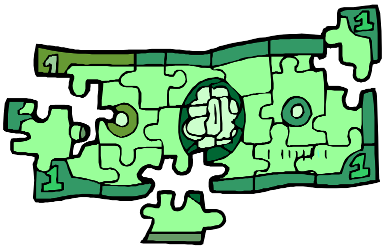
UNIVERSITY OF PIRAEUS

Department of Financial and Banking Management

<u>MSC THESIS</u>

Testing for Potential Portfolio Performance and Index Efficiency: An Application to the Equity Home Bias Puzzle.



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ABSTRACT

The object is this thesis is twofold. First to provide a survey of existing literature on testing procedures for portfolio efficiency, and second to investigate a well-known puzzle of the modern finance literature, the home equity bias puzzle. Several multivariate tests of portfolio efficiency are discussed in the theoretical part. In the empirical part, selected tests are implemented to test the efficiency of international portfolios of institutional investors. The results show that most portfolios were efficient over the period 1992-1999, as a result of good performance in most national equity markets, whereas our initial belief was that EMU portfolios should have been more efficient than non-EMU portfolios in the 90s, as a result of increased foreign equity holding.

1. Introduction

Mean-Variance efficiency and the related concept of performance evaluation have been of significant interest to financial economists ever since the originating work of *Markowitz (1952)*. After the introduction of the Capital Asset Pricing Model (*CAPM*) by *Sharpe (1964)* and *Lintner (1965)*, much of the development has been in the area of testing asset pricing models. The first tests of asset pricing theories, conducted by *Black, Jensen and Scholes (1972), Blume and Friend (1973)* and *Fama and MacBeth (1973)* were basically of univariate nature. But soon afterwards, multivariate statistical approaches and techniques emerged. As a result of *Roll's (1977)* critique that the CAPM is essentially untestable because the market portfolio is unobservable, researchers focused on tests of the mean-variance efficiency of the underlying portfolio under consideration.

The object is this thesis is twofold. First, to provide a survey of existing literature on multivariate testing procedures for portfolio efficiency, and second, to investigate a well-known puzzle of the modern finance literature, the home bias puzzle, as an application to tests of portfolio efficiency.

In the first (theoretical) part, the testing methodologies of *Jobson and Korkie (1982), Gibbons, Ross and Shanken (1989)* and *MacKinlay and Richardson (1991)* are discussed extensively, under the assumption that a riskless asset exists. Tests under the *Black (1972)* or zero-beta version of *CAPM* are discussed too, despite not being used in our empirical work.

In the empirical part, the *Equity Home Bias Puzzle,* i.e. the observation that investors are biased towards domestic assets despite the benefits of international diversification, is explored. In particular, we are interested in testing whether efficiency of equity portfolios of institutional investors in EMU countries has increased in the later part of the 1990s as a result of the launch the launch of the Euro and the abolishment of a series of investment barriers which used to restrict the holdings of foreign equity by institutional investors like pension funds and life insurance companies. However, our results do not sustain this statement sufficiently, as most EMU portfolios were efficient over the whole period, while most non-EMU portfolios were efficient as well, benefiting by the good performance of equity markets worldwide.

The remainder of this thesis is organized as follows. Section 2 is a literature review of the most popular tests of potential performance and portfolio efficiency. In most cases, an outline of the derivation of the test statistics is provided, along with discussion of their statistical properties and the economic intuition behind them. Section 3 discusses briefly recent literature on the equity home bias. Section 4 describes the data and the methodology utilized for the empirical part. Section 5 presents the empirical results and section 6 concludes. Data and numerical results are included in the appendices.

2. A literature review on tests of Mean-Variance efficiency

In this section, a synopsis of the available literature on the subject of testing methodologies for Mean-Variance (*MV*) efficiency of portfolios is made. Such tests were initially developed out of the necessity to evaluate asset pricing theories, like the *CAPM*. But since the testability of such theories is equivalent to testing whether the market portfolio is MV efficient, the same methods can be used to test the efficiency of a single portfolio[1]. In the first subsection, the statistical framework of development of such testing procedures is set. The second subsection is a survey of tests of *MV* efficiency under the assumption that investors can unlimitedly borrow and lend at a risk-free rate of return, or alternatively that the *traditional or Sharpe-Lintner version of CAPM* is used as generating process for asset returns. Finally, the third subsection presents corresponding tests under restricted borrowing (absence of riskless rate of return), or equivalently under the specification that the *zero-beta or Black version of CAPM* holds. A compact reference for subsections 2.2 and 2.3 is chapter 5 from *Campbell et al (1997).* For a more formal presentation of the analytics of efficient frontier and tests of the CAPM, we suggest *Huang and Litzenberger (1988)*, chapters 3 and 10.

2.1 Three asymptotically equivalent test procedures

This section discusses the most commonly used test procedures where an appropriate finite sample statistic is unavailable: the ***Likelihood Ratio*** *(LR)*, the ***Wald*** *(W)* and the ***Lagrange Multiplier*** *(LM)* tests. These are asymptotic tests, and as such their properties depend on large samples asymptotic results. All three are developed within the framework of *Maximum Likelihood (ML)* estimation and use the asymptotic normality of *ML* estimators. A more detailed analysis of this topic can be found in any standard econometrics textbook, such as *Greene (2000)*. Also in *Buse (1982)* you can find an elegant and intuitively appealing diagrammatic interpretation of these three tests.

Consider, in a multivariate context, the *ML* estimation[2] $\hat{\underset{\sim}{\theta}}$ of a set of parameters $\underset{\sim}{\theta}$, and the $q \times 1$ vector of restrictions under the null hypothesis H_0: $c(\underset{\sim}{\theta}) = \underset{\sim}{\theta}_0$. The basic intuition behind each of the three tests follows.

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[1] According to *Roll (1977)*.

[2] Henceforth, a tilde (~) under a small letter will denote a vector, and under a capital letter will denote a matrix.

Likelihood ratio test. If the restriction $c(\underset{\sim}{\theta}) = \underset{\sim}{\theta}_0$ is valid, then imposing it should not lead to a large reduction in the log-likelihood function. Stated differently, the likelihood ratio $\lambda = \dfrac{\hat{L}_R}{\hat{L}_U}$ (where $\hat{L}_R$ and $\hat{L}_U$ are the values of the likelihood function under the restriction of H_0 and under no restriction, respectively) should be close to zero (and non-negative). The *LR* test quantifies the deviation of *–2lnλ* from zero and rejects the null hypothesis if the value of the statistic is large enough.

Wald test. If the restriction of the null is valid, then $c(\underset{\sim}{\theta}) - \underset{\sim}{\theta}_0$ should be close to zero since the *ML* estimator is *consistent*. This test is based on $c(\underset{\sim}{\theta}) - \underset{\sim}{\theta}_0$ and rejects the null if it is significantly different from zero. Its formula is

$$W = [c(\underset{\sim}{\theta}) - \underset{\sim}{\theta}_0]'[Var(c(\underset{\sim}{\theta}))]^{-1}[c(\underset{\sim}{\theta}) - \underset{\sim}{\theta}_0] \qquad \textbf{(2-1)}$$

Lagrange Multiplier test (or "*efficient score*" or "*score*" test). If the restriction of the null is valid, then the restricted estimator should be near the point that maximizes the log-likelihood function. Therefore, the slope of the log-likelihood function at the value of the restricted estimator should be near zero. This test is based on that slope exactly, rejecting the null hypothesis when the slope is significantly different from zero. The formula for this test is

$$LM = \left[\frac{\partial \ln L(\underset{\sim}{\theta}_0)}{\partial \underset{\sim}{\theta}}\right]' [I(\underset{\sim}{\theta}_0)]^{-1} \left[\frac{\partial \ln L(\underset{\sim}{\theta}_0)}{\partial \underset{\sim}{\theta}}\right] \qquad \textbf{(2-2)}$$

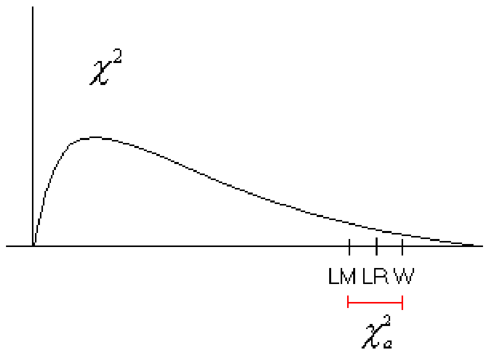
where *I* is the information matrix.

Under the null hypothesis, all three statistics have the same limiting χ^2 distribution, with degrees of freedom equal to the number of restrictions, *q*. That is to say, *the three tests are asymptotically equivalent under the null.* The *LR* test requires calculation of both the *restricted* and the *unrestricted estimators*, whereas the *W* test requires only the *unrestricted estimators* and the *LM* test only the *restricted estimators.* In some problems, either set of estimators may be easier to compute than the other. As a consequence, the choice among the three tests is typically made on the basis of ease of computation.

However, *despite following asymptotically the same distribution (under the null), they can behave rather differently in a small sample*. A strong result, first shown by *Berndt and Savin (1977)* is that a systematic numerical inequality exists between the *Likelihood Ratio*, the *Wald* and the *Lagrange Multiplier* tests. Specifically, **W≥LR≥LM**[3]. As a result, for a specified size the *Wald* test will tend to reject more often (followed by the *LR* and the *LM*) the null hypothesis. As a consequence, **there will always be a significance level for which the three tests will yield conflicting inference!**

Such a case is presented in the figure on the right (the red line defines the area where conflicting inference can be made). If the significance level of the test is set between the true significance level of the *W* and the *LR* statistic, *W* will reject and the other two will not reject, while if it is set between the *LM* and *LR* true significance level, *W* and *LR* will reject and LM will not reject[4].

A case like this would be disturbing for two reasons. First, because two researchers employing the same set of data, the same estimation technique (*Maximum Likelihood*), the same significance level but different asymptotically equivalent testing procedures, may reach conflicting decisions with regard to the truth of the null hypothesis. Second, because if the researcher has subjective preferences regarding the truth of the null hypothesis, he/she can judiciously choose the test procedure that is more likely to provide supporting evidence.



In such a case, the conflict may be resolved if one testing criterion is shown to be more *powerful* than the others. Unfortunately, **no test procedure has been found uniformly most powerful under all alternatives**, up to date. However, under a specified alternative, one test may be more powerful and thus be favored against the others. Therefore, for multivariate tests of linear restrictions to produce unambiguous results, more information about the alternative hypothesis is required *(Berndt and Savin (1977))*. Another method to resolve such a controversy would be to make corrections for the size (and possibly the power as well) of the tests, by adjusting the degrees of freedom of the statistics, like *Evans and Savin (1982)* do in the case of testing for the coefficients of the classical linear regression model[5]. Finally, another suggested method would be to use more than one testing procedures and compare and analyze the results they provide. We follow this principle for the empirical part of this thesis project.

In the next two subsections, we discuss various tests of MV efficiency that have appeared in the finance literature. They were developed in the

[3] A case where the equality survives even for small samples is when the likelihood function is of quadratic form, like in the linear multivariate regression model with normal disturbances with **known** covariance matrix (see *Buse (1982)* p. 156). However, in empirical applications the covariance matrix is seldom known a priori.

[4] Clearly no conflict arises when *W* does not reject or when *LM* rejects the hypothesis.

[5] Correction of this sort were commonly made for tests of financial models, see e.g. *Jobson and Korkie (1982)*.

statistical framework of the three equivalent testing procedures we just described.

2.2 Tests of MV efficiency under the Sharpe-Lintner CAPM

Under the assumption of existence of a risk-free rate, we can utilize the following model. Assume that asset returns follow a multivariate *Normal* distribution, and excess returns are independently and identically distributed (*IID*) through time. Excess asset returns are defined as returns in excess of a reference risk-free rate (such as the Treasury Bill rate). Under these assumptions, asset returns can be described by the *excess-return market model*:

$$\underset{\sim}{z}_t = \underset{\sim}{\alpha} + \underset{\sim}{\beta} z_{pt} + \underset{\sim}{\varepsilon}_t , \qquad t = 1, 2, \ldots, T \qquad \textbf{(2-3)}$$

$$E\,\underset{\sim}{\varepsilon}_t = 0 \qquad\qquad E\left(z_{pt}\right) = \mu_p$$

$$E\,\underset{\sim}{\varepsilon}_s \underset{\sim}{\varepsilon}_t' = \underset{\sim}{\Sigma}, \quad \text{if } s = t \qquad\qquad Var\left(z_{pt}\right) = \sigma_p^2$$

$$E\,\underset{\sim}{\varepsilon}_s \underset{\sim}{\varepsilon}_t' = 0 \quad \text{if } s \neq t \qquad\qquad Cov\left[z_{pt}, \underset{\sim}{\varepsilon}_t\right] = 0$$

where

$\underset{\sim}{z}_t$: $N\times1$ vector of excess asset returns for time period t

z_{pt} : excess market (or single portfolio) return for period t

$\underset{\sim}{\varepsilon}_t$: $N\times1$ disturbance vector

$\underset{\sim}{\alpha}$, $\underset{\sim}{\beta}$: $N\times1$ parameter vectors

$\underset{\sim}{\Sigma}$: $N\times N$ disturbance covariance matrix[6]

In the presence of a riskless asset, the Sharpe-Lintner model sets the following condition for a particular portfolio to be MV efficient:

$$E\left[\underset{\sim}{z}_t\right] = \underset{\sim}{\beta} E\left(z_{pt}\right) \qquad \textbf{(2-4)}$$

Taking the expectation of **(2-3)** and combining with **(2-4)**, we get the restriction $\underset{\sim}{a} = 0$, which can be stated in the form of a null hypothesis:

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[6] We further assume that $N \leq T - 2$ in order that $\hat{\Sigma}$ is non-singular.

$$H_0 : \underset{\sim}{a} = 0 \qquad \textbf{(2-5)}$$

Then we need estimators with desirable statistical properties for the parameters of model **(2-3)**, such as *Maximum Likelihood* (*ML*) estimators, in order to proceed with testing procedures for H_0. Making the extra assumption of *normality* for the residuals of **(2-3)**, we can derive the ML estimators[7] of $\underset{\sim}{\alpha}$, $\underset{\sim}{\beta}$ and $\underset{\sim}{\Sigma}$. These estimators will be *consistent*, *asymptotically efficient*, *asymptotically normal* and *invariant* (*Greene (2000)*, p. 127)).

In order to construct a *Wald* test for H_0, we just need the unrestricted estimators of the model. For the *Likelihood Ratio* and the *Lagrange Multiplier* tests we would need estimators for the restricted version of the model as well (under the restriction of H_0). The analytics of the derivation of the estimators and their distributions can be found in *Campbell et al (1997)*[8]. *Gibbons, Ross and Shanken (1989)* followed this procedure to derive the *GRS* statistic. In *Campbell et al (1997)* you can also find the standard forms[9] of the *Wald* and *Likelihood Ratio* statistic under the specified context. Other researchers however applied alternative methodologies so as to derive operational statistics. *Jobson and Korkie (1982)* developed *Wald*, *LR* and *LM* statistics without using the estimates from a regression like **(2-3)**, but instead used the efficient set constants and techniques from multivariate analysis. *MacKinley and Richardson (1991)* assumed a regression model, but proceeded taking *GMM* estimations instead of *ML*. In the remainder of this subsection, we present the former three most popular testing procedures for *MV* efficiency, under the Sharpe-Lintner context of CAPM.

2.2.1 The JK statistics

An important contribution to the literature on performance evaluation has been the work of *Jobson and Korkie (1982)* –henceforth *JK*– who extended the set of evaluation techniques by proposing a performance measurement procedure with reasonable statistical properties which utilized the efficient set constants of *Merton (1972)* and *Roll (1977)*. The distinctive feature of their technique is its comparison of the maximum attainable Sharpe performance (which *JK* call *potential performance*) of an asset set with the potential performance of an asset subset. This technique can be used for the quantification of the performance contribution made by additional assets, the efficiency evaluation of a portfolio or market index, and for tests of multifactor asset pricing models.

Given a population of N assets with $N \times 1$ vector of excess asset returns $\underset{\sim}{\mu}$ and $N \times N$ covariance matrix $\underset{\sim}{\Sigma}$, the $N \times 1$ vector of risky asset proportions $\underset{\sim}{w}$ is obtained from minimization of the Lagrangian

$$L = \underset{\sim}{w}' \underset{\sim}{\Sigma} \underset{\sim}{w} - \lambda_1 \left(\underset{\sim}{w}' \underset{\sim}{\mu} - \mu_p \right) - \lambda_2 \left(\underset{\sim}{w}' \underset{\sim}{i} - 1 \right) \qquad \textbf{(2-6)}$$

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[7] Naturally, in a linear regression model, the *ML* estimators of the model parameters will be the same as the *OLS* estimators. In the discussion that follows we will refer to *OLS* estimators or *ML* estimators remaining faithful to the expression of the corresponding authors.

[8] We would also suggest *Huang and Litzenberger (1988)*, for alternative estimators to *ML*, such as *GLS* estimators.

[9] Without corrections for degrees of freedom.

where λ_1 and λ_2 are the Lagrange multipliers, μ_p is excess return for a portfolio p and $\underset{\sim}{i}$ is a $N \times 1$ vector of ones.

The first extremum conditions provide the asset weights vector

$$\underset{\sim}{w_q} = \underset{\sim}{\Sigma}^{-1} \underset{\sim}{\mu} \Big/ \underset{\sim}{e}' \underset{\sim}{\Sigma}^{-1} \underset{\sim}{\mu} \qquad \textbf{(2-7)}$$

of the familiar tangency portfolio (*q*) in mean-standard deviation space. The mean excess return and variance of the tangency portfolio will then be

$$\mu_q = a / b \qquad \textbf{(2-8)}$$

$$s_q^2 = a / b^2 \qquad \textbf{(2-9)}$$

where *a* and *b* are the efficient set constants[10]

$$a = \underset{\sim}{\mu}' \underset{\sim}{\Sigma}^{-1} \underset{\sim}{\mu} \qquad \textbf{(2-10)}$$

$$b = \underset{\sim}{e}' \underset{\sim}{\Sigma}^{-1} \underset{\sim}{\mu} \qquad \textbf{(2-11)}$$

The Sharpe measure of performance for any portfolio *p* and the tangency portfolio *q* will be respectively

$$Sh_p = \mu_p / \sigma_p \qquad \textbf{(2-12)}$$

$$Sh_q = \mu_q / \sigma_q = \left(a / b\right)(\sqrt{a} / b) = \sqrt{a} \qquad \textbf{(2-13)}$$

Thus, the square of the reward to variability ratio of the tangency portfolio (or the maximum attainable Sharpe ratio) is given by the efficient set constant[11] "*a*". *JK* refer to the performance measure *a* as the **potential performance** of an one-period buy-and-hold strategy for a portfolio composed from a population of *N* assets and use it in the development of test statistics of performance evaluation.

JK developed a notional as well as technical framework for testing the efficiency of portfolios, under two approaches. First, they suppose a subset Γ_1 of N_1 assets from the population Γ of *N* assets, and denote the potential performance of asset sets Γ_1 and Γ as a_1 and a respectively. Under the first approach, testing for *potential performance*[12], they try to answer a first question of interest, *whether the potential performances of the two asset sets are identical*. This question can be answered through testing the hypothesis $H_{01}\!:a_1 = a$, a test for the comparative potential performance hypothesis. This hypothesis determines if the N_1 assets are jointly efficient with respect to the complete set on *N* assets or not, or equivalently, if there is efficiency loss for the portfolio of N_1 assets compared to the portfolio of *N* assets. Consequently, H_{01} *determines if the case of the inefficiency is due to the selection of assets.*

Under the second approach, testing for *portfolio or index efficiency*, *JK* attempt to answer a second question of interest, *whether the portfolio performance* μ_p^2 / σ_p^2 *is equivalent to the potential performance.* This can be tested with the

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[10] As defined by *Merton (1972)* and *Roll (1977)*.

[11] This *a* should not be confused with the intercept of the linear return generating regression, or Jensen measure of performance, that is used in the development of other efficiency testing procedures.

[12] *JK (1989)* interpret tests of potential performance as tests of *intersection.*

hypothesis $H_{02}{:}\,\mu_p^2 / \sigma_p^2 = a$. *This hypothesis determines if the inefficiency is due to incorrect selection of weights for the N assets*. It can be utilized to determine the relative efficiency of a single portfolio or a market index.

Therefore, under these two approaches, *JK* not only test for efficiency, but also distinguish the cause of the inefficiency (incorrect selection of assets or weights), were the null hypothesis rejected[13].

JK develop tests statistics for hypotheses H_{01} and H_{02}, using relation **(2-10)**. After partitioning the population of *N* assets into two mutually exclusive and exhaustive subsets of N_1 and N_2 assets, the mean vector $\underset{\sim}{\mu}$ and the covariance matrix $\underset{\sim}{\Sigma}$ can be partitioned accordingly as

$$\underset{\sim}{\mu} = \begin{bmatrix} \underset{\sim}{\mu_1} \\ \underset{\sim}{\mu_2} \end{bmatrix} \quad \text{and} \quad \underset{\sim}{\Sigma} = \begin{bmatrix} \underset{\sim}{\Sigma_{11}} & \underset{\sim}{\Sigma_{12}} \\ \underset{\sim}{\Sigma_{21}} & \underset{\sim}{\Sigma_{22}} \end{bmatrix}$$

Similar partitions can be made for the sample estimators of statistics $\underset{\sim}{\mu}$ and $\underset{\sim}{\Sigma}$. Then the null hypothesis relation H_{01} combined with **(2-10)** and bearing in mind the above partition can be restated as

$$H_{01}{:}\,\underset{\sim}{\mu_1'}\,\underset{\sim}{\Sigma_{11}}^{-1}\,\underset{\sim}{\mu_1} = \underset{\sim}{\mu'}\,\underset{\sim}{\Sigma}^{-1}\,\underset{\sim}{\mu} \qquad \textbf{(2-15)}$$

After some manipulation using results from multivariate analysis, H_{01} can further be restated as

$$H_{01}{:}\,\left(\underset{\sim}{\mu_2} - \underset{\sim}{\Sigma_{21}}\,\underset{\sim}{\Sigma_{11}}^{-1}\,\underset{\sim}{\mu_1}\right) = 0 \qquad \textbf{(2-16)}$$

JK use the form of H_{01} in relation **(2-16)** to develop the following test statistics[14], assuming additionally *multivariate normality* for the *T* excess return observations.

The **Wald** statistic

$$\phi_{1,1} = (T-N)\left(\frac{\hat{\alpha} - \hat{\alpha}_1}{1+\hat{\alpha}_1}\right) \overset{a}{\sim} \chi^2_{N-N_1} \qquad \textbf{(2-17)}$$

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[13] We note that the potential performance *a* in either case cannot be exceeded, except if the portfolio is actively managed.

[14] All distributions are assumed to hold under the null hypothesis (generally they don't hold under the alternative). Also $\overset{a}{\sim}$ denotes asymptotic convergence to distribution.

The **Likelihood Ratio** statistics[15]

$$\phi_{1,2} = (T - \tfrac{N}{2} - \tfrac{N_1}{2} - 1)\ln\left(\frac{1+\hat{\alpha}}{1+\hat{\alpha}_1}\right) \overset{a}{\sim} \chi^2_{N-N_1} \qquad \textbf{(2-18)}$$

and[16]

$$\phi_{1,3} = \frac{(T-N)}{(N-N_1)}\left(\frac{\hat{\alpha}-\hat{\alpha}_1}{1+\hat{\alpha}_1}\right) \sim F_{N-N_1,T-N} \qquad \textbf{(2-19)}$$

The **Lagrange Multiplier** or "*score*" statistic[17]

$$\phi_{1,4} = \mathrm{T}\,\frac{(\hat{\alpha}-\hat{\alpha}_1)}{(1+\hat{\alpha})(1+\hat{\alpha}_1)} \overset{a}{\sim} \chi^2_{N-N_1} \qquad \textbf{(2-20)}$$

Hypothesis H_{02} [18] can be regarded as a special case of H_{01}, since the potential performance of a single asset *p* equals its (squared) Sharpe performance, μ_p^2 / σ_p^2. Thus, the statistics for H_{02} can be directly obtained from the statistics of H_{01} for $N_1 = 1$ and $\alpha_1 = \hat{\mu}_p^2 / \hat{\sigma}_p^2$, where $\hat{\mu}_p^2$ and $\hat{\sigma}_p^2$ are the sample mean and sample variance respectively of the portfolio of interest *p*. Hence, the corresponding statistics for H_{02} are[19]:

$$\phi_{2,1} = (T-N)\left(\frac{\hat{\alpha} - \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}}{1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}}\right) \overset{a}{\sim} \chi^2_{N-1} \qquad \textbf{(2-21)}$$

$$\phi_{2,2} = (T - \tfrac{N}{2} - \tfrac{5}{2})\ln\left(\frac{1+\hat{\alpha}}{1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}}\right) \overset{a}{\sim} \chi^2_{N-1} \qquad \textbf{(2-22)}$$

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[15] These LR are approximations for the standard LR statistic, that are assumed to improve small sample performance. See *Rao (1973)*, pp. 555-556 for technical details.

[16] Originally *JK* had incorrectly stated the denominator degrees of freedom as ***T-N-1*** and also had stated that the statistic follows asymptotically an F distribution. In fact the F distribution is exact for every *T>N* and the denominator degrees of freedom are ***T-N***. The correction first appeared in *JK (1985)*. It appears also in *JK (1989)*.

[17] The score statistic was developed by Rao. See *Rao (1973)*, pp.415-420. *JK* suggest an adjustment of Rao's statistic for improvement of small sample properties. The proof is in the appendix of *JK (1982)*.

[18] *JK (1982)* state that hypothesis H_{02} is also equivalent to a hypothesis of equal Treynor measures $\left(\frac{\mu_1}{\beta_1} = \frac{\mu_2}{\beta_2} = \cdots = \frac{\mu_N}{\beta_N}\right)$ and a hypothesis of equal Jensen measures $(\alpha_1 = \alpha_2 = \cdots = \alpha_N)$.

[19] Statistic $\phi_{2,2}$ is not perfectly equivalent with $\phi_{1,2}$ because the preferred Bartlett correction is different for $N_1 = 1$. However, *Campbell et al (1997)* report this statistic with $(T - \frac{N}{2} - 2)$ instead of $(T - \frac{N}{2} - \frac{5}{2})$.

$$\phi_{2,3} = \frac{T-N}{N-1}\left(\frac{\hat{\alpha} - \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}}{1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}}\right) \sim F_{N-1,T-N} \qquad \textbf{(2-23)}$$

$$\phi_{2,4} = T\frac{\left(\hat{\alpha} - \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)}{(1+\hat{\alpha})\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)} \overset{a}{\sim} \chi^2_{N-1} \qquad \textbf{(2-24)}$$

The intuition behind the formulas is that when the potential performance of the subset of assets or single portfolio is significantly below the potential performance of the full set of assets, the statistics take a rather large value and tend to reject the null hypothesis of efficiency of the subset of assets or portfolio.

JK (1982) additionally perform Monte Carlo simulations to assess the properties of all of their statistics, under both H_{01} and H_{02}. They note the relative superiority of the ***LR*** statistics (with respect to distribution fitting and size) over the rest, with the ***F*** version of the ***LR*** performing better than its χ^2 counterpart in smaller samples. However, with respect to power, the results are less strongly in favor of these two statistics.

The results of *JK (1982)* were developed under the assumption that a riskless asset is available, and therefore excess returns can be calculated by subtracting the riskless rate of return from real returns. *JK (1985)* generalize these results, under a ***multifactor version of CAPM*** and for the case where a riskless rate does not exist and the zero-beta rate has to be estimated from the data instead (***zero-beta or Black's CAPM***). They find that the essence of potential performance can be used for testing a multifactor model and that the results for portfolio efficiency hold when the riskless rate is substituted by the zero-beta rate. They propose their ***F*** statistic for testing efficiency under the zero-beta context, as well. However, in *JK (1989)* they prefer to suggest ***Shanken's (1985,1986)*** statistics[20] for the zero-beta case.

Finally, *JK (1985)* suggest that the power of the ***F*** test increases when the number of assets N is kept small relative to the number of sample observations ***T***. When the portfolio ***p*** to be examined for efficiency is a market index composed of many assets, they further claim that adding ***p*** to the N-asset set (thus forming a *N+1* asset set) further improves the power of the test.

2.2.1 The GRS statistic

Another test statistic for portfolio efficiency, with the advantage of a tractable finite sample distribution under both the null and the alternative hypothesis, was developed by *MacKinlay (1987)* and *Gibbons, Ross and Shanken (1989) – GRS* henceforth. Under the common *Normality* and *IID* assumptions for asset returns, *GRS* construct a Wald statistic to test the hypothesis of zero intercepts ($\underset{\sim}{a} = 0$) for an excess return regression model. But instead of an asymptotic χ^2 statistic, they derive an exact Hotelling[21] T^2 statistic (a multivariate generalization of Student's t).

Specifically, taking ***unbiased estimators*** for the 2-parameter excess return model, we can show that

$$\underset{\sim}{\hat{a}} \sim N_N\left(\underset{\sim}{a}, \frac{1}{T}\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)\underset{\sim}{\Sigma}\right) \qquad \textbf{(2-25)}$$

$$(T-2)\underset{\sim}{\hat{\Sigma}} \sim W_N(T-2, \underset{\sim}{\Sigma}) \qquad \textbf{(2-26)}$$

where $\underset{\sim}{\hat{a}}$, $\underset{\sim}{\Sigma}$ are the true model parameters, $\hat{\mu}_p^2$ and $\hat{\sigma}_p^2$ are the sample mean and variance of the actual portfolio, N_N denotes a *N-variate Normal* distribution and the notation $(T-2)\underset{\sim}{\hat{\Sigma}} \sim W_N(T-2, \underset{\sim}{\Sigma})$ indicates that the $(N \times N)$ matrix $(T-2)\underset{\sim}{\hat{\Sigma}}$ follows a *Wishart* distribution[22] with *(T-2)* degrees of freedom and covariance matrix $\underset{\sim}{\Sigma}$. Then since $\underset{\sim}{\hat{a}}$ and $\underset{\sim}{\hat{\Sigma}}$ are independent, according to the definition of *Rao (1973)*, p. 541,

$$T\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)^{-1}\underset{\sim}{\hat{\alpha}}'\,\underset{\sim}{\hat{\Sigma}}^{-1}\,\underset{\sim}{\hat{\alpha}} \sim T^2(N, T-2, \delta) \qquad \textbf{(2-27)}$$

where $T^2(N, T-2, \delta)$ denotes a non-central *Hotelling* distribution with degrees of freedom N, T-2 and δ, where

$$\delta = T\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)^{-1}\underset{\sim}{\alpha}'\,\underset{\sim}{\Sigma}^{-1}\,\underset{\sim}{\alpha} \qquad \textbf{(2-28)}$$

is the *non-centrality parameter* of the distribution. To proceed, we make use of theorem 6.3.1 in *Muirhead (1982)*, p. 211.

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[20] Which we will refer to in subsection 3.3.

[21] *Muirhead (1982)* discusses the properties of the T^2 distribution.

[22] This distribution is a multivariate generalization of the X^2 distribution. In *Muirhead (1982)* one can find a discussion its properties.

***Theorem**. Let* $\underset{\sim}{x}$ *be* $N_m\left(\underset{\sim}{\mu},\underset{\sim}{\Sigma}\right)$ *and* $\underset{\sim}{A}=n\underset{\sim}{S}$ *be* $W_m(n,\underset{\sim}{\Sigma})\ (n\geq m)$*, with* $\underset{\sim}{x}$ *and* $\underset{\sim}{\Sigma}$ *independent, and put* $T^2=\underset{\sim}{x}'\underset{\sim}{S}^{-1}\underset{\sim}{x}$*. Then*

$$\frac{T^2}{n}\cdot\frac{n-m+1}{m}$$

is $F_{m,n-m+1}(\delta)$*, where* $\delta=\underset{\sim}{\mu}'\underset{\sim}{S}^{-1}\underset{\sim}{\mu}$.

For $\underset{\sim}{x}=\sqrt{T}\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)^{-\frac{1}{2}}\underset{\sim}{\hat{a}}$, $\underset{\sim}{S}=(T-2)\underset{\sim}{\hat{\Sigma}}$, $m=N$ and $n=(T-2)$ we have

$$\left(\frac{T}{T-2}\right)\left(\frac{T-N-1}{N}\right)\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)^{-1}\underset{\sim}{\hat{\alpha}}'\underset{\sim}{\hat{\Sigma}}^{-1}\underset{\sim}{\hat{\alpha}}\sim F(N,T-N-1,\delta) \qquad \textbf{(2-29)}$$

where $F(N,T-N-1,\delta)$ denotes a non-central *F* distribution, with *N* degrees of freedom in the numerator and *T-N-1* in the denominator and non-centrality parameter δ as defined above. If we use the standard –biased– *OLS* estimator for the covariance matrix $\underset{\sim}{\Sigma}$ instead of the unbiased estimator, the formula takes the form

$$GRS=\frac{T-N-1}{N}\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)^{-1}\underset{\sim}{\hat{\alpha}}'\underset{\sim}{\hat{\Sigma}}^{-1}\underset{\sim}{\hat{\alpha}} \qquad \textbf{(2-30)}$$

which **under the null** hypothesis (that $\underset{\sim}{a}=0$) follows a ***central F*** distribution (as the non-centrality parameter δ becomes zero). This statistic can be used to test the null hypothesis of zero regression intercepts (which is equivalent to an hypothesis of efficiency of the examined portfolio) given estimations of μ_p, σ_p, $\underset{\sim}{\alpha}$ and $\underset{\sim}{\Sigma}$. The first two parameters can be estimated directly from the data; the remaining require additionally the assumption of a return generation process, like the CAPM.

Alternatively, *GRS (1989)* prove in the appendix of their paper that

$$\underset{\sim}{\hat{\alpha}}'\underset{\sim}{\hat{\Sigma}}^{-1}\underset{\sim}{\hat{\alpha}}=\frac{\hat{\mu}_q^2}{\hat{\sigma}_q^2}-\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2} \qquad \textbf{(2-31)}$$

That is, they express the quadratic form $\underset{\sim}{\hat{\alpha}}'\underset{\sim}{\hat{\Sigma}}^{-1}\underset{\sim}{\hat{\alpha}}$ as a simple function of the Sharpe ratios of the tangency ***q*** and the actual portfolio ***p***. Under this transformation, the statistic takes the form

$$GRS = \frac{T-N-1}{N}\frac{\left(\frac{\hat{\mu}_q^2}{\hat{\sigma}_q^2}-\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)}{\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)} \qquad \textbf{(2-32)}$$

which is distributed as $F(N, T-N-1)$ under the null.

This interpretation is both operational[23] *and economically intuitive.* If the *Sharpe ratio* of the portfolio under examination is significantly less than the Sharpe ratio of the tangency portfolio, the test statistic will take a rather large value and indicate towards the rejection of the hypothesis of efficiency.

A third interpretation for the GRS test was offered by Britten-Jones (1999) as a test of the restriction that the weights of the tangency portfolio are proportional to the weights of the test portfolio. Such a test can be implemented using a standard *OLS F-test* to test a set of linear restrictions on regression coefficients. Furthermore, extensions were provided, that enable the testing of any hypothesis that can be expressed as a linear restriction on efficient portfolio weights[24], including hypotheses for individual coefficient restrictions (with Standard *t-tests*).

The main advantage of this test is that it follows an ***exact small sample distribution***. As a result, it does not suffer from size deviations that have been observed in asymptotic tests. *GRS* document this, making use of the fact that the three asymptotic tests (*Wald, Likelihood Ratio and Lagrange Multiplier*) are monotonic transformations of their test (and each other). Naturally, they verify the *Berndt and Savin (1977)* result.

Another advantage of this statistic is that it has the desirable property of being the ***uniformly most powerful invariant test***[25] (see *Muirhead (1982)*, pp. 212-215)). It can also be shown that it can be regarded as a *Likelihood Ratio test*. However, the power performance of this test in practical applications has been questioned. First by *MacKinley (1987)*, who used this statistic to test asset pricing theories. He concluded that one can see a substantial increase in the power of the tests when using a specific

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[23] For it only requires the sample mean return and variance of the actual portfolio and the mean return and variance of the tangency portfolio, which is provided by the solution of the efficient frontier. Consequently, we can interpret this statistic under the philosophy of Jobson and Korkie.

[24] A direct application of this is to test the potential performance of a subset of assets relative to a full set of assets, in a *Jobson and Korkie (1982)* fashion.

[25] A uniformly most powerful invariant test is most powerful among all the tests that preserve the original parameter space of the class of models under consideration. If knowledge or assumptions reduce the parameter space, then the uniformly most powerful in the larger parameter space often will be less powerful than tests that work within the smaller parameter space. *Affleck-Graves and McDonald (1990)* examine alternative tests that are found more powerful than the GRS tests, imposing restrictions on the elements of the covariance matrix Σ (parameter reduction).

alternative[26], while with an unspecified alternative hypothesis an important determinant of the power is the type of deviation present; the tests can have reasonable power if the deviation is random across assets, but if the deviation is a result of missing factors the tests are quite weak. Moreover, he investigated whether there are power gains when measuring returns more frequently (for example, on weekly instead of monthly intervals). Such power gains are likely, but then the *normality* and *independence* assumptions are strained, a fact that makes the results questionable.

In practice, the evaluation of the power of the test is not an easy task, because it depends on the distribution of the test under the alternative hypothesis. This distribution is a *non-central F*, which depends on the *non-centrality parameter* δ as in **(2-28)**. From **(2-28)** we can see that δ depends on the returns of the actual portfolio (p), or in other words the F distribution under the alternative is conditional on the returns of p. The power increases as δ increases[27]. Furthermore, combining **(2-28)** and **(2-31)**, we can express δ as a function of the Sharpe ratios:

$$\delta = T\frac{\left(\frac{\mu_q^2}{\sigma_q^2}-\frac{\mu_p^2}{\sigma_p^2}\right)}{\left(1+\frac{\mu_p^2}{\sigma_p^2}\right)} \qquad \textbf{(2-33)}$$

Using this relation, one need only specify the difference in the squared Sharpe ratios for the tangency portfolio and the market portfolio. Setting specific values for the means and variances of the tangency and the actual portfolio, *Campbell et al (1997)* find in their simulations that generally the test's power increases with N decreasing and T increasing, but such power gains would not be feasible in practice because with N decreasing, the Sharpe ratio of the tangency portfolio declines, affecting accordingly δ and the value of the power. Hence they state that the choice of N is sensitive and generally it should be kept small (perhaps not larger than 10).

GRS (1989) present some power results too, under the specific case that the off-diagonal elements of the covariance matrix of residuals are constant. Under this parameterization, they find that δ is approximately proportional to N and T. However, this is not adequate to determine the impact of changing N and T, for these two parameters affect not only the non-centrality parameter but also the degrees of freedom. In conclusion, they suggest that an empiricist, who has to decide on the ideal level of N and T, should keep N roughly a third to one half of T.

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[26] Such as the zero beta version of CAPM or a multifactor model against the traditional CAPM. The increase of power for a multivariate test of linear restrictions when more information about the alternative hypothesis is available was pointed out too by *Berndt and Savin (1977)*, p. 1272.
[27] According to *GRS (1989)*, p. 1131.

Finally, *GRS (1989*) show that their test procedure is applicable to test the potential performance of a subset of N_1 assets from a set of N assets. In such a case, the form of the statistic is

$$GRS_{pp} = \frac{T-N-N_1}{N} \frac{\left(\frac{\hat{\mu}_q^2}{\hat{\sigma}_q^2} - \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2} \right)}{\left(1 + \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2} \right)} \qquad \textbf{(2-34)}$$

which under the null hypothesis follows a central F distribution with N degrees of freedom in the numerator and $T\text{-}N\text{-}N_1$ in the denominator.

2.2.3 The GMM test

The testing procedures discussed previously based on the assumptions that asset returns were *jointly Normal* and *IID* through time. Under these assumptions, *ML* estimators can be derived for financial models. However, the *normality, homoskedasticity* and *independence* assumptions in financial price data have been frequently questioned. Hence, an interesting area of research is to investigate whether the available tests, which depend on the rather binding normality assumptions for returns, are robust to the violation of these assumptions.

The main drawback of *ML* estimation requires complete specification of the distribution of the observed random variable. If the correct distribution is somewhat other than what we assume, then the likelihood function is misspecified and the desirable properties of *ML* estimators may not hold (*Greene (2000)*).

Many previous studies have considered the distribution of price changes in speculative series. Early work (for example *Bachelier (1900)*) suggested that the distribution should be *Normal*[28]; later however, researchers such as *Mandelbrot (1963)* and *Fama (1965)* provided evidence against normality, indicating that the price change distributions for many speculative series were *leptokurtic.* This gave rise to alternative distributions for the description of asset returns, such as the *stable Paretian*, the *mixture of normals* and the *Student t.*

However, despite the concerns about non-normality raised by these previous studies, most studies of asset pricing theories have either explicitly or implicitly assumed normality of the residuals in the market model regression. A possible explanation for this is that many asset pricing tests have used monthly data, and it has

[28] Actually, from a statistical perspective, the tests require that asset returns **conditional** on the factor portfolios be IID and multivariate normal. Additionally, it is normality of the market model residuals

been argued that the normal distribution provided a good working approximation for **monthly** returns (*Fama (1976)*).

Affleck-Graves and McDonald (1989) examined the robustness of the *GRS* test to deviations from normality with monthly returns. They looked into the size and power characteristics of the *GRS* test at various levels of departure from normality. Their empirical results show that the *test is sensitive to certain large deviations from normality*, with both its size and power increasing. However, they note that the level of non-normality necessary to produce a substantial misspecification of the test is not typical in random samples of security data.

Additionally, *Huang and Litzenberger (1988)* mention that the fact that the disturbance terms in the linear regression model **(2-3)** are heteroskedastic and serially correlated is a major econometric problem and leads to inefficiency of *OLS* relative to *Generalized Least Squares (GLS)* estimators. This indicates that tests based on *OLS* estimators would be less powerful than tests based on *GLS* estimators.

MacKinley and Richardson (1991) –MR henceforth– employ the *Generalized Method of Moments (GMM)* framework to develop alternative tests of *MV* efficiency under weak distributional assumptions. *Within the GMM context, the distribution of returns conditional on the market returns can be both serially dependent and conditionally heteroskedastic*. The only statistical assumptions required are that excess asset returns are *stationary* and *ergodic*[29] with *finite fourth moments*.

The *GMM* estimation procedure, unlike *ML*, does not rely on a distributional assumption; instead, it only requires the specification of certain *moment conditions*. When the system of moment equations is *exactly identified*, that is when the number of moment conditions is equal to the number of unknown model parameters, the *GMM* estimators can be obtained simply by equating the moment conditions to zero (like in the classical method of moments framework). But there are cases in which there are more moment conditions than parameters, so the system is overdetermined and doesn't yield a unique solution. In such a case, to use all the information in the sample it is necessary to devise a way to reconcile the conflicting estimates that will emerge from the overdetermined system. This is possible by minimizing a criterion function that weighs the conflicting estimates with a certain weighting matrix *(Greene 2000)*. The sampling theory for GMM estimation, as well as the optimal GMM weighting matrix was developed by *Hansen (1982)*.

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instead of normality of returns that is critical in both Fama and MacBeth type tests and multivariate tests of theories.

[29] The notion of ergodicity implies that the time average of a time series over a period of *T* observations converges in mean square to the corresponding ensemble average as *T* approaches infinity.

MR use the excess return market model of **(2-3)**, but without making the assumption of normal disturbances; instead they only assume that excess asset returns $\underset{\sim}{z}_t$ are *stationary* and *ergodic* with *finite fourth moments*.

To obtain a *GMM* estimator for $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$**,** they set the following moment condition

$$g_T(\underset{\sim}{\delta}) = E\left[f_t(\underset{\sim}{\delta}) \right] = \frac{1}{T}\sum_{t=1}^{T} f_t(\underset{\sim}{\delta}) = 0 \qquad \textbf{(2-35)}$$

where[30]

$$f_t(\underset{\sim}{\delta}) = \begin{bmatrix} 1 \\ z_{pt} \end{bmatrix} \otimes \underset{\sim}{e}_t \qquad \textbf{(2-36)}$$

z_{pt}, $\underset{\sim}{e}_t$ as defined in **(2-3)** and $\otimes$ denotes the Kronecker product.

The *GMM* estimation procedure involves selecting an estimator to set linear combinations of the moment condition to zero. For this purpose, a weighting matrix $\underset{\sim}{W}$ is needed[31], so that $\underset{\sim}{W}\, g_T(\hat{\underset{\sim}{\delta}}) = 0$. *Hansen (1982)* has shown that the optimal weighting matrix for this purpose is $\mathbf{W}^*$, where

$$\underset{\sim}{W}^* = \underset{\sim}{D}_0' \, \underset{\sim}{S}_0^{-1} \qquad \textbf{(2-37)}$$

$$\underset{\sim}{D}_0 = E\left[\frac{\partial g_T(\underset{\sim}{\delta})}{\partial \underset{\sim}{\delta}'} \right] \qquad \textbf{(2-38)}$$

$$\underset{\sim}{S}_0 = \sum_{k=-\infty}^{+\infty} E\left[f_t(\underset{\sim}{\delta})\, f_{t-k}(\underset{\sim}{\delta})' \right] \qquad \textbf{(2-39)}$$

Under the *GMM* framework, *MR* propose two alternative ways of testing. The first approach is to estimate the *unrestricted system* and then test the hypothesis $\underset{\sim}{a} = 0$ using the *unrestricted estimates*. The second approach is to substitute the restrictions $\underset{\sim}{a} = 0$ in **(2-36)**, estimate the *restricted system* and then test the overidentifying restrictions.

Under the first approach, the system is *exactly identified* (with *2N parameters* and *2N orthogonality conditions*), and the *GMM* estimators of the parameters can be obtained directly by setting the sample moments to zero. The estimators that will be

[30] The rows of the vector of the moment conditions are sometimes called *orthogonality conditions*.

[31] Clearly, the weighting matrix is not necessary for an exactly identified case for purposes of estimation. However, we need to define it at this point because it resembles the asymptotic variance of the *GMM* estimator, which will be used to construct the *GMM* statistic.

derived by this method will naturally be to *OLS* estimators. Further, to develop a *Wald* statistic, we need to know the *asymptotic covariance matrix* and the *asymptotic distribution* of the vector of *GMM* estimators, $\hat{\underset{\sim}{\delta}}$. *Hansen (1982)* showed that $\hat{\underset{\sim}{\delta}}$ is *distributed asymptotically normally* with mean $\underset{\sim}{\delta}$ and *asymptotic covariance matrix*[32] $\left[\underset{\sim}{D_0}' \underset{\sim}{S_0}^{-1} \underset{\sim}{D_0}\right]^{-1}$. Hence, we can use the following *Wald* statistic to test the null hypothesis of efficiency of a portfolio:

$$\vartheta = T\,\underset{\sim}{\hat{a}}' \left[\underset{\sim}{R}\left[\underset{\sim}{D_0}' \underset{\sim}{S_0}^{-1} \underset{\sim}{D_0}\right]^{-1}\underset{\sim}{R}'\right]^{-1}\underset{\sim}{\hat{a}} \qquad \textbf{(2-40)}$$

where $\underset{\sim}{R} = I_{\mathbf{N}} \otimes \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $\underset{\sim}{R}\hat{\underset{\sim}{\delta}} = \underset{\sim}{\hat{\alpha}}$. Under the null hypothesis, ϑ is distributed asymptotically as χ^2_N.

Under the second approach, we can substitute the restrictions[33] $\underset{\sim}{a} = 0$ in **(2-36)**, and then obtain the GMM estimators of $\underset{\sim}{\delta}$ (which has been degenerated to $\underset{\sim}{\beta}$ after setting $\underset{\sim}{a} = 0$), in a somewhat different manner from before[34]. After setting the restriction of the null hypothesis, the system becomes *overidentified*, as we have *2N* equations and *N* unknown parameters. Thus we cannot set all the sample moment conditions to zero at the same time. But we can set to zero the optimal linear combination of moments, $\mathbf{W}^{*} g_T(\hat{\underset{\sim}{\delta}}) = 0$. Then according to *Hansen (1982),* $\hat{\underset{\sim}{\beta}} \overset{\alpha}{\sim} N(\underset{\sim}{\beta}, \left[\underset{\sim}{D_0}' \underset{\sim}{S_0}^{-1} \underset{\sim}{D_0}\right]^{-1})$ and the corresponding *Wald* statistic to test the *N* overidentifying restrictions ($\underset{\sim}{a} = 0$) of the model will be:

$$\vartheta' = T g_T(\hat{\underset{\sim}{\beta}})' \mathbf{S}_{\mathbf{T}}^{-1} g_T(\hat{\underset{\sim}{\beta}}) \overset{\alpha}{\sim} \chi^2_N \qquad \textbf{(2-41)}$$

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32 In practice $\underset{\sim}{D_0}$ and $\underset{\sim}{S_0}$ will be unknown, but the asymptotic results still hold for *consistent estimators* of them, denoted as $\underset{\sim}{D_T}$ and $\underset{\sim}{S_T}$. Additionally, an assumption with respect to $\underset{\sim}{S_0}$ is necessary in order to reduce the summation to a finite number of terms and permit construction of a consistent estimator. Estimation procedures for $\underset{\sim}{D_T}$ and $\underset{\sim}{S_T}$ can be found, for instance, in *Newey and West (1987a)*.

33 Certainly, a restricted GMM test could be carried to test a nolinear hypothesis, such as $\alpha = \gamma(1-\beta)$.

34 Apparently, this GMM estimator will be different from its OLS counterpart, because it uses a weighting matrix in the process.

For the unrestricted case, ***MR (1991)*** **develop a more operational formula for the GMM test, under the assumption of contemporaneous conditional heteroskedasticity** (and no serial correlation[35]) for the residuals of the excess return generating process. In compact form, the expression of the unrestricted GMM test is

$$\vartheta = T\,\underset{\sim}{\hat{a}}'\,\underset{\sim}{\hat{\Omega}}^{-1}\,\underset{\sim}{\hat{a}}' \qquad \textbf{(2-42)}$$

where $\underset{\sim}{\hat{\alpha}}$ is the standard *OLS* estimator (or *ML* estimator, since both are equivalent to the *GMM* estimator in the *exactly identified* case) of the intercept of the model **(2-36)** and $\underset{\sim}{\hat{\Omega}}$ is the asymptotic *GMM* estimator of the covariance matrix (which is essentially different from the corresponding *ML* estimator). Under the *heteroskedasticity* assumption, *MR* prove that the asymptotic variance of $\hat{a}$ can be expressed as

$$\mathrm{var}(\underset{\sim}{\hat{a}}) = \underset{\sim}{\Omega} = \left(1+\frac{\mu_p^2}{\sigma_p^2}\right)\underset{\sim}{\Sigma} + \frac{1}{\sigma_p^2}\underset{\sim}{\Psi} \qquad \textbf{(2-43)}$$

where $\underset{\sim}{\Sigma}$ is the $N \times N$ covariance matrix of the residuals of the regression model and $\underset{\sim}{\Psi}$ is a $N \times N$ **correction factor** matrix that captures the effect of heteroskedasticity. The (i, j)th element of Ψ has the form

$$\left[\mu_p^2\,\sigma\left(e_{it}e_{jt}, z_{pt}^2\right) - 2\mu_p\left(1+\frac{\mu_p^2}{\sigma_p^2}\right)\sigma\left(e_{it}e_{jt}, z_{pt}\right)\right]$$

where $\sigma(\cdot)$ denotes the covariance operator.

In applications $\underset{\sim}{\Omega}$ can be replaced by its consistent estimator $\underset{\sim}{\hat{\Omega}}$, without loss of the asymptotic result (that $\theta \overset{\alpha}{\sim} \chi_N^2$). If we examine formulas **(2-42)** and **(2-43)**, we can see that what distinguishes the *GMM Wald* statistic from a *standard Wald* (under the normality assumption), is the asymptotic covariance matrix of $\underset{\sim}{\hat{a}}$ provided by **(2-43)**, having in mind that the *GMM* estimates of $\underset{\sim}{\hat{a}}$ are the same with the *ML* estimates. Moreover, if we focus on formula **(2-43)**, we can see that $\mathrm{var}(\underset{\sim}{\hat{a}})$ is a sum of two factors: $\left(1+\frac{\mu_p^2}{\sigma_p^2}\right)\underset{\sim}{\Sigma}$, which is the asymptotic covariance matrix of $\underset{\sim}{\hat{a}}$ in the trivial case (under the *normality* assumption), and a correction factor that incorporates the effect of heteroskedasticity. In the absence of *heteroskedasticity*, the correction matrix is zero and the *GMM* test

[35] Their results can be generalized to allow for **serial correlation** using, for example, the technique of *Newey and West (1987a)*.

is equivalent to the standard Wald test and asymptotically equivalent to other tests developed under the normality framework. By this regard, *other tests can me considered special cases*[36].

Furthermore, ***MR* explore a specific case where *heteroskedasticity* can be accommodated under the assumption that the asset returns follow the *Student t* distribution** (instead of *Normal*). This assumption is realistic and appealing, both theoretically and empirically. One empirical stylized fact is that returns have fatter tails and are more peaked than one would expect from a normal distribution. This is consistent with returns coming from a multivariate *Student t*. This distribution has appeared in the finance literature by authors such as *Blattberg and Gonedes (1974)*.

In the multivariate *Student t* case, the regression equations are the same as for the multivariate normal case except that the conditional variance of the error term is no longer independent of the regressor (portfolio returns). Using relationship **(2-43)**, *MR* derive the formula for the asymptotic covariance matrix of $\underset{\sim}{\hat{a}}$:

$$\underset{\sim}{\Omega} = \left[1 + \frac{(v-2)}{(v-4)}\frac{\mu_p^2}{\sigma_p^2}\right]\underset{\sim}{\Sigma} \qquad \textbf{(2-44)}$$

Hence, using estimations for the parameters of the above formula, the *GMM* statistic takes the form

$$\vartheta = T\,\underset{\sim}{\hat{a}}'\left[\left(1 + \frac{(v-2)}{(v-4)}\frac{\mu_p^2}{\sigma_p^2}\right)\underset{\sim}{\hat{\Sigma}}\right]^{-1}\underset{\sim}{\hat{a}} \qquad \textbf{(2-45)}$$

Furthermore, exploiting the result **(2-31)** of *GRS (1989)*, we get

$$\theta = T\left(\frac{\hat{\mu}_q^2}{\hat{\sigma}_q^2} - \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)\Big/\left(1 + \frac{(v-2)}{(v-4)}\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right) \qquad \textbf{(2-46)}$$

where *v* are denoted the degrees of freedom of the *t* distribution. This is clearly less than the usual *Wald* statistic,

$$\omega = T\left(\frac{\hat{\mu}_q^2}{\hat{\sigma}_q^2} - \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)\Big/\left(1 + \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right) \qquad \textbf{(2-47)}$$

The exact relationship is

$$\vartheta = \left[\left(1 + \frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)\Big/\left(1 + \frac{(v-2)}{(v-4)}\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)\right]\omega \qquad \textbf{(2-48)}$$

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[36] However, the cost of the more general framework is the loss of sample results for the test distribution (compared to the exact *F* tests of *GRS (1989)* and *JK (1982,1985)*).

Under the *Student t* assumption, *MR* examined a case where the bias of the common test statistic can be quantified. An appropriate test statistic in an environment where returns are multivariate *Student t* can be expressed as the usual statistic scaled by a factor that accounts for the *conditional heteroskedasticity*. The magnitude of the impact of the contemporaneous conditional heteroskedasticity will depend on the degrees of freedom of the *Student t* and the sample Sharpe ratio of the portfolio being tested. As the degrees of freedom become large, the *Student t* is well approximated by the *Normal* distribution, and the conditional heteroskedasticity will be reduced. Asymptotically, the conditional heteroskedasticity will vanish.

MR assess the degree that ω overstates ϑ. They find that the misspecification can be as large as 35%, and that the error in the size of the test increases as the number of assets *N* increases and decreases as the degrees of freedom of the *Student t* increase. Therefore, the *GMM* statistic is more appropriate when the distribution of returns is a *Student t* distribution instead of normal.

Working with real data, *MR* compare the performance of the *GMM* statistic developed for the exactly identified case (in **(2-40)**) with the standard *Wald* –from **(2-47)**– and *F* –or *GRS*– statistics. They find that both the *Wald* and *F* statistic understate the *GMM* statistic, resulting in lower probability to reject the null hypothesis. They also show with simulations that this result cannot be attributed to poor small sample performance of the *GMM* statistic. Therefore, in that case the standard tests understate the size compared to the *GMM* tests. This proves that the biases introduced by violations of the *IID* and *joint normality* assumptions can be in either direction. Nevertheless, the *GMM* test improves inference by adjusting the size of the tests for serial correlation and/or conditional heteroskedasticity that may be present in the data.

Finally, it is worth adding that the *GMM* test for portfolio efficiency by *MR (1991)* is a *Wald* test in construction. *Newey and West (1987b)* suggest counterparts for the *Likelihood Ratio* and the *Lagrange Multiplier* tests as well. Just as in the *ML* case, all three statistics converge in the same limiting χ^2 distribution. Even more convenient is the result that **under linearity in the orthogonality conditions and the constraints** (such as for model **(2-3)**), **all the test statistics are numerically equivalent** (*Newey and West (1987b)*, p. 785). That means that the “disturbing” numerical inequality pointed out by *Berndt and Savin (1977)* within the *ML* context, does not hold under the *GMM* context; as a consequence, a choice of a test statistic depends even more on computational convenience.

2.3 Tests of MV efficiency under the Black CAPM

In the absence of a risk-free asset, we move to the ***zero-beta*** or ***Black*** version of the ***CAPM***. The expected rate of return on the zero-beta portfolio[37], or ***zero-beta return***, is treated as unobservable and hence becomes an unknown model parameter. Defining the ***zero-beta portfolio*** return as γ, the ***Black*** version is:

$$E[\underset{\sim}{r}_t] = \underset{\sim}{i}\gamma + \underset{\sim}{\beta} E[r_{pt} - \gamma] = (\underset{\sim}{i} - \underset{\sim}{\beta})\gamma + \underset{\sim}{\beta} E[r_{pt}] \qquad \textbf{(2-49)}$$

where $\underset{\sim}{i}$ is a vector of ones.

$\underset{\sim}{r}_t$ is provided by the ***real-return market model***:

$$\underset{\sim}{r}_t = \underset{\sim}{\alpha} + \underset{\sim}{\beta} r_{pt} + \underset{\sim}{\varepsilon}_t \qquad \textbf{(2-50)}$$

$E\,\underset{\sim}{\varepsilon}_t = 0$

$E\,\underset{\sim}{\varepsilon}_s \underset{\sim}{\varepsilon}_t' = \underset{\sim}{\Sigma} \quad \text{if } s = t$

$E\,\underset{\sim}{\varepsilon}_s \underset{\sim}{\varepsilon}_t' = 0 \quad \text{if } s \neq t$

$E(r_{pt}) = \mu_p$

$Var(r_{pt}) = \sigma_p^2$

$Cov(r_{pt}, \underset{\sim}{\varepsilon}_t) = 0$

where:

$\underset{\sim}{r}_t$: *N×1* vector of real asset returns for time period t

r_{pt} : real market (or single portfolio) return for period t

$\underset{\sim}{\varepsilon}_t$: *N×1* disturbance vector

$\underset{\sim}{\alpha}$, $\underset{\sim}{\beta}$: *N×1* parameter vectors

$\underset{\sim}{\Sigma}$: *N×N* disturbance covariance matrix

By comparing the unconditional expectation of **(2-50)** with **(2-49)**, the testable implication for the Black version becomes apparent:

$$\underset{\sim}{\alpha} = (\underset{\sim}{i} - \underset{\sim}{\beta})\gamma \qquad \textbf{(2-51)}$$

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[37] The ***zero-beta portfolio*** is a portfolio with zero correlation with the market portfolio. Hence, its return can play the part of the return of the riskless asset in the ***Lintner-Sharpe CAPM***.

This implication is more complicated to test than the zero-intercept restriction of the Sharpe-Lintner version because the parameters $\underset{\sim}{\beta}$ and γ enter in a *nonlinear* fashion.

Given the *IID* and *joint normality* assumption of returns, the Black version can be estimated and tested using the *ML* approach. The *ML* estimators for the unrestricted model can be derived in the same manner as the unrestricted estimators in the *Sharp-Lintner* model. For the restricted model, we can also take *ML* estimators[38], except that calculations are more demanding; an explicit solution for the *ML* estimators from the system of equations in *Campbell et al (1997)*, p. 199, cannot be obtained. Researchers used various techniques to overcome this problem and derive statistics for *MV* efficiency of portfolios. Below we present the approaches of *Gibbons (1982)*, *Kandel (1984)* and *Shanken (1985,1986).*

Gibbons (1982) first suggested an algorithm that, making use of an iterating procedure, **provides (approximately) the restricted estimators and enables the implementation of a *Likelihood Ratio* test**

$$G = T\left[\ln\left|\underset{\sim}{\hat{\Sigma}_R}\right| - \ln\left|\underset{\sim}{\hat{\Sigma}_U}\right|\right] \qquad \textbf{(2-52)}$$

which follows asymptotically a X^2_{N-1} distribution under the null. $\left|\underset{\sim}{\hat{\Sigma}_R}\right|$ and $\left|\underset{\sim}{\hat{\Sigma}_U}\right|$ are the determinants of the contemporaneous covariance matrices estimated from the restricted and the unrestricted model respectively. This statistic is equivalent to the standard *Likelihood Ratio* statistic for the *Sharpe-Lintner* model, with the difference that the degrees of freedom for the limiting χ^2 distribution have been reduced by one, because of the extra parameter (*γ*) that needs to be estimated.

The main drawbacks of this testing procedure are the tedious calculations[39] required and the poor finite sample properties. Nevertheless, Gibbons managed to reject his hypothesis of the *zero-beta CAPM* (indication of sufficient power for the statistic). However, *Jobson and Korkie (1982)* objected to Gibbon's results, attributing the seemingly sufficient power of his statistic to non-conformity to the theoretical χ^2 distribution. Instead, they proposed the Bartlett's correction[40] to adjust the *LR* statistic for the excess skewness that it exhibits when *N* is

[38] In this case, we can't take *OLS* estimators alternatively, because the *OLS* methodology cannot be used under a non-linear constraint. See *Huang and Litzenberger (1988)*, p. 314.

[39] The author himself admitted that his algorithm is only suggestive and not desirable due to the computational considerations. For the actual calculations he used a one-step Gauss-Newton procedure instead.

[40] Which they used for their χ^2 *LR* statistic.

relatively large compared to *T*. In fact with the same set of data and using this correction they ended up with different inference than Gibbons.

Kandel (1984) integrated Gibbons's work, by **deriving the exact form of the *ML* estimators for the constrained model and suggesting a *LR* test with good finite sample performance**. The *ML* estimators can be obtained directly as follows[41]: First maximize the likelihood function conditional on a given estimate of γ, to get the *ML* estimators of $\underset{\sim}{\Sigma}$ and $\underset{\sim}{\beta}$; then substitute these estimates in the original likelihood function to produce the ***concentrated likelihood function*** (as a function of γ only); finally maximize the concentrated likelihood function to produce a *ML* estimator of γ[42]. The *LR* statistic proposed by *Kandel (1984)* is

$$K = T \ln \left[\frac{\hat{\mu}_U - \hat{\gamma}}{\hat{\sigma}_U^2} \Big/ \frac{\hat{\mu}_R - \hat{\gamma}}{\hat{\sigma}_U^2} \right] \qquad \textbf{(3-53)}$$

where $\hat{\mu}_U$ and $\hat{\mu}_R$ are the unrestricted and restricted estimates of the expected excess rate of return on the ***minimum variance portfolio***, respectively, $\hat{\sigma}_U^2$ and $\hat{\sigma}_U^2$ the unrestricted and restricted estimates of the variance of the ***minimum variance portfolio***, respectively, and $\hat{\gamma}$ is the ML estimate of the zero beta rate of return. Under the null hypothesis, **this statistic is asymptotically distributed as** χ^2_{N-2}. Kandel provided also a meaningful geometric and economic interpretation for his statistic.

Finally, *Shanken (1985)* **develops a "cross-sectional regression test" (CSRT) for portfolio efficiency under the zero-beta CAPM, based on the Hotelling T^2 statistic**[43]. He uses a *GLS* estimator for the zero-beta rate of return and derives an asymptotic approximation for the distribution of the test as well as a useful small-sample bound. *Shanken (1986)* generalizes these results for tests of *potential performance*.

In their original expressions, the Shanken statistics are residual sums of squares from a *GLS* cross sectional regression. *Jobson and Korkie (1989)*, in a unified survey of multivariate *F* tests for *MV* efficiency, intersection and spanning, suggest the following alternative formulas[44] for the Shanken statistics:

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[41] See also *Huang and Litzenberger (1988)*.

[42] For the explicit algebraic procedure that derives $\hat{\gamma}$, see *Kandel (1984)*, pp. 580-581 (Theorem 1).

[43] Naturally, inference can be facilitated by the result $F_{n,m-n+1} \equiv (m-n+1/mn)T^2_{n,m}$.

[44] *JK (1989)* claim that these expressions allow the relationships among the tests to be seen more easily.

$$\varphi_z = \frac{T-N-1}{N-2}\frac{(\hat{a}_z - \hat{a}_{z1})}{(1+\hat{a}_{z1})} \qquad \textbf{(2-54)}$$

$$\varphi'_z = \frac{T-N-1}{N-N_1-1}\frac{(\hat{a}_z - \hat{a}_{z1})}{(1+\hat{a}_{z1})} \qquad \textbf{(2-55)}$$

where $\hat{a}_z$ and $\hat{a}_{z1}$ can be defined in the same fashion as $\hat{a}$ and $\hat{a}_1$ in *Jobson and Korkie (1982)*, except that excess returns are calculated by subtracting Shanken's *GLS* estimator of the zero-beta rate from real returns. **In large samples, φ_z and φ'_z are distributed under the null hypothesis, that $\hat{a}_z = \hat{a}_{z1}$, as $F_{N-2,T-N-1}$ and $F_{N-N_1-1,T-N-1}$ respectively**. Another alternative form of the statistic for portfolio efficiency, along with its geometry, is provided by *Roll (1985)*.

3. The Equity Home Bias Puzzle

Important literature has been devoted to the study of the benefits of international diversification. The weak correlation between national financial markets and the idea that the world financial market is more complete than national markets justify two essential motives for international diversification: the search for higher returns and the reduction of risk (*Coen (2001)*).

Even though the benefits of international diversification have been recognized quite early[45], it has been observed that investors hold a substantially larger proportion of their wealth portfolios in domestic assets than standard portfolio theory would suggest. The literature has described this phenomenon as *Home Bias* and considers it as one of the major puzzles in international macroeconomics (*Obstfeld and Rogoff (2000)*). The puzzle has received more attention in the case of equity portfolios –*Equity Home Bias*–, but it has been observed in other classes of assets as well. In the absence of this bias, investors would optimally diversify domestic risk using foreign assets. *French and Poterba (1991)* first highlighted the extent of the equity home bias puzzle at the end of the 1980s, and estimated the percentages of aggregate stock-market wealth invested in domestic assets at the beginning of the 1990s well above 90% for USA and Japan and around 80% for the UK and Germany. **While recent years have witnessed an increase in international diversification, holdings of domestic assets are far too high to be consistent with the standard theory of portfolio selection.**

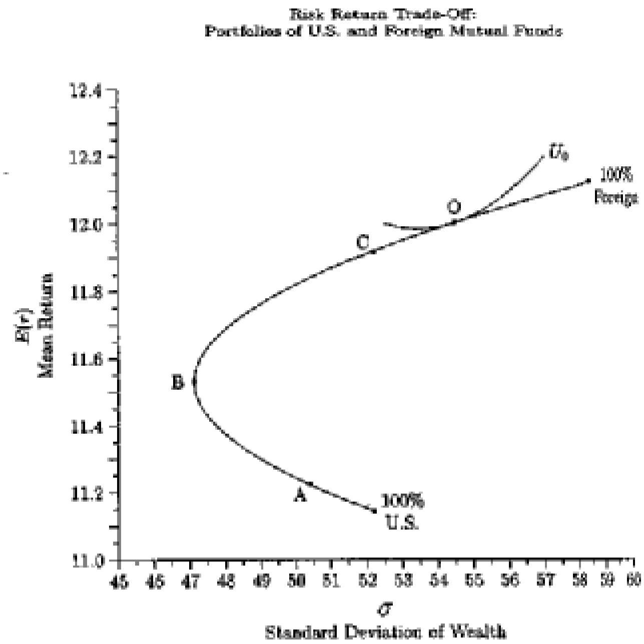
Lewis (1998) provides a thorough survey of the phenomenon and evaluates existing explanations for it. She also distinguishes the Equity Home Bias in the finance literature from the Consumption Home Bias[46] in the macroeconomics literature. We will focus on the equity version of the Home Bias. *In the remainder of this section we will describe the Equity Home Bias Puzzle and discuss in brief the main explanations that have been suggested in the literature.*

The figure below is taken from *Lewis (1998)*. She plots in a familiar mean return-standard deviation space a simplified version of the efficient frontier, where she places an artificial US mutual fund (measured by the S&P 500 index) and an artificial non-US international fund. Moving from the southwest to the northeast, the

[45] See *Tesar and Werner (1995)* for early references, or *Grauer and Hakansson (1987)* for the gains of international portfolio diversification for an American investor.

[46] The Consumption Home Bias concerns the lack of risk sharing observed in consumption comovements among countries.

line plots the mean returns and standard deviations from holding an increasing proportion of foreign stocks (for the US fund). With 100% domestic equity holdings, it is clear that the US fund is suboptimal, whereas at point C, with some proportion of foreign assets held, it could achieve much higher return at the same level of risk. Hence, point C would be more preferable according to portfolio theory. Recent estimates put the share of US holdings of foreign equity at around 8%, or point A, which is still suboptimal (like any point below the global minimum variance portfolio at B).



Further, using a simplified version of the International CAPM model[47] and assuming that domestic investors have access to two risky assets, domestic equity and foreign equity, finds that the optimal portfolio weight for the foreign asset is about 40%, while the observed share of foreign equity holding for an American investor is about 8% only. **These numbers suggest the strong presence of a home equity bias.**

One set of explanations for the home bias puzzle maintains that domestic investors have alternative methods of diversification of risk that provide the same benefits as holding foreign stock. According to this line of reasoning, other assets may provide the diversification potential without requiring domestic investors to look abroad for securities with requisite attributes. This sounds more plausible for larger and more developed markets, like the US market, where an investor can mimic the benefits of international diversification through domestic diversification (across sectors for example). An intuitive support for this theory is the fact that home bias is more pronounced for larger countries, which constitute a larger proportion of the world market and thus offer better investment opportunities domestically than smaller countries. However, this argument does not hold up empirically, because the country effect has been recognized as the most important factor of explaining the source of risk in stock investment, far beyond the sector (or industry) effect.

[47] The standard Sharpe-Lintner model modified to include foreign securities.

Others maintain that the lack of marketable trade in certain parts of wealth can affect the importance of diversification opportunities. *Baxter and Jermann (1997)* note that ignoring non-marketable wealth can have important implications for the standard home bias intuition. In particular, human capital is a significant fraction of wealth and may behave differently than financial wealth. Under a variety of assumptions about human capital measurement, *Baxter and Jermann (1997)* show that domestic human capital returns are highly correlated with the domestic stock market returns but negatively correlated with foreign stock returns. Since human capital is non-marketable, the observation implies that domestic investors should not only hold the foreign stock, but also short the domestic portfolio to put more of their wealth into the foreign stock. Therefore, the home bias puzzle is even more pronounced than thought. *Consequently, avenues of diversification other than holding foreign stocks do not appear to explain the puzzle and may even worsen it* (*Lewis (1998)*).

An alternative explanation for the equity home bias puzzle is that the gains from diversifying are insufficient to warrant the costs involved with diversifying the portfolio. Such costs include transaction costs in cross-border investment, taxes and other barriers that erode the potential gains.

While transaction costs and taxes are evident, it is hard to believe that they can completely offset the apparent gains and thus mitigate international investment. In fact, *Lewis (1998)* finds that the gains appear enormous, of the order of 20%-100% of permanent consumption, while the costs apparently do not keep domestic investors from turning over foreign securities at a rate of 3 to 7 times the turnover rate of domestic securities.

Other less "tangible" costs of international diversification have been mentioned as sources of the home bias puzzle. These are costs related to asymmetric information, differential access to markets and individual perception of risk by investors. Asymmetric information implies that different classes of investors, such as institutional and non-institutional investors, have different access to information and at a different cost. The same implication is made for differential access to markets, which is usually easier and cheaper for big players like institutional investors. In fact *Coen (2001)* asserts that this is the main explanation of the home bias puzzle, as asymmetric information plays an important role in risk perception. Thus, if domestic investors are convinced that they have to pay more to hold a foreign asset (even if this extra cost is more assumed than real), everything else being equal, they tend to prefer to hold more domestic assets. However, the growth of such investment instruments as mutual funds, tends to diminish this sort of cost too and therefore enhance the opportunities of international investment and accordingly diversification.

Trade barriers such as capital controls are unlikely to explain the presence and volume of home bias in equity investment either, especially as over the last three decades deregulation of financial markets and relaxation of capital controls have brought about increased opportunities for international investment, both in developed and emerging markets. One cannot even blame the foreign investment restrictions imposed on institutional investors (such as mutual funds, pension funds and life insurance companies), because *Tesar and Werner (1995)* find that actual investment positions in five G-7 countries are still substantially lower than the current limitations on foreign asset holdings for institutional investors.

Further, *Tesar and Werner (1995)* observe that the capital flows on international equity transactions tend to be higher than capital flows on domestic transactions. This suggests that investors frequently adjust the composition and size of their international portfolios[48]. If the relevant costs where indeed substantial, we would expect to find that investors engage in a buy-and-hold strategy and we should see fewer transactions on international securities in comparison with transactions on domestic securities.

To sum up, **the above explanations, despite appealing, have failed to explain the puzzle at a satisfactory degree**. In other words, **the home bias in portfolio investment remains a puzzle**.

However, the aim of this project is not to investigate the explanatory power of alternative factors to the equity home bias. Instead, **the aim of this project is to test whether the advent of EMU has led to increased foreign equity holdings, and thus more efficient portfolios, for European pension funds and life insurance companies, compared to non-European institutional investors.**

Hardouvelis et al (2001) refer to the gradual abolition of barriers to intra-EU investments and the launch of the common currency. First, investment barriers were lifted during the 1990s in an increased effort of EU governments to harmonize the regulatory framework of financial markets. Second, the launch of the common currency eliminated currency risk and lifted all remaining restrictions on the currency composition of institutional investors like pension funds and life insurance companies[49].

Furthermore, *Hardouvelis et al (2001)* present evidence with regard to the extent of the anticipated introduction of the single currency on cross border equity flows. Relative to the early 1990s, by mid 1998 cross border equity flows had nearly tripled to around $120-$140 billion. Estimates of the total rebalancing of equity portfolios from domestic to pan-European portfolios are in the region of $1.5 trillion[50] (more than one third of market capitalization).

Finally, Table 1 reports actual foreign equity holdings of pension funds and life insurance companies as a percentage of total equity holdings. Data are from Intersec Research[51]. **The increase of foreign equity holding for most EMU countries is apparent for both types of institutional investors, while for non-EMU countries the corresponding increase is rather moderate**. Foreign equity holdings of pension funds in EMU countries almost doubled on average in the period under examination, from 22.5% of total equity holdings in 1992 to 42.2% in 1999, while foreign equity holdings of pension funds in non-EMU countries increased more conservatively, from 16.2% to 23.8% over the same period. The portfolio reallocation towards foreign equity is even stronger for life insurance companies. Life insurance companies in EMU countries more than doubled their foreign equity holdings, from 11.5% of total equity holdings in 1992 to 25.9% in 1999. The corresponding numbers for non-EMU life insurance companies are 8.6% in 1992 to 13.5% in 1999. **These**

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[48] However, there is no evidence to determine what part of such high capital flows is the result of arbitrage activity of institutional investors.

[49] In most countries national regulations imposed limits on the share of foreign assets in their portfolios. For example, in France pension funds were restricted by a 95% currency matching rule between assets and liabilities. In Germany the corresponding restriction was 80%.

[50] Euromoney, August 1998.

portfolio shifts towards foreign equity suggest that risk sharing has increased dramatically among EMU investors, while non-EMU investors seem more reluctant to follow the same pattern.

Consequently, we are justified to expect that equity portfolios of institutional investors from EMU countries were more efficient in the 1990s than equity portfolios of their counterparts outside EMU, as a result of increased foreign equity holding. This would indicate that the equity home bias has ceased to exist within EMU. This accordingly would imply that **the existence of the bias previously should be attributed to factors that disappeared in the process: currency risk and legislative trade disincentives.** That would be an indirect way to explain the equity home bias, for the countries examined.

Table 1

Equity holdings of Pension Funds and Life Insurance Companies

	Pension Funds		Life Insurance Companies	
	1992	**1999**	**1992**	**1999**
EMU countries				
Austria	-	38.9	6.5	25.0
Belgium	50.0	80.8	7.1	58.8
France	14.3	37.3	7.9	19.4
Germany	18.3	31.7	5.2	34.7
Ireland	56.5	69.2	0.0	48.5
Italy	0.0	10.9	33.0	12.8
Netherlands	56.3	76.0	23.3	31.3
Spain	6.3	59.4	3.3	9.0
Average EMU	**22.5**	**42.2**	**11.5**	**25.9**
Non-EMU countries				
Australia	26.2	25.5	30.1	21.6
Canada	29.1	24.1	38.4	39.3
Denmark	18.0	50.4	13.7	61.2
Hong Kong	-	62.8	-	12.6
Japan	22.9	35.4	10.0	26.7
Norway	0.0	25.1	20.8	62.8
Sweden	12.5	11.1	37.0	36.0
Switzerland	33.3	42.6	15.9	23.7
UK	28.1	32.4	17.5	29.0
USA	7.8	15.9	2.9	3.2
Average non-EMU	**16.2**	**23.8**	**8.6**	**13.5**

The table reports the actual foreign equity holdings of pension funds and life insurance companies as a percentage of total equity holdings in 1992 and 1999. Both private and

[51] These data were used also in the study of *Hardouvelis et al (2001)*. They are provided fully in Appendix A.

public pension funds are considered. Data are from Intersec Research Corp. The averages for EMU and non-EMU are weighted averages, with the proportion of each country's total market capitalization to the total market capitalization of EMU and non-EMU set of countries considered used as weight respectively.

4. Data and Methodology

The available set of *data* is on asset allocation of pension funds and life insurance companies (from *Intersec Research Corporation*) for 1992-1999 for a number of both EMU and non-EMU countries[52]. Specifically, the data consist of the *weights* of international portfolios composed of two assets, a domestic and a foreign (rest of the world) equity index for 20 countries for the 8-year reference period. The portfolio weights are on yearly basis; to proceed with our analysis, we used *interpolation* in order to match the frequency of the portfolio weights with the frequency of the asset returns (monthly)[53]. These 20 countries mentioned are 10 EMU and 10 non-EMU countries. The 10 EMU countries are the countries that joined the single European currency on January 1, 1999, except Luxemburg. However, data for Finland and Portugal were available only since 1997, so we had to exclude them from the analysis. Therefore, we ended up with 8 EMU countries: Austria, Belgium, France, Germany, Ireland, Italy, Netherlands and Spain. The 10 non-EMU countries examined are Australia, Canada, Denmark, Hong Kong, Japan, Norway, Sweden, Switzerland, the United Kingdom and the United States of America[54]. We carry on the analysis and obtain empirical results for these 18 countries.

Additionally, we collected data on asset returns for the 18 countries and for the time period mentioned above, on monthly frequency; that is, from January 1992 to December 1999. For each monthly period we assigned the observed value of the first day of the respective month. For each country, we assumed as return of the domestic asset the return of the domestic equity market. We worked with total (i.e. dividend-adjusted) stock market returns. As return of the foreign asset for each country we used the total return of the World equity market.

Because we have assumed international portfolios, the currency return has to be taken into consideration. In order that our analysis is meaningful, we have denominated both returns in the local currency of each country's investor[55]. Thus, while the return on the domestic asset will be the proportional change of the domestic index between two consecutive periods, the return on the foreign asset needs to be adjusted for the change in the currency rate, since the available values of the world index are in US dollars. Taking logarithmic returns, the domestic and foreign asset returns will be respectively

[52] The full data set is included in appendix A.

[53] A consequence of the interpolation technique is that some observations, at the beginning or the end of the observation period are lost. Being more interested in the latest dates (closer and after the launch of Euro), we assumed that the weights hold at the end of each year, hence sacrificed the 11 first observations. To make up for that, we assumed that the weights for these first months were the same as for the 12th month of the sample (December of 1992)- after all, the foreign index weights for most countries in the first periods of the sample were very close to zero. We believe that these arbitrary assumptions do not affect the results much. Besides, the interpolation technique has been widely used in financial research to increase the number of available observations.

[54] The 1992 weights are missing for pensions funds of Austria and for pensions funds and insurance companies of Hong Kong. Nevertheless, the available data was considered sufficient (at least 60 monthly observations, after the interpolation) to carry on the analysis. In the place of the missing observations for these cases we assumed that the weights of 1993 held in 1992 as well.

[55] For EMU countries, we preferred to express returns in the local currency before the advent of the Euro, because the Euro only arrived towards the end of the period we examine.

$$r_{C,t} = \ln \frac{P_{C,t}}{P_{C,t-1}} \qquad \textbf{(4-1)}$$

$$r_W = \ln \frac{P_{W,t}}{P_{W,t-1}} + \ln \frac{S_t}{S_{t-1}} \qquad \textbf{(4-2)}$$

where r_C denoted the domestic (country) return, r_W the world return, S the spot exchange rate of local currency vis-à-vis the US dollar and *t* is the time subscript.

However, the world return needs additionally to be adjusted so as to eliminate the component of each country to the world index; this can be managed by subtracting the return of each country's equity index, weighted by the country's stock market capitalization as proportion to the world's stock market capitalization, from the return of the world index. Finally we multiplied the result by the ratio of the World capitalization including the country by the World capitalization excluding country[56]. This adjustment is important for large countries that constitute a large proportion of the world economy. The formula used for this purpose was

$$r_{W-C} = \frac{MV_W}{MV_W - MV_C} r_W - \frac{MV_C}{MV_W - MV_C} r_C \qquad \textbf{(4-3)}$$

where r_{W-C} is the world return excluding country, r_W the world return, r_C the country return, MV_W the world capitalization (market value) and MV_C the country capitalization.

Next, we calculate excess returns for each country as domestic returns (from **(4-1)**) minus the 1-month *euro-currency* interest rate (as at one period earlier). We assumed the euro-currency interest rate as the appropriate risk-free rate, because institutional investors can usually borrow at lower rates in the offshore market, compared to other investors who can only borrow domestically. For the foreign asset (or rest-of-the-world asset), we yielded the excess return in the same fashion, by subtracting the domestic euro-currency interest rate. This is meaningful because we have converted the foreign return in domestic currency.

For some countries the euro-currency interest rate was not available for the whole or part of the period; namely, Austria, Ireland, Australia, Hong Kong, Norway and Sweden. For these countries we constructed *synthetic euro-currency rates*, according to the *Covered Interest Rate Parity*:

$$\ln \frac{F_t}{S_t} = i_C - i_\$ \qquad \textbf{(4-4)}$$

where F_t is the 1-month Forward exchange rate of domestic currency vis-à-vis the US dollar, S_t is the spot exchange rate of domestic currency vis-à-vis the US dollar, i_C is the 1-month Euro-currency interest rate of the country we are interested in and $i_\$$ is the 1-month Euro-dollar interest rate. From **(4-4)** it is easy to create i_C synthetically.

Data on stock market returns, dividend yields, market capitalizations, spot and forward exchange rates and euro-currency interest rates were retrieved from *DataStream International*.

With portfolio weights and (excess) asset returns available, we can proceed with our testing procedure. Optimal portfolios are to be constructed and their performance is to be compared with the performance of the observed portfolios, with tests (discussed in the section 2) over the time period of

[56] To take into account the effect that the "new" world index consists of one asset less.

question for all the countries. *The main question of interest is whether allocation tends towards efficient allocation.*

We utilize the technique of *Jobson and Korkie (1982)*[57] and test the portfolio efficiency hypothesis by comparing the realized portfolio performance, as measured by its Sharpe ratio, with its potential performance, which is measured by the efficiency constant *a,* from relation **(2-10)**. For this purpose, we implement the statistics of subsection 2.2, under the assumption of existence of a risk-free rate of return (and consequently working with excess returns). It would be interesting to test without this assumption, on the zero-beta context. Unfortunately, this is not possible for most of the statistics of subsection 2.3 follow distributions with *N-2* degrees of freedom; as a result, they cannot be implement for the 2-asset case[58].

Another compromise we had to make was to calculate the realized performance of portfolios based on *average* weights; otherwise, in some cases we found higher performance for the actual portfolio than for the optimal (obviously because the changing weights were equivalent to beneficial rebalancing strategies, something that is not consistent with traditional portfolio theory) and obtained negative values for the test statistics. Therefore, we eventually *test the efficiency of average weights*.

Additionally, we compute the optimal portfolio weights, using formula **(2-7)** and perform the tests under two assumptions: (a) that short-selling is allowed (b) that short-selling is not allowed. This distinction is meaningful, because institutional investors usually are imposed to restrictions, such as the short-selling restriction. Consequently, in a case where the optimal portfolio involved extreme short-selling, a large deviation between optimal and realized performance could be detected by the statistics and the efficiency null hypothesis would be rejected. However, the information content of such a rejection would be poor, because the theoretical optimal performance could not be approached under short-selling restriction. To state it differently, a portfolio found inefficient with unlimited short-selling allowed, could be found efficient after prohibiting short-selling. For this reason, we test the efficiency hypothesis under these two alternative assumptions and compare the results, in order to determine if the short-selling restriction is crucial to inference.

In the empirical part of this project, we utilize the following statistics of portfolio efficiency.

First, the *JK* $\phi_{2,2}$ and $\phi_{2,3}$ statistics. *JK (1982)* showed with their Monte Carlo simulations that their statistical properties are superior to the properties of the other two. Assuming *a* as in **(2-10)** and $a_1 = \frac{\mu_p^2}{\sigma_p^2}$, we can test the hypothesis $a = a_1$ with[59]

$$\phi_{2,2} = (T - \tfrac{N}{2} - \tfrac{5}{2})\ln\left(\frac{1+\hat{\alpha}}{1+\hat{a}_1}\right) \sim \chi^2_{N-1} \qquad \textbf{(4-5)}$$

$$\phi_{2,3} = \left(\frac{T-N}{N-1}\right)\left(\frac{\hat{\alpha}-\hat{a}_1}{1+\hat{a}_1}\right) \sim F_{N-1,T-N} \qquad \textbf{(4-6)}$$

We implement also the *GRS* statistic from **(2-32)**,

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[57] An alternative technique would be to regress the (excess) asset returns with the (excess) portfolio returns and use the estimates of the regression parameters to compute the relevant statistics.

[58] With the exception of the Gibbon's statistic, which is distributed as χ^2_{N-1}, but has the poorest statistical properties. Additionally, *Jobson and Korkie (1985)* claimed that their statistics from *JK (1982)* could be implemented in the zero beta case as well, but in *JK (1989)* they suggest instead Shanken's statistics as the most appropriate for this purpose.

[59] We repeat the –operational– expressions of the statistics as a summary and in order that an immediate comparison among them is easier.

$$GRS = \frac{T-N-1}{N}\left(\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}\right) \sim F_{N,T-N-1} \qquad \textbf{(4-7)}$$

and the *GMM* statistic of *MacKinlay and Richardson (1991)* as in **(2-46)**, assuming that the returns follow the *Student t* instead of the *Normal* distribution, and therefore *heteroskedasticity* is present. The distributional assumption can be viewed as strong, in fact as rigid as the assumption of normality. However, it discharges us from the need of running a regression in order to implement the *GMM* test, and allows us to introduce it in the *JK* context. From the analysis of *Blattberg and Gonedes (1974)* it is implied that 5 and 10 are appropriate values for the degrees of freedom of the t distribution. *MR* use these values in their Monte Carlo experiments. Therefore, relation **(2-46)** takes the expressions

$$\vartheta_5 = T\left(\frac{(\hat{a}-\hat{a}_1)}{1+3\hat{a}_1}\right) \qquad \textbf{(4-8)}$$

$$\theta_{10} = T\left(\frac{(\hat{a}-\hat{a}_1)}{1+\frac{4}{3}\hat{a}_1}\right) \qquad \textbf{(4-9)}$$

for $v = 5$ and $v = 10$ respectively.

We implement the above tests for $N = 2$ assets and $T = 60$ monthly observations. The number of observations is consistent with most empirical studies of the past, like *JK (1982)*, *Gibbons (1982)* and *MR (1991)*. The number of asset on the contrary is not typical in the literature; most studies have assumed *N* equal to 5, 10 or 20 assets. However, most researchers have stated that the statistical properties of the statistic (especially the power) improve as *N* decreases[60].

We start the sample period in January 1992, and roll it forward by one month successively, as far as the data availability permits. Therefore, we obtain inference from January 1997 up to December 1999. **We expect to find that portfolios of institutional investors from EMU countries become increasingly efficient as we approach the end of the period under observation**, because the impact of EMU on diversification opportunities should have been felt even before the introduction of the common currency since European stock market integration was a gradual process rather than a one-off event. After all, the launch of the euro did not come as a surprise but was widely anticipated at least since February 1992, when the Maastricht Treaty was signed. The expectation of the future elimination of currency-related barriers on asset allocation ought to have affected the investment potential prior to 1999. This effect should be stronger the higher the probability of euro occurring and the closer the time span to the launch of the euro.

RATS 4.31 econometric software was used for the calculations. In the next section we present the main empirical findings.

5. Empirical results

In this section we present the empirical results of our study. The testing methodologies were described theoretically in section 2, and summarized in section 4, along with the general design of the testing methodology.

We tested the efficiency of portfolios of two types of institutional investors (pension funds and life insurance companies) from 18 countries (8 EMU and 10 non-EMU countries), for the time period January 1992 up to December 1999. We set the estimation period at 5 years (or 60 monthly observations), consistent with most previous studies, and rolled it forward by one month consecutively, to produce results for the remaining 36 months of the sample. Hence, we calculated the test statistics from January 1997 to December 1999 and examined how portfolio performance changes over time.

For every one of the 36 portfolios, we calculated the actual and optimal Sharpe ratios, the optimal weights, the values of the test statistics and their significance levels, under two cases: Short-selling permitted and short-selling not permitted. We made this distinction in order to assess the impact of short-selling on the composition of the

[60] For example, *JK (1985)*, *GRS (1989)*, *Campbell et al (1997)*.

optimal portfolios and thus the relative efficiency of the actual portfolios, since short-selling is generally not allowed for institutional investors. If extreme short-selling increased too much the potential performance of a portfolio, then the actual performance would appear rather poor relative to its potential and the statistics would reject the efficiency hypothesis. This would be incorrect inference if truly short-selling were not allowed, and therefore the "feasible" potential performance were more modest.

We preferred not to present any tables with results, due to their large volume. Instead we plotted the results, for the 36 time points they were available[61]. For each of the 36 portfolios (of 2 types of institutional investors for each of the 18 countries), we plotted the actual and optimal Sharpe ratios, the optimal weights, the values of the test statistics and their significance levels, both under "short-selling" and "no short-selling", so that immediate comparisons can be made. All these

[61] We remind that the results for the first 11 months (23 months for pensions funds of Austria and pension funds and life insurance companies of Hong Kong) may be considered less reliable, due to the assumption we made for equal weights to make up for the missing observations and the cost of the interpolation technique. See also section 4.

graphs are included in Appendix B. In the remainder of this section, we discuss the more interesting and striking results.

The first impression after an inspection of the results is that most portfolios are efficient and the rejections are relatively few. In fact, the only case with persistent rejections (even with short-selling disallowed) is Japan. Prohibition of short-selling seems to improve performance, but not by much (with a likely exception for the USA). Cases of extreme short-selling are observed unless we restrict short-selling, but usually not with a large impact on potential performance. Finally, the type of investor doesn't seem to be that distinctive, since for most cases pension funds –PFs henceforth– seem to have similar performance with life insurance companies –LICs henceforth. Next, we commend on the results more thoroughly.

For most countries we don't observe not even a single rejection, using any of the statistics and for either institutional investor type at the common significance levels (5% and 10%). For PFs, we notice most rejections for Japan. Several rejections occur for Netherlands and USA, some for Austria, Belgium and Ireland and a few for Italy, Hong Kong and Switzerland. After restricting short-selling, several rejections occur only for Japan, and a few for Italy and Netherlands. For LICs, most of the rejections happen again in the case of Japan, several in Austria, Netherlands and USA and a few in Belgium and Hong Kong. With short-selling not allowed, rejections persist only in Japan on a large scale and in very few cases in Austria and Hong Kong.

The choice of the statistic affects the true significance level of the value of the statistic (p-value), but surprisingly inference doesn't change, because performance is generally sufficient so that most of the portfolios are found efficient by all statistics for the majority of the time periods. Specifically, the *JK* statistics have the lowest p-values, and consequently reject more often. Both $\varphi_{2,2}$ and $\varphi_{2,3}$ (denoted as *JK2* and *JK3* in the graphs) have almost identical values and p-values[62]. The *GRS* and the two *GMM* statistics (under the

[62] The other two *JK* statistics that are not included in the displayed results produced similar values too.

assumption of returns following *Student t* with 5 and 10 degrees of freedom respectively, denoted as *GMM5* and *GMM10* in the graphs) have notably higher p-values, on average by as much as 20%, and are quite similar.

In fact the GRS and the GMM statistic only yield rejections for PFs of Japan, USA, Netherlands, and Belgium and LICs of Japan, USA, Netherlands and Austria (ranked by order of frequency). If we prohibit short-selling, rejections appear only for Japan.

Statistically speaking, it was expected for the *GMM* statistics to have lower values (and thus higher significance levels), according to the analysis of *MacKinlay and Richardson (1991)*. It was less expected for the *GRS* and the *JK* statistics to differ so much. The literature does not compare their sizes for the $N = 2$ asset case, to our knowledge. However, the discrepancy can be explained intuitively. If we compare relations **(4-6)** and **(4-7)**, we can see that $\varphi_{2,3}$ and GRS are monotonic transformation of each other. Specifically, $\varphi_{2,3} / GRS = \frac{T-N}{N-1} / \frac{T-N-1}{N} > 1$. In our case we have $T = 60$ and

$N = 2$, so the ratio is 2.035. However, the ratio between the critical values of $F_{58,1}$ and $F_{57,2}$ for a size of 5% is just 1.268. This means that the value of the test increases more than the critical value of the F distribution, when the degrees of freedom change from *(N,T-N-1)* to *(N-1,T-N)*. The impact of this is that $\varphi_{2,3}$ will always reject more often than *GRS*, when working with the same data and the same size. In fact, this is more pronounced the smaller is *N*[63]. This may indicate that one of the tests is less appropriate than the other, especially when *N* is kept small, even though both follow exact distributions. In that case, we are forced to incline towards the *GRS* test, because it has been proven in the literature that it has better statistical properties.

The choice of the significance level doesn't seem to affect inference much either. If we set it to 10% instead of 5%, we get a few more rejections for the countries that already had rejections at 5%, and get a few also for PFs of France and UK, and LICs of France, Ireland, Italy, Switzerland and UK (where we didn't notice any rejections at the 5%

[63] This can be verified if you set a higher value for *N* and repeat the calculations.

significance level), but only with the *JK* statistics and under short-selling allowed. However, generally the change of the size of the tests from 5% to 10% doesn’t influence inference remarkably.

If we focus on the optimal weights graphs, we see quite pronounced short-selling, if we permit it. Austria, Hong Kong and Japan are the most striking cases of large-scale short-selling in the domestic asset, whereas Ireland, Netherlands, UK and USA are the most characteristic cases with high short-selling in the foreign asset. The above apply for both PFs and LICs.

If we compare the results according to the investor type, we don’t see notable differences between the performance of PFs and LICs from the same country. The only exception is Netherlands, were PFs seem to do evidently worse than LICs. Clearly because they insist on holding a very large proportion of foreign equity, while portfolio theory suggested the contrary (as indicated by the optimal weights graph). Since performance patterns are almost identical for PFs and LICs, we will proceed with our examination by country, referring to both types of institutional investor for the country in question each time.

However, as we have already mentioned in section 3 and section 4, we are more interested in the time path of the performance; specifically, **we want to examine if portfolios of institutional investors from EMU countries become increasingly efficient as we approach the end of the period under observation,** as a result of the anticipation of the arrival of the Euro and the favorable consequences for inter-European investment.

Focusing accordingly on the countries that joined EMU on January 1, 1999, we notice the following. *For larger countries, namely Germany, France, Italy and Spain, the portfolios of institutional investors are efficient for the whole period.* However, *what is more striking is that from marginally efficient they become increasingly efficient over time*, as indicated by the rise of the significance levels of the values of the statistics. However, statistical theory dictates that p-values even marginally larger than the preset significance levels are sufficient to accept the null hypothesis. As a result, this finding is not informative statistically. But it can be considered informative by economic intuition, because it indicates that actual performance gradually moves closer to the potential performance. This picture is not distorted after imposing a short-selling restriction, because no large-scale short-selling is present in the optimal portfolios in these cases.

For smaller EMU countries, namely Austria, Belgium, Ireland and Netherlands, the picture is notably different. The portfolios of institutional investors from these countries are inefficient in large part of the 1997-1999 period: Specifically, in most of 1997 and 1999 for Austria and most of 1998 for the other three countries. This can be explained by examining the optimal weights' graphs. It seems that Belgium, Ireland and Netherlands should not invest so heavily in foreign equity, as long as domestic equity yielded higher returns, as indicated by the composition of the optimal portfolio[64]. Austria is a special case,

[64] This phenomenon, that investors prefer foreign assets when domestic assets yield higher performance, could be even characterized as reverse home bias, or foreign bias.

because it joined EU no sooner than 1995, therefore anticipations of participation of the country in a market with a common currency might not have been so strong. Consequently, even though investment in foreign equity was quite pronounced, it seems that is should have been even higher, as indicated by an optimal portfolio consisting strictly of foreign equity.

In conclusion, we find that some improvement in portfolio efficiency is observed in 4 of the 8 EMU countries examined, but the evidence is not strong by statistical aspect. Consequently, **we cannot determine if increased foreign equity holdings in portfolios of institutional investors in EMU countries had as a result more efficient equity portfolios, or equivalently that currency risk and other trade disincentives explain sufficiently the presence of equity home bias within EMU countries in the past.**

Next, we will discuss results from three countries that are members of EU, but chose not to enter EMU, namely Denmark, Sweden and UK. The portfolios of these countries remain efficient for the period in question. Their foreign equity holdings remain quite stable, and it seems that this strategy is prudent, since the optimal portfolios consist almost solely of home equity. The fact that foreign equity holdings do not increase much in these countries could be further interpreted as weak anticipation by investors that their countries will join EMU (at least in the short-term).

Portfolio performance of institutional investors from Australia, Canada, Hong Kong[65], Norway and Switzerland display the same pattern more or less. Their foreign equity holdings remain stable (and rather low) more or less, and their portfolios remain efficient for the whole period.

Japan is the only case where the efficiency hypothesis could be rejected for the whole period. Even after imposing a short-selling restriction efficiency doesn't improve much. The graph of optimal portfolio weights is informative, suggesting holding only foreign equity. As a result, it seems that the intention and attempt of Japanese portfolio managers to increase foreign equity holding, as

[65] For Hong Kong performance drops sharply for some time, perhaps as a result of the Asian Tigers crisis.

indicated by actual weights in Appendix A, in order to improve total performance was not enough. Even more informative is the graph with the Sharpe ratios; the performance of Japanese institutional investors is surprisingly low, below 10% for most periods. *This result is consistent with the bad performance of the Japanese stock market and the stagnation of Japanese economy in the 90s.*

Finally, a special case is USA too. For if we allow short-selling, the equity portfolios of American institutional investors are inefficient for the last half for the period for which we have results. However, without short-selling the portfolios become efficient. This is the only case where efficiency is affected so much by the short-selling constraint. The reason can be found after inspection of the Sharpe ratios and the optimal weights graphs. Even though the actual performance is above 25% for the whole period, it is far below the optimal performance with short-selling, which reaches 60%! The graph of the optimal weights confirms also the higher performance of the domestic equity. *This should* *not come as a surprise though, having in mind that the US economy (and the dollar) was at its peak during the 90s.* As a result, even the traditionally conservative placing in foreign equity by American investors could be regarded as imprudent for the period in question; they shouldn't bother at all to invest abroad, given such a high a domestic performance.

Consequently, **we can conclude that portfolios of institutional investors in non-EMU countries (with the exception of Japan) were efficient for the whole period, despite the fact that their foreign equity holdings didn't increase much.** This can be explained in the light of the consideration that stock markets outside EMU (especially the American stock market) realized very high equity returns during the period we examine. **Therefore, non-EMU institutional investors had no reason and motive to increase their foreign equity holdings; they acted rationally according to portfolio theory, by placing their assets where they could yield the higher returns. Under this perspective, investors**

from non-EMU countries cannot be considered "biased".

Evidently, since no home bias is observed in that case, we are unable to determine if bias of the past could be attributed to currency risk or any other factor. However, we have to admit that this outcome can be regarded as circumstantial and extremely delicate. *Otherwise, if the stock markets did worse worldwide, would investors outside EMU be motivated to seek higher returns and risk reduction by investing more heavily on foreign equity? We have no clue as to that.* One can argue, that earlier literature detected strong home bias, and that investor behaviour and culture do not change easily. However, we should take into account that we do not live in a static world, and that integration of the world economy (starting from Europe) advances fast. Therefore, *it remains an interesting area of research to look into the home bias puzzle*.

To sum up, the evidence from EMU equity portfolios shows that in most cases the portfolios were efficient over the whole period. A tendency for performance improvement during the last part of the 1990s was detected, yet the indication is not strong enough to determine if increased foreign equity holdings had as a result more efficient equity portfolios, a fact that could be attributed to the anticipated elimination of currency risk and foreign investment barriers. Non-EMU portfolios were found efficient too despite low foreign equity placements, due to the good performance in most national equity markets.

6. Conclusion

The object of this study was twofold. First, to provide a survey of existing literature on testing procedures for portfolio efficiency, and second, to investigate a well-known puzzle of the modern finance literature, the home bias puzzle.

In the first part, the most popular tests of Mean-Variance efficiency were reviewed, both under the Sharpe-Lintner and the Black version of CAPM and under the potential portfolio performance and portfolio (or index) efficiency approaches. The work of researchers such as *Jobson and Korkie (1982, 1985, 1989)*, *Gibbons (1982)*, *Kandel (1984)*, *Shanken (1985,1986)*, *Gibbons, Ross and Shanken (1989)* and *MacKinley and Richardson (1991)* was discussed and compared.

In the second part, the Equity Home Bias Puzzle was explored, as an application to portfolio efficiency tests. In particular, **we were interested in testing whether efficiency of equity portfolios of institutional investors in EMU countries has increased in the later part of the 1990s, as a result of the launch the Euro and the abolishment of a series of investment barriers. This would imply that the bias should be attributed to factors that disappeared within EMU: currency risk and legislative trade disincentives.**

However, the empirical results do not sustain this statement sufficiently. **A tendency for performance improvement during the last part of the 1990s was detected in EMU equity portfolios, yet the indication is not strong enough to determine if increased foreign equity holdings had as a result more efficient equity portfolios. At the same time, most non-EMU equity portfolios were found efficient too despite low foreign equity placements, due to the good performance in most national equity markets.** Nevertheless, these results are useful and encouraging for further research on the subject.

Finally, some interesting extensions for future research are suggested. First, it would be appropriate to implement the same tests with data for a broader period, in order that the patterns and trends across time become more distinctive and the results became more reliable in general. Empirical research on economic phenomena for short periods is difficult and its results can be questionable and unrobust for statistical reasons as well as due to the effect of business cycles. It would be useful also to test efficiency of portfolios consisted of a broader class of assets, so that the diversification potential became more pronounced. Furthermore, it would be interesting to test for portfolio efficiency assuming unavailability of a riskless asset, since inference can be sensitive to the choice of the risk-free rate of return. Finally, the implementation of semi-parametric procedures, like GMM, under a more general context could provide useful results, when used in conjunction with more traditional methods.

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Appendix A

Actual foreign equity holdings of pension funds and life insurance companies as a percentage of total equity holdings. Both private and public pension funds are considered. Data are from Intersec Research Corporation.

PENSION FUNDS

	1992	1993	1994	1995	1996	1997	1998	1999
AUSTRIA	-	67.0%	61.7%	47.1%	83.2%	83.8%	76.9%	38.9%
BELGIUM	50.0%	56.8%	58.0%	57.2%	56.9%	58.1%	63.3%	80.8%
FINLAND	-	-	-	-	-	2.4%	3.7%	41.0%
FRANCE	14.3%	5.8%	5.9%	11.0%	13.8%	13.8%	15.0%	37.3%
GERMANY	18.3%	18.8%	21.3%	19.4%	24.3%	30.9%	22.7%	31.7%
IRELAND	56.5%	60.0%	58.1%	59.9%	57.8%	54.5%	55.6%	69.2%
ITALY	0.0%	6.3%	4.3%	3.9%	4.9%	5.7%	6.0%	10.9%
NETHERLANDS	56.3%	56.8%	56.8%	56.8%	55.1%	57.2%	57.6%	76.0%
PORTUGAL	-	-	-	-	-	24.0%	28.5%	40.2%
SPAIN	6.3%	0.0%	0.0%	20.0%	33.3%	21.7%	36.4%	59.4%
AUSTRALIA	26.2%	26.5%	26.7%	21.8%	25.2%	39.0%	30.9%	25.5%
CANADA	29.1%	30.5%	30.4%	35.6%	36.0%	37.8%	27.0%	24.1%
DENMARK	18.0%	25.5%	25.6%	24.6%	21.0%	30.1%	22.2%	50.4%
HONG KONG	-	58.8%	62.1%	66.3%	65.5%	65.5%	82.6%	62.8%
JAPAN	22.9%	23.0%	21.2%	25.9%	31.1%	36.9%	36.5%	35.4%
NORWAY	0.0%	0.0%	3.9%	0.0%	0.0%	0.0%	14.9%	25.1%
SWEDEN	12.5%	3.9%	4.2%	3.8%	6.2%	4.2%	11.1%	11.1%
SWITZERLAND	33.3%	24.6%	27.2%	29.3%	29.3%	35.4%	50.8%	42.6%
UK	28.1%	29.9%	27.8%	29.4%	31.4%	28.0%	28.3%	32.4%
USA	7.8%	11.7%	14.7%	15.5%	15.8%	16.6%	14.8%	15.9%

LIFE INSURANCE COMPANIES

	1992	1993	1994	1995	1996	1997	1998	1999
AUSTRIA	6.5%	3.9%	17.1%	12.5%	15.5%	20.8%	24.8%	25.0%
BELGIUM	7.1%	5.3%	5.3%	14.1%	14.8%	27.0%	37.8%	58.8%
FINLAND	-	-	-	-	-	13.8%	5.7%	22.0%
FRANCE	7.9%	7.4%	9.2%	9.0%	9.1%	9.2%	9.4%	19.4%
GERMANY	5.2%	14.5%	19.9%	18.9%	18.3%	20.9%	26.3%	34.7%
IRELAND	0.0%	2.2%	29.2%	36.4%	35.1%	25.1%	37.3%	48.5%
ITALY	33.0%	10.5%	10.1%	39.1%	39.8%	40.4%	33.0%	12.8%
NETHERLANDS	23.3%	31.3%	31.3%	31.3%	22.7%	22.9%	27.9%	31.3%
PORTUGAL	-	-	-	-	-	0.0%	0.0%	0.0%
SPAIN	3.3%	10.5%	10.5%	11.7%	6.3%	12.4%	13.5%	9.0%
AUSTRALIA	30.1%	18.9%	27.3%	27.2%	26.0%	21.2%	24.6%	21.6%
CANADA	38.4%	30.1%	29.6%	24.6%	45.7%	43.2%	43.0%	39.3%
DENMARK	13.7%	21.4%	14.1%	15.8%	16.4%	20.3%	29.6%	61.2%
HONG KONG	-	18.5%	23.1%	23.1%	24.0%	13.2%	29.4%	12.6%
JAPAN	10.0%	21.6%	20.4%	18.2%	20.2%	22.1%	26.5%	26.7%
NORWAY	20.8%	23.9%	22.8%	28.0%	35.3%	37.3%	53.4%	62.8%
SWEDEN	37.0%	33.5%	34.0%	36.4%	35.6%	35.1%	38.0%	36.0%
SWITZERLAND	15.9%	16.2%	16.2%	16.3%	16.6%	16.8%	20.7%	23.7%
UK	17.5%	18.0%	23.3%	22.9%	27.5%	26.5%	26.9%	29.0%
USA	2.9%	2.7%	2.9%	3.0%	3.1%	3.4%	3.1%	3.2%

Appendix B

In this appendix we provide the numerical evidence of this study. We have plotted graphs for

- The Sharpe ratios of the actual and the tangency (optimal) portfolio
- The optimal weights of the tangency portfolio
- The values of the test statistics
- The significance level of these statistics

for the 36 monthly periods that these results are available (January 1997 – December 1999), for 18 countries (8 EMU and 10 non-EMU), 2 types of institutional investors (pensions funds and life insurance companies) and both under short-selling allowed (left column graphs) and short-selling disallowed (right column graphs). *JK2* and *JK3* denote statistics $\phi_{2,2}$ and $\phi_{2,3}$ respectively by *Jobson and Korkie (1982, 1985)*, *GRS* denotes the statistic suggested by *Gibbons, Ross and Shanken (1989)* and *GMM5*, *GMM10* the GMM statistic by *MacKinlay and Richardson (1991)*, under the assumption of Student t distribution for returns with 5 and 10 degrees of freedom, respectively.