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Forecasting Probabilities of Default based on Macro-adjusted Historical Rating Migration Matrices.

by

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Dedicated to my Mother.

Abstract

Forward Looking Probabilities of Default remain a significant research topic not only for academics but also for Financial Institutions because of its contribution to the capital adequacy of Financial Institutions. This thesis presents a methodology of Forecasting Probabilities of Default, based on macro – adjusted historical rating matrices, motivated both by International Literature which is associated with Probabilities of Default and the introduction of International Financial Reporting Standard 9 (IFRS 9), which was entered into force from 1st January of 2018, in Greece. For this purpose, we examined some macroeconomic variables aiming to detect which of them are related with corporate default rates. As a result of this research, GDP, Inflation and Unemployment Rate seem to be significantly associated with corporate default rates. Taking into consideration the expected future values of these macroeconomic variables, based upon the estimation that ECB does, we explicitly presented our methodological approach.

Key Words: Probability of Default; Forecasting Probabilities of Default; Point in Time; Through the Cycle; Migration Matrices; Merton; Vasicek; IFRS; IFRS 9 ; Forward Looking; Stages; GDP; Inflation.

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1. Introduction

Given the fact that the estimation of Forward-Looking Probabilities of Default ("PD") consists one of the key challenges in Credit Risk measurement within the Banking Industry and considering the enforcement of IFRS 9 from the 1st January of 2018, this thesis presents a methodology of forecasting Probabilities of Default (PD thereafter), based on macro – adjusted historical rating matrices. It aims to link the usage of migration matrices with the identification of term structures of PDs, using a market practical approach. This approach allows FIs to include and monitor Forward-Looking aspects not only for estimating the future PDs outcomes, but also for single rating migrations as well as to define future default rates that incorporate the probability of migrating through performing rating grades.

For these purposes, the presented methodology is based on structural models and particularly on Merton's theory (1974) and Vasicek's model (1976 & 2002), for the definition of forward-looking PD estimates, using the concatenation of forward-looking migration matrices.

In more detail, the second chapter presents an explicit review of the pertinent international literature which has created the framework around the topic of Credit Risk Models. Following this, an analytical reference to the International Financial Reporting Standards and IFRS9 in particular is included.

In the fourth chapter, we present the dataset, that is used in our econometric study, so as to identify which of the "candidate" macroeconomic variables are meaningfully associated with Greek corporate default rates. In the fifth chapter we explicitly present our econometric study and consequently our results. According to our results, GDP and Inflation appear to be the most significant macroeconomic variables. This finding is aligned with the relevant results that were officially presented by the Bank of Greece through the recent working paper of Petropoulos et al. (2018).

Taking into consideration the results of our econometric study and based on the estimations that European Central Bank (ECB thereafter) has published regarding GDP and Inflation, we procced with the practical application of our proposed methodological approach. Finally, the last chapter contains the main conclusions of this thesis

2 Theoretical Background

2.1.1 Approaching the Term of Risk

Risk is defined as the probability of loss, damage or any other negative occurrence that is caused by external or internal vulnerabilities, while it may be avoided through preemptive actions.
Under the financial aspect, risk is the probability that an actual return on an investment will be lower than the expected return. This is also one of the major threats of **Financial Institutions (thereafter "FIs")**, namely the decrease of FI's return for its owners.

According to Saunders and Cornett (2014) **Financial Risks** faced by FIs can be divided into the following (non-exhaustive) categories:

Interest Rate Risk:
The Risk incurred by a FI, when the maturities of its assets and liabilities are mismatched.

Credit Risk:
It is Risk that the Cash Flow (thereafter "CF") from Loans and Securities which are included in FIs' portfolio may not be paid either in full or partially.

Liquidity Risk:
It is the Risk, which can be caused by a sudden surge in liability withdrawals, triggering the liquidation of FIs' assets in a very short period.

Foreign Exchange Risk:
It is the Risk that is brought about when changes in Exchange Rates can affect the value of an FIs' assets and liabilities, which are expressed in nondomestic currencies.

Country/Sovereign Risk:
It is the Risk that can be brought about when the repayments from foreign borrowers may be not executed, because of restrictions, intervention or interference from foreign governments.

Market Risk

It is the Risk, which can be brought about from assets and liabilities in an FIs' trading book because of changes in interest rates, exchange rates or other rates.

Off- balance – sheet Risk:

It is the Risk, which is caused because of FIs' activities related to their potential assets and liabilities held off the balance sheet.

Technology Risk:

It is the Risk, which is brought about when FIs' technological investments do not produce anticipated cost savings.

Operational Risk:

It is the Risk that existing technology, auditing, monitoring and other support systems could break down or fail to function.

Insolvency Risk:

The Risk that an FI may not have enough capital to offset a sudden decline in the value of its assets.

2.1.2 Credit Risk

As it is mentioned above, Credit Risk is the Risk of Default on Debt from Loans and Securities held by FIs; that arises from borrowers, who are not capable of repaying either completely or partially its obligations against FIs. Another approach could be the uncertainty, which surrounds the borrowers' ability to service its debts.

In a general approach, FIs which provide long-term Loans and Securities are more exposed to Credit Risk than are others that provide short-term Loans and Securities. For instance, a Depository Institution is more exposed to this Risk than it is the Money Market Mutual Fund. Practically, all FIs face Credit Risk and they also do not have a way to differentiate the obligations which will not be serviced. However, in the "arsenal" of FIs there are probabilistic assessments of the likelihood of default. Thus, the borrowers pay a spread over the interest rate that consists a proportion of their default probability so as the lenders hedge this uncertainty.

According to Moody's (2003) and under regular economic conditions, a default event is a deceptively rare occasion and its probability is around 2%, in an annual reference. Despite this, there is a substantial variation among the default probabilities across the different kind of borrowers. For instance, in the sector of firms, the probability ratio of an entity with an AAA rating defaulting is only about 0,0002% annually. The probability ratio of a single A rated entity is of about 10 in 10,000 or 0,001% annually. So, it is five times higher than the odds of AAA. At the bottom of this rating dimension, the odds of a CCC rated entity's is almost 4%, namely it is 200 times the odds of an AAA-rated firm.

Even though, the above-mentioned probabilities do not seem large, there are in fact extremely important, mainly for three principal reasons. The first one is that these probabilities can be easily increased without a special warning. Secondly, the margins in lending and especially in corporate lending are very tight. Thus, a very small error in the calculation of default risk can conclude to the underestimation of the cost of lending. However, the extremely significant reason is the fact that a considerable number of the lenders are also borrowers with a considerable leverage ratio.

Credit Risk not only includes the loss of the Principal, Interests and the disruption of CF, but also the increase of collection cost. The last one consists of the hidden component of Credit Risk, which also has to be taken into consideration as it introduces a considerable cost for FIs.

The Loans and Securities, which are held by FIs, are displayed as Assets in the Financial Statements of FIs. The amount of the lending, which is going to be repaid, consists of the "Exposure", which is exactly what FIs expect to lose in the fact that the borrower defaults. For the most FIs, Credit Risk belongs among the principal threats, which are mainly supplied by the different kinds of Loans.

Under the historical aspect and taking into consideration the recent experience from the Global Financial Crisis, it has been proved that Credit Risk is the dominant factor during the biggest Banking Recessions, particularly, in the case of the Subprime Mortgage Crisis. Thus, the FIs have turned their attention to identify, measure and control Credit Risk, ensuring simultaneously the adequacy of Capital so that they can be capable of covering the possible losses. Moreover, they also collect information, maintaining Databases related to borrowers, whose assets are included into the FIs' portfolios. These Databases are updated over time, providing to the FIs essential information in their Credit Risk Management Strategy as they affect the Return and Risks of their Loan Portfolios.

However, as it is mentioned by Heider et. all, (2009), there is always a trade-off between cost of holding capital and amount of risk hedged, while they highlighted the fact that FIs aim to maximize their profit.

To further broaden the study, it should be mentioned that the higher levels of Credit Risk seem to be associated with higher levels of borrowing costs. Thus, measures of borrowing costs such as yields can be used so as to infer Credit Risk Levels based on assessments by market participants.

Lastly, in this chapter we refer to circumstances which can give rise to credit losses. The most common of these conditions are the following:

1. An entity may be not able to pay back asset-secured fixed or floating charge debt.
2. An entity does not pay the wages to its employees, when due. Consequently, the employees do not pay their obligations when due.
3. An entity fails to pay a trade invoice, when due.
4. A borrower fails to pay the payments of a loan, credit card or the payments of other credit means.
5. An issuer of bond (either the issuer is an entity or a government) fails to pay the coupon or the coupon and principal, when due.
6. A government provides bankruptcy protection to an insolvent borrower.

7. An insurance entity does not pay its obligations.

2.2. Literature Review

As Credit Risk is deemed as the predominant risk category, especially for FIs, various advanced methods have been extensively developed and introduced into the financial field, for the purposes of modeling and measuring the Exposure in Credit Risk.

The primary method is based on the **Structural Models,** which are founded on the framework of **Merton (1974).** The Merton Model is the first Structural Model and extensively referred to as the cornerstone in this sector. Structural Models are based on a value process of the counterparty. The latter one, is the stock price process, where the entity is in default, if the process is less than some threshold, with most of the times being a specific proportion of the entity's debt. These classes of Models aim to provide an explicit relationship between default events and capital structure.

Structural Models do not observe the market value of an entity's assets. The annual report of FIs presents only the accounting aspect of their assets, while for any publicly listed FI, the market value of equity is totally observable and measurable.

The second class of structural models is the so-called **Reduced Form Models**, pioneered by Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997) and Duffies and Singleton (1999). Reduced Form Models take into consideration the timing of default as an unpredictable factor and present the default event as an unexpected event whose probability is driven by a stochastic process.

These two approaches use different methods to incorporate the effect of the macroeconomic environment on the **Probability of Default (thereafter "PD").** The first one measures the cyclical impact on the PD, incorporating systematic risk factors into the specification of the process, driving the variation in the firms' asset values, while the Reduced Form Models are used to approach the default events using Poisson distribution with time-varying default intensity. The intensity function consists of macroeconomic variables and provides an explicit relationship between default events and capital structure.

As it has already been mentioned, the fundamental model in the sector of Structural Models is the Merton's one (1974). This approach assumes that an entity has a certain amount of zero-coupon debt that will become due at a future time T. This entity defaults, if the value of its assets is less than the promised debt repayment at time T. The equity of the entity is a European Call Option, on the assets of the firm with maturity T and a strike price equal to the Face Value of the debt. The model can be used to estimate either the risk-neutral probability that the company will default, or the credit spread on the debt.

Examining in more details **Merton's Model,** at first, we should provide its assumptions. The first one, is that there are no transactions costs, taxes, or any other problem with indivisibilities of assets. The second one is that there is a sufficient number of investors with similar, equal wealth levels, such that each investor has the belief that he can buy and sell as much of an asset as he wants at the market price. Moreover, the third assumption is that a borrowing market exists and lends at the same rate of interest.

Following the above assumptions, we should add that short sales of all assets, with full use of the proceeds are allowed, while trading in assets takes place continuously in time. The Modigliani-Miller theorem (thereafter M&M) is another assumption.

The M&M theorem, which is also called Capital Structure Irrelevance Principle, consists an influential theory in economics, creating the basis for modern thinking on capital structure. According to this theorem in an efficient market, the value of an entity is unaffected by the way that this entity is financed, given the absence of taxes, asymmetric information, agency and bankruptcy cost. The significant part of this theory is the existence of a world without taxes.

Lastly, the term structure is flat and known with certainty. For example, the price of a riskless discount bond with promised payment of \$1,00 at time t in the future is given by the following equation:

$$P(t) = \exp[-rt] \quad (1)$$

where:
r is the (instantaneous) riskless rate of interest, the same for all time.

The dynamics for the value of the firm, V, through time can be described by a diffusion type stochastic process with stochastic differential equation

$$dV = (aV - C)dt + \sigma V dz \quad (2)$$

where:

a is the instantaneous expected rate of return on the firm per unit time
C is the total dollar payouts by the firm per unit time to either its shareholders or liabilities - holders, for instance dividends or interest payments
σ^2 is the instantaneous variance of the return on the firm per unit of time
dz is a standard Gauss – Wiener process.

Suppose that at time t a firm has assets A_t financed by equity E_t, and zero-coupon debt $D_t$of face amount K, maturing at time T > t, with a capital structure given by the balance sheet relationship:

$$A_t = E_t + D_t \quad (3)$$

In practice a debt maturity T is chosen such that all debts are mapped into a zero-coupon bond. In the case $A_t > K$, the firms' debtholders can be paid the full amount K, while shareholders' equity still has value $A_t - K$. On the other hand, the firm defaults on its debt at T, if $A_t < K$ in which case debtholders have the first claim on residual assets A_t and shareholders are left with nothing. Thus, the value of equity at time T can be written as following:

$$E_T = \max(A_T - K, 0) \quad (4)$$

which is exactly the payoff of a European Call Option, written on underlying assets A_t with Strike Price K and maturity time at T.

The Black and Scholes Option Pricing Model (1973) can be applied under the corresponding modeling assumptions.

Assuming that the asset value follows the Geometric Brownian Motion (thereafter "GBM"), with Risk Neutral Dynamics given by the stochastic differential equation:

$$\frac{dA_t}{A_t} = rdt + \sigma_A \, dW_t \quad (5)$$

where:

W_t is a standard Brownian motion under risk neutral measure
R denotes the continuously compounded risk - free interest rate
σ_A is the asset return volatility

Regarding the asset, denoted as A_t, it ought to be mentioned that it grows at the risk-free rate under the risk – neutral measure and thus has drift r in the above equation, implicitly assuming the continuously tradability of corporate assets.

Applying the formula of Black and Scholes for European Call Option, we conclude to the following equation:

$$\mathrm{E_t} = \mathrm{A_t}\, \Phi(\mathrm{d_+}) - \mathrm{K}e^{-\mathrm{r(T-t)}}\, \Phi(\mathrm{d_-}) \quad (6)$$

where:
$\Phi(\cdot)$ is the N(0,1) cumulative distribution function

$$d_{+} = \frac{\ln\left(\frac{A_t}{K}\right)+\left(r+\frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{(T-t)}} \quad (7)$$

$$d_{-} = \frac{\ln\left(\frac{A_t}{K}\right)+\left(r-\frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{(T-t)}} \quad (8)$$

Under the above mentioned, a credit default occurs, when the call option matures out of the money, with a risk – neutral probability.
Thus:

$$\mathrm{P}(\mathrm{A_T} < K) = \Phi(-\,\mathrm{d}_{-}) \quad (9)$$

The default risk, that the debtholders face, can be hedged by purchasing a European Put Option, written on the same underline asset, namely A_T, with strike price K. In case of A_T<K, the Put option worth K-A_T, while if the $A_T > K$, the option worth nothing.

If we combine the above two positions, we will conclude to the following risk- free position, which guarantees a payoff of K for the debtholders.

$$D_t + P_t = Ke^{-r(T-t)} \quad (10)$$

where:

$\mathrm{P_t}$ is the put option price at time t and it can be estimated by using the Black-Scholes Model for European put option. Thus:

$$P_t = K\mathrm{e}^{\mathrm{r(T-t)}}\,\Phi(-\mathrm{d}_{-}) - \mathrm{A_t}\,\Phi(-\mathrm{d}_{+}) \quad (11)$$

Given that the corporate debt is deemed as a risky bond, it ought to be valued at a risk premium. We denote with (s) the continuously compounded credit risk spread, thus the bond price D_t is given by the following equation:

$$D_t = Ke^{-r(r+s)(T-t)} \quad (12)$$

Placing the equation (8), (9) & (10) into an equation system, we can export a formula for the risk premium, s :

$$\mathrm{s} = -\,\frac{1}{\mathrm{T-t}}\,\ln[\Phi(\mathrm{d}_{-}) - \frac{\mathrm{A_t}}{\mathrm{K}}\,\mathrm{e}^{\mathrm{r(T-t)}}\Phi(-\mathrm{d}_{+}) \quad (13)$$

The above formula can be used, when the return volatility and asset return are available.

So, at this point, the question is, how we can extract the A_t and σ_A. The answer in this query will be given by the GBM and Ito's Lemma. Specifically, using the GBM for equity price E_t and applying also Ito's Lemma, we can show that volatility can be satisfied by the following equation:

$$A_t \sigma_A \frac{\theta E_t}{\theta A_t} = E_t \sigma_E \quad (14)$$

And using the Black – Scholes Call Option Delta:

$$\mathrm{A_t} \sigma_\mathrm{A} \, \Phi\, (\mathrm{d}_+) = \mathrm{E_t} \, \sigma_\mathrm{E} \quad (15)$$

The equity price and the volatility return can be observed from the equity market, while $\mathrm{A_t}$ & σ_A can be estimated by solving simultaneously the equations (6) and (15). Finally, we use the $\mathrm{A_t}$ & σ_A into the formula (13), so as to obtain the credit risk premium (s).

Another significant model, which was introduced on August 1976, by **Oldrich Vasicek**, and was revised one year later, is the **One Factor Short Rate Model**, known also as the **Vasicek (1976) Model**. This model belongs to the earliest stochastic models of interest rates structure and it describes movements of interest rates as led by only one source of risk, the market risk. The Vasicek Model is based on an arbitrage argument similar to that of Black and Scholes (1973) and it is formulated in continuous time, while some implications for discrete interest rate series are also noted.

Specifically, the aforementioned model introduces a method of modeling the short - term interest rate assuming that the short-term interest rate follows a stochastic process that depends on parameters of a Wiener process. The drift rate and variance rate. The short-term interest rate is described as Ornstein – Uhlenbeck process:

$$dr(t)= k(\theta-r(t))dt +\sigma dW(t) \quad (16)$$

where

k, θ, σ are positive constants
$r(t)$ is the Short-term Rate at time t
σ is the Short Rate Volatility
and $W(t)$ is a Brownian motion at time t

The drift term exhibits mean reversion which means that over the time r(t) will converge to the mean reversion level b with speed a. b can also be thought of as the long- term interest rate level. When r(t) roams above the long-term rate, r(t) is pulled down, while when it drifts below the long-term, r(t) is pushed up.

$$r(t) > b \rightarrow dr(t) = k\,(\theta- r(t)) < 0 \quad (17)$$

$$r(t) > b \rightarrow dr(t) = k\,(\theta - r(t)) > 0 \quad (18)$$

$$\text{Mean Reverting Drift: } dr(t) = k(\theta - r(t)) \quad (19)$$

This can also be expressed under the economic aspect of Zeytun and Gupta (2007). Specifically, high interest rates tend to slow down an economy and lead borrowers to demand less capital. Consequently, the interest rates are reduced to a balance level in the long-term period. On the other hand, low interest rates instigate a higher level of demand for funds resulting to increases of interest rate levels.

The basic assumptions of this model are that the r(t) is a time constant function and that also follows Markov process. Thus, the future prices of r(t) are totally independent of past prices. Moreover, Vasicek's Model assumes the efficient market and consequently there are no transaction costs, investors are rational and have the same information while there is no arbitrage opportunity.

Regarding the second part of the above equation (16), this inserts the instantaneous variability, which can bring about unexpected events. Moreover, the Model postulates a constant risk premium λ.

The solution of this stochastic differential equation is:

$$r(t) = r(s)\, e^{-k\,(t-s)} + \theta\left(1 - e^{-k\,(t-s)}\right) + \sigma_r \int_s^t e^{-k(t-u)}\, dW(u) \quad (20)$$

and it is conditionally on F(s), normally distributed with

$$E(r(t) \,|\, F(s)) = r(s)e^{-k\,(t-s)} + \theta\left(1 - e^{-k\,(t-s)}\right) \quad (21)$$

and

$$V(r(t) \,|\, F(s)) = \frac{\sigma^2}{2k}\left(1 - e^{-2k(t-s)}\right) \quad (22)$$

$$\text{So, } r(t) \,|\, F(s) \sim N\left((\theta + (r(t) - \theta)e^{-k(t-s)}, \frac{\sigma^2}{2k}\left(1 - e^{-2k(t-s)}\right)\right) \quad (23)$$

Solving the equation as to Bond Price:

$$\frac{\theta B}{\theta t} + \frac{\sigma_r^2}{2}\frac{\theta^2 B}{\theta_r^2} + (k\left(\theta - r(t)\right) - \lambda\,\sigma_r\,\frac{\theta B}{\theta r} - rB = 0 \quad (24)$$

Boundary Condition B (T,T) = 1   (25)

Alternatively, the Bond Price can be also estimated by calculating the discounted expected terminal value of the bond regarding Q is

$$B(t,T) = E_Q(e^{-\int_t^T r(t)} \mid F(t)) \quad (26)$$

Consequently, the result of the above equation is

$$B(t,T) = e^{a(t,T)r(t)+b(t,T)} \quad (27))$$

where

$a(t,T) = \frac{1}{k}(e^{-(T-t)k} - 1) \quad (28)$

$b(t,T) = \frac{\sigma^2}{4k^3}\left(1 - e^{-2(T-t)k}\right) + \frac{1}{k}\left(\theta - \frac{\lambda\sigma}{k} - \frac{\sigma^2}{k^2}\right)\left(1 - e^{-(T-t)k}\right) - (\theta - \frac{\lambda\sigma}{k} - \frac{\sigma^2}{k^2})(T-t) \quad (29)$

Under the measure P, the Bond Price is given by the following equation:

$$\frac{dB}{B} = (r(t) + \frac{\lambda\sigma}{k}(e^{-(T-t)k} - 1))dt + \frac{\sigma}{k}(e^{-(T-t)k} - 1)dWt \quad (30)$$

The term structure is given by the following equation:

$$R(t,T) = -\frac{1}{T-t}\{\frac{1}{k}(e^{-(T-t)k}-1)r(t) + \frac{\sigma^2}{4k^3}(1 - e^{-2(T-t)k}) + \frac{1}{k}(\theta - \frac{\lambda\sigma}{k} - \frac{\sigma^2}{k^2})(1 - e^{-(T-t)k}) - (\theta - \frac{\lambda\sigma}{k} - \frac{\sigma^2}{2k^2})(T-t)\} \quad (31)$$

The infinitive maturity interest rate is constant and does not rely on r(t)

$$R(t,\infty) = \lim_{T\to\infty}(R,T) = \theta - \frac{\lambda\sigma}{k} - \frac{\sigma^2}{2k^2} \quad (32)$$

$$\text{Thus, } R(t,T) = R(t,\infty) + \frac{1-e^{-(T-t)k}}{(T-t)k}(r(t) - R(t,\infty) + \frac{\sigma^2}{4(T-t)k^3}(1 - e^{-(T-t)k})^2 \quad (33)$$

The shape of the term structure can be positive, negative and also concave.

In the first case $r(t) < R(t,\infty) - \frac{\sigma^2}{4k^2}$ (34)

In the second case $r(t) > R(t,\infty) - \frac{\sigma^2}{2k^2}$ (35)

while in the case of concave r(t) takes other values

The yield to maturity R(t,T) follows normal distribution

$$R(t,T) \mid F(s) \sim N(\mu_R(), \sigma_R()) \quad (36)$$

Where:

$$\mu_R() = (1-e^{-\kappa(t-s)})(R(t,\infty) + \frac{1-e^{-\kappa T}}{\kappa T}(\theta - R(t,\infty)) + \sigma_r^2 \frac{(1-e^{-\kappa T})^2}{4\kappa^3 T} + e^{-\kappa(t-s)} R(s,T) \quad (37)$$

$$\sigma_R() = (\frac{1-e^{-\kappa T}}{\kappa T})^2 (1 - e^{-2\kappa(t-s)}) \frac{\sigma r^2}{2\kappa} \quad (38)$$

Following that, in the late 1980's, one of the most important and also fundamental approaches in the field of Credit Risk Analysis, was developed. In 1984, **Kealhofer, McQuown and Vasicek** established the **KMV Corporation**, which was named by the proprietors' names. They studied and advanced the formula of **Distance to Default (thereafter "DD")**, building also a Default Database, which included more than 3.400 listed firms and more than 40.000 non-listed firms.

Relying on this Database they created a mapping from the DD to the **Expected Default Frequency (thereafter "EDF")**. Thus, they introduced the **KMV Model**, which is used to estimate the default probability of a firm based on the Merton framework and predicting **one-year PDs**. This is also known as EDF.

The EDF is the fundamental quantity of the model and is essentially the probability that the firm will default within a year, based on the methodology of the KMV Model. According to this Model, the equity of a firm is a European call option on the underlined value of the firm with a strike price which equals to the face value of the firm's debt. The model recognizes that neither the volatility, nor the underlying value of the firm can be observed.

A default event is defined when the firm's market net worth is below zero. The net worth of the firm is estimated as the net worth of assets minus liabilities. The **Default Point (thereafter "DP")** is proposed since the fact that default occurs before their market value equals the total liability. Generally, the default takes place when the firm's value is estimated to lie between the firm's total liability and short-term liability.

Given that frequently the firm's market value is not possible to be observed, the Black – Scholes Merton Option Pricing Model has to be utilized, assuming that the total market value follows GBM.

Thus:

$$dV = \mu V dt + \sigma_v V dW \quad (39)$$

where:

V is the firm's value
μ is the expected return of the firm's worth,
σ_v is the variation of the firm's value
dW is a standard Weiner process.

Under the above assumptions, we deem the firm's assets value as a call option, written on the firms and with strike price the firm's debt. Hence, in the case that the value of asset is greater than the debt, the call option is exercised, otherwise, namely in the case that the asset is lower than the debt the call option will be discarded.

Using the Black – Scholes Merton Option Pricing Model, we can formulate the relationship between firm's equity value, total value as well as liabilities.

$$E = VN\,(d_1) - De^{-rT}N(d_2) \quad (40)$$

where:
E is the firm's equity value
V is the firm's total value
D is the firm's total liabilities
N(.) is the CDF of standard normal distribution

$$d_1 = \frac{ln\frac{V}{D} + \left(r + \frac{1}{2} + \sigma_v\ \right)^2\ T}{\sigma_v\sqrt{T}} \quad (41)$$

$$d_2 = d_1 - \sigma_v\sqrt{T} \quad (42)$$

Given that in this formula, neither the value of the firm, nor the variation can be observed, the Black – Scholes Merton Option Pricing Model is required to be applied both to the volatility σ_v and the equity σ_E.

$$\sigma_E = \frac{V}{E}\ N\,(d_1)\sigma_v \quad (43)$$

Observing the 40th and 43th equation, we can notice that all the variables are observable, except from the firm's value V and volatility $\sigma_v -$. Thus, combining these equations into a system of equations, both variables can easily be assessed.

As mentioned above, the default event occurs, when the firm's market worth is between the total liability and short-term liability. The Model estimates the DP with both of the **Short-term Debt (thereafter "STD")** and **Long-term**

Debt (thereafter "LTD"), which means that the DP is equal to STD plus the fifty percent (50%) of the LTD.

$$DP = STD + 50\%\ LTD \quad (44)$$

The Distance to Default is estimated using the following equation:

$$DD = \frac{E(V_1) - DPT}{E(V_1)\sigma_v} \quad (45)$$

where:

$\mathrm{E}(V_1) = V(1 + g)$ (46), is the expected value of the firm's assets a year ago and
g is the expected market growth rate

Finally, in order to "translate" the default distance into the PD, the KMV Corporation collected the information from United States companies' Default History, creating a database with default information, which, can be easily used, so as to get the expected default frequency by mapping the DD to **Distance to Default Frequency (thereafter "DDF")**.

We should consider that, the model's assumptions can be deduced from the value of equity, the volatility of equity and other observable variables by applying an iterative, repetitive procedure to solve a system of nonlinear equations. Furthermore, the model specifies that the probability of default is the normal cumulative density function of a Z-score, relying on the firm's underlying value, the firm's volatility and the face of the firm's debt.

Approximately, during the same period Geske (1979) presents a defaultable coupon bond to be a compound option on a firm's assets. According to this approach default is brought about at coupon dates and explicitly results from the equity holder's strategic decision. Specifically, it depends on one of the following cases. The first one is that the equity holder decides to repossess the debt contract immediately and the other one is that he receives the coupon payment maintaining a claim on the assets of the firm.

The essential Model in the field of Credit Risk Models, which links macroeconomics condition to PD, was developed in 1997 by **Wilson**, presenting, the **Reduced Form Model** as an interpretation of the connection between the economic state and the rating transitions. The main characteristic of the rating transition matrix are the probabilities, which provide the opportunity to move between different rating classes. In order to estimate the PD, he uses a logit model (an autoregressive process) applies as a proxy variable for the risk factor and uses several macroeconomic variables, such as GDP and inflation. The empirical applications of this model provide that the PD includes cyclical movements. These movements are widely observable during recessions, when the PD increases dramatically

Following Wilson's approach, many models have been presented in the international literature so as to embody the impact of macroeconomic variables on the PD and the component of transition matrices. Among to the most significant approaches belong Belkin, Forest and Suchower (1998); Kim (1999), Nickell et al. (2000); Bangia et al. (2002), which are presented below.

Belkin et al (1998) based their work on the Merton framework so as to generate a standard normally distributed cycle index and suggested a method to estimate transition probabilities based on this normal distributed cycle index.

According to their work, they apply the Credit Metrics view, as this was described by Gupton et al. (1997). According to them the ratings transition matrices resulted from the "binning" of a standard normal random variable "X", which measures changes in creditworthiness. This variable splits into two parts. The first one is an idiosyncratic component "Y", unique to a borrower, while the second one is a systematic component "Z", which is shared by all borrowers.

The aforementioned systematic component measures the "credit cycle". This means that the values of default rates and of the end-of-period risk ratings cannot be predicted (using historical average transition rates) by the initial mix of credit grades. In expansion periods, "Z" will be positive, implying for each initial credit rating, a lower than average default rate and a higher than average ratio of upgrades to downgrades of default rates. In the prospect of a different situation, namely in periods of economic constrictions the reverse will be true.

As also referred in their paper, they described a method of estimating Z from the separate yearly transition matrices, which are summarized by S&P and Moody's. Conversely, they introduced an approach of estimating transition matrices dependent on an assumed value for Z.

Following the assumption that X follows the normal distribution and also under the condition of an initial credit rating G at the beginning of a year, one partitions X values into a set of disjoint bins (X_g^G, X_{g+1}^G) (In order to simplify the references, the indices G and g represent sequences. They define the bins so that the probability that X falls within a given interval equals the corresponding historical average transition rate), defined the bins as follows:

$$\mathrm{P}(\mathrm{G},\mathrm{g}) = \Phi\left(\mathrm{X}_{\mathrm{g}+1}^{\mathrm{G}}\right) - \Phi\left(\mathrm{X}_{\mathrm{g}}^{\mathrm{G}}\right) \quad (47)$$

Where:

P(G,g) denotes the historical average G-to-g transition probability
and
Φ (.) represents the standard normal cumulative distribution function

Regarding the default bin, it has been noticed that it has a lower threshold of -∞ , while the AAA bin has an upper threshold of +∞. The remaining thresholds are fit to the observed transition probabilities.

The model assumes that there are N ratings categories (including the default one.) Then there are (N-1) initial grades, which introduce all the ratings except of the default. For each of those grades, they observe (N-1) historical average transition rates. Regarding the other (Nth) value derives under the statistic condition that the probabilities sum to 1. Moreover, we determine (N-1) threshold values defining the bins and thus, we can solve for all of the bin boundaries.

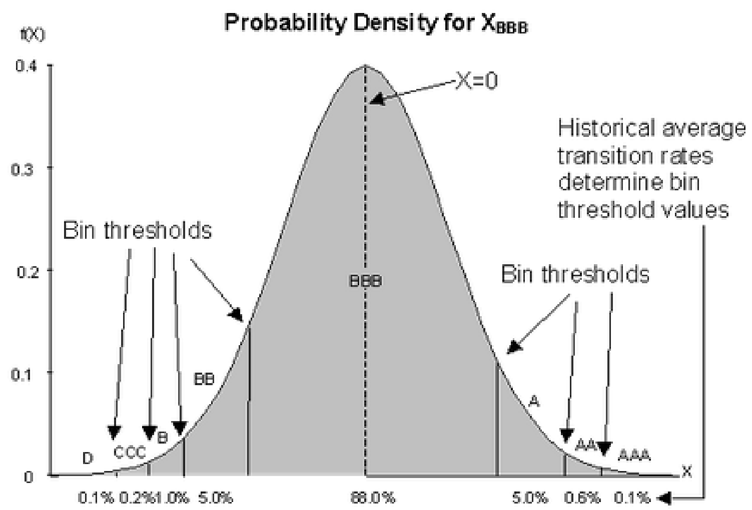

Chart 1: Relationship between continuous credit index X and Rating Transitions.
Historical Average Transitions Rates Determine Bin Thresholds
Source: Belkin et al (1998)

As it has already been mentioned, they decompose X separated into two parts, the first one is the idiosyncratic component Y which is unique to a borrower while the second one is the systematic component Z which shared by all borrowers.

Thus, X can also be present as following:

$$X = \sqrt{1-\rho}\, Y + \sqrt{\rho}\, Z \quad (48)$$

Suppose that Y and Z are not only unit normal random variables but mutually independent as well. The parameter r (ρ>0) represents the correlation between Z and X. Thus, Z explains a fraction r of the variance of X.

The observed transition rates will deviate from the norm (Z =0). Then we can estimate a value of Z in order that the probabilities associated with the bins defined above best approximate the given year's observed transition rates. We estimate that value of Z for year t, Zt and also we define Zt in order to minimize the weighted, mean-squared discrepancies between the observed transition probabilities and the model transition probabilities

Thus, we determine:

$$\Delta(x_{g+1}^{G}, x_{g}^{G}, Z_t) = \Phi\left(\frac{x_{g+1}^{G} - \sqrt{\rho} Z_t}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_{g}^{G} - \sqrt{\rho} Z_t}{\sqrt{1-\rho}}\right) \quad (48)$$

The above equation, namely the equation, is the model value for the G-to-g transition rate in year t. Thus, for a fixed ρ and t, the least-squares problem takes the following form:

$$\min_{Z_t} \sum_{G} \sum_{g} \frac{n_{t,G}\left[P_t(G,g) - \Delta(x_{g+1}^{G}, x_{g}^{G}, Z_t)\right]^2}{\Delta(x_{g+1}^{G}, x_{g}^{G}, Z_t)\left(1 - \Delta(x_{g+1}^{G}, x_{g}^{G}, Z_t)\right)} \quad (49)$$

where:

Pt(G,g) represents the G-to-g transition rate observed in year t
$n_{t,G}$ is the number of transitions from initial grade G observed in that year

Inverting the above method, namely the method of imputing the Zt from the transition matrices, we will be able to define transition matrices from the values of Zt. This can be estimated as follows Based on Zt, we estimate the probability of a G to g transitions as:

$$P_t(G,g) = \Phi\left(\frac{x_{g+1}^{G} - \sqrt{\rho}\, Z_t}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_{g}^{G} - \sqrt{\rho}\, Z_t}{\sqrt{1-\rho}}\right) \quad (50)$$

For a good year, Z_t =1, for a bad year Z_t =-1, while for an average year Z_t =0

It was in 1999 that Kim following the steps of Belkin et al. (1998) and Wilson (1997) applied an ordered probit model so as to measure the migration conditional probabilities. He also creates a credit cycle index taking into consideration interest rates, real GDP growth and unemployment.

Nickell et al. (2000) extended the ordered probit model, presenting the dependence of rating transition probabilities not only on the economic cycle but on industry, the country domicile and the stage of the business cycle as well. With other words, they proposed another similar model, where transitions matrices based on the industry, the domicile of the obligor and on the stage of the business circle by employing ordered probit models, concluding that the business cycle dimension is critical in explaining variations in transition probabilities.

In contrast to Kim (1999), the economic state variable is modeled as a discrete variable instead of a continuous macroeconomic index. They used the GDP growth to divide the economic state into three states, high, medium and low growth periods. The transition matrices are then estimated separately for these three stages of the economy. The migration probabilities estimated conditional on the medium state are interpreted as the unconditional or Through the Cycle (thereafter "TTC") probabilities. The empirical results mentioned that the economic cycle variable is the most significant factor to explain the variation in transition probabilities and especially the movements in the PD. They also observed that lower rating classes are more affected by the economic cycle, than higher rating classes.

Another interesting study is performed by Bangia et al. (2002), who also proposed a method to estimate transition matrices conditional on different discrete economic states, concluding that the loss distribution and also the economic capital of a synthetic bond portfolio could vary considerable in different economic environments. However, and as also it is referred in the paper, unlike Nickell et al.(2000), Bangia et al. (2002) used a structural approach to derive the systematic shifts in transition probabilities. Moreover, they separated the economic cycle into two stages; the recession and the expansion, by following the classification, which is provided by The National Bureau of Economic Research of US's Economic State.

At the first step, they estimated the transition matrix unconditional of the economic state from rating statistics provided by Standard & Poor's and thereafter separately for the two economic stages. In order to examine, whether the recession and expansion matrices differ, they compared these two transition matrices with the unconditional migration matrix. The empirical analysis indicated a distinct difference between the recession and expansion transition matrices. The most striking distinction between theses matrices is the PDs, which increase significantly in periods of recession.

During the same period, namely in 2002, **Vasicek** published the paper with the title "**The Distribution of Loan Portfolio Value**", providing a significant development in the sector of Credit Risk. According to his work, he assumes that the asset value of a given obligor is given by the effect of a systematic and idiosyncratic factor. Moreover, he assumes a correlated Gaussian default structure. Thus, if a certain random variable X_i falls below a threshold, a default event is triggered for an obligor. Regarding X_i, it has to be noticed that this variable follows the normal distribution.

Let A_i be the value of the i-th borrower's assets, described by the process

$$dAi = \mu i\ Ai\ dt + \sigma i\ Ai\ dxi \quad (51)$$

The asset value at time T is presented as:

$$\log Ai\ (T) = \log Ai + \mu iT - \frac{1}{2}\sigma_i^2\ T + \sigma_i\sqrt{T}X_i \quad (52)$$

where X is a standard normal variable.

The probability of default is given by:

$$P_i = P\ [\ A_i\ (T) < B_i\ ] = P[X_i < c_i\ ] = \Phi(c_i) \quad (53)$$

where $c_i = \frac{\log B_i - logA_i - \mu_i T + \frac{1}{2}\sigma_i^2 T}{\sigma_i\sqrt{T}}$ (54)

and Φ is the cumulative normal distribution function.

Let L_i be the gross loss on a loan (i) and when Li -1 the obligor defaults and when Li=0 obligor does not default.

The asset value, at time t of obligor i, is given by the following equation:

$$X_{it} = Y_t\sqrt{\rho} + Z_{it}\sqrt{1-\rho} \quad (55)$$

where:

Y is the systematic component
Z is the idiosyncratic component
P is the asset correlation between two different obligors
X and Z_i are mutual independent and follow the standard normal distribution.

The model uses three inputs, so as to estimate the PD. The first input is the through the cycle PD (thereafter "PD ^{TTC}) for each specific class of assets, while the second one is the portfolio common factor, like an economic index over the interval (0,T) given by S. Lastly, the asset correlation ρ is the final component.

Thus, the systematic factor, in which a firm is exposed to, is displayed by the term $Y_t\sqrt{\rho}$ (56) and the idiosyncratic risk is presented by the term $Z_{it}\sqrt{(1-\rho)}$ (57) .

As it mentioned before, an obligor defaults, if X_i falls below a threshold. This threshold is presented by the following condition:

$$X_i < C \quad (58)$$

where c is the function of PD^{TTC}

Integrating over Y into the equation: $X_{it} = Y_t\sqrt{\rho} + Z_{it}\sqrt{1-\rho}$ (59), denotes the unconditional probability of default by p*, which is the PD^{TTC}

$$P(X_i < c) = \Phi^{-1}(c) = p^* \quad (60)$$

$$P(X_i < c \mid Y) = P(S\sqrt{\rho} + Z_i\sqrt{1-\rho} < c \mid Y) = P(Z_i < \frac{c - S\sqrt{\rho}}{\sqrt{1-\rho}} \mid Y) \quad (61)$$

$$\text{Thus, } P(X_i < c \mid Y) = \Phi\left(\frac{c - S\sqrt{\rho}}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\Phi^{-1}(\rho*) - Y\sqrt{\rho}}{\sqrt{1-\rho}}\right) \quad (62)$$

When the common term is fixed, the probability of loss on any loan is:

$$p(Y) = P[\, Li=1 \mid Y \,] = \Phi\left(\frac{\Phi^{-1}(p) - Y\sqrt{\rho}}{\sqrt{1-\rho}}\right) \quad (63)$$

The portfolio loss conditional on Y converges by the law of large numbers to its expectation p(Y) as $n \to \infty$. Then

$$P[\, L \leq x \,] = P[\, p(Y) \leq x \,] = P[Y \geq p^{-1}(x)] = \Phi(-p^{-1}(x)) \quad (64)$$

And the cumulative distribution function of loan losses on a very large portfolio is in the limit

$$P[\, L \leq x \,] = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right) \quad (65)$$

The distribution function losses proportion is consisted by two parameters, default probability (p) and asset correlation (ρ). Regarding the cyclical level of a portfolio loss rate, this is leaded by two components, the stochastic common factor S and the asset correlation.

The common factor S represents aggregate macro-financial conditions that can easily be extracted from observable economic information. Aggregate credit risk relies on the stochastic common factor S, as during good economic times, the expected loss rate tends to below the long-term average, while during periods of recession, the expected loss rate is expected to be above the long-term average.

In 2005, Koopman et al. introduced an interesting approach to model the systematic risk factor. Using Hodrick – Prescott filter[1] , they tried to decompose U.S business failure rates into two components. The first component was an autoregressive while the second one was a time-varying cycle. Following, these components are interpreted as systematic risk factors. According to the empirical results, the default rate consists of long-run movements and cyclical pattern. Moreover, the extracted credit cycle has similar length as the economic cycles.

Regarding the above international literature, it has to be noticed, that none of these approaches take into consideration the terms of **Point in Time (thereafter PIT) and Through the Cycle (thereafter TTC)** calculation from Bank Rating Model PDs. This is because the Basel II framework, which introduced the above notion, was not in place at this period.

Analytically, the terms PIT and TTC are based on two different approaches that both describe the behavior of the PD. There is no specific definition either for PIT nor TTC. However, there is a common approach of describing them. In particular, PIT PD is mentioned as a rating system that follows the business cycle and changes over the time. As far as the TTC PD is concerned, it is mentioned as a rating system which follows a longer horizon and is almost unaffected by the macroeconomic conditions.

The official frame of reference of these terms was done by Moody's and S&P in 1995. Specifically, in November of 1995 Moody's for the first time referred to the TTC rating in a report on the copper industry. A year later, in 1996 S&P followed Moody's approach, presented its reference to TTC rating through its paper "Factoring Cyclicality into Corporate Ratings".

Following the above events, the first mention of PIT took place by Federal Reserve. Specifically, Mr. Treacy and Mr. Carey conducted a survey related to the Risk Rating System of the Large US Banks. In their article, which was published in 1998, is referred that the FIs did not update their rating criteria so as they took into consideration the current condition of their borrowers. From their sample of banks that they studied only the 25% were rating their borrowers' risk over a year, 25% were rating over a longer horizon such as the life of loan, while the rest 50% had a specific period in their minds. Moreover, they explicitly highlighted that in contrast to FIs approach, S&P and Moody's were proceeding with TTC rating.

However, approaching the present events, it should be mentioned that the first formal reference to this problem was done by the Basel Committee.

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[1] Hodrick – Prescott Filter is a macroeconomic mathematical tool. It is explicitly used in real business cycle theory in order t to remove the cyclical effect of a time series data.

Specifically, this was taken place in January 2000 by the Basel Committee, through the Basel Committee's Discussion Paper of 2000. This paper contains a survey by G-10 related to the internal rating system, referring for the first time the PIT and TTC under the aspect of risk rating time horizons.

During the progress of this survey, many FIs expressed their concern related to the potential inconsistencies created when mapping between PIT rating and TTC rating.

In the very next year, namely in January 2001, the Basel Committee proceeded with the first official distinction among the above-mentioned terms. According to this distinction there are PIT ratings that estimate default risk over a short period, usually a year and TTC ratings, which estimate it over a longer period of almost five and more years. Specifically, this is implied on the paragraph 53 of the Consultative Document for the Internal Rating Based Approach and it is quoted as follows:

§ 57, page 12 Basel (2001)

"Some banks distinguish their rating system on the basis of whether it estimates the probability of a borrower's default on a "point in time" or "through the cycle" approach. In a "point in time" process, an internal rating reflects an assessment of the borrower's current condition and/or most likely future condition over the course of the chosen time horizon. As such, the internal rating changes as the borrower's condition changes over the course of the credit/business cycle. In contrast, a "through the cycle" process requires assessment of the borrower's riskiness bases on a worst-case, "bottom of the cycle scenario" (i.e., its condition under stress). In this case, a borrower's rating would tend to stay the same over the course of the credit/business cycle".

PIT is a measure that estimates the value of PD, capturing the available information at a specific time. It is estimated as the PD for the next year. According to Engelmann and Rauhmeier (2011) PIT is a measure, which tends to be in an opposite position of the economic cycle. Its primary competitive superiority is that PIT is highly responsive of the external variables, however, its advantage is also a main disadvantage due to the high volatility it presents. Moreover, FIs seem to prefer using PIT both in pricing and management purposes as PIT needs less data for their estimation.

On the other hand, TTC is a measure independent of the economic cycle. TTC's cycle refers to a business cycle in the economy, and hence, if the current situation presents downturn or upturn behavior, this does not affect TTC. Its main advantage is that it is a stable measure and its main disadvantage is its low responsiveness to external variables. With other words, TTC's main benefit and drawback are the exactly opposite of PIT.

The philosophy of PIT and TTC in modeling the PD is relevant to a procyclical consequence. This prospect is arising from the minimum capital adequacy estimation.

To further broaden this study and based on the work of Engelmann and Rauhmeier (2006), it ought to be mentioned that the primary issue with the estimation of TTC is the lack of data. Given the fact that, PD was first inserted in Basel II, namely in the year of 2004, data strictly expands back until today. Thus, the ordinary approach in the calculation of TTC is to use an average of PIT PD over a full business cycle.

In 2008 Aguais et al. proposed a method, which gives the opportunity to move between the Through the Circle PD and Point in Time PD (thereafter "PIT PD"). This was achieved by using a credit cycle index, which is based on the Merton Model. According to their paper, they use the approach of distance-to-default so as to transform a TTC PD into a PIT PD. The distance-to-default is related to the distance between the expected value of the assets and the default point, as this is exported from the Merton Model. The PD is determined as the standard normal cumulative distribution function of the negative value of distance-to-default.

Aguais et al. having been based on Belkin et al (2008) proceeded to expose a credit cycle index form default rates. This index is positive when economic condition is better than historical average and negative when the economic condition is worse than historical average.

In the following Figure, the conversion from PIT to TTC is illustrated.

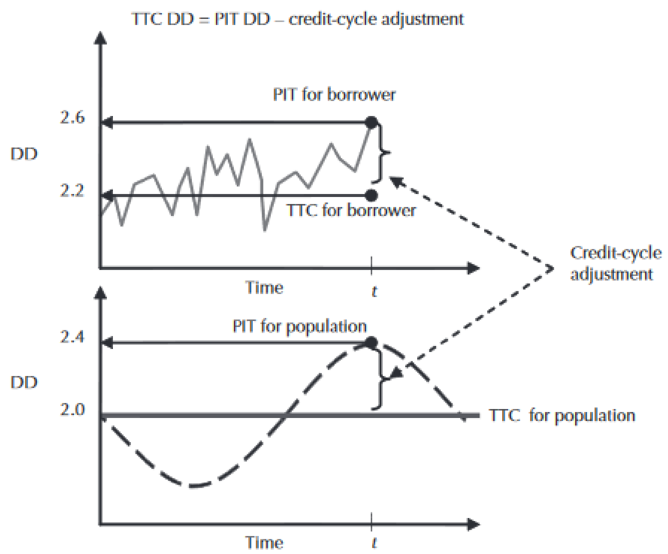


Chart 2: Relationship between PIT and TTC default – distance
Source: Aguais et al. (2008)

In some cases, the DD reflects current economic condition and it is deemed as **point in time DD (thereafter DD^{PIT})**. While, when the current condition is at a historic norm, it is deemed as **through the cycle (thereafter DD^{TTC}).**

As it can be deduced from the above figure, the DD^{TTC} is obtained if the current cyclical component is subtracted from DD^{PIT}. Respectively, if the current cyclical component is added to the DD^{TTC}, the DD^{PIT} can be easily obtained.

Another significant part of their work, which should be presented, it is deriving credit indices for economic sectors. According to them, it is a standard approach for deriving latent risk factors. At first, someone can obtain performance measures for a representative sample of obligors. Secondly, he should summarize them. Following that, he should scale the results in line with the CreditMetrics model's assumption and particular that the annual changes in systematic risk factors include unit variance.

The aforementioned scaling can be easily achieved by introducing a correlation parameter, ρ(s), into the index, as the following equation presents:

$$Z(s,t) = \frac{DD^{PIT}(s,t)}{\sqrt{\rho(s)}} \quad (66)$$

$$DD^{PIT}(s,t) = SUMM\left(DD^{PIT}(f,t)\right) \quad (67)$$

$$\rho(s) = Var\left(DD(s,t)\right) - DD(s,t-1)) \quad (68)$$

where:

Z(s,t) = unit – variance, credit index for sector s
ρ(s) is the correlation or scaling factor for sector s
DD^{PIT}(s,t) = summary PIT DD for sector s
SUMM – summarization operator (eg. mean, weighted mean)
f= obligor index
DD^{PIT}(f,t) = PIT DD for obligor f at time point t
Var = variance computed across all history

At this point, a "normal" Z value is needed for a sector (Z_n(s)) to be used in estimating the TTC PD (PD^{TTC}) for each related obligor. A possible solution to this need may be the past average value of Z (s, t). However, in most PD models, the relationship between PD and DD (and therefore Z) is non-linear. Hence, this solution of Zn(s) entails an average PD^{TTC}, which falls below the historical average PD^{PIT}, default rate. Therefore, it can instead define Zn(s) as the value usually slightly below the Z(s, t) average. This approach can be formulated as follows:

$$Z_n(s)= \frac{F^{-1}(PD(s))}{\sqrt{\rho(s)}} \quad (69)$$

where:

F^{-1} is the invert of the PD function for sector and
PD(s) is the long-run past average default rate for sectors s

Additionally, Aguais et al. introduced another index, which is used in deriving PIT PDs for agency rating. This index is called "agency Z" and it is estimated, using the following equation:

$$Z_{Agency}(t)= \text{AVG} \left(\frac{\Phi^{-1}(EDF(r,t))- \Phi^{-1}(EDF(r))+\Phi^{-1}(PD(r))}{\sqrt{\rho}}\right) \quad (70)$$

where:

AVG is the average across all alphabetic agency rating
Φ^{-1} is the inverse standard normal distribution function
EDF (r,t) is te median MKMV EDF for obligors with rating r, at time t
EDF (r) is the historical average of median EDFs for the rating r
PD(r) is the idealized, agency historical average default rate for rating r

The Z_{Agency} is applied in transforming each long- run average PD into an estimation of its current PD^{PIT}.

As it is also underlined in this paper, this index is different from the sector Zs. Sector Z measure general credit conditions, for instance a geographic region or global industry. On the other hand, the Z_{Agency} measures the average creditworthiness of companies within each agency rating, relative to its respective, long-run, historical average. Hence, if migrations in agency ratings is closely linked to the overall credit cycle, agency Zs remain constant.

However, as it is shown in the following figure, the agency Zs fluctuate widely.

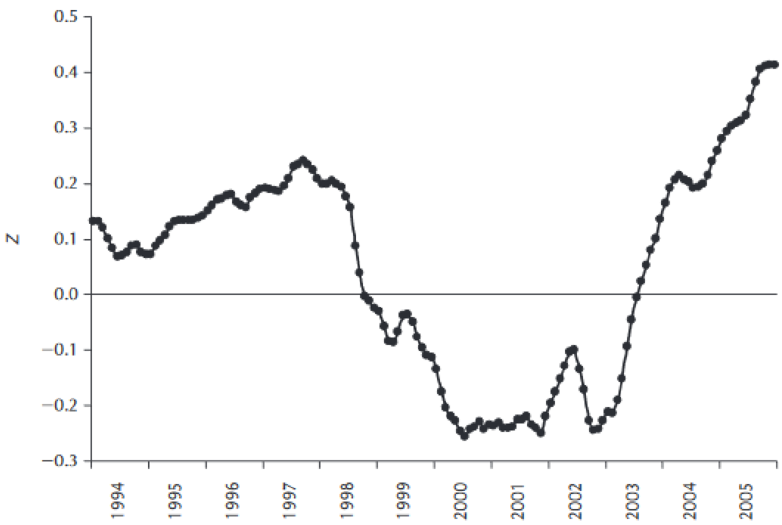


Chart 3: Normalized agency Z factor derived from S&P rated firms
Source: Moody's KMV and Standard and Poor's (2006)

This fluctuation implies that agency ratings migrations explain a minority share of changes in credit conditions.

The transmutation between PIT and TTC PDs is proceeded with using the below formulas:

$$DD^{PIT}(i,t) = DD(i,t) + \big(1 - \delta(f)\big)\sum_j \beta(i,s)\big(Z\,(s,t) - Z_n(s)\big) \quad (71)$$

$$DD^{TTC}(i,t) = DD(i,t) - \delta\,(f)\sum_j \beta(i,s)\big(Z\,(s,t) - Z_n(s)\big) \quad (72)$$

where:

β is the beta coefficients measuring the loading of each index on each obligor or account and factors

δ(f) indicates the degree from 0 to 100% that an obligor's or account's DD measure is PIT

Finally, we can note that referring to Aguas et al. work, is that their model is only for 100% PIT model and not for hybrids, as this is approached by Carlehed and Petrov (2012).

Following, in 2012, Carlehed and Petrov introduced another significant model. According to their paper, they presented the quantitative degree of PIT-ness of an internal hybrid PD model and suggested a method of measuring it.

The specific method consists of three steps. The first one is the description of the economic circle under the condition of a standard normal variable Z. Following, the Rating Model PD is decomposed into two parts; the economic one and the internal part, which does not stand on the economic circle. Lastly, the TTC can be easily estimated by averaging over all states of economy.

Analytically, the economic circle, as it approaches in the first step, it is indicated by the value of Z. At first, a Bank Portfolio is considered, which is a subset of a large global portfolio and it is divided into sectors, where each one includes counterparties with same characteristics under the aspect of how they can be affected by the general economic events.

Using the Merton Model for each obligor i:

$$X_i = \sqrt{r}\, Z + \sqrt{1-\rho}\varepsilon_i \quad (73)$$

where:

Z is the economic cycle effect and it follows the normal distribution
ε_i is the idiosyncratic risk factor, which also follows the normal distribution.

Assuming that ε_I are mutual independent and independent of Z and that ρ is the correlation between X_i *and* Z, it can easily be obtained the following equation:

$$p_i\,(Z) = P[\,X_i < B_i \,|\, Z] = \Phi(\frac{B_i - \sqrt{\rho}\,Z}{\sqrt{1-\rho}}) \quad (74)$$

where:

Φ is the cumulative distribution function of normal distribution and

B_i is obligor constant

Appling the Merton Model into the portfolio, the equation is converted into the following one:

$$\mathrm{p_p(Z)} = \Phi\,(\frac{\mathrm{B} - \sqrt{\rho}\mathrm{Z}}{\sqrt{1-\rho}}) \quad (75)$$

where:

ρ is the correlation of the average portfolio to the economic cycle
B is the constant, corresponding to the long-term default frequency Φ(B) for the sector and
Z is a time series of historical values

Inverting the equation 74 can be easily obtained Z:

$$Z = \frac{B-\Phi^{-1}(d_t)\sqrt{1-\rho}}{\sqrt{\rho}} \quad (76)$$

Based on Gordy (2000), Breeden (2008) and Kupiec (2009) and applying the method of moments, it can be obtained:

$$\mathrm{E}[\,\Phi^{-1}(\mathrm{p})] = \frac{\mathrm{B}}{\sqrt{1-\rho}} \quad (77)$$

and

$$\mathrm{V}[\,\Phi^{-1}(\mathrm{p})] = \frac{\rho}{1-\rho} \quad (78)$$

Following, they assume the transformation d → Φ^{-1}, and they define m as the mean as well as σ as the standard deviation. They solve for B and ρ and they get B≈ $m/\sqrt{1-\sigma^2}$ (79)

and

ρ ≈ $\sigma^2/(1+\sigma^2)$ (80).

Inserting these into the equation A: $Z_t = \frac{m-\Phi^{-1}(d_t)}{\sigma}$ (81)

In order to calculate obligator's TTC PD, they proceeded to derive the formula in two steps. The first one, is the unrealistic approach of a 100% PIT model and the second one is a general approach of the model.

Regarding the unrealistic case, they assume that the rating models estimates a PIT PD, p_i, at the time t, where (i) is the obligor and this value is known. Given that Z is known, they invert

$$p_i\,(Z) = P[\,X_i < B_i\,|\,Z] = \Phi\left(\frac{B_i-\sqrt{\rho}\,Z}{\sqrt{1-\rho}}\right) \quad (82)$$

and calculate for each obligor in the portfolio.

Thus, $B_i = \sqrt{\rho}\,Z_t + \sqrt{1-\rho}\,\Phi^{-1}(p_i)$ (83)

and

finally $\mathrm{q_i} = \Phi\left[\sqrt{\rho}\mathrm{Z_t} + \sqrt{1-\rho}\,\Phi^{-1}\,(\mathrm{p_i})\right]$ (84)

Assume (a) is a real and positive number, $0 \leq a \leq 1$, which is called "degree of PIT" of model and has not any financial mean.

As it is obvious from all the previous references, many studies examined macroeconomic variables and economic cycle into PD modeling so as ensuring that the relationship among the economic component and Credit Risk is taken into consideration. In 2014, Gavalas and Spyropoulos having studied the previous literature, they examined the linkages among the credit risk management and the macroeconomic components under the approach of four different business cycle scenarios (mixed economic state, the average, the boom and also the contraction economic state) concluding that changes in Credit Risk are explained by the components of industry, location, as well as the changes in economic cycle.

In order to revise the above international literature review that it was explicitly presented in the above sections, we are going to proceed with referring to the most significant models. We have to begin from the fundamental theory of Merton, who set up the foundation in the field of Credit Risk. According to his model, an equity defaults whether the value of its assets is less than the promised debt repayment at time T. However, the most significant point is how this model prices the entity's equity. It assumes that the equity is a European Call Option applying the Black and Scholes theory.

Similar with Merton, in 1976, Vasicek presented an innovative approach, describing the movements of interest rates as led by only one factor of risk, particular market risk. Vasicek's theory belongs to the earliest and most important stochastic models of interest rates structure. However, its main drawback is the fact, that the interest rate can become negative, while another significant disadvantage is the poor fitting to the current terms structure of interest.

Following in 1984, KMV model advanced the DD formula, also creating a default database. Moreover, it created a mapping from database to the EDF, thus this model is used to estimate the default probabilities of a firm based on the Merton framework and predicting one – year PD. The main drawback of this model is that it contains much subjective estimation of input parameters based on accounting data. For instance, the setting of the default point. Furthermore, for some entities, whose true market value is not assessable from accounting data, the estimated default probability could be far from reality. Another significant point is that the employed accounting data suffers from an infrequent updating problem and usually is released with a time lag and possible accounting manipulations.

Belkin et al. (1998) based on the Merton theory presented a one – parameter representation of credit risk and transition matrices in the form of a single normal distributed systematic factor (Z). They proved that the historical data of the systematic factor consists a description of the past credit conditions as well as that Z significantly affects the migration probabilities. Even if Belkin et al. introduced the concept of systematic factor, they did not take into consideration the effect of macroeconomic variables in default probabilities. Thus, a year later, in 1999, Kim advanced their methodology. In particular, he created a credit cycle index taking into consideration macroeconomic variables such as GDP and unemployment.

We also have to mention the Vasicek theory of 2002, which points out that the distribution of the loan portfolio, can be used in order to present the loan loss behavior of large portfolios. The loan loss can be realized loss on loan maturity prior to the horizon date. This approach belongs to the recent fundamental approach, consisting the base form many of the recent approaches. However, we have to underline that Vasicek's theory assumes that all loan in the same portfolio have the same maturity, the same probability of default and the same correlation of the obligors' assets. Moreover, their approach can only find apply in large portfolios with many small obligors.

In 2012 Petrov et al. based on Nickel et al. work, who proposed an approach where transition matrices depend on industry, domicile of the obligor and on the stage of business cycle, and they developed an approach for PIT and TTC probabilities of default decomposing in credit risk classification system, primarily for corporations. It has to be mentioned that before this methodology, no one has presented the concept on PIT and TTC estimation from the bank rating model PDs, probably because Basel II was not in place.

3. International Financial Reporting Standards (IFRS)

International Financial Reporting Standards (thereafter IFRS) is the International Accounting Framework under which financial information is not only organized but reported as well. It has come from the pronouncements of the London based International Accounting Standards Board (thereafter IASB). IFRS began as an endeavor to harmonize accounting rules across the European Union. However, the value of harmonization gradually led this idea to be converted globally to a significant and attractive approach, which was gradually adopted around the world.

Today, IFRS is the mandatory accounting framework in more than 120 countries and it requires from businesses and financial institutions to report, not only their financial results, but also their financial position, using the same framework and accounting and financial rules. Thus, the financial reporting, from businesses and financial institutions as well, are characterized by a significant level of uniformity, making in this way, easier any effort of comparison and contrast among entities' financial position and results.

The standards that were issued by the International Accounting Standards Committee (thereafter IASC)[2], which is the predecessor of IASB are still within use and they are well known as International Accounting Standards (IAS), while standards issued by the IASB are called IFRS. IAS were issued during the period of 1973 to 2001 by the Board of the IASC.

On the 1st of April 2001, the International Accounting Standards Board (thereafter IASB) took over from the IASC the responsibility of setting International Accounting Standards. At its first meeting the new Board adopted IAS 39, which has been issued by the IASC in March of 1999. From there and then, the IASB has been continuing to develop standards, including all the current progresses, calling the new standards International Financial Reporting Standards.

As it was already mentioned above, the main purpose of IFRS is to report the financial results and the financial position of businesses and financial institutions, making easier any comparison and any contrasting among

[2] The International Accounting Standards Committee (IASC) was founded in June 1973 in London and was replaced by the International Accounting Standards Board on 1 April 2001. It was responsible for developing the International Accounting Standards and promoting the use and application of these standards

entities. This can be executed through the financial statements, which are a structured representation of the financial positions and performance of any entity.

The main purpose of financial statements is to provide, and display information related to the financial position, performance and cash flows of entities. These pieces of information are indeed useful and significant to a wide range of users, helping them to take economic decisions. Moreover, the financial statements are also able to provide all the necessary data regarding the results of the stewardship management.

An important component related to IFRS is its features. According to the IFRS Foundation, the main features of IFRS are the following:

Fair presentation and compliance with IFRS:

Fair presentation requires the faithful representation of the effects of the transactions, other events and conditions in accordance with the definitions and recognition criteria for assets, liabilities, income and expenses set out in the Framework of IFRS.

Going concern:

Financial statements are presented on a going concern basis unless management either intends to liquidate the entity or to cease trading or has no realistic alternative but to do so.

Accrual basis of accounting:

An entity shall recognize items as assets, liabilities, equity, income and expenses when they satisfy the definition and recognition criteria for those elements in the Framework of IFRS.

Materiality and aggregation:

Every material class of similar items ought to be presented separately. Items that are of a dissimilar nature or function shall be presented separately unless they are immaterial.

Offsetting:

Offsetting is generally forbidden by the IFRS. However certain standards require offsetting when specific conditions are satisfied (such as in case of the accounting for defined benefit liabilities in IAS 19 and the net presentation of deferred tax liabilities and deferred tax assets in IAS 12).

Frequency of reporting:

IFRS requires that at least annually a complete set of financial statements is presented. However listed companies generally also publish interim financial statements (for which the accounting is fully IFRS compliant) for which the presentation is in accordance with IAS 34 Interim Financing Reporting.

Comparative information:

IFRS requires entities to present comparative information in respect of the preceding period for all amounts reported in the current period's financial statements. In addition, comparative information shall also be provided for narrative and descriptive information if it is relevant to understanding the current period's financial statements. The standard IAS 1 also requires an additional statement of financial position (also called a third balance sheet) when an entity applies an accounting policy retrospectively or makes a retrospective restatement of items in its financial statements, or when it reclassifies items in its financial statements. This for example occurred with the adoption of the revised standard IAS 19 (as of 1 January 2013) or when the new consolidation standards IFRS 10-11-12 were adopted (as of 1 January 2013 or 2014 for companies in the European Union).

Consistency of presentation:

IFRS requires that the presentation and classification of items in the financial statements is retained from one period to the next unless it is apparent, following a significant change in the nature of the entity's operations or a review of its financial statements, that another presentation or classification would be more appropriate having regard to the criteria for the selection and application of accounting policies in IAS 8 or an IFRS standard requires a change.

Based on the references of the IFRS Foundation and its papers, which are every so often published, it is easy to present the financial statements of IFRS. These statements are the following:

The Statement of Financial Position

The Statement of Comprehensive Income, which separates statements comprising an Income Statement and separately a Statement of Comprehensive Income, so as to reconcile Profit or Loss on the Income statement to total comprehensive income

The Statement of Changes in Equity (thereafter SOCE)

The Cash Flow Statement and notes, which include the summary of the significant accounting policies

The IFRS have been adopted and are in use in many countries around the world such as in the South Korea in the European Union, in India, Hong Kong, Australia, Malaysia, Pakistan, GCC countries, Russia, Chile, Philippines, South Africa, Singapore and Turkey. However, it is not used by the United States.

It is a firm belief that IFRS are expected to be adopted globally, helping investors, analysts and any other user of financial statements, to extract high quality financial, accounting and economic information related to the entity, which is under examination. In this way, costs like the cost of comparing alternative investments in different countries, can almost be vanished. Without any question, IFRS is a great tool for Financial Institutions, giving them the opportunity not only to easily analyze and examine their exposure to credit risk, but also to take globally different financial positions.

However, many different and skepticist opinions have been also expressed. Mr. Ray J. Ball, who is an accounting researcher and Davidson Professor at the Graduate School of Business of the University of Chicago has expressed the view that the enforcement of the standards could possibly be lax, and the regional differences in accounting could easily become obscured behind a label. He also expressed his doubt about the fair value emphasis of IFRS and the influence of accountants from non-common-law regions, where losses have been recognized in a less timely manner.

Regarding the European Union, it has to be mentioned that in 2002 the Union took the decision that from the 1st of January 2005 the IFRS would have applied for the consolidated accounts of the EU listed companies. In order to be approved for use in the EU, standards must have been endorsed by the Accounting Regulatory Committee (thereafter ARC), which includes representatives of member state governments and is advised by a group of accounting experts known as the European Financial Reporting Advisory Group.

3.1 International Financial Reporting Standards 9 (IFRS 9)

International Financial Reporting Standard 9 (thereafter IFRS 9) is promulgated by IASB, replacing IAS 39. Among the main reasons of this replacement or renewal is the need of recognition and measurement of financial assets, liabilities and contracts of buy or sale of non-financial items. Moreover, many users of financial statements have expressed the belief that the requirements in IAS 39 brought about difficulties in the field of application, and interpretation.

In March of 2008, the paper “Reducing Complexity in Reporting Financial Instruments” provided the result of a work, which has begun in 2005, between IASB and the US National Standard Setter. This cooperation aimed to improve and simplify the report of financial instruments. Through this paper were provided many possible approaches in order for the accounting of financial instruments to be improved and simplified as well. As a result, in November of 2008, the IASB added the aforementioned work into its active agenda.

In April 2009, the IASB under the pressure of global financial crisis, the significant conclusions of G20 and the recommendations of the Financial Stability Board, proceeded with the announcement of a timetable for replacing IAS 39. Specifically, the IASB aimed to revise the accounting standards for financial instruments, focusing them to address the deficiencies, which were believed to have taken place to the magnitude of the crisis.

During the same year, the IASB issued the first part of IFRS 9, which not only covered the classification but also the measurement of financial assets. This was aimed to replace a specific section of IAS 39 and particular, the section which was related to the asset classification and measurement.

In the very next year, the IASB issued another part of IFRS 9, so as to cover the classification and the measurement of financial liabilities and also the aspect of applying fair value option and embedded derivatives.

It may worth to be mentioned, that some of the elements of IFRS 9 faced a very strict critique, by some key IASB constituents. Only two different approaches were permitted by the model for classifying debt instrument assets. The first one was the fair value with all changes in fair value reported in profit and loss (thereafter FVPL), while the second approach was the amortized cost. The above approaches brought about an important deviation from Financial Accounting Standard Board (thereafter FASB) decisions, which would also have a category of fair value with certain changes in fair value reported in other comprehensive income (thereafter FVOCI).

Additional, there were many concerns related to the criteria for the amortized cost category. The specific criteria were very stringent, bringing about the concern that they would force many financial instruments to be reported at fair value even if they had the ability to be accounted at amortized cost. Thus, in 2012, the IASB proceeded with issuing a draft, suggesting amendments related to the classification and measurement of financial instruments.

IASB and FASB also cooperated in order to develop a model for impairment of financial assets. In 2003, IASB issued a draft suggesting an impairment model, while FASB took the decision to suggest an alternative model. During the same year, IASB worked independently developing a hedge accounting model and issuing the portion of the IFRS 9.

The next year, namely in 2014, IASB issued the final version of IFRS 9, in which included hedge accounting, impairment, and the amended classification and measurement guidance, bringing about a positive response in the European market.

As it mentioned above, IFRS 9 replaced IAS 39. The replacement took place in three main phases, as it is mentioned by IFRS Foundation.

Phase 1: Classification and Measurement of Financial Assets & Liabilities

The section of classification and measurement of financial assets issued in November 2009 by IASB. According to this issue, the financial assets are classified accordingly to the business model within which they are held and their contractual cash flow characteristics of the financial assets. Moreover, financial assets as well as cash flows have to be estimated by performing the Business Model Test and the Solely Payments of Principal and Interests Test.

According to the paragraphs 4.1 – 4.4, from IFRS 9 as it was issued from IFRS Foundation, an entity has to classify a financial asset at amortized cost, in the case where the asset is held within a business model, which aims not only to hold the asset, but also to collect cash flows. These cash flows constitute solely payments of principal and interest on principal outstanding. Differently, an entity ought to classify a financial asset at fair value through other comprehensive income, in the case that it is held within a business model. The objective of the business model is to achieve both the collecting of cash flows, which are solely payments of principal and interest on the principal amount outstanding and selling financial assets and the cash flows.

In the case that a financial asset is not estimated at amortized cost or at fair value through other comprehensive income, it must be estimated at fair value through profit or loss. Moreover, an entity, at initial recognition, may irrevocably designate a financial asset as estimated at fair value through profit or loss.

Almost a year late from the issue of the classification and measurement of financial assets, namely in October 2010, the requirements of financial liabilities was added in IFRS 9. Most of these requirements are related to the fair value of option for financial liabilities, which were changed so as to address own credit risk.

As it is explicitly stated in paragraph 4.2.1, a financial liability ought to be classified at amortized cost. The exceptions of the above framework are set up in the paragraphs 4.2.1a – 4.2.1e, and accordingly to these mentioned distinctions, a financial liability, such as a derivative, which is liability, estimates at fair value through profit and loss. Another exception is the case of a financial liability that results when a transfer of a financial asset does not qualify for derecognitions or when continuing involvement approach applies.

Moreover, in the case of financial guarantee contracts and after the initial recognition, the issuer of such a contract has to assess it at the higher of the amount of the loss allowance and at the higher of the amount initially recognized in the case the cumulative amount of income is recognized according to the framework of IFRS 15.

The above approach should be also followed in the case of a commitment, which provides a loan at a below market interest rate. However, in this case, it must be added that the issuer has to estimate it at the higher of contingent consideration as recognized by acquirer in a business combination which is relevant to IFRS 3. Such contingent should consequently be assessed at fair value with changes recognized in profit or loss.

In the case that a financial liability is not estimated at amortized cost or at fair value through other comprehensive income, it should be estimated at fair value through profit or loss. Moreover, an entity, at initial recognition, may irrevocably designate a financial liability as estimated at fair value through profit or loss.

Phase 2: Impairment Methodology

In July 2014, the IASB updated the IFRS 9 adding the impairment requirements relevant to the accounting for an entity's Expected Credit Losses (“ECL”) on its financial assets and commitment to extend credit. Due to these requirements, the threshold that was in IAS 39 for recognition of credit losses is reduced at a significant point. Thus, according to the IFRS 9, it is not necessary to recognize credit losses only when a default occurs, but an entity accounts for ECL and proceeds to the necessary changes in those ECL. The amount of ECL is updated at each reporting date in order not only to initially reflect and display any change in credit risk but also to provide timely information related to ECL.

Phase 3: Hedge Accounting

In 2013 the IASB updated the IFRS 9, adding the requirements of hedge accounting. The main purpose of this update was the alignment of the accounting treatment with risk management activities. As a result, the entities are capable of reflecting better these activities in their financial statements. This change also constitutes a great tool for non-financial entities, as it makes more achievable for them to use hedge accounting. The change also permits the use of hedge accounting for components of instruments and groups of contracts, easing also the hedge effectiveness test. Moreover, they enhance the disclosures of hedges and risk management.

From the above-mentioned phrases, the most significant one, which is related not only to business process but banking institutions too, is the impairment methodology, as it composes the appropriate ground for the calculation of ECL.

Taking everything into consideration, we can reach to the conclusion that the strongest weakness of IAS 39, is the mechanism of impairment calculation, which is related to financial assets and the accounting treatment of the loss allowances. This weakness has also become stronger by the global financial crisis, intensifying the concerns related with replacing the IAS 39.

The aforementioned mechanism was based on the Incurred Loss Model. According to this model the loss allowances have to be recognized after the occurrence of default event. However, with the replacement of IAS 39, this was also changed, and it was replaced by the ECL Model. Thus, the credit losses and consequently loss allowances should be recognized based upon the expectations, namely before a certain adverse event. Specifically, the

ECL Model is applied to debt instruments, which are recorded at amortized cost or fair value through other comprehensive income, such as loans, loans commitments and financial guarantee contracts.

Under the framework of IFRS 9, the impairment of financial assets is measured as 12-month expected credit losses or lifetime expected credit losses, relying on the case of a significant change in credit risk associated with the given asset since initial recognition. Thus, the requirements of IFRS 9 have brought about an increase in the overall level of loss allowances.

According to the IFRS Foundation, the 12-month time horizon is deemed to be an appropriate compromise among a reliable estimation of expected credit losses, the implementation and operational costs, which are associated with the implementation of the described system. The main purpose of this is that a 12-month horizon has already been used by many institutions for the calculation of credit risk capital requirements.

The “foundation stone” in the field of ECL estimation is a weighted average of credit losses and according to the IFRS Foundation this can occur through various scenarios with a certain probability. From this procedure cannot be excluded the time value of money, as well as the interest rate, with which ECL ought to be discounted. Furthermore, all relevant and supportable information has to be taken into consideration which is related with current, historical and future conditions.

As it was mentioned above, IFRS 9 brought about an increase in the level of provisions. This may be one of the most significant impacts of the replacement of IAS 39, as the costs will undoubtedly affect the accounts of Profit & Losses of Financial Institutions.

Meanwhile, a decrease will be also recorded in the Tier 1 capital. Under the aspect of Regulators, Tier 1 capital belongs to the core measure of Financial Institutions’ strength and it consists of the core capital and non-cumulative preferred stocks. As a consequence of this decrease, a reduction will also take place in the capital adequacy ratio of Banks.

As far as the estimation of increase in provisions are concerned, these differ significantly. Basing on the references of IFRS Foundation in 2013, the increase was estimated about 25% - 60%. On the other hand, two years later the chairman of IASB, Mr. Hoogervorst proceeded with the announcement that the expectation was almost about 35% - 50%. However, reading recent assessments from IFRS Foundation and the European Bank Authority, the increase of loss allowances is estimated about 20%.

The contribution of the forward-looking feature of the IFRS 9 is also significant, as it takes into consideration the macroeconomic forecasts, and hence it induces early credit losses estimates. Moreover, it also causes a moderation in profit and loss fluctuations, which come from the business cycle.

On the other hand, a controversial matter is the procyclicality of provisions. Many different opinions have been expressed about this topic. In 2016, Novotny – Farkas, published a paper with the title "The Interaction of IFRS 9 Expected Loss Approach with Supervisory Rules and Implications for Financial Stability" which examined this matter and concluded that a loan loss accounting model is procyclical by its nature. However, IFRS 9 could possibly moderate the effect of the features of IAS 39, which deteriorated procyclicality.

Before the replacement of the IAS 39, Regulatory Expected Losses were estimated at a higher level than the provisions of IAS 39, due to the framework of Basel. Because of the IFRS 9, this is expected to be changed, turning the provision greater than the Regulatory Expected Losses. A modest opinion, related to this topic was expressed in 2005 by Mrs. Stothers, from the Office of the Superintendent of Financial Institutions (thereafter OSFI). Specifically, she mentioned that turning over the relationship between the provision according to IFRS 9 and Basel expected losses may not be so obvious, due to the several methodological differences that exist and relate with the estimation of credit risk parameters.

3.2 The Three Stages of IFRS 9

According to the IFRS 9, the Credit Risk differentiates into three different Stages. At the initial recognition a financial instrument is included in the first Stage, which is also mentioned as Stage 1 and under the condition that is not credit impaired. At the reporting date, if there is an important increase in Credit Risk, since the initial recognition, the loan is transferred into the second Stage; Stage 2. Consequently, the requirements for Stage 2 will be applied. Finally, if the instrument meets the criteria for Credit Impairments it is transferred into Stage 3.

IFRS 9 Stages

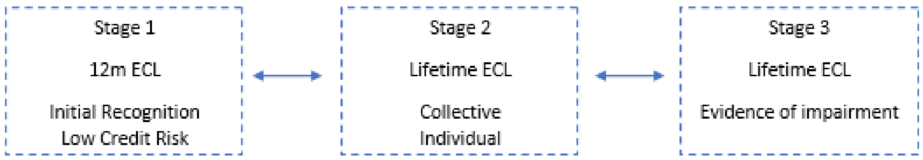


Chart 4: The stages of IFRS 9
Source: Created by the Author

Stage 1 Fully Performing Exposure

As mentioned above, the first Stage includes exposures, which do not present a significant increase in credit risk, from the initial recognition and exposures, which also present low credit risk at the reporting date. At this Stage, the 12 months ECLs are estimated in the sector of profit or loss and are deemed as the result from the potential default events, within the 12 months after the reporting date. Specifically, for lending exposure in this Stage, the estimations are based on the following mathematic type:

$$12month\ ECL = EAD * PD\ 12month * LGD \quad (85)$$

where:

EAD is the Exposure at Default at time t,

PD 12month is the Probability of Default for the next 12 months

and LGD is the Loss Given Default of the lending exposure

The recognition of ECLs in the period of a year, reflecting that the yield on the entities includes a return to cover those credit losses, which are expected from the time of the initial recognition, moderating the overestimation in the interest revenues from the IAS 39.

Stage 2 Underperforming Exposures – Significant Increase in Credit Risk (SICR)

Stage 2 comprises instruments with a significant, noticeable deterioration in credit risk and more specifically, a significant Increase in Credit Risk (thereafter SICR) and no objective evidence of impairment since the reporting date.

Analyzing the second Stage extensively, it should be mentioned that in order to estimate the case that a financial asset displays a SICR, the entity has to compare the Residual Lifetime PD estimated at reporting date in contrast to the Structure of Residual Lifetime PD estimated at initial recognition.

Moreover, according to the paragraph 5.5.3 of IFRS 9, an entity should assess expected lifetime credit losses for a financial asset in case of a significant increase in credit risk it is observed. As it is also referred in paragraph A26 of BCBS Guideline on Credit Risk and Accounting for Expected Credit Losses as well as in paragraph 5.5.9 of IFRS 9, at each reporting date, the entity has to perform the SICR assessment, comparing the risk of a default occurring over the remaining expected lifetime of the instrument with the expected risk of a default as this has been estimated at origination. It is significant to be mentioned that such analysis will not be performed comparing changes in the amount of expected credit losses. Regarding to the information, which is considered in this analysis, it should highlight that these data should also contain forward looking information when these data are reasonable and supportable as well.

The framework of IFRS 9 also includes two practical means as simplifications to the analysis of significant increase in credit risk.

The first one is the Low Credit Risk. Analytically, when an asset displays a low-credit risk at reporting date, an entity should estimate the case that credit risk has not increased at a significant level. Furthermore, it is deemed that the application of this simplification would entail a low-quality implementation of IFRS 9 and its expected credit losses model. Consequently, the entity has to estimate the case that there has been a SICR for all its financial assets regardless of their credit risk level at each reporting date.

The second one is the 30 Days past due rebuttable presumption, where an entity may consider as a rebuttable presumption that an important increase in credit risk has occurred when a financial asset is more than 30 days-past-due. Expanding the study, it should highlight that in the same way as the low credit risk expedient, it is considered that the reliance on the 30 days past due rebuttable presumption as a primary indicator of transfer to Stage 2 implies a low-quality implementation of an ECL model. Thus, such criterion will be only considered as a backstop, and a relevance analysis shall be performed for those portfolios where this assumption is not indicative of significant increase in credit risk.

A financial asset should be estimated at an individual basis so as to determine whether a SICR has been occurred since initial recognition. If so, it will be allocated in Stage 2. Such assessment has to include both risk management information, as well as additional forward-looking information, according to the characteristics of the instrument and the market conditions. In order to articulate this assessment, the entity shall carry out a comparison between Residual Lifetime PD at reporting date against Residual Lifetime PDs estimated at initial recognition.

The residual Lifetime PD at reporting date shall be calculated from the observed rating or scoring. The Residual Lifetime PDs at initial recognition shall be estimated from the information of rating or scoring considered when the exposures was granted, taking into account additional and relevant credit aspects included in the decision model.

As far as the lifetime expected credit losses are concerned, which (is defined in the second Stage, where an important change in credit risk occurs), the relevant estimation follows the below formula:

$$\text{Lifetime ECL} = \sum_{t=1}^{n} \left(\frac{\text{EAD}* \, \text{PD}_{t_i} * \text{SRt}_{n-1} * \text{LGD}}{(1+r_i)^{t_i}} \right) \quad (86)$$

where:

EAD is the Exposure at Default at time t,

PD_{ti} is the Probability at Default for each cash flow,

SRt_{n-1} is the Survival Rate, i.e. the cumulative probability of non-default at time t-1

LGD_t is the loss given default,

r is the exposure’s Effective Interest Rate and

t is the cash flow’s period (t=1,…,M).

The main difference between the lifetime ECLs and 12 months ECLs relates to the estimation of lifetime PD as well as 12 months PD, as the 12 months ECL is measured as cash shortfalls over the entire expected lifetime of the entities, scaled by the 12 months PD

Stage 3 Non-performing Loan

The last Stage includes entities, for which objective evidence indicates impairment at the reporting date.

In Stage 3 a financial asset will be deemed as credit-impaired, and subsequently allocated to Stage 3 under the framework of IFRS 9, when an event with a detrimental impact on the estimated future cash flows has been occurred and the asset is flagged as doubtful. The credit exposure, at this Stage is the similar to the exposure deemed to be individually impaired under IAS 39, while Stage 1 and Stage 2 credit exposures are replaced by the exposures that are collectively assessed for impairment under IAS 39. For instance, financial assets that are disclosed under the label 'Financial assets past due, but not impaired' in banks' financial statements would largely fall into Stage 2 under IFRS 9. Thus, the recognition of lifetime ECLs will indeed occur earlier than under IAS 39, namely already when there would be a significant increase in credit risk, but before the actual default.

According to the principles of IFRS 9, when an entity defines credit-impairment for the purposes of assessing its associated ECL, this definition shall be consistent with that implemented for internal credit risk management purposes and should also take into consideration quantitative indicators except from the rebuttable presumption of 90 days past due, which could not be refuted in any case by the entity. Moreover, a financial asset is credit-impaired in case of one or more events which have a detrimental impact on the estimated future cash flows of that financial asset have been occurred.

The difference between the second and the third Stage, is related to the approach of calculating the interest revenues. At the first and the second Stage, the interest recognition and impairment are decoupled. The interest revenue is calculated on the gross carrying amount, while at the last Stage, the interest revenues are calculated on the adjusted amortized cost, vs. the gross carrying amount net of the impairment.

Transition from the first to the last Stage is not possible, given the demand of not aligning the assumption of the significant increase in credit risk. A careful observation, in the above figure, provides the statement that the framework of the new standards and especially the model of Expected Loss, is almost an intermediate approach between the IAS 39 incurred loss model and the Fair Value Approach. On the one hand, IFRS 9 ignores the market interest rate changes while at the same time it recognizes ECLs.

As far as the impairment estimation in Stage 3, is concerned the formula which is used is the following:

$$\text{Stage 3 Impairment} = \text{EAD} \times \text{LGD} \quad (87)$$

where:

EAD is the Exposure at Default

LGD is the loss given default

Another significant point of mentioning, is that IFRS 9 does not refer to the concept of "forbearance". However, there are many references to "modified financial assets". Specifically, IFRS 9 determines that a modified financial asset should be also estimated for SICR. Regarding the initial recognition date, this will be relied on the case that the original financial asset was derecognized or not.

Moreover, in case that the modification of a financial asset has a consequence of the derecognizing of the original financial asset, the new asset would be estimated so as to determine whether it is credit-impaired at origination. In those instances, namely in case that the modification does not cause a derecognizing of the original asset, the modified financial asset should be evaluated for SICR, comparing simultaneously the risk of a default, which occurred not only at the reporting date, but also at initial recognition.

Additionally, IFRS 9 explicitly states that a customer (borrower) in most of cases has to demonstrate not only a consistent but also a good payment behavior over a period of time. In this way, it will be deemed that the credit risk has been decreased.

The framework of IFRS 9 does not refer to the concept of "cure or probation period". However, there is a reference that the impairment model of IFRS 9 should be symmetric in the sense that when the indication of significant increase in credit risk is no longer met, the facility should be transferred to Stage 1.

Another important point of mention is the conditions, according to which the classification of a financial asset into each Stage takes place.

Regarding the conditions which participate in the classification of Stage 3, it should be mentioned that:

- The modification made a total or partial cancellation by write-offs of the debt and that cancellation represents an amount equal or higher than the provision under Stage 2 that would correspond to that financial asset.

- Approval of the use of embedded forbearance clauses for a debtor who is "doubtful" or who would be considered as "doubtful" without the use of these clauses.

- The forbearance is based on an inadequate payment schedule (it is considered to be an inadequate payment schedule when several defaults are observed or would have occurred without a new modification).

- There are some contractual covenants that defer the payment (i.e. grace period) by a period longer than 2 years.

In case that none of the aforementioned conditions are met, the forborne financial asset will be transferred into the 2nd Stage, till the "cure criteria" are met and consequently it will be also moved into the 1st Stage.

A forborne financial asset that was classified into Stage 3 according to the previous will be transmitted into Stage 2 when all the following conditions are met:

- 1 year has passed since the forbearance measures were extended.
- The financial asset, following the forbearance measures, does not have any past-due amount at the end of the probation period

- The debtor has paid, via its regular payments in accordance with the post-forbearance conditions, a total amount equal to the one that was previously past-due (if there were any past-due amounts) or that has been written-off (if there were no past-due amounts) under the forbearance measures.

- None of the financial assets to the debtor is more than 90 days past-due at the end of the probation period.

According to the previous criteria, a financial asset will be transferred into the 1st Stage from the 2nd one, when all the following conditions are met:

The financial asset is no longer considered as "doubtful when"

- A 2 year probation period has been passed from the date the forborne financial asset was considered as performing.

- Regular payments of more than an insignificant aggregate amount of principal or interest have been made since the classification into Stage 2. In addition, the debtor shall have paid, through regular payments, an amount equal to the one that was past-due or written-off when the forbearance was carried out. As a consequence, the existence of a grace period may defer the accomplishment of this requirement.

- None of the financial assets to the debtor is more than 30 days past-due at the end of the probation period.

In the case the above conditions will not be met at the end of the probation period, the financial asset will not be moved into the 1st Stage, but it will remain in the 2nd one. Only when all the conditions will be met, the movement to the 1st Stage will be executed.

This assessment takes place on at least a quarterly basis. Meanwhile, it should be highlighted that the abovementioned criteria consist additional requirements. This means that forbearance flag acts just as an additional objective indicator of SICR.

The final case, which should be referred, is the movement to the 3rd Stage during the probation period from 2nd Stage to 1st Stage. This can take place if any of the following situations will be observed:

- Additional forbearance measures are extended
- Forborne financial asset becomes more than 30 days-past-due

3.3 Forward-Looking Information

Another important reference, which is presented by IFRS 9 is the macroeconomic forward-looking information. Macroeconomic forward-looking information ought to be incorporated by defining a range of different future scenarios of the relevant data identified, with their associated probabilities of occurrence, using a maximum of 5 scenarios but mainly 3.

According to the IFRS 9 principles, there are two key areas where macroeconomic scenario forecasts may be utilized. The first one is the case of assessing, if an instrument has undergone a significant increase in credit risk, i.e. "Staging". The preferred criteria involve performing staging according to the comparison of the average probability among all scenarios weighted by the probability of occurrence of each scenario between origination and reporting date. The second one is referred to the estimation of Lifetime ECL. Specifically, when estimating lifetime expected credit losses once an instrument has been passed from the first Stage to the second one, the preferred approach in order to introduce the forward-looking information, when measuring expected credit losses is to use the average of expected losses within each scenario weighted by their probability of occurrence. However, a single probability weighted scenario can be used so as to compute a single expected credit loss value in some cases.

IFRS 9 explicitly states that not only past and current information need to be used, but specifically that with forward-looking nature, disregarding if the provisioning criteria are applied at an individual or collective level as well.

The forward-looking information, which has to be taken under consideration, has to include some factors. These factors are specific not only to the borrower, but also to all macroeconomic data, that may have a significant impact in the financial instrument credit risk, using both internal and external sources. This information has to be used so as to adjust the historical information of credit risk patterns that is used to perform the estimations.
In cases, where market information, such as CDS market price is available, the entity should also consider its potential ability to improve credit risk assessments and expected losses prediction, especially when it can be demonstrated that this information includes forward-looking prospects.

The incorporation of forward-looking information is an essential part in order to accomplish a full implementation of IFRS 9 requirements. In this way it is highlighted that its incorporation should be based on the fact that the cost could be undue, particularly for those cases, where incorporating such information improves the estimates precision.

The use of a range of potential future scenarios, which was designed using all relevant forward-looking information at the reporting date, should be taken into consideration, not only in the assessment of SICR processes, but in the ECL processes as well.

The principles of IFRS 9 state that a minimum of two forward-looking scenarios ought to be used. However, the standard does not require to perform an overwhelmingly complex estimation, which will not yield significantly better estimates, given that the interpretation of the number and type of economic scenarios will differ as a function of portfolio complexity.

4. Data Sample - Econometric Framework

4.1 Introduction

The main purpose of this chapter is to investigate which Macroeconomic Variables are strongly associated with PDs of Greek Corporate Portfolios. For this purpose, we take into consideration, not only the literature presented in the second chapter, but also some of the most recent papers in the sector of Banking, such as Louzis et al. (2010), Papadopoulos et al. (2016), Petropoulos et al. (2018) and the Official Reports of European Central Bank (thereafter ECB), which embody all the recent empirical research in the relevant area considering at the same time the effect of the Global Financial Crisis of 2007 – 2008.

To achieve this goal, we use a well-established econometric model, which is essentially the Auto-Regressive ("AR") Model

4.2 Data Sample

The estimations are based on a dataset that comprises actual yearly data of Corporate Performing Loans and Corporate Non-performing Loans on a quarterly frequency, covering the period from the first quarter of 2004 until the first quarter of 2018[3]. Default Rates are estimated using these quarterly data, submitted by Greek Commercial Banks to the Bank of Greece. Furthermore, the dataset includes a series of candidate Macroeconomic Variables such as Gross Domestic Product Growth (thereafter GDP), Inflation, Unemployment Rate and House Price Index, covering the period mentioned above.

The aforementioned Macroeconomic Variables have been exported from the DataStream (Thomson Reuters) Platform, while the Default Rates have been taken from publicly available in the web site of Bank of Greece.

4.2.1 Corporate Default Rates

The Default rate is defined as the percentage of Performing Loans (thereafter PLs), which becomes Non-Performing Loans (thereafter NPLs). It can be estimated upon two different bases. The first one is the basis, which does not take into consideration the respective curing, namely the transition from

[3] A possible drawback of our study may be the size of dataset. However, the available data of Greek corporate default rate begins from 2004. Thus, the dataset is limited between Q_1 2004 and Q_1 2018.

NPLs to PLs and the second one is the net basis that includes the respective cure rates. It is obvious from the above that the Default Rates consist a crucial and significant measure of NPLs, as they represent the rate of new NPL formation.

Using the publicly available date mentioned above, the Corporate Default Rates are estimated using the following equation:

$$\text{Default Rate} = \frac{\sum \text{NPLs}_{t+1} - \sum \text{NPLs}_t}{\sum \text{PL}_t} \; x \; 100 \quad (88)$$

where [4]

NPLs is the total amount of Non-performing Loans at time (t+1) and t

PL is the total amount of Performing Loans at time t

The chart below presents the quarterly change (%) [5]in Greek Corporate Default Rates, covering the period from the last quarter of 2006 until the first quarter of 2018.

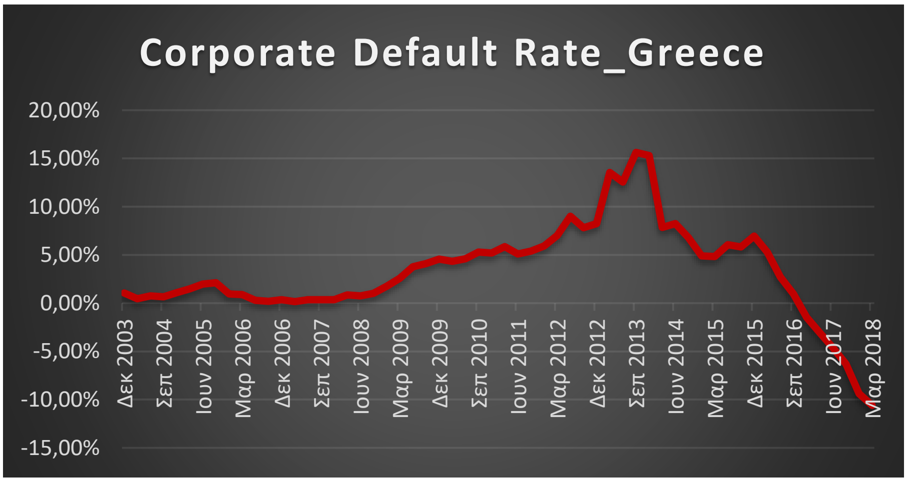


Chart 5: Greek Default Rates of Corporate Portfolios
Source: Created by the Author based on data of Bank of Word

[4] Non-Performing Loans (NPLs): Based on the definition of IMF (International Monetary Fund), a loan is considered as NPL, when payments of interest as well as principal are past due:
a) by 90 days
b) more
c) at least 90 days of interest payments have been capitalized, refinanced or delayed by agreement, or payments are less than 90 days overdue.

5 The quarterly change (%) in Greek Corporate Default Rates presents a highly fluctuation from 2012 until 2015. Especially, the highly fluctuation is presented in 2013 due to the political instability and in 2015 because of the capital controls that imposed on banks

As the chart shows, the default rates present a fluctuation, which is explicitly associated with the recent economic crisis. In particular, the data is characterized by a period of low default rates followed by a rapid growth in default rates after 2008. However, in the last quarter of 2016, the default rates have inserted in a negative default rate scale. This stems from for two main reasons. The first one is because of write offs, which have been extensively performed by Greek FIs and secondly because a significant part of corporate loans has reverted into Stage 2 or 1 (as it is described in section 3.3).

4.2.2 Macroeconomic Variables

a. Gross Domestic product (GDP)

GDP is a Monetary Measure of the Market Value of all the Final Goods and Services that are produced in a period of time. It is estimated mainly on quarterly and yearly basis. In 2017, the GDP of Greece was estimated at $198,49 billion, presenting an increase of 2% from the previous year.[6]

In algebraic form the GDP can be expressed by the following equation GDP = C + I + G + NX (89)

where:

C is Consumption and it is the largest component of GDP in the economy

I is Investments (public & private)

G is Government Spending

NX is the Balance of Goods and Services and it is estimated by the following equation (X – IM) (90)

X is Exports

IM is Imports

The following, chart presents the evolution of Greek GDP, in a yearly basis, covering the period from 1960 until 2017.

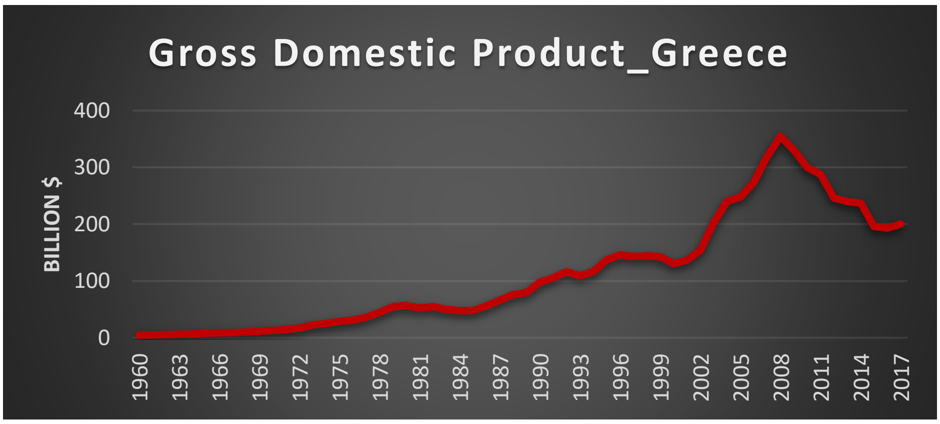


Chart 6: Gross Domestic Product of Greece
Source: Created by the Author based on data of Bank of Word

[6] According to the official report of ECB, the growth of Greek GDP for 2018 is estimated at 2,4%.

Juxtaposing the two above charts, it can be highlighted that during the period of 2007 – 2016, the sharply rise of PDs due to the economic crisis and sovereign debt is tied to the decrease in GDP. This decrease is estimated about 25%.

b. Inflation

Inflation is the sustained increase in the Price Level of Goods and Services in an Economy over a Period of Time and it belongs among the principal Macroeconomic Variables. According to the following chart, which presents the Greek Inflation in an annual basis, from 1960 until 2017, the recent rate of inflation is in negative levels. Thus, the Greek Economy is dealing with Deflation, which is the opposite situation of Inflation, Namely, it is a decrease in the price levels and it is generally associated with a potential cost in Social and Economic Sector.

Chart 7: Inflation of Greece
Source: Created by the Author based on data of Bank of Word

c. Unemployment Rate

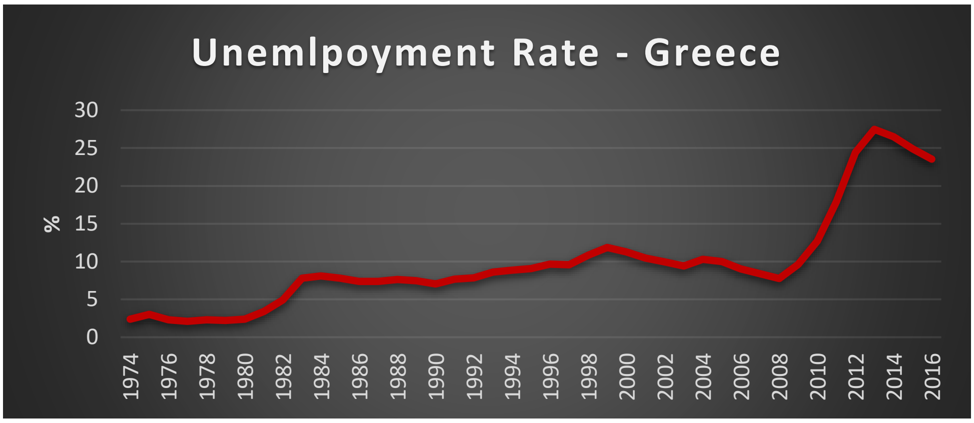


Chart 8: Unemployment Rate of Greece
Source: Created by the Author based on data of Bank of Word

The Unemployment Rate is an indicator of unemployed people as a percentage of the total available labor force. Its estimation is calculated by dividing the number of unemployed individuals by all individuals currently in the labor force and it is expressed as a percentage.

In Greece, the unemployment rate presents a significant increase during the period of 2007 till 2013, while today, it shows a mild decrease. Particularly, in 2013 the Unemployment Rate was estimated around 28%, while in 2016 it was about 24%.

4.3 Econometric Framework

As it has already been mentioned, we will use an Autoregressive Model (thereafter AR) to find out which of the candidate macroeconomic variables are strongly associated with the PDs of the Greek Corporate Portfolios.

The AR Model was chosen based on Petropoulos et al. (2018). This paper examines the relation between default rates and macroeconomic variables not only in corporate portfolios, but also in residential and consumer portfolios, using three different methodological approaches.

The first methodological approach is the Markov regime switching models, the second one is the Bayesian model averaging, while the last one is the Linear least Squares Regression and particularly the study of an AR(1) model. According to their results, the corporate portfolios are significantly associated with GDP and Consumer Price Index (thereafter CPI).

An AR Model is a representation of a type of random process, which predicts future behavior, based on past behavior. It is applied for forecasting, when there is a correlation between values in a time series and the values that precede and succeed them, consisting a widely used model in economics.

As far as the process, is concerned, it is a linear regression of data in current series against past values in the same series. It has to be highlighted that in an AR models the variable Y_t not only depends on a set of predictor variables, namely Xs, but also on its own previous value and on a stochastic term. Hence, AR model is in the form of a stochastic differential equation. Moreover, it includes degrees of randomness. That means that someone is capable of predicting future trends using past data, yet, he is never going to get 100% accuracy. AR models are also known as Markov models, transition models and conditional models. An AR(p) model is a Model, where specific lagged values of Y_t are used as predictor variables.

The value for (p) is called the order. For example, an AR(1) is a “first order autoregressive process.” The outcome variable in a first order AR process at some point in time t is associated with time periods that are one period apart, for example the value of the variable at (t – 1). A fourth order AR process would be related to data four periods apart.

The general equation of an AR(p) Model is the following:

$$\mathbf{Y_t = con + \boldsymbol{\varphi}_1 Y_{t-1} + \ldots + \boldsymbol{\varphi}_p Y_{t-p} + \varepsilon_t} \qquad \mathbf{(91)}$$

where:

$\Upsilon_{t-1}, \ldots, \Upsilon_{t-p}$ are the lags, namely the past series values

ε_t is the error term which is assumed to be randomly distributed (follows a white noise procedure)

Con, which is the constant term, is defined by the following equation:

$constant = (1 - \sum_{i=1}^{p} \varphi_i)\,\mu$ (92) ,

where μ is the mean of the process

5. Results and Discussion

5.1 Our Main Findings

In this chapter, we present the results of the econometric study, in order to reach a conclusion regarding the main question that was established in the previous chapter.

The first test, which has taken place, is related with the stationarity of the variables. This test is carried out using a Unit Root Test[7], to identify whether or not the variables are Non-Stationary.

The null hypothesis in the Unit Root Test, is defined as the presence of a unit root and the alternative one is either stationarity, trend stationarity or explosive root depending on the test used.

Unit Root Test: Null Hypothesis: Ho: Non -Stationarity

H_1: Stationarity

Note: Unit Root Test uses Dickey – Fullers estimation method

The result of the Unit Root Test is the Non-Stationarity of variables; thus, we proceed with the estimation of their first differences of them in order to overcome the issue of Non-Stationarity. Then, we examine again the variables for stationarity, using the same criterion. The result of the second Unit Root Test is the stationarity of the variables. Thus these variables are modified into I(1).

Observing the first chart, we can notice that during the year of 2013, default rates present a high fluctuation related to the political instability. Thus, we decided to add a dummy variable.

Dummy Variables take the value 0 or 1 to indicate the presence or the absence of a categorical effect, which is expected to affect the outcome. These variables are mainly used as "devices", so as to sort data into mutually

[7] Unit Root Test tests the case that a time series variable is stationarity or non- stationarity. A time series variable contains a stationarity variable, whether a change in a time does not arise a change in the shape of the distribution. The main way of applying the Unit Root Test is through the augmented Dickey – Fuller Test. This test examines if a unit root presents in an autoregressive model. It is developed in 1979 by the statisticians David Dicker and Wayne Fuller

exclusive categories, for instances invest / not- invest. As far as the econometric time series analysis, is concerned, dummy variables can be used to point out the occurrence of events. Hence, a dummy variable can be considered as a “Truth Value” represented by a numerical value 0 or 1.

Thus, we proceeded with adding a dummy variable related to the year of 2013.

$$y_{2013} = \text{Dummy}=1 \text{ , if it belongs to } 2013$$

Similarly, due to the capital controls on banks imposed in 2015 and their effect in default rates, we also added another dummy variable related to 2015.

$$y_{2015} = \text{Dummy}=1 \text{ , if it belongs to } 2015$$

Following the above steps, we proceeded with the estimation of the AR(1) Model, using at the same time the heteroscedasticity and autocorrelation consistent covariance estimator (HAC) to account for heteroscedasticity and autocorrelation.

AR(1) Model:

As we have already mentioned in previous section, we use the Auto-Regressive (“AR”) Model, basing on the paper of Petropoulos et al. (2018), which examines the relation between default rates and macroeconomic variables not only in corporate portfolios, but also in residential and consumer portfolios. For our objective, we focus on their results, which are associated with the Greek corporate portfolios. Taking into consideration the work of Petropoulos et al., we take a lag in the variables of GDP, Inflation and Default Rate, as in this way the dependent variable responds better in the time - shifts of the aforementioned independent variables. The AR(1) equation, according to which we apply the econometric study, is presented below:

$$\boldsymbol{\Delta DR_t = \beta_0 + \beta_1 \Delta DR_{t-1} + \beta_2 \Delta GDP(G)_{t-1} + \beta_3 \Delta Inf_{t-1} + \beta_4 \Delta Unemp_t + \beta_4 \Delta House_t + \beta_6 Y_{13} + \beta_7 Y_{15} + \varepsilon_t} \quad \textbf{(93)}$$

Where:

ΔDRt is the first difference of Default Rate at time t
ΔDR_{t-1} is the first difference of Default Rate at time t-1
$\Delta GDP(G)_{t-1}$ is the first difference of GDP growth at time t-1
ΔInf_{t-1} is the first difference Inflation at time t-1
$\Delta Unemp_t$ is the first difference of Unemployment Rate at time t
$\Delta House_t$ is the first difference of House Index at time t
Y_{13} is the dummy variable 2013
Y_{15} is the dummy variable 2015
ε_t is the error term

The results of the OLS are summarized in the following table:

Table 1:
OLS regression results for testing the explanatory power of the macroeconomic variables

	Coefficient	**Std. error**	**t-ratio**	**p-value**
const	-0,0145045	0,00402798	-3,601	0,0009***
ΔDr (-1)	0,122661	0,155666	-1,862	0,0709*
ΔGDPG (-1)	-0,116216	0,0677868	-1,714	0,0197**
ΔInf (-1)	-0,165208	0,265134	-2,134	0,0399**
ΔUnempl.	0,0177805	0,00550759	3,228	0,0331**
ΔHouse	0,00332702	0,00335441	0,9918	0,3295
Y_{13}	0,344916	0,0113708	3,033	0,0051***
Y_{15}	0,0236219	0,0117760	2,008	0,0541*
R^2	0,598314			
Durbin-Watson	1,557993			

Note: * (**) and *** indicates significant results at the 10 (5) and 1 per cent level respectively. Source: Created by the Author using GRETL software

Given the fact the we have already used the HAC estimator[8] and we have estimated an AR(1) Model, there is no need to check for autocorrelation effect, as this has already been overcome by the aforementioned actions. However, we observe that the Durbin – Watson criterion is almost near to the value of 2, signifying the likelihood of an autocorrelation effect. In order to further clarify the matter of autocorrelation effect, we proceed with the

[8] Heteroscedasticity & Autocorrelation consistent covariance matrix estimator. It is described by Newey and West (1987) and it is used to overcome autocorrelation and heteroskedasticity effect in the error terms of the models. It is often applied to time series data.

Breusch – Godfrey test[9]. This test tests for the presence of serial correlation, which has not been contained in the under-consideration model.

Breusch-Godfrey: Null Hypothesis: Ho: No Autocorrelation

H_1: Autocorrelation

The p-value of Breusch - Godfrey test is estimated at 0,0543, indicating that there in is no autocorrelation effect[10].

Before we get to a conclusion, we also test the model for Heteroscedasticity, carrying out the White's Test. This is a statistical test that examines if the variance of the errors in a model is constant; that is for homoskedasticity.

White's Test: Null Hypothesis: Ho: Homoskedasticity

H_1: No Homoskedasticity

The p-value of White Test is 0,0035, indicating that there is no heteroscedasticity.

According to the above econometric results, the default rates are negatively linked with Inflation and GDP. That means that when Inflation and GDP increase, default rates follow different direction, namely they decrease. Consequently, when Inflation and GDP decrease, default rates increase. This conclusion corresponds to the charts presented above. If we observe them, we will find out that during the increasing period of increasing Default Rates, Inflation and GDP were decreasing. The relation between Default Rates and GDP can be deemed predictable, while queries can be arisen from the relation between default Rate and Inflation. In this case, the predictable result will be that both variables will follow the same path, i.e., when Inflation increases, similarly Default Rates have to also increase. However, in our case this relation follows a different path. This is because of the effect of Deflation. Greece from 2013 until 2016 suffered from Deflation. Deflation is the opposite situation of Inflation, particularly, it is a continuing decrease in price

[9] It is a test, which is described by Breusch – Godfrey (1978) and it is used to test for an autocorrelation effect.

[10] The p-value is marginal up the threshold of acceptance the no autocorrelation hypothesis.

levels and if it is demand driven it can incur serious negative effects on economic growth and social welfare. Moreover, we have to mention that their p-values indicate that both are statistically significant variables at the 5 per cent level.

One the other hand, a positive relation is noticed among default rates, unemployment rate and house price index. This means that these variables follow the same direction. However, the variable of house price index does not seem to be significant, as its p-value is estimated at 0,3295. As far as the unemployment rate is concerned, this is a strongly statistically significant indicator at the 5 per cent level.

Similarly, the dummy variables are positive and significant. The Y_{13} is significant at 10%, while the other one at 1%. This positive relationship points out that even if all the other indicators remain stable, default rates are higher due to the political instability of 2013 and the capital controls of 2015.

Regarding the ΔDr, it should be highlighted that its positive coefficient indicates that the increase of default rates at time (t-1) brings about an increase at time (t).

R^2 is estimated at 0,598314, indicating that the dependent variable, namely the 1st difference in the default rate (ΔDR_t) is explained by the independent variables at almost 60%. Moreover, the p-value (F) is estimated at 0,0011, indicating that the model can be considered as an overall significant model.

Taking all the above into consideration, we get into the conclusion that the macroeconomic indicators, which are significantly associated with corporate default rates are GDP, Inflation and Unemployment Rate. Our results can be deemed similar to the results of the study of Petropoulos et al. According to this study, the macroeconomic indicators, which are significantly associated with corporate default rates are GDP and Inflation. However, they achieved a better R^2, which is estimated almost at 0,95. This difference is brought about the quality of sample, since our study has been based on Default Rates, which were estimated by public available data.

5.2. Our Proposed Methodology

Taking into consideration the results from the econometric study, which are presented in the previous chapter, we introduce a methodology which can practically be applied by FIs in order to estimate forward looking probabilities of default. This approach can be applied on corporate portfolios and it is based on the Vasicek model (1976 & 2002) and it embodies many recent approaches such as those adopted by Aguais et al. (2008) and Petrov et al. (2012).

5.2.1 Literature on the Methodology of Projection PDs through Migration Matrices.

The proposed methodology for the projection of PDs through Migration Matrices is mainly derived from the Vasicek (1976 & 2002) theory.

The theoretical approach, which addresses the PIT – TTC transformations is based upon the classical idea of a firm asset return and a default barrier. Particularly, the default event of an entity i in a period t, is caused when the value of entity's asset falls below a certain threshold, which is linked to the debt of the entity. Thus, the probability of default of the borrower i can be stated as follows:

$$\mathrm{PD}_{\mathrm{i,t}} = \mathrm{P}(\mathrm{r}_{\mathrm{i,t}} < \mathrm{R}^{\mathrm{D}}_{\mathrm{i,t}}) \quad (93)$$

In Vasicek's (2002) approach, the asset return $r_{i,t}$ of an obligor is assumed normally distributed and presented as a weighted sum of a **Single Systematic Factor Z_i** and an **Idiosyncratic Risk Factor $\varepsilon_{i,t}$**. The first one is common for all obligors and the other one, namely the idiosyncratic factor is obligor-specific. These two factors are **Independent Standard Normal Distributed Variables**, meaning that the asset return have also to be Gaussian Distributed Variable.

These factors can be mathematically specified as Standard Normal Variables. Thus: $\mathrm{r}_{\mathrm{i,t}} = \mathrm{Z}_{\mathrm{t}}\sqrt{\rho} + \varepsilon_{\mathrm{i,t}}\sqrt{1-\rho}$ (94)

where $Z_t \sim N(0,1)$, $\varepsilon_{i,t} \sim N\,(0,1)$ and correlation coefficient $\rho \in [0,1]$, which is also referred to as R^2 in Credit Portfolio Models.

Given the fact that, the Z_t factor is defined as One Single Macroeconomic Factor, which affects all obligors in the consideration portfolio and the PD_{TTC} is defined as the Expected Default Rate without Macroeconomic Effect, PD_{TTC} can be specified as following:

$$PD_{TTC} = \mathrm{P}(\mathrm{r}_{\mathrm{i,t}} < \mathrm{R}^{\mathrm{D}}_{\mathrm{i,t}}) = \Phi(\mathrm{R}^{\mathrm{D}}_{\mathrm{i,t}}) \quad (95)$$

where Φ is the

Cumulative Standard Normal Distribution Function

Inverting the function (), the certain threshold is specified as a PD_{TTC} function.

$Ri, tD \ = \ \Phi^{-1} \ (PD_i)$ (96)

where Φ^{-1} is the Inverse Cumulative Standard Normal Distribution Function.

Following the framework of Vasicek (2002), PD_{PIT} is presented by the following equation:

$$PD_{PIT} \ = \Phi \ (\frac{\Phi^{-1} \ (\mathrm{PD_{TTC}}) - \mathrm{Z}\sqrt{\rho}}{\sqrt{1-\rho}}) \quad (97)$$

Solving the above equation into the factor Z:

$$Z_{\mathrm{t}} = \ (\frac{\Phi^{-1} \ (\mathrm{PD_{TTC}}) - \Phi^{-1}(\mathrm{PD_{PIT}})\sqrt{1-\rho}}{\sqrt{\rho}}) \quad (98)$$

5.2.2 Our Methodology.

Based mainly on the aforementioned theory, the proposed methodology presents a functional approach to the implementation of the prospective cases in historically observed migration matrices.

On the one hand, this approach can be used to determine the lifetime PDs through the 12 months PD expressed by the rating model and on the other, through the consideration of migration between historically observed rating classes.

We present a table, which indicates the steps of our methodology.

Table 2: Methodology Steps.

1st Step	Collect Corporate Historical Default Rates	
2nd Step	2a. Migration Matrices Historical 3-factor Extraction	2b. Econometric Regression (GDP, Inflation, Unemployment. Rate)
3rd Step	3a. TTC Migration Matrices	3b. PD Model
4th Step	Fit Migration Matrices (3 years)	
5th Step	Forward Looking PDs	

Source: Created by the Author

The 1st and 2ndb steps, have already been presented. The first one, is described in the 4th Chapter. and particularly in paragraph 4.2.1, while the other one, is presented in the 5th Chapter

The process of determination of the term structure is therefore founded in the following steps:

1st Observation of one-year Migration Matrices

2nd Removal of the effect of Macroeconomic Variables from observed matrices

3rd Determination of the TTC Matrices (Starting from Annual Matrices with the Systemic Effect removed)

4th Estimation of the Forward-Looking Factors, through the preparation of models, which link the historical changes in the Risk of Portfolios to the performance of one or more Macro-Economic Factors

5thConditioning of the Through the Cycle Matrix, through the combination of the TTC Matrices determined as in 3rd step with the effects of the application of future cases through models as defined in 4th step.

Annual Migration Matrices

1st Selection of all Credit Position, namely the Performing and Non-Performing Loans at the beginning of the year

2nd Observation at the year-end of all selected positions

3rd Positions no more in the credit portfolio are labelled as ‘expired’;

4th Positions with no rating are labelled as ‘unrated’;

5th Default class includes all positions that during the year have been classified as Non-Performing

Following, we present an indicative PIT Migration Matrix, which constitutes the starting point of our methodology. As it is referred in the footnote, this matrix is a “dummy” due to the lack of publicly available data. Each column (vertical raw) and raw characterized by Ri, i = 1…8, which indicate the variable Rating Bands, while the last raw and column present the Default Rating Band. That means that as moving from R1 to Default Rating Band, the quality of Rating Band gets worse and worse. The percentage, which presented in each cell, points out the percentage of obligors, who move among the Rating Bands and belongs in each Rating band. For instance, in the first cell the 71,91% of the obligors remain in the R1 Rating Band, while the 26,60% transmits from R1 to R2 Rating Band etc.

Table 2. PIT Migration Matrix [11]

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	71,91%	26,60%	1,12%	0,11%	0,02%	0,00%	0,10%	0,10%	0,05%
R2	2,56%	81,78%	10,54%	4,33%	0,16%	0,03%	0,08%	0,36%	0,16%
R3	0,02%	6,46%	87,09%	2,95%	2,13%	0,09%	0,03%	0,75%	0,48%
R4	0,01%	3,37%	5,08%	86,23%	2,21%	0,97%	0,11%	1,17%	0,86%
R5	0,00%	0,19%	4,85%	7,00%	83,32%	1,04%	1,06%	1,51%	1,03%
R6	0,00%	0,15%	0,80%	6,01%	8,07%	77,41%	4,65%	1,68%	1,24%
R7	0,03%	0,43%	0,22%	1,45%	6,34%	4,23%	84,23%	1,78%	1,29%
R8	0,08%	2,21%	11,75%	15,89%	11,22%	4,65%	4,74%	22,70%	26,76%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Source: Created by the Author

Removal of the Macroeconomic Effect

For the purpose of deriving the TTC Matrices, the studied matrices are subject to Macroeconomic Past Effect Removal.

Under this framework, the identified approach involves the application of Merton and Vesical Models, as these have been explicitly referred and presented above. The approach is relied on the assumption that the credit quality of a position Yi is conditioned by the Macroeconomic Situation, which is synthesized by Z index.

$$Y_i = \beta_i * Z + \alpha_i * \varepsilon_i \quad (99)$$

where Yi and Z ~ Normal Distribution

The Default Rate conditioned to a given case PD_{PIT} is related to the Average Default Rate PD_{TTC} through the equation:

$$PD_{PIT(t)} = \Phi\left(\frac{\Phi^{-1}(PD_{TTc} - \sqrt{R} * Z_t}{\sqrt{1-\rho}}\right) \quad (100)$$

where ρ is the correlation between two positions. It is depending on the same factor of Credit Quality and it is estimated as following:

$$\rho = \frac{var\,(Z_t)}{1 + var\,(Z_t)} \quad (101)$$

[11] This migration matrix is a "dummy", due to lack of publicly available data. It has been constructed using the two following principals: a) stationarity of statuses at the diagonal of the matrix and b) increasing default rate as the rating deteriorates. Matrices are squared, not symmetric and the highest transition probabilities are along the diagonal. A transition probability has two subscripts. The first one is for the initial state and the other one is for the final credit state. The matrices have the Markov properties: a) time invariance, b) T relies only on the credit states, c) T does not change with time.

The dependence on the macroeconomic scenario is summarized by the indicator Z, corresponding to the portfolio average distance to default conditioned to the specific case identified.

Z can be easily extracted by the above equation Thus:

$$Z_t = \frac{\Phi^{-1}(PD_{TTC}) - \Phi^{-1}(PD_{PIT})\sqrt{1-\rho}}{\sqrt{\rho}} \quad (102)$$

Through the Vasicek formula, the macroeconomic effect can be easily removed from observed matrices, in order to obtain TTC matrices. The following operating steps can be applied:

1st Step: Transformation of PIT matrices in cumulative PIT matrices.
Through this transformation, each element of the array corresponds to the probability of remaining in the starting class or worsen their status:

$$F_{PiT}\,(x \rightarrow y) = \sum_{z=L}^{y} P_{PIT}(x \rightarrow z) \quad (103)$$

$F(x \rightarrow y)$ is the probability of migrating from class to class x y or worse $P_{PiT}(x \rightarrow y)$ is the probability of migrating from class to class x y.

2nd Step: Application of Vasicek Methodology to in each cell of the cumulative PIT matrix, in order to obtain the cumulative TTC matrix:

$$F(x \rightarrow y) = \Phi(\Phi{-1}(FPiT(x \rightarrow y))\ \sqrt{1-R} - \sqrt{R}\,Z) \quad (104)$$

3rd Step: The TTC matrix can be derived from the cumulative TTC matrix:

$$P(x \rightarrow y) = F_{TTC}(x \rightarrow y) - F_{TTC}(x \rightarrow\ (y-1)) \quad (105)$$

Where $P(x \rightarrow y)$ corresponds to the TTC probability of migrating from class to class x y.

Table 3. PIT Migration Matrices with Macroeconomic effect Removal

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	71,91%	26,60%	1,12%	0,11%	0,02%	0,00%	0,10%	0,10%	0,05%
R2	2,56%	81,78%	10,54%	4,33%	0,16%	0,03%	0,08%	0,36%	0,16%
R3	0,02%	6,46%	87,09%	2,95%	2,13%	0,09%	0,03%	0,75%	0,48%
R4	0,01%	3,37%	5,08%	86,23%	2,21%	0,97%	0,11%	1,17%	0,86%
R5	0,00%	0,19%	4,85%	7,00%	83,32%	1,04%	1,06%	1,51%	1,03%
R6	0,00%	0,15%	0,80%	6,01%	8,07%	77,41%	4,65%	1,68%	1,24%
R7	0,03%	0,43%	0,22%	1,45%	6,34%	4,23%	84,23%	1,78%	1,29%
R8	0,08%	2,21%	11,75%	15,89%	11,22%	4,65%	4,74%	22,70%	26,76%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Source: Created it by Author

Estimation of TTC Matrix

The TTC Matrices is developed as a simple average of the annual matrices with macroeconomic effect removed, using the formula:

$PD_{TTC} = \text{mean}\ (PD_{\ TTC})$ (106)

Table 4. TTC Matrix

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Source: Created it by Author

Estimation of Forward-Looking Elements

Through the application of estimated models, it is therefore obtained a prospective default rate as a function of a linear combination of the macro-economic variables:

$$DR_t \ = \ \beta_o \ + \ \sum_{j=1}^{n} \beta_j \ x_{jt\,W} \quad (107)$$

Following, the estimated default rate is transformed into the Z parameter using the aforementioned equation.

$$Z_{t+n} \ = \frac{\Phi^{-1}(PD_{TTC}) - \Phi^{-1}(PD_{PIT\,(t+n)})\sqrt{1-\rho}}{\sqrt{\rho}} \quad (108)$$

<u>Conditioning of the TTC Matrices</u>

For each segment identified the TTC matrices are built. These matrices are the starting point for identifying the PIT forward looking matrices on the basis of the scenario identified. Projected matrices are obtained through the following procedure:

1. Transformation of TTC matrices in cumulative TTC matrix. Through this transformation, each element of the array corresponds to the probability of remaining in the starting class or worsen their status

$$F_{TTC}\ (x \to y) \ = \ \sum_{Z=L}^{y} P_{TTC}(x \to z) \quad (109)$$

where:

$F_{TTC}\ (x \to y)$ is the probability of migrating from class to class x y or worse and
P_{TTC} is the probability of migrating from class x to class y.
L is the worst rating class

2. Following the calculation of the cumulative probability, the Vasicek approach is applied to each cell of the cumulative TTC matrix, in order to derive the forward looking PIT cumulative migrations:

$$F_{PIT}\,(x \to y) \ = \Phi\left(\frac{\Phi^{-1}(F_{TTC}\,(x \to y)\,) - \sqrt{R}\,Z}{\sqrt{1-R}}\ \right) \quad (110)$$

Where $F_{PIT}\,(x \to y)$ is the Point in Time probability of migration from class x to y or worse.

3. The PIT transition matrix is calculated from the cumulative PIT using the following formula.

$$P_{PIT}\ (x \to y) \ = \ F_{PIT}\,(x \to y) \ - F_{PIT}\,(x \to (y-1))$$

where $P_{PIT}\ (x \to y)$ corresponds to the probability of migration from rating class x to rating class y and forms the cells of the Forward-Looking PIT transition matrix.

Based on table 4, which consists the starting point for identifying the PIT Forward Looking Matrices on the basis of the scenario identified, we estimate the conditioned PIT Matrices (Table 5), using the equation below.

$$PD_{PIT_{ijT}} = \Phi\left[\frac{\Phi^{-1}(PD_{TTC_{idef}} + \ldots + PDTTCij) + \sqrt{R} * ZT}{\sqrt{1-R}}\right] - PDPITidefT + \ldots + PDPITijT$$

Table 5. PIT Matrix

Baseline scenario

Conditioned PIT 2018

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,09%	11,81%	1,64%	0,23%	0,09%	0,04%	0,03%	0,05%	0,02%
R2	27,16%	63,54%	7,60%	0,90%	0,35%	0,16%	0,13%	0,10%	0,05%
R3	5,65%	22,98%	66,74%	2,27%	1,05%	0,53%	0,30%	0,32%	0,15%
R4	2,76%	14,79%	21,44%	54,60%	2,77%	1,32%	1,16%	0,87%	0,30%
R5	1,94%	5,26%	25,42%	14,52%	43,25%	4,35%	3,43%	1,36%	0,49%
R6	1,40%	2,89%	16,29%	19,56%	10,60%	39,42%	7,14%	1,89%	0,81%
R7	1,28%	5,07%	17,43%	11,85%	13,00%	7,38%	38,69%	3,70%	1,60%
R8	1,19%	7,41%	17,93%	13,48%	7,89%	4,71%	7,33%	25,03%	15,03%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Conditioned PIT 2019

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,22%	11,71%	1,62%	0,22%	0,09%	0,04%	0,03%	0,04%	0,02%
R2	27,36%	63,44%	7,53%	0,89%	0,34%	0,16%	0,13%	0,10%	0,05%
R3	5,71%	23,11%	66,60%	2,25%	1,04%	0,52%	0,30%	0,32%	0,15%
R4	2,80%	14,90%	21,51%	54,45%	2,74%	1,31%	1,14%	0,85%	0,29%
R5	1,97%	5,31%	25,55%	14,54%	43,12%	4,31%	3,39%	1,34%	0,48%
R6	1,42%	2,92%	16,40%	19,62%	10,61%	39,29%	7,08%	1,87%	0,80%
R7	1,30%	5,12%	17,54%	11,88%	13,01%	7,38%	38,52%	3,66%	1,57%
R8	1,20%	7,48%	18,03%	13,51%	7,90%	4,71%	7,33%	24,94%	14,90%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Conditioned PIT 2020

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,27%	11,68%	1,62%	0,22%	0,08%	0,04%	0,03%	0,04%	0,02%
R2	27,42%	63,41%	7,51%	0,89%	0,34%	0,16%	0,13%	0,10%	0,05%
R3	5,74%	23,16%	66,55%	2,24%	1,03%	0,52%	0,29%	0,32%	0,15%
R4	2,81%	14,94%	21,53%	54,40%	2,73%	1,30%	1,14%	0,85%	0,29%
R5	1,98%	5,33%	25,59%	14,55%	43,07%	4,30%	3,38%	1,33%	0,47%
R6	1,43%	2,93%	16,44%	19,64%	10,61%	39,25%	7,06%	1,86%	0,79%
R7	1,31%	5,14%	17,58%	11,90%	13,02%	7,38%	38,47%	3,65%	1,56%
R8	1,21%	7,51%	18,07%	13,53%	7,90%	4,71%	7,32%	24,91%	14,85%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Source: Created it by Author

With regard to the forecast period over which there are no reliable estimates of macroeconomic variables, the PIT matrix adopted correspond to the average Through the Cycle matrix. The determination of the term structure will therefore be based on the Markov chain approach through the concatenation of the identified forward looking matrices:

TTC Matrices for 2019 are created as follows:

$$M_{ij}^{FDR}\ (2016)\ X\ M_{ij}^{FDR}\ (2017)\ X\ M_{ij}^{FDR}\ (2018)\ = M_{ij}^{3Y}$$

Table 6: Matrix Concatenation process

	2018 Default	2019 Default	2020 Default	2021 Default	2022 Default	2023 Default	2024 Default	2025 Default	2026 Default	2027 Default	2028 Default	2029 Default	2030 Default	2031 Default	2032 Default
R1	0,02%	0,05%	0,09%	0,45%	0,95%	1,55%	2,22%	2,93%	3,67%	4,43%	5,20%	5,98%	6,76%	7,54%	8,33%
R2	0,05%	0,12%	0,20%	0,69%	1,31%	2,01%	2,75%	3,52%	4,30%	5,08%	5,87%	6,67%	7,46%	8,24%	9,02%
R3	0,15%	0,33%	0,50%	1,26%	2,13%	3,03%	3,91%	4,78%	5,63%	6,47%	7,30%	8,11%	8,91%	9,70%	10,48%
R4	0,30%	0,67%	1,00%	2,07%	3,20%	4,29%	5,30%	6,26%	7,17%	8,05%	8,89%	9,71%	10,52%	11,31%	12,09%
R5	0,49%	1,07%	1,55%	2,91%	4,26%	5,50%	6,61%	7,63%	8,58%	9,48%	10,34%	11,17%	11,97%	12,76%	13,53%
R6	0,81%	1,65%	2,32%	3,94%	5,48%	6,83%	8,02%	9,08%	10,06%	10,97%	11,84%	12,66%	13,46%	14,24%	15,01%
R7	1,60%	2,94%	3,82%	5,55%	7,10%	8,43%	9,60%	10,64%	11,59%	12,48%	13,33%	14,14%	14,92%	15,69%	16,44%
R8	15,03%	19,02%	20,28%	21,71%	22,88%	23,88%	24,78%	25,60%	26,36%	27,08%	27,77%	28,44%	29,09%	29,72%	30,34%
Default	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%

Source: Created by the Author

Lifetime PD Chart

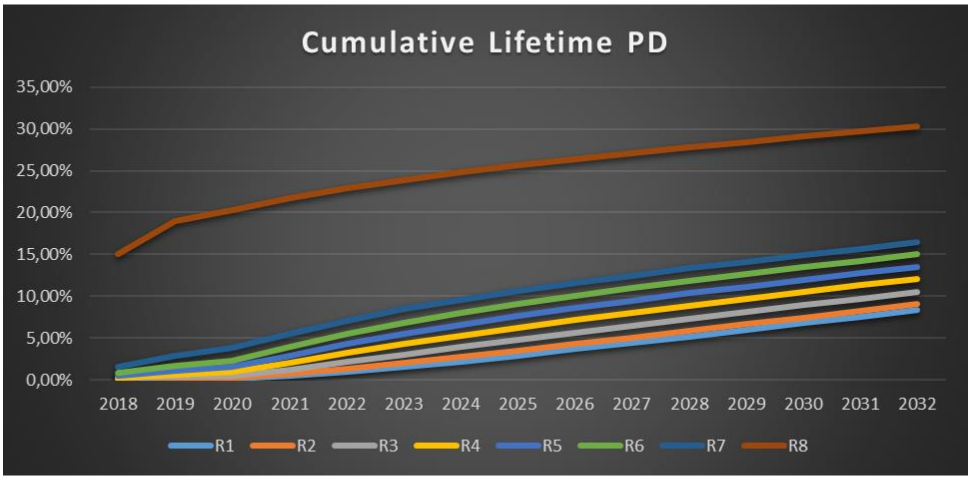


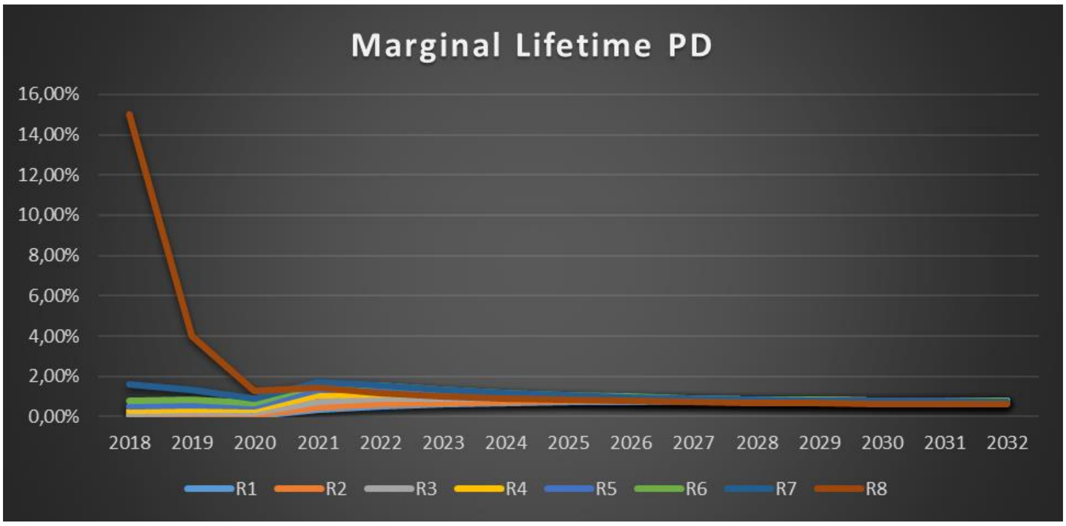
Chart 9: Lifetime PD Chart
Source: Created it by Author

Given that the previous matrix and chart include the percentage of the obligators that they have already defaulted, we proceed with removing this percentage and providing only the percentage of obligators that belong to each rating class.

Table 7. Marginal Lifetime PD

	2018 Default	2019 Default	2020 Default	2021 Default	2022 Default	2023 Default	2024 Default	2025 Default	2026 Default	2027 Default	2028 Default	2029 Default	2030 Default	2031 Default	2032 Default
R1	0,02%	0,03%	0,04%	0,35%	0,51%	0,60%	0,66%	0,71%	0,74%	0,76%	0,77%	0,78%	0,78%	0,78%	0,78%
R2	0,05%	0,07%	0,08%	0,48%	0,63%	0,70%	0,74%	0,76%	0,78%	0,79%	0,79%	0,79%	0,79%	0,79%	0,78%
R3	0,15%	0,18%	0,17%	0,76%	0,88%	0,89%	0,89%	0,87%	0,85%	0,84%	0,82%	0,81%	0,80%	0,79%	0,78%
R4	0,30%	0,37%	0,33%	1,07%	1,14%	1,08%	1,02%	0,96%	0,91%	0,87%	0,85%	0,82%	0,80%	0,79%	0,78%
R5	0,49%	0,58%	0,49%	1,35%	1,36%	1,23%	1,11%	1,02%	0,95%	0,90%	0,86%	0,83%	0,81%	0,79%	0,77%
R6	0,81%	0,84%	0,66%	1,62%	1,54%	1,35%	1,19%	1,06%	0,98%	0,91%	0,86%	0,83%	0,80%	0,78%	0,76%
R7	1,60%	1,34%	0,88%	1,73%	1,55%	1,33%	1,16%	1,04%	0,95%	0,89%	0,85%	0,81%	0,79%	0,77%	0,75%
R8	15,03%	3,99%	1,26%	1,43%	1,17%	1,01%	0,90%	0,82%	0,76%	0,72%	0,69%	0,67%	0,65%	0,63%	0,62%
Default	100,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Source: Created it by Author



6. Conclusions

To summarize the points, which are have been explicitly presented in this thesis, we have to point out the analytical reference to the relevant international literature that is associated with credit risk and in particular with Probabilities of Default. We presented the most significant models, in the area of Credit Risk, starting from the fundamental model of Merton (1974). Then, we presented the KMV Model (1984), Vasicek's theory (1976 & 2002) and many of the recent academic references in the field of credit risk such as Aguais et al. (2008) and Petrov et al. (2012).

Moreover, we examined the IFRS framework, highlighting IFRS 9, which was entered into force from the 1st of January 2018 in Greece, introducing the three stages, and the concept of Forward-Looking Probability of Default.

Taking all the above into consideration, we proposed a practical approach which aims to provide the usage of migration matrices for the identification of term structures of Probabilities of Defaults, using macro – adjusted historical rating matrices. This approach gives the advantage to the FIs to include and monitor Forward-Looking aspects not only for future Probability of Default outcomes, but also on single rating migrations as well as to define future default rates that also incorporate probability of migrating through performing rating grades.

Particularly, we leveraged methods linking migration matrices to macro states (z-factor, removal of macro history) and re-application (of macro forecasts). We utilized standard econometric techniques in order to identify the significant (with respect to PD evolution) macro variables creating a macro model, proving that GDP, Inflation and Unemployment Rate constitute significant macroeconomic variables.

Analytically, we proved that default rates are negatively linked with Inflation and GDP. Namely, when Inflation and GDP increase, default rates decrease. Consequently, when Inflation and GDP decrease, default rates increase. Under economic aspect, this means that when economy improves, default rates decreasing, reflecting the economic improvement.

On the other hand, the relation between Unemployment and default rates was proved to be positive. This means that when Unemployment Rate increases, similarly default rates increase or in other words when economy degrades, default rates increase, reflecting in this way the bad economic climate.

Taking into consideration these significant macroeconomic variables, we relied on the actual Greek Banking system historical default rate data and combined them with plausible dummy migration matrices we applied all this methodology on ECB's Stress Test used macro forecasts for the Greek Economy. Next, based on all the above, we estimated illustrative Lifetime PD curves, cumulative and marginal per each rating band, as per the dummy rating matrix we worked on.

The results of the above application point out that the future default rates are expected to decrease due to improved estimates of the economic fundamentals of the country.

Finally, the methodology presents an indicative way of incorporating the Forward -Looking Information of the progress of the economy into the estimation of future default rates by credit rating band. This approach can be applied in practice by banks not only in the estimation of Lifetime PD Curves but also in Lifetime Forecast for loans of Stage 2, during the classification under the aspect of IFRS 9.

6.1 Further Research

Further research could also be carried out in the area of relevant econometric models. For instance, the relation between default rates and macroeconomic variables could be examined using the Bayesian Model Average Model. This is a method, which is able to handle the bound of short dataset of default rates. Moreover, it provides the opportunity to perform multivariate modeling containing all potential predictor using different weight. Another component of further research is the choice of independent variables, for example we can include variables such as the balance of goods and services and the balance of current account.

References

Aguais S., Forest Jr. L. R., King L., Lennon C and Lordkipanidze B. (2008), "Designing and Implementing a Basell II compliant PIT - TTC rating Framework In the Basel Handbook", 2nd edn., p. 267-297, Risk Books, London.

Altman E., Resti A. and Sironi A. (2004), "Default Recovery Rates in Credit Risk Modeling: A Revie of the Literature and Empirical Evidence", Economic Notes, Vol. 33, No. 2, pp. 183 – 208.

Anthony Saunders and Marcia Millon Cornet, 2014, "Financial Institution Management – A Risk Approach", McGRAW – HILL, 8th Edition International Edition.

Asberg P. and Shahnazarian H. (2008), "Macroeconomic Impact on Expected Default Frequency", Sveriges Riksbank, Working Paper No. 219.

Athanasoglou P., Brissimis S and Delis M. (2006), "Bank-Specific, Industry -Specific and Macroeconomic Determinants of Bank Profitability", Journal of International Financial Markets, Institutions & Money, Vol.18, pp. 121 - 136

Bangia A., Diebold F. and Schuermann T. (2000), "Rating Migration and the Business Cycle with Application to Credit Portfolio Stress Testing", Penn Institute for Economic Research, Working Paper.

Basel Committee on Bank Supervision (2000), "Range of Practice in Banks? Internal Rating Systems", Discussion Paper January, Basel, Switzerland

Basel Committee on Bank Supervision (2001, January), "The New Basel Capital Accord", Report, Bank International Settlements, Basel, Switzerland.

Basel Committee on Bank Supervision (2005), "Amendment to the Capital Accord to incorporate Market Risks", Bank International Settlements, Basel, Switzerland.

Beerbaum D. (2015), "Significant Increase in Credit Risk Accounting to IFRS 9: Implications for Financial Institutions", International Journal of Economics & Management Science, Vol. 4, No. 9

Belkin B., Forest L. and Suchower S. (1998b), "A One-Parameter Representantion of Credit Risk and Transition Matrices", CreditMetrics Monitor, 3rd Quarter, pp. 46-56

Beneish M., Miller B. and Yohn T. (2015), "Macroeconomic Evidence on the Impact of Mandatory IFRS Adoption on Equity and Debt Markets", Journal Account, Vol. 34, pp. 1-27

Bharath A. T. and Shumway T. (2008), “Forecasting Default with Merton Distance to Default Model”, The Review of Finance Studies, Vol. 21, No. 3, pp. 1339 – 1369.

Black F. and Schole M. (1973), “The Pricing of Option and Corporate Liabilities”, Journal of Political Economy, Vol. 81, No. 3, pp. 637 - 654

CreditMetrics Manual (1997), JP Morgan.

Chan-Lau J and Sy A. (2006), “Distance -to-Default in Banking: A Bridge Too Far?”, International Monetary Fund, Working Paper No. 06/215.

Chatterjee S. (2015), “ Modelling Credit Risk”, Bank of England 2015.

Cihak, M. (2007). Introduction to Applied Stress Testing, International Monetary Fund, Working Paper No. 07/59.

Cohen B. H., Edwards G. A. (2007), “The New Era of Expected Credit Loss Provisioning”, BIS Quarterly Review.

Crosbie P. (2003), “Modeling Default Risk”, Moody’s | K.M.V

Duffies D. and Singleton K. (1999), “Modeling Term Structure of Defaultables Bonds”, The Review of Financial Studies, Vol. 12, No. 4, pp. 687-720.

ECB, “Adverse macro-financial scenario for the 2018 EU-wide banking sector stress test”, ECB Public 2018 January.

Farkas Z. (2017), “The Interaction of the IFRS 9 Expectedd Loss Approach with Supervisory Rules and Implications for Financial Stability ”,Accounting in Europe, Vol. 13, No.2, . 197-227.

Florian H., Hoerova M. and Holthausen C. (2009), “The Role of Counterparty Risk”, European Central Bank, Working Paper No.1126/09.

Geske R. (1979), “The Valuation of Compound Options”, Journal of Financial Economics, Vol. 7, No.1, pp. 63-81.

Gupton M., Finger C. and Batia, M. (1997), “CreditMetrics Technical Document”, JP Morgan.

Hodrick R and Prescott, E. C. (1997), “Postwar U.S. Business Cycles: An Empirical Investigation”, Journal of Money, Credit, and Banking No. 29, pp. 1-16.

International Accounting Standard 39 (2011), IASB London

International Financial Reporting Standards 9 (2014), IFRS Foundation

Jarrow, R., Lando, D. and Turnbull, S. (1997), "A Markov Model for the Term Structure of Credit Risk Spreads", Review of Financial Studies Vol.10, pp. 481-523.

Jarrow, R. and Turnbull, S. (1995). "Pricing derivatives on financial securities subject to credit risk", Journal of Finance Vol.50, pp. 53-85.

Jeffrey M. Wooldridge, 2009, "Introductory Econometrics A modern approach, South – Western

Jorge A., Chan L. and Amadou N.) 2015), " Distance to Default in Banking: A Brige Too Far?", IMF, Working Paper WP/06/215

Kaiser, R. and Maravall, A. (1999), "Estimation of the Business Cycle: A Modified Hodrick-Prescott Filter", Spanish Economic Review No.1 pp. 175-206.

Kealhofer S. (1998), "Portfolio Management of Default Risk", KMV Corporation, Revision 2.1.

Kealhofer S., Kurbat M (2002) "The Default Prediction Power of the Merton Approach, Relative to Debt Rating and Accounting Variables", KMV LLC, Revised 2002.

Kim, J. (1999). Conditioning the transition matrix. Special Report, Risk: Credit Risk (October).

Koopman S., Lucas A. and Klaassen P. (2005), "Empirical Credit Cycles and Capital Buffer formation", Journal of Banking and Finance, Vol.29, No.12, pp 3159 -3179

Leventis S., Dimitropoulos E. and Anandarajan A. (2001), "Loan Loss Provisions, Earnings Management and Capital Management under IFRS: The Case of EU Commercial Banks", Journal of Finance, Vol. 40, pp.103 – 122.

Louzis D., Vouldis A. ans Metaxas V. (2012), "Macroeconomic and Bank - Specific Determinants of Non- Performing Loans in Greece: A Comparative Study of Mortgage, Business and Consumer Loan Portfolio.", Journal of Banking and Finance, Vol.36. pp.1012-1027

Merton R. C. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rate", Journal of Finance, Vol. 29, No. 2, pp. 449 – 470.

Nelson C.R. and Siegel A.F(1987), "Parsimonious Modeling of Yield Curves", The Journal of Business Vol.60,pp. 473-489.

Nickell, P., Perraudin, W. and Varotto, S (2000), "Stability of rating transitions", Journal of Banking and Finance Vol. 24, pp. 203-22.

Papadopoulos G., Papadopoulos S., Sager T. (2016), "Credit Risk Stress Testing for EU15 Banks: A Model Combination Approach", Bank of Greece, Working Paper No. 203.

Petropoulos A., Siakoulis V., Mylonas D., Klamarias A. (2018), "A Combined Statistical Framework for Forecasting Default Rates of Greek Financial Institutions' Credit Portfolios", Bank of Greece, Working Paper No.243.

Petrov A., Carlehed M. (2012), "A Methodology for Point-in-Time – Through-the-Time Probability of Default Decomposition in Risk Classification System", Journal of Risk Model Validation, Vol. 6, No.3, pp. 3-25

Saunders A (2009), "Analytical methods for hedging systematic credit risk with linear factor portfolios. Journal of Economic Dynamics & Control", Vol. 33, pp. 37-52.

Skoglund J. (2016), "Credit Risk Term-Structures for Lifetime Impairment Forecasting: A Practical Guide", SAS Institute Inc. 2016

Skoglund J. and Chen W. (2016), "The Application of Credit Risk Models to Macroeconomic Scenario Analysis and Stress Testing", Journal of Credit Risk, Vol. 12, No.2

Vasicek O. (2002), "Loan portfolio value", Risk, pp.160-162

Yang B. H. & Du Z. (2015), "Stress Testing and Modeling of Rating Migration under the Vasicek Model Framework – Empirical Approach and Technical Implementation, Journal of Risk Model Validation, Vol. 9

William F Treacy & Mark Carey (2000), "Credit Risk Rating Systems at Large US Bank", Journal of Banking & Finanace Vol. 24, No.9, pp. 167-201

Wilson, T. (1997a), "Portfolio credit risk I", Risk Vol. 10, No.9, pp. 111-117.

Wilson, T. (1997b), "Portfolio credit risk II" Risk Vol. 10, No. 10, pp. 56-61.

Wooldridge, Jeffrey M. 2009, "Introductory Econometrics A modern approach, South – Western.

Zeytun S. and Cupta A. (2007), "A Comparative Study of the Vasicek and the CIR Model of the Short Rate", Institut Techno – und – Wirtschaftsmathematik, No. 124/2007.

Appendix

Application of Proposed Methodology.

This application is developing using the *Expected Values* of GDP and Inflation for 2018, 2019 and 2020, *based on the Official Report of ECB.*

Migration Matrices Calculation

PD PIT Matrices[12]

PIT 31-12-2011 a 31-12-2012

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	71,91%	26,60%	1,12%	0,11%	0,02%	0,00%	0,10%	0,10%	0,05%
R2	2,56%	81,78%	10,54%	4,33%	0,16%	0,03%	0,08%	0,36%	0,16%
R3	0,02%	6,46%	87,09%	2,95%	2,13%	0,09%	0,03%	0,75%	0,48%
R4	0,01%	3,37%	5,08%	86,23%	2,21%	0,97%	0,11%	1,17%	0,86%
R5	0,00%	0,19%	4,85%	7,00%	83,32%	1,04%	1,06%	1,51%	1,03%
R6	0,00%	0,15%	0,80%	6,01%	8,07%	77,41%	4,65%	1,68%	1,24%
R7	0,03%	0,43%	0,22%	1,45%	6,34%	4,23%	84,23%	1,78%	1,29%
R8	0,08%	2,21%	11,75%	15,89%	11,22%	4,65%	4,74%	22,70%	26,76%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

PIT 31-12-2012 a 31-12-2013

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	68,39%	28,90%	2,19%	0,24%	0,00%	0,02%	0,09%	0,14%	0,03%
R2	2,00%	76,91%	14,48%	5,39%	0,50%	0,02%	0,09%	0,41%	0,19%
R3	0,01%	5,55%	78,51%	10,90%	3,01%	0,17%	0,02%	1,12%	0,70%
R4	0,00%	3,09%	4,01%	79,93%	8,41%	1,34%	0,18%	1,87%	1,17%
R5	0,00%	0,08%	4,50%	6,78%	77,06%	4,66%	2,67%	2,57%	1,66%
R6	0,00%	0,02%	0,38%	5,06%	6,20%	72,96%	9,83%	3,38%	2,17%
R7	0,02%	0,21%	0,10%	0,66%	7,03%	6,28%	79,88%	3,79%	2,03%
R8	0,03%	2,30%	8,90%	10,79%	7,65%	2,58%	4,55%	23,85%	39,35%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

PIT 31-12-2013 a 31-12-2014

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	40,26%	46,25%	9,66%	2,32%	0,56%	0,00%	0,00%	0,88%	0,08%
R2	7,90%	60,89%	23,31%	4,13%	1,65%	0,63%	0,47%	0,56%	0,47%
R3	0,13%	10,56%	62,10%	17,64%	5,54%	1,16%	0,47%	1,40%	1,01%
R4	0,00%	2,17%	5,94%	57,28%	22,57%	4,39%	2,56%	2,30%	2,79%
R5	0,00%	0,07%	2,07%	2,56%	49,74%	22,64%	16,22%	2,68%	4,01%
R6	0,00%	0,03%	0,13%	1,88%	2,61%	41,59%	45,70%	2,15%	5,93%
R7	0,26%	3,34%	8,22%	3,63%	2,04%	3,45%	55,35%	13,13%	10,57%
R8	0,00%	0,95%	3,95%	7,56%	6,48%	2,91%	2,44%	16,03%	59,68%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

PIT 31-12-2014 a 31-12-2015

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	75,01%	20,86%	2,88%	0,46%	0,52%	0,02%	0,14%	0,00%	0,11%
R2	44,44%	32,72%	16,91%	1,68%	1,98%	1,11%	0,69%	0,17%	0,31%
R3	12,70%	20,38%	52,91%	2,21%	3,95%	4,10%	1,57%	1,22%	0,95%
R4	4,72%	15,11%	53,51%	8,76%	4,89%	4,04%	4,00%	3,55%	1,41%
R5	2,44%	7,21%	35,18%	31,87%	3,04%	5,38%	8,04%	5,16%	1,67%
R6	0,82%	2,42%	32,06%	34,32%	7,76%	4,00%	10,59%	5,93%	2,10%
R7	0,12%	1,36%	37,36%	28,22%	13,48%	2,37%	9,97%	5,55%	1,57%
R8	0,77%	1,49%	5,43%	4,62%	4,23%	3,89%	8,70%	36,23%	34,63%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

[12]These migration matrices are "dummy", due to lack of publicly available data. They have been constructed using the two following principals: a) stationarity of statuses at the diagonal of the matrix and b) increasing default rate as the rating deteriorates. Matrices are squared, not symmetric and the highest transition probabilities are along the diagonal. A transition probability has two subscripts. The first one is for the initial state and the other one is for the final credit state. The matrices have the Markov properties: a) time invariance, b) T relies only on the credit states, c) T does not change with time.

PIT 31-12-2015 a 31-12-2016

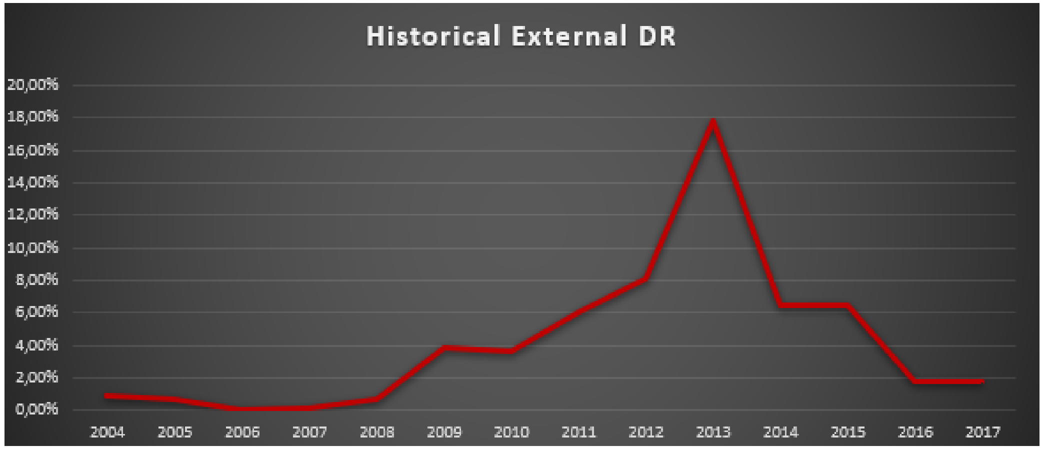
	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,42%	14,95%	4,99%	0,65%	0,31%	0,24%	0,18%	0,15%	0,10%
R2	12,75%	65,47%	16,85%	1,97%	1,13%	0,59%	0,57%	0,43%	0,23%
R3	2,08%	5,66%	82,57%	2,82%	2,21%	1,67%	1,24%	1,18%	0,57%
R4	1,30%	2,51%	12,15%	69,39%	2,42%	3,80%	4,28%	3,20%	0,95%
R5	1,52%	3,62%	19,10%	8,39%	44,06%	7,95%	8,72%	4,95%	1,68%
R6	1,98%	2,39%	15,34%	11,64%	8,35%	39,90%	10,42%	7,13%	2,85%
R7	1,00%	1,77%	10,34%	10,46%	7,00%	8,12%	43,75%	11,40%	6,15%
R8	0,34%	0,85%	9,05%	6,20%	4,08%	5,62%	10,76%	40,62%	22,48%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

PIT 31-12-2016 a 31-12-2017

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,45%	14,34%	5,40%	0,82%	0,33%	0,24%	0,15%	0,17%	0,10%
R2	13,32%	65,40%	16,55%	2,00%	1,04%	0,56%	0,50%	0,42%	0,21%
R3	2,18%	5,96%	82,57%	2,97%	2,07%	1,47%	1,12%	1,11%	0,54%
R4	1,75%	2,60%	12,75%	69,29%	2,36%	3,55%	3,95%	2,80%	0,96%
R5	1,92%	4,05%	20,02%	8,14%	43,54%	7,70%	8,65%	4,41%	1,56%
R6	2,45%	2,75%	17,04%	11,52%	8,35%	38,96%	9,59%	6,60%	2,73%
R7	1,41%	1,93%	12,19%	10,62%	6,88%	7,61%	42,04%	11,65%	5,67%
R8	0,40%	0,91%	9,56%	7,03%	4,21%	5,25%	10,87%	42,18%	19,58%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

<u>Parameters Calculation</u>

The next step is to calculate the parameters. In particular, the main goal is to estimate the Z factor, which will help us to neutralize the above PD PIT Matrices from the effect of the Past Macroeconomic Components.



$$Z = \frac{-\sqrt{1-R} * \Phi{-}1(PDPIT) + \Phi{-}1(PDTTC)}{\sqrt{R}}$$

$$x = \Phi^{-1}(DR_{series})$$

$$R = \frac{Var(x)}{1+Var(x)}$$

TTC DR	phi_1_DR_TTC	phi_1_DR_PIT	Var_DR_ext	R	rad_R	rad_1_R
4,16%	-1,731973703	-2,35345	0,45016399	31,04%	0,55716	0,83041
		-2,45726				
		-3,35279				
		-3,09023				
		-2,45726				
		-1,77076				
		-1,79408				
		-1,55059				
		-1,40038				
		-0,92340				
		-1,51647				
		-1,51965				
		-2,10321				
		-2,10321				

Anno	Z
2004	-0,39908
2005	-0,55381
2006	-1,88854
2007	-1,49721
2008	-0,55381
2009	0,46938
2010	0,43462
2011	0,79754
2012	1,02142
2013	1,73233
2014	0,84839
2015	0,84366
2016	-0,02611
2017	-0,02611

Neutralization of Economic Factor

Having estimated the Z factor for each year, we proceed with the assessing of the neutralization TTC Matrices. In order to estimate the neutralization TTC matrices, we use the following equation:

$$PD_{TTC_{ijT}} = \Phi\left[\sqrt{1-R} * \Phi^{-1}(PD_{PIT_{idef}} + \ldots + PDPITij) + \sqrt{R} * ZT\right] - PDTTCidefT + \ldots + PDTTCijT$$

TTC 31-12-2011 a 31-12-2012

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	85,33%	13,78%	0,62%	0,07%	0,02%	0,00%	0,06%	0,07%	0,05%
R2	14,69%	77,33%	5,27%	2,21%	0,09%	0,02%	0,04%	0,22%	0,13%
R3	0,96%	23,56%	72,12%	1,46%	1,09%	0,05%	0,01%	0,42%	0,32%
R4	0,40%	16,72%	11,21%	68,86%	1,10%	0,49%	0,05%	0,62%	0,54%
R5	0,16%	3,18%	18,02%	12,91%	63,25%	0,52%	0,53%	0,79%	0,64%
R6	0,00%	2,86%	5,53%	17,10%	13,08%	57,51%	2,30%	0,86%	0,75%
R7	1,04%	4,46%	1,43%	6,33%	15,10%	6,90%	63,04%	0,91%	0,78%
R8	1,94%	11,84%	23,42%	18,05%	9,68%	3,56%	3,42%	14,17%	13,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

TTC 31-12-2012 a 31-12-2013

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	91,35%	8,13%	0,42%	0,04%	0,00%	0,00%	0,02%	0,03%	0,01%
R2	22,98%	71,89%	3,80%	1,11%	0,09%	0,00%	0,02%	0,08%	0,04%
R3	1,76%	34,29%	60,30%	2,65%	0,61%	0,03%	0,00%	0,21%	0,13%
R4	0,69%	27,22%	12,07%	57,17%	1,96%	0,27%	0,04%	0,36%	0,22%
R5	0,00%	4,98%	28,19%	15,34%	48,98%	1,10%	0,58%	0,51%	0,31%
R6	0,00%	2,40%	8,34%	25,00%	13,26%	47,50%	2,39%	0,69%	0,41%
R7	2,41%	5,80%	1,53%	6,83%	25,48%	11,10%	45,69%	0,78%	0,39%
R8	3,17%	21,44%	23,68%	14,44%	7,18%	2,08%	3,35%	12,95%	11,71%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

TTC 31-12-2013 a 31-12-2014

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	60,56%	31,20%	5,64%	1,44%	0,38%	0,00%	0,00%	0,68%	0,10%
R2	24,21%	56,83%	13,96%	2,43%	1,02%	0,40%	0,32%	0,40%	0,42%
R3	2,11%	26,67%	54,77%	10,48%	3,25%	0,70%	0,29%	0,91%	0,82%
R4	0,16%	11,25%	13,16%	54,28%	13,75%	2,55%	1,50%	1,39%	1,97%
R5	0,00%	1,41%	9,91%	6,61%	53,47%	14,78%	9,52%	1,59%	2,71%
R6	0,00%	0,84%	1,52%	8,62%	6,81%	47,52%	29,58%	1,26%	3,84%
R7	3,22%	12,14%	15,13%	5,00%	2,58%	4,06%	43,58%	7,75%	6,55%
R8	0,00%	7,00%	11,37%	13,04%	8,41%	3,33%	2,63%	14,83%	39,39%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

TTC 31-12-2014 a 31-12-2015

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	84,86%	12,34%	1,81%	0,32%	0,39%	0,01%	0,13%	0,00%	0,13%
R2	63,83%	22,34%	9,96%	1,00%	1,22%	0,73%	0,49%	0,14%	0,31%
R3	31,66%	22,59%	37,17%	1,29%	2,30%	2,43%	0,97%	0,81%	0,78%
R4	17,92%	22,85%	43,07%	5,29%	2,87%	2,35%	2,35%	2,20%	1,09%
R5	12,19%	14,88%	37,07%	21,75%	1,83%	3,19%	4,69%	3,13%	1,27%
R6	6,39%	7,99%	41,86%	25,27%	4,80%	2,40%	6,20%	3,56%	1,54%
R7	2,04%	7,05%	50,19%	20,59%	8,34%	1,41%	5,83%	3,37%	1,20%
R8	6,14%	5,50%	12,12%	7,35%	5,69%	4,65%	9,11%	28,21%	21,24%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

TTC 31-12-2015 a 31-12-2016

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	73,84%	15,31%	6,92%	1,21%	0,64%	0,57%	0,49%	0,49%	0,53%
R2	16,86%	56,80%	17,60%	2,71%	1,77%	1,05%	1,16%	1,09%	0,96%
R3	4,39%	7,19%	74,07%	3,21%	2,75%	2,31%	1,95%	2,27%	1,85%
R4	3,12%	3,73%	13,15%	60,47%	2,46%	4,11%	5,25%	5,06%	2,65%
R5	3,51%	5,04%	19,06%	7,28%	37,37%	7,52%	9,37%	6,85%	4,00%
R6	4,24%	3,34%	15,94%	10,28%	7,06%	34,08%	10,39%	8,80%	5,87%
R7	2,58%	2,84%	11,80%	9,79%	6,12%	6,88%	37,62%	12,09%	10,28%
R8	1,19%	1,74%	11,36%	6,17%	3,80%	5,02%	9,23%	34,51%	26,98%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

TTC 31-12-2016 a 31-12-2017

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	73,87%	14,59%	7,32%	1,49%	0,69%	0,56%	0,40%	0,56%	0,52%
R2	17,43%	56,69%	17,40%	2,79%	1,67%	1,00%	1,02%	1,08%	0,91%
R3	4,55%	7,48%	74,06%	3,44%	2,64%	2,08%	1,80%	2,18%	1,78%
R4	3,87%	3,68%	13,53%	60,44%	2,45%	3,91%	4,95%	4,49%	2,68%
R5	4,15%	5,41%	19,60%	7,01%	36,99%	7,34%	9,45%	6,25%	3,81%
R6	4,96%	3,67%	17,19%	10,04%	7,01%	33,46%	9,72%	8,27%	5,68%
R7	3,32%	2,90%	13,37%	9,71%	5,94%	6,41%	36,22%	12,46%	9,68%
R8	1,34%	1,80%	11,81%	6,89%	3,86%	4,65%	9,27%	36,08%	24,30%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

TTC matrix calculation

The following step is to create a final matrix, which is derived as the simple average of the above annual matrices.

$$PD_{TTC_{ij}} = mean(PD_{TTCij\,T})$$

TTC MATRIX

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Macroeconomic Factors estimated by Linear Regression

Using Expected Values of GDP and Inflation for 2018, 2019 and 2020, based on the Official Report of ECB and through the application of estimated models, it is therefore obtained a prospective default rate as a function of a linear combination of the macroeconomic variables: $DR_t = \beta_0 + \sum_{j=1}^{n} \beta_j x_{jt} w$. Then, the estimated Default Rate is transformed into the factor Z, using the following equation: $Z = \frac{-\sqrt{1-R} * \Phi^{-1}(PD_{PIT}) + \Phi^{-1}(PD_{PIT})}{\sqrt{R}}$

Macroeconomic Factors Estimated by Linear Regression

Target		intercept	GDPg (t)	ΔINFL (t)	ΔUnempl (t)
ΔDR (DR(t)-DR(t-1)	Coefficients	-0,014	-0,116	-0,165	0,0177805

DR_2018	DR_TTC	hi_1_DR_TT	R	rad_R	rad_1_R
1,77%	4,16%	-1,7319737	31,04%	0,557156	0,830408

	Baseline scenario (ECB)				
	GDP growth %	ΔINFL %	ΔUnempl %	Conditioned ΔDR	Conditioned DR
2018	2,4	0,9	-2,1	-0,48%	1,294%
2019	2,5	1	-1,6	-0,50%	1,275%
2020	2,4	1,1	-1,7	-0,50%	1,268%

$$DR_{T+1} = DRT + \Delta DR_{T+1}$$

	Baseline scenario phi_1_DR_PIT	Baseline scenario Z
2018	-2,227963753	-0,2120499
2019	-2,233761999	-0,2206918
2020	-2,235795403	-0,2237225

$$Z = \frac{-\sqrt{1-R} * \Phi^{-1}(PD_{PIT}) + \Phi^{-1}(PD_{TTC})}{\sqrt{R}}$$

Using the above data and in particular, the values of Z for 2018,2019 and 2020, we rebuilt the TTC Matrix, which in this way absorbs the "macro-information" of the future years.

TTC MATRIX

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Projection of the TTC Matrix

The above matrix consists the starting point for identifying the PIT Forward Looking Matrices on the basis of the scenario identified.

$$PD_{PIT_{ijT}} = \Phi\left[\frac{\Phi^{-1}(PD_{TTC_{idef}} + \ldots + PDTTCij) + \sqrt{R} * ZT}{\sqrt{1-R}}\right] - PDPITidefT + \ldots + PDPITijT$$

Baseline scenario

Conditioned PIT 2018

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,09%	11,81%	1,64%	0,23%	0,09%	0,04%	0,03%	0,05%	0,02%
R2	27,16%	63,54%	7,60%	0,90%	0,35%	0,16%	0,13%	0,10%	0,05%
R3	5,65%	22,98%	66,74%	2,27%	1,05%	0,53%	0,30%	0,32%	0,15%
R4	2,76%	14,79%	21,44%	54,60%	2,77%	1,32%	1,16%	0,87%	0,30%
R5	1,94%	5,26%	25,42%	14,52%	43,25%	4,35%	3,43%	1,36%	0,49%
R6	1,40%	2,89%	16,29%	19,56%	10,60%	39,42%	7,14%	1,89%	0,81%
R7	1,28%	5,07%	17,43%	11,85%	13,00%	7,38%	38,69%	3,70%	1,60%
R8	1,19%	7,41%	17,93%	13,48%	7,89%	4,71%	7,33%	25,03%	15,03%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Conditioned PIT 2019

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,22%	11,71%	1,62%	0,22%	0,09%	0,04%	0,03%	0,04%	0,02%
R2	27,36%	63,44%	7,53%	0,89%	0,34%	0,16%	0,13%	0,10%	0,05%
R3	5,71%	23,11%	66,60%	2,25%	1,04%	0,52%	0,30%	0,32%	0,15%
R4	2,80%	14,90%	21,51%	54,45%	2,74%	1,31%	1,14%	0,85%	0,29%
R5	1,97%	5,31%	25,55%	14,54%	43,12%	4,31%	3,39%	1,34%	0,48%
R6	1,42%	2,92%	16,40%	19,62%	10,61%	39,29%	7,08%	1,87%	0,80%
R7	1,30%	5,12%	17,54%	11,88%	13,01%	7,38%	38,52%	3,66%	1,57%
R8	1,20%	7,48%	18,03%	13,51%	7,90%	4,71%	7,33%	24,94%	14,90%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Conditioned PIT 2020

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,27%	11,68%	1,62%	0,22%	0,08%	0,04%	0,03%	0,04%	0,02%
R2	27,42%	63,41%	7,51%	0,89%	0,34%	0,16%	0,13%	0,10%	0,05%
R3	5,74%	23,16%	66,55%	2,24%	1,03%	0,52%	0,29%	0,32%	0,15%
R4	2,81%	14,94%	21,53%	54,40%	2,73%	1,30%	1,14%	0,85%	0,29%
R5	1,98%	5,33%	25,59%	14,55%	43,07%	4,30%	3,38%	1,33%	0,47%
R6	1,43%	2,93%	16,44%	19,64%	10,61%	39,25%	7,06%	1,86%	0,79%
R7	1,31%	5,14%	17,58%	11,90%	13,02%	7,38%	38,47%	3,65%	1,56%
R8	1,21%	7,51%	18,07%	13,53%	7,90%	4,71%	7,32%	24,91%	14,85%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Markovian Chain - HP: first 3 years use of conditioned PIT migration matrices by baseline scenario, starting from 2021 use of TTC migration matrix

Conditioned PIT 2018 (baseline scenario)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,09%	11,81%	1,64%	0,23%	0,09%	0,04%	0,03%	0,05%	0,02%
R2	27,16%	63,54%	7,60%	0,90%	0,35%	0,16%	0,13%	0,10%	0,05%
R3	5,65%	22,98%	66,74%	2,27%	1,05%	0,53%	0,30%	0,32%	0,15%
R4	2,76%	14,79%	21,44%	54,60%	2,77%	1,32%	1,16%	0,87%	0,30%
R5	1,94%	5,26%	25,42%	14,52%	43,25%	4,35%	3,43%	1,36%	0,49%
R6	1,40%	2,89%	16,29%	19,56%	10,60%	39,42%	7,14%	1,89%	0,81%
R7	1,28%	5,07%	17,43%	11,85%	13,00%	7,38%	38,69%	3,70%	1,60%
R8	1,19%	7,41%	17,93%	13,48%	7,89%	4,71%	7,33%	25,03%	15,03%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Conditioned PIT 2019 (baseline scenario)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,22%	11,71%	1,62%	0,22%	0,09%	0,04%	0,03%	0,04%	0,02%
R2	27,36%	63,44%	7,53%	0,89%	0,34%	0,16%	0,13%	0,10%	0,05%
R3	5,71%	23,11%	66,60%	2,25%	1,04%	0,52%	0,30%	0,32%	0,15%
R4	2,80%	14,90%	21,51%	54,45%	2,74%	1,31%	1,14%	0,85%	0,29%
R5	1,97%	5,31%	25,55%	14,54%	43,12%	4,31%	3,39%	1,34%	0,48%
R6	1,42%	2,92%	16,40%	19,62%	10,61%	39,29%	7,08%	1,87%	0,80%
R7	1,30%	5,12%	17,54%	11,88%	13,01%	7,38%	38,52%	3,66%	1,57%
R8	1,20%	7,48%	18,03%	13,51%	7,90%	4,71%	7,33%	24,94%	14,90%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Conditioned PIT 2020 (baseline scenario)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	86,27%	11,68%	1,62%	0,22%	0,08%	0,04%	0,03%	0,04%	0,02%
R2	27,42%	63,41%	7,51%	0,89%	0,34%	0,16%	0,13%	0,10%	0,05%
R3	5,74%	23,16%	66,55%	2,24%	1,03%	0,52%	0,29%	0,32%	0,15%
R4	2,81%	14,94%	21,53%	54,40%	2,73%	1,30%	1,14%	0,85%	0,29%
R5	1,98%	5,33%	25,59%	14,55%	43,07%	4,30%	3,38%	1,33%	0,47%
R6	1,43%	2,93%	16,44%	19,64%	10,61%	39,25%	7,06%	1,86%	0,79%
R7	1,31%	5,14%	17,58%	11,90%	13,02%	7,38%	38,47%	3,65%	1,56%
R8	1,21%	7,51%	18,07%	13,53%	7,90%	4,71%	7,32%	24,91%	14,85%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2021 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2022 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2023 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2024 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2025 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2026 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2027 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2028 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2029 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2030 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2031 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2032 (TTC matrix)

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	78,30%	15,89%	3,79%	0,76%	0,35%	0,19%	0,18%	0,31%	0,22%
R2	26,67%	56,98%	11,33%	2,04%	0,98%	0,53%	0,51%	0,50%	0,46%
R3	7,57%	20,30%	62,08%	3,76%	2,11%	1,27%	0,84%	1,13%	0,95%
R4	4,36%	14,24%	17,70%	51,09%	4,10%	2,28%	2,36%	2,35%	1,53%
R5	3,33%	5,82%	21,97%	11,82%	40,32%	5,74%	5,69%	3,19%	2,12%
R6	2,60%	3,52%	15,06%	16,05%	8,67%	37,08%	10,10%	3,91%	3,02%
R7	2,43%	5,87%	15,57%	9,71%	10,59%	6,13%	38,66%	6,23%	4,81%
R8	2,30%	8,22%	15,62%	10,99%	6,44%	3,88%	6,17%	23,46%	22,92%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Cumulative PIT matrix

Following the calculation of the cumulative probability, the Vasicek Approach is applied to each cell of the cumulative TTC Matrix, in order to derive the Forward Looking PIT cumulative migrations:

$$F_{PIT}\,(x \to y) \;= \Phi(\frac{\Phi^{-1}(F_{TTC}\,(x\to y)\,) - \sqrt{R}\,Z}{\sqrt{1-R}}\;)$$

2019

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	77,57%	18,00%	3,47%	0,49%	0,19%	0,09%	0,07%	0,07%	0,05%
R2	41,27%	45,42%	10,64%	1,40%	0,54%	0,26%	0,21%	0,14%	0,12%
R3	15,07%	31,12%	47,23%	3,29%	1,41%	0,71%	0,47%	0,37%	0,33%
R4	9,28%	23,10%	28,46%	31,27%	3,33%	1,62%	1,40%	0,87%	0,67%
R5	5,95%	14,30%	33,09%	16,26%	20,34%	4,22%	3,44%	1,33%	1,07%
R6	4,36%	10,89%	26,06%	21,42%	10,55%	16,91%	6,32%	1,83%	1,65%
R7	4,73%	12,33%	26,55%	15,33%	12,24%	6,74%	16,33%	2,81%	2,94%
R8	5,07%	13,80%	24,01%	14,14%	7,41%	4,19%	5,48%	6,89%	19,02%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2020

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	72,07%	21,37%	5,11%	0,74%	0,28%	0,14%	0,11%	0,09%	0,09%
R2	48,73%	36,35%	11,70%	1,67%	0,64%	0,31%	0,25%	0,16%	0,20%
R3	24,39%	33,07%	35,35%	3,61%	1,47%	0,74%	0,53%	0,36%	0,50%
R4	16,97%	27,36%	29,08%	18,96%	3,09%	1,52%	1,30%	0,73%	1,00%
R5	11,93%	21,34%	33,44%	14,11%	10,60%	3,26%	2,71%	1,07%	1,55%
R6	9,40%	18,17%	29,77%	18,20%	8,20%	8,08%	4,45%	1,42%	2,32%
R7	10,00%	18,71%	29,59%	14,48%	9,08%	4,87%	7,65%	1,81%	3,82%
R8	10,29%	18,33%	24,93%	11,85%	5,58%	3,02%	3,41%	2,30%	20,28%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2021

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	62,57%	24,81%	8,57%	1,63%	0,74%	0,41%	0,38%	0,45%	0,45%
R2	48,85%	31,14%	13,77%	2,57%	1,16%	0,65%	0,58%	0,59%	0,69%
R3	30,84%	30,58%	27,83%	4,41%	2,04%	1,16%	0,98%	0,92%	1,26%
R4	23,80%	27,26%	26,38%	12,28%	3,28%	1,83%	1,74%	1,37%	2,07%
R5	18,71%	23,83%	29,53%	11,14%	6,45%	2,91%	2,79%	1,74%	2,91%
R6	15,88%	21,62%	28,05%	13,71%	6,15%	4,70%	3,88%	2,07%	3,94%
R7	16,35%	21,61%	27,63%	11,76%	6,44%	3,69%	4,78%	2,18%	5,55%
R8	15,75%	19,65%	22,61%	9,17%	4,25%	2,44%	2,68%	1,73%	21,71%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2022

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	56,38%	26,17%	11,14%	2,38%	1,12%	0,63%	0,61%	0,62%	0,95%
R2	47,79%	28,84%	14,92%	3,20%	1,50%	0,86%	0,82%	0,76%	1,31%
R3	34,74%	28,89%	23,61%	4,78%	2,26%	1,32%	1,22%	1,05%	2,13%
R4	28,67%	26,89%	24,02%	9,00%	3,16%	1,83%	1,82%	1,40%	3,20%
R5	24,12%	24,92%	26,28%	9,12%	4,64%	2,48%	2,51%	1,67%	4,26%
R6	21,39%	23,41%	25,88%	10,71%	4,85%	3,23%	3,15%	1,91%	5,48%
R7	21,65%	23,16%	25,36%	9,67%	4,90%	2,88%	3,37%	1,91%	7,10%
R8	20,00%	20,23%	20,48%	7,40%	3,42%	2,01%	2,17%	1,42%	22,88%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2023

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	52,16%	26,64%	12,97%	2,96%	1,40%	0,80%	0,79%	0,73%	1,55%
R2	46,49%	27,74%	15,61%	3,63%	1,71%	0,99%	0,97%	0,84%	2,01%
R3	37,07%	27,79%	21,14%	4,90%	2,32%	1,36%	1,32%	1,08%	3,03%
R4	32,06%	26,50%	22,11%	7,27%	2,96%	1,73%	1,76%	1,32%	4,29%
R5	28,24%	25,31%	23,71%	7,71%	3,71%	2,12%	2,19%	1,50%	5,50%
R6	25,78%	24,25%	23,77%	8,69%	3,98%	2,48%	2,56%	1,65%	6,83%
R7	25,83%	23,90%	23,23%	8,11%	3,94%	2,34%	2,59%	1,62%	8,43%
R8	23,18%	20,43%	18,69%	6,21%	2,88%	1,70%	1,81%	1,23%	23,88%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2024

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	49,16%	26,74%	14,24%	3,39%	1,60%	0,93%	0,92%	0,81%	2,22%
R2	45,27%	27,14%	16,05%	3,91%	1,84%	1,08%	1,07%	0,90%	2,75%
R3	38,42%	27,06%	19,63%	4,88%	2,31%	1,36%	1,35%	1,07%	3,91%
R4	34,38%	26,16%	20,62%	6,27%	2,75%	1,62%	1,65%	1,23%	5,30%
R5	31,26%	25,36%	21,73%	6,72%	3,17%	1,86%	1,93%	1,35%	6,61%
R6	29,13%	24,58%	21,92%	7,32%	3,38%	2,05%	2,14%	1,45%	8,02%
R7	29,01%	24,19%	21,42%	6,97%	3,31%	1,97%	2,12%	1,41%	9,60%
R8	25,49%	20,43%	17,25%	5,39%	2,51%	1,49%	1,56%	1,09%	24,78%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2025

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	46,97%	26,67%	15,09%	3,70%	1,75%	1,02%	1,01%	0,86%	2,93%
R2	44,21%	26,75%	16,32%	4,09%	1,93%	1,13%	1,13%	0,93%	3,52%
R3	39,17%	26,56%	18,66%	4,82%	2,27%	1,34%	1,34%	1,05%	4,78%
R4	35,93%	25,87%	19,48%	5,66%	2,57%	1,52%	1,55%	1,16%	6,26%
R5	33,41%	25,26%	20,23%	6,01%	2,82%	1,67%	1,72%	1,24%	7,63%
R6	31,60%	24,64%	20,41%	6,38%	2,97%	1,77%	1,85%	1,29%	9,08%
R7	31,34%	24,24%	19,95%	6,15%	2,90%	1,72%	1,81%	1,26%	10,64%
R8	27,14%	20,34%	16,14%	4,82%	2,25%	1,33%	1,39%	0,99%	25,60%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2026

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	45,32%	26,52%	15,66%	3,92%	1,85%	1,08%	1,08%	0,90%	3,67%
R2	43,31%	26,46%	16,48%	4,21%	1,98%	1,16%	1,17%	0,94%	4,30%
R3	39,54%	26,18%	18,01%	4,73%	2,23%	1,31%	1,33%	1,03%	5,63%
R4	36,95%	25,60%	18,60%	5,25%	2,43%	1,43%	1,47%	1,10%	7,17%
R5	34,90%	25,09%	19,08%	5,51%	2,58%	1,53%	1,57%	1,15%	8,58%
R6	33,36%	24,56%	19,20%	5,73%	2,67%	1,59%	1,65%	1,18%	10,06%
R7	32,99%	24,16%	18,78%	5,56%	2,61%	1,55%	1,61%	1,16%	11,59%
R8	28,27%	20,21%	15,27%	4,41%	2,07%	1,22%	1,26%	0,92%	26,36%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2027

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	44,05%	26,33%	16,01%	4,08%	1,92%	1,12%	1,13%	0,92%	4,43%
R2	42,54%	26,20%	16,55%	4,28%	2,02%	1,18%	1,19%	0,95%	5,08%
R3	39,68%	25,87%	17,54%	4,65%	2,19%	1,29%	1,30%	1,01%	6,47%
R4	37,57%	25,35%	17,93%	4,96%	2,32%	1,37%	1,39%	1,06%	8,05%
R5	35,89%	24,89%	18,21%	5,14%	2,41%	1,42%	1,46%	1,09%	9,48%
R6	34,57%	24,42%	18,25%	5,26%	2,46%	1,46%	1,50%	1,11%	10,97%
R7	34,13%	24,01%	17,87%	5,13%	2,41%	1,42%	1,47%	1,08%	12,48%
R8	29,03%	20,05%	14,60%	4,12%	1,93%	1,14%	1,17%	0,87%	27,08%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2028

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	43,05%	26,13%	16,23%	4,18%	1,97%	1,15%	1,16%	0,94%	5,20%
R2	41,89%	25,97%	16,57%	4,31%	2,03%	1,19%	1,20%	0,96%	5,87%
R3	39,66%	25,60%	17,18%	4,57%	2,15%	1,26%	1,28%	1,00%	7,30%
R4	37,93%	25,11%	17,40%	4,76%	2,23%	1,31%	1,34%	1,02%	8,89%
R5	36,53%	24,68%	17,54%	4,86%	2,28%	1,35%	1,38%	1,04%	10,34%
R6	35,37%	24,23%	17,50%	4,93%	2,31%	1,36%	1,40%	1,05%	11,84%
R7	34,88%	23,83%	17,16%	4,82%	2,26%	1,33%	1,37%	1,03%	13,33%
R8	29,51%	19,88%	14,08%	3,90%	1,83%	1,08%	1,11%	0,84%	27,77%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2029

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	42,23%	25,92%	16,34%	4,24%	2,00%	1,17%	1,18%	0,94%	5,98%
R2	41,32%	25,74%	16,54%	4,33%	2,04%	1,20%	1,21%	0,96%	6,67%
R3	39,54%	25,35%	16,90%	4,49%	2,12%	1,24%	1,26%	0,98%	8,11%
R4	38,08%	24,89%	16,99%	4,60%	2,16%	1,27%	1,30%	1,00%	9,71%
R5	36,89%	24,47%	17,01%	4,66%	2,19%	1,29%	1,32%	1,00%	11,17%
R6	35,87%	24,04%	16,92%	4,68%	2,20%	1,30%	1,33%	1,00%	12,66%
R7	35,35%	23,63%	16,60%	4,58%	2,15%	1,27%	1,30%	0,98%	14,14%
R8	29,78%	19,71%	13,67%	3,74%	1,76%	1,04%	1,06%	0,81%	28,44%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2030

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	41,55%	25,71%	16,38%	4,27%	2,01%	1,18%	1,19%	0,95%	6,76%
R2	40,81%	25,52%	16,49%	4,33%	2,04%	1,20%	1,21%	0,96%	7,46%
R3	39,35%	25,12%	16,66%	4,43%	2,09%	1,23%	1,24%	0,97%	8,91%
R4	38,10%	24,66%	16,65%	4,48%	2,11%	1,24%	1,26%	0,98%	10,52%
R5	37,06%	24,25%	16,59%	4,50%	2,12%	1,25%	1,27%	0,98%	11,97%
R6	36,15%	23,83%	16,46%	4,50%	2,11%	1,25%	1,27%	0,97%	13,46%
R7	35,59%	23,43%	16,15%	4,41%	2,07%	1,22%	1,25%	0,95%	14,92%
R8	29,90%	19,54%	13,35%	3,62%	1,70%	1,00%	1,02%	0,78%	29,09%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2031

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	40,97%	25,49%	16,36%	4,29%	2,02%	1,18%	1,20%	0,95%	7,54%
R2	40,35%	25,30%	16,40%	4,32%	2,03%	1,19%	1,21%	0,95%	8,24%
R3	39,12%	24,90%	16,45%	4,37%	2,06%	1,21%	1,23%	0,96%	9,70%
R4	38,03%	24,45%	16,37%	4,39%	2,06%	1,21%	1,23%	0,96%	11,31%
R5	37,10%	24,04%	16,25%	4,38%	2,06%	1,21%	1,23%	0,95%	12,76%
R6	36,26%	23,63%	16,09%	4,36%	2,05%	1,21%	1,23%	0,95%	14,24%
R7	35,69%	23,23%	15,79%	4,28%	2,01%	1,18%	1,20%	0,93%	15,69%
R8	29,92%	19,37%	13,08%	3,52%	1,66%	0,98%	0,99%	0,77%	29,72%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

2032

	R1	R2	R3	R4	R5	R6	R7	R8	Default
R1	40,45%	25,27%	16,32%	4,29%	2,02%	1,18%	1,20%	0,95%	8,33%
R2	39,92%	25,08%	16,31%	4,30%	2,02%	1,19%	1,20%	0,95%	9,02%
R3	38,86%	24,68%	16,27%	4,32%	2,03%	1,19%	1,21%	0,95%	10,48%
R4	37,88%	24,23%	16,13%	4,31%	2,03%	1,19%	1,21%	0,94%	12,09%
R5	37,04%	23,83%	15,97%	4,29%	2,02%	1,19%	1,21%	0,93%	13,53%
R6	36,25%	23,42%	15,78%	4,25%	2,00%	1,18%	1,20%	0,92%	15,01%
R7	35,67%	23,03%	15,50%	4,17%	1,96%	1,15%	1,17%	0,91%	16,44%
R8	29,86%	19,20%	12,86%	3,45%	1,62%	0,95%	0,97%	0,75%	30,34%
Default	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Cumulative & Marginal Lifetime PD

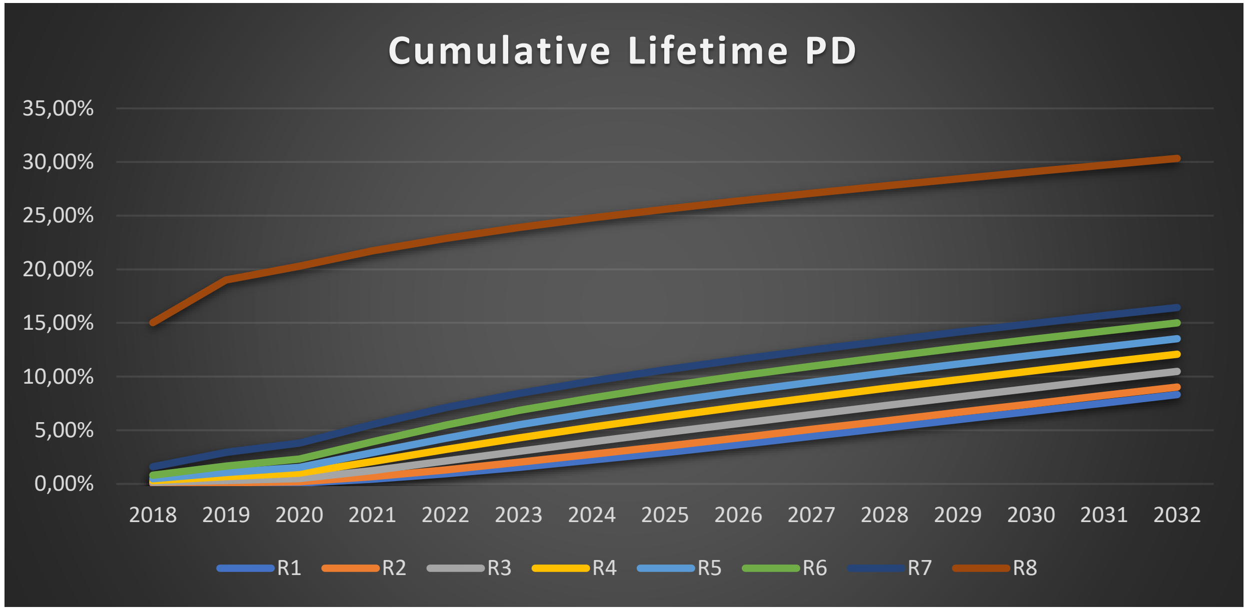
The PIT transition matrix is calculated from the cumulative PIT using the following formula.

$$P_{PIT}\ (x \rightarrow y)\ =\ F_{PIT}\ (x \rightarrow y)\ -F_{PIT}\ (x \rightarrow (y-1))$$

TTC Matrices for 2019 are created as follows:

$$\mathrm{M}_{\mathrm{ij}}^{\mathrm{FDR}}\ (2016)\ \mathrm{X}\ \mathrm{M}_{\mathrm{ij}}^{\mathrm{FDR}}\ (2017)\ \mathrm{X}\ \mathrm{M}_{\mathrm{ij}}^{\mathrm{FDR}}\ (2018)\ =\mathrm{M}_{\mathrm{ij}}^{3\mathrm{Y}}$$

	2018 Default	2019 Default	2020 Default	2021 Default	2022 Default	2023 Default	2024 Default	2025 Default	2026 Default	2027 Default	2028 Default	2029 Default	2030 Default	2031 Default	2032 Default
R1	0,02%	0,05%	0,09%	0,45%	0,95%	1,55%	2,22%	2,93%	3,67%	4,43%	5,20%	5,98%	6,76%	7,54%	8,33%
R2	0,05%	0,12%	0,20%	0,69%	1,31%	2,01%	2,75%	3,52%	4,30%	5,08%	5,87%	6,67%	7,46%	8,24%	9,02%
R3	0,15%	0,33%	0,50%	1,26%	2,13%	3,03%	3,91%	4,78%	5,63%	6,47%	7,30%	8,11%	8,91%	9,70%	10,48%
R4	0,30%	0,67%	1,00%	2,07%	3,20%	4,29%	5,30%	6,26%	7,17%	8,05%	8,89%	9,71%	10,52%	11,31%	12,09%
R5	0,49%	1,07%	1,55%	2,91%	4,26%	5,50%	6,61%	7,63%	8,58%	9,48%	10,34%	11,17%	11,97%	12,76%	13,53%
R6	0,81%	1,65%	2,32%	3,94%	5,48%	6,83%	8,02%	9,08%	10,06%	10,97%	11,84%	12,66%	13,46%	14,24%	15,01%
R7	1,60%	2,94%	3,82%	5,55%	7,10%	8,43%	9,60%	10,64%	11,59%	12,48%	13,33%	14,14%	14,92%	15,69%	16,44%
R8	15,03%	19,02%	20,28%	21,71%	22,88%	23,88%	24,78%	25,60%	26,36%	27,08%	27,77%	28,44%	29,09%	29,72%	30,34%
Default	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%


Cumulative Lifetime PD
35,00%
30,00%
25,00%
20,00%
15,00%
10,00%
5,00%
0,00%
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032
R1
R2
R3
R4
R5
R6
R7
R8

<u>Marginal Lifetime PD</u>

	2018 Default	2019 Default	2020 Default	2021 Default	2022 Default	2023 Default	2024 Default	2025 Default	2026 Default	2027 Default	2028 Default	2029 Default	2030 Default	2031 Default	2032 Default
R1	0,02%	0,03%	0,04%	0,35%	0,51%	0,60%	0,66%	0,71%	0,74%	0,76%	0,77%	0,78%	0,78%	0,78%	0,78%
R2	0,05%	0,07%	0,08%	0,48%	0,63%	0,70%	0,74%	0,76%	0,78%	0,79%	0,79%	0,79%	0,79%	0,79%	0,78%
R3	0,15%	0,18%	0,17%	0,76%	0,88%	0,89%	0,89%	0,87%	0,85%	0,84%	0,82%	0,81%	0,80%	0,79%	0,78%
R4	0,30%	0,37%	0,33%	1,07%	1,14%	1,08%	1,02%	0,96%	0,91%	0,87%	0,85%	0,82%	0,80%	0,79%	0,78%
R5	0,49%	0,58%	0,49%	1,35%	1,36%	1,23%	1,11%	1,02%	0,95%	0,90%	0,86%	0,83%	0,81%	0,79%	0,77%
R6	0,81%	0,84%	0,66%	1,62%	1,54%	1,35%	1,19%	1,06%	0,98%	0,91%	0,86%	0,83%	0,80%	0,78%	0,76%
R7	1,60%	1,34%	0,88%	1,73%	1,55%	1,33%	1,16%	1,04%	0,95%	0,89%	0,85%	0,81%	0,79%	0,77%	0,75%
R8	15,03%	3,99%	1,26%	1,43%	1,17%	1,01%	0,90%	0,82%	0,76%	0,72%	0,69%	0,67%	0,65%	0,63%	0,62%
Default	100,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%