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Μοντελοποίηση της τιμής του ηλεκτρισμού σε συνεχή χρόνο και αποτίμηση παραγώγων σε ηλεκτρισμό

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Abstract

In this dissertation our main purpose was to examine the relative efficiency of the double-exponential mean-reverting process (MRDEJP) compared to two others, the simple mean-reverting (MRP) and the mean-reverting with Gaussian jump (MRGJP), in a continuous time setting. The idea came up from Lucia and Schwartz (2002) who actually used data from the same area, the Nord Pool market, but only estimated the mean-reverting process in discrete time, testing also its efficiency in the futures market. Escribano et al. (2002) had estimated the Gaussian jump process but only in a discrete time setting. We just tried to go further to that by incorporating a new jump model, the mean-reverting double exponential jump process (MRDEJP) and also test the other two in continuous time. The performance of the models was estimated under both econometric and financial metrics. The estimation of the parameters was made with the Maximum Likelihood Estimation method. We then tried to compare the three models by comparing the root mean-squared errors of various futures and forward prices of different maturity. We finally estimated the implicit market price of risk for each model separately. We found that the use of jump models is necessary. Whether the Gaussian or the double-exponential jump is generally better is ambiguous.

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2 Introduction

In this dissertation our main aim will be to asses the efficiency of three popular diffusion models in a continuous time setting. The data set that will be used will be the average hourly spot price of electricity taken by Nord Pool, the Nordic Power Market. This assessment will be done under both econometric and financial metric. This approach has already been done by Dotsis, Psychoyios and Skiadopoulos (2007) in their effort to capture the dynamics of various volatility indices as well as by Lucia and Schwartz (2002) using data from the Nordic power market.

The aim of this dissertation was actually inspired by the paper of Lucia and Schwartz (2002) and Escrinbano et al. (2002) who used data from Nord Pool, although earlier ones, and assessed the performance of the mean-reverting and the Gaussian-jump process for the spot price and the natural logarithm of the spot price in discrete time. In this paper we will try to expand the research in a way. We will first try to incorporate the double exponential jump-diffusion models in the data set and secondly apply the mean-reverting and the Gaussian jump process again since our approach will be a continuous time setting. We will then compare the results that Lucia and Schwartz (2002) came up with since they also tried to price futures and forwards whereas Escribano, Peña and Villaplana (2002) did not. The reason we will do that is double: Firstly the period that their data were taken is only a part of the period we will examine and secondly their estimation of the parameters was done by assuming a discrete time setting. So it will be challenging to look at the efficiency of the continuous time model in a broader dataset than Lucia and Schwartz (2002) used.

The models we are going to use will all have a common characteristic, the mean-reversion of the spot prices. The first model will be a simple mean-reverting model which will be notated from now on as MRP (mean-reverting process). The second model will be a mean-reverting Gaussian jump process (MRGJP) and the third one will be a mean-reverting double exponential jump process (MRDEJP). Initially there was a thought of using a stochastic volatility model. The reason this did not go further was the fact that in that case the parameters would become numerous and the models would be unstable. Besides, the aim of this dissertation is to find the best fitting model with as less parameters as possible, in order not to lose its creditability.

Now let's have a look at the former bibliography concerning Nord Pool market. As we said before, Lucia and Schwartz (2002) made an attempt to catch the dynamics of the spot electricity price using the simple mean reverting model in a continuous time setting. They also calculated the Mean Errors of the pricing models. They did that in two ways. They initially made the simplifying assumption that the market price of risk is zero and then after estimating the market price of risk they re-calculated the ME. They came up with some interesting results which we are actually going to compare and extend in the chapters to follow. Another attempt to model Nord Pool data set was made by Escribano, Peña and Villaplana (2002). These researchers used the discrete time approach. Among the models they used, it was the simple mean-reverting model and the mean-reverting with Gaussian jump model. For the parameter estimation they used the MLE and their effort was focused to find the best fitting model using only the econometric method. So they actually didn't use their conclusions for the futures market. Only Villaplana (2003) tried to include a double-exponential jump model in his research but the data set was different.

Within the econometric framework, maximum likelihood estimation is used to estimate the parameters. In case that the conditional density function cannot be directly calculated, we use the characteristic function to get a closed-form solution being followed by a Fourier inversion in order to get the likelihood function which we are trying to maximize.

Within the financial metric, we compare the effectiveness of the three models. Under the objective probability measure the futures price equals the expected value of the spot price. This could stand in the case that we assume that the objective measure is the same with the risk-neutral measure. We will see further in the text what has been said about this assumption and if actually stands in our case. Under the assumption of the existence of market price of risk the futures price equals the expected value of the spot price under a risk adjusted metric. The prices in both cases are compared with the market price of futures for different maturities. This latter approach helps us compare our results with the ones of Lucia and Schwartz (2002).

Before doing all these we first get to know the special features of the electricity market in Section 3. The Nord Pool power market is fully presented in Section 4. Apart from the spot market we also describe the financial market and explain the main contracts which are being traded. In Section 5 we make a short review of the previous bibliography that has to do with attempts to model electricity prices. In Section 6 we present some basic statistical functions that help us interpret their evolution through time and make their understanding easier. In Section 7 we actually get into the main part of the dissertation by describing the processes that will be used for our research and the features they are trying to capture. Section 8 describes the parameter estimation procedure and Section 9 demonstrates the results and discusses them. In Section 10 we present the relative efficiencies of the models in their effort to calculate the different futures/forwards prices. Finally, in the last section we conclude and sum up the work done. In the appendices the reader can find some mathematical proofs.

3 Characteristics of Electricity Prices

Developing predictive models for electricity prices is a relatively new area of application. Until recently, electricity was a monopoly in most countries often, government owned, and if not, highly regulated. As such, electricity prices reflected the government's social and industrial policy. That is why any price forecasting had no scientific interest since the prices had nothing to do with the free market. In this respect, it tended to be over the longer term, taking a view on fuel prices, technological innovation and generation efficiency. This changed dramatically, however, during the 1990's. Following the examples of structural reforms and market liberalizations in the U.S., Britain, Norway, Chile, Argentina and Australasia in the early 90's, other European countries such as Spain, Germany, Finland, Sweden and Denmark followed suit a few years later, as well as various regions in North and South America and this trend has continued so that power sector reform has now become a major issue worldwide.

Ownership in the sector has generally become private rather than public, competitive markets have been introduced for wholesale trading and retail markets gradually liberalized to erode local franchises. Typically the industry has been split up into separate companies for generation, transmission, local distribution and retail supply of electricity power. Transmission and distribution are network services and, as natural monopolies, are regulated. Generation is progressively deregulated as competition develops between a sufficient number of companies. Retail suppliers buy from the wholesale market and sell to customers. Industrial and commercial customers have generally been the first to receive full market liberalization. The residential sector has been opened in many countries, but often quite slowly, and in some cases not at all. All of this structural change has been motivated by a faith in the ability of competitive forces to create a more efficient and enterprising industry, than either the public sector or regulated monopolies could deliver. Of course, it is the prospect of business risk that should drive the efficiency gains, and the major new element of risk is wholesale price uncertainty.

One of the important consequences of this restructuring is that prices are now determined according to the fundamental economic rule of supply and demand. Supply, as mentioned above, is provided by generators and demand is represented by industrial consumers, power marketers and distributors buying electricity in the pool to sell it to end-users. There is a "market pool" where bids and offers are placed by consumers to buy electricity and generators to sell it for the next day. The equilibrium prices are defined as the intersection of the aggregate demand and supply curves for each hour (or half-hour) of the day. These new deregulated prices have been characterized in all markets by having an extremely high volatility. Even when compared with financial markets (stocks, bonds) or with other commodities, the behavior of electricity prices is still regarded as quite complex and volatile. The deregulating processes have been accompanied by the introduction of competitive wholesale electricity markets and power derivative contracts, both OTC and exchange-traded, providing a variety of contract provisions to meet the needs of the electricity market participants. In fact the more experienced markets now include futures and options markets (for instance, electricity futures contracts are traded in different markets, Sidney Futures Exchange, New Zealand Futures and Options Exchange, Eltermin (Nord Pool), NYMEX and others).

At the same time, deregulation of the energy industry has paved the way for a considerable amount of trading activity, both in the spot and derivative markets. It has provided utilities with new opportunities but challenges as well. The volume risk they have been used to analyze and account for in their revenue projections is now augmented by price risk. This price risk has forced the industry to go into financial practices such as hedging, use of derivative contracts and, quite importantly, to identify and price the options embedded in energy contracts that have been written for decades. Accordingly, the scheduling of plant operation and maintenance has become the subject of even increased scrutiny and concern.

Electricity may be considered as a flow commodity strongly characterized by its very limited storability and transportability. Both limits to the possibilities of “carrying” electricity across time and space turn out to be crucial in explaining the behavior of electricity spot and derivative prices as compared to other commodities. In other words arbitrage across time and space, which is based on storability and transportation, is seriously limited, if not completely eliminated, in electricity markets. If the links across time and space provided by arbitrage break down, we would expect spot prices to be highly dependent on temporal and local supply and demand conditions. The limits of the arbitrage are also expected to affect decisively the relationship between spot and derivative prices.

The non-storability of electricity makes electricity delivered at different times and on different dates to be perceived by users as distinct commodities. In other words, prices are strongly dependent on the electricity needs (demand) and their determinants in every precise moment (this is to say, business activity, temporal weather conditions, and the like). Distinguishing between on-peak and off-peak electricity prices, or among prices corresponding to different time periods, such as seasons, is indeed important in power markets (such distinctions determine, for instance, derivative contractual terms). The non-storability of electricity is also likely to affect derivative pricing significantly, notably influencing on the shape of the forward curve and its behavior.

Transportation constraints for electricity come in the form of capacity limits in the transmission lines and transportation losses, which can make impossible or uneconomical the transmission of electricity along certain regions. These limitations make electricity contracts and prices highly local, strongly dependent on the local determinants of supply and demand (such as characteristics of the local generation plants, and local climate weather conditions together with their derived uses of electricity). That is why electricity has always been treated as a commodity with special features making the modeling of its price really challenging.

4 Nord Pool-The Nordic Power Market

During the 90's, the Nordic European countries started a gradual deregulating process in their respective electricity sectors based on opening them as much as possible to competition. In the resulting Nordic electricity system, the transmission network is owned and operated by a number of independent transmission system operators, whose activity is subject to regulation and control by public authorities. This guarantees a non-discriminatory access to the grid to all market participants in the new electricity market. The new Nordic wholesale electricity market combines both over the counter bilateral contracting and trading via the Nordic Power Excange, Nord Pool ASA.

Established in January 1993, and first covering only the Norwegian market, the Nord Pool is currently the world's only common multinational market which also includes Sweden since January 1996, Finland since June 15, 1998 and the western part of Denmark (Jutland and Funen) since July 1, 1999. It is owned by the two national grid companies, Statnett SF in Norway (50%) and Affärsverket Svenska Kraftnät in Sweden (50%). These companies, as implied by their names, are the owners of the electricity networks in Norway and Sweden. At present Nord Pool has more than 70 employees with headquarters in Oslo and branch offices in Stockholm, Sweden in Fredricia, Denmark and Helsinki, Finland. In 2006 representative offices were opened in Amsterdam and Berlin. Basically, Nord Pool organizes two markets, a "physical market" (Elspot) and a "financial market" (Eltermin and Eloption), and also provides clearing services.

4.1 Electricity spot market

Elspot is Nord Pool's physical day-ahead market for Norway, Sweden Finland, Denmark and North Eastern Germany. This market works in a really special way which we will thoroughly explain.

Before explaining the procedure we should clear out a couple of notations that might come up. First of all a NOK is a Norwegian Krown the currency of Norway, the SEK is the Swedish Krown and the DKK is the Danish Krown. Additionally, a MWh (Megawatt-hour) is a measurement of electricity power. For example one kilowatt-hour is the power that a 40-watt light bulb produces if it is on for 25 hours (1 Megawatt-hour = 1.000 Kilowatt-hours = 1.000.000 watt-hours).

Now let's get into the market and see how it works. Every day until 12:00 at noon there is a "pool" open, where market participants place their bids and offers for electricity power for the next day starting at 00:00 and ending at 23:00. A bid and an offer have three parts, a price, a load in electric power and a time period. So, every participant that needs electricity makes a bid to buy a specific load in MWh at a specific price during a specific hour of the following day. On the other hand each electricity supplier makes his offer to sell a specific load of electric power, at a specific price during a specific hourly period of the day. The hourly periods are set to be the sharp hours, e.g. 11:00-12:00, 17:00-18:00 etc.

All these happen until 12:00 at noon, when the "pool" is closed. After that the price calculation for each one of the 24 hours of the following day is made. These are actually the equilibrium prices between the demand and supply. So for each hour a demand and a supply curve are drawn, according to the bids and offers. The point of the intersection is the fair price where the electric power will be sold at this specific hour on the following day. This price is also called the system price and its average for each day is used as a settlement price at the Nord Pool's financial market. So after this procedure is done both counterparties of the deal have a contract that he will buy/sell a specific load of MWh, at a specific price and at a specific hour of the following day. So it is like having a forwards contract with expiration period of a few hours.

In order to make clear the way the spot market works let's illustrate an example: Let's assume that today is Monday and that we belong to a big company which wants to buy 100 MWh of electricity power for the time period 14:00-15:00 on Tuesday. So, before 12:00 at noon of Monday we submit our bid that we want to buy 100 MWh, at the price of 300 NOK/MWh for the period 14:00-15:00. After 12:00 all bids are aggregated and depending on the demand and supply level at that specific time period an equilibrium price is set. Afterwards we sign a contract with the supplier that he will deliver to us 100 MWh from 14:00-15:00 at the price that is agreed. We should of course make clear that the final delivery price might be a bit different from the system price due to cost of carry or other fees. However from now on we will simply forget this detail in order to make our research clearer. The process mentioned above can also take place on weekends.

In order for the market to be protected Nord Pool Spot AS has set a technical upper limit for the bids which is known as the max price. The max prices per MWh are 2,000 EUR/16,500 NOK/15,000 DKK/18,000 SEK.

The respective national system operators have established different methods for handling bottlenecks situations (i.e. situations when the required electricity flow between any two given areas exceeds the capacity limits of the transmission lines), depending on the specific involved areas. Bottlenecks between any two countries as well as internal bottlenecks in Norway are all managed using the pricing mechanism in the spot market, implying price adjustments for the involved areas. In essence, when the required power flow between two or more areas exceeds the capacity limits two or more zone prices are calculated besides the system price. Internal bottlenecks in the other countries are managed directly by the national grid operators and the cost of the regulation is financed through tariffs for power transmission.

An electric power system must be continuously balanced. In order to handle any unpredictable differences between the planned and the real exchange during delivery, once the Elspot market is closed, the national system operators have additionally set up regulating or balance markets from which the required upward or downward regulation is obtained on short notice.

4.2 Electricity financial market

The Nord Pool's financial market is also known as the Eltermin and Eloption. The products traded at Nord Pool's financial market comprise of Nordic, German and Dutch power derivatives and are used for trading and risk management purposes. Besides its clearing function, which is currently conducted by a separate business area called the Nordic Electricity Clearing (NEC), Nord Pool guarantees settlement and delivery of all trades made at the market, by entering into the contracts as a legal counterparty for both the buyer and the seller. NEC also offers clearing services of standardized OTC bilateral financial contracts registered in the market for that purpose.

The derivatives which are traded in Nord Pool financial market are futures, forwards, contracts for difference (CfDs) and European options. But before presenting all these products analytically, we should first clear out some terms that will be presented below.

4.2.1 Payoff function of futures and forward contracts

Since September 1995, none of the above mentioned contracts entails physical delivery of electricity power; a cash settlement takes place throughout the delivery period, starting at the due day of each contract.

Let us now make a small pause and explain how a delivery period is defined. Let's assume that we have a futures/forward contract which has a timeline like the one in figure 1. The hypothetical futures/forward contract starts trading at time $t = 0$.

Figure 1: Timeline of a life of a futures/forward contract. Date t is the starting date of the contract, T is the maturity date, and $[T_1, T_2]$ is the delivery period of the contract.

At time T this contract finishes trading. So date T is the maturity date of the contract. How big T will be depends on the type of contract and it varies from one day to five years.

The period from t to T is called the trading period of the contract. As we already said, there is a cash settlement that is taking place when the contract expires. This cash settlement starts on time T_1 and ends on time T_2. The time period $[T_1, T_2]$ is called the delivery period of the contract which is most of the time smaller than the trading period. The lengths of both periods depend, of course, on the kind of the contract. We should also note that T_1 usually starts immediately after T. This is not always the case and we will see in which cases this does not happen.

We will next describe the payoff function of the electricity futures and forwards. First of all we should point out to the reader that these futures/forwards have a special feature which changes their payoff function compared to the one of the commodity futures. The payoff of usual commodity or index futures/forwards ends at their maturity date. From that date on no extra profit or loss is realized and the contract seizes to exist. On the other hand electricity futures/forwards keep giving a payoff even after their maturity date. This payoff goes on for a certain time period, which is the delivery period that we described above. We should keep that in mind looking at the payoff function.

We assume that the interest rates are non-stochastic and constant. This assumption helps us ignore the differences in pricing futures and forwards.

Suppose we stand at time t and we take a long position in an electricity futures/forward contract with price $F_t(T_1,T_2)$, where $[T_1,T_2]$ is the delivery period of the contract. Let's assume that the maturity date of that contract is T. This means that at this date the trading of the contract stops and that a final price is settled. Its delivery period $[T_1,T_2]$ usually starts the next day after the maturity date. At the maturity date of the contract the owner realizes a payoff, either a profit or a loss. This payoff at the maturity date T is given by the following equation:

$$f_T = F_T(T_1,T_2) - F_t(T_1,T_2) \qquad (4.1)$$

where $F_t(T_1,T_2)$ is the purchase price of the contract at time t and $F_T(T_1,T_2)$ is the final price at time T, the maturity date.

As we said before, although the contract has a final price at time T, it still provides a payoff during the delivery period $[T_1,T_2]$ and seizes to exist at time T_2. We will now describe the concept of the total payoff which is realized at the end of the delivery period, using a simple example. We will provide mathematical notation afterwards.

The general rule about the payoff of every contract is that after every hour of the delivery period the owner of the contract is credited or debited the difference of the spot price of that specific hour and the final futures price, which has been settled at the maturity date of the contract.

So, suppose we own a day contract which has a maturity date on 31/1/2006 and the day of the delivery is the 1/2/2006. As we will describe in more details below, a day contract is a futures contract with a delivery period of one day. So in our case the delivery period $[T_1,T_2]$ is the 24-hour period of day 1/2/2006. We bought this contract on the 25/1/2006 paying [illegible]. Let's assume that the closing price of the contract was 42 €. So at the maturity date we had a profit of 42-30 = 12 €. This is what we explained mathematically above. Suppose the spot price for the time period of 00:00-01:00 a.m. of the 1/2/2006 is 35 €. At 01:00 we get an extra payoff, which equals 35-42 = -7 €. Totally, counting from the time of the purchase of the contract on the 25/1/2006 we have a total payoff of (42-30)+(35-42) = 35-30 = 5 €. So we have a profit of 5 € in total. After each hour we make the same calculations. Let's have a look at table 1 where we see the total payoff structure of a day contract. We conclude that, typically, the payoff of every hour is the difference between the spot price of that hour (35 € for the first hour of the delivery period) and the price of the contract at its purchase date (42 €). The total payoff at the end of the day is the sum of the payoff at the maturity date (12 €) and the payoff at the delivery period (-7 €) for the first hour of the delivery period):

$$\begin{aligned} f_{T_2} &= f_T + f_{T_2-T_1} \\ &= \sum_{s=T_1}^{T_2}\left(P_s - F_t\left(T_1,T_2\right)\right) \\ &= \sum_{s=T_1}^{T_2} P_s - \left(T_2 - T_1\right) F_t\left(T_1,T_2\right) \end{aligned} \tag{4.2}$$

where P_s is the price at time s, f_T the payoff of the contract at time T and $f_{T_2-T_1}$ the payoff of the contract at the delivery period., $\left[T_1,T_2\right]$ is the delivery period (0:00 of 1/2/2006 until 0:00 of 2/2/2006), translated in days, and $F_t\left(T_1,T_2\right)$ is the price of the contract at the purchase time t.

The above payoff is true if we assume a discrete time setting. In this thesis we assume a continuous time setting and so equation 4.2 becomes:

$$\begin{aligned} f_{T_2} &= f_T + f_{T_2-T_1} \\ &= \int_{T_1}^{T_2}\left(P_s - F_T\left(T_1,T_2\right)\right)ds \\ &= \int_{T_1}^{T_2} P_s ds - \left(T_2 - T_1\right) F_t\left(T_1,T_2\right) \end{aligned} \tag{4.3}$$

Hourly periods		Spot price at each hourly period of 1/2/2006	Payoff realized at end of each period
0:00	1:00	35 €	(42-30)+(35-42) = 35-30 = 5 €
1:00	2:00	36 €	(42-30)+(36-42) = 36-30 = 6 €
2:00	3:00	33 €	(42-30)+(33-42) = 33-30 = 3 €
3:00	4:00	30 €	(42-30)+(30-42) = 30-30 = 0 €
4:00	5:00	31 €	(42-30)+(31-42) = 31-30 = 1 €
5:00	6:00	25 €	(42-30)+(25-42) = 25-30 = -5 €
6:00	7:00	39 €	(42-30)+(39-42) = 39-30 = 9 €
7:00	8:00	41 €	(42-30)+(41-42) = 41-30 = 11 €
8:00	9:00	44 €	(42-30)+(44-42) = 44-30 = 14 €
9:00	10:00	45 €	(42-30)+(45-42) = 45-30 = 15 €
10:00	11:00	48 €	(42-30)+(48-42) = 48-30 = 18 €
11:00	12:00	50 €	(42-30)+(50-42) = 50-30 = 20 €
12:00	13:00	52 €	(42-30)+(52-42) = 52-30 = 22 €
13:00	14:00	46 €	(42-30)+(46-42) = 46-30 = 16 €
14:00	15:00	47 €	(42-30)+(47-42) = 47-30 = 17 €
15:00	16:00	48 €	(42-30)+(48-42) = 48-30 = 18 €
16:00	17:00	51 €	(42-30)+(51-42) = 51-30 = 21 €
17:00	18:00	45 €	(42-30)+(45-42) = 45-30 = 15 €
18:00	19:00	40 €	(42-30)+(40-42) = 40-30 = 10 €
19:00	20:00	33 €	(42-30)+(33-42) = 33-30 = 3 €
20:00	21:00	31 €	(42-30)+(31-42) = 31-30 = 1 €
21:00	22:00	30 €	(42-30)+(30-42) = 30-30 = 0 €
22:00	23:00	22 €	(42-30)+(22-42) = 22-30 = -8 €
23:00	0:00	31 €	(42-30)+(31-42) = 31-30 = 1 €
Total payoff			213 €

Table 1: Payoff of a day contract, which was initially bought at 30 € and its final price was 42 €. We can see the payoff of each hour separately and in total at the end of the day. At the end of the day the buyer has a profit of 213 € without taking into account the time value of money.

or to make it look better:

$$f_{T_2} = (T_2 - T_1)\left[\tilde{P}(T_1, T_2) - F_t(T_1, T_2)\right] \qquad (4.4)$$

where $\tilde{P}(T_1, T_2) = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} P_s ds$ is the average of the spot price, P_s, during the delivery period $[T_1, T_2]$.

In the next paragraphs we will see a thorough description of the contracts an investor will find in Nord Pool financial market.

4.2.2 Futures day contracts

These contracts refer to a delivery period of one day. The differences between these contracts and the ones mentioned in spot market are two. The first one is that these refer to a whole day period and not just to an hourly one. The second difference is that most of them trade for more than one day ahead the delivery period. More specifically, every Friday nine day contracts are being traded. Every day a contract expires it is not replaced by any other until the following Friday. So depending on the day somebody enters the market, he could see nine contracts, six or even three on Thursdays. Examples of some day contracts that an investor could see on the 20/1/2006 are the ones illustrated in table 2. Seven of those contracts start trading on that date. We should note that the observation date is on Friday so the number of day contracts is the maximum that one could see at any day of the week.

4.2.3 Futures week contracts

These futures have a delivery period of one week and a time to maturity of six weeks. In table 2 we can see the week contracts which were traded on the 20/1/2006.The delivery period of the first one is the following week of the one we stand (21/1/2006 – 27/1/2006). The delivery period of the second contract is the second week (28/1/2006 – 3/2/2006) and so on. Every time a week contract expires there is another one that takes its place. So anytime we enter the market we can see six different week contracts being traded simultaneously. The maturity day of the futures is the last working day of the week, mostly Friday, and the trading period starts on the following Saturday and ends on next Friday.

4.2.4 Forward month contracts

The delivery period of this contract is one calendar month and the time to maturity is six months. Every time a month contract expires another one takes its place. So every time an investor enters the financial market he can see six different month contracts being traded simultaneously. The maturity date of these contracts is always the last weekday of each month and their first day of the delivery period is the first day of the following month. Examples of some month contracts that an investor could see on the 20/1/2006 are the ones illustrated in table 2. The first one for example is the forward contract of February-06, because it has a delivery period that expands from 1/2/2006 to 28/2/2006. As we can see it started trading on the 1/8/2006, six months before the beginning of the delivery period, as we said before.

4.2.5 Forward quarter contracts

A forward quarter contract, as can be easily concluded by its name has a delivery period of three months. At the beginning of each year there are eleven open quarter contracts in the market. Each contract corresponds to each quarter of the next three years, except the one that starts and its contract has expired. The dates for each quarter contract are 1/1-31/3, 1/4-30/6, 1/7-30/9 and 1/10-31/12. During the year, every time a quarter contract expires it is not replaced by any other. So till the 31/3 we have eleven contracts, till the 30/6 we have ten contracts, till the 30/9 we have nine contracts and till the 31/12 we have eight contracts. At the beginning of the year the contract that corresponds to the first quarter of that year, expires and four other contracts start negotiating with delivery periods of two years after. If we take a look at table 2 we will see the eleven contracts that were traded on the 20/1/2006. The first three of them have delivery periods during 2006, the other four have delivery periods during 2007 and the last four have delivery periods during 2008.

4.2.6 Forward year contracts

The longer lasting forward contracts are the year ones. Until the middle of 2006 these contracts had three years time to maturity. From the 1/8/2006 they have five years time to maturity. Every time a year contract expires another one takes its place. So until the 15/6/2006 an investor could always see three open year contracts which had a delivery period that corresponds to each one of the following three years. For example, looking at table 2, we can see that on the 20/1/2006 three year contracts were traded with delivery periods the years 2007, 2008 and 2009. From the 15/6/2006 year contracts for 2010 and 2011 were added. From that date on the year contracts that an investor could enter are five.

Table 2: Examples of futures day and week contracts that an investor could see at the close of the trading on the 20/1/2006, as we can see by the first column. The second column indicates the closing price in Norwegian Kroner of the futures price. The third and fourth columns show the start and the maturity date of each contract whereas the last columns show the delivery period of each contract.

Futures day contracts				
Observation dates	**Futures price (in NOK)**	**Start date**	**Maturity date**	**Delivery period**
20/1/2006	331,91	13/1/2006	20/1/2006	21/1/2006
20/1/2006	314,03	13/1/2006	20/1/2006	22/1/2006
20/1/2006	381,60	20/1/2006	20/1/2006	23/1/2006
20/1/2006	373,65	20/1/2006	23/1/2006	24/1/2006
20/1/2006	389,55	20/1/2006	24/1/2006	25/1/2006
20/1/2006	425,33	20/1/2006	25/1/2006	26/1/2006
20/1/2006	382,63	20/1/2006	26/1/2006	27/1/2006
20/1/2006	382,63	20/1/2006	27/1/2006	28/1/2006
20/1/2006	382,63	20/1/2006	27/1/2006	29/1/2006

Futures week contracts					
				Delivery period	
Observation dates	**Futures price (in NOK)**	**Start date**	**Maturity date**	**First day**	**Last day**
20/1/2006	382,63	12/12/2005	20/1/2006	21/1/2006	27/1/2006
20/1/2006	407,28	19/12/2005	27/1/2006	28/1/2006	3/2/2006
20/1/2006	396,71	27/12/2005	3/2/2006	4/2/2006	10/2/2006
20/1/2006	379,61	2/1/2006	10/2/2006	11/2/2006	17/2/2006
20/1/2006	371,11	9/1/2006	17/2/2006	18/2/2006	24/2/2006
20/1/2006	364,75	16/1/2006	24/2/2006	25/2/2006	4/3/2006

Table 2 (continued): Examples of forwards month and quarter contracts that an investor could see at the close of the trading on the 20/1/2006, as we can see by the first column. The second column indicates the closing price of the futures price Norwegian Kroner. The third and fourth columns show the value (date that the contracts starts being negotiated) and the maturity date of each contract whereas the last columns show the delivery period of each contract.

Forward month contracts					
				Delivery period	
Observation dates	**Futures price (in NOK)**	**Start date**	**Maturity date**	**First day**	**Last day**
20/1/2006	387,96	1/8/2005	31/1/2006	1/2/2006	28/2/2006
20/1/2006	359,74	1/9/2005	28/2/2006	1/3/2006	31/3/2006
20/1/2006	344,63	3/10/2005	31/3/2006	1/4/2006	30/4/2006
20/1/2006	333,90	1/11/2005	28/4/2006	1/5/2006	31/5/2006
20/1/2006	326,75	1/12/2005	31/5/2006	1/6/2006	30/6/2006
20/1/2006	312,04	2/1/2006	30/6/2006	1/7/2006	31/7/2006

Forward quarter contracts					
				Delivery period	
Observation dates	**Futures price (in NOK)**	**Start date**	**Maturity date**	**First day**	**Last day**
20/1/2006	335,89	2/1/2004	31/3/2006	1/4/2006	30/4/2006
20/1/2006	333,50	2/1/2004	30/6/2006	1/7/2006	31/7/2006
20/1/2006	352,58	2/1/2004	30/9/2006	1/10/2006	31/10/2006
20/1/2006	362,92	1/1/2005	31/12/2006	1/1/2007	31/1/2007
20/1/2006	312,83	1/1/2005	31/3/2007	1/4/2007	30/4/2007
20/1/2006	300,11	1/1/2005	30/6/2007	1/7/2007	31/7/2007
20/1/2006	327,54	1/1/2005	30/9/2007	1/10/2007	31/10/2007
20/1/2006	338,75	1/1/2006	31/12/2007	1/1/2008	31/1/2008
20/1/2006	289,38	1/1/2006	31/3/2008	1/4/2008	30/4/2008
20/1/2006	280,48	1/1/2006	30/6/2008	1/7/2008	31/7/2008
20/1/2006	312,44	1/1/2006	30/9/2008	1/10/2008	31/10/2008

Table 2 (continued): Examples of forward year contracts that an investor could see at the close of the trading on the 20/1/2006, as we can see by the first column. The second column indicates the closing price of the futures price Norwegian Kroner. The third and fourth columns show the value (date that the contracts starts being negotiated) and the maturity date of each contract whereas the last columns show the delivery period of each contract.

Year contracts					
				Delivery period	
Observation dates	Futures price (in NOK)	Start date	Maturity date	First day	Last day
20/1/2006	325,95	2/1/2004	31/12/2006	1/1/2007	31/12/2007
20/1/2006	305,28	1/1/2005	31/12/2007	1/1/2008	31/12/2008
20/1/2006	304,64	1/1/2006	31/12/2008	1/1/2009	31/12/2009

Cfd name and reference area	Cfd definition
Norway	Oslo area price *minus* system price
Sweden	Stockholm area price *minus* system price
Finland	Helsinki area price *minus* system price
Denmark West	Aarhus area price *minus* system price
Denmark East	Copenhagen area price *minus* system price
SYGER	Phelix price Germany *minus* system price

Table 3: Cfd names and definition in Nord Pool financial market

4.2.7 Contracts for Difference (CfDs)

The reference price for the Nordic forward and futures contracts is the system price. Actual physical-delivery purchase costs are determined by actual area prices. An area price differs from the system price when there are constraints in the transmission grid.

CfDs allow investors to hedge against this area price risk. A perfect hedge using forward or futures instruments is possible only in situations when there is no transmission grid congestion in the market area, that is, area prices are equal to the system price. Hedging in forwards or futures therefore implies a basis risk equal to the difference between the area price at the member's physical location and the system price. Contracts for Difference were introduced to provide the possibility for a perfect hedge even when the markets are split into one or more price areas.

A CfD is a forward contract with reference to the difference between the area price and the Nord Pool Spot system price. A list of the CfDs that are traded in the Nord Pool financial market is illustrated in table 3. The delivery period of a CfD is a month, a quarter or a year. Its payoff function resembles the one of the futures and forward contracts. The cocept is exactly the same with the only difference that the settlement price is not the average of the system price but the average difference between the area price and the system price.

Suppose we stand at time t and we take a long position in a CfD with price $D_t(T_1,T_2)$, where $[T_1,T_2]$ is the delivery period of the contract. Let's assume that the maturity date of that contract is T. This means that at this date the trading of the contract stops and that a final price is settled. Its delivery period $[T_1,T_2]$ usually starts the next day after the maturity date. At the maturity date of the contract the owner realizes a payoff. This payoff at the maturity date T is given by the following equation:

$$d_T = D_T(T_1,T_2) - D_t(T_1,T_2) \tag{4.5}$$

where $D_t(T_1,T_2)$ is the purchase price of the contract at time t and $D_T(T_1,T_2)$ is the final price at time T, the maturity date.

In spite of the fact that the contract has a closing price at time T, it still provides a payoff during the delivery period $[T_1,T_2]$ and seizes to exist at time T_2. Suppose P_s^A and P_s is the area and system price in time s respectively and $D_T(T_1,T_2)$ is the CfD price at the time to maturity T, d_T the payoff of the CfD at time T and $d_{T_2-T_1}$ the payoff of the CfD at the delivery period. The payoff function of a CfD is described by the following equation:

$$\begin{aligned}
\widehat{d}_{T_2} &= \widehat{d}_T + \widehat{d}_{T_2-T_1} \\
&= \int_{T_1}^{T_2} \left(P_s^A - P_s - D_T(T_1,T_2)\right) ds \\
&= \int_{T_1}^{T_2} \left(P_s^A - P_s\right) ds - (T_2 - T_1) D_T(T_1,T_2) \\
&== (T_2 - T_1)\left[\breve{P}(T_1,T_2) - D_t(T_1,T_2)\right]
\end{aligned} \tag{4.6}$$

where $\breve{P}(T_1,T_2) = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2}\left(P_s^A - P_s\right) ds$ is the average of the difference of the area spot price and the system price during the delivery period $[T_1,T_2]$.

The market price of a CfD during the trading period reflects the market's prediction of the price difference during the delivery period. The market price of a CfD can be positive, negative or zero. CfDs trade at positive prices when the market expects a specific area price to be higher than the system price, (that is, the selected market area is in a net import situation). CfDs will trade at negative prices if the market anticipates an area price below the system price.

4.2.7 European options

Underlying assets of the European options in Nord Pool are the forward quarter and year contracts. The maturity of the option is before the start of the delivery period of the electricity forwards. The payoff of an option is dependent only on the price of the electricity forward contract. European options are actually futures options.

Electricity forward contracts are tradable assets and so it is easy to create a replicating portfolio for the European options. Suppose we have a European call forward option with strike price K and maturity at T. The payoff at the time of its maturity is the following:

$$c_T = \max\left(F_T\left(T_1, T_2\right) - K, 0\right) \quad (4.7)$$

where $F_T\left(T_1, T_2\right)$ is the forward contract price at time T with delivery period $\left[T_1, T_2\right]$.

The payoff of a European put forward option at the time of maturity is the following:

$$p_T = \max\left(K - F_T\left(T_1, T_2\right), 0\right) \quad (4.8)$$

where $F_T\left(T_1,T_2\right)$ is the forward price at time T with delivery period $\left[T_1,T_2\right]$.

5 Literature Review

Due to the fact that the majority of the new wholesale spot markets are imperfect and inefficient and the emergent power exchanges incomplete, in the financial sense, and insufficiently liquid, the need for careful and detailed modeling of prices has become an essential aspect of risk management in the industry. If competition were so efficient that prices reflected marginal costs, even in peak periods of demand, then there would be a complete price convergence of electricity with the underlying fuel costs (gas, etc), and spot price modeling would be relatively straightforward to specify. If the futures market were liquid and complete, as well, then forward prices would have a simple dynamic structure similar to other financial products from the forward markets. Neither of these situations prevails in most electricity markets, however, with the result that price models are considerably richer in structure than is seen in most other commodities.

In the last few years a great variety of publications concerning electricity prices modeling and valuation of electricity derivatives has been written. In this section we will try to give a short review of the models considered so far in the literature.

Since, due to the non-storability of electricity, spot products cannot be used for hedging purposes, the electricity market is a highly incomplete market and pure arbitrage option pricing methods fail for most structured products. Previous work had been focused on either of the two followi

- Stochastic models based on futures prices
- Stochastic models based on spot prices

5.1 Stochastic models based on futures prices

This group of models has a special feature. Instead of modeling the spot prices and deriving the futures prices, the futures prices themselves are being modeled. This approach has its routes in Black's methodology (1976), which in the case of electricity was used by Deng-Johnson-Sogomonian (1998). Their aim was to evaluate spark spread and locational options. For this reason they assume that the relevant price processes follow geometric Brownian motion processes and then under the more reasonable assumption that they follow mean reverting processes. These models are mostly bivariate. The first variable is, of course, the electricity futures price while the second one is the futures price of the generating fuel. By the term generating fuel we mean the fuel that is used to produce electricity, mainly gas and coal.

$$dF_e / F_e = \mu_e dt + \sigma_e dB^1 \tag{5.1}$$

$$dF_g / F_g = \mu_g dt + \sigma_g dB^2 \tag{5.2}$$

where B^1 and B^2 are two Wiener processes with instantaneous correlation ρ and F_e, F_g are the futures price processes of electricity and the appropriate generating fuel. μ_e, μ_g, σ_e, and σ_g are assumed to be constants.

$$dF_e / F_e = \kappa_e \left(\mu_e - \ln F_e\right) dt + \sigma_e(t) dB^1 \tag{5.3}$$

$$dF_g / F_g = \kappa_g \left(\mu_g - \ln F_g\right) dt + \sigma_g(t) dB^2 \tag{5.4}$$

where $\sigma_e(t)$ and $\sigma_g(t)$ are functions of time t, μ_e and μ_g are the long-term means, κ_e and κ_g are the mean-reverting coefficients and B^1 and B^2 are two Wiener processes with instantaneous correlation ρ.

Such models have the advantage that the market can be considered as being complete making it possible to construct replicating portfolios and use the risk-neutral asset for the option valuation procedure. Consequently, there is only one fair price for this derivatives and does not depend on the assumptions made. On the other hand futures prices cannot reveal information about price behavior on an hourly or even daily basis.

5.2 Stochastic models based on spot prices

These models aim at capturing the hourly or even daily price behavior by fitting their model to historical spot price data trying to find the best one for each dataset. Obviously, in this case we cannot use no-arbitrage arguments and so additional assumptions have to be made. The main assumption is the existence of an equivalent martingale measure Q^*:

$$F_{t,T} = E^*\left[S_T \middle| \Im_t\right] \tag{5.5}$$

This can be achieved after calibrating a market price of risk which will allow for the change of the drift of the stochastic process according to the Girsanov's theorem. Such an approach is done by Burger, Klar, Müller and Schindlmayr (2004), Villaplana (2003), Cartea and Figueroa (2004) etc. Due to the unique feature of electricity price there is not a unique probability measure, under which there will be a fair pricing of the electricity derivatives. That's why the market price of risk always depends on the assumptions authors will make.

There are also cases where a zero market price of risk is assumed. Lucia and Schwartz (2002) actually assume that when they try to estimate the MSE of the futures and forward contracts. This is done basically to make calculation easy and since there is not a unique market price of risk this assumption could stand for certain markets with sufficient liquidity like the Nord Pool.

5.2.1 One–factor and two-factor models

One way to distinguish models based on spot prices is by looking at the factors that are being used. So, we have one-factor and two-factor models. Examples of one-factor models are the ones below:

$$\begin{aligned} S_t &= g(t) + X_t \\ \ln S_t &= g(t) + X_t \end{aligned} \qquad (5.6)$$

where X_t is a non-observable variable whereas $g(t)$ is a deterministic factor which, as will be analyzed later, includes the behavioral trend of the historical spot data using various functions.

Apart from the above mentioned models Villaplana (2003), Schwartz and Smith (2000) and Burger et al. (2004) introduce the two-factor models:

$$\begin{aligned} S_t &= g(t) + X_t + \xi_t \\ \ln S_t &= g(t) + X_t + \xi_t \end{aligned} \qquad (5.7)$$

The factor (X_t) of equation (5.7) captures the short-term variations mostly characterized by mean reversion and very high volatility. The second factor, ξ_t, represents the long-term dynamics observed in the spot market. Although these factors are not directly observed they can be estimated by the use of spot and futures prices. Intuitively, movements in prices for [illegible] futures contracts provide information about the equilibrium price level and differences between the prices for the short- and long-term contracts provide information about short term variations in price.

Because it is not quite clear what the authors might mean by saying "long-term factor" let us think of an example. In the case that we take data from Nord Pool market we must always have in mind that most power plants are hydroelectric. This means that prices are highly correlated to the level of water reservoirs. The more water there is in the reservoir the more electricity can be produced. This means a greater supply and consequently a lower price. So if one year the level of raindrops is much different from the average of all years the equilibrium price will be higher or lower than usual. This change, that might not be a part of a deterministic trend, could be modeled by the long-term factor we saw above.

5.2.1.1 Modelling the short-term factor

The mean reversion characteristic and the very high volatility of the short-term factor were easily observable by the majority of the researchers who actually studied the spot price data for electricity. As a result there have been various stochastic models in the literature.

The simplest model of all was introduced by Johnson-Barz (1999), Bhanot (2000), Karesen-Husby (2000), Knittel-Roberts (2001) and Lucia-Schwartz (2002) that studied the simplest approach of a mean-reverting short-term factor:

$$dX_t = -kX_t dt + v_X dZ_X \quad (5.8)$$

with $(k > 0)$ the mean reversion speed coefficient and dZ_X is a Wiener process. They tested this approach using Nord Pool data. They managed to get closed-form solutions for forwards and futures but their [illegible] explain the existence of spikes in the spot data. These spikes, which were actually abnormal price changes, have their source in the inelasticity of supply of electricity which does not allow excess or low demand to be absorbed in a normal way. So they suggested that jumps should be included in the spot price models to capture this phenomenon. Escribano, Peña and Villaplana (2002), Villaplana (2003), Geman and Roncoroni (2003) and Cartea and Figueroa (2005) were among the researchers who used jump processes to model the spikes of the spot data. The two different approaches concerning the jump factor had to do with the distribution of the jump intensity process:

$$dX_t = -kX_t dt + v_X dZ_X + J(\gamma, \delta^2) d\Pi(\lambda_t) \qquad (5.9)$$

$$dX_t = -kX_t dt + v_X dZ_X + J_u(\eta_u) dN(\lambda_{t,u}) - J_d(\eta_d) dN(\lambda_{t,d}) \qquad (5.10)$$

where $d\Pi(\lambda_t)$, $dN(\lambda_{t,u})$ and $dN(\lambda_{t,d})$ are three Poisson processes with probability of occurrence of jumps given by λ_t, $\lambda_{t,d}$ and $\lambda_{t,u}$ respectively. $J_u(\eta_u)$, $J_d(\eta_d)$ is a double exponential process and $J(\gamma, \delta^2)$ is a random variable normally distributed with mean γ and standard deviation δ all trying to capture the jump intensity process. The Gaussian assumption imposes symmetry in the jump distribution, while the exponential assumption allows for asymmetry and separates positive and negative jumps.

A further diversification at these kinds of models is about the assumption made about λ_t. There are two trends, one supporting the fact that jump intensity coefficient should be constant through time (λ) and another supporting the necessity of modeling a time-dependent coefficient. Knittel and Roberts (2001), Escribano, Peña and Villaplana (2002) and Villaplana (2003) tried to capture the seasonality in the jump intensity modeling λ_t with the help of dummy variables:

$$\lambda_t = L_1 \cdot winter_t + L_2 \cdot fall_t + L_3 \cdot spring_t + L_4 \cdot summer_t \qquad (5.11)$$

This last equation models the time-dependent intensity process for the jumps by means of four dummies; $winter_t$ is a dummy variable that takes value 1 if the observation is in December, January and February and zero otherwise; $fall_t$ takes value 1 if the observation is in September, October and November and zero otherwise; $spring_t$ takes value 1 if the observation is in March, April and May and zero otherwise; $summer_t$ takes value 1 if the observation is in June, July and August and zero otherwise. On the other hand Cartea and Figueroa (2005), Lucia and Schwartz (2002) etc decided to assume that the jump intensity λ would be constant. Escribano et al. (2002) compared the two approaches used data from five different power exchange markets. They concluded that if we assume that the jump intensity is time-dependent we don't actually improve the model fitness and so it would be better if we made the simple assumption that λ remains constant through time.

5.2.1.2 Modelling the long-term factor

Apart from the different views for the short-term factor a lot of argument has been made about the proper modeling of the long-term factor of the above two-factor model although the alternatives are not so many as in the case of the short-term factor. Lucia and Schwartz (2002) and Villaplana (2003) modeled the long-term factor as following an arithmetic Brownian motion, meaning:

$$d\xi_t = \mu_\xi dt + v_\xi dZ_\xi \qquad (5.12)$$

An alternative approach of modeling this factor is to consider that it might follow a mean reverting process which in the case of electricity seems to have more sense. As a result the process modeled by Villaplana (2003) looks like that:

$$d\xi_t = k_\xi\left(\bar{\xi} - \xi\right)dt + v_\xi dZ_\xi \qquad (5.13)$$

with κ_ξ being the mean-reversion speed coefficient and $\bar{\xi}$ the long-term equilibrium price.

5.2.1.3 Modeling the seasonality factor

At these kinds of models, especially in the case of electricity price modeling, the above mentioned factors are always accompanied by a deterministic factor which tries to capture the seasonality of the spot price process. This is manly achieved with the use of sinusoidal functions and dummy variables which distinguish the prices between weekdays and weekends. So for example Escribano, Peña and Villaplana (2002) modelled $g(t)$ in the way presented below:

$$g(t) = c_0 + b \cdot t + c_1 \cdot \sin\left((t + c_2) \cdot 2\pi/365\right) + c_3 \cdot \sin\left((t + c_4) \cdot 4\pi/365\right) + d \cdot wkd_t \qquad (5.14)$$

where wkd_t is a dummy variable that takes value one if the observation is in weekday and zero otherwise (weekend). With this general formulation for the sinusoidal function they allow for the possibility of having two cycles per year, with one year and half-year period. In the case of one annual cycle we should have $c_3 = c_4 = 0$.

Nomikos and Soldatos (2007) using Nord Pool data proposed a slightly different deterministic component than the previous one excluding the linear and the constant factor:

$$g(t) = c_1 \cdot \sin\left((t + c_2) \cdot 2\pi/365\right) + c_3 \cdot \sin\left((t + c_4) \cdot 4\pi/365\right) + d \cdot wkd_t \qquad (5.15)$$

On the other hand Lucia and Schwartz (2002), who also made their research in Nord Pool, gave two alternatives of the deterministic function. So the additional terms, excluding the constant, try to capture, respectively, the variation in the level of prices between working and non-working days and the seasonal evolution of prices throughout the year. Their first suggestion was about the model:

$$g(t) = c_0 + d \cdot wkd_t + \sum_{i=2}^{12} \beta_i M_{it} \qquad (5.16)$$

where wkd_t is a dummy variable that takes value one if the observation is in weekday and zero otherwise (weekend) and M_{it} is a dummy variable that takes value one if the observation belongs to the i-th month of the calendar and zero otherwise. In this case the d parameters try to capture the changes on the level of the variable for holidays and weekends and for the different months of the year, respectively, with respect to the general long-run mean (assumed constant) for the working days during January.

Their second suggestion of the deterministic function takes the following form:

$$g(t) = c_0 + d \cdot wkd_t + c_1 \cdot \cos\left((t + c_2) \cdot 2\pi/365\right) \qquad (5.17)$$

where wkd_t is a dummy variable that takes value 1 if the observation is in weekday and zero otherwise (weekend). Here the coefficient tries to capture the changes on the level of the variable for holidays and weekends. The cosine function is supposed to reflect the seasonal pattern in the evolution of the relevant variable throughout the year, hence it has annual periodicity.

5.2.2 More sophisticated models

5.2.2.1 Assuming stochastic volatility

Stochastic volatility models have also been a very popular approach of modeling the spot price data (Duffie-2000, Knittel and Roberts-2001, Deng-1999, Escribano-2001, Escribano, Peña and Villaplana-2002). These models as indicated from their name, have a second uncertainty factor which comes from the time-dependent volatility since the Black-Scholes assumptions are loosened due to the fact of volatility clustering which is being observed in the spot price t-plots. One of the most popular approaches for modeling conditional volatility is the GARCH model and its extensions. A GARCH(1,1) model used is like the following:

$$h_t = \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot h_{t-1} \tag{5.18}$$

The nonnegative parameters ω, α, β characterize the dynamics of the volatility following a GARCH(1,1) process ($\omega > 0, \alpha, \beta \geq 0$). The nonnegativity restrictions are needed to guarantee that the conditional variance is positive and also ω has to be strictly positive for the process not to degenerate. If $\alpha + \beta \neq 1$ then the variance reverts back to its unconditional mean $\sigma^2 = \omega / (1 - \alpha - \beta)$.

In continuous time the above model is called Mean-Reverting Square-Root Process (MRSRP) and its mathematical specification is :

$$dv_t = k_v \cdot (\theta_v - v_t) dt + \sigma v_t^{1/2} dZ_v \tag{5.19}$$

where θ_v is a long-run average to which volatility reverts and k_v is a parameter that determines how fast v reverts to θ_v. Small k_v means fast mean-reversion. According to this model the probability density function of volatility is the non-central chi-square $\chi^2(2cV_t; 2q+2, 2u)$ with $2q+2$ degrees of freedom and a parameter of non-centrality $2u$. The advantage of the latter model in comparison with a simple mean-reverting model is the fact that according to that volatility cannot take negative values which is empirically true.

5.2.2.2 Regime-switching models

There has always been a lot of argument about the best way spikes should be modeled. The main effort of modeling these abnormalities, as already seen, above, has been made with the use of jump diffusion processes which present some limitations. Firstly, it is assumed that all shocks affecting the price series die out at the same rate. In reality, however, two types of shocks exist implying different reversion rates; large disturbances, which diminish rapidly due to economic forces, and moderate ones, which might persist for a while. This happens because the return to the long-run mean after a jump is motivated by the mean reversion coefficient that stands for the rest of the process. Secondly, the model assumptions for jump intensity (constant or seasonal) are convenient for simulating the distribution of prices over several periods of time but restrictive for actual short-term predictions for a particular time. The Poisson assumption for jumps may be valid empirically, but only provides the average probability of jumps for particular transitions.

Therefore an alternative modeling framework has been suggested by many authors the so-called regime-switching model. This can replicate the price discontinuities, observed in practice, and could detach the effects of mean-reversion and spike reversal, aliased in jump diffusion.

Ethier and Mount (1999) assume two latent market states and an AR(1) price process under both the regular and the abnormal regime and constant transition probabilities:

$$P_t - \mu_{S_t} = \phi\left(P_{t-1} - \mu_{S_{t-1}}\right) + \varepsilon_t \tag{5.20}$$

where $\varepsilon_t \sim N\left(0, \sigma^2_{S_t}\right)$. This retains however the misspecification and also imposes stationarity in the irregular spike process.

The model suggested by Huisman and Mahieu (2001) allows an isolation of the two effects assuming three market regimes: a regular state with mean-reverting price, a jump regime that creates the spike and finally a jump reversal regime that ensures with certainty reversion of prices to their previous normal level.

Deng (1999) introduced this kind of model using U_t a continuous-time two-state Markov chain:

$$dU_t = 1_{U_t=0} \cdot \delta(U_t) dN_t^{(0)} + 1_{U_t=1} \cdot \delta(U_t) dN_t^{(1)} \qquad (5.21)$$

where $N_t^{(i)}$ is a Poisson process with arrival intensity $\lambda^{(i)}$ (I = 0,1) and $\delta(0) = -\delta(1) = 1$. He next defines the corresponding compensated continuous-time Markov chain M(t) as

$$dM_t = -\lambda(U_t)\delta(U_t)dt + dU_t \qquad (5.22)$$

Another regime-switching model that has been proposed by Nomikos-Soldatos (2007) is the following:

$$\begin{aligned} P_t &= g(t) + \exp(X_t + Y_t) \\ dX_t &= k_1(\varepsilon - X_t)dt + \sigma_X dW_X \\ dY_t &= -k_2 Y_t dt + J(\mu_{J_i}, \sigma_{J_i}^2)dq(l_i) \end{aligned} \qquad (5.23)$$

$\varepsilon = \varepsilon_w$ if water reservoir levels are above seasonal average regime, $R_t = w$

$\varepsilon = \varepsilon_d$ if water reservoir levels are below seasonal average regime, $R_t = d$

where X_t and Y_t are non-observable variables and $g(t)$ is a seasonal function. In addition ε is a continuous time Markov chain assuming values for $i = w, d$ .The variable R_t determines the current state and is a random variable that follows a Markov chain with two possible states $R_t = \{w, d\}$, which are assumed to be observable based on whether water levels in the reservoirs are above or below their seasonal average at time t.

5.3 Estimation of parameters

The estimation of parameters has always been the most important part of model analysis since, whether a model is proper for each dataset is determined by the prices of parameters.

The most popular approach followed by Escribano-Peña-Villaplana (2002) and Villaplana (2003) is the use of Maximum Likelihood Estimators. Lucia-Schwartz (2002) estimated the parameters with the help of nonlinear least square methods. Although the latter method gives a good approximation of the parameters it is generally admitted that MLE gives better estimators in the sense of efficiency and consistency. The spot price data were used for these methods and the parameters that were estimated were the ones under the objective probability measure. The risk-neutral parameters were calibrated using the futures market after implementing the already estimated parameters (Burger et al. (2004), Villaplana-2003).

Finally a really interesting but questionable approach of estimation is the one used by Cartea-Figueroa (2005). There were many different algorithms used in order to estimate the mean-reversion speed and the various parameters concerning the jumps. The mean-reversion was estimated using a linear regression. More specifically they regressed the returns on the price series. In order to estimate the parameters of the jump component they filtered the data using an interesting code with the help of which they gradually took filtered out the jumps from the series of the returns. The problem with this method is that the parameters might not be the best approximation of the real ones. The advantage of this process is its simplicity and its quick calculation. We have actually used this method in order to estimate the initial values of our parameters.

6 Data Set Description

6.1 Spot data description

The Elspot data set was obtained from the Nord Pool's FTP server files and consisted of twenty-four hour time series, one for each hour during a day, of daily system prices, in Norwegian Kroner (NOK) per MWh for the fifteen year period starting 1/1/1993 and ending 31/12/2007. An example of this sample is displayed in table 3. There we can see, for example, that the price that electricity was sold at 09:00 on 22/05/1998 was 123.83 Norwegian Crowns (NOK). This price was settled the previous day, 21/05/1998 at 12:00 at noon.

The twenty-four hour price time series turned out to be highly correlated pair-wise. The linear correlation coefficients between any two hourly series during the sample period lie all above 0.85 (with a mean value of 0.97), and are always above 0.96 for any two consecutive hours. The 24 x 24 correlation matrix for our sample is displayed graphically in Figure 1. The cells in the grids each correspond to an element of the correlation matrix. The shade of the cell indicates the strength of the correlation: the whiter the grid gets, the stronger the correlation between the two hourly periods, which this grid stands for, is. Looking at the graph we observe that all trading periods fall naturally into groups: the prices in different trading periods within each group are highly correlated with each other, yet the correlations between prices in different groups are lower.

Date	07:00	08:00	09:00	10:00	11:00	12:00	13:00
22/05/1998	120.70	124.16	123.83	123.48	122.03	120.89	120.95

Table 3: This table shows the price for each hour separately from 07:00-13:00 for the 22/5/1998. The prices are in Norwegian Kroner. Those prices are the ones in which electricity was sold at that specific day and those specific hours and was settled the day before.

Figure 2: Intra-day correlation structure of the spot (system) price series for the period 1/1/1993 – 31/12/2007. The brighter the grid gets, the stronger the correlation between the corresponding hours. The data were taken from Nord Pool electricity market.

Another special feature of electricity prices is the fact that the departure from normality is extreme. Figure 2 shows a normality test for the electricity spot price from 1/1/1993 to 31/12/2007. If the returns were indeed normally distributed the graph would be a straight line. We can clearly observe this is not the case, as evidenced from the fat tails.

The Nord Pool uses the arithmetic average of all hourly prices for a given day as the reference price in the cash-settlement calculations at expiration for the derivatives' market. The way this happens is thoroughly described from equation (4.1). In accordance to this practice, we generated a new time series for this underlying variable by calculating the arithmetic average of the 24 available data for each day. Table 1 shows the descriptive statistics for the average price and other related time series and Figures 2 and 3 plot them for the complete sample period.

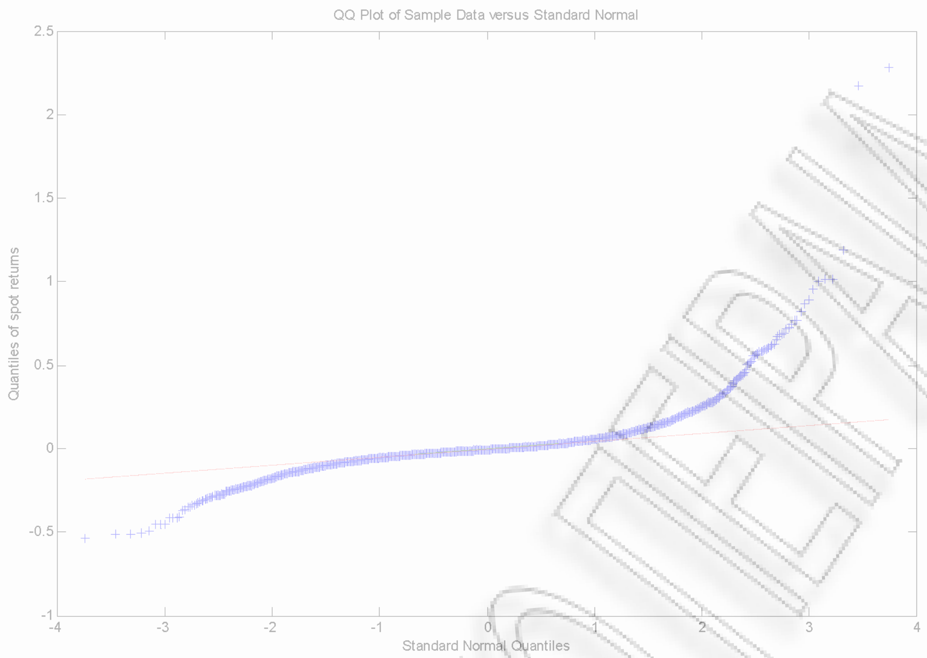

Figure 3: Normal probability test for returns of electricity prices from 1/1/1993 to 31/12/2007. The data were taken from the Nord Pool electricity market.

A first look to the average price series in Figure 2 reveals a quite erratic behavior of the system price. With a mean value of 191.43 NOK, it reached maximum and minimum values of 831.41 and 14.80 NOK, respectively, during the sample period. Nonstationarity tests have been conducted for both the price and the log-price series using the augmented Dickey-Fuller t-test for a unit root (see Table 1). The presence of a unit root is rejected in both cases at the 1% significance level.

Electricity prices are highly volatile, as measured by standard volatility measures. The standard deviation of the daily changes in log-prices is 0.1, which translates into an annualized volatility of 191%. Observing the data we could say that there exists a significant difference between cold and warm seasons. More specifically standard deviations of log-price changes show that warm seasons are much more volatile than cold seasons (0.087 or 166% annualized and 0.128 or 244% annualized, respectively). This is an expected result since the log transformation is expected to increase comparatively the volatility during periods with consistently lower prices and to contribute to the significant difference in the standard deviations of changes in log prices between warm and cold seasons. Note that warm seasons had a daily mean price about 17% lower than the mean for cold seasons during the sampling period (see Table 1). We should also notice that changes in log-prices are not directly interpreted as changes in prices.

The standard deviation of the price itself is 102.78 for warm and 97.744 for cold seasons, indicating a significantly higher stability of the mean price for warm seasons as compared to cold seasons, during the sample period. Changes in prices, in turn, show very similar standard deviations for both seasons

Table 4: This table displays descriptive statistics for the average hourly system price and other related time series. The data were taken from Nord Pool Spot Market for the period 1/1/1993-31/12/2007. Panel A displays the results for the whole data set, panel B for warm seasons and panel C for cold seasons. Warm seasons include observations for any date t from May 1 to September 30, both inclusive. So cold seasons include observations for any date t from October 1 to April 30. Under the heading ADF, the last column shows the value of the augmented Dickey-Fuller test t-statistic for the null hypothesis of a unit root, using twenty one lags in the relevant model, i.e. the statistic is the usual t-ratio for the ϕ coefficient in the model: $\Delta y_t = \mu + \phi y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + u_t$ with p = 21 and $\Delta y_t \equiv y_t - y_{t-1}$ where y_t stands for P_t and $\ln P_t$, alternatively. Critical values are: -3.458 at the 1%, -2.871 at the 5% and -2.594 at the 10% level.

Series	Autocorrelation Coefficient of Lag 1	2	3	4	5	7	14	21	28	35	ADF
Panel A: All Seasons											
P_t	0.984	0.972	0.966	0.958	0.952	0.946	0.904	0.879	0.853	0.813	-4.849
$P_t - P_{t-1}$	-0.121	-0.177	0.036	-0.053	-0.096	0.262	0.256	0.284	0.287	0.235	
$\ln P_t$	0.984	0.969	0.960	0.954	0.950	0.952	0.914	0.885	0.858	0.825	-4.452
$\ln P_t - \ln P_{t-1}$	-0.047	-0.180	-0.073	-0.093	-0.159	0.402	0.370	0.366	0.341	0.324	

Descriptive stats

Series	Number of Observations	Mean	Median	Minimum	Maximum	Standard Deviation	Skewness	Kurtosis
Panel A: All Seasons								
P_t	5,475	191.430	173,560	14.797	831.410	101.380	1.398	6.668
$P_t - P_{t-1}$	5,474	0.044	-0.897	-264.290	440.590	17.873	2.731	106.276
$\ln P_t$	5,475	5.114	5.157	2.694	6.723	0.557	-0.645	4.219
$\ln P_t - \ln P_{t-1}$	5,474	0.000	-0.005	-0.774	1.190	0.1[illegible]	1.114	[illegible]
Panel B: Warm Seasons								
P_t	2,295	170.930	148.800	14.797	646.690	102.7[illegible]	1.274	5.515
$P_t - P_{t-1}$	2,294	0.070	-0.753	-321.880	133.040	17.7[illegible]	-2.[illegible]	[illegible]
$\ln P_t$	2,295	4.951	5.003	2.694	6.472	0.6[illegible]	-0.577	3.318
$\ln P_t - \ln P_{t-1}$	2,294	0.000	-0.005	-1.153	0.701	0.12[illegible]	[illegible]	[illegible]

Table 4 (Continued): This table displays descriptive statistics for the average hourly system price and other related time series. The data were taken from Nord Pool Spot Market for the period 1/1/1993-31/12/2007. Panel A displays the results for the whole data set, panel B for warm seasons and panel C for cold seasons. Warm seasons include observations for any date t from May 1 to September 30, both inclusive. So cold seasons include observations for any date t from October 1 to April 30. Under the heading ADF, the last column shows the value of the augmented Dickey-Fuller test t-statistic for the null hypothesis of a unit root, using twenty one lags in the relevant model, i.e. the statistic is the usual t-ratio for the ϕ coefficient in the model: $\Delta y_t = \mu + \phi y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + u_t$ with p = 21 and $\Delta y_t \equiv y_t - y_{t-1}$ where y_t stands for P_t and $\ln P_t$, alternatively. Critical values are: -3.458 at the 1%, -2.871 at the 5% and -2.594 at the 10% level.

Series	Number of Observations	Mean	Median	Minimum	Maximum	Standard Deviation	Skewness	Kurtosis
				Panel C: Cold Seasons				
P_t	3,180	206.220	186.080	33.442	831.410	97.744	1.668	7.954
$P_t - P_{t-1}$	3,179	0.076	-0.954	-264.290	440.590	20.100	3.193	113.410
lnP_t	3,180	5.231	5.226	3.510	6.723	0.440	0.094	3.153
$lnP_t - lnP_{t-1}$	3,179	0.000	-0.005	-0.774	1.190	0.087	[illegible]	[illegible]

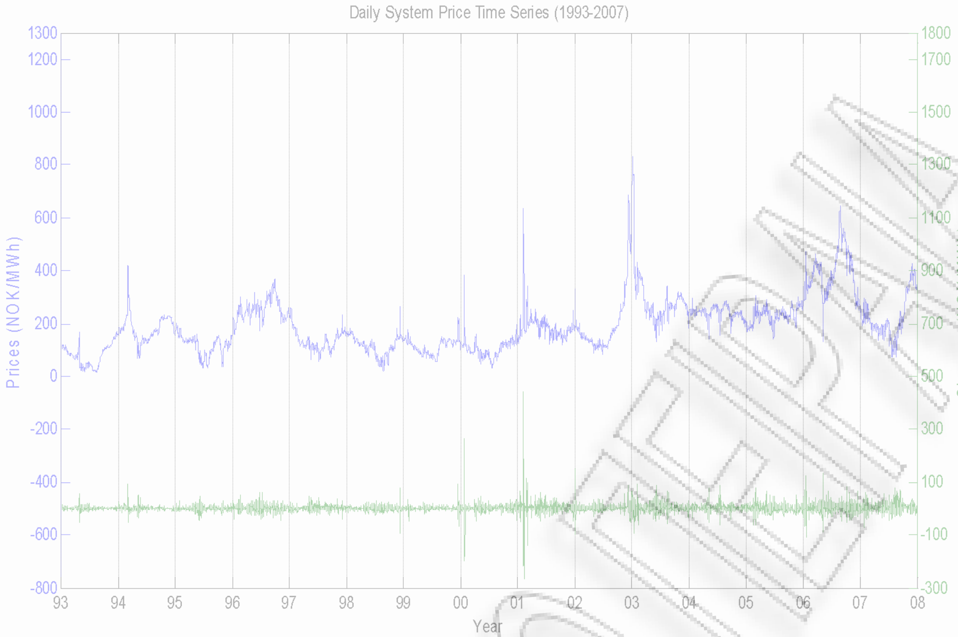

Figure 4: The figure plots the average hourly system price series (above) as well as its average daily changes for the period 1/1/1993-31/12/2007 (below). The left y-axis corresponds to the system prices whereas the right y-axis corresponds to the daily changes of them.

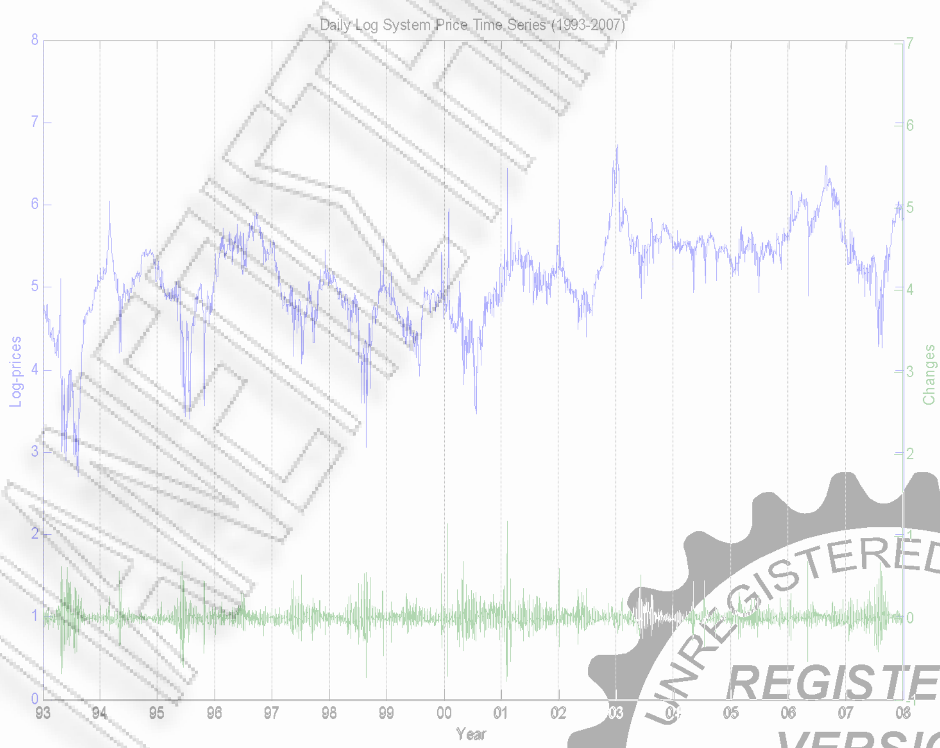

Figure 5: The figure plots the natural log of the average daily system price series for the period 1/1/1993–31/12/2007 (above) together with its average daily changes (below). The left y-axis corresponds to the logarithm of the system prices whereas the right y-axis corresponds to the daily changes of them.

Extreme electricity prices are relatively frequent in Elspot. This fact is reflected in the kurtosis coefficient, which is 6.663 for the whole sample. This is significantly different from three which is the kurtosis for a normal distribution. This practically means that extreme high and low values have a greater probability to occur than that dictated by a normal distribution with the same variance. The kurtosis estimates are similar between cold and warm seasons (7.954 and 5.515 respectively). The positive sign of the skewness estimates for the price series reveals that high extreme values are more probable than low ones. The log-price series shows a lower leptokurtosis than the price series (the kurtosis estimate is 4.219).

An interesting question to be answered is whether such extreme values are results of jumps in prices. By the notion of jump we mean the abnormally large variations in price. Looking at the kurtosis coefficient of changes in prices and changes in log-prices we notice that extreme variations are quite frequent especially during cold seasons. Specifically the kurtosis coefficient for daily price changes and daily log-price changes are 106.27 and 20.649, respectively, showing extremely leptokurtic distributions for both changes.

The kurtosis in changes during cold seasons is higher than the one of the whole sample. The excess kurtosis is more than one and a half times higher in cold seasons than in warm seasons for absolute changes in prices and it is about three and a half times higher for changes in log-prices. All these facts probably have to do with the shape of the supply curve which exacerbates the jumps in prices as a result of jumps in demand which cannot be followed by equal supply jumps due to the inelastic nature of it. Many of these jumps are due to temporary shocks in demand and as a result prices return rapidly to previous levels, causing spikes in the behavior of spot prices. We should also notice that there is a significative degree of asymmetry for prices (1.398) and log-prices (-0.645).

Looking at the system price we observe that it displays some signs of predictability. For instance, note that changes in spot prices and log-spot prices are positively autocorrelated at several lags multiples of seven (Table 3), meaning that there is a weekly pattern that spot prices and log-spot prices follow. This means that we could predict the electricity price from one week to another taking advantage of this pattern.

Figure 6 shows that there exists a regular intra-day pattern which is repeated every day with slight differentiations depending on the day we are looking at. More specifically, as can be easily seen by Figure 7, the price curve has a slightly different form for weekdays and weekends. This is absolutely logical if we remember that prices mainly depend on demand of electricity which changes depending on the business activity of that day. What is strange about this graph is that there seem to be higher prices for specific hours at weekends, which based on demand and supply rules should not happen. This might be an indication that prices are not totally determined by a supply and demand curve and that there are other factors that determine the equilibrium price.

There seems to be a general rule of seasonal fluctuations, which can be noticed by looking at Figures 4 and 5. More specifically, with the exception of a couple of years, the mean, the minimum, the maximum and the skewness coefficient are bigger for cold seasons than warm seasons. The latter one means that in winter we have a greater chance of having bigger price jumps. This regular pattern has mainly to do with seasonal changes in climate along the year which strongly determine heating needs and also with day-light length which influences lighting needs. We should also note the fact that except from demand, climate changes influence supply side. This happens because most of the electricity is produced from hydroelectric power plants. A large part of inflows in the reservoirs of these power plants is comes from snow melting during the warm seasons so it is predictable that electricity prices will sustain more demand shocks during the summer than winter.

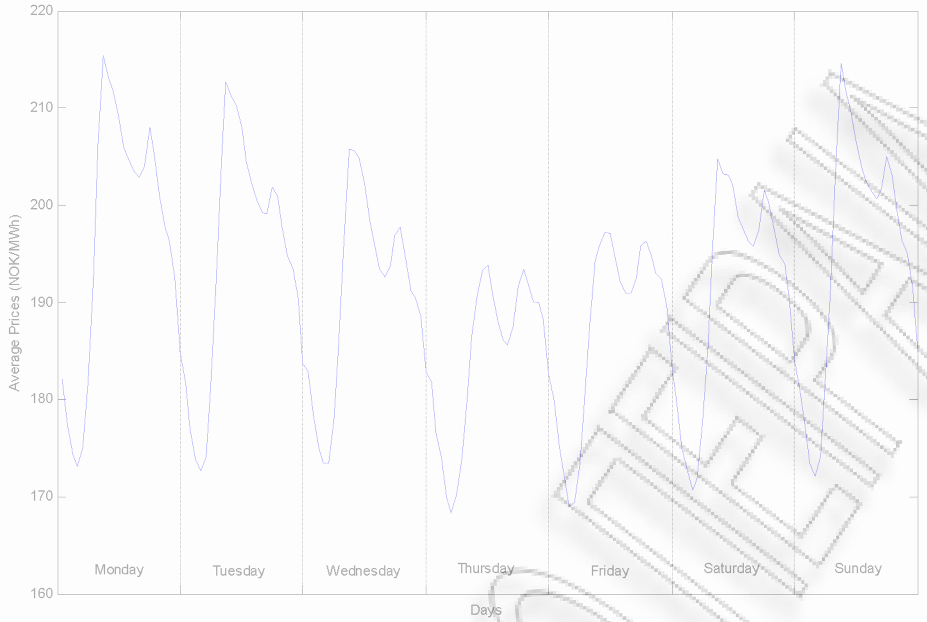

Figure 6: The figure displays the mean of the hourly prices throughout the week during the period 1/1/1993-31/12/2007. So for every day we see an average 24-hour series based on the above mentioned sample.

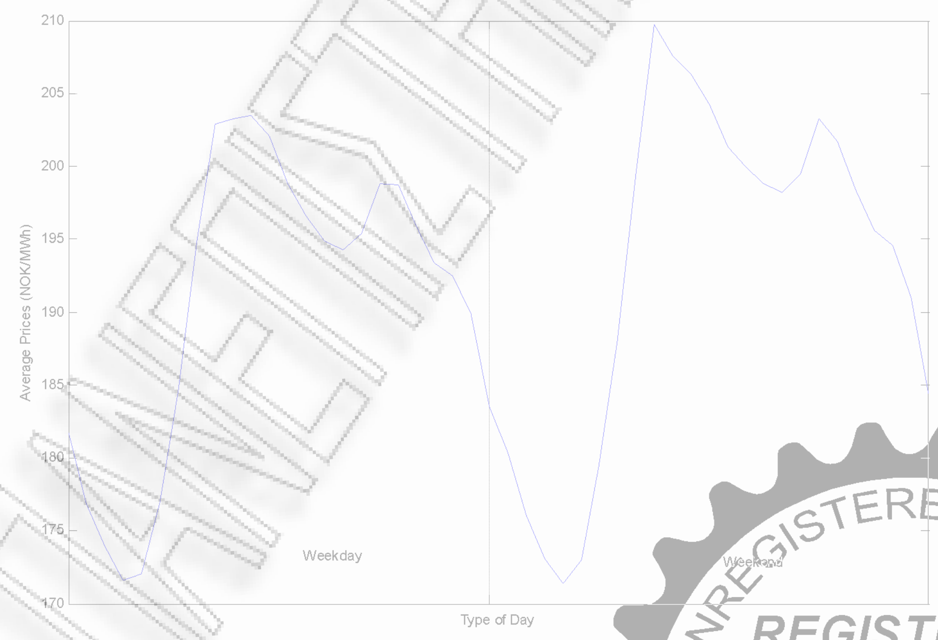

Figure 7: The figure displays the mean of the hourly prices separately for weekdays and weekends during the period 1/1/1993-31/12/2007. So for example if we look at the left part of the diagram we observe a 24-hour price series for every day that is supposed to be a weekday.

We should also note the fact that except from demand, climate changes influence supply side. This happens because most of the electricity is produced from hydroelectric power plants. A large part of inflows in the reservoirs of these power plants is comes from snow melting during the warm seasons so it is predictable that electricity prices will sustain more demand shocks during the summer than winter.

6.2 Futures/forwards data description

In this dissertation we used futures and forwards data from Nord Pool financial market in our effort to evaluate our models in terms of futures and forward pricing. So the prices we used were from the period 1/1/2006 – 31/12/2006. The prices were from futures day and week contracts and forward month, quarter and year contracts. Some descriptive statistics from all these contracts are illustrated in table 5.

As we can see the mean price of the quarter and year contracts is much smaller than the mean price in the other contracts with shorter delivery period. We assume that this happened because the market might have thought that in the longer term the electricity prices would be lower than in the near future.

Month contracts seem to be the most volatile of all forward contracts since it was in the month contracts that the minimum and the maximum prices of all forwards were observed. The same thing happens with the day futures which are the most volatile among all the futures.

It is also worth noticing that the volume for forward contracts is much bigger than the one in futures. This might mean that the forward market is deeper than the futures market and that maybe the market forward prices would be closer to the fair prices. This remains to be seen in the section where we try to price each contract separately according to our models.

Table 5: This table illustrates some descriptive statistics of the futures and forward contracts for the period 1/1/2006 – 31/12/2006. The contracts are separated according to their delivery period. Every descriptive statistic is shown for each type of contract separately and totally for the whole sample.

	Number of Contracts Traded per Day			Delivery Period (in days)			Futures/Forwards Prices (in NOK)			Daily Trading Volume (in GWh)		
Type of Contract	**Mean**	**Minimum**	**Maximum**	**Mean**	**Minimum**	**Maximum**	**Mean**	**Minimum**	**Maximum**	**Mean**	**Minimum**	**Maximum**
Futures												
Day	5.43	2.00	11.00	1.00	1.00	1.00	392.34	178.88	645.62			
Week	6.00	6.00	6.00	7.00	7.00	7.00	410.45	260.76	667.80			
All	5.72	2.00	11.00	4.00	1.00	7.00	401.40	178.88	645.62	30.99	0.00	200.04
Forwards												
Month	6.00	6.00	6.00	30.29	28.00	31.00	419.75	251.06	703.58			
Quarter	9.50	8.00	11.00	91.45	90.00	92.00	363.31	260.76	673.60			
Year	4.11	3.00	5.00	365.20	365.00	366.00	350.47	287.79	469.05			
All	6.54	3.00	11.00	162.31	28.00	366.00	377.84	251.06	703.58	[illegible]	0.00	[illegible]
All contracts	6.21	2.00	11.00	98.99	1.00	366.00	387.26	178.8	703.58	1,724.18	0.00	9,163.36

7 Models Specifications

As it has been previously said we will use three diffusion and jump-diffusion models in order to evaluate the dynamics of the electricity spot prices. Looking at Lucia and Scwartz (2002), Villaplana (2003), Elliot et al. (2000), Escribano et al (2002) and others, who used a deterministic seasonal function along with a stochastic process, we will follow the same way for our modelling purposes. The first one will try to catch the weekly and yearly trend that obviously exists in spot prices and the second one will try to explain the mean-reversion and the spikes in spot prices. The general model which we are going to use in this dissertation will be the following:

$$P_t = X_t + g(t) \tag{7.1}$$

where P_t is the electricity price at time t, $g(t)$ is the seasonal function, which represents the absolutely predictable part of the process, and X_t is the non-observable part of the process which we are going to model. The form of the seasonal function will be the following:

$$g(t) = c_0 + b \cdot t + c_1 \sin\left(\left(t + c_2\right)\frac{2\pi}{7}\right) + c_3 \sin\left(\left(t + c_4\right)\frac{2\pi}{365}\right) \tag{7.2}$$

where the first sinusoidal factor will try to explain the weekly seasonality and the second one will try to capture the yearly trend. The idea for the weekly seasonality factor came from the working paper of Ru[illegible] (2001) who found this trend in electricity spot price data. We were also triggered by the fact that this factor has not been widely used for this purpose. As we have already said we wanted to do something more than Lucia and Schwartz (2002) who assumed an annual and semiannual period.

The processes which will be tested in this dissertation will be the following:

7.1 Mean-reverting process (MRP)

$$dX_t = -kX_t dt + vdZ_t \tag{7.3}$$

Here we assume that the diffusion stochastic process X_t follows a stationary mean-reverting process, or Ornstein-Uhlenbeck process, with a zero long-run mean and a speed of adjustment k and that dZ_t is a Brownian motion. If we substitute (7.2) and (7.3) in (7.1) we get the dynamics of the spot price:

$$dP_t = k\left(a(t) - P_t\right)dt + vdZ_t \tag{7.4}$$

where $a(t) = \frac{1}{k}\frac{dg(t)}{dt} + g(t)$ is the long-term mean to which the process reverts. The extraction of the previous equation is provided in Appendix A.

We will also consider the case in which there is a market price of risk. In that case we switch from the objective measure that governs the real world to an equivalent risk-adjusted measure. According to Girsanov's theorem, we may change the probability measure by simply changing the drift of the process. The formulas in which we are going to conclude will be used for futures/forwards pricing purposes. Taking into account the non-tradable nature of X_t, standard arbitrage arguments with two derivative assets allow us to obtain the risk-neutral process for X_t. This is given by the equation:

$$dX_t = k\left(\alpha^* - X_t\right)dt + vdZ_t \tag{7.5}$$

with $\alpha^* \equiv -{}^{\phi v}\!/_{k}$ where ϕ stands for the market price of risk linked to the variable X_t. In this dissertation we assume that ϕ is constant. In general, it could be a function of variable X_t and t.

We can now derive the process of the spot price P_t. If we substitute (7.2) and (7.5) in (7.1) we get the dynamics of the spot price:

$$dP_t = k\left(a^*(t) - P_t\right)dt + vdZ_t^* \quad (7.6)$$

where $a^*(t) = \frac{1}{k}\frac{dg(t)}{dt} + \alpha^* + g(t)$.

The derivation of the market price of risk and the equation (7.6) is shown in Appendix B.

7.2 Mean-reverting process with Gaussian jumps (MRGJP)

$$dX_t = -kX_t dt + vdZ_t + Jdq_t \quad (7.7)$$

where $J \sim N\left(\gamma, \delta^2\right)$. Let dZ_t be a Brownian motion and dq_t a Poisson process with intensity λ which shows the number of jump arrivals per year and J is the jump amplitude. The Poisson process can be explained as follows:

- $\Pr\{dq_t = 1\} = \lambda dt$
- $\Pr\{dq_t = 0\} = 1 - \lambda dt$
- $\Pr\{dq_t > 1\} = 0$

Finally dZ_t, dq_t, J are all assumed to be mutually independent processes.

7.3 Mean-reverting process with double exponential jumps (MRDEJP)

$$dX_t = -kX_t dt + vdZ_t + Jdq_t \tag{7.8}$$

where the jump size is drawn from an asymmetric double exponential distribution, according to Kou (2002):

$$f(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y \leq 0\}} \tag{7.9}$$

where $p, q \geq 0$ and $p + q = 1$ represent the probabilities of the upward and downward jump, respectively, and $1/\eta_1, 1/\eta_2$ are the mean sizes of the upward and downward jumps, respectively.

The dynamics of the price for the two jump models, the MRGJP and the MRDEJP are given by the equation:

$$dP_t = k\left(a(t) - P_t\right)dt + vdZ_t + Jdq_t \tag{7.10}$$

with $a(t)$ being the same long-term mean as previously.

We should also illustrate the MRGJP and the MRDEJP in the case that we assume the existence of a market price of risk. For both jump processes we assume that the market price of jump risk is zero. So the jump parameters do not change when we switch to a risk-adjusted measure. This assumption is made by most of the authors who tried to include market price of risk in their research, like Villaplana (2003) Carte and Figueroa (2004) and many others. They assumed that the jump risk premia are captured by the jump size risk and so the initially estimated parameter λ could be the same in the risk-adjusted measure. This is something which makes our effort to approach the futures and forward time series much easier. Besides the more parameters we need to estimate the more unstable the model could become.

So the only change will be again in the drift of the processes. The dynamics of the variable X_t can be described in this case by the following equation:

$$dX_t = k\left(\alpha^* - X_t\right)dt + vdZ_t + Jdq_t \qquad (7.11)$$

with $\alpha^* \equiv -\phi v / k$ where ϕ stands for the market price of risk linked to the variable X_t.

So the spot price dynamics for the MRGJP and the MRDEJP if we switch to a risk-adjusted measure is described by the following equation:

$$dP_t = k\left(a^*(t) - P_t\right)dt + vdZ_t + Jdq_t$$

where $a^*(t) = \frac{1}{k}\frac{dg(t)}{dt} + \alpha^* + g(t)$. The transformation of the price dynamics were done following the approach from Lucia and Schwartz (2002), Cartea and Figueroa (2004) and Villaplana (2003). Their results will also be used when we illustrate the futures/forward formulas.

As it can be easily observed from figure 4 there is a mean-reverting trend. That explains the use of a mean-reverting model. Mean-reversion is a basic characteristic of the electricity spot prices. As a result most of the previous models used were, among others, mean-reverting models. Johnson-Barz (1999), Bhanot (2000), Karesen-Husby (2000), Knittel-Roberts (2001) and Lucia-Schwartz (2002) have used these kinds of models using data from

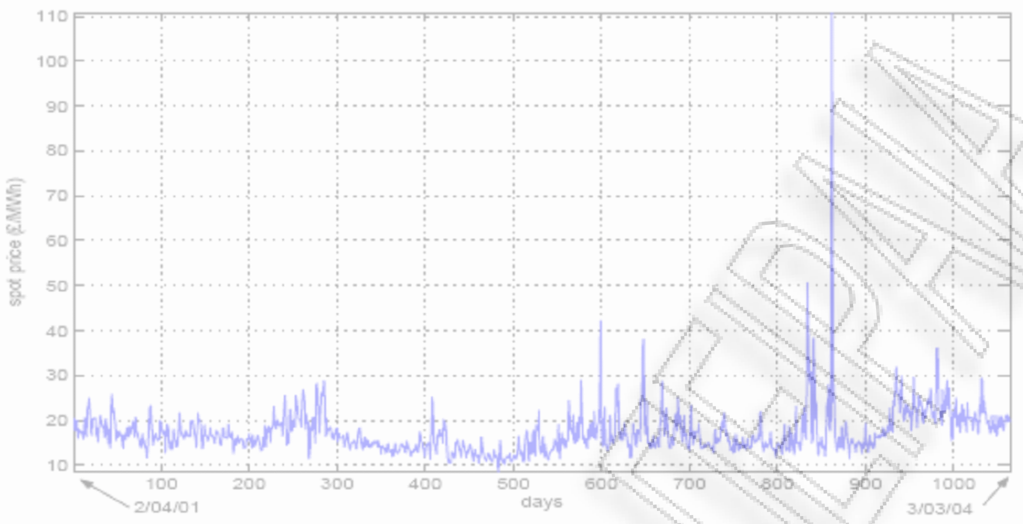

Figure 8: Averaged hourly electricity prices in Power Exchange Market of England and Wales from 2/04/2001 to 3/03/2004. This diagram was taken by Cartea and Figueroa (2004).

Nord Pool, Argentina, Australia, New Zealand and Spain ranging from 1993 to 2000.

The reason that we use the simple mean-reverting model is to get a full picture of the contribution of the jump components. It is also the nature of the price series from Nord Pool that encourages us to do so. By just looking at figures 4 and 8 and comparing the price series from Nord Pool and England & Wales market we can see that the spikes in the first market do not so intense compared to the ones in the second. As we have mentioned before this is absolutely normal due to the existence of hydroelectric factories which can sustain abnormal increases of demand of electricity. But this initial guess is left to be seen in the estimation process.

Finally because of the outliers observed in the price data pattern it is essential that a jump diffusion process is included in the spot price modeling. Jump diffusion processes have been widely used in previous literature. Gaussian jumps were used by Escribano, Peña and Villaplana (2002), Villaplana (2003), Geman and Roncoroni (2003) and Cartea and Figueroa

(2005). The process we are using is a double exponential jump-diffusion model proposed by Kou (2002).

This model allows for asymmetry and separates positive and negative jumps. We are mainly concerned in the comparison between the two jump models. We expect the MRDEJP to be more efficient than the MRGJP in the sense that it divides the upward jumps from the downward ones.

8 The Econometric Methodology

8.1 Empirical parameter estimation

In this section we will use empirical estimation techniques to estimate some of the parameters of the proposed model as Dafas (2004) and Cartea and Figueroa (2004) did with their models. In particular in the first section we will estimate the parameters of the seasonal function with curve fitting method. In the second section we will use Ordinary Least Squares (OLS) to estimate the mean reversion rate. Finally, in the last section we will use an iterative method to estimate the parameters of the jump process. It is important to note that these methods are just attempts to estimate the parameters of the model and act as a prologue to the Maximum Likelihood estimation technique.

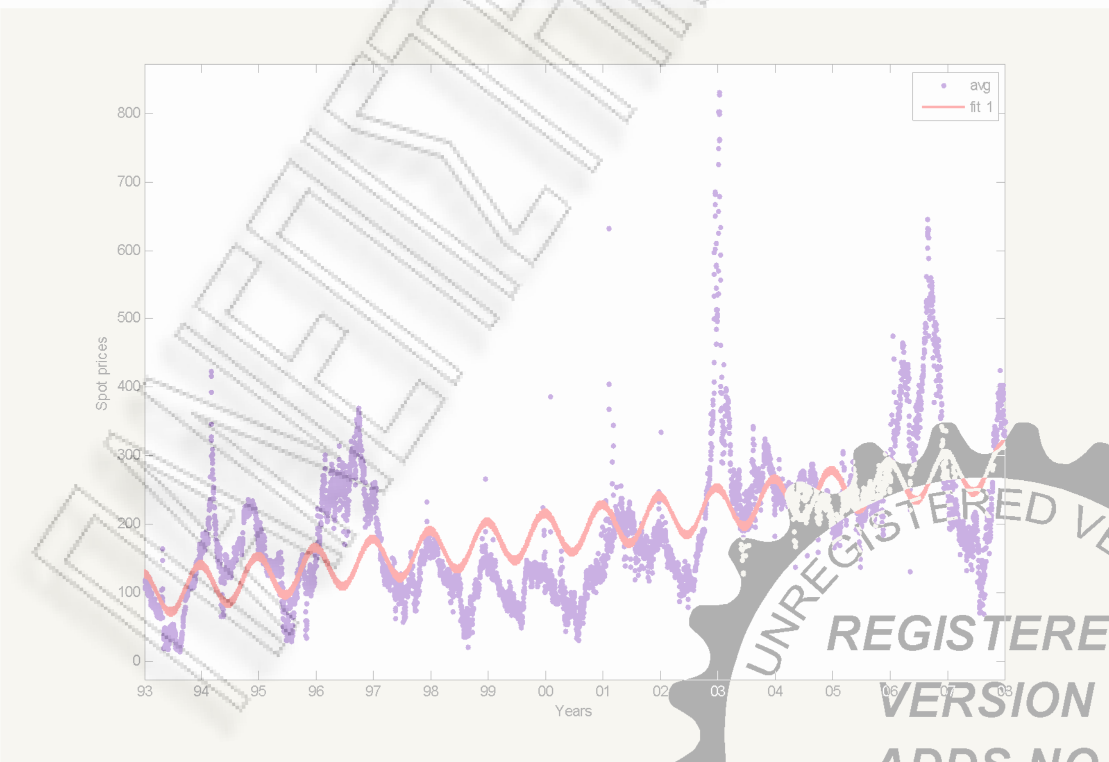

Figure 9: Fitting a seasonal function to the average hourly price series for the period 1/1/1993-31/12/2007. This function is given by the formula $g(t)=c_0+b\cdot t+c_1\sin\left(\left(t+c_2\right)\frac{2\pi}{7}\right)+c_3\sin\left(\left(t+c_4\right)\frac{2\pi}{365}\right)$ **and the parameters were estimated with the help of Ordinary Least Squares method.**

8.1.1 Estimation of the seasonal function

The estimation of the seasonal function parameters is done by fitting the seasonal function on the time series of electricity spot prices using the OLS method. This function has the form given from Figure 8. The initial estimates of the parameters with their t-statistics are given in Table 2. As we can see all the initial parameters are statistically significant.

8.1.2 Estimation of the mean-reversion rate

The mean-reversion rate can be easily estimated via simple linear regression of ΔP_t and P_t according to Cartea-Figueroa (2004) and Dafas (2004), who used it. Before doing so, we will assume that the mean is constant and that there is no jump in the process. This continuous time model in discrete form will be of the form presented in equation (8.1).

$$\Delta P_t = k\left(m - P_t\right)\Delta t + v\varepsilon_t \tag{8.1}$$

where ε_t is normally distributed with zero mean and variance Δt, more precisely ε_t is a white noise process. For performing OLS we transform (8.1) into:

$$P_{t+1} - P_t = km\Delta t - kP_t\Delta t + v\varepsilon_t \tag{8.2}$$

The drift initial estimates are found by minimizing the OLS objective function:

$$\left(\hat{k},\hat{m}\right) = \arg\min\sum_{i=1}^{n-1}\left(P_{t+1} - P_t - km\Delta t + kP_t\Delta_t\right) \tag{8.3}$$

The diffusion parameter initial estimate $\hat{v}$ is found as a standard deviation of residuals. After using the above mentioned method we extract the initial parameters of our estimation procedure. The results are shown in Table 6.

8.1.3 Estimation of the jump parameters

The most difficult part of the estimation of the initial parameters was the one for the jump models. The objective of the algorithm used, was to separate the Brownian increments from the jump part of the data series. The algorithm which will follow has also been used by Cartea and Figueroa (2005) and Dafas (2004).

So we extract the jumps from the original series of returns by writing a numerical algorithm that filters returns with values greater than three times the standard deviation of the returns of the series at that specific iteration. On the second iteration, the standard deviation of the remaining series (stripped from the first filtered returns) is again calculated; those returns which are now greater than three times this last standard deviation are filtered again. The process is repeated until no further returns can be filtered. This algorithm allows us to estimate the cumulative frequency of jumps, the mean and the standard deviation of jumps. In order to estimate the parameters of the double-exponential model we separate the upward and the downward jumps. We calculate the probability of upward and downward jumps by dividing the number of upward and downward jumps, respectively to the total number of jumps. We should also point out that we re-estimated the volatility of the diffusion part, without the jumps this time. The results of the estimation are also presented in table 4.

Table 6: In this table we present the initial estimation which were done in order make our Maximum Likelihood Estimation much more reliable and accurate. The data we used for the estimation of the models are the average hourly system prices from Nord Pool ranging from 1/1/1993 to 31/12/2007. Under each parameter there are the corresponding t-statistics in parentheses for the mean-reverting model (MRP: $dP_t = k\left(a(t) - P_t\right)dt + vdZ_t$), the Gaussian-jump model (MRGJP: $dP_t = k\left(a(t) - P_t\right)dt + vdZ_t + Jdq_t$) and the double-exponential jump model (MRDEJP: $dP_t = k\left(a(t) - P_t\right)dt + vdZ_t + Jdq_t$). The mean-level has the following form: $a(t) = dg(t)/(kdt) + g(t)$ where the seasonal function is $g(t) = c_0 + b \cdot t + c_1 \sin\left((t + c_2)2\pi/7\right) + c_3 \sin\left((t + c_4)2\pi/365\right)$. The jump size for the last two models is J and $E(dq) = \lambda dt$ where dq is the standard Poisson counter for the occurrence of jumps.

Parameters	Initial Estimation		
	Simple mean-reverting	Mean-reverting jump diffusions	
	MRP	MRGJP	MRDEJP
b	0,03 (48,53)	0,03 (48,53)	0,03 (48,53)
c_0	97,53 (43,69)	97,53 (43,69)	97,53 (43,69)
c_1	-5,60 (-3,55)	-5,60 (-3,55)	-5,60 (-3,55)
c_2	148,40 (484,78)	148,40 (484,78)	148,40 (484,78)
c_3	31,33 (19,84)	31,33 (19,84)	31,33 (19,84)
c_4	102,50 (34,88)	102,50 (34,88)	10[illegible] (34,88)
k	5,73 (6,55)	5,73 (6,55)	5,73 (6,55)
v	340,07 (-)	181,17 (-)	181,17 (-)
λ	-	21,75 (-)	21,75 (-)
γ	-	49,93 (-)	-
δ	-	38,07 (-)	-

Table 6 (Continued): In the MRGJP $\ln J \sim N(\gamma, \delta)$ while in the MRDEJP the probability density function of the jump amplitude is given by $f(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y \leq 0\}}$ where p, q are the probabilities of an upward and downward jump respectively and $1/\eta_1, 1/\eta_2$ are the mean sizes of the upward and downward jumps, respectively.

Parameters	Initial Estimation		
	Simple mean-reverting	Mean-reverting jump diffusions	
	MRP	MRGJP	MRDEJP
η_1	-	-	0,02 (-)
η_2	-	-	0,02 (-)
p	-	-	0[illegible]

8.2 Maximum Likelihood Estimators

The parameters of the various models are estimated by conditional Maximum Likelihood Estimation. The reason we are using this method is the fact that for large samples, MLE is the best method of estimation, because the estimated parameters are consistent, asymptotically normal and asymptotically efficient. However the MLE requires a complete specification of the conditional density function, which for nonlinear models may not have an explicit expression.

The log-likelihood function, which we want to maximize, is given from the expression:

$$L = \sum_{t=1}^{T} \log\left(f\left(P_t \middle| P_{t-1}, \Theta \right) \right) \qquad (8.4)$$

where $f\left(P_t \middle| P_{t-1}, \Theta \right)$ is the conditional density function of the process P_t and Θ is the set of parameters. We should note that discretization of continuous-time stochastic differential equations might introduce an estimation bias. However this is small in our case, where the data exist in a daily frequency. In the case of the mean-reverting model the density function can be extracted easily. However in the case of the jump models the density function can be derived via the corresponding conditional characteristic function.

In order to do that, following Duffie, Pan and Singleton (2000) we impose an affine structure on the on the coefficients. More specifically we consider the stochastic differential equation:

$$P_t = P_0 + \int_0^t \mu(P_s, s)\, ds + \int_0^t \sigma(P_s, s)\, dZ_s + q_t \qquad (8.5)$$

imposing an affine structure means that:

$$\mu(P_t,t)=K_0(t)+K_1(t)P_t \qquad (8.6)$$

$$\sigma(P_t,t)\sigma(P_t,t)'=H_0(t)+\sum_{k=1}^{n}H_1^k(t)\cdot P_k \qquad (8.7)$$

$$\lambda(P_t,t)=l_0(t)+l_1(t)\cdot P_t \qquad (8.8)$$

Notice that given an initial condition X_0, the vector $\theta=(\mathrm{K}_0,\mathrm{K}_1,\mathrm{H}_0,\mathrm{H}_1,l_0,l_1)$ can be used to determine a transform $\Psi_\theta:\mathbb{C}^n\times[0,\infty)\times[0,\infty)\times D\rightarrow\mathbb{C}$ of X_T conditional on $(\Im_t)_{0\le t\le T}$ defined by:

$$\begin{aligned}\Psi_\theta(u,t,T,P_t)&=E^\theta\left[\exp(u\cdot P_T)\middle|\Im_t\right]\\&=\exp\left(A(u,t,T)+B(u,t,T)\cdot P_t\right)\end{aligned} \qquad (8.9)$$

where $\mathrm{A}(\cdot),\mathrm{B}(\cdot)$ satisfy the complex ordinary differential equations:

$$\begin{aligned}\frac{\partial A(u,t,T)}{\partial t}&=-K_0(t)c-\frac{1}{2}c'H_0(t)c-l_0(\varphi(c)-1)\\\frac{\partial B(u,t,T)}{\partial t}&=-K_1(t)'c-\frac{1}{2}c'H_1(t)c-l_1(\varphi(c)-1)\end{aligned} \qquad (8.10)$$

with boundary conditions $A(u,T,T)=0,B(u,T,T)=u$. By setting $u=is$ we get the conditional characteristic function:

$$\phi_\theta(s,X_T|X_t)=E^\theta\left[\exp(is\cdot X_T)\middle|X_t\right]=\Psi_\theta(is,t,T,X_t) \qquad (8.11)$$

The Fourier inversion of the characteristic function gives us the conditional density function, $f(P_t|P_{t-1},\Theta)$:

$$f(P_t|P_{t-1},\Theta)=\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[e^{-isP}\ [illegible]\ (P_t,t,T,s,\Theta)\right]ds \qquad (8.12)$$

with Re denoting the real part of complex numbers. Afterwards we try to maximize the conditional likelihood function to get the estimators.

8.3 Estimation of the mean-reverting process

Under the mean-reverting process the electricity spot price at time t, given the price at time $t-1$ is distributed normally:

$$P_t \left| P_{t-1} \sim N(a(t)+k\left(a(t-1)-P_{t-1}\right)\exp(-k\Delta t), \frac{v^2}{2k}\left(1-\exp(-2k\Delta t)\right)\right. \tag{8.13}$$

As a result, the conditional density function for this process is the following:

$$f(P_t | P_{t-1};\Theta)=\exp\left[-\frac{\left(P_t-a(t)-\left(P_{t-1}-a(t-1)\right)e^{-k\Delta t}\right)^2}{\frac{\sigma^2}{k}(1-e^{-2k\Delta t})}\right]\times\frac{1}{\sqrt{\pi\frac{\sigma^2}{k}\left(1-e^{-2k\Delta t}\right)}} \tag{8.14}$$

which we will use to construct the log-likelihood function which takes the form:

$$K(\Theta)=-\frac{n}{2}\log\left(\frac{\pi v^2}{k}\right)-\frac{\left(P(0)-g(0)\right)}{v^2}-\frac{n-1}{2}\left(1-\exp(-2k\Delta t)\right)$$
$$-\sum_{k=1}^{n-1}\frac{\left(P_t-g(t)-\exp(-k\Delta t)\left(P_{t-1}-g(t-1)\right)^2\right)}{\frac{v^2}{k}\left(1-\exp(-2k\Delta t)\right)} \tag{8.15}$$

with $\Theta=(b,c_0,c_1,c_2,c_3,c_4,d,k,v)$ being the set of parameters and v the annualized volatility.

8.4 Estimation of the mean-reverting Gaussian jump process

In this process we assume that the spot price follows a mean-reverting process augmented by jumps, whose size is log-normally distributed with mean γ and variance δ^2, according to Merton (1976). The conditional spot price is distributed normally as follows:

$$P_t | P_{t-1} \sim \lambda N\left(P_{t-1} + k\left(a(t) - P_{t-1}\right)\Delta t + \gamma, v^2 + \delta^2\right) + (1-\lambda) N\left(P_{t-1} + k\left(a(t) - P_{t-1}\right)\Delta t, v^2\right) \quad (8.16)$$

and thus the conditional density function is:

$$\begin{aligned} f\left(P_t \mid P_{t-1}\right) &= \lambda \cdot \Delta t \cdot \exp\left[-\frac{\left(P_t - P_{t-1} - k\left(a(t) - P_{t-1}\right)\Delta t - \gamma\right)^2}{2\left(v^2 \Delta t + \delta^2\right)}\right] \times \frac{1}{\sqrt{2\pi\left(v^2 \Delta t + \delta^2\right)}} \\ &+ (1 - \lambda \cdot \Delta t)\exp\left[-\frac{\left(P_t - P_{t-1} - k\left(a(t) - P_{t-1}\right)\Delta t\right)}{2v^2 \Delta t}\right] \times \frac{1}{\sqrt{2\pi v^2 \Delta t}} \end{aligned} \quad (8.17)$$

Equation (8.17) approximates the true Poisson-Gaussian density with a mixture of normal distributions. Previous evidence by Ball and Torous (1983), and Escribano et al. (2002) have shown that the approximation of a Poisson process with a Bernoulli process leads to similar results and is much easier to estimate. Actually the assumption made here is that in each time interval (day) either only one jump occurs or no jump occurs. Since the limit of the Bernoulli process is governed by a Poisson distribution, we can approximate the likelihood function for the Poisson-Gaussian model using a Bernoulli mixture of the normal distributions governing the diffusion and jump shocks.

8.5 Estimation of the mean-reverting double-exponential jump process

The difference of this process with the latter is that this time the jump size is drawn from an asymmetric double exponential distribution described by the equation (7.7). In this case we cannot directly take a density function of closed form. Therefore we will use the Duffie et al. (2000) approach as was described in section 7.1. So, the conditional density function takes the following form, as was calculated by Psychoyios (2005):

$$f\left(P_t \mid P_{t-1};\Theta\right) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[e^{-isP_t} F\left(P_t, \Delta t; s, \Theta\right)\right] ds \qquad (8.18)$$

$$F\left(P_t, \Delta t; s, \Theta\right) = \exp\left(A\left(\Delta t; s\right) + B\left(\Delta t; s\right) P_t\right) \qquad (8.19)$$

$$A\left(\Delta t; s\right) = a\left(\Delta t; s\right) + z_1\left(\Delta t, s\right) + z_2\left(\Delta t, s\right) \qquad (8.20)$$

$$B\left(\Delta t; s\right) = ise^{-k\Delta t} \qquad (8.21)$$

$$a\left(\Delta t; s\right) = -\lambda\Delta t + isa(t)\left(1 - e^{-k\Delta t}\right) - s^2 v^2 \left(\frac{1 - e^{-2\kappa\Delta t}}{4k}\right) \qquad (8.22)$$

$$\begin{aligned} z_1\left(\Delta t, s\right) &= \frac{\lambda p}{2k}\left(2iArcTan\left(s/\eta_1\right) - 2iArcTan\left(e^{-k\Delta t}\, s/\eta_1\right) - \ln\left(1 + s^2/\eta_1^2\right) + \ln\left(e^{2k\Delta t} + s^2/\eta_1^2\right)\right) \\ z_2\left(\Delta t, s\right) &= \frac{\lambda q}{2k}\left(-2iArcTan\left(s/\eta_2\right) + 2iArcTan\left(e^{-k\Delta t}\, s/\eta_2\right) - \ln\left(1 + s^2/\eta_2^2\right) + \ln\left(e^{2k\Delta t} + s^2/\eta_2^2\right)\right) \end{aligned} \qquad (8.23)$$

9 MLE Results

Table 6 shows the MLE results for the Nordic system price series on the left and the initial parameter estimation, as described in the previous section, on the right. For each one of the processes we can see the t-statistics, the maximised log-likelihood function values (unstandardised and standardised), the Akaike Information Criterion and the Bayes Information Criterion.

Now let's filter some interesting results from the parameter estimations. Firstly almost all of the parameters are statistically significant at a 1% level of significance. The only parameters that are statistically insignificant are some of them related to the deterministic seasonal function. At that point we should note that we assume that in case the MLE are significant the same holds for the initial estimation. Besides this is not the main issue of the dissertation because we have already said that the MLE method is much more reliable than any other and so since the final estimated parameters are statistically significant we should not bother from where we started. To end this discussion we should notice that in spite of the fact that there were some initial estimations made, we also tried some other initial values especially in the case of the two jump models. By keeping a couple of parameters constant we changed all the others. In all the cases the fit was not as good as the one we got using the parameters estimated with the OLS and other methods.

Secondly we could observe that the mean-reverting process (MRP) is the worst fit for both series while the mean-reverting double-exponential jump process (MRDEJP) is the best one. We can be driven to this conclusion by looking at the log-likelihood values, the AIC and the BIC. The smaller the AIC and BIC the best is the fit of the corresponding model.

Another important conclusion is the fact that the mean-reverting speed becomes a bit smaller if we include jumps in the model. Moreover the mean-reverting speed is smaller if we assume that the jumps are being produced by a double-exponential rather than a Gaussian process.

The volatility of the prices, which is explained by the Brownian motion, is also affected by the choice of the model. With the addition of jumps the volatility becomes smaller whereas between the two jump models the MRDEJP leaves less volatility explained by the Brownian motion. In all the cases the initial estimations was much different than the final ones.

The jump intensity parameter, which notifies the number of jumps per year, is bigger than the one which we initially estimated for both MRGJP and MRDEJP. Also, by comparing the two jump models we can see that the jump intensity in the case of the MRDEJP is much bigger than in the case of MRGJP.

Another interesting result is the fact that, in the MRDEJP, the mean of the upward and downward jumps are equal. We remind that the mean sizes of them are given by the formulas $1/\eta_1, 1/\eta_2$. Another remark we could make is the fact that the probability of an upward jump ($p = 0.62$) is bigger than the probability of a downward one ($q = 1 - p = 0.38$).

Moreover, we could compare the mean and the standard deviation of the jumps in the case of the MRGJP and the MRDEJP. This will be done by using the formulas introduced by Kou (2002). Let's just repeat that the probability density function of the jump amplitude is given by:

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y \leq 0\}}$$

where p, q are the probabilities of an upward and downward jump respectively and $1/\eta_1, 1/\eta_2$ are the mean sizes of the upward and downward jumps, respectively. In the case of the mean of all the jumps, this is given by the formula:

$$E(Y) = \frac{p}{\eta_1} - \frac{q}{\eta_2} \tag{9.1}$$

whereas the variance is given by the formula:

$$Var(Y) = pq\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right)^2 + \left(\frac{p}{\eta_1^2} + \frac{q}{\eta_2^2}\right) \tag{9.2}$$

The mean of the jumps is $E(Y) = 2.18$ while the variance is $Var(Y) = 82.64$ and the standard deviation is $\sigma = 9.09$. Both the mean and the standard deviation of the jumps are smaller in the case of the MRDEJP.

Table 7: This table displays a comparison between the initial estimation using OLS and other methods we made in section 8 and the final estimation using the MLE method. The data which was used for the estimation of the models are the average hourly system prices from Nord Pool ranging from 1/1/1993 to 31/12/2007. Under each parameter there are the corresponding t-statistics in parentheses for the mean-reverting model (MRP: $dP_t = k\left(a(t) - P_t\right)dt + vdZ_t$), the Gaussian-jump model (MRGJP: $dP_t = k\left(a(t) - P_t\right)dt + vdZ_t + Jdq_t$) and the double-exponential jump model (MRDEJP: $dP_t = k\left(a(t) - P_t\right)dt + vdZ_t + Jdq_t$).

Parameters	Final Estimation witn MLE			Initial Estimation with OLS and other methods		
	Simple mean-reverting	Mean-reverting jump diffusions		Simple mean-reverting	Mean-reverting jump diffusions	
	MRP	MRGJP	MRDEJP	MRP	MRGJP	MRDEJP
b	0.03 (-14.07)	-0.01 (4.63)	-0.08 (-5.15)	0.03 (48.53)	0.03 (48.53)	0.03 (48.53)
c_0	97.37 (35.95)	115.25 (5.62)	104.51 (3.80)	97.53 (43.69)	97.53 (43.69)	97.53 (43.69)
c_1	-5.62 (-7.80)	3.72 (37.73)	-3.28 (-23.97)	-5.60 (-3.55)	-5.60 (-3.55)	-5.60 (-3.55)
c_2	148.40 (1,541.78)	-2.77 (-25.80)	147.2 (8953)	148.40 (484.78)	148.40 (484.78)	148.40 (484.78)
c_3	31.37 (22.87)	55,80 (6,7)	44,57 (7,83)	31,33 (19,84)	31,33 (19,84)	31,33 (19,84)
c_4	102.48 (19.02)	88.05 (12.17)	96.14 (15.85)	102.50 (34.88)	102.50 (34.88)	[illegible] [illegible]
k	6.87 (3.97)	4.52 (8.54)	2.75 (7.64)	5.73 (6.55)	[illegible] [illegible]	5.73 (6.55)
v	425.71 (28.31)	142.69 (49.92)	87.84 (91.26)	340.07 (-)	[illegible] [illegible]	181.17 (-)
λ	-	59.17 (5.22)	283.53 (21.07)	-	[illegible] (-)	[illegible] (-)
γ	-	8.31 (34.21)		-	49.93 (-)	-
δ	-	38.83 (27.35)		-	38.07 (-)	-

Table 7 (Continued): The mean-level has the following form: $a(t)=dg(t)/(kdt)+g(t)$ where the seasonal function is $g(t)=c_0+b\cdot t+c_1\sin\left((t+c_2)2\pi/7\right)+c_3\sin\left((t+c_4)2\pi/365\right)$. The jump size for the last two models is J and $E(dq)=\lambda dt$ where dq is the standard Poisson counter for the occurrence of jumps. In the MRGJP $\ln J\sim N(\gamma,\delta)$ while in the MRDEJP the probability density function of the jump amplitude is given by $f_Y(y)=p\eta_1 e^{-\eta_1 y}1_{\{y\geq 0\}}+q\eta_2 e^{\eta_2 y}1_{\{y<0\}}$ where p,q are the probabilities of an upward and downward jump respectively and $1/\eta_1, 1/\eta_2$ are the mean sizes of the upward and downward jumps, respectively. The statistics that show the goodness of fit are the maximized log-likelihood functions L (standardised and unstandardised), the Akaike Information Criterion and the Bayes Information Criterion. Estimation has been performed using the conditional Maximum Likelihood Estimation.

Parameters	Final Estimation with MLE			Initial Estimation with OLS and other methods		
	Simple mean-reverting	Mean-reverting jump diffusions		Simple mean-reverting	Mean-reverting jump diffusions	
	MRP	MRGJP	MRDEJP	MRP	MRGJP	MRDEJP
η_1	-	-	0.11 (24.65)	-		0.02 (-)
η_2	-	-	0.11 (23.51)	-		0.02 (-)
p	-	-	0.62 (31.57)	-		[illegible] (-)
L	-28.534	-21.435	-20.803	-	-	-
$L/n-1$	-5.21	-3.92	-3.80	-	[illegible]	-
AIC	57,085	42,892	41,629	-	[illegible]	-
BIC	57,138	42,965	41,708	-	[illegible]	-

10 Pricing performance

In this section we will estimate the pricing performance of the various processes estimated above. This estimation will be done by calculating the mean errors (ME) and the root mean squared errors (RMSE) between the pricing models and the actual prices for futures/forwards of different time to maturity. This will be initially done assuming a zero market price of risk, following the methodology of Lucia and Schwartz (2002). Afterwards we will make the same calculations under an equivalent probability measure under which we estimate a mean market price of risk for each model separately. Lucia and Schwartz (2002) do the same thing in their paper. We then look at each contract separately, and calculate the errors for each one using a different model each time.

For every process there is a different pricing model. The data we are going to use are from the electricity financial market and they are futures and forward prices from 1/1/2006 – 31/12/2006. We should denote that the year was chosen randomly. The observation dates were the last working day of each week. In our research we consider all the closing prices of the futures and forward contracts that were being traded at that date. No differentiation is made between the electricity futures and forwards when we try to price them because we assume that we are under a constant interest rate environment, where their prices are equal.

10.1 Forward pricing models

The price at time t of the forward expiring at time T with a delivery period $[T_1, T_2]$ is obtained using the expected value of the spot price at expiry under an equivalent Q-martingale measure conditional on the information set available up to time t. This expected value is denoted by $E_t^Q\left[P_T \middle| \Im_t\right]$, where $\Im_t = \sigma\left(P_s : s \leq t\right)$ is the natural filtration until time t.

So we have to calculate the expectation under an equivalent Q-martingale measure. In a complete market this measure is unique, ensuring only one arbitrage-free price of the forward. However, in incomplete markets (such as the electricity markets) this measure is not unique. Thus we are left with the difficult task of selecting an appropriate measure for the particular market in question.

We are going to make two assumptions. Initially we will assume that the market price of risk is zero. This means that we assume that the market is liquid enough to provide us with an appropriate probability measure. Cartea and Figueroa (2004) actually mentioned that if a market is liquid enough one could assume, not necessarily wrongly, that the objective probability measure is the right one to price the various financial contracts. Lucia and Schwartz (2002) also tested this possibility for Nord Pool by initially making the same assumption as we do. The period we are testing is eight years after the one they tested and so it is not pointless to test the same hypothesis.

Afterwards, we will make the assumption that there is a market price of risk and we will see what changes, if any, take place in the pricing performance of our models. Lucia and Schwartz (2002) did exactly the same thing. They found that there was no big difference between the two assumptions. The models with the zero market price of risk approached the futures/forward prices almost as good as the ones with the implicit market price of risk.

Table 8 shows the expected values of the spot prices according to the three different models which we estimated earlier. The expected values are illustrated under the objective probability measure, which we initially assume that is risk-neutral, and the risk-adjusted probability measure. The formulas for the MRP were derived in the appendices. The formulas for the two jump models were taken from Villaplana (2003).

The contracts which are negotiated in Nord Pool financial market are separated, as we saw in Section 4 according to their delivery period. So we have day, week, month, quarter and year contracts.

Now, let's assume that $\Im_t = \sigma(P_s : s \le t)$ is the natural filtration generated by the price process and $F_t(P_t; T_1, T_2)$ is the futures/forward price where T_1 and T_2 represent, respectively, the first day and the last day of the delivery period. So the futures/forward price will be:

$$F_t(P_t; T_1, T_2) = E^Q\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} P_T dT \,|\, \Im_t\right] = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} E_t^Q(P_T) dT \qquad (10.1)$$

where $E_t(P_T)$ is the expected value of the spot price for each model calculated in table 8 under the risk adjusted probability measure. In order to avoid confusion we will denote by $E_t(P_T)$ the expected value under the zero-market-price-of-risk-assumption probability measure. We repeat that this assumption seems wrong and the reason we are making it, is to denote the difference that the change of drift would make in the case of electricity futures/forwards. The above formula was taken by Lucia and Schwartz (2002), who used it in a discrete time setting and Burger (2004) who also provided the formula in the case of the continuous time setting.

Table 8: Expected value of the spot price according to the estimated models where MRP: $dP_t = k\left(a(t)-P_t\right)dt + vdZ_t$, MRGJP: $dP_t = k\left(a(t)-P_t\right)dt + vdZ_t + Jdq_t$ and MRDEJP: $dP_t = k\left(a(t)-P_t\right)dt + vdZ_t + Jdq_t$. The mean-level has the following form: $a(t) = dg(t)/(kdt) + g(t)$ where the seasonal function is $g(t) = c_0 + b\cdot t + c_1\sin\left((t+c_2)2\pi/7\right) + c_3\sin\left((t+c_4)2\pi/365\right)$. The jump size for the last two models is J and $E(dq) = \lambda dt$ where dq is the standard Poisson counter for the occurrence of jumps. In the MRGJP $\ln J \sim N(\gamma,\delta)$ while in the MRDEJP the probability density function of the jump amplitude is given by $f(y) = p\eta_1 e^{-\eta_1 y}1_{\{y\geq 0\}} + q\eta_2 e^{\eta_2 y}1_{\{y\leq 0\}}$ where p, q are the probabilities of an upward and downward jump respectively and $1/\eta_1, 1/\eta_2$ are the mean sizes of the upward and downward jumps, respectively. The upper half of the table has the pricing formulas assuming a zero market price of risk and the second half has the formulas under the risk-adjusted equivalent measure. The drift adjustment $\alpha^* = -\phi v/k$ where ϕ is the market price, v is the spot price volatility and k is the mean-reversion rate.

Models	**Expected mean values of the spot price under the objective probability measure**
1. MRP	$E_t(P_T) = g(T) + \left(P_t - g(t)\right)e^{-k(T-t)}$
2.MRGJP	$E_t\left(P_T\right) = g(T) + \left(P_t - g(t)\right)e^{-k(T-t)} + \lambda\frac{\gamma}{k}(1-e^{-k(T-t)})$
3.MRDEJP	$E_t(P_T) = g(T) + \left(P_t - g(t)\right)e^{-k(T-t)} + \frac{\lambda}{k}\left(1-e^{-k(T-t)}\right)\left[\frac{p}{\eta_1} - \frac{q}{\eta_2}\right]$
Models	**Expected mean values of the spot price under the risk adjusted probability me**
1. MRP	$E_t^Q(P_T) = g(T) + \left(P_t - g(t)\right)e^{-k(T-t)} + \alpha^*\left(1-e^{-k(T-t)}\right)$
2.MRGJP	$E_t^Q\left(P_T\right) = g(T) + \left(P_t - g(t)\right)e^{-k(T-t)} + \lambda\frac{\gamma}{k}(1-e^{-k(T-t)}) + \alpha^*\left(1-e^{-k(T-t)}\right)$
3.MRDEJP	$E_t^Q(P_T) = g(T) + \left(P_t - g(t)\right)e^{-k(T-t)} + \frac{\lambda}{k}\left(1-e^{-k(T-t)}\right)\left[\frac{p}{\eta_1} - \frac{q}{\eta_2}\right] + \alpha^*\left(1-e^{-k(T-t)}\right)$

10.2 Pricing performance: Results and discussion

Our next step will be to calculate the mean errors (ME) and the root mean squared errors (RMSE). If we assume that we stand on date t the formula for the ME is the following:

$$ME = \frac{1}{N}\sum_{t=1}^{N}\frac{F_{t,i}^{M}\left(P_t;T_1,T_2\right)-F_t^{A}\left(T_1,T_2\right)}{F_t^{A}\left(T_1,T_2\right)} \quad (10.3)$$

where $F_{t,i}^{M}\left(P_t;T_1,T_2\right)$ is the model price, with $i = MRP, MRGJP, MRDEJP$ are the three models we are investigating and $F_t^{A}\left(T_1,T_2\right)$ is the market (actual) futures and forward price for five different delivery periods. These are day, week, month, quarter and year. We also remind that date t varies from 1/1/2006 to 31/12/2006.

The formula for the calculation of the RMSE is the following:

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\frac{F_{t,i}^{M}\left(P_t;T_1,T_2\right)-F_t^{A}\left(T_1,T_2\right)}{F_t^{A}\left(T_1,T_2\right)}\right)^2} \quad (10.4)$$

The above formulas were the ones for the percentage estimation of the ME and RMSE. We also calculated the errors in NOK for each model.

The parameters we used were the ones estimated in the previous section. The results of the pricing performance of the models are shown in table 9. In contrast to Lucia and Schwartz (2002) who estimated the pricing performance of the MRP for Nord Pool forward/futures contracts we also estimated the performance of the MRGJP and the MRDEJP.

Looking at the results we extract some really interesting conclusions. First of all looking at each line we can see that the two jump models are better in predicting the daily futures price with one day to delivery and get worse as the delivery period expands going to extreme levels of error.

This is not the case for the mean-reverting process. Here the forward year contract seems to be a lot better than the forward quarter contract estimation. Let us remind that Lucia and Schwartz (2002) found that the year contract pricing was better for the MRP than any other contract. We actually found, that the MRP is better in approaching the actual futures and forward prices looking at the average of the ME of each contract. Specifically, the ME of the MRP for all the models is about -15%, whereas for the MRGJP the ME is -40% and for the MRDEJP IS -46%. So, if we assume that the market price of risk is zero, the metrics estimation shows something completely different from the econometrics estimation. We remind that under the econometrics estimation we found that the MRDEJP is the best fit in the spot data.

For the estimation of the day futures seems that it is irrelevant whether we use the Gaussian jump or the double-exponential jump model since their pricing performance is very close to each other.

Another interesting part of our estimation, as we have already seen, is the fact that the ME for every model and every contract are negative, meaning that our models tend to underestimate the price behavior of the futures/forwards of our sample. Since our models are supposed to be equal to the average of the expected of spot price, we could go further and say that

In general our pricing results are much worse than the ones Lucia and Schwartz (2002) found using a discrete time setting. The fact that we assume a zero market price of risk should not affect the comparison, since Lucia and Schwartz did the same thing. So, either our sample had a much different behavior than the one they used, or the fact that they used a discrete time setting might have helped them to approach the futures/forward curve in a better way. Of course it is always the case of the seasonal function which is slightly different.

So, after looking at the pricing results of all the models we conclude that the assumption of a zero market price of risk does not stand. The errors are sometimes quite big and so the change of the drift of the processes seems necessary.

So, after looking at the pricing results of all the models we conclude that the assumption of a zero market price of risk does not stand. The errors are sometimes quite big and so the change of the drift of the processes seems necessary.

This time we will use the formulas that are at the lower half of table 8. We will now calibrate the futures formulas in order to find the implicit market price of risk, ϕ, by minimizing the percentage RMSE for each observation date. We then calculate the mean of all observation dates for each model and we find the market price of risk. We make the simplifying assumption that ϕ remains constant throughout the sample period.

The implicit ϕ is for all the delivery periods and models negative in contrast to Lucia and Schwartz (2002) who found in all cases a positive market price of risk. For the whole sample it takes value -3.86 for the MRP, -8.00 for the MRGJP and -10.00 for the MRDEJP. We should here remind the reader that the market price of risk is considered as compensation per unit standard deviation. The fact that the market price of risk is always negative shows us that forward prices are upward-biased predictors of the spot price, or to be more specific the expected mean of the spot price is lower than the actual futures price.

Table 9: The table displays the results for the mean errors (ME) for each model using the formula $ME = \frac{1}{N}\sum_{t=1}^{N}\frac{F_{t,i}^{M}(P_t;T_1,T_2) - F_t^{A}(T_1,T_2)}{F_t^{A}(T_1,T_2)}$ and the root mean squared errors (RMSE) using the formula $RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\frac{F_{t,i}^{M}(P_t;T_1,T_2) - F_t^{A}(T_1,T_2)}{F_t^{A}(T_1,T_2)}\right)^2}$ assuming a zero market price of risk. $F_{t,i}^{M}(P_t;T_1,T_2)$ is the calculated price of each futures at date t where $i = MRP, MRGJP, MRDEJP$ is the model we use each time and T_1, T_2 is the first and the last day of the delivery period respectively. The delivery period is of one day, one week, one month, one quarter and one year. $F_t^{A}(T_1,T_2)$ is the actual price for the same futures/forwards according to the Nord Pool financial market. The parameters used for the models are the estimated ones of Section 9. The upper half of the table is used for the RMSE estimation whereas the lower half is used for the ME estimation.

	RMSE (percentage) with zero Market Price of Risk						RMSE (in NOK) with zero Market Price of Risk					
	Day	Week	Month	Quarter	Year	Total	Day	Week	Month	Quarter	Year	Total
MRP	0.10	0.28	0.58	0.65	0.60	0.44	39.96	117.06	254.05	239.33	214.69	173,02
MRGJP	0.09	0.22	0.49	0.67	0.70	0.43	35.53	89.36	214.08	242.20	246.37	165,51
MRDEJP	0.08	0.18	0.44	0.80	0.97	0.49	32.95	71.74	190.55	283.80	338.57	183,52

	ME (percentage) with zero Market Price of Risk						ME (in NOK) with zero Market Price of Risk					
	Day	Week	Month	Quarter	Year	Total	Day	Week	Month	Quarter	Year	Total
MRP	-0.04	-0.24	-0.51	0.64	-0.60	-0.15	-18.56	-99.85	-218.47	-233.79	-212.02	-156,54
MRGJP	-0.03	-0.18	-0.45	-0.67	-0.70	-0.40	-11.72	-74.78	-189.71	-238.77	-245.00	-152,01
MRDEJP	-0.02	-0.15	-0.40	-0.78	-0.96	-0.46	-7.33	-58.93	-167.35	-278.71	-334.65	-169,40

Table 10: The table displays the implicit market price of risk as it was estimated using futures and forward data from the period 1/1/2006-31/12/2006. Assuming the existence of a market price of risk it also shows the results for the mean errors (ME) for each model using the formula $ME = \frac{1}{N}\sum_{t=1}^{N}\frac{F_{t,i}^{M}(P_t;T_1,T_2) - F_t^A(T_1,T_2)}{F_t^A(T_1,T_2)}$ **and the root mean squared errors (RMSE) using the formula** $RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\frac{F_{t,i}^{M}(P_t;T_1,T_2) - F_t^A(T_1,T_2)}{F_t^A(T_1,T_2)}\right)^2}$. $F_{t,i}^{M}(P_t;T_1,T_2)$ **is the calculated price of each futures at date** t **where** $i = MRP, MRGJP, MRDEJP$ **is the model we use each time and** T_1, T_2 **is the first and the last day of the delivery period respectively. The delivery period is of one day, one week, one month, one quarter and one year.** $F_t^A(T_1,T_2)$ **is the actual price for the same futures/forwards according to the Nord Pool financial market. The parameters used for the models are the estimated ones of Section 9.**

Implicit Market Price of Risk	ϕ
MRP	-3.86
MRGJP	-8.00
MRDEJP	-10.00

	RMSE (in percentage) with Market Price of Risk						RMSE (in NOK) with Market Price of Risk					
	Day	Week	Month	Quarter	Year	Total	Day	Week	Month	Quarter	Year	Total
MRP	0.082	0.093	0.217	0.134	0.126	0.130	31.27	43.43	98.66	54.98	43.24	54.32
MRGJP	0.083	0.083	0.180	0.107	0.076	0.106	31.43	35.70	76.62	42.88	25.99	42.52
MRDEJP	0.081	0.071	0.133	0.098	0.141	0.105	30.98	29.79	58.38	37.24	49.14	41.11

	ME (in percentage) with Market Price of Risk						ME (in NOK) with Market Price of Risk					
	Day	Week	Month	Quarter	Year	Total	Day	Week	Month	Quarter	Year	Total
MRP	0.014	-0.050	-0.131	0.015	0.081	-0.071	2.69	-24.28	-63.87	1.89	26.07	-11.50
MRGJP	0.015	-0.038	-0.098	0.013	0.022	-0.017	3.36	-17.80	-46.88	0.02	5.94	-11.07
MRDEJP	0.016	-0.027	-0.052	0.025	-0.066	-0.021	4.44	-12.21	-27.44	4.79	-24.49	-54.91

This last conclusion can also be seen in the ME of our models which are mostly negative. This actually shows that the expected futures price is lower than the actual and since the expected futures price in a risk-adjusted measure equals the expected mean value of the spot price of the same time period then we see what we expected according to the previous paragraph.

It is also worth noticing that jump models experience the larger percentage reduction (MRDEJP about 79% and MRGJP about 75%) and the MRP has a reduction of 70%. More accurately the RMSE for the MRP in the whole sample is 13%, for the MRGJP is 10.6% and for the MRDEJP is about 10.5%. The last two fits were better than the ones Lucia and Schwartz (2002) found, using their discrete time models, which only assumed a simple mean-reversion of the spot price without jumps. The RMSE for the two jump models is almost the same but the big improvement is obvious if we compare them with the errors of the simple mean-reverting model. So, from that point of view the need to incorporate jumps in our models is unquestionable. Whether we use one jump model or the other depends on the contracts we would like to price.

Looking at the RMSE of the models we can now see that the error of all three models day contracts is almost the same. It is also worth noticing that in the case of MRP year contracts are much better approached than medium-term contracts like month and quarter ones. This conclusion was one of the basic conclusions that Lucia and Schwartz came up with. It is also interesting that the month contracts still have quite big errors (21.7% for the MRP, 18% for the MRGJP and 13.3% for the MRDEJP) compared to the errors of the other types of contracts. We also notice that the year contracts are the only ones where the MRGJP fits the best whereas in the other cases the MRDEJP does the best job.

So, in general, if we look at the total pricing performance we could say that we are indifferent between the two jump models but if we want to focus in a specific contract then our choice depends on what contract we focus.

11 Conclusions

In this dissertation we tried to capture the special features of the electricity prices. These are basically the existence of spikes, the seasonality and the mean-reversion. We then tried to indicate those characteristics in our dataset. The first two were obvious by simply looking to the spot price time series whereas the third one was not distinguishable because of the non-constant mean-reversion level.

In this paper our goal was to test the efficiency of the two jump-models along with the mean-reverting characteristic compared to the simple mean-reverting model. This comparison was done in a continuous time setting. Lucia and Schwartz (2002) and Escribano et al. (2004) estimated the MRP and the MRGJP in a discrete time setting. We made a small change in the seasonal function by incorporating a sinusoidal factor which tried to capture the weekly seasonality. We evaluated the models in two ways, firstly with econometric methodology and secondly by comparing their pricing performance in five different futures and forward contracts.

The three models that we used were a simple mean-reverting model, a mean-reverting with Gaussian jump model and a mean-reverting with a double-exponential jump model. At first by comparing the Likelihood function we conclude that the MRDEJP has a better fit comparing to the other two and the MRGJP is better than the simple MRP. This makes sense by looking at the data graphs which are full of jumps. It is also obvious by looking at the parameters, that the mean-reverting speed becomes a bit smaller if we include jumps in the model both for the spot and futures prices. It is also interesting to look at the great number of jumps per year if we separate them into upward and downward ones.

We then tried to evaluate the pricing formulas under two assumptions. Initially, we assumed that the market price of risk is zero. The models got only a bit close in the case of day futures. In general the MRDEJP was better explaining the futures and forward curve but the errors were too big to be ignored.

So our next step was to calibrate the models under the risk-adjusted probability measure in order to find the market of risk, ϕ, for each model and every delivery period. We made the simplifying assumption that ϕ remained constant throughout the investigating period. The results were more than encouraging. The errors became very minor; in most cases the models approached the futures and forward curve much better than it was approached by Lucia and Schwartz who also did the same thing.

Unlike the econometrics approach, MRDEJP proved as good as the MRGJP in the explanation of the futures/forward behavior. The final conclusion which seemed quite interesting was the fact that the market price of risk for every model was negative, maybe implying that the futures prices are actually bigger than the expected average spot price of the same period.

12 Appendices

Appendix A

Mean-reverting model at the objective probability measure

$$P_t = g(t) + X_t \tag{12.1}$$

$$dX_t = -kX_t + vdZ_t \tag{12.2}$$

Next we will try to model the dynamics of the spot price P_t:

Recalling that $X_t = P_t - g(t)$ we can write:

$$d\big(P_t - g(t)\big) = -\kappa\big(P_t - g(t)\big)dt + vdZ_t \tag{12.3}$$

$$dP_t - dg(t) = \kappa\big(g(t) - P_t\big)dt + vdZ_t \tag{12.4}$$

$$dP_t = k\left(\frac{1}{k}\frac{dg(t)}{dt} + g(t) - P_t\right)dt + vdZ_t \tag{12.5}$$

$$dP_t = \kappa\big(a(t) - P_t\big)dt + vdZ_t \tag{12.6}$$

where,

$$a(t) = \frac{1}{k}\frac{dg(t)}{dt} + g(t) \tag{12.7}$$

Suppose we form the following function:

$$f(P_t, t) = P_t \cdot \tag{12.8}$$

Using Ito's Lemma in function (12.8) we take the following expression:

$$df\left(P_t,t\right)=kP_te^{kt}dt+e^{kt}dP_t \qquad (12.9)$$

$$df(P_t,t)=kP_te^{kt}dt+e^{kt}\left[k\left(a\left(t\right)-P_t\right)dt+vdZ_t\right] \qquad (12.10)$$

$$df\left(P_t,t\right)=e^{kt}ka\left(t\right)dt+ve^{kt}dZ \qquad (12.11)$$

Integrating from 0 to t in both sides we obtain the following expression:

$$P_te^{kt}=P_0+\int_0^t ka\left(s\right)e^{ks}ds+\int_0^t \sigma e^{ks}dZ_s \qquad (12.12)$$

We are now going to solve the first integral:

$$\int_0^t ka\left(s\right)e^{ks}ds=\int_0^t k\left(\frac{1}{k}\frac{dg\left(s\right)}{ds}+g\left(s\right)\right)e^{ks}ds$$

$$=\int_0^t e^{ks}\frac{dg\left(s\right)}{ds}ds+\int_0^t ke^{ks}g\left(s\right)ds=e^{kt}g\left(t\right)-g\left(0\right) \qquad (12.13)$$

If we substitute (12.13) into (12.12) we will have:

$$P_te^{kt}=X_0+g\left(0\right)+e^{kt}g\left(t\right)-g\left(0\right)+v\int_0^t e^{ks}dZ_s \qquad (12.14)$$

$$P_t=g\left(t\right)+X_0e^{-kt}+v\int_0^t e^{ks}dZ_s \qquad (12.15)$$

Using the last relationship we can find the conditional mean and variance of P_t:

$$E_0(P_t)=E\left(P_t\middle|\Im_0\right)=g\left(t\right)+X_0e^{-kt}$$

$$=g\left(t\right)+\left(P_0-g\left(0\right)\right)e^{-kt} \qquad (12.16)$$

Appendix B

Mean-reverting model at the risk-adjusted probability measure

In this appendix we will illustrate the way the market price of risk, in the case of a non-tradable asset, is derived.

The initial mean-reverting process is described by equation (12.2):

$$dX_t = -kX_t + vdZ_t$$

In order to begin we assume that the process of a non-tradable asset is given by:

$$\frac{d\theta}{\theta} = \mu dt + vdW \qquad (12.17)$$

Let two derivatives being written on it. Their prices are:

$$\begin{aligned} \frac{df_1}{f_1} &= m_1 dt + \sigma_1 dW \Rightarrow \Delta f_1 = m_1 f_1 \Delta t + \sigma_1 f_1 \Delta W \\ \frac{df_2}{f_2} &= m_2 dt + \sigma_2 dW \Rightarrow \Delta f_2 = m_2 f_2 \Delta t + \sigma_2 f_2 \Delta W \end{aligned} \qquad (12.18)$$

We then form a portfolio using those two derivatives (we remind that the underlying asset is not tradable).

$$\Pi = n_1 f_1 + n_2 f_2 \qquad (12.19)$$

If we write the instant change of the value of the portfolio and replace on the equation we get equations (12.18) we get the result:

$$\Delta\Pi = n_1 \Delta f_1 + n_2 \Delta f_2$$

$$= n_1\left(m_1 f_1 \Delta t + \sigma_1 f_1 \Delta W\right) + n_2\left(m_2 f_2 \Delta t + \sigma_2 f_2 \Delta W\right)$$

$$= \left(n_1 m_1 f_1 + n_2 m_2 f_2\right)\Delta t + \left(n_1 \sigma_1 f_1 + n_2 \sigma_2 f_2\right)\Delta W \qquad (12.20)$$

In order to make our portfolio risk-free we should eliminate the source of uncertainty, ΔW. So we will choose quantities that make the second factor of the above sum equal to zero. These are:

$$\begin{aligned} n_1 &= \sigma_2 f_2 \\ n_2 &= -\sigma_1 f_1 \end{aligned} \qquad (12.21)$$

Then, if we substitute (12.21) into (12.20) we get:

$$\Delta\Pi = \left(m_1 \sigma_2 f_1 f_2 - m_2 \sigma_1 f_1 f_2\right)\Delta t \qquad (12.22)$$

Since the portfolio is risk-free it should give the owner the risk-neutral interest rate:

$$\Delta\Pi = r\Pi\Delta t \qquad (12.23)$$

If we substitute equation (12.19) and (12.22) in (12.23) we get:

$$\left(m_1 \sigma_2 f_1 f_2 - m_2 \sigma_1 f_1 f_2\right)\Delta t = r\left(\sigma_2 f_1 f_2 - \sigma_1 f_1 f_2\right)\Delta t \Rightarrow$$

$$\frac{m_1 - r}{\sigma_1} = \frac{m_2 - r}{\sigma_2} = \phi \qquad (12.24)$$

So the risk-neutral drift is given by the equation: $r = m - \phi\sigma$. In the case of mean-reverting process, as we can see by (12.2), the [illegible] m equals $-kX_t$. If we take that into consideration we conclude that:

$$r = -kX_t \qquad (12.25)$$

So in the case of the risk-adjusted probability measure the dynamics of the non-observable variable X_t are described by the following differential equation:

$$dX_t = k\left(\alpha^* - X_t\right) + vdZ_t \tag{12.26}$$

where $\alpha^* \equiv {}^{-\phi v}\!/_{k}$ is the adjustment that is made in the drift of the process because of the switch of the measure.

Using equation (12.26) and (12.1) we derive:

$$d\left(P_t - g(t)\right) = k\left(\alpha^* - P_t + g(t)\right)dt + vdZ_t^*$$

$$dP_t - dg(t) = k\left(\alpha^* - P_t + g(t)\right)dt + vdZ_t^*$$

$$dP_t = k\left(\frac{1}{k}\frac{dg(t)}{dt} + \alpha^* + g(t) - P_t\right)dt + vdZ_t^*$$

$$dP_t = k\left(a^*(t) - P_t\right)dt + vdZ_t^* \tag{12.27}$$

with

$$a^*(t) = \frac{1}{k}\frac{dg(t)}{dt} + \alpha^* + g(t) \tag{12.28}$$

Assuming that $f\left(P_t, t\right) = P_t e^{kt}$ and using Ito's Lemma:

$$df\left(P_t, t\right) = kP_t e^{kt}dt + e^{kt}dP_t$$

$$df\left(P_t, t\right) = kP_t e^{kt}dt + e^{kt}\left(k\left(a^* \ldots P_t\right)dt + vdZ_t^*\right)$$

$$df\left(P_t, t\right) = kP_t e^{kt}dt + e^{kt}ka^*(t) \ldots - ke^{kt}P_t dt + ve^{kt}dZ_t^*$$

$$df\left(P_t, t\right) = ka^*(t)e^{kt}dt + ve^{kt}dZ \ldots$$

Integrating from 0 to t in both members we derive the next expression:

$$P_t e^{kt} = P_0 + \int_0^t k a^*(s) e^{\kappa s} ds + \int_0^t v e^{ks} dZ_s^* \qquad (12.29)$$

We will now try to calculate the first integral:

$$\int_0^t k\left(\frac{1}{k}\frac{dg(s)}{ds} + \alpha^* + g(s)\right) e^{ks} ds$$

$$= \int_0^t \frac{dg(s)}{ds} e^{ks} ds + \int_0^t k\alpha^* e^{ks} ds + \int_0^t k f(s) e^{ks} ds$$

$$= \int_0^t \frac{dg(s)}{ds} e^{\kappa s} ds + \alpha^* \int_0^t \left(e^{\kappa s}\right)' ds + \int_0^t \left(e^{\kappa s}\right)' f(s) ds$$

$$= \int_0^t \frac{dg(s)}{ds} e^{ks} ds + \alpha^*\left(e^{kt} - 1\right) + e^{kt} g(t) - g(0) - \int_0^t \frac{dg(s)}{ds} e^{ks} ds$$

$$= \alpha^*\left(e^{kt} - 1\right) + e^{kt} g(t) - g(0) \qquad (12.30)$$

Combining (12.29) and (12.30) we obtain:

$$P_t e^{kt} = P_0 + \alpha^*\left(e^{kt} - 1\right) + e^{kt} g(t) - g(0) + \int_0^t v e^{ks} dZ_s^*$$

$$P_t = g(t) + X_0 e^{-kt} + \alpha^*\left(1 - e^{-kt}\right) + \int_0^t v e^{k(s-t)} dZ_s^* \qquad (12.31)$$

We will now calculate the fair forward price by first calculating the conditional mean:

$$E_0^Q[P_t] = g(t) + X_0 e^{-kt} + \alpha^*\left(1 - e^{-kt}\right) \qquad (12.32)$$

Using the martingale approach we will evaluate the value of the forward price:

$$v_0\left(P_T, T\right) = e^{-rT} E_0^Q\left[P_T - F_0\left(P_0, T\right)\right] \quad (12.33)$$

Keeping in mind that at the beginning of a forward contract its value is zero we take the following result:

$$F_0\left(P_0, T\right) = E_0^Q\left(P_T\right) = g\left(T\right) + \left(P_0 - g\left(0\right)\right)e^{-kT} + \alpha^*\left(1 - e^{-kT}\right) \quad (12.34)$$

with $\alpha^* \equiv {}^{-\phi v}/_{k}$.

13 References

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