets, the expected rate of return for the portfolio including a risk-free asset is the weighted average of the two returns.

$$E(R_p) = w_{RF}R_F + (1-w_{RF})E(R_i)$$

Using the general formula for standard deviation though, we end up to the equation:

$$\sigma_p = (1-w_{RF})\sigma_i$$

which shows that the standard deviation of the portfolio including the risk-free asset is the linear proportion of the standard deviation of the risky asset or portfolio of assets.

Assumptions of Capital Market Theory

Capital Market Theory extends portfolio theory and develops a model for pricing all risky assets. As a theory, it is built on a number of assumptions some of which derive from the already presented Markowitz portfolio model:

1. There are no transaction costs, meaning no costs of buying or selling any asset.
2. Assets are infinitely divisible which means that investors could take any position in any investment, regardless of the size of their wealth.
3. There is no personal income tax involved in the theory.
4. An individual cannot affect the price of a stock by his buying or selling action. While no single investor can affect prices by an individual action, investors in total determine prices by their actions.
5. Investors are expected to make decisions solely in terms of expected values and standard deviations of the returns on their portfolios.
6. The theory assumes an infinite number of short sales allowed.
7. Investors can borrow or lend any amount of money at the risk-free rate of return.
8. All investors are assumed to be concerned with the mean and variance of returns, Markowitz efficient investors who want to target points on the efficient frontier depending on his/her risk-return utility function.
9. All investors have homogenous expectations with respect to the necessary inputs to the portfolio decision.
10. All investors are assumed to define the relevant period in exactly the same manner.
11. All assets are marketable.
12. There is no inflation or any change in interest rates.

Although not all these assumptions conform to reality, they are simplifications that permit the development of the CAPM, which is useful for financial decision making because it quantifies and prices risk.

A Description of Equilibrium

Assumption 9 above states that all investors have homogenous (identical) beliefs about the expected distributions of returns offered by all assets and all perceive the same efficient set. Therefore they will try to hold some combination of the risk-free asset, R_F, and portfolio *M ("market portfolio")*, in which all assets are held according to their market value weights. If V_i is the market value of the *i*th asset, then the percentage of wealth held in each asset (w_i) is equal to the ratio of the market value of the asset to the market value of all assets. Mathematically,

$$w_i = \frac{V_i}{\sum_{i=1}^{N} V_i}$$

Each investor will have a utility-maximizing portfolio that is a combination of the risk-free asset and a portfolio of risky assets that is determined by the line drawn from the risk-free rate of return tangent to the investor's efficient set of risky assets. The straight line will be the efficient set for all investors. This line is called the *"capital market line"* and represents a linear relationship between portfolio risk and return.

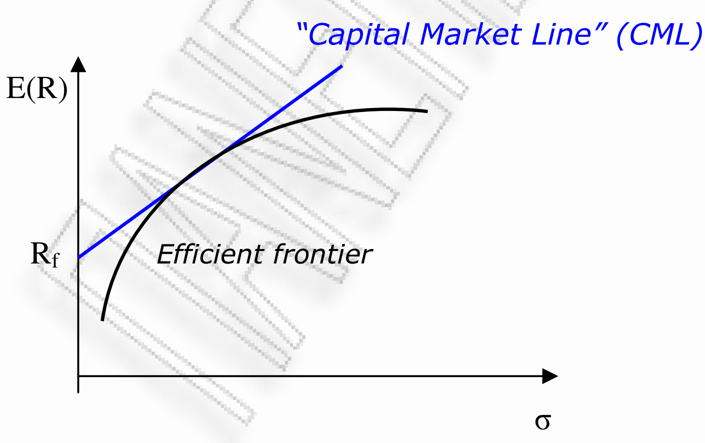


Exhibit (6): *The Capital Market Line*

The slope of the above line (CML) is: $[E(R_M) - r_F]/\sigma_M$

Therefore the equation for the capital market line is:

$$E(R_p) = R_f + \frac{E(R_M) - R_f}{\sigma_M}\sigma(R_p)$$

It provides a simple linear relationship between the risk and return for efficient portfolio of assets. The term $[E(R_M) - r_F]/\sigma_M$ can be thought of as the market price of risk for all efficient portfolios. It is the extra return that can be gained by increasing the level of risk on an efficient portfolio by one unit. The second right-hand side of this equation is simply the market price of risk times the amount of risk in a portfolio. The second term represents that element of required return that is due to risk. Thus the expected return on an efficient portfolio is:

(Expected return) = (Price of time) + (Price of risk) x (Amount of Risk)

Although this equation establishes the return on an efficient portfolio, it does not describe equilibrium returns on non-efficient portfolios or on individual securities.

The Market Portfolio

The portfolio as noted earlier is one that includes all risky assets such as stocks, bonds, real estate, etc. Therefore it is a completely diversified portfolio which means that all the risk unique to individual assets included is diversified away. Specifically, the unique risk of any single asset is offset by the unique variability of all the other assets in the portfolio. This unique risk is also called *unsystematic risk*. This implies that only *systematic risk* remains in the market portfolio. This kind of risk can only change over time if and when there are changes in the macroeconomic variables that affect the valuation of all risky assets. Examples of such variables would be interest rate volatility, corporate earnings variability, etc.

A Risk Measure for the CML

One of the basic points in the Markowitz portfolio theory as presented earlier was the fact that the relevant risk to consider when adding a security to a portfolio is its average covariance with all other assets in the portfolio. After noting the relevant importance of the market portfolio, one can simply derive that the only consideration when adding any individual risky asset is its average covariance with all the risky assets in the M portfolio, or simply, the asset's covariance with the market portfolio. This covariance is the relevant risk measure for an individual risky asset. Furthermore one can describe the rates of return on all individual risky assets in relation to the returns for the market portfolio using the following linear model:

$$R_{it} = a_i + b_i R_{Mt} + e_{it}$$

Where:

R_{it} = return for asset *i* during period *t*

a_i = constant term for asset *i*

b_i = slope coefficient for asset *i*

R_{Mt} = return for the market portfolio during period *t*

e_{it} = random error term for asset *i* during period *t*

Acting by the same way, the variance of returns for a risky asset could be described as: $\boldsymbol{Var(R_{it}) = Var(b_i R_{Mt}) + Var(e_{it})}$

The term $Var(b_i R_{Mt})$ is the variance of return related to the variance of the market return, or the systematic risk. On the other hand, the term $Var(e_{it})$ is the residual variance of return for the individual asset that is not related to the market portfolio and arises from the unique features of the asset.

Therefore:

$Var(R_{it})$ = Systematic Variance + Unsystematic Variance

Derivation of the CAPM

The Capital Asset Pricing Model indicates what should be the expected or required rates of return on risky assets. This statement helps a rational investor value an asset by providing an appropriate discount rate to use in a valuation model. Alternatively, one can compare the estimated rate of return on a potential investment to the required rate of return implied by the CAPM and determine consequently whether the asset is undervalued, overvalued or properly valued. In order to do so, the creation of a ***Security Market Line (SML)*** is introduced to represent the relationship between risk and the expected or required rate of return on an asset.
The slope of the capital market line as presented before is: $[E(R_M) - R_F]/\sigma_M$
Equating this with the slope of the opportunity set at tangency point M as presented above derives the security market line which states that the required return on any asset is equal to the risk-free rate of return plus a risk-premium.

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$$

The risk premium is the price of risk multiplied by the quantity of risk. The price of risk is the slope of the line, the difference between the expected rate of return on the market portfolio and the risk-free rate of return. The quantity of risk is also called ***beta (β_i)***. It is the covariance between returns on the risky asset and the market portfolio divided by the variance of the market portfolio: $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\operatorname{cov}(R_{i,}R_M)}{Var(R_M)}$

It is best viewed as a standardized measure of systematic risk and this is because it relates the covariance to the variance of the market portfolio. The risk-free asset has a beta of zero because its covariance with the market portfolio is zero. The market portfolio has a beta of 1 because the covariance of the market portfolio with itself is identical to the variance of the market portfolio. Stocks can be characterized as more or less risky than the market, according to whether their beta is larger or smaller than 1.

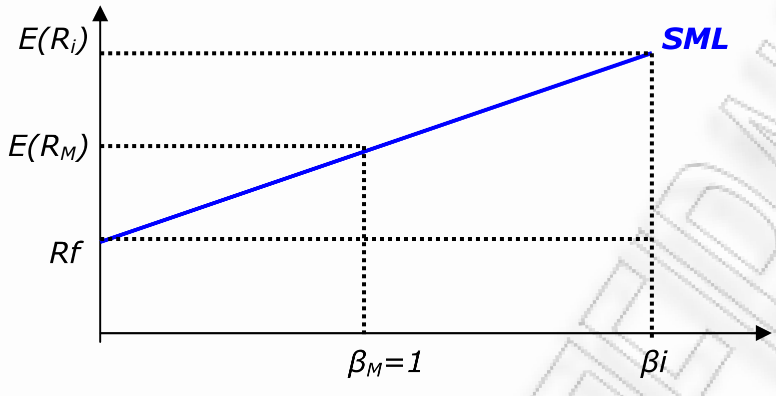


Exhibit (7): *The Security Market Line*

The exhibit above shows the security market line which replaces the covariance of an asset's returns with the market portfolio as the risk measure with the standardized measure of systematic risk (beta). Thus, the expected required rate of return for a risky asset is determined by the risk-free asset plus a risk premium for the individual asset. In turn, the risk premium is determined by the systematic risk of the asset and the market risk premium $E(R_M)$ - $R_{F.}$

Properties of the CAPM

In equilibrium, every asset must be priced so that its risk-adjusted required rate of return falls exactly on the Security Market Line (SML). This means that assets that do not lie on the mean-variance efficient set, will lie exactly on the SML because not all the variance of the asset's return is of concern to risk-averse investors. As described before, investors can always diversify away all risk except the covariance of an asset with the market portfolio which is the risk of the economy as a whole (undiversifiable). Having mentioned the use of beta to the model, one can go further on to express total risk partitioned into two parts as:

$$\sigma_j^2 = b_j^2 \sigma_{\mathrm{M}}^2 + \sigma_e^2$$

The variance is total risk; it can be partitioned into systematic risk (first half on the right hand of equation) and unsystematic risk (other half).
A second important property of the CAPM is that the measure of risk for individual assets is linearly additive when the assets are combined into portfolios. Thus, the beta of the portfolio is the weighted average of the betas of the individual securities in it:

$$\boldsymbol{\beta_p = a\beta_x + b\beta_y}$$

For the above equation the portfolio is assumed to be consisted of 2 securities X, Y with betas β_x & β_y proportionally. All that is needed to measure the systematic risk of portfolios is the betas of the individual assets.

The estimation of beta has drawn attention to many academics and researchers in the past and it has been documented that many different beta estimates can be obtained for one stock depending on factors such as the choice of the market index, the length of the estimation period and the sample period. This paper, as stated before, is interested in investigating only the impact of the return interval on the beta estimate. A brief discussion of previous studies on this impact is presented right below.

C. "WHY BETA SHIFTS AS THE RETURN INTERVAL CHANGES"
Financial Analysts Journal (1983)
Gabriel Hawawini (USA)

In his article, Hawawini (1983) presents a *simple model* to explain why estimates of beta depend upon the length of the return measurement interval. The model also predicts the direction and strength of the variations in estimated betas. To present this, he uses betas for the 4-year period of January 1970 to December 1973 estimated on the basis of 50 monthly returns, 1,009 daily returns and various combinations of weekly returns (triweekly, biweekly, weekly) taking as a proxy for market returns the S&P 500. Returns are measured as the logarithm of investment relatives whereas all betas are statistically significant at 5% level of significance. The model estimates beta as follows:

$$\beta_i(T) = \beta_i(1)\,\frac{T + (T-1)\dfrac{p_{im+1} + p_{im-1}}{p_{im}}}{T + 2(T-1)p_{mm-1}}$$

where $\beta_i(T)$ is the security's i estimated beta over return intervals of T–day length, $\beta_i(1)$ is the security's i estimated beta over return intervals of 1-day length, p_{im-1}, p_{im} , p_{im+1}, the intertemporal cross-correlation coefficient of one lag behind (-1), no lag (0) and one lag ahead (+1) respectively between the security and the market returns measured over 1-day interval and p_{mm-1} the autocorrelation coefficients of one lag behind (-1) on the market daily returns. The author names the ratio $(p_{im+1}+ p_{im-1})/ p_{im}$ *"q–ratio"* of a given security to include the importance of friction in the trading process meaning delays (lags) in the response of securities' prices to new information.

From the above equation, one can draw conclusions on how beta is affected by intertemporal cross-correlations as T (in days) varies. More specifically, beta will be invariant to the length of return interval in the extreme situation when the intertemporal cross-correlations between the security and market returns are zero or when the market provides a zero correlation with itself.

Hawawini (1983) in his article goes on further to predict the direction and strength of a beta shift. In order to achieve this, he measures how $\beta_i(T)$ changes with a small change in variable T (in days). As the measurement length interval is shortened, securities with q-ratios larger than the market's will see their betas decrease whereas securities with q-ratios smaller than the market's will see their betas increase. The decrease will be faster for the first group of securities as the difference between the security's q-ratio and the market's larger is. Respectively, the increase will be faster for securities of the latter situation as the q-ratios of those get smaller relative to the market. Not only does the author confirm the appliance of q-ratios on the data he presents but he also goes on to present a faster way to tell if beta will shift upward or downward regardless of the use of q-ratio. More specifically, he uses the "market value of shares outstanding" ("MVSO") as a proxy for a security's relative market thinness stating that securities with large "MVSO" will have an increasing beta when the return interval is shortened whereas those with small values will generally have a decreasing beta.

Finally Hawawini (1983) examines the implications of his results for risk-adjusted measures of portfolio performance like the *Treynor ratio*, the estimation of the *Security Market Line* of Sharpe & Lintner and the market performance of securities with biased betas. He concludes that any equation or computation incorporating beta will be affected by the length of the return interval used and much attention has to be given to the fact that securities may appear to be less or more risky than they truly are depending on the interval used.

"FRICTION IN THE TRADING PROCESS AND THE ESTIMATION OF SYSTEMATIC RISK" (Journal of Financial Economics – 1983a)
K. Cohen, G. Hawawini, S. Maier, R. Schwartz, D. Whitcomb (USA – France)

This paper contains theoretical analysis of the bias of the market model beta parameter due to friction in the trading process and presents several propositions from which consistent beta estimates are obtained and the effect of different interval length measurements is derived. After indicating some related observations and arguments by former empirical studies, Cohen et al (1983) makes a distinction between *"observed return"* and *"true return"* and uses many leads and lags of the market's return to derive a consistent estimator of true beta as a measure of systematic risk. This can be found by the formula:

$$\hat{\beta}_i = \frac{\beta_i + \sum_{n=1}^{N} \beta_{i+n} + \sum_{n=1}^{N} \beta_{i-n}}{1 + \sum_{n=1}^{N} \rho_{m,m+n} + \sum_{n=1}^{N} \rho_{m,m-n}}$$

where betas of the security *i* are obtained by separate regressions using the OLS method and serial correlations of market returns are used with a lag and lead of *n*. The paper goes on to examine the case of increasing differencing interval over which returns are used and introduces the term of *"asymptotic estimator"*. It is a consistent OLS estimator of the *"true"* beta based on non-overlapping (no lag or lead) boundless measurement interval periods (L) which can be defined as: $\beta_i^* = \lim_{L \to \infty} \beta_i(L)$.

An important implication stated first by Scholes-Williams (1977) but proved here is that the bias presented in such articles is positive for some securities and negative for others leaving the overall weighted average beta bias zero with the weights being the percentage of each security in the market index ($\sum_i x_i \beta_i = 1$). Lastly, the paper using price-adjustment delay variables proves that securities with relatively short price-adjustment delays have their betas overestimated by the OLS method whereas those with lengthy delays will be underestimated by the same method.

The above proposition series as summarised above are contrasted with past related analyses of Scholes-Williams (1977) and Dimson (1979). The original work of the first ones measuring the impact of non-synchronous trading on beta measurement gives the exact same results as the ones presented in this article if one lead and lag of the market's return is used (N=1). This paper regards several assumptions used in their work as restricting leading to a loss of valuable information when estimating OLS betas. Dimson's estimator (1979 pg.204) $\beta_i = b_i + \sum_{n=1}^{N} b_{i+n} + \sum_{n=1}^{N} b_{i-n}$ appears to be insufficient according to the authors since it's based on multiple regressions to estimate betas, a methodology inconsistent with the coefficients presented here.

In conclusion, the article emphasizes the basic result that OLS "observed" beta can be thought of as a consistent estimator of "true" beta using small price-adjustment delays as interval length measurement increases with no bounds $(l \to \infty)$ and the magnitude of the bias depends on the relative magnitude of these delays.

"ESTIMATING BETAS FROM NONSYNCHRONOUS DATA"
(Journal of Financial Economics - 1977)
M. Scholes & J. Williams (USA)

After developing early notes by Fama (1965) and Fisher (1966), Scholes & Williams (1977) went on to explain an econometric problem in the market model due to nonsynchronous trading of securities and build a methodology to measure the bias and correct it. The paper starts with the realization that given the availability of daily returns of securities traded and their use in estimating security returns through the capital asset pricing model (CAPM), an econometric problem of errors in variables including betas and alphas exists. In particular, most securities trade at discrete time intervals with prices observed only at points of actual trades and not at all times. CAPM theory on the other hand is based on normally distributed returns assumptions (for risky securities and the market index) and thus ordinary least squares estimators of both alphas and betas are biased and inconsistent when measured this way. More specifically, it is shown that variances and covariances of reported returns differ from corresponding variances and covariances of true returns. Securities trading very infrequently have estimators biased upward for alpha and downward for beta while the remaining ones are biased in the opposite directions leaving the overall measured alphas and betas equal to true alphas and betas.

The authors, assuming that non-trading periods are distributed independently and identically over time, make a distinction using their theory between single securities and relatively large portfolios. The properties derived contrast sharply each category. For single securities measured variances closely approximate true variances whereas for portfolios measured variances typically understate the true values and this phenomenon is more intense for portfolios with less frequently weighted securities.

In order to verify their theoretical arguments, they used daily returns from all stocks listed on the New York (NYSE) and American Stock Exchanges (ASE) between January 1963 and December 1975. The calculations included 251 days of trade for 13 years with an average of 2,305 securities each year. From the above data, five (5) equal-numbered portfolios of securities were constructed, selected by the *"trading volume"* of each security, meaning the number of shares of a security traded during the year. In case a single security was not traded during a given day, no return was included in that portfolio for both that day and the subsequent trading day.

The results of the research were then contrasted with the ordinary least square estimators of alpha and beta as calculated by the market model theory. The authors proposed a consistent estimator of beta by the following equation to correct the bias: $\hat{\beta}_i = \frac{(\hat{\beta}_i^{-1} + \hat{\beta}_i^{o} + \hat{\beta}_i^{+1})}{(1 + 2\hat{\rho}_{1m})}$,

where the numerator presents estimates of the parameter derived from the simple regression between the observed security return and the corresponding market index return with one lag, matching, and one lead respectively and the denominator the first-order serial correlation coefficient of market returns.

Low-volume securities portfolio generates larger beta than the corresponding least squares estimate whereas as we are moving to higher level-volume portfolios this is reversely true. This result holds when the value-weighted market portfolio is weighted most with securities traded relatively frequently. The above relationship is partly explained by the apparent relationship between true betas and trading volume. Thus, larger consistent estimates of beta are associated with larger trading volumes. Throughout all these results standard errors of betas are statistically more significant when examining portfolios trading at lower levels of volume and when estimating alphas (they can not be verified through this research).

Finally, the authors summarize the need for this research which derives from the use of daily data used at the capital asset pricing model of estimating returns, address the problem of infrequent trading by securities and consequent bias when calculating ordinary least square estimators and therefore specify this bias by constructing consistent estimators of true beta and alpha.

"RISK MEASUREMENT WHEN SHARES ARE SUBJECT TO INFREQUENT TRADING" (Journal of Financial Economics – 1979) E. Dimson (England)

Dimson (1979) introduces his article referring to the infrequent–trading problem which causes beta estimates to be severely biased. He attributes the intervaling effect to the tendency of the mean value of beta of the market model to rise as the differencing interval is increased and presents a number of past related studies to point the general need for a proper solution. He summarizes these studies to 3 major approaches: the lagged market returns method supported by authors like Ibbotson (1975) & Pogue-Solnik (1974), the trade-to-trade method by Marsh (1979) & Schwert (1977) and finally the Scholes-Williams method (1977). After setting the weak points of each method, he proposes his own, the "*Aggregated Coefficients (AC) method"*, which is a development of the lagged market returns method.

It assumes that changes in value of a security are derived from the market model where the security and market returns population follow a serial and cross-serial independence distribution. This process generates observed returns whose covariance with the market, $cov(R_{it},R_{mt})$, is positively related to its trading frequency. Thus, regression based on simulation results generates for frequently traded shares upward biased estimates and for infrequently traded shares downward biased beta estimates. The aggregated coefficients (AC) method is based on the following multiple regression of observed returns on lagging, matching and leading market returns:

$$R_{it} = a_i + \sum_{k=-L}^{L} \hat{\beta}_{i+k} R_{mt+k} + u_{it}$$

where R_{it} is the security return for period t, a_i is the time independent alpha (constant), R_{mt+k} is the returns on preceding, contemporaneous & leading market returns where L is the number of non-synchronous terms (measuring the degree of thinness) used in the regression and u_{it} is the error term (variable).

A consistent estimate of systematic risk is obtained by aggregating the slope coefficient from the regression and is expressed as:

$$\hat{\beta}_i = \sum_{k=-L}^{L} \hat{\beta}_{i+k}$$

Dimson (1979) notices that as *L*, the number of non-synchronous terms is increased taking positive values, the bias of the AC estimator is decreased but the method starts to lack efficiency since the beta coefficients with leads & lags face estimation error. Therefore, he proposes that the maximum number of leads & lags should be accompanied by a positive cross-sectional variance of the β_{i+k}. To test the AC method empirically, Dimson (1979) tracked down from the London Share Price Database monthly returns of companies listed on the London Stock Exchange between January 1955 and December 1974. Then he formed a sample of 421 companies which appeared throughout all these years on that list and took their returns to form 10 deciles according to their trading frequency. He first run the simple regression to confirm the theoretical bias of beta estimates due to infrequent trading and then went on to apply his AC method using five lags and leads. He noticed that a more even distribution of estimated betas was formed in comparison with the simple regression with a reduced beta range for all deciles (frequent to infrequent). These results show that the use of lagged and leading terms into the market model improves beta estimates but the number of these terms varies according to the empirical work of the researcher. At least 1 leading term and 4 lagged terms are required according to the author to explain a quarter or more of the cross-sectional variance of coefficient estimates since the lagged ones are much more statistically important than the rest.

Finally, Dimson (1979) summarizes the empirical results of the AC method, distinguishes it from others dealing the infrequent-trading problem and suggests its use in situations when the times of the transactions are unknown.

"RISK MEASUREMENT WHEN SHARES ARE SUBJECT TO INFREQUENT TRADING" (Journal of Financial Economics – 1983)
D. Fowler & C. H. Rorke (Canada)

The hereby article (referred to as a "*comment*" by the authors) is a theoretical approach on methods first developed by Scholes & Williams (1977) and later by Dimson (1979) to calculate a consistent estimate of beta so as to cope with the thin-trading problem. More specifically, it shows that Dimson's estimator is not consistent with that of Scholes & Williams and itself not efficient at all. Therefore, a corrected version is suggested.

After examining the theory behind the basic regression models by Dimson (1979) and Scholes-Williams (1977), the authors extended the latter to calculate a consistent estimate of beta using two period returns instead of one as the original theory stated. The Scholes-Williams estimator (1977) is still expressed by the same equation but the ordinary least square estimators $\beta_i^{-1}, \beta_i^{0}, \beta_i^{+1}$ become ${}_2\beta_i^{-1}, {}_2\beta_i^{0}, {}_2\beta_i^{+1}$ to include the two-period estimate. Thus, the extended Scholes-Williams estimator (1977) is given by the following equation:

$$\widehat{\beta}_i = \frac{\widehat{\beta}_i^{-2} + \widehat{\beta}_i^{-1} + \widehat{\beta}_i^{0} + \widehat{\beta}_i^{+1} + \widehat{\beta}_i^{+2}}{1 + 2\widehat{p}_{1m} + 2\widehat{p}_{2m}}$$

which includes a beta estimate with a lead and lag of 2 at the nominator and a second order serial correlation coefficient of market returns at the denominator. Relating the above extended approach with the multiple regression proposed by Dimson (1979), the authors concluded that theoretically the latter was inconsistent with the former and therefore incorrect. Fowler et al (1980–Journal of Business Administration 12 no 1 p.77-90) had actually tested the empirical results by Dimson's estimator (1979) on the Toronto Stock Exchange and found out that it was generally inferior to Scholes & Williams'(1977) and frequently inferior to the simple regression OLS estimates.

"ADJUSTING FOR THE INTERVALLING EFFECT BIAS IN BETA"
(Journal of Banking and Finance – 1985)
W. Fung, R. Schwartz, D. Whitcomb (USA)

The authors used daily-returns data from the Paris Bourse to test the Cohen-Hawawini-Maier-Schwartz-Whitcomb ("CHMSW") model (1983) of the intervalling-effect bias in OLS beta estimates. From a selected sample of 52 traded stocks, they used data of 803 daily closing prices and corresponding index values (the benchmark used is the "C.A.C index") to follow *a 3-pass regression procedure*[*], similar to the one proposed by the "CHMSW" model (1983), to estimate the magnitude of the bias.

The first pass is actually the market model estimate of security betas, with the security and market returns measured as continuously compounded returns [ln (P_t/P_{t-1})] over each of the 32 differencing measurement intervals formed. The results acquired were consistent with those of any other past related study: As the interval (measured in days) is lengthened, average OLS beta estimate is increased and so does the average R^2. This is because the C.A.C index is a value weighted market index consisting of relatively large stocks and average beta rises as the measurement interval is lengthened.

In the second-pass part, Fung et al (1985) tested CHMSW's (1983) view that the bias diminishes asymptotically to zero as the differencing interval increases, using three alternative functional forms:

- Power function: $f(L) = L^{-n}$
- Log function: $f(L) = \ln(1+L^{-n})$
- Exponential function: $f(L) = \exp(-L^{n})$

where L is the measurement interval length in days and n is the sample size. Each of these functions converges to an asymptote but their efficiency varies. The first-pass OLS betas are regressed using all three above functions but the empirical results are the same for all: significantly negative beta estimates for stocks that trade less frequently than the Paris index and positive or insignificantly negative beta estimates for traded stocks that trade more frequently.

Finally at the third-pass part, the authors use the "value of shares outstanding", as an empirical proxy for every security's price adjustment delay. The second-pass slope coefficients, which measure the intervalling-effect bias, are then regressed on the values of shares outstanding. The empirical results show that there is a positive and significant relationship between these two parameters. Thus, small security betas decrease and large security betas increase as the measurement interval is shortened.

What is also important for the above theory produced is the choice of the sample-efficient adjusted beta. The alternatives comprise the estimated adjusted beta (second-pass regression), the inferred asymptotic beta & the Scholes-Williams (1977) adjusted beta. The first two are derived by the CHMSW model (1983). Empirical results of all three procedures show that the Scholes-Williams (1977) beta is inefficient since it's inconsistent with the asymptotic beta (which is used as a benchmark) whereas the other two come relatively close concerning their results.

Summarizing, the article introduces a 3-pass regression procedure on daily data acquired by the Paris Bourse to test the implications of the Cohen-Hawawini-Maier-Schwartz-Whitcomb model (1983) and shows that the bias diminishes asymptotically to zero as the measurement interval increases and uses as a proxy the security's value of shares outstanding.

"ADJUSTING FOR BETA BIAS: AN ASSESSMENT OF ALTERNATIVE TECHNIQUES: A NOTE" (Journal of Finance – 1986)
T. McInish & R. Wood (USA)

The paper attributes OLS beta bias to 2 sources of bias: thin trading & price adjustment delays and tests the effectiveness of previous related works by Scholes-Williams (1977), Dimson (1979), Fowler-Rorke-Jog (1980, 1981), Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980, 1983). McInish & Wood (1986) recognize in their work these studies are inadequately presented, since they are based on simulated data which provide an unmeasured level of true bias. Therefore, they propose a linear programming model to develop portfolios with equal risk (betas) to test the techniques presented above.

Their model is based on data for 958 sample firms listed on the NYSE, for the period September 1971 to February 1972. Prices were adjusted to include dividend existence and capitalization changes. The model uses *"LTIME"*, the average time from last trade to closing time, as a proxy for thin-trading measurement. In order to isolate the beta bias, 5 portfolios are created that maximize the difference in *"LTIME"* across portfolios while holding risk and return determinants approximately equal. These risk attributes include: financial leverage (Debt/ Assets ratio), Payout ratio (Dividend/ Earnings ratio), Book Value of assets and the average stock price of each security. The model is based on 2 assumptions:

1. according to CAPM, ex post returns are good estimates of ex ante returns
2. portfolios with equal expected returns should have equal betas

According to their theoretical approach, OLS mean beta estimates should reveal the true beta bias magnitude for each security, while betas proposed by the Scholes-Williams, Dimson, Fowler-Rorke-Jog & CHMSW techniques should equal 1 (one) for each portfolio. The empirical results showed that the OLS betas were biased as expected, with "thick" traded portfolios having betas above 1 while "thin" trading betas below 1 when using daily returns. All alternative techniques reduce the amount of bias but the amount of reduction is 29 % in the best case. Furthermore, Scholes-Williams (1977) and Dimson's (1979) methods achieve a greater reduction than the CHMSW method (1983).

Summarizing, McInish & Wood (1986) investigate in this paper the extent of beta bias for NYSE stocks and the effectiveness of past proposed techniques through a linear programming model but find that each of these techniques contributes to beta bias reduction but the extent is maximum 29 % compared to the OLS beta estimate. They emphasize the need for further theoretical and empirical work on this.

"THE RELATION BETWEEN THE RETURN INTERVAL AND BETAS" Implications for the Size Effect (Journal of Financial Economics – 1989)
P. Handa, S.P Kothari, C. Wasley (USA)

The article examines both theoretically and empirically the behavior of beta as a function of the return measurement interval and the sensitivity of beta estimation to the size effect. Theory presented prior to the article stated that beta changes as the return interval is increased due to non-synchronous trading and trading frictions that distinguish true betas from observed betas. On the other hand, related research had shown that firm size has an incremental explanatory power.

Concerning the first relation, beta as a measure of systematic risk is sensitive to return interval because the covariance of securities returns with the market return (the numerator in beta equation) and the market return variance (the denominator in beta equation) do not change proportionally as the return interval changes. Two implications for the above view can be stated:

1. risk & return are positively cross-sectionally related irrespective of the interval used
2. the larger the return interval, the larger the difference between betas of high and low risk securities

Furthermore, taking into account the standard errors of measured betas when using longer intervals, the authors find that these will be greater. To test the above theoretical approach, Handa et al (1989) used a sample of all stocks listed on the Center for Research in Security Prices monthly tape (NYSE) for the period 1926 – 1982. Then they formed 20 portfolios according to their market values using equally-weighted buy-and-hold returns. The portfolios were flexible in a sense that each year securities were ranked according to their initial market values (first trading day of the year) but any changes were taken into account. Eight intervals were used to test the effect of different return interval on beta estimates: 1 day, 1 week, 1 month, 2 months, 3 months, 4 months, 6

months & 1 year. Over all these intervals, equal weighted sample mean returns were used as the market portfolio proxy in the regressions. The results confirmed the theoretical analysis summarized before: beta estimates are indeed sensitive to return intervals.
However, the authors attribute this sensitivity to 3 factors:

1. Lack of proportionate change in the numerator of the standard beta equation with the denominator.
2. Larger standard errors of betas estimated using longer interval measurements
3. Measured returns are not serially uncorrelated due to frictions in the trading process and infrequent trading

In order to measure the effect of firm size on beta measurement, the authors tested monthly versus annual betas. Evidence showed that annual betas were better in explaining variation in annual returns on the portfolios than monthly returns.
(R^2 = 50 % in the first situation compared to R^2 = 37 % in the second)
Then they tested annual returns versus monthly returns and firm size. The results showed that:

- Firm size alone explains return variation beyond monthly betas.
- The effectiveness of firm size diminishes when annual beta is added.
- Return – variation annual beta has significant incremental explanatory power, but firm size and monthly beta don’t.

"THE INTERVALLING EFFECT BIAS IN BETA: A NOTE"
(Journal of Banking and Finance – 1992)
A. Corhay (Netherlands)

Corhay (1992) wishes to examine the intervalling effect bias in estimated betas showing that they tend to converge to their asymptotic values depending on the measurement interval used. To test the empirical evidence of his approach, he used daily continuously compounded returns of 250 sample securities listed on the spot market of the Brussels Stock Exchange from January 1977 to December 1985. The whole test-period was divided into 3 adjacent periods each comprising of 3 years. For each period, he formed 10 value-weighted portfolios and used a finite set of 30 differencing interval lengths.

The author based portfolio return measurement on the market model to estimate portfolio beta and subsequently used methods proposed by Hawawini (1980) and Cohen et al (1983) to examine the speed of convergence of betas to their asymptotic value as differencing interval is lengthened.

His results, all in all, tend to confirm the asymptotic behavior of security betas as presented in the past by Hawawini (1980) & Cohen et al (1983). More specifically, the whole 250-security-sample demonstrates no intervalling effect with an average beta approximately closing to one, since the sample taken stands for the entire BSE market. But average portfolios betas on the other hand, reflect an intervalling effect bias with the last one being large for small intervals and decreasing as the interval is lengthened. Judging from the F-test statistic values obtained when analyzing the variance between the individual betas of the portfolios, Corhay (1992) concluded that small market-value firms appear to have on average lower beta coefficients than larger firms. Finally values of the standard errors of beta estimates reveal that small-firm portfolio betas show a greater volatility than that of betas of the large firm portfolios, and this volatility tends to increase continuously with the length of the differencing interval.

"COMPOUNDING PERIOD LENGTH AND THE MARKET MODEL"
(Journal of Economics and Business – 1994)
G. Frankfurter, W. Leung, P. Brockman (USA)

In this paper, Frankfurter et al (1994) investigated the effect of the compounding period length for which rates of return are calculated on the estimation of alpha and beta parameters of the market model. The analytical model presented, points out the functional relationship between the alpha – beta estimates and the investment horizon:

$$a_{ic} = n*a_{id} + c$$

where a_{ic} & a_{id} are the corresponding a estimates of the market model of longer and shorter compounding periods and c is a constant.

$$\beta_{ic} = \beta_{id}*c$$

where β_{ic} & β_{id} are the corresponding β estimates of the market model of longer and shorter compounding periods and c is a constant.

To test the model, Frankfurter et al (1994) used daily return data for 1297 stocks listed on the Center for Research in Security Prices (CRSP) NYSE-AMEX daily file for the period: 01/19/1976 to 11/30/1987. The model uses the CRSP equal value weighted index adjusting for dividends as a proxy for the market index specification. From the above-mentioned file, daily, weekly, monthly, quarterly, semiannual & annual compounding periods were formed using the market model to draw conclusions on the beta – alpha estimates. Results showed that average beta estimates differed very little from one period to another, whereas average alpha estimates varied at a great degree (maximum 40 times). Nevertheless, mean beta values are of the same order of magnitude, regardless of the holding period, so any comparison is misleading. Consequently, regression results based on differencing compounding periods will significantly differ from each other.

Above all, the paper supports the view that the methodology used is totally dependent on the relative validity and efficiency of the market model which acts as a benchmark of comparison for actual returns. If the market model, for some reasons, generates biased estimates, then the so longed "true" returns are a myth. All in all, the market model should reflect the investors' practices. The objective of each investor should therefore specify the proper length of the compounding period.

"ESTIMATION OF RISK ON THE BRUSSELS STOCK EXCHANGE: METHODOLOGICAL ISSUES AND EMPIRICAL RESULTS"
(Global Finance Journal – 1997)
F. Beer (USA)

The purpose of the above titled article is to assess models of beta estimation by Vasicek (1973), Scholes & Williams (1977) and Dimson (1979) and their implied ability to do so in a relatively thin market like the Belgian market. In an older study by Hawawini & Michel (1974), it was concluded that Belgian stockholders tended to support the thinness of their stock market since they hold securities for a long period of time avoiding often changing the synthesis of their personal portfolios. As a result, 40 % of securities are quoted half of the period. The main characteristic of this specific market though is that all securities are traded at the same moment every day once a day and that to avoid speculation by the few large players, a maximum and a minimum price change limit is set.

The author formed a sample of 181 Belgian securities for the period starting January 1974 to December 1986 and grouped them according to their market capitalization. Therefore, she formed 10 groups of security returns, adjusted for capital changes and computed the beta estimate of each one as the mean of its constituent's betas. Then she went on to specify a suitable market index by constructing an equally-weighted index containing all the securities in the market. Results were acquired using first the OLS method of beta estimation, then the Scholes & Williams (1977) method and finally Dimson's (1979). In comparison with the first two, Dimson's method (1979) provided better results.

The results confirmed the intervalling effect bias in the Belgian stock market all in all and the biased results when using the OLS method to estimate it. The model proposed by Scholes & Williams (1977) did not decrease the bias, while Dimson's (1979) only succeeded to a small degree. As a result, the paper recommends the use of the OLS method in thin markets like the Belgian and points out the need for further theoretical and empirical research.

"THE IMPACT OF THE RETURN INTERVAL ON THE ESTIMATION OF SYSTEMATIC RISK" (Pacific-Basin Finance Journal – 1997)
T. Brailsford, T. Josev (Australia)

The model proposed by Brailsford & Josev (1997) provides a prediction of the size and direction of change in the estimation of systematic risk as measured by the beta, as a result of different return intervals used. This kind of effect is widely known as "interval effect" and the authors initiate their study by referring to past articles on this topic starting from Blume (1971) to Frankfurter et al (1994). Initial theoretical and empirical results supported the view that betas of thinly traded securities rise as the return interval is lengthened whereas betas of frequently traded securities fall. Moreover mean R^2 rises as return interval is lengthened with the largest increase observed in thinly traded securities. This time though, the authors choose to examine the effect of return interval in the Australian Stock Exchange. Their study is based on little evidence of markets outside U.S.

To test their model, they formed 2 extreme portfolios, a sample of 15 thinly traded stocks and a sample of 15 frequently traded stocks for the period covering January 1988 to December 1992. The stocks were ranked according to their market capitalization and selected due to their continuous listing on the market index over the sample period. For each of the firms selected, daily, weekly and monthly prices adjusted for both capitalization and dividend effect were derived and a continuously compounded return series for each interval was calculated.

The results indicated a strong presence of zero returns (meaning no change in price due to no new added information or no trading) for thinly traded firms compared to frequently traded ones. Moreover, the return interval effect on beta estimates when using the OLS method was confirmed. As the return measurement interval lengthens, the mean beta estimates for the thin portfolio rises. However the effect is not the same for both portfolios when using daily or weekly interval returns. A significant difference in mean beta estimates in this situation exists. Also, the standard error of mean beta estimates rise for both portfolios as the return interval is lengthened, a conclusion constituent with that of Handa et al (1989) and Frankfurter et al (1994). The same property exists for mean R^2 values for both portfolios as well.

After presenting the method and results of examining the Australian Stock Exchange, the authors went on to test the predictive ability of their model based on the model proposed by Hawawini (1983). Evidence derived, supported the implication of this proxy model and confirmed its assumption that the serial cross-correlation coefficients for lead and lag orders higher than one are equal to zero. The Hawawini model performs quite well in approximating OLS beta estimates when forecasting weekly betas using daily returns or monthly betas using weekly returns but not sufficiently well when forecasting monthly betas using daily returns. Especially such ability is much seen when predicting OLS betas for the upper extreme portfolio of frequently traded securities.

"ESTIMATING SYSTEMATIC RISK: THE CHOICE OF RETURN INTERVAL AND ESTIMATION PERIOD"
(Journal of Financial and Strategic Decisions – 2000)
P. Daves, M. Ehrhardt, R. Kunkel (USA)

The study addresses the difficult task held by financial managers when estimating the systematic risk of a firm or a security. The Capital Asset Pricing Model (CAPM) is a useful tool in their hands when estimating betas but managers need to specify the proper estimation period and return interval for their estimation. Of note though is the fact that such a decision implies a trade-off between precision and bias. The more lengthened the estimation period, the smaller is the standard error of the estimated beta because of including more observations and the more the likelihood that the value of such beta will change in time because of changes in structural characteristics of the firms exists.

This study examines four return intervals (daily, weekly, two-weekly and monthly returns) and eight estimation periods ranging from 1 year to 8 years between 1982 and 1989. Security returns are obtained from the CRSP NYSE/ AMEX databases and then listed into 4 samples according to their return intervals. Thus, using daily returns each firm's beta is estimated 8 times, repeating for weekly, two-weekly and monthly return intervals. Test results show that choosing among individual return intervals; shorter return intervals are associated with a greater precision in beta estimation (smaller S_β) and consequently financial managers should use daily returns if given the choice.

The author uses the market model to estimate the beta of a firm:

$$R_{it} = a_i + \beta_i R_{Mt} + \varepsilon_{it}$$

where R_{it} is the rate of return on stock i in period t, R_{Mt} is the market return as calculated via the CRSP database, a_i is the intercept, b_i is the security beta and ε_{it} is the error term for security i in period t

And the standard error of the estimated beta S_β as follows:

$$S_\beta = 1/(N-1)^{0.5} * (S_\varepsilon/S_M)$$

where S_ε is the standard deviation of the estimated errors, S_M is the standard deviation of market returns and N denotes the number of observations.

Concerning the estimation period, results show that increasing it, one can obtain a greater precision in estimation period or equivalently a decreasing S_β, but such reduction is 91 % seen during a 3-year estimation period. Additionally less than 50 % of the firms examined experience a significant shift in beta over the 3-year proposed estimation period.

Finally, the author concluded emphasizing the need for a proper estimation of the return interval and the estimation period by financial managers when estimating beta and proposed a daily return interval and a 3-year period to profit best from the implied trade-off between precision and bias, based on his empirical results.

"SOME ESTIMATION ISSUES ON BETAS: A PRELIMINARY INVESTIGATION ON THE ISTANBUL STOCK EXCHANGE" (2003)
A. Odabasi (TURKEY)

The author of this paper reports on the findings of an investigation on the Istanbul Stock Exchange on beta stability and three main factors that affect it:

- The estimation period
- The return interval
- Diversification

Although little related research has been conducted for emerging markets in the past, the author emphasizes the need for further study on the grounds of the volatile nature of such markets.

The study uses a sample of 100 continuously Istanbul Stock Exchange (ISE) listed stocks over the period between January 1992 and December 1999. The daily returns obtained adjusted for capitalization and dividends are calculated as follows:

$\mathbf{R_{it} = (P_{it} - P_{t-1}) / P_{t-1}}$ where P_{it} denotes the stock price in period t.

As a proxy for market performance, a relatively value-weighted index, the ISE 100 index is used. Furthermore, beta coefficients β_i are calculated via the market model.

Concerning the effect of the return interval on beta estimation, empirical results using weekly and monthly returns over annual, 2-year and 4-year estimation periods show that the mean beta and the R^2 increase as the return interval lengthens. The standard error of mean beta estimates also increases as the return interval lengthens.

The effect of the estimation period on beta estimates is measured through correlation coefficients over pairs of estimated periods. In the case of weekly returns, a 2-year estimation period seems to produce a more stable beta based on the highest value of the correlation coefficient, whereas using monthly returns, a 4-year estimation period is better according to the empirical results of the study.

Finally, to examine the effect of diversification on beta estimates, the author constructed portfolios of different sizes for different return intervals and computed the correlation coefficient between portfolios betas made up of exactly the same securities but for consecutive years. Results showed that the correlation coefficient increases as the portfolio gets larger for every estimation period used.

Summarizing, the study based on a 100 ISE stock sample wishes to examine the effect of the estimation period chosen, the return interval used and the diversification on beta estimates and comments on differences between the empirical results and finance theory as presented in the past.

"ESTIMATING BETAS IN THINNER MARKETS: THE CASE OF THE ATHENS STOCK EXCHANGE"
(International Research Journal of Finance and Economics – 2008)
G. Diacogiannis, P. Makri (GREECE)

The paper examines the presence and magnitude of the intervalling effect bias in ordinary least squares (OLS) beta estimates for securities traded on the Main Market of the Athens Stock Exchange (ATSE) during the period 2001 – 2004 and tests the efficiency of certain past related models to estimate these betas. The paper wishes to accomplish 3 tasks: First, it examines the presence of the intervalling effect bias and its relation to the capitalization of the firms. Second, it appraises the relative ability of the Hawawini model (1980) to estimate "true" betas and third, it estimates betas using models proposed by Scholes & Williams (1977) and Cohen et al (1983a) and compares the results with those of the OLS method. The importance of studying this specific market is the fact that it's a small emerging one while the specific period of study is one reflecting the thin-trading problem since share prices experienced a deep decrease during that period.

The authors made a sample of 187 continuously listed firms on the main market of the ATSE during the period starting January 2001 to December 2004. The choice of the 4-year period was not without a plan since a trade-off between precision and bias is implied. Using a lengthened sample period means having greater precision in beta estimation since more observations are included but on the other hand during that period certain fundamentals of firms may have changed (e.g. divisions, recapitalizations).

The above mentioned sample was then sorted on the basis of market capitalization at December 29 2000. The 30 lowest ranked stocks were selected to comprise the low-cap portfolio while the 30 highest ranked stocks were selected to comprise the high-cap portfolio. Security betas were typically estimated via the standard market model (denotes & assumptions stated earlier in the theory section):

$$R_{it} = a_i + \beta_i R_{Mt} + \varepsilon_{it}$$

while the rate of return for each security was continuously compounded calculated by:

$$R_{it} = ln(P_{it} + d_t) - ln(P_{it-1})$$

where P_{it}, P_{it-1} are the last traded prices for security *i* in period *t* & *t-1* correspondingly, adjusted for capital changes and d_t is the dividend for security *i* announced during period *t*.

For each security in the high-cap portfolio and low-cap portfolio its zero returns (attributable to no change in price due to no new information or no trading) were calculated as a percentage of its total daily, biweekly and monthly return observations. Then for each portfolio and return interval, mean percentage of zero returns and standard deviations were computed showing that the low-cap portfolio experienced higher percentages of zero returns than the high cap portfolio for all three intervals and also as the measurement interval is lengthened, this percentage was decreasing for both portfolios. Results indicate that market capitalization is a good proxy for the frequency of trading.

The authors went on to examine the return interval effect on beta estimates under OLS using two portfolios (high-cap, low-cap) and three measurement intervals (daily, biweekly, monthly). Results show that mean beta estimates for both the high-cap and low-cap portfolios increase as the return interval lengthens, something that contradicts findings from Brailsford & Josev (1997) that for the high-cap portfolio, the mean beta estimates decline as the return interval lengthens. The difference between the beta estimates is significant only for the return interval comparison of daily-to-monthly at 1 % level of significance, a finding that indicates the intervalling-effect bias in betas when daily returns instead of monthly returns are used.

Similar results are obtained when examining the mean R^2 values. Both portfolios experience an increase in mean R^2 values as the return interval lengthens but for the high-cap portfolio, mean R^2 from daily or biweekly returns is significantly different at 1 % level of significance from that calculated using monthly returns. Therefore, the authors show their preference to the use of monthly returns for OLS beta estimation. Concerning the ranges of betas (between maximum and minimum values) and mean standard error of beta estimates, examination of results show that values are increased as the return measurement interval is lengthened for both portfolios.

Another observation shows that for the low-cap portfolio there is only a statistical difference in beta estimates when comparing daily-to-monthly return intervals, a finding that supports the view that the rate of change in beta is greater for the low-cap portfolio and consequently the magnitude of the intervalling-effect bias is inversely related to the market capitalization of the firms.

The authors went on to test the Hawawini (1983) model to estimate betas for biweekly and monthly return intervals using OLS betas estimated from daily returns. Results indicate that the Hawawini (1983) model is efficient to estimate security betas for high-cap portfolios when using longer return intervals. Concerning its ability to predict the change in beta from the change in return interval, the performance is better in the case of the low-cap portfolio, since it estimates a decreasing beta when using near-term return intervals (daily-biweekly, biweekly-monthly) while mixed results are obtained in the case of high-cap portfolios.

Finally, the authors estimate betas using models proposed by Scholes & Williams (1977) with the standard one lead & one lag assumption and Cohen et al (1983a) with 2,3,4 leads and lags as opposed to those via the OLS method only on daily data. Results show that the above-mentioned methods do not decrease the bias of the OLS method and consequently the authors conclude that the OLS method may be the best method in the case of an infrequently traded market like the ATSE.

SUMMARIZING PAST RELATED ARTICLES

Financial economists have long been interested in the risk-return relation and the use of the Capital Asset Pricing Model (CAPM) as a tool to estimate security returns. Nevertheless, a serious econometric problem of errors in variables causes “measured” returns to differ from “true” returns. ***Scholes & Williams (1977)***, based on early notes by Fama (1965) and Fisher (1966), tried to explain this type of problem and built a methodology to overpass it. They attributed this anomaly to the non-synchronous trading of securities and found that OLS estimators of both alphas and betas are biased. More specifically, securities trading very infrequently have beta estimators biased downward and securities trading very frequently have beta estimators biased upwards. For the purposes of their research, they proposed a consistent estimator of beta with one lag, matching and one lead for the simple regression between the observed security return and the corresponding market index on data taken from the NYSE and ASE for the period 1963 to 1975. Their results showed that OLS beta estimates for thin securities are biased downward and for securities traded frequently are biased upwards.

Dimson (1979) referred to the infrequent-trading problem which causes beta estimates to be biased and attributed it to the tendency of the mean value of beta to rise as the differencing interval is increased. He proposed his own method, the Aggregated Coefficients Method which generates returns whose covariance with the market is positively related to its trading frequency. It is actually a multiple regression of security returns on lagging, matching and leading market returns and estimates beta according to the degree of thinness of the security or/and the market. He used monthly returns of firms on the London Share Price Database between 1955 and 1974. Results showed that the use of lagged and leading terms into the market model improves beta estimates but the number of these terms varies according to the nature of the study.

Hawawini (1983) presented a simple model to explain why estimates of beta depend upon the length of the return measurement interval using data from the S&P 500 for the period 1970 to 1973. Results showed that betas are unaffected by the intertemporal cross-correlations between the security and market returns over one-day interval when these are zero or when the market provides a zero auto-correlation. The model also predicts the direction and the strength of the variations in estimated betas.

Cohen et al (1983a) developed a theoretical model to analyze the friction in the trading process that causes "observed" returns to vary from "true" returns. It uses many leads and lags of the market's return to derive a consistent estimator of beta as opposed to one lead and lag of Scholes & Williams model (1977). Furthermore it introduced the term of the "asymptotic estimator" as a consistent estimator using small price-adjustment delays as the interval length measurement lengthens boundless and showed that the magnitude of this bias depends on the relative magnitude of these delays. Findings of this work show that securities with relatively short price-adjustment delays will have their betas overestimated by the OLS method whereas those with lengthy delays will be underestimated by the same method.

Fowler & Rorke (1982) examined the theory behind the basic regression models of Scholes & Williams (1977) and Dimson (1979) and their relative ability to decrease the bias in estimating betas. They concluded that Dimson's estimator is not consistent with that of Scholes & Williams (1977) and itself not expected to yield consistent beta estimates. Therefore, a corrected version is proposed.

Fung, Schwartz & Whitcomb (1985) used daily data from the Paris Bourse during the period January 1977 – April 1980 to identify and correct for the intervalling effect bias in OLS beta estimates. They followed a 3-pass regression to test the beta bias via the Cohen-Hawawini-Schwartz-Whitcomb model (1983) and concluded that average OLS beta estimate and average R^2 increase as the interval is lengthened. Furthermore, they showed that stocks that trade less frequently than the Paris index (used as a proxy) experience significantly negative betas and those trading more frequently positive or insignificantly negative betas. Finally findings showed that there is a significant positive relationship between intervalling effect-bias and market value of shares outstanding.

McInish & Wood (1986) recognize that OLS beta estimates are subject to two sources of bias: thin trading and price adjustment delays. They propose a linear programming model to test the ability of past studies by Scholes & Williams (1977), Dimson (1979), Fowler-Rorke-Jog (1980) and Cohen et al (1983) using data from the NYSE for the period September 1971 to February 1972. Findings showed that "thick" traded portfolios had betas above 1 while "thin" traded portfolios had betas below 1 when daily data used. Each of the past related study reduced the amount of bias but this was 29 % the maximum.

Handa, Kothari & Wasley (1989) examined the behavior of beta as a function of the return interval and the sensitivity of its estimate to the firm's size. They used a sample of stocks listed on the NYSE for the period 1926-1982 and formed 20 portfolios over 8 different return intervals. Findings show that portfolio betas change with relevance to the return measurement interval possibly because the covariance of securities return with the market return and the market return variance do not change proportionally as the interval changes.

Corhay (1992) examined the intervalling effect bias in estimated betas using methods proposed by Hawawini (1980) and Cohen et al (1983) and showed that betas tend to converge to their asymptotic values with a speed that changes relevant to the measurement interval used. He used daily returns from a sample of 250 securities listed on the Brussels Stock Exchange from 1977 to 1985 and sorted them out in a way that 30 differencing interval lengths were used to examine 10 value-weighted portfolios. Findings show that the intervalling effect bias is quite large for small intervals and decreasing as the interval is lengthened leaving the whole-sample beta approximately closely to one. Furthermore small market-value firms appear to have on average lower beta coefficients than larger firms.

Frankfurter et al (1994) investigated the effect of the compounding period length for which rates of return are calculated on the estimation of alpha and beta parameters of the market model and used daily return data from the CRSP NYSE-AMEX file for the period 1976-1987 to examine this relationship. Over 6 different compounding periods based on the market model, the authors showed that average beta estimates differed very little from one period to another whereas average alpha estimates varied at a great degree. Above all, the market model used must be based on the proper length of the compounding period to yield unbiased results.

Beer (1997) assessed the relative ability of models proposed by Vasicek (1973), Scholes & Williams (1977) and Dimson (1979) to estimate betas in a relatively thin market like the Belgian. Using a sample of 181 securities during 1974-1986, she formed 10 groups according to market capitalization and confirmed the intervalling-effect bias in the market. Results showed that the above alternative methods did not decrease the beta bias effectively and consequently proposed the OLS method as the best technique for such a market.

The model proposed by ***Brailsford & Josev (1997)*** provides a description of the size and direction of change in beta estimation as a result of different return intervals. The authors used daily, weekly and monthly returns to examine the intervalling-effect bias in the Australian Stock Exchange for the period 1988 to 1992 and therefore formed two extreme portfolios: a sample of 15 very thinly traded stocks and a sample of 15 very frequently traded stocks. For the first one, mean beta estimates rise as the return measurement interval lengthens while for the latter falls. Finally, testing the Hawawini model (1983), evidence shows that it performs quite well in approximating OLS beta estimates especially for the portfolio of frequently traded stocks.

The choice of the return interval and the estimation period on estimating betas was examined by ***Daves, Ehrhardt & Kunkel (2000)***. In their study based on returns obtained from the CRSP NYSE/AMEX, they tested 4 return intervals and 8 estimation periods. Results showed that shorter return intervals are associated with a greater precision in beta estimation as shown by smaller values of S_β and therefore daily return intervals are proposed. Concerning the estimation period, a 3-year estimation period is proposed for the same reasons.

A recent study on the Istanbul Stock Exchange by ***Odabasi (2003)***, wishes to examine the effect of the estimation period, the return interval and diversification on beta stability. A sample of 100 continuously stocks listed on the ISE over the period 1992 to 1999 was used and findings show that mean beta and R^2 rise as the return interval lengthens. Also, diversification as measured by the correlation coefficient between portfolios of different sizes seems to be positively correlated with beta stability.

Diacogiannis and Makri (2008) examined the presence and magnitude of the intervalling-effect bias in OLS beta estimates for the Athens Stock Exchange during 2001-2004 and its relation to the market capitalization of the firms. Therefore, they formed a low-cap and a high-cap portfolio of 30 stocks each and calculated the zero returns as a percentage of daily, weekly and monthly observations. Findings show that market capitalization seems to be a good proxy for measuring the frequency of trading. Testing the effect of return interval on beta estimates under OLS method, the authors used 3 different intervals to show that mean beta estimates increase for both portfolios as the interval is lengthened, something that contradicts findings of Brailsford & Josev (1997). Testing afterwards the Hawawini model, it appears to perform better in the case of the low-cap portfolio when using near-term intervals but mixed results are derived in the case of the high-cap portfolio. Finally, using models proposed by Scholes & Williams (1977) and Cohen et al (1983a), the authors conclude that the OLS method seems to be the appropriate method to estimate betas in a relatively thin market like this, since the above alternative ones do not decrease beta bias effectively.

A review with the main points of each study is presented at the table below.

AUTHOR(S)/ YEAR	METHODOLOGY	DATA	RESULTS
Scholes & Williams (1977)	Built a consistent estimator of beta	Daily returns from NYSE & ASE (1963-1975)	OLS beta estimates for thin securities biased downward and thick securities upwards.
Dimson (1979)	Aggregated Coefficients Method	London Share Price Database (1955-1974)	The introduction of lagged and leading terms into the market model decreases beta bias.
Hawawini (1983)	Simple model (only one lag & one lead)	S&P 500 (1970-1973)	Security beta estimates change as the return measurement interval is lengthened.
Cohen-Hawawini-Maier-Schwartz-Whitcomb (1983)	Analytical model (uses many leads and lags of the market's return)	Theoretical work	Introduced the term of "asymptotic estimator" of true beta and showed that the bias in OLS beta is cross-sectionally distributed around zero & depends on the magnitude of a security's price adjustment delays.
Fowler-Rorke (1983)	Used corrected versions of Scholes & Williams model (1977) and Dimson model (1979)	Theoretical work	Dimson's model (1979) is not consistent with that of Scholes & Williams (1977) and itself not efficient at all. A corrective method is proposed that yields consistent betas.
Fung-Schwartz-Whitcomb (1985)	3-pass regression procedure	Daily returns from the Paris Bourse (1977-1980)	As the interval is lengthened, average OLS beta estimate is increased. A positive and significant relationship between intervalling effect bias and market value of shares outstanding exists.
McInish-Wood (1986)	Linear programming model	Daily data from the NYSE (1971-1972)	Each past chosen model reduces the amount of beta bias but at 29% the maximum.
Handa-Kothari-Wasley (1989)	Examined portfolios betas over different return intervals	A sample of all stocks listed on CRSP monthly tape (1926-1982)	A security's beta is sensitive to the return interval used to estimate it.
Corhay (1992)	Formed 10 value-weighted portfolios over 30 differencing interval lengths	Daily returns of 250 firms listed on the Brussels Stock Exchange (1977-1985)	The intervalling effect bias is large for small intervals and decreasing as the interval is lengthened leaving the overall sample beta equal to one.
Frankfurter-Leung-Brockman (1994)	Analytical model showing the effect of investment horizon on alphas-betas	Daily return data for 1297 stocks listed on the CRSP NYSE-AMEX daily file (1976-1987)	Regression results based on differencing compounding periods will significantly differ from each other.
Beer (1997)	Assessed models of S&W (1977) & Dimson (1979) on beta estimation	A sample of 181 securities listed on the Brussels Stock Exchange (1974-1986)	Alternative methods did not decrease the beta bias effectively. The OLS method is proposed to measure betas in such a thin market.
Brailsford-Josev (1997)	Formed 2 extreme portfolios (very thin-very thick)	Australian Stock Exchange (1988-1992)	As the return interval is lengthened, mean beta of thin portfolio rises while that of the thick one falls.
Daves-Ehrhardt-Kunkel (2000)	Examined 4 return intervals & 8 estimation periods	Security returns from the CRSP NYSE/AMEX database (1982-1989)	Greater precision in estimating betas when daily returns and a 3-year estimation period used.
Odabasi (2003)	Examined impact of estimation period, return interval & diversification on beta stability	A sample of 100 Istanbul Stock Exchange listed stocks (1992-1999)	Mean beta and R^2 rise as the return interval is lengthened. Diversification and beta stability seem to be positively correlated. Estimation period varies according to interval used.
Diacogiannis-Makri (2008)	Formed a low-cap & a high-cap portfolio to estimate the intervalling effect bias using 3 different return intervals	A sample of continuously listed firms on the Athens Stock Exchange (2001-2004)	Mean beta estimates increase for both portfolios as the return interval is lengthened. The Hawawini model (1977) performs well only in the case of the low-cap portfolio for near-term intervals. OLS method seems to be the appropriate for this market.

D. Data and Research Methodology

Empirical results as presented briefly above conclude that different beta estimates are generated using different return intervals over the sample period. Mean beta estimates and mean R^2 values increase as the return interval is lengthened. Scholes & Williams (1977) and Dimson (1979) demonstrated that because securities are not traded with the same regularity, beta estimates for securities trading infrequently are biased downward and beta estimates for securities trading very frequently are biased upward. The reason stated by most authors is the fact that the whole impact of information is not immediately reflected into prices because of the so called "price adjustment delays" (Cohen et al – 1980). Findings based on different beta estimators showed that this impact is reduced as the return interval lengthens because prices of stocks have the time to incorporate any additional information. Therefore long return intervals could be used when measuring betas as less bias is introduced.

The present study employs a time-series sample of security returns listed on the main market of the Athens Stock Exchange (ATSE). Beta estimation becomes more of an issue for an emerging market like this since it is characterized by light volume and a relatively thin market capitalization compared to the developed and mature markets of Western Europe and U.S. Damodaran (2002) argues that in many emerging markets, the companies and the market itself experience significant changes over short periods of time and thus the implications of such studies are of particular importance.

The sample period covers January 2002 through to December 2006. In order to avoid data problems due to listing and delisting of securities, the securities have been selected on the basis of their continuous presence on the whole sample period. The choice of this period was not without a plan, since the purpose of the paper is to balance between precision and bias. More specifically, the paper wishes to avoid changing structural and fundamental characteristics of the firms chosen by not restricting the period of study but at the same time it wishes to gather as much information as possible in order to extract precise results concerning the beta estimates. Therefore, a 5-year period of study was selected. Past related studies on similar emerging markets used analogous study periods (e.g. Odabasi-2003 for Turkey used 5-year data also).

The next step was to rank the firms with continuous presence according to their market-capitalization as of December 31 of 2001 and choose the 60 ones with the smaller values. We assume that low-capitalized firms reflect the phenomenon of thin trading at best. From this point on, the range of our study is strictly confined to these 60 firms which form a so called *"low-cap study portfolio"* as presented at Tables 1 & 2 below.

Table 1:

Presentation of the "Low-Cap Study Portfolio"

NAME	***Code***	***NAME***	***Code***
EMPORIKOS DESMOS PR	EMPO	VIS-CONTAINER CR	BISK
IDEAL GROUP PR	INTP	KORDELLOS CH BROS	KOR
PRAXITELIO HOSPITAL PR	KORP	MATHIOS	MATI
ANEK LIN.PR 1990 ISS.	ANLE	KNIT.FAC.MAXIM CM PTDS.	MAXI
J BOUTARIS & SON HLDG PR	MPOP	DUROS	DOUR
BIOSSOL PR	BIOP	MEVACO METALLURGICAL	MEVA
N LEVENTERIS PR	LEBP	ZAMPA	ZAMP
VIS-CONTAINER PR	BISP	MULTIRAMA	ATHH
XYLEMPORIA PR	XYLP	TECHNICAL PUBS.	TECP
ELVIEMEK LD.DEV.LOGIST. PARKS	ELBI	IMPERIO ARGO GROUP	IMP
TRIA ALPHA PR	AAAP	MICROLAND	MICR
EMPORIKOS DESMOS CR	EMKO	EKTER	EKT
TRIA ALPHA CR	AAAK	N VARVERIS-MODA BAGNO	MODA
SAOS AOY.SHPC.OF SMTE.	GALI	IKONA-IHOS	IKO
ANEK LIN.PR 1996 ISS.	ANLO	PIPE WORKS CR	TZKA
CHATZIKRANIOTIS MILLS	TYRN	SFAKIANAKIS CB	SFA
XYLEMPORIA CR	XYLK	VIVERE ENTERTAINMENT	VIVE
PERSEFS	PERS	VOGIATZOGLOU SYSTEMS	VOSY
MINERVA KNITWEAR	MIN	KARMOLEGOS	KARA
N LEVENTERIS CR	LEBK	YALCO-CONSTANTINOU	YALC
E PAIRIS	EPA	FIERATEX	FIER

VARANGIS	VAR	PLIAS CONSUMER GOODS CB	PAPK
PRAXITELIO HOSPITAL CR	KORK	FG EUROPE	BIMK
FLR MLS C SARANTOPOULOS	SARA	SHEET STEEL	XALI
BIOSSOL CR	BIOK	DIONIC	DION
PARNASSOS ENTERPRISES	PARN	NAFPAKTOS TEX.INDS.	NAYP
C ROKAS PR	ROKP	ELTRAK CR	ELTK
LANAKAM CB	KASK	ELVE	ELVE
ELFICO	ELFK	PHILIPPOS NAKAS	NAKA
FINTEXPORT	FINA	MESOHORITIS BROTHERS	MESH

Table 2:

The "Low-Cap Study Portfolio" based on market capitalization as of December 31 2001

Code	Market Cap as at 31/12/2001 (€ million)	Code	Market Cap as at 31/12/2001 (€ million)	Code	Market Cap as at 31/12/2001 (€ million)
EMPO	0,07	SARA	17,38	VIVE	27,66
INTP	1,50	BIOK	18,75	VOSY	27,70
KORP	1,97	PARN	19,00	KARA	27,96
ANLE	2,24	ROKP	19,06	YALC	28,38
MPOP	3,08	KASK	19,38	FIER	28,56
BIOP	3,13	ELFK	19,76	PAPK	28,88
LEBP	3,63	FINA	19,85	BIMK	29,04
BISP	3,89	BISK	20,44	XALI	29,31
XYLP	4,37	KOR	20,99	DION	29,60
ELBI	4,95	MATI	21,94	NAYP	29,70
AAAP	7,94	MAXI	22,37	ELTK	29,78
EMKO	7,98	DOUR	23,21	ELVE	29,90
AAAK	11,13	MEVA	23,63	NAKA	30,05
GALI	12,29	ZAMP	24,81	MESH	30,58
ANLO	13,52	ATHH	25,65	SFA	27,64
TYRN	14,53	TECP	25,70	KORK	17,16
XYLK	15,46	IMP	25,80	TZKA	27,30
PERS	15,64	MICR	26,55	VAR	17,14
MIN	15,69	EKT	26,82	EPA	16,63
LEBK	15,87	MODA	26,96	IKO	27,01

Examining the low-cap portfolio serves us to test the intervaling-effect bias better because such firms reflect the *"thin-trading problem"*. This refers to firms that experience at a great degree zero daily return changes due to no new information or zero trading. According to Brailsford & Josev (1997) a high percentage of zero returns is suggestive of a thinly-traded firm. They additionally state that low capitalized firms would have a higher percentage of zero returns compared to high capitalized firms. Table 3 presented below clearly shows the magnitude of the thinness of the stocks chosen with the numbers showing the percentage of zero changes in daily returns for all stocks during the 5-year period. On average, the "low-cap study portfolio" faces 32.96% (st.dev. 17.86%) zero return changes for the study period on a daily basis, a finding similar to that of past studies that indicates that these stocks face important non-trading problems.

Table 3:

Measuring the thinness of the portfolio using the trading volume as percentage of zero daily moves during the 5-year period

Code	% of zero daily trading	Code	% of zero daily trading	Code	% of zero daily trading
EMPO	69,59%	SARA	21,81%	VIVE	27,19%
INTP	88,10%	BIOK	41,17%	VOSY	23,27%
KORP	77,34%	PARN	22,12%	KARA	17,97%
ANLE	65,28%	ROKP	31,72%	YALC	22,96%
MPOP	70,89%	KASK	33,64%	FIER	19,05%
BIOP	56,91%	ELFK	31,87%	PAPK	32,49%
LEBP	33,33%	FINA	23,35%	BIMK	24,58%
BISP	79,42%	BISK	28,80%	XALI	33,56%
XYLP	48,31%	KOR	16,67%	DION	22,12%
ELBI	71,81%	MATI	17,97%	NAYP	21,27%
AAAP	50,15%	MAXI	26,73%	ELTK	22,04%
EMKO	30,72%	DOUR	14,52%	ELVE	18,13%
AAAK	49,54%	MEVA	15,67%	NAKA	18,28%
GALI	26,73%	ZAMP	33,33%	MESH	30,95%
ANLO	37,86%	ATHH	22,04%	KORK	42,70%
TYRN	20,35%	TECP	17,74%	SFA	31,34%
XYLK	31,41%	IMP	26,04%	MICR	30,18%
PERS	22,20%	EKT	17,97%	MODA	26,57%
MIN	26,27%	IKO	32,03%	TZKA	23,27%
LEBK	22,27%	EPA	15,98%	VAR	20,28%

For each security in the sample daily, weekly and monthly prices for each of the firms in the sample were extracted from *DataStream* and adjusted for capitalization changes but not for dividends. The weekly stock returns are computed using the closing value for the Tuesday of each week. The monthly stock returns are computed using the closing value for the first working day of each month.

The study examined continuously compounded return series calculating the return on each security as: $R_{it} = ln(P_{it}) - ln(P_{it-1})$ where *ln* is the natural logarithm operator, P_{it} is the last traded price for security *i* in period *t* and P_{it-1} is the last traded price for the same security with one lag in the period of study.

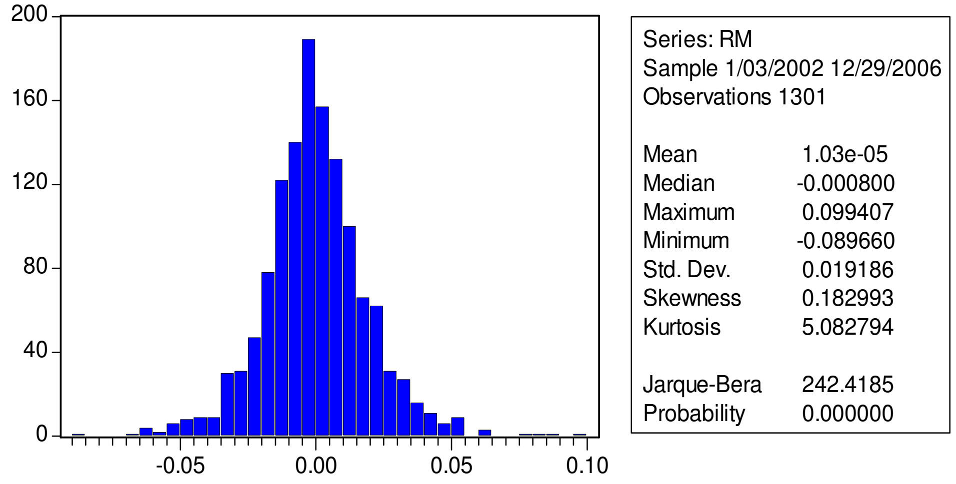
Saniga, McInish & Gouldey (1981) and Roden (1985) have shown that the estimation of beta depends among others on the specification of the market index. In particular Roll and Ross (1994) emphasize the importance of the selection of an efficient market portfolio to improve the quality of the results. The present study does not use a published market value weighted index (e.g the *FTSE Small-CAP 80 Index*) as the market index, but uses as a proxy the weighted market index composed of the 60 thinly-traded securities chosen, adjusted for capitalization changes but not for dividend payments [similar to study by Corhay (1992)]. Thus the Market Index created is mainly a price index, weighted to give greater influence to higher capitalized firms.

Market return series based on daily, weekly and monthly return intervals are subsequently calculated by:

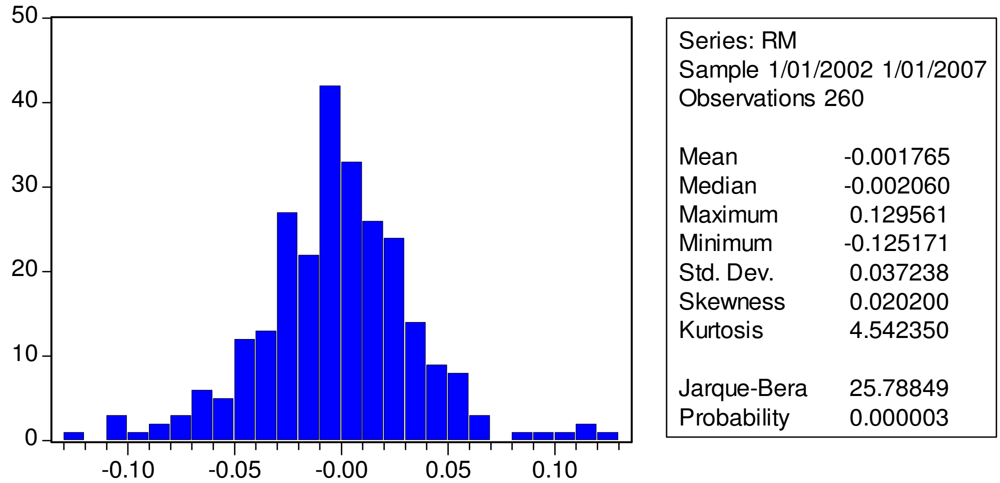
$$R_{mt} = \frac{\sum_{1}^{60} p_{it} w_i}{\sum_{1}^{60} w_i}$$

where w_i is the weight, being the market capitalization of each firm and p_{it} stands for the daily, weekly and monthly return series for each of the 60 securities and are attributed at graphs shown below.

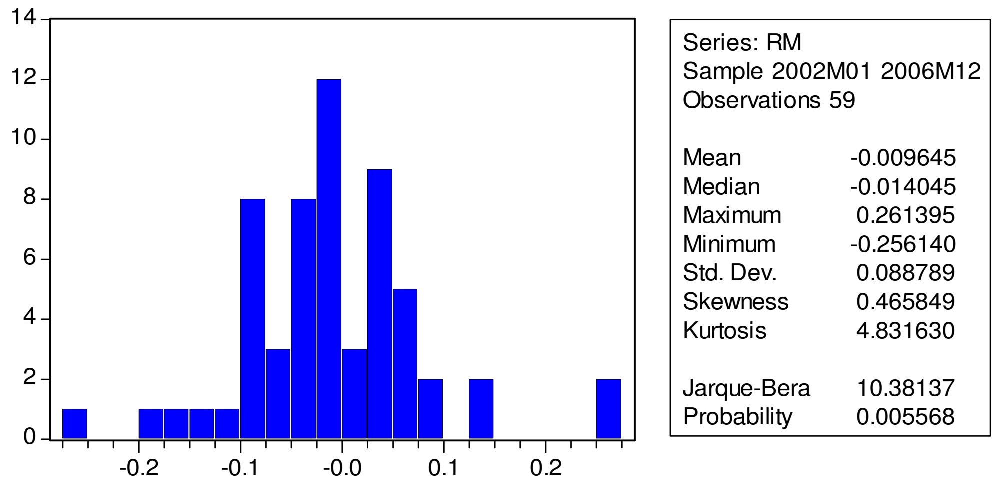
Graph 1: *Daily market return series (R_M) for 1301 observations*



Graph 2: *Weekly market return series (R_M) for 260 observations*



Graph 3*: Monthly market return series (R_M) for 59 observations*



Portfolio theory states that security beta is estimated via the standard market model with the OLS method: $R_{it} = a_i + b_i R_{mt} + e_{it}$ where:

R_{it} is the return on security *i* in period *t*

R_{mt} is the return on the market index in period *t*

a_i is the constant term for security i

b_i is the sensitivity of returns of security *i* to the market index returns measured as $cov(R_i,R_M)/Var(R_M)$.

e_{it} is the error term which we assume to be normally distributed with zero mean, constant variances, zero $cov(R_{mt},e_{it})$ and $cov(e_{it},e_{it-1})$.

The estimates of a_i and b_i are obtained using an ordinary least square regression but as stated before are dependent on the length of the measurement interval.

In our study, we compute the beta of our portfolio as the mean of its security betas and the standard deviation accordingly.

The results obtained by applying the OLS technique of the security returns on the market index are reported on Table 4 below.

Table 4[*]:

Summary Statistics of beta estimates and R^2 for three different return intervals

Summary Statistics	Daily	Weekly	Monthly
mean betas	0.904173	0.941028	0.992196
stdev of beta estimates	0.386756	0.387435	0.446598
mean standard error	0.074692	0.122868	0.202300
max beta	1.555486	1.676175	2.030074
min beta	0.055436	-0.168102	-0.032365
range	1.500050	1.844277	2.062439
skewness	0.386756	0.388893	0.041528
kurtosis	-0.177122	2.944428	2.774677
mean Rsq	0.127064	0.215010	0.324590

[*]*OLS betas are estimated over the period 01/01/2002 to 31/12/2006. Daily estimates are based on 1301 observations, weekly estimates are based on 260 observations and monthly estimates are based on 60 observations*.

Inspection of the Table 4 above shows that the mean beta estimate increases as the return interval lengthens. Indeed, the mean beta estimate using daily returns rises from 0.904173 to 0.941028 using weekly returns (+ 4%) and to 0.992196 (+ 10%) using monthly returns. Such a finding seems to support the presence of the intervalling-effect bias which is caused by friction in the trading process and is consistent with previous evidence from the same market (Diacogiannis, Makri - 2008) and other markets: (Brailsford, Josev – 1997) for the Australian market and Odabasi (2008) for the Turkish market.

Table 5*:

Testing the difference between mean beta estimates for pairs of returns

	Daily	***Weekly***	***Monthly***
Daily		0,036855	0,088023
		(0,521474)	(1,154083)
Weekly	0,036855		0,051168
	(0,521474)		(0,67037)
Monthly	0,088023	0,051168	
	(1,154083)	(0,67037)	

**The difference between the mean beta estimates is not significant for any pair at 5 % significance level. Numbers in parentheses show the confidence level at which the null hypothesis (equity of mean betas) can be rejected.*

Table 5 above proves the lack of the intervaling-effect bias when using daily instead of monthly returns or weekly instead of daily returns or any other combination. The difference between the mean beta estimates is not statistically significant at 5 % significance level for any pair chosen. Such a finding seems to contradict evidence from a similar study by Diacogiannis & Makri (2008) on the same market but on a different time period which supported the presence of the intervalling-effect bias in estimated betas when using daily returns instead of monthly returns for a low-cap portfolio. Analytical statistical results for all 3 pairs of data obtained are presented at Tables 6, 7 & 8 below.

Table 6:

Test for Equality of Mean Betas between Monthly & Weekly Series

Method		df	Value	Probability
t-test		118	0,67037	0,5039
Anova F-statistic		(1, 118)	0,449397	0,5039
Analysis of Variance				
Source of Variation		df	Sum of Sq.	Mean Sq.
Between		1	0,078545	0,078545
Within		118	20,62381	0,174778
Total		119	20,70236	0,173969
Category Statistics				
Variable	Count	Mean	Std. Dev.	Std. Err.of Mean
MONTHLY	60	0,992196	0,446598	0,057656
WEEKLY	60	0,941028	0,387435	0,050018
All	120	0,966612	0,417096	0,038076

Table 7:

Test for Equality of Means between Daily & Weekly Series

Method		**df**	**Value**	**Probability**
t-test		118	0,521474	0,603
Anova F-statistic		(1, 118)	0,271935	0,603
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,040748	0,040748
Within		118	17,68149	0,149843
Total		119	17,72224	0,148926
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	60	0,904173	0,386756	0,04993
WEEKLY	60	0,941028	0,387435	0,050018
All	120	0,922601	0,38591	0,035229

Table 8:

Test for Equality of Means between Daily & Monthly Series

Method		**df**	**Value**	**Probability**
t-test		118	1,154083	0,2508
Anova F-statistic		(1, 118)	1,331908	0,2508
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,232438	0,232438
Within		118	20,59279	0,174515
Total		119	20,82523	0,175002
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	60	0,904173	0,386756	0,04993
MONTHLY	60	0,992196	0,446598	0,057656
All	120	0,948184	0,418332	0,038188

The mean R^2 values increase as the return measurement interval lengthens, a finding that is consistent with previous studies (Cohen et al - 1980). The mean R^2 of the portfolio is ranged from 12.70 % (daily returns) to 32.45 % (monthly returns). Of importance though is the fact that the mean R^2 obtained using daily or weekly returns is statistically different from that calculated using monthly returns at 5% significance level, a finding that supports the presence of the intervaling-effect. More specifically, Table 9 below shows that the difference between the mean R^2 using daily returns and the mean R^2 using weekly returns is significant at 5% significance level. The same condition exists if we compare daily with monthly returns or monthly with weekly returns at 5% significance level.

Table 9:
Testing the difference between mean R^2 values for pairs of returns

	Daily	**Weekly**	**Monthly**
Daily		0,087946	0,197526
		(4,500446)	(7,842443)
Weekly	0,087946		0,10958
	(4,500446)		(3,992467)
Monthly	0,197526	0,10958	
	(7,842443)	(3,992467)	

**The difference between mean R^2 is significant for any pair chosen at 5% significance level. Numbers in parentheses show the confidence level at which the null hypothesis (equity of mean R^2) can be rejected.*

Analytical statistical results for all 3 pairs of data obtained are presented at Tables 10, 11, 12, 13 & 14 below.

Table 10:

Test for Equality of Mean R^2 values between Weekly & Monthly Series

Method		**df**	**Value**	**Probability**
t-test		118	3,992467	0,0001
Anova F-statistic		(1, 118)	15,93979	0,0001
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,360235	0,360235
Within		118	2,666771	0,0226
Total		119	3,027006	0,025437
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
WEEKLY	60	0,215010	0,122575	0,015824
MONTHLY	60	0,324590	0,173709	0,022426
All	120	0,2698	0,15949	0,014559

Table 11:

Test for Equality of Mean R^2 values between Weekly & Daily Series

Method		**df**	**Value**	**Probability**
t-test		118	4,500446	0
Anova F-statistic		(1, 118)	20,25402	0
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,232037	0,232037
Within		118	1,351847	0,011456
Total		119	1,583883	0,01331
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	60	0,127064	0,088814	0,011466
WEEKLY	60	0,215010	0,122575	0,015824
All	120	0,171037	0,115369	0,010532

Table 12:

Test for Equality of Mean R^2 values between Monthly & Daily Series

Method		**df**	**Value**	**Probability**
t-test		118	7,842443	0
Anova F-statistic		(1, 118)	61,50391	0
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	1,170503	1,170503
Within		118	2,245701	0,019031
Total		119	3,416204	0,028708
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	60	*0,127064*	*0,088814*	*0,011466*
MONTHLY	60	*0,324590*	*0,173709*	*0,022426*
All	120	0,225827	0,169433	0,015467

Table 13:

Test for Equality of Mean R^2 values between all Series

Method		df	Value	Probability
Anova F-statistic		(2, 177)	33.20512	0.0000
Analysis of Variance				
Source of Variation		df	Sum of Sq.	Mean Sq.
Between		2	1.175184	0.587592
Within		177	3.132159	0.017696
Total		179	4.307343	0.024063
Category Statistics				
Variable	Count	Mean	Std. Dev.	Std. Err.of Mean
RSQ_DAILY	60	0.127064	0.088814	0.011466
RSQ_MON	60	0.324590	0.173709	0.022426
RSQ_WEEKLY	60	0.215010	0.122575	0.015824
All	180	0.222221	0.155124	0.011562

Table 14*:

Correlation Matrix

Correlation Matrix	RSQ_DAILY	RSQ_MON	RSQ_WEEKLY
RSQ_DAILY	1.000000	0.089836	0.924288
RSQ_MON	0.089836	1.000000	0.106417
RSQ_WEEKLY	0.924288	0.106417	1.000000

**The matrix shown above shows the intensity of the relationship between daily & weekly results (0.924288 very close to 1) while the intensity between daily & monthly (0.089836) along with weekly and monthly (0.106417) results is of particular weakness.*

Examining the range of betas at Table 4 above, meaning the difference between the maximum and minimum betas, it can be seen that it increases as the return interval is lengthened, a finding that is consistent with the study of Brailsford and Josev (1997). The range for daily data is very close to 1.50 and increases to 1.84 (+22.9%) for the weekly data and to 2.06 (+37.5%) for the monthly data.

In addition, the standard error of mean beta estimates rises as the return interval is lengthened, consistent with findings from Handa et al (1989) and Frankfurter et al (1994). Such a result is not unexpected since the number of observations used in the OLS regression decreases as the length of the return interval increases from daily to weekly and from weekly to monthly, given the fixed sample period of 5 years.

Skewness and Kurtosis are 2 measures of asymmetry and show how the series of data are characterized in terms of location and variability with the respect to the optimum prices of the normal distribution (Skewness=0 & Kurtosis=3). According to Table 4 above, monthly data appear to have a value for skewness (0.041528) closer to the optimum 0 which CAPM theory when measuring betas assumes whereas daily and weekly data skewness is 0.386756 and −0.388893 respectively. Negative values for skewness show that the data set is skewed left whereas positive values show that the data set is skewed right. Our values are very close to zero for all 3 sets and can be assumed to follow a normal distribution. Weekly data on the other hand seem to have a value of kurtosis closer to 3 (2.944428) followed by monthly data (2.774677) while the daily data are far lower (-0.177122). A positive kurtosis shows a peaked distribution while a negative a flat one.

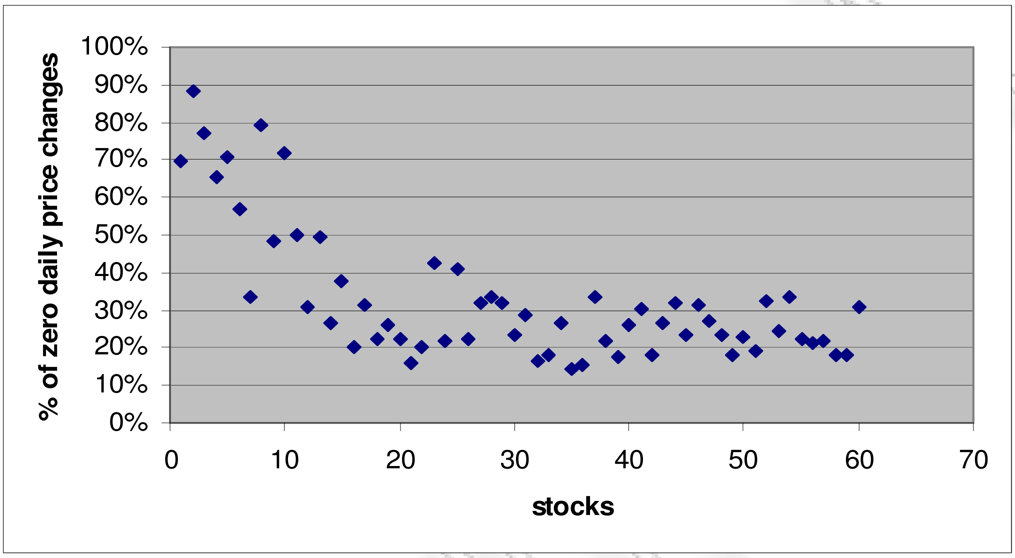
Summarizing, Tables 4, 5, 9 presented above state clearly that mean beta estimates and mean R^2 increase as the return measurement interval lengthens just like any other past related article finds, but differences for means are statistically significant at 5% only between mean R^2 and not between mean beta pairs. It seems that the intervalling effect bias is not supported in the case of our research judging by this criterion. The reason seems to be on the one hand the extremely thin trading of this type of stocks (for some stocks over 70% of zero trading in the period studied) and on the other hand the delay of the stock prices to incorporate any additional information. Such a condition causes cross serial correlation in the security returns and autocorrelation in the market returns. This has as an effect the estimates of mean betas and variances of betas to be downward biased and consequently the values of t-statistics to be rather low and biased. Table 5 above confirms the downward bias of the t-statistics when measuring the statistical difference between mean beta estimates. Similar findings with biased mean betas were also found in similarly thin and immature markets like the Belgian, conducted by Beer (1997).

Also because of the presence of autocorrelation in the market returns, the variance of the market index seems to be downward biased and so is the covariance of the security returns with the market returns. Since the market index is equally weighted, extremely thin stocks are not so correlated with the market index (since they don't trade that often) but dominate the index causing the correlation of the other stocks to be downward biased. This in part explains the low values of mean R^2 at Table 4 above. Concluding, we can say that the presence of certain stocks that are regarded as extremely thin in our sample make the results of our research biased and consequently defy the presence of the intervalling effect bias for this market.

- ***Adjusted try to estimate betas of thinly traded stocks listed in the Athens Stock Exchange for the period 2002-2006***

Given the bias of the results when dealing with the sample of the 60 stocks chosen according to the market capitalization of 31/12/2001, we decided to go on the same procedure but at this time to place stricter rules concerning the measure of thinness. Stocks experiencing zero daily changes more than 50% of the time during the 5-year period of study are excluded from the sample taken, seen as extremely thin and consequently as causing biased results. The diagram below gives us a clear view of the 13 stocks that are regarded as unacceptable by the new rules.

Diagram**: *Measuring the thinness of each stock in the sample separately



** 13 stocks of the sample are rejected as extremely thin for the 2nd try to estimate mean betas in the market*

***Table 15**:**

Summary Statistics of beta estimates and R^2 for three different return intervals

2nd TRY	Daily	Weekly	Monthly
mean betas	0.993894	1.013271	1.032268
stdev of beta estimates	0.294211	0.307867	0.400219
mean standard error	0.066189	0.109825	0.187286
max beta	1.482911	1.623085	1.985838
min beta	0.449526	0.457692	0.347439
range	1.033384	1.165394	1.638398
skewness	0.294211	0.047998	0.356330
kurtosis	0.001354	1.974421	2.520242
mean Rsq	0.158389	0.256770	0.353870

**OLS betas are estimated over the period 01/01/2002 to 31/12/2006. Daily estimates are based on 1301 observations, weekly estimates are based on 260 observations and monthly estimates are based on 60 observations.*

Inspection of the Table 15 above confirms the tendency of mean betas and mean R^2 to rise as the return interval lengthens just as the 1st try finds. The mean standard error and the range also rise as the interval lengthens for the same reasons as before. What is more important though is to estimate the statistical difference between mean beta estimates and mean R^2 values for every possible pair of interval. Results for the second time confirm the absence of the intervalling effect bias when estimating mean betas of thin stocks of the Athens Stock Exchange during the period 2002-2006. More specifically, the difference between mean beta estimates is not statistically significant for any pair chosen at 5%, as the Table 16 states.

Table 16**:

Testing the difference between mean beta estimates for pairs of returns

2nd TRY	Daily	Weekly	Monthly
Daily		0,019377	0,038374
		(0,311958)	(0,529624)
Weekly	0,019377		0,018997
	(0,311958)		(0,257920)
Monthly	0,038374	0,018997	
	(0,529624)	(0,257920)	

**The difference between the mean beta estimates is not significant for any pair at 5 % significance level. Numbers in parentheses show the confidence level at which the null hypothesis (equity of mean betas) can be rejected.*

Analytical statistical results for all 3 pairs of data obtained are presented at Tables 17, 18 & 19 below

Table 17:

Test for Equality of Mean Betas between Monthly & Weekly Series

Method		**df**	**Value**	**Probability**
t-test		92	**0,25792**	0,797
Anova F-statistic		(1, 92)	0,066523	0,797
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,00848	0,00848
Within		92	11,72803	0,127479
Total		93	11,73651	0,126199
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
MONTHLY	47	1,032268	0,400219	0,058378
WEEKLY	47	1,013271	0,307867	0,044907
All	94	1,022769	0,355245	0,036641

Table 18:

Test for Equality of Means between Daily & Weekly Series

Method		**df**	**Value**	**Probability**
t-test		92	**0,311958**	0,7558
Anova F-statistic		(1, 92)	0,097318	0,7558
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,008824	0,008824
Within		92	8,341731	0,090671
Total		93	8,350555	0,089791
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	47	0,993894	0,294211	0,042915
WEEKLY	47	1,013271	0,307867	0,044907
All	94	1,003582	0,299651	0,030907

Table 19:

Test for Equality of Means between Daily & Monthly Series

Method		**df**	**Value**	**Probability**
t-test		92	**0,529624**	0,5976
Anova F-statistic		(1, 92)	0,280502	0,5976
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,034605	0,034605
Within		92	11,34983	0,123368
Total		93	11,38444	0,122413
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	47	0,993894	0,294211	0,042915
MONTHLY	47	1,032268	0,400219	0,058378
All	94	1,013081	0,349876	0,036087

On the other hand mean R^2 values experience statistically significant changes between all possible pairs of intervals for 95% statistical level. Such a result is not unexpected given the fact that the new market index is weighted and re-calculated according to the 47 stocks of the new sample that comprise it. Analytical results for the tests of equality are presented below.

Table 20:
Testing the difference between mean R^2 values for pairs of returns

2nd TRY	Daily	Weekly	Monthly
Daily		0,098381	0,195481
		(4,979920)	(7,497748)
Weekly	0,098381		0,097100
	(4,979920)		(3,428195)
Monthly	0,195481	0,097100	
	(7,497748)	(3,428195)	

The difference between mean R^2 is significant for any pair chosen at 5% significance level. Numbers in parentheses show the confidence level at which the null hypothesis (equity of mean R^2) can be rejected.

Table 21:

Test for Equality of Mean R^2 values between Weekly & Monthly Series

Method		**df**	**Value**	**Probability**
t-test		92	**3,428195**	0,0009
Anova F-statistic		(1, 92)	11,75252	0,0009
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,221569	0,221569
Within		92	1,734468	0,018853
Total		93	1,956037	0,021033
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
WEEKLY	47	0,35387	0,160173	0,023364
MONTHLY	47	0,25677	0,109774	0,016012
All	94	0,30532	0,145026	0,014958

Table 22:

Test for Equality of Mean R^2 values between Weekly & Daily Series

Method		**df**	**Value**	**Probability**
t-test		92	**4,97992**	0
Anova F-statistic		(1, 92)	24,7996	0
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,227452	0,227452
Within		92	0,843788	0,009172
Total		93	1,071241	0,011519
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	47	0,158389	0,079327	0,011571
WEEKLY	47	0,25677	0,109774	0,016012
All	94	0,207579	0,107325	0,01107

Table 23:

Test for Equality of Mean R^2 values between Monthly & Daily Series

Method		df	Value	Probability
t-test		92	**7,497748**	0
Anova F-statistic		(1, 92)	56,21623	0
Analysis of Variance				
Source of Variation		**df**	**Sum of Sq.**	**Mean Sq.**
Between		1	0,898005	0,898005
Within		92	1,469619	0,015974
Total		93	2,367624	0,025458
Category Statistics				
Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Err.of Mean**
DAILY	47	0,158389	0,079327	0,011571
MONTHLY	47	0,35387	0,160173	0,023364
All	94	0,256129	0,159557	0,016457

E. Summary and Conclusions

The present work investigates the intervalling-effect bias in OLS estimated betas for stocks listed on the main market of the Athens Stock Exchange for the period January 2002 to December 2006. To achieve this, a sample of the 60 most thinly traded stocks at December 31 2001 was formed according to market capitalization and named thereafter *"low-cap study portfolio"*. Daily, weekly and monthly data for the above stocks were then derived for the 5-year selected period to test the impact of different return intervals on the estimation of beta. More specifically, mean beta estimates and mean R^2 values increase as the return interval lengthens, consistent with findings from earlier studies [Cohen et al (1980) & Brailsford, Josev (1997)]. Although the change in mean beta as we use longer return intervals is not statistically significant at 5% significance level, the change in mean R^2 values is statistically significant for any pair of intervals chosen. Findings seem to contradict those of a previous study by Diacogiannis & Makri (2008) on the same market but on a different study period which showed that a statistically significant change in mean betas exists only for the case of the low-cap portfolio created (between daily and monthly returns) and not for the high-cap one.

Even when we followed the same procedure for a new 47-stock sample excluding certain extremely thin stocks (over 50% of zero daily changes during the 5-year period), the intervalling effect bias was not supported by findings from the differences of mean beta estimates for every pair of interval chosen. Therefore judging from the results of this research on 60 thinly traded stocks of the Athens Stock Exchange during the period 2002-2006, no differences in beta estimates is found when using 3 different return intervals (daily, weekly, monthly) and thus one may use either interval to estimate the systematic risk of such stocks as measured by the beta of the stock.

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