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“OPTION VALUATION WITH EDGEWORTH BINOMIAL TREES”

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Abstract

The purpose of this thesis is the Option Valuation using Edgeworth Binomial Trees. Mark Rubinstein in 1998 proposed an Edgeworth Expansion to transform a standard binomial density into a unimodal standardized discrete density, which is evaluated at equally-spaced points with prespecified skewness and kurtosis – the skewness being non-zero and the kurtosis being greater than three.

At the beginning, there is an introduction about the various option pricing models, followed by the Option Pricing theory. Furthermore, the Edgeworth Binomial Tree and its main advantages compared to the Cox-Rubenstein-Ross (CRR) Binomial Model is described, as well as the theory behind the construction of the tree and where the Edgeworth Expansion is applied.

The Empirical study is next, with the evaluation of the Edgeworth Binomial Tree compared to the CRR Binomial model, using real market option data from the Meta Platforms stock, for a period of 5 months, with 4 different maturity samples.

Finally, after the In-the-sample and Out-of-sample comparison of the models, we have the conclusion with the results of this comparison.

Keywords: Option, CRR Model, Edgeworth Model, Edgeworth Binomial Trees, Option Pricing, Parameter Estimation, Forecasting Ability, Skewness, Kurtosis

Περίληψη

Ο σκοπός αυτής της διπλωματικής εργασίας είναι η Αποτίμηση Δικαιωμάτων χρησιμοποιώντας τα Διωνυμικά Δέντρα Edgeworth (Edgeworth Binomial Trees). O Mark Rubinstein το 1998 πρότεινε τη χρήση μιας Edgeworth Κατανομής για να μετατρέψει μια τυπική διωνυμική κατανομή σε ομοιόμορφη τυποποιημένη διακριτή κατανομή, με προκαθορισμένη ασσυμετρία και κυρτότητα – η ασσυμετρία είναι μη μηδενική και η κυρτότητα πάνω από 3.

Στην αρχή έχουμε μια εισαγωγή για τα διάφορα μοντέλα αποτίμησης δικαιωμάτων, και ακολουθεί η θεωρία της αποτίμησης δικαιωμάτων. Επιπλέον παρουσιάζεται το μοντέλο των Διωνυμικών Δέντρων Edgeworth και τα προτερήματά του σε σχέση με το Διωνυμικό Μοντέλο των Cox-Rubenstein-Ross (CRR). Επίσης παρουσιάζεται η κατασκευή του Διωνυμικού Δέντρου και πως εφαρμόζεται η επέκταση Edgeworth.

Ακολουθεί η Εμπειρική μελέτη, όπου συγκρίνεται το μοντέλο των Διωνυμικών Δέντρων Edgeworth με το Διωνυμικό Μοντέλο των Cox-Rubenstein-Ross χρησιμοποιώντας πραγματικά δεδομένα δικαιωμάτων της μετοχής της Meta Platforms για ένα διάστημα 5 μηνών με συνολικά 4 δείγματα τα οποία έχουν διαφορετικές ληκτότητες.

Τέλος, μετά τη παρουσίαση των αποτελεσμάτων Εντός Δείγματος και Εκτός Δείγματος, έχουμε τα συμπεράσματα της σύγκρισης.

Λέξεις Κλειδιά: Δικαιώματα, Διωνυμικό Δέντρο, Μοντέλο Edgeworth, Διωνυμικά Δέντρα Edgeworth, Αποτίμηση Δικαιωμάτων, Εκτίμηση Παραμέτρων, Προβλεπτική Ικανότητα, Ασσυμετρία, Κυρτότητα, Κύρτωση

Contents

1 Introduction

1.1 Problem Description

Financial derivatives are one of the three main categories of financial instruments, the other being equity, like stocks/shares and debt, like bonds and mortgages.

The derivatives in the last 50 years have become increasingly important in finance, and the derivatives market is huge – in 2020 the derivative's gross market value (based on the contracts) is estimated at 11,6 trillion dollars, while the low estimate of the notional value (based on the strike price) is estimated at 558,5 trillion dollars, with a high estimate of 1 quadrillion dollars.

Examples of financial derivatives are future contracts, forward contracts, warrants, swaps and options.

There are two types of options: a *call option* gives the holder the right but not the obligation to buy the underlying asset, while a *put option* gives the holder the right but not the obligation to sell the underlying asset. The fact that the buyer of the right is not obligated to exercise the right at the end of the contract, is what distinguishes options from forwards and futures. For this reason, there is a cost for these rights – the *option price* or *premium*.

There are many pricing models to calculate the option price, such as closed form models like the Black-Scholes formula, and lattice models (trees), like the Binomial Options Pricing model. While these two models are very popular, they have certain assumptions and limitations that do not apply to the real market world – for this reason there are many expansions to these models. For example, there are trees with more than one state variables, trees that improve the tree's convergence, trees with a titl parameter, and trees that take in account the skewness and kurtosis of the underlying asset's returns.

1.2 Historic Overview

The year 1973 is probably the most important year regarding Option Trading, it's the year that the **Black-Scholes model** was published, which provided legitimacy to the **Chicago Board Options Exchange**, which opened on April 26, 1973.

Option trading and various efforts to model the option prices are much older – the oldest option-like instrument was dated back to Ancient Greece when Thales of Miletus predicted - speculated using his astronomy observation skills, that olive presses will be needed at a specific time as he foresaw a good year for olives – olive oil's supply in Ancient Greece was like the crude oil of our times.

So, Thales of Miletus agreed with the olive press owners that he could rent the olive presses at a specific time in the future with a discount – when the olive presses were needed, he exercised his right to rent the presses and he made a profit as he rented the presses out at a much higher price.

In late 17^{th} century in London, the first organized option market traded both puts and calls, which were called refusals. In the United States over the counter option trading dates back to the 19^{th} century.

In 1900 Louis Bachelier in his thesis, Theory of Speculation, used the concept known as Wiener process or Brownian motion to model stock option prices, this model is the foundation of the Black-Scholes model and other financial models.

Bachelier's work was close to the mathematics of the eventual Black-Scholes model, but it had some problems that various authors in the 60s tried to fix.

The Black-Scholes model's name excludes **Robert Merton**'s name, his contribution was as significant as **Fisher Black**'s and **Myron Scholes**'.

The model was first introduced in the paper *The Pricing of Options and Corporate Liabilities* by Black and Scholes in 1973, Merton presented his contribution to the model in the paper titled *Theory of Rational Option Pricing* – in this paper he named the model "Black-Scholes Model". It was particularly innovative because the option price was determined by known parameters.

1.3 Binomial Historic Overview

In 1979, Cox, Ross and Rubinstein developed the **Binomial Options Pricing Model**, which was published on their paper titled *Option pricing: A simplified approach*. The model introduces the binomial tree that models the movement of the underlying asset's price. This model is considered the dominant model in option valuation, as long as the option price is determined by known parameters. This model does not include speculative opportunities. The model's key assumption is that the price of the underlying asset will either go up or down. These those two states have two probabilities, q and 1-q which are the risk-neutral probabilities.

The Jarrow and Rudd extension of the CRR model is known as the Equal Probability Binomial Tree– the up and down moves in their binomial tree have equal probabilities, 50% each.

Boyle in 1988 in his *A lattice framework for option pricing with two state variables* article proposed an extension to the Binomial model developed by Cox, Ross and Rubinstein when there are two underlying state variables, and it is mentioned that the procedure he proposes can be extended to situations that involve a higher number of state variables.

Hull and White in 1990, in their *Valuing derivative securities using the explicit finite difference method* paper introduce a modification to the explicit finite difference method for valuing derivatives.

Kunitomo and Ikeda in 1992 in *Pricing options with curved boundaries* provide a general valuation for European options, whose payoff is restricted by curved boundaries contractually set on the underlying asset price process when it follows Brown's geometric motion.

Leisen and Reimer in 1996 in *Binomial models for option valuation: Examining and improving convergence* compared different tree approaches with respect to their speed and convergence behavior, and prove a general theorem which allows an easy derivation of the order of convergence with particular models under consideration.

Broadie and Detemple in 1996 in *American option valuation: new bounds, approximations, and a comparison of existing methods* developed upper and lower bounds on the prices of American call and put options written on an asset with dividend payments. They provided two approximations of the option price, one based on the lower bound, termed

LBA and one based on both bounds, termed LUBA. The advantage of this valuation is its accuracy compared to the binomial tree in terms of speed, as it provides the accuracy of a 1000-step binomial tree with the computational speed of a 50-step binomial tree.

In 1997, Heston and Zhou in their article titled *On rate of convergence of discrete-time contingent claims*, describe the speed of convergence of discrete-time multinomial option prices. They show that the smoothness of the option's payoff function affects the rate of the convergence, but it's much lower than what was believed because the option payoff functions are usually of all-or-nothing type and not continuously differentiable. They propose two methods of improving the accuracy: the first method is an adjustment of the discrete-time solution prior to maturity and smoothing of the payoff function which brings solutions that that converge to their continuous-time limit at the maximum possible rate enjoyed by smooth payoff functions. The second method is an intuitive approach that systematically derives multinomial models by matching the moments of a normal distribution.

In 1998, Mark Rubinstein in his *Edgeworth Binomial Trees* article, which is presented in this dissertation, used an Edgeworth expansion to transform the standard binomial density into a unimodal standardized discrete density, in order to take in account the skewness and kurtosis of the underlying asset's returns.

In 1999, Yisong "Sam" Tian developed in his *A flexible binomial option pricing model* article a flexible binomial model with a tilt parameter, which alters the shape of the CRR binomial tree, which is symmetric. The tilt parameter λ, shifts the tree upward when it is positive and downward when it is negative. The rate of the convergence is improved as the binomial tree can be recalibrated through the tilt parameter, to position the nodes relative to the strike price of the option, or even the barrier, if exists.

S.G. Kou in 2003, in his *On pricing of discrete barrier options* article discusses the barrier options, which are options that are activated or extinguished when the underlying asset's price crosses a certain level. The pricing of these options is difficult, and Kou extends a 1997 approach by Broadie, Glasserman and Kou to barrier discretion covering most cases and providing an easier proof.

In the same year, Walsh's *The rate of convergence of the binomial tree scheme* article studies the detailed convergence of the binomial tree. As the scheme is first order, Walsh shows the exact constants and finds that it is possible to modify Richardson extrapolation to get a method of order three-halves. The delta which is used in hedging converges at the same rate, and this is proven by embedding the tree scheme in the Black-Scholes diffusion model by means of Skorokhod embedding. Walsh also points that this technique can apply to more general cases.

In 2004, Francine and Mark Diener in their *Asymptotic of the Price Oscillations of a European Call Option in a Tree Model* article compute the oscillatory behavior of the binomial tree model using asymptotics of Laplace integrals, giving explicitly the first terms of the asymptotics.

The 2006 article *Smooth convergence in the binomial model* by Chang and Palmer adds an additional parameter to the binomial model, the parameter λ, and they show that the binomial price of a European call converges to the Black-Scholes price with a factor of 1/n and they give a formula for the factor of 1/n in the error extension. They demonstrated that convergence is smooth in Tian's (1999) flexible model and to the new center binomial model they propose, by making specific options for λ.

Mark S. Joshi in 2007 in his *Achieving Higher Order Convergence for the Prices of European Options in Binomial Trees* article introduces another family of binomial trees. In Joshi's family of trees, he demonstrates the presence of complete asymptotic expansions at the costs of European straightforward options and estimates and the initial three conditions. In other explicit cases, a tree with third sequence convergence is fabricated and the notion of Leisen and Reimer (1996) demonstrates that their tree has second-order convergence.

1.4 Thesis Description

The purpose of this thesis is to present the Edgeworth Binomial Trees approach which was presented by Mark Rubinstein in 1998.

One of the most significant differences among the pricing models lies in their specification of the higher order moments of the risk-neutral distribution.

The Black-Scholes-Merton formula assumes that the standardized risk-neutral distribution of the logarithm of the underlying asset's returns is **normal** with **zero skewness** and **kurtosis 3**.

Jarrow and Rudd in 1982 developed a way of approximating the present value of a derivative using an Edgeworth expansion allowing different values of skewness and kurtosis.

Mark Rubinstein's Edgeworth Binomial Trees (1998) simplifies their approach by applying an Edgeworth Expansion to the discretized risk-neutral probabilities, in order to valuate European and American derivatives with risk-neutral returns that have **non-zero skewness** and **greater than 3 kurtosis**.

In Chapter 3 we show how we transform the standarized binomial density into an Edgeworth Density, how the Edgeworth Density affects the probabilities of the binomial tree and therefore the option prices, and how we construct the Edgeworth Binomial Tree.

Our empirical study in Chapter 4 will compare this model to the Cox-Ross-Rubinstein model using real market data of options with an underlying asset that matches the aforementioned skewness and kurtosis pair. Finally in the last chapter we have the conclusions, followed by the references and the Matlab algorithms used in the empirical study.

2 Option Theory and Binomial Trees

The *Edgeworth Binomial Trees* is a method for evaluating the *option price*. Presented in this chapter is the basic theory about options, option pricing and Binomial Trees.

2.1 Option Theory

Option are one of the most widespread types of financial derivatives. Options are traded both on exchanges and over-the-counter market (OTC). The underlying assets can be stocks, bonds, interest rates, market indices and other.

There are two types of options:

A **call option** gives the buyer of the option the **right to buy** the underlying asset at a specific date for a specific price.

A **put option** gives the buyer the option the **right to sell** the underlying asset at a specific date for a specific price.

The seller of the call option has the obligation to sell the underlying asset if the buyer exercises the right, and the seller of the put option has the obligation to buy the underlying asset if the buyer exercises the right. For this reason, there is a certain cost of buying an option.

The **buyer** of the option has the **long position**, and the **seller** of the option has the **short position**.

The date that is specified in the contract is named *expiration date* or *maturity date*. The price that is specified in the contract is named *strike price* or *exercise price*.

The price of the option is named *option price* (call price / put price) or *premium*.

Depending of when the option buyer can exercise the right, there are two main styles of options: *European options* which can be exercised only on the expiration date, and *American options* which can be exercised at any time up to the expiration date. There are also other Exotic option styles, such as the Bermudan option which can be exercised on specific dates on or before expiration, and Asian options which the payoff is determined by the average of the underlying asset price, over a specific period.

Summing up, the main **attributes** of an option are:

1. The underlying asset of the option
2. The current price of the underlying asset
3. The Strike Price
4. The type (call or put)
5. The maturity and the style (European/American)
6. The position (long or short)
7. The premium

2.1.1 Option Pricing

The premium – the option price – is very complex to calculate – there are plenty of pricing models that are used and can be used:

- Analytic models like the Black-Scholes formula
- Lattice models like the Binomial option pricing model
- Monte Carlo methods
- Finite difference methods

and others. The **Edgeworth Binomial Tree** which is the theme of this thesis, is an expansion of the original Cox Ross Rubinstein Binomial option pricing model.

The option price is affected by six factors:

1. The current stock price S_0
2. The strike price K
3. The time to maturity T
4. The volatility of the stock price σ
5. The risk-free rate ρ
6. and the dividends that are expected to be paid

The effects of these factors to the option price are shown in the next table.

	European Call	European Put	American Call	American Put
Current Stock Price	+	-	+	-
Strike Price	-	+	-	+
Time to Maturity	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Amount of future Dividends	-	+	-	+

Table 1 - Effects of Factors to Option Prices

The + indicates that when a variable increases, the premium also increases or stays the same, while the – indicates that when that variable increases, the premium decreases (or stays the same). The ? means that the relationship is uncertain.

2.1.2 Option Positions

In every contract there are two sides: One investor has to take the long position, to buy the option and one investor has to take the short position – to sell the option. Adding these sides to the Call and Put options, there are 4 main option positions:

1. **Long Call** (buying an option to buy the underlying asset)
2. **Short Call** (selling an option to sell the underlying asset)
3. **Long Put** (buying an option to sell the underlying asset)
4. **Short Put** (selling an option to buy the underlying asset)

The seller of the option has a profit upfront (the premium) but may face liabilities later, and the buyer of the option has losses upfront, but has potential profits in the future.

The payoff is calculated as follows (c = call premium, p = put premium):

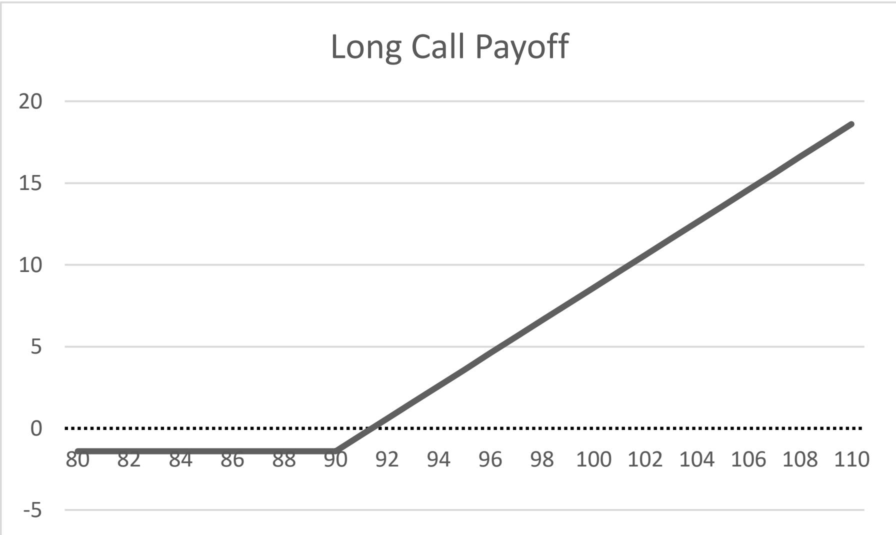
Long Call Payoff: $\max(S_0 - K, 0) - c$

Short Call Payoff: $-\max(S_0 - K, 0) + c = \min(K - S_0, 0) + c$

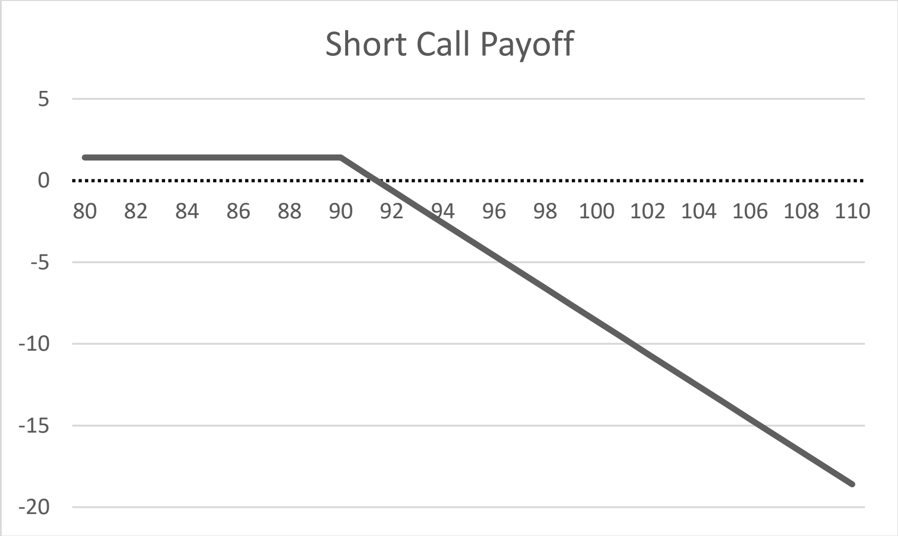
Long Put Payoff: $\max(K - S_0\ ,0) - p$

Short Put Payoff: $-\max(K - S_0\ ,0) + p = \min(S_0 - K, 0) + p$

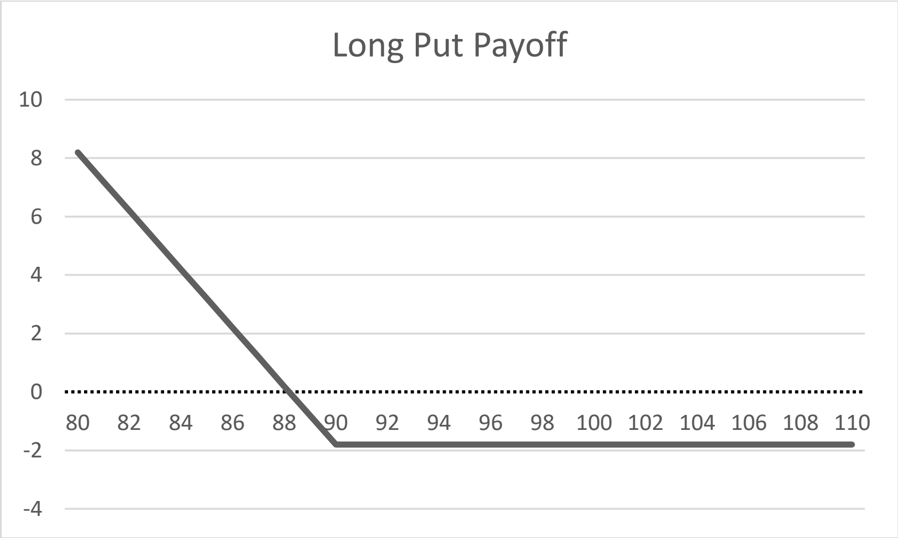
For an option with Strike Price 90, call premium 1,4 and put premium 1,8, we have the following four payoff charts:



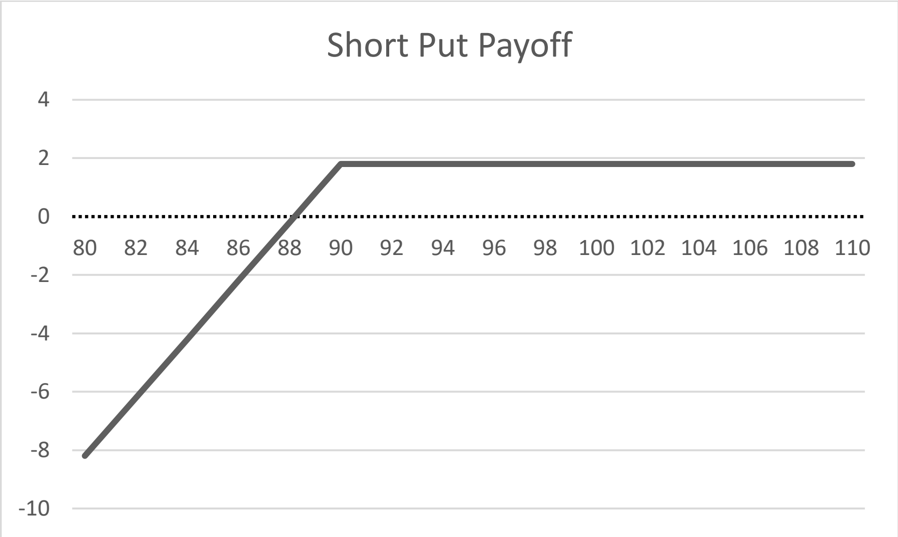
Graph 1 - Long Call Payoff



Graph 2 - Short Call Payoff



Graph 3 - Long Put Payoff



Graph 4 - Short Put Payoff

It is apparent from the payoff charts that the profit of the Long Short position is the loss of the Short Long position, and vice versa. Same for the Long Put and Short Put positions.

If S is the stock price and K is the strike price, options are referred as in the money, at the money or out of the money based on the following chart:

	Call Option	**Put Option**
In the Money	S>K	S<K
At the Money	S=K	S=K
Out of the Money	S<K	S>K

Table 2 - Moneyness

This means that the option is exercised only when it is **In the Money**. This option classification will be included in the sample used in Chapter 4, the empirical study.

An investor may choose one of the aforementioned 4 positions based on his assessment of the market. If for example wants take a position on a stock with the strike price equal to the current price, for simplicity's sake:

- If he estimates that the price of the stock will rise a lot by the maturity date, he may take a long position on a call. That way he will profit a lot from the rise and in the worst-case scenario, he will have a set loss – the call premium.
- If he estimates that the price of the stock will rise but not much by the maturity date, he may take a short position on a put. This way he will profit instantly from the put premium that will receive, but if the price fells and the buyer of the option exercises the right, his losses may be much more than if he had a long call position
- If he estimates that the price of the stock will drop a lot, he can take a long put position. This way he'll profit a lot from the drop and in the event of the stock rising above the strike price, he will have a set loss
- If he estimates that the price of the stock will slightly drop, he can take a short call position. He will have an instant profit from the call premium, but he has the risk of the stock rising a lot, which in turn bring him a lot of loses

In addition to these 4 basic strategies involving the 4 main positions, there a lot of different and more complicated strategies involving more or less risk – a trading strategy that involves taking positions in two or more options of the same type (for example two or more calls) is named Spread strategy, while the strategies that involve combinations of options on the same stock are named straddles, strips, straps and strangles.

For example, if an investor believes that the stock will move a lot from the strike price K, but doesn't know if the price will rise or fall, he can take a straddle position, which is a long call on K and a long Put also on K.

Regarding the spreads, a **bull spread** can be created by buying a call (or put) with a low strike price and selling a call (or put) with a high strike price, while a **bear spread** can be created by buying a put (or call) with a high strike price and selling a put (or call) with a low strike price.

A **butterfly spread** involves buying calls (or puts) with a low and high strike price and selling two calls (or puts) with an intermediate strike price. A **calendar spread** involves selling a call (or put) with a short time to maturity and buying a call (or put) with a longer time to maturity.

Regarding the combinations, a **straddle** combination involves taking a long position in a call and a long position in a put with the same strike price and time to maturity. A **strip** consists of a long position in one call and two puts with the same strike price and time to maturity. A **strap** consists of a long position in two calls and one put with the same strike price and time to maturity, while a **strangle** consists of a long position in a call and a put with different strike prices and the same time to maturity.

In the Option Market, usually these strategies are available as a single item with reduced transaction costs comparing to the investor building the strategy himself.

2.1.3 Black Scholes Merton Option Pricing Model

The first model for pricing derivatives is the Black-Scholes-Merton model (BSM model).

It uses five variables-inputs to determine the fair price of an option:

1. Volatility of returns of the underlying asset (**σ**)
2. Price of the underlying asset (**S_0**)
3. Strike price (**K**)
4. Risk free rate (**r**)
5. Time to maturity (**ttm**)

This model has some assumptions:

1. The underlying asset **does not pay any dividends** or returns
2. The option can be only **exercised** on its **maturity** date
3. The underlying asset's prices follow a **lognormal distribution**, and as the prices cannot take a negative value, they are bounded by zero
4. The price of the underlying asset follows a **random walk**
5. There are no transaction costs
6. The interest rates are constant and hence **risk-free**.
7. The stock price returns are **normally distributed**
8. There are **no arbitrage** opportunities

The Black-Scholes-Merton formulas for European Call (c) and Put (p) options are the following:

$$c = S_0 N(d1) - Ke^{-rT} N(d2)$$

$$p = Ke^{-rT} N(-d2) - S_0 N(-d1)$$

With

$$d1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

And

$$d2 = d1 - \sigma\sqrt{T}$$

The N(x) in the call and put option formulas is the cumulative probability distribution function for a variable with a standard normal distribution.

2.2 Binomial Trees

A popular and useful technique for pricing an option involves the construction of a **binomial tree**. A binomial tree is a diagram that represents the different possible paths that may be followed by the stock price over the life of an option. There is the assumption that the stock price follows a random walk, and in each time steps there is a probability of the stock price moving up by a certain percentage amount, and a probability of moving down by another certain percentage amount.

As the time steps become smaller, the model is the same as the Black-Scholes-Merton model.

2.2.1 One Step Binomial Tree

We will assume that the derivatives are priced using the no-arbitrage argument: the price of the derivative is set at the same level as the value of the replicating portfolio, so that no trader can make a risk-free profit by buying one and selling the other. With that in mind, we will consider a stock with

- Stock price S_0
- Option price f
- Time to Maturity T

During the life of the option, the price can either

- move **up** from S_0 to S_0d where u>1, or
- move **down** from S_0 to S_0d where d<1.

So, if the stock moves up the payoff from the option is f_u and if the stock moves down the payoff is f_d.

With that data, we can construct the simple one step tree.

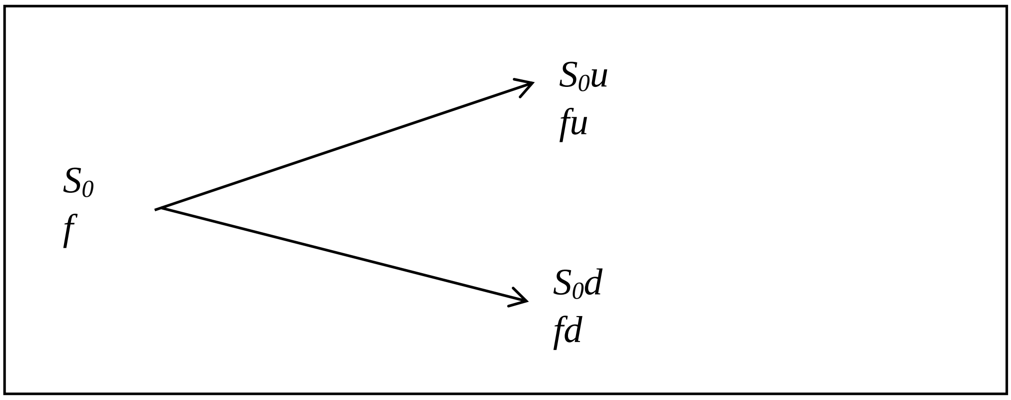


Figure 1 - One Step Binomial Tree

We'll also consider a portfolio consisting of a long position in Δ shares, and a short position in one option. We'll have to calculate the value of Δ that makes the portfolio risk-free.

If the stock price moves up, the value of the portfolio at the end of the life of the option is

$$S_0 u\Delta - f_u$$

If the stock price moves down, the value of the portfolio becomes

$$S_0 d\Delta - f_d$$

These two portfolios are equal when

$$S_0 u\Delta - f_u = S_0 d\Delta - f_d$$

Or

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

And in this case, the portfolio is riskless and because there is no arbitrage, it can earn the risk-free rate which we will denote by r.

This Δ, the delta, is the number of units of the stock we should hold for each option shorted, in order to create the riskless portfolio.

The present value of the portfolio is

$$(S_0 u\Delta - f_u)e^{-rT}$$

with a cost of setting the portfolio of

$$S_0\Delta - f$$

Following, we have

$$S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT}$$

Which becomes

$$f = S_0\Delta(1 - ue^{-rT}) + f_u e^{-rT}$$

If we substitute from the $\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$ equation, we have

$$f = \frac{S_0(f_u - f_d)}{S_0 u - S_0 d}(1 - ue^{-rT}) + f_u e^{-rT}$$

Which becomes

$$f = \frac{f_u(1 - de^{-rt}) + f_d(ue^{-rT} - 1)}{u - d}$$

And finally becomes

$$f = e^{-rT}[pf_u + (1 - p)f_d$$

With

$$p = \frac{e^{rT} - d}{u - d}$$

As p we denote the probability of the stock going up, while (1-p) is the probability of the stock going down.

2.2.2 Two Step Binomial Tree

We can extend the one step binomial tree to a two-step binomial tree with the same assumptions as before.

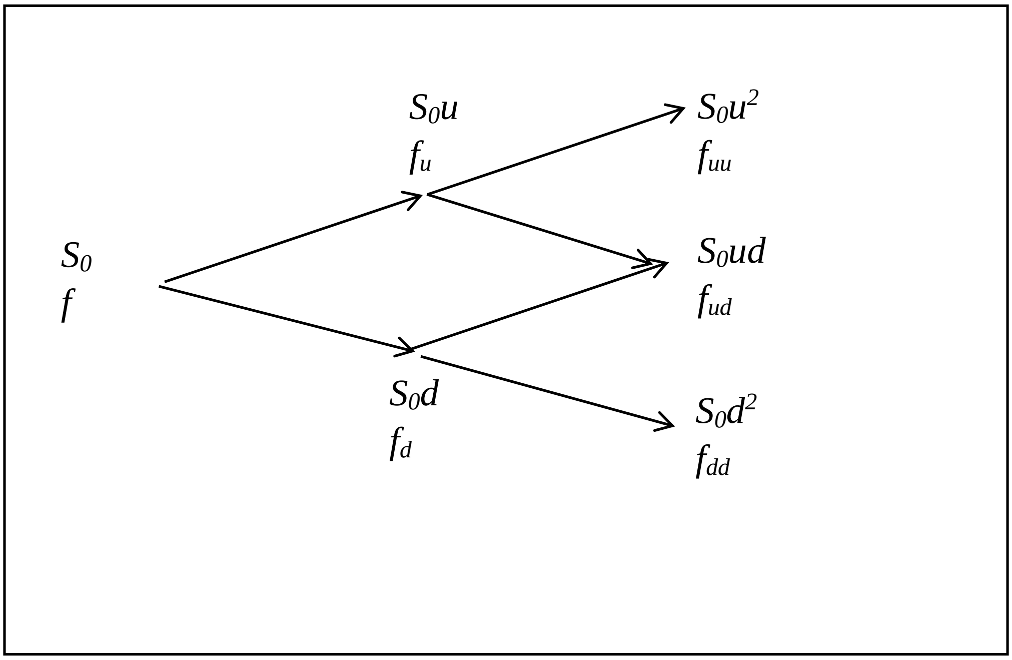


Figure 2 - 2 Step Binomial Tree

The stock price is initially S_0, and during each time step it either moves up *u* times its value or moves down *d* times its value.

For example, after two up movements the value becomes $S_0*u*u = S_0*u^2$

or after two down movements the value becomes $S_0*d*d = S_0*d^2$

or after one up and one down movement the value becomes S_0*u*d.

Because instead of the time to maturity T we now have a time step Δt, the equations of the one step binomial tree become:

$$f = e^{-r\Delta\tau}[pf_u + (1-p)f_d$$

With

$$p = \frac{e^{r\Delta\tau} - d}{u - d}$$

If we transform the option price equation to get f_u and f_d, we have:

$$f_u = e^{-r\Delta\tau}[pf_{uu} + (1-p)f_{ud}$$

$$f_d = e^{-r\Delta\tau}[pf_{ud} + (1-p)f_{dd}$$

$$f = e^{-r\Delta\tau}[pf_u + (1-p)f_d$$

and if we substitute the first two equations into the last, we get

$$f = e^{-r\Delta\tau}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}]$$

Which is consistent with the principle of the risk-neutral valuation. The variables p^2, 2p(1-p) and $(1-p)^2$ are the probabilities of the upper, middle and lower final nodes of the two-step binomial tree.

As we add more steps to the binomial tree, the risk-neutral valuation still holds, and the option price is always equal to its expected payoff discounted at the risk-free rate.

In subsection 2.1.1 we mentioned the factors that affect the price of the option. All of them appear as is in the above equations – the volatility appears in the form of the up and down movements (*u* and *d*), and it will explained in detail in subsection 2.2.4.

The implied volatility of an option is the value of the volatility that when it is the input of an option pricing model such as the Black-Scholes model, it returns a theoretical value equal to the market price of that option.

2.2.3 Implied Binomial Trees

While the Black-Scholes formula and the original CRR Binomial Trees are the most popular models for calculating the option price, they have certain limitations and disadvantages. One of their assumptions is that the volatility of the underlying asset is constant for the duration of the option.

Mark Rubinstein in his 1994 article “Implied Binomial Trees” develops a method for inferring risk-neutral probabilities.

In order to create the implied binomial tree, there must be a prior guess of the risk-neutral probabilities, for example an average of some Black-Scholes implied volatilities of some at the money call options.

We then denote the nodal underlying asset prices at the end of the tree from lowest to highest by S_j for j = 0,…n, and also denote the ending nodal derived risk-neutral probabilities by P_j where S_j*P_j=1.

For example, if p' is the risk-neutral probability of an up move over each step, then $P_j' \frac{n!}{j!(n-j)!} * p'^j (1-p')^{n-j}$.

For a *n* large enough, the probability distribution will be approximately lognormal. The solution of the implied binomial tree will be described in the next chapter, as the Edgeworth Binomial Trees are an expansion of the Implied Binomial Trees.

2.2.4 Volatility in Binomial Trees

The three parameters that are necessary to construct a binomial tree with time step Δt are u, d, and p. When we specify u and d, we must choose a p so that the expected return is the risk-free rate r.

We showed before that

$$p = \frac{e^{r\Delta\tau} - d}{u - d}$$

And the parameters u and d must be chosen to match the volatility of the stock or any other underlying asset.

The volatility of the stock σ is defined so that the standard deviation of its return in a short period of time Δt is $\sigma\sqrt{\Delta t}$. The variance of the return in time Δt is $\sigma^2 \Delta t$.

The variance of a variable X is defined as $E(X^2) - [E(X)]^2$, where E is the expected value.

During a time step of length Δt, there is a probability that the stock will provide a return of u-1 and a probability 1-p that it will provide a return of d-1.

Therefore, the stock's volatility is matched if

$$p(u-1)^2 + (1-p)(d-1)^2 - [(p(u-1) + (1-p)(d-1)]^2 = \sigma^2 \Delta\tau$$

If we substitute p from the first equation, the above equation simplifies to

$$e^{r\Delta t}(u+d) - ud - e^{2r\Delta t} = \sigma^2 \Delta\tau$$

A solution to this equation if we ignore higher powers of Δt is:

$$u = e^{\sigma\sqrt{\Delta t}}$$

and

$$d = e^{-\sigma\sqrt{\Delta t}}$$

The values of u and d match the volatility in the risk-neutral world. The formulas are the same if we match the volatility of the real world.

3 Edgeworth Binomial Trees

In this chapter we will explain the steps to create the Edgeworth Binomial Tree, which is an expansion of the Implied Volatility Trees.

3.1 Edgeworth Densities

In order to create an Edgeworth Density, we will first create a standardized binomial density b(x).

If the number of points is **n+1**, then at each **point j=0,…,n** the random variable x equals $\frac{(2j)-n}{\sqrt{n}}$

with associated probability $b(x) = \frac{n!}{j!(n-j)!} * \left(\frac{1}{2}\right)^n$

For example,

If **n=1** then x equals **-1 or 1** with equal probability **½.**

If n=4 then x equals -2, -1, 0, 1 or 2 with probabilities 1/16, 1/4, 3/8, 1/4 and 1/16 respectively.

This distribution has a mean of zero and variance of 1.

Given prespecified skewness ξ and kurtosis **κ**, we can transform this standardized binomial density b(x) into f(x) which is an approximately standardized density with the desired skewness and kurtosis, which we will refer as **Edgeworth Density**.

So

$$f(x) = b(x) * [1 + \frac{1}{6}\xi(x^3 - 3x) + \frac{1}{24}(\kappa - 3)(x^4 - 6x^2 + 3) + \frac{1}{72}\xi^2(x^6 - 15x^4 + 45x^2 - 15)]$$

If skewness is zero and kurtosis is 3, $f(x) = b(x)$.

Because this expansion is only an approximation, we have to rescale the probabilities so they sum to 1, replacing $f(x_j)$ with $f(x_j)/\sum_j f(x_j)$.

Then with this rescaled density, we calculate its mean

$$M \equiv \sum_j f(x_j)\, x_j$$

and its variance around that mean

$$V^2 \equiv \sum_j f(x_j)\,(x_j - M)^2$$

Finally, we replace the x_j with the standardized zero mean, standard deviation one random variable $\frac{(x_j)-M}{V}$.

This continues to leave the variables (x_j) equally spaced. The resulting skewness and kurtosis will approximate the target levels, but with a larger **n** the approximation improves.

In the following figure we can get an idea of the flexibility of the Edgeworth density comparing to the standardized binomial density:

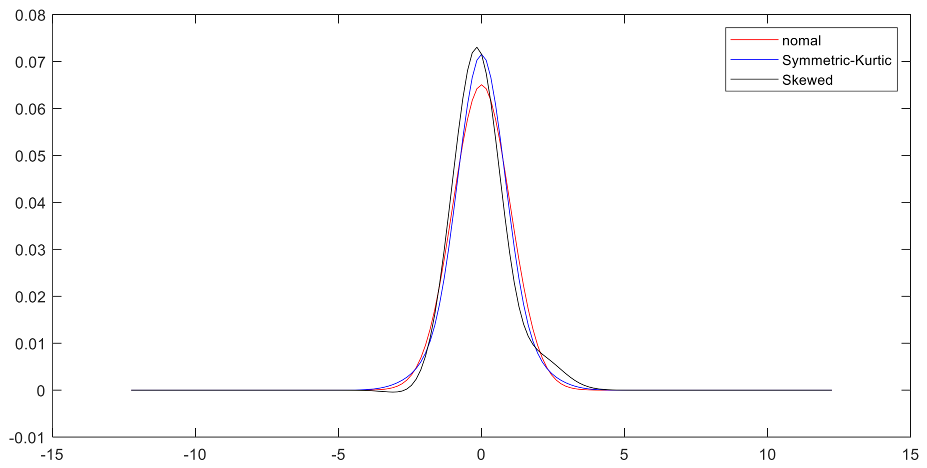


Figure 3 - Comparison of densities

Normal	ξ=0	κ=3
Symetric-Kurtic	ξ=0	κ=3.8
Right-Skewed	**ξ=-0.86**	**k=3.62**

The Right-Skewed example is chosen based on the returns of an S&P500 company that we'll use in our numerical analysis.

3.2 Edgeworth Densities and Option Values

The application of the Edgeworth densities to the risk-neutral distribution for European derivatives valuation requires the standardized distribution to be transformed to have the risk-neutral mean and standard deviation.

We will denote

$$P_j \equiv f(x_j)$$

as the risk-neutral probability associated with price S_j on expiration date of the underlying asset.

S_j is constructed from x_j from the following equation:

$$S_j = Se^{\mu t+\sigma\sqrt{t*x_j}}$$

and

$$\left(\frac{r}{d}\right)^t = \sum_j P_j\left(\frac{S_j}{S}\right)$$

where

S ≡ current price of the underlying asset

r ≡ annualized risk free rate

d ≡ annualized payout return of the underlying asset

t ≡ time to maturity in years

μ ≡ annualized risk neutral expectation of the logarithm of $\left(\frac{S_j}{S}\right)$

σ ≡ annualized risk neutral volatility of the logarithm of $\left(\frac{S_j}{S}\right)$

By replacing S_j in the second equation, we have:

$$\left(\frac{r}{d}\right)^t = \sum_j P_j S e^{\mu t+\sigma\sqrt{t*x_j}} = \left(\sum_j P_j S e^{\sigma\sqrt{t*x_j}}\right) e^{\mu t}$$

$$log\left(\frac{r}{d}\right)^t = \log\left(\sum_j P_j S e^{\sigma\sqrt{t*x_j}}\right) + \mu t$$

$$\mu = \log\left(\frac{r}{d}\right) - \log(\frac{\Sigma_j P_j S e^{\sigma\sqrt{t*x_j}}}{t})$$

This formula for μ is similar to the one which is commonly used if Sj/S conforms to a risk-neutral lognormal distribution.

Now the discrete density P_j for j = 0, …, n defined on the points of the S_j now has the mean, the standard deviation, and approximately the desired skewness and kurtosis that we want. To value European calls with striking prices Ki, i = 1, …, m, we have to calculate:

$$C(K_i) = \Sigma_j * P_j * \max(0, \Sigma_j - K_i)/r^t$$

3.3 Constructing the Edgeworth Binomial Tree

There is a way to modify the binomial tree model while keeping its main advantages which are:

- Binomial price moves
- Recombining nodes
- Ending nodal values organized from lowest to highest
- Constant risk-free and payout returns and
- All paths leading to the same ending node **having the same risk-neutral probability**

The last point means that if you stand at a node at the end of the binomial tree and try to go backwards, there will be many paths from the beginning of the tree to that node, and each of these paths **has the same probability**.

However, the modified binomial tree is different from the standard tree in an important way – the move sizes doesn't have to be constant.

It allows the local volatility of the underlying asset return to vary with changes in the underlying asset price and time.

In addition, it can be shown that given the ending risk-neutral distribution, the riskless and payout returns, and with the above assumptions, there exists a unique consistent binomial tree, which preserves the property that there are no arbitrage opportunities in the interior of the implied tree (all risk-neutral move probabilities, although they may be different at each node, are non-negative).

The steps to construct the Edgeworth Binomial Tree are the following:

Step Zero

We start at the end of the tree where there is a node corresponding from each S_j, for j=1, 2, …, n. To each of these nods there is a risk-neutral probability P_j.

We calculate the risk neutral probability of a single path to that node:

$$P = \frac{P_j}{\frac{n!}{j!\,(n-j)!}}$$

The two adjacent ending nodes have as path probability the nodal value pairs (P^+,S^+) and (P^-,S^-), so the pair (P,S) will be the pair prior to that node.

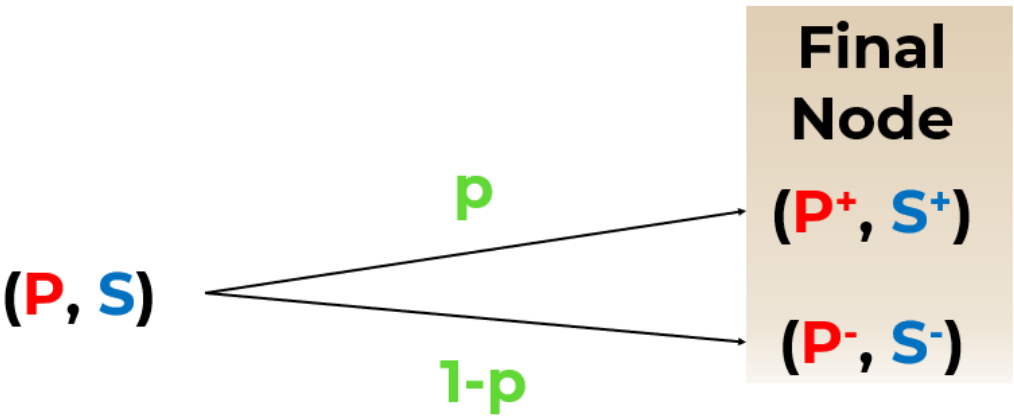


Figure 4 - Construction of the Edgeworth Binomiaal Tree Nodes from the end of the tree

Step One

The path probability of the previous node P must equal to the sum of P^+ and P^-.

This is because having arrived at the previous node, there are two moves that may happen, one ending at S^+ with probability P^+ and one ending at S^- with probability P^-.

Step Two

With that in mind, the probability of moving to S^+ (moving up) must be $p = \frac{P^+}{P}$, and so the probability of moving to S^- (moving down) must be $1 - p = \frac{P^-}{P}$.

Step Three

Taking in account the previous steps, S must be the risk-neutral expectation of S^- and S^+ discounted at the risk-free return over the period with a correction for payouts.

Under the assumption of constant riskless and payout returns, then the single-move riskless return is $r \equiv r^{\frac{t}{n}}$ and the single-move payout return is $\delta \equiv d^{\frac{t}{n}}$

With those steps we can fill the binomial tree from the expiration step to one step earlier than the expiration. The same procedure is done working backwards in order to fill the entire tree.

3.4 Valuation of the Edgeworth Tree

Now that we have constructed the Edgeworth Binomial tree, we can value derivatives.

The value of a European call is the risk neutral expected payoff discounted back to the present, with the risk-free rate, as the European call can be exercised only in maturity.

For American options, a tree is needed to infer the current value of options with earlier maturities in a way that is consistent with the values of the longer maturity options. For example, the current values of three-month options can be calculated from the underlying asset nodal values and probabilities half-way through the six-month option tree.

Then the option payoff is evaluated using these three-month underlying asset values weighted by their associated risk neutral probabilities, and discounted back to the present using the risk-free rate for 3 months.

4 Empirical Study

In this chapter will perform an empirical study by using real market data. We will compare 2 Lattice models, the CRR model and the Edgeworth Model.

Regarding the Edgeworth model, we will check the performance of the model estimating only the volatility, and then volatility, skewness and kurtosis.

CRR	Edgeworth 1	Edgeworth 2
σ	σ	σ, ξ, κ

Table 3 - Models and their Parameters

Summary of the steps:

1) We will obtain the data from our source (Refinitiv Datastream) by creating the tickers, as we'll explain in the next chapter
2) Using the Matlab algorithms, we will input the data and will compute the point estimation of our parameters
3) Regarding the In-sample efficiency of the models, we'll compare the models' residuals in order to see which performs best
4) Regarding the Out-of-sample efficiency, we'll use the estimated parameters to asses the forecasting ability of the models.

4.1 Data of the Empirical Study

For the purpose of our empirical study, we obtained **daily** American Option data from **Refinitiv Datastream**, with the **Meta Platforms (META)** stock as underlying asset for a period of 5 months: from March 24, 2022 to August 24, 2022.

Meta was chosen because it's an S&P500 American company with significant Market Value, and with more than 10000 options available in the database. Also, there is no stock split which would need the obtained data to be adjusted to that split.

The 5 year's returns of the stock also have a desired skewness and kurtosis:

Meta Stock Returns **Skewness**: -0,86253

Meta Stock Returns **Kurtosis**: 3,6231

These two values will be later used as inputs in our Edgeworth 1 model.

For each day, in the **Mixed Maturity sample** we got 9 different options with different **moneyness** and different **maturity**.

Of these 9 options, 3 of them have a strike price near the current stock price (**At the Money**), 3 have a strike price more than the current price (**Out of the Money**) and 3 have a strike price below the current price (**In the Money**). Regarding the maturities, there are 3 options with short-term maturities (1,5 to 2,5 months), 3 with medium-term maturities (4-6 months) and 3 with long-term maturities (close to a year, between 11 and 13 months).

In the Specific Maturity samples, for each maturity (Short-Term: 1,5-2,5 Months, Medium-Term: 4-6 Months and Long-Term: 11-13 Months), we got one option with At the Money strike price, 4 with Out of the Money strike price (in increments of 10) and 4 In the Money (also with increments of 10).

An example of the tickers for the first few days can be seen in the following table:

Date	Short Atm	Short Otm	Short ITM	Mid Atm	Mid Otm
24/3/2022	FB$0522220C	FB$0522240C	FB$0522200C	FB$0822220C	FB$0822240C
25/3/2022	FB$0522220C	FB$0522240C	FB$0522200C	FB$0822220C	FB$0822240C
28/3/2022	FB$0522225C	FB$0522245C	FB$0522205C	FB$0822220C	FB$0822240C
29/3/2022	FB$0522230C	FB$0522250C	FB$0522210C	FB$0822230C	FB$0822250C
30/3/2022	FB$0522230C	FB$0522250C	FB$0522210C	FB$0822230C	FB$0822250C
31/3/2022	FB$0522220C	FB$0522240C	FB$0522200C	FB$0822220C	FB$0822240C
1/4/2022	FB$0522225C	FB$0522245C	FB$0522205C	FB$0822220C	FB$0822240C

Date	Mid ITM	Long Atm	Long Otm	Long ITM
24/3/2022	FB$0822200C	FB$0323220C	FB$0323240C	FB$0323200C
25/3/2022	FB$0822200C	FB$0323220C	FB$0323240C	FB$0323200C
28/3/2022	FB$0822200C	FB$0323220C	FB$0323240C	FB$0323200C
29/3/2022	FB$0822210C	FB$0323230C	FB$0323250C	FB$0323210C
30/3/2022	FB$0822210C	FB$0323230C	FB$0323250C	FB$0323210C
31/3/2022	FB$0822200C	FB$0323220C	FB$0323240C	FB$0323200C
1/4/2022	FB$0822200C	FB$0323220C	FB$0323240C	FB$0323200C

Table 4 - Sample of the Tickers

From these tickers we obtained:

1) The Market Price of the Option
2) The Implied Volatility
3) The Maturity date, in order to calculate the Time to Maturity

We also obtained

1) META's stock price
2) United States' 10 Year Treasury Note, to use as our risk-free rate

4.2 Parameters Estimation

From our sample we can compute the point estimation of the parameter of a random variable, such as the volatility. The point estimation of a parameter is a value calculated based on the sample data and represents the actual value of the population relative parameter – a statistic.

We can use the Levenberg Marquardt algorithm which assumes that we have a model and a set of parameters to estimate. The algorithm detects the values of the parameters for which the squares of the differences between the theoretical values of each model, and the actual market values.

The procedure of minimizing square errors is illustrated by the following equation:

$$\theta = \arg\min \sum_{i=0}^{N} (f_i^{market} - f_i^{model})^2$$

θ is the estimated parameter and N is the number of our in-sample days.

The Levenberg-Marquardt algorithm was developed in the early 1960's to solve nonlinear least squares problems.

Least squares problems arise in the context of fitting a parameterized mathematical model to a set of data points by minimizing an objective expressed as the sum of the squares of the errors between the model function and a set of data points. If a model is linear in its parameters, the least squares objective is quadratic in the parameters.

This objective may be minimized with respect to the parameters in one step via the solution to a linear matrix equation. If the fit function is not linear in its parameters, the least squares problem requires an iterative solution algorithm. Such algorithms reduce the sum of the squares of the errors between the model function and the data points through a sequence of well-chosen updates to values of the model parameters.

In Matlab, we can easily use the **lsqnonlin** function to get the estimated parameters as well as the residuals.

Regarding the volatility, we can use as our initial point the average of all the obtained implied volatilities of our options.

In our 3rd model where we have 3 parameters, we will also use the calculated skewness and kurtosis.

Parameter	Initial Value	Lower Bound	Upper Bound
σ (volatility)	0,48	0,2	0,7
ξ (skewness)	-0,86253	-1	-0,1
κ (kurtosis)	3,623176	3,05	7.9

Table 5 - Initial parameters and bounds

4.3 In the Sample Efficiency of the Models

4.3.1 Mixed Maturity Sample

As explained in Section 4.1, in this sample for each day we have 9 different options: one with strike price at the money, one in the money and one out of money – we also have 3 different maturities (short-term, medium-term and long-term).

CRR Model Results

In the CRR model we estimate one parameter, the stock price volatility σ. We use the average of our 990 implied volatilities as a starting point (0,48).

The average value of the daily estimates of σ is 0,4686 with a standard deviation of 0,059.

The minimum estimate of σ is 0,3804 and the maximum estimate of σ is 0,7.

Edgeworth Model 1 Results

In our first Edgeworth Model, we also estimate one parameter, the stock price volatility σ, with the same starting point as before (0,48).

The average value of the daily estimates of σ is 0,5675 with a standard deviation of 0,0565.

The minimum estimate of σ is 0,4659 and the maximum estimate of σ is 0,7.

Edgeworth Model 2 Results

In our second Edgeworth Model, in addition to the stock price volatility **σ**, we also estimate the skewness ξ and the kurtosis **κ**. Starting point for volatility is the same, **0,48**, while the starting point of the skewness is -0,86253 and the starting point of the kurtosis 3,623176. Note that the kurtosis has a lower bound of 3.1, as the kurtosis has to be greater than three. Same for the skewness, the skewness has to be non-zero, so a non-zero value close to the initial value was chosen (-0,2 as the upper bound).

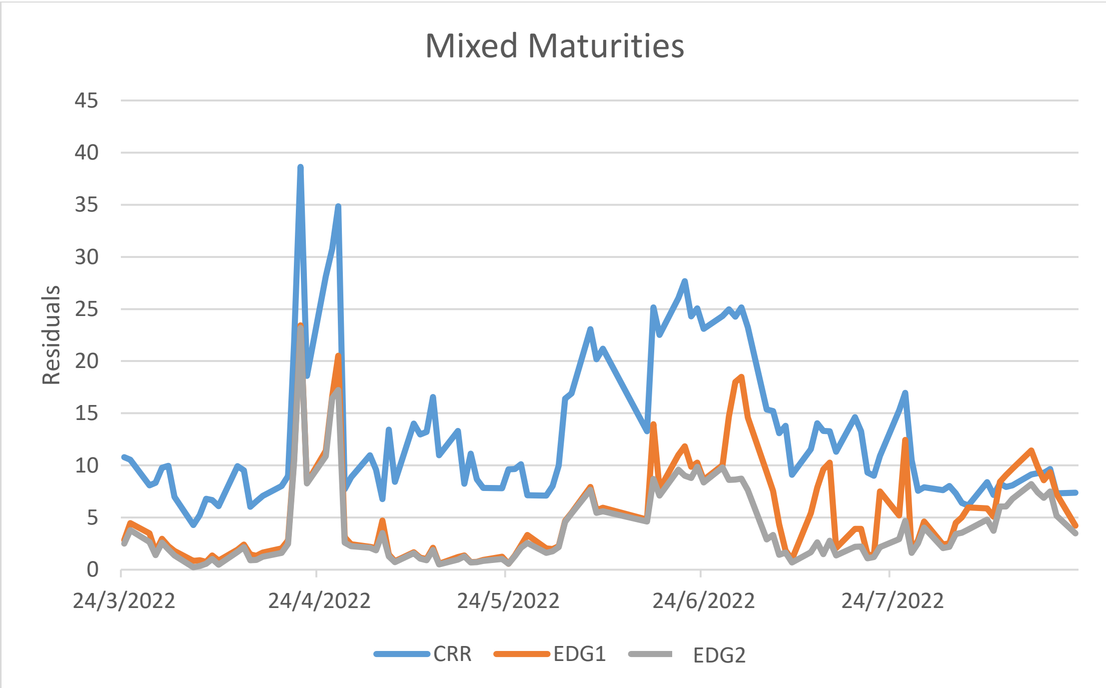
Regarding **volatility** σ, τhe average value of the daily estimates of σ is 0,51763 with a standard deviation of 0,0629.

The minimum estimate of σ is 0,41 and the maximum estimate of σ is 0,7.

Regarding **skewness**, the average value is -0,8365 with a standard deviation of 0,233. The minimum skewness is -1 (the lower bound) and the maximum skewness is -0,1 (the upper bound).

Finally, the average value of **kurtosis** is 3,407 with a standard deviation of 0,60. The minimum kurtosis is 3,05 (the lower bound) and the maximum kurtosis is 5,50.

Total Model Results



Graph 5 - Mixed Maturities In The Sample Performance

As we can see, the 2nd Edgeworth model performs better than the other two models. The 1st Edgeworth model performs better than the CRR model until late August, where it performs similarly to the CRR model.

4.3.2 Short-Term Maturity Sample

For our Short-Term Maturity sample, we'll also use 9 different options for each day -, this time we'll have one At the Money option, 4 In the Money options and 4 Out of Money options.

The maturity of these options we'll be the same for each day, ranging from 1,5 months to 2,5 months.

CRR Model Results

In the CRR model we estimate one parameter, the stock price volatility σ. We use the average of our 990 implied volatilities as a starting point (0,5119).

The average value of the daily estimates of σ is 0,4935 with a standard deviation of 0,08.

The minimum estimate of σ is 0,3917 and the maximum estimate of σ is 0.7.

Edgeworth Model 1 Results

In our first Edgeworth Model, we also estimate one parameter, the stock price volatility σ, with the same starting point as before (0,5119).

The average value of the daily estimates of σ is 0,5797 with a standard deviation of 0,084.

The minimum estimate of σ is 0,4644 and the maximum estimate of σ is 0,7.

Edgeworth Model 2 Results

In our second Edgeworth Model, in addition to the stock price volatility **σ**, we also estimate the skewness **ξ** and the kurtosis **κ**. Starting point for volatility is the same, **0,48**, while the starting point of the skewness is -0,86253 and the starting point of the kurtosis 3,623176. Note that the kurtosis has a lower bound of 3.1, as the kurtosis has to be greater than three. Same for the skewness, the skewness has to be non-zero, so a non-zero value close to the initial value was chosen (-0,2 as the upper bound).

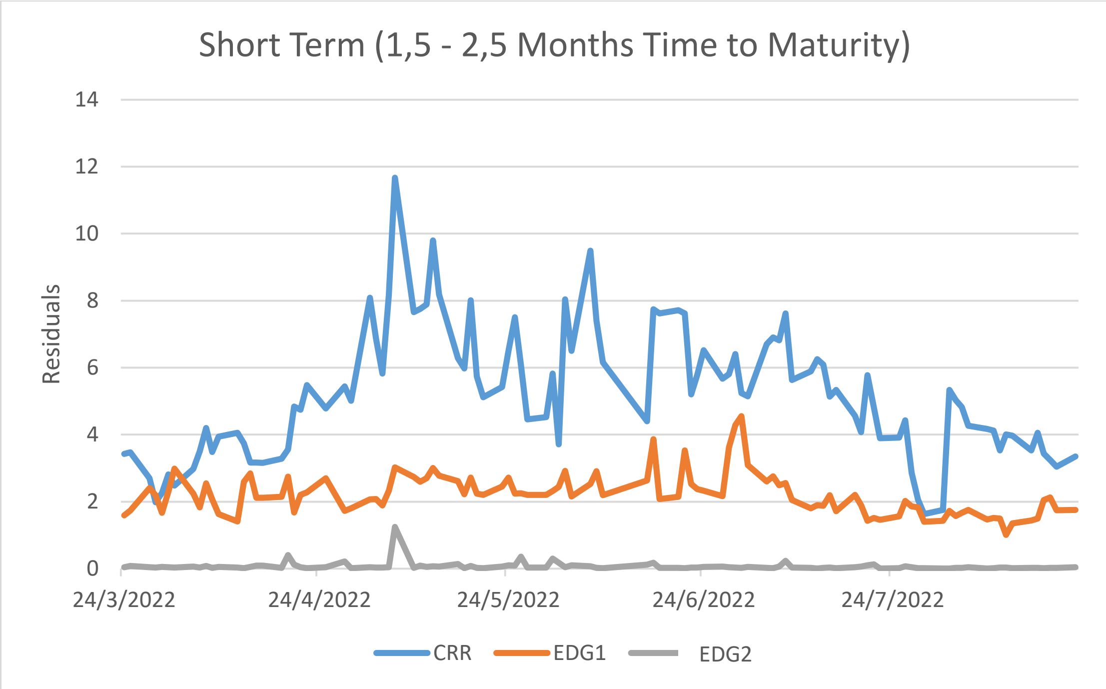
Regarding volatility σ, τhe average value of the daily estimates of σ is 0,5278 with a standard deviation of 0,084.

The minimum estimate of σ is 0,4063 and the maximum estimate of σ is 0,7.

Regarding skewness, the average value is -0.68 with a standard deviation of 0,1446. The minimum skewness is -0,93 and the maximum skewness is -0,1 (the lower bound).

Finally, the average value of kurtosis is 3,7486 with a standard deviation of 0,290. The minimum kurtosis is 3,05 (the lower bound) and the maximum kurtosis is 4,3766.

Total Model Results



Graph 6 - Short-Term Maturities In The Sample Performance

In this short-term sample, the 2nd Edgeworth Model is the best of all three as it has the smaller residuals. The simple 1st Edgeworth model also generally performs better than the simple CRR model.

4.3.3 Medium-Term Maturity Sample

Same as the Short-Term Maturity sample regarding the moneyness of the options, this time the maturity is between 5 months and 7 months.

CRR Model Results

In the CRR model we estimate one parameter, the stock price volatility σ. We use the average of our 990 implied volatilities as a starting point (0,4868).

The average value of the daily estimates of σ is 0,4797 with a standard deviation of 0,062.

The minimum estimate of σ is 0,3809 and the maximum estimate of σ is 0,7.

Edgeworth Model 1 Results

In our first Edgeworth Model, we also estimate one parameter, the stock price volatility σ, with the same starting point as before (0,4868).

The average value of the daily estimates of σ is 0,5772 with a standard deviation of 0,062.

The minimum estimate of σ is 0,4619 and the maximum estimate of σ is 0,7.

Edgeworth Model 2 Results

In our second Edgeworth Model, in addition to the stock price volatility **σ**, we also estimate the skewness **ξ** and the kurtosis **κ**. Starting point for volatility is the same, 0,4868, while the starting point of the skewness is -0,86253 and the starting point of the kurtosis 3,623176.

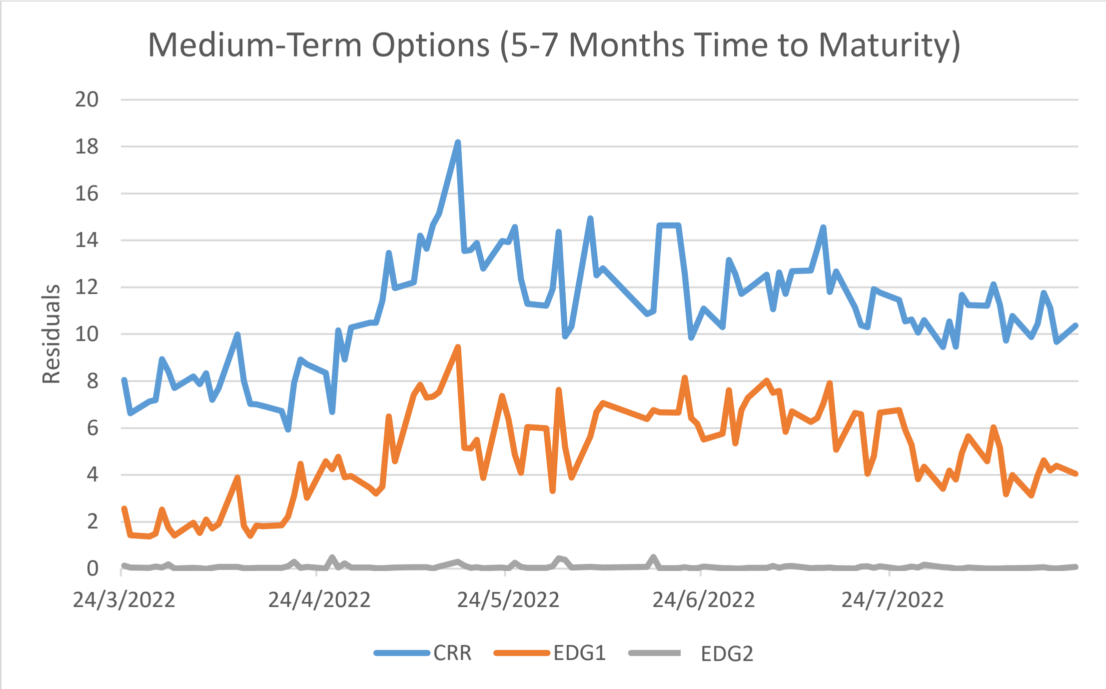
Regarding volatility σ, τhe average value of the daily estimates of σ is 0,5275 with a standard deviation of 0,07.

The minimum estimate of σ is 0,4 and the maximum estimate of σ is 0,7.

Regarding skewness, the average value is -0.8384 with a standard deviation of 0,166. The minimum skewness is -1 (the lower bound) and the maximum skewness is -0,1.

Finally, the average value of kurtosis is 3,69 with a standard deviation of 0,39. The minimum kurtosis is 3,05 (the lower bound) and the maximum kurtosis is 4,35.

Total Model Results



Graph 7 - Medium-Term Maturities In The Sample Performance

Similarly to the Short-Term Options, the 2nd Edgeworth model is the best of the three. In addition to that, the 1st Edgeworth model also performs better than the simple CRR model in the whole sample.

4.3.4 Long-Term Maturity Sample

This time the maturity of the options is between 11 and 13 months. The moneyness of each day's 9 options is the same as Short-Term's and Medium-Term's.

CRR Model Results

In the CRR model we estimate one parameter, the stock price volatility σ. We use the average of our 990 implied volatilities as a starting point (0,456).

The average value of the daily estimates of σ is 0,4572 with a standard deviation of 0,055.

The minimum estimate of σ is 0,369 and the maximum estimate of σ is 0,7.

Edgeworth Model 1 Results

In our first Edgeworth Model, we also estimate one parameter, the stock price volatility σ, with the same starting point as before (0,456).

The average value of the daily estimates of σ is 0,5532 with a standard deviation of 0,0533.

The minimum estimate of σ is 0,4519 and the maximum estimate of σ is 0,7.

Edgeworth Model 2 Results

In our second Edgeworth Model, in addition to the stock price volatility **σ**, we also estimate the skewness **ξ** and the kurtosis **κ**. Starting point for volatility is the same, 0,456, while the starting point of the skewness is -0,86253 and the starting point of the kurtosis 3,623176.

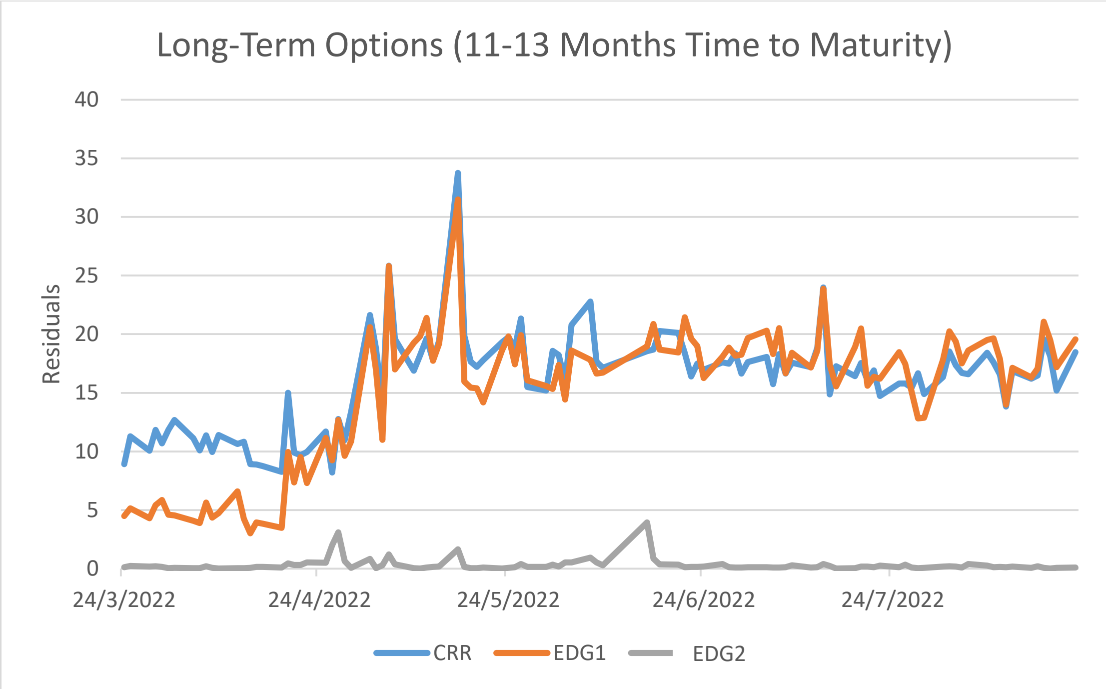
Regarding volatility σ, τhe average value of the daily estimates of σ is 0,52 with a standard deviation of 0,06.

The minimum estimate of σ is 0,407 and the maximum estimate of σ is 0,7.

Regarding skewness, the average value is -0.9204 with a standard deviation of 0,158. The minimum skewness is -1 (the lower bound) and the maximum skewness is -0,1.

As for the average value of kurtosis, it is 3,507 with a standard deviation of 0,35. The minimum kurtosis is 3,05 (the lower bound) and the maximum kurtosis is 4,9.

Total Model Results



Graph 8 - Long-Term Maturities In The Sample Performance

In the Long-Term option sample, while the 2nd Edgeworth model is still the best of the three, the 1st Edgeworth model and the CRR model perform similarly, with approximately the same residuals after the 1st month of the sample.

4.4 Out of the Sample Efficiency of the Models

In order to evaluate the Out of Sample efficiency of the models, we will use the estimated values of each model on a certain day, and then using these values we'll calculate the option price for a period of 15 working days.

In our case, we'll use the May 6, 2022 calculated date (the volatility σ for all 3 models, and skewness and kurtosis for the 2nd Edgeworth model), in order to calculate the option price from May 9, 2022 to May 27, 2022.

Then we'll get the residuals of the difference between the calculated option value, and the market option value.

The calculated inputs are in the following table:

May 6, 2022

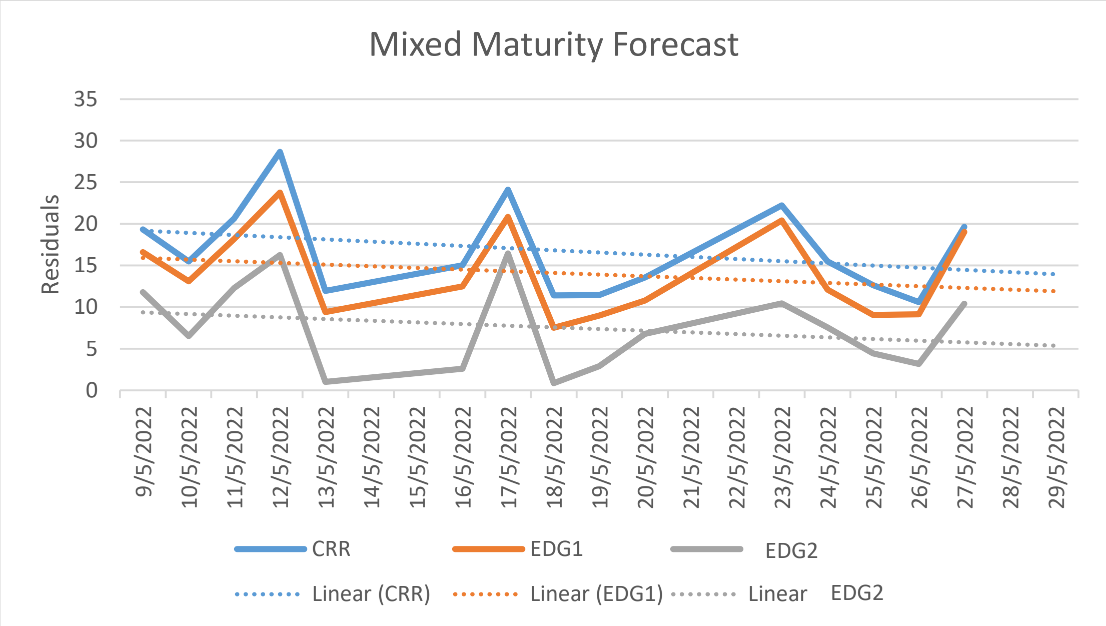
	CRR	Edgeworth 1	Edgeworth Model 2		
Estimated Variable	σ	σ	σ	ξ	κ
Mixed Maturity	0,4494	0,5503	0,4855	-0.80	3,05
Short-Term Maturity	0,4557	0,5398	0,5008	-0,93	4,37
Medium-Term Maturity	0,4631	0,5614	0,4959	-0,81	3,27
Long-Term Maturity	0,4428	0,5405	0,5052	-1	3,36

Table 6 - May 6, 2022 Calculated Inputs

4.4.1 Forecasting Results

In the following charts we can see how the 3 models fared regarding their forecasting ability.

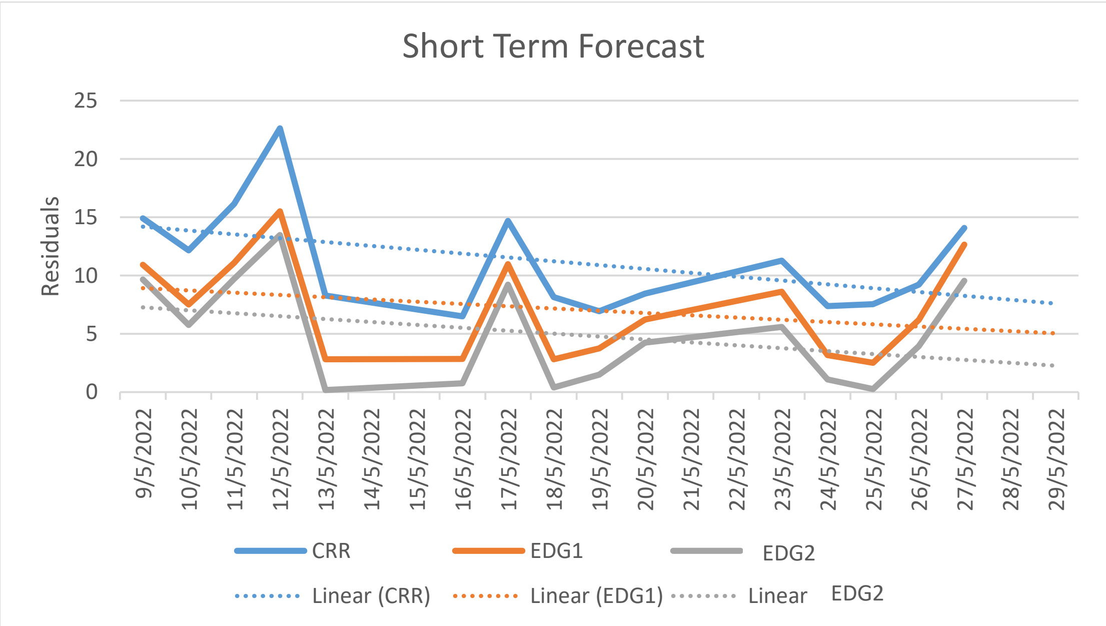
Mixed Maturity



Graph 9 - Mixed Maturity Out of Sample Forecast

In the mixed maturity sample, the 2nd Edgeworth performs better than the other two. The 1st Edgeworth model performs better than CRR but only slightly. The trend of the residual for all models is negative.

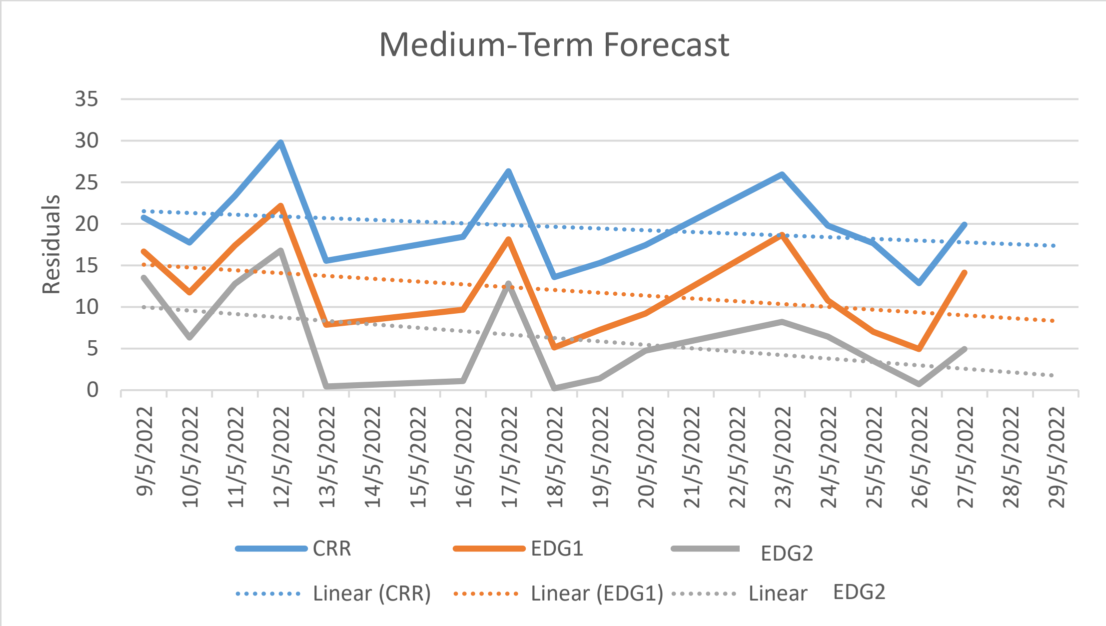
Short-Term Maturity



Graph 10- Short-Term Maturity Out of Sample Forecast

Regarding the options with a time to maturity of 1,5-2,5 months, we have similar results as the Mixed Maturity sample – the 2nd Edgeworth model is better than the other two models, and the 1st Edgeworth model is better than the CRR – the residuals trend is s till negative for all 3 models.

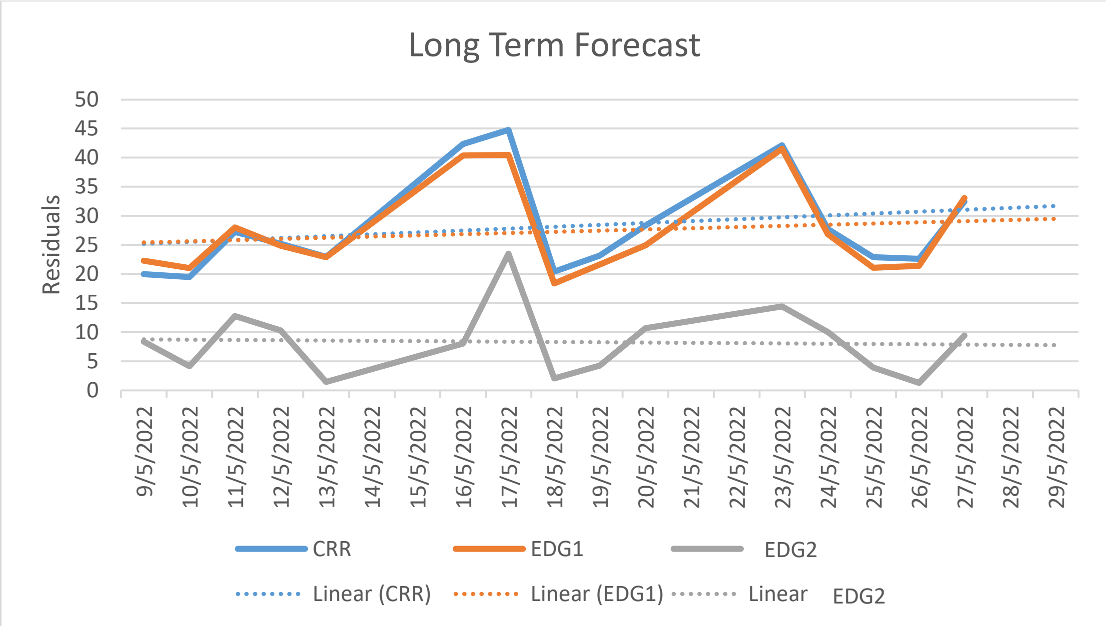
Medium-Term Maturity



Graph 11 - Medium-Term Maturity Out of Sample Forecast

Same as the Mixed Maturity and the Short-Term Maturity, the 2nd Edgeworth model performs better than the other two, and the 1st Edgeworth model also performs better than the CRR model. The trend remains negative for all models.

Long-Term Maturity



Graph 12- Long-Term Maturity Out of Sample Forecast

Finally, regarding the Long-Term Maturity, while the 2nd Edgeworth model outperforms the other two, this time the 1st Edgeworth model and the CRR model perform the same, and while the 2nd Edgeworth Model's trend remains negative, the trend of the other two models is positive.

5 Conclusions

Options are one of the most popular financial instruments, but the calculation of the option's price is very complex – various lattice models were introduced in Chapter 1, and in this thesis, we compared the Edgeworth Binomial Tree to the Cox-Ross-Rubinstein Binomial Tree (CRR Tree).

There were two efficiency tests: One In-The-Sample test to check which model has the least residuals compared to the real market price of the option, and one Out-of-The-Sample test to check the forecasting ability of the models.

In those tests we also split the Edgeworth Binomial Tree into two – the 1st where we compute the point estimation of the stock's volatility, and the 2nd where we compute the point estimations of the stock's volatility, and also the stock return's skewness and kurtosis.

There were also four different samples with different maturities.

In all *In-the-sample* tests the 2nd Edgeworth Binomial Tree performed better than the 1st Edgeworth Model and the CRR Tree model, while the 1st Edgeworth model performed better than the CRR Tree Model in all but the Long-Term maturity sample where the two models perform similarly, as the skewness and kurtosis that we use inputs for the 1st Edgeworth model will not be representative of the values in one year.

Likewise, in all *Out-of-sample* test, the 2nd Edgeworth Binomial Tree performed better than the other two models, with the 1st Edgeworth Model performing better than the CRR in all but the Long-Term maturity sample, where the 1st Edgeworth Model and the CRR model perform the same.

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7 Appendix – Matlab Codes

Creating the Edgeworth Densities

```
function[x, bx]=EdgDenEE(n,ks,kei)
pt=n+1;
for j = 0:n
    ji=j+1;
    z(ji)=((2*j)-n)/sqrt(n);
    prob(ji)=(factorial(n)/    ( factorial(j)*(factorial(n-j))) )
*(1/2)^n;

    ee1=(1/6)* ks * (   (z(ji)^3)  -          (3*z(ji))  );

    ee2=(1/24)*(kei-3)*((z(ji)^4)-(6*(z(ji)^2))+3);

    ee3= (1/72) *(ks^2) *    ((z(ji)^5)      -(10*(z(ji)^3))  +
(15*(z(ji))) );
    ee3t(ji)=ee3;
    fx(ji)=(1+ ee1  +ee2  +ee3)*prob(ji);
    fx2(ji)=(1+ ee1  +ee2 )*prob(ji);
    fxk(ji)=(1+ ee2 )*prob(ji);
end


plot(z, prob, 'r', z, fxk, 'b', z, fx, 'k')
legend('nomal','Symmetric-Kurtic','Skewed')
```

CRR Binomial Tree Calculation

```
function[price, lattice]=LatticeEurCall(S0,K,r,T,sigma,N)
tic
deltaT=T/N;
u=exp(sigma*sqrt(deltaT));
d=1/u;
p=(exp(r*deltaT)-d)/(u-d);
lattice=zeros(N+1,N+1);
for i=1:N
    lattice(i+1,N+1)=max(0,S0*(u^i)*(d^(N-i))-K);
    lattice;
end
for j=N-1:-1:0
    for i=0:j
        lattice(i+1,j+1)=exp(-r*deltaT)*(p*lattice(i+2,j+2)+(1-
p)*lattice(i+1,j+2));
    end
    lattice;
end

price=lattice(1,1);
```

Edgeworth Binomial Tree Calculation

```
function[price, lattice]=LatticeEW(S0,K,r,T,sigma,N,ks,kei)
%megalh for gia graph
n=N;
EC=zeros(N,N);
BigP=zeros(N,N);
SmallP=zeros(N,N);
pt=n+1;
dt=T/n;
for j = 0:n
    ji=j+1;
    z(ji)=((2*j)-n)/sqrt(n);
    prob(ji)=(factorial(n)/    ( factorial(j)*(factorial(n-j))) )
*(1/2)^n;

    ee1=(1/6)* ks * (   (z(ji)^3)  -          (3*z(ji))  );

    ee2=(1/24)*(kei-3)*((z(ji)^4)-(6*(z(ji)^2))+3);

    ee3= (1/72) *(ks^2) *    ((z(ji)^5)      -(10*(z(ji)^3))  +
(15*(z(ji))) );
    ee3t(ji)=ee3;
    fx(ji)=(1+ ee1  +ee2  +ee3)*prob(ji);
    fx2(ji)=(1+ ee1  +ee2 )*prob(ji);
    fxk(ji)=(1+ ee2 )*prob(ji);
end
for j = 1:n+1
    p1(j)=fx(j)/sum(fx);
    SmallP(j,n+1)=p1(j)/prob(j); %Small Probability P-
    BigP(j,n+1)=SmallP(j,n+1); % Big Probability P+
    Fmean(j)=p1(j)*z(j); %Then, using this rescaled density, calculate its
mean M ≡ Σjf(xj)xj
end
FmeanS=sum(Fmean);
for j = 1:n+1
   PyM(j)=p1(j)*((z(j)-FmeanS)^2);
end
V2=sum(PyM); %and its variance around that mean V2 ≡ Σjf(xj)(xj - M)2
for j = 1:n+1
      x(j) = (z(j) - FmeanS) / sqrt(V2);
      Pe(j) = p1(j)*exp(sigma*sqrt(T)*x(j));
end
mu=r-(1/T)*log(sum(Pe));
S=zeros(n+1,n+1);
for j = 1:n+1
    S(j,n+1)=S0*exp(mu*T+sigma*sqrt(T)*x(j));  %page 6 of EBT
end
S;
for j=n:-1:1
    for i=1:j
                BigP(i, j) = BigP(i, j + 1) + BigP(i + 1, j + 1);
                SmallP(i, j) = BigP(i, j + 1) / BigP(i, j);
                S(i, j) = (SmallP(i, j)*S(i, j + 1) + (1.0 - SmallP(i,
j))*S(i + 1, j + 1))*exp(-r*dt);
    end
end

for i=1:n+1
```

```
    EC(i,n+1)=max(S(i,n+1) - K, 0); %European Call
    AC(i,n+1)=max(S(i,n+1) - K, 0); %American Call
    EP(i,n+1)=max(K - S(i,n+1), 0);
    AP(i,n+1)=max(K - S(i,n+1), 0);
end
for j=n:-1:1
    for i=1:j
        Pr=SmallP(i,j);
             EC(i, j) = exp(-r*dt)*(Pr*(EC(i, j + 1)) + (1 - Pr)*(EC(i + 1,
j + 1)));
                   EP(i, j) = exp(-r*dt)*(Pr*(EP(i, j + 1)) + (1 -
Pr)*(EP(i + 1, j + 1)));
                   AC(i, j) = max(S(i, j) - K, exp(-r*dt)*(Pr*(AC(i, j +
1)) + (1 - Pr)*(AC(i + 1, j + 1))));
                   AP(i, j) = max(K - S(i, j), exp(-r*dt)*(Pr*(AP(i, j +
1)) + (1 - Pr)*(AP(i + 1, j + 1))));

    end
end
price=EC(1,1);
```

Calculation of Least Square Errors – CRR

```
function[CRR_lsqd]=CRRLSQD(x)
global S0; %Τρέχουσα τιμή
global Strike; %Τιμή εξάσκησης-Strike Price
global rate; %Εγχώριο επιτόκιο, στην περίπτωση μας USD
global TTM; %Χρόνος για τη λήξη του δικαιώματος
global imp_vol; %Implied volatility- (Τεκμαρτή Μεταβλητότητα)
global mktprice; %αγοραία τιμή του δικαιώματος
global k; %βοηθητική μεταβλητή
global numofoptions;
CRR_lsqd=zeros(1,9);
for j=1:numofoptions
 CRR_lsqd(j)=mktprice(k,j)-
LatticeEurCall(S0(k),Strike(k,j),rate(k),TTM(k,j),x(1),150);
End
```

Calculation of Least Square Errors – Edgeworth 1

```
function[CRR_lsqd]=EDWLSQD(x)
global S0; %Τρέχουσα τιμή
global Strike; %Τιμή εξάσκησης-Strike Price
global rate; %Εγχώριο επιτόκιο, στην περίπτωση μας USD
global TTM; %Χρόνος για τη λήξη του δικαιώματος
global imp_vol; %Implied volatility- (Τεκμαρτή Μεταβλητότητα)
global mktprice; %αγοραία τιμή του δικαιώματος
global k; %βοηθητική μεταβλητή
global kei;
global ks;
global numofoptions;
CRR_lsqd=zeros(1,numofoptions);
for j=1:numofoptions
 CRR_lsqd(j)=mktprice(k,j)-
LatticeEW(S0(k),Strike(k,j),rate(k),TTM(k,j),x(1),150,ks,kei);
end
```

Calculation of Least Square Errors – Edgeworth 3

```
function[CRR_lsqd]=EDWLSQD(x)
global S0; %Τρέχουσα τιμή
global Strike; %Τιμή εξάσκησης-Strike Price
global rate; %Εγχώριο επιτόκιο, στην περίπτωση μας USD
global TTM; %Χρόνος για τη λήξη του δικαιώματος
global imp_vol; %Implied volatility- (Τεκμαρτή Μεταβλητότητα)
global mktprice; %αγοραία τιμή του δικαιώματος
global k; %βοηθητική μεταβλητή
global kei
global ks
global numofoptions;
CRR_lsqd=zeros(1,numofoptions);
for j=1:numofoptions
 CRR_lsqd(j)=mktprice(k,j)-
LatticeEW(S0(k),Strike(k,j),rate(k),TTM(k,j),x(1),150,x(2),x(3));
End
```

In-Sample Residual Calculation – CRR

```
function[x,resnorm,residual,exitflag]=EdgResidualsCRR(~)
clear all
global S0; %Τρέχουσα τιμή
global Strike; %Τιμή εξάσκησης-Strike Price
global rate; %Εγχώριο επιτόκιο, στην περίπτωση μας USD
global TTM; %Χρόνος για τη λήξη του δικαιώματος
global imp_vol; %Implied volatility- (Τεκμαρτή Μεταβλητότητα)
global mktprice; %αγοραία τιμή του δικαιώματος
global k; %βοηθητική μεταβλητή
global kei
global ks
global numofoptions
numofoptions=9;
kei=3.623176; %Kurtosis
ks=-0.86253; %Skewness
S0=zeros(95);
Strike=zeros(95,numofoptions);
rate=zeros(95);
TTM=zeros(95,numofoptions);
imp_vol=zeros(95,numofoptions);
mktprice=zeros(95,numofoptions);
parameter=zeros(95,3);
res=zeros(95,1);
exit=zeros(95,1);
%Εισάγω τις τιμές
S0=xlsread('META_Options.xlsx','Price','B4:B112');
Strike=xlsread('META_Options.xlsx','Strike','B4:J112');
rate=xlsread('META_Options.xlsx','rate','C4:C112');
TTM=xlsread('META_Options.xlsx','ttm','B4:J112');
imp_vol=xlsread('META_Options.xlsx','impvol','B4:J112');
mktprice=xlsread('META_Options.xlsx','marketprice','B4:J112');
CRR_put_matrix=zeros(95,9);
for i=1:108 %108
 %Θέτω αρχική τιμή στην παράμετρο, που θέλω να εκτιμήσω. Διαλέγω την
 % πιο ρεαλιστική των δεδομένων (μέσος των implied volatilities)
 x0=[0.48];
 lb=[0.001];
 ub=[2];
```

```
 k=i;
 [x,resnorm,residual,exitflag]=lsqnonlin(@CRRLSQD,x0,lb,ub);
 %parameter(i)=x;
 parameter(i,:)=x;
 res(i)=resnorm;
 exit(i)=exitflag;
 %Αποθηκεύω τις τιμές για κάθε ένα από τα 9 δικαιώματα με την
 %εκτιμώμενη παράμετρο στο μοντέλο
        for j=1:numofoptions

EdgeCallMatrix(i,j)=LatticeEurCall(S0(i),Strike(i,j),rate(i),TTM(i,j),x(1),
150);
        end
 pricedata=(EdgeCallMatrix);
xlswrite('META_Options.xlsx',pricedata,'CRR_Vol','D4:L112');
xlswrite('META_Options.xlsx',res,'CRR_Vol','B4:B112');
xlswrite('META_Options.xlsx',parameter,'CRR_Vol','A4:A112');
end
```

In-Sample Residual Calculation – Edgeworth 1

```
function[x,resnorm,residual,exitflag]=EdgResiduals_Edg_Vol(~)
clear all
global S0; %Τρέχουσα τιμή
global Strike; %Τιμή εξάσκησης-Strike Price
global rate; %Εγχώριο επιτόκιο, στην περίπτωση μας USD
global TTM; %Χρόνος για τη λήξη του δικαιώματος
global imp_vol; %Implied volatility- (Τεκμαρτή Μεταβλητότητα)
global mktprice; %αγοραία τιμή του δικαιώματος
global k; %βοηθητική μεταβλητή
global kei
global ks
global numofoptions
numofoptions=9;
kei=3.623176; %Kurtosis
ks=-0.86253; %Skewness
S0=zeros(95);
Strike=zeros(95,numofoptions);
rate=zeros(95);
TTM=zeros(95,numofoptions);
imp_vol=zeros(95,numofoptions);
mktprice=zeros(95,numofoptions);
parameter=zeros(95,3);
res=zeros(95,1);
exit=zeros(95,1);
%Εισάγω τις τιμές
S0=xlsread('META_Options.xlsx','Price','B4:B112');
Strike=xlsread('META_Options.xlsx','Strike','B4:J112');
rate=xlsread('META_Options.xlsx','rate','C4:C112');
TTM=xlsread('META_Options.xlsx','ttm','B4:J112');
imp_vol=xlsread('META_Options.xlsx','impvol','B4:J112');
mktprice=xlsread('META_Options.xlsx','marketprice','B4:J112');
CRR_put_matrix=zeros(95,9);
for i=1:108 %108
 %Θέτω αρχική τιμή στην παράμετρο, που θέλω να εκτιμήσω. Διαλέγω την
 % πιο ρεαλιστική των δεδομένων (μέσος των implied volatilities)
 x0=[0.48]; %Meta: [0.48,-0.86253,3.623176]
 lb=[0.001];
```

```
 ub=[2];
 k=i;
 [x,resnorm,residual,exitflag]=lsqnonlin(@EDWLSQD,x0,lb,ub);
 parameter(i,:)=x;
 res(i)=resnorm;
 exit(i)=exitflag;
 %Αποθηκεύω τις τιμές για κάθε ένα από τα 9 δικαιώματα με την
 %εκτιμώμενη παράμετρο στο μοντέλο
        for j=1:numofoptions

EdgeCallMatrix(i,j)=LatticeEW(S0(i),Strike(i,j),rate(i),TTM(i,j),x(1),150,k
s,kei);
        end
 pricedata=(EdgeCallMatrix);
xlswrite('META_Options.xlsx',pricedata,'2_EDG_Vol','D4:L112');
xlswrite('META_Options.xlsx',res,'2_EDG_Vol','B4:B112');
xlswrite('META_Options.xlsx',parameter,'2_EDG_Vol','A4:A112');
end
```

In-Sample Residual Calculation – Edgeworth 3

```
function[x,resnorm,residual,exitflag]=EdgResiduals_Edg_Vol_Sk_Kurt(~)
clear all
global S0; %Τρέχουσα τιμή
global Strike; %Τιμή εξάσκησης-Strike Price
global rate; %Εγχώριο επιτόκιο, στην περίπτωση μας USD
global TTM; %Χρόνος για τη λήξη του δικαιώματος
global imp_vol; %Implied volatility- (Τεκμαρτή Μεταβλητότητα)
global mktprice; %αγοραία τιμή του δικαιώματος
global k; %βοηθητική μεταβλητή
global kei
global ks
global numofoptions
numofoptions=9;
kei=3.623176; %Kurtosis
ks=-0.86253; %Skewness
S0=zeros(95);
Strike=zeros(95,numofoptions);
rate=zeros(95);
TTM=zeros(95,numofoptions);
imp_vol=zeros(95,numofoptions);
mktprice=zeros(95,numofoptions);
parameter=zeros(95,3);
res=zeros(95,1);
exit=zeros(95,1);
%Εισάγω τις τιμές
S0=xlsread('META_Options.xlsx','Price','B4:B112');
Strike=xlsread('META_Options.xlsx','Strike','B4:J112');
rate=xlsread('META_Options.xlsx','rate','C4:C112');
TTM=xlsread('META_Options.xlsx','ttm','B4:J112');
imp_vol=xlsread('META_Options.xlsx','impvol','B4:J112');
mktprice=xlsread('META_Options.xlsx','marketprice','B4:J112');
CRR_put_matrix=zeros(95,9);
for i=1:108 %108
 %Θέτω αρχική τιμή στην παράμετρο, που θέλω να εκτιμήσω. Διαλέγω την
 % πιο ρεαλιστική των δεδομένων (μέσος των implied volatilities)
 x0=[0.48,-0.86253,3.623176];
 lb=[0.001,-1,3.1];
```

```
 ub=[2,-0.2,inf];
 k=i;
 [x,resnorm,residual,exitflag]=lsqnonlin(@EDWLSQD3,x0,lb,ub);
 parameter(i,:)=x;
 res(i)=resnorm;
 exit(i)=exitflag;
 %Αποθηκεύω τις τιμές για κάθε ένα από τα 9 δικαιώματα με την
 %εκτιμώμενη παράμετρο στο μοντέλο
        for j=1:numofoptions

EdgeCallMatrix(i,j)=LatticeEW(S0(i),Strike(i,j),rate(i),TTM(i,j),x(1),150,x
(2),x(3));
        end
 pricedata=(EdgeCallMatrix);
xlswrite('META_Options.xlsx',pricedata,'3_EDG_Vol_3','G4:P112');
xlswrite('META_Options.xlsx',res,'3_EDG_Vol_3','E4:E112');
xlswrite('META_Options.xlsx',parameter,'3_EDG_Vol_3','A4:C112');
end
```

Out-of-Sample Forecasting Calculation – CRR

```
function[pricedata,pricedata_squares,res]=forecastCRR(~)
clear all
sigma=xlsread('META_Options.xlsx','1_CRR_Vol','A35'); % Sigma Τελευταίας
ημέρας
S0=zeros(15);
Strike=zeros(15,9);
rate=zeros(15);
TTM=zeros(15,9);
mktprice=zeros(15,9);
%Εισάγω τις τιμές
S0=xlsread('META_Options.xlsx','Price','B36:B50');
Strike=xlsread('META_Options.xlsx','Strike','B36:J50');
rate=xlsread('META_Options.xlsx','rate','C36:C50');
TTM=xlsread('META_Options.xlsx','ttm','B36:J50');
mktprice=xlsread('META_Options.xlsx','marketprice','B36:J50');
res=zeros(15,1);
CRR_call_matrix=zeros(15,9);
CRR_call_matrix_squares=zeros(15,9);
for i=1:15
 for j=1:9
 CRR_call_matrix(i,j)=abs(mktprice(i,j)-
LatticeEurCall(S0(i),Strike(i,j),rate(i),TTM(i,j),sigma,150));
 CRR_call_matrix_squares(i,j)=(abs(mktprice(i,j)-
LatticeEurCall(S0(i),Strike(i,j),rate(i),TTM(i,j),sigma,150))^2);
 end
 pricedata=[CRR_call_matrix];
pricedata_squares=[CRR_call_matrix_squares];
end
for i=1:15
 res(i)=(sum(CRR_call_matrix_squares(i,:)));
end
xlswrite('META_Options.xlsx',pricedata,'7_forecastCRR','B2:M16');
xlswrite('META_Options.xlsx',pricedata_squares,'7_forecastCRR','B18:M32');
xlswrite('META_Options.xlsx',res,'7_forecastCRR','B34:B48');
End
```

Out-of-Sample Forecasting Calculation – Edgeworth 1

```
function[pricedata,pricedata_squares,res]=forecastEDG1(~)
clear all
sigma=xlsread('META_Options.xlsx','2_EDG_Vol','A35'); % Sigma τελευταίας
ημέρας
kei=3.623366; %Kurtosis default
ks=-0.86253; %Skewness default
S0=zeros(15);
Strike=zeros(15,9);
rate=zeros(15);
TTM=zeros(15,9);
mktprice=zeros(15,9);
%Εισάγω τις τιμές
S0=xlsread('META_Options.xlsx','Price','B36:B50');
Strike=xlsread('META_Options.xlsx','Strike','B36:J50');
rate=xlsread('META_Options.xlsx','rate','C36:C50');
TTM=xlsread('META_Options.xlsx','ttm','B36:J50');
mktprice=xlsread('META_Options.xlsx','marketprice','B36:J50');
res=zeros(15,1);
EDW_call_matrix=zeros(15,9);
EDW_call_matrix_squares=zeros(15,9);
for i=1:15
 for j=1:9
 EDW_call_matrix(i,j)=abs(mktprice(i,j)-
LatticeEW(S0(i),Strike(i,j),rate(i),TTM(i,j),sigma,150,ks,kei));
 EDW_call_matrix_squares(i,j)=(abs(mktprice(i,j)-
LatticeEW(S0(i),Strike(i,j),rate(i),TTM(i,j),sigma,150,ks,kei))^2);
 end
 pricedata=[EDW_call_matrix];
pricedata_squares=[EDW_call_matrix_squares];
end
for i=1:15
 res(i)=(sum(EDW_call_matrix_squares(i,:)));
end
xlswrite('META_Options.xlsx',pricedata,'8_forecastEDG1','B2:M16');
xlswrite('META_Options.xlsx',pricedata_squares,'8_forecastEDG1','B18:M32');
xlswrite('META_Options.xlsx',res,'8_forecastEDG1','B34:B48');
end
```

Out-of-Sample Forecasting Calculation – Edgeworth 3

```
function[pricedata,pricedata_squares,res]=forecastEDG3(~)
clear all
sigma=xlsread('META_Options.xlsx','3_EDG_Vol_3','A35'); % Sigma Τελευταίας
Ημέρας
kei=xlsread('META_Options.xlsx','3_EDG_Vol_3','C35') %Kurtosis Τελευταίας
Ημέρας
ks=xlsread('META_Options.xlsx','3_EDG_Vol_3','B35') %Skewness Τελευταίας
Ημέρας
S0=zeros(15);
Strike=zeros(15,9);
rate=zeros(15);
TTM=zeros(15,9);
mktprice=zeros(15,9);
%Εισάγω τις τιμές
S0=xlsread('META_Options.xlsx','Price','B36:B50');
Strike=xlsread('META_Options.xlsx','Strike','B36:J50');
rate=xlsread('META_Options.xlsx','rate','C36:C50');
```

```
TTM=xlsread('META_Options.xlsx','ttm','B36:J50');
mktprice=xlsread('META_Options.xlsx','marketprice','B36:J50');
res=zeros(15,1);
EDW_call_matrix=zeros(15,9);
EDW_call_matrix_squares=zeros(15,9);
for i=1:15
 for j=1:9
 EDW_call_matrix(i,j)=abs(mktprice(i,j)-
LatticeEW(S0(i),Strike(i,j),rate(i),TTM(i,j),sigma,150,ks,kei));
 EDW_call_matrix_squares(i,j)=(abs(mktprice(i,j)-
LatticeEW(S0(i),Strike(i,j),rate(i),TTM(i,j),sigma,150,ks,kei))^2);
 end
 pricedata=[EDW_call_matrix];
pricedata_squares=[EDW_call_matrix_squares];
end
for i=1:15
 res(i)=(sum(EDW_call_matrix_squares(i,:)));
end
xlswrite('META_Options.xlsx',pricedata,'9_forecastEDG3','B2:M16');
xlswrite('META_Options.xlsx',pricedata_squares,'9_forecastEDG3','B18:M32');
xlswrite('META_Options.xlsx',res,'9_forecastEDG3','B34:B48');
end
```