THESIS

The role of VIX index as investor fear gauge

Thesis submitted at the University of Piraeus in partial fulfilment of the requirements for the M.Sc. in Financial and Banking Management

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Table of Contents

Figures and Tables

Figures

Tables

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ABSTRACT

This thesis focuses on the VIX index, also called the investors' fear gauge. The VIX index is a volatility index that is supposed to express the market expectations about the 30-day volatility. It has been created by CBOE (Chicago Board Option Exchange) in 1993. Soon after its creation, CBOE also created several derivatives written on VIX. The investors' fear gauge has drawn both academics' and practitioners' attention ever since it was first introduced. Nowadays the index has become really important especially for investors. It can be used as a measure of market's predictions about the S&P 500 fluctuations in 30 days ahead. It can also be used to hedge the risk of investments by taking the opposite position of which it has open and, for speculation reasons, as an investor can bet on the increase or the decrease of the index.

In this thesis, we study the relation between the VIX index and the futures contract written on it. We present historical volume data and we then concentrate on the causality between VIX spot and VIX futures. The controversial issue of whether the price of the VIX spot can predict the price of the VIX future (and vise versa) is extensively analyzed. In order to support our assumptions and verify our results, we use two econometric methods. The Linear Engle-Granger cointegration and the resulting model, which is known as a vector error correction model (VECM).

Introduction

This thesis focuses on the VIX index, and on the causality issues between spot and futures prices and the probable predictability of VIX index to the realized volatility.

The VIX index was created by the CBOE (Chicago Board Option Exchange) in 1993 and it aims at estimating the implied volatility of S&P 500. More precisely, VIX is an index of implied volatility of 30-day options on the S&P 500 calculated from a wide range of call and puts (www.cboe.com). The index itself and the derivatives written on it have drawn great attention not only from academics but traders as well, for various reasons. One of those reasons is that the index is forward looking and it is widely used as a measure of market risk. Because of this use, the index is also called "fear gauge" (Whaley, 2008).

Nowadays, the VIX is really important especially for investors. It is used not only as a measure of how much the market thinks the S&P 500 will change in the 30 days (www.cboe.com), but also for hedging the risk of investments in the stock market by taking the positions to the VIX products (derivatives or ETFs). Furthermore, it can be used for speculation reasons, as an investor can bet on the increase or the decrease of the index.

Both the issues that we plan to study (causality and predictability) have captured many researchers' attention.

This thesis structure will be as follows. In the First Chapter, we present the index and the way it has been calculated. VIX was based on S&P 100 till 2003, since then is based on the S&P 500. Furthermore, we review the basics of the derivatives written on the VIX and its ETFs. Moreover, we present details about the main reason of the popularity of this index and its uses (hedging, speculation and forecasting). Furthermore, we have collected some data on the amount of derivative contracts traded per day and present charts which compare the closing prices of VIX index with the closing prices of S&P 500. In the last part of the chapter, we develop our study on the relation between VIX and the stock market.

In Chapter 2, we are conducting a literature review on the causality between the VIX spot and the VIX futures. We have studded the summary statistics of the index and its futures and in the second part of this chapter we have also examined if there is a long run equilibrium between the VIX spot price and the VIX future price (Enger Granger cointegration test).

In Chapter 3 we conduct another econometric test, the vector error correlation model (VECM) which has been suggested also from Enger and Granger. Finally, the work's findings are summarized and discussed.

CHAPTER 1

1.1 Definition of the term index in stock exchange

An index is a statistical measure which describes the changes in a portfolio of stocks of a specific sector of market. It is thus a mathematical object used by investors and financial managers to describe and compare specific features of a market.

Mr. Charles Dow created the first index ever in May of 1896 and the most widely known until now. Nowadays it is known as the Dow Jones Industrial Average[1] (ticker symbol "DJIA"). Since 1896 many other indices have been created for example S&P 100, S&P 500, Nasdaq Composite index etc. Other types of indices are capitalisation indices[2], fixed income indices[3], residential property indices[4], sector indices[5], strategy indices[6] and volatility indices, which are analyzed in detail in the next paragraphs.

1.2 Volatility Indices and the VIX index

Volatility indices measure the market's prospect of volatility based on the prices of options. The most widely used volatility indices is the VIX index of Chicago Board Option Exchange (CBOE).

The VIX index is more or less, like the Nasdaq Composite index. Their major difference is that the VIX measures volatility, that is the unexpected moves either up or down of an index, and not price. The VIX index (volatility index) was created in 1993 by the CBOE. The index is an instant measure of implied market volatility. It is calculated by using the mean of real time S&P 500 options (SPX).

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[1] The Dow Jones Industrial Average (DJIA) contains 30 of the largest and most influential companies in the U.S.

[2] Capitalisation indices represent the sum of the market capitalisations of the companies making up the index (www.asx.com.au)

[3] Fixed income indices measure the performance of the bond market and the short-term money market. (www.asx.com.au)

[4] Property indices are similar to other asset performance indices in that they measure changes in the market value of the asset class in question, in this case residential property. (www.asx.com.au)

[5] Sector indices enable investors to benchmark the performance of a specific stock market sector or industry. (www.asx.com.au)

[6] Strategy indices track the performance of a specific investment strategy (www.asx.com.au)

When Whaley developed the VIX index, had two things in mind (Whaley ,2008). The first one was that, the index had to become a barometer of implied short-term (30- days) market volatility. He also wanted to make the comparison between the then-current level of VIX with the historical ones, so the minute by minute values were calculated using index option prices of 1986. The choice of this specific date was not random. 1987 was the period of the worst stock market crash since, the Great Depression, the worst depression that the world had ever faced until then. This was very important since inventors were able to report the level of market's worries during a very difficult period for the economy. The second reason for the creation of the index was the intention of creating an index which could be used as an underline asset, for futures and options contracts on volatility. The importance of such trading assets was recognized soon after they were first launched (futures May 2004 and options February 2006).

1.3 The "Investor Fear Gauge"

The VIX index is also called "investors fear gauge" (Whaley,2008). It has been named like that because, as already mentioned, it is forward looking and shows what market guesses for the 30-days volatility. The index became popular because many investors want to hedge their investments. Investors can buy puts on the index in order to avoid loses from a potential drop in the price. Figures 1, 2 and 3 show the negative correlation between the prices of the index and the prices of S&P 500). So, as investors became more worried about the potential prices decreases they ask for more puts so they push the put prices up. The raise of the prices of puts increase the implied volatility (ceteris paribus)[7] so the price of the index is increased as well. The VIX is thus an index that reflects the price of the portfolio insurance (investors fear gauge).

[7] The price of the put is an increasing function of implied volatility and as a result a "1-1" function.

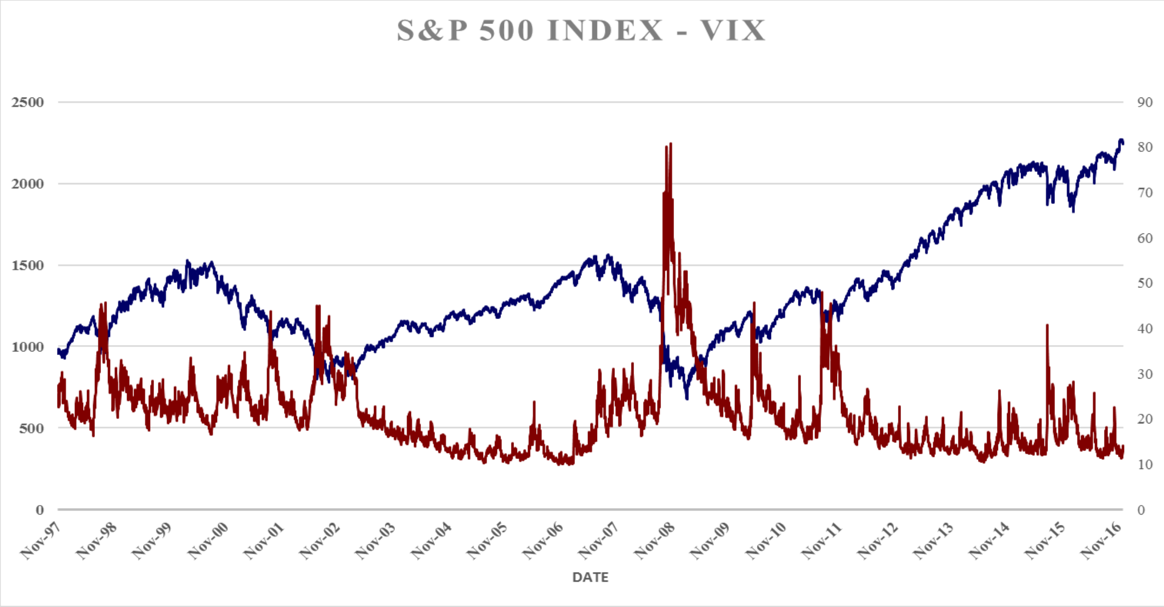

Figure 1:

Daily closing prices of S&P 500 Index (blue line) and VIX index (red line) from 28th of November 1997 to 18th of October 2016. Correlation coefficient: (-0.704274)

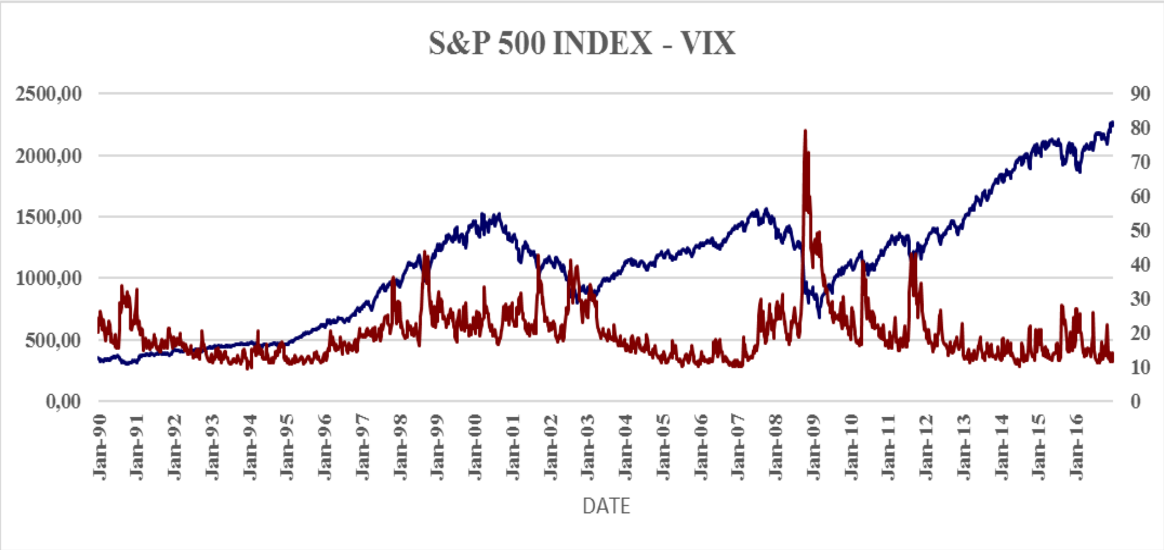

Figure 2:

Weekly closing prices of S&P 500 Index (blue line) and VIX Index (red line) from 1st January of 1990 to 28th of November of 2016. Correlation coefficient: (-0.685823)

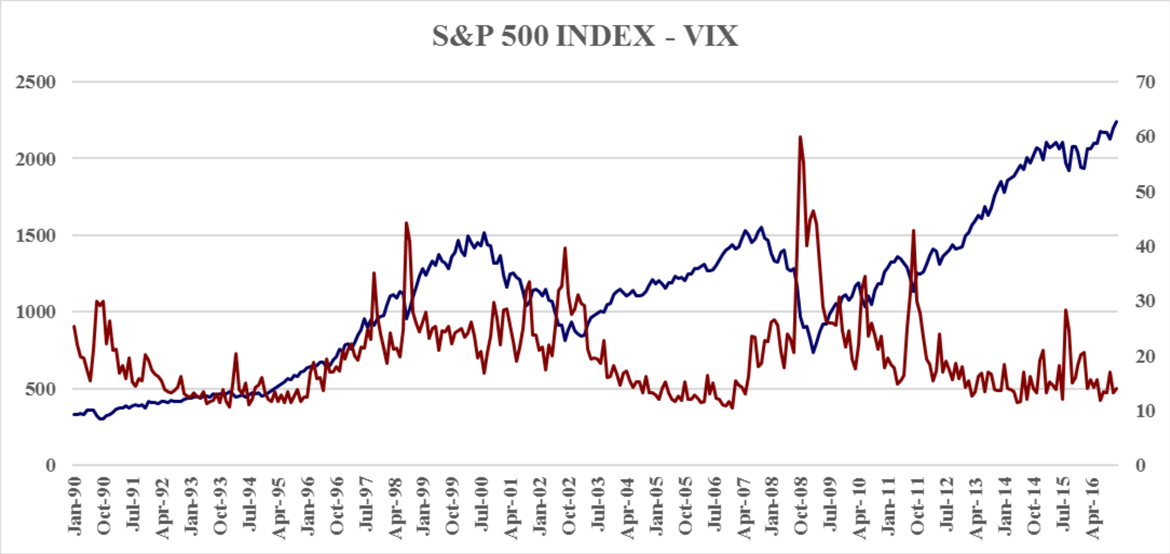

Figure 3:

Monthly closing prices of S&P 500 Index (blue line) and VIX Index (red line) from 31st of January of 1990 to 30th of September of 2016. Correlation coefficient: (-0.638207)

1.4 Change in the way of calculation

The VIX index was designed to measure the market's beliefs of 30-day volatility implied by at-the-money S&P 100 option prices (ticker symbol "OEX"). The index was based on S&P 100 because the OEX options were the most actively-traded index options in the US. In 1992, the OEX options accounted for 75% of the total index options volume (Whaley, 1993). So, for the index to be useful and trustworthy, it had to be based on a deep and active index option market. Another very important feature of the original way of calculation for the VIX index is the fact that it was based on the prices of only eight at-the-money index calls and puts. The fact that they used at-the-money calls and puts was reasonable. Because, at-the-money options were by far the most actively traded ones. Investors tended to buy at-the-money options because they were more likely to be exercised.

Ten years later in 2003, CBOE in association with Goldman Sachs, updated the VIX to reflect a new way to measure implied volatility. The new way is to the one that has been widely used by academics and practitioners. Firstly, the new VIX is not based on S&P 100 but on S&P 500 (ticker symbol "SPX"). Since the introduction of the index market's conditions have changed (While,2008). In 1993 SPX option market was about one-fifth as active as the OEX option market, as time passed SPX options became more popular (Figure 4).

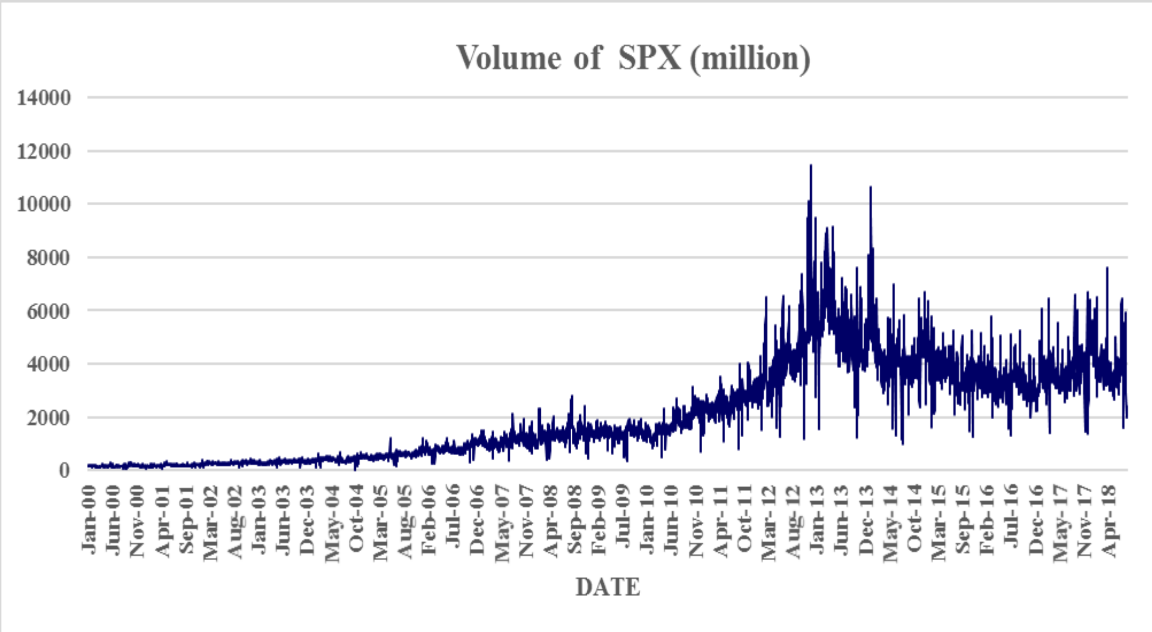

Figure 4

Daily volume of SPX from 2nd January of 1990 to 7th January of 2016.

It is still uncertain why this shift in investors' preferences took place. Some possible answers could be that the S&P 500 index is better known, future contracts are actively traded and if someone thinks more practically, S&P 500 options are European-style (Whaley, 2008). Being European style means that they can be exercised only on the expiration date this feature makes them easily valued. As that OEX options became less popular in the passage of time, SPX options became more popular so they exceeded OEX options. As a result, VIX's way of calculation had to be changed because it had to be based, as it was mentioned before, on a deep and active option market. It is of minor importance which specific reasons lead the investors to move toward SPX options. But, it is worth to be mentioned that contrary to the early 90's when both index calls and index puts were of the same importance for the investors, things changed as well. As the index option market started to be deluged with portfolio insurers, out-of-the money and at-the-money puts became very popular. The above, had as a result the change of the way of calculation of the index. The new VIX was not only based on S&P 500 but it included out of the money options as well.

The formula that it is used to calculate the VIX is as follows:

$$\boldsymbol{\sigma}^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

Where:

$$\frac{VIX}{100} \Rightarrow VIX = \sigma * 100$$

T Time to expiration

F Forward index level desired from index option prices

K_0 First strike below the forward index level, F

K_i Strike price of the ith out-of-the-money option; a call if $K_i > K_0$; and a put if $K_i < K_0$; both put and call if $K_i = K_0$.

ΔK_i Interval between strike prices-half the difference between the strike on either side of K_i:

$$\Delta K_i = \frac{K_{(i+1)} - K_{i-1}}{2}$$

Note: ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lowest strike.

R Risk-free interest rate to expiration

$Q(K_i)$ The midpoint of the bid-ask spread for each option with strike K_i

1.5 Historical Background

After the introduction of the index in 1993 and the change of its calculation in 2003, the first exchange-traded VIX futures contracts had been launched in 2004 (all-electronic CBOE Futures ExchangeSM, CFE®). It was only two years after the launch of the futures that the first options on the VIX index were created (in February of 2006). The options written on the index have been considered by many investors and academics as the most successful product of the CBOE.

Then in 2008, CBOE advanced the use of the VIX methodology to estimate expected volatility of specific commodities and foreign currencies (www.cboe.com).

For example, the CBOE U.S. Energy Sector ETF Volatility Index (VXXLESM) CBOE Emerging Markets ETF Volatility Index (VXEEMSM) etc. Each of these volatility indices are calculated using exchange traded funds (ETF's).

The development of the index did not stop at that point. In 2014, CBOE improved the VIX Index, it included a series of SPX WeeklysSM (ticker symbol "SPXW"). SPX WeeklySM are options based on S&P 500 with weekly expiration. Nowadays the CBOE bargains tree type of SPWX, the SPXW Friday weeklys, the SPXW Wednesday and SPXW Monday Weeklys. SPXW can be used on targeted buying, selling or spreading strategies. Specifically, SPXW Weeklys may help investors to take advantage of market events, like government reports and announcements. The SPXW was introduced in 2005. Weekly options are now available on too many of indexes, ETFs, ETNs and equities. They have become very popular and actively-traded especially as a hedging tool. They considered so useful because the track the index better than the monthly ones.

As mentioned, the importance of the index was recognized soon after it was launched. But what exactly makes an index important and what is its precise meaning? The answer is mainly the comparison of its current price with previous ones.

In order to understand and take full advantage of the index an investor can study **Figure 2**. It shows week-ending levels of S&P 500 and VIX from the beginning of January 1990 through November 2016. Many observations are outstanding. It is obvious that VIX reached one of its highest levels in October 20, 2008 (the higher one since October 1987, which according to Whaley 2008, is the only time VIX ever exceeded 100).

On October of 2008 another global financial crisis occurred; soon after the bankruptcy of Lehman on September of 2008. Another interesting phenomenon is that the index is quite volatile with many peaks. Some indicative dates are, early 1991 when U.S.A forces attacked Iraq and mid-1990 when Iraq invaded Kuwait. Then two sharp spikes occurred on October of 1997 and 1998. On 1997 occurred a sell-off which resulted in the drop of 555 points of Dow (Whaley, 2008). The 1998 spike occurred in a period of general anxiety. Investors were afraid of a possible drop in the prices and wanted to secure their investments. On 2001 and 2002 happened two other spikes, 2001 was because of Enron bankruptcy and 2002 was because many internet companies bankrupt (e.g Webvan, Exodus Communications, and Pets.com). Another spike started at the end of 2009 and mainly on 2010, the "European crisis" as it has been characterized. A crisis caused because of the failure of many European countries to recompense or refinance their government debt or even to bail out banks with great debt under their national supervision and they needed help from third parties such as other Eurozone countries, the International Monetary Fund (IMF), or the European Central Bank (ECB). Furthermore, on September of 2011, happened a sharp drop in stock prices of stock exchanges across the United States, Middle East, Europe and Asia. This was because of worries of contagion of the European sovereign debt crisis to Spain and Italy, as well as worries over France's credit rating downgrade. Of course, in the aftermath of each spike, the VIX index returns to normal levels.

Finally, it is worth to be mentioned that although the weekly closing levels of S&P 500 and VIX appear to move on opposite directions, there are also times when an increase in stock prices is go together with an increase in volatility. Characteristic examples of such cases are those of January 1999 when the VIX was rising while the level of S&P 500 was rising as well. The same pattern appears in early 1995 the June and July of 1997 and December of 1999. Those examples make it clear that investors become worried even when market is going well. This is also apparent since the correlation, between the closing prices of S&P 500 and VIX, is about -0,7.

1.6 Derivatives and ETFs written on the index

1.6.1 Futures

In 2004, the CBOE introduced the first futures on the VIX index. Futures contracts are agreements between two parties to buy or to sell an asset at a certain time (expiration time) in the future for a certain price (strike price). Future contracts are traded on exchanges. Each future is written on a specific underline asset. The underline of the VIX future is the VIX index.

In July 2015, the CBOE Future Exchange introduced VIX weekly futures. In general, weekly futures have the same contract specifications as the monthly expiring contracts. The advantage of weekly expirations to standard monthly futures expirations is the fact that they offer volatility exposures that track the performance of the VIX

Index more precisely. Weeklys VIX futures on VIX are a different product from futures on the VXSTSM Index[8].

1.6.2 Options

In 2006, CBOE introduced for the first time options written on the VIX index. Options are products that are traded in exchanges and in the over the counter market. There are two categories of options. The first one are the call options and the second one the put options. A European call option gives the holder the choice (not the obligation) to buy the underlying asset at a fixed date and at a certain price. On the other hand, a European put option gives the holder the choice (not the obligation) to sell the underlying asset at a fixed date and at a certain price. Both call and put options are written on an underlying asset. The VIX options are written on the VIX index. Investors who believe in the increase of the market volatility buy VIX calls. On the other hand, many investors use VIX options as hedging tools. An investor can choose among many strategies (bull[9], bear[10] etc.) that consist of calls and puts.

VIX options have some distinct characteristics which make them different from the other index options. Some of those characteristics are the following. The pricing of VIX option is based on forward VIX value. The settlement takes place every Wednesday. Furthermore, there is a negative correlation to the stock index and the option have a high volatility of volatility.

On October 8, 2015 VIX Weeklys options began trading at Chicago Board Options Exchange, Incorporated (CBOE®). Today, SPX Weeklys account for one-third of all SPX options traded, and average over a quarter of a million contracts traded per day. SPX Weeklys currently represents approximately 30% of all SPX options volume.

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[8] VXSTSM index, (CBOE Short-Term Volatility IndexSM), is an index of implied 9-days volatility.

[9] Bull spread is a speculative strategy for which the investors buy one option with the low strike price and sell one option with the high strike.

[10] Bear spread is a speculative strategy for which the investors sell one option with the low strike price and buy one option with the high strike

1.6.3 ETFs

It is well known that many ETFs trade the VIX index. An ETF (exchange traded fund), is a fund that tracks an index, a commodity, a bundle of bonds, or a basket of assets like an index fund. Mutual funds are different from ETF though. ETF is traded exactly like a stock on a stock exchange. ETFs experience price changes throughout the day as they are bought and sold. ETFs in general have higher daily liquidity and lower fees than mutual funds This is the main reason that especially individual investors use them as an alternative to mutual index funds. The first S&P 500 VIX ETF was launched in 2010 by Source UK Services[11]. In January of 2011, ProShares[12] launched the first VIX Short-Term Futures ETF and VIX Mid-Term Futures ETF.

1.7 Usefulness of the index

The VIX index has drawn great attention since it has first been first launched. The index is commonly used by investors for three specific reasons.

Its first use and the purpose of its creation is to make available an instantaneous measure of how much the market thinks the S&P 500 will vary in the following 30 days as it will be studied in the following chapters. The second main use and perhaps the most interesting one, is for hedging the risk of investments. Something like that is feasible because as it has been shown the VIX index has a negative correlation with the S&P 500 (Figures 1, 2, 3). Finally, the index can be used for speculation reasons. Many investors make their living by using speculation strategies. So, an investor can bet on the increase or the decrease of the index (direction) or even on the spread (the investor bets on the magnitude of the change).

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[11] Source UK Services Ltd., or simply Source, is a specialist British-based provider of exchange-traded funds (ETFs) and exchange-traded commodities (ETCs).

[12] ProShares is a leading provider of exchange traded funds (ETFs) designed to help investors reduce volatility, manage risk and enhance returns.

1.8 VIX relation to the stock market

The fact that the VIX index increases when the market drops is why it has been known as “investors fear gauge”. If someone looks the **Figure 1** carefully, he will find out that the change in VIX increases at a higher absolute rate when the stock market falls than when it increases. To prove the above proposition, it is enough to regress the daily rate of change of VIX, $RVIX$, the rate of change of S&P 500 index, $RSPX_t$, and the rate of change of S&P 500 index conditional on the market going down and 0 otherwise, $RSPX^-$, we have used the same symbols as Robert E. Whaley in his paper in 2008, so the regression model is as follows:

$$RVIX_t = \beta_0 + \beta_1 RSPX_t + \beta_2 RSPX_t^- + \varepsilon_t$$

In order for our proposition to be true, the intercept term should not be significantly different from 0, and the slope coefficients should be significantly less than 0. It turns out, our predictions are true. The estimated relation between the rate of change of VIX and the rate of change in SPX is

$$RVIX_t = -0.0014 - 3.4104 RSPX_t - 1.1466 RSPX_t^-$$

where the number of observations used in the regression is 6801 and the regression R-squared is 50.9%. All the regression coefficients are significantly different from zero at the level of 5% apart from the intercept term.

The intercept in the regression is -0.0014 and it is not significantly different from zero. Not being significantly different from zero means that if the S&P does not vary over the day, the VIX will change but this change will be insignificant. Something like that is not surprising. While investors think the value of stocks will increase over the time in order to be compensated for putting their money at risk, volatility is not. When looking the graph of VIX someone can notice that volatility tends to follow a mean reverting process, when VIX is high, it tends to go back down and when it is down it goes up again.

The estimated slope coefficients are both negative as expected, and significant. It is clear that they mirror not only the opposite relation between VIX and S&P 500 but also the asymmetry of their movement. To understand the coefficients, someone can consider the followings:

If S&P increases by 100 basis points, the VIX is expected to fall by

$$RVIX_t = -3.4104(.01) = -3.4104\%$$

And if S&P 500 decreases by 100 basis points the VIX is expected to rise by

$$RVIX_t = -3.4104(.01) - 1.1466(.01) = -4.557\%$$

The demand of portfolio insurance makes the relation between rates of change in the S&P 500 and VIX asymmetric. VIX is more a gauge of investors' fear of a possible downside than a barometer of investors' optimism of a possible upside. It is of great importance though, to point out that the above results express correlation and not causality.

Chapter 2

2.1 VIX and Futures

The data for this study cover the period from March 26, 2004 to December 21, 20016. Between March 26, 2004 and March 8, 2006, only four future contracts were available each trading day. Since then the number of contracts has increased. Each contract has different expiration day.

We have constructed ten different VIX future prices series, each with rolling contracts. Each F*i* consists of the i^{th} nearest to maturity futures contract. This contract rolls to the new i^{th} nearest to maturity contract when the current expires. For example, F1 consists of the first nearest to maturity futures contract and rolls to the new F1 when the current contract expires. The data was collected from Bloomberg.

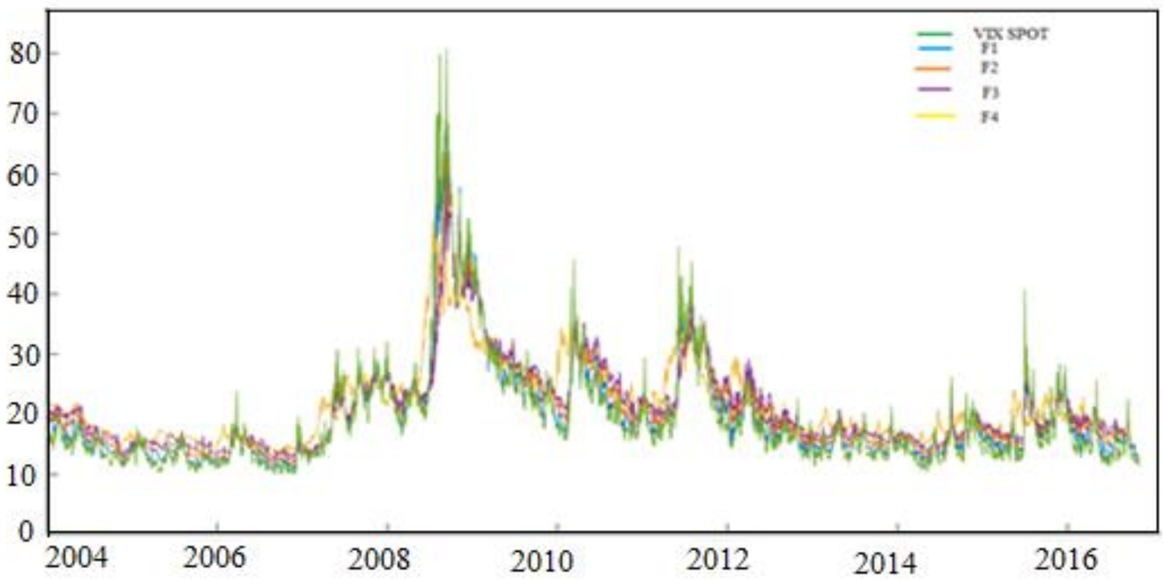

Figure 5

VIX spot and the four VIX futures contracts that have been created in 2004. F1 is the price of the nearest to maturity futures contract, F2 the second nearest to maturity and goes on. The sample period is from March 26, 2004 to December 21,2016.

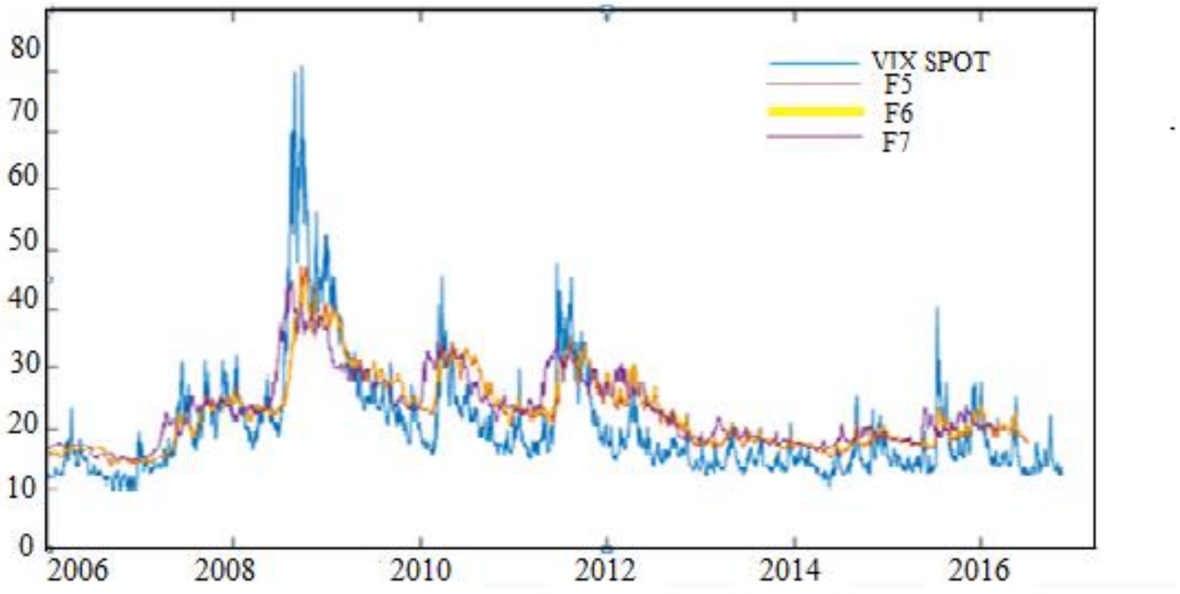

Figure 6

VIX spot and the tree VIX futures contracts that have been created in 2006. The sample period is from March 9, 2006 to August 17,2016.

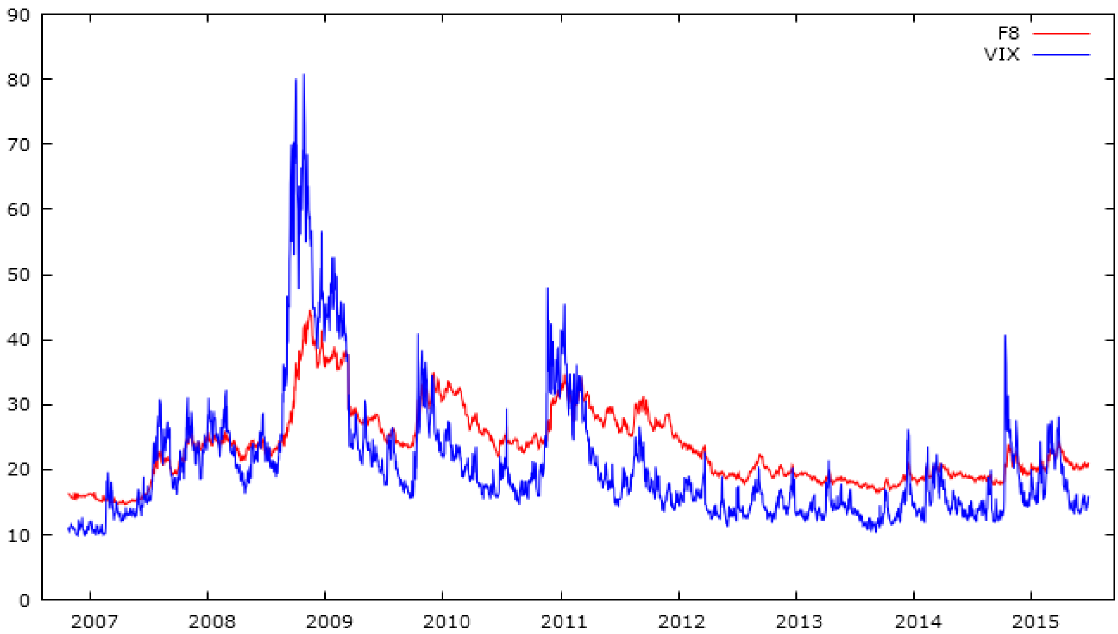

Figure 7

VIX spot and the two VIX futures contracts that have been created in 2006. The sample period is from October 23, 2006 to May 18,2016.

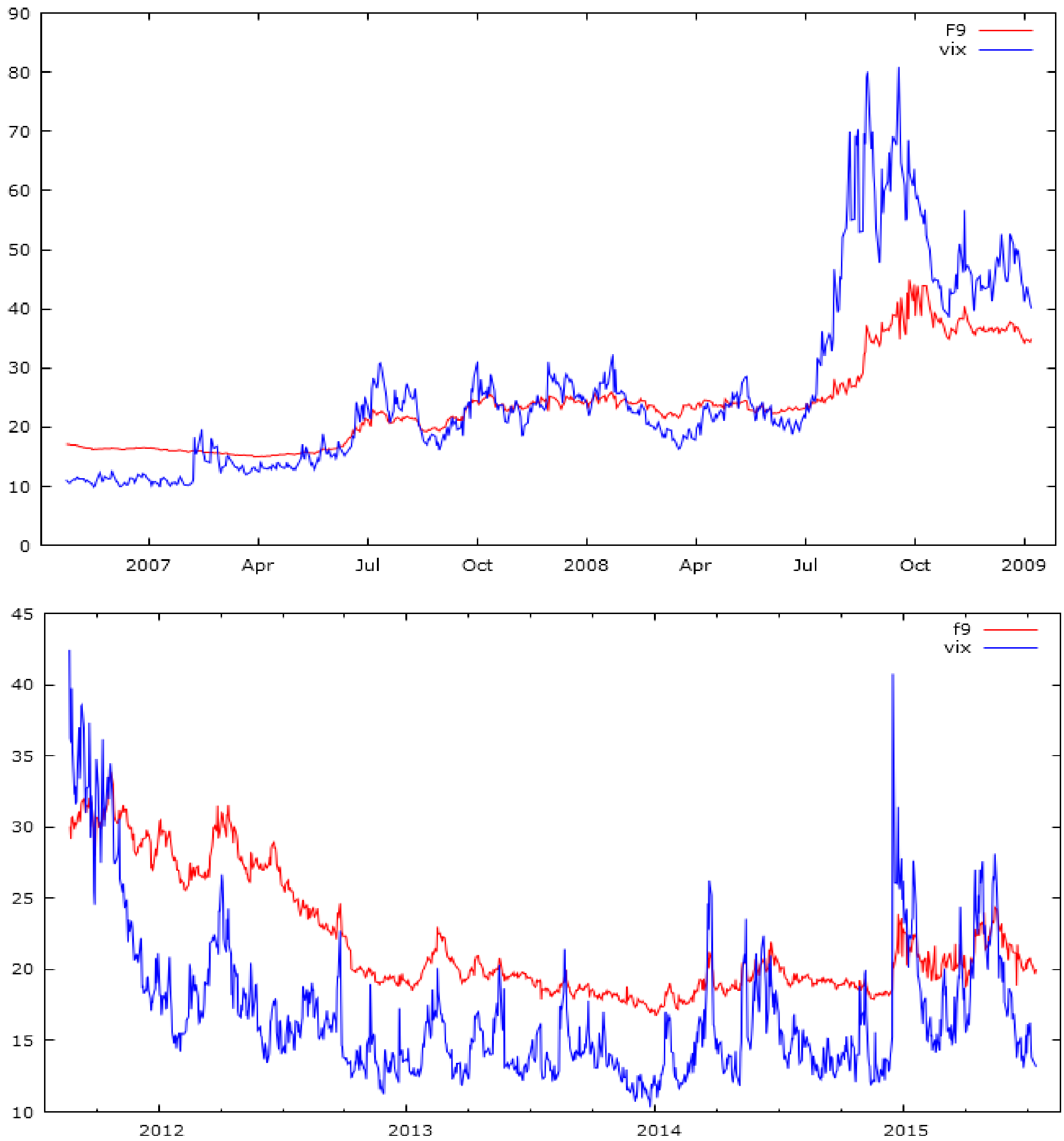

Figure 8

VIX spot and the two VIX futures contracts that have been created in 2006. The sample period is from October 23, 2006 to May 18,2016. We showed the diagrams of F9 with VIX in two different plot because in 2010 in the market was traded only eight futures.

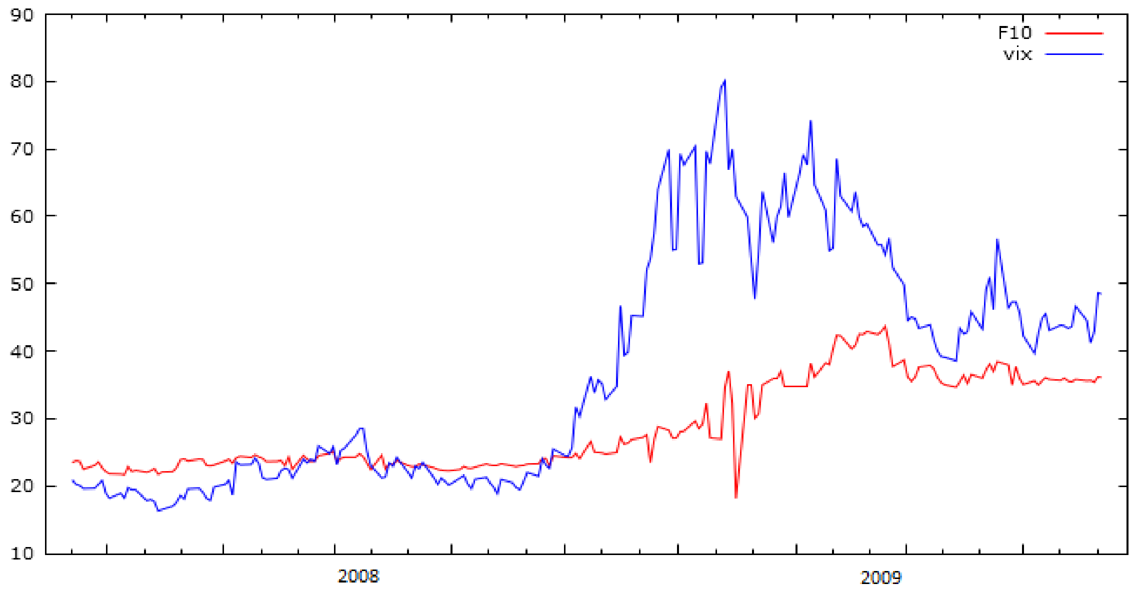

Figure 9

VIX spot and a VIX futures contracts that have been created in 2008. The sample period is from April 22, 2008 to February 18,2009.

Figures 5, 6 ,7, 8, 9 plot the daily prices of VIX since March 23, 2004 as well as the ten constructed contracts. Looking at the plot makes it clear that VIX usually moves in the same direction with its futures prices. Though, VIX is more volatile in the crises period, (the last quarter of 2008 and the first quarter of 2009). At the same period S&P 500 felled sharply (Figure 2). Furthermore, we observe that in turbulence periods VIX prices are higher from futures prices while when the economy is blossom or at least goes better futures prices are higher from those of the index. It is expected that the price of VIX is higher when the economy is turbulence because it is an index that expresses the market's expectations. This may suggest that when the traders are worried that portfolio insurance with long S&P 500 options is very expensive; on contrary futures market is more stable with higher liquidity, and hence more attractive (Shu and Zhang, 2012). From the other hand, when the economy is blossom or at least goes better, future prices are higher; this is maybe due to the fact that investors cannot be sure that nothing bad will happen to the economy in the future, so they demand some premium in order to sell futures. Another possible explanation is that money invested today will not have the same value one, two or ten months ahead.

Table I presents the summary statistics of VIX and VIX futures prices. Looking at the Table I, we can see that the mean of the VIX prices and the mean of the VIX futures prices are more or less the same, but the index has a higher standard deviation and range as well. This has also been observed by Shu and Zhang 2008 2008 and Anthropelos, Bouras and Malmpanzi, 2017. Furthermore, futures which have longer maturity are more stable than those with shorter time to maturity. It is obvious from the Table I that the standard deviation from returns decreases with time to maturity (this is not the case for F9 and F10). Nevertheless, this is reasonable since the nearest to maturity futures contract tracks the changes in the spot VIX closer. Both the spot and the futures VIX have excess kurtosis and are positively skewed apart from the last nearest to maturity contract F10.

Similar time-series has been used and analyzed by Shu and Zhang (2012), where the sample period is from March 2004 to May 2009 and also by Anthropelos, Bouras and Malmpanzi (2017). The important feature of our longer data set is that it not only includes the financial crises of 2008 and 2010 but also, one more time series F10.

If we compare our results (summary statistics) with those of the Shu and Zhang (2012) and Anthropelos, Bouras and Malmpanzi (2017), we notice that levels have higher mean of those of Shu and Zhang and lower of those of Anthropelos, Bouras and Malmpanzi (2017) and lower standard deviation from both. This has to do with the fact that our data set is larger and apart from the period of crises we have included the period after the crises when the economy has started to improve. So, it is reasonable that our results have higher mean of those of Shu and Zhang that haven't included the crisis of 2008 when the index had reached very high prices but it would also be reasonable that are lower of those that Anthropelos, Bouras and Malmpanzi have estimated since we have included prices of the index six year after the pick of the crises, prices that they are obviously lower. Furthermore, the smaller standard deviation may be a result of our larger data set and to the fact that we have included many years before and after the crisis, periods that things for the economy are smoother.

As far as the returns are concerned, they also have lower standard deviation also because of the period covered from our data set. It is worth to be pointed out that because of the high kyrtosis, the tails are thinner, fact that has been caused from the financial crisis of 2008.

TABLE I

Summary statistics of levels and returns of VIX spot and VIX Futures Contracts

	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
Panel A: Summary statistics of levels											
Mean	19,16	19,66	20,53	21,13	21,60	22,65	22,92	23,30	23,38	22,53	28,76
Median	16,24	16,93	18,28	19,22	19,88	21,50	21,95	22,62	22,65	20,73	24,82
Standard deviation	9,16	8,39	7,53	6,94	6,50	6,40	6,14	5,96	5,72	5,57	6,60
Kurtosis	8,79	6,56	4,50	3,41	2,26	1,00	0,56	0,39	0,60	1,57	-1,14
Skewness	2,58	2,28	1,89	1,65	1,42	1,07	0,93	0,86	0,92	1,31	0,61
Range	70,97	58,00	51,28	44,52	39,33	33,80	31,18	30,68	29,72	34,69	25,56
Minimum	9,89	9,95	11,62	12,35	12,93	13,54	14,08	14,32	14,73	10,25	18,14
Maximum	80,86	67,95	62,90	56,87	52,26	47,34	45,26	45,00	44,45	44,94	43,70
Panel B: Summary statistics of levels											
Mean	-0,0001	-0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0002
Median	-0,0025	-0,0025	-0,0014	-0,0010	-0,0006	-0,0003	-0,0003	0,0000	0,0000	0,0000	0,0000
Standard deviation	0,0307	0,0228	0,0157	0,0121	0,0102	0,0096	0,0090	0,0083	0,0083	0,0112	0,0183
Kurtosis	3,9542	5,1337	3,4713	3,1173	3,3106	3,2499	2,9822	3,2388	4,1259	9,9661	3,9074
Skewness	0,6958	0,8514	0,6237	0,5263	0,4811	0,5055	0,3900	0,4646	0,5076	0,3940	-0,6089
Minimum	-0,1523	-0,1280	-0,0064	-0,0608	-0,0476	-0,0419	-0,0421	-0,0398	-0,0472	-0,0657	-0,0754
Maximum	0,2154	0,1566	0,0988	0,0775	0,0642	0,0639	0,0495	0,0469	0,0593	0,0861	0,0586

We have also calculated the summary statistics having taken out of the sample the outliers. In order to get read of them, we have taken out of the sample 5% of the higher and the lower prices of the returns. Comparing the results of TABLE I with those of TABLE II, we can easily see that they have great differences. Those differences indicate that outliers have important meaning, so it is important to be taken into consideration. Shu and Zhang (2012), Anthropelos, Bouras and Malmpanzi (2017) have also included outliers on their data sets.

TABLE II

Summary statistics of levels and returns of VIX spot and VIX Futures Contracts

	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
Panel A: Summary statistics of levels											
Mean	19,14	19,66	20,53	21,13	21,60	22,65	22,93	23,30	23,39	22,53	28,84
Median	16,21	16,95	18,25	13,22	19,87	21,50	21,95	22,75	22,69	20,70	24,88
Standar deviation	9,10	8,32	7,52	6,93	6,50	6,40	6,14	5,96	5,72	5,55	6,54
Kurtosis	8,65	6,35	4,48	3,39	2,22	0,98	0,54	0,37	0,58	1,53	-1,16
Skewness	2,56	2,25	1,89	1,65	1,42	1,07	0,93	0,86	0,92	1,31	0,61
Range	75,14	57,70	51,28	44,52	39,33	33,80	31,18	30,68	29,72	28,95	22,04
Minimum	5,73	10,25	11,62	12,35	12,93	13,54	14,08	14,32	14,73	14,93	21,66
Maximum	80,86	67,95	62,90	56,87	52,26	47,34	45,26	45,00	44,45	43,88	43,70
Panel B: Summary statistics of levels											
Mean	-0,0006	-0,0006	-0,0003	-0,0002	-0,0001	-0,0001	-0,0001	0,0000	0,0000	0,0001	0,0020
Median	-0,0022	-0,0021	-0,0014	-0,0010	-0,0005	-0,0003	-0,0003	0,0000	0,0000	0,0000	0,0005
Standard deviation	0,0227	0,0162	0,0117	0,0091	0,0076	0,0073	0,0067	0,0063	0,0062	0,0070	0,0146
Kurtosis	-0,0739	0,3841	0,3269	0,1968	0,1626	0,0407	0,0242	0,3681	0,0514	0,5451	1,5299
Skewness	0,3091	0,5004	0,4405	0,3529	0,2843	0,2294	0,2199	0,2427	0,2217	0,2163	0,4311
Minimum	-0,0519	-0,0371	-0,0271	-0,0218	-0,0186	-0,0182	-0,0132	-0,0153	-0,0152	-0,0197	-0,0382
Maximum	0,0633	0,0516	0,3499	0,0266	0,0213	0,0204	0,0189	0,0181	0,0173	0,0219	0,0572

2.2 Linear Cointegration Granger Test and Error Correction Mechanism

The long run equilibrium relationship between spot VIX and VIX future prices is given from the following equation:

$$F_t = b_0 + b_1 S_t + v_t \tag{1}$$

Where:

- F_t: the VIX future prices
- S_t: the spot VIX price

It is well known that the above equation cannot be tested by ordinary least squares if at least one of the variables is not stationary. So, the first step in time series testes is to test if the variables are stationary. The null hypothesis of the unit root test used, is that the variable is not stationary. The models used for the unit root tests are the following:

$$\Delta F_{i,t} = a_0 + a_1 F_{i,t-1} + \sum_{j=1}^{p} c_j \Delta F_{i,t-j} + v_{i,t} \quad \text{(without trend)}$$

$$\Delta F_{i,t} = a_0 + a_1 F_{i,t-1} + a_2 t + \sum_{j=1}^{p} c_j \Delta F_{i,t-j} + v_{i,t} \quad \text{(with trend)},$$

$$i = 0,1,2,3,4,5,6,7,8,9,10$$

Where , $i = 0,1,2,3,4,5,6,7,8,9,10$ stand for the spot VIX and the ten nearest to maturity VIX Future contracts prices respectively. Having run Augmented Dicey-Fuller unit root tests on spot VIX and ten VIX futures prices indexes. All t-statistics are below 1% critical values, as a result the null hypothesis cannot be rejected for any of the ten indexes. Since the null hypothesis cannot be rejected all the time series of VIX and futures prices are not stationary.

A nonstationary time series which has stationary first difference, is said to be integrated to order 1, it is denoted as I(1). Having run Augmented Dicey-Fuller unit root tests on the first differences of VIX and Fi. The null hypothesis is rejected for all the indexes at the 1% significance level, so there is no unit root problem. In conclusion all the ten indexes are I(1) processes.

In 1987 Enger and Granger proved that if we have two I(1) (nonstationary) processes and their liner combination is I(0) (stationary), the two time series are cointegrated. From the economical perspective, two time series are said to be cointegrated if they have a long-term or else equilibrium relationship between them. One way to test if two time series are cointegrated is to conduct test statistics from the residuals of their regression. Let v_t denote the estimated residuals from equation (1), a test for no cointegration is given from a test for unit root of those residuals. The ADF regression equation is:

$$\Delta\hat{v}_t = a * \hat{v}_{t-1} + \sum_{j=1}^{p} \Delta\hat{v}_{t-1} + e_t$$

Test statistics is a t-ratio test for a=0 (t-test). Indicative, the critical values are -3.34 for 5% confidence interval and -3.04 for 10% confidence interval. Significant negative test statistics suggest cointegration (rejection of the unit root hypothesis). Table IV and V presents the Enger-Granger cointegration results for different pairs of time series. The tables below and specifically the significant level that the Enger Granger cointegration test gives, indicate that there is not a specific pattern followed. So, we can conclude that the time series are not linear combined between them.

TABLE III

Cointegration Test for pairs of VIX and VIX Futures Rolling Contracts

	F0		**F1**		**F2**		**F3**		**F4**		**F5**		**F6**		**F7**		**F8**		**F9**		**F10**	
F1	-7,62	***	NA		-6,51	***	-5,42	***	-4,51	***	-3,70	***	-3,42	**	-3,06	*	-3,19	**	-2,53	*	-	
F2	-7,18	***	-6,87	***	NA		-4,81	***	-4,01	***	-3,08	**	-		-		-		-2,78	**	-	
F3	-5,48	***	-5,89	***	-4,95	***	NA		-5,87	***	-3,73	***	-3,17	**	-3,02	*	-		-3,02	**	-3,07	**
F4	-4,40	***	-5,09	***	-4,22	***	-5,93	***	NA		-4,76	***	-3,63	***	-3,21	**	-2,81	*	-3,24	***	-3,12	**
F5	-3,72	***	-3,98	***	-3,36	**	-3,82	***	4,82	***	NA		-3,85	***	-3,55	***	-2,90	*	-3,66	***	-3,43	***
F6	-3,63	**	-4,10	***	-3,26	**	-3,42	**	-3,78	***	-3,93	***	NA		-4,57	***	-3,78	***	-3,73	***	-3,79	***
F7	-3,48	**	-3,71	***	-3,12	**	-3,32	**	-3,38	**	-3,67	***	-4,62	***	NA		-4,55	***	-4,61	***	-4,19	***
F8	-3,32	**	-3,90	***	-2,94	*	-2,88	*	-3,03	**	-3,04	**	-3,82	***	-4,54	***	NA		-5,57	***	-4,44	***
F9	-3,18	**	-2,91	**	-2,97	**	-3,20	***	-3,37	***	-3,72	***	-3,77	***	-4,61	***	-5,49	***	NA		-4,97	***
F10	-		-		-		-3,02	**	-3,05	**	-3,28	**	-3,57	***	-3,90	***	-4,10	***	-4,74	***	NA	

TABLE IV

Cointegration Test for pairs of VIX and VIX Futures Rolling Contracts

	F0		F1		F2		F3		F4		F5		F6		F7		F8		F9		F10	
F1	-7,62	***	NA		-6,51	***	-5,42	***	-4,51	***	-3,70	***	-3,42	**	-3,06	*	-3,19	**	-		-	
F2	-7,18	***	-6,87	***	NA		-4,81	***	-4,01	***	-3,08	**	-		-		-		-		-	
F3	-5,48	***	-5,89	***	-4,95	***	NA		-5,87	***	-3,73	***	-3,17	**	-3,02	*	-		-		-3,07	**
F4	-4,40	***	-5,09	***	-4,22	***	-5,93	***	NA		-4,76	***	-3,63	***	-3,21	**	-2,81	*	-		-3,12	**
F5	-3,72	***	-3,98	***	-3,36	**	-3,82	***	4,82	***	NA		-3,85	***	-3,55	***	-2,90	*	-3,09	*	-3,43	***
F6	-3,63	**	-4,10	***	-3,26	**	-3,42	**	-3,78	***	-3,93	***	NA		-4,57	***	-3,78	***	-2,99	*	-3,79	***
F7	-3,48	**	-3,71	***	-3,12	**	-3,32	**	-3,38	**	-3,67	***	-4,62	***	NA		-4,55	***	-4,07	***	-4,19	***
F8	-3,32	**	-3,90	***	-2,94	*	-2,88	*	-3,03	**	-3,04	**	-3,82	***	-4,54	***	NA		-5,09	***	-4,44	***
F9	-5,20	***	-4,02	***	-3,46	**	-3,18	**	-3,16	*	-3,50	**	-3,26	**	-5,75	***	-5,29	***	NA		-4,97	
F10	**-**		-		-		-3,02	**	-3,05	**	-3,28	**	-3,57	***	-3,90	***	-4,10	***	-4,74	***	NA	

The difference of the two tables is on F9, the first table includes the prices of F9 from 23 October of 2006 till 18 March of 2009, while the second table includes F9 prices from 22 August of 2011 till 22 April of 2016.

In 1987 Enger and Granger stated that two cointegrated time series can be represented by an error correction mechanism (ECM). This ECM includes not only the last period's equilibrium error but also the lagged values of the first differences of each variable. Wahab and Lashgariin, 1993 created the following models:

$$\Delta F_{i,t} = d_f + a_f \hat{v}_{i,t-1} + b_f \Delta S_{t-1} + c_f \Delta F_{i,t-1} + u_{f,t}$$

$$\Delta S_t = d_s + a_s \hat{v}_{i,t-1} + b_s \Delta F_{t-1} + c_s \Delta S_{t-1} + u_{s,t}, i = 1,2,3,,5,6,7,8,9 \qquad (2)$$

Where Δ represents the first difference of the variables, and $\hat{v}_{i,t} = F_{i,t} - a_0 - a_1 S_t$, the error from the linear regression between $F_{i,t}$ and S_t. The two random errors are symbolized with $u_{f,t}$ and $u_{s,t}$. The ECM is denoted with the coefficient a. This term measures how fast current prices return to their long run equilibrium. So, if a_f is significant, the current VIX futures price will return to the equilibrium. If we consider $\hat{v}_{i,t-1}$ to be a positive, this means that $F_{i,t-1}$ is too high[13] and as we would guess a to be negative, the term $a_f \hat{v}_{i,t-1}$ is negative as well.

So, $\Delta F_{i,t}$will be negative so to return to the equilibrium. If F_{t-1} is above the equilibrium, it will start decreasing, in order to restore it. In the same way, if the previous futures price is below the equilibrium, it will start to rise in the near future, in order for the equilibrium to be restored. If we consider an efficient market we would assume that both the spot prices and the future prices will respond to information simultaneously, so there will be no adjustment in the next period, therefore, a will be insignificant. Coefficients b represents the lead-lag relationships. If b_s is significant and b_f is not, it is said that futures prices cause spot prices (futures market lead spot market). On the other hand, if b_s is insignificant and b_f is not, spot prices cause futures prices (spot market leads futures market). Last, if both b_s and b_f are insignificant, there

[13] $F_{i,t-1}$ above equilibrium level $a_0 + a_1 S_{t-1}$

is no Granger causality between futures and spot prices. In conclusion, tests for cointegration and error correction mechanism are useful in testing for price discovery.

Chapter 3

3.1 Empirical results

3.1.1 Linear Granger Test Results

Having run equation (2) enables us to test the causality between spot VIX prices and the nine nearest to maturity VIX futures prices. The null hypothesis of the test is that the independent variable cannot predict the dependent one. Those results are presented on Table V and Table VI. Panel A presents the results of Enger-Granger linear tests on VIX and F1 (the group of the nearest to maturity futures contracts) prices.

From the results of the model we have indications that VIX spot prices do not lead the VIX Futures prices because the null hypothesis fails to be rejected as $\beta_{1,f}$ is insignificant even at 10 % level of significance (p-value: 0.2631), the economical explanation of that fact is that any change in VIX price reveals little information in forecasting the next period's VIX futures price. Contrary, β_s is significant at the level of 5% and even at the level of 1% (p-value: 0.0007), this means that the alternative hypothesis fails to be rejected, thus VIX futures prices disclose useful information on forecasting the next periods spot VIX price. The error correction term a_f is negative and significant at 5% and 1% confidence level. The fact that a_f is negative suggests that if the futures price at the moment is above the equilibrium, it is expected to fell in the next period and the distance from the equilibrium will be eliminated. From the other hand a_s is also significant but positive.

It is important to point out that as an error correction term we have used futures equilibrium errors ($\hat{v}_{I,t} = F_{I,t} - a_0 - a_1 S_t$). If a_s is positive and $\hat{v}_{I,t}$ is positive as well (indication of too high futures prices) then it is likely that spot prices will rise in the next period. From our results we have indications that even in the absence of cost of carry (VIX itself is not a tradable asset) there is a long run equilibrium between the index and its futures. It is obvious that both VIX futures and VIX spot respond to any deviation from the equilibrium. As far as the autocorrelation coefficient (c_f) is concerned, is significant at 5% and even at 1% confidence level. This mean that futures prices of F1 can be predicted by their historical prices. Panel B and G which represents the relationship between F2 and VIX spot and F7 and VIX spot respectively, indicates the same relation with the only difference on the significance level which is 5% and not 1%.

Panels C, D, E, F, H represent the relationship between F3 and VIX, F4 and VIX, F5 and VIX, F6 and VIX and F8 and VIX respectively. For those groups things are different. We have evidence that historical VIX spot prices can predict the futures prices of next period with confidence level 5% except of F8 with VIX that is for 10% (p-value: 0.0518). Those findings are in accordance with those of Konstantinidi and Skiadopoulos (2011) and Shu and Zhang (2012).

From our results we can see that VIX future prices lead the VIX spot prices. This has to do with the fact that VIX futures market attracts more institutional traders, which are better informed that the individual ones, Shu and Zhang (2012). Furthermore, we should point out, that there are indications of price discovery in the spot market, in the majority of our results we have evidence that VIX historical prices can predict future ones. Something like that, comes in contrast with the market efficiency. Those indications may be due to the existence of non-linear relations and of the fact that if the gain tends to decrease, the trader will respond very fast.

One the other hand, we cannot be sure that there is not a bi-directional relation between VIX futures and VIX itself because that type or relation may be a result of moments of higher order that cannot be captured by the model used.

Finally, as far as the F9 group is concerned from 25 October to 2006 till 6 January of 2009 we have detected that F9 Granger cause VIX for 1% confidence interval. We also detected that both F9 and VIX historical prices can predict their future ones respectively. The speed of adjustment to the long run equilibrium is quicker for futures. From the error correction model of F9 group of futures from 28 August of 2011 till 15 July of 2015 we have indications that VIX Granger causes F9 and F9 historical prices can predict the future ones. The speed of adjustment to the equilibrium, is still faster for the futures. The contradictive resulted of the two groups of F9 may have to do with the fact that the model is based on the mean of the sample and F9 may have considerable tails.

Last, we have conducted an ECM for F10 while we did not had indications that F10 with VIX are cointegrated. As we expected, both a_s and a_f are negative (-

0.02445118 and -0.11225161) giving as further indications that there is no long run equilibrium between the futures and the index.

TABLE V

Regressions on VIX spot and VIX Futures Prices with an ECM

DEPENDENT	δ	γ	β	α	R^2
Panel A: Relation between F1 and F0					
F1	-0,0030	-0,1382	0,0251	-0,0912	0,023
std.error	0,022	0,031 ***	0,022	0,031 ***	
VIX	-0,0020	-0,0210	-0,1466	0,0918	0,024
std.error	0,031	0,032	0,043 ***	0,022 ***	
Panel B: Relation between F2 and F0					
F2	-0,0015	-0,0673	-0,0200	-0,0331	0,016
std.error	0,015	0,028 **	0,014	0,006 ***	
VIX	-0,0012	-0,0689	-0,1287	0,0359	0,020
std.error	0,031	0,028 **	0,057 **	0,013 ***	
Panel C: Relation between F3 and F0					
F3	-0,0009	0,0244	-0,0358	-0,0201	0,011
std.error	0,012	0,028	0,011 ***	0,004 ***	
VIX	-0,0006	-0,1467	0,0850	0,0242	0,019
std.error	0,032	0,028 ***	0,075	0,012 **	
Panel D: Relation between F4 and F0					
F4	-0,0009	0,0610	-0,0375	-0,0157	0,011
std.error	0,010	0,029 **	0,009 ***	0,004 ***	
VIX	-0,0010	-0,1464	0,0901	0,0192	0,018
std.error	0,032	0,029 ***	0,089	0,011 *	
Panel E: Relation between F5 and F0					
F5	0,0010	0,0433	-0,0264	-0,0144	0,010
std.error	0,011	0,031	0,009 ***	0,003 ***	
VIX	0	-0,1514	0,1219	0,0137	0,017
std.error	0,037	0,031 ***	0,105	0,012	
Panel F: Relation between F6 and F0					
F6	0,0010	0,0458	-0,0201	-0,0118	0,008
std.error	0,010	0,029	0,008 **	0,003 ***	
VIX	-0,0003	-0,1662	0,2119	0,0145	0,018
std.error	0,038	0,029 ***	0,109 *	0,011	
Panel G: Relation between F7 and F0					
F7	0,0018	0,0045	-0,0113	-0,0120	0,008
std.error	0,010	0,028	0,007	0,003 ***	
VIX	0,0033	-0,1706	0,2712	0,0126	0,018
std.error	0,038	0,028 ***	0,112 **	0,011	
Panel H: Relation between F8 and F0					
F8	0,0022	-0,0333	-0,0155	-0,0141	0,014
std.error	0,011	0,030	0,008 *	0,003 ***	
VIX	0,0025	-0,1589	0,0491	0,0024	0,023
std.error	0,042	0,030 ***	0,012	0,013	

TABLE VI

Regressions on VIX spot and VIX Futures Prices with an ECM

DEPENDENT	δ	γ	β	α	R^2
Panel J: Relation between F9 and F0 (25/10/2006-06/01/2009)					
F9	0,0432214	0,0182032	-0,45113	-0,0398679	0,214661
st.error	0,0370739	0,0143161	0,0393174 ***	0,0117671 ***	
VIX	0,0515209	-0,195877	0,327784	0,0365495	0,04287
st.error	0,11447	0,0442025 ***	0,121397 ***	0,0363322 ***	
Panel K: Relation between F9 and F0 (24/08/2011-15/07/2015)					
F9	-0,0113849	0,0383518	-0,276312	-0,0134595	0,059286
st.error	0,0158283	0,0117324 ***	0,367086 ***	0,0053879 **	
VIX	-0,0254099	-0,0263131	-0,18635	0,0300476	0,009875
st.error	0,050221	0,0376707	0,117865	0,0172996 *	

3.1.2 Forecasting

The results of our study indicate, that the futures written on VIX does not seem to reveal any information on the next periods price of the index itself. In order to support those results further, we did a forecasting exercise. We run some regressions, the dependent variable is each one of VIX futures groups (Fi) and the independent one is VIX then we made an in the sample forecast.

From our results we have indications that none of our time series (Fi) reveal any information for the index. This came in contrast with the results of ECM which gave indications that F1, F2, F7 and F9 till 2009 may reveal some information about the index. We strongly believe that those conflicting results may be due to the fact that a simple linear model as OLS cannot give unbiased results.

For the rest of our time series, the results support further the founding of our survey. As we can see from the plots, the futures have not got good predictability on VIX. The root mean squared error is grater that 4.0000 for all the Fi apart from F9. It is worth to be mentioned that the nearest to maturity contracts have better predictability from the distanced ones (smaller root mean squared error). This is not the case for F9 and F10, but this may be due to smaller data set of those groups (small number of observations). Moreover, Theil's U which is another statistic measure of the predictability is grater than one for all the time series so we have another indication about the bad predictability of futures on VIX.

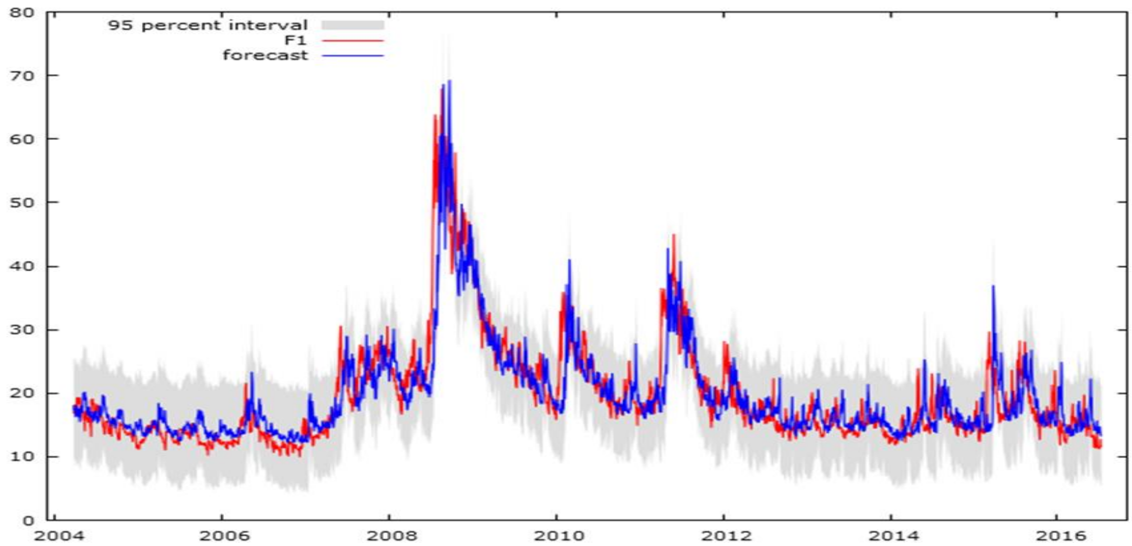

Figure 10

The forecasting ability of F1 to VIX, with confidence interval 5%. The root mean squared error of the above regression is 4.0408. Theil's U: 3.1828

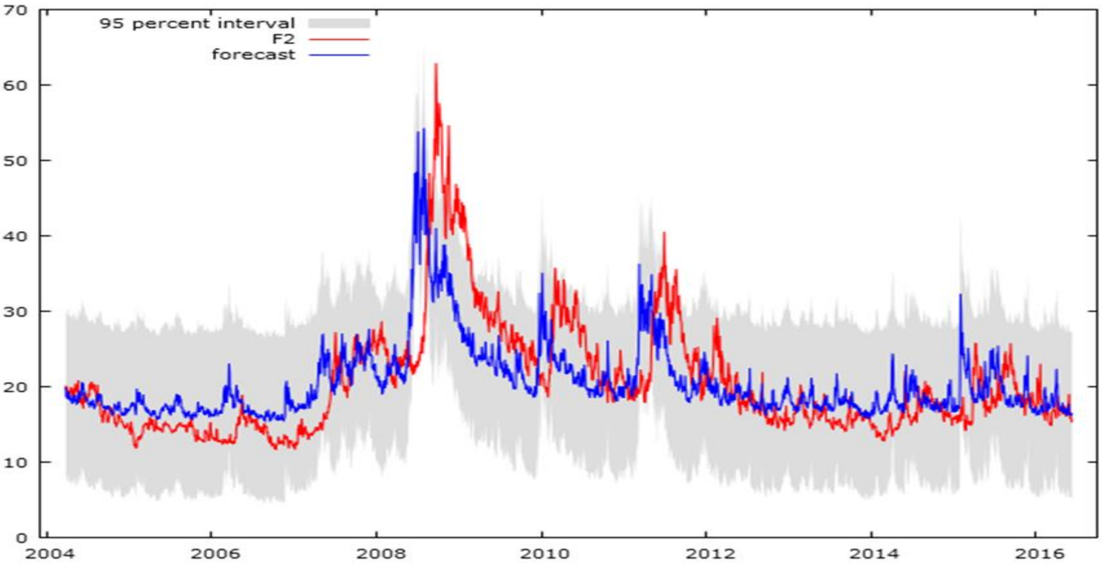

Figure 11

The forecasting ability of F2 to VIX, with confidence interval 5%. The root mean squared error of the above regression is 5.6007. Theil's U: 6.6481

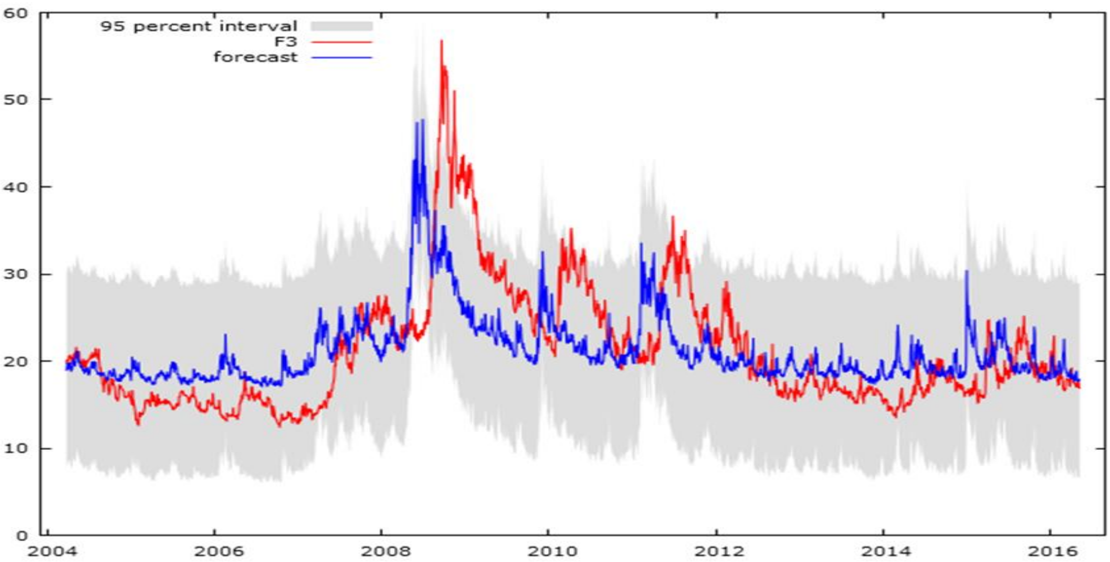

Figure 12

The forecasting ability of F3 to VIX, with confidence interval 5%. The root mean squared error of the above regression is 5.6736. Theil's U: 8.6361

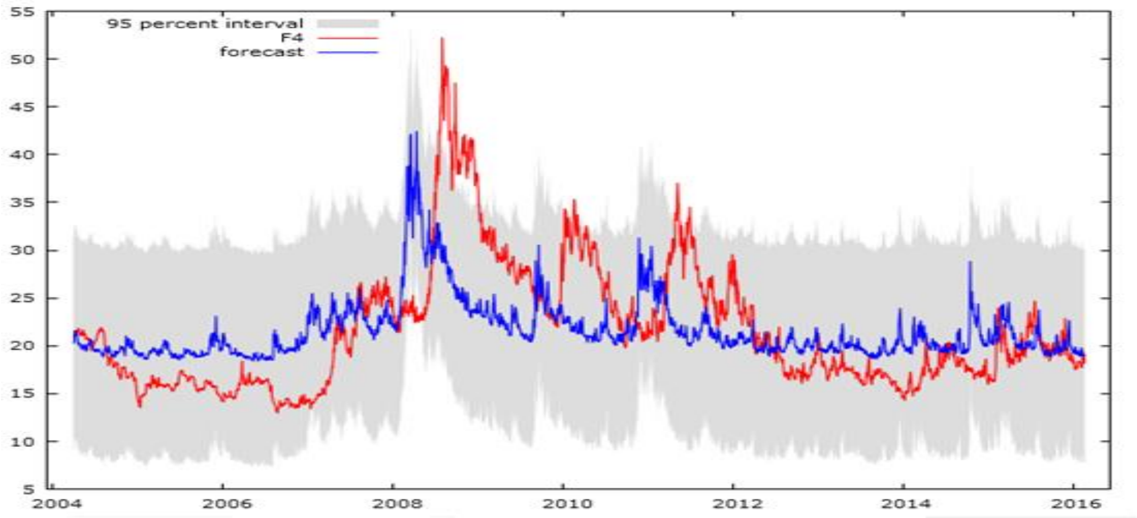

Figure 13

The forecasting ability of F4 to VIX, with confidence interval 5%. The root mean squared error of the above regression is 5.6839. Theil's U: 10.186

Table VII

Root mean squared error of F5 with VIX	5,9636
Root mean squared error of F6 with VIX	5,8924
Root mean squared error of F7 with VIX	5,8217
Root mean squared error of F8 with VIX	5,6887
Root mean squared error of F9 with VIX, from 23/10/2006 till 06/01/2009	3,2285
Root mean squared error of F9 with VIX, from 22/08/2011 till 15/07/2015	2,9692
Root mean squared error of F10 with VIX	5,1437

TABLE VIII

Theil's U F5 with VIX	11,322
Theil's U F6 with VIX	12,147
Theil's U F7 with VIX	12,999
Theil's U F8 with VIX	12,113
Theil's U F9 with VIX from 23/10/2006 till 06/01/2009	4,1208
Theil's U F9 with VIX from 22/08/2011 till 15/07/2015	5,413
Theil's U F10 with VIX	2,056

3.2 Conclusion

The purpose of this study is to examine in detail the Granger causality of VIX index and the derivatives written on it. As VIX and its derivatives have become very popular particularly as a hedging tool it is important and interesting to examine if the index prices can predict the futures' prices and vise versa and if any of them (index or futures) can be forecasted by their own historical patterns.

In this study, we use Enger-Granger cointegration test with the error correction model in order to examine if there is a lead lag relation between the VIX and its futures. We have worked in the same framework of those of Shu and Zhang (2012) and Michail Anthropelos, Chris Bouras and Evangelos S. Malmpanzi (2017) but for the first time (to the best of our knowledge) we used such an extended data sample 2004 till 2016 and we also study one more time series (F10). The findings of our test suggest that the two futures which are closer to maturity F1 and F2 as well as two distanced, F7 as well as F9 from 25/10/2006 till 06/01/2009 can predict the prices of the index while in the rest of the futures we have indications than the index can predict the its futures' prices.

We have also indications that the historical prices of the index can revel information about next period's prices. Those results seem to come in contrast with the efficient market. This may has to do with the fact that the relationship between the time series may be not perfectly linear and also if the gain tends to decrease, the traders will respond very fast. We have also indications that futures react to any upcoming information faster than the index, maybe as a result of the fact that futures market is more attractive to institutional traders.

In the last part of our study we do an in the sample forecast using OLS in order to support our findings. From this exercise we have indications that the futures prices cannot predict the index's ones for any of our time series. This came in contrast with the results given from the ECM but still OLS is a linear model and cannot give unbiased results for this particular study.

One the other hand, we cannot be sure that there is not a bi-directional relation between VIX futures and VIX itself because that type or relation may be a result of moments of higher order that cannot be captured by the model used.

Finally, we have also empirical results that in turbulence periods VIX prices are higher of those of the futures while this is not the case when the economy is blossom.

This may has to do with the fact that in crises periods investors are warred and want to hedge their investments so going long in S&P 500 options is expensive in contrary in a good period for the economy, investors cannot be sure that nothing bad will happen so they ask for premium in order to sell options.

Appendix 1

If someone wants to run a regression he has to check if the auxiliary variables of the regression are not stationary $I(0)$[14], if not he has to use the first differences of those variables. It is of grate importance for the auxiliary variables to be stationary. In case of non-stationarity, any deviation from equilibrium will not be temporary. This of course is the safe way which has been used in many regressions of time series ever since Granger &Newbold published their paper about the problem of spurious regression. Apparently, that technic cannot be considered flowless. The need of using the levels and not the first differences of the variables "created" the meaning of cointegration.

A: Cointegration

In most of cases, the linear combination of two variables which are I(1) is also I(1). In general, variables with different orders of integration are combined, their combination order of integration equals the largest. So, if $X_{i,t} \sim I(d_i)$ for $i = 1,2, \dots, k$, we have k variables each integrated of order d_i , so that

$$z_t = \sum_{i=1}^{k} a_{i X_{i,t}}$$

The integration order of z_t is $I(max\ d_i)$. Solving the above requisition with respect to $X_{1,t}$, we have:

$$X_{1,t} = \sum_{i=2}^{k} b_i X_{i,t} + z_t'$$

Where $b_i = -\frac{a_i}{a_1}$, $z_t' = \frac{z_t}{a_1}, i = 2, \dots, k$. The equation can be considered as a new regression where z_t' is a disturbance term. This disturbance term has two unwanted properties: first it is not stationary in most of the cases and secondly is autocorrelated since all the X_i are $I(1)$.

Let's consider an example:

$$y_t = b_1 + b_2 x_{2t} + b_3 x_{3t} + u_t$$

The sample regression function of the above equation will be written as follows:

$$y_t = \hat{b}_1 + \hat{b}_2 x_{2t} + \hat{b}_3 x_{3t} + \hat{u}_t$$

[14] Integration is when in a one variable context, y_t is $I(d)$ if its (d-1)th difference is $I(0)$. That is $\Delta d y_t$is stationary→ y_t is $I(1)$ if Δy_t is $I(0)$.

If we solve the above requisition with respect to $\widehat{u_t}$ we have:

$$\hat{u}_t = y_t - \hat{b}_1 - \hat{b}_2 x_{2t} - \hat{b}_3 x_{3t}$$

We have expressed the residuals like a linear combination of our auxiliary variables. In most of the regressions the combination of non stationary variables will itself be non stationary but this is not very convenient. The perfect case would be the residuals to be $I(1)$. This is the case when the variables are cointegrated.

Back in 1987, Engle and Granger proposed the following definition about cointegration. Let x_t be a $k \times 1$ vector of variables integrated of order (d, b):

- All x_t are $I(d)$
- There is at least one vector of coefficients c such that $c' x_t \sim I(d - b)$

In reality most of the financial variables have one unit root so they are $I(1)$. Having this in mind, a set of variables is considered cointegrated if their linear combination is stationary.

It has been observed that many time series may not be stationary, but they may be related in the long run. A cointegration relationship is also a long term or else equilibrium phenomenon because it is possible that cointegrated variables may seem unrelated in the short run but this is not be true for the long run.

In this point it is important to distinct the meaning of spurious relations with cointegrated ones. The spurious regression problem is appeared when totally unrelated time series may appear to be related using traditional testing procedures. And from the other hand, we face genuine relationships which arise when the time series are cointegrated.

B: Engle and Granger test

Engle and Granger in 1987, recommended that «If a set of variables are cointegrated, then there exists a valid error correction representation of the data, and vice versa». To put it different, if two variables are cointegrated there must be some force that will make the equilibrium error to go back to zero.

Engle and Granger in 1987, also suggested a two-step model for cointegration analysis. For example, let's say that we have an independent variable x_{1t} and a dependent one y_t.

First of all, it should be estimated the long-run equilibrium equation:

$$y_t = a_0 + a_1\, x_{1,t} + u_t$$

We run a OLS regression and we have:

$$y_t = \hat{a}_0 + \hat{a}_1 x_{1t} + \hat{u}_t$$

We solve the above equation with respect to $\widehat{u_t}$ and we have:

$$\hat{u}_t = y_t - \hat{a}_0 - \hat{a}_1 x_{1t}$$

In practice a cointegration test is a test which examines if the residuals have not got unit root. To examine this, we run a ADF test on the residuals, but we use the MacKinnon (1991) critical values. If the hypothesis of the existence of cointegration cannot be rejected, the OLS estimator, is said to be super-consistent. This means that for a very big sample it is not necessary to include $I(0)$ variables in our model.

The only important thing from the above test is the stationarity of the residuals, if they are stationary (no unit root) we can move to the second step. So, we save the residuals from the OLS and we prosed to the second step.

The second step we use the unit root process for the stationarity of the residuals to the next equation:

$$\Delta\hat{u}_t = \delta_2 \hat{u}_{t-1} + \sum_{i=1}^{r-1} b_i \Delta\hat{u}_{t-1} + e\_t$$

The above equation does not have constant term because of the fact that, the residuals have been calculated with the method of ordinary lest squares, so they have zero mean. The test suggested from the Engle Granger is a little bit different from those of the one of Dickey-Fuller. The hypothesis of this test is:

- H_0: $\delta_2 = 0$ (no cointegration)
- H_1: $\delta_2 < 0$ (cointegration)

The null hypothesis can be rejected only when $t\delta_2 < \tau$ (τ is the critical value of Engle-Granger table).

The Engle- Granger Test can be also used for more than two variables. The process is alike the one we have described.

In conclusion the cointegration process is a way to estimate the long run relation between two or even more variables. Engle and Granger in 1987 proved that if two variables are cointegrated, then they have a long run relation equilibrium, while in short run this may not be true. To check if their is a short run disequilibrium we can use an Error Correction Mechanism (ECM). The equilibrium error can be used to combine the long run with the short run with the help of ECM.

The equation of this model is:

$$\Delta y_t = \varphi_0 + \sum_{j=1} \varphi_j \Delta y_{t-j} + \sum_{h=0} \theta_h \Delta x_{t-1} + b\hat{u}_{t-1} + \varepsilon_t$$

Where:

- u_{t-1}: is the equilibrium error
- b: is the short run coefficient which has to be between 0 and -1.
- ΔY_t and ΔX_t: are the first differences of Y_t and X_t which are not stationary

We now can now use ordinary least squares since all the variables are $I(0)$.

It is important to point out that long run equilibrium is tested trough the p-value of coincidence b. If b is significant then x causes y in the long run. Furthermore, the coefficient b measures the speed of adjustment to the long run equilibrium. The higher this coefficient the faster the return to the equilibrium.

C: Granger causality

It is really useful, in finance to study if a variable or group of variables can forecast another variable. We consider two variables y_t and x_t. In general, a variable or a group of variables (y_t) can forecast another variable or even a group of variables $(x)_t$, we say that y_t Granger-cause x_t. Otherwise, it is said that y_t fails to Grange-cause x_t. It should be pointed out, that Granger-causality does not imply true causality. It only implies forecasting ability.

D: Theil's U

Theil's U is a statistical measure of the forecasting ability of a model. Actually, it compares the results of the whole forecast with the ones of the minimal historical data. Theil's U measure, squares the deviations in order to eliminate large errors.

$$U = \sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{\hat{Y}_{t+1} - Y_{t+1}}{Y_t}\right)^2}{\sum_{t=1}^{n-1}\left(\frac{Y_{t+1} - Y_t}{Y_t}\right)^2}}$$

Where Y_t is the actual value and $\hat{Y}_t$ is the forecasted one.

When Theil's U is less than 1 the forecast is very good for than we would expect, when is 1 is as good as we would expect and when is grater than 1 the forecast is really bad.

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