



University Of Piraeus

**Department of Financial and
Banking Management**

**Postgraduate Program in Financial
and Banking Management**

Thesis

Subject:

**“The Fama and French model with co-
Skewness and co-kurtosis”**

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*To my parents for their support
To Marita for inspiration
To Dr Diakogiannis for his guidance*

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Preface

The purpose of this thesis is by using the methodology proposed by Daniel Chi-Hsiou Hung, Mark Shackleton and Xinzhong Xu (2004) to examine the significance of an extended model based on Fama and French multifactor model for asset pricing

This thesis examines determinants of the cross-section of portfolio returns from CAPM beta and other strategies based on value and size. It does so using two recent developments in the literature, using higher order asset pricing models that encompass systematic risks above the traditional CAPM beta covariance. The second development is the use of the up and down markets theory in the model. It is quite common that the returns on stocks do not follow a normal distribution but have skewness and kurtosis. This has not been widely researched in the past literature especially with the use of the Fama and French factors of value and size.

It is important to note that the thesis consists of two main parts. The first part is theoretical and consists of all the notions and knowledge that is needed for the reader to understand the second part of the thesis. The second part consists of the empirical procedure that is needed in order to examine the model proposed.

The first part begins with the presentation of the Markowitz theory which is the benchmark and the initiative for all models and theories that contributed to the evolution of the science of finance. The presentation of this theory is essential because all the basic notions of finance are presented here. The notions that are analyzed here are, expected return, variance, portfolio variance, portfolio return, the risk free asset and diversification. Prior to Markowitz's work, investors focused on assessing the risks and rewards of individual securities in constructing their portfolios. Standard investment advice was to identify those securities that offered the best opportunities for gain with the least risk and then construct a portfolio from these. Markowitz was the first to understand and present the benefits of portfolio making in the risk and return involved in investment strategies. Furthermore, he was the first to model these theoretical findings in a mathematical way.

The next chapter presents the Market Model which has increased usage in finance. The basic notions of this model are presented here. Also it is important to note

here that the emphasis in the presentation of the market model is given to the mathematical part of it.

The next chapter is the presentation of the most important model in finance theory the Capital Asset Pricing Model widely known as CAPM. The standard form of the general equilibrium relationship for asset returns was developed separately by Sharpe, Lintner and Mossin. Hence, it is often referred to as the Sharpe-Lintner-Mossin form of the capital asset pricing model. This model has been derived in several forms involving different degrees of rigor and mathematical complexity. There is a trade-off among these derivations. The more complex forms are more rigorous and provide a framework within which alternative sets of assumptions can be examined. Here the basic form of CAPM is presented followed by a thorough analysis of all notion involved in the model such as the beta coefficient, the Capital Market Line the Security Market Line and so on. The presentation of the model is very important for this thesis as the model that is being examined is actually a derivation of the CAPM.

The next chapter is a presentation of all the past tests on CAPM. These tests show the fact that the CAPM in principle fails to explain the returns of common stocks and portfolios. This happens because of important theoretical and methodological problems that are presented in detail in this chapter. Furthermore within this chapter there is a detailed presentation of the major empirical tests in CAPM and their finding are discussed. All in all, this chapter contains the criticism on the CAPM that occurred throughout the years.

The last chapter of the first part of this thesis is the presentation of the major work and the impact of the theory of Fama and French. The purpose of Fama and French is to evaluate the joint role of market beta, size (ME), E/P, leverage, book to market equity in the cross section analysis of average returns of stocks. Their research find out that the CAPM is not supported meaning that the average returns are not positively correlated with market betas. Their result are that beta doesn't seem to contribute to cross section of average return of common stocks and the combination of size factor and book to market equity seems to absorb the effect of E/P and leverage in the explaining of average returns of stocks. This chapter contains all the basic notions of their theory. Size and book to

market factors are analyzed thoroughly and the finding of their empirical research are presented.

The next chapter is the first of the second part of this thesis. After the presentation of all the needed theoretical background for the reader in order to understand the research done, an analysis of the methodology used is presented. This chapter contains the presentation of the data that are used in this thesis and then the methodology is presented in great detail. The beta decile portfolios formation, the size and book to market portfolios formations, the regressions that are performed, the theory and the derivations of the proxies for skewness and kurtosis are analyzed.

The next chapter contains the empirical results of this study. The regressions, both time series and cross-sectional are presented and also an analysis and explanation of the results is performed.

Finally, the last chapter of this thesis contains the conclusions and the recommendations for further research.

It is important to thank the PhD students Mr. Antonis Antipas, Mr. Theodore Stamatiou and Emily Tzagarakis and Mr. Athanasios Haremis for their help in the completion of this project.

1. Portfolio Theory

1.1 Theoretical presentation

Modern portfolio theory (MPT)—or portfolio theory—was introduced by Harry Markowitz with his paper "Portfolio Selection," which appeared in the 1952 Journal of Finance. Thirty-eight years later, he shared a Nobel Prize with Merton Miller and William Sharpe for what has become a broad theory for portfolio selection.

Prior to Markowitz's work, investors focused on assessing the risks and rewards of individual securities in constructing their portfolios. Standard investment advice was to identify those securities that offered the best opportunities for gain with the least risk and then construct a portfolio from these. Following this advice, an investor might conclude that railroad stocks all offered good risk-reward characteristics and compile a portfolio entirely from these. Intuitively, this would be foolish. Markowitz formalized this intuition. Detailing mathematics of diversification, he proposed that investors focus on selecting portfolios based on their overall risk-reward characteristics instead of merely compiling portfolios from securities that each individually has attractive risk-reward characteristics. In a nutshell, investors should select portfolios not individual securities.

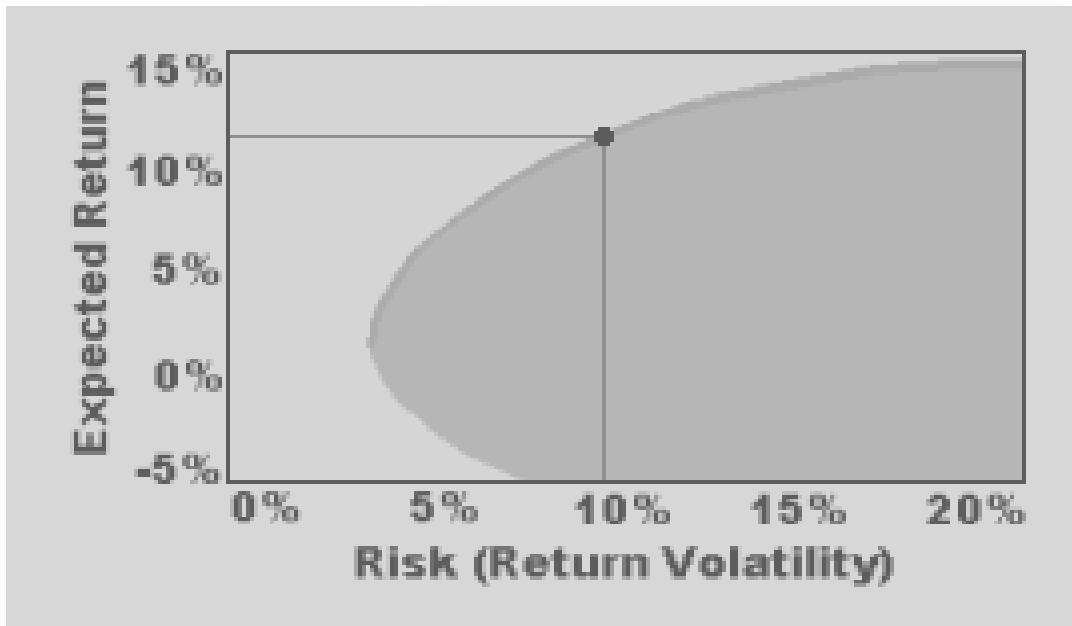
In his article Markowitz analyses the stage of the portfolio selection procedure that starts with the relevant beliefs for future returns and ends with the selection of assets that will compose the portfolio. Initially he considers the rule that states that the investor wants to maximize the future expected returns. This rule is rejected by Markowitz as a guide to investment decision. Next, Markowitz examines the rule that investors find desirable to have high expected returns and undesirable to have variance in these returns. This rule is for Markowitz more suitable as a guide for the formation of portfolio and the behavior of the investor.

A basic rule for the choosing and formation of a portfolio is that the investor must maximize the present value of future returns. Since there is uncertainty in future, expected returns, there must be a risk estimation for that returns. Or there must be a variation of the rate that we calculate future values according to the risk involved.

A rule of expecting high present value for future returns must be rejected. This happens because such a lure does not imply the diversification which intuitively is very important. It is accepted then a diversified portfolio is more desirable than an undiversified one. Markowitz states that a rule that does not imply diversification cannot be accepted as a guide for portfolio selection

The first assumption of the Markowitz model is that investors are risk averse. In other words on two given assets with the same return and different risk an investor should choose the one with the lower risk and vice versa. The thing that is different among investors is the exact trade-off of risk and return and it is relevant to the individual risk aversion characteristics. The immediate result of the risk aversion is that an investor will never invest in a portfolio if there exist a portfolio with more favorable risk-return trade-off. The theory of Markowitz sets a universe of risky portfolios and tries to find out the optimal portfolios for individual risk aversion characteristics of investors.

The explanation of the notion of optimal portfolio is twofold. At first it can be defined as the portfolio that in any level of volatility has the highest expected return. Secondly it can be defined as the portfolio that in any level of expected return is the one with the lowest volatility. Any of these two definitions produce a set of optimal portfolios. The first definition produces an optimal portfolio for each level of risk and the second definition produces an optimal portfolio for each level of expected return. The fact is that the portfolios that investors get by using these two different definitions are exactly the same and they can be plotted in the following diagram. The curve that is produced is called the efficient frontier and shows the optimal portfolios at any level of risk or return.



On the vertical axis is the expected returns and on the horizontal the risk. The portfolios that are possible are on the inside of the curve (the shaded region) and those that the risk return combination is not possible is on the outside part of the curve. On the shaded part of the curve are the optimal portfolios for each combination of risk and return. It is obvious from the shape that an investor cannot get a better risk return trade-off than the ones that are on the grey line. That is why these portfolios are called optimal. Also it is important to state at this point that the portfolios that are on the efficient frontier are those with the best possible diversification. The portfolios that are not well diversified are in the middle of the shaded region of possible portfolios.

The model of Markowitz assumes that the risk-return preference of any individual investor can also be described by a quadratic utility function. The immediate and obvious result of this assumption is that only the risk and return matter to the investor. Hence, risk factors such as skewness and kurtosis and others are not taken under consideration. It is also very important to note as a theoretical problem that the theory of Markowitz uses a historical parameter the volatility for risk while return is an expected variable in the future.

1.2 Mathematical presentation

The mathematical presentation of the theory is following but at first it is important to note that the portfolio return is the proportion-weighted combination of the constituent assets' returns and portfolio volatility is a function of the correlation of the component assets. The change in volatility is non-linear as the weighting of the component assets changes.

In general:

Expected return:

$$E(R_p) = \sum_i w_i E(R_i)$$

Where R is return.

Portfolio variance:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_{ij} = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Portfolio volatility:

$$\sigma_p = \sqrt{\sigma_p^2}$$

For a two asset portfolio:

Portfolio return:

$$E(R_p) = w_A E(R_A) + (1 - w_A) E(R_B) = w_A E(R_A) + w_B E(R_B)$$

Portfolio variance:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B$$

For a three asset portfolio, the variance is:

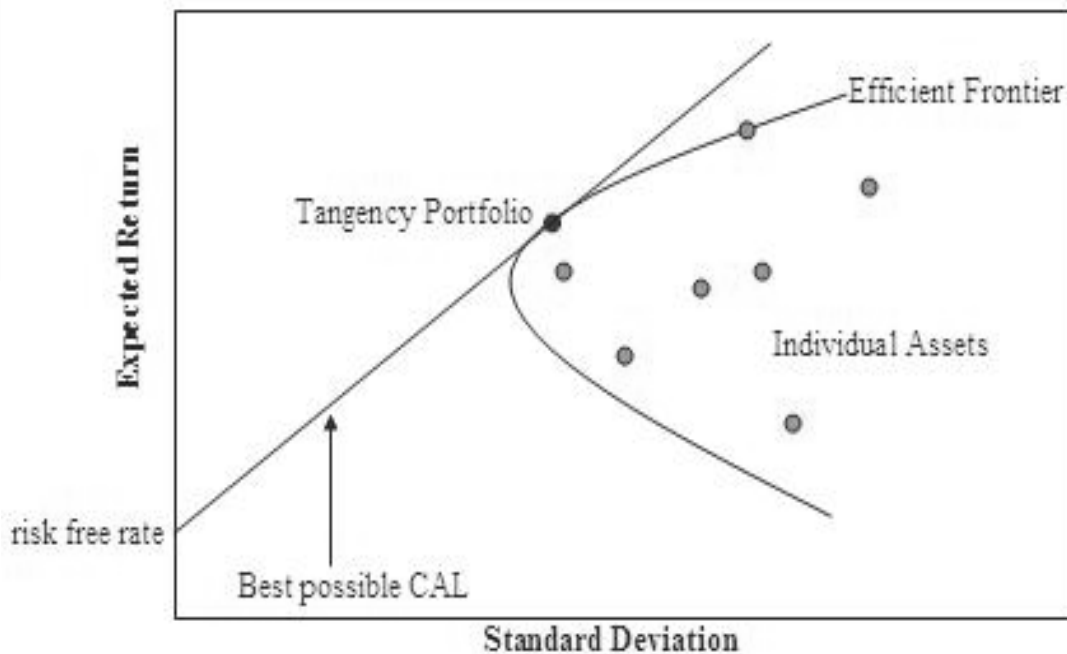
$$w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B + 2w_A w_C \rho_{AC} \sigma_A \sigma_C + 2w_B w_C \rho_{BC} \sigma_B \sigma_C$$

One problem that occurs from the above mathematical presentation is that as the number of asset is increased it becomes very difficult in a computational aspect to calculate risk and return. This is now solved by specific software or my modeling using matrices from the algebra theory.

1.3 Diversification

A very important notion of the Markowitz model is diversification. Diversification is the act of investors who reduce their exposure to risk by holding a diversified portfolio of assets. In other words diversified portfolios have reduced risk for the same amount of expected return. It is proven mathematically that for diversification to work the assets must not have a correlation coefficient of 1, in other words they must not be perfectly correlated.

Another important notion of the Markowitz theory is the efficient frontier. Every possible combination of expected return and standard deviation for every asset can be plotted in a risk return space and the all these combinations form a certain region in the risk-return space. The line across the upper left edge of this formatted region is called the efficient frontier. The portfolios that are in this line have the lowest risk for a given return or the highest return for a given amount of risk.



The region above the frontier is unachievable by holding risky assets alone. No portfolios can be constructed corresponding to the points in this region. Points below the frontier are suboptimal. A rational investor will hold a portfolio only on the frontier.

1.4 The risk-free asset

The risk free asset is the asset that has theoretically at least no risk. The return of the investor that holds the risk free rate is consequently small as it pays no premium for its risk. It is usually proxied by short-term investments in Government securities which they have no risk. The most usual proxy for the risk free rate is the 90 days Treasury bill issued every three months from the United States Government. Another feature of the risk free asset is that it has no correlation with the market index.

2. The Market Model

Although the single-index model was developed to aid in portfolio management, a less restrictive form of it the Market Model has been used widely in finance. The Market Model is actually the same to the single-index model. The only difference is that the assumption $\text{cov}(e_i, e_j) = 0$ is not made. $\text{cov}(e_i, e_j) = 0$.

The mathematical presentation of the model is as follows

Consider a simple regression of asset i 's excess return on the market portfolio's excess return, where $t = 1, \dots, T$:

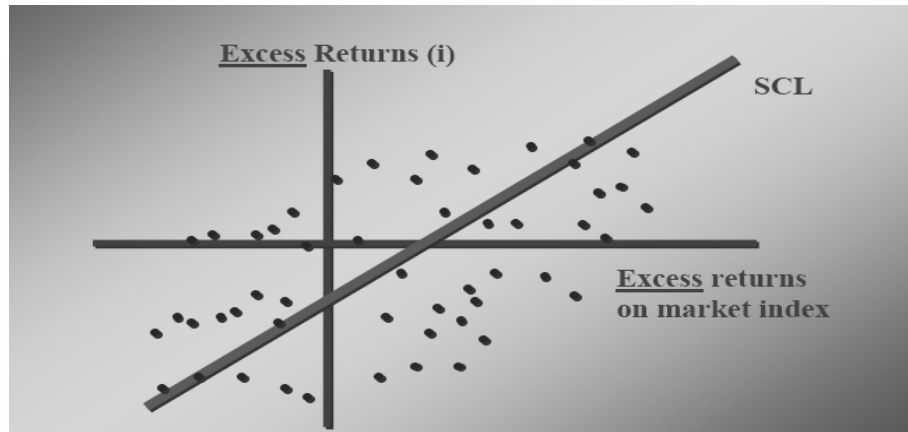
$$\underbrace{R_{it} - R_{ft}}_{\text{Excess return of stock } i} = \alpha_i + \beta_i \underbrace{(R_{Mt} - R_{ft})}_{\text{Realized market risk premium}} + \underbrace{\epsilon_{it}}_{\text{Firm specific return/residual}}$$

If we take the expected value of both sides of the regression,

$$m_i - R_f = \alpha_i + \beta_i(m_M - R_f)$$

Comparing the above equation with the CAPM restriction provides a testable hypothesis on the intercept:

$$\alpha_i = 0$$



The market regression decomposes the variance of an asset's return into two components: market related; and non-market related.

The market model/regression says that:

$$R_{it} - R_f = \alpha_i + \beta_i(R_{mt} - R_f) + \epsilon_{it}$$

Let

$$r_{it} \equiv R_{it} - R_f$$

And

$$r_{mt} \equiv R_{mt} - R_f$$

The market regression becomes:

$$r_{it} = \alpha_j + \beta_i r_{mt} + \epsilon_{it}$$

Using the previous equation, we can decompose the variance of r_{it} :

$$\begin{aligned} \text{Var}[r_{it}] &= \text{Cov}[r_{it}, r_{it}] = \text{Cov}[\alpha_j + \beta_i r_{mt} + \epsilon_{it}, \alpha_j + \beta_i r_{mt} + \epsilon_{it}] \\ &= \text{Cov}[\beta_i r_{mt}, \beta_i r_{mt}] + \text{Cov}[\epsilon_{it}, \epsilon_{it}] \\ &= \underbrace{\beta_i^2 \text{Var}[r_{mt}]}_{\text{market-related variance}} + \underbrace{\text{Var}[\epsilon_{it}]}_{\text{non-market related variance}} \end{aligned}$$

Where we have used the basic OLS assumption:

$$\text{Cov}[r_{mt}, \epsilon_{it}] = 0$$

Total variance = market-related variance + non-market-related variance

Since the model does not make the assumption that all covariances between stocks are due to the common covariance with the market, however, it does not lead to the simple expression of portfolio risk that arise under using the single-index model.

3. CAPM

3.1 Theoretical presentation

CAPM (Capital Asset Pricing Model) is the model with which the equilibrium required return ($E(R)$) is derived. Also with the use of that model we can find the equilibrium price of a stock and which portfolios should investors hold in equilibrium.

In other words CAPM is the model that predicts optimal portfolio choices and the relationship between risk and expected return. CAPM also is important because it underlies much of the modern capital theory and it is used widely in real-world financial decision making.

The theoretical background of CAPM derives from the work of Sharpe, Litterman and Mossin. Sharpe was awarded for his work with the Nobel Prize in 1990. The basic theory however that CAPM is based is the theory of Harry Markowitz which was enhanced with additional simplifying assumptions

The assumptions that the CAPM is based are the following

1. the Market is in competitive equilibrium
2. there is a single period investment horizon
3. all assets are tradable
4. there are no frictions
5. Investors are rational mean-variance optimizers
6. Investors have homogeneous expectations

Although some assumptions may be relaxed CAPM still holds. If many assumptions are relaxed generalized versions of CAPM applies. However, CAPM is an important approximation of reality despite the criticism which will be presented later on this thesis.

The first assumption is that the market is in competitive equilibrium. This means that the demand is equal to the supply. Also the supply of securities is fixed at least in the short run. If demand is larger than the supply for a particular security then the excess demand drives up the price and reduces expected return. The reverse happens when the demand is smaller than the supply for a particular security. Also the term competitive

means that all investors take prices as given and that no investor can manipulate the market and finally that there are no monopolists.

The second assumption is that there is a single period horizon. This means that all investors agree on a certain horizon for their investment decisions. This assumption also insures that all investors are facing the same problem at least in terms of time.

The third assumption is that all assets are tradable. This assumption in principle includes all financial assets (including international stocks), real estate and human capital. Also the fact that all assets are tradable insures that every investor has the same assets to invest which means that every investor is able to invest in all the assets of the world the so-called "Market Portfolio"

The fourth assumption is that there are no frictions. This means that there are no taxes, no transaction costs (no bid-ask spread). Also there is the same interest rate for lending and borrowing. At last all investors can borrow or lend unlimited amounts, in other words, there are no margin requirements. As it is obvious this assumption is the most difficult to stand true in the real world and offers an apparent reason for criticism on the CAPM.

Finally the last two assumptions state that investors are rational mean-variance optimizers with homogeneous expectations. This means that investors choose efficient portfolios that are consistent with their risk-return preferences. Also investors have the same views about expected return, variances and covariances (and hence correlation).

3.2 Risk and Diversification

The risk of a portfolio comprises systematic risk and unsystematic risk which is also known as idiosyncratic risk. Systematic risk refers to the risk common to all securities – i.e. market risk. Unsystematic risk is the risk associated with individual assets. Unsystematic risk can be diversified away to smallest levels by including a greater number of assets in the portfolio. (specific risk average out); systematic risk (within one market) cannot. Depending on the market, a portfolio of approximately 30-40 securities in developed markets will render the portfolio sufficiently diversified to limit exposure to systematic risk only.

A rational investor should not take on any diversifiable risk as only non-diversifiable risk are rewarded within the scope of this model. Therefore, the required return on an asset, that is, the return that compensates for risk taken, must be linked to its riskiness in a portfolio context as opposed to its “stand alone risk”. In the CAPM context, portfolio risk is represented by higher variance. In other words the beta of the portfolio is the defining factor in rewarding the systematic exposure taken by the investor.

3.3 Market Portfolio

After the presentation of assumptions that CAPM is based on the next step in understanding this model is the presentation of the market portfolio and the equilibrium tangency portfolio. From the portfolio theory we know that all investors should have a (positive or negative) fraction of their wealth invested in the risk-free security and the rest of their wealth invested in the tangency portfolio. The tangency portfolio is the same for all investors (homogeneous expectations). Also we know that in equilibrium supply is equal to demand. After all the above we can sum up to the conclusion that the tangency portfolio must be the portfolio of all existing risky assets, the “market portfolio”.

In order to define the market portfolio we have the following

P_i = price of one share of risky security i

N_i = number of shares outstanding for risky security i

M = Market Portfolio. The portfolio in which each risky security i has the following weight:

$$\Omega_{iM} = (P_i * N_i) / (\sum_i P_i * N_i) = \text{market capitalization of security } i / \text{total market capitalization}$$

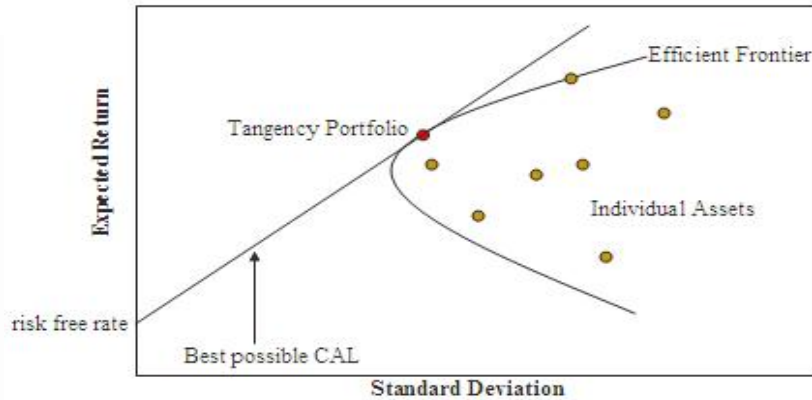
From the above we conclude that the market portfolio is consisting of all assets. However an investor can invest in the market portfolio if he buys a few shares of every security weighted by their market capitalization.

3.4 Capital Market Line

It is known from the financial theory that the capital market line with the highest Sharpe ratio is the CAL with respect to the tangency portfolio. Also as it is shown before, in equilibrium the tangency portfolio is the market portfolio. The market's portfolio capital asset line is the one called capital market line. The capital market line is very useful because it gives the risk-return combinations that are achieved by forming portfolios from the risk free security and the market portfolio. The mathematical expression of the above statement is the following

$$E(R_p) = R_f + \frac{[E(R_M) - R_f]}{\sigma_M} \sigma_p$$

The graphic presentation of the Capital Market Line is the following



3.5 The Required Return on Individual Stocks

CAPM is most famous for its prediction concerning the relationship between risk and return for individual securities:

$$E[R_i] = R_f + \beta_i * [E[R_M] - R_f]$$

$$0 = (E(R_i) - R_f)\sigma_M - (E(R_M) - R_f) \frac{\text{cov}(R_i, R_M)}{\sigma_M}$$

$$\text{Where } \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$

The model predicts that expected return of an asset is linear its 'beta'. The beta coefficient is the measure of sensitivity of a security or a portfolio to the movement of its market index. For example if beta for a stock is 1,2 this means that when the market index rises by 1% the stock will rise by 1,2%. This linear relation is called the Security Market Line (SML).

Deriving CAPM Equation using FOC

The market portfolio is the tangency portfolio and therefore it solves:

$$\max_{w_1 \dots w_i \in R} SR_P = \frac{E(R_P) - R_f}{\sigma_p}$$

where

$$E(R_P) = \sum_i w_i E(R_i) + (1 - \sum_i w_i) R_f$$

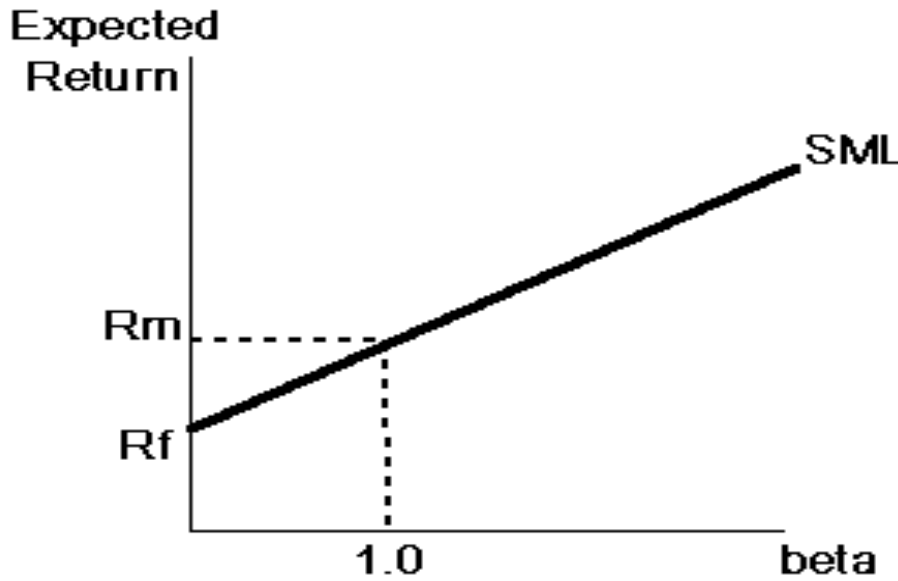
$$\sigma_p = \sqrt{\sum_{i,j} w_i w_j \text{cov}(R_i, R_j)}$$

The first-order condition (FOC) is:

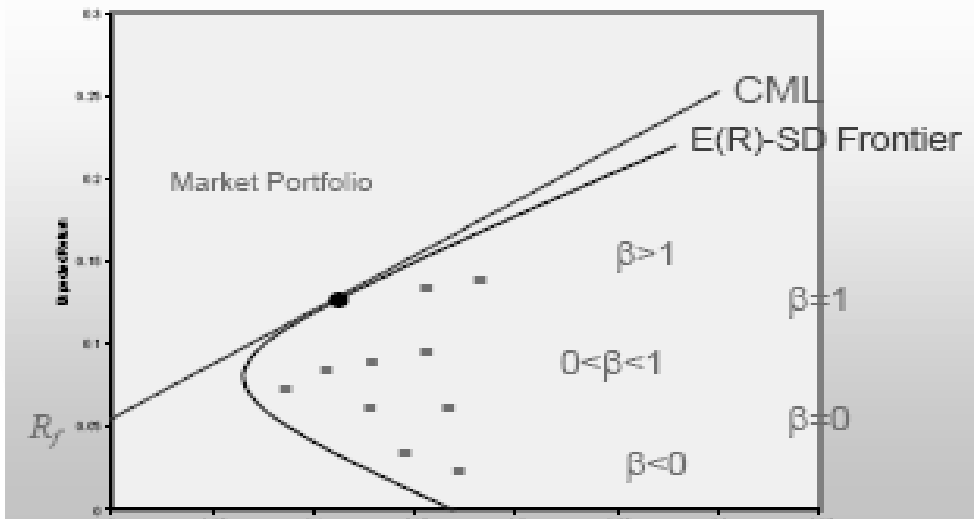
$$0 = \frac{\partial}{\partial w_i} SR_P \quad \text{that is,}$$

$$0 = (E(R_i) - R_f) \sigma_M - (E(R_M) - R_f) \frac{\text{cov}(R_i, R_M)}{\sigma_M}$$

The security market line is the following



The following graph relates the location of individual securities or portfolios with respect to the return-standard deviation frontier to their betas coefficients.



3.6 Characteristics of Betas

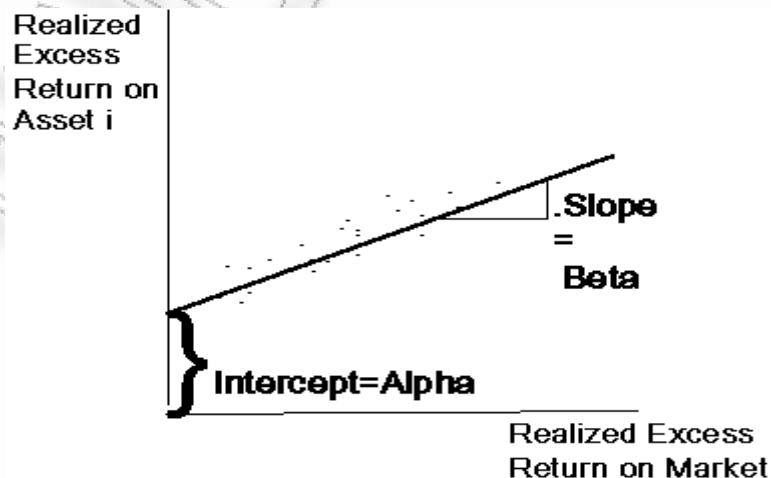
To get a deeper insight into risk, consider the estimation of the *beta* coefficient from an ordinary least squares regression:

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + \varepsilon_{it}$$

In this regression, the *beta* is the ratio of the covariance to the variance of the market return. The *alpha* is the intercept in the regression. This is not the CAPM equation. This is a regression that allows us to estimate the stock's *beta* coefficient. The CAPM equation suggests that the higher the *beta*, the higher the expected return. Note that this is the only type of risk that is rewarded in the CAPM. The *beta* risk is referred to in some text books as *systematic* or *non-diversifiable* or *market* risk. This risk is rewarded with expected return. There is another type of risk which is called *non-systematic* or *diversifiable*, *non-market* or *idiosyncratic* risk. This type of risk is the residual term in the above time-series regression.

$$R_{it} - R_{ft} = \alpha_i + \underbrace{\beta_i (R_{mt} - R_{ft})}_{\text{Nondiversifiable or Systematic Risk}} + \underbrace{\varepsilon_{it}}_{\text{Diversifiable, Nonsystematic or Idiosyncratic Risk}}$$

The asset's *characteristic line* is the line of the best fit for the scatter plot that represents simultaneous excess returns on the asset and on the market.



This is just the fitted values from a regression line. As mentioned above, the *beta* will be the regression slope and the *alpha* will be the intercept. The error in the regression, *epsilon*, is the distance from the line (predicted) to each point on the graph (actual).

The CAPM implies that the *alpha* is zero. So we can interpret, in the context of the CAPM, the *alpha* as the difference between the expected excess return on the security and the actual return. The *alpha* for Franklin would have been $-.10$ whereas the *alpha* for both the Dow and the Salomon Bonds were zero.

3.7 Implications of CAPM

1. The market portfolio is the tangent portfolio.
2. Combining the risk-free asset and the market portfolio gives the portfolio frontier.
3. The risk of an individual asset is characterized by its co variability with the market portfolio.
4. The part of the risk that is correlated with the market portfolio, the systematic risk, cannot be diversified away.
 - Bearing systematic risk needs to be rewarded.
5. The part of an asset's risk that is not correlated with the market portfolio, the non-systematic risk, can be diversified away by holding a frontier portfolio.
 - Bearing nonsystematic risk need not be rewarded.
6. For any asset i :

$$E[R_i] - R_f = \beta_{iM} * [E[R_M] - R_f]$$

where

$$\beta_{iM} = \sigma_{iM} / \sigma_M^2.$$

Given the premium of market portfolio, the riskless rate and assets' market betas, the previous equation determines the premium of all assets. We thus have an asset pricing model — the CAPM. The relation between an asset's risk premium and its market beta is called the "Security Market Line" (SML).

The Capital Asset Pricing Model has been derived under a set of very restrictive assumptions. The test of the model is how well it described reality. The key test is: How well it describes the behavior of returns in the capital markets. A presentation of these tests will be taken up in the next chapter. On the other hand, even if the standard CAPM model explains the behavior of expected returns it does not explain the behavior of individual investors. Individual investors hold nonmarket and quite often very small portfolios.

4. Tests and Criticism on CAPM

4.1 Early Empirical Tests

Tests of the CAPM are based on three implications of the relation between expected return and market beta implied by the model. First, expected returns on all assets are linearly related to their betas, and no other variable has marginal explanatory power. Second, the beta premium is positive, meaning that the expected return on the market portfolio exceeds the expected return on assets whose returns are uncorrelated with the market return. Third, in the Sharpe – Lintner version of the model, assets uncorrelated with the market have expected returns equal to the risk free interest rate, and the beta premium is the expected market return minus the risk free rate. Most tests of these predictions use either cross-section or time-series regressions. Both approaches date to early tests of the model.

4.2 Tests on Risk Premiums

The early cross-section regression tests focus on the Sharpe – Lintner model's predictions about the intercept and slope in the relation between expected return and market beta. The approach is to regress a cross-section of average asset returns on estimates of asset betas. The model predicts that the intercept in these regressions is the risk free interest rate, R_f , and the coefficient on beta is the expected return on the market in excess of the risk free rate, $E(R_M) - R_f$. Two problems in these tests quickly became apparent. First, estimates of beta for individual assets are imprecise, creating a measurement error problem when they are used to explain average returns. Second, the regression residuals have common sources of variation, such as industry effects in average returns. Positive correlation in the residuals produces downward bias in the usual ordinary least squares estimates of the standard errors of the cross-section regression slopes. To improve the precision of estimated betas, researchers such as Blume (1970), Friend and Blume (1970), and Black, Jensen, and Scholes (1972) work with portfolios, rather than individual securities. Since expected returns and market betas combine in the

same way in portfolios, if the CAPM explains security returns it also explains portfolio returns.⁴ Estimates of beta for diversified portfolios are more precise than estimates for individual securities. Thus, using portfolios in cross-section regressions of average returns on betas reduces the critical errors in variables problem. Grouping, however, shrinks the range of betas and reduces statistical power. To mitigate this problem, researchers sort securities on beta when forming portfolios; the first portfolio contains securities with the lowest betas, and so on, up to the last portfolio with the highest beta assets. This sorting procedure is now standard in empirical tests.

Fama and MacBeth (1973) propose a method for addressing the inference problem caused by correlation of the residuals in cross-section regressions. Instead of estimating a single cross-section regression of average monthly returns on betas, they estimate month-by-month cross-section regressions of monthly returns on betas. The times series means of the monthly slopes and intercepts, along with the standard errors of the means, are then used to test whether the average premium for beta is positive and whether the average return on assets uncorrelated with the market is equal to the average risk free interest rate. In this approach, the standard errors of the average intercept and slope are determined by the month-to-month variation in the regression coefficients, which fully captures the effects of residual correlation on variation in the regression coefficients, but sidesteps the problem of actually estimating the correlations. The effects of residual correlation are, in effect, captured via repeated sampling of the regression coefficients. This approach also becomes standard in the literature.

Jensen (1968) was the first to note that the Sharpe – Lintner version of the relation between expected return and market beta also implies a time-series regression test. The Sharpe – Lintner CAPM says that the average value of an asset's excess return (the asset's return minus the risk free interest rate, $R_{it} - R_{ft}$) is completely explained by its average realized CAPM risk premium (its beta times the average value of $R_{Mt} - R_{ft}$). This implies that "Jensen's alpha," the intercept term in the time-series regression, (Time Series Regression)

$$R_{it} - R_{ft} = a_t + \beta_{iM} (R_{Mt} - R_{ft}) + \varepsilon_{it}$$

is zero for each asset.

The early tests firmly reject the Sharpe – Lintner version of the CAPM. There is a positive relation between beta and average return, but it is too “flat”. Recall that, in cross-section regressions, the Sharpe – Lintner model predicts that the intercept is the risk free rate and the coefficient on beta is the expected market return in excess of the risk free rate, $E(RM) - R_f$. The regressions consistently find that the intercept is greater than the average risk free rate (typically proxied as the return on a one-month Treasury bill), and the coefficient on beta is less than the average excess market return (proxied as the average return on a portfolio of U.S. common stocks minus the Treasury bill rate). This is true in the early tests, such as Douglas (1968), Black, Jensen and Scholes (1972), Miller and Scholes (1972), Blume and Friend (1973), and Fama and MacBeth (1973), as well as in more recent cross-section regression tests, like Fama and French (1992).

The evidence that the relation between beta and average return is too flat is confirmed in time series tests, such as Friend and Blume (1970), Black, Jensen, and Scholes (1972), and Stambaugh (1982). The intercepts in time series regressions of excess asset returns on the excess market return are positive for assets with low betas and negative for assets with high betas.

The hypothesis that market betas completely explain expected returns can also be tested using time-series regressions. In the time-series regression described above (the excess return on asset i regressed on the excess market return), the intercept is the difference between the asset's average excess return and the excess return predicted by the Sharpe – Lintner model, that is, beta times the average excess market return. If the model holds, there is no way to group assets into portfolios whose intercepts are reliably different from zero. For example, the intercepts for a portfolio of stocks with high ratios of earnings to price and a portfolio of stocks with low earning-price ratios should both be zero. Thus, to test the hypothesis that market betas suffice to explain expected returns, one estimates the time-series regression for a set of assets (or portfolios), and then jointly tests the vector of regression intercepts against zero. The trick in this approach is to choose the left-hand-side assets (or form portfolios) in a way likely to expose any shortcoming of the CAPM prediction that market betas suffice to explain expected asset returns.

In early applications, researchers use a variety of tests to determine whether the

intercepts in a set of time-series regressions are all zero. The tests have the same asymptotic properties, but there is controversy about which has the best small sample property. Gibbons, Ross and Shanken (1989) settle the debate by providing an F-test on the intercepts that has exact small sample properties. They also show that the test has a simple economic interpretation. In effect, the test constructs a candidate for the tangency portfolio T in Figure 1 by optimally combining the market proxy and the left-hand-side assets of the time series regressions. The estimator then tests whether the efficient set provided by the combination of this tangency portfolio and the riskfree asset is reliably superior to the one obtained by combining the risk free asset with the market proxy alone. In other words, the Gibbons, Ross, and Shanken statistic tests whether the market proxy is the tangency portfolio in the set of portfolios that can be constructed by combining the market portfolio with the specific assets used as dependent variables in the time series regressions.

Enlightened by this insight of Gibbons, Ross, and Shanken (1989), one can see a similar interpretation of the cross-section regression test of whether market betas suffice to explain expected returns. In this case, the test is whether the additional explanatory variables in a cross-section regression identify patterns in the returns on the left-hand-side assets that are not explained by the assets' market betas. This amounts to testing whether the market proxy is on the minimum variance frontier that can be constructed using the market proxy and the left-hand-side assets included in the tests.

An important lesson from this discussion is that time-series and cross-section regressions do not, strictly speaking, test the CAPM. What is literally tested is the whether a specific proxy for the market portfolio (typically a portfolio of U.S. common stocks) is efficient in the set of portfolios that can be constructed from it and the left-hand-side assets used in the test. One might conclude from this that the CAPM has never been tested, and prospects for testing it are not good because: 1) the set of left-hand-side assets does not include all marketable assets, and 2) data for the true market portfolio of all assets are likely beyond reach (Roll, 1977, more on this later). But this criticism can be levelled at tests of any economic model when the tests are less than exhaustive or they use proxies for the variables called for by the model.

The bottom line from the early cross-section regression tests of the CAPM, such

as Fama and MacBeth (1973), and the early time-series regression tests, like Gibbons (1982) and Stambaugh (1982), is that standard market proxies seem to be on the minimum variance frontier. That is, the central predictions of the Black version of the CAPM, that market betas suffice to explain expected returns and that the risk premium for beta is positive, seem to hold. But the more specific prediction of the Sharpe – Lintner CAPM that the premium per unit of beta is the expected market return minus the riskfree interest rate is consistently rejected.

The success of the Black version of the CAPM in early tests produced a consensus that the model is a good description of expected returns. These early results, coupled with the model's simplicity and intuitive appeal, pushed the CAPM to the forefront of finance.

4.3 Recent Tests

Starting in the late 1970s, empirical work appears that challenges even the Black version of the CAPM. Specifically, evidence mounts that much of the variation in expected return is unrelated to market beta.

The first blow is Basu's (1977) evidence that when common stocks are sorted on earnings-price ratios, future returns on high E/P stocks are higher than predicted by the CAPM. Banz (1981) documents a size effect; when stocks are sorted on market capitalization (price times shares outstanding), average returns on small stocks are higher than predicted by the CAPM. Bhandari (1988) finds that high debt-equity ratios (book value of debt over the market value of equity, a measure of leverage) are associated with returns that are too high relative to their market betas. Finally, Statman (1980) and Rosenberg, Reid, and Lanstein (1985) document that stocks with high book-to-market equity ratios (B/M, the ratio of the book value of a common stock to its market value) have high average returns that are not captured by their betas.

There is a theme in the contradictions of the CAPM summarized above. Ratios involving stock prices have information about expected returns missed by market betas. On reflection, this is not surprising. A stock's price depends not only on the expected

cash flows it will provide, but also on the expected returns that discount expected cash flows back to the present. Thus, in principle the cross-section of prices has information about the cross-section of expected returns. (A high expected return implies a high discount rate and a low price.) The cross-section of stock prices is, however, arbitrarily affected by differences in scale (or units). But with a judicious choice of scaling variable X , the ratio X/P can reveal differences in the cross-section of expected stock returns. Such ratios are thus prime candidates to expose shortcomings of asset pricing models – in the case of the CAPM, shortcomings of the prediction that market betas suffice to explain expected returns (Ball, 1978). The contradictions of the CAPM summarized above suggest that earnings-price, debt-equity, and book-to-market ratios indeed play this role. Fama and French (1992) update and synthesize the evidence on the empirical failures of the CAPM. Using the cross-section regression approach, they confirm that size, earnings-price, debt-equity, and book-to-market ratios add to the explanation of expected stock returns provided by market beta. Fama and French (1996) reach the same conclusion using the time-series regression approach applied to portfolios of stocks sorted on price ratios. They also find that different price ratios have much the same information about expected returns. This is not surprising given that price is the common driving force in the price ratios, and the numerators are just scaling variables used to extract the information in price about expected returns.

Fama and French (1992) also confirm the evidence (Reinganum, 1981, Stambaugh, 1982, Lakonishok and Shapiro, 1986) that the relation between average return and beta for common stocks is even flatter after the sample periods used in the early empirical work on the CAPM. The estimate of the beta premium is, however, clouded by statistical uncertainty (a large standard error). Kothari, Shanken, and Sloan (1995) try to resuscitate the Sharpe – Lintner CAPM by arguing that the weak relation between average return and beta is just a chance result. But the strong evidence that other variables capture variation in expected return missed by beta makes this argument irrelevant. If betas do not suffice to explain expected returns, the market portfolio is not efficient, and the CAPM is dead in its tracks. Evidence on the size of the market premium can neither save the model nor further doom it.

The synthesis of the evidence on the empirical problems of the CAPM provided

by Fama and French (1992) serves as a catalyst, marking the point when it is generally acknowledged that the CAPM has potentially fatal problems. Research then turns to explanations. One possibility is that the CAPM's problems are spurious, the result of data dredging – publication-hungry researchers scouring the data and unearthing contradictions that occur in specific samples as a result of chance. A standard response to this concern is to test for similar findings in other samples.

Chan, Hamao, and Lakonishok (1991) find a strong relation between book-to-market equity (B/M) and average return for Japanese stocks. Capaul, Rowley, and Sharpe (1993) observe a similar B/M effect in four European stock markets and in Japan. Fama and French (1998) find that the price ratios that produce problems for the CAPM in U.S. data show up in the same way in the stock returns of twelve non-U.S. major markets, and they are present in emerging market returns. This evidence suggests that the contradictions of the CAPM associated with price ratios are not sample specific.

5. Fama and French

5.1 Introduction

The Capital Asset Pricing Model has set the way in which investors realize the expected return and risk. It is needless to say that CAPM is the most widely used model in investment decision making not only in theory but also in business practice. The basic idea behind the model is that the market portfolio is efficient in terms of expected return and variance based on the theory of Markowitz.

On the other hand CAPM faced a big amount of criticism by researchers. The size effect of Banz (1981) is one of them. Banz finds that market equity (ME= price times number of stocks) contributes in a major way in cross sectional analysis of average returns that occurs from market betas. The average returns of stocks that have low ME (small size) are too big according to their beta estimation and the contrary happens in the examination of stocks with big ME. Also Bhandari (1988) finds that leverage helps explain the cross section of average stocks returns in tests that include size as well as market betas. Stattman (1980), Rossenber, Reid, and Lanstein (1985) find that the average returns of USA stocks are positively correlated with the BE/ME (BE= book value of common equity and ME= market value of common equity). Chan, Hamao and Lakonishock (1991) also find an important role in the cross section of Japanese stocks that the ration BE/ME plays. Basu (1983) shows that the E/P ratio helps in cross sectional analysis of average return of common stocks in researches that include size (ME) and market beta. Finally Ball (1978) argues that E/P ratio is a good proxy that includes all factors that have a certain effect on the expected returns. E/P is likely to be higher for stocks with larger risk and expected return regardless the source of risk.

The purpose of Fama and French is to evaluate the joint role of market beta, size (ME), E/P, leverage, book to market equity in the cross section analysis of average returns of stocks. Their research find out that the CAPM is not supported meaning that the average returns are not positively correlated with market betas. Their result are that beta doesn't seem to contribute to cross section of average return of common stocks and

the combination of size factor and book to market equity seems to absorb the effect of E/P and leverage in the explaining of average returns of stocks.

Fama and French in their study use data of stocks return, excluding financial stocks, from NYSE, AMEX and NASDAQ indices and yearly data of income and balance sheets. The financial firms are excluded because the high leverage that is common for those firms doesn't have the same meaning in an economic point of view with common firms for who the high leverage indicate some distress. In order to verify that the accounting variables are known before the returns that they explain, Fama and French matched the accounting data in the end of year $t-1$ with returns from July of year t to June of year $t+1$.

Fama AND French use the cross sectional regression of Fama and Mac Beth (1973) in estimating the market betas. They regress every month the returns of stocks and the variables that they expect that explain the expected returns. As size (ME), leverage and BE/ME have been precisely calculated for every stock they don't use portfolios on the above regressions. Fama and French estimate the betas for stocks portfolios and then assign a portfolio beta to every stock of the portfolio. This allows them to use stock in their research. More specifically in June of every year they sort the stocks by size forming ten size portfolios. In order to have a differentiation in betas that is not related to the size they sort each size portfolio in ten beta based portfolios by using the pre-ranking betas of each stock. Those pre-ranking betas have been estimated in 24-60 monthly returns in the 5 years before time t . after they assign betas of beta-size portfolios in June they calculate the equal weighted returns of the portfolios for the next 12 months from July to June. In the end they have created post ranking monthly returns from July 1963 to December 1990 for the 100 portfolios (ten size portfolios that each is divided to ten pre-ranking beta portfolio). Next they calculate betas using the whole sample of expected returns (post-ranking) for each and every one of the 100 portfolios. Finally they assign every individual stock with the post-ranking betas of the size-beta portfolios that have calculated for the whole period of 330 months. Those are the betas that will be used for the cross-sectional regressions of Fama and French.

Fama and French show that when the common stocks portfolios are created based on size only then the average return is positively correlated with market betas and

negatively correlated with size (ME). The problem is that the betas of these portfolios are almost perfectly correlated with the size so test on these portfolios are not capable of separating the effect of beta and size in average returns. On the other hand when they divide the ten size portfolios to ten beta portfolios based on pre-ranking betas they manage to find a strong relation between size and average return but no relation between beta and average return. More specifically they do monthly regressions among returns of common stocks and the size variable (ME) in portfolios that are sorted by size. In their regressions they use the $\ln(\text{ME})$. Their results show that during their examination period size helps in the cross-sectional analysis of the average returns of common stocks. They find that the average slope of monthly regressions is -0.15% with a t-statistic of -2.58 which is statistically important.

The regressions of returns with beta show that the market beta does not explain the average returns of common stocks. The average slope of the monthly regressions is 0.15% with a t-statistic of only 0.46 which means that is statistically unimportant. When Fama and French do monthly regression to monthly returns using both variables ME and beta in size-beta sorted portfolios they conclude that the ME is helpful in cross-sectional analysis with a t-statistic of -3.41 . The average slope for beta is negative with a t-statistic of 1.21 . Finally they conclude that market beta don't have any explanatory power in average returns when they use for their regressions various combinations of beta with variables such as ME, BE/ME, leverage and E/P.

Fama and French also built portfolios based on the BE/ME in the same way they constructed the size-beta portfolios. The results of their monthly regression show that there is a strong relation between the Book to Market Value and the average returns of common stocks. The average slope of monthly regression is 0.5% with a t-statistic of 5.71 . The BE/ME variable does not replace the ME variable. When both variables are used in regression their t-statistics are for BE/ME 4.44 and for ME -1.99 which show that both variables have explanatory power in the model.

Fama and French formatted the portfolios based on the leverage in the same way they had constructed the beta and the size portfolios. They use 2 leverage variables A/ME (fixed assets / current market equity), which is the current leverage, and A/BE (Fixed assets / accounting value of equity), which is the accounting leverage. In their regressions

they use again the logarithmic ratios of $\ln(A/ME)$ and $\ln(A/BE)$. The regression of the returns with the above leverage variables for the time period of July 1963 to December 1990 give opposite results.

High current leverage is being related to higher average returns. The average slopes of $\ln(A/ME)$ are always positive and over 4 standard errors from 0. On the other hand high accounting leverage is being related to lower average returns. The average slopes of $\ln(A/BE)$ are always negative and over 4 standard errors from 0. The average slopes of these leverage variables are opposite in the sign but very close in absolute number. Also the difference $\ln(A/ME) - \ln(A/BE)$ is equal to the $\ln(BE/ME)$. Fama and French regressions show that the average slopes of the book to market equity variable are very close to the results given by the regressions on the two leverage variables. The conclusion is that the relative distress that is proxied by the book to market equity variable can be translated as a problem occurred by the leverage and the difference between A/ME and A/BE .

5.2 E/P

The regressions of Fama and French when the E/P ratio is used alone have the following results. The average slope of the E/P ratio is 0.57% every month with 2.28 standard errors from 0. This confirms the fact that firms with negative earnings have higher returns. The average slope for firms with positive E/P ratios is 4.72% with 4.75 standard errors from 0. This confirms the fact that the average returns are getting higher when the E/P ratio is positive.

When the variables of size and book to market value are added to the regressions of E/P ratio the average slope of E/P is reduced from 4.72% to 0.87% (with a t-statistic of 1.23). The results of Fama and French show that the relation between E/P ratio when it is positive and the average returns is due mostly to the positive correlation between the E/P and $\ln(BE/ME)$. In other words, firms with high E/P ratio have also high book to market value ratios.

5.3 RESULTS

The results of the model are the following

1. the regression of returns to beta show that the beta variable alone does not have explanatory power on average returns of common stocks
2. the opposite effects of current leverage (meaning the Fixed Income/ Accounting Value of Equity) in the expected returns can be explained by the book to market equity.
3. the relation between the E/P ratio and the expected return is absorbed by the combination of size and book to market equity.

5.4 Expected returns, size and book to market value

From the two dimension analysis of expected returns when each one the ten size portfolios are being divided to ten book to market value portfolios, we conclude the following. At first, in a size portfolio expected returns are raised by using the BE/ME. More specifically the difference of the returns of the portfolios with the highest and those with the lowest BE/ME is 0.99% every month. Secondly, there is a negative relation between expected return and size. The difference in a book to market portfolio is 0.58%. at last we conclude that by adjusting to size, book to market affect in a major way the expected returns and vice-versa.

5.5 The intersection between the size and the book to market value.

The monthly mean of the correlations among $\ln(\text{ME})$ and $\ln(\text{BE/ME})$ IS -0.26. Firms with low present value are more likely to have lower prospects as a result of low prices and high BE/ME. On the contrary, firms with big size have better prospects, higher prices, lower BE/ME and lower returns. From the regressions that Fama and French did we conclude that stocks with a low ME are more likely to have a high BE/ME ratio and stocks with a high BE/ME tens to have low ME. On the other hand there must not be an

exaggeration in the relationship between size and book to market value. The correlation (-0.26) between $\ln(\text{ME})$ and $\ln(\text{BE}/\text{ME})$ is not so big and also the slopes of the two variables regressions show that $\ln(\text{ME})$ and $\ln(\text{BE}/\text{ME})$ are both very important on the cross-sectional analysis of stock returns.

5.6 Means of FM slopes for subperiods

By studying the regression and the FM slopes for the time period of 1963-1990 we conclude that size has a negative effect in cross-sectional average returns and that book to market value has a positive effect and finally the effect of beta is zero. By creating same regression for subperiods 1969-1977 and 1977-1990, it seems that again the role of beta is not economically important. The FM slope of beta is slightly positive for the period of 1963-1977 (0.10% per month, $t\text{-statistic}=0.25$) and it is positive for the second subperiod (-0.44 per month $t\text{-statistic}=-1.17$). Also there is an indication that the size effect is not so robust for the second subperiod. On the other hand the relation between the BE/ME and the expected return is so robust that it is easily visible in both subperiods. The slopes of $\ln(\text{BE}/\text{ME})$ are all above 2.95 standard errors from 0 and the subperiods slopes (0.36 and 0.36 in respect) coincide with the slope of the whole period (0.35). In conclusion we can say that from all the tested variables the book to market value is the most robust in explaining cross-sectional average returns.

5.7 Conclusion

The capital asset pricing model of **Sharpe** (1972), **Lintner** (1965) and **Black** (1972) has been the most used model for researcher to compute the expected return and risk. **Black, Jensen and Scholes** (1972) and **Fama and MacBeth** (1973) find out that, as it is predicted by the model, there is a positive relation between the expected return and the beta for the period 1926-1968 according to CRSP NYSE data. On the other hand **Reinganum** (1981) and **Lakonishok and Shapiro** (1986), **Fama and French** (1992) find that this positive relation between beta and expected return does not exist for the

more recent period of 1963-1990. This has as a result that their model does not support the CAPM model.

The main conclusion is that two easily computed variables, size and book to market value can describe the cross-sectional analysis of average returns. In order to use this model we have to know

- If it applies constantly in time
- If it derives from rational asset-pricing

At first, despite the fact that BE/ME is considered as a measure for the prospects of returns of stocks there is no solid proof that it's explanatory power is getting worse in time. The relation between BE/ME and the expected return is very robust for the time period of 1963-1990. The same happens for the sub-periods of 1963-1976 and 1977-1990. Secondly, firms with high BE/ME tend to have smaller earnings in respect to those with a low BE/ME. Also, small firms have large periods of low earnings during the 1980s in contrast to big firms. Size and book to market value represent the risk factors in returns, in respect to the prospect of earnings, which is priced rationally in expected returns.

These results have a practical meaning for the formation of a portfolio and the computation of returns by investors that have as a primary goal long term investments. If the asset pricing is rational, size and book to market value represent risk. In this case results declare that portfolios such mutual funds or pension funds can be priced by doing a comparison of them with some other benchmark portfolios that have similar size and book to market value. If there is an irrational asset pricing and size and book to market value do not represent risk these results could also be used in portfolio pricing and the computation of expected returns within a different investment strategy.

6. Methodology

6.1 Data

The Data that are used on this thesis are collected from the Datastream database. The examination of the model is being held on USA stocks. The period of the empirical research is 20 years starting at January 1986 and ending at December 2006. All stocks of NYSE COMPOSITE index (which contain 3218 stocks) that have observed monthly prices, market value and book to market value within that twenty years are selected.

It is important to note here that market value is the price per stock multiplied by the number of stocks for each month. Market value for each month was available for all stocks by the datastream database.

Also, Book to Market value is the book value per share divided by the market value. As a proxy for the book to market value the price to book value is being used. Actually it is the same thing. Note that,

$$\frac{\text{Book Value}}{\text{Market Value}} = \frac{\text{Book value} \times \text{No of stocks}}{\text{price} \times \text{No of stocks}} = \frac{\text{Book Value}}{\text{price}}$$

From the above equation we can conclude that the Book to Market value is equal to the 1/price to book value ratio. Price to Book value was also available for each month by the Datastream database.

After the exclusion of stocks that have no price, market value and book to market value in the examined time period we sum up to 629 stocks that are shown on appendix A.

It is important to note here that stocks prices are used for finding the returns, with which, regressions are performed and Market value and book to market value are used for the formation of Fama and French factors.

The risk free rate is proxied by the 90 days Treasury bill (second market) which is widely used in the literature. The risk free rate for each month is taken from the Datastream database.

Finally the index returns that are used are from the NYSE COMPOSITE index and are also taken by the Datastream database. This index is the most adequate for the purposes of this thesis because it contains all the examined stocks and also it is the most representative index for USA stock market as it contains a very large amount of stocks.

6.2 Methodology

The methodology of this study is quite similar to the one used in the article of Daniel Chi-Hsiou Hung, Mark Shacleton and Xinzhong Xu CAPM, Higher Co-moment and Factor Models of UK Stock Returns (2004) with the proper arrangements and modifications. The main purpose is to examine the impact of higher co-moment on the Fama and French model.

6.3 Higher Co-moments

As well as pricing the first co-moment of stock returns with the market return (beta), Kraus and Litzenberger (1976) were the first to suggest that higher co-moments may also be priced. If market returns are not normal (but skewed or leptokurtic), investors are also concerned about portfolio skewness and kurtosis. If investors' preferences contain portfolio skewness and kurtosis measures, each stock's contribution to systematic skewness (co-skewness) and kurtosis (co-kurtosis) may determine a stock's relative attractiveness and therefore required return.

In order to add the higher co-moments, in other words the skewness and kurtosis in the model the following factors are estimated

$$(R_m - r_f)^2 \quad \text{and}$$

$$(R_m - r_f)^3$$

R_m : the return of the market index

R_f : the return of the risk free rate asset

The above factors are proxies for skewness and kurtosis used in the same way as in the article of Daniel Chi-Hsiou Hung, Mark Shacleton and Xinzhong Xu. They are calculated simply with the use of Microsoft excel in the same way that the CAPM factor of $(R_m - r_f)$ is calculated.

6.4 Up and down markets

When testing the CAPM in cross-section, Pettengill, Sundaram and Mathur (1995) refined the cross-sectional regression to incorporate information relating to the sign of the market realisation in the period. Using a dummy variable that is one for positive and zero for negative excess market returns in the period, they augmented the cross-sectional regression to allow for the fact that the realised market premium can be negative within a particular period. In this thesis this methodology is inserted and in rolling time series regressions. Two dummy variables are added the one $D^+ = 1$ in up markets meaning $(R_m - r_f) > 0$ and $D^+ = 0$ in down markets meaning $(R_m - r_f) < 0$. The exact opposite happens with the other Dummy variable D^- . Then all the monthly market returns are multiplied by the Dummy variables in respect to their sign. For an up market the dummy variable D^+ is used and vice versa.

6.5 Beta decile portfolios formation

After the above calculations the beta decile portfolios are calculated. The calculation method is described below. For each stock, a beta is estimated from a rolling time-series regression of historical stock returns on the market returns. This regression is being held for the first 60 months of the time period. This is done according to the methodology of Fama and McBeth (1973) and also in order to calculate the pre-ranking betas that are needed for the formation of the ten beta decile portfolios.

After the ten beta decile portfolios are estimated, a new rolling time series regression is performed in order to re-estimate the portfolios for each year. In other words the beta decile portfolios need to be rearranged every year after the 60 first months. This

is done by a regression at first between the 12th and 72th month then between 24th and 84th month and so on. The regression type is the following

$$R_i - r_f = a + b(R_m - r_f) + u_i$$

Where,

R_i : the return of an individual stock.

r_f : the return of the risk free rate asset.

a : the stable term of the regression.

b : the beta coefficient

$(R_m - r_f)$: the excess market return.

u_i : the residual term of the regression

and was performed on E-views.

After the regressions are completed a set of data is made that consist of 16 beta rankings of all 629 stocks. The first beta ranking is the one that occurred from the first regression which is for 60 months from year 1986 to year 1991 and the last is the one occurred from the regression of the last 60 month of the time period.

After the formation of the 16 beta rankings each ranking is divided in ten deciles the first being the one with the stocks with the smallest beta, the last with the stocks with the biggest beta. All in all $16 \times 10 = 160$ portfolios are formatted from the ten beta deciles and the 16 different beta rankings.

Following the previous procedure the returns of the stocks of the largest beta portfolio and the smallest beta portfolio are estimated in all the 16 different rankings monthly and finally two times series are constructed. For the first beta decile, the first 12 numbers of the time series are the 12 first return averages of the second beta ranking (of the first beta decile) the second 12 returns are the respected returns (in respect to the month) of the second ranking and so on until the last 11 returns of the 16th beta ranking. The same happens of course for the 10th beta ranking. This is done in order to find the monthly average returns of all stocks that form the first and the 10th beta decile, meaning the stocks that have small and big beta, but since the small and big beta decile portfolio

is re-estimated every year every twelve monthly average returns are taken from the different beta rankings.

After this procedure is done two time series are constructed the one consisting of the monthly portfolio returns of the first beta decile the second of the last beta decile. Four different time-series regression models are conducted to compare their ability to explain the profits from Beta (long the highest and short the lowest decile beta portfolios), Size (long the largest and short the smallest decile size portfolios) and Value (long the highest and short the lowest decile book-to-market ratio portfolios) strategies. Following consecutive formation periods, the monthly return differences (denoted as Rdt) between the two extreme deciles are taken as the observations of the dependent variable. The independent variables are the excess market returns, the square and the cube of excess market returns, the returns of the Fama-French small-minus-big portfolio (SMB) and the returns of the high-minus-low book-to-market ratio portfolio (HML). The number of monthly portfolio returns for the beta sort is 179 during February 1992 to December 2006 and 179 for both the size and book-to-market ratio sorts during February 1992 to December 2006¹.

6.6 Regressions

The time series regressions that are constructed by the previous procedure are the following.

The first regression consists of all the independent variables meaning the exceeding market return the dummy variables for up and down market the proxies for skewness and kurtosis and the Fama and French factor proxies of size and value.

¹ See CAPM, Higher Co-moment and Factor Models of UK Stock Returns DANIEL CHI-HSIU HUNG, MARK SHACKLETON AND XINZHONG XU Journal of Business Finance & Accounting, 31(1) & (2), January/March 2004,

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + b_3 D^+ (Rm - rf)^2 + b_4 D^- (Rm - rf)^2 + b_5 D^+ (Rm - rf)^3 + b_6 D^- (Rm - rf)^3 + b_7 D^+ SMB + b_8 D^- SMB + b_9 D^+ HML + b_{10} D^- HML + u_i$$

where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

B₁₋₁₀: the beta coefficients

D⁺: The Dummy variable for up and markets

D⁻: the dummy variable for down markets

(Rm - rf): the excess market return.

(Rm - rf)²: the proxy for skewness

(Rm - rf)³: the proxy for kurtosis

SMB: Small minus Big, the returns of size portfolios²

HML: High minus Low, the returns of the Book to Market value portfolios³

u_i: the residual term of the regression

The second regression consist of the market variable the dummy variables for up and down markets and the Fama and French factor proxies for size and value.

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + b_7 D^+ SMB + b_8 D^- SMB + b_9 D^+ HML + b_{10} D^- HML + u_i$$

The third regression consist of the market variable the dummy variables for up and down markets and the proxies for skewness and kurtosis

², ³ see The SMB and HML portfolios on this chapter

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + b_3 D^+ (Rm - rf)^2 + b_4 D^- (Rm - rf)^2 + b_5 D^+ (Rm - rf)^3 + b_6 D^- (Rm - rf)^3 + u_i$$

The fourth regression consist of the market variable the dummy variables for up and down markets. It is actually the standard CAPM enhanced with the Dummy variables.

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + u_i$$

After these four rolling time series regressions are conducted four more rolling time series regressions are being held on the same way and the same data. The only difference is that no up and down markets are inserted in the model. This is done in order to see the explanatory power of this enhancement of the model. The four new rolling time series regressions are the following

The first regression consists of all the independent variables meaning the exceeding market return, the proxies for skewness and kurtosis and the Fama and French factor proxies of size and value.

$$Rdt = a + b_1 (Rm - rf) + b_3 (Rm - rf)^2 + b_5 (Rm - rf)^3 + b_7 SMB + b_9 HML + u_i$$

where,

R_i: the return of an individual stock.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

B_{1, 3, 5, 7, 9}: the beta coefficients

(Rm - rf): the excess market return.

(Rm - rf)²: the proxy for skewness

(Rm - rf)³: the proxy for kurtosis

SMB: Small minus Big, the returns of size portfolios⁴

HML: High minus Low, the returns of the Book to Market value portfolios⁵

u_i : the residual term of the regression

The second regression consists of the market variable and the Fama and French factor proxies for size and value.

$$R_{dt} = a + b_1(R_m - r_f) + b_2SMB + b_3HML + u_i$$

The third regression consist of the market variable and the proxies for skewness and kurtosis

$$R_{dt} = a + b_1(R_m - r_f) + b_2(R_m - r_f)^2 + b_3(R_m - r_f)^3 + u_i$$

The fourth regression is the standard CAPM model.

$$R_{dt} = a + b_1(R_m - r_f) + u_i$$

6.7 The SMB and HML portfolios

The Fama and French variables are constructed in the way described in their article “Eugene F. Fama; Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, The Journal of Financial Economics.”

The factors that are considered on this thesis are size and value. Size is defined as the market capitalization of equity while value is defined as the ratio of book value to market value. Following Fama and French (1993), six value-weighted portfolios are constructed (from combinations of small, big firms and low, medium, high book to

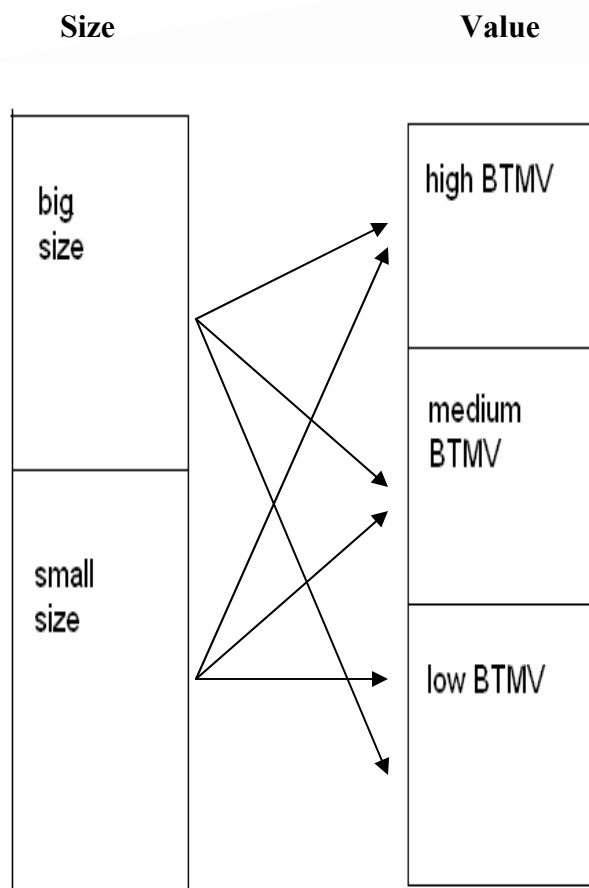
⁴, ⁵ see The SMB and HML portfolios on this chapter

market values) from the intersections of the two size and the three value (book-to-market ratio) groups.

The exact procedure that was followed in order for the six portfolios to be formed is described in detail below. For the year 1992 to 2006 15 rankings of market value were formed from the average monthly market values of each stock. Then these ranking were divided in two deciles the first with the big market value stocks the second with the small market value stocks. This was done for all the 15 different ranking. Then for each year the stocks that consisted the big market value portfolio were divided in three deciles the first being the one with high book to market value, the second the one with medium book to market value and finally the third with the low book to market value. The same procedure was followed with the small market value decile portfolio and for all the 15 different rankings. This is done in order to rearrange every year the six portfolios as the market values and the book to market value are not stable during the whole time period. After the formation of the portfolios the first 12 returns of the stocks consisting each portfolio were selected in order to form the first year time series, then he second twelve returns of all portfolios were selected in order to form the second year time series and so on until the last year time series is formatted.

After the completion of the above procedure 6 time series with monthly returns are constructed in respect to the six Fama and French portfolios. Then, the return on the so-called size factor portfolio (SMB) is the monthly return difference between the simple average of the returns on the three small stock portfolios and the simple average of the returns on the three big stock portfolios. We also construct the return on the value factor portfolio (HML) as the monthly return difference between the simple average of the returns on the two high value portfolios and the simple average of the returns on the two low value portfolios.

Below is a graphical presentation of the formation of the six portfolios



6.8 Cross sectional regressions

The cross sectional regressions are being held in order to estimate the risk price of all betas that come up from the rolling time series regressions or in other words to examine the explanatory power of the model.

For tests of risk premia in the cross section, a two-pass methodology is applied. Firstly, monthly returns of thirty stock beta sorted portfolios are obtained during the whole time period from January 1986 to December 2006. Portfolio risk factors for each month during the 179-month period from January 1992 to December 2006 are then estimated from rolling time-series regressions of previous 60 monthly portfolio returns on the excess value weighted market returns, the square and the cube of excess market returns, the returns of the Fama-French small-minus-big portfolio (SMB) and the returns

of the high-minus-low book-to-market ratio portfolio (HML). Secondly, the cross sectional regressions of excess portfolio returns on risk factors of the twenty portfolios is performed for the whole time period. All reported slope coefficients and adjusted R-squared are values across the 179 cross-section months. The t-statistics are the mean divided by the standard error of the slope coefficient. The regressions are held both for the model with up and down markets and the model without this enhancement.

The first regression that is conducted contains all the betas coefficient in up and down markets that occurred from the rolling time series regressions of the returns of the 30 beta decile portfolios with the market excess return, the factors for skewness and kurtosis and the Fama and French factors. After the completion of this procedure two cross sectional regression are being performed.

The first regressions consists of all the beta coefficients and includes the theory of up and down markets. The type of the regression is the following

$$R_i = a + \lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3 + \lambda_4 b_4 + \lambda_5 b_5 + \lambda_6 b_6 + \lambda_7 b_7 + \lambda_8 b_8 + \lambda_9 b_9 + \lambda_{10} b_{10} + u_i$$

Where,

R_i : the average return of the time period of 179 month of the thirty beta decile portfolios

a : the stable term of the regression

λ_{1-10} : the cross-sectional regression coefficients

b_1 : the beta coefficient of excess market returns in up markets

b_2 : the beta coefficient of excess market returns in down markets

b_3 : the beta coefficient of the skewness proxy in up markets

b_4 : the beta coefficient of the skewness proxy in down markets

b_5 : the beta coefficient of the kurtosis proxy in up markets

b_6 : the beta coefficient of the kurtosis proxy in down markets

b_7 : the beta coefficient of the SMB proxy in up markets

b_8 : the beta coefficient of the SMB proxy in down markets

b_9 : the beta coefficient of the HML proxy in up markets

b_{10} : the beta coefficient of the HML proxy in down markets

u_i : the residual term of the regression

The second regression consists of all the beta coefficients without though the use of up and down markets theory.

The type of the regression is the following

$$R_i = a + \lambda_1 b_1 + \lambda_3 b_3 + \lambda_5 b_5 + \lambda_7 b_7 + \lambda_9 b_9 + u_i$$

Where,

R_i : the average return of the time period of 179 month of the thirty beta decile portfolios

a : the stable term of the regression

$\lambda_{1,3,5,7,9}$: the cross-sectional regression coefficients

b_1 : the beta coefficient of excess market returns

b_3 : the beta coefficient of the skewness proxy

b_5 : the beta coefficient of the kurtosis proxy

b_7 : the beta coefficient of the SMB proxy

b_9 : the beta coefficient of the HML proxy

u_i : the residual term of the regression

7. Time series regressions.

7.1 Regression 1.

The first regression consists of all the independent variables meaning the exceeding market return the dummy variables for up and down market the proxies for skewness and kurtosis and the Fama and French factor proxies of size and value. The type of the regression is the following.

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + b_3 D^+ (Rm - rf)^2 + b_4 D^- (Rm - rf)^2 + b_5 D^+ (Rm - rf)^3 + b_6 D^- (Rm - rf)^3 + b_7 D^+ SMB + b_8 D^- SMB + b_9 D^+ HML + b_{10} D^- HML + u_i$$

Where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

b_{1-10} : the beta coefficients

D^+ : The Dummy variable for up and markets

D^- : the dummy variable for down markets

$(Rm - rf)$: the excess market return.

$(Rm - rf)^2$: the proxy for skewness

$(Rm - rf)^3$: the proxy for kurtosis

SMB: Small minus Big, the returns of size portfolios⁶

HML: High minus Low, the returns of the Book to Market value portfolios⁷

u_i : the residual term of the regression

The results are presented on the following panel.

^{6, 3} see The SMB and HML portfolios on this chapter

Dependent Variable: Rdt
 Method: Least Squares
 Sample: 1 179
 Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a				
b ₁	-0.0074	0.008229	-0.89947	0.3697
b ₂	2.968296	0.864797	3.432363	0.0008
b ₃	0.640375	0.890329	0.719256	0.473
b ₄	-39.1892	22.46189	-1.7447	0.0829
b ₅	-35.1282	23.45089	-1.49795	0.136
b ₆	356.7024	145.2564	2.455673	0.0151
b ₇	-185.825	150.9797	-1.2308	0.2201
b ₈	0.233884	0.228431	1.023871	0.3074
b ₉	0.434053	0.203446	2.1335	0.0435
b ₁₀	0.277099	0.222341	1.246281	0.2144
	-0.79369	0.229819	-3.45354	0.0007
R-squared				
Adjusted R-squared	0.774086	Mean dependent var		0.010114
S.E. of regression	0.760639	S.D. dependent var		0.079705
Sum squared resid	0.038996	Akaike info criterion		-3.59126
Log likelihood	0.25547	Schwarz criterion		-3.39538
Durbin-Watson stat	332.4173	F-statistic		57.5646

0

From the above panel we conclude that the market excess coefficient in up markets has a high beta coefficient of 2.968296 which is statistically significant with a t- statistic of 3.43. Another finding is that the kurtosis in down markets is statistically significant with a coefficient of 356.7024 and a t- statistic of 2.455679. As for the Fama and French factor an important finding is that they are statistically significant only in down markets. The coefficients are 0.43 and -0.7936 for the SMB and the HML factors in respect with t- statistics of 2.1335 and -3.45354 in respect. As it is shown on the above panel all the

other coefficients are statistically insignificant. Finally a high R-squared (0.774086) is found which means that the model has a significant explanatory power.

7.2 Regression 2.

The second regression consist of the market variable the dummy variables for up and down markets and the proxies for skewness and kurtosis

The type of the regression is the following

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + b_3 D^+ (Rm - rf)^2 + b_4 D^- (Rm - rf)^2 + b_5 D^+ (Rm - rf)^3 + b_6 D^- (Rm - rf)^3$$

where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

$b_{1,2,3,4,5,6}$: the beta coefficients

D^+ : The Dummy variable for up and markets

D^- : the dummy variable for down markets

$(Rm - rf)$: the excess market return.

$(Rm - rf)^2$: the proxy for skewness

$(Rm - rf)^3$: the proxy for kurtosis

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent Variable: Rdt

Method: Least Squares

Sample: 1 179

Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a	-0.00475	0.008386	-0.56598	0.5721
b ₁	2.61105	0.853221	3.060228	0.0026
b ₂	0.896678	0.913795	0.981268	0.3278
b ₃	-28.8559	21.91091	-1.31696	0.1896
b ₄	-20.2553	23.89973	-0.84751	0.3979
b ₅	275.4325	140.1546	1.965205	0.051
b ₆	-71.9613	152.8837	-0.47069	0.6385
R-squared				
Adjusted R-squared	0.751985	Mean dependent var		0.010114
S.E. of regression	0.743333	S.D. dependent var		0.079705
Sum squared resid	0.040381	Akaike info criterion		-3.54261
Log likelihood	0.280463	Schwarz criterion		-3.41797
Durbin-Watson stat	324.0637	F-statistic		86.91754
				0

This model consists of all the factors except the Fama and French factors of size and value. The market beta coefficient is also statistically significant with a t- statistic of 3.060228 and an also high price of 2.61105 as the above model. Also the kurtosis in down markets is marginally insignificant when the Fama and French factors are not used in the model.

7.3 Regression 3.

The third regression consist of the market variable the dummy variables for up and down markets and the Fama and French factor proxies for size and value. The regression type is the following.

The type of the regression is the following

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + b_7 D^+ SMB + b_8 D^- SMB + b_9 D^+ HML + b_{10} D^- HML + u_i$$

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

$b_{1,2,7,8,9,10}$: the beta coefficients

D^+ : The Dummy variable for up and markets

D^- ; the dummy variable for down markets

$(Rm - rf)$: the excess market return.

SMB: Small minus Big, the returns of size portfolios

HML: High minus Low, the returns of the Book to Market value portfolios

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent Variable: Rdt

Method: Least Squares

Sample: 1 179

Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a	-0.00213	0.005078	-0.41984	0.6751
b ₁	2.078746	0.171003	12.15619	0
b ₂	2.04783	0.180284	11.35893	0
b ₇	0.006908	0.230866	0.029922	0.9762
b ₈	0.433931	0.204424	2.1227	0.0425
b ₉	0.059097	0.217693	0.271469	0.7864
b ₁₀	-0.80615	0.235707	-3.42014	0.0008
Adjusted R-squared	0.745963	Mean dependent var		0.010114
S.E. of regression	0.737102	S.D. dependent var		0.079705
Sum squared resid	0.040868	Akaike info criterion		-3.51862
Log likelihood	0.287272	Schwarz criterion		-3.39398
Durbin-Watson stat	321.9168	F-statistic		84.17796
				0

In this regression where the Fama and French factors are included and the proxies for skewness and kurtosis are excluded the factors of size and book to market value are statistically significant in down markets as in the first regression with coefficients of 0.4339 and -0.80615 and t-statistics of 2.1227 and -3.42014 in respect.

7.4 Regression 4.

The fourth regression consist of the market variable the dummy variables for up and down markets. It is actually the standard CAPM enhanced with the Dummy variables.

The type of the regression is the following

$$Rdt = a + b_1 D^+ (Rm - rf) + b_2 D^- (Rm - rf) + u_i$$

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

$b_{1,2}$: the beta coefficients

D^+ : The Dummy variable for up and markets

D^- : the dummy variable for down markets

$(Rm - rf)$: the excess market return.

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent Variable: Rdt
 Method: Least Squares
 Sample: 1 179
 Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a	-0.00186	0.00506	-0.36673	0.7143
b ₁	2.065465	0.168881	12.23027	0
b ₂	1.87395	0.175102	10.70203	0
R-squared				
Adjusted R-squared	0.726878	Mean dependent var		0.010114
S.E. of regression	0.723774	S.D. dependent var		0.079705
Sum squared resid	0.041891	Akaike info criterion		-3.49088
Log likelihood	0.308854	Schwarz criterion		-3.43746
Durbin-Watson stat	315.4333	F-statistic		234.2
				0

The result of this regression is that the coefficient betas for up and down markets are statistically significant with a price of 2,065465 for up market beta and a t-statistic of 12.23027 and a price of 1,877395 and a t-statistic of 10.70203 for down markets. Also the R-squared is quite high with a price of 0.72878. A reason that this might happen is that there exists the phenomenon of missing variables.

7.5 Regression without up and down markets theory

After the above regressions four more regression follows this time without the up and down market Dummy variables. It is important to include these regression in the research in order to find out whether the Up and down markets theory adds to the model explanatory power and if it affect the performance of the other factors.

7.6 Regression 1.

The first regression consists of all the independent variables meaning the exceeding market return, the proxies for skewness and kurtosis and the Fama and French factor proxies of size and value

The type of the regression is the following

$$Rdt = a + b_1(Rm - rf) + b_3(Rm - rf)^2 + b_5(Rm - rf)^3 + b_7SMB + b_9HML + u_i$$

where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

$b_{1, 3, 5, 7, 9}$: the beta coefficients

$(Rm - rf)$: the excess market return.

$(Rm - rf)^2$: the proxy for skewness

$(Rm - rf)^3$: the proxy for kurtosis

SMB: Small minus Big, the returns of size portfolios

HML: High minus Low, the returns of the Book to Market value portfolios

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent
Variable:Rdt
Method: Least Squares

Sample: 1 179
Included observations:
179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a	-0.00137	0.003626	-0.37703	0.7066
b ₂	1.672643	0.127316	13.13772	0
b ₃	1.241266	1.506889	0.823727	0.4112
b ₅	71.42868	19.66806	3.631709	0.0004
b ₇	0.28268	0.159502	1.772267	0.0781
b ₉	-0.25885	0.157679	-1.64165	0.1025
R-squared	0.754492	Mean dependent var		0.010114
Adjusted R-squared	0.747396	S.D. dependent var		0.079705
S.E. of regression	0.04006	Akaike info criterion		-3.56395
Sum squared resid	0.277627	Schwarz criterion		-3.45711
Log likelihood	324.9731	F-statistic		106.3323
Durbin-Watson stat	1.915599	Prob(F-statistic)		0

The result of this regression are quite interesting as we can see that the Fama and French factors if the up and down markets theory is not included in the model loose the statistical significance that they showed in the above regressions. On the other hand the excess market return coefficient, the market beta is statistically important with a price of 1.672643 and a t- statistic of 13.13772. the R-squared of the model is still quite high with a price of 0.754492

7.7 Regression 2.

The second regression consists of the market variable and the Fama and French factor proxies for size and value.

The type of the regression is the following

$$Rdt = a + b_1(Rm - rf) + b_7SMB + b_9HML + u_i$$

where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

$b_{1,7,9}$: the beta coefficients

$(Rm - rf)$: the excess market return.

SMB: Small minus Big, the returns of size portfolios

HML: High minus Low, the returns of the Book to Market value portfolios

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent Variable: B1

Method: Least Squares

Date: 07/19/07 Time: 14:20

Sample: 1 179

Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.00106	0.003227	-0.32709	0.744
B2	2.003326	0.092207	21.72628	0
B5	0.223165	0.163373	1.365985	0.1737
B6	-0.35237	0.160702	-2.19266	0.0297
R-squared	0.734375	Mean dependent var		0.010114
Adjusted R-squared	0.729822	S.D. dependent var		0.079705
S.E. of regression	0.04143	Akaike info criterion		-3.50754
Sum squared resid	0.300376	Schwarz criterion		-3.43631
Log likelihood	317.9246	F-statistic		161.2749
Durbin-Watson stat	1.903086	Prob(F-statistic)		0

On the above regression the market beta is statistically significant and the price of it is 2.003326 with a quite high t-statistic of 21.72628 and also the book to market factor proxy of HML is significant to the explanation of the model with a price of -2.19266 but the SMB factor is insignificant.

7.8 Regression 3.

The third regression consist of the market variable and the proxies for skewness and kurtosis

The type of the regression is the following

$$Rdt = a + b_1(Rm - rf) + b_2(Rm - rf)^2 + b_3(Rm - rf)^3 + u_i$$

Where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

$b_{1, 2, 3}$: the beta coefficients

$(Rm - rf)$: the excess market return.

$(Rm - rf)^2$: the proxy for skewness

$(Rm - rf)^3$: the proxy for kurtosis

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent Variable: B1
 Method: Least Squares
 Date: 07/19/07 Time: 14:19
 Sample: 1 179
 Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000397	0.003553	0.111706	0.9112
B2	1.630135	0.126463	12.89017	0
B3	1.173578	1.494618	0.785203	0.4334
B4	72.79692	19.51199	3.730882	0.0003
R-squared	0.747934	Mean dependent var		0.010114
Adjusted R-squared	0.743613	S.D. dependent var		0.079705
S.E. of regression	0.040359	Akaike info criterion		-3.55993
Sum squared resid	0.285043	Schwarz criterion		-3.4887
Log likelihood	322.6138	F-statistic		173.0876
Durbin-Watson stat	1.909875	Prob(F-statistic)		0

In this regression, the market beta is statistically significant with a price of 1.630135 and a t-statistic of 12.89017 which is quite high. The skewness proxy is unimportant but an important finding is that the kurtosis factor which has explanatory power in the model with the up and down market theory in use has a high significance here too. The price of the coefficient is 72.79692 and the t-statistic is 3.730882.

7.9 Regression 4.

The fourth regression is the standard CAPM model.

The type of the regression is the following

$$Rdt = a + b_1(Rm - rf) + u_i$$

Where,

Rdt: the return difference of the high minus the low beta portfolio deciles.

rf: the return of the risk free rate asset.

a: the stable term of the regression.

b_1 : the beta coefficients

$(Rm - rf)$: the excess market return.

u_i : the residual term of the regression

The result of this regression can be seen on the next panel.

Dependent Variable: B1

Method: Least Squares

Date: 07/19/07 Time: 14:23

Sample: 1 179

Included observations: 179

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000739	0.003156	0.234074	0.8152
B2	1.972116	0.091018	21.66743	0
R-squared	0.726209	Mean dependent var		0.010114
Adjusted R-squared	0.724662	S.D. dependent var		0.079705
S.E. of regression	0.041824	Akaike info criterion		-3.4996
Sum squared resid	0.309611	Schwarz criterion		-3.46399
Log likelihood	315.2143	F-statistic		469.4776
Durbin-Watson stat	1.907347	Prob(F-statistic)		0

The result of this regression is that the coefficient beta is statistically significant with a price of 1.972116 with a t-statistic of 21.66743. Also the R-squared is quite high with a price of 0.726209. A reason that this might happen is that there exists the phenomenon of missing variables.

РАНЕШЕШМО РЕПАА

8. Cross-sectional regressions

8.1 Regression 1

The first regressions consists of all the beta coefficients and includes the theory of up and down markets.

The type of the regression is the following

$$R_i = a + \lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3 + \lambda_4 b_4 + \lambda_5 b_5 + \lambda_6 b_6 + \lambda_7 b_7 + \lambda_8 b_8 + \lambda_9 b_9 + \lambda_{10} b_{10} + u_i$$

Where,

R_i : the average return of the time period of 179 month of the thirty beta decile portfolios

a : the stable term of the regression

λ_{1-10} : the cross-sectional regression coefficients

b_1 : the beta coefficient of excess market returns in up markets

b_2 : the beta coefficient of excess market returns in down markets

b_3 : the beta coefficient of the skewness proxy in up markets

b_4 : the beta coefficient of the skewness proxy in down markets

b_5 : the beta coefficient of the kurtosis proxy in up markets

b_6 : the beta coefficient of the kurtosis proxy in down markets

b_7 : the beta coefficient of the SMB proxy in up markets

b_8 : the beta coefficient of the SMB proxy in down markets

b_9 : the beta coefficient of the HML proxy in up markets

b_{10} : the beta coefficient of the HML proxy in down markets

u_i : the residual term of the regression

The result of the regression can be seen in the following panel

Dependent Variable: Ri
 Method: Least Squares

Sample: 1 30

Included observations: 30

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a	0.0007720	0.0022100	-0.3494550	0.7306000
λ_1	0.0039870	0.0016920	2.3573080	0.0293000
λ_2	0.0015730	0.0021100	0.7452430	0.4652000
λ_3	0.0001370	0.0001560	0.8816020	0.3890000
λ_4	0.0000088	0.0001910	0.0462030	0.9636000
λ_5	0.0000042	0.0000210	-0.2012480	0.8426000
λ_6	0.0000072	0.0000198	-0.3618300	0.7215000
λ_7	0.0049390	0.0022320	2.2126470	0.0394000
λ_8	0.0029300	0.0037050	0.7907630	0.4388000
λ_9	0.0049220	0.0025670	1.9177970	0.0703000
λ_{10}	0.0001970	0.0017310	0.1140270	0.9104000
R-squared	0.899398	Mean dependent var		0.008757
Adjusted R-squared	0.84645	S.D. dependent var		0.003621
S.E. of regression	0.001419	Akaike info criterion		-10.00112
Sum squared resid	3.83E-05	Schwarz criterion		-9.487347
Log likelihood	161.0168	F-statistic		16.9863
Durbin-Watson stat	1.800356	Prob(F-statistic)		0

From this cross sectional regression we can conclude that the excess market premium on up markets is statistically significant with a price of 0.0039870 and a t-statistic of 2.357308. Also the Fama and French factors are significant only in up markets. Another important finding of this regression is that the R-squared is quite high with a price of 0.899398. also the proxies for skewness and kurtosis are not statistically significant.

8.2 Regression 2

The second regression consists of all the beta coefficients without though the use of up and down markets theory.

The type of the regression is the following

$$R_i = a + \lambda_1 b_1 + \lambda_3 b_3 + \lambda_5 b_5 + \lambda_7 b_7 + \lambda_9 b_9 + u_i$$

Where,

R_i : the average return of the time period of 179 month of the thirty beta decile portfolios

a : the stable term of the regression

$\lambda_{1, 3, 5, 7, 9}$: the cross-sectional regression coefficients

b_1 : the beta coefficient of excess market returns

b_3 : the beta coefficient of the skewness proxy

b_5 : the beta coefficient of the kurtosis proxy

b_7 : the beta coefficient of the SMB proxy

b_9 : the beta coefficient of the HML proxy

u_i : the residual term of the regression

The result of the regression can be seen in the following panel

Dependent Variable: Ri
Method: Least Squares

Sample: 1 30
Included observations: 30

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a	-0.0010230	0.0014910	-0.6862030	0.4992000
λ_1	0.0054700	0.0013890	3.9364560	0.0006000
λ_3	0.0002390	0.0002620	0.9125740	0.3705000
λ_5	-0.0000112	0.0000244	-0.4601970	0.6495000
λ_7	0.0084270	0.0022070	3.8185700	0.0008000
λ_9	0.0036390	0.0022910	1.5886910	0.1252000
R-squared	0.8826110	Mean dependent var		0.0087570
Adjusted R-squared	0.8581540	S.D. dependent var		0.0036210
S.E. of regression	0.0013640	Akaike info criterion		-10.1801300
Sum squared resid	0.0000446	Schwarz criterion		-9.8998880
Log likelihood	158.7019000	F-statistic		36.0895300
Durbin-Watson stat	2.0763830	Prob(F-statistic)		0.0000000

On this regression we can see that the excess market premium is statistically significant and also the skewness and kurtosis factors are insignificant. The important finding here is that the Fama and French factor of size explains the model as it is statistically significant but on the other hand it does not happen the same with the book to market value factor.

9. Conclusion

Many empirical papers have found that the CAPM is only moderately significant once exposed to Fama French factors. This is to say that once time series regressions are used to compute betas, in cross-sectional regressions these betas produce average slope coefficients (market risk premia) that are insignificant. In contrast, Fama French factors remain highly significant in explaining the cross-section of stock returns.

However, once the methodology of Pettengill et al. (1995) is adopted to separate up and down markets and thus allocate a negative realized risk premium to the down markets, beta becomes highly significant in explaining the cross-section of returns. Cross-sectional regression tests of the CAPM that ignore this methodology risk rejecting the CAPM when it might hold. Furthermore the market beta in these cases remains significant when exposed to higher co-moments and Fama French factors and contributes to the explained variance.

Overall, Fama French factors remain significant even when the Pettengill et al. (1995) methodology is adopted in time series regressions. On the other hand in cross-sectional regressions, one of the Fama French factors, size, itself reacts differently to the experience of up and down markets (with different slope coefficients) in particular the size effect seems to manifest itself through anomalous higher returns for smaller stocks in the up markets. The other factor, value, does not react like this and reacts almost symmetrically across up and down market, in other words it is statistically insignificant. Another important finding is that the Fama and French factors are statistically significant in down markets in time series regressions.

The above conclusion, in other words, the fact that in time series regression the Fama and French factors are significant only in down markets is a matter of further research. Also other factors that are very popular in recent researches regarding the modifications and alternatives of CAPM such as the momentum may be included in the model.

Another significant matter for further research is the potential of the combination of the model used in this thesis with the model proposed by Dr G.P. Diakogiannis. The

objective of the research done by Mr. G.P. Diakogiannis is twofold. First, it derives a three dimensional risk-return relation based upon a portfolio which is not a member of the minimum variance boundary. Diakogiannis shows in a theoretical point of view that a portfolio lies inside the boundary portfolio set if and only if the expected return on any security under consideration is expressed as a linear function of its systematic risk and an additional risk associated with moving inside the boundary portfolio set. Secondly, using the previous theoretical results he questions the validity of using an exact linear relation for expected return and beta when the proxy used lies inside the boundary portfolio set.

His analysis emphasizes an essential implication: where the CAPM is well defined and where market portfolio proxies are inefficient, CAPM regressions are essentially misspecified because of three sources of misspecification. The first source of misspecification arises because the use of the CAPM for inefficient portfolios inappropriately and incorrectly ignores a non-zero addend in the restriction. The second source of misspecification arises from the, above mentioned, existence of infinitely many “zero beta” portfolios, and at all expected returns, *for any* inefficient market portfolio proxy. Thus, the identification of a correct “market risk premium,” “excess return,” or beta coefficient, is extremely unlikely. On the other hand, the identification of “zero relations” that induce a zero R^2 becomes possible. The third source of misspecification arises from the use of unadjusted betas, while adjusting the betas is required for inefficient proxies.

If a proper combination of the model with the model of Diakogiannis is done one can show that all these factors used are in fact a special case of the generalized model of Diakogiannis.

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APPENDIX A
Stocks used in the model

РАНЕЕ НЕ ПЕРП

21ST CENTURY IN.GP.
3M
A G EDWARDS
AAR
ABBOTT LABS.
ABM INDS.
ADVANCED MICRO DEVC.
AFLAC
AGL RES.
AIR PRDS.& CHEMS.
AIRGAS
ALASKA AIR GROUP
ALCOA
ALEXANDER'S
ALLEGHANY
ALLEGHENY EN.
ALLETE
ALLIANT ENERGY CORP.
ALLTEL
ALPHARMA
ALTRIA GROUP
AMER.ELEC.PWR.
AMER.GREETINGS 'A'
AMERICAN EXPRESS
AMERICAN FINL.GP.OHIO
AMERICAN INTL.GP.
AMERICAN LD.LEASE
AMERICAN STS.WATER
AMERON INTL.
AMETEK
AMPCO PITTSBURGH
AMR (AMERICAN AIRLINES)
AMREP
ANADARKO PETROLEUM
ANALOG DEVICES
ANGELICA
ANHEUSER-BUSCH COS.
AON
APACHE
APPLERA APPD.BIOS.
APPLIED INDL.TECHS.
AQUA AMERICA
AQUILA
ARCHER-DANLS.-MIDL.
ARROW ELECTRONICS

ARTHUR J GALLAGHER
ASHLAND
AT&T
ATMOS ENERGY
ATWOOD OCEANICS
AUTOMATIC DATA PROC.
AVERY DENNISON
AVISTA
AVNET
AVON PRODUCTS
BALDOR ELECTRIC
BALL
BANCORPSOUTH
BANK OF AMERICA
BANK OF HAWAII
BANK OF NEW YORK CO.
BARD C R
BARNES GROUP
BAUSCH & LOMB
BAXTER INTL.
BB & T
BEAR STEARNS
BECTON DICKINSON
BELO
BEMIS
BERKLEY W R
BIG LOTS
BLACK & DECKER
BLACK HILLS
BOEING
BOWATER
BOWNE & CO
BRADY 'A'
BRE PROPERTIES
BRIGGS & STRATTON
BRINKER INTL.
BRINKS
BRISTOL MYERS SQUIBB
BROWN SHOE
BROWN-FORMAN 'A'
BROWN-FORMAN 'B'
BRT REALTY TRUST
BRUNSWICK
BRUSH ENGD.MATERIALS
BUCKEYE PARTNERS LP.
BURL.NTHN.SANTA FE C

CA
CABOT
CAL.WATER SER.
CAMPBELL SOUP
CAP.TST.'A'
CAPSTEAD MGE.
CARDINAL HEALTH
CARLISLE COS.
CARPENTER TECH.
CASCADE
CASCADE NAT.GAS
CASTLE A M & CO
CATERPILLAR
CDI
CENTERPOINT EN.
CENTEX
CENTURYTEL
CH EN.GP.
CHECKPOINT SYS.
CHEMED
CHEMTURA
CHESAPEAKE
CHEVRON
CHIQUITA BRANDS INTL.
CHITTENDEN
CHUBB
CHURCH & DWIGHT CO.
CIGNA
CINCINNATI BELL
CIRCUIT CITY STORES
CITY NATIONAL
CKE RESTAURANTS
CLARCOR
CLEAR CHL.COMMS.
CLECO
CLEVELAND CLIFFS
CLOROX
CMS ENERGY
CNA FINANCIAL
COACHMEN INDS.
COCA COLA
COCA COLA ENTS.
COEUR D'ALENE MNS.
COLGATE-PALM.
COMMERCIAL MTLs.
COMPUTER SCIS.

CONAGRA FOODS
CONOCOPHILLIPS
CONSOLIDATED EDISON
CONSTELLATION BRANDS 'A'
CONSTELLATION EN.
CON-WAY
COOPER COS.
COOPER INDS.
COOPER TIRE RUB.
CORNING
COUNTRYWIDE FINL.
CPI
CRANE
CROWN HDG.
CROWN HDG.
CSS INDS.
CTL.ILL.LT.PF.4 50%
CTL.VMT.PUB.SVS.
CULLEN FO.BANKERS
CULP
CUMMINS
CURTISS WRIGHT
CVS CAREMARK
CVS CAREMARK
CYPRESS SEMICON.
DEERE
DELUXE
DIEBOLD
DILLARDS 'A'
DOLLAR GENERAL
DOMINION RES.
DONALDSON
DONNELLEY R R & SONS
DOVER
DOW CHEMICALS
DOW JONES & CO
DOWNEY FINANCIAL
DPL
DREW INDS.
DRS TECHS.
DTE ENERGY
DU PONT E I DE NEMOURS
DUCOMMUN
DUKE ENERGY
DYCOM INDS.
EASTMAN KODAK

EATON
EATON VANCE NV.
ECOLAB
EDISON INTL.
EDO
ELI LILLY
EMERSON ELECTRIC
EMPIRE DST.ELEC.
EMULEX NEW
EN.EAST
ENERGEN
ENNIS
ENTERGY
ENZO BIOCHEM
EQUIFAX
EQUITABLE RESOURCES
ESTERLINE TECHS.
EXELON
EXXON MOBIL
FAIRCHILD 'A'
FAMILY DOLLAR STORES
FANNIE MAE
FEDERAL REALTY INV.TST.
FEDERAL SIGNAL
FEDEX
FERRO
FIRST ACCEP.
FIRSTENERGY
FLEETWOOD ENTS.
FLORIDA ET.CST.INDS.
FLORIDA ROCK INDS.
FLOWSERVE
FMC
FOOT LOCKER
FORD MOTOR
FOREST CITY ENTS.'A'
FOREST LABS.
FORTUNE BRANDS
FPL GROUP
FRANK.RES.
FREMONT GEN.
FRONTIER OIL
FULLER 'H' 'B'
FURMANITE
GANNETT
GAP

GATX
GENCORP
GENERAL DYNAMICS
GENERAL ELECTRIC
GENERAL MILLS
GENERAL MOTORS
GENESCO
GENUINE PARTS
GEORGIA GULF
GERBER SCIEN.
GETTY REALTY
GLATFELTER
GOODRICH
GOODYEAR TIRE & RUB.
GRACO
GRAINGER W W
GREAT PLAINS EN.
GREIF 'A'
GROUP STRATEGIES
GRUBB & ELLIS
GT.ATL.& PAC.
GT.NTHN.IRON ORE
H & R BLOCK
HALLIBURTON
HANDLEMAN
HARRIS
HARSCO
HARTMARX
HASBRO
HAVERTY FRTR.COS.
HAVERTY FRTR.COS.'A'
HAWAIIAN ELEC.INDS.
HEALTH CARE REIT.
HEALTHCARE PR.INVRS.
HEALTHSOUTH
HECLA MINING
HEICO
HEINZ HJ
HELMERICH PAYNE
HERCULES
HESS
HEWLETT-PACKARD
HEXCEL
HILLENBRAND INDS.
HILTON HOTELS
HNI

HOLLY
HOME DEPOT
HONEYWELL INTL.
HORMEL FOODS
HOST HOTELS & RESORTS
HRPT PROPERTIES TRUST
HUBBELL 'A'
HUBBELL 'B'
HUMANA
IDACORP
IKON OFFICE SLTN.
ILLINOIS TOOL WKS.
INDYMAC BANCORP
INGERSOLL-RAND
INTEGRYS ENERGY GROUP
INTERNATIONAL BUS.MACH.
INTERPUBLIC GP.
INTL.FLAV.& FRAG.
INTL.PAPER
INTL.RECTIFIER
INTL.SHIPHLDG.
IOMEGA
JACOBS ENGR.
JO-ANN STORES
JOHNSON & JOHNSON
JOHNSON CONTROLS
JP MORGAN CHASE & CO.
K2
KANSAS CITY STHN.PF.4%
KAYDON CP.
KEITHLEY INSTRUMENTS
KELLOGG
KELLWOOD
KENNAMETAL
KEYSPAN
KIMBERLY-CLARK
KIRBY
KROGER
KV PHARM.'B'
LACLEDE GP.HLDG.
LAMSON & SESSION
LA-Z-BOY CHAIR
LEE ENTERPRISES
LEGG MASON
LEGGETT&PLATT
LENNAR 'A'

LEUCADIA NATIONAL
LIMITED BRANDS
LINCOLN NAT.
LIZ CLAIBORNE
LL&E ROYALTY TRUST
LOEWS
LONE STAR TECH.
LONGS DRUG STRS.
LOUISIANA PACIFIC
LOWE'S COMPANIES
LSI
LUBRIZOL
LUBY
M&T BK.
MANITOWOC
MARCUS
MARSH & MCLENNAN
MARSHALL & ILSLEY
MASCO
MASTEC
MATTEL
MCCORMICK & CO
MCCORMICK & CO NV.
MCDERMOTT INTL.
MCDONALDS
MCGRAW-HILL
MDC HDG.
MDU RES.GP.
MEADWESTVACO
MEDIA GENERAL
MEDTRONIC
MELLON FINL.
MERCK & CO.
MEREDITH
MERRILL LYNCH & CO.
MESA ROY TRUST.
MESABI TRUST
MICRON TECHNOLOGY
MILACRON
MILLIPORE
MINE SAFETY APP.
MOLSON COORS BREWING 'B'
MOOG 'A'
MOOG 'B'
MOTOROLA
MURPHY OIL

MYERS INDS.
MYLAN LABORATORIES
NACCO INDS.'A'
NAT.PRESTO INDS.
NATIONAL CITY
NATIONAL FUEL GAS
NATIONAL SEMICON.
NATIONWIDE HEALTH PROPS.
NBTY
NEW JERSEY RES.
NEW YORK TIMES 'A'
NEWELL RUBBERMAID
NEWMARKET
NEWMONT MINING
NICOR
NIKE 'B'
NISOURCE
NL INDUSTRIES
NOBLE ENERGY
NORDSTROM
NORFOLK SOUTHERN
NORTH EUR.OIL TRUST
NORTHEAST UTILITIES
NORTHROP GRUMMAN
NORTHWEST NTRL.GAS
NUCOR
OCCIDENTAL PTL.
OFFICEMAX
OGE EN.
OIL-DRI AMER.
OLD NATIONAL BANCORP
OLIN
OMNICARE
OMNICOM GP.
ONEOK
OSHKOSH TRUCK 'B'
OVERSEAS SHIPHLDG.
OWENS & MINOR
OXFORD INDS.
PALL
PAR PHARMACEUTICAL RES.
PAR TECHNOLOGY
PARK ELECTROCHEM
PARKER DRILLING
PARKER-HANNIFIN
PENN VA.

PENN.REIT.
PENNEY JC
PENTAIR
PEP BOYS-MANNY
PEPCO HOLDINGS
PEPSICO
PERKINELMER
PERMIAN BASIN RTY.TST.
PFIZER
PG & E
PHILLIPS V HEUSN
PIEDMONT NATGS.
PIER 1 IMPORTS
PINNACLE WEST CAP.
PITNEY BOWES PF.\$2.12
PLAYBOY ENTS.'A'
PNC FINL.PF.C \$1.6
PNM RES.
POGO PRODUCING
POPE & TALBOT
POTLATCH
PPG INDUSTRIES
PPL
PRE PAID LEGAL SVS.
PREC.CASTPARTS
PROCTER & GAMBLE
PROGRESS ENERGY
PROGRESSIVE OHIO
PROTECTIVE LIFE
PUB.SER.ENTER.GP.
PUBLIC STORAGE
PUGET ENERGY
PULTE HOMES
QUAKER CHEMICAL
QUANEX
QUESTAR
RADIOSHACK
RAYMOND JAMES FINL.
RAYTHEON 'B'
REGAL BELOIT
REGIONS FINL.NEW
REX STORES
RITE AID
RLI
ROBERT HALF INTL.
ROCKWELL AUTOMATION

ROGERS
ROHM & HAAS
ROLLINS
ROWAN COS.
RPC
RPM INTL.
RUBY TUESDAY
RUDDICK
RUSS BERRIE
RYDER SYSTEM
RYERSON
RYLAND GROUP
SABINE ROYALTY TST.
SAFECO
SAFEQD.SCIENTIFICS
SAN JUAN BASIN REAL.TST.
SARA LEE
SCANA
SCHAWK 'A'
SCHERING-PLOUGH
SCHLUMBERGER
SEALED AIR
SEMCO ENERGY
SENSIENT TECHS.
SEQUA 'A'
SEQUA 'B'
SERVICE CORP.INTL.
SERVICEMASTER
SHERWIN-WILLIAMS
SJW
SKYLINE
SLM
SMITH (AO)
SMITH INTL.
SMUCKER JM
SNAP-ON
SONOCO PRDS.
SOUTHERN
SOUTHWEST AIRLINES
SOUTHWEST ENERGY
SOUTHWEST GAS
SPARTON
SPRINT NEXTEL
SPX
ST.JUDE MED.
STANDARD MTR.PRDS.

STANDARD PACIFIC
STANDARD REGISTER
STANDEX
STANLEY WORKS
STARRETT LS
STARWOOD HTLS.& RSTS. WORLDWIDE
STATE STREET
STEPAN
STERLING BANC.
STEWART INFO.SVS.
STH.JERSEY IND.
STRIDE RITE
STRYKER
STURM RUGER & CO
SUNOCO
SUNTRUST BANKS
SUPERIOR IND.INT.
SUPERVALU
SWIFT ENERGY
SYMS
SYNOVUS FINL.
SYSCO
T R C
TARGET
TECHNITROL
TECO ENERGY
TEJON RANCH DEL.
TEKTRONIX
TELEFLEX
TEMPLE INLAND
TENET HLTHCR.
TENNANT
TENNECO
TERADYNE
TEREX
TERRA INDS.
TESORO
TEXAS INDS.
TEXAS INSTS.
TEXAS PAC.LD.TST.
TEXTRON
THE HERSHEY COMPANY
THE TRAVELERS COS.
THERMO FISHER SCIENTIFIC
THOMAS & BETTS
TIDEWATER

TIMKEN
TJX COS.
TNSC.REAL.INV.
TODD SHIPYARDS
TOLL BROS.
TOOTSIE ROLL
TORCHMARK
TORO
TOTAL SYSTEM SERVICES
TRIBUNE
TRI-CONTINENTAL
TRINITY INDS.
TXU
TYLER TECHS.
TYSON FOODS 'A'
UGI
UIL HDG.
UNIFIRST
UNION PACIFIC
UNIONBANCAL
UNISOURCE EN.
UNISYS
UNIT
UNITED TECHNOLOGIES
UNIVERSAL
UNIVERSAL HEALTH SVS.'B'
UNUM GROUP
URS
URSTADT BIDDLE PROPS.
US BANCORP
UST
UTD.INDUSTRIAL
V F
VALERO ENERGY
VALHI
VALMONT INDS.
VALSPAR
VARIAN MED.SYS.
VERIZON COMMS.
VIAD
VISHAY INTERTECH.
VOLT INFO.SCI.
VORNADO REALTY TST.
VULCAN MATERIALS
WACHOVIA
WAL MART STORES

WALGREEN
WALT DISNEY
WASH.RL.EST.INV. SHRE.BENEFIT INT.
WASHINGTON MUTUAL
WAUSAU PAPER
WEATHERFORD INTL.
WEINGARTEN REALTY INVRS.
WEIS MARKETS
WELLS FARGO & CO
WENDY'S INTL.
WEST PHARM.SVS.
WESTAR EN.
WESTWOOD ONE
WEYERHAEUSER
WGL HDG.
WHIRLPOOL
WHITE MOUNTAINS IN.GP.
WILLIAMS COS.
WILLIAMS SONOMA
WILMINGTON TRUST
WINNEBAGO INDS.
WINTHROP REALTY TRUST
WISCONSIN ENERGY
WMS INDUSTRIES
WOLVERINE WWD.
WORTHINGTON INDS.
WRIGLEY WILLIAM JR.
WYETH
XCEL ENERGY
XEROX
ZAPATA
ZENITH NAT.IN.

ENVIRONMENTAL PROTECTION AGENCY

APPENDIX B
Characteristics of beta decile and Fama & French portfolios

DATE

AVERAGE HIGH

AVERAGE LOW

SMB

HML

	BETA	BETA		
1/2/1992	0.125851689	0.043867405	0.080677129	0.020932558
1/3/1992	0.030082632	-0.011946538	0.007172682	0.008927874
1/4/1992	-0.059474689	-0.006544804	-0.007454677	-0.002579112
1/5/1992	-0.018001332	0.010756824	-0.021330354	0.008173852
1/6/1992	0.008477744	0.012592174	0.005877892	0.002917005
1/7/1992	-0.068629294	-0.015944122	-0.031741375	-0.017635148
1/8/1992	0.056729109	0.04026666	-0.002660207	-0.009668938
1/9/1992	-0.048262737	-0.000161371	0.003530263	-0.023473809
1/10/1992	0.006017389	0.003292871	-0.005332923	0.014608403
1/11/1992	0.080574428	-0.01032042	-0.009021128	-0.015255588
1/12/1992	0.094370764	0.011909	0.026270807	-0.009445768
1/1/1993	0.01334993	0.014720015	0.015855226	0.036985072
1/2/1993	0.057661921	0.035436122	0.01631296	0.014124974
1/3/1993	-0.012548173	0.026910226	0.007179401	0.02715761
1/4/1993	0.025333248	0.045336702	0.004004193	-0.009439154
1/5/1993	-0.009310832	0.002806535	-0.003160035	-0.002842874
1/6/1993	0.065508663	-0.003513727	-0.000748655	-0.031471606
1/7/1993	-0.002997641	0.028348687	0.011505262	0.020986882
1/8/1993	0.007857145	0.01823076	0.006765638	0.025421391
1/9/1993	0.0503086	0.011745558	-0.003519155	-0.021410012
1/10/1993	0.023572591	0.028387565	0.024817695	0.005150342
1/11/1993	0.02011809	-0.011692462	0.021174439	-0.005234041
1/12/1993	-0.005709342	-0.013262424	0.006197612	-0.010199543
1/1/1994	0.034995845	-0.000451988	0.011730164	-0.000549865
1/2/1994	0.101589694	-0.010299054	0.010920662	0.005547547
1/3/1994	-0.017081932	0.000455122	0.014899686	-0.013873782
1/4/1994	-0.050381761	-0.019293078	-0.002326499	0.002656451
1/5/1994	0.003383619	-0.009325687	-0.009928591	0.000298124
1/6/1994	-0.009736971	-0.012059637	-0.018594132	-0.002644778
1/7/1994	-0.068208323	-0.002251631	0.003219063	-0.019040674
1/8/1994	0.048589361	0.026229853	-0.004260706	0.025636169
1/9/1994	0.053886712	0.024638798	0.010519297	-0.016919137
1/10/1994	-0.031092079	-0.022698168	0.013062332	-0.01498982
1/11/1994	0.018848899	-0.022566007	-0.018288038	-0.003388134
1/12/1994	-0.069778103	-0.028214153	-0.001793055	0.003249181
1/1/1995	0.021973706	0.008148404	0.002470155	0.000361277
1/2/1995	0.01669511	0.014297236	-0.021709306	-0.001281558
1/3/1995	0.08407244	0.007266636	0.000261279	-0.010639684
1/4/1995	0.022484067	-0.000484518	-0.014900557	-0.031633942
1/5/1995	0.042833905	0.009483383	0.008397387	-0.000661869
1/6/1995	0.06580801	0.015341422	-0.011621448	0.019980773
1/7/1995	0.037712067	0.009740577	0.001379001	-0.027122255
1/8/1995	0.08002198	0.018178712	0.00818221	-0.02293073
1/9/1995	0.037404432	0.022975522	0.016202042	0.011447051
1/10/1995	0.013326579	0.003460462	-0.008818841	0.009505976
1/11/1995	-0.034437131	-0.025077632	-0.017847022	0.00313399
1/12/1995	0.050159316	-0.003609523	-0.013491922	-0.008972513
1/1/1996	0.01191093	0.030036549	0.015388071	0.002141755
1/2/1996	0.033236271	0.014847629	-0.017307865	-0.00588938

1/3/1996	-0.007996094	0.0012024	0.005100892	-0.028654805
1/4/1996	0.045849388	0.023348794	0.008380105	-0.018136003
1/5/1996	0.077189937	0.011185869	0.016044944	-0.026046104
1/6/1996	0.057903025	0.022503751	0.03571514	-0.027608011
1/7/1996	-0.036066232	0.004185142	-0.017991974	0.009657193
1/8/1996	-0.128222149	0.011338272	-0.00160234	-0.009598274
1/9/1996	0.050188029	0.007260141	0.014182974	0.00673762
1/10/1996	0.067295202	-0.005221645	-0.024071351	-0.022559687
1/11/1996	-0.00697609	0.008423229	-0.001326108	0.011860276
1/12/1996	0.149358174	0.005744376	-0.010877084	0.010674896
1/1/1997	-0.044028913	0.034172414	0.03371939	0.007129093
1/2/1997	0.09763644	0.002057087	-0.008078323	-0.026569644
1/3/1997	-0.011433541	-0.002698349	-0.018521176	-0.003875022
1/4/1997	-0.066549788	-0.003732548	0.002073542	-0.018335739
1/5/1997	0.058483483	-0.009268012	-0.025201461	-0.025350765
1/6/1997	0.126058388	0.038056374	0.019531818	-0.020007667
1/7/1997	0.078433676	0.020232605	-0.003806326	-0.011170329
1/8/1997	0.100590881	0.016135336	-0.01201451	-0.005016606
1/9/1997	-0.033424668	0.048707866	0.053327891	0.017777685
1/10/1997	0.113580394	0.028181392	0.015981202	0.006132264
1/11/1997	-0.059751092	0.011102529	0.008965084	-0.004306968
1/12/1997	0.007458717	-0.023307895	-0.034651184	0.008896057
1/1/1998	-0.016509558	0.022265531	0.007022741	0.016471922
1/2/1998	0.033188164	-0.035502239	-0.017486641	-0.037800917
1/3/1998	0.072280246	0.013243649	0.010619459	-0.007804653
1/4/1998	0.067618948	0.041214877	-0.010098132	-0.001911292
1/5/1998	0.016624466	-0.017515081	0.005027585	-0.013394333
1/6/1998	-0.09006602	-0.00797258	-0.015396749	0.007906032
1/7/1998	0.018101587	-0.007140678	-0.021385117	-0.028597702
1/8/1998	-0.134967867	-0.042686624	-0.022620721	-0.018696016
1/9/1998	-0.260782824	0.013899435	-0.004177216	0.01337981
1/10/1998	-0.050983976	0.048776485	0.010576571	0.018203645
1/11/1998	0.328196586	-0.051443146	-0.056868411	-0.075438104
1/12/1998	0.031405319	0.003870009	-0.016054116	-0.014817242
1/1/1999	0.067146794	0.009518637	0.010808036	-0.000973633
1/2/1999	-0.024294023	-0.067204131	-0.024330478	-0.037485165
1/3/1999	-0.061335258	-0.046596927	-0.041508551	-0.020541317
1/4/1999	0.069433309	-0.024372208	-0.023443696	-0.006137904
1/5/1999	0.193593189	0.052126856	0.008386612	0.04183196
1/6/1999	-0.032538924	0.029997681	0.036825239	0.014523312
1/7/1999	0.063805207	-0.027509474	0.009043887	-0.026317918
1/8/1999	-0.042453454	0.010323186	0.027277164	-0.022782134
1/9/1999	-0.036559398	-0.032865818	-2.95259E-05	-0.017280625
1/10/1999	-0.099470621	-0.027442442	0.026653127	-0.027196771
1/11/1999	0.056980508	-0.030891022	-0.049635274	-0.029750829
1/12/1999	0.014221189	-0.043971608	0.014648749	-0.02984818
1/1/2000	0.021440786	-0.026924822	0.004281745	-0.013622155
1/2/2000	-0.004947462	0.032212036	-0.010514104	-0.04397926
1/3/2000	-0.007385641	-0.049570742	0.019870245	-0.081493024
1/4/2000	0.184758791	0.001064684	-0.06160612	0.009838681

1/5/2000	0.037794144	0.076780318	-0.003556281	0.015789765
1/6/2000	-0.015710654	0.013584521	-0.030757861	-0.013540355
1/7/2000	-0.051895023	-0.002734774	0.041035796	-0.061586573
1/8/2000	-0.034050106	0.031298556	0.006664949	0.043095563
1/9/2000	0.103187966	0.027928739	-0.000460454	-0.012373903
1/10/2000	-0.062703166	0.060786126	-0.007689341	0.019565266
1/11/2000	0.006386612	-0.011738013	-0.028481268	-0.021138345
1/12/2000	-0.081455894	0.022574516	0.00637816	0.010391321
1/1/2001	0.102207638	0.00954237	-0.007225285	0.009870217
1/2/2001	0.143076164	-0.035450846	0.051669261	0.058691799
1/3/2001	-0.139602826	0.042049743	0.020513436	0.015104409
1/4/2001	-0.109315404	0.003917908	0.010414042	0.019939767
1/5/2001	0.212539825	-0.010701648	-0.002855456	-0.011174395
1/6/2001	0.013567418	0.035619505	0.036161508	0.023868255
1/7/2001	-0.06271539	0.00176968	0.017229678	0.008844122
1/8/2001	-0.008779724	-0.012365619	-0.007936315	-0.006695786
1/9/2001	-0.108084903	0.027534306	0.025049201	0.020036941
1/10/2001	-0.256787488	-0.013606687	-0.014299215	-0.034672329
1/11/2001	0.155025926	0.012982453	-0.009635553	-0.012161231
1/12/2001	0.082428121	-0.022930191	0.000888854	0.009899954
1/1/2002	0.097852825	0.036597312	0.028674056	0.027443471
1/2/2002	0.025904317	0.027747589	-0.000346971	-0.001379819
1/3/2002	-0.008981355	0.03204512	0.004730462	-0.014686442
1/4/2002	0.091774748	0.04688508	0.041468943	0.0273158
1/5/2002	-0.038020986	0.031151878	0.05059096	0.022014093
1/6/2002	-0.08825802	-0.013854629	-0.019701074	-0.022644224
1/7/2002	-0.115879125	0.008118613	0.049319192	0.010877899
1/8/2002	-0.184356894	-0.045562766	-0.006447324	-0.042551389
1/9/2002	0.032554338	0.010773975	-0.010498212	-0.002294722
1/10/2002	-0.150275367	-0.020173204	0.026758501	-0.033588169
1/11/2002	0.136596835	-0.014529885	-0.022002831	-0.000641315
1/12/2002	0.200121943	-0.017562314	-0.010671084	0.033322808
1/1/2003	-0.132267647	0.01924654	0.016128702	-0.003496592
1/2/2003	-0.077077843	-0.017934001	-0.019387661	-0.009411238
1/3/2003	-0.019334393	0.003798419	0.002657891	-0.026786401
1/4/2003	0.028392524	0.040894211	0.004176717	-0.011394464
1/5/2003	0.137203933	0.02441069	0.019910134	0.023054389
1/6/2003	0.216335568	0.039205548	0.002915454	0.041944197
1/7/2003	0.027807407	0.028215709	0.022411458	0.011851607
1/8/2003	0.047821689	0.011996572	0.027040443	0.015346619
1/9/2003	0.126796574	0.023775323	0.013999414	0.00372431
1/10/2003	-4.69491E-05	0.014424228	-0.000226791	0.013565602
1/11/2003	0.103850972	0.021667212	0.007847118	0.01977375
1/12/2003	0.045998993	0.038245201	0.022101311	0.007777559
1/1/2004	0.038509003	0.027118227	0.00950391	0.027813034
1/2/2004	0.058660664	0.014598439	0.007551097	0.002045944
1/3/2004	0.034041191	0.010327996	0.001369538	-0.008533525
1/4/2004	-0.016137876	0.01669564	0.011085976	0.002275229
1/5/2004	-0.053080633	-0.052383499	-0.025509763	-0.015757062
1/6/2004	0.019591349	0.015394212	0.000515549	0.002961832

1/7/2004	0.029405512	0.024202672	0.038304595	0.015449708
1/8/2004	-0.045371014	-0.002513932	-0.003927741	0.00509987
1/9/2004	-0.042240873	0.023941906	0.006330354	-8.67086E-06
1/10/2004	0.065651547	0.021688864	0.021210981	-0.011408958
1/11/2004	0.009228431	0.018428179	-0.007857983	-0.004577533
1/12/2004	0.115043756	0.047303489	0.029421808	0.008784692
1/1/2005	0.010262572	0.001957914	-0.008019553	0.001369756
1/2/2005	-0.0289294	-0.009401989	-0.005145721	-0.011458646
1/3/2005	0.044993634	0.014698406	-0.004461753	-0.010468595
1/4/2005	-0.044697922	-0.021076756	-0.004200893	0.000688155
1/5/2005	-0.039059989	0.004173033	-0.02567946	-0.01952922
1/6/2005	0.090423377	0.027582742	0.016723351	0.004283472
1/7/2005	0.016274526	0.012358887	0.015405046	0.009511099
1/8/2005	0.101744249	0.00402539	0.007974254	-0.005333699
1/9/2005	-0.004589332	-0.012631921	-0.004121872	0.006414491
1/10/2005	0.021295898	-0.010441408	-0.004782602	-0.017534022
1/11/2005	-0.060341993	-0.035307744	-0.009426827	-0.004303198
1/12/2005	0.109185653	0.02373051	0.008978839	-0.006702174
1/1/2006	-0.006270059	0.000315885	-0.007662234	0.011824066
1/2/2006	0.103172595	0.01852904	0.038795518	-0.031737926
1/3/2006	0.002909589	-0.000409871	-0.017668919	-0.001767309
1/4/2006	0.012519035	-0.003050121	0.01531197	-0.017283837
1/5/2006	0.006784566	-0.005554012	-0.002403757	-0.010107684
1/6/2006	-0.049289445	0.003687238	-0.014996032	0.004946665
1/7/2006	-0.037613837	-0.012991292	-0.008813098	-0.00037616
1/8/2006	-0.071568923	-0.002590879	-0.020707618	0.006155223
1/9/2006	0.041660921	0.033852489	0.014590077	-0.013188467
1/10/2006	0.000913789	-0.00499595	-0.007906438	0.007100463
1/11/2006	0.058715498	0.024795803	0.017492125	0.000748957
1/12/2006	0.041240735	0.00236974	0.013903892	0.006415131

AVERAGE RETURN

0.016278424

VARIANCE

0.006462488

St. deviation

0.0803896

No OF STOCKS

63

AVERAGE RETURN

0.006163988

VARIANCE

0.000579999

St. deviation

0.024083167

No OF STOCKS

62

AVERAGE RETURN

0.002372161

VARIANCE

0.000410085

St. deviation

0.020250552

AVERAGE RETURN

-0.003168888

VARIANCE

0.000408815

St. deviation

0.020219185