University of Piraeus
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"Comparison of Estimation For Conditional Value at Risk"

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Abstract

Value at Risk (V@R) is one of the most popular risk assessment tools in the world of investment and risk management. Conditional value at risk (CV@R) or Expected Shortfall (ES) is a technique often used to reduce the probability that a portfolio will incur large losses and is performed by assessing the likelihood (at a specific confidence level) that a specific loss will exceed the V@R.

This thesis studies the ES notion and compares its estimation methods. The goal of the thesis is to analyze the techniques of V@R and ES estimations and apply the techniques of 1) historical and 2) monte Carlo simulation method.

The empirical study concerns the assessment of alternatives ES methods in a real mixed portfolio and the comparison of their results. We used a portfolio with historical data and estimated the one-day 99% V@R, one-day 95% V@R such as one-day 99% ES and one-day 95% ES in order to compare their results.

Using different ways of estimation for two portfolios, we came to a conclusion in which, Historical Simulation is this simulation in which we have the underestimation of V@R and ES contrary to Monte Carlo Simulation.

Key words: Value-at-Risk, Conditional Value-at-Risk, Expected Shortfall, Historical Simulation, Monte Carlo Simulation

Contents

CHAPTER 1: V@R and CV@R

The indicative reference of this chapter is Hull C. John (2012) chapter 21 in order to define Value at Risk (V@R) and Conditional Value at Risk (CV@R) or Expected Shortfall (ES). We will cite the uses of these measures in the Basel III framework, capital market and solvency and also highlight the differences between V@R and CV@R.

1.1 Value-at-risk (V@R) and Conditional value-at-risk (CV@R) as a risk measures

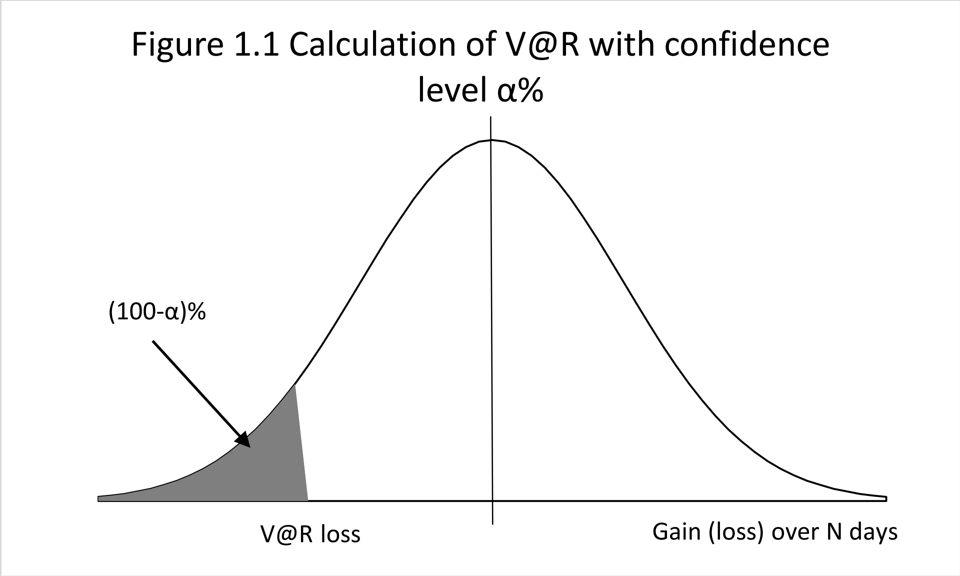
1.1.1 The V@R measure

V@R has become an industry standard in the world of investment since the 1990s due to its ability to combine the risks of such different assets as equities, bonds, commodities, currencies, and options.

Using V@R it is simply like asking: "I am α percent certain that the loss of the portfolio will not be higher than V euros in the next N days".

The variable V is defined as the V@R of the portfolio. As we said above V@R is a risk assessment tool which has two parameters: the time horizon (N days) and the confidence level (α%). The probability that the loss level will exceed is (100-α)%. With this information, V@R is defined as the loss level that has (100-α)% probability not to be exceeded over the next N days. For example, when N=10 and α=95, V@R is the fifth percentile of the distribution of the gain in the value of the portfolio over the next 10 days. Usually, the confidence level that is used is 99% or 95%.

V@R is widely applied in finance for quantitative risk management for many types of risks due to its simplicity. It is very convenient for investors to compress all the risks for all the market variables that exist in every single portfolio into a single number. See Figure 1.1 for graphical representation.



In spite of its problems that will be analyzed below, V@R is the most popular measure of risk in the world of investment and risk management. Unlike traditional methods of risk measurement, V@R takes into account the leverage, the various correlations and the current position of the portfolio. The leverage and the correlations are very important factors for the measurement of VaR in portfolios with large positions in financial derivatives. Therefore, the V@R is a way to see potential future risks with great precision. In parallel, the VaR methodology can be used widely for measuring and other types of risks. There are mainly four methods for its estimation that are: **Historical and Monte Carlo Simulation**, which shall be described in the second chapter.

1.1.2 The time horizon

As mentioned above, V@R has two parameters: the time horizon N that is measured in days and the confidence level X. In practice, based on Hull C. John (2012) chapter 12, analysts set N=1. And this happens because it is difficult to estimate the behavior of market variables in periods of more than one day due to the lack of data. Financial firms, typically use one day whereas institutional investors and nonfinancial corporations may use longer periods. Usual approximation is

$$\textbf{V@R of N-day equals V@R of one day multiplied by } \sqrt{N} \qquad \textbf{(1.1)}$$

When the changes in the value of portfolio have independent identical normal distributions with mean zero, this formula is true.

1.1.3 The CV@R measure or expected shortfall

CV@R is defined as an extension of V@R, which measures the scale of expected losses, once the V@R breakpoint has been exceeded. In other words, expected shortfall is the expected loss during an N-day period conditional that an outcome in the (100-α)% left tail of the distribution occurs. It is calculated by taking a weighted average of the V@R estimate and the expected losses beyond V@R.

In the search for a suitable alternative to V@R, the expected shortfall has been characterized as a suitable risk measure to dominate V@R. Specifically, the V@R tells you that the loss will not be greater than a certain amount over a certain period with α% probability. The expected shortfall tells you what to average loss will be over a certain period given the V@R has been breached. A CV@R estimate is always higher than a V@R estimate.

When V@R asks the question: "How bad can things get?" , CV@R asks: "If things do get bad, how much can the company expect to lose?"

The relationship between V@R and CV@R is illustrated in Figure 1.2.

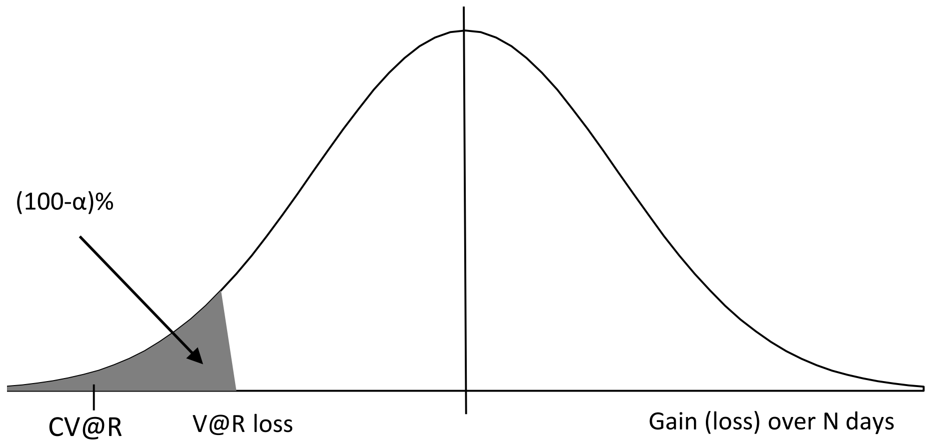


Figure 1.2 Calculation of CV@R (Expected Shortfall) with confidence level α%

1.2 Usages of V@R

V@R is a risk measure that is used globally at the following sectors according to Georgios Ntragas (2007):

- ***Financial Institutions*:** Banks with large portfolios have immediate need of correct management of various risks. Institutions, which are daily confronted with multiple sources of financial risks and complex financial instruments are now using integrated risk management systems.

- ***Regulatory authorities*:** The supervision of financial institutions requires a minimum capital as a reserve against the financial risks. The Basel Committee, the Federal Central Bank of the United States, the Securities and Exchange Commissions in U.S as well as in Greece and the regulatory authorities of the European Union have adopted the V@R method as a commonly accepted measure of risk measurement.
- ***Non-financial Corporations:*** An integrated risk management system is useful to any enterprise that is exposed to financial risks. Multinational enterprises, for example, have inputs and outputs in many currencies, making it vulnerable to opposing changes in exchange rates.
- ***Asset Managers:*** With the V@R measure, investors have the ability to measure the potential risks at assets.

1.2.1 V@R in the Basel III framework

The current framework contained in Basel II Capital Accord has established V@R as the official measure of market risk. As the Basel Committee on Banking Supervision (BCBS) has not yet recommended a particular V@R methodology (such as historical or Monte Carlo simulation), the adoption of the most appropriate V@R approach becomes a matter of importance to be decided.

The 2007 crisis highlights the weakness in the regulation measure taken by Basel II Committee. It was a responsibility to fill these gaps and give some recommendation about risk measurement. Published in December 2010, the main goal was to strengthen financial institutions in order to secure bank liquidity and decrease the bank leverage.

According to the Basel III framework, in constructing V@R models estimating potential quarterly losses, institutions may use quarterly data or convert shorter horizon period data to a quarterly equivalent using an analytically appropriate method supported by empirical evidence (Basel Committee on Banng Supervisor 2006).

The Basel III Committee agreed to replace V@R with the ES for the internal model-based approach. Also recommends using 97.5% confidence level instead of using 99% level of confidence like for the V@R in order to stay consistent. The 10-day returns must be used, that is calculated by the approximation we presented above (formula 1.1). In addition, the length of the sample period for the calculation must be at least one year. Besides that, the bank is still free to choose between models based on variance-covariance, historical simulations or Monte Carlo simulations.

1.2.2 V@R in the regulation of capital market

We understand the importance of V@R considering that Hellenic Capital Market Commission (which is responsible for the regular operation of the capital market in Greece) has imposed specific regulation about V@R.

The picture below shows a small part of the resolution that was adopted by the Hellenic Capital Commission about the calculation of V@R for all corporations in Greece and is published on their official page, which is http://www.hcmc.gr/en_US/web/portal/home.

Άρθρο 3
Συνολική έκθεση σε κίνδυνο

1. Η Εταιρεία υπολογίζει τη συνολική έκθεση σε κίνδυνο σε καθημερινή βάση και τηρεί τα όρια για τη συνολική έκθεση σε διαρκή βάση. Ανάλογα με την επενδυτική στρατηγική που ακολουθείται, η Εταιρεία διενεργεί, όπου είναι αναγκαίο, υπολογισμούς και κατά τη διάρκεια της ημέρας.
2. Η Εταιρεία επιλέγει την κατάλληλη μεθοδολογία για τον υπολογισμό της συνολικής έκθεσης σε κίνδυνο, μετά από αξιολόγηση των χαρακτηριστικών κινδύνου που απορρέουν από την επενδυτική πολιτική, ιδίως όταν γίνεται χρήση παράγωγων χρηματοπιστωτικών μέσων.
3. Η Εταιρεία χρησιμοποιεί για τον υπολογισμό της συνολικής έκθεσης σε κίνδυνο προηγμένη μεθοδολογία μέτρησης κινδύνων, όπως η μέθοδος υπολογισμού της δυνητικής ζημίας (Value-at-Risk, VaR), διενεργώντας παράλληλα ελέγχους προσομοίωσης ακραίων καταστάσεων (stress tests), όταν:
(α) χρησιμοποιεί πολύπλοκες επενδυτικές στρατηγικές σε βαθμό που δεν αποτελούν αμελητέο τμήμα της επενδυτικής πολιτικής,
(β) έχει έκθεση σε μη τυποποιημένα παράγωγα χρηματοπιστωτικά μέσα (exotic derivatives) σε βαθμό που δεν μπορεί να χαρακτηρισθεί ως αμελητέα,
(γ) η προσέγγιση βάσει των υποχρεώσεων (Commitment Approach) δεν καλύπτει επαρκώς τον κίνδυνο αγοράς του χαρτοφυλακίου.
4. Η χρήση οποιασδήποτε μεθοδολογίας μέτρησης κινδύνων και υπολογισμού της συνολικής έκθεσης σε κίνδυνο δεν απαλλάσσει την Εταιρεία από την υποχρέωσή της να θεσπίσει όρια διαχείρισης κινδύνων και κατάλληλα μέτρα για την τήρησή τους.

The regulation about V@R defined as below:

1. The company calculates V@R on a daily basis and follows the limits for the total risk exposure on an ongoing basis. Depending on the investment strategy that followed, if it is necessary, the company makes calculations and during the day.
2. The company selects the appropriate methodology for the calculation of V@R, after assessment of the risk profile arising from the investment policy, especially when using derivative financial instruments.
3. For calculating the total risk, the company uses exposure advanced risk measurement methodology, such as the method of calculation of V@R, by conducting parallel audits stress tests, when:
    a) using complex investment strategies to the extent that they do not constitute a negligible portion of the investment policy,
    b) has exposure to non-standardised financial instruments derivatives (exotic derivatives) to an extent that can not be regarded as negligible,
    c) the approach on the basis of commitments (Commitment Approach) does not sufficiently cover the market risk of the portfolio.
4. Using any methodology for risk measurement and calculation of V@R does not absolve the company from its obligation to adopt risk management limits and appropriate measures to follow them.

1.2.3 V@R in Solvency

Risk management tools, similar with those that are used in other sectors of finance (such as banking), are more and more applied to pension funds. Nowadays, pension funds also calculate V@R which originates from the banking industry (Franzen D. 2010). In banking regulation, the confidence level is 99% and the horizon is about 10 days. When V@R first used in pension funds in 2005, the horizon was extended for them to one year in order to be responding to pension funds' longer-term investment horizon. Also, the confidence level applied is usually lower than this of the banking regulation. V@R is also used in a risk budgeting approach. Risk budgeting approach was more recently developed for pension funds and is used for large funds.

1.3 Comparative analysis of V@R and CV@R

V@R has become a standard measure used in financial risk management due to its simplicity and facility in use. However, many authors claim that V@R has several problems. To alleviate these problems, the use of CV@R is proposed.

1.3.1 V@R benefits and drawbacks

V@R is a single number measuring risk that is defined by a specific confidence level. One of its benefits is that someone can choose between two distributions by comparing their V@Rs for the same confidence level. Hence, V@R is superior to the standard deviation. Differently from standard deviation, V@R focuses on a specific part of the distribution specified by the confidence level and that why V@R has been popular in risk management. Also, another benefit of V@R is its stability of estimation procedures. It is not affected by very high tail losses, which are usually difficult to measure, considering that it disregards the tail.

One of the main drawbacks of V@R is that it is a nonconvex and discontinuous function for discrete distributions. For example, in the financial sector, V@R is a nonconvex and discontinuous function concerning portfolio positions when returns have discrete distributions. Furthermore, it provides no information beyond the confidence level. This means that V@R may increase dramatically with a small increase in confidence level (α%). So, to estimate the risk in a tail, one may calculate a lot of V@Rs with different confidence levels.

Last but not least, the measure of V@R is not subadditive. Subadditive holds that adding, for example, the risk of Asset A and the risk of Asset B will not result in an overall risk that is greater than the sum of the two risks together.

An example of subadditivity is following below:

Example of subadditivity

A bank has two €10 million one-year loans. The probabilities of default are in the following table.

Outcome	**Probability**
Neither loan defaults	95%
Loan 1 defaults; Loan 2 does not default	2,5%
Loan 2 defaults; Loan 1 does not default	2,5%
Both loans default	0%

If the loan does not default, a profit of €0,3 million is made.

Let's begin with Loan 1. This loan has 2,5% chance of defaulting. In the event of a default, the loss is distributed between zero and €10 million. So, there is a 2,5% chance that a loss greater than zero will be incurred, and this means that there is a 1,25% chance that a loss greater than €5 million is incurred. We have supposed that there is no chance of a loss greater than €10 million. The loss level that has a probability of 1% of being exceeded is €6 million. This arises from the fact that if a loss is made, there is a 40% chance that the loss will be greater than €6 million. Because the probability of a loss is 2,5%, the probability of a loss greater than €6 million is $40\% \times 2{,}5\% = 1\%$. The one-year 99% V@R is, therefore, €6 million. The same applies to Loan 2 and the 99% V@R is €6 million too.

To continue, we consider a portfolio of the two loans. There is a 5% probability that a default will occur. The V@R, in this case, is €7,7 million. This is because, there is a 5% chance of one of the loans defaulting and so, there is a 20% chance that the loss on the loan that defaults is greater than €8 million. The probability of a loss from a default being greater than €8 million is therefore $20\% \times 5\% = 1\%$. But, in the event that one loan defaults, a profit of €0,3 million is made on the other loan, so the one-year 99% V@R is €7,7 million.

If we consider the two loans separately, we can see that the total V@R is $6 + 6 = €12$ million. The total V@R after they have been combined in the portfolio is €7,7 million which is €4,3 million smaller. This shows the condition of subadditivity.

1.3.2 CV@R benefits and drawbacks

Unlike the V@R, CV@R quantifies the 'tail risk'. Tail risk is the problem of V@R that disregards any loss beyond the V@R level. For example, if $\bar{L}$ is a loss then the constraint CV@R $\leq \bar{L}$ ensures that the average of (1-α)% highest losses does not exceed $\bar{L}$.

Moreover, CV@R has several mathematical properties. It is a convex and continuous function of discrete distributions (CV@R also called "coherent risk measure" (Uryasev et al. 2010)). In financial setting, CV@R of a portfolio is a convex function of portfolio positions.

The bad news about CV@R is that it is more sensitive than V@R to estimation errors. If we don't have a good model for the tail of the distribution, it is possible that CV@R value may be misleading, and this happens because the accuracy of CV@R estimation is strongly affected by the accuracy of tail modeling. For instance, historical data often do not provide enough information about tails. Furthermore, Yamai and Yosiba (2005) have shown that expected shortfall requires a larger sample size than V@R for the same level of accuracy.

1.3.3 Comparison of V@R and CV@R

V@R and CV@R measure different parts of the distribution. Depending on what is needed, one may be preferred over the other. A trader using V@R as a potential loss measure may prefer V@R over CV@R because V@R is less restrictive, the firm's owner may prefer CV@R because it is more conservative with the same confidence level. (Uryasev et al. 2010). If a good model of tail is available, then CV@R can be accurately estimated. As cited above, CV@R has superior mathematical properties and can be easily used in statistics.

When comparing the stability of estimation of V@R and CV@R, we should choose appropriate confidence level for them, avoiding comparison of them for the same level of confidence level (α%) because V@R and CV@R refer to different parts of the distribution. Finally, CV@R can be optimized and constrained with convex and linear programming methods, whereas V@R is difficult to optimize since is a nonconvex distribution.

Considering all above, CV@R is a more accurate risk measure than V@R owning the fact that when V@R asks the question: "*How bad can things get?*" , CV@R asks: "*If things do get bad, how much do we expect to lose?*"

CHAPTER 2: Estimation methods of V@R and CV@R

The indicative reference of this chapter is Hull C. John (2012) Chapter 21, in order to define the methods of estimations of V@R and CV@R or Expected Shortfall (ES). These methods are 1) Historical and 2) Monte Carlo Simulation as has also mentioned above.

2.1 Historical Simulation

Historical simulation is one popular way of estimating V@R. Suppose that we want to calculate V@R for a portfolio using daily data, a 99% confidence level and 501 trading days (we use 501 trading days in order to create 500 scenarios as we will see below).

In applying Historical simulation, four steps are involved:

I. Identify the market variables (risk factors) affecting the portfolio such as interest rates, equity prices, commodity prices.
II. Select a sample of actual daily risk factor prices or changes over a given period such as 501 days. All prices are measured in the domestic currency.
III. Apply those daily changes to the current value of the risk factors or prices, and revalue the portfolio as many times as the number of days in the sample.
IV. Construct the histogram of the portfolio and identify the V@R that separate the first percentile of the distribution in the left tail assuming that we use a 99% confidence level.

Historical simulation has its limitations. One limitation is that it heavily relies on a particular set of historical data. Historical data may capture periods of extremely high or extremely low volatility and may not accurately represent future outcomes. Another limitation is data availability. For example, one year of data corresponds to only 250 trading days on average and 250 scenarios. By contrast, Monte Carlo simulations usually involve a large number of simulations (as we will see below). Working in small samples of historical data may leave gaps in the distributions of the risk factors.

Let’s describe this process with more details. Data are collected on movements in the market variables over the most 501 days as we cited above. This provides us 500 alternative scenarios about what will happen between today and tomorrow. The first day that we have data is denoted as Day 0, the second day as Day 1, and so on. Scenario 1 is where the percentage changes in the values of all variables are the same as they were between Day 0 and Day 1, Scenario 2 is where they are the same as between Day 1 and Day 2, and so on. For each scenario, we calculate also the euro change in the value of portfolio between today and tomorrow. This defines a probability distribution for daily loss in the value of the portfolio. At these 500 scenarios, the 99^{th} percentile of the distribution can be estimated as the fifth highest loss. The V@R is the loss when we are at this 99^{th} percentile point. In other words, we are 99% certain that the loss will not be greater than the V@R estimation if the changes in market variables in the last 501 days are a representative sample of what will happen between today and tomorrow.

Algebraically, we define as u_i the value of a market variable on Day *i* and suppose that today is Day *n*. The *i*th scenario in the historical simulation approach assumes that the value of the market variable tomorrow will be

$$\text{Value under } i^{th} \text{ scenario} = u_n \frac{u_i}{u_{i-1}}$$

2.1.1 Example in historical simulation

Suppose that an investor in Greece owns, on October 6, 2017, a portfolio worth €10 million consisting of investments in four stock indices: the Dow Jones Industrial Average (DJIA) in the US, the FTSE 100 in the UK, the CAC 40 in France, and the Nikkei 225 in Japan. Table 2.1 shows the value of the investment in each index on October 6, 2017.

Table 2.1 Investment portfolio used for V@R calculations

Index	**Portfolio Value (€000s)**
DJIA	€4.000
FTSE 100	€2.000
CAC40	€3.000
Nikkei 225	€1.000
Total	€10.000

Table 2.2 shows also a part of 501 days of historical data on the closing prices of the four indices in their currency.

Table 2.2 Data on stock indices for historical simulation

Day	*Date*	*DJIA ($)*	*FTSE-100 (£)*	*CAC-40 (€)*	*Nikkei(¥)*
0	13/10/2015	17.081,890	6.342,280	4.643,380	18.234,740
1	14/10/2015	16.924,750	6.269,610	4.609,030	17.891,000
2	15/10/2015	17.141,750	6.338,670	4.675,290	18.096,900
3	16/10/2015	17.215,970	6.378,040	4.702,790	18.291,800
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
499	5/10/2017	22.775,390	7.507,990	5.379,210	20.690,710
500	6/10/2017	22.773,670	7.522,870	5.359,900	20.659,635

The values of the FTSE 100, CAC 40, and Nikkei 225 are adjusted for exchange rate changes so that they are measured in euros (as we have also supposed an investor in Greece).

For example, the FTSE 100 was £7.522,870 on October 6, 2017, when the exchange rate was 0,89803 EUR per GBP. It was £6.342,280 on October 13, 2015, when the exchange rate was 0,7466 EUR per GBP. When measuring in EUR, if the index is set to £7.522,870 on October 6, 2017, it is

$$£6.342{,}280 \times \frac{0{,}7466\ €/£}{0{,}89803\ €/£} = 5.272{,}815€$$

on October 13, 2015. An extract from the data after exchange rate adjustments have been made is shown in Table 2.3.

Table 2.3 Data on stock indices for historical simulation after exchange rate adjustments

Day	Date	DJIA	FTSE-100	CAC-40	Nikkei
0	13/10/2015	16.582,69	5.272,82	4.643,38	22.074,18
1	14/10/2015	16.556,10	5.177,49	4.609,03	21.664,02
2	15/10/2015	16.625,16	5.188,01	4.675,29	21.739,11
3	16/10/2015	16.657,37	5.220,73	4.702,79	22.016,19
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
499	5/10/2017	22.737,72	7.465,94	5.379,21	20.721,66
500	6/10/2017	22.773,67	7.522,87	5.359,90	20.659,64

Table 2.4 shows the values of the market variables on October 7, 2017, for the scenarios considered. Scenario 1 (the first row in Table 1.4) shows the values of market variables on October 7, 2017, assuming that their percentage changes between October 6 and October 7, 2017, are the same as they were between October 13 and October 14, 2015; Scenario 2 (the second row in Table 1.4) shows the values of market variables on October 7, 2017, assuming these percentage changes are the same as those between October 14 and October 15, 2015; and so on. In general, Scenario I assumes that the percentage changes in the indices between October 6 and October 7 are the same as they were between Day i-1 and Day i for $1 \leq i \leq 500$. The 500 rows in Table 1.4 are the 500 scenarios considered.

Table 2.4 Scenarios generated for October 7, 2017, using data in Table 2.3

Scenario number	DJIA	FTSE 100	CAC 40	Nikkei 225	Portfolio Value (€000s)	Loss (€000s)
1	22.737,158	7.661,375	5.399,846	21.050,780	10.071,700	71,700
2	22.868,665	7.507,622	5.283,937	20.588,272	9.966,660	-33,340
3	22.817,791	7.475,721	5.328,557	20.399,631	9.965,087	-34,913
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
499	22.786,993	7.439,983	5.343,977	20.588,997	9.967,973	-32,027
500	22.809,675	7.465,937	5.379,210	20.721,660	10.004,998	4,998

Therefore the value of the DJIA under Scenario 1 is

$$22.773{,}67€ \times \frac{16.556{,}099€}{16.582{,}685€} = 22.737{,}158€$$

Similarly, the values of the FTSE 100, the CAC 40, and the Nikkie 225 are 7.661,375€, 5.399,846€, and 21.050,780€, respectively. Therefore the value of the portfolio under Scenario 1 is (in €000s)

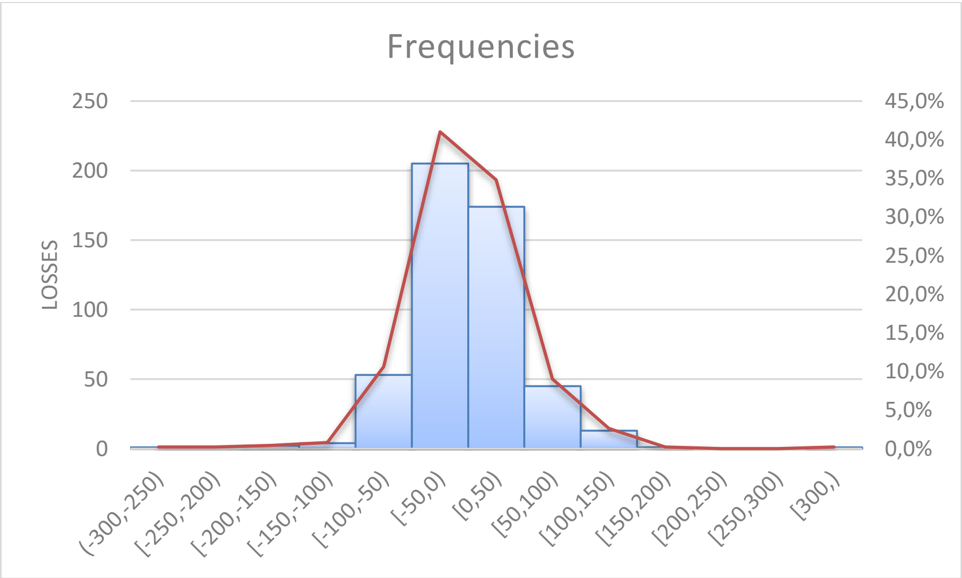
$$4.000 \times \frac{22.737{,}158}{22.773{,}670} + 2.000 \times \frac{7.661{,}375}{7.522{,}870} + 3.000 \times \frac{5.399{,}846}{5.359{,}900} + 1.000 \times \frac{21.050{,}780}{20.659{,}635} = €10.071{,}700$$

The portfolio, therefore, has a gain of €71.700 under Scenario 1. A similar calculation is made for the others scenarios. A histogram of the losses is shown in Figure 2.1. The descriptive statistics for the losses of the portfolio are also shown in Table 2.5.

Table 2.5 Descriptive statistics of losses for the scenarios considered between October 6 and October 7, 2017

Descriptive statistics	
Mean (€000s)	-0,954
Standard Error	2,267
Median (€000s)	-3,788
Standard Deviation	50,693
Sample Variance	2.569,806
Kurtosis	6,270
Skewness	0,189
Range (€000s)	619,831
Minimum (€000s)	-284,958
Maximum (€000s)	334,873

Figure 2.1 Histogram of losses for the scenarios considered between October 6 and October 7, 2017



The losses that have already arisen from the 500 different scenarios are now ranked. An extract of the results of this is shown in Table 2.6. The worst scenario is number 89. The one day 99% V@R can be estimated as the fifth worst loss (as we have a 99% confidence level and 500 scenarios). This is €125.144 and -1,2514%. The one day 95% V@R can be estimated also as the 25th worst loss (as we have a 95% confidence level and 500 scenarios). This is €86.335 and -0,8634%.

As we have already seen in Chapter 1.1.2 (formula 1.1), the ten-day 99% V@R is usually calculated as $\sqrt{10}$ times the one-day 99% V@R (under circumstances). In this case, the ten-day V@R would therefore be

$$\sqrt{10} \times 125.144 = €395.740{,}075$$

Table 2.6 Losses ranked from highest to lowest for 500 scenarios

Scenario number	Loss (€000s)	Loss (%)
176	-334,873	-3,3487%
63	-162,746	-1,6275%
25	-137,244	-1,3724%
243	-127,850	-1,2785%
89	-125,144	-1,2514%
229	-116,110	-1,1611%
182	-112,763	-1,1276%
177	-111,316	-1,1132%
70	-109,797	-1,0980%
46	-108,844	-1,0884%
392	-108,774	-1,0877%
72	-104,885	-1,0488%
8	-104,699	-1,0470%
296	-103,573	-1,0357%
84	-100,809	-1,0081%
181	-98,271	-0,9827%
251	-97,213	-0,9721%
496	-95,115	-0,9511%
190	-94,290	-0,9429%
275	-93,880	-0,9388%
45	-93,296	-0,9330%
87	-89,813	-0,8981%
127	-89,045	-0,8904%
184	-87,068	-0,8707%
156	-86,335	-0,8634%
297	-85,642	-0,8564%
131	-84,656	-0,8466%
47	-84,193	-0,8419%

99% V@R

95% V@R

356	-81,937	-0,8194%
76	-80,409	-0,8041%
.	.	.
.	.	.
.	.	.

Each day the V@R estimate in our example would be updated using the most recent 501 days of data. For example, we can wonder about what happens on October 7, 2017 (Day 501). We should find new values for all market variables and calculate a new value for the portfolio. We should go then through the procedure we have summarized to calculate a new V@R. Data on the market variables from October 14, 2015, to October 7, 2017 (Day 1 to Day 501) are used in the calculation. This gives us the required 500 observations on the percentage changes in market variables (the October 13, 2015, Day 0, values of the market variables are no longer used). Similarly, on the next trading day October 8, 2017 (Day 502), data from October 15, 2015, to October 8, 2017 (Day 2 to Date 502) are used to determine V@R, and so on.

In practice, a real financial's portfolio is, of course, more complicated than the one we analyzed here. It may consist of thousands or more positions. These positions can be in forward contracts, options, and other derivatives. The V@R is calculated at the end of each day on the speculation that the portfolio will remain the same over the next business day. We can understand here that sometimes, should be considered hundreds or even thousands of market variables in a V@R calculation.

In order to calculate expected shortfall with historical simulation, we should average the five observations of the worst losses, as have already ranked above. More exactly, in our example, the five worst losses (€000s) are from scenarios 176, 63, 25, 243 and 89 (see Table 2.6 above). The average for these scenarios is €177.571 and -1,7757% and this is the estimation of the expected shortfall for the 99% confidence level. The expected shortfall for a confidence level of 95% is also €115.750 and -1,1575%.

In this part of the chapter, we will do just the same analysis for the same indices but we suppose that the whole portfolio consists of one index each time. So, we will calculate V@R and ES for four different portfolios (one portfolio for each index) and then we will compare the results of them with the V@R and ES of the initial portfolio that consist of four indices.

To begin with, Tables 2.7-2.10 show the data of four indices in their currency separately, such as their value after exchange rate adjustments (our currency is euros as we are in Greece).

Table 2.7 Data on **DJIA** for historical simulation after exchange rate adjustments

Day	Date	DJIA	Exchange Rate EUR/USD	Adjusted DJIA
0	13/10/2015	17.081,890	1,13872	16.582,685
1	14/10/2015	16.924,750	1,14745	16.556,099
2	15/10/2015	17.141,750	1,13765	16.625,159
3	16/10/2015	17.215,970	1,13494	16.657,368
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
499	5/10/2017	22.775,390	1,17106	22.737,722
500	6/10/2017	22.773,670	1,17300	22.773,670

Table 2.8 Data on **FTSE 100** for historical simulation after exchange rate adjustments

Day	Date	FTSE-100	Exchange Rate EUR/GBP	Adjusted FTSE-500
0	13/10/2015	6.342,280	0,7466	5.272,815
1	14/10/2015	6.269,610	0,7416	5.177,492
2	15/10/2015	6.338,670	0,73501	5.188,007
3	16/10/2015	6.378,040	0,73508	5.220,727
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
499	5/10/2017	7.507,990	0,89300	7.465,937
500	6/10/2017	7.522,870	0,89803	7.522,870

Table 2.9 Data on **CAC-40** for historical simulation without exchange rate adjustments (as this index values in €).

Day	Date	CAC-40
0	13/10/2015	4.643,380
1	14/10/2015	4.609,030
2	15/10/2015	4.675,290
3	16/10/2015	4.702,790
.	.	.
.	.	.
.	.	.
499	5/10/2017	5.379,210
500	6/10/2017	5.359,900

Table 2.10 Data on **Nikkei** for historical simulation after exchange rate adjustments

Day	Date	Nikkei	Exchange Rate EUR/YEN	Adjusted Nikkei
0	13/10/2015	18.234,740	136,3625	22.074,178
1	14/10/2015	17.891,000	136,4	21.664,017
2	15/10/2015	18.096,900	135,3155	21.739,109
3	16/10/2015	18.291,800	135,58	22.016,186
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
499	5/10/2017	20.690,710	112,81300	20.721,660
500	6/10/2017	20.659,635	112,64450	20.659,635

Moreover, we calculate the 500 scenarios for each index as we analyze above. An extract of these scenarios is shown in Table 2.11.

Table 2.11 Scenarios generated for October 7, 2017, using data in Tables 2.7-2.10

Scenario number	DJIA	FTSE 100	CAC 40	Nikkei 225
1	22737,16	7661,375	5399,846	21050,78
2	22868,67	7507,622	5283,937	20588,272
3	22817,79	7475,721	5328,557	20399,631
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
499	22786,99	7439,983	5343,977	20588,997
500	22809,67	7465,937	5379,21	20721,66

Then, we calculate the value of each portfolio such as the losses of them. We can see the results in the following tables (Tables 2.12-2.15).

Table 2.12 Portfolio’s value and losses of **DJIA**

Scenario number	DJIA	Portfolio Value (€000s)	Losses (€000s)
1	22737,16	3993,587	-6,412957
2	22868,67	4016,685	16,6851
3	22817,79	4007,749	7,74945
.	.	.	.
.	.	.	.

.	.	.	.
499	22786,99	4002,34	2,34008
500	22809,67	4006,324	6,32389

Table 2.13 Portfolio's value and losses of **FTSE 100**

Scenario number	FTSE 100	Portfolio Value (€000s)	Losses (€000s)
1	7661,375	2036,822	36,8223
2	7507,622	1995,946	-4,053698
3	7475,721	1987,465	-12,53477
.	.	.	.
.	.	.	.
.	.	.	.
499	7439,983	1977,964	-22,03612
500	7465,937	1984,864	-15,13608

Table 2.14 Portfolio's value and losses of **CAC-40**

Scenario number	CAC-40	Portfolio Value (€000s)	Losses (€000s)
1	5399,846	3022,358	22,3583
2	5283,937	2957,483	-42,51715
3	5328,557	2982,457	-17,54278
.	.	.	.
.	.	.	.
.	.	.	.
499	5343,977	2991,088	-8,912089
500	5379,21	3010,808	10,808

Table 2.15 Portfolio's value and losses of **Nikkie**

Scenario number	Nikkie	Portfolio Value (€000s)	Losses (€000s)
1	21050,78	1018,933	18,9328
2	20588,27	996,5458	-3,454224
3	20399,63	987,4149	-12,58514
.	.	.	.
.	.	.	.
.	.	.	.
499	20589	996,5809	-3,419131
500	20721,66	1003,002	3,00225

In conclusion, the losses that have already arisen from the 500 different scenarios for each portfolio are now ranked and the V@R has also calculated (as the fifth highest loss of the losses of each portfolio). An extract of the results of this is shown in Tables 2.16-2.19.

Table 2.16 Losses ranked from highest to lowest for 500 scenarios for **DJIA**

Scenario number	Ranked Losses (€000s)	Losses (%)	
176	-223,046	-5,5762%	
177	-98,404	-2,4601%	
229	-96,294	-2,4074%	
56	-75,813	-1,8953%	
65	-75,665	-1,8916%	← 99% V@R
67	-75,467	-1,8867%	
47	-74,762	-1,8690%	
63	-72,947	-1,8237%	
251	-72,353	-1,8088%	
46	-68,423	-1,7106%	
55	-68,372	-1,7093%	
465	-66,892	-1,6723%	
296	-66,759	-1,6690%	
147	-66,672	-1,6668%	
79	-66,051	-1,6513%	
231	-64,38692	-1,6097%	
76	-63,49187	-1,5873%	
23	-61,27269	-1,5318%	
42	-53,0764	-1,3269%	
83	-52,64498	-1,3161%	
182	-51,82424	-1,2956%	
234	-51,74983	-1,2937%	
58	-49,1392	-1,2285%	
166	-48,4986	-1,2125%	
121	-47,82099	-1,1955%	← 95% V@R
93	-47,29101	-1,1823%	
401	-47,21381	-1,1803%	
378	-47,14849	-1,1787%	
16	-46,94483	-1,1736%	
90	-46,3892	-1,1597%	
.	.	.	
.	.	.	
.	.	.	

Table 2.17 Losses ranked from highest to lowest for 500 scenarios for **FTSE 100**

Scenario number	Ranked Losses (€000s)	Losses(%)
384	-70,756	-3,5378%
179	-64,268	-3,2134%
85	-59,737	-2,9868%
180	-59,402	-2,9701%
204	-59,360	-2,9680%
176	-59,264	-2,9632%
86	-56,018	-2,8009%
44	-52,227	-2,6113%
152	-51,135	-2,5568%
249	-50,364	-2,5182%
291	-49,620	-2,4810%
72	-47,052	-2,3526%
74	-46,818	-2,3409%
89	-45,426	-2,2713%
84	-45,286	-2,2643%
92	-44,87976	-2,2440%
347	-42,97176	-2,1486%
178	-42,97012	-2,1485%
246	-41,83111	-2,0916%
445	-41,30683	-2,0653%
78	-41,14139	-2,0571%
115	-40,42586	-2,0213%
245	-38,98608	-1,9493%
50	-36,29476	-1,8147%
270	-36,17003	-1,8085%
181	-36,13205	-1,8066%
400	-35,87238	-1,7936%
312	-35,58222	-1,7791%
496	-34,85134	-1,7426%
66	-34,80375	-1,7402%
.	.	.
.	.	.
.	.	.

99% V@R (row 204, -2,9680%)

95% V@R (row 270, -1,8085%)

Table 2.18 Losses ranked from highest to lowest for 500 scenarios for **CAC-40**

Scenario number	Ranked Losses (€000s)	Losses (%)	
392	-119,371	-3,9790%	
175	-101,547	-3,3849%	
127	-96,474	-3,2158%	
106	-95,112	-3,1704%	
45	-91,884	-3,0628%	99% V@R
71	-90,131	-3,0044%	
87	-87,618	-2,9206%	
89	-87,028	-2,9009%	
25	-80,813	-2,6938%	
496	-76,553	-2,5518%	
181	-76,400	-2,5467%	
182	-76,135	-2,5378%	
8	-74,006	-2,4669%	
86	-73,854	-2,4618%	
156	-72,043	-2,4014%	
51	-68,62508	-2,2875%	
229	-67,69892	-2,2566%	
7	-66,89462	-2,2298%	
243	-66,70111	-2,2234%	
95	-65,74868	-2,1916%	
76	-64,42805	-2,1476%	
356	-61,83441	-2,0611%	
70	-58,09005	-1,9363%	
68	-58,06529	-1,9355%	
178	-57,68378	-1,9228%	95% V@R
275	-56,3188	-1,8773%	
54	-53,22502	-1,7742%	
92	-52,80201	-1,7601%	
117	-52,48154	-1,7494%	
189	-52,13712	-1,7379%	
.	.	.	
.	.	.	
.	.	.	

Table 2.19 Losses ranked from highest to lowest for 500 scenarios for **Nikkie**

Scenario number	Ranked Losses (€000s)	Losses(%)
272	-68,139	-6,8139%
84	-66,261	-6,6261%
69	-57,546	-5,7546%
185	-50,086	-5,0086%
96	-45,647	-4,5647%
34	-36,288	-3,6288%
72	-34,781	-3,4781%
126	-34,648	-3,4648%
176	-34,373	-3,4373%
307	-31,977	-3,1977%
129	-31,407	-3,1407%
184	-27,150	-2,7150%
205	-26,461	-2,6461%
334	-25,471	-2,5471%
171	-25,431	-2,5431%
131	-24,7736	-2,4774%
187	-24,67682	-2,4677%
485	-24,26018	-2,4260%
220	-24,1358	-2,4136%
242	-24,05034	-2,4050%
125	-23,69108	-2,3691%
104	-22,88895	-2,2889%
63	-22,70881	-2,2709%
274	-22,33421	-2,2334%
481	-21,73752	-2,1738%
74	-21,23465	-2,1235%
390	-20,71012	-2,0710%
75	-20,68352	-2,0684%
46	-20,1247	-2,0125%
388	-19,83009	-1,9830%
.	.	.
.	.	.
.	.	.

99% V@R (row 96)

95% V@R (row 481)

As a result, the 99% V@R and 99% ES of four indices (at four different portfolios) are shown in Table 2.20.

Table 2.20 99% V@R and 99% ES for four indices.

Index	Value-at-Risk	Losses (%)	Expected Shortfall	Losses (%)
DJIA	-75,66536	-1,8916%	-113,8444	-2,8461%
FTSE 100	-59,35967	-2,9680%	-62,70453	-3,1352%
CAC-40	-91,8841	-3,0628%	-100,8778	-3,3626%
Nikkie	-45,64665	-4,5647%	-57,53569	-5,7536%

Now, we want to check the **benefits of diversification**. In the example above we have just considered:

I. The 99% V@R for the portfolio of DJIA is €75.665 and ES is €113.844.
II. The 99% V@R for the portfolio of FTSE 100 is €59.359 and ES is €62.704.
III. The 99% V@R for the portfolio of CAC-40 is €91.884 and ES is €100.877.
IV. The 99% V@R for the portfolio of Nikkie is €45.646 and ES is €57.535.

For the first measure (V@R) we can see that the amount (€75.665 +€59.359 +€91.884 + €45.646) = €272.554, is bigger than the amount of V@R that have already calculated above for the portfolio of all four indices together which is €125.144. This represents, the benefits of diversification, even though that the measure of **V@R is not subadditive** (as we have also mentioned above).

For the second measure (ES) we can see that the amount (€113.844 +€62.704 +€100.877 +€57.535)= €334.960 is also bigger than the amount of ES that has already calculated above for the portfolio of all four indices together which is €177.571 and it is expected, as the measure of **ES is subadditive**.

Respectively, the 95%V@R and 95% ES of four indices (at four different portfolios) are shown in Table 2.21.

Table 2.21 95% V@R and 95% ES for four indices.

Index	Value-at-Risk	Losses (%)	Expected Shortfall	Losses (%)
DJIA	-47,82099	-1,1955%	-72,47301	-1,8118%
FTSE 100	-36,17003	-1,8085%	-48,94846	-2,4474%
CAC-40	-57,68378	-1,9228%	-77,38960	-2,5797%
Nikkie	-21,73752	-2,1738%	-33,23686	-3,3237%

We will check again **the benefits of diversification**. In the example above we have just considered:

I. The 95% V@R for the portfolio of DJIA is €47.821 and ES is €72.473
II. The 95% V@R for the portfolio of FTSE 100 is €36.170 and ES is €48.948.
III. The 95% V@R for the portfolio of CAC-40 is €57.683 and ES is €77.389.
IV. The 95% V@R for the portfolio of Nikkie is €21.737 and ES is €33.236.

As we can see again, the first measure (V@R) has a total amount of (€47.821 +€36.170 +€57.683 + €21.737) = €163.411, and it is bigger than the amount of V@R that have already calculated above for the portfolio of all four indices together which is €86.335 This represents also, the benefits of diversification, even though that the measure of **V@R is not subadditive** (as we have also mentioned above).

For the second measure (ES) we can see that the amount (€72.473 +€48.948 +€77.389 +€33.236)= €232.046 is also bigger than the amount of ES that has already calculated above for the portfolio of all four indices together which is €115.750 and it is again expected, as the measure of **ES is subadditive**.

2.2 Monte Carlo Simulation

Monte Carlo is a mathematical technique that generates random variables for modeling uncertain situations (The Economic Times). This technique was introduced during World War II. Today, it is used in a large variety of fields such as biology, physical science, artificial intelligence, statistics and quantitive finance.

Monte Carlo is based on probability theory in order to construct the simulation process. It contains repeated trials of the values of uncertain inputs based on a known probability distribution and a known process in order to construct a probability distribution for the output. In detail, each uncertain input in the problem is supposed to be a random variable with a known probability distribution. The output of the model, after a large number of iterations, is also a probability distribution.

We can think the Monte Carlo simulation like scenario analysis that we have described above. Instead of having 500 scenarios (as we used above), the simulation process generates thousands or ten of thousands of scenarios. The more scenarios we have, the better we understand the nature of the problem.

Rather than defining the probability distribution of the risk factor (in this case, the risk factor is the return of an index), the Monte Carlo simulation method exports the distribution of the indices returns using a stochastic process. We assume that indices prices follow a special type of stochastic process known as **Geometric Brownian Motion** that is described by the following equation:

$$S_{t+\Delta t} = S_t \, e^{(k\Delta t + \sigma \varepsilon_t \sqrt{\Delta t})} \qquad \textbf{(2.1)}$$

where S_t is the index price at the time t, e is the natural log, Δ_t is the time increase (that is expressed as portion of a year in term of trading days, for example one trading day will yields $\Delta_t = 1/252$ of a trading year), $k = \mu - (\frac{\sigma^2}{2})$ is the expected return (which equals annualised mean return μ minus half of the annualised variance of return σ^2, and ε_t is the randomness at time t that is introduced to randomise the change in index. In detail, the variable ε_t is a random number, generated from a standard normal probability distribution, which has a mean of zero and a standard deviation of one. We can rearrange equation (2.1) with equation (2.2) as follows:

$$\boldsymbol{R_{t+\Delta_t} = \ln\left(\frac{S_{t+\Delta_t}}{S_t}\right) = k\Delta_t + \sigma\varepsilon_t\sqrt{\Delta_t}} \quad \textbf{(2.2)}$$

So, the main key in Monte Carlo simulation is to generate the future returns according to equation (2.2). The number of runs is defined by us, normally we used upwards of 10.000. So, in each simulation we have the following four steps in order to calculate V@R and ES:

- ✓ **Step one** calculates the parameters in the Geometric Brownian Motion process
- ✓ **Step two** generates normally distributed random numbers
- ✓ **Step three** applies the normally distributed random numbers into the Geometric Brownian Motion process in order to yield the simulated asset returns
- ✓ And **the final step** is to calculate V@R which is again the observation of the 1% (for 99% V@R) or 5% (for 95% V@R) of the worst scenarios, such as we have also described in the historical simulation. For the calculation of ES, we average again the 1%(for 99% V@R) or 5% (for 95% V@R) of the worst scenarios, that has also explained in historical simulation above.

2.2.1 Example in Monte Carlo simulation

We will use the same data with the example in historical simulation in order to compare the results. So we have again an investor in Greece, who owns, on October 6, 2017, a portfolio worth €10 million consisting of investments in four stock indices: the Dow Jones Industrial Average (DJIA) in the US, the FTSE 100 in the UK, the CAC 40 in France, and the Nikkei 225 in Japan. We have also a part of 501 days of historical data on the closing prices of the four indices in their currency. The values of the FTSE 100, CAC 40, and Nikkei 225 are adjusted for exchange rate changes so that they are measured in euros (as we have also supposed an investor in Greece) just as we calculated them in historical simulation above. Now, we calculate the returns of the four indices. The total return of an index is the following as described in equation 2.3:

$$\textbf{Return} = \frac{\textbf{Price}_{t+1} - \textbf{Price}_t}{\textbf{Price}_t} \qquad \textbf{(2.3)}$$

An extract of the returns of the four indices is shown in the Table 2.22 below.

Table 2.22 Historical returns of the four indices

HISTORICAL RETURNS				
Day	DJIA	FTSE 500	CAC 40	Nikkei
1	-0,160%	-1,808%	-0,740%	-1,858%
2	0,417%	0,203%	1,438%	0,347%
3	0,194%	0,631%	0,588%	1,275%
4	-0,100%	-0,750%	0,027%	-1,035%
5	0,137%	0,229%	-0,643%	0,904%
6	-0,384%	0,128%	0,456%	1,881%
7	-0,237%	-1,473%	2,281%	-2,059%
8	0,091%	0,785%	2,529%	1,914%
9	0,248%	-0,282%	-0,538%	0,712%
10	-0,441%	-0,764%	-1,022%	-1,584%
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
498	0,258%	0,089%	-0,078%	-0,041%
499	0,059%	1,114%	0,298%	0,343%
500	0,158%	0,763%	-0,359%	-0,299%

The first step for the Monte Carlo simulation, as mentioned above, is to calculate the parameters in the Geometric Brownian Motion for all indices. The results of the calculation of these parameters are shown in the Table 2.23 below. We should remind there the equation of the Geometric Brownian Motion (equation 2.1) as was presented above:

$$S_{t+\Delta t} = S_t \, e^{(k\Delta t + \sigma \varepsilon_t \sqrt{\Delta t})}$$

Table 2.23 Parameters of Geometric Brownian Motion

Geometric Brownian Motion				
	DJIA	FTSE 500	CAC 40	Nikkei
Number of observations	500	500	500	500
Min daily return	-5,5762%	-3,9109%	-8,0425%	-14,5408%
Max daily return	2,7174%	3,6675%	4,1439%	7,3121%
Share price now (S0)	22.773,670	7.522,870	5.359,900	20.659,635
Number of trading days per year	252	252	252	252
Time increment (Δt) for one day	0,00396825	0,0039683	0,0039683	0,00396825
Average daily return	0,0667%	0,0763%	0,0351%	0,0003%
Daily standard deviation	0,8056%	1,0212%	1,1253%	1,6311%
Annualized mean return for one year (μ)	16,8147%	19,2293%	8,8363%	0,0778%
Annualized standard deviation (σ)	12,7888%	16,2109%	17,8640%	25,8933%
Expected return (k)	15,9969%	17,9153%	7,2407%	-3,2745%

The next step is to generate normally distributed random numbers. We have also generated 100.000 of these numbers, as we decided to have 100.000 simulations in order to have better results. Because, the more simulations, the better results with respect to accuracy. An extract of these numbers is presented also in Table 2.24 below:

Table 2.24 Normally distributed random numbers

Normally Distributed Random Numbers				
	DJIA	FTSE 500	CAC 40	Nikkie
1	-2,95462	-2,95462	-2,95462	-2,95462
2	-0,50279	-0,50279	-0,50279	-0,50279
3	-1,16968	-1,16968	-1,16968	-1,16968
4	0,612893	0,612893	0,612893	0,612893
5	0,134376	0,134376	0,134376	0,134376
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
99.996	-1,27338	-1,27338	-1,27338	-1,27338
99.997	1,462139	1,462139	1,462139	1,462139
99.998	-0,27676	-0,27676	-0,27676	-0,27676
99.999	-0,80006	-0,80006	-0,80006	-0,80006
100.000	-0,35434	-0,35434	-0,35434	-0,35434

Then, we use these numbers to the equation of the Geometric Brownian Motion (equation 2.1) in order to refund us the indices returns for 100.000 simulations. After all this process, we follow the same way to calculate V@R and ES. More exactly, we take into consideration the weights of each index in our portfolio and calculate the value of this in each simulation. An extract of the returns and the weighted yield of the portfolio, after the use of the Geometric Brownian Motion, that consists of the four indices, is shown in Table 2.25 below:

2.25 Returns of indices and final yield of the portfolio

SIMULATED RETURNS					
	DJIA	FTSE 500	CAC 40	Nikkei	Weighted average
Number of simulations	40%	20%	30%	10%	
1	-2,3168%	-2,9461%	-3,2962%	-4,8323%	-2,9880%
2	-0,3416%	-0,4423%	-0,5371%	-0,8331%	-0,4695%
3	-0,8788%	-1,1234%	-1,2875%	-1,9209%	-1,1546%
4	0,5572%	0,6970%	0,7184%	0,9867%	0,6765%
5	0,1717%	0,2083%	0,1800%	0,2062%	0,1850%
.	.	.	.	.	.
.	.	.	.	.	.

.	.	.	.	.	.
99.996	-0,9624%	-1,2293%	-1,4042%	-2,0900%	-1,2611%
99.997	1,2414%	1,5642%	1,6741%	2,3719%	1,5488%
99.998	-0,1595%	-0,2115%	-0,2827%	-0,4644%	-0,2374%
99.999	-0,5811%	-0,7459%	-0,8716%	-1,3180%	-0,7749%
100.000	-0,2220%	-0,2908%	-0,3700%	-0,5910%	-0,3170%

All things considered, the weighted yields of the portfolio are now ranked from smallest to highest. An extract of the results of this is shown in Table 2.26. The worst yield is at the 8.573rd simulation. The one day 99% V@R can be estimated as the 1000th worst yield (as we have a 99% confidence level and 100.000 simulations). This is -2,3369%. The one day 95% V@R can be estimated also as the 5000th worst yield (as we have a 95% confidence level and 100.000 simulations). This is -1,6446%.

Table 2.26 Yields ranked from lowest to highest for 100.000 simulations for the portfolio

Number of simulation	**Ranked Average**	
8.573	-4,3922%	
62.275	-4,1619%	
81.706	-4,0148%	
12.335	-3,9513%	
78.457	-3,9350%	
.	.	
.	.	
.	.	
85.989	-2,3369%	← 99% V@R
.	.	
.	.	
.	.	
6.364	-1,6446%	← 95% V@R
.	.	
.	.	
.	.	
52.440	4,1723%	
41.905	4,2368%	
80.130	4,3616%	
55.461	4,4079%	
1.311	4,4101%	

In order to calculate the 99% expected shortfall with Monte Carlo simulation, we should average the 1000 worst yields, as have already ranked above. More exactly, the average for these yields is -2,6724% and this is the estimation of the expected shortfall for the 99% confidence level. The expected shortfall for a confidence level of 95% is also -2,0655%.

In all this process in which we calculate V@R and ES with the Monte Carlo simulation, we have assumed that the random variables are uncorrelated. So, it is necessary to repeat all the process, **taking into account the correlation of the random variables**.

There is a matrix-based methodology that can be used to a large number of correlated normal samples (as we have four indices). Individual measures of dependency are collected into the correlation matrix as shown in Table 2.27. A correlation matrix summarizes the dependency between all the four variables. The diagonal part of this matrix is always 1, as each variable is always perfect correlated with itself. The calculation of correlation is presented below in equation 2.4:

$$\boldsymbol{\rho_{x,y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}} \qquad \textbf{(2.4)}$$

Where $\rho_{x,y}$ is the correlation between variables x and y, $\sigma_{x,y}$ is the covariance between x and y and $\sigma_x\sigma_y$ is the standar deviation of x and y correspondingly. Also the covariance of of two variables is calculated as it is shown in equation 2.5 below:

$$\boldsymbol{\sigma_{x,y} = \frac{\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})}{N}} \qquad \textbf{(2.5)}$$

Where x_i is the i-th observation of the first variable, $\bar{x}$ is the expected value of the variable x and for y_i and $\bar{y}$ the same for variable y.

Table 2.27 Correlation Matrix of four indices

Correlation Matrix

	DJIA	*FTSE 500*	*CAC 40*	*Nikkei*
DJIA	1	0,26542	0,01366	0,00874
FTSE 500	0,26542	1	-0,01672	0,10268
CAC 40	0,01366	-0,01672	1	0,00918
Nikkei	0,00874	0,10268	0,00918	1

Also, the Variance-Covariance matrix that includes the covariance of the variables is also presented below as the Table 2.28. The name of this matrix is Variance-Covariance and not only covariance because the diagonal part of the matrix which presents the covariance of its variable with itself, always represents the variance as we can see in equation 2.6

$$\sigma_{x,x} = \sigma_x \quad (2.6)$$

Table 2.28 Variance-Covariance matrix of four indices

Variance-Covariance Matrix

	DJIA	*FTSE 500*	*CAC 40*	*Nikkei*
DJIA	0,0000649024	0,0000218356	0,0000012382	-0,0000019211
FTSE 500	0,0000218356	0,0001042828	-0,0000019211	0,0000171030
CAC 40	0,0000012382	-0,0000019211	0,0001266365	0,0000016844
Nikkei	0,0000011484	0,0000171030	0,0000016844	0,0002660566

The next step in order to find correlated random variables is to perform the Cholesky decomposition. It is an operation on the **correlation matrix** that essentially takes the square root of the matrix. This is shown algebraically in equation 2.7. The problem in the case of matrix, Σ, is to find a matrix M, which, when multiplied by itself, produces Σ.

$$MM^T = \Sigma \quad (2.7)$$

Excel does not include a function to calculate a new matrix, so we used a code in Matlab in order to produce this matrix. The code is presented below:

<u>Matlab code for Cholesky decomposition</u>

```
>>M=[1 0.265416459 0.013657809 0.008739396;0.265416459 1 -0.01671701 0.102678642;0.013657809 -0.01671701 1 0.009176615;0.008739396 0.102678642 0.009176615 1]

>>M =

   1.0000    0.2654    0.0137    0.0087

   0.2654    1.0000   -0.0167    0.1027

   0.0137   -0.0167    1.0000    0.0092

   0.0087    0.1027    0.0092    1.0000


>> n=length(M);

>> L=zeros(n,n);
```

```
>> for i=1:n
L(i,i)=sqrt(M(i,i)-L(i,:)*L(i,:)');
for j=(i+1):n
L(j,i)=(M(j,i)-L(i,:)*L(j,:)')/L(i,i);
end
end
>> format long
>> L
L =
  1.000000000000000    0                   0                   0
  0.265416459000000    0.964133861707959   0                   0
  0.013657809000000   -0.021098747912912   0.999684103649661   0
  0.008739396000000    0.104092456914744   0.011257030561607   0.994465516060734
```

So, the matrix of the Cholesky decomposition is the Table 2.29 below:

Table 2.29 Matrix of the Cholesky decomposition

	Cholesky			
	DJIA	*FTSE 500*	*CAC 40*	*Nikkei*
DJIA	1	0	0	0
FTSE 500	0,2654164590	0,9641338617	0	0
CAC 40	0,013657809	-0,021098748	0,999684104	0
Nikkei	0,008739396	0,104092457	0,011257031	0,994465516

Now, we generate the random variables as we have also do it in the process that the random variables are not correlated. We multiply the Cholesky decomposition by these random variables. We can see this process algebraically in equation 2.8.

$$\boldsymbol{\varphi = M\varepsilon} \qquad \textbf{(2.8)}$$

Where ε is the uncorrelated normally distributed random variables, M is the matrix of the Cholesky decomposition and φ is the correlated normally distributed random variables.

After this process, we use these correlated random variables to the equation of the Geometric Brownian Motion (equation 2.1) in order to refund us the indices returns for 100.000 simulations. Then, we follow the same way to calculate V@R and ES. More exactly, we take into consideration the weights of each index in our portfolio and calculate the value of this in each simulation. An extract of the returns and the weighted yield of the portfolio, after the use of the Geometric Brownian Motion, that consists of the four indices, is shown in Table 2.30 below:

Table 2.30 Returns of indices and final yield of the portfolio (with correlated random variables)

SIMULATED RETURNS (with correlated random variables)					
	DJIA	FTSE 500	CAC 40	Nikkei	**Weighted average**
Number of simulations	40%	20%	30%	10%	
1	-3,0019%	-3,8145%	-4,2531%	-6,2194%	-3,8616%
2	-0,4582%	-0,5901%	-0,6999%	-1,0691%	-0,6182%
3	-1,1501%	-1,4672%	-1,6664%	-2,4700%	-1,5004%
4	0,6993%	0,8771%	0,9169%	1,2744%	0,8577%
5	0,2029%	0,2478%	0,2235%	0,2693%	0,2247%
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
99.996	-1,2576%	-1,6035%	-1,8167%	-2,6878%	-1,6375%
99.997	1,5804%	1,9940%	2,1477%	3,0584%	1,9811%
99.998	-0,2237%	-0,2929%	-0,3724%	-0,5944%	-0,3192%
99.999	-0,7666%	-0,9811%	-1,1307%	-1,6936%	-1,0114%
100.000	-0,3041%	-0,3949%	-0,4848%	-0,7573%	-0,4218%

All things considered, the weighted yields of the portfolio are now ranked from smallest to highest. An extract of the results of this is shown in Table 2.31. The worst yield is the 8.573rd simulation. The one day 99% V@R can be estimated as the 1000th worst yield (as we have a 99% confidence level and 100.000 simulations). This is -3,0230%. The one day 95% V@R can be estimated also as the 5000th worst yield (as we have a 95% confidence level and 100.000 simulations). This is -2,1315%.

Table 2.31 Yields ranked from lowest to highest for 100.000 simulations for the portfolio

Number of simulation	**Ranked Average**	
8.573	-5,6698%	
62.275	-5,3732%	
81.706	-5,1838%	
12.335	-5,1020%	
78.457	-5,0810%	
.	.	
.	.	
.	.	
85.989	-3,0230%	99% V@R
.	.	
.	.	
.	.	
6.364	-2,1315%	95% V@R
.	.	
.	.	
.	.	
52.440	5,3596%	
41.905	5,4427%	
80.130	5,6034%	
55.461	5,6630%	
1.311	5,6658%	

In order to calculate the 99% expected shortfall with Monte Carlo simulation, we should average the 1000 worst yields, as have already ranked above. More exactly, the average for these yields is -3,4550% and this is the estimation of the expected shortfall for the 99% confidence level. The expected shortfall for a confidence level of 95% is also -2,6735%.

2.3 Indicative references

As we have also seen above (Chapter 1), V@R has several drawbacks. We can see some papers that reinforce this aspect. Artzner et al. (1997,1999) have shown that Value at risk ignores any loss beyond the value at risk level, such as it is not subadditive, that is the violation of one of the axioms of coherence. Furthermore, Yamai and Yoshiba (2002) have shown two more disadvantages. The first one is that rational investors hoping to maximize expected utility may be fooled by the information offered by Value at risk. The second one is that Value at risk is not easy to be used when investors want to optimize their portfolios. That's why Artznet et al. (1999) introduced a new measure of risk named Expected Shortfall.

Both V@R and ES have a relationship each other. The Expected shortfall has also its disadvantages. For example, Expected shortfall needs a larger sample than Value at risk for the same level of accuracy, as shown in Yamai and Yoshiba (2002).

Nevertheless, the Expected shortfall has been extensively applied in a lot of fields. Some applications include: operational risk in Taiwanese commercial banks (Lee and Fang 2010); reward-risk stock selection criteria (Rachev et al. 2007); extreme daily changes in US Dollar London inter-bank offer rates (Krehbiel and Adkins 2008); Shanghai stock exchange (Li and Li 2006, Fan et al. 2008a); exchange rate risk of CNY (Wang and Wu 2008); cash flow risk measurement for Chinese non-life insurance industry (Teng and Zhang 2009); financial risk associated with US movie box office earnings (Bi and Giles 2007, Bi,G. and Giles 2009); and extreme dependence between European electricity markets (Lindstrom and Relang 2012).

Furthermore, there are some applications where Expected shortfall has been shown to be better than Value at risk. Kerkhof and Melenberg (2004) provide "*evidence that tests for expected shortfall with acceptable low levels have a better performance than tests for Value at risk in realistic financial sample sizes*"; Yamai and Yoshiba (2005) show "*how the tail risk of Value at risk can cause serious problems in certain cases, cases in which Expected shortfall can serve more aptly in its place*". In one more application, Oh and Moon (2006) show that "*Expected shortfall values are much bigger than Value at risk values, which means that Value at risk measure can underestimate tail-related risks as well*". Acerbi and Tasche (2002) show that Expected shortfall has advantages relative to Value at risk. Liang and Park (2007) also prove that Expected shortfall is superior to Value at risk as a downside risk measure.

A remarkable use of expected shortfall has also shown by Inui and Kijima (2005). This paper proves that any coherent risk measure is given by a combination of expected shortfalls and an expected shortfall gives the minimum value among coherent risk measures. As for the minimum value, Tasche (2002) also points that expected shortfall has been characterized as the smallest coherent and law invariant risk measure to prevail Value at risk.

In respect of the advantages of expected shortfall, Rockafellar and Uryasev (2002) have shown that "*expected shortfall provides optimization short-cuts which, through linear programming techniques, make practical many large-scale calculations that could otherwise be out of reach*". This paper also shows the numerical efficiency and stability of the calculations of the expected shortfall with an example of index tracking. Expected shortfall and its minimization formula were first developed by Rockafella and Uryasev (2000). In this paper, has been demonstrated the numerical effectiveness, through several case studies that include portfolio optimization and options hedging. In one more application, Peracchi and Tanase (2008) have extended the concept of the expected shortfall to the case when auxiliary information about the outcome is available in the form of a set of predictors. In this study have been used a set of Monte Carlo experiments in order to secure the accuracy of the estimators.

Moreover, in the most recent years M.B. Righi, P.S. Ceretta (2015) investigate whether there is a pattern in terms of the model advantage of ES estimation taking into consideration the asset classes, the estimation windows, and the significance level. They use 17 different estimation models of three classes: the unconditional, conditional, and quantile/expectile regressions. Regarding empirical results, they found that there are distinctions between asset classes.

Inglesias M. (2015) has also used Value at Risk and Expected Shortfall in order to analyze extreme movements of the main stocks traded in the Eurozone in the 2000-2012 period. The results are helpful for a future risk-averse investor who want to invest in the Eurozone. The main results of this analysis are two. The first one is that they can classify firms by economic sector according to their difference at the V@R estimation values in five of the seven countries that have been analyzed. This means that there are sectors in general where companies have very high or very low estimated V@R values. The second one is that they find differences according to the geographical situation of where the stocks are traded in two countries: 1) all Irish firms have a high estimation of V@R values in all sectors, 2) in Spain all firms have a very low estimation of V@R values in all sectors too. All these results are also supported by the study of ES of all firms.

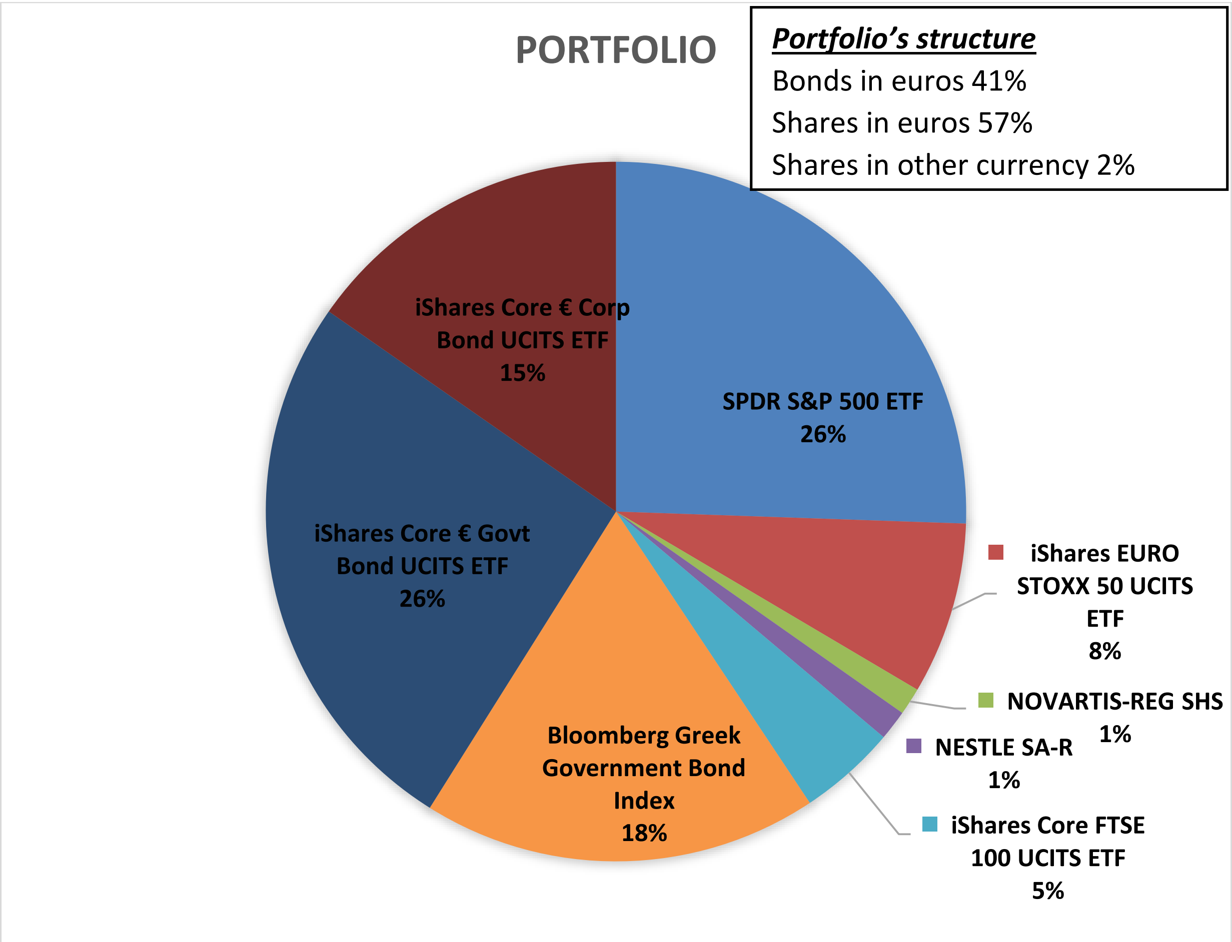
Then Du Z., Escanciano J. C. (2015), proposed some simple tools for evaluation of ES forecasts. They propose "*backtests for ES based on cumulative violations, which are the natural analog of the commonly used backtests for V@R. They establish the asymptotic properties of the tests, and investigate their finite sample performance through some Monte Carlo simulations.*" An empirical application to three major stock indices shows that V@R is unresponsive to extreme events such as those experienced during the recent financial crisis, while ES provides a more accurate description of the risk involved.

To sum up, a last one paper that has been taking into account is this of Frey and McNeil (2002) in which has been summarised how all standard models may be remade as Bernoulli mixture models. It has been shown that "*the tail of the portfolio loss distribution is driven essentially by the mixing distribution in the Bernoulli mixture representation, and that Vale at risk and Expected shortfall may be estimated in large portfolios by calculating quantiles and conditional tail expectations for this mixture distribution and scaling them appropriately*".

CHAPTER 3: V@R and ES analysis of a multi-asset portfolio

The aim of this chapter is to estimate V@R and ES with historical and Monte Carlo simulation at a multi-asset portfolio. The portfolio of our analysis is the following in Table 3.1:

Table 3.1 The portfolio for the analysis (the amounts in €)



But let's see the meaning of a multi-asset portfolio and why an investor would prefer this and not a single asset portfolio.

3.1 Multi-asset portfolios

A multi-asset portfolio contains a blend of investments for different assets classes and offers to investors the opportunity to reach income with the potential for capital growth. Investing in different assets also diversifies risk across these investments and this strategy may appear attractive to more careful investors. However, this is not mean that the value of investments and the income from them couldn't go down as well as up.

The three main assets classes in which investors typically invest are fixed income, equities, and real estate:

- Fixed income

A bond can provide a fixed level of income. More exactly, government bonds usually carrying the lowest level of underlying risk and offer the lowest yields in general, this means that investors may not meet their income needs.

Higher yields require investment in bonds that have greater credit risk, this means that these bonds may increase the income of investors but also they increase the risk of losing part of the investment.

Extra income can also be achieved with bonds that take longer to mature. These bonds offer generally higher yields.

- Equities

Equities are popular to investors who want to take a greater risk. Stocks that offer high dividend yields are concentrated in a small number of industries.

- Real estate

Real estate has offered higher yields than global equities or bonds over the past decade (UBS 2011).

Investing in a single asset portfolio has the potential to gain income, but for many investors, exposure to several multi-asset classes may be safer and better. A multi-asset portfolio gives us the opportunity to invest in assets that perform well and diversify the portfolio. It gives the opportunity also to investors to benefit from manager's skill at selecting the best asset class and this asset class that is responding to given market conditions.

However, when a manager makes a change there is no guarantee that it is the right change at the right time and this will prove beneficial.

Investors are also reminded that the past performance is not a guide to future performance (UBS 2011).

To sum up, a skilled manager should choose a multi-asset portfolio that would generate the required level of income without taking more risk than it is necessary and think about what the right mix of assets is to prevailing market conditions.

An example of one real multi-asset portfolio as presented in the official site of Alpha Asset Management A.E.D.A.K (www.alphamutual.gr) is the following:

- Cash 0,9%
- Government Bonds 50,1%
- Equities 43,0%
- Equity ETF 0,5%
- Treasury Bills 5,6%

3.2 Exchange-Traded Funds (ETFs)

As we have also presented above, our portfolio for analysis consists of a large number of ETFs. In this part, we will introduce the concept of ETFs.

Exchange-traded funds are some of the most popular and innovative new securities to hit the market since the introduction of the mutual fund (www.investinganswers.com). The first ETF was the Standard and Poor's Deposit Receipt (SPDR) which was first on the market in 1993.

ETFs are securities that resemble index funds but can be bought and sold during the day just like common stocks. More simply, ETFs are funds that track indices like the NASDAQ-100 Index, S&P 500, Dow Jones, etc. So, when you buy shares on an ETF, you buy shares of a portfolio that has the yield and return of its native index.

ETFs shares trade exactly like stocks. Unlike index mutual funds, which are priced after market closing, ETFs are priced continuously any time the market is open. For example, you can buy shares in the morning and sell them in the afternoon.

They have created a series of benefits that make them a better choice that traditional mutual funds for many reasons such as lower costs, better tax efficiency, liquidity and more. Let's see the advantages of ETFs in more details:

Single transactions: Everyone can purchase an ETF with one easy, single transaction. Basically, you are purchasing a mini portfolio, not a basket of stocks, like you do with an index.

Low cost: Since there is only one transaction per trade as referred above, commissions are usually lower on an ETF as opposed to that of an index, which requires a basket of stocks and multiple trades. ETFs have no back-end loans like traditional mutual and index funds. Furthermore, ETFs have minimal expanse ratios which make them more affordable for the investors. They are also more accessible for small investors as they can purchase as little as one share of the ETF of their choice, whereas most mutual funds have minimum investment requirements.

Tax-Advantages: Taxes are lower for ETFs than for traditional mutual funds due to the structure of each trade. When a gain is realized in a mutual fund trade, capital gain taxes are incurred immediately. In ETFs, the individual capital gains are not realized until the assets are sold with the entire fund. Therefore ETFs are an investment that can be characterized as "tax friendly" investment.

<u>Derivatives:</u> Many ETFs consist of options, swaps, and future contracts. So when the investor wants to hedge his ETFs with call and puts, some funds have this flexibility.

<u>Immediate Dividends:</u> With most ETFs, dividends are immediately reinvested back into the fund. In the case of traditional funds, the time frames may vary.

3.3 Analysis of the portfolio

In this part, we use the portfolio that has also presented at the beginning of chapter 3 (Table 3.1) in order to calculate Value at risk and Conditional value at risk or Expected shortfall and compare their results. The ways of estimation as have also analyzed in chapter 2 is 1) Historical simulation and 2) Monte Carlo simulation.

3.3.1 Historical simulation of portfolio

To begin with, Table 3.2 shows a part of 501 days of historical data on the closing prices of the assets in their currency.

Table 3.2 Data of assets of the portfolio for historical simulation

Day	*Date*	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R (CHF)	NOVARTIS-REG SHS (CHF)
0	14/12/2015	162,740	126,2477	5,7981	121,2340	31,7406	202,900	71,55	82,20
1	15/12/2015	161,910	125,9475	5,9399	120,6620	32,7743	205,030	73,10	84,40
2	16/12/2015	167,360	125,9228	5,9827	120,6367	32,8272	208,030	73,20	84,50
3	17/12/2015	168,160	126,2234	6,0244	121,1266	33,4306	204,860	73,55	85,20
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
499	3/1/2018	257,860	130,8327	7,5424	122,3749	35,6215	270,470	83,38	83,50
500	4/1/2018	261,000	130,8972	7,5673	122,5527	36,2193	271,610	83,30	83,06

The values of the Bloomberg Greek government bond, iShares-Core € Corp Bond UCITS ETF, iShares Core FTSE 100 UCITS ETF, iShares Core € Govt Bond UCITS ETF, iShares EURO STOXX 50 UCITS ETF, SPDP S&P ETF (SPY), NESTLE SA-R (CHF), NOVARTIS-REG SHS (CHF) are now adjusted for exchange rate changes so that they are measured in euros (as we have also supposed an investor in Greece). The only assets that are not in euros are the two shares of Nestle and Novartis. So, an extract from the data after exchange rate adjustments have been made is shown in Table 3.3.

Table 3.3 Data on assets for historical simulation after exchange rate adjustments

Day	***Date***	**Bloomberg greek government bond**	**iShares-Core € Corp Bond UCITS ETF**	**iShares Core FTSE 100 UCITS ETF**	**iShares Core € Govt Bond UCITS ETF**	**iShares EURO STOXX 50 UCITS ETF**	**SPDP S&P ETF (SPY)**	**NESTLE SA-R**	**NOVARTIS-REG SHS**
0	14/12/2015	162,740	126,2477	5,7981	121,2340	31,7406	202,900	65,77206	75,56203
1	15/12/2015	161,910	125,9475	5,9399	120,6620	32,7743	205,030	67,01667	77,37629
2	16/12/2015	167,360	125,9228	5,9827	120,6367	32,8272	208,030	67,1768	77,54699
3	17/12/2015	168,160	126,2234	6,0244	121,1266	33,4306	204,860	67,39483	78,06988
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
499	3/1/2018	257,860	130,8327	7,5424	122,3749	35,6215	270,470	83,45514	83,57525
500	4/1/2018	261,000	130,8972	7,5673	122,5527	36,2193	271,610	83,30000	83,0600

Table 3.4 shows the values of the market variables on January 5, 2018, for the scenarios considered. Scenario 1 (the first row in Table 3.4) shows the values of market variables on January 5, 2018, assuming that their percentage changes between January 4 and January 5, 2018, are the same as they were between December 14 and December 15, 2015; Scenario 2 (the second row in Table 3.4) shows the values of market variables on January 5, 2018, assuming these percentage changes are the same as those between December 15 and December 16, 2015; and so on. In general, Scenario I assumes that the percentage changes in the indices between January 4 and January 5 are the same as they were between Day i-1 and Day i for $1 \leq i \leq 500$. The 500 rows in Table 3.4 are the 500 scenarios considered as we have also described in Chapter 2.1.1.

Table 3.4 Scenarios generated for January 5, 2018, using data in Table 3.3

Scenario number	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
1	259,668858	130,58594	7,7524395	121,97454	37,398807	274,4613	84,8763	85,05429
2	269,785436	130,87157	7,6217944	122,52696	36,277782	275,5842	83,49904	83,24324
3	262,24761	131,20975	7,6200562	123,05039	36,885175	267,47116	83,57036	83,62006
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
499	262,752635	130,99193	7,5508177	122,38735	36,279691	275,28426	83,47024	84,76567
500	264,178236	130,96183	7,5922491	122,73078	36,827159	272,7548	83,14515	82,54793

Then, we calculate the value of the portfolio such as its losses with the way that we have also analyzed in Chapter 2.1.1. We can see the results in the following tables (Table 3.5).

Table 3.5 Portfolio’s value and losses

Scenario number	Portfolio's Value	Gain/Loss (in €)
1	11.649.676,89	51.392,95
2	11.717.909,52	119.625,58
3	11.601.753,54	3.469,60
4	11.522.182,38	-76.101,56
.	.	.
.	.	.
.	.	.
498	11.577.721,54	-20.562,40
499	11.653.570,06	55.286,12
500	11.657.790,62	59.506,68

The losses that have already arisen from the 500 different scenarios are now ranked. An extract of the results of this is shown in Table 3.6. The worst scenario is number 128. The one day 99% V@R can be estimated as the fifth worst loss (as we have a 99% confidence level and 500 scenarios). This is €137.149,83 and -1,1638%. The one day 95% V@R can be estimated also as the 25th worst loss (as we have a 95% confidence level and 500 scenarios). This is €66.917,92 and -0,5678%.

Table 3.6 Losses ranked from highest to lowest for 500 scenarios for portfolio

Scenario number	Loss (in €)	Loss (%)
128	- 316.182,49	-2,6830%
22	- 217.807,19	-1,8482%
35	- 182.397,14	-1,5477%
14	- 144.642,72	-1,2274%
38	- 137.149,83	-1,1638%
20	- 125.848,71	-1,0679%
119	- 117.209,98	-0,9946%
179	- 116.552,44	-0,9890%
36	- 113.834,24	-0,9659%
275	- 108.913,74	-0,9242%
11	- 105.536,55	-0,8955%
31	- 100.635,71	-0,8539%
34	- 91.211,42	-0,7740%
120	- 88.935,97	-0,7547%
153	- 85.125,85	-0,7223%
73	- 83.849,76	-0,7115%
118	- 81.381,89	-0,6906%
25	- 78.524,46	-0,6663%
4	- 76.101,56	-0,6458%
373	- 75.791,87	-0,6431%
402	- 75.540,22	-0,6410%
129	- 73.014,45	-0,6196%
246	- 72.184,56	-0,6125%
18	- 68.479,73	-0,5811%
379	- 66.917,92	-0,5678%
181	- 64.816,93	-0,5500%
13	- 64.704,08	-0,5490%
375	- 64.434,40	-0,5468%
347	- 62.797,76	-0,5329%
407	- 62.301,49	-0,5287%
.	.	.
.	.	.
.	.	.

99% V@R (row 38)

95% V@R (row 379)

99% ES with **equal weights**= -1,69401%

99% ES with **unequal weights**= -2,16840%

95% ES with **equal weights**= -0,95165%

95% ES with **unequal weights**= -2,14201%

In order to calculate expected shortfall with historical simulation, we should average the five observations of the worst losses, as have already ranked above. More exactly, in our example, the five worst losses are from scenarios 128, 22, 35, 14 and 38 (see Table 3.6 above). The average for these scenarios is €199.635,876 and -1,69401% and this is the estimation of the expected shortfall for the 99% confidence level. The expected shortfall for a confidence level of 95% is also €112.150,816 and -0,95165%.

Another way to calculate ES is the following. Unlike the simple average of the losses as we have also used in our analysis, we prefer to choose weights for each loss in order to make a weighted average. But these weights aren't random. We prefer, the smallest observation of the five worst losses (for 99% V@R) to have the highest weight, the next observation a smaller weight and so on. So Expected shortfall will be calculated as follows:

$$ES = \sum_{i=1}^{n} w_i r_i \quad \textbf{(3.1)}$$

Where $\sum_{i=1}^{n} w_i = 1$

$w_i = x^i \quad x \in (0,1)$

So, $$\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} x^i = 1 \quad \textbf{(3.2)}$$

But, equation 3.2 is a geometric progression, so:

$$\sum_{i=1}^{n} x^i = \frac{x(1-x^n)}{1-x} = 1 \quad \textbf{(3.3)}$$

In this way, we find the suitable x and finally find the suitable weight for each loss. Then, use the equation 3.1 in order to find the Expected Shortfall.

The 99% ES that we calculate in this way is -2,16840% and the 95% ES is -2,14201% and it is a better estimation than this of equal weights.

In this part of the chapter, we will do just the same analysis for the same assets but we suppose that the whole portfolio consists of one asset each time. So, we will calculate V@R and ES for eight different portfolios (one portfolio for each asset) and then we will compare the results of them with the V@R and ES of the initial portfolio that consist of all assets.

To begin with, Tables 3.7-3.14 show the data of all assets in their currency separately, such as their value after exchange rate adjustments (our currency is euros as we are in Greece).

Table 3.7 Data on Bloomberg greek government bond for historical simulation

Day	*Date*	Bloomberg greek government bond
0	14/12/2015	162,740
1	15/12/2015	161,910
2	16/12/2015	167,360
3	17/12/2015	168,160
.	.	.
.	.	.
.	.	.
499	3/1/2018	257,860
500	4/1/2018	261,000

Table 3.8 Data on iShares-Core € Corp Bond UCITS ETF for historical simulation

Day	*Date*	iShares-Core € Corp Bond UCITS ETF
0	14/12/2015	126,2477
1	15/12/2015	125,9475
2	16/12/2015	125,9228
3	17/12/2015	126,2234
.	.	.
.	.	.
.	.	.
499	3/1/2018	130,8327
500	4/1/2018	130,8972

Table 3.9 Data on iShares-Core € Corp Bond UCITS ETF for historical simulation

Day	*Date*	iShares Core FTSE 100 UCITS ETF
0	14/12/2015	5,798084
1	15/12/2015	5,939947
2	16/12/2015	5,982731
3	17/12/2015	6,024449
.	.	.
.	.	.
.	.	.
499	3/1/2018	7,542411
500	4/1/2018	7,567289

Table 3.10 Data on iShares Core € Govt Bond UCITS ETF ETF for historical simulation

Day	*Date*	iShares Core € Govt Bond UCITS ETF
0	14/12/2015	121,2340
1	15/12/2015	120,6620
2	16/12/2015	120,6367
3	17/12/2015	121,1266
.	.	.
.	.	.
.	.	.
499	3/1/2018	122,3749
500	4/1/2018	122,5527

Table 3.11 Data on iShares EURO STOXX 50 UCITS ETF for historical simulation

Day	*Date*	iShares EURO STOXX 50 UCITS ETF
0	14/12/2015	31,7406
1	15/12/2015	32,7743
2	16/12/2015	32,8272
3	17/12/2015	33,4306
.	.	.
.	.	.
.	.	.
499	3/1/2018	35,6215
500	4/1/2018	36,2193

Table 3.12 Data on iShares SPDP S&P ETF (SPY) for historical simulation

Day	*Date*	SPDP S&P ETF (SPY)
0	14/12/2015	202,900
1	15/12/2015	205,030
2	16/12/2015	208,030
3	17/12/2015	204,860
.	.	.
.	.	.
.	.	.
499	3/1/2018	270,470
500	4/1/2018	271,610

Table 3.13 Data on NESTLE SA-R (CHF) for historical simulation after exchange rate adjustments

Day	*Date*	NESTLE SA-R (CHF)	*Exchange rate (EUR/CHF)*	*Adjusted price*
0	14/12/2015	71,55	1,08130	65,772059
1	15/12/2015	73,10	1,07840	67,016671
2	16/12/2015	73,20	1,07950	67,176802
3	17/12/2015	73,55	1,07785	67,394832
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
499	3/1/2018	83,38	1,17735	83,455137
500	4/1/2018	83,30	1,17629	83,3

Table 3.14 Data on NOVARTIS-REG SHS (CHF) for historical simulation after exchange rate adjustments

Day	*Date*	NOVARTIS-REG SHS (CHF)	*Exchange rate (EUR/CHF)*	*Adjusted price*
0	14/12/2015	82,2	1,08130	75,56203
1	15/12/2015	84,4	1,07840	77,37629
2	16/12/2015	84,5	1,07950	77,54699
3	17/12/2015	85,2	1,07785	78,06988
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
499	3/1/2018	83,5	1,17735	83,57525
500	4/1/2018	83,06	1,17629	83,06

Then, we calculate the 500 scenarios for each asset as we analyzed in Chapter 2.1.1. An extract of these scenarios is shown in Table 3.15 below.

Table 3.15 Scenarios generated for January 5, 2018, using data in Tables 3.7-3.14

Scenario number	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
1	259,6688583	130,585939	7,752440	121,9745	37,39881	274,4613	84,8763	85,054292
2	269,7854364	130,871569	7,621794	122,527	36,27778	275,5842	83,49904	83,243236
3	262,2476099	131,209747	7,620056	123,0504	36,88518	267,4712	83,57036	83,620063
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
499	262,7526353	130,991934	7,550818	122,3873	36,27969	275,2843	83,47024	84,765666
500	264,1782363	130,961832	7,592249	122,7308	36,82716	272,7548	83,14515	82,547931

Then, we calculate the value of each portfolio such as the losses of them and we ranked these losses from biggest to smallest. We can see an extract of the results in the following tables (Table 3.16-3.24).

Table 3.16 The value of each portfolio

Scenario number	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
1	2.106.909,4	1.771.083,1	535.510,8	2.974.762,3	952.972,0	2.994.123,3	164.692,2	149.623,7
2	2.188.993,7	1.774.957,0	526.486,3	2.988.235,1	924.406,8	3.006.373,1	162.019,7	146.437,8
3	2.127.833,0	1.779.543,6	526.366,3	3.001.000,8	939.884,0	2.917.867,1	162.158,1	147.100,7
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
499	2.131.930,7	1.776.589,5	521.583,5	2.984.830,2	924.455,4	3.003.101,0	161.963,9	149.116,0
500	2.143.497,8	1.776.181,2	524.445,4	2.993.205,9	938.405,6	2.975.506,9	161.333,1	145.214,7

Table 3.17 Losses ranked from highest to lowest for 500 scenarios for Bloomberg greek government bond

Scenario number	Ranked Losses	Losses (%)
22	-125.882,626	-5,9443%
128	-117.176,295	-5,5332%
35	-89.548,413	-4,2285%
36	-79.304,780	-3,7448%
275	-67.770,244	3,2002%
21	-58.171,585	-2,7469%
246	-56.912,652	-2,6875%
119	-52.447,099	-2,4766%
120	-50.351,315	-2,3776%
72	-48.713,374	-2,3003%
89	-48.277,587	-2,2797%
12	-43.741,875	-2,0655%
38	-41.030,917	-1,9375%
352	-40.016,833	-1,8896%
274	-39.938,465	-1,8859%
83	-37.191,458	-1,7562%
79	-34.043,012	-1,6075%
37	-33.820,191	-1,5970%
240	-33.703,050	-1,5915%
26	-31.778,778	-1,5006%
20	-31.133,217	-1,4701%
25	-30.906,532	-1,4594%
73	-30.792,280	-1,4540%
286	-28.582,590	-1,3497%
61	-27.926,405	-1,3187%
247	-27.639,129	-1,3051%
14	-25.601,820	-1,2089%
33	-25.577,835	-1,2078%
64	-24.653,788	-1,1642%
125	-24.583,699	-1,1609%
.	.	.
.	.	.
.	.	.

99% V@R (row 275)

95% V@R (row 61)

99% ES with **equal weights**= -4,5302%

99% ES with **unequal weights**= -5,3714%

95% ES with **equal weights**= -2,41612%

95% ES with **unequal weights**= -5.30061%

Table 3.18 Losses ranked from highest to lowest for 500 scenarios for iShares Core € Corp Bond UCITS ETF

Scenario number	Ranked Losses	Losses (%)
19	-19.741,972	-1,1120%
141	-14.068,748	-0,7925%
384	-13.707,663	-0,7721%
264	-12.585,138	-0,7089%
373	-7.040,756	-0,3966%
223	-6.127,082	-0,3451%
295	-5.929,986	-0,3340%
379	-5.763,234	-0,3246%
375	-5.684,057	-0,3202%
302	-5.547,993	-0,3125%
179	-5.156,125	-0,2904%
153	-5.124,061	-0,2886%
213	-5.053,708	-0,2847%
365	-5.026,151	-0,2831%
60	-4.927,121	-0,2775%
225	-4.768,490	-0,2686%
331	-4.756,090	-0,2679%
300	-4.734,120	-0,2667%
197	-4.733,446	-0,2666%
212	-4.696,030	-0,2645%
465	-4.581,121	-0,2580%
39	-4.445,164	-0,2504%
492	-4.394,835	-0,2476%
56	-4.384,248	-0,2470%
21	-4.320,160	-0,2433%
1	-4.222,068	-0,2378%
14	-4.220,192	-0,2377%
36	-4.032,951	-0,2272%
392	-3.990,548	-0,2248%
272	-3.950,265	-0,2225%
.	.	.
.	.	.
.	.	.

99% V@R (row 373)

95% V@R (row 21)

99% ES with **equal weights**= -0,7564%

99% ES with **unequal weights**= -0,9333%

95% ES with **equal weights**= -0,3769%

95% ES with **unequal weights**= -0,91783%

Table 3.19 Losses ranked from highest to lowest for 500 scenarios for iShares Core FTSE 100 UCITS ETF

Scenario number	Ranked Losses	Losses (%)
22	-18.080,184	-3,4589%
128	-16.396,687	-3,1368%
11	-15.025,971	-2,8746%
35	-14.186,039	-2,7139%
129	-13.282,021	-2,5409%
57	-12.900,234	-2,4679%
327	-12.835,523	-2,4555%
425	-12.458,311	-2,3834%
38	-11.987,174	-2,2932%
31	-11.912,842	-2,2790%
120	-10.495,725	-2,0079%
365	-10.263,439	-1,9635%
14	-10.195,181	-1,9504%
20	-10.095,597	-1,9314%
118	-9.717,286	-1,8590%
122	-9.045,681	-1,7305%
266	-8.366,546	-1,6006%
46	-8.336,407	-1,5948%
103	-8.166,040	-1,5622%
67	-7.522,399	-1,4391%
219	-7.494,716	-1,4338%
224	-7.491,464	-1,4332%
32	-7.485,627	-1,4320%
489	-7.050,536	-1,3488%
190	-6.915,035	-1,3229%
91	-6.624,587	-1,2673%
135	-6.531,413	-1,2495%
45	-6.503,190	-1,2441%
93	-6.237,400	-1,1933%
179	-6.233,935	-1,1926%
.	.	.
.	.	.
.	.	.

99% V@R (row 129)

95% V@R (row 190)

99% ES with **equal weights**= -2,9450%

99% ES with **unequal weights**= -3,2175%

95% ES with **equal weights**= -2,0486%

95% ES with **unequal weights**= -3,19765%

Table 3.20 Losses ranked from highest to lowest for 500 scenarios for iShares Core € Govt Bond UCITS ETF

99% ES with **equal weights**= -0,7428%

99% ES with **unequal weights**= -0,8591%

Scenario number	Ranked Losses	Losses (%)
223	-31.009,945	-1,0375%
379	-20.377,360	-0,6818%
375	-19.970,225	-0,6682%
141	-19.868,728	-0,6648%
179	-19.785,910	-0,6620%
373	-19.631,636	-0,6568%
197	-19.603,963	-0,6559%
19	-18.416,798	-0,6162%
213	-17.663,144	-0,5910%
257	-17.086,599	-0,5717%
178	-15.934,871	-0,5331%
242	-15.658,528	-0,5239%
212	-15.521,550	-0,5193%
384	-15.229,428	-0,5095%
224	-15.169,332	-0,5075%
272	-14.794,962	-0,4950%
492	-14.296,046	-0,4783%
1	-14.100,396	-0,4718%
465	-14.077,644	-0,4710%
300	-14.057,040	-0,4703%
153	-13.944,974	-0,4666%
85	-13.494,841	-0,4515%
232	-13.200,192	-0,4416%
392	-13.037,970	-0,4362%
237	-12.721,330	-0,4256%
51	-12.385,615	-0,4144%
302	-12.342,267	-0,4129%
225	-11.926,758	-0,3990%
273	-11.902,915	-0,3982%
295	-11.750,389	-0,3931%
.	.	.
.	.	.
.	.	.

99% V@R (row 179)

95% V@R (row 237)

95% ES with **equal weights**= -0,5603%

95% ES with **unequal weights**= -0,85498%

Table 3.21 Losses ranked from highest to lowest for 500 scenarios for iShares EURO STOXX 50 UCITS ETF

99% ES with **equal weights**= -4,6391%

99% ES with **unequal weights**= -6,3142%

Scenario number	Ranked Losses	Losses (%)
128	- 79.606,012	-8,6255%
38	-39.287,054	-4,2568%
11	-34.634,989	-3,7528%
22	-30.322,868	-3,2855%
35	-30.222,943	-3,2747%
91	-27.779,055	-3,0099%
129	-26.168,596	-2,8354%
153	-25.868,485	-2,8029%
402	-24.315,843	-2,6347%
118	-24.052,196	-2,6061%
73	-22.042,475	-2,3883%
20	-21.846,650	-2,3671%
134	-21.840,069	-2,3664%
46	-21.477,977	-2,3272%
31	-21.133,304	-2,2898%
28	-19.423,013	-2,1045%
120	-18.267,798	-1,9794%
119	-18.226,023	-1,9748%
32	-17.266,714	-1,8709%
67	-16.895,138	-1,8306%
375	-16.793,480	-1,8196%
135	-16.656,510	-1,8048%
36	-16.119,231	-1,7466%
14	-16.071,497	-1,7414%
71	-15.875,413	-1,7201%
15	-15.323,998	-1,6604%
19	-14.711,418	-1,5940%
45	-14.642,128	-1,5865%
347	-14.344,878	-1,5543%
92	-14.204,786	-1,5391%
.	.	.
.	.	.
.	.	.

99% V@R (scenario 35)

95% V@R (scenario 71)

95% ES with **equal weights**= -2,6966%

95% ES with **unequal weights**= -6,24385%

Table 3.22 Losses ranked from highest to lowest for 500 scenarios for SPDP S&P ETF (SPY)

99% ES with **equal weights**= -2,6524%

99% ES with **unequal weights**= -3,0290%

Scenario number	Ranked Losses	Losses (%)
128	-106.399,352	-3,5909%
18	-73.899,503	-2,4941%
14	-71.087,399	-2,3992%
179	-70.919,340	-2,3935%
11	-70.647,400	-2,3843%
4	-70.003,944	-2,3626%
20	-63.604,621	-2,1466%
34	-56.445,805	-1,9050%
31	-53.400,121	-1,8022%
129	-53.067,241	-1,7910%
347	-52.576,047	-1,7744%
407	-46.195,918	-1,5591%
3	-45.151,024	-1,5238%
25	-44.790,533	-1,5117%
181	-42.594,927	-1,4376%
402	-41.823,795	-1,4115%
35	-39.885,267	-1,3461%
38	-38.543,066	-1,3008%
309	-38.043,566	-1,2839%
22	-37.971,253	-1,2815%
45	-37.421,833	-1,2630%
201	-37.421,538	-1,2630%
13	-37.376,173	-1,2614%
75	-35.455,164	-1,1966%
15	-32.523,724	-1,0977%
55	-32.349,617	-1,0918%
27	-32.247,359	-1,0883%
73	-29.594,266	-0,9988%
118	-28.082,169	-0,9478%
98	-27.718,328	-0,9355%
.	.	.
.	.	.
.	.	.

99% V@R (row of scenario 11)

95% V@R (row of scenario 15)

95% ES with **equal weights**= -1,7513%

95% ES with **unequal weights**= -3,01097%

Table 3.23 Losses ranked from highest to lowest for 500 scenarios for NESTLE SA-R

Scenario number	Ranked Losses	Losses (%)
42	-6.527,245	-4,0383%
77	-5.361,596	-3,3171%
38	-4.886,342	-3,0231%
14	-4.109,980	-2,5428%
223	-3.989,941	-2,4685%
360	-3.647,778	-2,2568%
375	-3.475,787	-2,1504%
119	-3.325,562	-2,0575%
22	-3.265,798	-2,0205%
454	-3.025,123	-1,8716%
19	-2.916,025	-1,8041%
266	-2.880,478	-1,7821%
45	-2.865,432	-1,7728%
36	-2.834,176	-1,7535%
237	-2.813,453	-1,7406%
151	-2.796,999	-1,7305%
215	-2.771,781	-1,7149%
189	-2.756,104	-1,7052%
20	-2.749,389	-1,7010%
34	-2.654,713	-1,6424%
33	-2.587,162	-1,6006%
369	-2.542,264	-1,5729%
11	-2.542,248	-1,5728%
323	-2.433,231	-1,5054%
35	-2.393,131	-1,4806%
118	-2.308,398	-1,4282%
401	-2.290,470	-1,4171%
373	-2.259,882	-1,3982%
91	-2.214,913	-1,3703%
480	-2.205,723	-1,3646%
.	.	.
.	.	.
.	.	.

99% V@R (→ scenario 223)

99% ES with **equal weights**= -3,0780%

99% ES with **unequal weights**= -3,5645%

95% V@R (→ scenario 35)

95% ES with **equal weights**= -2,0330%

95% ES with **unequal weights**= -3,53000%

Table 3.24 Losses ranked from highest to lowest for 500 scenarios for NOVARTIS-REG SHS

99% ES with **equal weights**= -3,5820%

99% ES with **unequal weights**= -3.6468%

Scenario number	Ranked Losses	Losses (%)
35	-5.392,164	-3,6903%
27	-5.338,592	-3,6537%
28	-5.210,718	-3,5662%
38	-5.179,306	-3,5447%
296	-5.048,838	-3,4554%
231	-4.585,162	-3,1380%
33	-4.406,834	-3,0160%
454	-4.066,743	-2,7832%
20	-4.006,125	-2,7418%
151	-3.874,704	-2,6518%
453	-3.732,826	-2,5547%
211	-3.713,106	-2,5412%
266	-3.648,529	-2,4970%
15	-3.571,262	-2,4441%
264	-3.378,024	-2,3119%
22	-3.320,771	-2,2727%
118	-3.194,196	-2,1861%
14	-3.122,982	-2,1373%
91	-3.112,444	-2,1301%
237	-3.104,682	-2,1248%
36	-3.003,939	-2,0559%
375	-2.910,093	-1,9916%
299	-2.886,206	-1,9753%
401	-2.884,639	-1,9742%
120	-2.883,211	-1,9732%
309	-2.882,844	-1,9730%
119	-2.880,035	-1,9711%
174	-2.820,802	-1,9305%
493	-2.726,179	-1,8658%
212	-2.659,370	-1,8200%
.	.	.
.	.	.
.	.	.

99% V@R (row 296)

95% V@R (row 120)

95% ES with **equal weights**= -2,6165%

95% ES with **unequal weights**= -3.62776%

As a result, the 99% V@R and 99% ES (calculated with the equal weights) of all assets are shown in Table 3.25.

Table 3.25 99% V@R and 99% ES for eight portfolios

Portfolios	99% Value-at-risk	99% Expected Shortfall
Bloomberg greek government bond	-3,2002%	-4,5302%
iShares-Core € Corp Bond UCITS ETF	-0,3966%	-0,7564%
iShares Core FTSE 100 UCITS ETF	-2,5409%	-2,9450%
iShares Core € Govt Bond UCITS ETF	-0,6620%	-0,7428%
iShares EURO STOXX 50 UCITS ETF	-3,2747%	-4,6391%
SPDP S&P ETF (SPY)	-2,3843%	-2,6524%
NESTLE SA-R	-2,4685%	-3,0780%
NOVARTIS-REG SHS	-3,4554%	-3,5820%

In this part, we want to check the benefits of diversification. In the example above we can observe that for 99% V@R, the amount €217.788,05, **is bigger** than the amount of V@R that we have already calculated above for the portfolio of all assets together, which is €137.149,834. This represents, the benefits of diversification, even though that the measure of **V@R is not subadditive**.

Also, for ES, we can observe that the amount of 278.576,26€ **is also bigger** than the amount of ES that we have also calculated above for the portfolio of all assets together which is €199.635,876. And this is expected as the measure of **ES is always subadditive**.

Moreover, this analysis should be done again for 95% V@R and 95% ES in order to check again the benefits of diversification.

Table 3.26 95% V@R and 95% ES for eight portfolios

Portfolios	95% Value-at-risk	95% Expected Shortfall
Bloomberg greek government bond	-1,3187%	-2,4161%
iShares-Core € Corp Bond UCITS ETF	-0,2433%	-0,3769%
iShares Core FTSE 100 UCITS ETF	-1,3229%	-2,0486%
iShares Core € Govt Bond UCITS ETF	-0,4256%	-0,5603%
iShares EURO STOXX 50 UCITS ETF	-1,7201%	-2,6966%
SPDP S&P ETF (SPY)	-1,0977%	-1,7513%
NESTLE SA-R	-1,4806%	-2,0330%
NOVARTIS-REG SHS	-1,9732%	-2,6165%

In this part, we want to check the benefits of diversification. In the example above we can observe that for 95% V@R, the amount 105.558,41€, **is bigger** than the amount of V@R that we have already calculated above for the portfolio of all assets together, which is 66.917,916€. This represents, the benefits of diversification, even though that the measure of **V@R is not subadditive**.

Also, for ES, we can observe that the amount of 169.199,55 € **is also bigger** than the amount of ES that we have also calculated above for the portfolio of all assets together which is 112.150,816€. And this is expected as the measure of **ES is always subadditive.**

3.3.2 Monte Carlo simulation of portfolio

We will use the same portfolio with this in historical simulation in order to compare the results. So we have again an investor in Greece, who owns, on January 5, 2018, a portfolio worth €11.598.283,94 consisting of investments in eight different assets. We have also a part of 501 days of historical data on the closing prices of these assets in their currency. Their values are adjusted for exchange rate changes so that they are measured in euros (as we have also supposed an investor in Greece) just as we calculated them in historical simulation above. Now, we calculate the returns of them. An extract of the returns of the assets of the portfolio is shown in the Table 3.26 below.

Table 3.26 Historical returns of all assets of the portfolio

Day	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
1	-0,510%	-0,238%	2,447%	-0,472%	3,256%	1,050%	1,892%	2,401%
2	3,366%	-0,020%	0,720%	-0,021%	0,161%	1,463%	0,239%	0,221%
3	0,478%	0,239%	0,697%	0,406%	1,838%	-1,524%	0,325%	0,674%
4	0,059%	0,141%	-0,824%	0,300%	-1,385%	-2,363%	-0,634%	-0,482%
5	-0,553%	0,012%	-0,292%	-0,093%	-1,385%	0,825%	-0,540%	-0,398%
6	0,114%	-0,188%	0,796%	-0,322%	0,045%	0,907%	-1,353%	-1,012%
7	-0,710%	-0,089%	2,596%	-0,236%	2,247%	1,238%	2,626%	2,068%
8	0,259%	0,007%	0,245%	0,007%	-0,068%	-0,393%	0,573%	1,115%
9	0,150%	0,010%	0,956%	0,015%	0,903%	1,067%	1,913%	1,788%
10	0,443%	0,051%	-0,640%	0,127%	-0,793%	-0,709%	-0,331%	-0,332%
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
498	0,004%	-0,018%	0,851%	-0,238%	-0,576%	-0,377%	-0,263%	-0,507%
499	0,672%	0,072%	-0,218%	-0,135%	0,167%	1,353%	0,204%	2,054%
500	1,218%	0,049%	0,330%	0,145%	1,678%	0,421%	-0,186%	-0,617%

The first step for the Monte Carlo simulation, as mentioned above, is to calculate the parameters in the Geometric Brownian Motion for all assets. The results of the calculation of these parameters are shown in the Table 3.27 below. We should remind there the equation of the Geometric Brownian Motion (equation 2.2) as was presented above in Chapter 2.

$$R_{t+\Delta_t} = \ln\left(\frac{S_{t+\Delta_t}}{S_t}\right) = k\Delta_t + \sigma\varepsilon_t\sqrt{\Delta_t} \quad (2.2)$$

Table 3.27 Parameters of Geometric Brownian Motion

Geometric Brownian Motion								
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
Number of observations	500	500	500	500	500	500	500	500
Min daily return	-5,9443%	-1,1120%	-3,4589%	-1,0375%	-8,6255%	-3,5909%	-4,0383%	-3,6903%
Max daily return	5,5009%	0,7477%	3,5765%	0,6956%	4,0188%	2,4377%	4,6864%	4,9681%
Number of trading days per year	252	252	252	252	252	252	252	252
Time increment (Δt) for one day	0,003968 254	0,003968 254	0,003968 254	0,003968 254	0,003968 254	0,003968 254	0,003968 254	0,003968 254
Average daily return	0,1000%	0,0074%	0,0572%	0,0024%	0,0327%	0,0607%	0,0525%	0,0258%
Daily standard deviation	1,0513%	0,1537%	0,8811%	0,2361%	1,1180%	0,6853%	1,0223%	1,1720%
Annualised mean return for one year (μ)	25,2095 %	1,8527%	14,4031%	0,6155%	8,2399%	15,2965%	13,2227%	6,4991%
Annualised standard deviation (σ)	16,6895 %	2,4406%	13,9871%	3,7472%	17,7484%	10,8781%	16,2278%	18,6054%
Expected return (k)	23,8168 %	1,8229%	13,4249%	0,5453%	6,6649%	14,7049%	11,9060%	4,7683%
Number of iteration of trials	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000

The next step is to generate normally distributed random numbers. We have also generated 100.000 of these numbers, as we decided to have 100.000 simulations in order to have better results. An extract of these numbers is presented also in Table 3.28 below:

Table 3.28 Normally distributed random numbers

	Random Standard Normal							
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
1	-2,95462	-2,95462	-2,95462	-2,95462	-2,95462	-2,95462	-2,95462	-2,95462
2	-0,50279	-0,50279	-0,50279	-0,50279	-0,50279	-0,50279	-0,50279	-0,50279
3	-1,16968	-1,16968	-1,16968	-1,16968	-1,16968	-1,16968	-1,16968	-1,16968
4	0,61289	0,61289	0,61289	0,61289	0,61289	0,61289	0,61289	0,61289
5	0,13438	0,13438	0,13438	0,13438	0,13438	0,13438	0,13438	0,13438
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
99.996	-1,27338	-1,27338	-1,27338	-1,27338	-1,27338	-1,27338	-1,27338	-1,27338
99.997	1,46214	1,46214	1,46214	1,46214	1,46214	1,46214	1,46214	1,46214
99.998	-0,27676	-0,27676	-0,27676	-0,27676	-0,27676	-0,27676	-0,27676	-0,27676
99.999	-0,80006	-0,80006	-0,80006	-0,80006	-0,80006	-0,80006	-0,80006	-0,80006
100.000	-0,35434	-0,35434	-0,35434	-0,35434	-0,35434	-0,35434	-0,35434	-0,35434

Then, we use these numbers to the equation of the Geometric Brownian Motion (equation 2.2) in order to refund us the indices returns for 100.000 simulations. After all this process, we follow the same way to calculate V@R and ES. More exactly, we take into consideration the weights of each asset in our portfolio and calculate the value of this in each simulation. An extract of the returns and the weighted yield of the portfolio, after the use of the Geometric Brownian Motion, that consists of eight assets, is shown in Table 3.29 below:

Table 3.29 Returns on assets and final yield of the portfolio

SIMULATED RETURNS									
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS	
Number of simulation	18,259%	15,307%	4,507%	25,770%	7,957%	25,547%	1,394%	1,260%	**Weighted Average**
1	-3,0118%	-0,4470%	-2,5501%	-0,6953%	-3,2770%	-1,9663%	-2,9731%	-3,4440%	**-1,7604%**
2	-0,4341%	-0,0701%	-0,3897%	-0,1165%	-0,5357%	-0,2862%	-0,4667%	-0,5704%	**-0,2670%**
3	-1,1352%	-0,1726%	-0,9773%	-0,2739%	-1,2813%	-0,7432%	-1,1485%	-1,3520%	**-0,6732%**
4	0,7389%	0,1015%	0,5933%	0,1468%	0,7117%	0,4783%	0,6738%	0,7373%	**0,4125%**
5	0,2358%	0,0279%	0,1717%	0,0339%	0,1767%	0,1504%	0,1846%	0,1764%	**0,1211%**
.	.	.	.	.	.	.	.	.	**.**
.	.	.	.	.	.	.	.	.	**.**
.	.	.	.	.	.	.	.	.	**.**
99.996	-1,2442%	-0,1885%	-1,0687%	-0,2984%	-1,3972%	-0,8142%	-1,2545%	-1,4735%	**-0,7364%**
99.997	1,6317%	0,2320%	1,3416%	0,3473%	1,6612%	1,0603%	1,5419%	1,7326%	**0,9298%**
99.998	-0,1965%	-0,0353%	-0,1906%	-0,0632%	-0,2830%	-0,1313%	-0,2357%	-0,3055%	**-0,1293%**
99.999	-0,7466%	-0,1158%	-0,6517%	-0,1867%	-0,8681%	-0,4899%	-0,7706%	-0,9188%	**-0,4481%**
100.000	-0,2780%	-0,0472%	-0,2589%	-0,0815%	-0,3697%	-0,1845%	-0,3150%	-0,3964%	**-0,1766%**

All things considered, the weighted yields of the portfolio are now ranked from smallest to highest. An extract of the results of this is shown in Table 3.30. The worst yield is the 8.573rd simulation. The one day 99% V@R can be estimated as the 1000th worst yield (as we have a 99% confidence level and 100.000 simulations). This is -1,3742%. The one day 95% V@R can be estimated also as the 5000th worst yield (as we have a 95% confidence level and 100.000 simulations). This is -0,9638%.

Table 3.30 Yields ranked from lowest to highest for 100.000 simulations for the portfolio

99% ES with **equal weights**= -1,5732%

99% ES with **unequal weights**= -2,4577060088%

95% ES with **equal weights**= -1,2133%

95% ES with **unequal weights**= -2,457706124%

Number of simulation	Ranked Average
8.573	-2,5929%
62.275	-2,4564%
81.706	-2,3692%
12.335	-2,3315%
78.457	-2,3218%
.	.
.	.
.	.
85.989	-1,3742%
.	.
.	.
.	.
6.364	-0,9638%
.	.
.	.
.	.
80.130	2,5976%
55.461	2,6251%
1.311	2,6263%

99% V@R (row 85.989, -1,3742%)

95% V@R (row 6.364, -0,9638%)

In order to calculate the 99% expected shortfall with Monte Carlo simulation, we should average the 1000 worst yields, as have already ranked above. More exactly, the average for these yields is -1,5732% and the average for unequal weights is -2,457706 and this is the estimation of the expected shortfall for the 99% confidence level. The expected shortfall for a confidence level of 95% is also -1,2133% with equal weights for the losses and -2,4577061 for unequal weights of losses.

In all this process in which we calculate V@R and ES with the Monte Carlo simulation, we have assumed that the random variables are uncorrelated. So, it is necessary to repeat all the process, **taking into account the correlation of the random variables**.

As we have also presented above we need the correlation matrix in order to make the Cholesky decomposition to make the correlated random variables. The correlation matrix is shown below (Table 3.31).

Table 3.31 Correlation Matrix for all assets

Correlation Matrix								
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
Bloomberg greek government bond	1	0,082520	0,295204	-0,027742	0,335774	0,255700	0,123808	0,173268
iShares-Core € Corp Bond UCITS ETF	0,082520	1	0,119207	0,760226	0,063303	0,015542	0,124310	0,083610
iShares Core FTSE 100 UCITS ETF	0,295204	0,119207	1	0,048927	0,797511	0,573464	0,542223	0,493962
iShares Core € Govt Bond UCITS ETF	-0,027742	0,760226	0,048927	1	-0,016025	-0,025455	0,120109	0,017240
iShares EURO STOXX 50 UCITS ETF	0,335774	0,063303	0,797511	-0,016025	1	0,632149	0,527026	0,536501
SPDP S&P ETF (SPY)	0,255700	0,015542	0,573464	-0,025455	0,632149	1	0,387065	0,395316
NESTLE SA-R	0,123808	0,124310	0,542223	0,120109	0,527026	0,387065	1	0,541899
NOVARTIS-REG SHS	0,173268	0,083610	0,493962	0,017240	0,536501	0,395316	0,541899	1

Now, we use this matrix in order to make the Cholesky decomposition as we have also mentioned in Chapter 2.2.1. The Matlab code which makes the Cholesky matrix is the following:

<u>Matlab code for Cholesky decomposition</u>

```
>> M=[1 0.082519688 0.295203791 -0.027741676 0.335773517 0.255700236 0.123807529
0.173267556;0.082519688 1 0.119207016 0.760225735 0.633029 0.015541683 0.12431025
0.83609624;0.295203791 0.119207016 1 0.048927089 0.797511151 0.573464145 0.542223069
0.493961544;-0.27741676 0.760225735 0.048927089 1 -0.016024849 -0.025454617 0.120108674
0.017239817;0.335773517 0.0633029 0.797511151 -0.016024849 1 0.63214853 0.527026488
0.536501215;0.255700236 0.015541683 0.573464145 -0.025454617 0.63214853 1 0.387064954
0.39531599;0.123807529 0.12431025 0.542223069 0.120108674 0.527026488 0.387064954 1
```

0.541898612;0.173267556 0.083609624 0.493961544 0.017239817 0.536501215 0.39531599 0.541898612 1]

M =

1.0000	0.0825	0.2952	-0.0277	0.3358	0.2557	0.1238	0.1733
0.0825	1.0000	0.1192	0.7602	0.6330	0.0155	0.1243	0.8361
0.2952	0.1192	1.0000	0.0489	0.7975	0.5735	0.5422	0.4940
-0.2774	0.7602	0.0489	1.0000	-0.0160	-0.0255	0.1201	0.0172
0.3358	0.0633	0.7975	-0.0160	1.0000	0.6321	0.5270	0.5365
0.2557	0.0155	0.5735	-0.0255	0.6321	1.0000	0.3871	0.3953
0.1238	0.1243	0.5422	0.1201	0.5270	0.3871	1.0000	0.5419
0.1733	0.0836	0.4940	0.0172	0.5365	0.3953	0.5419	1.0000

```
>> n=length(M);
L=zeros(n,n);
for i=1:n
L(i,i)=sqrt(M(i,i)-L(i,:)*L(i,:)');
for j=(i+1):n
L(j,i)=(M(j,i)-L(i,:)*L(j,:)')/L(i,i);
end
end
format long
>> L
```

L =

Columns 1 through 6

1.000000000000000	0	0	0	0	0
0.082519688000000	0.996589434567908	0	0	0	0

0.295203791000000 0.095171479829490 0.950682444986912 0 0 0

-0.277416760000000 0.785798095301610 0.058942921432255 0.549624441509377
0 0

0.335773517000000 0.035716788582984 0.731043599158461 0.010858909942224
0.592821892729241 0

0.255700236000000 -0.005577643614751 0.524372045785326 0.034488632607106
0.274580010236183 0.763568326346603

0.123807529000000 0.114484147009181 0.520446121631907 0.061527318448216
0.169071466784833 0.045304204628740

0.173267556000000 0.069548840208615 0.458821177216985 -0.029817570926935
0.237413631984083 0.061089076741798

Columns 7 through 8

0	0
0	0
0	0
0	0
0	0
0	0
0.816258333845086	0
0.284983276363352	0.782574454546551

So, the matrix of the Cholesky decomposition is the Table 3.32 below:

Table 3.32 Matrix of the Cholesky decomposition

Cholesky								
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
Bloomberg greek government bond	1	0	0	0	0	0	0	0
iShares-Core € Corp Bond UCITS ETF	0,082520	0,996589	0	0	0	0	0	0
iShares Core FTSE 100 UCITS ETF	0,295204	0,095171	0,950682	0	0	0	0	0
iShares Core € Govt Bond UCITS ETF	-0,277417	0,785798	0,058943	0,549624	0	0	0	0
iShares EURO STOXX 50 UCITS ETF	0,335774	0,035717	0,731044	0,010859	0,592822	0	0	0
SPDP S&P ETF (SPY)	0,255700	-0,005578	0,524372	0,034489	0,274580	0,763568	0	0
NESTLE SA-R	0,123808	0,114484	0,520446	0,061527	0,169071	0,045304	0,816258	0
NOVARTIS-REG SHS	0,173268	0,069549	0,458821	-0,029818	0,237414	0,061089	0,284983	0,782574

Then, we generate the random variables as we have also do it in the process that the random variables are not correlated. We multiply the Cholesky decomposition by these random variables. We can see this process algebraically in equation 2.8 (Chapter 2.2.1).

After this process, we use these correlated random variables to the equation of the Geometric Brownian Motion (equation 2.1) in order to refund us the returns of each asset for 100.000 simulations. Then, we follow the same way to calculate V@R and ES. More exactly, we take into consideration the weights of each asset in our portfolio and calculate the value of this in each simulation. An extract of the returns and the weighted yield of the portfolio, after the use of the Geometric Brownian Motion, that consists of eight assets, is shown in Table 3.33 below:

Table 3.33 Returns on assets and final yield of the portfolio (with correlated random variables)

SIMULATED RETURNS (with correlated random variables)									
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS	
Number of simulation	18,259%	15,307%	4,507%	25,770%	7,957%	25,547%	1,394%	1,260%	**Weighted Average**
1	-6,0835%	-0,8962%	-5,1244%	-1,3850%	-6,5435%	-3,9684%	-5,9598%	-6,8683%	-3,5399%
2	-0,9568%	-0,1465%	-0,8278%	-0,2339%	-1,0916%	-0,6269%	-0,9750%	-1,1531%	-0,5698%
3	-2,3513%	-0,3504%	-1,9965%	-0,5470%	-2,5745%	-1,5358%	-2,3308%	-2,7076%	-1,3777%
4	1,3760%	0,1946%	1,1273%	0,2899%	1,3893%	0,8936%	1,2933%	1,4476%	0,7817%
5	0,3755%	0,0483%	0,2888%	0,0653%	0,3253%	0,2415%	0,3204%	0,3322%	0,2020%
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
99.996	-2,5681%	-0,3821%	-2,1782%	-0,5957%	-2,8051%	-1,6771%	-2,5417%	-2,9493%	-1,5033%
99.997	3,1518%	0,4543%	2,6155%	0,6886%	3,2777%	2,0511%	3,0199%	3,4272%	1,8104%
99.998	-0,4842%	-0,0774%	-0,4317%	-0,1278%	-0,5890%	-0,3188%	-0,5154%	-0,6262%	-0,2960%
99.999	-1,5784%	-0,2374%	-1,3487%	-0,3734%	-1,7526%	-1,0320%	-1,5794%	-1,8460%	-0,9299%
100.000	-0,6464%	-0,1011%	-0,5677%	-0,1642%	-0,7615%	-0,4246%	-0,6732%	-0,8070%	-0,3900%

All things considered, the weighted yields of the portfolio are now ranked from smallest to highest. An extract of the results of this is shown in Table 3.34. The worst yield is the 8.573rd simulation. The one day 99% V@R can be estimated as the 1000th worst yield (as we have a 99% confidence level and 100.000 simulations). This is -2,7720%. The one day 95% V@R can be estimated also as the 5000th worst yield (as we have a 95% confidence level and 100.000 simulations). This is -1,9556%.

Table 3.34 Yields ranked from lowest to highest for 100.000 simulations for the portfolio

Number of simulation	**Ranked Average**
8573	-5,1958%
62275	-4,9242%
81706	-4,7507%
12335	-4,6758%
78457	-4,6566%
.	.
.	.
.	.
85989	-2,7720%
.	.
.	.
.	.
6364	-1,9556%
.	.
.	.
.	.
52440	4,9042%
41905	4,9803%
80130	5,1275%
55461	5,1821%
1311	5,1846%

99% V@R: 85989, -2,7720%

95% V@R: 6364, -1,9556%

99% ES with **equal weights**= -3,1676%

99% ES with **unequal weights**= -4,9262021%

95% ES with **equal weights**= -2,4520%

95% ES with **unequal weights**= -4,9262023%

In order to calculate the 99% expected shortfall with Monte Carlo simulation, we should average the 1000 worst yields, as have already ranked above. More exactly, the average for these yields is -3,1676% with equal weights and -4,9262021% with unequal weights and this is the estimation of the expected shortfall for the 99% confidence level. The expected shortfall for a confidence level of 95% is also -2,4520% for equal and -4,9262023% with unequal weights.

3.3.3 Monte Carlo simulation with different parameters

In this part of the analysis, we will calculate again V@R and ES with the same way for the same data, but we will change some of the parameters in order to detect the differences.

3.3.3.1 Monte Carlo simulation changing parameter of μ

We collect again the same data with the same portion of our portfolio. So, the returns are the same. The change is about the calculation of parameter μ of the Brownian motion. In our analysis above, we observe μ as the annualized mean return of the 500 returns of our assets. Now, we choose to have as μ the annualized return of the 50 first returns of the assets in our portfolio. So, our new table of parameters of Brownian Motion is formed as follows:

Table 3.35 Parameters of Geometric Brownian Motion

Geometric Brownian Motion								
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS
Number of observations	500	500	500	500	500	500	500	500
Min daily return	-5,9443%	-1,1120%	-3,4589%	-1,0375%	-8,6255%	-3,5909%	-4,0383%	-3,6903%
Max daily return	5,5009%	0,7477%	3,5765%	0,6956%	4,0188%	2,4377%	4,6864%	4,9681%
Number of trading days per year	252	252	252	252	252	252	252	252
Time increment (Δt) for one day	0,003968254	0,00396825	0,0039683	0,0039683	0,0039683	0,0039683	0,0039683	0,0039683
Average daily return	0,1000%	0,0074%	0,0572%	0,0024%	0,0327%	0,0607%	0,0525%	0,0258%
Daily standard deviation	1,0513%	0,1537%	0,8811%	0,2361%	1,1180%	0,6853%	1,0223%	1,1720%

Annualised mean return for one year (μ)	-0,1171%	0,0017%	0,1198%	0,0366%	-0,0772%	-0,0383%	0,0158%	-0,2371%
Annualised standard deviation (σ)	16,6895%	2,4406%	13,9871%	3,7472%	17,7484%	10,8781%	16,2278%	18,6054%
Expected return (k)	-1,5098%	-0,0281%	-0,8584%	-0,0337%	-1,6522%	-0,6300%	-1,3009%	-1,9679%
Number of iteration of trials	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000

We use the same random numbers, first the uncorrelated random variables, so the new returns of assets and the yield of the portfolio are calculated again with only change, the parameter of μ in the Geometric Brownian Motion. An extract of these returns and yields is shown in Table 3.36.

Table 3.36 Returns on assets and final yield of the portfolio (with uncorrelated random variables and different μ)

	SIMULATED RETURNS								
	Bloomberg greek government bond	iShares-Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS-REG SHS	**Weighted Average**
Number of simulation	18,259%	15,307%	4,507%	25,770%	7,957%	25,547%	1,394%	1,260%	
1	-3,1123%	-0,4544%	-2,6067%	-0,6976%	-3,3100%	-2,0272%	-3,0255%	-3,4707%	**-1,8022%**
2	-0,5346%	-0,0774%	-0,4464%	-0,1188%	-0,5687%	-0,3470%	-0,5191%	-0,5971%	**-0,3089%**
3	-1,2357%	-0,1799%	-1,0340%	-0,2762%	-1,3143%	-0,8040%	-1,2009%	-1,3787%	**-0,7151%**
.	.	.	.	.	.	.	.	.	**.**
.	.	.	.	.	.	.	.	.	**.**
.	.	.	.	.	.	.	.	.	**.**
99.998	-0,2970%	-0,0427%	-0,2473%	-0,0655%	-0,3160%	-0,1922%	-0,2881%	-0,3322%	**-0,001712**
99.999	-0,8471%	-0,1231%	-0,7083%	-0,1890%	-0,9011%	-0,5507%	-0,8230%	-0,9455%	**-0,4899%**
100.000	-0,3785%	-0,0546%	-0,3156%	-0,0838%	-0,4027%	-0,2453%	-0,3674%	-0,4231%	**-0,2185%**

So, with the same process, as we have analyzed in details at chapter 2.2, we find the ranked table of portfolio's worst yields as shown in Table 3.37 below, in order to calculate V@R and ES.

Table 3.37 Yields ranked from lowest to highest for 100.000 simulations for the portfolio

99% ES with **equal weights**= -1,6150%

99% ES with **unequal weights**= -2,4989041919%

Number of simulation	**Ranked Average**
8573	-2,6348%
62275	-2,4982%
81706	-2,4110%
12335	-2,3734%
78457	-2,3637%
.	.
.	.
.	.
85.989	-1,4161%
.	.
.	.
.	.
6.364	-1,0056%
.	.
.	.
.	.
80.130	2,5558%
55.461	2,5832%
1.311	2,5845%

99% V@R

95% V@R

95% ES with **equal weights**= -1,2552%

95% ES with **unequal weights**= -2,498904309%

And then, we run just the same, taking into consideration the correlation of the random variables. An extract of the returns and yields of the portfolio calculated with the correlated random variables is shown in Table 3.38.

Table 3.38 Returns on assets and final yield of the portfolio (with correlated random variables and different μ)

	SIMULATED RETURNS (Correlated)								
	Bloomberg greek government bond	iShares- Core € Corp Bond UCITS ETF	iShares Core FTSE 100 UCITS ETF	iShares Core € Govt Bond UCITS ETF	iShares EURO STOXX 50 UCITS ETF	SPDP S&P ETF (SPY)	NESTLE SA-R	NOVARTIS- REG SHS	**Weighted Average**
Number of simulation	18,259%	15,307%	4,507%	25,770%	7,957%	25,547%	1,394%	1,260%	
1	-6,1840%	-0,9035%	-5,1810%	-1,3873%	-6,5765%	-4,0293%	-6,0122%	-6,8950%	-3,5818%
2	-1,0573%	-0,1538%	-0,8845%	-0,2362%	-1,1246%	-0,6877%	-1,0274%	-1,1798%	-0,6117%
3	-2,4518%	-0,3578%	-2,0531%	-0,5493%	-2,6075%	-1,5966%	-2,3833%	-2,7343%	-1,4195%
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
99.998	-0,5847%	-0,0847%	-0,4884%	-0,1301%	-0,6220%	-0,3797%	-0,5679%	-0,6529%	-0,3379%
99.999	-1,6789%	-0,2447%	-1,4054%	-0,3757%	-1,7856%	-1,0929%	-1,6318%	-1,8727%	-0,9718%
100.000	-0,7469%	-0,1085%	-0,6244%	-0,1665%	-0,7945%	-0,4854%	-0,7256%	-0,8338%	-0,4319%

So, with the same process, as we have analyzed in details at chapter 2.2, we find again the ranked table of portfolio's worst yields as shown in Table 3.39 below, in order to calculate V@R and ES.

Table 3.39 Yields ranked from lowest to highest for 100.000 simulations for the portfolio

99% ES with **equal weights**= -3,2095%

99% ES with **unequal weights**= -4,9674003495%

Number of simulation	Ranked Average
8573	-5,2377%
62275	-4,9661%
81706	-4,7926%
12335	-4,7177%
78457	-4,6985%
.	.
.	.
.	.
85.989	-2,8138%
.	.
.	.
.	.
6.364	-1,9975%
.	.
.	.
.	.
80.130	5,0856%
55.461	5,1402%
1.311	5,1428%

99% V@R (85.989; -2,8138%)

95% V@R (6.364; -1,9975%)

95% ES with **equal weights**= -2,4938%

95% ES with **unequal weights**= -4,967400581%

3.4 Comparison of estimations for V@R and ES

In this part of chapter 3, we will compare all the ways of estimation for V@R and ES. The portfolios that have observed are 14. These are:

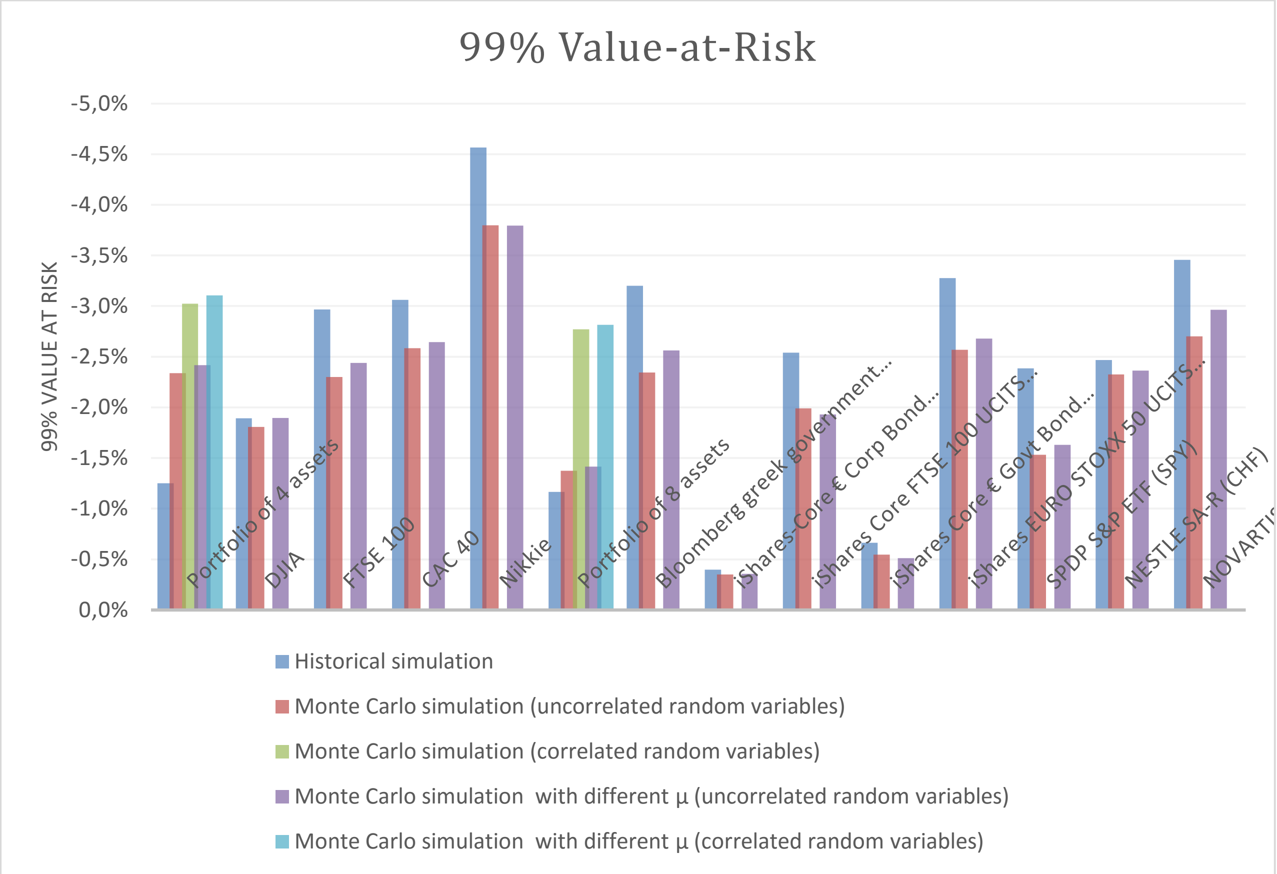
1) portfolio consists of DJIA, FTSE 100, CAC 40, Nikkie
2) portfolio of DJIA
3) portfolio of FTSE 100
4) portfolio of CAC 40
5) portfolio of Nikkie
6) portfolio consists of Bloomberg Greek government bond, iShares-Core € Corp Bond UCITS ETF, iShares Core FTSE 100 UCITS ETF, iShares Core € Govt Bond UCITS ETF, iShares EURO STOXX 50 UCITS ETF, SPDP S&P ETF (SPY), NESTLE SA-R (CHF), NOVARTIS-REG SHS (CHF)
7) portfolio of Bloomberg greek government bond
8) portfolio of iShares-Core € Corp Bond UCITS ETF
9) portfolio of iShares Core € Govt Bond UCITS ETF
10) portfolio of iShares Core FTSE 100 UCITS ETF
11) portfolio of iShares Core € Govt Bond UCITS ETF
12) portfolio of iShares EURO STOXX 50 UCITS ETF
13) portfolio of SPDP S&P ETF (SPY)
14) portfolio of NESTLE SA-R (CHF)
15) portfolio of NOVARTIS-REG SHS (CHF)

Then, the ways of estimation that have also presented above are:

1) Historical Simulation
2) Monte Carlo Simulation with uncorrelated random variables
3) Monte Carlo Simulation with correlated random variables
4) Monte Carlo Simulation with different μ with uncorrelated random variables
5) Monte Carlo Simulation with different μ with correlated random variables.

First, let's see the comparison of estimation for the calculation of 99% Value at Risk in Figure 3.1 below:

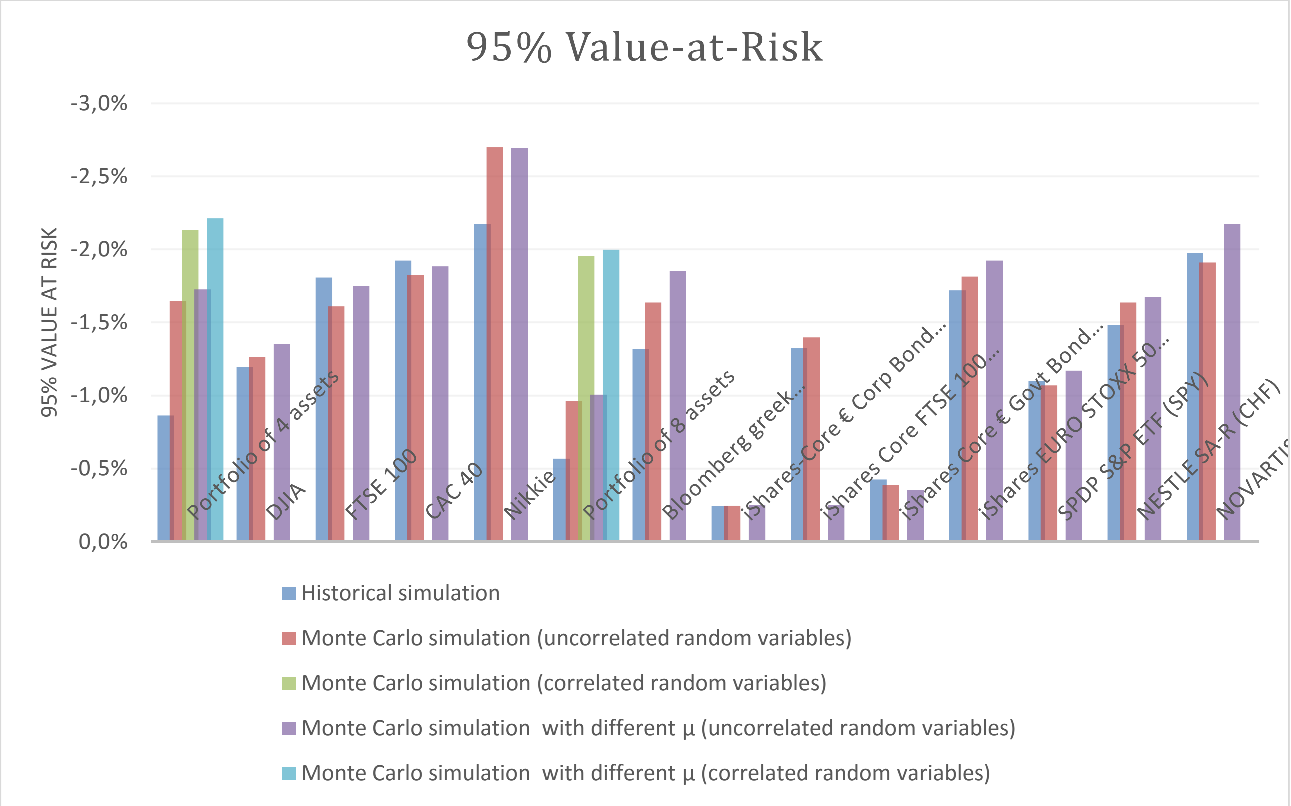
Figure 3.1 Comparison of estimations for 99% V@R



As we can see at Figure 3.1 above, in this case, the change of μ does not affect a lot the estimation of 99%V@R as the two bars are almost at the same level for all assets, with this one of the different μ overestimating V@R. Moreover, in the two multi-assets portfolios we can observe that the correlation of random variables gives us the overestimation of 99% V@R. On the grounds that Monte Carlo estimation underestimates the 99% V@R in the most of the cases, we can say that this estimation has the better results. The difference of the estimation for two methods is observed because of the heavier tail of the normal distribution that we have assumed for the Monte Carlo simulation. However, in multi-assets portfolios, we observe that Historical simulation is this simulation which underestimates the 99% V@R, so historical simulation is the simulation that gives us the underestimation we need and this may happens because of the benefit of diversification that we have already analyzed above.

Then, we will do just the same process for 95% V@R. Figure 3.2 shows the comparison of estimations for 95% V@R.

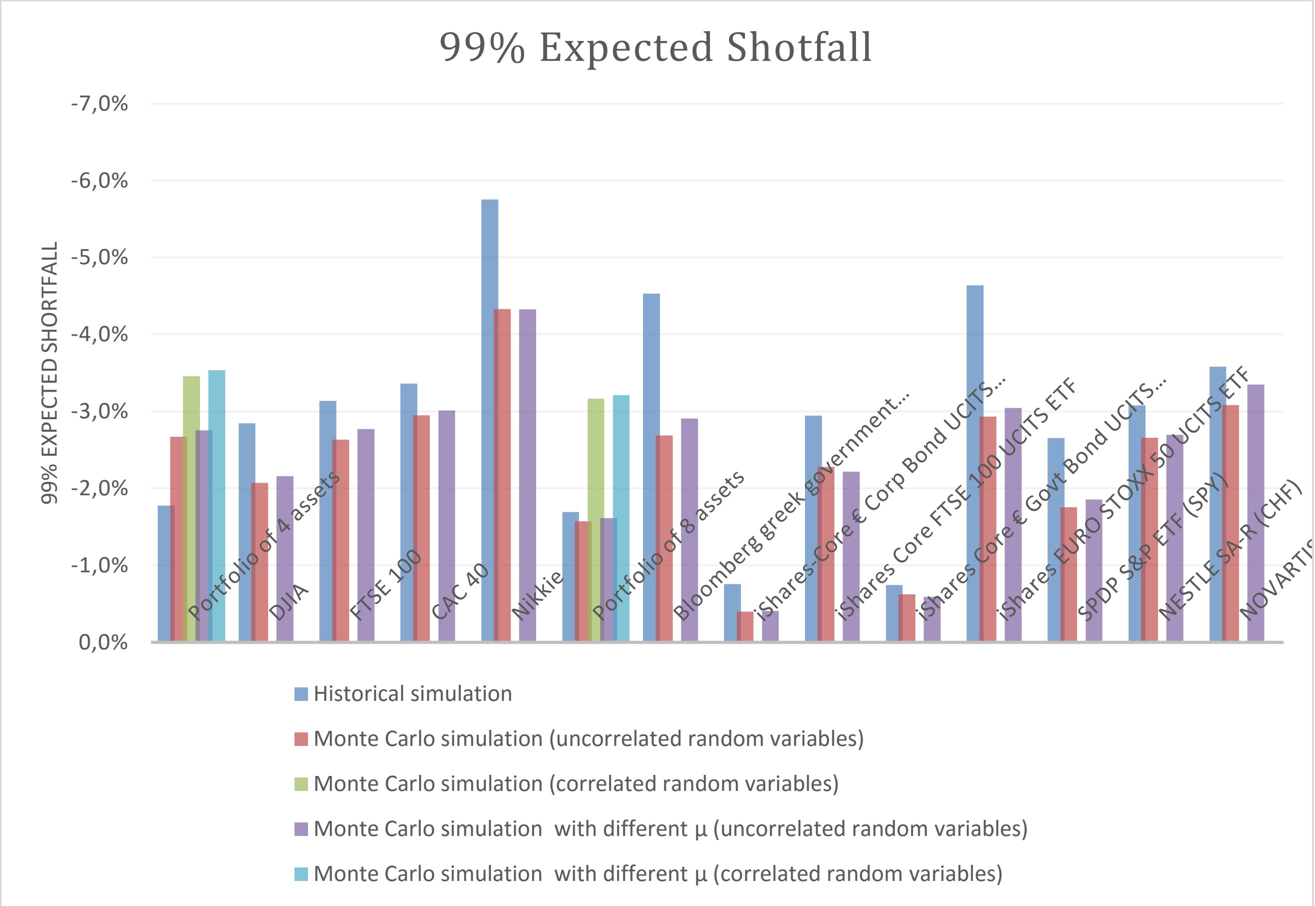
Figure 3.2 Comparison of estimations for 95% V@R



As we can see again at Figure 3.2 above, the change of μ does not affect a lot the estimation of 95% V@R as the two bars are almost at the same level for all assets with this one of the different μ overestimating again V@R. Moreover, in the two multi-assets portfolios we can observe that the correlation of random variables gives us the overestimation of 95% V@R. But, on the contrary from the estimation of 99% V@R, in this case, Historical simulation is this simulation which underestimates 95% V@R in the majority of the portfolios and not only in the multi-asset portfolios as we observed above in Figure 3.1.

Now we want to compare the estimations for calculation of 99% Expected Shortfall and 95% Expected Shortfall. To begin with, Figure 3.3 gives us the opportunity to compare the ways of estimations about 99% ES.

Figure 3.3 Comparison of estimations for 99% ES



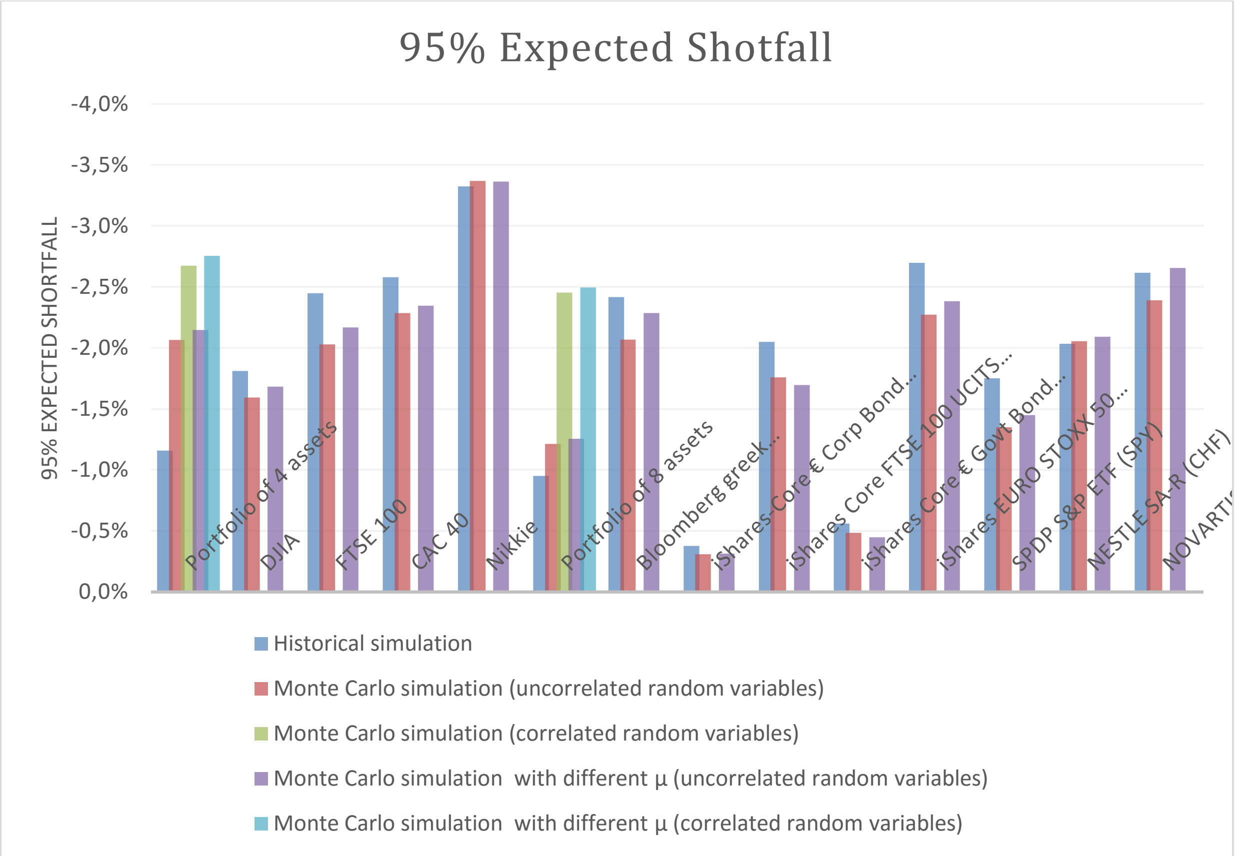
As we can see again at Figure 3.3 above, the change of μ does not affect a lot the estimation of 99% ES as the two bars are almost at the same level for all assets with this one of the different μ overestimating again ES. Moreover, in the two multi-assets portfolios we can observe that the correlation of random variables gives us the overestimation of 99% ES. On the grounds that Monte Carlo estimation underestimates the 99% ES in the most of the cases, we can say that this estimation has the better results. However, in multi-assets portfolios, we observe that Historical simulation is this simulation which underestimates the 99% ES against Monte Carlo simulation with correlated random variables and this may happens because of the benefit of diversification that we have already analyzed above. The difference of the estimation for two methods is observed because of the heavier tail of the normal distribution that we have assumed for the Monte Carlo simulation.

We also check the kurtosis for losses in these two portfolios at Historical Simulation in order to check that Monte Carlo Simulation has heavier tail than Historical Simulation. We observed that for the first portfolio (portfolio with four assets) the kurtosis is 6,27 and it is bigger than 3, which is the kurtosis for the normal distibution, and for the second portfolio

(portfolio with eight assets) the kurtosis is 7,84, also bigger than 3. This means that Historical simulation gives us leptokurtic distributions.

Then, we will do just the same for the estimation of 95% ES in Figure 3.4 below.

Figure 3.4 Comparison of estimations for 95% ES



We can observe again at Figure 3.4 above that, the change of μ does not affect a lot the estimation of 95% ES as the two bars are almost at the same level for all assets with this one of the different μ overestimating again ES. Moreover, in the two multi-assets portfolios we can observe that the correlation of random variables gives us the overestimation of 95% ES. On the grounds that Monte Carlo estimation underestimates the 95% ES in the most of the cases, we can say that this estimation has the better results. So, with the above analysis, we prefer Monte Carlo simulation against Historical Simulation. However, in multi-assets portfolios, we observe that Historical simulation is this simulation which underestimates the 95% ES as at the 99% V@R and this may happens because of the benefit of diversification that we have already analyzed above. The difference of the estimation for two methods is observed because of the heavier tail of the normal distribution that we have assumed for the Monte Carlo simulation.

To conclude, with the above analysis, we observe that in the most cases the best solution to estimate such V@R such as ES is **the Monte Carlo simulation**. This is an expected outcome because as we have also mentioned in the analysis of the previous chapters Monte Carlo simulation gives better results as for accuracy. And this is happening because we can think the Monte Carlo simulation like scenario analysis in which instead of having fo example 500 scenarios, the simulation process generates thousands or ten of thousands of scenarios. However, if we want to focus only **in portfolios, Historical Simulation** is this simulation wich underestimates V@R and ES.

Conclusion

Value at Risk (V@R) is one of the most popular risk assessment tools in the world of investment and risk management. Conditional value at risk (CV@R) or Expected Shortfall (ES) is a technique often used to reduce the probability that a portfolio will incur large losses and is performed by assessing the likelihood (at a specific confidence level) that a specific loss will exceed the V@R. This thesis studies the ES notion and compares its estimation methods.

In the first part, we define V@R and ES and give examples to show their applicability in practical issues. Also, we analyze the usage of these measures and cite examples in the Basel III framework, capital market, and solvency. Moreover, these two concepts are compared in order to understand their differences such as their benefits and drawbacks. The main benefit of ES that makes it to outweight from V@R is its subadditivity.

The second part analyses the alternative techniques of V@R and ES estimation. The techniques applied in this study are 1) Historical and 2) Monte Carlo simulation method.

The empirical study concerns the assessment of alternatives ES methods in a real mixed portfolio and the comparison of their results. Mixed portfolios are usually portfolios consisted of shares and bonds and these portfolios will be presented extensively in this part. An example of a portfolio like this is the portfolio that we have also used in our analysis which consists of: 18,259% Bloomberg Greek government bond, 15,307% iShares-Core € Corp Bond UCITS ETF, 4,507% iShares Core FTSE 100 UCITS ETF, 25,770% iShares Core € Govt Bond UCITS ETF, 7,957% iShares EURO STOXX 50 UCITS ETF, 25,547% SPDP S&P ETF (SPY), 1,394% NESTLE SA-R (CHF), 1,260% NOVARTIS-REG SHS (CHF). We used this portfolio with historical data and estimated the one-day 99% V@R, one-day 95% V@R such as one-day 99% ES and one-day 95% ES in order to compare their results.

We estimated V@R and ES of the portfolio with Historical Simulation, Monte Carlo Simulation with uncorrelated and correlated random variables and Monte Carlo Simulation with different μ, again with uncorrelated and correlated random variables. We came to a conclusion that the best simulation which underestimates V@R and ES is the Monte Carlo Simulation in single asset portfolios and Historical Simulation in multi-asset portfolios.

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