UNIVERSITY OF PIRAEUS

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"ASSET ALLOCATION WITH DIFFERENT COVARIANCE/CORRELATION ESTIMATORS"

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1. Introduction

The roots of modern investment theory can be traced back to Markowitz theory first developed in 1952. The core idea of his theory was that investors should hold mean-variance efficient investment portfolios, which could comprise of assets, such as bonds, stocks and cash. However, in the past, Markowitz theory was not widely used. Instead, most investment managers focused on uncovering securities with high expected returns and theoretical research on investments has concentrated on modeling expected returns.

When the Stock Market Crash of 1987 incurred, Markowitz's theory gained great in popularity, since those following the strategy managed largely to avoid an overcommitment to equities immediately before the Crash. After that, professional investors are rediscovering the importance of portfolio risk management. The recent interest in asset allocation methods, including international diversification, has also spurred interest in risk measuring. Another factor is the increased use of sophisticated quantitative methods in the investment industry, together with increased computing power. In short, there is an increased emphasis on risk control in the investment management industry. Correlation matrices of financial returns play a crucial role in several branches of modern finance such as investment theory, capital allocation and risk management. Furthermore, financial correlation matrices are the key input parameters to Markowitz's classical portfolio optimization problem.

Great importance to risk control also gives the continuous growth of the hedge fund industry. Over the last few years institutional investors and high net worth individual investors invest in alternative strategies as hedge funds. The difference can be illustrated from a comparison between traditional and alternative investments: from January 2000 to December 2003, the MSCI World Index lost on average 5.9% per year, while the CSFB-Tremont hedge fund index gained on average 6.8% per year, with much lower volatility. Despite some high-profile losses, investors are committing more assets to the alternative investment industry. This fact gives great importance to measuring of variances and covariances/correlations of hedge fund returns, because there may be, if not measured accurately, important impacts in terms of asset allocation, pricing and portfolio construction and in risk measurement (VaR, CVaR).

In addition, focusing on forecasting the second moments, rather than expected returns, may be more useful in asset allocation terms for two reasons. Firstly, there are several studies which examine the importance of the forecasts of mean returns for mean-variance optimization (Michaud (1989), Best and Grauer (1991), Chopra and Ziemba (1993), Winston (1993)) and there is a general consensus that expected returns are notoriously difficult to predict and that the optimization process is very sensitive to differences in expected returns. As Chopra and Ziemba conclude (1993), errors in means are approximately ten times as important as errors in variances and covariances considered together for a low risk tolerance, and for a higher risk tolerance they become twenty-one times. At the same time, there is a common impression that variances and covariances of returns are much easier to estimate from historical data in order to manage the allocation of the assets in the portfolio, without suggesting that, although future covariances are more easily predictable than future mean returns, the difficulties should be understated. Like Merton (1980) states, variances and covariances of returns can typically be estimated with far greater precision than the expected returns. This gives rise to the possibility that the second moments pose fewer problems in the context of portfolio optimization through a mean-variance analysis and that reduction in volatility may lead to better optimization results.

Second, one could say that, with today's stock market structure, returns and variances are no longer sufficient in order to have a good asset allocation. Stock market crises in Europe and USA since the year 2000 and fragility of the international stock markets sparked the interest of researchers in understanding and modeling the markets' rising volatilities in order to prevent against crises. Therefore correlations between assets need to be determined precisely in order to perform an optimal asset allocation. Otherwise, if correlations that increase during periods of high volatility are falsely disregarded, the allocation process will be biased and the portfolio will not be diversified enough as correlations increase during high volatility episodes, which might lead to considerable losses.

Also, as mentioned before, covariances and correlations of returns play a central role in derivatives pricing, optimal portfolio selection, and risk management. These applications motivate an extensive literature on volatility modelling. Starting with Engle (1982), researchers have fit a variety of autoregressive conditional heteroskedasticity (ARCH), generalized ARCH (Bollerslev (1986)), exponential ARCH (Nelson (1991)), and stochastic volatility models to asset returns. This literature has centred on evaluating the statistical performance of volatility models and in extent, the statistical performance of predicting covariances and correlations of assets.

The remainder of the master thesis is organised as follows: Section 2 gives a first impression of our study combined with a review of the previous relative literature. Section 3 describes the theoretical background of our study; the different estimators of covariances and performance measures. Section 4 presents the data set for our study, Section 5 provides the results and the empirical analysis and Section 6 concludes.

2. Literature Review

The subject of our study is to test whether the use of different covariance – correlation estimators than the historical covariance matrix that is widely used, would help in portfolio optimization through the mean-variance analysis. In other words, if an investor would like to use the mean-variance analysis in order to invest in assets like stocks or indices, would it be of some help to use more sophisticated estimators for the covariance matrix of the returns of his portfolio? The procedure that we follow to answer this question is the following. First, we define seven different universes of data. Second, for each universe, we use fifteen years of data to estimate the parameters of each covariance matrix estimator and next we compute the covariance matrix of each estimator. Third, we estimate the mean-variance efficient frontiers of the seven universes. The expected returns are estimated by the sample historical average, because they are not a subject to our study and although their estimation might not be the best, it is sufficient for our purposes. Next, according to the created mean-variance efficient frontiers, we define three portfolios for each universe with different expected returns: the conservative, the average and the aggressive portfolio. We compute the realized returns and the forecasted standard deviations for each portfolio of each universe for all the four estimators of covariance matrices and finally, we compare the performance according to the estimators used. The metrics of performance are the Informational ratio, the Conditional Sharpe Ratio and the Certainty Equivalent. Our methodology will be further analyzed in Section 4.

Previous literature – to our knowledge – does not approach the answer to this question the way we did. There are many papers that use and compare different covariance or correlation estimators in order to assess risk. Most of them do not use asset allocation terms in order to compare the performance of the covariance – correlation estimators, but they use other metrics like mean squared errors and correlations between forecasted and realized values. The ones that use the mean-variance framework in order to find out the optimal estimator avoid defining different levels of expected returns by using the minimum variance portfolio.

Here we are going to report shortly some of these which use the mean-variance framework. Firstly, ***Amenc et al (2001)*** attempt to rehabilitate the importance of active asset allocation in the investment process by using implicit factors as an estimator of the covariance matrix (like Chan et al (1999)) and then comparing the minimum variance portfolio with value-weighted and equally value-weighted portfolios. They review the benefits of traditional and alternative style management and provide evidence that optimal strategic and tactical asset allocation strategies are likely to significantly enhance the risk-adjusted performance of a multi-style multi-class portfolio. More specifically, they consider the following two investment universes: a portfolio invested only in hedge funds (AI only) and an equity-oriented portfolio invested in traditional equity indices and equity-related alternative indices (AI/TI). The results of the AI and the AI/TI investment universes are that the ex-post volatility of the minimum variance portfolio generated using implicit factor based estimation techniques is lower in both universes than the volatility of the other two portfolios. This indicates that optimal variance minimization can achieve lower portfolio volatility. In addition, differences in mean returns are not statistically significant, suggesting that the improvement in terms of risk control does not necessarily come at the cost of lower expected returns.

Second, ***Fleming et al (2000)*** examine if standard volatility models have low explanatory power by using conditional mean-variance analysis to assess the value of volatility timing to short-horizon investors. They consider an investor who uses a mean-variance optimization rule to allocate funds across four asset classes: stocks, bonds, gold, and cash. The investor's objective is to maximize expected return (or minimize volatility) while matching a target volatility (or expected return). They also use the minimum variance portfolio and, in order to estimate the conditional covariance matrix, they employ a general non-parametric approach developed by Foster and Nelson (1996). The estimator is a weighted rolling average of the squares and cross-products of past return innovations that nests most ARCH, GARCH and stochastic volatility models as special cases. Allowing for daily rebalancing, the solution to the portfolio problem is a dynamic trading strategy that specifies the optimal asset weights as a function of time, strategy which requires estimates of both the conditional expected returns and the conditional covariance matrix. They treat expected returns as constant and let the variation in the portfolio weights be driven purely by changes in the conditional covariance matrix. They find that the volatility timing strategies outperform the unconditionally efficient static portfolios that have the same target expected return and volatility. This finding is robust to estimation risk and transaction costs.

Third, ***Chan et al (1999)*** compare the performance of different methods of forecasting variances and covariances and then they apply their forecasts in portfolio optimization, using the global minimum variance portfolio. The different risk models are evaluated on a statistical basis and on a more practical, economic basis. The forecasting models were a full historical model, a constant correlation model, and factor models ranging from one to ten factors. They find that factor models generate a slight improvement in the ability to predict future covariances compared to forecasts based on historical covariances. Since the models' covariance forecasts move in the same direction as the realized covariances, they help for portfolio risk optimization. They find that a few factors (such as the market, size and the book-to-market value of equity) capture the general structure of return covariances and that a three-factor model is adequate for selecting the minimum variance portfolio.

Next, ***Elton et al (2006)*** investigate the ability of several techniques to forecast correlation coefficients between securities. One distinguishing feature of this research is that they forecast the average level of correlations separately from pair-wise differences from the overall average and find that this two-step approach improves the forecast of correlations. In exploring differences in pair-wise correlations from the average level of correlations, they examine several alternative methods for forming groups and forecasting correlations within and between groups than the two extreme methods of grouping: (1) each firm is a group and (2) all firms are in one group. They find that grouping firms by either industry or several of the firm characteristics that have been shown to be part of the return generating process, improves forecasts compared to most suggestions contained in the literature. Finally, preparing forecasts based on a simple weighted average of three forecasts namely those obtained by grouping into 30 industries and grouping by Size and Beta and historical pair-wise correlations, provides forecasts that outperforms all compared forecasting techniques. This outperformance is robust whether performance is measured by (1) minimum squared forecast error and (2) minimum future variance of portfolios selected. They examine the performance by using global minimum variance portfolios.

Another article is the one of ***Jagannathan and Ma (2002)***, which examines the ability of alternative estimates of the correlation matrix to produce the minimum variance portfolio. The authors show that constraining portfolio weights to be nonnegative is equivalent to using the sample covariance matrix after reducing its large elements and then form the optimal portfolio without any restrictions on portfolio weights. This shrinkage helps reduce the risk in estimated optimal portfolios even when they have negative weights in the population. They also find that once the non-negativity constraint is imposed, minimum variance portfolios constructed using the sample historical covariance matrix perform as well as those constructed using covariance matrices estimated using more sophisticated estimators, like factor models and shrinkage estimators. They compare the risk of the minimum variance portfolio when imposing upper and lower bounds with the risk when covariance matrix estimates are obtained from (1) the sample historical covariance matrix, (2) Sharpe's one-factor model, (3) Ledoit's (1999) optimal shrinkage estimator, (4) Fama and French's three-factor model and (5) Connor and Korajcyzk's five-factor model plus a three-factor version. They find all models produce about the same risk. Of all models, the Ledoit shrinkage procedure works best, but not significantly so. Using weights of one-half on the estimates produced by the single index model and one-half on the historical pair-wise estimate works as well as Ledoit's optimal shrinkage estimator.

Also, another approach of estimating covariances and correlation matrices is *shrinkage*, which is supposed to be an evolution in covariance matrix estimation with large dimensions, like ***Ledoit and Wolf (2002)***. They have developed a flexible method for imposing some structure into a large dimensional estimation problem, namely the problem of estimating the covariance matrix of a large number of stock returns. The crux of the method is to shrink the unbiased but very variable sample covariance matrix towards the biased but less variable single-index model covariance matrix and to thereby obtain a more efficient estimator. They also compared the performance of the shrinkage method to that of various previously suggested estimators for the covariance matrix of stock returns. The previously suggested estimators were (1) identity, (2) constant pairwise correlation, (3) generalized inverse of the sample covariance matrix, (4) single-index covariance matrix of Sharpe, (5) industry factors, (6) principal components, (7) shrinkage towards identity and (8) shrinkage towards market. Performance was measured in terms of out-of-sample standard deviation of minimum variance stock portfolios, where the estimated covariance matrix is the input of the portfolio selection method of Markowitz (1952). The proposed method improved upon all the other estimators included in the study.

Finally, a very useful article was that of ***Giamouridis and Vrontos (2007)***. We could say that our study is in the same spirit with this article as far as the methodology is concerned, because the goals are different. This paper studies the impact of modelling time-varying covariances-correlations of hedge fund returns in terms of hedge fund portfolio construction and risk measurement. The authors use a variety of static and dynamic covariance-correlation prediction models and compare the optimized portfolios' out-of-sample performance. The authors start with the case where the hypothetical investor is concerned with the volatility of the portfolio. They compare the performance of five different methods of forecasting variances and covariances/correlations. The methods are: (1) the sample historical covariance matrix, (2) an implicit factor model, (3) an implicit factor GARCH model, (4) a full-factor multivariate GARCH model and (5) a regime switching dynamic correlations model. The different models are evaluated, out-of-sample, in a case study which examines the portfolio risk, realized return, risk-adjusted realized return and tail-risk. They construct optimal hedge fund portfolios. Two portfolios are constructed: a conservative (minimum variance portfolio) and an aggressive (15,5% annual expected return). The empirical performance of the covariance prediction models is assessed on several grounds. First, the authors examine the realized returns of the constructed portfolios. Second, they compare the return per unit of risk (Conditional Sharpe Ratio). Next, they set out to incorporate transaction costs and finally they investigate the capacity of the different covariance prediction models to assess tail-risk. They find that dynamic covariance/correlation models construct portfolios with lower risk and higher out-of-sample risk-adjusted realized return. The tail-risk of the constructed portfolios is also lower. Using a mean-CVaR (conditional value at risk) framework, they show that dynamic covariance-correlation models are also successful in constructing portfolios with minimum tail-risk.

As we can conclude from the literature presented before, most researchers use factor models to evaluate risk. In our study, using factor models would be of no use, because the portfolios we construct include six assets maximum. This is because we use indices and not stocks, where each index contains a large number of stocks. Instead, we are using two multivariate GARCH models, which, according to the literature, are supposed to outperform the widely used and considered "traditional" estimators as the sample historical covariance matrix and the exponential weighted moving average. These methods are fully analysed in next section (Section 3).

3. Methodology

In Section 3 we present the theory behind our study and fully explain the metrics and methods we use to fulfill our study. We start by defining the mean-variance analysis introduced by Markowitz (1952, 1959), an analysis which is widely used from investors. We next present the covariance matrix estimators which we are going to compare and finally we present the measures of performance of our constructed mean-variance portfolios.

3.1 Terminology of Mean-Variance Analysis

The theory of mean-variance analysis first developed by Markowitz (1952, 1959) has the following hypotheses: (1) the investors evaluate stocks or portfolios by expected return and risk that are connected to portfolios, (2) the investors are risk averse and (3) between two portfolios with the same return, an investor would prefer the one with the minimum risk and between two portfolios with the same risk, an investor would prefer the one with the maximum return.

The mathematical problem is expressed as follows:

We consider securities whose returns $r' = (r_1, r_2, ..., r_n)$ during the forthcoming period have expected values $\mu' = (\mu_1, \mu_2, ..., \mu_n)$ and a covariance matrix $C = [\sigma_{ij}]$. An investor is to select a portfolio with weights $w' = (w_1, w_2, ..., w_n)$. The realized return $R = r'w$ on the portfolio has expected value and variance, respectively:

$$E = \mu' w \tag{1}$$

$$V = w'Cw \tag{2}$$

The portfolio is to chosen subject to the following constraints (when we want to minimize risk):

$$E = k \tag{3}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{4}$$

$$w_i \geq 0,\ i = 1, 2, ..., n \tag{5}$$

The minimum risk is obtained by taking the first derivative of the following quadratic equation with respect to w and solving the system:

$$\min(w'Cw) \tag{6}$$

$$\frac{\partial w'Vw}{\partial w} = (V + V')w = 2Vw = 0 \quad (7)$$

A portfolio is feasible if it satisfies the constraints. An EV combination is feasible if it is the E and V of a feasible portfolio. A feasible EV combination (E_0, V_0) is inefficient if there is another feasible EV combination (E_1, V_1) such that either

$$(i) \quad E_1 > E_0 \ \ and \ \ V_1 \leq V_0 \quad (8)$$

or

$$(ii) \quad V_1 < V_0 \ \ and \ \ E_1 \geq E_0 \quad (9)$$

A feasible EV combination is efficient if it is not inefficient. A feasible portfolio is efficient or inefficient in accordance with its EV combination.

By the above method, we plot the mean-variance frontier for efficient portfolios, where its points show the combination of maximum expected return for the minimum expected risk.

To make the above method more clear, we are going to use the simple example of a portfolio that consists of two stocks. In this case, the expected return and the risk of the portfolio are given by the next equations:

$$E(R) = w_1 R_1 + w_2 R_2 \quad (10)$$

$$\sigma^2(R) = w_1^2 \sigma^2(R_1) + w_2^2 \sigma^2(R_2) + 2 w_1 w_2 Cov(R_1, R_2) \quad (11)$$

where the risk of the portfolio can be also estimated by the following equation:

$$\sigma^2(R) = w_1^2 \sigma^2(R_1) + w_2^2 \sigma^2(R_2) + 2 w_1 w_2 \rho_{12} \sigma(R_1) \sigma(R_2) \quad (12)$$

Where ρ_{12} is the correlation between stocks 1 and 2 and $\sigma(R_1)$, $\sigma(R_2)$ are standard deviations of stocks 1 and 2.

This is where our study fits. To estimate the risk of the portfolio, one needs to estimate either the covariance matrix or the correlation matrix. Without this estimation, the mean-variance efficient frontier cannot be computed.

In the next section, we are going to present the covariance matrix estimators that we are going to use and compare, in order to decide which predicts best the covariance of the assets that constitute the selected portfolios.

3.2 Estimation Methods of Covariance – Correlation Matrices

The main subject of our study is to estimate the covariance – correlation matrices of returns of the portfolios constructed, in order to make out if a more sophisticated estimator would improve mean-variance analysis. As mentioned before, using factor models would be of no use, because the portfolios we construct include six assets maximum. Instead, we are using two multivariate GARCH models, which, according to the literature, are supposed to outperform the widely used and considered "traditional" estimators as the sample historical covariance matrix and the exponential weighted moving average. The models that we are going to use are the following.

3.2.1 Historical Covariance Matrix (HCM) Model

The sample covariance matrix from historical data has the structure of the following matrix:

$$V = \begin{pmatrix} V(r_1) & Cov(r_1,r_2) & \cdots & Cov(r_1,r_n) \\ Cov(r_2,r_1) & V(r_2) & & Cov(r_2,r_n) \\ \vdots & & \ddots & \vdots \\ Cov(r_n,r_1) & Cov(r_n,r_2) & \cdots & V(r_n) \end{pmatrix}$$

where

$$V = [\sigma_{ij}] = \frac{1}{n-1}\sum_{i=1}^{n}\sum_{j=1}^{n}(r_i - \bar{r})(r_j - \bar{r}) \tag{13}$$

where n is the sample size and $\bar{r}$ is a $n \times 1$ vector of the averages of each return vector. This model involves low specification error and high sampling error.

The n-period "historic" correlation at time t between two return series is:

$$\rho_{ij} = \frac{Cov(r_i, r_j)}{\sigma(r_i)\sigma(r_j)} \tag{14}$$

3.2.2 Exponentially Weighted Moving Average (EWMA) Model

Exponentially weighted moving average (EWMA) model is proposed by JP Morgan's RiskMetrics system. The model's forecast for tomorrow's volatility can be seen as a weighted average of today's volatility and today's squared return. The degree of weighing decrease is expressed as a constant *smoothing factor λ*, a number between 0 and 1. The recent returns matter more for tomorrow's variance than distant returns as λ is less than one and therefore gets smaller when the lag gets bigger. In other words, the model applies weighting factors which decrease exponentially. The weighting for each day decreases exponentially, giving much more importance to recent returns while still not discarding older observations entirely. According to the size of λ, we have a big smoothing effect when λ is high and a small smoothing effect when λ is low.

The smoothing factor λ may be expressed as a percentage, so a smoothing factor of 10% is equivalent to λ=0.1. In our case, since we use monthly data, according to JP Morgan's RiskMetrics we should have $\lambda = 0.97$.

The volatility and covariance estimation with use of the EWMA model have the following formulas (proposed by JP Morgan's RiskMetrics system):

$$\sigma_t^2(r_i) = \lambda\sigma_{t-1}^2(r_i) + (1-\lambda)r_{i,t-1}^2 \tag{15}$$

$$Cov_t(r_i, r_j) = \lambda\sigma_{ij,t-1} + (1-\lambda)r_{i,t-1}r_{j,t-1} \tag{16}$$

where $\sigma_{ij,t}$ the covariance, $\sigma_{i,t}^2$, $\sigma_{j,t}^2$ the variances of assets i, j and $r_{i,t}$, $r_{j,t}$ their returns.

The EWMA approach can be used for one-step-ahead forecasting. It has the attractive feature that relatively little data need to be stored. At any given time, we only need to remember the current estimate of the variance rate and the most recent observation of the value of the returns. When we get a new observation of the value of the returns, we calculate a new daily percentage change and use the above equation to update our estimate of the variance and covariance. The old estimates can then be discarded. The disadvantages of the method are that it cannot predict a sudden structural break and that that prediction depends on the choice of λ.

The EWMA correlation formula is:

$$\rho_{ij,t} = \frac{\lambda\sigma_{ij,t-1} + (1-\lambda)R_{i,t-1}R_{j,t-1}}{\sqrt{[\lambda\sigma_{i,t-1}^2 + (1-\lambda)R_{i,t-1}^2][\lambda\sigma_{j,t-1}^2 + (1-\lambda)R_{j,t-1}^2]}} \tag{17}$$

where, as before, $\sigma_{ij,t}$ the covariance, $\sigma_{i,t}^2$, $\sigma_{j,t}^2$ the variances of assets i, j and $R_{i,t}$, $R_{j,t}$ their returns.

3.2.3 GARCH Models

We use two GARCH models: (1) the Constant Conditional Correlation (CCC) GARCH Model (1990) and (2) the Flexible Multivariate GARCH (FlexM) Model (2003).

Before presenting the GARCH Models, we are going to refer to the GARCH (1,1) model which is used from both methods to estimate the diagonal coefficients of the covariance matrix.

3.2.3.1 GARCH (1,1) Model

The GARCH models capture very important features of returns, are flexible and accommodate specific aspects of individual assets. These models were firstly proposed by Engle (ARCH - 1982) and were extended by Bollerslev (GARCH – 1986). They offer a statistical theory that establishes the distinction between conditional and unconditional volatility. The general idea is to add a second equation to the regression model (autoregressive moving average models – ARMA). This equation refers to the conditional variance, and the first equation is the so-called conditional mean equation. In normal GARCH, we assume that ε_t is distributed conditionally normally with conditional variance. This leads to a leptokurtic distribution. The coefficients a_i measure the persistence of returns, while the coefficients β_i measure the persistence of variance.

GARCH (1,1) is the model most often met in the finance literature and it is sufficient for short term variance forecasting. The (normal) GARCH (1,1) model is:

$$\begin{gathered} r_t = \mu + \varepsilon_t \\ \varepsilon_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = w + a r_{t-1}^2 + \beta \sigma_{t-1}^2 \\ \alpha + \beta < 1 \end{gathered} \tag{18}$$

where *β* (GARCH) coefficient is similar to the *λ* coefficient in the EWMA model and it can be interpreted as a decay rate. Furthermore, high *α* (ARCH) coefficient means that volatility reacts fast to changes in the market and high *β* coefficient means that volatility is "persistent" – the effects of previous shocks decay slowly. The coefficients *α, β* are calculated through maximum likelihood. In addition, the EWMA model is a particular case of GARCH (1,1), where ω = 0, $a = 1 - \lambda$ and $\beta = \lambda$.

The multivariate GARCH models that we are going to use are different: CCC is a non-linear combination of a univariate GARCH model and FLEXM is a generalization of the univariate standard GARCH model. In terms of computational feasibility, they are both easy and fast to estimate, using simple optimization functions of Matlab.

3.2.3.2 Constant Conditional Correlation (CCC) GARCH Model

The Constant Conditional Correlation GARCH Model was proposed by Bollerslev (1990). It is a non-linear combination of univariate GARCH models and a simple multivariate conditional heteroskedastic time series model. The model has time varying conditional variances and covariances, but constant conditional correlations. Thus, the conditional covariances are proportional to the product of the corresponding conditional standard deviations. This restriction highly reduces the number of unknown parameters and simplifies estimation. Despite all these, the assumption that the conditional correlations are constant may seem unrealistic in many empirical applications.

As far as conditional correlations is concerned, let r_t denote the $n \times 1$ time series vector of returns with time varying conditional covariance matrix H_t, i.e.

$$\begin{aligned} r_t &= E(r_t \mid I_{t-1}) + \varepsilon_t \\ Var(\varepsilon_t \mid I_{t-1}) &= H_t \end{aligned} \tag{19}$$

where I_{t-1} is the σ-field generated by all the available information up through time $t-1$ and H_t is almost surely positive definite for all t.

Each conditional variance h_{iit} is modelled by a separate univariate GARCH (1,1) model with parameters wii, aii, and βii, respectively:

$$h_{iit} = w_i + a_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1}, \quad i = 1,\dots,n \tag{20}$$

The CCC model is defined as:

$$H_t = D_t R D_t = \left(\rho_{ijt} \sqrt{h_{iit} h_{jjt}} \right) \tag{21}$$

where

$$D_t = diag\left(h_{11t}^{1/2}, \dots, h_{nnt}^{1/2} \right) \tag{22}$$

and

$$R_t = \left(\rho_{ijt}\right) \tag{23}$$

is a symmetric positive definite matrix with $\rho_{iit} = 1, \forall i$.

As mentioned before, the full conditional covariance matrix H_t is partitioned as $H_t = D_t C D_t$, where

$$D_t = diag(\sqrt{h_{it}}) = \begin{pmatrix} \sqrt{h_{1t}} & 0 & \cdots & 0 \\ 0 & \sqrt{h_{2t}} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{h_{nt}} \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & & \rho_{2n} \\ \vdots & & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix}$$

3.2.3.3 Flexible Multivariate GARCH (FLEXM) GARCH Model

The Flexible Multivariate GARCH (FLEXM) Model was proposed by Ledoit, Santa-Clara and Wolf (2003). It is a new approach to estimating time-varying covariance matrices in the framework of the diagonal-vech version of the multivariate GARCH (1,1) model. In this framework belong the BEKK models, which have a high number of unknown parameters, even after imposing several restrictions (e.g. diagonal BEKK, scalar BEKK). Consequently, these models are rarely used when the number of series is larger than 3 or 4; they are not computationally feasible because the parameters interact in a way that is too intricate for optimization algorithms to converge. Contrary, the FLEXM model is numerically feasible for large scale problems and according to the article of ***Ledoit et al (2003)***, outperforms the diagonal BEKK GARCH model. This is the reason why we are going to use FLEXM instead of another generalization of the univariate standard GARCH model.

As far as conditional correlations is concerned, the most general multivariate GARCH-style model commonly considered is defined by

$$E(r_t \mid I_{t-1}) = 0$$

$$h_{ij,t} = Cov(r_{i,t}, r_{j,t} \mid I_{t-1}) = w_{ij} + a_{ij} r_{i,t-1} r_{j,t-1} + b_{ij} h_{ij,t-1} \qquad (24)$$

where I_{t-1} denotes the conditioning information set available at time $t-1$ and $r_{i,t}$ denotes the realization of the i^{th} return ($i=1,\ldots,n$) at time t. The parameter values satisfy $a_{ij}, b_{ij} \geq 0 \;\; \forall i, j = 1,\ldots,n$ and $w_{ii} > 0 \;\; \forall i = 1,\ldots,n$.

The basic idea of the model proceeds in two steps: firstly to obtain each set of coefficient estimates $\hat{w}_{ij}$, $\hat{a}_{ij}$ *and* $\hat{\beta}_{ij}$ separately for every *(i,j)*. This can be achieved simply by estimating a two-dimensional or one-dimensional GARCH (1,1) model (for $I \neq j$ or $i = j$ respectively) which is computationally feasible using a traditional method such as maximum likelihood. We bring together the outputs of these separate estimation procedures into matrices

$$\hat{W} = [\hat{w}_{ij}]_{i,j=1,\ldots,n},\; \hat{A} = [\hat{a}_{ij}]_{i,j=1,\ldots,n} \; and \; \hat{B} = [\hat{\beta}_{ij}]_{i,j=1,\ldots,n}$$

However, the above coefficients matrices are generally incompatible with each other, in the sense that they yield conditional covariance matrices that are not positive semi-definite. Therefore, the second step is to transform the estimated parameter matrices in such a way that they yield conditional covariance matrix that are guaranteed to be positive semi-definite.

Analytically, each conditional variance h_{iit} is modelled by a separate univariate GARCH (1,1) model with parameters *wii*, *aii*, and *βii*, respectively.

The model assumes that the conditional covariance of variables r_i and r_j depends on its lagged value and on past realizations of the product $r_i r_j$ only. Also, the first equation assumes that the variables have zero conditional mean, which can always be justified by taking them to be residuals coming from some regression model.

The quasi maximum likelihood that is used to get the estimators of the coefficients $\hat{w}_{ij}, \hat{a}_{ij}$ and $\hat{b}_{ij}$ is different for diagonal and non-diagonal coefficients. For diagonal coefficients, we estimate a univariate GARCH (1,1) process for every one of the variables by quasi maximum likelihood, and we get consistent estimators of $\hat{w}_{ii}, \hat{a}_{ii}$ and $\hat{b}_{ii}$.

For off-diagonal coefficients, we get parameter estimates $\hat{w}_{ii}, \hat{a}_{ii}$ and $\hat{b}_{ii}$ from diagonal coefficients estimation. We can use them to construct conditional variance estimates $\hat{h}_{ii,t}$. In the second stage, we use these estimates to specify quasi-likelihood functions for the off-diagonal elements.

3.3 Performance Measures

3.31 Transformed Sharpe Ratio

This ratio was developed by William Forsyth Sharpe in 1966. Sharpe originally called it the "reward-to-variability" ratio in before it began being called the Sharpe Ratio by later academics and financial professionals.

The Sharpe ratio or Sharpe index or Sharpe measure or reward-to-variability ratio is a measure of the mean excess return per unit of risk in an investment asset or a trading strategy. Since its revision made by the original author in 1994, it is defined as:

$$S = \frac{E(r - r_f)}{\sigma} = \frac{E(r - r_f)}{\sqrt{Var(r - r_f)}} \tag{25}$$

where r is the asset return, r_f is the return on a benchmark asset, such as the risk free rate of return, $E[r - r_f]$ is the expected value of the excess of the asset return over the benchmark return, and σ is the standard deviation of the excess return (Sharpe 1994).

The Sharpe ratio is used to characterize how well the return of an asset compensates the investor for the risk taken. Investors are often advised to pick investments with high Sharpe ratios.

Here we are not using the conventional Sharpe Ratio, but another metric which will help us measure the return per unit of risk. This is in the same spirit with Sharpe ratio but it does not use the excess return on the nominator; it uses the realized return. So it is a metric that gives the realized return per unit of risk. We use this metric instead of the traditional measure because we have international portfolios and we don’t face the problem of asset allocation as a US investor, for example, in order to know which risk free we could use. The metric is defined as:

$$SR = \frac{\text{Average Realized Return}}{\text{StDev of Realized Returns}} \tag{26}$$

Where average realized return is calculated for each efficient portfolio with specified expected return. We rank the SRs' from the higher to the lower and prefer the higher SR; the higher return per unit of risk.

We use this metric to make out which estimator of covariance is the best. It is the metric that it is more closely to the traditional Sharpe Ratio and it would be easily used by an investor. The problem with this metric is that it gives about the same results for all estimators and does not give a clear advantage to one of them, although there is a slight favor to historical covariance matrix. To overcome this fact, we use another, more sophisticated metric, the Conditional Sharpe Ratio.

3.3.2 Conditional Sharpe Ratio (CSR)

The Conditional Sharpe Ratio is proposed by Giamouridis and Vrontos (2006). It is a metric that helps compare the realized return per unit of forecasted risk. Giamouridis and Vrontos defined this measure, which is similar in spirit to the Sharpe Ratio, by standardizing the realized returns with the risk of the portfolio when it is constructed one period before (forecasted standard deviation). It is calculated through:

$$CSR_{t+1} = \frac{realized(r_{t+1})}{\sqrt{Var_t(r)}} \tag{27}$$

where $Var_t(R_p)$ is determined as the minimum variance that equals the expected return for that period (time varying risk) computed from the efficient frontier.

In contrast to the Sharpe Ratio we defined before, CSR may be more useful in measuring the realized return per unit of risk and making out which estimator is optimal compared to the others (which estimator calculates the lower forecasted standard deviation), because it uses the forecasted standard deviation in the denominator. The portfolio optimization will generally arrive at a different minimum variance for each covariance prediction model and expected return; as a result, the realized return will not be comparable across models since it will represent portfolios bearing different risk. In our study, this metric helps to differentiate the different covariance estimators, because the previous metric gives us about the same results.

3.3.1 Certainty Equivalent (CE)

The concept of expected utility (dating back to Bernoulli – 1738) is used in economics to describe the behaviour of an investor choosing between several lotteries and alternatives. Its applications include not only portfolio choice, but insurance and game theory as well. A certainty equivalent of a risky outcome is a sure-thing lottery which yields the same utility as a random lottery. If the investor is risk-averse the outcome of the certainty equivalent will be less than the expected outcome of the random lottery. In other words, the certainty equivalent is that amount of wealth such that the investor is indifferent between receiving it for sure at the horizon, and having his current wealth today and the opportunity to invest it optimally up to the horizon.

We use this metric as follows: we take the Constant Absolute Risk Aversion Utility Function (CARA)

$$U(R) = -\exp(-\gamma R) \tag{28}$$

where R are the realized returns and γ measures the investor's constant absolute risk aversion. The higher γ, the more risk averse is the investor. We chose a risk aversion of γ=10, because we are supposed to be risk averse investor.

Next, we calculate the Certainty Equivalent by using the equation

$$U(CE) = E[U(R)] \tag{29}$$

and solving for CE, after having calculated the average price of the utilities of the realized returns. After some mathematics, we get that the CE is equal to

$$CE = -\frac{\ln(-E[U(R)])}{\gamma} \tag{30}$$

According to what CE is bigger, it is the best strategy to follow. This is because the higher the CE is, the higher the amount of wealth that we receive for sure at the horizon. In other words, if we make an investment of 100 index units, the more index units we get for sure at the end of the horizon, the more riskless and safer my strategy is. So, CE can be a metric of whether different covariance estimators improve asset allocation.

4. Data

The data we are going to use in our analysis are indices and risk free rates. They range from September 1988 to May 2007 (monthly data – 224 observations). We use the following indices: CAC40 (France), DAX30 (Germany), ATHEX (Greece), FTSE100 (United Kingdom), DOW JONES and NASDAQ (United States) and risk free rates: 3-month FR PIBOR/EURIBOR (France), 3-month BD FIBOR/EURIBOR (Germany), 3-month GR Treasury Bill Rate/EURIBOR (Greece) and 3-month US Treasury Bill Rate (United States). The sample which is used for the estimation of models' parameters is from September 1988 to September 2003 (181 observations of monthly data). The out-of-sample set is from October 2003 to May 2007 (43 observations of monthly data).

Let's say some for the indices and risk free rates used. The CAC40, which takes its name from Paris Bourse's early automation system Cotation Assistée en Continu (Continuous Assisted Quotation), is a French stock market index. The index represents a capitalization-weighted measure of the 40 most significant values among the 100 highest market caps on the Paris Bourse. Its base value of 1,000 was set on 31 December 1987. As of 1 December 2003, the index has become a free float weighted index. Interestingly, although CAC40 is composed of French companies, about 45% of their shares are owned by foreign investors. German investors share the largest part of it at 21%. Japanese, American and British investors are also important owners — this percentage is unusually high. The explanation can be in the fact that CAC40 companies, or multinational, are more international than any other European market. Many of these companies conduct business outside of France (63% of the CAC40 companies' employees are outside of France).

DAX30 (Deutsche Aktien Xchange 30, former Deutscher Aktien-Index 30) is a Blue Chip stock market index consisting of the 30 major German companies trading on the Frankfurt Stock Exchange. Prices are taken from the electronic Xetra trading system.

The FTSE 100 Index (pronounced footsie) is a share index of the 100 most highly capitalised companies listed on the London Stock Exchange, begun on 3 January 1984. The index was developed with a base level of 1000 on that date. Component companies must meet a number of requirements set out by the FTSE Group, including having a full listing on the London Stock Exchange with a Sterling or Euro dominated price on SETS, and meeting certain tests on nationality, free float, and liquidity. Trading lasts from 08:00-16:29 (when the closing auction starts), and closing values are taken at 16:35 (mainly though the closing Value of the FTSE100 is at 16:36). The highest value of the index to date was 6950.6, set on 30 December 1999. The index is seen as a barometer of success of the British economy and is the leading share index in Europe. It is maintained by the FTSE Group, a now independent company which originated as a joint venture between the Financial Times and the London Stock Exchange (hence the abbreviation Financial Times Stock Exchange is a bit misleading considering it is actually an index company not actually a stock exchange at all). According to the FTSE Group's website the FTSE100 companies represent about 80% of the UK share market. As of 29 December 2006 the 6 largest constituents of the index were BP, Royal Dutch Shell, HSBC Holdings, the Vodafone Group, the Royal Bank of Scotland Group and GlaxoSmithKline, which were each valued at more than £60 billion.

The Dow Jones Industrial Average (also called the DJIA, Dow 30, or informally the Dow industrials, the Dow Jones or The Dow) is one of several stock market indices created by Wall Street Journal editor and Dow Jones & Company co-founder Charles Dow. Dow compiled the index as a way to gauge the performance of the industrial component of America's stock markets. It is the oldest continuing U.S. market index, aside from the Dow Jones Transportation Average, which Dow also created. Today, the average consists of 30 of the largest and most widely held public companies in the United States. The "industrial" portion of the name is largely historical — many of the 30 modern components have little to do with heavy industry. To compensate for the effects of stock splits and other adjustments, it is currently a scaled average, not the actual average of the prices of its component stocks — the sum of the component prices is divided by a divisor, which changes over time, to generate the value of the index.

The NASDAQ (acronym for National Association of Securities Dealers Automated Quotations system) is an American stock market. It was founded in 1971 by the National Association of Securities Dealers (NASD), who divested themselves of it in a series of sales in 2000 and 2001. It is owned and operated by The Nasdaq Stock Market, Inc., the stock of which was listed on its own stock exchange in 2002.

NASDAQ is the largest electronic screen-based equity securities market in the United States. With approximately 3,200 companies, it lists more companies and on average trades more shares per day than any other U.S. market.

ATHEX is a share index of the 60 most highly capitalised companies (blue chips) listed on the Athens Stock Exchange, begun on 31 December 1980. The index was developed with a base level of 100 on that date. Component companies must meet a number of requirements, including stock prices denominated in Euros. The index is calculated every 30 minutes. Trading lasts from 08:00-16:29 (when the closing auction starts), and closing values are taken at 16:35.

The 3-month Paris Interbank Offered Rate (PIBOR) is an arithmetic average of rates given by eight representative bodies calculated after eliminating four extreme rates. Monthly data are averages of daily rates.

The 3-month Frankfurt Interbank Offered Rate (FIBOR) is taken as a reference rate for variable rate bonds in Germany. The rate is derived from the method of computing interest on the basis of 365/360 days. The rates are non-weighted averages calculated from daily rates.

The United States 3-month Treasury bill rate is the rate of Treasury securities. Treasury securities are government bonds issued by the United States Department of the Treasury through the Bureau of the Public Debt. They are the debt financing instruments of the U.S. Federal government, and are often referred to simply as Treasuries. There are four types of treasury securities: Treasury bills, Treasury notes, Treasury bonds, and Savings bonds. All of the Treasury securities (besides savings bonds) are very liquid and are heavily traded on the secondary market.

The Greek 3-month Treasury bill rate is the rate of Treasury securities. Treasury securities are government bonds of short maturity, which are sold to the investors in value lower than their final nominal value. There are issues of 13, 26 and 52 weeks duration.

5. Results

The objective of our study and our contribution to the literature has to do with the asset allocation of assets in a portfolio, where covariances – correlations of returns are forecasted with different estimators. If not measured accurately, there may be important impacts in terms of asset allocation, pricing and portfolio construction and selection and in risk measurement (VaR, CVaR) and management.

In fact, as mentioned before, we prefer on focusing on forecasting the second moments, rather than expected returns for two reasons. Firstly, there are several studies which examine the importance of the forecasts of mean returns for mean-variance optimization (Michaud (1989), Best and Grauer (1991), Chopra and Ziemba (1993), Winston (1993)) and there is a general consensus that expected returns are notoriously difficult to predict and that the optimization process is very sensitive to differences in expected returns. As Chopra and Ziemba conclude (1993), errors in means are approximately ten times as important as errors in variances and covariances considered together for a low risk tolerance, and for a higher risk tolerance they become twenty-one times. At the same time, there is a common impression that variances and covariances of returns are much easier to estimate from historical data in order to manage the allocation of the assets in the portfolio, without suggesting that, although future covariances are more easily predictable than future mean returns, the difficulties should be understated. Like Merton (1980) states, variances and covariances of returns can typically be estimated with far greater precision than the expected returns. This gives rise to the possibility that the second moments pose fewer problems in the context of portfolio optimization through a mean-variance analysis and that reduction in volatility may lead to better optimization results.

Second, one could say that, with today's stock market structure, returns and volatilities are no longer sufficient in order to have a good asset allocation. Stock market crises in Europe and USA since the year 2000 and fragility of the international stock markets sparked the interest of researchers in understanding and modeling the markets' rising volatilities in order to prevent against crises. Therefore correlations between assets need to be determined precisely in order to perform an optimal asset allocation. Otherwise, if correlations that increase during periods of high volatility are falsely disregarded, the allocation process will be biased and the portfolio will not be diversified enough as correlations increase during high volatility episodes, which might lead to considerable losses.

Summarizing, our study is important for the literature because it tries to improve asset allocation in a portfolio in a way that could be easily used from any investor. This is by using different estimators of covariance matrix and trying to figure out which is the best estimator (between the estimators we study) that can be used to estimate and predict the covariances and the correlations between the assets of the portfolio.

For our problem we define seven universes. These are:

1. Universe of all risky assets (indices) European and United States' (CAC40, DAX30, ATHEX, FTSE100, DOW JONES, NASDAQ).
2. Universe of European risky assets – indices (CAC40, DAX30, ATHEX, FTSE100).
3. Universe of United States' risky assets – indices (DOW JONES, NASDAQ).
4. Universe of French index and risk free rate (CAC40 & FR PIBOR/EURIBOR).
5. Universe of German index and risk free rate (DAX30 & BD FIBOR/EURIBOR).
6. Universe of Greek index and risk free rate (ATHEX & GR Treasury Bill Rate/EURIBOR).
7. Universe of United States' indices and risk free rate (DOW JONES, NASDAQ & US Treasury Bill Rate).

Our data set covers 17 years and 5 months period for indices and risk free rates. Our data will have monthly rebalancing frequency (monthly data), so the total observations that we will have will be 224.

The objective of this study is to examine which is the best estimator of covariance matrices to be used in asset allocation. This is achieved through an investment exercise which compares the empirical out-of-sample performance of the different methods of forecasting covariance matrix presented in section 3.

5.1 Preliminary Analysis of data

Firstly, we start with the preliminary analysis of the data. We work on monthly data of the indices and the risk free rates mentioned above. The sample in question covers the period from September 1988 to May 2007.

Because we have 3-month risk free rates we must convert them to monthly. We do this through the equation

$$r_{m_2} = \left[\left(1 + \frac{r_{m_1}}{m_1} \right)^{\frac{m_1}{m_2}} - 1 \right] m_2 \qquad (31)$$

where the compounding frequencies m_1 and m_2 equal $m_1 = 4$ (3-month risk free rate) and $m_2 = 12$ (monthly risk free rate). We then get the annualized monthly risk free rate and divide by 12 to obtain the monthly risk free rate.

The Figures 1-2 present the index prices and the risk free rates for our sample.

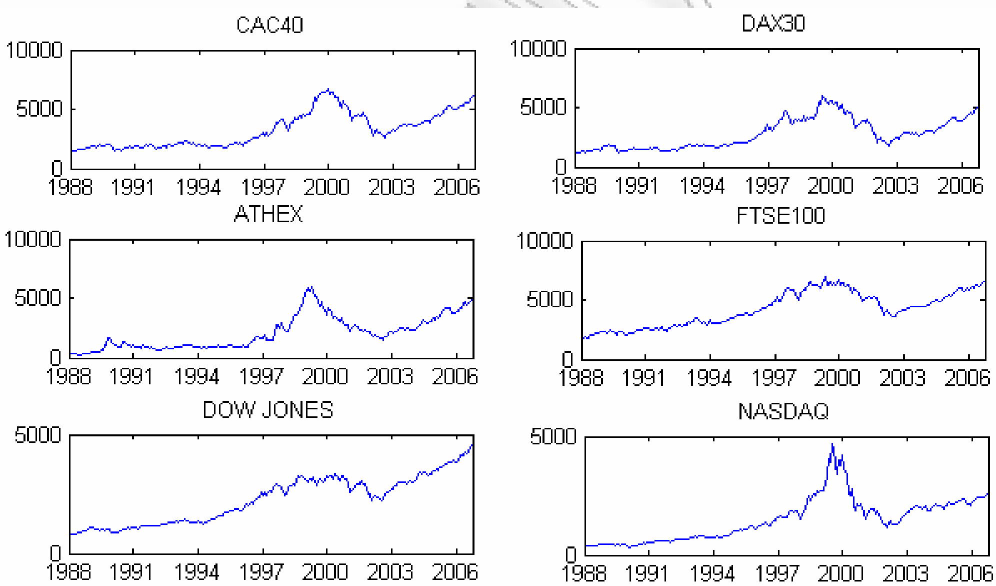


Figure 1: Prices of indices CAC40, DAX30, ATHEX, FTSE100, DOW JONES and NASDAQ covering the period from September 1988 to May 2007.

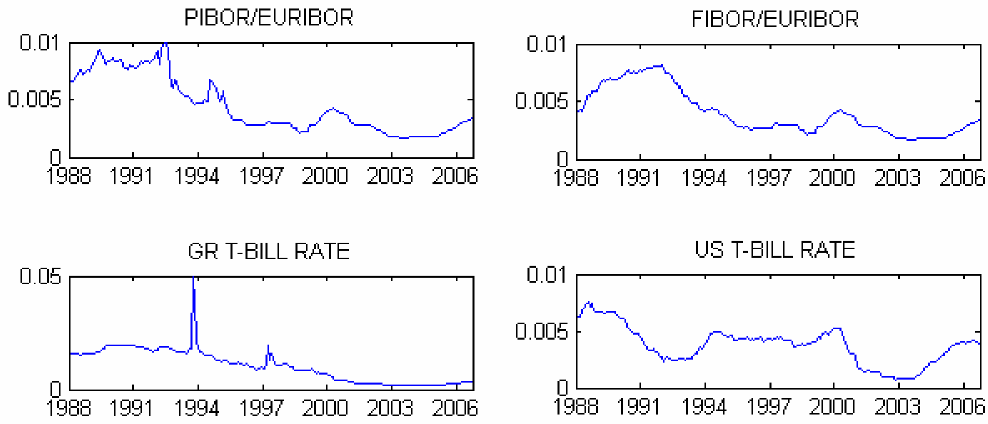


Figure 2: Risk free rates of 3-month PIBOR/EURIBOR, 3-month FIBOR/EURIBOR, 3-month GR T-BILL RATE and 3-month US T-BILL RATE covering the period from September 1988 to May 2007.

Then we calculate the returns from the index prices only, as far as the risk free rates are returns by their construction. The returns are calculated from the following equation (percentage returns):

$$R_t = \frac{p_t - p_{t-1}}{p_{t-1}} \tag{32}$$

where p_t is the index price at time t. We plot the returns for each portfolio constructed, so we finally have seven figures. Figure 3 present the returns of the indices.

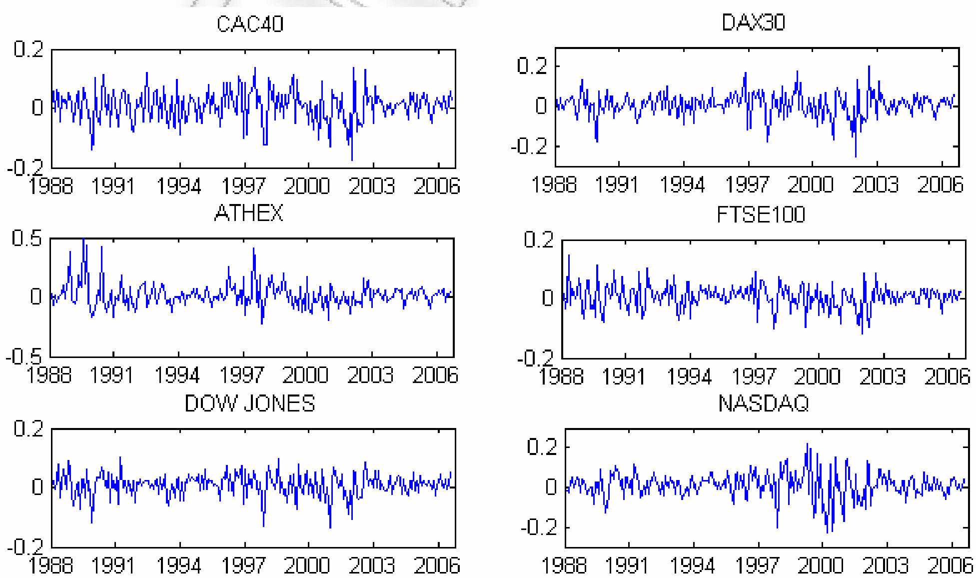


Figure 3: Returns of indices CAC40, DAX30, ATHEX, FTSE100, DOW JONES and NASDAQ covering the period from September 1988 to May 2007.

From the figures of returns (Figure 3) we can result, although there are based on monthly data, that the returns tend to be normal. Also we observe that there are stretches of time that volatility is relatively high and others that is relatively low, i.e. volatility values tend to cluster together in time, with more or less smooth transitions from higher to lower volatility and conversely. So, since volatility does not remain constant over time, we can result that the monthly estimation of covariance matrices will capture better the true values of variances and covariances.

Next, we present in Table 1 the summary statistics of index returns and risk free rates from September 1988 to May 2007.

Panel A: Index Returns and Risk Free Rates descriptive statistics

INDEX	Mean	StDev	Variance	IQR	Kurtosis	Skew	Min	Max	JB	Pvalue
CAC40	0.0080	0.0544	0.0030	0.0749	3.3259	-0.3950	-0.1749	0.1341	6.82	0.0344
DAX30	0.0084	0.0619	0.0038	0.0686	5.0442	-0.5928	-0.2542	0.2028	52.12	0.001
ATHEX	0.0173	0.1012	0.0102	0.1012	8.3023	1.5570	-0.2223	0.5063	352.90	0.001
FTSE100	0.0066	0.0410	0.0017	0.0465	3.7282	-0.1939	-0.1196	0.1443	6.3531	0.0399
DOW JONES	0.0086	0.0387	0.0015	0.0433	4.2662	-0.6492	-0.1362	0.1029	30.699	0.001
NASDAQ	0.0109	0.0679	0.0046	0.0731	4.3858	-0.3957	-0.2290	0.2198	23.768	0.0014
Minimum	0.0066	0.0387	0.0015	0.0433	3.3259	-0.6492	-0.2542	0.1029		
Maximum	0.0173	0.1012	0.0102	0.1012	8.3023	1.5570	-0.1196	0.5063		

RISK FREE RATE	Mean	StDev	Variance	IQR	Kurtosis	Skew	Min	Max	JB	Pvalue
PIBOR/EURIBOR	0.0044	0.0024	0.0000	0.0039	2.0380	0.7026	0.0017	0.0100	27.1879	0.001
FIBOR/EURIBOR	0.0040	0.0020	0.0000	0.0028	2.3434	0.8249	0.0017	0.0082	29.5603	0.001
GR T-BILL RATE/EURIBOR	0.0103	0.0073	0.0001	0.0136	7.1523	1.0823	0.0017	0.0532	205.5728	0.001
US T-BILL RATE	0.0037	0.0017	0.0000	0.0020	2.5888	0.0448	0.0007	0.0076	1.6604	0.3919
Minimum	0.0037	0.0017	0.0000	0.0020	2.0380	0.0448	0.0007	0.0076		
Maximum	0.0103	0.0073	0.0001	0.0136	7.1523	1.0823	0.0017	0.0532		

Panel B: Index Returns and Risk Free Rates correlations

INDEX / RISK FREE	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1) CAC40	1.0000									
(2) DAX30	0.8424	1.0000								
(3) ATHEX	0.3786	0.3371	1.0000							
(4) FTSE100	0.7371	0.6897	0.2571	1.0000						
(5) DOW JONES	0.6416	0.6479	0.2381	0.7268	1.0000					
(6) NASDAQ	0.5794	0.6054	0.2481	0.5778	0.6142	1.0000				
(7) PIBOR/EURIBOR	-0.0444	-0.0482	0.0203	0.0290	-0.0026	-0.0179	1.0000			
(8) FIBOR/EURIBOR	-0.0682	-0.0655	-0.0090	0.0023	-0.0280	-0.0261	0.9627	1.0000		
(9) GR T-BILL RATE/EURIBOR	-0.0228	-0.0178	0.0050	0.0317	0.0097	0.0435	0.7571	0.7401	1.0000	
(10) US T-BILL RATE	0.0594	0.0500	0.1142	0.0659	0.0509	-0.0089	0.5133	0.4290	0.4938	1.0000

Table 1: Summary statistics of index returns and risk free rates from September 1988 to May 2007. The summary statistics include mean, standard deviation (StDev), variance, interquartile range IQR (75%-25%), kurtosis, skewness (Skew), Jarque-bera statistic for normality test and its corresponding p-value, minimum and maximum values of returns and minimum and maximum values of each statistic among the indices and the risk free rates except the Jarque-Bera Statistic (JB) and its p-value (Panel A) and correlations between indices and risk free rates (Panel B).

The summary statistics include mean, standard error of mean, median, standard deviation, variance, interquartile range (75%-25%), kurtosis, skewness, minimum and maximum and minimum and maximum values of each statistic among the indices and the risk free rates (Panel A) and correlations (Panel B). Panel B also contains correlations that are not needed to calculate, because of the universes that we have chosen, but we calculate them in order to have a general idea of the correlation between the indices and the risk free rates.

In Panel A, we observe that the returns of the six indices are different enough. CAC40 and DAX30 have about the same average return and volatility, where DAX30 exhibits a slightly higher average return and volatility. ATHEX has the highest average return, but also has the highest volatility, in contrast to NASDAQ that has a similar average return but much lower volatility. FTSE100 has the lowest average return but not the lowest volatility, which has DOW JONES combined with a much higher average return than FTSE100. The risk free rates are not that different, excluding the GR T-BILL RATE/EURIBOR, which has the highest average return with a big difference from the next average return (0.59%), but also has a very high volatility. The other three risk free rates are almost the same, with PIBOR/EURIBOR having the next bigger average return and volatility and US T-BILL RATE the lower average return and volatility. Differences in the higher moments are also present. As for the skewness, which is the measure of the distributions' asymmetry of returns, the indexes' returns all have negative values, although for the risk free rates is not the same, where they all are positive. As for the kurtosis, which measures the heaviness of tails compared to a measure of three for the normal distribution, we find that the indices returns exhibit excess kurtosis (larger than 3), therefore their distributions have fatter tails than the normal one. Agreeing with the skewness, risk free rates exhibit low kurtosis, except the GR t-bill rate. We are starting to wonder if risk free rates follow a normal distribution and we perform the Jarque-Bera statistic for 5% significance level. The results show that neither index nor risk free rate is normal, except possibly US t-bill rate, which has a JB statistic of 1.6604. We can conclude that most indices and risk free rates are not symmetric and fat-tailed.

In Panel B, we observe that in general, indices exhibit high correlation between them and very low correlation with risk free rates, which exhibit high correlation between them except the US T-BILL RATE which exhibits a medium correlation. An exception is ATHEX, which exhibits low correlation with the other indices.

Our preliminary analysis of the data concludes that our sample can serve our purpose, because of the existence of variation and variety in average returns, volatilities and correlations that will help us extract the best possible results. In other words, we could say that there is a fair amount of heteroskedasticity in the data.

5.2 Results and Performance

Our methodology is as follows.

Firstly, we are forming the following seven universes:

1. Universe of all risky assets (indices) European and United States' (EU & US) (CAC40, DAX30, ATHEX, FTSE100, DOW JONES, NASDAQ).
2. Universe of European risky assets – indices (EU) (CAC40, DAX30, ATHEX, FTSE100).
3. Universe of United States' risky assets – indices (US) (DOW JONES, NASDAQ).

4. Universe of French index and risk free rate (CAC40 & FR PIBOR/EURIBOR).
5. Universe of German index and risk free rate (DAX30 & BD FIBOR/EURIBOR).
6. Universe of Greek index and risk free rate (ATHEX & GR Treasury Bill Rate/EURIBOR).
7. Universe of United States' indices and risk free rate (DOW JONES, NASDAQ & US Treasury Bill Rate).

The universes that we finally have are seven: three with risky assets and four with combinations of risky and risk free assets. We suppose that there are no transaction costs, because the mean-variance weights that we obtain from the mean-variance optimization do not project great differences and the transaction costs would be about the same for any universe. Therefore, their calculation does not change significantly our results.

We can treat risk free rates as risky assets and construct the mean-variance portfolios, because we have time series of the risk free rates, which exhibit variance and covariance with the risky assets (they are small but exist). Because we have 3-month risk free rates we must convert them to monthly. We do this through the equation (31):

$$r_{m_2} = \left[\left(1 + \frac{r_{m_1}}{m_1} \right)^{\frac{m_1}{m_2}} - 1 \right] m_2$$

where the compounding frequencies m_1 and m_2 equal $m_1 = 4$ (3-month risk free rate) and $m_2 = 12$ (monthly risk free rate). We then get the annualized monthly risk free rate and divide by 12 to obtain the monthly risk free rate and form the corresponding universe.

The objective of this study is to examine if more sophisticated covariance matrix estimators will advance asset allocation. To achieve this, we form an investment exercise which compares the empirical out-of-sample performance of mean-variance efficient frontiers produced from different covariance matrix estimators for all universes. The setup of our experiment is as follows. Beginning at September 2003, we want to forecast the expected return and covariance matrix for each universe. In order to forecast the expected return we use as an estimator the average of returns until time t:

$$E_{t+1}(R) = \frac{1}{t}\sum_{i=1}^{t} R_i \tag{33}$$

In order to forecast the covariance matrix of returns, we use the history of data covering the period September 1988 to October 2003 (181 return observations for each asset) to estimate the parameters of the HCM, EWMA, FLEXM and CCC covariance matrices for each universe.

After calculating expected returns and covariance matrix of returns, we use them as inputs to the mean-variance analysis and we plot the mean-variance efficient frontier of each universe at time t+1. Allowing for monthly rebalancing, at time t+1 the estimation period grows by one data point and we repeat the same forecasting procedure for time t+2 and so on, until the dataset is exhausted, in order to utilize all available information. The parameters of the covariance matrices models are computed every month. This exercise produces 43 out-of-sample observations that cover the period from November 2003 to May 2007.

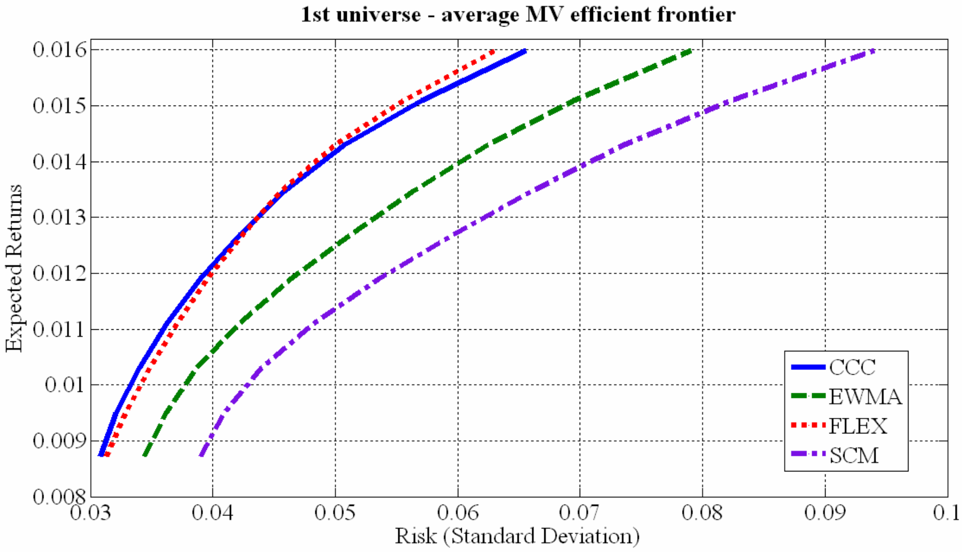
After producing the 43 out-of-sample observations for each universe, we plot the "average" mean-variance efficient frontier of each universe. The "average" mean-variance efficient frontier is an efficient frontier which presents the "average" efficient portfolios – the portfolios which correspond to levels of expected return which exists throughout the procedure of producing the 43 out-of-sample observations. According to the range of expected returns of each "average" efficient frontier, we define 3 portfolios with different expected returns, concerning on the results of the mean-variance efficient frontier: (1) a conservative, (2) an average and (3) an aggressive portfolio. The returns of the 3 portfolios are monthly. For each mean-variance efficient frontier that we form, the range of the expected returns is not the same for any universe and any portfolio. This is because of the differences in average returns, as we have seen in the preliminary analysis, resulting that we cannot take the same three expected returns for all portfolios. So the three different portfolios have the following expected returns:

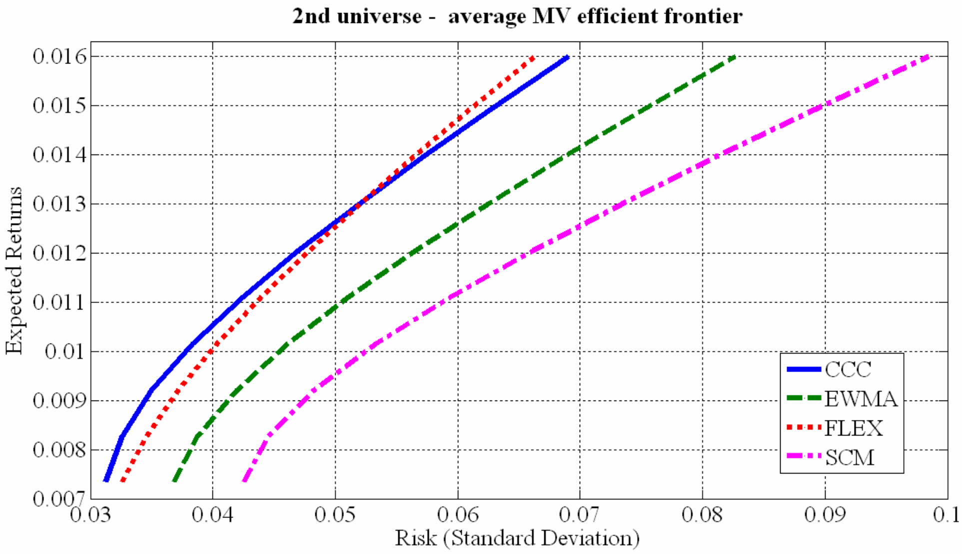
- 1st universe: all risky (EU and US) assets
  1. conservative portfolio with expected return 0.91%
  2. average portfolio with expected return 1.22%

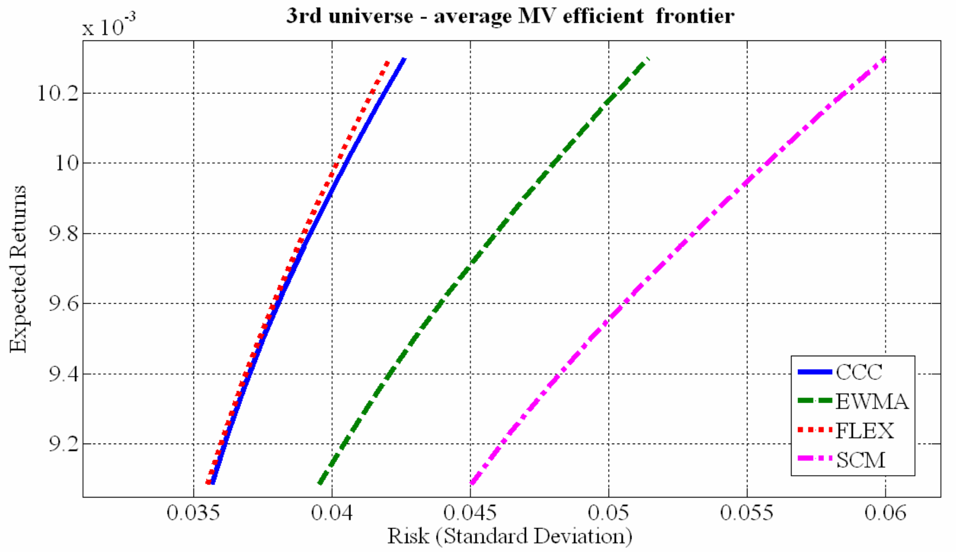
3. aggressive portfolio with expected return 1.6%

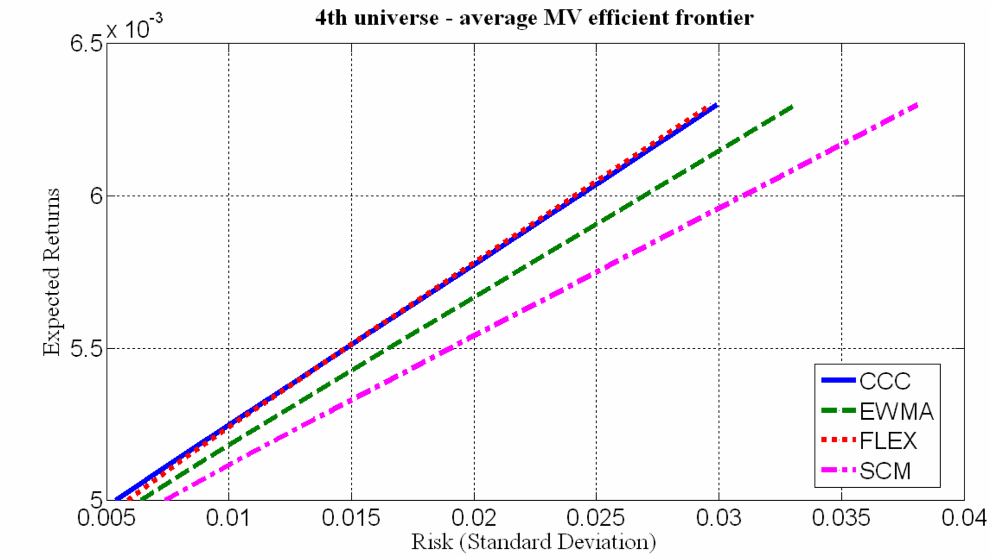
- 2nd universe: all EU risky assets
    1. conservative portfolio with expected return 0.73%
    2. average portfolio with expected return 1.22%
    3. aggressive portfolio with expected return 1.6%
- 3rd universe: all US risky assets
    1. conservative portfolio with expected return 0.91%
    2. average portfolio with expected return 0.97%
    3. aggressive portfolio with expected return 1.03%
- 4th universe: CAC40 & FR PIBOR/EURIBOR
    1. conservative portfolio with expected return 0.5%
    2. average portfolio with expected return 0.56%
    3. aggressive portfolio with expected return 0.63%
- 5th universe: DAX30 & BD FIBOR/EURIBOR
    1. conservative portfolio with expected return 0.44%
    2. average portfolio with expected return 0.5%
    3. aggressive portfolio with expected return 0.63%
- 6th universe: ATHEX & GR T-BILL RATE/EURIBOR
    1. conservative portfolio with expected return 1.22%
    2. average portfolio with expected return 1.41%
    3. aggressive portfolio with expected return 1.6%
- 7th universe: DOW JONES, NASDAQ & US T-BILL RATE
    1. conservative portfolio with expected return 0.5%
    2. average portfolio with expected return 0.63%
    3. aggressive portfolio with expected return 1.03%

The resulting "average" efficient frontiers for each universe for all four estimation models of covariance matrices are presented in figure 4.

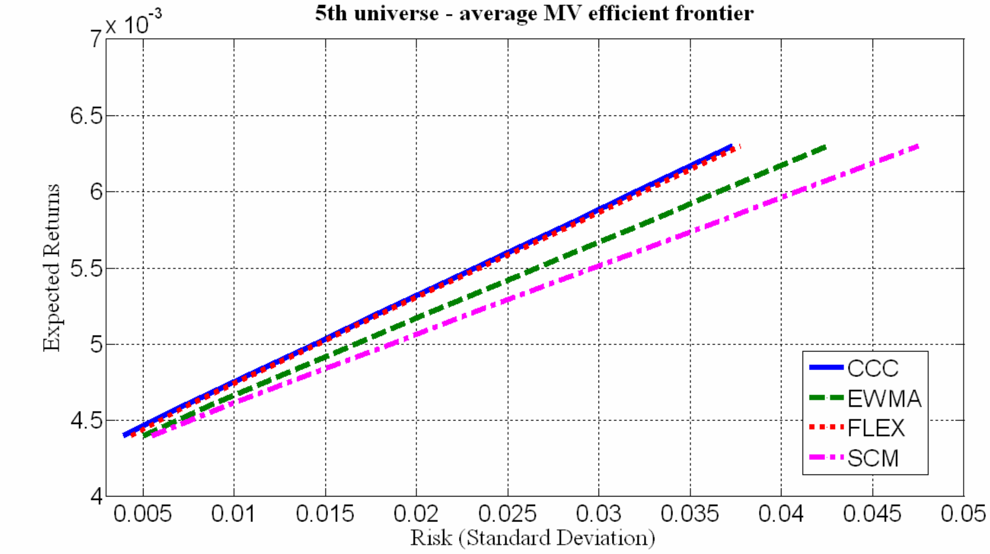

1st universe - average MV efficient frontier
Expected Returns
Risk (Standard Deviation)
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0.009
0.008
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
CCC
EWMA
FLEX
SCM


2nd universe - average MV efficient frontier
Expected Returns
Risk (Standard Deviation)
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0.009
0.008
0.007
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
CCC
EWMA
FLEX
SCM

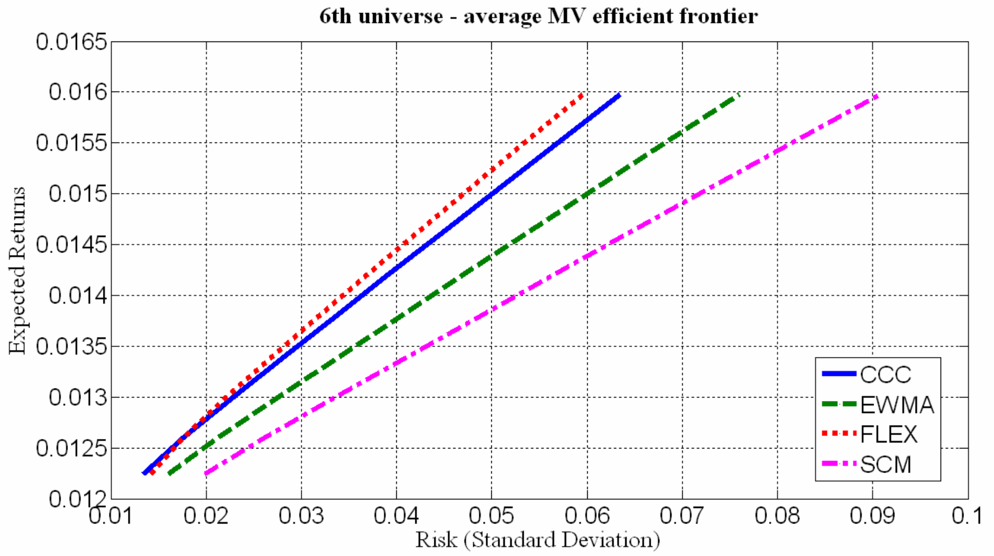

3rd universe - average MV efficient frontier
x 10-3
Expected Returns
Risk (Standard Deviation)
10.2
10
9.8
9.6
9.4
9.2
0.035
0.04
0.045
0.05
0.055
0.06
CCC
EWMA
FLEX
SCM



4th universe - average MV efficient frontier
x 10^-3
6.5
6
5.5
5
Expected Returns
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
Risk (Standard Deviation)
CCC
EWMA
FLEX
SCM



5th universe - average MV efficient frontier
x 10^-3
7
6.5
6
5.5
5
4.5
4
Expected Returns
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
Risk (Standard Deviation)
CCC
EWMA
FLEX
SCM



6th universe - average MV efficient frontier
0.0165
0.016
0.0155
0.015
0.0145
0.014
0.0135
0.013
0.0125
0.012
Expected Returns
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Risk (Standard Deviation)
CCC
EWMA
FLEX
SCM

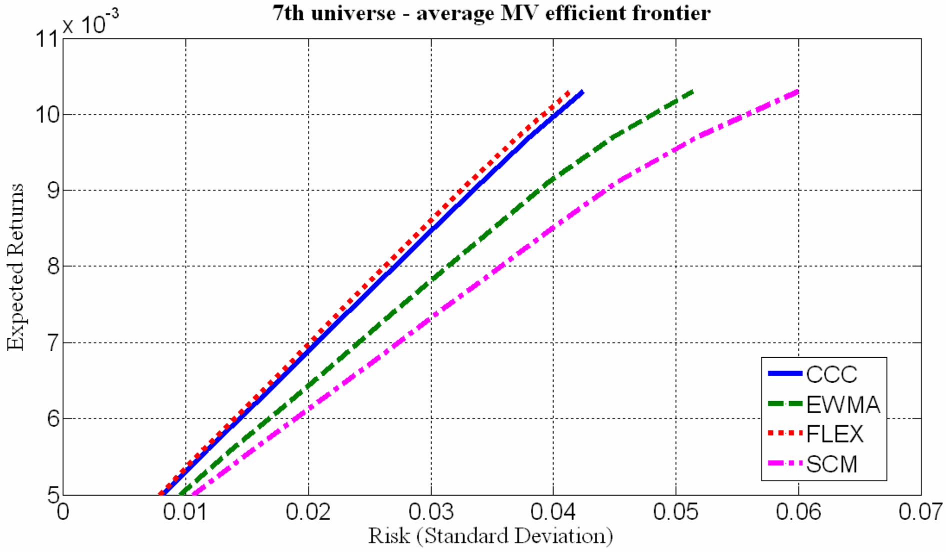


Figure 4: "Average" mean-variance efficient frontiers for all seven universes: (1) all risky assets, (2) EU risky assets, (3) US risky assets, (4) CAC40 & PIBOR/EURIBOR, (5) DAX30 & FIBOR/EURIBOR, (6) ATHEX & GR T-BILL RATE and (7) DOW JONES, NASDAQ & US T-BILL RATE. The "average" mean-variance efficient frontiers of each estimation model of covariance matrix are plotted for the 43 out-of-sample observations covering the time period from November 2003 to May 2007.

The mean-variance efficient frontiers we produce are consistent with theory. When we have only risky assets, we can see that they form a convex figure, when we have one risky and one risk free asset they are almost straight lines that exhibit very low, almost zero, standard deviation for the lower expected return (compared to mean-variance efficient frontiers that are formed from only risky assets) and when we have two risky assets and one risk free, we can see that the effect of the risk free asset decreases and the mean-variance efficient frontier exhibits convexity for higher risk (standard deviation).

As we can see from the mean-variance efficient frontiers, the method that exhibits the lower risk (standard deviation) for the same expected returns must be the Flexible Multivariate GARCH Model. Constant Conditional Correlation GARCH Model is the next best model, although there are many cases where the two models perform almost the same. Exponentially Weighted Moving Average Model comes third and Historical Covariance Matrix Model exhibits the higher risk.

We plot the weights of the mean-variance efficient portfolios that were constructed for universes 4, 5 and 6 through the above procedure in order to have a general idea of the weights movement. The following plots are presenting the weights that we put on the risky asset of universes 4, 5, 6 (CAC40, DAX30, ATHEX). The weights for each portfolio are same for any estimation model of covariance matrix used for the universes that include a risky asset and a risk free rate, say universes 4, 5 and 6. The weight that we put on the risk free asset is the remaining weight, since we have no short sales constraint.

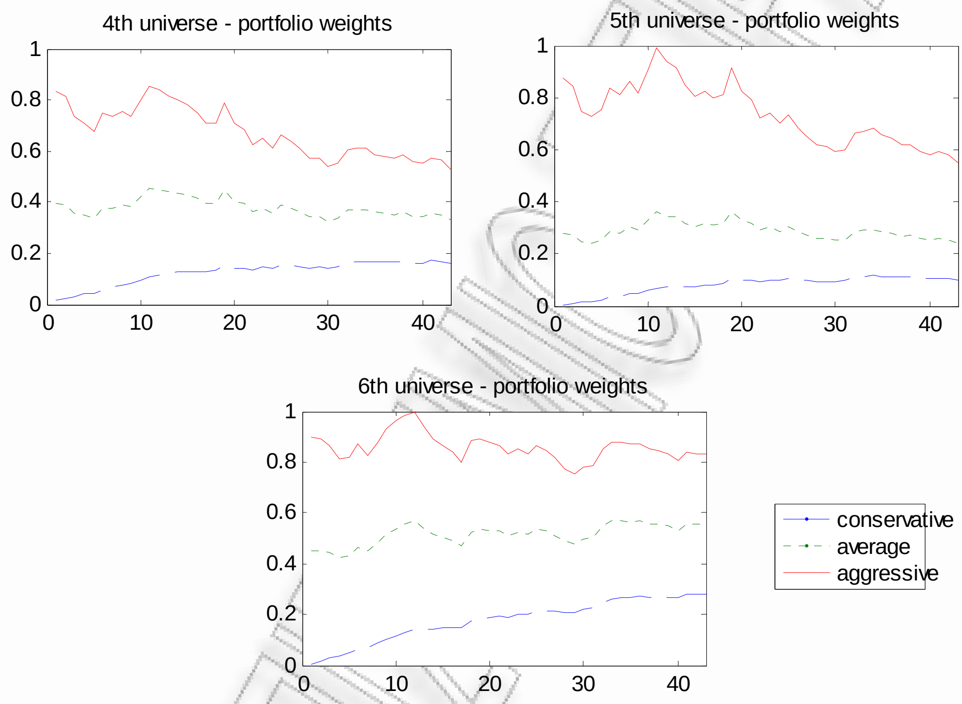


Figure 5: Plot of weights of the mean-variance efficient portfolios that were constructed for universes 4, 5 and 6 through the above procedure in order to have a general idea of the weights movement. The following plots are presenting the weights that we put on the risky asset of every of the above universes (CAC40, DAX30, ATHEX). The weight that we put on the risk free asset is the remaining weight, since we have no short sales constraint.

After estimating the efficient frontiers, we calculate the realized returns using the weights obtained from the mean-variance optimization for the 3 expected returns for the three portfolios of each universe, as we presented them before. Using the realized returns and the forecasted standard deviations for each portfolio of each universe, we calculate the SR, the Conditional Sharpe Ratio and the Certainty Equivalent for each portfolio. The results are presented in Table 2.

Panel 1: 1st universe

Conservative Portfolio: monthly expected return 0.91% (annualized 10.92%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0126	0.0398	0.5724	0.3168	0.0102
EWMA	0.0127	0.0352	**0.5781**	0.3699	**0.0103**
CCC	0.0120	0.0314	0.5208	**0.3949**	0.0093
FLEXM	0.0116	0.0320	0.5021	0.3718	0.0089

Average Portfolio: monthly expected return 1.22% (annualized 14.64%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0141	0.0558	0.5059	0.2498	**0.0102**
EWMA	0.0141	0.0482	**0.5076**	0.2999	**0.0102**
CCC	0.0131	0.0402	0.4476	**0.3333**	0.0087
FLEXM	0.0126	0.0406	0.4186	0.3055	0.0080

Aggressive Portfolio: monthly expected return 1.6% (annualized 19.20%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0193	0.0940	0.4990	0.2009	0.0113
EWMA	0.0193	0.0790	0.4990	0.2456	0.0113
CCC	0.0193	0.0656	0.4990	0.2997	0.0113
FLEXM	0.0193	0.0631	0.4990	**0.3046**	0.0113

Panel 2: 2nd universe

Conservative Portfolio: monthly expected return 0.73% (annualized 8.76%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0116	0.0425	0.5510	0.2739	0.0093
EWMA	0.0116	0.0368	0.5510	0.3233	0.0093
CCC	0.0117	0.0312	**0.5546**	**0.3830**	**0.0094**
FLEXM	0.0116	0.0326	0.5502	0.3636	0.0093

Average Portfolio: monthly expected return 1.22% (annualized 14.64%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0166	0.0673	**0.5486**	0.2451	**0.0117**
EWMA	0.0165	0.0575	0.5466	0.2922	0.0116
CCC	0.0166	0.0478	0.5477	**0.3574**	**0.0117**
FLEXM	0.0166	0.0485	0.5468	0.3434	0.0116

Aggressive Portfolio: monthly expected return 1.6% (annualized 19.20%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**

HCM	0.0206	0.0985	**0.5089**	0.2068	**0.0117**
EWMA	0.0204	0.0827	0.5046	0.2496	0.0116
CCC	0.0204	0.0690	0.5060	0.3047	0.0116
FLEXM	0.0204	0.0663	0.5051	**0.3109**	0.0116

Panel 3: 3rd universe

Conservative Portfolio: monthly expected return 0.91% (annualized 10.92%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0108	0.0451	0.4258	0.2356	0.0076
EWMA	0.0108	0.0396	0.4258	0.2786	0.0076
CCC	0.0108	0.0357	0.4258	0.3133	0.0076
FLEXM	0.0108	0.0355	0.4258	**0.3163**	0.0076

Average Portfolio: monthly expected return 0.97% (annualized 11.64%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0100	0.0518	0.3565	0.1864	0.0061
EWMA	0.0100	0.0449	0.3565	0.2237	0.0061
CCC	0.0100	0.0386	0.3565	0.2596	0.0061
FLEXM	0.0100	0.0384	0.3565	**0.2604**	0.0061

Aggressive Portfolio: monthly expected return 1.03% (annualized 12.36%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0093	0.0600	0.2955	0.1453	0.0044
EWMA	0.0093	0.0514	0.2955	0.1765	0.0044
CCC	0.0093	0.0426	0.2955	0.2079	0.0044
FLEXM	0.0093	0.0421	0.2955	**0.2087**	0.0044

Panel 4: 4th universe

Conservative Portfolio: monthly expected return 0.5% (annualized 6.00%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0038	0.0074	1.0308	0.5163	0.0038
EWMA	0.0038	0.0064	1.0308	0.5892	0.0038
CCC	0.0038	0.0054	1.0308	**0.7262**	0.0038
FLEXM	0.0038	0.0059	1.0308	0.6391	0.0038

Average Portfolio: monthly expected return 0.56% (annualized 6.72%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0069	0.0214	0.7262	0.3181	0.0064
EWMA	0.0069	0.0186	0.7262	0.3783	0.0064

CCC	0.0069	0.0167	0.7262	0.4191	0.0064
FLEXM	0.0069	0.0166	0.7262	**0.4235**	0.0064

Aggressive Portfolio: monthly expected return 0.63% (annualized 7.56%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0105	0.0381	0.6313	0.2742	0.0091
EWMA	0.0105	0.0331	0.6313	0.3292	0.0091
CCC	0.0105	0.0299	0.6313	0.3605	0.0091
FLEXM	0.0105	0.0297	0.6313	**0.3660**	0.0091

Panel 5: 5th universe

Conservative Portfolio: monthly expected return 0.44% (annualized 5.28%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0035	0.0055	1.1537	0.6323	0.0035
EWMA	0.0035	0.0050	1.1537	0.6883	0.0035
CCC	0.0035	0.0039	1.1537	**0.9006**	0.0035
FLEXM	0.0035	0.0043	1.1537	0.7922	0.0035

Average Portfolio: monthly expected return 0.5% (annualized 6.00%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0064	0.0185	0.6784	0.3420	0.0059
EWMA	0.0064	0.0166	0.6784	0.3973	0.0059
CCC	0.0064	0.0144	0.6784	0.4500	0.0059
FLEXM	0.0064	0.0145	0.6784	**0.4629**	0.0059

Aggressive Portfolio: monthly expected return 0.63% (annualized 7.56%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0126	0.0475	0.5254	0.2702	0.0097
EWMA	0.0126	0.0426	0.5254	0.3193	0.0097
CCC	0.0126	0.0373	0.5254	0.3538	0.0097
FLEXM	0.0126	0.0377	0.5254	**0.3658**	0.0097

Panel 6: 6th universe

Conservative Portfolio: monthly expected return 1.22% (annualized 14.64%)

Model	**Return**	**Risk**	**SR**	**CSR**	**CE**
HCM	0.0056	0.0199	0.6843	0.2750	0.0053
EWMA	0.0057	0.0160	0.6839	0.3366	0.0053
CCC	0.0057	0.0134	0.6839	**0.4059**	0.0053
FLEXM	0.0057	0.0143	0.6839	0.3790	0.0053

Average Portfolio: monthly expected return 1.41% (annualized 16.92%)

Model	Return	Risk	SR	CSR	CE
HCM	0.0122	0.0546	0.5629	0.2171	0.0098
EWMA	0.0122	0.0453	0.5629	0.2657	0.0098
CCC	0.0122	0.0377	0.5629	0.3234	0.0098
FLEXM	0.0122	0.0356	0.5629	**0.3315**	0.0098

Aggressive Portfolio: monthly expected return 1.6% (annualized 19.20%)

Model	Return	Risk	SR	CSR	CE
HCM	0.0189	0.0908	0.5232	0.2020	0.0120
EWMA	0.0189	0.0760	0.5232	0.2477	0.0120
CCC	0.0189	0.0635	0.5232	0.3008	0.0120
FLEXM	0.0189	0.0596	0.5232	**0.3072**	0.0120

Panel 7: 7th universe

Conservative Portfolio: monthly expected return 0.5% (annualized 6.00%)

Model	Return	Risk	SR	CSR	CE
HCM	0.0045	0.0106	**0.7348**	0.4189	**0.0043**
EWMA	0.0044	0.0095	0.7221	0.4678	0.0042
CCC	0.0039	0.0081	0.6261	**0.4927**	0.0037
FLEXM	0.0039	0.0078	0.6051	0.4912	0.0037

Average Portfolio: monthly expected return 0.63% (annualized 7.56%)

Model	Return	Risk	SR	CSR	CE
HCM	0.0064	0.0214	**0.5265**	0.2955	**0.0057**
EWMA	0.0063	0.0190	0.5143	**0.3378**	0.0055
CCC	0.0053	0.0163	0.4202	0.3219	0.0045
FLEXM	0.0051	0.0158	0.4010	0.3171	0.0043

Aggressive Portfolio: monthly expected return 1.03% (annualized 12.36%)

Model	Return	Risk	SR	CSR	CE
HCM	0.0093	0.0600	**0.2955**	0.1453	**0.0044**
EWMA	0.0093	0.0514	**0.2955**	0.1765	**0.0044**
CCC	0.0089	0.0425	0.2777	**0.1956**	0.0038
FLEXM	0.0085	0.0414	0.2652	0.1880	0.0034

Table 2: It presents out-of-sample results of the mean-variance portfolio construction method calculated for the three different portfolios (conservative, average and aggressive) of all universes. These include mean values of realized returns (Return), of portfolio standard deviation (Risk), the Sharpe Ratio (SR) as defined in Section 3, Conditional Sharpe Ratio (CSR) and the values of Certainty

Equivalent (CE). the data that are presented are monthly. The first row of each table presents the monthly expected return and the annualized expected return is presented in brackets.

We next test for the statistical significance of Returns and CSR. The tests we perform are two: we test if the differences of the means of each metric are statistically significant and we tests if the values of the metrics are statistically significant. More specifically, the first test (named hereafter ***ttest2***) performs a *t*-test of the null hypothesis that values of different metrics are independent random samples from normal distributions with equal means and unequal and unknown variances (Behrens-Fisher problem), against the alternative that the means are not equal. The result of the test is returned in h. h = 1 indicates a rejection of the null hypothesis at the 5% significance level. h = 0 indicates a failure to reject the null hypothesis at the 5% significance level. The second test (named hereafter ***ttest***) performs a t-test of the null hypothesis that data in the vector x are a random sample from a normal distribution with mean 0 and unknown variance, against the alternative that the mean is not 0. The result of the test is returned in h. h = 1 indicates a rejection of the null hypothesis at the 5% significance level. h = 0 indicates a failure to reject the null hypothesis at the 5% significance level. The results of the tests for statistical significance are presented in Table 3 at the end of the article.

After finishing the estimation process and having found the performance measures, let's comment on results. In terms of realized returns, the model which gives the higher realized returns is most of the times the HCM model, with EWMA model following. FLEXM and CCC models give about the same realized returns. The differences between the means of the realized returns are not statistically significant at 5% for any case. This fact could be generalized in terms of Sharpe Ratio and Certainty Equivalent, but this testing has not be done in order to be absolutely sure for the accuracy of the results. The means of the realized returns are almost all statistically significant. As we can observe, the differences in average realized returns are very small. This is mostly because we have defined certain levels of expected return based on the "average" mean-variance efficient frontiers. Since the differences between the means are not statistically significant, one could say that there is no difference from using the one or the other model in order to estimate covariance matrices, since we would get the same return.

The SR metric and the CE metric could also suggest the same opinion, because its results are similar. The HCM model has the higher SR and CE, with EWMA, CCC and FLEXM following (the GARCH models give almost same results). Despite these facts, the differences between the values of Returns and between the values of SRs are very small. On the other hand, we can see that Risk (standard deviation) does not have the same values for all models and the differences between the means are statistically different. This gives rise to the suspicion that maybe Returns, SR and CE are not the suitable metrics for us to result if the use of more sophisticated metrics would advance the asset allocation. Returns, SR and CE are traditional metrics which are based to realized returns and as mentioned before, realized returns are not statistically different for our portfolios. Furthermore, except from mean-variance analysis, they do not use at all the more sophisticated estimators to evaluate the performance.

Therefore, we are going to use a more sophisticated estimator. The CSR performance measure gives us a different ranking for the models: FLEXM model is higher than the others most of the times and CCC follows. Next ranks EWMA and finally HCM model. Of course CSR is only one metric in contrast to the others that are three, but the results that we extract are logical: we have about the same average realized expected return for all models, but the more sophisticated models estimate much smaller risk. This result could have been helpful for an investor which might not invest if the risk he measured was very high related to his expected return.

6. Conclusion

The objective of our study and our contribution to the literature has to do with the asset allocation of assets in a portfolio, where covariances – correlations of returns are forecasted with different estimators. If not measured accurately, there may be important impacts in terms of asset allocation, pricing and portfolio construction and selection and in risk measurement (VaR, CVaR) and management.

Summarizing, our study is important for the literature because it tries to improve asset allocation in a portfolio in a way that could be easily used from any investor. This is by using different estimators of covariance matrix and trying to figure out which is the best estimator (between the estimators we study) that can be used to estimate and predict the covariances and the correlations between the assets of the portfolio.

To approach the goal of our study, we form an investment exercise which compares the empirical out-of-sample performance of mean-variance efficient frontiers produced from different covariance matrix estimators for all universes. The setup of our experiment is as follows. Beginning at September 2003, we want to forecast the expected return and covariance matrix for each universe. In order to forecast the expected return we use as an estimator the average of returns until time t. In order to forecast the covariance matrix of returns, we use the history of data covering the period September 1988 to October 2003 (181 return observations for each asset) to estimate the parameters of the HCM, EWMA, FLEXM and CCC covariance matrices for each universe.

For our problem we define seven universes. These are:

1. Universe of all risky assets (indices) European and United States' (CAC40, DAX30, ATHEX, FTSE100, DOW JONES, NASDAQ).
2. Universe of European risky assets – indices (CAC40, DAX30, ATHEX, FTSE100).
3. Universe of United States' risky assets – indices (DOW JONES, NASDAQ).
4. Universe of French index and risk free rate (CAC40 & FR PIBOR/EURIBOR).
5. Universe of German index and risk free rate (DAX30 & BD FIBOR/EURIBOR).

6. Universe of Greek index and risk free rate (ATHEX & GR Treasury Bill Rate/EURIBOR).
7. Universe of United States' indices and risk free rate (DOW JONES, NASDAQ & US Treasury Bill Rate).

After calculating expected returns and covariance matrix of returns, we use them as inputs to the mean-variance analysis and we plot the mean-variance efficient frontier of each universe at time t+1. Allowing for monthly rebalancing, at time t+1 the estimation period grows by one data point and we repeat the same forecasting procedure for time t+2 and so on, until the dataset is exhausted, in order to utilize all available information. The parameters of the covariance matrices models are computed every month. This exercise produces 43 out-of-sample observations that cover the period from November 2003 to May 2007.

After producing the 43 out-of-sample observations for each universe, we plot the "average" mean-variance efficient frontier of each universe. The "average" mean-variance efficient frontier is an efficient frontier which presents the "average" efficient portfolios – the portfolios which correspond to levels of expected return which exists throughout the procedure of producing the 43 out-of-sample observations. According to the range of expected returns of each "average" efficient frontier, we define 3 portfolios with different expected returns, concerning on the results of the mean-variance efficient frontier: (1) a conservative, (2) an average and (3) an aggressive portfolio. The returns of the 3 portfolios are monthly. For each mean-variance efficient frontier that we form, the range of the expected returns is not the same for any universe and any portfolio. This is because of the differences in average returns, as we have seen in the preliminary analysis, resulting that we cannot take the same three expected returns for all portfolios.

After all the analysis, we can conclude that the traditional metrics of performance that we use (Returns, SR and Certainty Equivalent) show as the best estimator the HCM model. In market terms, the use of more sophisticated estimators makes no difference, only in terms of risk. The risk decreases very much when the sophisticated estimators insert to the mean-variance analysis, without that fact imposing an analogous in magnitude decrease in realized return. This is shown by the Conditional Sharpe Ratio metric, which captures the higher return per unit of estimated risk.

Concluding, the use of more sophisticated estimators has been shown that it can improve asset allocation, not in terms of return, but in terms of return per unit of risk.

References

Alexander C., Dimitriu A. (2004). *The art of investing in hedge funds: Fund selection and optimal allocations.* ISMA Centre Discussion Papers in Finance. Can be downloaded from papers.ssrn.com.

Amenc N., Martellini L.(2001). *It's time for asset allocation.* Journal of Financial Transformation.

Bauwens L., Laurent S., Rombouts J. (2006). *Multivariate GARCH models: a survey*. Journal of Applied Econometrics, Vol.21, Issue 1, p.79-109.

Best M., Grauer R. (1991). *On the sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results.* The Review of Financial Studies, Vol. 4, No 2, pp. 315-342.

Bollerslev T. (1990). *Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized Arch Model.* The Review of Economics and Statistics, Vol. 72, No. 3, pp. 498-505.

Bollerslev T. (1986). *Generalized autoregressive conditional heteroskedasticity.* Journal of Econometrics 31, pp. 307-327.

Bollerslev T., Engle R., Wooldridge J. (1988). *A capital asset pricing model with time-varying covariances.* The Journal of Political Economy, Vol. 96, No 1, pp.116-131.

Brennan M., Schwartz E., Lagnado R. (1997). *Strategic Asset Allocation.* Journal of Economic Dynamics and Control 21, pp.1377-1403.

Campbell J. (2000). *Asset Pricing At the Millennium.* The Journal of Finance, Vol. 55, No 4, p.1515-1567.

Chan L., Karceski J., Lakonishok J. (1999). *On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model.* The Review of Financial Studies, Vol.12, No 5, pp.937-974.

Chopra V., Ziemba W. (1993). *The effect of errors in means, variances, and covariances on optimal portfolio choice.* Journal of Portfolio Management; Vol.19, No 2.

Colacito R., Engle R. (2004). *Multiperiod Asset Allocation with Dynamic Volatilities.* New York University. Can be downloaded from papers.ssrn.com.

Colacito R., Engle R. (2006). *Testing and Valuing Dynamic Correlations for Asset Allocation.* Journal of Business & Economic Statistics, Vol. 24, No. 2.

Dahlquist M., Harvey C. (2001). *Global Tactical Asset Allocation.* Emerging markets quarterly.

DeRoon F., Nijman T. (2001). *Testing for mean-variance spanning: a survey.* Journal of Empirical Finance 8 111–155.

Engle R., Kroner K. (1993). *Multivariate Simultaneous Generalized ARCH.* Econometric theory.

Engle R. (1982). *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.* Econometrica, Vol. 50, No. 4, pp. 987-1007.

Elton E., Gruber M., Spitzer J. (2006). *Improved estimates of correlation and their impact on the optimum portfolios.* European Financial Management.

Fleming J., Kirby C., Ostdiek B. (2001). *The economic value of volatility timing.* The Journal of Finance.

Giamouridis D., Vrontos I. (2007). *Hedge Fund Portfolio Construction: A Comparison of Static and Dynamic Approaches.* Journal of Banking and Finance, Vol.31, No 1, pp 199-217.

Jagannathan R.,Ma T. (2003). *Risk Reduction in Large Portfolios: A Role for Portfolio Weight Constraints*. Journal of Finance.

Ledoit O., Santa-Clara P., Wolf M. (2003). *Flexible Multivariate GARCH modeling with an application to international stock markets.* The Review of Economics and Statistics, 85(3): 735–747.

Ledoit O., Wolf M. (2003). *Honey, I shrunk the sample covariance matrix*. Can be downloaded from papers.ssrn.com.

Ledoit O., Wolf M. (2003). *Improved estimation of the covariance matrix of stock returns with an application to portfolio selection.* Journal of Empirical Finance 10 603-621.

Markowitz H. (1952). *Portfolio Selection*. The Journal of Finance 7, pp. 77-91.

Markowitz H., Lacey R., Plymen J., Dempster M. A. H., Tompkins R. G. (1994). *The General Mean-Variance Portfolio Selection Problem [and Discussion].* Philosophical Transactions: Physical Sciences and Engineering, Vol. 347, No. 1684, Mathematical Models in Finance, pp. 543-549.

Pafka S., Kondor I. (2004). *Estimated correlation matrices and portfolio optimization.* Physica A 343, pp. 623-634.

Papamastoraki I. (2004). *The role of real estate when choosing the optimal portfolio.* Master Thesis, University of Piraeus.

Simaan Y. (1993). *What Is the Opportunity Cost of Mean-variance Investment Strategies?* Management Science Vol. 39, No. 5.

Suleimann R. (2003). *Should Stock Market Indexes Time Varying Correlations Be Taken Into Account? A Conditional Variance Multivariate Approach.* Can be downloaded from ideas.repec.org.

Teräsvirta T. (2006). *An Introduction to Univariate GARCH Models.* SSE/EFI Working Papers in Economics and Finance, No. 646. Can be downloaded from ideas.repec.org.

Tew B., Reid D., Witt C. (1991). *The Opportunity Cost of a Mean-Variance Efficient Choice.* The Financial Review, Vol.26 No.1, pp. 31-43.

Topaloglou N., Vladimirou H., Zenios S.(2002). *A Dynamic Stochastic Programming Model for International Portfolio Management.* Journal of Banking and Finance.

Tsui K., Gao Y. (1999). *CCC in a bivariate GARCH model: evidence from the stock markets of China*. Can be downloaded from www.sciencedirect.com.

Vrontos I., Dellaportas P., Politis D. (2003). *A full-factor multivariate GARCH model.* Econometrics Journal, Vol. 6, pp. 312-334.

http://www.kevinsheppard.com/research/ucsd_garch/ucsd_garch.aspx

Table 3: In Panel 1 we present the results of ttest and ttest2 for the realized returns of the mean-variance efficient portfolios of each universe. We perform a *t*-test of the null hypothesis that values of different metrics are independent random samples from normal distributions with equal means and unequal and unknown variances (Behrens-Fisher problem), against the alternative that the means are not equal. The result of the test is returned in h. h = 1 indicates a rejection of the null hypothesis at the 5% significance level. h = 0 indicates a failure to reject the null hypothesis at the 5% significance level. The second test we perform is a t-test of the null hypothesis that data in the vector x are a random sample from a normal distribution with mean 0 and unknown variance, against the alternative that the mean is not 0. The result of the test is returned in h. h = 1 indicates a rejection of the null hypothesis at the 5% significance level. h = 0 indicates a failure to reject the null hypothesis at the 5% significance level. In Panel 2 we present the results of the above tests for the conditional sharpe ratios of the mean-variance efficient portfolios of each universe.

Panel 1: ttest and ttest2 for Returns

Ttest

1st universe

Model	**10.92%**	**14.64%**	**19.20%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

2nd universe

Model	**8.76%**	**14.64%**	**19.20%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

3rd universe

Model	**10.92%**	**11.64%**	**12.36%**
HCM	1	1	0
EWMA	1	1	0
CCC	1	1	0
FLEXM	1	1	0

4th universe

Model	**6.00%**	**6.72%**	**7.56%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

5th universe

Model	**5.28%**	**6.00%**	**7.56%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

6th universe

Model	**14.64%**	**16.92%**	**19.20%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

7th universe

Model	**6.00%**	**7.56%**	**12.36%**
HCM	1	1	0
EWMA	1	1	0
CCC	1	1	0
FLEXM	1	1	0

Ttest2

1st universe

10.92%					**14.64%**				**19.20%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

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2nd universe

8.76%					**14.64%**				**19.20%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

3rd universe

10.92%					**11.64%**				**12.36%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

4th universe

6.00%					**6.72%**				**7.56%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

5th universe

5.28%					**6.00%**				**7.56%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

6th universe

14.64%					**16.92%**				**19.20%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	

FLEXM	0	0	0	-	0	0	0	-	0	0	0	-
7th universe												
6.00%					**7.56%**				**12.36%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 2: ttest and ttest2 for CSR

Ttest

1st universe

Model	**10.92%**	**14.64%**	**19.20%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

2nd universe

Model	**8.76%**	**14.64%**	**19.20%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

3rd universe

Model	**10.92%**	**11.64%**	**12.36%**
HCM	1	1	0
EWMA	1	1	0
CCC	1	1	0
FLEXM	1	1	0

4th universe

Model	**6.00%**	**6.72%**	**7.56%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

5th universe

Model	**5.28%**	**6.00%**	**7.56%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

6th universe

Model	**14.64%**	**16.92%**	**19.20%**
HCM	1	1	1
EWMA	1	1	1
CCC	1	1	1
FLEXM	1	1	1

7th universe

Model	**6.00%**	**7.56%**	**12.36%**
HCM	1	1	0
EWMA	1	1	0
CCC	1	1	0
FLEXM	1	1	0

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Ttest2

Panel 1:1st universe

10.92%					**14.64%**				**19.20%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 2:2nd universe

	8.76%				**14.64%**				**19.20%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 3: 3rd universe

	10.92%				**11.64%**				**12.36%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 4: 4th universe

	6.00%				**6.72%**				**7.56%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 5: 5th universe

	5.28%				**6.00%**				**7.56%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	1	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 6: 6th universe

	14.64%				**16.92%**				**19.20%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	

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FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

Panel 7: 7th universe

6.00%					**7.56%**				**12.36%**			
Model	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM	HCM	EWMA	CCC	FLEXM
HCM	-				-				-			
EWMA	0	-			0	-			0	-		
CCC	0	0	-		0	0	-		0	0	-	
FLEXM	0	0	0	-	0	0	0	-	0	0	0	-

ΠΑΝΕΠΙΣΤΗΜΙΟ ΠΕΙΡΑΙΑ