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DEPARTMENT OF ECONOMICS

THE IMPLICATIONS OF FINANCIAL RISK FOR HOUSEHOLD CONSUMPTION UNDER MACROPRUDENTIAL POLICY RULES

Ph.D. Thesis

Alexandros P. Bechlioulis

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<u>Defense Committee</u>
Professor Manthos Delis
Professor Gikas Hardouvelis
Professor Angelos Kanas
Professor Apostolis Philippopoulos
Professor Emmanouil Tsiritakis
Associate Professor Claire Economidou
Professor Emeritus Sophocles N. Brissimis

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Alexandros P. Bechlioulis

Abstract

This dissertation studies the impact of consumer debt non-payment on household consumption decisions in the presence of macroprudential policy rules, which take the form of borrowing constraints or imposition of penalty premiums on the borrowing interest rate.

Chapter 2 develops a dynamic equilibrium model that analyzes the role of default in consumption decisions. The default is studied in the context of two-period overlapping processes of consumer behavior assuming that penalty costs are imposed on borrowers if they are delinquent in the first period and are subsequently refinanced by banks. From the analytical solution of the household intertemporal optimization problem, an augmented Euler equation for consumption is obtained, as a function *inter alia* of an expected default factor which determines, in a static equilibrium context, the optimal value of the percentage of debt repaid.

Chapter 3 studies delinquent and non-performing loans in consumer credit markets and their implications for consumer behavior. By introducing endogenously non-payment of debt in the intertemporal optimization problem of the representative household, an augmented consumption Euler equation is derived analytically which features a risk factor in terms of expected non-performing debt and delinquent debt. The presence of the risk factor differentiates the estimated values of the preference parameters and enhances the model's structure in comparison to the benchmark representative agent model with full debt repayment, which seems to be an incomplete description of consumer behavior.

Chapter 4 analyzes the assumption of multiplicative non-separable (Cobb-Douglas) consumer preferences as a key assumption for analyzing the interdependence of consumption and leisure choices. The consumer utility maximization problem under these preferences is solved and a simultaneous system of two equations is derived, corresponding to a static and an inter-temporal equation of consumption and leisure choice. The system is estimated with GMM to obtain consistent estimates of the consumer's preference parameters, of which the relative weight of consumption in the utility function is found to be much higher than that commonly assumed in DSGE model calibration exercises.

Chapter 5 examines whether the consumption behavior of a borrowing constrained household is affected by debt non-payment. From the household's intertemporal maximization problem a two equation model is derived consisting of augmented forms of the standard consumption Euler equation and static labor supply equation, and use nonlinear GMM to estimate its equations. The results show that the credit constrained household tends to be more patient against future consumption needs under debt non-payment. An estimate of the household's borrowing limit, i.e. its debt-payment-to-income ratio, is also obtained. Our results are found to be robust to a number of specification tests.

Περίληψη

Η παρούσα διατριβή εξετάζει την επίδραση στην καταναλωτική συμπεριφορά από τη μη πληρωμή οφειλών από καταναλωτικά δάνεια, όταν υφίστανται μακροπροληπτικοί εποπτικοί κανόνες με τη μορφή περιορισμών στο δανεισμό ή επιπλέον κόστους επιβάρυνσης των επιτοκίων δανεισμού.

Το Κεφάλαιο 2 αναπτύσσει ένα υπόδειγμα δυναμικής ισορροπίας, το οποίο αναλύει το ρόλο της αθέτησης πληρωμών για τις καταναλωτικές αποφάσεις. Εξετάζεται η αθέτηση στο πλαίσιο αλληλοεπικαλυπτόμενων διαδικασιών καταναλωτικής συμπεριφοράς δύο περιόδων, με την υπόθεση της επιβολής ποινής επί του επιτοκίου δανείων κατά την πρώτη περίοδο στην περίπτωση υπερημερίας, ενώ ταυτόχρονα υφίσταται επαναχρηματοδότηση από τις τράπεζες. Από την αναλυτική επίλυση του προβλήματος διαχρονικής αριστοποίησης του νοικοκυριού, προκύπτει μια διευρυμένη εξίσωση Euler για την κατανάλωση που αποτελεί συνάρτηση μεταξύ άλλων ενός προσδοκώμενου παράγοντα αθέτησης, η οποία συνάρτηση προσδιορίζει, στο πλαίσιο της στατικής ισορροπίας, τη βέλτιστη τιμή του ποσοστού εξόφλησης των οφειλών.

Το Κεφάλαιο 3 πραγματεύεται την υπερημερία και τα μη εξυπηρετούμενα δάνεια καταναλωτικής πίστης και τις επιπτώσεις τους στην καταναλωτική συμπεριφορά. Εισάγοντας ενδογενώς τη μη πληρωμή οφειλών στο διαχρονικό πρόβλημα βελτιστοποίησης του αντιπροσωπευτικού νοικοκυριού, εξάγεται μια διευρυμένη εξίσωση κατανάλωσης Euler, η οποία περιλαμβάνει ένα παράγοντα κινδύνου σε όρους προσδοκώμενων μη εξυπηρετούμενων οφειλών και υπερήμερων οφειλών. Η παρουσία του παράγοντα κινδύνου διαφοροποιεί τις εκτιμημένες τιμές των παραμέτρων των προτιμήσεων του καταναλωτή και βελτιώνει τη δομή του υποδείγματος σε σχέση με το βασικό υπόδειγμα αντιπροσωπευτικού καταναλωτή με πλήρη εξόφληση των οφειλών, το οποίο φαίνεται να είναι μια ατελής περιγραφή της καταναλωτικής συμπεριφοράς.

Το Κεφάλαιο 4 αναλύει την υπόθεση των πολλαπλασιαστικών μη διαχωρίσιμων (Cobb-Douglas) καταναλωτικών προτιμήσεων, ως βασικό στοιχείο για την ανάλυση της αλληλεξάρτησης των επιλογών μεταξύ κατανάλωσης και ελεύθερου χρόνου. Το πρόβλημα μεγιστοποίησης χρησιμότητας του καταναλωτή λύνεται κάτω από αυτές τις προτιμήσεις και εξάγεται ένα ταυτόχρονο σύστημα δύο εξισώσεων που αντιστοιχούν σε μια στατική και μια διαχρονική εξίσωση επιλογής μεταξύ κατανάλωσης και ελεύθερου χρόνου. Το σύστημα εκτιμάται με GMM με σκοπό την απόκτηση συνεπών εκτιμήσεων των παραμέτρων των

καταναλωτικών προτιμήσεων, εκ των οποίων η σχετική βαρύτητα της κατανάλωσης στην συνάρτηση χρησιμότητας εκτιμάται ότι είναι αρκετά μεγαλύτερη σε σχέση με τις τιμές της παραμέτρου αυτής που χρησιμοποιούνται στα Δυναμικά Στοχαστικά Υποδείγματα Γενικής Ισορροπίας.

Το Κεφάλαιο 5 εξετάζει αν η καταναλωτική συμπεριφορά του νοικοκυριού όταν αυτό υπόκειται σε πιστοληπτικό περιορισμό επηρεάζεται από τη μη πληρωμή των δανειακών οφειλών του. Από το πρόβλημα μεγιστοποίησης του νοικοκυριού εξάγεται ένα υπόδειγμα δύο εξισώσεων, το οποίο αποτελείται από μια διευρυμένη εξίσωση κατανάλωσης Euler και μια στατική εξίσωση προσφοράς εργασίας, και χρησιμοποιείται η μη γραμμική μέθοδος GMM για την εκτίμηση των εξισώσεών του. Τα αποτελέσματα δείχνουν ότι το νοικοκυριό με πιστοληπτικό περιορισμό τείνει να είναι περισσότερο υπομονετικό έναντι μελλοντικών καταναλωτικών αναγκών όταν δεν εξοφλεί πλήρως τις δανειακές του υποχρεώσεις. Επιπλέον, γίνεται εκτίμηση του ορίου δανεισμού του νοικοκυριού, δηλαδή του λόγου των πληρωμών των οφειλόμενων δόσεων του δανείου προς το εισόδημά του. Τα αποτελέσματα αποδεικνύονται ισχυρά βάσει ενός αριθμού στατιστικών ελέγχων.

Table of Contents

List of Figures

List of Tables

1. Introduction

This dissertation studies the impact of debt non-payment to household consumption decisions in a representative agent framework featuring macroprudential policy rules. The proposed framework for examining debt non-payment as a key feature of consumer behavior goes a long way towards addressing the following problem also pointed out in Goodhart and Tsomocos (2009) for DSGE models in general: standard representative agent models of consumption or DSGE models do not include the possibility of debt non-payment and as such they are not properly micro-founded in that their assumptions are at odds with human behavior in this respect; they are also of little use for analyzing real data and in particular periods of financial crises. Thus non-payment of debt should be explicitly incorporated into the micro-foundations of these models so that they can offer an appropriate framework for monetary and macroprudential policy analysis.

The two following chapters focus on the implications of debt non-payment for optimal consumption decisions by assuming penalty costs on the borrowing interest rate if unpaid debt is refinanced. The third chapter analyzes the interdependence between consumption and leisure choices, and the final chapter examines the role of debt non-payment to household behavior under a borrowing constraint in terms of income.

Consumer default and optimal consumption decisions

This chapter examines theoretically the optimal consumption decisions of households in a micro-founded framework that assumes agent heterogeneity and debt default. A dynamic partial equilibrium model is set up. The household sector consists of two different sub-sectors: borrowing households and saving households. For the former, by solving analytically the optimization problem, an augmented relative to the literature consumption Euler equation is derived by including an expected default factor. Default is studied in the context of an infinite number of overlapping processes of consumer behavior with periodicity two. In this respect, borrowers were assumed not to repay all of their debt in a given period. A percentage of unpaid debt comes to default in the same period, for cash flow reasons, while the other part is refinanced by banks in the following period and a penalty premium is added onto the interest rate of the loan. Again, a proportion of refinanced debt is not repaid in that period but also defaults. The optimal value of this variable in a static equilibrium is shown to depend on a number of determinants, specifically the time preference rate, the borrowing interest rate, the penalty premium added on the interest rate and the percentage of unpaid debt which is refinanced by banks. Finally, an ordering by size is provided for the discount factor of the different types of households: borrowers who do not repay all of their loans have the lowest discount factor, followed in turn by borrowers who fully repay their loans and by savers.

Are household consumption decisions affected by past due unsecured debt? Theory and evidence

The consumption Euler equation has become the mainstay of much macroeconomic research over the past thirty years and is now an essential element of nearly every DSGE model. Being an intertemporal first-order condition derived by solving the lifetime utility maximization problem of the representative household, it directly provides a basis for estimating the two important preference parameters of the household: the elasticity of intertemporal substitution in consumption and the discount factor. To estimate these parameters, the large majority of existing papers use simulation techniques, only few estimate the parameters directly from the data, and there is none that does both things simultaneously: estimate the Euler equation from real aggregate data and admit the possibility of default on unsecured consumer debt. These aspects are analyzed in this chapter using macro data for the US economy.

The non-payment of consumer debt is introduced endogenously in the above optimization problem, allowing households to stop payment through two possible routes: become delinquent and/or file an application for bankruptcy, in which case the loan moves into the non-performing state until the court decides on the application. By adopting this framework, an augmented consumption Euler equation is derived analytically including a risk factor, which reflects the percentage of consumer net borrowing expected to become non-performing. This equation is estimated with macroeconomic data and nonlinear GMM yielding consistent estimates of the household's preference parameters. Estimation biases which are likely to be due to measurement errors and unobserved heterogeneity are relatively small and are accounted for by specifying a parametric process for the errors of the Euler equation. Comparing the literature with current estimates non-negligible differences are found, especially with papers using calibration. Based on these results, a conclusion is obvious that the general specification of the Euler equation advanced in this chapter, without losing empirical tractability, is found to improve the model's structure relative to the standard representative agent model commonly used in policy assessments in the context of DSGE models.

The link between consumption and leisure under Cobb-Douglas preferences: Some new evidence

In this chapter, the links between consumption and leisure are examined by solving the consumer utility maximization problem under multiplicative non-separable (Cobb-Douglas) preferences. The strategy involves estimating a static and an inter-temporal equation of consumption and leisure choice and testing the restriction inherent in these equations, which concerns the relative weight of consumption in the utility function. The empirical results for the US economy provide strong support for the above non-separability of preferences and suggest that consumers derive about three fourths of their satisfaction from current consumption and only the remaining one fourth from their current leisure time. In this respect, the choice in many DSGE models to rely, among other parameters, on a "standard value" for the share of consumption in utility would seem unwarranted in view of the estimates presented in this chapter.

Consumer debt non-payment and the borrowing constraint: Implications for consumer behavior

The empirical failure of the rational expectations - permanent income model of consumption in its simple form (see e.g., Campbell and Mankiw, 1989) has led researchers to test whether the presence of constraints in household behavior can account for this failure (see e.g., Jappelli et al., 1998). On the other hand, standard representative agent models of consumption, which are widely used in the literature, by not including the possibility of debt non-payment, are of little use for analyzing real data.

In this chapter, a partial equilibrium framework is developed that helps analyze the household's non-durable consumption decisions. The focus is on two main features of consumer behavior: consumers do not fully repay their loan obligations in a given period and also are subject to a limit on loan payments in terms of income, i.e., the debt payment to income (DPTI) ratio. The household's intertemporal maximization problem is solved by assuming either multiplicative non-separable or additive separable preferences and a two-equation model consisting of a consumption Euler equation and a static labor supply equation is obtained. The model's equations are estimated by using US macro data and nonlinear GMM.

2. Consumer default and optimal consumption decisions

2.1. Introduction

It is widely recognized that the recent financial market turmoil has been associated with the severest economic contractions since the Great Depression. In the years leading up to the global financial crisis, a combination of factors, including low interest rates and lax lending standards, fueled a rapid increase in household leverage, as measured by the ratio of debt to personal disposable income, which for US households exceeded 130% in 2007 (Glick and Lansing, 2009). Countries with very high increases in this ratio prior to the crisis, such as Denmark (with a ratio of 199%) and Ireland (191%), experienced the largest declines in real consumption (-6.3% and -6.7% from the second quarter of 2008 to the first quarter of 2009, respectively; see Glick and Lansing, 2010). In addition, other research (Mian and Sufi, 2010) has shown that the top 10% leverage growth counties in the US experienced an increase in the household default rate of 12 percentage points (from the second quarter of 2006 through the second quarter of 2009) and also the sharpest decline in durables consumption.

Against these stylized facts, this chapter studies the optimal consumption decisions of households in a micro-founded framework by setting up a dynamic equilibrium model featuring two main characteristics: agent heterogeneity and debt default. As to agent heterogeneity, Tobin (1980) indicated that the population of households is not distributed randomly between debtors and creditors. Thus, debtors are frequently young families acquiring homes and consumer durables through borrowing; given the difficulty of borrowing against future income, they are liquidity-constrained and have a high marginal propensity to consume. Middle-aged families, on the other hand, are usually savers accumulating wealth. In our model, household heterogeneity is incorporated, as in Iacoviello (2005); Agenor et al. (2013); Gelain et al. (2013); Suh (2014); Kannan et al. (2012), where the household sector consists of two types of households: saving (unconstrained or patient) households and borrowing (constrained or impatient) households[1]; it is assumed that the former do not lend directly to the latter. Instead, as in Kannan et al. (2012), it is considered that financial intermediaries take deposits from saving households, offering a deposit rate, and lend to borrowing households, charging a borrowing rate. Heterogeneity is also distinguished by the

[1] Most dynamic stochastic general equilibrium (DSGE) models do not include agent heterogeneity, but assume that there is one representative household, the saving household.

household discount factor, where the usual assumption is that the discount factor of borrowers is smaller than the respective discount factor of savers.

As to the second characteristic, most models in the literature consider that default happens in a single step (Dubey et al., 2005; Chang and Sundaresan, 2005; Goodhart et al., 2009). In Goodhart et al. (2009), households choose what percentage of outstanding debt they will repay, determining the level of the unpaid part. Einav et al. (2012) present a standard consumer theory framework where heterogeneous borrowers decide how much to borrow, taking decisions about whether to continue making loan payments or to default. They use a model to derive a set of linear estimating equations that capture purchasing of goods, borrowing, and repayment decisions. They put emphasis on the larger loans which cause higher monthly payments and also have a higher probability of default. Similarly, in Chatterjee et al. (2007), default risk will vary with the size of the loan and household's specific characteristics. Moreover, in Dubey et al. (2005), default can either occur for strategic reasons[2] or be due to ill fortune[3]. However, default on the debt is a possibility only for the business sector and not for the household sector in DSGE frameworks.

The existence of default affects banks' lending guidelines and induces them to follow a stricter path, especially for the cases of strategic default (Huber et al., 2012). To discourage borrowers from defaulting, financial intermediaries impose costs, either directly through penalty on the interest rate for defaulters or indirectly by raising the lending interest rates for borrowers with a past record of bankruptcy. As noted by Chatterjee et al. (2007), households for whom the period begins with a record of bankruptcy cannot get new loans. However, with an exogenous probability, their bad credit rating is expunged after a certain period, since an individual's credit history is kept only for a finite number of years. Borrowers enter again the credit market, being charged a pecuniary cost which affects their financial constraint. In de Walque et al. (2010), the cost of default affects both the utility function as a non-pecuniary burden (social stigma, reputation cost) and the budget constraint as a pecuniary burden, although these authors' analysis refers to default of firms. In a similar vein, Einav et al. (2012) mention that default affects utility of households due to potential costs associated with default, such as a constant per-loan indirect cost. It is argued (Dubey et al., 2005) that banks

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[2] Guiso et al. (2009) determine the reasons why households may strategically default on their debts, such as economic and moral incentives. Also, borrowers may have the incentive to use their borrowed money to finance consumption or simply to hide loan proceeds from their creditors, defaulting on their promise to repay (Manove et al., 2001).

[3] Baltensperger (1975) argues that the decision to default means that it is possible that the borrowing household's income is not sufficient to cover its debt.

should charge penalties, irrespective of the cause of default, since it is difficult to study the strategic decision. Strategic defaulters have every incentive to disguise themselves as agents who are not able to repay their contractual obligations (Guiso et al., 2009).

Our model extends the analysis of loan default to two periods. In a given period, some of the unpaid loans of households come to default, while the rest is delinquent debt refinanced by banks in the following period and carrying a penalty cost. A percentage of delinquent debt is recovered in the second period but the other part also comes to default.

The section contains several novel features. First, unlike typical DSGE models, the household sector is split into two different sub-sectors, borrowing and saving households. Borrowers always borrow while savers always save. As a matter of fact, this dichotomy explains why optimal consumption decisions of these two groups are essentially independent of each other. For borrowing households, an augmented Euler equation is derived, by solving analytically the household maximization problem, which determines the household optimal consumption, as a function *inter alia* of an expected default variable. Second, as mentioned above, default is studied in the context of an infinite number of overlapping processes of consumer behavior with periodicity two. Thus, borrowers do not repay all of their debt in a given period. From the unpaid debt, a part comes to default in the same period, while the other part is refinanced in the following period by banks, who charge a penalty premium on the interest rate; again, a proportion of refinanced debt is not repaid in that period but also comes to default. Third, the percentage of debt repaid is a basic decision variable in the household's optimization problem. The optimal value of this variable in a static equilibrium, in which there is optimal consumption smoothing, is shown to depend on a number of determinants, specifically the time preference rate, the borrowing interest rate, the penalty premium added on the interest rate and the percentage of unpaid debt which is refinanced by banks. Finally, an ordering by size is provided for the discount factor of the different types of households: borrowers who do not repay all of their loans have the lowest discount factor, followed in turn by borrowers who fully repay their loans and by savers.

The chapter is organized as follows. In Section 2, the borrowing household's optimal consumption decisions are analyzed in the framework of an augmented Euler equation for consumption, which includes an expected default variable in addition to other variables already in the literature. Further, the determinants of the household decision to repay its debt obligations are analyzed in a static equilibrium context. In Section 3, the behavior of the saving household is briefly examined and the two types of households are combined in the same model. This section also provides an ordering by size of discount factors (or time preference rates) of households. Section 4 concludes and suggests some directions for further work.

2.2. Optimal decisions for borrowing households

2.2.1. The model

In this section a dynamic equilibrium model is developed, which determines the optimal consumption decisions for borrowing households. The analysis is carried out in discrete time. In the model, it is assumed that there is a population of infinitely-lived homogeneous[4] households that consume a consumption bundle. The representative household does not save but borrows from banks (the loan supply of which is perfectly elastic at the prevailing lending rate) to support consumption smoothing[5], i.e. it is a borrowing household. The household derives utility from consuming goods and services and disutility from supplying labor to producers of goods and services. Further, it is assumed that the household can default on its loan[6]. The household can become delinquent (an earlier stage of default)[7] or come to default for reasons such as cash flow problems (due to negative income shocks or downturn of the business cycle) or failure to meet banks' minimum credit requirements. Also, according to Huber et al. (2012), if agents were allowed to borrow and the marginal utility of income was more than the marginal disutility of debt, this would be a good reason for them to borrow more and to default, applying strategic defaulting[8,9].

In the current period, the household buys consumer goods and services and works in the production process, earning income for labor services provided. At the end of each period, the household can obtain a loan from banks (one-period loan[10]), assuming that it does not face

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[4] Homogeneity means that households have the same preferences and do not face any idiosyncratic shocks or frictions.

[5] The notion of consumption smoothing usually describes an attempt to keep consumption similar in each period.

[6] Not all households are assumed to default. Thus, default is measured by the default rate, i.e. the ratio of the value of debt in default to the value of total debt. It is in this sense that the default rate refers to the representative household.

[7] In the household decision not to repay, delinquency is preferable, at least initially, since surviving via delinquency may help to avoid any costs associated with default (Athreya et al., 2012).

[8] In Guiso et al. (2009), it is reported that survey data for the US indicate that about one out of four households that default applies strategic defaulting.

[9] As discussed in the literature (e.g. Guerrieri and Iacoviello, 2013), one way to discourage defaulters is to configure an appropriate macro-prudential framework with well-planned and effective tools, such as default penalties and strict indicators (e.g. loan-to-value ratio, debt-to-income ratio), which would factor into default decisions.

[10] Roszbach (2004) notes that in financial markets with perfect information any optimal multi-period financial contract can be obtained by a sequence of one-period loan agreements, while under asymmetric information things become more intricate. Also, Besanko and Thakor (1987) develop a single-period credit model under

any credit constraint, in the sense that credit demand never exceeds the bank's credit requirements, which the household always observes. At the end of the current period, part of the previous period loan, including interest, is serviced, while the rest is not. The regular servicing of the loan is followed by an extension of a new one-period loan by the banks and the same process runs in the following period. In the case where part of the previous period's loan is not serviced, the household either is refinanced in the following period[11] (the proceeds of the new loan are used to repay the unpaid loan), being charged a penalty premium on the interest rate[12], or defaults. Again, only a proportion of households that renewed their loan for the next period are in a position to pay off their debt, while the rest also come to default. The above two groups of defaulters determine the household default rate. The debt in default is written off from banks' balance sheets.

The household is assumed to have a life-time utility function:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta^b)^j \, u\big(C_{t+j}^b, N_{t+j}^b\big) \tag{2.1}$$

where E_t denotes the expectation of the household, conditional on information available at time t, β^b is the subjective discount factor (capturing the idea of impatience)[13], which discounts future utility, with $0<\beta^b<1$, and u denotes utility which is related to real consumption (C_{t+j}^b) and working hours (N_{t+j}^b, expressed as a ratio to total available time per day). The utility function is assumed to be time separable and twice differentiable with respect to consumption and working hours. The marginal utility of consumption is positive and non-increasing, while that of working hours is negative and non-decreasing. Equation (2.1) simply states that the household is interested in life-time utility U_t obtained as the present discounted value of current and all future levels of expected utility.

The functional form of the utility function is of the constant relative risk aversion type:

$$u\big(C_t^b, N_t^b\big) = \frac{(C_t^b)^{1-\sigma}}{1-\sigma} - \frac{(N_t^b)^{1+\varphi}}{1+\varphi} \tag{2.2}$$

asymmetric information and assume that no borrower has an initial endowment and each must approach a lender for a one-period loan.

[11] It is assumed that banks refinance when they assess that households meet certain credit requirements set by them, e.g. debt-to-income threshold or credit history.

[12] The premium may be set by the financial regulator and its magnitude is determined by factors such as credit history and reputation of borrowers, search costs of banks and risk parameters.

[13] The impatient household values future utility less than present utility.

where $1/\sigma$, $1/\varphi$ is the inter-temporal elasticity of substitution of consumption and working time, respectively.

The household is also assumed to be subject to a sequence of budget constraints (in nominal terms), which describe its feasible choices through time. These constraints are of the form (for period *t*):

$$P_t C_t^b + \mu_t(1+i_{t-1}^L)[L_{t-1} - k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}] + \mu_t(1+i_{t-1}^L+f_{t-1})k_{t-1}$$
$$(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2} = W_t N_t^b + [L_t - k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}] \quad (2.3)$$

Equation (2.3) displays the household's receipts (inflows) and payments (outflows) stemming from its income-generating and financing activities. The left-hand side contains payments. The first term is consumption expenditure, where P_t is the price index of consumer goods and services. The second term involves the assumption that, concerning the loan contracted at the end of period *t-2*, i.e. L_{t-2}, the bank refinances only a part of the loan that has not been serviced at the end of period *t-1*, i.e. $k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}$, where i_{t-2}^L is the lending rate of period *t-2* in nominal terms, $1-\mu_{t-1}$ is the percentage of the loan (plus interest), i.e. $(1+i_{t-2}^L)L_{t-2}$, that has not been repaid at the end of period *t-1* $(0<\mu\leq 1)$ and k_{t-1} is the percentage of the unpaid loan at the end of *t-1* that is refinanced by the bank $(0<k\leq 1)$. The terms of that refinancing will be discussed below. Excluding this component, $L_{t-1} - k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}$ represents net borrowing of households at the end of period *t-1*, which is made for consumption purposes, and $\mu_t(1+i_{t-1}^L)[L_{t-1} - k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}]$ is the percentage of that borrowing (including interest) which is repaid at the end of period *t*, while the rest is not repaid.

Finally, the third term focuses on the unpaid part of the loan at the end of period *t-1* that is assumed to be refinanced by banks. The interest rate charged on that loan is the lending rate i_{t-1}^L augmented by a penalty premium f_{t-1}[14] and the proceeds of the loan are used to repay the unpaid loan. Again, as already indicated, only a proportion of households that renewed their loan are in a position to repay their debt at the end of period *t* (the rest coming to default) so that the amount of the loan paid back is $\mu_t(1+i_{t-1}^L+f_{t-1})[k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}]$.

[14] In Huber et al. (2012), the imposition of default penalties, in addition to preventing strategic bankruptcy from borrowers, is shown to also resolve the multiplicity of equilibria that is known to exist in closed economies, and in Dubey et al. (2005), default penalties are imposed on agents who fail to repay their debts, irrespective of the cause of default.

The right-hand side includes receipts. The first term reflects the income earned from labor, where W_t is the nominal wage rate and the second term refers to net borrowing in period *t*, namely the loan obtained (L_t) less the amount used by the household to repay its unpaid but refinanced by the bank loan.

Rearranging the terms of equation (2.3) and doing some simple algebra gives a simplified presentation of the budget constraint:

$$P_t C_t^b + [\mu_t + k_t(1-\mu_t)](1+i_{t-1}^L)L_{t-1} + \mu_t f_{t-1} k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2} = W_t N_t^b + L_t \quad (2.4)$$

This form of the budget constraint has an interesting reading. In particular, the second term is the proportion of capitalized debt that has been repaid by households or has not been repaid and has been refinanced by the banks. Also, the third term is the amount of the penalty paid on the previous period's capitalized debt that has been refinanced. The other terms are as in equation (2.3).

2.2.2. The household default rate

Given the analytical exposition of the budget constraint just presented, an expression for the household default rate can be written. The amount of household debt in default at the end of period *t* consists of two parts: a) the part of the net borrowing which has not been repaid by households and has not been refinanced by the banks and b) the part of previous period capitalized debt from refinancing plus the amount of the unpaid penalty, both of which have not been repaid by the households. Thus, the household default rate (d_t) is the ratio of the sum of the above two components of default divided by the value of capitalized debt:

$$d_t = \frac{(1-k_t)(1-\mu_t)(1+i_{t-1}^L)(L_{t-1}-B_{t-1}) + (1-\mu_t)(1+i_{t-1}^L+f_{t-1})B_{t-1}}{(1+i_{t-1}^L)L_{t-1}} \quad (2.5)$$

where $B_t = k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}$

2.2.3. The optimization problem

Now the problem facing the representative household is to choose its decision variables, i.e. consumption, working hours, loans and the percentage of debt repaid, to maximize utility given the state variables, i.e. wage rate, interest rate, price level, penalty premium on the interest rate and the percentage of unpaid debt which is refinanced. Thus, the problem is to maximize

$$\max_{C_t^b, N_t^b, L_t, \mu_t} E_t \sum_{j=0}^{\infty} (\beta^b)^j \left(\frac{(C_{t+j}^b)^{1-\sigma}}{1-\sigma} - \frac{(N_{t+j}^b)^{1+\varphi}}{1+\varphi} \right) \quad (2.6)$$

s.t.

$$P_t C_t^b + [\mu_t + k_t(1-\mu_t)](1+i_{t-1}^L)L_{t-1} + \mu_t f_{t-1} k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2} = W_t N_t^b + L_t$$

The Lagrangian of this problem is formed:

$$\mathcal{L} = \sum_{j=0}^{\infty} (\beta^b)^j \left\{ \begin{array}{c} \frac{(C_{t+j}^b)^{1-\sigma}}{1-\sigma} - \frac{(N_{t+j}^b)^{1+\varphi}}{1+\varphi} + \\ \lambda_{t+j} \left[\begin{array}{c} +W_{t+j} N_{t+j}^b + L_{t+j} - P_{t+j} C_{t+j}^b - \\ -[\mu_{t+j} + k_{t+j}(1-\mu_{t+j})](1+i_{t-1+j}^L)L_{t-1+j} - \\ -[\mu_{t+j} f_{t-1+j} k_{t-1+j}(1-\mu_{t-1+j})(1+i_{t-2+j}^L)L_{t-2+j}] \end{array} \right] \end{array} \right\} \quad (2.7)$$

where λ is the Lagrange multiplier.

First-order conditions for consumption, working hours, loans and the percentage of debt repaid are taken, as shown in Appendix A, eqs (A.1) to (A.4).

2.2.4. The consumption Euler equation

Further, by combining equations (A.1) and (A.5) from Appendix A and writing equation (A.1) one period forward, the Euler equation for the household optimal consumption is derived analytically; it is essentially an equilibrium relation[15], where for given values of the state (exogenous) and other decision variables, there is no tendency for the path of consumption to change:

$$\frac{(C_t^b)^{-\sigma}}{P_t} = \beta^b \frac{E_t\{(C_{t+1}^b)^{-\sigma}\}}{E_t\{P_{t+1}\}} (1+i_t^L) \left[1 + \frac{(1-E_t\{\mu_{t+1}\}) f_t k_t (1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t} \right] \quad (2.8)$$

The Euler equation determines the borrowing household's optimal consumption as a function *inter alia* of a component of the expected default rate, namely the expected amount of the unpaid penalty as a proportion of the value of capitalized debt (cf. eq. 2.5). This equation is the optimal solution to the representative household maximization problem.

Using a log-linear approximation to eq. (2.8) yields (cf. Gali, 2015):

[15] All endogenous (decision) variables are at their equilibrium values since all first-order conditions are satisfied.

$$c_t^b = E_t\{c_{t+1}^b\} - \frac{1}{\sigma}\left[i_t^L - E_t\{\pi_{t+1}\} - \rho^b + \frac{(1 - E_t\{\mu_{t+1}\})f_t k_t (1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \quad (2.9)$$

where $c_t^b = \ln C_t^b$, $E_t\{c_{t+1}^b\} = \ln E_t\{C_{t+1}^b\}$, $E_t\{\pi_{t+1}\} = \ln\left(\frac{E_t\{P_{t+1}\}}{P_t}\right)$, $\rho^b = -\ln \beta^b$,

This equation is augmented relative to that of the literature (see, among others: Gali, 2015; Romer, 2012) by including a default premium over the real interest rate[16]. The equation presents the relation between the current level of consumption and the next period's expected level of consumption[17], the real interest rate[18], the time preference rate and an expected default factor. Consumption decisions of the current period are affected negatively by the default factor, since any rise in next period's expected default decreases current consumption, so that the future consequences of default, either pecuniary (default penalty, inability of future borrowing) or non-pecuniary (bad reputation), are limited. Further, any policy move either by the monetary authorities who adjust the short-term nominal interest rate thus affecting the corresponding real rate in the short run, or by the supervisory authorities who raise the penalty premium on the interest rate affecting upwards the expected default rate, or by the commercial banks in their refinancing policy, alters the optimal consumption path. Thus, the Euler equation includes the degree of control over the household available to the financial system (central bank, financial regulator, commercial banks) in the most basic model, i.e. that of a closed economy without a public sector.

2.2.5. The steady-state

Using the Euler equation, which as already indicated is a dynamic equilibrium relation, to deduce the corresponding static equilibrium relation, i.e. one in which all variables are time-invariant[19], it is obtained the following expression for the optimal value of the decision variable μ, i.e. the percentage of the debt repaid:

$$\mu = 1 - \sqrt{(\rho^b - i^L)/fk} \quad (2.10)$$

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[16] As noted by Baltensperger (1975), it can be reasonably assumed that individuals expect that the rate of interest on a loan has to be adjusted upwards in the presence of default risk since in this case the interest rate should contain a risk premium.

[17] Most of the empirical literature on the Euler equation for consumption relies on a purely backward-looking specification of the optimal consumption decisions. Exceptions are Fuhrer and Rudebusch (2004) and Goodhart and Hofmann (2005), who allow for forward-looking elements.

[18] According to Etro (2009), optimal consumption is increasing over time if the real interest rate is larger than the rate of time preference. Consumption smoothing is optimal if they are equal.

[19] This relation corresponds to a situation of optimal consumption smoothing (see footnote 18 above).

As can be seen from this equation, the variable μ depends on four determining factors: the penalty premium on the interest rate *(f)*, the percentage of unpaid debt which is refinanced *(k)*, the real interest rate (i^L) and the rate of time preference (ρ^b). The penalty premium has a positive effect on μ, since its rise induces households to repay more in order to avoid incurring higher premiums. The percentage of unpaid debt which is refinanced has similarly a positive effect, since the higher level of obligations arising from an increase in k is expected to lead to a higher μ. The interest rate has also a positive impact on μ, reflecting the household's willingness to repay more when i^L is higher so as to mitigate the higher risk of future default. Finally, an increase in the time preference rate implies a lower discount factor[20] and hence its effect on μ will be negative.

By rearranging the terms of equation (2.10), we arrive at the following expression for the real interest rate:

$$i^L = \rho^b - (1-\mu)^2 fk \tag{2.11}$$

whereby the default factor is negatively associated with the real interest rate.

In equation (2.10), when $\mu = 1$, namely households repay all of their debt and there is no household default in the economy, the interest rate (real) equals the default-free rate of time preference ρ^w, i.e.

$$i^L = \rho^w, \tag{2.12}$$

a standard result in models without consumer default (cf. Barro and Sala-i-Martin, 2004). When, on the other hand, $0 < \mu < 1$, which signifies the existence of default, the real interest rate can be shown to vary in the following interval:

$$\rho^b - fk \; < \; i^L \; < \; \rho^b \tag{2.13}$$

Thus, the household borrowing cost (real interest rate) is bounded between a minimum value equal to the benefit in terms of increase in future utility minus the extra cost of the refinanced debt due to the imposition of the penalty premium on the interest rate, and a maximum value equal to the above benefit.

[20] In the next section, it is shown that borrowers who do not repay all of their loans have a lower discount factor compared with either borrowers who repay fully their loans or savers.

2.3. Optimal decisions for borrowing and saving households

In this section, it is assumed that the household sector consists of two different types of households, the borrowing and the saving household. The former is characterized by a lower inter-temporal discount factor, which generates an incentive for it to borrow, i.e. this household is financially constrained. As in Agenor et al. (2013) and Suh (2014), savers can hold financial assets and trade in asset markets, while borrowers do not participate in asset markets[21]. They have also the same inter-temporal elasticities of substitution. It is assumed a fraction α of households to be borrowers and the remaining fraction *1*-α to be savers.

The maximization problem of the borrowing household has been analyzed in section 2, while the behavior of the saving household is going to be examined briefly in the following subsection. Finally, the combination of the two types of households in the same model is going to be analyzed in the second subsection.

2.3.1. The optimization problem of the saving household

The saver's utility maximization problem presents no novelties but it is shown here for the sake of completeness. To ensure comparability of the results, our analysis uses the same general assumptions as in the case of the borrowing household. The only difference is that the saving household does not borrow but saves, investing its saving in deposits[22], the maturity of which is assumed to be one period. At the end of the current period, deposits including interest are withdrawn.

The saving household is further assumed to have a life-time utility function:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta^s)^j \, u\left(C_{t+j}^s, N_{t+j}^s\right) \qquad (2.14)$$

having as arguments only consumption and working hours; β^s is the discount factor of savers. As with borrowing households, the functional form of the utility function is of the constant relative risk aversion type:

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[21] Saving households will always save and borrowing households will always borrow. The household in solving the optimization problem will never choose to engage in both saving and borrowing. This is ensured by the condition that the saving interest rate is lower than the borrowing rate. In fact, the borrowing rate is a markup over the saving rate since banks which intermediate between borrowers and savers need to cover the costs of intermediation. However, the relation between these two rates need not concern us further here.

[22] It is assumed that bonds that may be issued by the private sector (firms) are perfect substitutes for deposits as a means of saving, i.e. they have the same rate of return.

$$u(C_t^s, N_t^s) = \frac{(C_t^s)^{1-\sigma}}{1-\sigma} - \frac{(N_t^s)^{1+\varphi}}{1+\varphi} \tag{2.15}$$

Also, the saving household is assumed to be subject to an infinite number of budget constraints, which have the following form (for period *t*):

$$P_t C_t^s + D_t = W_t N_t^s + (1 + i_{t-1}^D) D_{t-1} \tag{2.16}$$

where D_t are deposits at the end of period *t* and i_{t-1}^D is the deposit rate of the previous period.

The household's maximization problem consists in choosing its decision variables, i.e. consumption, working hours and deposits, so as to maximize utility subject to an infinite number of inter-temporal budget constraints of the form of eq. (2.16). The Lagrangian of this problem is set up by combining equations (2.14), (2.15) and (2.16):

$$\mathcal{L} = \sum_{j=0}^{\infty} (\beta^s)^j \begin{Bmatrix} \frac{(C_{t+j}^s)^{1-\sigma}}{1-\sigma} - \frac{(N_{t+j}^s)^{1+\varphi}}{1+\varphi} + \\ \lambda_{t+j}\left[W_{t+j} N_{t+j}^s + \left(1 + i_{t-1+j}^D\right) D_{t-1+j} - P_{t+j} C_{t+j}^s - D_{t+j} \right] \end{Bmatrix} \tag{2.17}$$

In Appendix B, the first-order conditions for consumption, working hours and deposits are presented (eqs B.1, B.2 and B.4). From these equations the standard Euler equation for consumption can be derived analytically, which is commonly encountered in the literature.

$$c_t^s = E_t\{c_{t+1}^s\} - \frac{1}{\sigma} \lfloor i_t^D - E_t \pi_{t+1} - \rho^s \rfloor \tag{2.18}$$

where ρ^s is the time preference rate of savers.

The long-run static equilibrium relation corresponding to this equation is:

$$i^D = \rho^s, \tag{2.19}$$

i.e. the (real) deposit rate equals the time preference rate of savers.

2.3.2. The optimization problem of borrowing and saving households

The Euler equations for consumption are next examined for both borrowing and saving households. The two equations are equations (2.9) and (2.18) of subsections 2.2.4 and 2.3.1 above. By noting that the optimization problems of the two households are essentially independent of each other, they can be combined in a weighted average sense in one representative Euler equation for total consumption. Thus, by multiplying equation (C.1) of

Appendix C by α[23] and equation (C.2) by $1\text{-}\alpha$ and by adding outcomes, the following result is obtained in terms of total household consumption:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\alpha\left[i_t^L - E_t\{\pi_{t+1}\} - \rho^b + \frac{(1 - E_t\{\mu_{t+1}\})f_t k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right]$$

$$-\frac{1}{\sigma}(1-\alpha)[i_t^D - E_t\pi_{t+1} - \rho^s]^{24} \qquad (2.20)$$

Overall consumption is a weighted average of consumption of both types of households (Wieland and Wolters, 2013), where the weights are the percentage of each type of households in the population. As noted by Suh (2014), the separation of the two Euler equations is useful to see how monetary policy and macro-prudential policy affect differently the inter-temporal consumption decisions of borrowing and saving households. From the analysis already presented above, it can be understood that monetary policy affects decisions of both households, while macro-prudential policy only affects decisions of borrowing households. This stems from the fact that macro-prudential policy tools, e.g. loan-to-value ratios and default penalties, affect borrowing households' behavior, in particular through loans, which are a basic determinant of the default rate (see equation 2.5 above).

2.3.3. An ordering by size of discount factors

A critical parameter in the household's optimization problem is the household's discount factor or, equivalently, its time preference rate[25]. The discount factor is a measure of how strongly consumer choices depend on expectations because it weighs current utility and expected utility from future choices. A change in expectations affects observable choices of consumers with higher discount factors more than choices of consumers with smaller discount factors. Therefore, an ordering by size of discount factors is required for all types of households, particularly in empirical calibrations of dynamic models of consumer behavior to make them relevant for optimal policy making.

The literature compares the discount factors of savers and borrowers who repay all of their debts. Because under optimal consumption smoothing, the time preference rate of each of these two groups is equal to the respective interest rate, and the borrowing rate is higher

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[23] Assuming that households do not move between the two groups, it implies that α remains constant over time.

[24] With no capital, and therefore no investment, the economy's aggregate resource constraint is simply $y_t = c_t$, i.e. all output must be consumed, and equation (20) can be interpreted as an aggregate demand equation.

[25] The discount factor and the time preference rate are connected with the following relation: $\rho = -ln\beta$, see also eq. (9) above.

than the saving rate, the borrowers' time preference rate is also higher than that of savers ($\rho^w > \rho^s$); the reverse is true for the discount factor, i.e. $\beta^s > \beta^w$.

In the current analysis, borrowing households that repay all of their debts and those that do not repay the debts fully are distinguished, and thus the possibility of default in the latter's decisions is admitted. The parameter μ in the model developed, where μ lies between 0 and 1, characterizes this subgroup of borrowers. Equation (2.11) above shows that these borrowers' time preference rate is the borrowing rate (real) plus the default premium $(1-\mu)^2 fk$, which is a positive quantity, and, therefore, is higher than the default-free time preference rate, i.e. $\rho^b > \rho^w$. This provides a unique ordering by size of time preference rates as follows:

$$\rho^s < \rho^w < \rho^b \tag{2.21}$$

or a reverse ordering for discount factors

$$\beta^s > \beta^w > \beta^b \tag{2.22}$$

These calculations assess the discount factor/time preference rate when there is a possibility of non-payment, and consequently a default risk, associated with borrowing households.

2.4. Conclusions

This chapter has examined the optimal consumption decisions of households in a micro-founded framework that assumes agent heterogeneity and debt default. A dynamic equilibrium model is set up in which the household sector consists of two different sub-sectors: borrowing households and saving households. For the former, by solving analytically the optimization problem, an augmented Euler equation is derived, which includes an expected default variable as an additional argument to those already in the literature.

In the model, default is studied in the context of an infinite number of overlapping processes of consumer behavior with periodicity two. In this respect, borrowers were assumed not to repay all of their debt in a given period. A percentage of unpaid debt comes to default in the same period, for reasons such as cash flow problems, while the other part is refinanced by banks in the following period and a penalty premium is added onto the interest rate of the loan. Again, a proportion of refinanced debt is not repaid in that period but also defaults.

Further, a static equilibrium relation for the proportion of debt repaid is obtained. In particular, according to the results, this decision variable was shown to depend on a number of determinants: the time preference rate, the borrowing interest rate, the penalty premium on the interest rate and the percentage of unpaid debt which is refinanced by banks. In addition, an ordering by size was provided for the discount factor, with borrowers who do not repay all of their loans having the lowest discount factor, followed in turn by borrowers who fully repay their loans and finally by savers.

The current chapter represents a promising line of research for incorporating macroprudential tools in one of the basic components of DSGE models, making the latter more appropriate for analyzing monetary and macro-prudential policies. The results of this chapter may generate interesting trade-offs in a typical DSGE model for the economy as a whole. This, however, remains an important task for future work.

3. Are household consumption decisions affected by past due unsecured debt? Theory and evidence

3.1. Introduction

Since the mid-1990s, the delinquency and the non-performing loans rates for consumer loans in the US have been fluctuating around 3.5 and 1.5 percent respectively. During the recent financial crisis (2007-2009) both rates have risen rapidly, the former to almost 5 percent and the latter by more than a 100 basis points exceeding 2.5 percent. Over the same period, the growth rate of consumption ranged from 0.5% to 1.5% except for the crisis period when it tumbled below -1%. Based on these empirical observations, this chapter answers the following questions: Is there any relationship between delinquency and non-performing consumer loans on the one hand, and household consumption on the other? Could such debt non-payment determine a different consumption behavior? Is the impact of debt non-payment important in order to differentiate the estimated preference parameters of households relative to the values reported in the literature? Finally, is the consumption Euler equation a feasible tool to estimate the household's preference parameters in the presence of past due unsecured debt?

This chapter tries to give an answer to these questions by developing a partial equilibrium model to assess the impact of delinquent and non-performing consumer debt in the unsecured credit market on consumption behavior[26].

The theoretical discussion of delinquency and non-performing consumer loans is part of the wider discussion of consumer default, its aspects, and the measures for its reduction and control.

A number of recent theoretical and empirical works highlight the role that endogenous default plays in the propagation of shocks to the economy compared to the absence of default frictions (Goodhart et al., 2009) or in influencing asset pricing (Dubey et al., 2005; Chang and Sundaresan, 2005). Einav et al. (2012) present a standard consumer theory framework where heterogeneous consumers decide how much to borrow, taking decisions about whether to continue making loan payments or to default. Nakajima and Rios-Rull (2014), by extending the model of Chatterjee et al. (2007) to address business cycles, try to answer the question whether the extended model can offer correct predictions about the cyclical behavior of

[26] Recent contributions to the quantitative literature on unsecured consumer debt have been discussed by Livshits (2015).

unsecured household credit and bankruptcies, and also whether a large increase in consumer credit affects the nature of business cycles.

Another strand of the literature attempts to model the link of bankruptcy, i.e. the insolvency of the debtor, and delinquency to economic factors. Athreya et al. (2012) integrate both bankruptcy and delinquency in a life-cycle model of consumption and savings where agents opt to become delinquent, since they view delinquency as less costly than bankruptcy. Both bankruptcy and delinquency carry costs. Bankruptcy eliminates debt with future heavy consequences, while delinquency does not automatically remove any debt and the borrower is exposed to higher future debt obligations that arise from penalties. Benjamin and Mateos-Planas (2014) propose an equilibrium model which distinguishes between default and delinquency (formal and informal default, respectively) allowing for household heterogeneity. They view informal default as a process of negotiation between the debtor and the creditor that may follow after debt repayments fail to be met. In a survey of credit card delinquency and bankruptcy, Stavins (2000) argues that there is likelihood of delinquency leading to bankruptcy if the factors that induce delinquent loans persist. Lenders' strategies against delinquency and bankruptcy should be reevaluated when economic conditions change.

Further, the increase in loan defaults in the recent financial market turmoil highlights the link between macroeconomic and financial shocks. In the last decade, more and more studies attempt to shed some light on the determinants that affect non-refundable loans of the banking sector, namely non-performing loans, in developing and developed countries. For instance, in a recent study for a number of South European countries for the period 2004-2008, Messai and Jouini (2013) argue that the minimization of non-performing loans is a necessary condition for improving economic growth, while in a recent study for Central, Eastern and South-Eastern Europe covering the period 1998–2011, Klein (2013) shows that an increase in non-performing loans has a significant impact on real GDP growth and other macroeconomic variables such as unemployment and inflation.

A handful of studies investigate the measures banks take to reduce non-payment of consumer debt. To discourage borrowers, financial intermediaries impose costs, either directly through a penalty on the interest rate or indirectly by raising the lending interest rate for borrowers with a past record of bankruptcy (Huber et al., 2016). As noted by Chatterjee et al. (2007), households for whom the period begins with a record of bankruptcy cannot get new loans. In a paper on defaults by firms, de Walque et al. (2010) assume that the cost of default affects both the utility function as a non-pecuniary burden (social stigma and reputation cost) and the budget constraint as a pecuniary burden. In a similar vein, Einav et al. (2012) report that default affects utility of households due to potential costs associated with default, such as a constant per-loan indirect cost. Gross and Souleles (2002) find that the sizable increase in the default rate in their dataset of credit card accounts from 1995 to 1997 is consistent with a decline in the cost of bankruptcy. Finally, Livshits et al. (2010) give some potential explanations for the increase in consumer bankruptcies and find that a decline in the cost of bankruptcy together with a reduction in the cost of lending through credit market innovations (such as the development and spread of credit scoring), closely matches the US experience. The former makes bankruptcy more attractive and the latter increases the probability of default.

To link non-payment of consumer debt to the theory of household behavior, the general guidelines of Chatterjee et al. (2007) are followed, who allow borrowers to default on their consumer loans. From the modeling point of view, the current chapter includes both delinquency and non-performing loans rates in the analysis of the consumption Euler equation, but in other respects the general discussion on the estimation of the household preference parameters is followed.

For the study of consumer behavior, modern economics mostly uses the consumption Euler equation, which reflects the relationship between the real interest rate and the consumption growth rate. It is assumed that economic agents smooth their consumption spending over a lifetime and thus maximize total utility. Hall (1978) in his seminal paper proposed that consumption dynamics could be modeled in a function derived from the intertemporal optimization problem of a fully rational and forward-looking representative consumer. The main attraction of this approach lies in the fact that it avoids solving explicitly the optimization problem, while at the same time it allows the estimation of the consumer preference parameters (Attanasio and Low, 2004). The Euler equation approach has been applied to both the micro- and macro-level of analysis. In the latter case, an output equation often generalizes the consumption Euler equation for the whole economy and as such it is currently at the core of modern macroeconomics.

A large number of theoretical and empirical works use calibration or simulation methods, attempting to fit the Euler equation to some aspects of the data rather than flat-out estimating the model to capture the values of the preference parameters.In Hall (1988), it is apparent that the relationship between the expected rate of growth of consumption and the expected real interest rate is governed by the intertemporal substitution aspect of preferences.

Alan and Browning (2010) using a novel structural estimation procedure for models of intertemporal allocation are able to account for measurement error in consumption and for heterogeneity in discount factors and coefficients of relative risk aversion. Alan et al. (2009) develop two alternative GMM estimators and argue that their Monte Carlo results suggest that such estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially in short panels. Finally, Attanasio and Low (2004) perform a Monte Carlo experiment to solve and simulate a simple life-cycle model under uncertainty, and show that in most situations the estimates obtained from the log-linearized equation are not systematically biased.

Another strand of the literature employs survey and non-survey (synthetic) techniques to construct micro-data to estimate the preference parameters from behavioral equations derived under the expected utility maximization framework (see e.g. Gourinchas and Parker, 2002; Campbell and Cocco, 2007; Attanasio and Borella, 2014).

Nevertheless, empirical estimates of the consumption equation with real data have been relatively rare (see e.g. Hall, 1988). Rosenblatt-Wisch (2008) used prospect theory in a stochastic optimal growth model to derive a stochastic Euler equation and tested its implications when GMM estimation was used with US aggregate macroeconomic time-series data. Schorfheide (2012) noted that although calibration is an attractive method for complicated structural models that are costly to solve repeatedly for different parameter values, we need to know how real data can be used to determine the model's parameters and to what extent the model is consistent with the data.

In this chapter, the consumption Euler equation is derived analytically by solving the lifetime utility maximization problem of the representative household, and the values of the two important preference parameters, the discount factor and the elasticity of intertemporal substitution in consumption are estimated. Along similar lines with Chatterjee et al. (2007), households smooth consumption by means of unsecured loans[27] on which they are allowed to default by filing for bankruptcy[28,29]. Our model introduces endogenously non-payment of

[27] We can get some impression of the importance of unsecured consumer debt by noting that the amount of consumer loans in all US commercial banks more than tripled in the period from 1995 to 2016 and now exceeds US$1.35 trillion.

[28] This is done under the rules laid down in Chapter 7 of the US Bankruptcy Code. The interesting thing with this Chapter and any Chapter of the Bankruptcy Code is that no individual may be a debtor under a particular Chapter unless he or she has, within 180 days before filing, received credit counseling from an approved credit counseling agency. In addition, the unpaid debts up to 90 days are classified in the banks' books either as delinquent if the period of non-payment is between 30 and 90 days or as non-performing if the period is more than 90 days. For these reasons our analysis does not include bankruptcy as an outcome of non-payment for the consumer since it occurs in a longer timeframe and we consider it as a result of the bank's decision to write off

consumer debts. There are two routes available to borrowers for stopping payment: (i) become delinquent and thus delay bankruptcy at least temporarily; in this case their debt is rolled over with a penalty on the interest rate paid, and (ii) file an application for bankruptcy, in which case the debt is marked as non-performing until the court decides on the application; non-performing debt stops accruing interest. By introducing non-payment of debt, we are able to arrive at a straightforward generalization of the well-known Euler equation for consumption that is augmented by a risk factor in terms of expected non-performing debt and delinquent debt. The extended model cannot be estimated as it stands since it lacks directly observable counterparts in the data and is therefore transformed to match the available aggregate US statistics for the unsecured credit market and for both the delinquency and non-performing consumer debt rates. The model is then estimated with nonlinear GMM to yield consistent estimates of the preference parameters. Estimation biases possibly due to the presence of measurement errors and unobserved heterogeneity are accounted for by specifying a parametric process for the errors of the Euler equation (see Arellano, 2002). The results from our model are compared with those of the benchmark representative agent model with full debt repayment, which seems to be an incomplete description of consumer behavior.

The chapter is structured as follows: in Section 2 the theoretical model of the borrowing household's consumption behavior is presented, which includes debt non-payment. Section 3 discusses the estimation methodology, data and empirical results, and Section 4 concludes.

3.2. Theoretical analysis

In this section, the household intertemporal utility maximization problem is set up by admitting the possibility of debt non-payment; its solution gives a more complete characterization of the household's optimal consumption decisions in the presence of past due unsecured debt. The analysis is carried out in discrete time. It is assumed that there is a population of infinitely-lived homogeneous[30] households that consume a consumption bundle. The household derives utility from consuming goods and services, and disutility from supplying labor to producers of goods and services. The representative household does not

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any non-performing debt assessed as non-refundable once a decision has been made by the court on the application for bankruptcy.

[29] As in Livshits et al. (2007), the focus is on Chapter 7 bankruptcies and we abstract from examining consumer durables bankruptcies.

[30] Homogeneity means that households have the same preferences and do not face any idiosyncratic shocks or frictions.

save but borrows from banks (the loan supply of which is perfectly elastic at the prevailing lending rate) to support consumption smoothing[31], i.e. it is a borrowing household. Household loans can become delinquent[32]or non-performing[33] for cash flow reasons, e.g. due to negative income shocks or downturn of the business cycle. Default per se is not examined since this is the result of the bank's decision to write off any non-performing debt, assessed as non-refundable, when a decision has been made by the court on the application for bankruptcy.

In the current period, the household buys consumer goods and services and works in the production process, earning income for labor services provided. At the end of each period, the household can obtain a loan from banks (one-period loan[34]), assuming that it does not face any credit constraint, in the sense that credit demand never exceeds the bank's credit requirements, which the household always observes. At the end of the current period, part of the previous period loan, including interest, is serviced, while the rest is not. The regular servicing of the loan is followed by an extension of a new one-period loan by the banks and the same process runs in the following period. In the case where part of the previous period's loan is not serviced, the household is refinanced in the following period[35]and the proceeds of the new loan are used to repay the unpaid loan, being charged a penalty premium on the interest rate[36]. This lasts for only one period and if the household does not repay again its delinquent debt, the loan is marked as non-performing, provided that an application for bankruptcy has been filed in court.

3.2.1. The model

The household is assumed to have a lifetime utility function:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta)^j \, u\big(C_{t+j}, N_{t+j}\big) \tag{3.1}$$

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[31] The notion of consumption smoothing usually describes an attempt to keep consumption similar in each period.
[32] Delinquent loans are those that are past due 30 days or more and still accruing interest.
[33] Non-performing loans are those that are 90 days or more past due and have stopped accruing interest. Banks consider the unpaid debt as non-performing once an application for bankruptcy has been filed with a court and the court has not yet decided on it.
[34] Roszbach (2004) notes that in financial markets with perfect information any optimal multi-period financial contract can be obtained by a sequence of one-period loan agreements, while under asymmetric information things become more intricate. Also, Besanko and Thakor (1987) develop a single-period credit model under asymmetric information and assume that no borrower has an initial endowment and each must approach a lender for a one-period loan.
[35] It is assumed that banks refinance when they assess that households meet certain credit requirements set by them, e.g. debt-to-income threshold or credit history.
[36] The magnitude of this premium is determined by factors such as credit history and reputation of borrowers, search costs of banks and risk parameters.

where E_t denotes the expectation of the household, conditional on information available at time t, β is the subjective discount factor (capturing the idea of impatience)[37], which discounts future utility, with $0<\beta<1$, and u denotes utility which is related to real consumption (C_{t+j}) and working hours (N_{t+j}, expressed as a ratio to total available time per day). The utility function is assumed to be time separable so that current period utility depends only on current consumption, and twice differentiable with respect to consumption and working hours. The marginal utility of consumption is positive and non-increasing, while that of working hours is negative and non-decreasing. Equation (3.1) simply states that the household is interested in lifetime utility U_t obtained as the present discounted value of current and all future levels of expected utility.

The functional form of the utility function is assumed to be of the constant relative risk aversion type:

$$u(C_t, N_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \tag{3.2}$$

where $1/\sigma$ and $1/\varphi$ are the elasticities of intertemporal substitution of consumption and working time, respectively.

The household is also assumed to be subject to a sequence of budget constraints (in nominal terms), which describe its feasible choices through time. These constraints are of the form (for period t):

$$P_t C_t + \mu_t(1 + i_{t-1})l_{t-1} + \mu_t(1 + i_{t-1} + f_{t-1})(1 - \mu_{t-1})(1 + i_{t-2})l_{t-2} = W_t N_t + l_t \tag{3.3}$$

where

$$l_t = L_t - (1 - \mu_t)(1 + i_{t-1})l_{t-1} \tag{3.4}$$

is net borrowing at the end of the period t.

Equation (3.3) displays the household's receipts (inflows) and payments (outflows) stemming from its income-generating and financing activities.

The left-hand side contains payments. The first term is consumption expenditure, where P_t is the price index of consumer goods and services. The second term involves the proportion μ_t $(0 < \mu \leq 1)$ of previous period net borrowing l_t that is repaid by the household, while the

[37] The impatient household values future utility less than present utility.

rest is delinquent debt refinanced by banks. The third term focuses on the unpaid part of the loan at the end of period *t-1* that is assumed to have been refinanced by banks. The interest rate charged on that loan is the lending rate i_{t-1} augmented by a penalty premium f_{t-1} and the proceeds of the loan are used to repay the unpaid loan. Again, as already indicated, only a proportion of delinquent loans are repaid by households at the end of period t, while the rest become non-performing.

The right-hand side includes receipts. The first term reflects the income earned from labor, where W_t is the nominal wage rate, and l_t , as defined above (eq. 3.4), refers to net borrowing in period t, namely the loan obtained L_t less the amount used by the household to repay its loan that is unpaid but refinanced by the bank.

The financing flows of the household are illustrated in Table 3.1 in a simplified schematic representation[38].

Table 3.1 The timing of the household's financing flows

t-2	*t-1*	*t*
. .	**Net borrowing (payment)** $\mu_{t-1}(1+i_{t-2})l_{t-2}$	**Net borrowing (payment)** $\mu_t(1+i_{t-1})l_{t-1}$
. .	**Delinquent loan/Refinancing** $(1-\mu_{t-1})(1+i_{t-2})l_{t-2}$	**Delinquent loan/Refinancing** $(1-\mu_t)(1+i_{t-1})l_{t-2}$
. .	. .	**Refinancing (payment)** $\mu_t(1+i_{t-1}+f_{t-1})(1-\mu_{t-1})(1+i_{t-2})l_{t-2}$
. .	. .	**Non-performing loan** $(1-\mu_t)(1+i_{t-1}+f_{t-1})(1-\mu_{t-1})(1+i_{t-2})l_{t-2}$
Net borrowing l_{t-2}	**Net borrowing** l_{t-1}	**Net borrowing** l_t

[38]The analysis of the borrowing household's financial flows takes place in period t. In order to obtain a clearer picture of these flows, details of the previous periods *t-1* and *t-2* are given in the Table so as to help an understanding of how the current period flows are generated. Thus we do not consider the period *t-2* as the initial period for the financial flows but rather as the starting period of a repeated process of evolution of these flows.

3.2.2. The optimization problem

Now the problem faced by the representative household is to choose its decision variables, i.e. consumption, working hours, net borrowing and the percentage of debt repaid so as to maximize utility given the state variables, i.e. wage rate, interest rate, price level and penalty premium on the interest rate. Thus, the problem is to maximize

$$\max_{C_t, N_t, l_t, \mu_t} E_t \sum_{j=0}^{\infty} (\beta)^j \left(\frac{(C_{t+j})^{1-\sigma}}{1-\sigma} - \frac{(N_{t+j})^{1+\varphi}}{1+\varphi} \right) \tag{3.5}$$

s.t.

$$P_t C_t + \mu_t (1 + i_{t-1}) l_{t-1} + \mu_t (1 + i_{t-1} + f_{t-1})(1 - \mu_{t-1})(1 + i_{t-2}) l_{t-2} = W_t N_t + l_t$$

The Lagrangian of this problem is formed:

$$\mathscr{L} = \sum_{j=0}^{\infty} (\beta)^j \left\{ \begin{array}{c} \frac{(C_{t+j})^{1-\sigma}}{1-\sigma} - \frac{(N_{t+j})^{1+\varphi}}{1+\varphi} + \\ \lambda_{t+j} \left[\begin{array}{c} W_{t+j} N_{t+j} + l_{t+j} - P_{t+j} C_{t+j} \\ -\mu_{t+j} (1 + i_{t-1+j}) l_{t-1+j} - \\ -\mu_{t+j} \left[\begin{array}{c} (1 + i_{t-1+j} + f_{t-1+j}) * \\ (1 - \mu_{t-1+j})(1 + i_{t-2+j}) l_{t-2+j} \end{array} \right] \end{array} \right] \end{array} \right\} \tag{3.6}$$

where λ is the Lagrange multiplier.

First-order conditions for consumption, working hours, net borrowing and the percentage of debt repaid are taken, as shown in the Appendix D, eqs. (D1) to (D4).

3.2.3. The Euler equation

By combining equations (D1) and (D4) from the Appendix D and writing equation (D1) one period forward, the Euler equation for the household's optimal consumption is derived. This is essentially an equilibrium relation[39], where for given values of the state (exogenous) and other decision variables, there is no tendency for the path of consumption to change:

$$\frac{(C_t)^{-\sigma}}{P_t} = \left\{ \begin{array}{c} \beta \frac{E_t\{(C_{t+1})^{-\sigma}\}}{E_t\{P_{t+1}\}} (1 + i_t) * \\ \left[1 + \frac{(1 - E_t \mu_{t+1})(1 + i_t + f_t)(1 - \mu_t)(1 + i_{t-1}) l_{t-1}}{(1 + i_t) l_t} \right] \end{array} \right\} \tag{3.7}$$

[39]All endogenous (decision) variables are at their equilibrium values since all first-order conditions are satisfied.

Using a log-linear approximation to equation (3.7) yields (cf. Gali, 2015):

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left[i_t \frac{\begin{matrix} -E_t\{\pi_{t+1}\} - \rho + \\ (1 - E_t\mu_{t+1})(1 + i_t + f_t)(1 - \mu_t)(1 + i_{t-1})l_{t-1} \end{matrix}}{(1 + i_t)l_t} \right] \tag{3.8}$$

where $c_t = \ln C_t$, $E_t\{c_{t+1}\} = \ln E_t\{C_{t+1}\}$, $E_t\{\pi_{t+1}\} = \ln\left(\frac{E_t\{P_{t+1}\}}{P_t}\right)$, $\rho = -ln\beta$.

The above equation, which determines the borrowing household's optimal consumption, is augmented relative to that of the literature (see *inter alia*: Gali, 2015; Romer, 2012) by including an additional term which represents the percentage of net borrowing expected by the household to become non-performing. Thus the household's consumption decision is influenced not solely by the real interest rate but also by a risk factor which proxies for the expected default rate of loans. In this respect, the risk-adjusted real interest rate, which is the sum of the real interest rate and the risk factor, incorporates the consequences of moving into the non-performing state – a state close to default – that are either pecuniary (inability to access future borrowing) or non-pecuniary (bad reputation).

3.3. Empirical results

In this section, the empirical relevance of the consumption Euler equation derived in the previous section is evaluated, which incorporates a risk factor in terms of expected non-performing loans. This equation is estimated by using aggregate quarterly data for the U.S. covering the period 1995Q2 to 2015Q3. Other studies have used panel data and simulation methods rather than estimating it directly to obtain the values of the preference parameters (e.g. Alan et al., 2009; Alan and Browning, 2010). Whatever the approach may be, the most important aspect of the empirical analysis is how the key parameters of the Euler equation can be identified based on the available statistical information.

Equation (3.8) can be seen to have some variables that lack directly observable counterparts in the data. A way forward would be to transform it to match the available data. Two of the model's variables that play an essential role in it are delinquent and non-performing loans which, when expressed as a ratio to total consumer loans, define the delinquency rate (*del)* and the non-performing loans rate (*npl*), respectively. On the basis of the theoretical results shown in section 3, these rates can be written as:

$$del_t = \frac{(1 - \mu_t)(1 + i_{t-1})l_{t-1}}{(1 + i_{t-1})L_{t-1}} \tag{3.9}$$

$$npl_t = \frac{(1-\mu_t)(1+i_{t-1}+f_{t-1})(1-\mu_{t-1})(1+i_{t-2})l_{t-2}}{(1+i_{t-1})L_{t-1}} \tag{3.10}$$

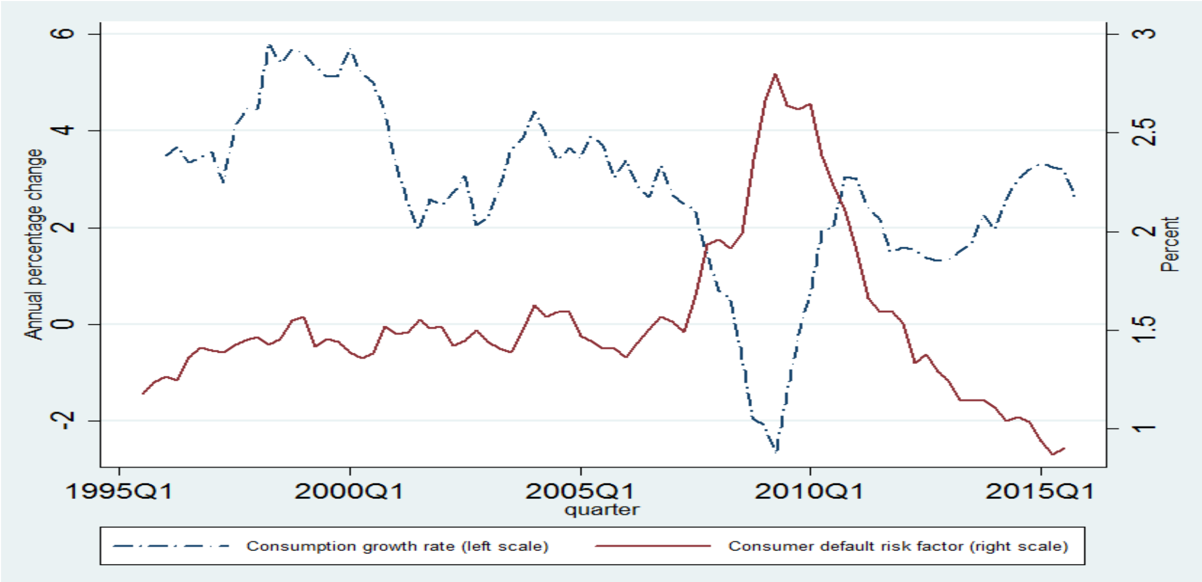
By combining eqs. (3.4), and (3.8) to (3.10), the following equation is obtained as the basis for estimation, which contains only observable variables:

$$c_t = c_{t+1} - \frac{1}{\sigma}\left[i_t - \pi_{t+1} - \rho + \frac{npl_{t+1}}{1 - del_t \dfrac{(1+i_{t-1})L_{t-1}}{L_t}}\right] \tag{3.11}$$

To derive eq. (3.11), it is assumed that future realizations of consumption, inflation and the non- performing loans rate do not differ systematically from their rational expectations values so that deviations from perfect foresight are only random.

To gain some insight on the importance of the last term in the brackets of equation (3.11), which as demonstrated in the theoretical section is a risk factor indicating the percentage of consumer net borrowing expected to become non-performing, its relationship to consumption growth is examined. Looking at equation (3.11), we notice that both the delinquency rate and the expected non- performing loans rate have a negative influence on current consumption. Figure 3.1 displays the growth rate of consumption and the above risk factor proxying for the expected default rate of consumer loans. As can be seen from the figure, the latter variable fluctuates without any obvious trend around 1.5 percent in the first half of the sample period, but rises during the financial crisis, peaking in 2009Q2 at 2.8. Thereafter it steadily declines, falling below 1 percent at the end of the sample. The consumption growth rate shows a fairly close association with the risk factor, whose persistence to high values seems to have contributed to declines in this rate.

Figure 3.1 Consumption growth rate and consumer default risk factor



Source: FRED database and our own calculations

The data employed for estimating the model given by equation (3.11) are seasonally adjusted (except for the interest rate). Sources of the data are the Federal Reserve Economic Data (FRED) and the Federal Reserve Bank of New York (FRBNY) databases. The interest rate i_t is the average of the commercial bank interest rate on credit card plans and the finance rate on personal loans. Inflation is the year-on-year percentage change of the implicit price deflator of personal consumption expenditure P_t. Consumption c_t is the log of personal consumption expenditure in billions of chained 2009 US$. Consumer loans L_t are at the end of quarter in billions of US$. Finally, the delinquency and non-performing loans rates, del_t andnpl_t, are the end-of-quarter ratios of delinquent and non-performing loans respectively to total consumer loans.

In Table 3.2, summary statistics of the variables used in the empirical analysis are reported.

Table 3.2 Descriptive statistics of the variables used in the empirical analysis

Variable	Mean	Std. dev.	Min.	Max.
Real personal consumption expenditures (C_t)	8,177.9	1,929.2	5,097.9	11,322.5
Interest rate (i_t)	12.89	1.38	10.7	15.13
Inflation rate (π_t)	2.26	1.08	-0.94	4.98
Delinquency rate(del_t)	3.34	0.6	2.01	4.85
Non-performing loans rate (npl_t)	1.44	0.36	0.85	2.65
Consumer loans (L_t)	754.8	246.1	464.3	1,206.6

Before proceeding to estimation, some issues of specification need to be addressed.

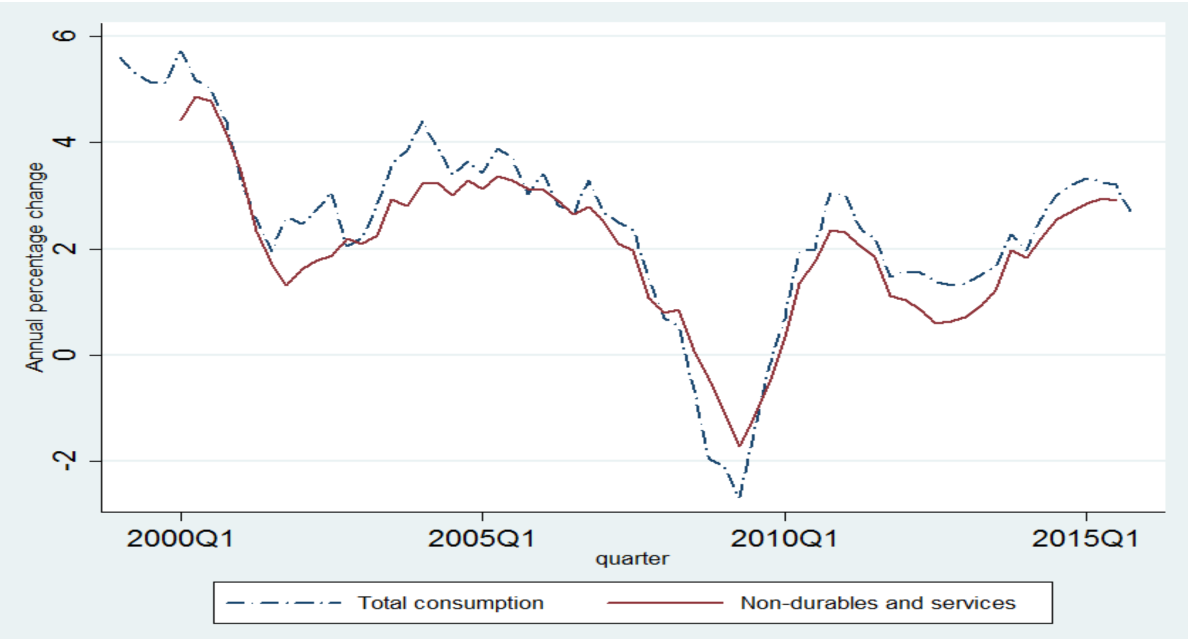
First, probably the most discussed problem with estimating the consumption Euler equation is the measurement error, which mainly arises from the log-linear approximation of the interest rate. The implication is that one may lose the ability to identify the discount factor that is mixed up with the higher-order terms in the Taylor approximation. However, Attanasio and Low (2004) reported that in a standard model and for a wide set of parameter values their simulation results indicate no systematic biases arising from estimating structural preference parameters with a log-linearized Euler equation. Second, an additional component of the measurement error is the error generated as a result of the rational expectations hypothesis, which is also made in this paper. In this respect, there may be persistent deviations from rationality that are not necessarily permanent but nevertheless would result in estimation biases if actual values of the variables were used (see e.g. Brissimis and Migiakis, 2016). Finally, a basic concern of the literature is that unobserved heterogeneity may lead to substantial biases in the estimation of the parameters, while one can in principle control for heterogeneity between groups of households in panel estimation by inserting group-specific intercepts or by introducing the so-called taste shifters, i.e. group-specific and time-varying characteristics, such as household income and working hours. Although estimates based on panel data could possibly alleviate the problem of heterogeneity, long panel data containing information on consumption are almost non-existent (Attanasio and Low, 2004)[40]. In the Euler equation that will be estimated and tested using aggregate consumption data, certain aspects of non-homogeneity are not likely to be much of a problem: (i) to benefit from a larger sample size, given that quarterly data availability on the expenditure on consumer non-durables and services begins in 2000, data on total personal consumption expenditure are employed. As Figure 3.2 shows, the growth rates of these two aggregates are similar in the post-2000 period. (ii) a reasonable assumption would be that borrowers of consumer loans come mainly from the younger working-age group (18-44)[41]. It is interesting to note that the number of persons in this group as a percentage of total population shows only a mild downward trend over the sample period decreasing at an annual average rate of 0.3 percent.

[40] The use of food consumption as a proxy for total consumption is common in the empirical literature for Euler equation estimation since panel survey data on food consumption are the longest available for the U.S., see e.g. Alan and Browning (2010).

[41] As noted by Modigliani, young people borrow to spend more than their income, middle-aged people save a lot and old people run down their savings.

(iii) the interest rate used is the borrowing rate for consumer loans[42]. In the empirical literature, the interest rate variable is selected according to the theoretical set up adopted. Thus, studies in which households are defined as saving households use an interest rate affecting the consumption-savings decision, see, e.g. Alan et al. (2009). On the other hand, given that in many empirical studies an output equation generalizes the consumption Euler equation to the whole economy, the choice of the interest rate in that context should reflect correctly the link between aggregate demand and the (short-term) policy interest rate, see, e.g. Fuhrer and Rudebusch (2004).

Figure 3.2 The growth rate of personal consumption expenditure (non-durables and services, total)



Source: FRED database

Turning to estimation, instrumental variables estimation is relied on to obtain consistent estimates of the model's parameters. The procedure used is nonlinear GMM, which is appropriate since equation (3.11) is a nonlinear model and the interest is in a single equation, the consumption Euler equation, obtained by solving the household's optimization problem. It does not require the complete specification and estimation of the structural equations for the other endogenous variables of the wider system of simultaneous equations. The endogenous

[42] Attanasio and Low (2004) note that to be able to estimate the intertemporal elasticity with a log-linear Euler equation and panel data, it is necessary to have an interest rate that varies over time and in a way that is partly predictable, if one wants to use instrumental variables techniques of the kind typically used in the empirical analysis of Euler equations.

variables of the Euler equation are themselves functions of the state variables of the optimization problem. For instance, the non-performing loans rate (eq. 3.10) can be seen to depend on household's net borrowing (l_t) and the percentage of debt repaid (μ_t). In the presence of endogenous variables, nonlinear GMM is known to yield consistent estimates of the parameters just as linear GMM does (see Davidson and MacKinnon, 2004). Below in Table 3.3 (line 1), estimates of the parameters of equation (3.11) are presented, namely the preference parameters of the representative household[43]. The instruments used in estimation are the lagged values of the real interest rate, real consumption and the change in consumer loans[44]. Both of the estimated preference parameters turn out to be significant. The value of the discount factor (0.912) appears to be on the low side relative to that found in the literature. The relevant tests, shown also in Table 3.3, indicate that our model suffers from both first-order serial correlation and non-normality of the residuals. The finding of serial correlation in the residuals implies that the GMM estimates are biased but they retain the property of consistency even though the errors are autocorrelated.

Above, it was discussed how the presence of measurement errors and unobserved heterogeneity may generate estimation biases in the preference parameters of the household. On the other hand, it has been argued that the lack of autocorrelation in measurement errors can be considered as unrealistic (Sargan, 1958). Our data do indicate first-order autocorrelation, thus suggesting that measurement errors may be an important issue. To obtain a consistent estimate of the variance matrix of the GMM estimators that would allow us to perform optimal inference regarding the preference parameters, we could alternatively specify a parametric process for the equation error or try to obtain a robust estimate of the variance matrix under more general assumptions (see Arellano, 2002). If the sample size is small, then the former may be a better idea than the latter, but even if the process for the errors is misspecified, the GMM estimates will still be consistent (Arellano, op. cit.). Following the first option, it is assumed that the error of the equation is described by a first-order autoregressive process and estimate again our model with non-linear GMM. The instruments used are the same as in the previous estimation[45].

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[43] The model is estimated with Stata.

[44] Angrist and Krueger (2001) noted that using fewer instruments in instrumental variables estimation reduces bias in coefficient estimates. In a similar vein, Arellano (2002), surveying Sargan's work, stressed that the improvements of the asymptotic variance matrix of the GMM estimators are usually very small after the first three or four instrumental variables have been added, while the estimates have large biases if the number of instrumental variables becomes too large.

[45] Sargan argued that when the errors are autocorrelated, it is probably not wise to use lagged values of a variable appearing in the relationship as instrument. However, since the variables used as instruments in our model are

The estimation results are also reported in Table 3.3 (line 2).

Table 3.3 Nonlinear GMM estimation of the consumption Euler equation

Models	*1/ σ*	*β*	*ρ*	*J-test*	*SK-test*	*Q-test*
Eq. 3.11	0.204 (5.41)	0.912 (130.11)	-	0.45	0.00	0.00
Eq. 3.11 with autocorrelated errors	0.230 (2.27)	0.908 (74.33)	0.59 (1.76)	0.74	0.07	0.16
Model with full debt Repayment	0.216 (2.27)	0.924 (71.02)	0.60 (1.47)	-	0.23	0.03

Notes: Columns 2, 3 and 4 of the Table present coefficient estimates with their t-values in parenthesis. Columns 5,6 and 7 show the p-value of the J-test for instrument exogeneity, the D'Agostino et al. (1990) skewness and kurtosis test for normality of the residuals and the Box-Pierce test for first-order autocorrelation of the residuals, respectively.

The J-test indicates that all instruments are exogenous and that there is no evidence against the correct specification of the model with autocorrelated errors[46]. Furthermore, the results of the SK- and Q-tests show that the hypothesis that the residuals are further autocorrelated can be rejected, while that of residual normality can be accepted. It is interesting to note that the correction of the residuals for first-order serial correlation has improved the J-test and has also remedied the violation of residual normality in the initial estimation.

All the estimated coefficients have the expected signs and are significant.

The elasticity of intertemporal substitution (EIS) of consumption, which is a key parameter measuring the response of expected consumption growth to changes in the expected real interest rate (adjusted for a risk factor in our model), is estimated at 0.230. In spite of the important role of this parameter in evaluating standard models of consumption behavior and policy effectiveness, there have been relatively few estimates of its magnitude ranging from zero to higher than one. Earlier studies reported low estimates of the EIS. Davies (1981) reviewing this literature suggested that a reasonable estimate for the EIS would be $1/\sigma = 0.25$. Consistent with the widely used representative-agent approach, other macro-studies also found the EIS estimated from aggregate consumption data to be of a small magnitude. For example, Hall (1988) reported that the elasticity is unlikely to be much above 0.1, while Campbell and Mankiw (1989) extending Hall's framework obtained the same

likely to be uncorrelated with the measurement error component of the error term, autocorrelation due to measurement error does not invalidate lags of these variables as instruments (cf, Vissing and Jorgensen, 2002).

[46]This means that any kind of misspecification that gives rise to correlations between the regressors and the residuals would lead to the rejection of the null hypothesis of correct specification.

result. A number of papers on DSGE modeling avoid estimating Euler equation parameters and set the EIS as unity (e.g. Schorfheide, 2000). Also, as noted by Guvenen (2006), observations on growth and aggregate fluctuations suggest a value of EIS close to one. Other papers estimate or calibrate the EIS at around 0.6 - 0.7 (e.g. Smets and Wouters, 2004; Kydland and Prescott, 1982), while Lucas (1990) argues that even an elasticity of 0.5 appears low when confronted with macro data. Thus, there is an apparent contradiction between the macroeconomics literature and econometric studies which both use the same aggregate data. An explanation for this contradiction was provided by Guvenen (2006): An individual's EIS appears to increase with his wealth level, and there is also substantial wealth inequality in the U.S. and accordingly the preferences of the wealthy are not revealed in consumption regressions. This result is reinforced by survey studies which provide evidence in this direction. Thus Attanasio et al. (2002) obtain elasticity values from UK data around 1 for stockholders and between 0.1 and 0.2 for non-stockholders. Barsky et al. (1997) using also survey data for the U.S. provide evidence of an average EIS of 0.18, with only 2.5 percent of households having an EIS greater than or equal to 1. Our estimate of the EIS obtained from aggregate data is broadly in line with the above evidence on the average elasticity.

The other important preference parameter of the representative household, namely the discount factor β, is estimated to be 0.908, a value which is generally lower than that used in calibrations of models, where there is no default on consumer debt. The majority of these papers set values not smaller than 0.94 (see e.g. Livshits et al. 2010; Alan et al., 2009; Gelain et al., 2013; Mendicino and Punzi, 2014; Rubio and Carrasco-Gallago, 2014). Estimates of this parameter using macro data are somewhat lower. For example, Balfoussia et al. (2011) estimate the New Keynesian model for the US economy and report a value of β equal to 0.92. To compare the specifications, the estimation of our model (eq. 3.11) is repeated by omitting the last factor in the Euler equation, which is the expected default rate proxy for consumer loans. The results are presented in Table 3.3 (line 3). They show β to be higher (0.924), confirming that if consumers evaluate future obligations at a risk-adjusted interest rate, which is higher than the market rate, they are less patient (i.e. they have a lower discount factor), spending less money now and thus avoiding the future consequences of the loan moving into the non performing state.

Papers that studied default in unsecured consumer debt have also set values for calibration higher than or equal to 0.94 (e.g. Davis et al., 2006; Livshits et al., 2010). An exception to this is the study by Chatterjee et al. (2007) which specifies a general equilibrium model with unsecured consumer credit, incorporating the main characteristics of the US consumer bankruptcy law, and estimates the value of the discount factor for four versions of the model between 0.887 and 0.919. An interesting comparison in this respect is offered by the fact that the 95 percent confidence interval for the discount factor estimated from our model is 0.884 to 0.932.

All in all, the extension of the consumption Euler equation to incorporate the hypothesis that unsecured consumer loans can become delinquent and/or non-performing and this in turn may affect consumption decisions appears to improve the specification of the representative agent model compared to the one with full debt repayment. Indeed, the presence of the risk term in the model lowers, as one would expect a priori, one of the most important preference parameters, namely the discount factor, implying that the household becomes less patient and finds it advantageous to trade away future consumption to current consumption. Moreover, the fact that in the model with full debt repayment (Table 3.3, line3) autocorrelation of the residuals remains, even after the correction for first-order serial correlation, indicates that this model is missing the persistence of the risk factor[47] when the latter is omitted from the specification.

3.4. Conclusions

The consumption Euler equation has become the mainstay of much macroeconomic research over the past thirty years and is now an essential element of nearly every DSGE model. Being an intertemporal first-order condition derived by solving the lifetime utility maximization problem of the representative household, it directly provides a basis for estimating the two important preference parameters of the household: the elasticity of intertemporal substitution in consumption and the discount factor. To estimate these parameters, the large majority of existing papers use simulation techniques, only few estimate the parameters directly from the data, and there is none that does both things simultaneously: estimate the Euler equation from real aggregate data and admit the possibility of default on unsecured consumer debt.

In this chapter, the non-payment of consumer debt in the above optimization problem is introduced endogenously, allowing households to stop payment through two possible routes: become delinquent and/or file an application for bankruptcy, in which case the loan moves

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[47] A simple first-order autoregressive model estimated for the risk factor indeed shows a very high degree of persistence over time (0.98).

into the non-performing state until the court decides on the application. By adopting this framework of analysis, an augmented Euler equation for consumption including a risk factor is derived analytically, which reflects the percentage of consumer net borrowing expected to become non-performing. This equation was estimated with macroeconomic data and nonlinear GMM yielding consistent estimates of the household's preference parameters. Estimation biases which are likely to be due to measurement errors and unobserved heterogeneity were relatively small and were accounted for by specifying a parametric process for the errors of the Euler equation. Comparing the literature with current estimates non-negligible differences are found, especially with papers using calibration. Based on these results, it is concluded that the general specification of the Euler equation advanced in this chapter, without losing empirical tractability, has improved the model's structure relative to the standard representative agent model commonly used in policy assessments in the context of DSGE models.

The current proposed framework for examining debt non-payment as a key feature of consumer behavior goes a long way towards addressing the following problem also pointed out in Goodhart and Tsomocos (2009) for DSGE models in general: standard representative agent models of consumption or DSGE models do not include the possibility of debt non-payment and as such they are not properly micro-founded in that their assumptions are at odds with human behavior in this respect; they are also of little use for analyzing financial crises. Thus non-payment of debt should be explicitly incorporated into the micro-foundations of these models so that they can offer an appropriate framework for monetary and macroprudential policy analysis.

4. The link between consumption and leisure under Cobb-Douglas preferences: Some new evidence

4.1. Introduction

An important task for economists is to study consumer preferences as revealed by his intra-temporal or inter-temporal choices and estimate a broad range of preference parameters that have an essential role in determining how the consumer behaves, i.e. how he decides about the level of consumption and leisure. An interesting aspect of this behavior is whether consumption and leisure choices are interdependent or not. The literature has generally paid little attention to this issue. In representative agent models, when preferences are assumed to be separable (either additive or multiplicative) or additive non-separable, interdependence is not a feature of the model. The only case in which consumption and leisure decisions are cross-dependent is when preferences are multiplicative non-separable (Cobb-Douglas preferences). The advantage of adopting this form of non-separable utility function is not so much that it is an important ingredient in explaining the co-movements in consumption and leisure but that it represents a better choice for the analysis of consumer behavior since it does not require, as other forms of the utility function do, any a priori constraint on the preference parameters.

Unfortunately, there have been very few empirical studies to date that have attempted to endogenize the link between consumption and leisure choices (Eichenbaum et al., 1988; Domeij and Flodén, 2006; Lopez-Salido and Rabanal, 2006). These studies, by solving the consumer maximization problem, obtained an aggregate labor supply equation and a consumption Euler equation. Eichenbaum et al. (1988) applied GMM estimation to the consumption equation, while they considered the labor supply equation as an exact relation among current wage, consumption, and leisure. They reported evidence against the over-identifying restrictions in the Euler equation and a non-sensible estimated value of the discount factor. Domeij and Flodén (2006) again estimated only the consumption equation by using synthetic micro-data or panel data. They did not test the validity of the instruments used and obtained a non-sensible value for the weight of consumption in the utility function. Their model was estimated by setting exogenously values for the inter-temporal elasticity of substitution and the discount factor. Finally, Lopez–Salido and Rabanal (2006) used Bayesian methods to estimate a DSGE model, but for the household sector of that model all parameters were fixed instead of being estimated.

The purpose of this chapter is to extend previous work, in particular that of Eichenbaum et al. (1988), in a number of ways. Unlike previous studies, we estimate, using aggregate quarterly data for the last twenty years, the simultaneous system of both the labor supply equation and the inter-temporal consumption equation and test the cross-equation restriction regarding the weight of consumption in the utility function. A number of specification tests are applied to establish the robustness of the results and the soundness of the specification and estimation procedures; they include an autocorrelation test for the residuals, the J-test for instrument exogeneity, the test for the normality of the residuals and finally a Wald-test for parameter stability. The empirical results presented in Section 2 indicate that all preference parameters are significantly estimated, have the correct sign and take plausible values. A notable result is that the estimated value of the weight of consumption in the utility function is much higher than both the value of this parameter estimated by Eichenbaum et al. (1988) and the values used in model calibrations by other researchers (e.g. Domeij and Flodén, 2006; Heathcote et al., 2008; Collard and Dellas, 2012).

The chapter is organized as follows. Section 2 develops the theoretical model of household decisions regarding consumption and leisure and presents the estimation methodology and empirical results, and Section 3 concludes.

4.2. Model and estimation results

In this section, the consumption-leisure framework is developed in which a representative consumer derives utility from consuming goods and leisure time. It is assumed that this agent is liquidity constrained and obtains loans to support consumption smoothing.

The consumer maximizes a lifetime utility function given by:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta)^j \, u\left(C_{t+j}, l_{t+j}\right) \tag{4.1}$$

where β is the discount factor, and u denotes utility which is related to real consumption (C_{t+j}) and leisure (l_{t+j}, expressed as the ratio of leisure time to total available time per period). The utility function is assumed to be twice differentiable with respect to consumption and leisure, the marginal utilities of which are positive and non-increasing.

Some problems of specification arise in the choice of the appropriate form of the utility function. Thus, the assumption of additive separable preferences between consumption and leisure appears quite restrictive (see e.g., Bennet and Farmer, 2000; Domeij and Flodén, 2006), while that of multiplicative separable and additive non-separable preferences implies the existence of non-trivial constraints on the preference parameters that are necessary to ensure positive non-increasing marginal utilities. For these reasons, it seems that the most appropriate form of the utility function without any a priori constraint is the Cobb-Douglas function, which incorporates multiplicative non-separable preferences as below:

$$U_t = \frac{\left(C_t^{\gamma}(l_t)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma} \tag{4.2}$$

where $1/\sigma$ is the intertemporal elasticitiy of the consumption-leisure composite good, and γ is the weight of consumption relative to leisure.

The consumer is also assumed to be subject to a sequence of budget constraints. The constraint for period *t* (in real terms) is:

$$C_t + (1 + i_{t-1})\frac{1}{P_t}L_{t-1} = w_t(1 - l_t) + \frac{1}{P_t}L_t \tag{4.3}$$

where i_t is the interest rate, P_t is the consumer price level, L_t is consumer loans and w_t is the real wage rate.

Next, the Lagrangian for the consumer maximization problem is set up:

$$\mathscr{L} = \sum_{j=0}^{\infty} \beta^j \left\{ \begin{array}{c} \dfrac{\left[(C_{t+j})^{\gamma}(l_{t+j})^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma} + \\ \lambda_{t+j}\left[\begin{array}{c} w_{t+j}(1 - l_{t+j}) + \dfrac{1}{P_{t+j}}L_{t+j} - \\ C_{t+j} - (1 + i_{t-1+j})\dfrac{1}{P_{t+j}}L_{t-1+j} \end{array}\right] \end{array} \right\} \tag{4.4}$$

where λ_{t+j} is the Lagrange multiplier.

By taking derivates with respect to consumption, leisure and loans, the following FOC are obtained:

$$\lambda_t = \gamma(C_t)^{\gamma(1-\sigma)-1}(l_t)^{(1-\gamma)(1-\sigma)} \tag{4.5}$$

$$\lambda_t = (1-\gamma)(C_t)^{\gamma(1-\sigma)}(l_t)^{(1-\gamma)(1-\sigma)-1}\ \frac{1}{w_t} \tag{4.6}$$

$$\lambda_t = \beta\lambda_{t+1}(1 + i_t)\frac{P_t}{P_{t+1}} \tag{4.7}$$

By combining eqs. (4.5) and (4.6), the static labor supply equation is derived, which corresponds to the optimal intra-temporal choice for consumption and leisure:

$$lnl_t = ln\left(\frac{1-\gamma}{\gamma}\right) + lnC_t - lnw_t \qquad (4.8)$$

Also, by combining eqs. (4.5) and (4.7), the following Euler equation is obtained which describes the optimal consumption-leisure inter-temporal choice of the representative household:

$$lnC_t = lnC_{t+1} + \frac{1}{\gamma(1-\sigma)-1}\begin{bmatrix} ln(1+i_t) - ln\frac{P_{t+1}}{P_t} + ln\beta + \\ (1-\gamma)(1-\sigma)ln\frac{l_{t+1}}{l_t} \end{bmatrix} \qquad (4.9)$$

The system of eqs. (4.8) and (4.9) suggests that consumption and leisure decisions are indeed interdependent. The reason for this originates from the fact that the labor supply plans of households have both an intra- and an inter-temporal dimension. These equations are estimated by using aggregate quarterly U.S. data for 1999Q1 - 2015Q4. The data are seasonally adjusted (except for the interest rate). Sources of the data are the Federal Reserve Economic Data (FRED) and the Organization for Economic Co-operation and Development (OECD) databases. The interest rate is the average of the commercial bank interest rate on credit card plans and the finance rate on personal loans. Inflation is defined in terms of the implicit price deflator of personal consumption expenditure. Consumption refers to non-durable goods and services consumption expenditure expressed in billions of chained 2009 US dollars. The wage variable measures average real weekly earnings before taxes and other deductions, of both private and public sector employees but not of self-employed persons.
In Table 4.1, summary statistics of the variables used in the empirical analysis are reported.

Table 4.1 Descriptive statistics

Variable	Mean	Std. dev.	Min.	Max.
Real consumption expenditure (bn of US$)	8,576.4	742.2	7,025.6	9,828.2
Interest rate (percent %)	12.41	1.13	10.7	15.05
Inflation rate (percent %)	1.89	0.97	-0.94	3.99
Real wage rate (US$)	4,370.7	55.13	4,212.0	4,485.0
Leisure time (ratio)	0.79	0.00	0.79	0.80

The parameters of the system of eqs. (4.8) and (4.9) are estimated consistently using single equation GMM subject to the theory restriction as regards the relative weight of consumption in the utility function. The instruments of choice for the two equations are shown in Table 4.2 below. Estimation biases that are likely to be due to measurement errors and unobserved heterogeneity across households, which usually afflict the estimated values, are accounted for by specifying a parametric process for the errors (cf. Arellano, 2002). Since the data do indicate the presence of autocorrelation, it is assumed that the errors follow a first-order autoregressive process with parameter ρ.

The estimation results under the cross-equation restriction that permits to identify the parameters of the simultaneous system of the two equations are reported in Table 4.2. All estimated coefficients have the anticipated sign, are statistically significant and take plausible values. The results of the SK- and Q-tests show that the hypothesis that the residuals are further autocorrelated can be rejected while that of residual normality can be marginally accepted. The J-test indicates that all instruments are exogenous. A Wald-test for the validity of the cross-equation restriction is further applied, the p-value of which is equal to 0.40. Thus the hypothesis that the parameter γ takes the same value across the two equations cannot be rejected.

Table 4.2 GMM estimation of the system's equations under a cross-equation restriction

Equation	*1/ σ*	*γ*	*β*	*ρ*	*J-test*	*SK-test*	*Q-test*
Eq. (4.8)	-	0.74 (58.14)	-	0.965 (67.71)	0.43	0.07	0.27
Eq. (4.9)	0.26 (2.07)	0.74 (58.14)	0.905 (177.78)	0.676 (10.50)	0.43	0.04	0.58

Notes:
Instruments for eq. (4.8): $l_{t-2}, l_{t-3}, l_{t-4}, \Delta l_{t-4}, c_{t-4}, w_{t-1}, \Delta w_{t-1}$
Instruments for eq. (4.9): $c_{t-2}, c_{t-3}, c_{t-4}, \Delta l_{t-4}, l_{t-4}, r_{t-1}, \Delta r_{t-1}$
Columns 2, 3, 4 and 5 of the Table present coefficient estimates with their t-values in parenthesis. Columns 6,7 and 8 show the p-value of the J-test for instrument exogeneity, the skewness and kurtosis test for normality of the residuals and the Box-Pierce test for higher order autocorrelation of the residuals, respectively. Finally, r_t refers to the real interest rate.

The value of the inter-temporal elasticity of the consumption-leisure composite good is estimated to be 0.26 which lies in the range 0.15 to 0.31 that Eichenbaum et al. (1988) obtained. Further, the discount factor is highly significant and its value is 0.905, which is lower compared to that of the majority of calibrated models, which set this parameter at values not smaller than 0.94 for liquidity constrained households.

The most notable finding in Table 4.2 is that the weight of consumption is estimated at 0.74. This value is more than four times the estimated values in Eichenbaum et al. (1988) which range from 0.12 to 0.18, while it is about twice as large as the values used in model calibrations (e.g. Domeij and Flodén, 2006; Heathcote et al., 2008; Collard and Dellas, 2012) which range from 0.33 to 0.39. The prior choice of the parameter values draws mainly on

Kydland and Prescott (1982), who have set this parameter equal to 1/3 on the grounds that “households’ allocation of time to nonmarket activities is about twice as large as the allocation to market activities”.

4.3. Conclusion

In this chapter, the links between consumption and leisure have been examined by solving the consumer utility maximization problem under multiplicative non-separable (Cobb-Douglas) preferences. The strategy involved estimating a static and an inter-temporal equation of consumption and leisure choice and testing the restriction inherent in these equations, which concerns the relative weight of consumption in the utility function. The empirical results provide strong support for the above non-separability of preferences and suggest that consumers derive about three fourths of their satisfaction from current consumption and only the remaining one fourth from their current leisure time. In this respect, the choice in many DSGE models to rely, among other parameters, on a "standard value" for the share of consumption in utility would seem unwarranted in view of the estimates presented in this chapter.

5. Consumer debt non-payment and the borrowing constraint: Implications for consumer behavior

5.1. Introduction

The empirical failure of the rational expectations - permanent income model of consumption in its simple form (see e.g., Campbell and Mankiw, 1989) has led researchers to test whether the presence of constraints in household behavior can account for this failure (see e.g., Jappelli et al., 1998). On the other hand, Goodhart and Tsomocos (2009) argued that the standard representative agent models of consumption, which are widely used in the literature, by not including the possibility of debt non-payment, are of little use for analyzing real data, in particular data from periods of financial crises.

Prompted by these criticisms and in light of the important microeconomic role of consumer credit through its link to household consumption fluctuations over the last two decades, in this chapter the focus is on the analysis of a borrowing constrained household, which only consumes non-durable goods and services[48] and obtains unsecured consumer credit from banks to support consumption smoothing, while at the same time it decides how much to repay. To eliminate the possibility of household insolvency, banks impose a constraint in terms of the debt payment capacity of the household.

Through this analysis, we aim to give answers to a number of interesting questions: is the household's consumption behavior affected by the non-payment of unsecured debt? If the answer is yes, is the impact of non-payment important to differentiate the household's preference parameters? Is the borrowing constraint a binding constraint? If so, how does it affect parameter values? And finally, if all the previous questions are meaningful, how may the type of household preferences in the utility function manifest as a different pattern of consumption behavior?

An extensive presentation of the quantitative literature on unsecured consumer debt and default is provided by Livshits (2015) who analyzes some important issues, such as the sources of the rise in personal bankruptcies, the importance of asymmetric information and the cyclical behavior of consumer debt. The most cited papers in this literature analyze calibrated models and do not provide analytical solutions. The models analyzed are the general equilibrium model of Chatterjee et al. (2007) with an endogenously determined risk-

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[48] Consumption expenditure on non-durable goods and services is about 90% of total US consumption between 1999 and 2015 – see Federal Reserve Economic Data (FRED) database.

free interest rate, and the partial equilibrium model of Livshits et al. (2007), where the interest rate is given. Along similar lines, Athreya et al. (2012) attempt to model the link of bankruptcy and delinquency to economic factors by integrating these forms of non-payment in a life-cycle model of consumption and savings.

In the literature surveyed by Livshits (2015) on unsecured consumer credit, there is no reference at all to borrowing constraints. Indeed, all theoretical and empirical work, which introduces borrowing constraints puts the emphasis on mortgage debt. Johnson and Li (2010) using mortgage and automobile loans micro-data claim that a household that was able to borrow in the past will not show the same borrowing capacity in the future, and this is consistent with consumption models that assume limited access to credit by setting leverage ratios which act as borrowing constraints. Greenwald (2016), investigating the macroeconomic implications of mortgage credit growth in a general equilibrium framework, finds that the inclusion of a "payment-to-income" constraint together with a "loan-to-value" constraint may generate substantial aggregate effects, while the relaxation of payment-to-income standards was shown to play a key role in the recent financial crisis. He also reports that a cap on the payment-to-income ratio but not on the loan-to-value ratio is quite an effective macroprudential policy tool. Further, a handful of studies analyze households' mortgage decisions in dynamic models with endogenously determined mortgage rates where these models are calibrated for different values of their consumer leverage ratios (see e.g., Nakajima, 2012; Campbell and Cocco, 2015). Finally, another strand of the literature, by assuming the loan-to-value ratio as a collateral constraint, considers DSGE models which through calibration present the implications of macroprudential or monetary policies for business cycles and welfare (see e.g., Iacoviello, 2005; Gelain et al., 2013; Mendicino and Punzi, 2014; Rubio and Carrasco-Gallego, 2014).

In this chapter, a partial equilibrium framework is developed in which the household's non-durable consumption decisions are analyzed. The focus is on two main features of consumer behavior: consumers do not fully repay their loan obligations in a given period and also are subject to a limit on loan payments in terms of income, i.e., the debt payment to income (DPTI) ratio. The household's intertemporal maximization problem is solved by assuming either multiplicative non-separable or additive separable preferences, and a two-equation model consisting of a consumption Euler equation and a static labor supply equation is obtained. The model's equations are estimated by using US macro data[49] and nonlinear GMM.

The analysis contributes to the literature on consumer behavior in a number of respects. First, the household's non-payment of unsecured debt is endogenized. Second, a consumer borrowing constraint in terms of income is introduced which includes an upper limit set by the financial regulator multiplied by a factor which is a function of consumer non-payment and conditions bank behavior. This constraint is shown to be binding. Its omission would lead to a non-reasonable - from an economic point of view - solution to the maximization problem[50]. Third, a consumption Euler equation is obtained which is augmented relative to the simple Euler equation in the literature by including an additional factor in terms of debt non-payment. Also, the specification of the static labor supply equation which is derived departs from the simple counterpart in the literature by incorporating a function of debt non-payment and DPTI. Fourth, the estimation results reveal that the household's discount factor, in conformity with a priori theoretical predictions, is lower than commonly used values of this parameter. Further, the assumption of multiplicative non-separable or additive separable consumer preferences does not differentiate its estimated value to a significant extent. As for the rest of the parameters, the estimated optimal credit limits implied by our model are seen to be consistent with observed bank credit limits, while the other parameters have quite different values to the commonly used calibration values in the literature. Fifth, our consumption Euler equation seems to fit real consumption data much better than the baseline model of the literature.

The remainder of the chapter is organized as follows. Section 2 sets up the theoretical framework. Section 3 solves the consumer maximization problem under multiplicative non-separable preferences, estimates the resulting model, and discusses the empirical results. Section 4 derives the theoretical model under additive separable preferences commonly used in the literature and presents the empirical results. Section 5 compares the performance of the new model specification to that of the model with full debt repayment and no borrowing constraint. Section 6 concludes. Theoretical derivations can be found in the Appendices.

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[49] The choice of US macro data is dictated by their availability, since it is impossible to combine consumption micro data with the respective loan market data (delinquency rate, non-performing loans rate, charge-off rate, borrowing interest rate).

[50] For instance, the Lagrange multiplier of the budget constraint in this case is equal to zero – see Appendices E to G below.

5.2. Theoretical framework

In this section, a partial equilibrium framework is set up which will help determine the optimal decisions for borrowing households as regards consumption and leisure. Our analysis is carried out in discrete time. The reason for assuming borrowing households is twofold. First, US households started in the 1980s to borrow more than in previous years in order to overcome the effects of the rising income inequality, as indicated by the increased Gini coefficient in the last 30 years (Brown, 2008). Second, there has been a sharp decrease of the personal savings rate after 1993 (Brown, 2008). The borrowing household derives utility from consuming both non-durable goods and services, and leisure. It obtains unsecured consumer credit to support consumption smoothing, i.e. it is liquidity constrained. The household debt is assumed not to be fully repaid in each period.

During the current period, the household buys consumer goods and services and works in the production process, earning income by supplying its labor. At the end of each period, the household obtains a one-period loan from banks[51], under certain restrictions on the amount granted. Namely, a credit constraint is applied to the amount of the loan in the sense that the ratio of the loan installment to income never exceeds an upper limit (the size of which depends on banks' policy, given a threshold which is set by the financial regulator, as will be discussed below). Finally, the household repays part of the previous period debt (including interest), while the non-serviced debt includes delinquent, non-performing or bankrupt debt[52]. The interest here is not in the individual stages of non-payment but rather in the sum of unpaid debt in a given time period.

The household is assumed to have a non-recursive life-time utility[53]:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta)^j \, u\left(C_{t+j}, l_{t+j}\right) \tag{5.1}$$

where U is total utility over time, β is the subjective discount factor of the borrowing household, and u denotes utility which is related to real consumption expenditure *(C)* and

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[51] Roszbach (2004) notes that in financial markets with perfect information any optimal multi-period financial contract can be obtained by a sequence of one-period loan agreements, while under asymmetric information things become more intricate.

[52] The unpaid debt is assumed to be classified in three non-overlapping categories: delinquent debt when it is overdue up to 90 days, non-performing debt when it is overdue for more than 90 days and up to the time of the court decision on the application for bankruptcy, and bankrupt debt when it is written off from banks' books, once the court decision has been taken.

[53] We prefer to use a non-recursive utility which incorporates preferences that are remarkably parsimonious, in the sense that behavior over time depends solely on the discount factor and the utility function.

leisure (l, expressed as the ratio of leisure time to total available time per period). The utility function is assumed to be twice differentiable with respect to consumption and leisure, the marginal utilities of which are positive and non-increasing.

It is assumed that the household is subject to a budget constraint (in real terms) for period t of the following form:

$$C_t + \mu_t(1 + i_{t-1})L_{t-1}\frac{1}{P_t} = w_t(1 - l_t) + L_t\frac{1}{P_t} \tag{5.2}$$

where C is real consumption expenditure on non-durables, μ is the percentage of consumer debt repaid in a given period, i is the interest rate on consumer loans, L is consumer loans, P is the consumer price level and w is the real wage rate. Equation (5.2) displays the household's inflows and outflows stemming from its income-generating and financing activities. An interesting aspect of this constraint is that the household is currently repaying only part of the previous period debt.

An important component of the household's maximization problem is the borrowing constraint. In Appendix G this constraint is shown to be binding. The constraint is:

$$(1 + i_{t-1})L_{t-1} \leq e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]}\,\overline{DPTI}\,W_t(1 - l_t) \tag{5.3}$$

where W is the nominal wage rate and $\overline{DPTI}$ is a macro-prudential policy tool, namely the upper limit of the debt payment-to-income ratio which is set by the financial regulator and measures the maximum level of the loan payment that can be made out of the household's income. The $\overline{DPTI}$ is adjusted by multiplying it with a factor, $e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]}$, which reflects banks' aversion toward risk in their credit policy. This factor is a negative function of debt non-payment[54] and, as it ranges from values less than or equal to one and higher than zero, it makes the $\overline{DPTI}$ more restrictive. The back story behind this function is that banks follow a more restrictive credit policy than suggested by $\overline{DPTI}$ to avoid taking excessive risk, which may cause either possible penalties by the financial regulator or even reputation costs.

To complete the theoretical framework, we need to determine the type of the household's preferences in the utility function. In the next section, the multiplicative non-separable utility function (Cobb-Douglas function) is used, which can be considered as the most appropriate form of utility, since it does not involve any a priori constraints on the

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[54] As noted by Maki (2000), borrowing limits seem to be associated with higher delinquencies.

parameters. Later, the assumption of additive separable preferences is employed for comparison purposes, which although quite restrictive[55] (see e.g., Bennet and Farmer, 2000; Domeij and Floden, 2006) is the most widely used in the literature. We avoid using multiplicative separable and additive non-separable preferences which imply some non-trivial constraints on the household's parameters to obtain positive non-increasing marginal utilities.

5.3. The maximization problem under multiplicative non-separable preferences

In this section, the household's maximization problem by assuming multiplicative non-separable preferences is solved, known as Cobb-Douglas preferences, which as mentioned above are the most appropriate type of preferences to use since they do not imply any a priori constraint on the parameters. These preferences are given by:

$$U_t = \frac{\left(C_t^{\gamma}(l_t)^{1-\gamma}\right)^{1-\eta} - 1}{1-\eta} \tag{5.4}$$

where $1/\eta$ is the inter-temporal elasticity of a consumption-leisure composite good and γ is the weight of consumption relative to leisure.

By combining eqs. (5.1) to (5.4), the Lagrangian of the household's maximization problem is set up:

$$\mathscr{L} = \sum_{j=0}^{\infty} \beta_1^j \left\{ \begin{array}{c} \dfrac{\left[\left(C_{t+j}\right)^{\gamma}\left(l_{t+j}\right)^{1-\gamma}\right]^{1-\eta} - 1}{1-\eta} + \\ \kappa_{t+j} \left[\begin{array}{c} w_{t+j}\left(1 - l_{t+j}\right) + \dfrac{1}{P_{t+j}} L_{t+j} - C_{t+j} \\ -\mu_{t+j}\left(1 + i_{t-1+j}\right) \dfrac{1}{P_{t+j}} L_{t-1+j} \end{array} \right] + \\ \kappa'_{t+j} \left[e^{\left[\frac{1-\mu_{t+j}}{(1-\mu_{t+j})-1}\right]} \overline{DPTI} W_{t+j}(1 - l_{t+j}) - \left(1 + i_{t-1+j}\right) L_{t-1+j} \right] \end{array} \right\} \tag{5.5}$$

where β_1 is the subjective discount factor of the household, and κ_{t+j} and κ'_{t+j} are the Lagrange multipliers of the budget constraint and the borrowing constraint, respectively.

By taking FOCs for consumption, leisure, loans and the percentage of debt repaid (see Appendix E) and by assuming rational expectations, an augmented form of the consumption Euler equation (eq. E7) is obtained, in which logs to both sides are applied:

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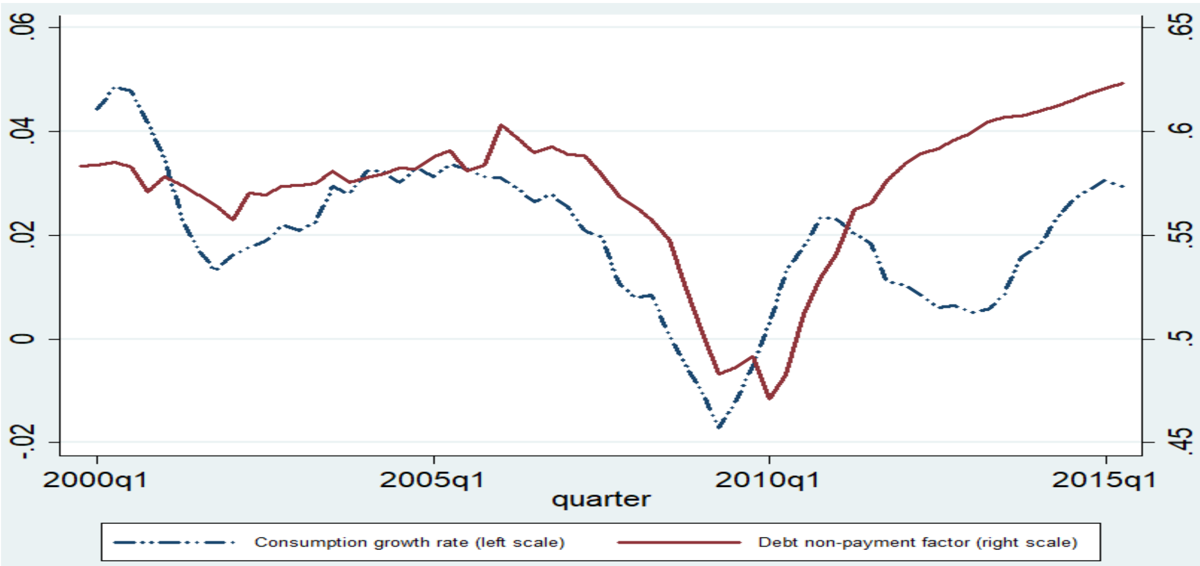
[55] This type of preferences is the most restrictive one since with these preferences it can be proved that the marginal utility of consumption does not depend on leisure, which is clearly a non-reasonable assumption.

$$c_t = c_{t+1} + \frac{1}{\gamma(1-\eta)-1}\left\{\begin{matrix} ln(1+i_t) - ln\frac{P_{t+1}}{P_t} + ln\beta_1 + (1-\gamma)(1-\eta)ln\frac{l_{t+1}}{l_t} + \\ ln\{1-(1-\mu_{t+1}) + [1-(1-\mu_{t+1})]^2\} \end{matrix}\right\} \quad (5.6)$$

where $c_t = lnC_t$

From this equation, it is obvious that the last term in the brackets is a new element relative to the simple consumption Euler equation of the literature. As can be seen, this term is a negative function of consumer debt non-payment. To get a better insight into this matter, the relationship of the above term to consumption growth is also presented. Figure 5.1 below shows that the growth rate of consumption correlates well with the new term (the simple correlation coefficient is 0.56). This overall correlation seems to have weakened after mid-2011, when consumers reached more normal levels of consumption growth and debt non-payment in the post-financial crisis era but continued to be more cautious some time thereafter.

Figure 5.1 Consumption growth rate and consumer debt non-payment factor



Source: FRED database and our own calculations

From the household maximization problem, an extended form of the standard static labor supply equation (see Appendix E) is also obtained. Thus, by applying logs to both sides of eq. (E8), the following equation is obtained:

$$lnl_t = ln\left(\frac{1-\gamma}{\gamma}\right) + c_t - lnw_t - ln\left[1 + (\mu_t)^2 e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]}\overline{DPTI}\right] \quad (5.7)$$

Eq. (5.7) includes an extra term relative to the standard equation, which is a negative function of consumer debt non-payment. This equation shows that the elasticity of leisure with respect to the term in brackets equals -1. Thus, there is a positive relationship between debt non-payment and leisure. Indeed, to repay more (a decrease in debt non-payment), the household should reduce current consumption. However, to be able to keep consumption high, it should increase hours worked to earn more income, and therefore have its leisure time reduced.

To estimate the parameters of the two equation model derived from the maximization problem aggregate quarterly US data are used, covering the period 1999Q1 to 2015Q3.[56] These data are seasonally adjusted (except for the interest rate). Sources of the data are the Federal Reserve Economic Data (FRED) and the Federal Reserve Bank of New York (FRBNY) databases. Consumption (c_t) is the log of personal consumption expenditure on non-durable goods and services in billions of chained 2009 US$. The interest rate (i_t) is the average of the commercial bank interest rate on credit card plans and the finance rate on personal loans. The price level (P_t) is the implicit price deflator of personal consumption expenditure. The percentage of debt repaid (μ_t) is the difference between one and the algebraic sum of the delinquency, nonperforming loans and charge-off rates, which are the end-of-quarter ratios of delinquent, nonperforming and bankrupt loans respectively to total consumer loans. The real wage rate (w_t) represents the average real weekly earnings before taxes and other deductions of both private and public sector employees but not of self-employed persons. Finally, the leisure variable (l_t) is the ratio of leisure time to total available time per quarter.

In Table 5.1, summary statistics of the variables used in the empirical analysis are reported.

Table 5.1 Descriptive statistics

Variable	Mean	Std. dev.	Min.	Max.
Real personal consumption expenditure (bn of US$)	2,139.4	182.9	1,756.4	2,444.6
Interest rate (percent %)	12.44	1.12	10.70	15.05
Price level (index, 2009=100)	95.93	9.28	80.45	109.7
Percentage of debt repaid (percent %)	92.26	2.14	86.07	95.42
Real wage rate (US$)	4,369.2	54.11	4,212.0	4,485.0
Leisure time (percent %)	79.31	0.33	78.67	80.39

Before turning to estimation, a brief discussion of the most important sources of the estimation bias which have been discussed in the literature (e.g., Angrist and Krueger, 2001;

[56] The Quarterly Trends for Consolidated U.S. Banking Organizations produced by the Federal Reserve Bank of New York discontinued reporting the NPL rate for consumer loans after the 3rd quarter of 2015.

Vissing and Jorgensen, 2002; Arellano, 2002) is in order. First, by using non-linear estimation in this paper, we avoid the measurement error resulting from the log-linear approximation of the real interest rate (Ludvigson and Paxson, 2001). Second, the unobserved heterogeneity problem is mitigated by using a borrowing interest rate for consumer loans, since borrowers come basically from the younger working-age group (18-44), which is the largest group of the population of labor force age. Third, the error generated through the rational expectations hypothesis still exists. However, the impact of the last two sources of estimation bias can be largely neutralized by specifying parametric processes for the system's equation errors (Arellano, 2002).

As regards estimation, nonlinear GMM[57] is relied on to obtain consistent estimates of the parameters, since the household's optimization problem includes both endogenous variables and some nonlinear relations between them. Our aim is to apply nonlinear GMM to both equations (5.6) and (5.7) under a cross-equation restriction involving the parameter γ, which is present in both equations. The Q-test applied, indicated that the model strongly suffers from serial correlation. Thus, it is assumed that the errors of eq. (5.6) follow a first-order autoregressive process with parameter ρ_1 and those of eq. (5.7) a second-order autoregressive process with parameters ρ_2 and ρ_3. Table 5.2 presents both the estimation results and the instruments used.

Table 5.2 Nonlinear GMM estimation under Cobb-Douglas preferences

<table>
<tr><th>Eq.</th><th>$1/\eta$</th><th>β_1</th><th>γ</th><th>$\overline{DPTI}$</th><th>ρ_1</th><th>ρ_2</th><th>ρ_3</th><th>J-test</th><th>SK-test</th><th>Q-test</th></tr>
<tr><td>(5.6)</td><td>0.223
(2.23)</td><td>0.505
(37.01)</td><td rowspan="2">0.678
(27.74)</td><td>-</td><td>0.95
(13.19)</td><td>-</td><td>-</td><td rowspan="2">0.50</td><td>0.82</td><td>0.21</td></tr>
<tr><td>(5.7)</td><td>-</td><td>-</td><td>0.427
(3.16)</td><td>-</td><td>0.38
(2.81)</td><td>0.58
(4.20)</td><td>0.74</td><td>0.80</td></tr>
</table>

Notes:
Instruments for eq. (5.6) : $c_{t-4}, r_{t-4}, \frac{l_{t-4}}{l_{t-5}}, \Delta\frac{l_{t-4}}{l_{t-5}}, \mu_{t-2}, lnw_{t-2}, \Delta_{t-4}lnw_{t-3}$
Instruments for eq. (5.7) : $c_{t-4}, r_{t-4}, \Delta r_{t-1}, \mu_{t-4}, \Delta\mu_{t-1}, lnw_{t-4}, \Delta lnw_{t-1}$
r_t refers to the real interest rate.
Columns 2 to 8 of the Table present coefficient estimates with their t-values in parenthesis. Columns 9, 10 and 11 show the p-value of the J-test for instrument exogeneity, the skewness and kurtosis test for normality of the residuals and the Box-Pierce Q-test for higher order autocorrelation of the residuals, respectively.

[57] It is known that nonlinear GMM yields consistent estimates of the parameters just as linear GMM does (see Davidson and MacKinnon, 2004).

Table II shows that our model does not suffer from serial correlation and is characterized by exogenous instruments and residual normality. All estimated coefficients have the expected sign, are statistically significant and obtain reasonable values.

The discount factor is estimated to be 0.505, a value which is significantly lower than that used or estimated[58] in the literature. The majority of papers that use calibration set values not smaller than 0.94 (Livshits et al. 2010; Mendicino and Punzi, 2014; Rubio and Carrasco-Gallago, 2014). Exceptions to this are very few papers that set values lower than 0.9, e.g., Aguiar and Gopinath (2006) and Chatterjee et al. (2007) who set the value between 0.8 and 0.9, Laibson et al. (2007) and Yue (2010) who choose a value around 0.7, Laibson (1996) who reports a value of 0.60, Nakajima (2012) who develops a macroeconomic model with temptation preferences setting values at 0.70 and 0.56 and, finally, Shapiro (2005) who argues that by deriving a consumption function for the case of log utility he can obtain an annual discount factor of about 0.23.

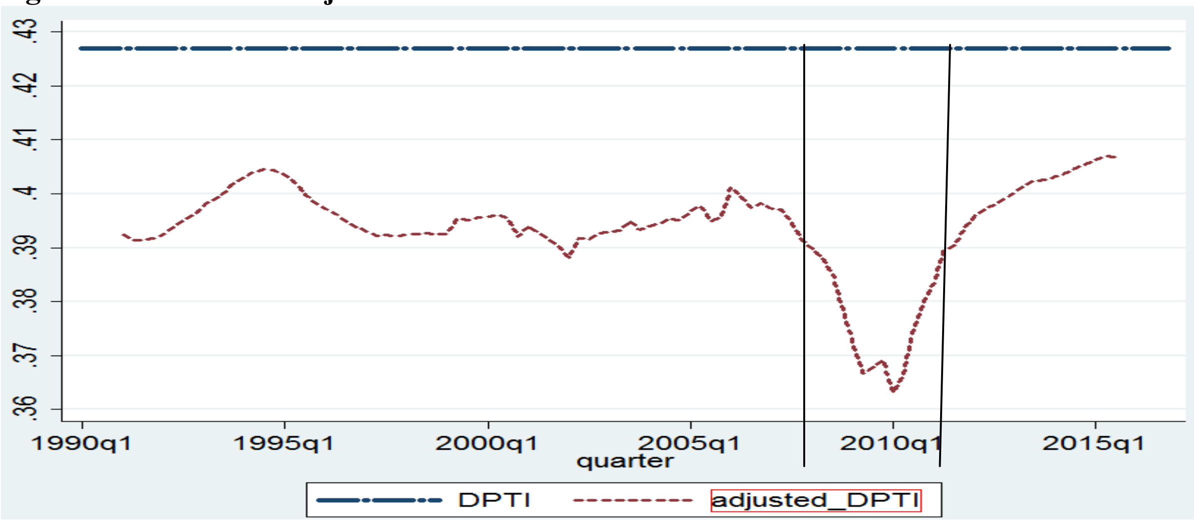
The inter-temporal elasticity of the consumption-leisure composite good is estimated to be 0.223, a value close to that of Heathcote et al. (2008) who set the value equal to 0.25, and of Wu (2005) who sets a value lower than 0.20. Our estimated value lies in the interval 0.15 to 0.31 that Eichenbaum et al. (1988) obtained. Note that Domeij and Floden (2005), Lopez-Salido and Rabanal (2006) and Collard and Dellas (2012) set a higher value, between 0.30 and 0.50.

The weight of consumption relative to leisure is estimated to be 0.678, a value which is close to Wu (2005) who sets the value equal to 0.6. However, the rest of the literature calibrates the parameter to 0.33 and 0.39 (Domeij and Floden, 2005; Heathcote et al., 2008; Collard and Dellas, 2012; Nakajima, 2012).

Finally, the most notable finding of Table 5.2 is the value of the $\overline{DPTI}$ parameter which is estimated to be 0.427. Figure 5.2 shows the estimated borrowing limit $\overline{DPTI}$ and the adjusted $\overline{DPTI}$.

[58] There is no estimate of this parameter below 0.9.

Figure 5.2 $\overline{DPTI}$ and adjusted $\overline{DPTI}$



Source: Our estimates and own calculations

As can be seen from the figure, $\overline{DPTI}$ is the upper threshold which applies when non-payment of debt is zero. The adjusted $\overline{DPTI}$ varies between 0.39 and 0.40 before the crisis, with a mean of around 0.395, i.e. 3 percentage points less than the threshold ($\overline{DPTI}$). Higher values than this mean are reported after the first quarter of 2015, when the adjusted $\overline{DPTI}$ increased to 0.41. The interesting thing about this variable is that the adjusted $\overline{DPTI}$ displays the lowest values in the period from 2008Q1 to 2011Q2 (i.e. a period which goes beyond the recent financial crisis) having tumbled more than 3 percentage points from the pre-crisis mean. It should be noted that the adjusted $\overline{DPTI}$ follows a similar pattern to the debt non-payment factor (see Figure 5.1), since both are negative nonlinear functions of debt non-payment.

The existing literature is concerned with DPTI ratios on mortgage loans. In particular, Campbell and Cocco (2015) report that for households who defaulted, mortgage payments were equal to 40% of the period income. Johnson and Li (2010), by measuring debt service ratios which include principal and interest payments on all automobile loans and all mortgage debt, including mortgages on primary residences, mortgages on other real estate and all forms of home equity debt, present a median ratio equal to 38%. Quercia et al. (2003) state that household payments to gross monthly income should not exceed 36%. In a survey of the euro area countries, Bankowska et al. (2015) provide information about household indebtedness, focusing mainly on the debt service-to-income (DSI) ratio, as an indicator of household vulnerability to indebtedness. They point out that a high DSI ratio indicates a risk of bankruptcy for the household, mainly when this ratio is higher than 40%. The Consumer Financial Protection Bureau (2017) stresses that evidence from studies of mortgage loans suggests that borrowers with a debt payment-to-income ratio higher than 43% are more likely to run into trouble, making monthly payments. Finally, the Federal National Mortgage Association (2017), known as Fannie Mae, reports on the basis of specific criteria DPTI ratios from 36% to 50% for several categories of borrowers, for example borrowers who do not have a credit score[59].

However, the vast majority of empirical studies on mortgage debt use a borrowing limit similar to DPTI, namely the Debt to Income ratio (DTI). [60] Since this borrowing limit refers to a multi-period loan, which is quite difficult to analyze in the context of an imperfect information model (see also footnote 51), the BIS formula mentioned in Table 5.3 below is applied to obtain DTI limits on consumer loans, under the assumptions shown in the notes section of Table 5.3.[61] The objective is to combine the estimated values of $\overline{DPTI}$ and adjusted $\overline{DPTI}$ with the calculated values of $\overline{DTI}$ so as to develop a more informed policy framework for banks. The first two limit the risk of debt non-payment while the latter also limits the loan level, for a given loan maturity.

Table 5.3 $\boldsymbol{DTI}$ threshold (in terms of yearly income)

Maturity (years)	$\overline{DTI}$	Adjusted $\overline{DTI}$ (Min)	Adjusted $\overline{DTI}$ (Mean)	Adjusted $\overline{DTI}$ (Max)
1	0.427	0.363	0.394	0.407
2	0.805	0.685	0.743	0.767
3	1.140	0.969	1.052	1.087
4	1.437	1.222	1.326	1.370
5	1.700	1.445	1.568	1.620
6	1.933	1.643	1.783	1.842

Notes:

1. To obtain the results of Table III, the BIS formula for the household sector is applied (see http://www.bis.org/statistics/dsr/dsr_doc.pdf). The borrowing rate and the values of $\overline{DPTI}$ and adjusted $\overline{DPTI}$ are taken from this chapter.
2. Loan maturities up to 6 years[62] are assumed and the mean of the borrowing interest rate over the sample period (0.124) is used, the estimated $\overline{DPTI}$ (0.427) and the minimum, mean and maximum of the adjusted $\overline{DPTI}$ (0.394, 0.363 and 0.407, respectively).
3. The second column reports the $\overline{DTI}$ for the given value of the estimated $\overline{DPTI}$. The following three columns present the values of $\overline{DTI}$ for the minimum, mean and maximum of the adjusted $\overline{DPTI}$, respectively.

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[59] Recently, in a May 30, 2017 notice, Fannie Mae changed the old DPTI limit of 45% to 50% in their automated underwriting system.

[60] The results of the existing literature on mortgage debt will not be discussed here, since the DTI ratios on mortgage debt and unsecured consumer debt have three major differences: the loan maturity, the borrowing interest rate and the average amount of the loan. Indeed, Maki (2000) noted that in spite of the fact that consumer debt is only about one third of mortgage debt, the required debt service payments on mortgage debt obligations are actually higher than those on consumer debt obligations because of the shorter maturity of consumer debt.

[61] The BIS formula links the DSR (DPTI ratio) to the DTI ratio, the borrowing interest rate and the average loan maturity.

[62] Since the mid-1980s, the average maturity of new car loans at finance companies is between 5 and 6 years (see FRED).

From Table 5.3 and by taking into account the average real wage rate, we can determine the average upper threshold of the consumer loan amount. For instance, by using a $\overline{DTI}$ equal to 1.933 (the highest value of Table 5.3) and the mean of the real annual wage rate over the sample period (approximately \$17,000), we obtain a maximum loan amount of \$33,000 that a household can obtain from banks, given the $\overline{DPTI}$ limit, the maturity of the loan and the borrowing interest rate.

5.4. The maximization problem under additive separable preferences

In this section, the household's maximization problem is presented by assuming additive separable preferences. This type of preferences has been noted as the most frequently used in the literature (see e.g., Iacoviello, 2005; Domeij and Floden, 2006; Gali, 2015). Contrary to the previous analysis, here the main assumption of separable impact of consumption and leisure on the utility implies the use of preference parameters that capture the separate intertemporal choices of both variables and not the combined ones. These preferences are given by:

$$u_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{l_t^{1-\varphi} - 1}{1-\varphi} \tag{5.8}$$

where $1/\sigma$ and $1/\varphi$ are the intertemporal elasticities of substitution for consumption and leisure, respectively.

By combining eqs. (5.1) to (5.3) and eq. (5.8), the Lagrangian of the household's maximization problem is established:

$$\mathscr{L} = \sum_{j=0}^{\infty} \beta_2^j \left\{ \begin{array}{c} \dfrac{C_t^{1-\sigma} - 1}{1-\sigma} + \dfrac{l_t^{1-\varphi} - 1}{1-\varphi} + \\ \lambda_{t+j} \begin{bmatrix} w_{t+j}\left(1 - l_{t+j}\right) + \dfrac{1}{P_{t+j}} L_{t+j} - C_{t+j} \\ -\mu_{t+j}\left(1 + i_{t-1+j}\right) \dfrac{1}{P_{t+j}} L_{t-1+j} \end{bmatrix} + \\ \lambda'_{t+j} \left[e^{\left[\frac{1-\mu_{t+j}}{(1-\mu_{t+j})-1}\right]} \overline{DPTI}\, W_{t+j}(1 - l_{t+j}) - \left(1 + i_{t-1+j}\right) L_{t-1+j} \right] \end{array} \right\} \tag{5.9}$$

where β_2 is the discount factor of the household under additive separable preferences and λ_{t+j} and λ'_{t+j} are the Lagrange multipliers of the budget constraint and the borrowing constraint, respectively.

By using the rational expectations hypothesis and taking FOCs for consumption, leisure, loans and the percentage of debt repaid (see Appendix F), we obtain augmented forms of the consumption Euler equation (eq. F7) and of the static labor supply equation (eq. F8), to which we apply logs in both sides:

$$c_t = c_{t+1} - \frac{1}{\sigma}\begin{bmatrix} ln(1+i_t) - ln\frac{P_{t+1}}{P_t} + ln\beta_2 + \\ ln\{1-(1-\mu_{t+1}) + [1-(1-\mu_{t+1})]^2\} \end{bmatrix} \tag{5.10}$$

and

$$lnl_t = \frac{\sigma}{\varphi}c_t - \frac{1}{\varphi}lnw_t - \frac{1}{\varphi}ln\left[1 + [1-(1-\mu_t)]^2 e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]}\overline{DPTI}\right] \tag{5.11}$$

As analyzed in the previous section, nonlinear GMM estimation of equations (5.10) and (5.11) is used under a cross equation restriction on the parameter σ, where it is assumed that the errors of eq. (5.10) follow a first-order autoregressive process with parameter ρ_1 and those of eq. (5.11) a third order autoregressive process with parameters ρ_2 , ρ_3 and ρ_4. The following Table reports the results.

Table 5.4 Nonlinear GMM estimation under additive separable preferences

Eq.	*1/ φ*	β_2	*1/ σ*	$\overline{DPTI}$	ρ_1	ρ_2	ρ_3	ρ_4	*J-test*	*SK-test*	*Q-test*
(5.10)	-	0.513 (69.83)	0.249 (2.24)	-	0.84 (13.06)	-	-	-	0.33	0.62	0.79
(5.11)	0.076 (2.11)	-		0.323 (0.60)	-	0.42 (4.07)	0.15 (1.31)	0.43 (4.44)		0.11	0.17

Notes:
Instruments for eq. (5.10) : c_{t-6} , l_{t-4} , r_{t-6} , $\Delta_{t-2}lnw_{t-4}$, μ_{t-4}
Instruments for eq. (5.11) : c_{t-1} , l_{t-1} , l_{t-2} , l_{t-3} , r_{t-4} , lnw_{t-2} , μ_{t-4} , $\Delta\mu_{t-4}$
r_t refers to the real interest rate.
Columns 2 to 9 of the Table present coefficient estimates with their t-values in parenthesis. Columns 10, 11 and 12 show the p-value of the J-test for instrument exogeneity, the SK-test for normality of the residuals and of the Box-Pierce Q-test for higher order autocorrelation of the residuals, respectively.

Table 5.4 shows that this model also passes at the 0.05 confidence level the three tests for instrument exogeneity, and normality and autocorrelation of the residuals. Contrary to the estimation under multiplicative non-separable preferences, the estimated values of the parameters of the maximization problem with additive separable preferences are not all statistically significant. In particular, the parameter of our interest, i.e. of the $\overline{DPTI}$, is only significant at the 0.55 level.

Before reviewing the literature regarding the size of the preference parameters under additive separable preferences, we need to stress that, as can be seen from Tables 5.4 and 5.2, we cannot fully compare the empirical results across the two different types of preferences. Indeed, under multiplicative non-separable preferences, where consumption and leisure choices are interdependent, the parameters to be estimated are the weight of consumption relative to leisure and the EIS for the consumption-leisure composite good. Without this interdependence, as is the case with additive separable preferences, we need to estimate separate EIS for consumption and leisure. Thus, the only household parameter value that we could directly compare is the discount factor, which is close to 0.51 for both cases.

The EIS for leisure ($1/\varphi$) is estimated to be 0.076. The existing literature sets this parameter between zero and one in calibration. Ludvigson (1996) sets values between zero and one. Pancaro (2010) sets the value equal to 0.13, such that in steady state the time allocated to leisure is equal to 80% of the total time endowment. Pijoan-Mas (2006) calibrates the value of the EIS of leisure to 0.35 so that in equilibrium the incomplete markets economy matches some statistics from data. Yum (2016) develops a dynamic general equilibrium model in which the value of EIS for leisure is calibrated to be 0.40. Heylen and Van de Kerckhove (2010) report that micro studies often reveal very low elasticities and, given their macro focus, they restrict the value of the EIS for leisure to be equal to 0.5.

The EIS for consumption ($1/\sigma$), which is crucial for evaluating standard models of consumption behavior, shows how the marginal rate of substitution between today and tomorrow's consumption reacts to changes in the interest rate, keeping lifetime utility constant (see Attanasio and Weber, 2010). Our estimated value equals 0.249. The literature values range between zero and one. In particular, Hall (1988) and Campbell and Mankiw (1989) report that this elasticity is unlikely to be well above 0.1. Attanasio et al. (2002) obtain elasticity values from UK data between 0.1 and 0.2 for non-stockholders. Barsky et al. (1997) using survey data for the U.S. provide evidence of an average EIS of 0.18. Davies (1981) reviewing the relevant literature suggests that a reasonable estimate for the EIS would be 0.25. Finally, a number of papers on DSGE modeling avoid estimating Euler equation parameters and set the EIS equal to one (e.g. Schorfheide, 2000; Iacoviello, 2005; De Walque et al., 2010) or even calibrate the value to 0.5 or more (Aguiar and Gopinath, 2006; Yue, 2010; Nakajima, 2012; Campbell and Cocco, 2015). To sum up, there seems to be inconsistency in the empirical literature, which estimates the EIS at around 0.1 - 0.3, while the majority of the calibrated models use values two or more times higher than the upper value of this range. This observation is in a similar spirit to Guvenen (2006) who reports that empirical studies using aggregate consumption data find the EIS to be close to zero, whereas calibrated models designed to match growth and aggregate fluctuations data typically require it to be close to one.[63]

5.5. Performance of the consumption Euler equation

In the previous sections, the theoretical and empirical analysis of the household sector model was completed by assuming either multiplicative non-separable or additive separable preferences and non-payment of unsecured debt for borrowing constrained households. In this section, a comparative analysis is presented of our specification and the baseline specification of the consumption Euler equation as regards the goodness of fit to the consumption data. An analytical derivation of the Euler equation under the assumption of full debt payment and no borrowing constraint is presented in Appendix H.

In Table 5.5, the estimated values of the parameters for all versions of the two equation system derived from the solution of the household optimization problem are reported.[64]

Table 5.5 A summary table of nonlinear GMM estimates

Eqs.	*Preferences*	*Full debt payment*	*Discount factor*	*Weight on consumption*	*EIS for consumption*	*EIS for leisure*	*EIS for composite good*	$\overline{DPTI}$
5.6-5.7	Multiplicative non-separable	No	0.505 (37.01)	0.678 (27.74)	-	-	0.223 (2.23)	0.427 (3.16)
E1-E2	Multiplicative non-separable	Yes	0.905 (177.78)	0.738 (58.14)	-	-	0.262 (2.07)	-
5.10-5.11	Additive separable	No	0.513 (69.83)	-	0.249 (2.24)	0.076 (2.11)	-	0.323 (0.60)
E3-E4	Additive separable	Yes	0.907 (139.62)	-	0.286 (2.12)	0.06 (2.02)	-	-

Notes: Columns 4 to 9 present coefficient estimates with their t-values in parenthesis.

[63] Guvenen provides an explanation for this apparent inconsistency, namely that there is a positive relationship between the individual's EIS and his wealth level and, because of the substantial wealth inequality in the U.S., the preferences of the wealthy are not revealed in consumption equations.

[64] The Table includes, in addition to the parameters of the consumption Euler equation, those of the labor supply equation. The reason is that both sets of parameters were estimated under a cross-equation restriction.

The parameter values of Table 5.5 are used as an input to calculate the fitted values of the consumption growth rate which we then compare to the actual values of this variable. The measure of closeness of the two series that we use is the Pearson correlation coefficient. This measure is calculated for alternative models which include the standard or augmented specifications of the Euler equation, under multiplicative non-separable or additive separable preferences and those including or excluding the parametric process for the errors. The results are presented in Table 5.6 below.

Table 5.6 Pearson correlation coefficient

Eq.	E1	E3	5.6	5.10	E1	E3	5.6	5.10
	With AR(1) error process				Without AR(1) error process			
	0.651 0.00 66	0.694 0.00 67	0.684 0.00 65	0.724 0.00 65	0.257 0.04 67	0.245 0.04 68	0.483 0.00 66	0.478 0.00 66

Notes: The first line of each cell shows the value of the Pearson correlation coefficient. The second line reports the p-value. The third line is the number of observations.

From this Table, a number of conclusions stand out. Considering the equations without the parametric process for the errors, we observe that our specification clearly outperforms the standard specification of the Euler equation, which is known in the literature to provide a remarkably poor fit to the data on consumption. Indeed, the Pearson correlation coefficient of our model is almost double its value in the standard model. This finding is consistent with the high correlation found between the growth rate of consumption and the debt non-payment factor, which is part of our theoretical specification (see Fig. 5.1). The difference between the Pearson correlation coefficients in the two models is substantially reduced if we compare the models inclusive of the autoregressive process for the equation errors. Nevertheless, our specification still performs better since its Pearson correlation coefficient is higher by about 4 percentage points than its counterpart from the standard model. Why is the performance gap with the standard Euler equation closed so much in the autocorrelation-corrected estimates of the model? The answer may lie in the following: the autoregressive process for the error appears to absorb the omitted variables bias that exists in the standard specification along with other possible sources of bias such as those mentioned in Section 5.3. A final remark is that the above conclusions hold true whether we use multiplicative non-separable or additive separable preferences, an assumption that is seen not to generate substantial differences in the size of estimated parameters.

5.6. Conclusions

This chapter investigated, in the context of a partial equilibrium model for the household sector, the implications of unsecured debt non-payment when the household is subject to a binding borrowing constraint. The model's equations were augmented forms of the standard consumption Euler equation and static labor supply equation. The equations were derived by solving analytically the household's intertemporal maximization problem on the assumption of either multiplicative non-separable or additive separable preferences. These equations were estimated by nonlinear GMM and the estimation results for the Euler equation were compared with those for the equation which assumes full debt payment and no borrowing constraint. Our equation was shown to clearly outperform the simple model.

There were four main findings in the paper. The estimated value of the subjective discount factor is almost half the value of the parameter that is either estimated or calibrated in the empirical literature. This shows that credit constrained households tend to be more patient against future consumption needs under debt non-payment. Further, an important parameter of interest was estimated in our empirical analysis, namely the DPTI on consumer loans. This parameter has not been estimated before and its value is close to values reported in the literature on mortgage finance. In addition, the consumption Euler equation obtained here had a much better goodness of fit to the real data of the growth rate of non-durables consumption than the simple form considered in the literature. Last, it is observed that the choice between multiplicative non-separable and additive separable household preferences makes no big difference as far as goodness of fit to real data is concerned but is important concerning the statistical significance of the parameters.

Future extension of this work may be the analysis of mortgage debt together with unsecured consumer debt considered in this paper. Incorporating both debt markets in a representative agent's framework may enhance the appropriateness of the framework for monetary and macroprudential policy analysis.

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Appendices

Appendix A

We take first-order conditions in equation (2.7) as follows:

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{C_t^b}} = (C_t^b)^{-\sigma} - \lambda_t P_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(\boldsymbol{C_t^b})^{-\boldsymbol{\sigma}}}{\boldsymbol{P_t}} \quad (A1)$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{N_t^b}} = -(N_t^b)^{\varphi} + \lambda_t W_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(\boldsymbol{N_t^b})^{\boldsymbol{\varphi}}}{\boldsymbol{W_t}} \quad (A2)$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\mu_t}} = -\lambda_t(1-k_t)(1+i_{t-1}^L)L_{t-1} - \lambda_t f_{t-1} k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2} + \beta^b \lambda_{t+1}$$

$$\mu_{t+1} f_t k_t \,(1+i_{t-1}^L)L_{t-1} = 0 \quad (A3)$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{L_t}} = \lambda_t - \beta^b \lambda_{t+1}[\mu_{t+1} + k_{t+1}(1-\mu_{t+1})](1+i_t^L) - (\beta^b)^2 \lambda_{t+2} \mu_{t+2} f_{t+1} k_{t+1}$$

$$(1-\mu_{t+1})(1+i_t^L) = 0 \quad (A4)$$

By re-arranging equation (A3), writing it one period forward and substituting in equation (A4), we finally get:

$$\lambda_t = \beta^b \lambda_{t+1}(1+i_t^L)\left[1 + \frac{(1-E_t\mu_{t+1}) f_t k_t (1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \quad (A5)$$

Appendix B

We take first-order conditions in equation (2.17) as follows:

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{C_t^s}} = (C_t^s)^{-\sigma} - \lambda_t P_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(\boldsymbol{C_t^s})^{-\boldsymbol{\sigma}}}{\boldsymbol{P_t}} \quad (B1)$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{N_t^s}} = -(N_t^s)^{\varphi} + \lambda_t W_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(\boldsymbol{N_t^s})^{\boldsymbol{\varphi}}}{\boldsymbol{W_t}} \quad (B2)$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{D_t}} = -\lambda_t + \beta^s \lambda_{t+1}(1+i_t^D) = 0 \quad (B3)$$

or

$$\lambda_t = \beta^s \lambda_{t+1}(1+i_t^D) \quad (B4)$$

Appendix C

From equation (2.9) we obtain:

$$lnC_t^b = l\,n(\alpha C_t) = l\,n(\alpha C_{t+1}) - \frac{1}{\sigma}\left[\frac{i_t^L - E_t\{\pi_{t+1}\} - \rho^b + (1 - E_t\{\mu_{t+1}\}) f_t k_t (1-\mu_t)(1+i_{t-1}^L) L_{t-1}}{(1+i_t^L) L_t}\right] \qquad (C1)$$

or

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left[\frac{i_t^L - E_t\{\pi_{t+1}\} - \rho^b + (1 - E_t\{\mu_{t+1}\}) f_t k_t (1-\mu_t)(1+i_{t-1}^L) L_{t-1}}{(1+i_t^L) L_t}\right] \qquad (C2)$$

From equation (2.18) we obtain:

$$ln\, C_t^s = l\,n((1-\alpha)C_t) = l\,n((1-\alpha)C_{t+1}) - \frac{1}{\sigma}\lfloor i_t^D - E_t\{\pi_{t+1}\} - \rho^s \rfloor \qquad (C3)$$

or

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\lfloor i_t^D - E_t\{\pi_{t+1}\} - \rho^s \rfloor \qquad (C4)$$

Appendix D

We take first-order conditions in equation (3.6) as follows:

$$\frac{\partial \mathcal{L}}{\partial C_t} = (C_t)^{-\sigma} - \lambda_t P_t = 0 \Rightarrow \lambda_t = \frac{(C_t)^{-\sigma}}{P_t} \qquad (D1)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -(N_t)^{\varphi} + \lambda_t W_t = 0 \Rightarrow \lambda_t = \frac{(N_t)^{\varphi}}{W_t} \qquad (D2)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} = -\lambda_t(1+i_{t-1})l_{t-1} - \lambda_t(1+i_{t-1}+f_{t-1})(1-\mu_{t-1})(1+i_{t-2})l_{t-2}$$

$$+\beta\lambda_{t+1}\mu_{t+1}(1+i_t+f_t)(1+i_{t-1})l_{t-1} = 0 \qquad (D3)$$

$$\frac{\partial \mathcal{L}}{\partial l_t} = 0$$

or

$$\lambda_t - \beta\lambda_{t+1}\mu_{t+1}(1+i_t) - \beta^2\lambda_{t+2}\mu_{t+2}(1+i_{t+1}+f_{t+1})(1-\mu_{t+1})(1+i_t) = 0$$

or, by re-arranging equation (D3), writing it one period forward and substituting it above, we finally get:

$$\lambda_t = \beta\lambda_{t+1}(1+i_t)\left[1 + \frac{(1 - E_t\mu_{t+1})(1+i_t+f_t)(1-\mu_t)(1+i_{t-1})l_{t-1}}{(1+i_t)l_t}\right] \qquad (D4)$$

Appendix E

We take first-order conditions in eq. (5.5) as below:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \gamma (C_t)^{\gamma(1-\eta)-1}(l_t)^{(1-\gamma)(1-\eta)} - \kappa_t = 0 \qquad (E1)$$

$$\frac{\partial \mathcal{L}}{\partial l_t} = (1-\gamma)(C_t)^{\gamma(1-\eta)}(l_t)^{(1-\gamma)(1-\eta)-1} - \kappa_t w_t - \kappa_t^{'} e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t = 0 \qquad (E2)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} = -\kappa_t(1+i_{t-1})L_{t-1}\frac{1}{P_t} + \kappa_t^{'}\frac{1}{(\mu_t)^2} e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t(1-l_t) = 0 \qquad (E3)$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\beta_1 \kappa_{t+1}\mu_{t+1}(1+i_t)\frac{1}{P_{t+1}} + \frac{1}{P_t}\kappa_t - \beta_1 \kappa_{t+1}^{'}(1+i_t) = 0 \qquad (E4)$$

$$\frac{\partial \mathcal{L}}{\partial k_t^{'}} = e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t(1-l_t) - (1+i_{t-1})L_{t-1} = 0 \qquad (E5)$$

By substituting eq. (E5) to eq. (E3), we obtain:

$$-\kappa_t(1+i_{t-1})L_{t-1}\frac{1}{P_t} + \kappa_t^{'}\frac{1}{(\mu_t)^2}(1+i_{t-1})L_{t-1} = 0$$

or

$$\kappa_t^{'} = \kappa_t(\mu_t)^2\frac{1}{P_t} \qquad (E6)$$

By substituting eq. (E6) and eq. (E1) to eq. (E4), we finally get the following Euler equation:

$$\frac{1}{P_t}\gamma(C_t)^{\gamma(1-\eta)-1}(l_t)^{(1-\gamma)(1-\eta)} = \beta_1\gamma(C_{t+1})^{\gamma(1-\eta)-1}(l_{t+1})^{(1-\gamma)(1-\eta)}(1+i_t)\frac{1}{P_{t+1}}[\mu_{t+1} + (\mu_{t+1})^2]$$

or

$$(C_t)^{\gamma(1-\eta)-1}(l_t)^{(1-\gamma)(1-\eta)} = \beta_1(C_{t+1})^{\gamma(1-\eta)-1}(l_{t+1})^{(1-\gamma)(1-\eta)}(1+i_t)\frac{P_t}{P_{t+1}}[\mu_{t+1} + (\mu_{t+1})^2] (E7)$$

Also, by substituting eq. (E6) to eq. (E2), we get the following labor supply equation:

$$(1-\gamma)(C_t)^{\gamma(1-\eta)}(l_t)^{(1-\gamma)(1-\eta)-1} - \kappa_t w_t - \kappa_t(\mu_t)^2\frac{1}{P_t} e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t = 0$$

or by substituting eq. (A1) to this equation, we end up with:

$$\frac{1-\gamma}{\gamma}\frac{C_t}{l_t}\kappa_t = \kappa_t w_t + \kappa_t(\mu_t)^2 e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIw_t$$

or

$$\frac{1-\gamma}{\gamma}\frac{C_t}{l_t} = \left[1 + (\mu_t)^2 e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTI\right] w_t \qquad (E8)$$

Appendix F

We take first-order conditions in eq. (5.9) as below:

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma} - \lambda_t = 0 \qquad (F1)$$

$$\frac{\partial \mathcal{L}}{\partial l_t} = l_t^{-\varphi} - \lambda_t w_t - \lambda_t' e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t = 0 \qquad (F2)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} = -\lambda_t (1+i_{t-1}) L_{t-1} \frac{1}{P_t} + \lambda_t' \frac{1}{(\mu_t)^2} e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t (1-l_t) = 0 \qquad (F3)$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\beta_2 \lambda_{t+1} \mu_{t+1} (1+i_t) \frac{1}{P_{t+1}} + \lambda_t \frac{1}{P_t} - \beta_2 \lambda_{t+1}' (1+i_t) = 0 \qquad (F4)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t'} = e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t (1-l_t) - (1+i_{t-1}) L_{t-1} = 0 \qquad (F5)$$

By substituting eq. (F5) to eq. (F3), we obtain:

$$-\lambda_t (1+i_{t-1}) L_{t-1} \frac{1}{P_t} + \lambda_t' \frac{1}{(\mu_t)^2} (1+i_{t-1}) L_{t-1} = 0$$

or

$$\lambda_t' = \lambda_t (\mu_t)^2 \frac{1}{P_t} \qquad (F6)$$

By substituting eq. (F6) and eq. (F1) to eq. (F4), we obtain the consumption Euler equation:

$$C_t^{-\sigma} = \beta_2 C_{t+1}^{-\sigma} (1+i_t) \frac{P_t}{P_{t+1}} [\mu_{t+1} + (\mu_{t+1})^2] \qquad (F7)$$

Also, by substituting eq. (F6) to eq. (F2), we get the following labor supply equation:

$$l_t^{-\varphi} - \lambda_t w_t - \lambda_t (\mu_t)^2 \frac{1}{P_t} e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIW_t = 0$$

or by substituting eq. (F1) to this equation, we finally get:

$$l_t^{-\varphi} - C_t^{-\sigma} w_t - C_t^{-\sigma} (\mu_t)^2 e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTIw_t = 0$$

or

$$l_t^{-\varphi} = C_t^{-\sigma} w_t \left[1 + (\mu_t)^2 e^{\left[\frac{1-\mu_t}{(1-\mu_t)-1}\right]} DPTI\right] \qquad (F8)$$

Appendix G

From eqs. (E1) and (F1), it is obvious that the Lagrange multipliers of the budget constraint under the two types of preferences considered in this paper (κ_t, λ_t) take nonzero values. Thus, the Lagrange multipliers of the borrowing constraints (κ_t', λ_t') are also nonzero, since on the basis of eqs. (E6) and (F6) they depend on the Lagrange multipliers of the budget constraint and the variables μ_t and P_t which are nonzero by definition.

Appendix H

In this Appendix, we solve the household's maximization problem under full debt payment and with no borrowing constraint, and estimate the resulting equations.

Multiplicative non-separable preferences

We start by assuming multiplicative non-separable preferences. Taking FOCs for consumption, leisure, loans and the percentage of debt repaid, we obtain the following log form of

(i) the consumption Euler equation:

$$c_t = c_{t+1} + \frac{1}{\gamma_2(1-\eta_2)-1}\left\{ln(1+i_t) - ln\frac{P_{t+1}}{P_t} + ln\beta_3 + (1-\gamma_2)(1-\eta_2)ln\frac{l_{t+1}}{l_t} + \right\} \quad (H1)$$

where $1/\eta_2$ is the inter-temporal elasticity of the consumption-leisure composite good, γ_2 is the weight of consumption relative to leisure and β_3 is the subjective discount factor, and

(ii) the static labor supply equation:

$$lnl_t = l\,n\left(\frac{1-\gamma_2}{\gamma_2}\right) + c_t - lnw_t \quad (H2)$$

By applying nonlinear GMM to equations (H1) and (H2) and assuming that the errors of the equations follow a first-order autoregressive process with parameter ρ_1' and ρ_2', respectively, we obtain the estimates presented in Table H-1:

Table H-1 Nonlinear GMM estimation under multiplicative non-separable preferences

Equation	$1/\eta_2$	β_3	γ_2	ρ_1'	ρ_2'	*J-test*	*SK-test*	*Q-test*
(H1)	0.262 (2.07)	0.905 (177.78)	0.738 (58.14)	0.68 (10.50)	-	0.43	0.07	0.27
(H2)	-	-			0.97 (67.71)		0.04	0.58

Notes:
Instruments for eq. (H1) : c_{t-2} , c_{t-3} , c_{t-4} , r_{t-1} , Δr_{t-1}, l_{t-4} , Δl_{t-4}
Instruments for eq. (H2) : c_{t-4} , l_{t-2} , l_{t-3} , l_{t-4} , Δl_{t-4} , lnw_{t-1} , Δlnw_{t-1}
r_t refers to the real interest rate.
Columns 2 to 6 of the Table present coefficient estimates with their t-values in parenthesis. Columns 7, 8 and 9 show the p-value of the J-test for instrument exogeneity, the SK-test for normality of the residuals and the Box-Pierce Q-test for higher order autocorrelation of the residuals, respectively.

Additive separable preferences

We continue by assuming additive separable preferences. We take again FOCs for consumption, leisure, loans and the percentage of debt repaid and thus obtain the standard form of

(i) the consumption Euler equation:

$$c_t = c_{t+1} - \frac{1}{\sigma_2}\left[ln(1+i_t) - ln\frac{P_{t+1}}{P_t} + ln\beta_3\right] \qquad (H3)$$

where $1/\sigma_2$ is the intertemporal elasticity of substitution for consumption and β_3 is the subjective discount factor, and

(ii) the static labor supply equation:

$$lnl_t = \frac{\sigma_2}{\varphi_2}c_t - \frac{1}{\varphi_2}lnw_t \qquad (H4)$$

where $1/\varphi_2$ is the intertemporal elasticity of leisure.

Similarly to the previous case, we apply nonlinear GMM to the two equations (H3) and (H4) assuming that the errors of the two equations follow a first- and a third-order autoregressive process with parameters ρ'_1, and ρ'_2 , ρ'_3 and ρ'_4 , respectively. The results are reported in Table H-2.

Table H-2 Nonlinear GMM estimation under additive separable preferences

Equation	*1/* φ_2	β_2	*1/* σ_2	ρ'_1	ρ'_2	ρ'_3	ρ'_4	*J-test*	*SK-test*	*Q-test*
(H3)	-	0.907 (139.62)	0.286 (2.12)	0.726 (11.83)	-	-	-	0.14	0.05	0.37
(H4)	0.060 (2.02)	-		-	0.39 (4.31)	0.20 (2.03)	0.41 (5.31)		0.10	0.46

Notes:
Instruments for eq. (H3) : c_{t-1} , c_{t-2} , c_{t-3} , c_{t-4} , l_{t-4} , $\Delta_{t-2}l_{t-4}$, r_{t-1} , $\Delta_{t-3}r_{t-1}$
Instruments for eq. (H4) : c_{t-4} , l_{t-1} , l_{t-2} , l_{t-3} , l_{t-4} , $\Delta_{t-2}l_{t-4}$, r_{t-4} , lnw_{t-2} , Δlnw_{t-1}
r_t refers to the real interest rate.
Columns 2 to 6 of the Table present coefficient estimates with their t-values in parenthesis. Columns 7, 8 and 9 show the p-value of the J-test for instrument exogeneity, the SK-test for normality of the residuals and the Box-Pierce Q-test for higher order autocorrelation of the residuals, respectively.