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CAPITAL STRUCTURE ARBITRAGE

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Abstract

This thesis presents a valuation model of financial derivatives related to securities with probability of default. Specifically, using the Merton model, this thesis studies an empirical (synthetic) valuation of CDS. Furthermore, it proves that the main driver of the pertinent arbitrage strategy should be to trade the CDS against the equity implied volatility and not the stock itself. The analysis shows that the key parameter is volatility. In order to maximize the sensitivity to volatility, the deep out of the money option implied volatilities are proposed. This thesis compares the theoretical price of CDS against its derived synthetic price and shows whether opportunities for arbitrage exist. Additionally, by conducting representative case studies, we describe the strategy we need to follow in order to hedge the volatility risk versus the risk that emerges from exposure to underlying equity itself.

Key Words

Merton model; Credit Default Swap; Capital Structure Arbitrage; Barrier options; Option pricing; Implied volatility; Vega Hedge.

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1. Introduction

The aim of this thesis is to indicate that the driver of an arbitrage strategy must be to trade the CDS against the equity implied volatility instead of trading against the stock itself. Although till now consider only delta hedging or trading the underlying equity itself against the CDS are effectual. Will prove that not only the delta hedge is not effectual but that is also widely ineffectual when compared to the Vega hedge. The efficacy of focusing the hedging strategy on the implied volatility will show it by illustrate case studies.

« A CDS or credit default swap is a financial derivative that help lenders manage credit risk, and as the recent crises have shown it is also a mean of speculating on the financial health of others.» (www.investopedia.com, 2017)

CDS contracts can be purchased by investors holding a bond, but can also be purchased by investors who do not own the bond. When the buyer does not hold a bond then CDS is called naked CDS. Naked because whoever buys the insurance without having a bond is like being insured to protect himself against a disaster that does not concern him and from which he will win if he does.
CDS market supporters claim that these contracts do not "move" markets, just reflect investors' fears and help them to cover against future risks.

In this thesis we will examine how the CDS responses to any movement of the implied volatility or to market capitalization. This is going to tested to all of the companies which belong to specify industries. The industries that will take place in our thesis is Bank (Money Center), Insurance (Life), Oil – Gas (Integrated), Telecom (Wireless), Utility (General) and Financial Services.
As summarized in Table 1, 799 companies have been separated by industry and region. Refers to all companies that are internationally active. The model is tested for the period between 2004 – 2011.

Industry	Number of firms	Region	Number of Firms
Bank (Money Center)	167	US	337
Insurance (Life)	55	Japan	58
Oil/Gas (Integrated)	25	Europe	309
Telecom (Wireless)	44	China	95
Utility (General)	43		
Financial Srvs	465		

Table 1. Classifications are those used by MrAswathDamodaran, Professor of Finance at the Stern School of Business at New York University. ***http://pages.stern.nyu.edu/~adamodar/***

From those companies there was data in Bloomberg from January 2004 until December 2011 only for the 157 across those industries and geographies. This time series includes the liquid pre-Lehman market, the Lehman default (September 2008) and the European crisis (2008). Hence the final companies which used for our purpose summarized in Table 2. The region with the fewer data in Bloomberg was Japan. Additional the Industry with the most data was the Bank (Money Center).

Industry	Number of Firms	Region	Number of Firms
Bank (Money Center)	60	US	40
Insurance (Life)	25	Japan	9
Oil/Gas (Integrated)	12	Europe	69
Telecom (Wireless)	12	China	14
Utility (General)	23		
Financial Srvs	32		

Table 2.*Final companies on which the thesis will be held.*

September 15, 2008 it is the date of the biggest bankruptcy in world history when Lehman Brothers applied for protection for creditors. Lehman Brothers Holdings Inc. was a global financial services firm. Bankruptcy was declared in 2008after the mass expense of most of its customers, its drastic decline in inventories, and the depreciation of assets by credit rating agencies. Lehman's bankruptcy petition is the largest in US history and is believed to have played an important role in the global economic crisis of the recent decade 2000.Thousands of small investors lost the money they had invested in CDS, since investors did not make it clear that CDS could lose any value in a potential bankruptcy of the issuing company.

Purpose of this work is to compare theoretical CDS spreads created by a structural framework, with empirical CDS spreads. The basic premise is that the actual CDS and the synthetic CDS should track each other. When they diverge, it is a mispricing between the equity and credit markets. That is the opportunity for arbitrage.

1.1 The problem of Capital Structure

«Capital Structure theory refers to a systematic approach to financing business activities through a combination of equities and liabilities.
Capital structure competing theories explore the relationship between equity financing, debt financing and the market value of the firm. According to the traditional theory, a company should aim to minimize its weighted average cost of capital (WACC) and maximize the value of its marketable assets. This approach suggests that the use of debt financing has a clear and identifiable limit. Any debt capital beyond this point will create company devaluation and unnecessary leverage. » (www.investopedia.com, 2017)

The Modigliani and Miller approach is one popular alternative to traditional capital structure theory. The MM approach has two central suggestions. The first tells that the value of the capital structure and the value of the company haven't any direct correlation. Lieu in, the value of the firm is dependent on expected future earnings. The second suggestion claims that financial leverage increases expected future earnings but not the

value of the firm. That is because leverage-based future earnings are offset by corresponding increases in the required rate of return.
Capital structure arbitrage come in for, since debt must be priced judicially to reflect the exact state of the company. There is no true market valuation for the most of the debt instruments. Moreover, there isn't any punctual market valuation of a company's assets. Potential arbitrage opportunities comes for in, whether market price of debt cannot be justified by its capital structure.

There are two types of conflicts. There are conflicts between the holders of the shares and managers. This conflict arise because managers don't hold all of the residual claim. Thereafter, the managers don't capture the entire gain from the activities which improve their profit, but they do bear the entire cost which can be caused from these activities. For instance, managers can invest not enough effort in managing firm resources and may be able to transfer firm resources to their personal benefit, e.g. plush offices, huge buildings, corporate jets etc. Keeping stable the manager's absolute investment in the firm, growth in the fraction of the firm financed by debt increase the manager's share of the equity and moderate the loss from the conflict between them. Moreover, as pointed out by Jensen (1986), « since debt commits the firm to pay out cash, it reduces the amount of "free" cash available to managers to engage in the type of pursuits mentioned above. This mitigation of the conflicts between managers and equity holders constitutes the benefit of debt financing. »(Jensen, 1986)

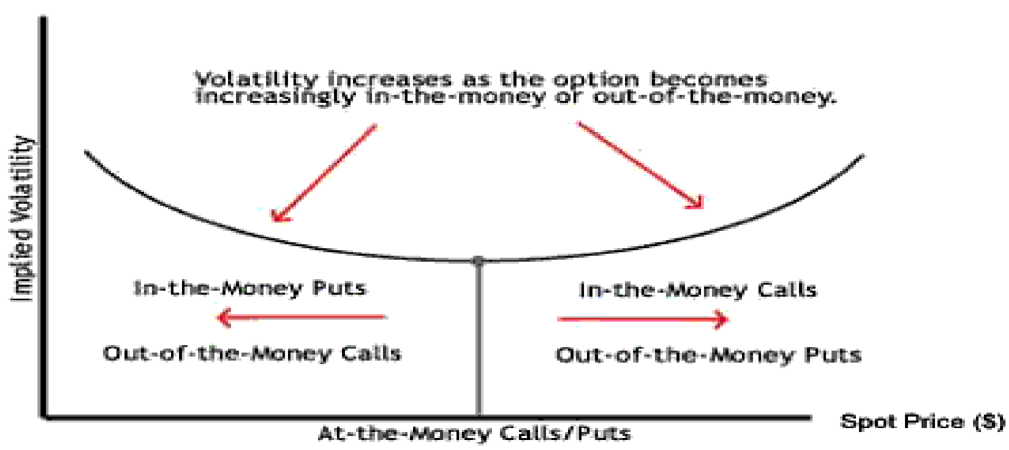
Conflicts between the debt holders and the equity holders rise because the debt contract does not give equity holders a motivation to invest optimally. More notably the debt contract provides that if an investment yields has large returns, well above the face value of the debt, equity holders capture most of that gain. On the other hand, if the investment fails, because of the limited liabilities, the holders of the debt bear these consequences. As a result, equity holders may benefit from investing in very risky projects. Such investments have as result a decrease in the value of the debt. The holders of equity stand this cost to debt holders, nevertheless, the debt is issued if the holders of the debt correctly forecast equity holders' future behavior. In this manner, the equity holders receive less for the debt than they would otherwise.

Capital structure arbitrage mentions to trading strategies that take advantage of the relative mispricing across different securities traded on the same capital structure.
We shall display that the main driver of the arbitrage strategy must be to trade the CDS against the equity volatility, instead of trading against the stock.

1.2 Review of related literature

« The modern theory of capital structure began with the celebrated paper. They pointed the direction that such theories must take by showing under what conditions capital structure is unnecessary. The theory stating that under certain basic assumptions, it doesn't matter how a company finances its projects be it debt, equity or a combination of the two. » (Modigliani, 1958)

« The Black–Scholes formula was developed in 1974 by Fisher Black, Robert Merton and Myron Scholes and is still widely used in 2017 .It gives a theoretical estimate of the price of European style options. Many empirical tests have shown that the Black–Scholes price is "fairly close" to the observed prices, despite that there are well-known discrepancies such as the "option smile". » (Merton, 1974)



« Merton was the first to publish a paper expanding the mathematical understanding of the options pricing model, and coined the term "Black–Scholes options pricing model". In 1974 extends the Black and Scholes (1973) derivative pricing formulation to propose a simple model of the firm that connect the credit risk with the capital structure of the firm. This model assessing the credit risk of a company by characterizing the company's equity as a call option on its assets. This gave rise to Capital Structure theory based on a Gaussian setting. Merton told that a firm would default whenever its assets value fell below the debt level at maturity. Default happens when the firm value hits the default barrier, and at the same time as the equity price goes to zero. » (Merton, 1974)

In 1976 Black and Cox showed a similar analysis with Black and Scholes, according to which all corporate securities could be valued in the same way as financial options. The purpose of this article was to make some generalizations in relation to this valuation method and then to specify them on specific clauses applicable to bond contracts. In particular, they examine the impact of security clauses, the existence of priority options and the restrictions on the financing of interest and dividend costs. Furthermore, they allowed default to happen at any time before or at maturity.

According to Schwartz's study (1992-1993) both creditors and companies themselves foresee that the costs arising from a reorganization or from a bankruptcy through court can be large. For this reason, in the event of bankruptcy, calculated not only the share that are entitled to the dissolution of a company but also the money they will escape if they avoid the court. Many companies that face debts was proposing out-of-court settlements to avoid these costs. In particular, they offered creditors an amount that is close to what they would anyway get if the court costs were incurred. In cases over 90% there was an agreement through private procedures, not through courts. What the companies that proposed the out-of-court settlement was trying to succeed is a successful bid to accept the other borrowers. This offer can certainly be accepted if the initial priority series is respected within the agreement. The main reason they fail was because the amount they offered to creditors was small and the amount they enjoy was greater than the initial agreement on the order of priority. Furthermore Longstaff and Schwartz introduced the stochastic interest rate.

The article of Leland and Toft (1996) examined the optimal capital structure of a firm that can select both the amount and the time of maturity of its debt. « Bankruptcy was determined endogenously rather than by the imposition of a positive net worth condition or by a cash flow constraint. The model predicted leverage, credit spreads and default rates. Leland introduced endogenous default, allowing default to be a strategic decision by the management. » (K.B., 1996) (H., 1994)

In 2001 Dufresne and Goldstein, « investigated the determinants of credit spread changes. From a contingent-claims or no-arbitrage standpoint, credit spreads obtain for two fundamental reasons, firstly there is a risk of default and secondly during the event of default, the holder of the bond receives sole a portion of the promised payments. Thus, they investigated how changes in credit spreads respond to proxies for both changes in the probability of future default and for changes in the recovery rate. Furthermore, they introduced the stationary leverage ratio. » (Goldstein, 2001)

Finger et al (2002) introduced the CreditCrades Model. « CreditCrades is a model for the calculation of portfolio credit risk. Among the main goals of this study was to create a transparent standard and demonstrate to the industry that market participants were able to manage credit risk at the portfolio level. Finger et al also made the default barrier uncertain.»(C., 2006) (Finkelstein V., 2002)

« Huang and Huang (2003) hypothesis that a structural credit risk model with stochastic asset volatility may solved the credit spread puzzle. »(Huang J. H. M., 2012)
« Huang (2005) assume an affine class of structural models with stochastic asset volatility and jumps. Based on regression analysis, Zhang, Zhou, and Zhu (2006) provided empirical evidence that a stochastic asset volatility model may improve the model performance.» (Huang J. Z. H., 2017)
Sepp (2006) equipped the underlying firm value process with stochastic variance and jumps. These approaches all somewhat resolved the problem of short term credit spread. More involved implementation and calibration schemes are needed to fit each model.

Huang and Zhou (2008) «drove a specification analysis of structural credit risk models, using term structure of credit default swap (CDS) spreads and equity volatility. This study provided consistent econometric estimation of the pricing model parameters and specification tests based on the joint behavior of time-series asset dynamics and cross-sectional pricing errors. » (Huang J. Z. H., 2017)

The first who talked about how sensitive to volatility is the Merton's model were Zeitsch and Birchall. They studied the Greek sensitivities delta, gamma and Vega and they were giving rise to a reassessment of the relative importance of model inputs, particularly for volatility. The power of Merton's theory is that not only is credit quality determined by the calculation of a firm's asset value using more responsive market implied volatilities in an effort to capture the unique characteristics of individual markets and their constituent credits. (Zeitsch P., 2003)

Helwege, Huang and Eom « investigated credit risk through five structural models. The Merton (1974), the Geske (1977), the Longstaff and Schawrtz (1995), Leland and Toft (1996) and Collin - Dufresne and Goldstein (2001). All these structural model have some significant differences, such as, the specification of the default boundary, the recovery rates, the interest rates and the coupons. »(Eom Y.H., 2004)

1.3 Description of thesis

This thesis aims to illustrate a new approach for recognizing and hedging capital structure arbitrage. This study searches to hedge the volatility risk, or Vega, instead of as usual from the underlying equity itself, or Delta. To maximize the sensitivity to volatility, is proposed the deep out-of-the-money (10 – delta) option implied volatilities. The results question the efficacy of the common arbitrage strategy of only executing the delta hedge.

In section 2 a description of the Merton model is represented and the way we adapt it to our own model. Furthermore, the risk neutral calibration of the default barrier is defined. This calculation was difficult to achieve and the use of Matlab programming langue required. The code which used is provided at the appendix at the end of this thesis. Furthermore, by JP Morgan's paper [25] about pricing the CDS we define the valuation of Credit Default Swap Spread approximation from default probabilities and produce the synthetic CDS. Also the need of the programming language Matlab was necessary, for the calculation of the probability of default.

In section 3 is highlighting the pros for calibrating the model to deep out-of the-money put volatilities or 10 delta put implied volatilities versus the at-the-money put implied volatilities or 50 delta put implied volatilities . Further, it turns out how the debt-holder’s payoff is related with the value of a put option. Also the outputs from the empirical performance are presented for four companies belonging to different industries. The default point, the market capitalization, the implied volatility and the liabilities of the company are presented. The charts are clearly show that during the period of the Lehman default, the variables that affected are market capitalization, implied volatility and default point. Additional charts with the synthetic Credit Default Swaps versus the traded five – year CDS are presented. The period of incorrect pricing and opportunities for arbitrage is between 2008-2009, the period of Lehman default.

In section 4 arguments between the Vega hedge and Delta hedge are provided. The most of the arguments are in favor of Vega hedge. We quote the correlation of CDS versus the volatility and versus the market capitalization. How the movement in the implied volatility corresponds to movement in the CDS and even more how the movement in the market capitalization moves the CDS. In addition regressions have been achieved. One of the regression is for market capitalization against the total asset of the company and other one for market capitalization against the asset volatility. The third regression is for CDS against the market capitalization and at last for CDS against the put implied volatility. The results are given in tables. The regression has been achieved with the help of Gretl. Also in this section the strategies you have to take in case of mispricing between the actual CDS and the synthetic CDS are provided. In any case what you have to do in order to be protected. At the end of the section case studies take place for different cases. The first case study deals with the fact where the synthetic CDS was assessed with lower price than the traded five – years CDS. The second case study deals the fact where the synthetic CDS was estimated much more than the traded five – year CDS. Both of the case studies by following the appropriate strategies results in profit.

2. The Merton Equity-Debt Model

The Merton model supposes a scenario where a company has a certain amount of zero coupon debt with time of maturity T. The value of the firm’s assets follows a lognormal diffusion process with a constant volatility. Additionally the firm has issued two classes of securities an equity which receives no dividends and a debt which is a pure discount bond where a payment of D is promised at the time of maturity T.

Therefore the equity of the company is a European call option on the assets of the company with maturity T and a strike price equal to the face value of the debt. In terms of investors, equity holders are long a call and debt holders are short a put.

If at time T, the firm’s asset value be superior to the promised payment, D, the lenders are paid the amount which they have promised and the shareholders receive all residual claims to the assets. On the other hand, if the asset value is less than the payment which they have promised, the firm defaults, the lenders receive a payment equal to the asset value, and the shareholders don’t get anything.

With the beneath assumptions holding, suppose there is a derivative security also trading in this market (CDS). This security will have a certain payoff at a specified date in the future, depending on the values taken by the stock up to that date. The derivative's price is completely determined at the current time and we do not know what path the stock price will take in the future.

« Assumptions:

1. There are no transactions costs, taxes or problems with indivisibilities of assets.
2. There are a sufficient number of investors with comparable wealth levels so that each investor believes that he can buy and sell as much of an asset as he wants at the market price.
3. There exists an exchange market for borrowing and lending at the same rate of interest.
4. Short – sales of all assets, with full use of the proceeds, is allowed.
5. Trading in assets takes place continuously in time.

6. The Modigliani – Miller theorem that the value of the firm is invariant to its capital structure obtains.
7. The Term – Structure is “flat” and known with certainty. The price of a riskless discount bond which promises a payment of one dollar at time τ in the future is

$$P(\tau) = \exp[-r\tau] \ ,$$

where, r is the instantaneous riskless rate of interest, the same for all time.

8. The dynamics for the value of the firm V, through time can be described by a diffusion – type stochastic process with stochastic differential equation.

$$dV = (\alpha V - C)dt + \sigma V dz \ ,$$

where,

- $\alpha$is the instantaneous expected rate of return on the firm per unit time.
- C is the total dollar payouts by the firm per unit time to either its shares holders or liabilities holders if positive and it is the net dollar received by the firm from new financing if negative.
- $\sigma^2$is the instantaneous variance of the return on the firm per unit time.
- $dz$is a standard Gauss – Weiner process.

The model can be used to estimate either the risk-neutral probability that the company will default or the credit spread on the debt.

The model assumes the price of traded assets follows a geometric Brownian motion with constant drift and volatility. Additionally, the model assumes stock prices follow a lognormal distribution because asset prices cannot be negative.

The Merton’s model requires five input variables.

- K: the strike price of an option
- S: the current stock price
- T: the time to expiration
- r: the risk-free rate
- σ: the volatility

It is known that equity is a market observable and total liabilities are obtained directly from an issuer’s balance sheet. The remaining unknown variables are then the value of the firm’s assets, V_t and the asset volatility, σ_V. » (Merton, 1974)

2.1 Description of the model

This model is a Structural form model. This kind of models are based on the model of Black and Scholes (1973) and Merton (1974). The structural model treats equity as a market option on the company's assets, and the exercise price is the company's debt. Over the years, the model has evolved using more and more new variables. This model studies the possibility of default when a bond expires and the payment fails.

In this study the desire is to improve the responsiveness of the default probability calculation.

Let S represent the value of the firm's equity and V the value of its assets. Let $S_0 \& V_0$ be the values of S and V today and let $S_T \& V_T$ be their values at time T.
In the Merton framework the payment to the shareholders at time T, is given by (a call option):

$$S_T = \max[V_T - B, 0] \quad , \qquad (1)$$

where, .

The underlying asset follows the following stochastic process, the driver noise being given by standard normal Weiner process dW:

$$dV_t = rV_t dt + \sigma_V V_t dW_t. \qquad (2)$$

The current equity price, therefore, can be immediately evaluated as a closed form solution of the Black-Scholes' PDE:

$$S_0 = V_0 N(d_1) - Be^{-rT} N(d_2) \quad , \qquad (3)$$

where,

- $d_1 = \left(\frac{\ln\left(V_0 * \frac{e^{-rT}}{B}\right)}{\sigma_V * \sqrt{T}} \right) + \frac{\sigma_V * \sqrt{T}}{2}$
- $d_2 = d_1 - \frac{\sigma_V * \sqrt{T}}{2}$

- N(…) is the Gaussian distribution.
- σ_V is the volatility of the asset value.
- r is the asset drift due to risk – neutral interest rate dynamics.

The first time within the model will hit the default barrier is defined as,

$$\tau = \min[t: V_t \le B'(t,T)], \qquad (4)$$

where, B'(t,T) is the instantaneous default point for the bond at time t.

We assume that the equity volatility exists and is given by,

$$dS \approx S\sigma_S dW. \qquad (5)$$

Additionally applying Ito's Lemma to equation (1) concludes,

$$dS(V_t,t) = \left[\mu V_t \frac{\partial S}{\partial V} + \frac{\partial S}{\partial t} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 S}{\partial V^2}\right] dt + \sigma V_t \frac{\partial S}{\partial V} dW. \qquad (6)$$

From equations (5), (6) results

$$\sigma_S = \sigma_V \frac{V_t}{S} \frac{\partial S}{\partial V_t} \ . \qquad (7)$$

The firm's asset V_t has to satisfy some boundary conditions,

1) $V_t \big|_{S=0} = B'(t,T)$ or $\lim_{S \to 0} V_t = B'(t,T)$ (8)
2) When $S >> B'(t,T)$ assume that,
   - $S/V_t \to 1$
   - $\partial S/\partial V_t \to 1$ (9)

These conditions represent the behavior of V_t near default and when well capitalized.

Taking the first order of equation (8) gives,

$$V_t \approx B'(t,T) + \frac{\partial V}{\partial S} S \ ,$$

$$V_t \approx S + B'(t,T) \ . \tag{10}$$

Substituting (10) into (7) yields,

$$\sigma_V = \sigma_S \frac{S}{S+B\prime(t,T)} \quad . \tag{11}$$

We have to define an initial estimate for the default point.

$$B'_1(t,T) = LGD * B \tag{12}$$

Where, LGD is the loss given default for senior unsecured debt and the market standard assumption is 60%.

Substituting (12) into (11) equation (8) can now be solved numerically by seeking an updated $B'_i(t,T)$ such that,

$$S = C^{BS}[B'_i(t,T), \sigma_V, T, B] \rightarrow 0 \tag{13}$$

Although C^{BS} is a non-negative, monotonically increasing function and finding a root of this function, is not possible. For that reason a tolerance is defined to represent the limit, $\lim_{S \to 0} V_t = B'(t,T)$. The Brent – Dekker algorithm then seeks a solution to that tolerance.

Brent – Dekker is an algorithm using in programming language of matlab to find the root for the function (13). Here, the algorithm searches the value of B'(t,T) for every day to within a accuracy of 1%. In the appendix there are the codes which used for this calculation.(Dekker, 1969)(Brent, 1973)

2.2 Synthetic CDS

« One of the most important insurance contracts for debt is the credit default swap contract. Credit default swaps have proved to be one of the most successful financial innovations of the 1990s. A CDS or credit default swap is a financial derivative that helps lenders manage credit risk. It is an instrument that provides insurance against a particular company defaulting on its debt. The company is known as the reference entity and a default by the company is known as a credit event. A difference between a CDS contract and a regular insurance policy is that the buyer of credit protection does not have to own the underlying instrument. Like most derivative instruments credit default swaps can be used for hedging, speculation and arbitrage. » (en.wikipedia.org, 2017)

« The buyer of the protection makes periodic payments to the seller at a predetermined fixed rate per year. The payments continue until the end of the life of the contract or until a credit event occurs. If a credit event occurs, the buyer of protection has the right to deliver a bond issued by the reference entity to the seller of protection in exchange for its face value.» (www.investopedia.com, 2017)

The credit events which are triggering the CDS are specified in the contract, but can include formal bankruptcy, failure to pay or restructuring.
«The amount of the payments made per year by the buyer is known as the CDS spread.
In a standard contract, payments are made quarterly or semiannually in arrears. If the reference entity defaults, there is a final accrual payment covering the period from the previous payment to the default date and payments then stop. Contracts are sometimes settled in cash rather than by the delivery of bonds. In this case there is a calculation agent who has the responsibility of determining the market price of bonds issued by the reference entity a specified number of days after the credit event. The cash settlement is then the face value of the bond less the post-default market price. » (www.investopedia.com, 2017)

With the use of some examples we will show how these contracts can be used for hedging. Firstly, if a holder of a bond has $1 million worth of bonds report to a company with a CDS spread of 200 BPS and wants to take away this exposure for 5 years, he must reach an agreement with a seller of a CDS of paying $5 thousands every quarter for this 5 years. The most certain is that the bond won't default and the company is just going to pay its quarterly insurance unnecessarily. But if the bond defaults during this 5 years the insurance will stop paying its quarterly payments and the seller will reimburse the protection buyer with the par value of the bond of $1 million. The total cost for the seller will be by how many quarterly payments the buyer has already paid in premiums and the recovery rate of the bond.

An investor by not possessing the reference entity, if he believes that the default risk is smaller than what is implied by the CDS spreads can speculate on the credit quality of a bond by selling protection. On the contrary if he believes that the company is going to default he can speculate by buying CDS protection.

CDS is also used in arbitrage type transactions. A negative relationship between a company's CDS spread and its stock price causes these opportunities. A decrease in stock price leads to an increase in probability of default. If the company's shares decrease in value the CDS spread increases.
If this relationship is weak an arbitrageur can take advantage of this mispricing and the arbitrager makes a profit when the price converge.

According to JPMorgan paper, « a credit default swap has two valuation legs, the fee and the contingent. For a par spread, the net present value of both legs must be equal. »(JPMorgan, 2001). The valuation of the fee leg is approximated by,

$$PV\ of\ No\ Default\ Fee = S_N \cdot Annuity_N = S_N \sum_{i=1}^{N} DF_i \cdot PND_i$$

where,

- S_N is the Par Spread for maturity N
- DF_i is the Riskless Discount Factor from T_0 to T_i
- PND_i is the No Default Probability from T_0 to T_i

The valuation of the contingent leg is approximated by,

$$PV\ of\ Contingent = Contingent_N = (1-R)\sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i)$$

where,

- R is the Recovery Rate of the reference obligation.
- $PND_{i-1} - PND_i$is the Probability of a Credit Event occurring during period T_{i-1} to T_i.

Therefore, for a par credit default swap,

$$Valuation\ of\ Fee\ Leg = Valuation\ of\ Contingent\ Leg \Rightarrow$$

$$S_N \sum_{i=1}^{N} DF_i \cdot PND_i = (1-R)\sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i) \Rightarrow$$

$$S_N = \frac{(1-R)\sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i)}{\sum_{i=1}^{N} DF_i \cdot PND_i}$$

« Once the recovery rate has been estimated the probability of default can be calculated from the prices of bonds issued by the reference entity or from the spreads quoted for other CDSs on the reference entity.
The CDS fair-value is obtained by equating the premiums paid by the protection buyer versus any contingent payout from a credit event paid by the protection seller. » (JPMorgan, 2001)

Consequently,

$$CDS_i = \frac{LGD * \sum_{i=1}^{n}(p_i - p_{i-1}) * DF_i}{\sum_{i=1}^{n} DF_i(1-p_i)} \qquad (14)$$

Where,

- $DF_i$is the discount factor to the i-th day.
- $p_i = 1 - PNDi$is the associated default probability.
- $LGD = 1 - R$is a constant which represents a degree of freedom that can be calibrated to reduce the pricing error between the synthetic CDS and the traded contract. The LGD is calibrated once, where 0<LGD<1 and then held constant.

The probability of default is the probability that the issuer's assets will be less than the book value of the issuer's liabilities when the debt matures.

That is,

$$p_t = P[V_t \leq B'(t,T) | V_{t=0} = V_t] \ ,$$
$$= P[lnV_t \leq lnB'(t,T) | V_{t=0} = V_t] \qquad (15)$$

From equation (2) we have that,

$$lnV_t = lnV_{t=0} + \left(r - \frac{\sigma_V^2}{2}\right) t + \sigma_V \sqrt{t} \varepsilon \ . \qquad (16)$$

Substituting (16) into (15) gives,

$$p_t = P\left[lnV_t + \left(r - \frac{\sigma_V^2}{2}\right) t + \sigma_V \sqrt{t} \varepsilon \leq lnB'(t,T)\right]$$

$$= P\left[-\frac{ln\frac{V_t}{B'(t,T)} + \left(r - \frac{\sigma_V^2}{2}\right) t}{\sigma_V \sqrt{t}}\right] \geq \varepsilon$$

Because $\varepsilon \sim N(0,1)$ we can define the default probability in terms of the cumulative normal distribution as,

$$p_t = N\left[-\frac{ln\frac{V_t}{B'(t,T)} + \left(r - \frac{\sigma_V^2}{2}\right) t}{\sigma_V \sqrt{t}}\right] \qquad (17)$$

Substituting (17) into (14) produces the synthetic CDS. The $CDS_i, i = 1,2, \dots, n$ form the synthetic CDS time series.

The aim is to replicate the CDS by defining p_t and model the liquid five – year CDS of which the maturity of the model, T, is set to five years also for the synthetic CDS.

Both the calculation of the synthetic CDS and the calculation of the default probabilities required the use of the programming language Matlab. The codes are presented in the appendix at the end of the thesis.

For the computation of the synthetic CDS for every day a sum of probabilities at the day i minus the probabilities at the day (i-1) is required. The maturity dates of CDS contracts are standardized to the International Monetary Market (IMM) dates such as March 20th, June 20th, September 20th and December 20th. Thus, in each IMM date has to subtract the current probability from the probability of the previous IMM date and multiply the results by the current IMM date discount factor.

3. Risk Neutral Calibration

« Risk neutral is a mindset where an investor is indifferent to risk when making an investment decision. The risk-neutral investor places himself in the middle of the risk spectrum. Risk-neutral measures have extensive application in the pricing of derivatives. Risk neutral investors would not be interested in insuring at a premium that exceeds the expected loss. Holders of the debt have bought the face value of the bond and have sold a put option on the assets of the company. Generally, as the volatility seller will demand a higher premium for the risk, an increased open interest in low delta strikes will quickly skew the implied volatility versus higher strikes. » (en.wikipedia.org, 2017)

3.1Asset volatility calibration

In a structural model volatility is the most important parameter. The model which dependences on volatility is a key reason that financial institutions haven't performed well historically in Merton models. This will be exacerbated if the model isn't fully calibrated risk - neutrally.

We will calibrate the below equation to a one-month 10-delta put implied volatility.

$$\sigma_V = \sigma_S \frac{S}{S + B'(t,T)}$$

Following Merton, the debt-holder's payoff define as Dt.

$$\begin{aligned} D_t &= \min[B, V_t] \\ &= B + \min[V_t - B, 0] \\ &= B - P^{BS}[V_t, \sigma_V, r, T, B] \quad , \qquad (18) \end{aligned}$$

where, P^{BS} is the value of a put option.

Huge declines in the price of the market capitalization will increase the price of the put option. The best indicator of such a decline will be the 10 – delta put implied volatilities.

Low delta puts will attract interest as a relatively inexpensive hedge. In the case where the stock price weakens, because of this position it will quickly increase in value. The holders of debt would be hedged against default risk if they bought back the put. However, because there isn't put option on the asset of the firm we can buy CDS protection as they have the same behavior. Therefore, CDS can be replace a put option on the assets of the firm.

To show this we have plotted through 2005 until 2011 the average 10-delta and 50-delta one-month put implied volatility for the companies listed in Table 2. The time series includes the liquid pre-Lehman market, the Lehman default and the European debt crisis.

As we can see in Figure 1 the average 10-delta one-month put implied volatility has higher volatilities point than the 50-delta through all the time. The peak difference occurred at the end of January 2009, having almost the double value difference. Throughout the duration the average of the 10 – delta put implied volatilities is higher than the average of the 50 – delta put implied volatilities.

The fact that low delta volatilities (10 – delta) can trade significantly higher and over a greater range of values than at – the – money (50 – delta) volatilities, will increase the variability and responsiveness of the default probability in equation (17). This can also affect the calculation of the synthetic CDS. Using the low delta put implied volatility, should therefore be the choice for calibration.

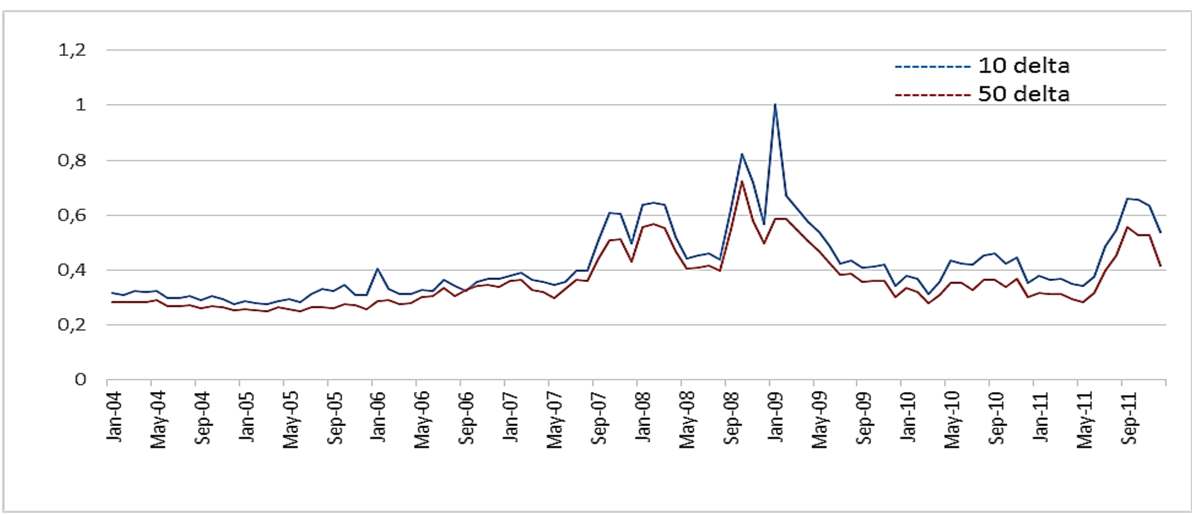


Figure 1. The average one month 10-delta versus 50-delta implied equity put volatilities for all the companies listed in Table 2. The data was monthly.

INDUSTRY	MEAN DELTA	MEAN VEGA
BANK (MONEY CENTER)	0.21	2.35
INSURANCE (LIFE)	0.26	7.18
OIL/GAS (INTEGRATED)	0.58	2.64
TELECOM (WIRELESS)	0.30	6.98
UTILITY (GENERAL)	0.22	3.57
FINANCIAL SRVS	0.15	3.12

REGION	MEAN DELTA	MEAN VEGA
US	0.36	2.19
EUROPE	0.25	3.12
CHINA		
JAPAN	0.25	12.36

Table 3. *Mean Delta was calculated by regressing the Market Capitalization against the total assets for each company. Mean Vega was calculated by regressing the Market Capitalization against the put implied asset volatility. All variables have been converted into USD.*

Regression analysis used to measure the power of the model. The dependent variable will be the time series of the market capitalization. The independent variables used will be the time series of the total assets for the first regression and the time series of the asset implied volatility for the second regression. The regressions will be the following:

$$MCap_i = \alpha_0 + \alpha_1 TotalAsset_i \quad , \quad MCap_i = \beta_o + \beta_1 AssetVol_i \ ,$$

where,

- α_0 , β_0 are constants
- α_1 is the delta of the of the company i
- β_1 is the Vega of the CDS of the company i

For all the companies the results was statistical significant with p-values less than 5%. CDS is significantly more sensitive to movements in the volatility than to market capitalization

3.1Empirical study data

In order to be able to do the empirical study, had to find data for all of the companies listed in Table 2, from January 2004 until December 2011. This Period includes the liquid pre – Lehman market, the Lehman default and the European crisis. The most of the data were sourced from Bloomberg. Also data from the Data Stream was received.
All the data are daily, excluded the weekend.

The data required and received from Bloomberg were as follows,

- The 1/month 10 – delta put implied volatility.
- The 1/month 50 – delta put implied volatility.
- The Market Capitalization of the company.
- The Total Asset of the company.
- The Total Liabilities of the company.
- The theoretical CDS price.

The data required and received from Data Stream were as follows,

- The exchange rates with US$.
- The 10 year treasury interest rate of US.
- The information needed for the put option which used in the case studies.

The 10 year treasury interest rate is the benchmark used to decide mortgage rates across the U.S. and is the most liquid and widely traded bond in the world.

The exchange rates that were needed were the following,

- UK £ / US $
- EURO € / US $
- JAPANESE YEN ¥ / US $
- CHINESE YUAN / US $
- SWISS FRANC / US $
- HUNGARIAN FORINT / US $

- SLOVAK KORUNA / UK £
- DANISH KRONE / EURO €
- HONG KONG $ / US $
- NORWEGIAN KRONE / US $

Some of the companies listed in Table 2, had data calculated in local currencies. So conversion to US dollars was necessary for several times. For this reason we took the exchange rates of these currencies with dollar.

All of these data were needed to,

- Calibrate the asset volatility.
- Show from charts, how these variables have reacted to Lehman default.
- Calculate the default barrier.
- Calculate the default probability.
- Calculate the value of the synthetic CDS and compare it with the theoretical value of the CDS.
- Regress the theoretical value of the CDS against the market capitalization and against the put implied volatility and find the delta and Vega hedge ratios.
- Show the correlation between the CDS versus the market capitalization and the CDS versus the volatility.

A problem that was detected, was that there were no data available for all of the variables of the companies, from all of the days of the time series. So some days have to been removed. In the case where the Matlab code was used these days have to been removed.

3.2Empirical Performance

To illustrate the model's performance, the output for four different credits from US, Europe and China are shown in Figures 2-5. The companies in question are JPMorgan from bank (Money Center) industry from US, Prudential Financial from insurance (Life) industry from United Kingdom, Credit Suisse Group from financial services (Non-bank & Insurance) industry from Switzerland and PetroChina Company from oil/gas (Integrated) industry from China.

There are four diagrams for each company. These concern the default point, the market capitalization, the asset volatility and the liabilities. All of them reported from 2004 until 2011.

The charts show that the market capitalization and liabilities affected by the default point. The spike in volatility happens because of Lehman default. This also reduces the default barrier, furthermore it reduces the default risk. As it is known by the Merton model an increase in volatility means higher profits. The more risk you take the more returns you get.

Figures 6 – 9 present the synthetic CDS which calculated using equation (16) versus the actual 5-year senior unsecured, CDS. For the four corporates there is a close agreement between the synthetic CDS and the traded contract. The outright magnitudes of the spreads are comparable. So both the equity and the debt markets are pricing efficiently.

For the capital structure, a declination between the synthetic and traded CDS is very important. Thereafter, it is needful to trade the capital structure completely to hedge this result by either buying or selling volatility on the underlying stock against the CDS as required.

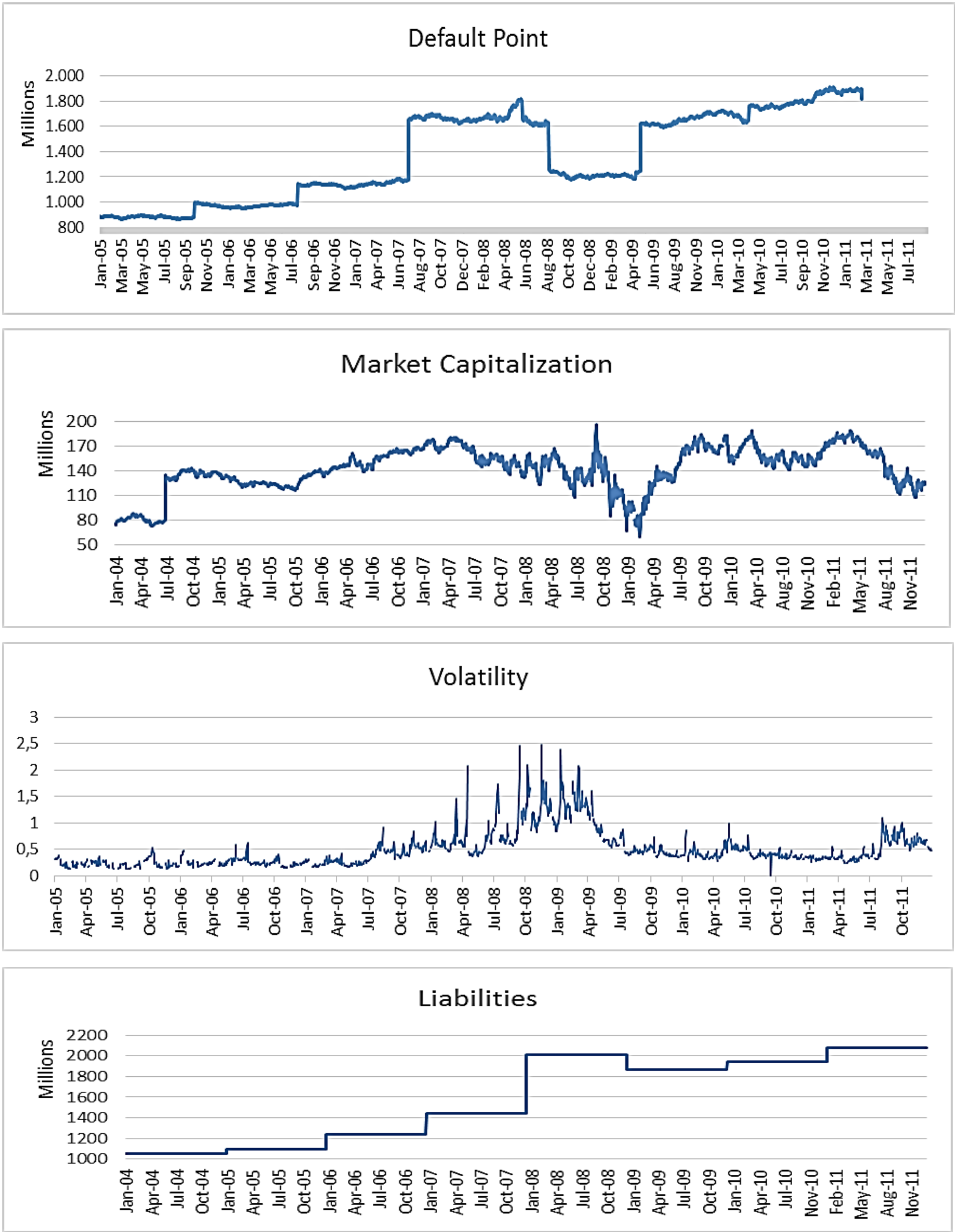


Figure 2. JPM time series for the default barrier, market capitalization, 10-delta put implied volatility and liabilities. All variables are in $US.

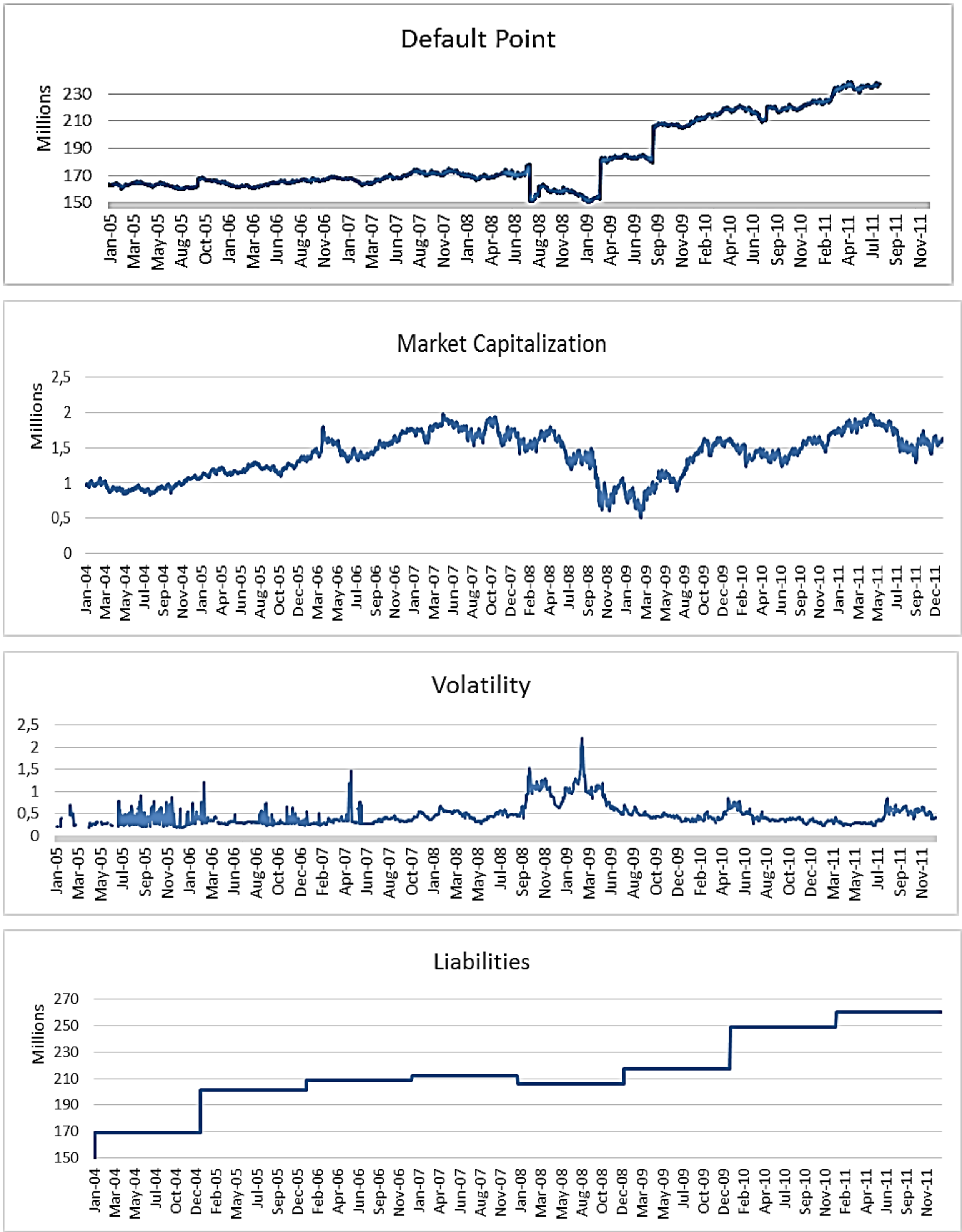


Figure 3. Prudential Financial time series for the default barrier, market capitalization, 10-delta put implied volatility and liabilities. All variables are in Euro €.

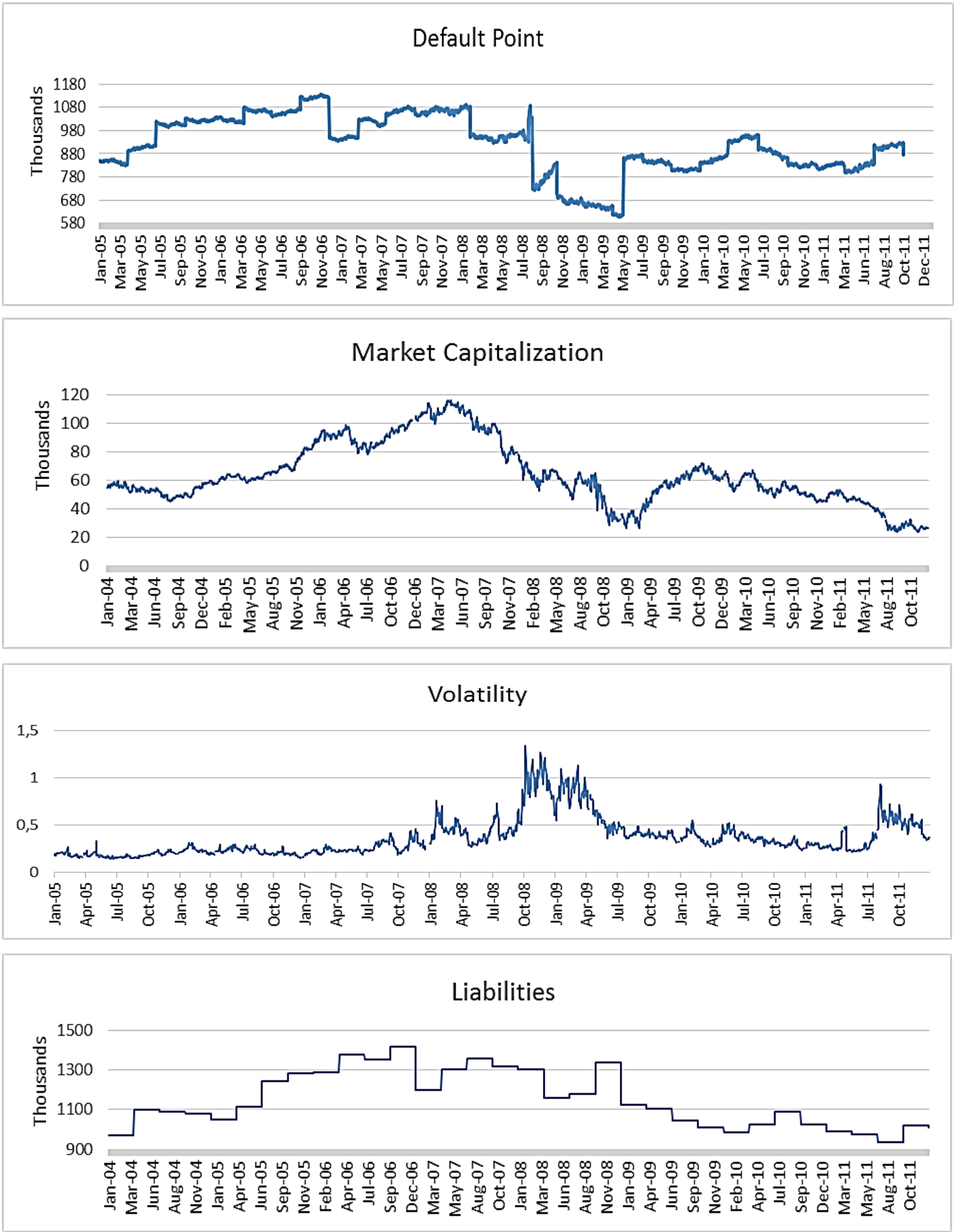


Figure 4. Credit Suisse time series for the default barrier, market capitalization, 10-delta put implied volatility and liabilities. All variables are in Swiss franc.

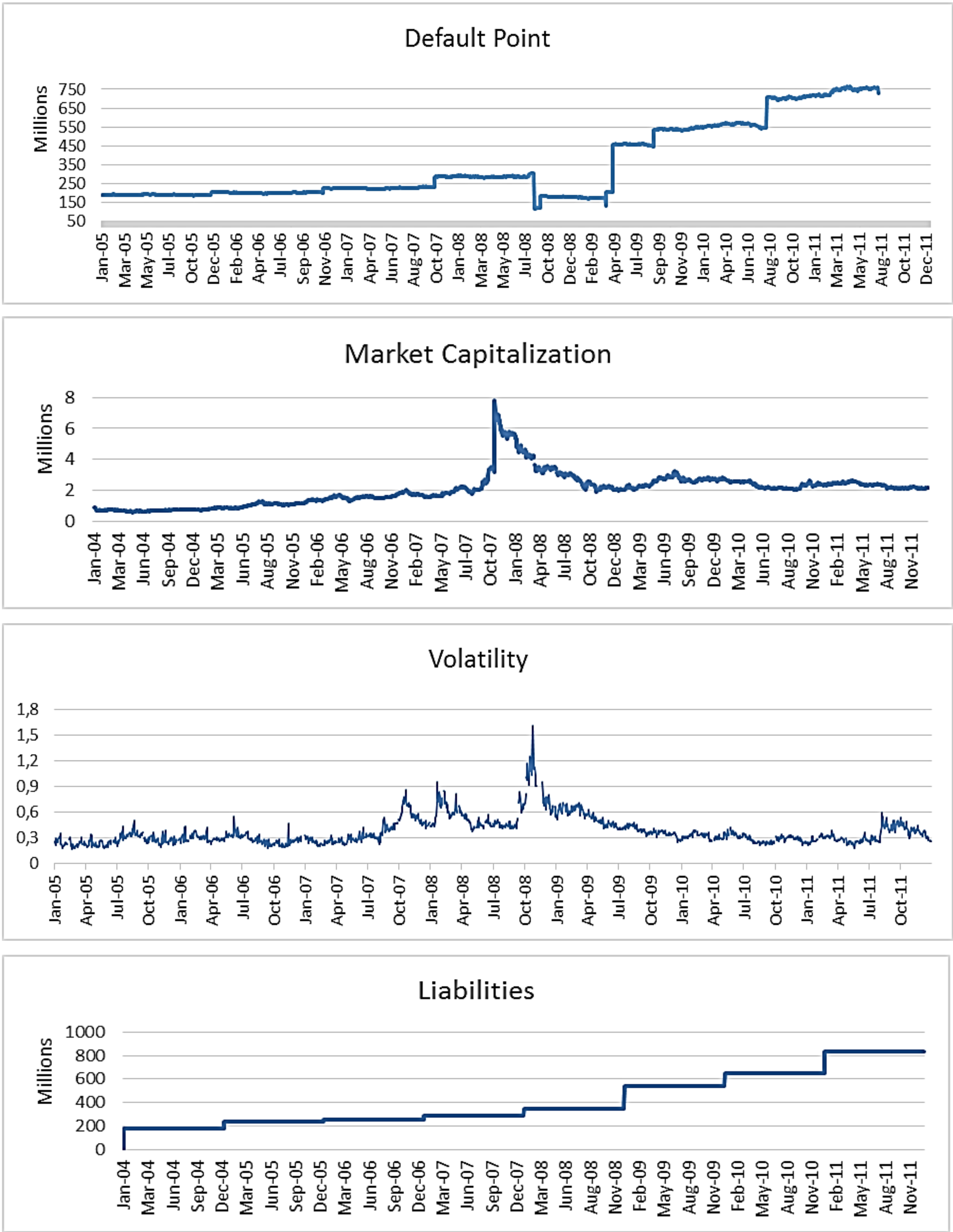


Figure 5. Petro China time series for the default barrier, market capitalization, 10-delta put implied volatility and liabilities. All variables are in Chinese Yuan.

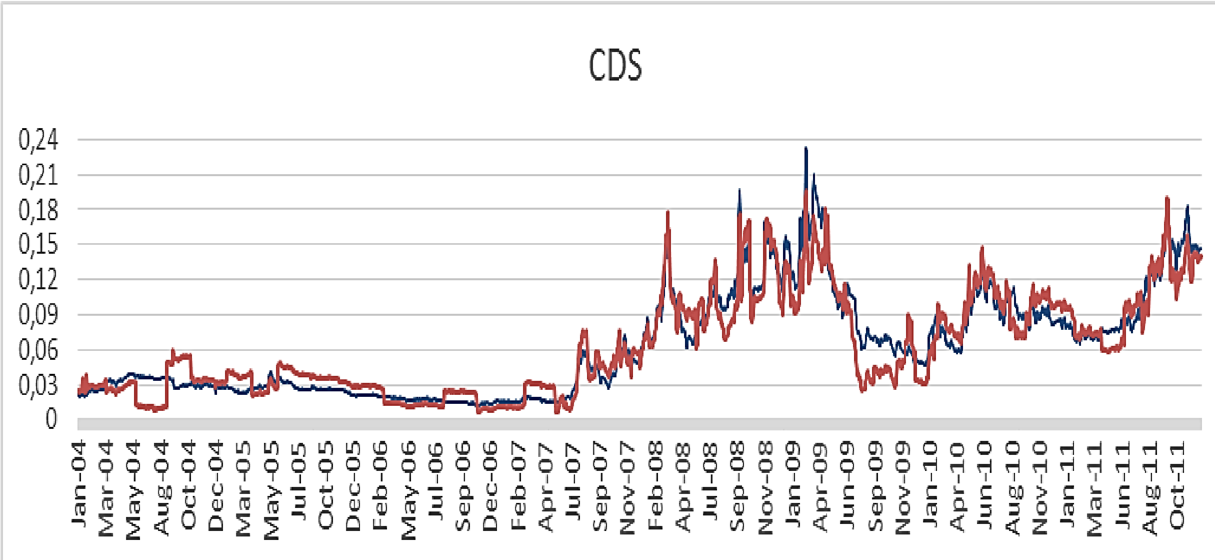


Figure 6. *Synthetic CDS* ***red line*** *versus traded five – year CDS blue line. From January 2004 to October 2009 for JP Morgan.*

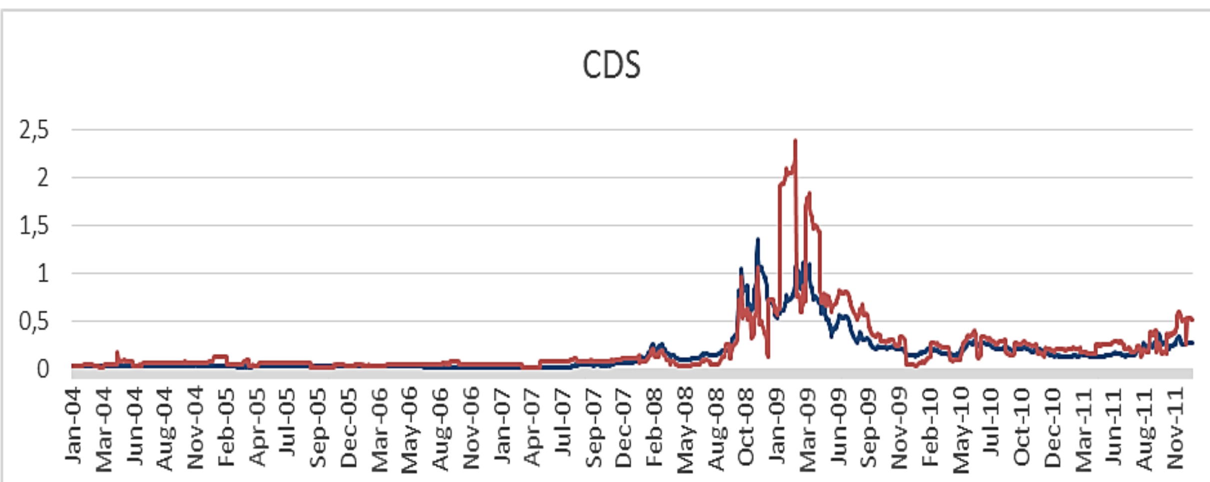


Figure 7.*Synthetic CDS* ***red line*** *versus traded five – year CDS blue line. From January 2004 to November 2001 for Prudential Financial.*

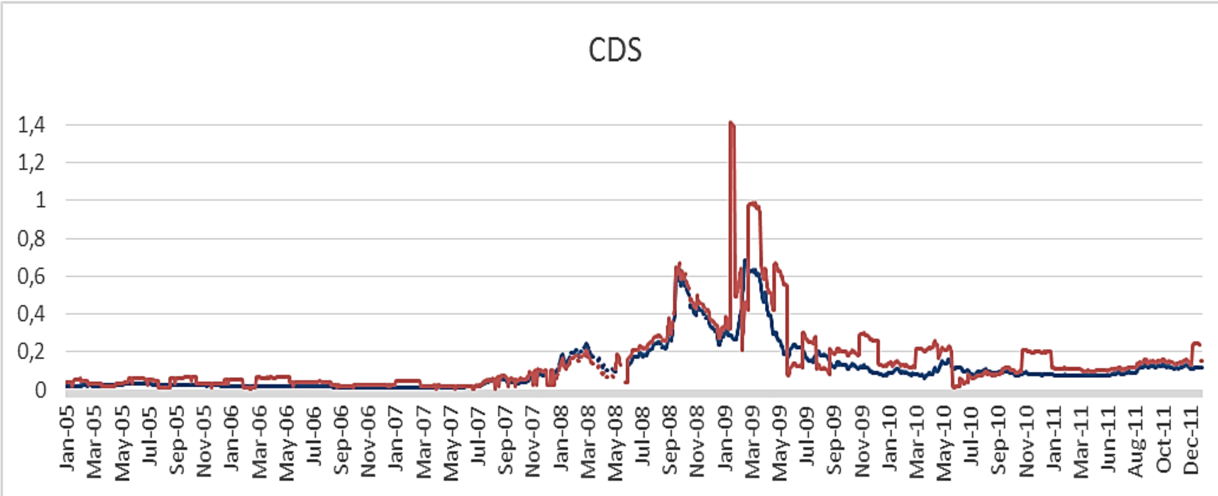


Figure 8. *Synthetic CDS* ***red line*** *versus traded five – year CDS blue line. From January 2005 to December 2011 for Credit Suisse.*

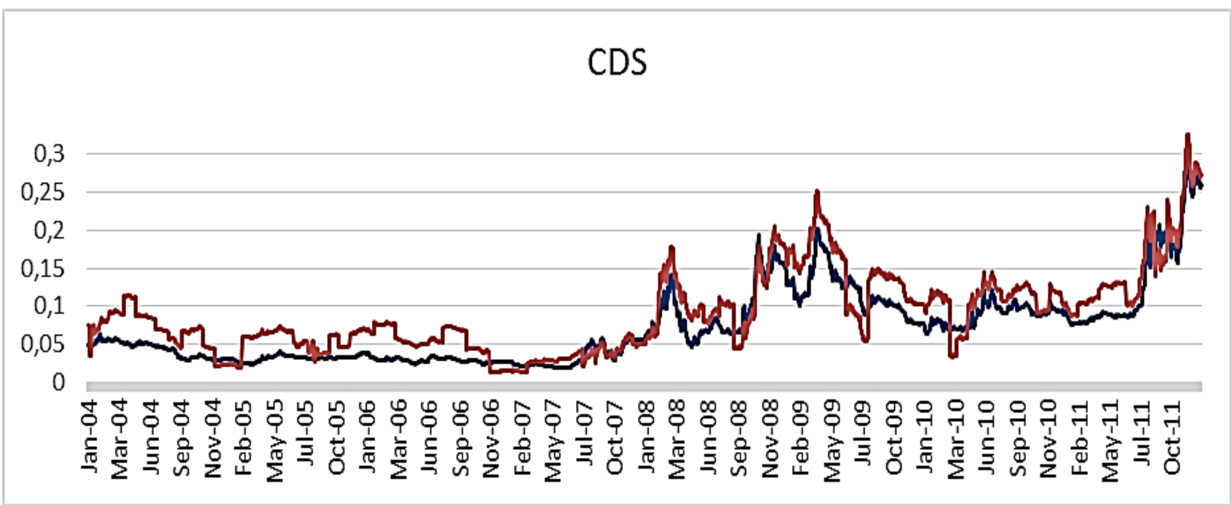


Figure 9. *Synthetic CDS* ***red line*** *versus traded five – year CDS blue line. From January 2004 to October 2011 for Petro China.*

4. Capital Structure Arbitrage

The most arbitrage studies have used a strategy of trading the CDS against the underlying equity itself using the below hedge ratio which is derived from the model.
Where,

$$\delta' = \frac{\partial CDS}{\partial S} \tag{19}$$

and CDS is the synthetic CDS calculated using equation (14).

This study is based on CDS with 5 – years maturity. This is because CDS with 5 – years maturity is the most liquid market. There isn't a significant difference in the correlation between the CDS and the implied volatility versus the CDS with the market capitalization. Absolute values are very close one to another. Hence, there is no empirical reason to favor one sensitivity over the other. The results present in tables 4 – 5.

INDUSTRY	CDS VS VOLATILITY	CDS VS MKT CAP
BANK (MONEY CENTER)	0.64	-0.52
INSURANCE (LIFE)	0.84	-0.77
OIL/GAS (INTEGRATED)	0.58	-0.56
TELECOM (WIRELESS)	0.36	-0.45
UTILITY (GENERAL)	0.66	-0.47
FINANCIAL SRVS	0.68	-0.74

Table 4.*Correlation of CDS versus implied volatility and CDS versus market capitalization. Classification by Industry. Calculated from January 2005 until December 2011 using 5Y CDS, 10-delta implied put volatilities and Market Capitalization.*

REGION	CDS VS VOLATILITY	CDS VS MKT CAP
US	0.69	-0.60
EUROPE	0.63	-0.54
CHINA		
JAPAN	0.39	-0.56

Table 5. *Correlation of CDS versus implied volatility and CDS versus market capitalization. Classification by Industry. Classification by Region. For China no CDS data was available.*

As we can see in Figure 10small changes in the price of the implied volatility corresponds to great changes in the price of the CDS, whereas large moves in the market capitalization do not affect the price of the CDS. Only large CDS moves, occur when the Market Capitalization drops below $20.000.

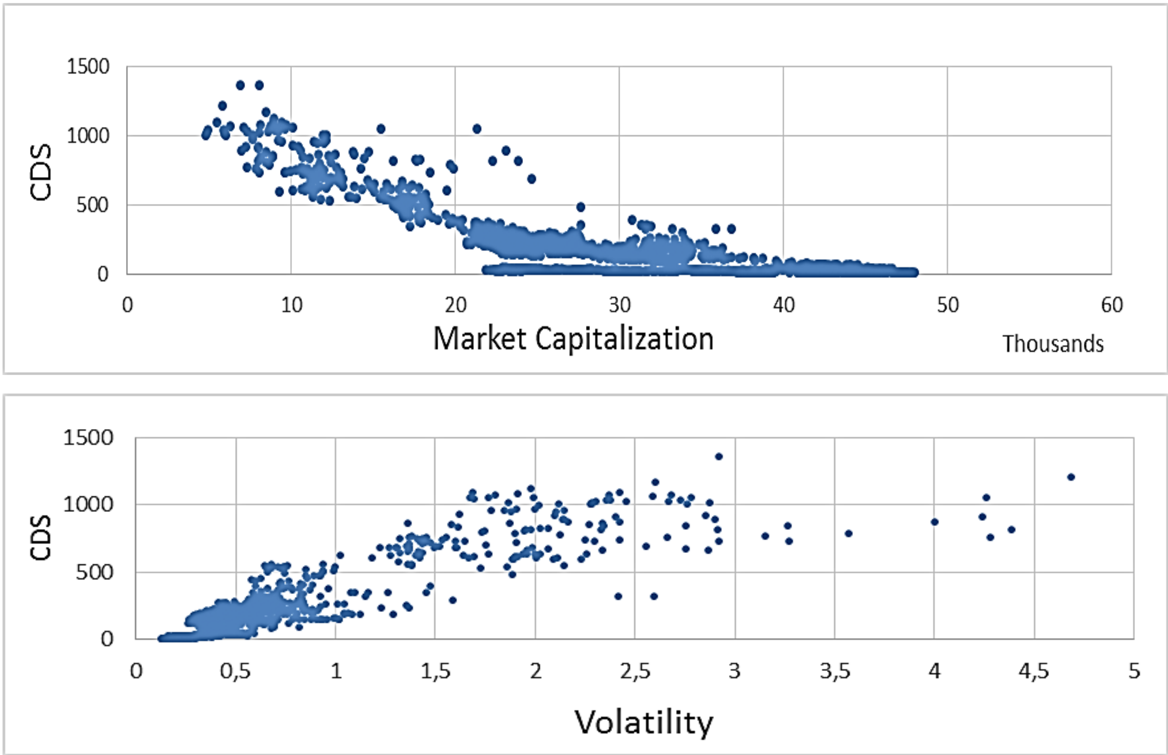


Figure 10.*Scatter plot for Prudential Financial. Five year CDS versus Market Capitalization and 10-delta implied volatility.*

The hedge ratio which be used to reflect both the theoretical and empirical sensitivity is,

$$v' = \frac{\partial CDS}{\partial \sigma} \tag{20}$$

Where σ is the 10-delta put implied volatility and CDS is the traded 5 year contract.

Using Gretl, we calculate hedge ratios. We regress the CDS against the 10-delta implied volatility and then against market capitalization. The results present in Table 6. The regression performed to all companies listed in Table 2.

INDUSTRY	Mean Delta	Mean Vega	REGION	Mean Delta	Mean Vega
BANK (MONEY CENTER)	-0,26	2.21	US	-0.43	1.49
INSURANCE (LIFE)	-0,16	6.16	EUROPE	-0.34	2.87
OIL/GAS (INTEGRATED)	-0,13	2.32	JAPAN	-0.03	9.69
TELECOM (WIRELESS)	-0,04	5.44	CHINA	0	0
UTILITY (GENERAL)	-0,17	2.45			
FINANCIAL SRVS	-0,56	1.54			

Table 6. *Mean Delta was calculated by regressing the CDS against the market capitalization for each company. Mean Vega was calculated by regressing the CDS against the implied volatility.*

Regression analysis will be used to measure the power of the model. The dependent variable will be the time series of the credit default swap spreads. The independent variables used will be the time series of the 10 – delta put implied volatility for the first regression and the time series of the market capitalization for the second regression. The regressions will be the following:

$$CDS_i = \alpha_0 + \alpha_1 \sigma_i \,, \quad CDS_i = \beta_o + \beta_1 S_i \,, \tag{21}$$

where,

- α_0 , β_0 are constants
- σ_i is the put implied volatility of the company i
- S_i is the market capitalization of the company i
- α_1 is the vega of the CDS of the company i
- β_1 is the delta of the CDS of the company i

For all the companies the results was statistical significant with p-values less than 5%.
CDS is significantly more sensitive to movements in the volatility than to market capitalization. For Japan the mean Vega is 3230 times greater than mean delta.
The delta hedge in all cases is negative so the delta hedge strategy is to sell CDS protection and shorting the stock. Although this is difficult to accomplish because a substantial part of the total market capitalization are needed. That position is impossible to exist in the market, either from the amount of shares required or the amount of capital needing to be held to execute it. The more effective hedge is to trading the volatility.

4.1 Vega-Hedging of CDS

One of the main points of the thesis is that the delta hedging strategies used by other authors don't work. The model is not sensitive to the delta, is sensitive to Vega. Furthermore, they use strategies like sell protection and short the stock. That doesn't make any sense because if the CDS trades down, you will win on the CDS leg but the stock will probably increase in price so you lose on that leg. If you sell protection on the CDS, that is a bullish trade so you should be long the stock i.e. buy the stock. If you short the CDS, you should short the stock i.e. sell the stock. Whereas the other authors would buy the stock. They all seem to blindly accept the hedge strategy from credit grades.

Bullish trade is the trade where investors believe « that a stock price will increase over time. Investors who buy calls believe that the stock price will rise and have paid for the right to purchase the stock at a specific price known as the exercise price or strike price. An investor who has sell a put has an obligation to buy the stock and, therefore, believes that the stock price will rise. » (www.investopedia.com, 2017)

The basic premise is that the actual CDS and the synthetic CDS should track each other. When they diverge, it is a mispricing between the equity and credit markets. That is the opportunity for arbitrage. There are three different cases for that.

In the first case, the synthetic CDS spiked wider. This means that the actual CDS should widen i.e. buy protection. Alternatively, the implied volatility might be oversold (the reason the synthetic CDS spiked). So you sell the volatility. The trade is then buy protection on the CDS looking for spread widening and sell the volatility looking for the volatility to trade down.

In second case the trade is the opposite. The synthetic CDS, has tightened faster than the actual CDS. This means the implied volatility has dropped. So the model is telling us that the actual CDS should continue to tighten. So the trade is to sell protection on the CDS. But the model might be wrong, so you buy the volatility as a hedge. Again you are looking for convergence between the synthetic and actual CDS.

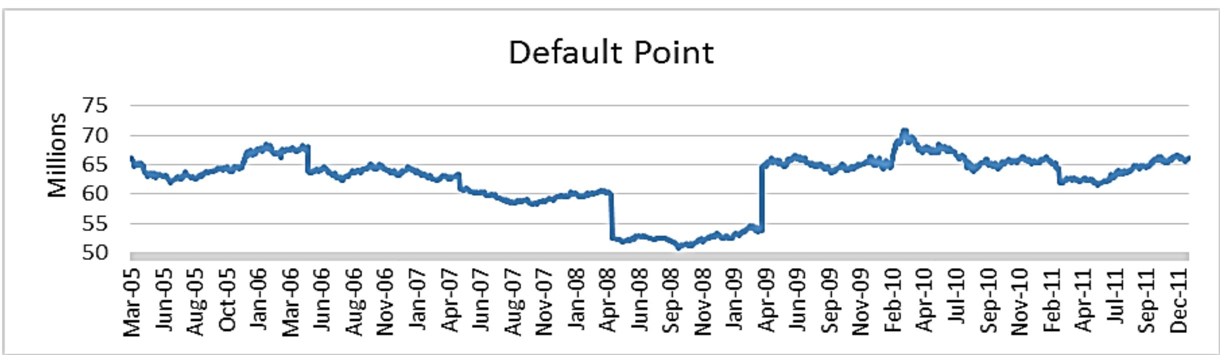
Finally, the third case is one where the model got it wrong. The CDS didn't tighten as predicted. Buying the options protects you as the volatility also spiked wider.

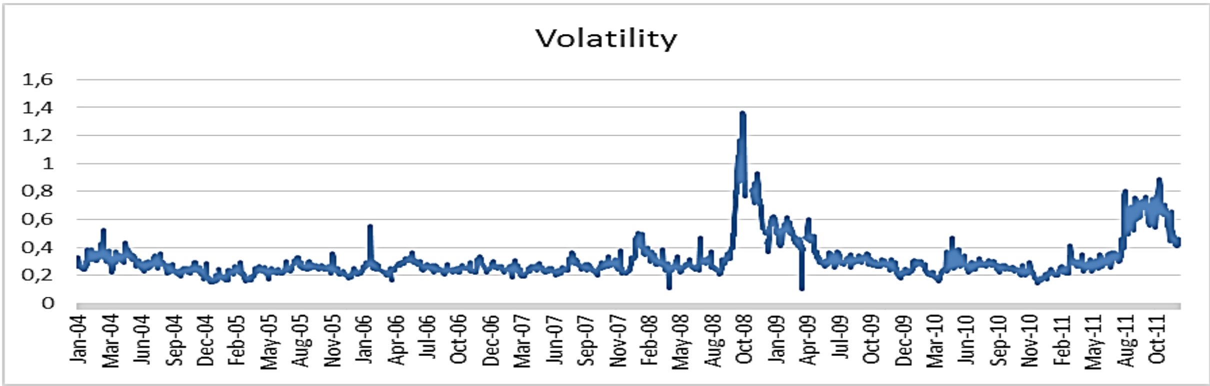
4.2 Case Studies

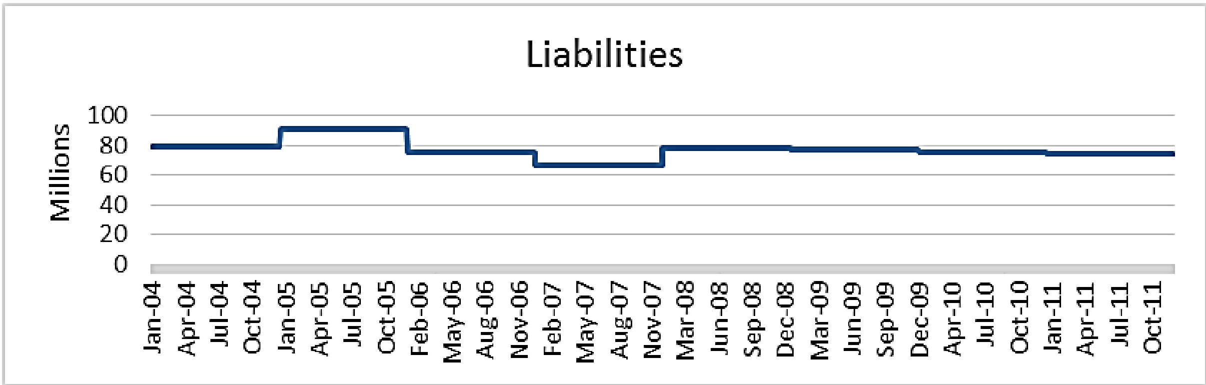
4.2.1 RWE case study

« RWE, is a German electric utilities company. The energy company supplies electricity and gas to more than 20 million electricity customers and 10 million gas customers, principally in Europe. RWE is the second largest electricity producer in Germany. »(en.wikipedia.org, 2017)

According to Figure 11, during the period of the Lehman default, there was a capital structure misalignment for RWE Company, in August 2008. The synthetic CDS, red line, had widened substantially further than the traded five – year CDS, blue line. This display that the implied volatility is richer versus the CDS. We expect the price of the traded CDS to widen, so the strategy that requires to take is to sell puts on the equity itself and buy protection on the CDS.







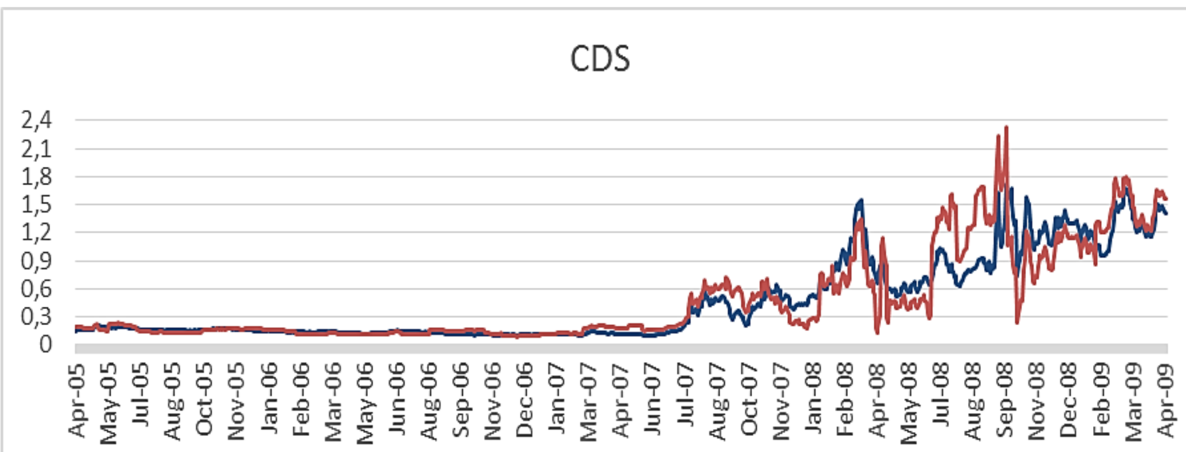


Figure 11. *Synthetic CDS* ***red line*** *versus traded five – year CDS blue line. From April 2005 to April 2009.*

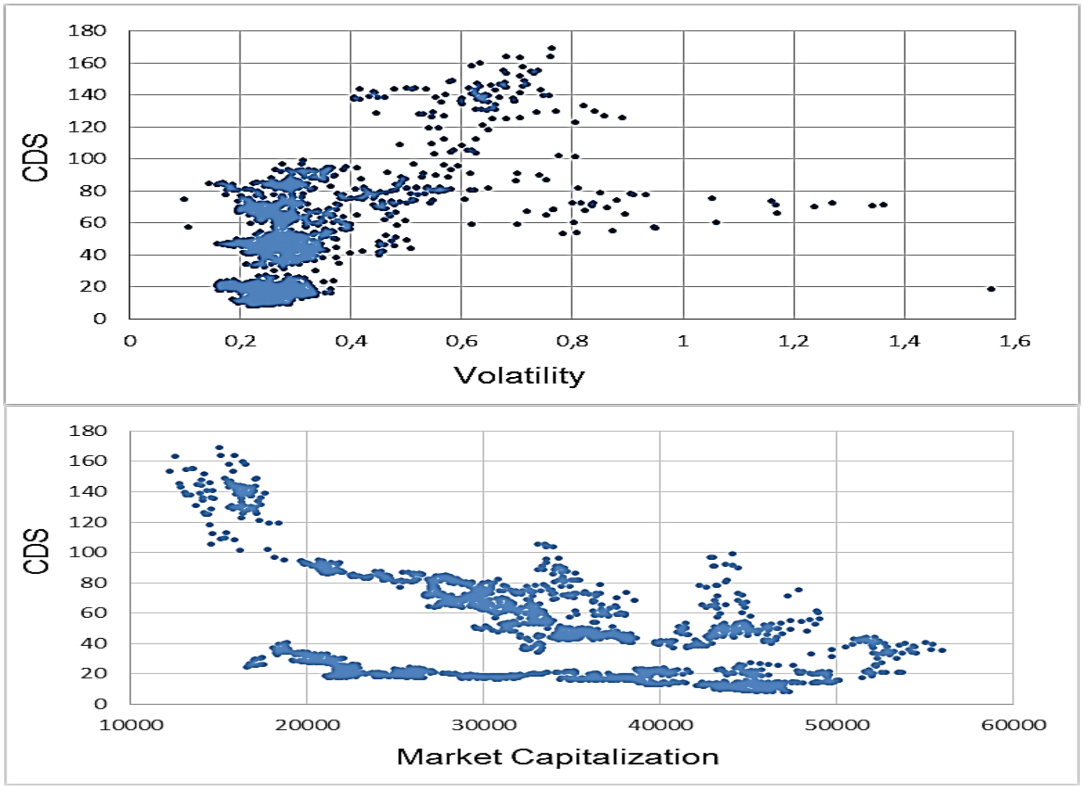


Figure 12. *Scatter plots of five – year CDS versus 10 – delta implied put volatility and five – year CDS versus market capitalization.*

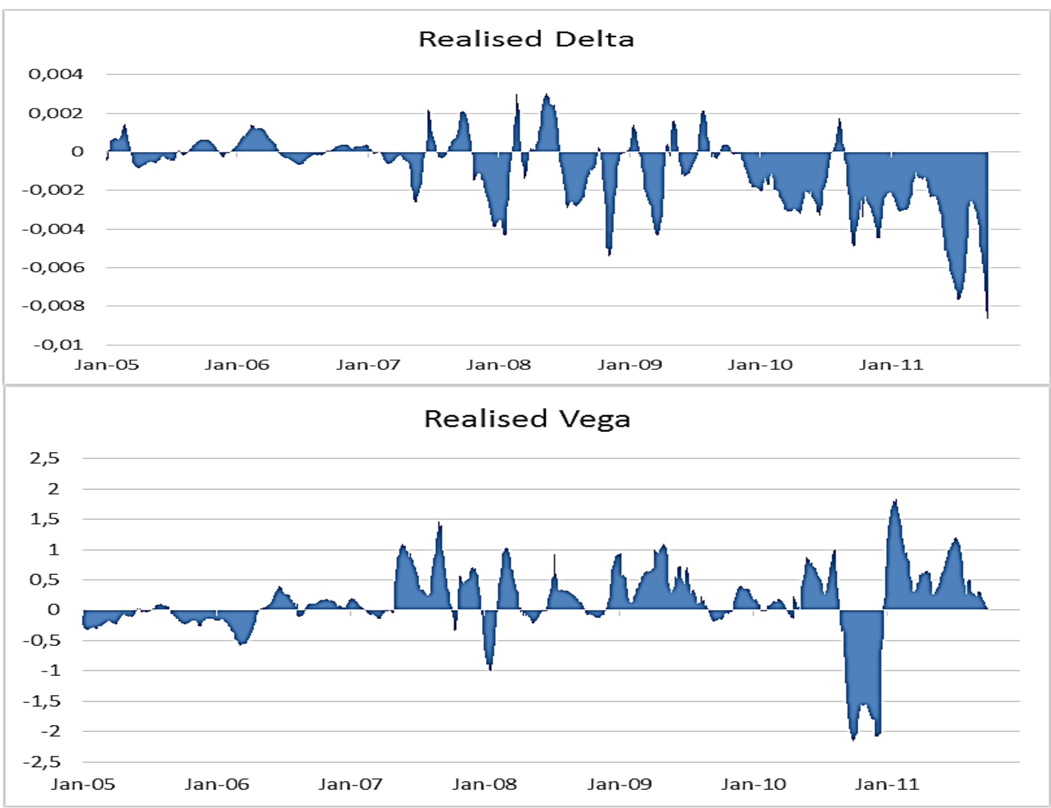


Figure 13. *Regressed 3 – month rolling hedge ratio*

In figure 12 the five – year traded CDS is plotted against the 10 – delta implied equity put volatility and against the market capitalization. These charts show that small movements in the implied volatility corresponds to large movements in the CDS, whereas large movements in the market capitalization do not move the CDS necessarily.

Figure 13 represents the 3 – month rolling hedge ratios which are calculated with the help of programming language Matlab. The code which used here is quoted at the appendixA. The rolling windows are used for monitoring the stability of the hedge.

The average hedge ratios from August 2008 are:

$$\frac{\Delta CDS}{\Delta S} = -0{,}0019, \qquad \frac{\Delta CDS}{\Delta \sigma} = 0{,}20$$

CDS DV01 is the change in the value of credit default swap in reaction to a one basis point increase in the underlying spread. More specifically, it measures the dollar present value changes for each basis point shift in the credit curve. The DV01 of a CDS is roughly equal to the DV01 of a par bond issued by the same reference entity. In calculation, this value is equal to the remaining life of the CDS times the notional principal amount (NPA) times one basis point:

$$\text{CDS DV01} = T \times NPA \times 0{,}0001$$

where, T is the remaining life of a swap.

Consequently, taking a standard five – year CDS contract with notional of $10million a quick calculation for RWE shows that,

DV01=$5000.

Thus,

$$V_{CDS} = 5000 \times 0{,}20 \approx 1.000$$

Where V_{CDS}, is the Vega of the CDS contract.

For late August 2008, seek liquid option contracts with the longest maturity profile. To satisfy this, choose the RWE 21 November 2008 puts at 54 with stock price at 53,329 €, risk free interest rate at 3,8% and implied volatility at 71,8% .

A quick calculation[1] from a Black-Scholes calculator shows that,

$$V_{OPTION} = 0,664$$

Where, V_{OPTION} is the Vega of the put option.

Each contract contains 100 options, which produces the following hedge:

$$Hedge = \frac{1.000}{0,664\ x\ 1,25} \approx 1.205$$

where, 1,25 is the foreign exchange spot rate for the Euro.

In this case, you need 1.205 option contracts. The idea is then that if the volatility moves against you, you are locally hedged and won't lose money. So you buy protection on the CDS and sell the options i.e. looking for convergence between the CDS and the options. So you want the CDS to widen and the volatility to drop.

The strategy that have to take is then:

- Buy $10 million RWE protection at 42.
- Sell 1.205 contracts of RWE 21 November 2008 puts at 4,347.

The revenue from selling the puts is:

- 1.205 x 100 x 3,29x 1,25 ≈ $495.556
- 1.205 x 100 x 4,347 x 1,25 = $654.767

Revenue = 654.767– 495.556= $159.211

Where 3,29 and 4,347 are the values of the put option calculated by the Black Scholes calculator for the end of the first week of October and on 21 November 2008.

By the end of the first week of October 2008, the five – year CDS referencing to RWE had widened to 73,11. The RWE 1 – month put implied volatility dropped from 71,8% to 39,25%.

[1] For Vega put calculation we used the calculator from the below site: http://www.soarcorp.com/black_scholes_calculator.jsp

Hence the P&L equals:

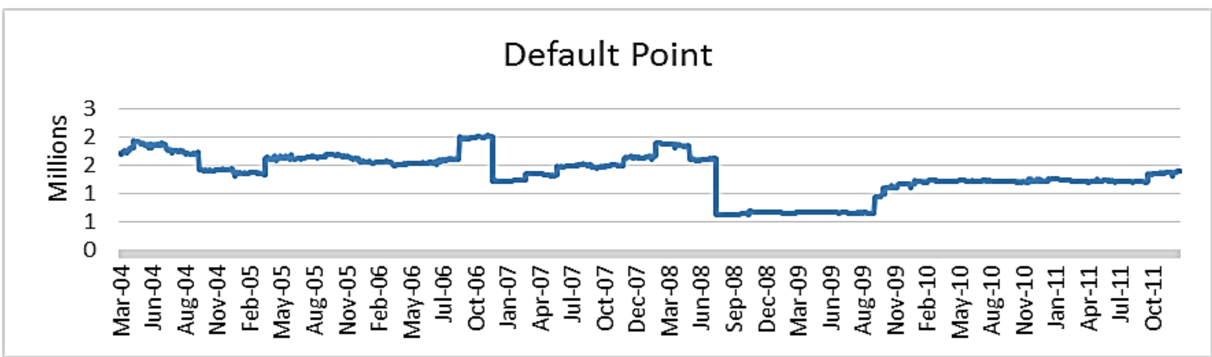
- CDS: 5.000 x 31,11 x 1,25 = $194.438 profit.
- Equity Volatility: 159.211 profit

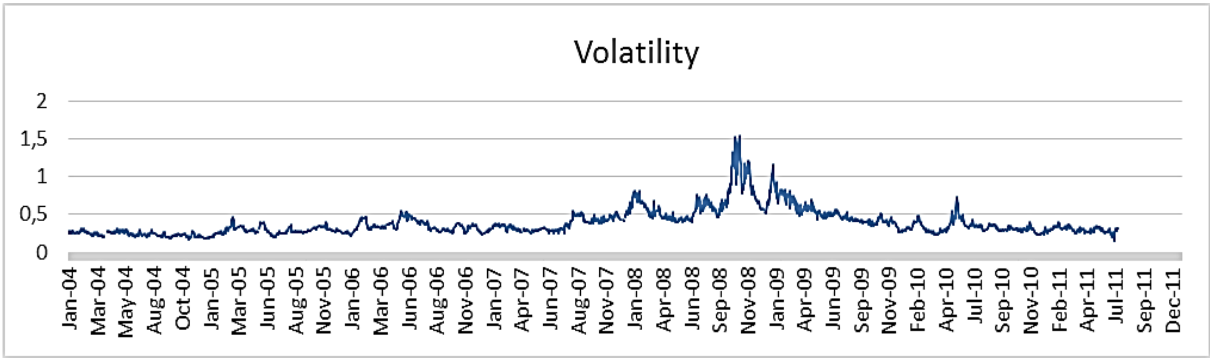
A net profit of $353.649 results

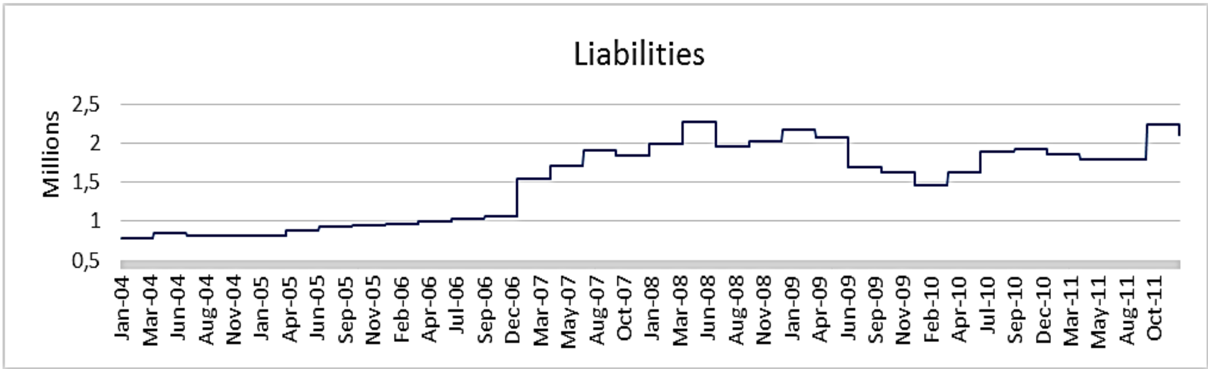
4.2.2 Deutsche Bank Case Study

« Deutsche Bank AG is a German global banking and financial services company, with its headquarters in the Deutsche Bank Twin Towers in Frankfurt. It has more than 100,000 employees in over 70 countries, and has a large presence in Europe, the Americas, Asia-Pacific and the emerging markets. As of June 2017 Deutsche Bank is the 16th largest bank in the world by total asset. » (en.wikipedia.org, 2017)

As shown in Figure 14, during the Lehman default, there was a capital structure misalignment for Deutsche Bank, in October 2008.
The synthetic CDS had dropped below the traded five – year CDS. This indicate that the implied volatility has also dropped.







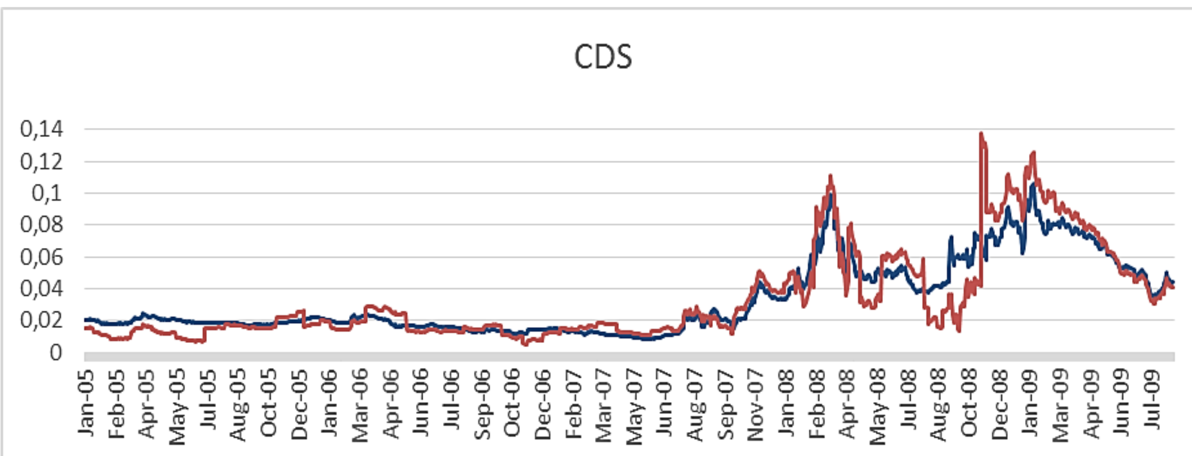


Figure 14. *Synthetic CDS* ***red line*** *versus traded five – year CDS blue line. From January 2005 to July 2009.*

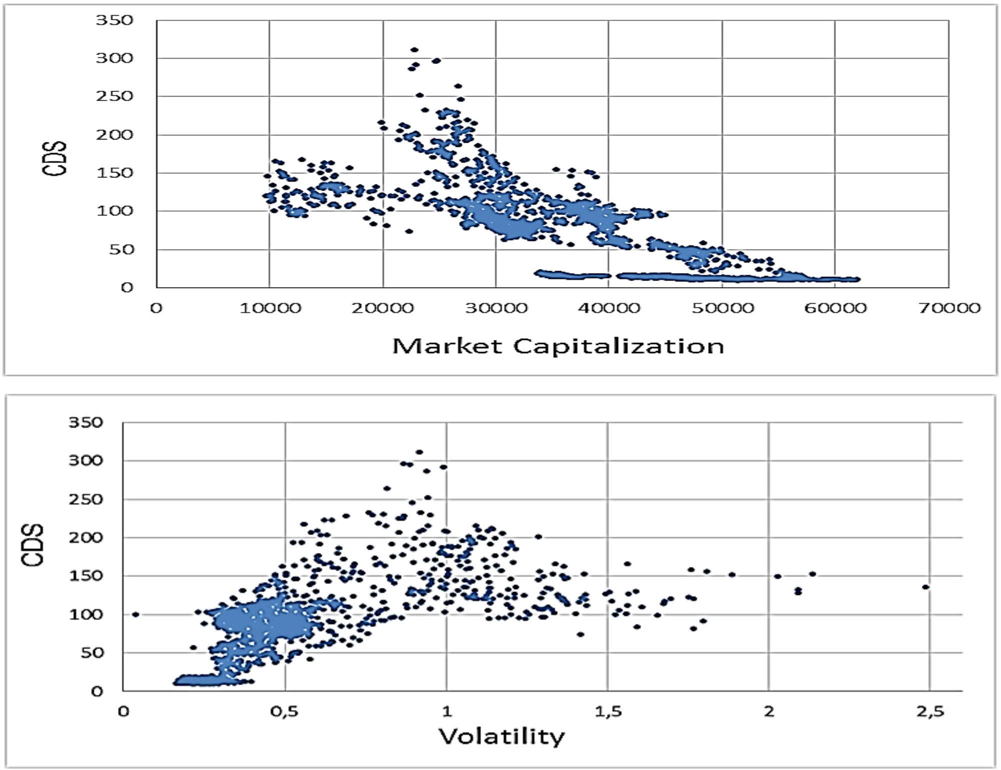


Figure 15. *Scatter plots of five – year CDS versus 10 – delta implied put volatility and five – year CDS versus market capitalization.*

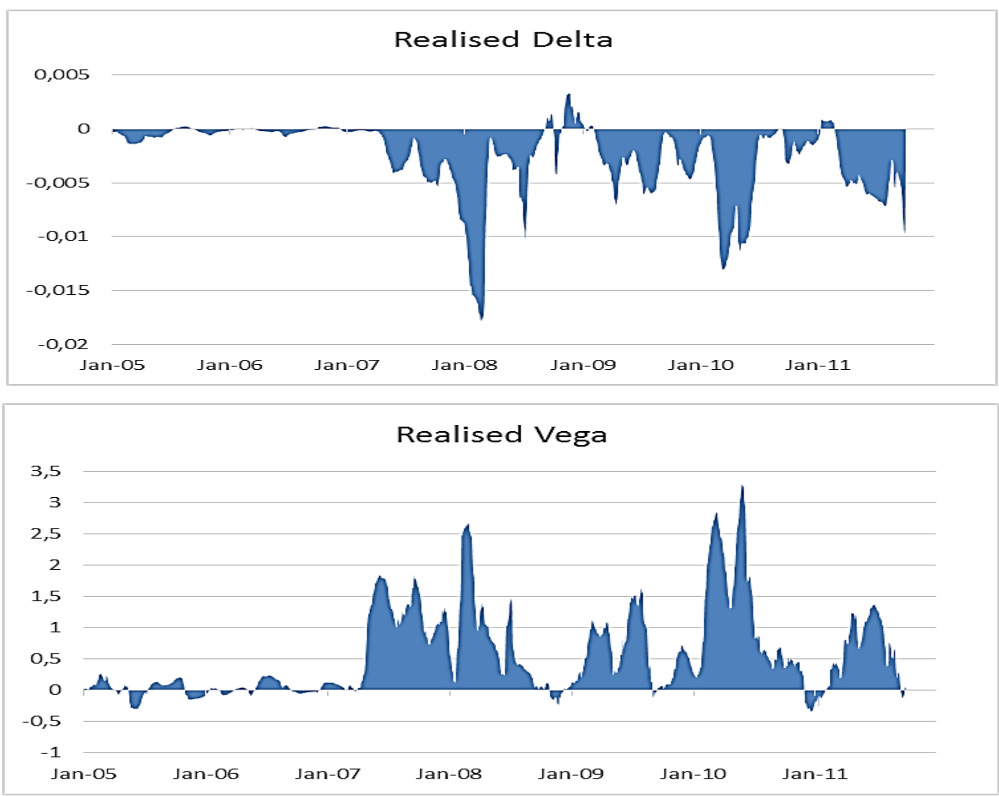


Figure 16. *Regressed 3 – month rolling hedge ratios.*

In figure 15 the five – year traded CDS is plotted against the 10 – delta implied equity put volatility and against the market capitalization. These charts show that small movements in the implied volatility corresponds to large movements in the CDS, whereas large movements in the market capitalization do not move the CDS necessarily.

Hence, the average hedge ratios are:

$$\frac{\Delta CDS}{\Delta S} = -0{,}00257, \quad \frac{\Delta CDS}{\Delta \sigma} = 0{,}55104$$

Consequently, taking a standard five – year CDS contract with notional of $10million a quick calculation for DBK shows that,

DV01=$5000.

Thus,

$$V_{CDS} = 5000 \text{ x } 0{,}55104 \approx \$2.755.$$

Where V_{CDS}, is the Vega of the CDS contract.

For early October 2008, seek liquid option contracts with the longest maturity profile. To satisfy this, choose the DBK16 January 2009 puts at 18,5 with stock price at 19,74, risk free interest rate at 3,8% and implied volatility at 77% .

A quick calculation[2] from a Black-Scholes calculator shows that,

$$V_{OPTION} = 3{,}655$$

Where, V_{OPTION} is the Vega of the put option.

Each contract contains 100 options, which produces the following hedge:

$$Hedge = \frac{2.755}{3{,}655\ x\ 1{,}27} \approx 594$$

where, 1,27 is the foreign exchange spot rate for the Euro.

In this case, you need 594 option contracts to buy.

[2] For Vega put calculation we used the calculator from the below site: http://www.soarcorp.com/black_scholes_calculator.jsp

The strategy that we will take is then:

- Sell$10 million protection on DBK protection at 120.
- Buy 594 contracts of DBK 16 January 2009 puts at 18,5.

By the end of the second week of December 2008, the five – year CDS referencing DBK had widened to 135,38.The DBK 1 – month put implied volatility had increased from 77% to 105,49%.

The revenue from buying the puts is :

- 594 x 100 x 7,057x 1,27 = $532.366
- 594 x 100 x 2,251 x 1,27 = $169.811

Revenue = 532.366 – 169.811 ≈ $362.555

Where $7,057 and $2,251 are the values of the put option on 16 December 2008 and on 1 October 2008, calculated by the Black Scholes calculator.

- CDS: $97.663 loss
- Equity options: $362.555 profit

Net Profit and Loss:$264.892 profit.

Deutsche bank is an important case study because it shows the effectiveness of the Vega hedge. If the trader only sold protections on CDS the result would be a significant loss. Likewise the Delta hedge would have been ineffectual. From October to 2008 until December 2008, the underline DBK stock price only moved from €22.3 to €18.

Information about the put options which used in these case studies are provided at the appendix B.

5. Conclusion

In this thesis the main purpose was to present a model that would compute the value of the CDS. This price of the CDS was referred to as the synthetic CDS. In particular, we aimed to compare the latter with the actual price of the CDS and to show which strategy we should take to avoid bankruptcy.

The traded security was considered as a call option with underlying the asset value of the firm and strike price its liabilities subject to a default point. For pricing the synthetic CDS a code in Matlab was used, where there were two dependent unknown variables for pricing the probability of default of the synthetic CDS; that is, the default point and the volatility of the asset value. The process followed was to estimate an initial price for the default point as the 60% of the total liabilities, then compute the volatility of the asset value, and repeat recursively the process until the latest values converge.

Arguments between Vega and Delta hedge were presented. Regression for calculating the mean Vega and Delta was performed. Additionally, a correlation matrix of CDS versus implied volatility and CDS versus market capitalization was presented. Moreover, scatter plots of CDS versus volatility and CDS versus market capitalization showed how CDS was affected. As discussed, most of them were in favor of the Vega hedge, i.e., it was shown that volatility is a more important parameter than the price of the underlying asset. In particular, large movements of the price of the underlying asset did not move the price of the CDS. Instead very small movements of the volatility moved a lot the price of the CDS.

Two case studies were illustrated to show the effectiveness of the Vega hedge. The first one concerned an underpriced CDS, in which the investors were expecting the actual price of the CDS to rise. Thus, the strategy they had to take was to buy CDS protection and to sell put options. In this case both of these actions were profitable.

The second case study was about an overpriced CDS, in which the investors were expecting the actual price of the CDS to fall. So the strategy they had to take was to sell CDS protection and to buy put options. Here, the hedging leg was crucial, since its resulting profit overcame the loss coming from the CDS leg.

In both case studies the hedge was successful and we ended up with a net profit.

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Appendix A

1. The first code which used in this thesis was for calculated the default point.

```
function [Bt,sigmaV,no] = Solusion( no,sigmaS,  S,  r,  B,  T)
% no = d(1,1)
% sigmaS = d(1,2)
% S = d(1,3)
% r = d(1,4)
% B = d(1,5)
% T = d(1,6)
% Start with Bt(1) = 0.6*B
Bt_old = 0.6 * B;

% Calculate first SigmaV(1)
sigmaV = sigmaS * S /(S + Bt_old);

% Get rout of brent for step (1)
 y = @(bt) blsprice(bt,B,r/100,T,sigmaV/100)-0.01;

Bt = brent(y,0,B);
Bt_old = Bt;

fori = 1: 1000
sigmaV = sigmaS * S /(S + Bt_old);
    y = @(bt) blsprice(bt,B,r/100,T,sigmaV/100) - 0.01;
try
Bt = brent(y,0,B);
catch
Bt=no;sigmaV = -1;
end
if  abs((Bt - Bt_old) / Bt_old) < 0.01
break;
end
Bt_old = Bt;
end
```

Furthermore, a loop has to be done for finding the default point for all the days. For this reason an other function was necessary.

```
function ret = Loop(d)

tmp = size(d);
days = tmp(1);
res = zeros(days,3);
fori = 1:days

no = d(i,1) ;
sigmaS = d(i,2);
   S = d(i,3);
   r = d(i,4);
   B = d(i,5);
   T = d(i,6);
   [bt0,sigmaV0] = Solusion(no, sigmaS,    S,  r,  B,  T);
res(i,:) = [i,bt0,sigmaV0];
end

ret = res;
```

In the command window we define a table d (n x 6) that contains all the known variables. We run the code with the command Loop(d).

2. An other code which used in this thesis is for finding the probabilities of default.

```
function ret = probi(d,out,t)

% Load arrays d & Out and call pi = probi(d,out)
tmp = size(d);
days = tmp(1);
i5 = 1275 % total data for 5 Years
iLast = tmp(1);% index if last day 2017 where we have +5 Years Data
res = zeros(days,1);

fori = 1:iLast

%no = d(i,1) ;
%sigmaS = d(i,2);
%S = d(i,3);
```

```
    r = d(i,4)/100;
%B = d(i,5);
%T = d(i,6);
Bt = out(i,2);
sigmaV = out(i,3)/100;
% t = (i5 +1 - i)/i5; % first day will be 1
Vt = d(i,7);%Waiting definition of Vt

%  pt
        a= ( log(Vt/Bt) + (r - sigmaV^2/2)*t ) / (sigmaV* sqrt(t));
pt = normcdf(-a);
res(i) = pt;
end
ret= res;
```

In the command window we define three tables d, out, t.

3. At last a code for calculate a 3 – month rolling window is used.

```
function [Alpha,Beta] = regr(X,Y)
% Create Y and X rows, must be same length
% Call the function [a,b] = regr(X,Y)
temp = size(Y);
steps=temp(1); % this function gives you the size of the colume vector
Alpha = zeros(steps,1);
Beta = zeros(steps,1);
d=65;%this is discrete window length you wish to regress over - Change as per
requirements
p=1; %number of regressors - Change as per requirements
adjustment=(d-1)/(d-p-1); % regressor adjustment for multiple regression
j=1; %increment level so in this case we are running a rolling regression with a fixed
window length of 24 data points that updates with 1 point increment. The loop will work
for j>1 as well.
for k=1; % this is for the dependant variable so 1=col 1 and so on..increase k as per
requirements
fori=1:j:steps-d;
        [bbint r rint stats]=regress(Y(i:64+i,k),[ones(65,1),X(i:64+i,1)]);
Alpha(i,k)=b(1);
Beta(i,k)=b(2);
R_Squared(i,k)=stats(1); %stats provides other information as well such as F-stat;p-
valueetc that can be added to this easily
%Adj_R_Squared(i)=1-(1-R_Squared(i))*adjustment; %Use this when the number of
regressors is greater than 1
error(:,i,k)=r(:,1); %if k>1 then the residulas will be 3D but are easily understood based
on the matrix structure
CI_Alpha(i,:,k)=bint(1,:);
```

```
CI_Beta(i,:,k)=bint(2,:);
ifCI_Alpha(i,1,k)*CI_Alpha(i,2,k)<0;
Significance_Alpha(i,k)=0;
else
Significance_Alpha(i,k)=1;
end
ifCI_Beta(i,1,k)*CI_Beta(i,2,k)<0;
Significance_Beta(i,k)=0;
else
Significance_Beta(i,k)=1;
end
end
end
```

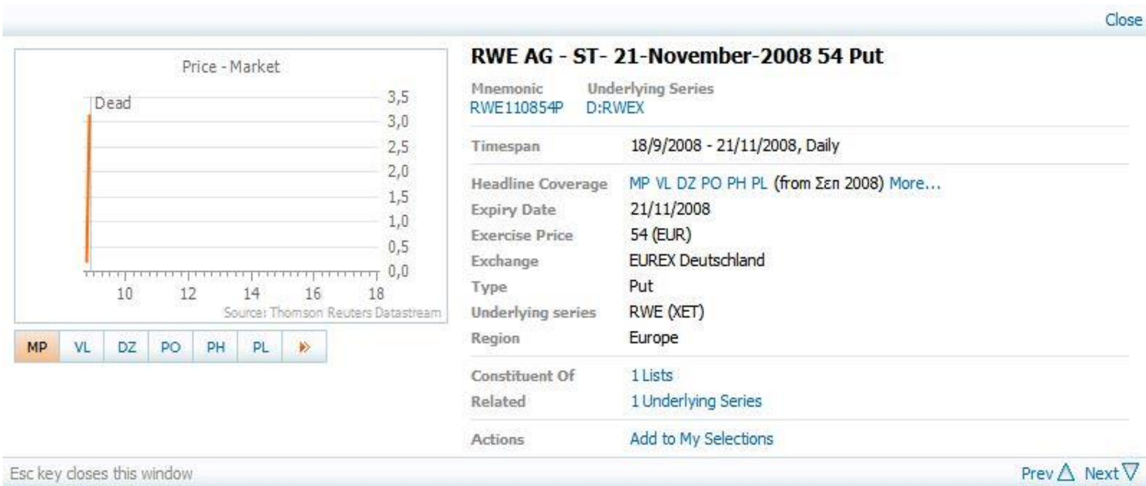
At the command window we define the variables X, Y and we regress the equation

$Y = a + bX$. With the help of the code.

We run the command [a,b]=regr(X,Y).

Appendix B

Informations about the put option which used in the first case study.



Informations about the put option which used in the second case study.