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Essays in Auction Theory
Applications in Corporate Finance

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Declaration of Authorship

I, *Lamprini Zarpala*, declare that this thesis “*Essays in Auction Theory: Applications in Corporate Finance*” and the work presented in it are my own and has been generated by me as the result of my own original research.

I confirm that:

- (i) This work was done wholly while in candidature for a research degree at this University;
- (ii) I have not submitted any part of this thesis for a degree or any other qualification at this University or any other institution;
- (iii) Where I have consulted the published work of others, this is always clearly attributed;
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- (v) I have acknowledged all main sources of help;
- (vi) Part of this work has been awarded in 2020 the 1st prize for its contribution to game-theoretic analysis by the academic journal *Games (Mdpi)*.
- (vii) Part of this work has been published in the academic journal *Annals of Finance(Springer)* in 2021.

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Abstract

This Ph.D. thesis is a collection of three publishable papers with a significant contribution to auction theory and its corporate finance applications. Each chapter of this thesis addresses each paper separately, apart from the *Chapter 1*.

In *Chapter 1*, we introduce and explain some essential auction-theoretic toolkits for the following chapters' analysis and discuss some seminal papers we used as a grounding base of our research.

Chapter 2 investigates how uniform-price auctions can be used for the pricing of corporate bonds. In order to have a realistic approach, we incorporate the investment mandates and the budget limits on the bidding strategies under the presence of a secondary market. To our knowledge, we are the first to address all these factors in one model and result to some interesting results presented both in section 2.4 and *Chapter 5*.

The next chapters examine the auction designs of portfolio trading. In *Chapter 3* we propose a two-stage auction design for the pricing of blind portfolios - a basket of unknown securities auctioned at a pre-determined execution price. The mechanism allows information release at the interim and this feature not only can reduce the asset managers' liquidation costs but also eliminate brokers' "winners' curse".

In *Chapter 4* we design a two-stage auction for a divisible portfolio auctioned either as parts or as a whole. Again the two-round setup admits an information release at the interim, which may offset the "winner's curse". Our main novelty is a new pricing rule for a core-selecting auction that extends the current Nearest-VCG in a dynamic two-round setup and can mitigate free-riding incentives.

The last chapter summarizes the results from previous chapters and concludes.

*To my mom and dad who always encouraged me to go on every adventure,
especially this one.*

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Chapter 1

An Overview on Auction Theory

1.1 Auction Basics

1.1.1 Introduction

The best description for an auction is the term market mechanism, which determines an outcome based on the information and answers to who will acquire the items and at what price. This market mechanism operates under specific rules, especially in environments where the market price is hard to be set [Krishna, 2010; Mochón and Sáez, 2015].

The history of auctions goes back to antiquity. Herodotus reports that auctions were used in Babylon as early as 500 b.c. for women on marriageable age and slaves [McNeal, 1988]. In recent years, auctions have become universal in the sense that they may be used for art paintings, bonds' issuance, spectrum rights, and auctions etc. [Krishna, 2010].

Both the evolution of auction theory¹ and the operational research have contributed to more sophisticated auction designs, which promote the efficiency in allocations and competitiveness in the seller's revenue. Additionally, auctions could be characterized as market barometers on the market price when the latter is hard to be determined [Mochón and Sáez, 2015].

¹Begun from William Vickrey [Vickrey, 1961] whose work was initially unrecognized.

1.1.2 Valuations, Bids and Prices

A seller auctions an object because he is uncertain about the object's *value* i.e., the maximum price that the bidders are willing to pay for it. Each bidder's valuation for the object can be different from the amount offered i.e., *bid*. [Krishna, 2010; Mochón and Sáez, 2015].

When the bidder makes a bid equal to his valuation, then he follows a *sincere strategy*. Any bidding below the valuation is called *underbidding*, while bidding above the valuation is an *overbidding*.

In the context of bidders' valuations, their preferences can be classified as follows:

- *Private Valuation*. It is a situation where no information can change the valuation. This means that no bidder knows the values of others, and any knowledge for others' would not change how much the object worths for him. Private valuations are associated with the "personal use" with no intentions to resale the object in order to gain profits.
- *Interdependent Valuation*. The valuations of other rivals may affect his valuation. If the bidder has the resale option, his valuation may change if the signals he receives show that other bidders estimate more or less the object. The term refers only to the structure of the values, and how these are affected by the signals of others - it does not refer to any statistical properties of this information.
- *Common Valuation*. It is a special case of interdependence, and it refers to the case of the value that derives from the market price of the item, which is unknown at the time of the auction. It is often called the *mineral rights model* [Milgrom and Weber, 1982] to illustrate that bidders might have different estimates on the amount of recoverable ore, its quality, and the prices that will based on private information, yet the final value is the same for all bidders. This means that a bidder's estimate might change if he could know the other bidders' estimates, as all of them are trying to estimate the same underlying value of the object.

1.2 Standard Single-Unit Auctions

Single-unit auctions are auctions where one seller sells a single item or object to one out of many bidders. There are two types concerning the revelation of price quotes:

- **Open auctions:** Bids are required to be public and adjustable. After some time, the auction matches buyers and sellers and determines the final price. Such auctions are considered the English auction, the clock auction, and the dutch auction. The analysis of those formats is beyond the scope of this thesis, yet the reader may find further information on Mochón and Sáez [2015].
- **Sealed-Bid auctions :** The bids are private and are opened simultaneously. The auctioneer receives the bid and rearranges them in a decreasing/ increasing order. If the format is the **first-price sealed-bid auction**, then the higher/lower bidder acquires the item and pays the seller his bid. Another format is the **second-price sealed-bid auction** (aka a Vickrey auction), in which the bidder pays only the second-highest/lowest bid price. Each format is analyzed in the following subsection.

In the subsection 1.1.2 we explain the notion of independent private and common valuation. In the following chapters, we focus on common-value auctions; hence we shed more light on those auctions. In a **common-value auction**, the bidders will take into account each other's information, and their value estimates will coincide to a single joint distribution. We shall use the concept of Bayesian Nash equilibrium to predict each bidder's strategic behavior, where all bidders have a common prior distribution of their valuation.

1.2.1 First-Price Sealed-Bid Auctions

Each bidder submits one bid in a first-price sealed-bid auction but does not learn about other bidders' bids. The highest/lowest bidder pays the price equal to the winning bid and acquires the item. A bidder's strategy is his bid as a function of his value. A bidder's bid should be large(low) enough to win the second-highest (lowest) bid. In this type of auction, no dominant strategy exists, in the sense that bidders tend to

bid below their values because a bid equal to their value implies zero surplus [Bichler, 2017; Mochón and Sáez, 2015].

If the values are considered as private information and only a common prior distribution with values exists, then a Bayesian Nash equilibrium might be a complex solution. Bidder's dilemma is between bidding high/low and win more often or bidding low/high and benefit from winning if it occurs. Also, the Bayesian Nash equilibrium strategy (explicitly defined in subsection 1.3.1) depends on risk aversion and the prior distribution [Bichler, 2017]. In particular, risk aversion causes an increase in equilibrium bids, since bidders buy insurance against the probability of losing [Krishna, 2010]. Thus, in the first-price sealed-bid auctions, the Bayesian Nash equilibrium is less robust to mistakes than the equilibrium of the second-price sealed-bid auction [Bichler, 2017]. In this thesis, we focus only on risk-neutral bidders. We apply the rule of first-price sealed-bid auction in Chapter 3 and Chapter 4.

1.2.2 Second-Price Sealed-Bid Auctions

Similar to the previous subsection 1.2.1, in the second-price sealed-bid auction, winning bids are revealed ex-post. This rule attributes the item to the highest/lowest bidder, yet the amount paid by the winner is the first rejected bid (second highest/lowest). The winner's payoff is determined by his valuation over the item minus the price paid (second highest/lowest bid) for the acquisition. The second highest/lowest bid associates the seller's payoff.

The most appealing property in this auction format is that bidding *sincerely* is a weakly dominant strategy. To explain it further, a bidder who bids less is no more likely to win the auction, but he pays the same price, the second-highest/lowest bid if he is the winner. Bidding more than the value of the item could lead to a negative payoff. The rule belongs to the class of VCG mechanisms and it is strategy-proof² [Vickrey, 1961]. For example if bidder 1 bids b_1 with a value v_1 , such as $b_1 > v_1$, and bidder 2 bids b_2 with value v_2 , such as $v_1 < b_2 < b_1$. The outcome of the auction attributes to bidder 1 the item, but bidder 1 suffers a loss [Bichler, 2017].

²Mechanisms with dominant strategies are referred in the literature as strategy-proof [Bichler, 2017; Borgers et al., 2015].

1.3 The Common Value Model

The common value model was introduced by Wilson [1969] and developed the first closed-form equilibrium analysis on the “**winner’s curse**” - the possibility to pay more than the true value of the object. The winner’s curse is a form of adverse selection. A bidder who wins in an auction against well-informed bidders must be apprised that the other unwillingness to bid higher is unfavorable information about the value of the item [Menezes and Monteiro, 2005; Milgrom, 2004].

The usual practice in a common value auction model is to refer to bidders’ types as bidders’ signals. If we assume that bidders’ types determine bidders’ preferences, in these models bidders’ preferences might be a function of the other bidders types/signals [Menezes and Monteiro, 2005].

1.3.1 Auctions as a Bayesian Games

In this section we borrow the example of Menezes and Monteiro [2005] to explain how auctions can be considered as games of incomplete information. Thus, the expected payoff and the strategies of each bidder i depends on his beliefs about others’ payoffs.

A *Bayesian Game* is defined as a tuple (I, X, F, S_i, π_i) , where

- a set of potential bidders is $I = \{1, 2, \dots, n\}$;
- a set $X = X_1 \times \dots \times X_n$ of possible types, where $X_i = [0, \bar{v}]$ is the type space of bidder $i \in I$ and a type x_i in this space is the value that the bidder i estimates for the object;
- $F(\cdot)$ is the probability distribution over X , depicting the probabilities attached to each combination of types where $F(\cdot) : [0, \bar{v}]^n \rightarrow [0, 1]$ with a density function $f(\cdot) : [0, \bar{v}]^n \rightarrow R$;
- $S_i = R_+$ is the set of strategies or bids for bidder $i \in I$ and $s_i : X_i \rightarrow S_i$ the decision of i .
- $\pi_i(s_i, s_{-i}, x_i, x_{-i})$ is the payoff function of i given his type v_i choosing a strategy $s_i \in S_i$ and $s_{-i}(x_{-i})$ to be the strategy followed by other bidders.

First Harsanyi [1967] introduced the *type* of a bidder that is a random variable that encompasses the information about bidder i 's payoff. The probability distribution $F(\cdot)$ of v_i is assumed to be common a priori among the bidders. For each bidder i , the value v_i is chosen randomly from X_i according to F and bidder i observes the realized type v_i . Then the bidders update their beliefs about other bidders' types based on F . The payoff π_i of bidder i will depend on his attitude towards risk and on the rules of the auction.

Next, we define the Bayesian Nash equilibrium, which is basically the same concept as a Nash equilibrium with the addition that players need to take expectations over opponents' types.

DEFINITION 1.1 (Bichler [2017]). *A Bayesian-Nash equilibrium is a list of decision functions $(s_1^*(\cdot), \dots, s_n^*(\cdot))$ such that $\forall i \in I, \forall x_i \in X$ and $\forall s_i \in S_i$:*

$$\mathbb{E}(\pi_i(s_i^*|s_{-i}^*, x_i)) \geq \mathbb{E}(\pi_i(s_i|s_{-i}^*, x_i))$$

for all $s_i^(x_i)$ and for all types x_i occurring with positive probability.*

In other words, each bidder chooses a strategy using a Bayesian decision function. We then can apply the concept of Nash equilibrium to these decision functions: each bidder forms a best response strategy of choosing the best Bayesian decision functions, based on the best response strategies of other bidders.

1.3.2 Auction as a Mechanism

This section will consider the underlying allocation problem by disregarding the auction formats and identifying the best allocation rule. One property that is desirable in every mechanism is *truthfulness*, and holds when agents truthfully disclose their preferences to the mechanism in the equilibrium. If this property can be attained for all the implemented social choice functions (allocative efficiency or revenue maximization for the auctioneer) and agents' preferences, then there is a subclass of such mechanisms called "*direct mechanisms*" [Bichler, 2017; Borgers et al., 2015]. In "direct mechanisms" the set of strategies ("bids") is the same as the set of bidders' types - that is, for all i , $S_i = X_i$. If $f(\mathbf{x})$ is the joint density of $\mathbf{x} = (x_1, x_2, \dots, x_N)$ and types are independently

distributed, $f(\mathbf{x}) = f_1(x_1) \times f_2(x_2) \cdots \times f_N(x_N)$ ³, then a “direct mechanism” is defined as follows [Krishna, 2010].

DEFINITION 1.2 (Borgers et al. [2015]; Krishna [2010]). *A direct mechanism consists of a pair of functions $\mathbf{A} : X \rightarrow \Delta$ and $\mathbf{P} : X \rightarrow \mathbb{R}^N$, where Δ is the set of all probability distributions over the set bidders I where $A_i(\mathbf{x})$ is the probability that i will get the object and $P_i(\mathbf{s})$ is the expected payment by i . The pair $(\mathbf{A}(\mathbf{x}), \mathbf{P}(\mathbf{x}))$ is the outcome of the mechanism at \mathbf{x} .*

The interpretation of a “direct mechanism” is that the only available action to each bidder is to announce his private information, i.e. his type. From subsection 1.3.1, where we define auctions as a Bayesian Game, then in “direct mechanism” we have $S_i = X_i$. The direct mechanism is *truthful* (or *incentive compatible*), if, for any type vector x , in the equilibrium of the game defined by the mechanism, every bidder’s i strategy is to announce his true type [Borgers et al., 2015; Shoham and Leyton-Brown, 2010].

Now we will state one of the most basic results in mechanism design, the *revelation principle* which shows that the outcome resulting from any equilibrium of any mechanism can be replicated by a truthful equilibrium of a direct mechanism.

PROPOSITION 1.1 (Krishna [2010]). *Given a mechanism and an equilibrium for that mechanism, there exists a direct mechanism in which (1) it is an equilibrium for each bidder to report his value truthfully, and (2) the outcomes are the same as in the given equilibrium of the original mechanism.*

Proof. See Krishna [2010], p.62. □

The *revelation principle* means that even though one might have thought *a priori* that a particular auction design problem calls for an arbitrary complex strategy space, in reality, one can only focus on truthful, direct mechanisms [Shoham and Leyton-Brown, 2010]. Suppose one can fix a mechanism and an equilibrium \mathbf{s} of the mechanism. Now instead of having bidders submit bids $s(x_i)$ and then applying the rules of the mechanism in order to determine the outcome, we would directly ask the bidders to “report” their types x_i and then reassure that the outcome remains the same as if they

³Similarly for $f_{-i}(\mathbf{x}_{-i})$ and $\mathbf{x}_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_N)$.

had submitted bids $s_i(x_i)$. In other words, a direct mechanism does the “equilibrium calculations” for the bidders automatically.

1.3.3 Correlation and Affiliation

An important assumption in the analysis of optimal auctions⁴ is that each bidder’s private information is independent of his competitors’ private information. Myerson [1981] in his influential paper⁵ presents an example indicating that if bidders’ private information is correlated, then the seller can design a mechanism that achieves the entire social surplus as if bidder’s information were fully public!

The notion of affiliation in information was firstly introduced by Milgrom and Weber [1982]. The assumption of positively affiliated signals attributes a strong positive correlation and roughly means that if a subset of X_i 's are all large, it is more likely that the remaining X_j 's are also large. In that sense, affiliation is stronger than correlation. The formal definition may be found in Milgrom [2004] at pages 182-183. Briefly, we give the intuition with an example.

Suppose the random variables X and Y with a joint distribution $f : [0, \bar{v}]^2 \rightarrow \mathbb{R}$. If X and Y are affiliated, then $x' \geq x$ and $y' \geq y$,

$$f(x', y)f(x, y') \leq f(x, y)f(x', y')$$

The main results of their paper are that ascending auctions lead to higher expected prices than sealed-bid second-price auctions, which in succession lead to higher expected prices than first-price auctions (under the risk-neutrality assumption). Their intuition is that the more the price paid depends on other’s information, the more closely the price is related to the winner’s information (due to affiliation). Thus the lower is the winner’s information rent, and so the winner’s expected surplus, the higher the expected price [Klemperer, 2003].

We use this notion of correlation in Chapter 3 to show how an update in public

⁴Revenue-maximizing auctions are also referred to as optimal auctions.

⁵A few years later Crémer et al. [1985] showed that Myersons’ result is very general.

information may affect positively or negatively bidder's valuations.

1.3.4 The symmetric model

Now we present the notion of symmetric bidders for common value auctions based on the definition given by Krishna [2010]. When signals are affiliated, symmetry concerns the valuations and the distribution of signals.

If all signals X_i are drawn from the same interval $[0, \bar{v}]$, then the valuations of the bidders are symmetric in the following sense

$$v_i(\mathbf{X}) = v(X_i, \mathbf{X}_{-i})$$

where v is a symmetric function in the last $N - 1$ components and it is the same for all bidders.

This means that from the perspective of a bidder i , the signals of the other bidders can be interchanged without affecting the value. In the example with $N = 3$ (three bidders), the value of bidder 1 depends on his signal and the signal of bidders 2 and 3. If the signals of the other bidders were interchanged, then the value would not be affected. Also we assume that the symmetric joint density function of the signals $f \rightarrow [0, \bar{v}]^N$ is affiliated. We define the following function

$$v(x, y) = \mathbb{E}[V_1 | X_1 = x, Y_1 = y]$$

which is the expected value of bidder 1 when the received signal of bidder 1 is $X_1 = x$ and the highest signal of other rival bidders is $Y_1 = y$. Since we have assumed symmetry among the participating bidders, this function is the same for all bidders with $v(x, y)$ to be strictly increasing in x and y .

1. Second-Price Auction

Following the Theorem 6 of Milgrom and Weber [1982], we will prove that in the symmetric equilibrium of second-price auction the *symmetric equilibrium strategies* are given by:

$$s(x) = v(x, x)$$

Suppose all other bidders $i \neq 1$ follow the strategy s . We denote with $s^{-1}(b)$ the value of s for which b is the equilibrium bid. If bidder 1 has a signal x and bids an amount b , his expected payoff is given by

$$\begin{aligned} \mathbb{E}(\pi_1|b, x) &= \int_0^{s^{-1}(b)} [v(x, y) - s(y)] f(y|x) dy \\ &= \int_0^{s^{-1}(b)} [v(x, y) - v(y, y)] f(y|x) dy, \end{aligned}$$

where $f(\cdot|x)$ is the conditional density function of the distribution $F(\cdot|X)$ for $Y_1 \equiv \max_{i \neq 1} X_i$ conditional on $X_1 = x$.

Since v is increasing in the first argument for all $y < x$, $v(x, y) - v(y, y) > 0$. Thus, $\mathbb{E}(\pi_1|b, x)$ is maximized by choosing b so that $s^{-1}(b) = x$ or equivalently, by choosing $b = s(x)$ [Krishna, 2010].

To explain further the bidding strategies mentioned above, assume that bidder 1 with a signal x bids an amount $s(x)$ such that he wins in the auction with this bid. If the highest competing bid that determines the price was $s(x)$ too, bidder 1 would “break even” because he would conclude that $Y_1 = x$ and the expected value of the object conditional on the new information is in accord with:

$$s(x) = v(x, x) = \mathbb{E}[V_1|X_1 = x, Y_1 = x]$$

At this stage we can calculate bidder’s 1 expected payment conditional on receiving a signal $X_1 = x$

$$\mathbb{E}[b(Y_1)|X_1 = x, x > Y_1] = \mathbb{E}[v(Y_1, Y_1)|X_1 = x, x > Y_1]$$

The seller’s expected revenue from the second-price auction in a symmetric environment is simply n times this expected payment [Menezes and Monteiro, 2005].

2. First-Price Auctions

Suppose that all bidders $j \neq i$ follow the increasing and differential strategy b . Evidently, bidder 1 does not pay for any bid less than $b(0)$ or more than $b(\bar{v})$. If bidder 1 has a signal x and bids an amount $b(z)$, then his expected payoff is defined by

$$\begin{aligned}\mathbb{E}(\pi_i(z, x)) &= \int_0^z (v(x, y) - b(z)) f(y|x) dy \\ &= \int_0^z v(x, y) f(y|x) dy - b(z) F(z|x)\end{aligned}$$

Applying the first-order conditions

$$(v(x, z) - b(z)) f(z|x) - b'(z) F(z|x) = 0$$

At a symmetric equilibrium, the optimal $z = x$, thus we obtain the differential equation which is only a necessary condition:

$$b'(x) = (v(x, x) - b(x)) \frac{f(x|x)}{F(x|x)}$$

s.t. $v(x, x) - b(x) \geq 0$, for all x ,

since otherwise a bid equal to zero would be better. By assumption $v(0, 0) = 0$ with a boundary condition $b(0) = 0$. The solution of the above differential equation results to the *symmetric equilibrium*.

The *symmetric equilibrium strategies* in a sealed-bid first-price auction are given by

$$b(x) = \int_0^x v(y, y) dL(y|x)$$

where,

$$L(y|x) = \exp\left(-\int_y^x \frac{f(t|t)}{F(t|t)} dt\right)$$

Further analysis on the proof is given by Krishna [2010] in the section 6.4.

1.3.5 The “Linkage” Principle

In many instances, the seller may have information that is potentially useful to the bidders. How should the seller react in these cases? Should he keep it hidden or reveal it publicly?

These questions have been answered in the remarkable paper of Milgrom and Weber [1982] in Theorems 8, 12 and 16. In Theorem 8 and 12, they have shown in second-price and English auctions “revealing information publicly raises seller’s revenues”, while in Theorem 16 they have proven that in the first-price auction, “revealing the seller’s information cannot lower, and may raise, the expected price”.

In a first-price auction, revealing the seller’s information connects the price to that information, even when the winning bidder’s reported signal is fixed. In the second-price auction, the price is connected to the evaluation of the second-highest bidder, and revealing information connects the price to that information as well. Last, the English auction, with linkages to all of their estimates, yields the highest expected price. In all of the three, revealing information, there is a “linkage” which raises prices. Milgrom [2004] renames the “linkage” principle with the term “publicity effect”.

Kagel and Levin [1986] confirmed the theory with experimental results. In the absence of “winner’s curse”, revealing public information may reduce uncertainty about the value of the item resulting in higher winning bids and an increase in the seller’s revenue. However, in the presence of a “winner’s curse”, the same public information generates lower average winning bids and reduces seller’s revenues. So, the response of public information is conditional on the presence or absence of “winner’s curse”, creating several practical applications.

The significance of the “linkage ” principle has been quite influential in practical auction designs. This can be demonstrated on FCC spectrum auctions, which included open-auction components. According to McMillan [1994], the option given to bidders to learn from others’ bids out-weighted the potential inherent risk-aversion and the possibility of collusive behavior.

As said by Milgrom [1989] another application of “linkage” principle arises in the weakly Treasury bill (T-bill) auction. Those brokers who bid in the T-bill auction have

estimates for the future prices to resell the bills to their customers. Thus, it can be assumed that their estimates are affiliated. This can suggest that a design, such as uniform auction, in which the price paid by each broker is linked to the bids made by others. Indeed, the theory predicts that it can generate a higher average price than discriminatory. In general, under this principle, the bidders are made worse off and the seller better off if the price paid by the buyer can be more effectively linked to exogenous variables that are affiliated with the bidder's private information.

1.3.6 The number of bidders

An essential part of the auction literature has been concerned with the properties of pure-common-value auctions as the number of bidders becomes large. Matthews [1984] in his model showed that as the number of bidders becomes large, the amount of information each bidder receives falls, but in a way that the first-price sale price does not, in general, converge to the true value. Matthews [1987] and McAfee and McMillan [1987] analyzed how the nature of bidders' risk aversion affects bidders' and the seller's preferences conditional on the revelation of the number of bidders.

Bulow and Klemperer [1996] discuss that when bidders are symmetric, the entry of an additional bidder is worth more to the seller in an ascending auction than the ability to set a reserve price, provided bidders with higher signals have higher marginal revenues. Keeping the number of bidders secret may robust ascending auctions and restrict collusions [Cramton and Schwartz, 2000]. Milgrom [2004] results that in symmetric models, the total value enjoyed by the bidders and the auctioneer is a concave function of the number of bidders. In particular, in a second-price auction, the entry of the last bidder causes a decline in the expected welfare.

Michael J. Fishman [1988] illustrated that bidders might influence the number of their rivals through their strategic behavior, in a sense that it can be profitable for a bidder to commit a high bid ("jump bid") to prohibit potential rivals from incurring the cost requires to enter the contest.

Another work by Kremer and Nyborg [2004] proves that when bidders are risk-averse, an ex-ante specified allocation rule in uniform auctions might have a considerable effect on the equilibrium price. The lower bound on prices is determined by the

number of participating bidders.

Overall, the increasing number of bidders affects the outcome of an auction and can increase the revenue of the seller [Bichler, 2017]. However, when it comes to Vickrey-Clarke-Groves mechanism [Clarke, 1971; Groves, 1973; Vickrey, 1961] which suffers from monotonicity problems, adding more bidders may reduce equilibrium revenues to the point that might reach zero [Milgrom, 2004].

1.4 Sealed-Bid Auctions for Homogeneous Items

Prior literature has focused on debate for the optimal design of multi-unit auctions for decades used in a real-world application. Friedman [1960] took the first steps to shape the proposition that the U.S. Treasury could decrease funding costs by using uniform price rather than discriminatory price auctions. Both auction formats are sealed-bid, with individual bidders submit demand schedules (collection of bids), and the securities are awarded in the order of descending price until supply equals demand.

Wilson [1979] in one of his seminal papers that was many years ahead of its time, analyzed “share auctions”, in which each bidder offers a schedule specifying a price for each possible fraction of the item (for example, a particular volume of Treasury notes), comparatively to unit auctions. He showed that in the “share auction” the selling price can be significantly lower if bidders are allowed to *submit bid schedules* rather than a single price bid. The seller may experience a reduction in the revenue due to two reasons: it may be that the seller obtains no advantage from the increased number of bidders, and secondly, the optimal strategies that are selected from the bidders may be disadvantageous for the seller.

Maskin and Riley [1989] extended the work of Myerson [1981] for optimal auctions in which bidders have downward-sloping demand curves, independently drawn from a one-parameter distribution, for quantities of homogeneous goods. They provide an exposition of revenue equivalence for the multi-unit case when bidders each demands no more than a single unit. Finally, Back and Zender [1993] point that bidders buying multiple units are concerned with marginal cost than price. This is very important

because the marginal cost in auctions is endogenous⁶ and determined by the demand schedules submitted by the bidders.

Here below, we analytically present those two formats, but first, we discuss the Vickrey auction [Vickrey, 1961], which has received a particular interest for its theoretical properties.

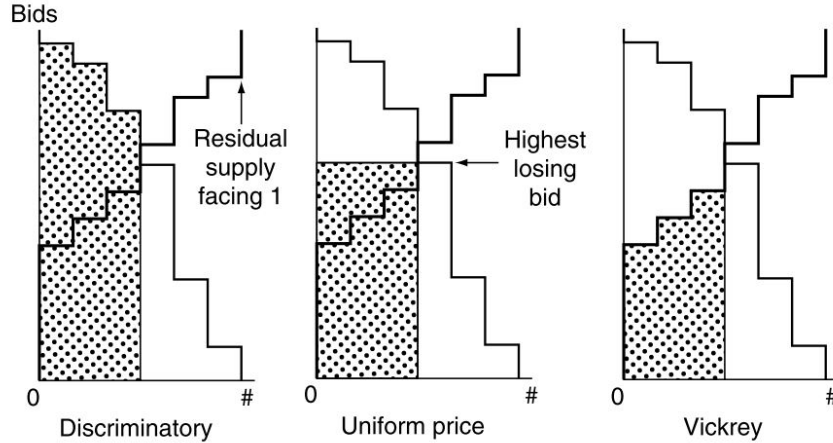


Figure 1.1: Pricing Rules for bidder i 's payments [Krishna, 2010].

1.4.1 Vickrey Auction

The Vickrey Auction, named from the seminal work of the Nobel prize winner Vickrey [1961], is a single-round auction for M identical items. This auction format bears some fundamental properties such as dominant strategy⁷ and sincere bidding (a bid equal to the valuation).

Following Krishna [2010], if a bidder wins in the Vickrey auction, he pays the opportunity cost for the items obtained. For instance, for the k^i winning units, he pays the k^i highest losing bids of the other bidders (not including his own). To compute this payment, we denote $\mathbf{c}_{-i} = (c_1^{-i}, c_2^{-i}, \dots, c_K^{-i})$ the vector of competing bids that bidder i is facing, which is the K -vector of the highest bids submitted by the rivals. This vector is obtained by rearranging the $(N - 1) \times M$ bids of all bidders except i in decreasing

⁶The supply curve faced by a bidder is the residual from the demands of other bidders, so his marginal cost depends on his competitor's strategies.

⁷A strategy that does at least as well as any other strategy for one bidder, no matter how rivals might play.

order and selecting K highest bids, which are the K winning bids if bidder i would not place a bid.

Thus to win one unit, bidder's i highest bid must defeat the lowest competing bid - $b_1^i > c_K^{-i}$, sequentially, to win a second unit, bidder's i second-highest bid must defeat the second-lowest competing bid - $b_2^i > c_{K-1}^{-i}$ and so on.

In Vickrey pricing rule the bidder is asked to pay c_K^{-i} for the first unit he wins (Figure 1.1). If the bidder wins k^i the amount that he pays is:

$$\sum_{k=1}^{k^i} c_{K-k^i+k}^{-i}$$

The basic principle of the Vickrey auction is the same as the VCG mechanism presented in the next chapter. When there is a single unit for sale, the **second-price sealed-bid auction** becomes a particular case of the Vickrey auction.

1.4.2 Uniform Price Auctions

Since the spectrum auction of 1994, one of the most popular designs used for the auction of homogeneous items is the *uniform-price* auction. This auction format has been used for spectrum auctions, suggested for the sale of Treasury Bills, and currently used in the electricity spot market around the globe [Menezes and Monteiro, 2005]. Uniform-price auctions' popularity stems from the fact that they can mitigate the price risk and reduce the transaction costs of bidding repeatedly [Milgrom, 2004]. In the following paragraphs, we will analyze the simplest form of uniform-price auctions, the sealed-bid. Other ascending auction formats that enforce uniformity by rule are out of the scope of our research.

In a uniform-price auction, all bidders pay the same "market-clearing" price for the acquired items, such that the total amount demanded is equal to the total amount supplied. Practically, market-clearing prices are calculated as the price would clear a Walrasian Market. The selection of the price can either be the highest-rejected-bid or the lowest-accepted-bid. We adopt the rule that the "market-clearing" price is the same as the highest-rejected-bid. Thus, if p^* is the "market-clearing" price and k^* the

items that bidder i wins, then his payment is equal to:

$$P_i^* = p^* \times k_i^*$$

Following Krishna [2010], we denote by c^{-i} the K -vector of bidder i 's rivals. Then by rearranging in decreasing order the $(N - 1)K$ bids b_k^j of bidders $j \neq i$ and the mechanism selects the first K of these. For example, c_1^{-i} the highest of the other bids and so on. Thus, the number of units that the bidder i wins is just the number of competing bids he defeats:

$$b_{k^i}^i > c_{K-k^i+1}^{-i}$$

and

$$b_{k^i+1}^i < c_{K-k^i}^{-i}$$

At any price p the residual S^{-i} supply that bidder i is facing equal to (Figure 1.1) :

$$S^{-i}(p^*) = K - \max\{k : c_k^{-i} \geq p^*\}$$

The highest losing bid is:

$$p^* = \max\{b_{k^i+1}^i, c_{K-k^i+1}^{-i}\}$$

These auctions create incentives for bidders to reduce demand to avoid driving up prices and exist Nash equilibria with meager prices. When a bidder wants to buy more than one unit, and when the units have declining marginal values, a bidder generally has an incentive to reduce his demand that is, to bid less than his value for some units. The incentive to reduce demand arises because the bids for the second and subsequent units in the highest-rejected-bid auction affect both the expected quantity the bidder acquires and the expected price he pays for each unit he buys if one of these subsequent bids does not win.

1.4.3 Discriminatory Price Auctions

In the discriminatory price auctions, each bidder pays an amount equal to the sum of his bids that count for winning. If K is the vector of bidder's i bids

$$(b_1^i, \dots, b_K^i), b_1^i \geq b_2^i \geq \dots \geq b_K^i$$

Then, the winning bids are among the K 's highest of $N \times K$ such as $\{b_k^i; \leq 1 \leq n, 1 \leq k \leq K\}$. Thus, if bidder i has k bids among the winning bids his payment is

$$\sum_{k=1}^{k^i} b_k^i.$$

The residual supply function that each bidder is competing in this pricing rule is

$$S^{-i}(p^*) = \max\{K - \sum_{j \neq i} b^j(p^*), 0\}.$$

It is a nondecreasing function of the price. Each bidder pays an amount equal to the area under his demand function up to the point where it intersects the residual supply curve Figure 1.1.

1.4.4 Discussion

Wilson [1979] showed that in a uniform-price auction, there are Nash equilibria that look very collusive, in a sense that prices may be much lower than if the item sold as an indivisible unit. The intuition is that bidders can implicitly agree to divide up the item at a low price with each bidder to bid aggressively for smaller quantities than her equilibrium “share” discouraging the others from bidding more. Thus, there are two ways to “ruin” the equilibrium.

One way is to run a discriminatory auction as bidding aggressively for small quantities is costly, as bidders submit flatter demand curves, which induce greater price competition at the margin. Anton and Yao [1992] use a private-value framework to prove that there is implicit coordination in discriminatory auctions if bidders' values are non-linear in the volume purchased [Klemperer, 2003]. Another way to “ruin” the low-price equilibrium is to include some randomness in the demand schedules [Klemperer, 2003].

Supporting Merton Miller's view on 1991 in New York Times that in a uniform auction “*You just bid what you think it's worth*” and compared to the result Milgrom and Weber [1982] on second-price versus first-price auctions, it has been argued that uniform auctions reduce the “winner's curse” relative to discriminatory auctions, and that generates more revenue.

Bikhchandani and Huang [1989] studied competitive bidding when the resale market signals the auction and showed that, there is a symmetric equilibrium in uniform auctions, which generates higher expected revenues than the symmetric equilibrium in discriminatory auctions. Also, in the cases where bidders have no signaling incentive⁸, uniform auctions result in a symmetric Nash equilibrium and generate strictly higher expected revenue for the auctioneer. The feature of the bid schedule (price and quantity) was not included in this model.

Back and Zender [1993] argue that discriminatory auctions are probably more profitable for a seller than uniform auctions, including Ausubel et al. [2014] for symmetric bidders with flat demands. Their result is relevant to Friedman’s argument. The uniform-price auction, even though non-cooperative equilibria exist, could be characterized as “collusive”. The device for this “collusive” outcome is the fact that marginal costs are very high for other bidders and thereby inhibit competition from them. This problem does not necessarily diminish as the number of bidders is increased. On the treasury experiment evaluating the spread of winning bids is very important. The assumption is that a greater spread will indicate that bidders are bidding their “truthfull valuations”. Last but not least, the presence of a pool of potential bidders is an essential aspect in auctions and is related to Friedman’s point that increasing the number of bidders will lead to higher prices for the Treasury.

A similar view has been expressed by Bikhchandani and Huang [1993] in their later work. They claim that in a uniform-price auction bidders are more likely to bid more aggressively; thus the average selling price in a uniform-price auction is higher than in a discriminatory auction. Competitive bidders usually keep an inventory of fixed income securities and place their bids based on their expectation of falling interest rates, which would increase the value of these securities. They have the incentive to bid higher than they would, if the secondary market buyers received no signal from the auction, in order to signal the market’s buyers that the bidders’ private information is very favorable. In uniform-price auctions, it is cheaper for bidders to bid high to signal their expectation that the interest rates will fall.

Nyborg and Sundaresan [1996] used a dataset of when-issued transactions to assess

⁸Only when the highest losing bid is announced.

the U.S. Treasury bills experiment with uniform auctions. They conclude that uniform auctions release more information than discriminatory reducing the pre-auction uncertainty and the “winner’s curse”. While for uniform-auctions, when-issued volatility falls after the auction and the outcome’s announcement, for discriminatory auctions, there is strategic behavior that increases the when-issued volatility.

Tenorio [1997] builds a model with two bidders who compete for three identical items, and are constrained with a single price for either bidding two items or bidding three items. He concludes that under uniform-price auction, there is a higher demand reduction than in the discriminatory auction.

Binmore and Swierzbinski [2000] assess the evidence of prior empirical research comparing discriminatory and uniform multi-unit auction, and found that neither empirical evidence nor auction theory, offer any constraining reason for preferring a uniform auction to a discriminatory auction. Ausubel et al. [2014] result in the same ambiguity, who find that the appealing properties of the second-price auction i.e., “truth-telling” and efficiency, do not exist in uniform-price auctions, and any equilibrium in this type of auction is ex-post inefficient.

Interestingly Ausubel et al. [2014] conclude that bid shading results generically in ex post inefficient allocations in the uniform-price and pay-as-bid auctions and that strong assumptions are required in settings with decreasing marginal utility to obtain a sharp ranking of multi-unit auctions.

Wang and Zender [2002] address the issue of pure common value auctions in which allocation is always efficient. Similar to Back and Zender [1993], they examine the revenue ranking between the uniform-price and discriminatory auctions. They result that equilibrium bid schedules in such auctions contain strategic aspects and consider the “winner’s curse”. In symmetry, the equilibria in uniform-price auction yields lower expected revenue than discriminatory auction. For risk averse bidders, the uniform-price auction gives higher expected revenues in the equilibrium than discriminatory.

However, Ausubel et al. [2014] overcome the methodological limitations of the above mentioned works, which compared one equilibrium (out of a multiplicity of equilibria) of uniform-price auction with one equilibrium of the discriminatory auction. They discuss the entire set of equilibria of uniform auctions in “Inefficiency Theorem”.

Kremer and Nyborg [2004] have shown that the tools for eliminating underpricing equilibria in uniform-price auctions are the bid discretization, and the tie-breaking rules holding the market price fixed. On top of that Burkett and Woodward [2020], showed for uniform auctions that the price selection (first-rejected or last-accepted bid) can also be a valuable tool for eliminating equilibria.

The theoretical ambiguity on the outcome rankings, between discriminatory and uniform-price auctions, has opened the doors for empirical research. Some studies conclude that discriminatory auctions do better than uniform auctions, such as Février et al. [2004], Kang and Puller [2008], Marszalec [2017], while other studies reach on reverse outcomes. The studies which merit uniform auctions are Armantier and Sbaï [2006, 2009] and Castellanos and Oviedo [2008].

Hortaçsu and Mcadams [2010] used Turkish treasury’s auction data, and concluded that discriminatory auctions produced more revenue *ex post*, than the uniform-price auction, and any switch from discriminatory auction to a uniform price or Vickrey auction would not significantly increase revenue. They show that there is no statistical difference between the two and rely on the bounding “best response” behavior.

Jehiel [2011] employs an analogy-based expectation equilibrium to model an auction framework, in which bidders receive incomplete information about the bids’ distribution in earlier auctions. There are two situations: either past bids are disclosed anonymously in asymmetric bidders or the distribution of the bids.

To conclude, we illustrate some experimental studies which investigate the effects of the uniform-price auction in finance. Goswami et al. [1996] provide experimental evidence that pre-play communication facilitates collusive equilibrium outcomes in uniform-price auctions. On the other hand, in discriminatory auctions, bidder strategies approximate the unique equilibrium outcome, producing a larger surplus for the auctioneer. Their results have significant implications on the design of Treasury auctions. If participants have many opportunities to communicate with uniform auctions, the Treasury will yield lower revenues. Kagel and Levin [2001] have found a material demand reduction under uniform pricing either in the case of static or dynamic auctions. Zhang [2009] compares uniform-price auctions with fixed-price offerings in Initial Public Offerings (IPO) using laboratory experiments and results that uniform-

price auctions outperform fixed-price offerings for raising revenues.

1.5 Introduction to Combinatorial Auctions

Now, we will introduce the concept of *combinatorial auctions* (CA) or *package* auctions related to Chapter 4. This format captures the market where the seller offers multiple items (usually heterogeneous but related) in a single auction. Each bidder is allowed to place bids for a combination of items, called “packages” (non-trivial subsets). It is suitable for items that are substitutes or complements as the *exposure problem*⁹ is reduced [Cramton et al., 2006; Mochón and Sáez, 2015].

The most important features that determine the auction environment in combinatorial auctions can be the number of bidders, the items being traded, the parties’ preferences expressed through strategies, and the type of private information the participants might have about preferences [Cramton et al., 2006; Mochón and Sáez, 2015]. In this context, the *winner determination problem* becomes complex.

Although there is a vast literature about the auction of substitutes, here in this thesis, we will be concerned with the case of complements. For complements, the bidders bear an *aggregation risk or exposure problem* which means that a bidder might be required to buy an item that no longer wants after the price adjustment, without receiving the super-additivity value of the whole package. Thus, a bidder has to decide either to bid aggressively despite the associated risk or to avoid bidding for complement packages, despite his willingness to acquire the packages at the current market prices [Milgrom, 2007]. In that sense, expressing preferences for a different combination of items can mitigate bidders’ exposure problem.

This market design has several applications in practice starting from “Spectrum auctions” by the FCC, transportation, communication networks, and finance [Milgrom, 2007; Nisan et al., 2007]. However, there are two potential caveats linked to combinatorial auctions: (a) the “free-rider” problem which occurs when diseconomies of scale dominate, and (b) it is computationally difficult (NP-hard) to solve the winner

⁹The risk associated with winning an inferior subset of high prices, when bidders compete aggressively for a particular combination of items.

determination problem [Rothkopf et al., 1998].

1.5.1 The Winner Determination Problem (WDP)

Now we will focus on the winner-determination problem in combinatorial auctions. It is an optimization problem that one may face in different multi-object auctions on various bidding languages.

Among all bids submitted by all bidders, the question that arises is how to compute the allocations which maximize the seller's revenue. It is the feasible combination of bids (labeled as winning or losing) that maximized the sum of the accepted bids under the constraint that each item can be allocated to at most one bidder [Cramton et al., 2006].

To give some mathematical intuition behind this notion, we represent a set of bidders with $I = \{1, 2, \dots, N\}$ and $J = \{1, 2, \dots, M\}$. The combinations of items or the packages are represented by $Q \subset J$. Each bidder i has a value for the package $v_i(Q)$, which is the maximal amount that bidder i is willing to pay for the package Q . Bidder i 's bid for that package is denoted as $b_i(Q)$ and $x_i(Q)$ is a binary variable, which equals to one if the bidder wins the package and equal to zero when he does not win. Then the WDP has the following formulation:

$$\max \sum_{i \in I} \sum_{Q \subseteq J} b_i(Q) x_i(Q)$$

subject to:

1. $\sum_{Q \ni \{j\}} \sum_{i \in I} x_i(Q) \leq 1, \forall j \in J$
2. $\sum_{Q \ni j} x_i(Q) \leq 1, \forall j \in J$
3. $x_i(Q) \in \{0, 1\}, Q \subseteq J, \forall j \in J$

The first restriction implies that each item is awarded to, at most, one bidder. The second restriction ensures that bids are mutually exclusive - each bidder obtains, at most, one winning bid. As the number of bidders and items increases, the possible

allocations may grow exponentially. This means that WDP may be NP-hard and can be solved with advanced optimization techniques [Mochón and Sáez, 2015].

1.5.2 Payment rules

The two seminal theoretical mechanisms that utilize package pricing are *pay-as-bid* or first-price sealed-bid [Whinston and Bernheim, 1986] and the *Vickrey-Clarkes-Glove (VCG)* or *generalized Vickrey auction* [Clarke, 1971; Groves, 1973; Vickrey, 1961]. The latter auction format has a dominant strategy equilibrium yet encounters caveats when it comes to applying in multi-object auctions. An outstanding issue is when the VCG outcome is outside the *core* (see the section below), creating an opportunity for the losing bidders to collude with the auctioneer for a better outcome. For this reason bidder-optimal core-selecting payment rules have been designed. Briefly, in the VCG mechanism, the payment rule for bidder i is calculated so as the total payoff of the seller and all bidders except i is the same as if bidder i had not participated in the auction [Milgrom, 2007]. The pay-as-bid payment rule gives payments to bidders equal to their bids for the acquired items, subject to the auctioneer’s payoff maximization. In the next paragraphs, we elaborate on each rule separately.

Pay-as-Bid

It can be asserted that it is the most widely used payment rule in practice. Some examples of this rule are industrial procurement, auctioning bus routes in London, or transportation auctions. They present a “strategical difficulty” in a sense that bidders need to decide not only on which packages to bid but also by how much to shade on bids, and no closed-form equilibrium strategy with general valuations exists [Bichler, 2017].

First Whinston and Bernheim [1986] developed a theory of first-price package auctions with complete information. Even though the assumption of full information is far from practice, their theory identifies some critical aspects. The first observation for this type of auction is that it has many Nash equilibria.

They focused on *profit-targeted strategies*¹⁰, which have two appealing characteristics: (a) they are simple, and (b) regardless of others bidders' strategies, each bidder's best reply will always include a profit-targeted strategy. These two characteristics imply that it is difficult to apply a collusive behavior in equilibrium in the first-price package auction, as no pure strategy profile can deter a bidder from bidding aggressively for additional units. This occurs because if a bidder uses a profit-targeted strategy, a marginal increase in his allocation will increase his bid offering to pay his marginal value for the additional unit. Thus, *unlike uniform auctions, in first-price package auctions bidders have no incentives to reduce demand* [Milgrom, 2004].

Vickrey-Clarke-Glove (VCG)

Under this rule, each bidder has monotonically non-increasing marginal values for the good and submits a sealed bid associated with his demand curve. The seller aggregates the individual demand curves and determines the clearing price for the units. Each bidder wins the quantity demanded at a clearing price. However, the price he pays or the clearing price for the units he won is the "opportunity cost". For instance, if a bidder wins 2 units and the highest rejected bids by his competitors are 12 and 14, the bidder pays 26 for these two units. Alternatively, the price that a bidder pays for his q^{th} unit is the clearing price that would have resulted if the bidder had restricted his demand to q units (all other bidders' behavior held fixed). Thus, the total payment of a winning bidder is computed by aggregating this payment over all items won (discrete units) or integrating this payment from 0 to the quantities won (continuous units) [Ausubel and Milgrom, 2006].

The WDP determines the winners, and the VCG price is computed as:

$$p_i^{VCG} = v_i(x^*) - [\omega(I) - \omega(I_{-i})]$$

where $\omega(I)$ is the objective value to the WDP of the valuations of all bidders, and $\omega(I_{-i})$ is the objective value to the WDP of all bidders except the winning bidder i . The optimal allocation is x^* and its respective valuation is $v_i(x^*)$ assuming that

¹⁰The strategy is characterized only the profit that the bidder bears from any winning bid.

it is a *direct revelation mechanism*¹¹. The payment that each bidder receives is his contribution on the increase on the total value of the auctioneer [Bichler, 2017].

The most appealing property of VCG mechanism is the one of dominant-strategy for bidders, who bid their true values for every combination of items. This property reduces the auction's costs by making it easier for bidders to determine their optimal bidding strategies and eliminate bidders' incentives to spend resources by learning about competitor's values. Additionally, it can impose some constraints, for example, the government seller in a spectrum auction may wish to limit the concentration of spectrum ownership. Finally, the average revenues are not less than from any other efficient mechanism [Ausubel and Milgrom, 2006].

These properties of VCG prompt economists to believe that they could design an efficient auction. The definition of efficiency in auctions includes allocations, revenues, costs of the various parties, and lastly, the incentives for a pre-auction investment [Ausubel and Milgrom, 2002].

However, when it comes to applying the VCG rule several shortcomings emerge [Ausubel and Baranov, 2020; Ausubel and Milgrom, 2002, 2006; Day and Raghavan, 2007; Milgrom, 2004]. The first is that VCG may yield low or zero revenues and “unfair” outcomes. For example, assume an item A and an item B and three bidders. Bidder 1 wants to acquire both items AB as a package, and bids 2, while bidder 2 is willing to pay 2 for A and bidder 3 pays 2 for B. The Vickrey outcome assigns both items to bidders 2 and 3 at the price of zero! Some other limitations to Vickrey's design are its vulnerability to shill bidding and collusion, even by losing bidders. Lastly, Vickrey suffers from monotonicity problems which means that the increased competition can reduce sellers' revenues.

VCG and the Core

To better understand the problems above of the VCG mechanism, we model the auction as a *cooperative (coalition) game*¹². The question that arises is “*how low must the*

¹¹A bidder needs to submit bids on all possible packages, the number of which is exponential in the number of items.

¹²The game is competition among the coalition of players rather than between individuals.

revenue be before it is considered unacceptably low?”. The threshold that answers this question is a payoff outcome that lies in the core [Ausubel and Milgrom, 2006].

The *core*, as old as Walras competition, is the set of allocations of objects and money with the property that no coalition can either be *weakly or strictly Pareto* optimal by trading on its own. In other words, it is the set of payoff profiles that correspond to the core allocations, usually called as *imputations* as the payoffs are imputed from the underlying allocation [Milgrom, 2007].

To illustrate the relationship between VCG outcome and the *core*, we introduce some notation from Milgrom [2007]. Suppose N is the set of bidders plus the seller. Let X denote the set of feasible allocations of the goods with typical element x , let x_j denote the goods allocated to bidder J with $v_j(x_j)$ to be the value of j 's allocation, which is weakly increasing. For any coalition $S \subset N$, we construct the value $\omega(S)$ of the coalition as the maximum value of the allocation of the objects among the coalition members S .

$$\omega(S) = \begin{cases} 0, & \text{if seller} \neq S \\ \max_{x \in X} \sum_{j \in S} v_j(x_j), & \text{if seller} \in S \end{cases}$$

We assume that the total payoff can be arbitrarily reallocated by the coalition S among themselves using side payments. Thus, the core of this utility game with a coalition value function ω is

$$\text{core}(N, \omega) = \left\{ \pi \geq 0 \mid \sum_{j \in N} \pi_j = \omega(N), (\forall S) \sum_{j \in S} \pi_j \geq \omega(S) \right\}$$

The core of a game with a single seller is always nonempty, because it includes the imputation at which the seller gets the whole value $\omega(N)$ and each bidder gets zero.

Now let's denote $\bar{\pi}$ the *Vickrey payoff vector* i.e the payoffs associated with the dominant strategy equilibrium of the generalized Vickrey auction. For bidders ($i \in N$) $\bar{\pi}_i = \omega(N) - \omega(N \setminus i)$ while the seller's payoff is $\bar{\pi}_0 = \omega(N) - \sum_{i \in N \setminus 0} \bar{\pi}_i$.

THEOREM 1.1 (Ausubel and Milgrom [2002]). *A bidder's Vickrey payoff $\bar{\pi}_i$ is i 's highest payoff over all points in the core. That is, for all $i \in N \setminus 0$: $\bar{\pi}_i = \omega(N) - \omega(N \setminus i) = \max\{\pi_i \mid \pi \in \text{Core}(N, \omega)\}$.*

THEOREM 1.2 (Ausubel and Milgrom [2002]). *The core contains a bidder Pareto-dominant point, if and only if, the Vickrey payoff vector $\bar{\pi}$ is the core. If $\bar{\pi}$ is in the core, then it is bidder Pareto-dominant.*

Unfortunately, the Vickrey outcome might be outside the core. This is better presented in examples 1 and 2 of Chapter 4. These outcomes might lead to several issues, such as non-monotonicity in seller's revenues caused by the number of bidders or the low seller's revenues or collusions among bidders.

Consequently, Day and Raghavan [2007] argue that the outcomes in combinatorial auctions must be in the core. One can find a vector of core prices that are close to VCG payments, meaning that these prices are minimal for the bidders i.e., *bidder-optimal*. The notion of *Bidder-Optimal-Core-Payments* minimizes the incentives to deviate from truthful bidding [Day and Cramton, 2012] and lie on the *bidder-optimal-frontier*.

DEFINITION 1.3 (Milgrom [2004]). *A payoff vector $\pi \in R^N$ is bidder-optimal if $\pi \in Core(N, \omega)$ and there exists no $\pi' \in Core(N, \omega)$ with $\pi'_{-0} > \pi_{-0}$. The set of such points is called the bidder-optimal frontier of the core.*

Like matching theory, it means that a payoff vector is in *Bidder-Pareto-Frontier* if there is no other payoff vector that is Pareto-preferred.

Core-Selecting Package Auctions

Day and Milgrom [2008] was the first to propose this auction format which several countries have adopted. For example, in the U.K. and other European Countries, it has been used for radio spectrum auctions. Similarly to first-price sealed-bid combinatorial auctions, the bidder-optimal core-selecting auctions are modeled as games with complete information.

The mechanism is a mapping of bids to an “efficient” allocation where the relevant payments are chosen with the single criterion to be included in the core. Simply, it allocates items similar to VCG auction, but replaces VCG payments with the core payments. Unlike VCG outcomes, in core selecting auctions no bidder can earn more by creating “shills”, and for any profile of rival's bids, each bidder has best-reply that is a semi-sincere strategy [Day and Milgrom, 2008].

Milgrom and Day [2013] showed that only if the VCG outcome is within the core a dominant strategy is provided and truthful reporting can be an ex-post equilibrium. Interestingly, Goeree and Lien [2016] prove a crucial negative result implying that “truly core-selecting auctions”¹³ do not exist. An independent private value setting creates an efficient equilibrium with the same expected revenues as VCG (revenue equivalence theorem).

In practice, the most used payment rule in core-selecting auctions is the *Nearest-VCG*, introduced by Day and Raghavan [2007] and Day and Cramton [2012]. The intuition behind this rule is to select a point in the bidder-optimal-frontier which will minimize bidders incentives for misreporting [Parkes et al., 2001]. This point is defined by minimizing the sum of square deviations from the VCG outcome i.e., Euclidean distance. However, Erdil and Klemperer [2010] support that the justification for the “Nearest-VCG” rule is not clear, as all points in the bidder-optimal-frontier minimize the sum of bidder’s incentives to deviate from truth-telling. Thus, they propose “reference rule” which selects points in the bidder-optimal-frontier which are close to a reference point defined by the auction designer (for instance, the loser’s bids). They prove that a reference rule always dominates the Nearest-VCG.

The Local - Global model. This model first appeared in Krishna and Rosenthal [1996]. In this setting, the auctioneer wishes to allocate multiple items which are auctioned simultaneously. Two types of bidders participate in the auction: the *local* and the *global*. The “local” bidders are interested only in one item and compete against “global”. On the other side, the “global” values multiple items, and his valuation exceeds the sum of each item separately. Krishna and Rosenthal [1996] use a setting of bidders who are unaffected by *positive synergies with independent private values in a second-price auction*, incorporating in this way bidders’ asymmetry. They prove the existence of symmetric equilibrium, reaching some comparative static results. They show that increasing the number of global bidders always results in less aggressive bidding by the global bidders. Additionally, when synergies are strong, the simultaneous auction is revenue superior to the sequential auction.

¹³Selection of core-outcome concerning true values than bids.

Rosenthal and Wang [1996] build a *simultaneous-auction model with synergies for bidders with common values* extending the aforementioned work of Krishna and Rosenthal [1996]. They simplify their assumption to the first-price sealed-bid auction. They assume that there are three neighbors of objects, and for each neighbor, there three interested groups of bidders: one group of local bidders and two of global. Objects values are perfectly correlated with two different states: one that all objects are highly valued and the other in which they are low-valued. Bidders are unaware of the state, and each of them receives a private signal. They construct two qualitatively different equilibrium. Few conclusions are drawn due to the limitations created by the “common value” assumption. The conclusions are that global bidders have become more aggressive, increasing the size of synergy. This improves efficiency, increasing the number of global bidders and decreasing the number of local bidders. Last, the seller’s revenue is increased on average but decreased in the presence of locals.

The most recent work of Ausubel and Baranov [2020] diverge from Goeree and Lien [2016] results. They prove in “local-local-global”, and according to the structure of specific pricing rules, the presence of correlations can materially affect (negatively and positively) the equilibrium performance of core-selecting auctions. Thus, the core-selecting auctions can perform reasonably well in nontrivial and empirically relevant settings. The reason is that correlations create an environment of complete information in which core-selecting auctions perform well.

From an experimental perspective, Bosshard et al. [2020] use some techniques to move beyond the single-minded dimensional domains¹⁴ like “local-local-global” and discuss the difficulties of scaling the BNE algorithm to high-dimensional auctions. They find that for a domain with four “local” bidders and two “global” bidders (LLLLGG) an accurate ϵ -BNEs for both the VCG-nearest and first-price payment rules illustrating the scalability of their algorithm.

¹⁴Bidders are interested only in a single specified bundle of items and get a specified scalar value if they get the whole bundle and get zero value for any other bundle [Nisan et al., 2007].

1.6 Other Topics

1.6.1 Early Literature

The game-theoretic aspect of auctions was firstly recognized by the Nobel prize winner Vickrey [1961]. His work contributes to the understanding of the “revenue equivalence theorem” in the different auction formats. The theorem states that the seller can expect equal profits on average from all the standard (and many non-standard) types of auctions and that buyers are also indifferent among them all.

Reichert Ortega [1968] extended Vickrey’s work. More specifically, he analyzed the competitive bidding process as a strategic decision problem under uncertainty incorporating essential elements of decision-making with the use of quantitative models.

Myerson [1981] and Riley and Samuelson [1981] have shown that Vickrey’s results about the equivalence in expected revenue of different auction mechanisms apply very generally and show how to derive in optimal auctions: *“Under the assumption of a given number of risk-neutral potential buyers of an object with privately known signals, i.i.d. to a common, strictly increasing, atom-less distribution, any auction mechanism in which (i) the object always goes to the buyer with the highest signal, and (ii) any bidder with the lowest-feasible signal expects zero surplus, yields the same expected revenue (and results in each bidder making the same expected payment as a function of her signal). Both for private-valued models and common-value models.”*

Hence all “standard” auctions, the ascending, the descending, the first-price sealed-bid, and the second-price sealed-bid, yield the same expected revenue under the conditions above.

Myerson’s result is the most available treatment and also develops the mathematics used to prove the revenue equivalence theorem. He makes a step further to derive conclusions for optimal auctions. However, his work seemed to have little relationship with the traditional price theory making it hard to be understood by many economists [Klemperer, 2003].

Next Bulow and Roberts [1989] simplified the analysis of optimal auctions by “*applying the usual logic of marginal revenue versus marginal cost*” cited as their own

words. They showed that under the assumption of the revenue equivalence theorem, the expected revenue from an auction equals the expected marginal revenue of the winning bidders. Bulow and Klemperer [1996] generalize this result and extended to common values, non-independent private information, and risk-aversion, developing a result about the value to an auctioneer of an additional bidder relative to the importance of constructing an optimal auction.

It particular to focus on bidders' "marginal revenues". In the case of independent private values, a bidder's "marginal revenue" is defined as the marginal revenue of this firm at the price that equals the bidder's actual value [Bulow and Klemperer, 1996].

Thus an optimal auction allocates the objects to the bidders with the highest marginal revenues, similarly to a price-discriminating monopolist who sells to the buyers with the highest marginal revenues (by equalizing the lowest marginal revenues sold to across different markets) - without selling below the price where marginal revenue equals marginal cost [Klemperer, 2003]. All the above statements indicate how to run an optimal auction in the general case.

1.6.2 Budget constraints

In many situations, bidders may encounter financial constraints, which is why the revenue equivalence might fail in practice. In this section, we present an overview of the current literature on how the presence of budget constraints may influence equilibrium behavior in first-price and second-price auctions.

We start our discussion with Che [1998], who studied the performance of the first price and second-price auctions for one object, where bidders are privately informed about their valuation and their ability to pay. Their symmetric equilibrium bid function depends continuously on the valuation and on the budget constraint. Their intuition is that auctions that generate lower bids perform better since budget constraints will bid for less. They conclude that second-price auctions yield lower expected revenues and lower social surplus than first-price auctions, all else equal. Thus, the second-price auction should not be used when such constraints exist.

Dobzinski et al. [2012] study the fundamental question of how to produce an efficient

allocation in an incentive-compatible¹⁵ way using the concept of dominant strategies. Their setting is not quasilinear; therefore they focus on *Pareto Optimal* allocations rules and identify those that are implementable in dominant strategies. Their result states that there is no deterministic, incentive compatible and *Pareto Optimal* auction for any finite number $m > 1$ of units of an indivisible good and any $n \geq 2$ number of player. They examine the case of a “clinch auction” [Ausubel, 2004] where budgets are publicly known, and they show that the (only) possible incentive-compatible and *Pareto-Optimal* is this Ausubel’s mechanism. They show that the combination of incentive-compatible and Pareto Optimality yields the impossibility of private budgets and public budgets’ uniqueness.

Another major work related to budget constraints is the one performed by Hafalir et al. [2012]. Their research introduces a mechanism similar to Vickrey’s, and proposes a weakly dominant understanding of budgets and values. They suggest that since the revenue is increasing in budgets and values, all kinds of equilibrium deviations from the valuations turn out to be beneficial to the auctioneer, showing that the ex-post Nash equilibrium of their mechanism is *Pareto Optimal* since all full winners’ values are above all full loser’s values. They consider a set of hard budget constraints meaning that bidders cannot spend more than their budgets, and they leave a gap for further research for soft-budget constraints in which bidders will be able to finance further budgets at some cost, for instance, through the modeling of marginal value up to some budget in a linear value/budget function.

Till now, we have reviewed the literature that is concerned with exogenous constraints. Gavious et al. [2002] study the case of endogenous bidding constraints, where privately informed bidders bear a cost of bidding that is an increasing function of their bids, and bids may be capped. They have shown that for a finite set of bidders with linear or concave cost functions, the setting of a bid cap is not profitable for the auctioneer who wants to maximize the average bid. If a sufficient number of bidders have convex cost functions, then effectively capping the bids is profitable for the auctioneer. In an ex-ante symmetric model, the bid caps lower the highly-valuated bid type but increase the middle-valuated bid types. When bidders face increasing marginal cost, they increase their average bid, respectively decreasing marginal costs associated with

¹⁵All bidders maximize their utility by reporting their private information truthfully.

a decrease in average bid.

Finally, the most recent work on the endogenous budget constraints is this of Ausubel et al. [2017] in which they perform some laboratory experiments to compare auctions with endogenous budget constraints. The setting includes an agent (the bidder) who wants to acquire an asset with a private benefit, more than profit maximization. The “principal” imposes a budget constraint on bids which limits the bidder’s discretion. From one side, the principal chooses the budget, and from the other, the agent chooses the bid based on their respective signals. The most interesting variable in the experiment is the principal’s budget choice, while the bidder’s choice is non-essential. Their results showed that when the auction format is a first-price, the principal provides low bidder budgets rather than a second-price auction. The endogenous choice of budgets regarding the realized efficiency and revenues is not expected to be greater in the first-price than in the second-price auction.

Chapter 2

Auctioning Corporate Bonds: A Uniform-Price with Investment Mandates

Abstract

There has been a rapid growth in the use of investment mandates for the management of fixed-income assets. In this chapter, we examine how the limits set in investment mandates can affect the bidding strategy during the issuance of a corporate bond. We apply the uniform-price auction and prove the existence of symmetric Bayesian Nash equilibrium. Under the presence of an exogenous secondary market, an expectation for higher yields on resale increases the demand of the bond. Moreover, the number of participating investors and the oligopolistic market power of each investment manager always affect the bidding strategy inversely.

Keywords - Bond Market, Auctions, Market Structure, Market Design, Risk Limits, Budget Limits

JEL Classification - D44, D47, D82, D81, G23

2.1 Introduction

Over the last decade, the global market size of corporate bonds has more than tripled, yet with a decline in the overall bond credit quality (25% in 2019 with non-investment grade) combined with the longer maturities. The new environment triggered the enforcement of quantitative regulatory limits and self-imposed rating-based investment mandates and policies. For these reasons, there has been a significant shift in passive investment management¹, with major institutional investors using external credit ratings for their investment decisions and asset allocation [Appel et al., 2016]. For example, corporate bond ETFs² which typically use passive investment tools have reached from USD 32 billion in 2008 to USD 420 billion in 2018 [Çelik et al., 2020].

This trend has also been expanded in the primary market of corporate bonds, where the demand for the new issuance may be included in bond indexes if specific criteria are satisfied. Dathan et al. [2020] show that issuers exploit this passive demand by issuing index-eligible bonds with favorable characteristics and a higher passive demand increases issuers' propensity towards a new issuance.

Surprisingly, more and more investors complain that the access in the primary markets is restricted only to “flippers”³ supporting that “allocations always come down to favors” [Bessembinder et al., 2020; Cornelli and Goldreich, 2001; Jenkinson and Jones, 2004]. Resting upon this premise, the Securities Exchange Commission (SEC) has launched an investigation on how large financial institutions handle the allocation of debt during the issuance and penalized in 2020 a big institution for such violation⁴. To this extent, the Securities and Exchange Board of India (Sebi) proposed in 2016 a uniform-price auction for the pricing of corporate bonds, which would help to deepen

¹These strategies refer to a buy-and-hold portfolio strategy for long-term horizons. They are implemented by benchmarking certain market indexes (e.g. Barclays U.S. Corporate Bond Index,i Shares Short-Term Corporate Bond ETF).

²Passive investment vehicles which track various market indexes.

³Immediately resale or “flip” the bonds to other broker-dealers at a profit (<https://www.sec.gov/news/press-release/2020-159>).

⁴<https://www.ft.com/content/55406aea-a30a-11e3-ba21-00144feab7de>
<https://www.thetradenews.com/ubs-agrees-to-pay-10-million-to-the-sec-to-resolve-bonds-sale-violation-charges/>

the market⁵.

The common practice for the pricing of newly issued corporate bonds is an open-price process called book-building. Briefly, the issuer assigns to the underwriter the competitive sale and the efficient allocation of the new issuance. The underwriter markets the offering to investors, asking for a non-binding indication of interest (IOIs) [Iannotta, 2010; Nikolova et al., 2020]. This pre-market information helps the underwriter measure the demand and adjust the offering’s price and coupon if needed. A vital feature of the process is that the allocation is left in the underwriter’s discretion, with the IOIs often being cheaper than the final price⁶. Due to this pre-play communication which reveals investors’ valuations, the issuance’s allocation weakly corresponds to bidding while the yield not at all [Habib and Ziegler, 2007].

In this study, we develop an exploitative model for the pricing of corporate bonds in the primary market. We adopt a common-value uniform-price auction mechanism, which allows each investment manager to act as a bidder and submit sealed-bid demand schedules. Each demand schedule specifies the desired share over a fully divisible bond at different yield levels complying with the bounds set by the auctioneer.

Uniform-price auctions have been widely used in U.K. and U.S. for the selling of Treasury securities, with much debate in auction theory to be around the optimal choice between uniform-price and discriminatory auctions. However, neither empirical research [Binmore and Swierzbinski, 2000; Nyborg and Sundaresan, 1996; Tenorio, 1997], nor auction theory [Back and Zender, 1993; Bikhchandani and Huang, 1993; Wang and Zender, 2002; Wilson, 1979] offer a constraining reason for preferring uniform to discriminatory auctions. Ausubel et al. [2014] have shown that the uniform-price auction creates demand reduction incentives to bidders, reversing its strategic simplicity and efficiency. This result favors small bidders over the large ones, and under certain assumptions, the uniform auction outperforms the discriminatory.

We encompass in our analysis the investment mandate’s parameters at which investment managers abide by the objectives of investment strategies. For the asset

⁵<https://economictimes.indiatimes.com/mf/mf-news/sebi-proposes-uniform-pricing-for-debt-securities/articleshow/64200207.cms?from=mdr>

⁶Commission Expert Group on Corporate Bonds, Analysis of European Corporate Bond Markets, November 2017.

allocation limits, we employ a budget limit in line with a risk limit. The budget limit is the available capital for investing in the new issuance, and the risk limit is how much risk is acceptable for the investment’s capital (e.g., invest only in investment-grade bonds). To our knowledge, we are the first to address these factors in a mechanism for the pricing of corporate bonds.

Prior literature in auction theory has studied little the topic of budget limits. Some works [Benoit and Krishna, 2001; Che, 1998] find that between the standard auction formats, the second-price auction yields lower revenues than first-price auction in the presence of financial constraint in a private-value model, and that the revenue of a simultaneous ascending auction is lower than the revenue of a sequential auction. Ausubel et al. [2017] study the budget constraint as an endogenous factor and result in their experiment that the budget choices yield higher revenues and efficiency for second-price auctions. Hafalir et al. [2012] proposes a mechanism for divisible goods similar to Vickrey’s with a good revenue outcome and optimality properties in which it is weakly dominated if budgets or values are understated. Dobzinski et al. [2012] show that when budgets are public information the “clinching auction” of Ausubel [2004] is individual-rational and dominant strategy incentive compatible.

We include the secondary market as an exogenous random variable in our model, and we assume that all investors have the sole purpose of resale. Ex-ante, all investors have a private valuation for the bond, based on signals received for the expected equilibrium yield on the secondary market. Thus, at the time of the auction, the exact value of the bond, i.e. the issuance’s yield, is unknown. Theoretical research on uniform-price auctions in treasury bills markets with resale has shown that the auctioneer’s expected revenues are higher versus discriminatory when there is an equilibrium [Bikhchandani and Huang, 1989]. Additionally, uniform auctions favor higher information release, reducing uncertainty before the auction [Nyborg and Sundaresan, 1996].

Henceforth, the yield of the issuance named as the *stop-out yield*, is a market-clearing yield at the point where aggregate demand equals the full face value of the issuance, and it is defined by the *first rejected bidding schedule*. The intuition is that investors want to acquire a share of the face value at the highest possible yield to maximize their return in the secondary market.

Interestingly, the number of participating investors and a constant factor that measures the effective demand for the bond at different yield levels always affect the bidding strategy inversely. Also, it seems that the maximum spread that an investor can earn from the resale determines his bidding. As anticipated, if stringent mandates are followed with restricted budget limits, the bidding strategy is reduced leading to a lower cost of capital for the issuer.

The chapter is organized as follows. The following section 2.2 contains a formal analysis and describes the model as a direct revelation mechanism. In the same section, we introduce the concept of risk limit. Section 2.3, studies bidders' incentives and provides the proof of a Bayes-Nash symmetric equilibrium for independent signals performing the respective comparative statics. The last section 2.4, discusses the outcomes of previous sections and concludes. All proofs are expanded on the Appendix 2.5.

2.2 Model

2.2.1 Preliminaries

We assume a single unit of perfectly divisible bond for sale with a face value equal to one, and n competitive bidders, defined as a finite set $\mathcal{I} = \{0, 1, 2, 3 \dots n\}$, with $n \geq 3$. All bidders are risk-neutral, and none of them is eligible to bid for the full face value of the bond.

Each bidder i has an upper-bound bidding stipulated by the investment mandate, defined as the *budget limit* $c_i \in [\underline{c}, \bar{c}]$, as well as a *risk limit* $r_i^\ell \in [\underline{r}, \bar{r}]$. The type of i is defined as $\tau_i = (c_i, r_i^\ell)$, with $\tau \in \mathcal{T}$, which attributes bidders' preferences $\mathcal{T} := [\underline{c}, \bar{c}] \times [\underline{r}, \bar{r}]$ to the eligible real intervals. Each type τ_i , is i.i.d. to a continuous joint cumulative function $F(\tau) = F_{cr^\ell}(c, r^\ell)$ commonly known to all bidders, and fully supported by $f(\tau) > 0$.

Ex-ante, the equilibrium yield of the secondary market, is an unknown random variable, $r^s \in [\underline{r}, \bar{r}]$, with a cumulative distribution $H(r^s)$ that is common knowledge to all bidders. Also, it is fully supported by a density function $h(r^s) > 0, \forall r^s \in [\underline{r}, \bar{r}]$.

The expectation over the secondary market is denoted as $\mathbb{E}[r^s]$.

All information for the bidding strategies $\tau_{-i} := (\tau_1, \dots, \tau_{i-1}, \tau_{i+1}, \dots, \tau_n)$ is summarized in a joint cumulative function $G(r^s, \tau_{-i}) = H(r^s) \times F(\tau_{-i})$, fully supported by the density function $g(r^s, \tau_{-i}) > 0$. Additionally, bidders receive an independent private signal s about the actual private value of the bond and other bidders' preferences. This information for each bidder i is embedded in $s_i \in \mathcal{S}$, where \mathcal{S} is the signal space with an infinite number of elements that allow each bidder's value to be a general function of all the signals.

The strategy of each bidder i is a bid schedule, such as

$$b_i(r, s_i | \tau_i) : \mathcal{S} \times [r, \bar{r}] \rightarrow [0, 1] \quad (2.1)$$

defined on the signals' space \mathcal{S} while $[r, \bar{r}] \in R_+^*$ is the domain of eligible yields set by the auctioneer. Each schedule specifies the quantity demanded at a specific yield based on the different realizations of private signals for the secondary market. Bid schedules are assumed to be continuously differentiable to the yield r and an increasing continuous function in the budget limit c and the risk limit r^ℓ .

The issuance is produced through a mechanism (α, \hat{r}) consisting of two components: an allocation rule α and a payment rule \hat{r} . The allocation is an increasing continuous function that takes bidding strategies and parcels out the issuance to each bidder. Also, it is a linear increasing and differentiable function on the budget limit c . The payment rule \hat{r} is the *stop-out yield*, common for all winners as resulted from the uniform-price rule. In other words, if $b(\cdot)$ is a strategy profile for each type, then α_i is a fraction on the issuance for each type, then α_i is a fraction on the issuance that bidder i acquires paying \hat{r} .

For a strategy profile $b(\cdot)$ the payoff function of a risk-neutral bidder i given the observed signal $s_i \in S$ is:

$$\mathbb{E}_{(r^s, \tau)}[\pi_i(b|s_i)] = \mathbb{E}_{\tau|s_i} \left[\left(\hat{r}(b) - \mathbb{E}[r^s] \right) \alpha_i(b) \right]. \quad (2.2)$$

2.2.2 Market Mechanism

In this section, we will elaborate on the mechanism that produces the outcomes of the auction. The process starts with the simultaneous submission of bids. Following the uniform pricing rule [Bikhchandani and Huang, 1993; Krishna, 2010; Wang and Zender, 2002], a step function re-indexes individual bidding schedules until the issuance size is fully covered.

After the auction is completed, bidders from $1, \dots, j - 1$, are called full winners and from $j + 1 \dots, n$ are called losers. A cutoff bidder $j + 1$ with a bid schedule $b_{j+1}(\cdot)$, defines the *stop-out yield* \hat{r} and is the first of losers.

In the case of excess demand, $D(b) > 1$, there is a bidder j called partial winner where his demand might be partially satisfied at the point of *stop-out yield*. [Hafalir et al., 2012].

DEFINITION 2.1. *Winning bidders receive the stop-out yield \hat{r} , defined as the highest losing bid in a magnitude set by the auctioneer, where*

$$\hat{r} = \min\{r \in [\underline{r}, \bar{r}] \mid D(b) \geq 1\} \quad (2.3)$$

and

$$r = \begin{cases} \Theta - \theta D(b) & , \text{ with } b, \theta > 0 \text{ and } \Theta > \theta D(b) \\ 0 & , \text{ otherwise} \end{cases} \quad (2.4)$$

where $D(b) = \sum_{i=1}^n b_i(\cdot | \tau_i)$ for $n \geq 2$, Θ is an exogenous parameter denoting the opportunity cost of the issuer from alternative funding sources and θ is an exogenous sensitivity factor of the yield towards a change on $D(b)$.

Equation (2.4) is the inverse demand function of the issuance. In our analysis, the parameter θ remains symmetric for all bidders, which means that all of them have equal market power over the yield's structure.

Here below, we define an allocation rule that specifies how the asset is allocated so that no bidder gets more than his demand curve i.e., bid schedule [Kremer and Nyborg, 2007].

DEFINITION 2.2. *An allocation rule is a mapping from the set of bid schedules' profiles $b_i(\cdot)_{i=1}^n$ to non-negative allocations $\alpha \in (0, 1)$, with $\alpha(0) = 0$, such that $D(b) \geq 1$. Non-winners receive $\{\alpha\}_{j+1}^n = 0$, $\forall j \in \mathcal{I}$ and there is the partial winner j where $a_j = \omega$ such as $\omega \in (0, c_j]$, for $c_j > 0$.*

We focus for tractability reasons on a direct revelation mechanism, which means that all bidders truthfully report their types in the bidding schedule. We assume that the risk limit is common knowledge and the bidder's type is reduced to the budget limit only. This means that the only available action to each bidder is to announce the budget limit to the auctioneer [Dobzinski et al., 2012; Hafalir et al., 2012].

LEMMA 2.1. *The auction admits to a direct revelation mechanism, where bidders truthfully report their budget limit.*

Proof. Under the assumption of bidders' risk neutrality the payoff is linear and the budget limit is always binding at the equilibrium. \square

2.2.3 The concept of risk limits

This section will clarify the notion of risk limits since the evaluation of the payoff is only meaningful on a risk-adjusted basis, which creates limitations in investment decisions.

The intuition behind this notion is that each asset manager complies with a set of instructions or agreed-on constraints to manage investor's wealth. For instance, the investment mandate of funds (such as pension funds, mutual funds, ETFs, etc.) due to their idiosyncratic structures differentiate their investment strategy from retail investors. This means that the asset manager must adhere to more rigorous guidelines limiting the fund's ability to grab opportunities outside mandates. Baghai et al. [2019] perform a textual analysis of mutual funds' mandates and identify that credit ratings play a crucial role. The mandates require investments in investment-grade securities, fixed minimum ratings, or certain rating agencies. In our analysis, we assume that bidders' mandates are horizontal and require only investment-grade bonds.

Now, we will explain how the risk limit affects the infimum bidding amount. Let us assume a bidder i who has to comply with an investment mandate with a supremum

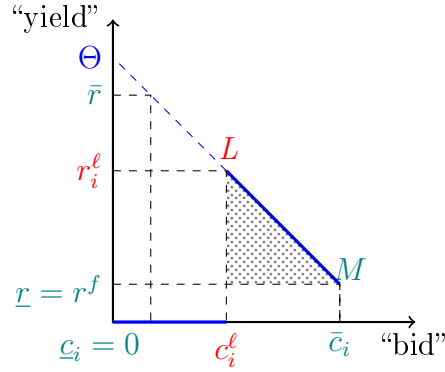


Figure 2.1: Mapping of budget and risk constraints on the inverse demand curve.

risk $r_i^\ell \in [r^f, \bar{r}]$, where r^f is the risk-free rate. We further assume that bidder i has a budget limit $c_i \in [0, \bar{c}]$, with c_i^ℓ to be the infimum bid associated with the risk limit r_i^ℓ .

Based on the instructions, bidder i will invest at least a bidding amount c_i^ℓ with a bidding strategy $b_i(\cdot|c^\ell)$, for an acceptable risk level r_i^ℓ (e.g. bonds with at least BBB^+ credit rating). This bidding strategy corresponds to point L , in Figure 2.1.

If the issuance concerns a bond with a lower probability of default (e.g. AA credit rating), then bidder i will increase his bidding amount. As the credit risk of the issuance is further reduced and $r^\ell = r^f$, the bidder will tend to invest all his budget $c^* = \bar{c}$ at the risk-free rate, submitting a bid equal to zero. This corresponds to point M in Figure 2.1, where the bidder becomes indifferent between investing in the bond and risk-free rate.

In reverse, bidder i will never bid for a “high-yield” bond i.e., \bar{r} because it is beyond his risk bounds. Thus, only the shaded area in Figure 2.1, expresses bidder’s willingness to participate in the auction.

2.3 Existence of symmetric equilibrium

This section proves the existence of symmetric Bayesian Nash equilibrium. Under the assumption that all bidders choose the same strategy b^* , we examine the auction from the bidder’s i point of view. The analysis from other bidders’ standpoints are symmetric. In a set of strategies $b_i^*(\cdot|\tau_i)$ of n bidders, bidder i maximizes the expected payoff for different realizations of signals. On this basis of equilibrium, information

updated through the signal space is the same and does not affect the outcome. Bidders privately observe the same signals before bid submission.

DEFINITION 2.3. *For each strategy $b_i \in \mathcal{B}$, where \mathcal{B} is the space of all strategies, there is an optimal strategy profile $b^* = (b_i^*, b_{-i}^*)$, which maximizes the expected payoff for all i , over the joint distribution $G(r^s, \tau)$ and the signal space \mathcal{S} . That is, for pure strategies for bidder i :*

$$\mathbb{E}_{(r^s, \tau)}[\pi_i(b_i^*, b_{-i}^* | s_i)] \geq \mathbb{E}_{(r^s, \tau)}[\pi_i(b_i, b_{-i}^* | s_i)]$$

Bidders types are independent and identically distributed in a probability function that is common knowledge to everyone, and we assume that risk limit r^{ℓ^*} is symmetric and common to all.

For our analysis, we assume that $y = y^{n-1}$ is a random variable that attributes the type profiles $(n-1)$ remaining bidders, and $f_{y|\tau_i}$ denotes the conditional density function of y given τ_i . Bidder i knows his type τ_i and that the highest value component-wise in y is τ .

The expected profit of bidder i is given by:

$$\mathbb{E}(\pi_i) = \alpha_i \int_{c^\ell}^{\bar{c}} \left[\hat{r}(b_i(\tau_i), b_{-i}(y)) - \mathbb{E}[r^s] \right] f_{y|\tau_i} dy \quad (2.5)$$

where α_i is the allocation rule (Definition 2.2), and $\bar{c} = \max_{j \in N/\{i\}} \bar{c}_j$, $c^\ell = \max_{j \in N/\{i\}} c_j^\ell$ respectively.

Let a minimum bid to participate in the auction defined upon the risk characteristics of the bond. Herein, we parametrize the minimum bid by $\lambda \in (0, 1)$. Hence, we assume that $b(c^\ell) = \lambda$ with an allocation equal to $\alpha(c^\ell)$.

Thus, bidder's decision problem is to choose a bid b to solve

$$\max_{b^*} \mathbb{E}[\pi_i(b_i^*, b_{-i}^* | s_i)],$$

if b_i^* solves this problem, then the strategy b_i^* is the best reply to $b_{-i} \dots b_n$.

THEOREM 2.1. *The n -tuple (b^*, \dots, b^*) is a symmetric Nash equilibrium under uniform-price auctions when bidders follow the same bidding strategy concerning their budget and risk limits. For $\xi = \frac{\theta}{\Theta - \mathbb{E}[r^s]}$ and $\xi < \frac{1}{\lambda n}$ the bidding strategy is*

$$b^*(c^*) = \lambda \frac{\alpha(c^\ell)}{\alpha(c^*)} + \frac{1}{\xi n} \left[1 - \frac{\alpha(c^\ell)}{\alpha(c^*)} \right], \quad (2.6)$$

with $c^* \in [c^\ell, \bar{c}]$.

Proof. See the Appendix 2.5.1. □

Alike the rationale of symmetric Cournot oligopoly, the more competitive bidders, the lower the equilibrium strategy. Also, the bidding strategy is directly affected by Θ , and it is used to calculate a spread from the resale in the secondary market. The parameter θ in ξ factor measures the response of stop-out yield to a bid's change. It attributes the oligopolistic effect of bidders upon the stop-out yield. As expected, the instructions set on the investment mandate directly impact the bidding strategy through the minimum bid λ and allocation $\alpha(c^\ell)$. Additionally, the ratio of the minimum allocation $\alpha(c^\ell)$ to symmetric allocation $\alpha(c^*)$ positively affects the equilibrium bid.

Next, we provide some basic comparative statics.

COROLLARY 2.1. *In symmetric equilibrium, the higher the oligopolistic power of bidders upon the stop-out yield, the lower their bid.*

Proof. The result follows trivially since the equilibrium bid depends on inversely to θ (recall that θ appears in the numerator of ξ). □

For an asymmetric θ , sophisticated investors with the strongest market power would demand a lower portion on the issuance, creating a residual supply for retail investors.

COROLLARY 2.2. *In symmetric equilibrium, the stop-out yield is given by,*

$$\hat{r} = \mathbb{E}[r^s] - (r^{\ell*} - \mathbb{E}[r^s]) \frac{\alpha(c^\ell)}{a(c^*)},$$

where $r^{\ell*}$ is the symmetric risk limit for a symmetric minimum bid $b^*(c^\ell)$.

Proof. See the Appendix 2.5.2. □

Since bidding strategies respond to investors' signals, the stop-out yield in the equilibrium reflects information about bond's value. From the investors' point of view, the degree of over-subscription and the market power of each participating investor is not among the stop-out-yield's determinants. The outcome is affected by the strictness of investment mandates that each bidder encounters, other things held constant. This means that investment mandates with low-risk acceptance and restricted budget limits can decrease the issuer's cost of capital. Inversely, an expectation for underpricing in the secondary market bounces the issuer's costs upwardly.

PROPOSITION 2.1. *In symmetric equilibrium, ceteris paribus, as the risk limit of bidders become strict (lower), i.e., r^ℓ goes to r^f , in the limit the equilibrium bid equals to λ .*

Proof. By the yield function, lower yields are associated with higher bids. I.e., for a decreasing sequence $(r_k^\ell)_{k \in N}$ corresponds to an increasing sequence $(c_k^\ell)_{k \in N}$. By letting c^ℓ to increase and given that $c^\ell < c^*$ then the ratio $\frac{\alpha(c^\ell)}{\alpha(c^*)}$ approaches to one. By equation (2.6) the result follows immediately. \square

From Proposition 2.1, it seems that a high demand guided by strict investment mandates would result in allocating the bond to a greater number of bidders, since any bidder who participates in the issuance in the limit would bid the minimum.

2.4 Conclusion

This study attempts to apply auction theory to the pricing of corporate bonds. The model is consistent with a statistical independent secondary market and the exogenous risk limits imposed by the investment mandates.

We prove the existence of symmetric Bayes-Nash equilibrium for a uniform-price auction, when investment managers have bounded budgets and comply with certain levels of risk. Each bidding strategy is inversely affected by the number of competitive bidders, which means that the investment managers will receive a smaller share of the bond (symmetric Cournot oligopoly) in an oversubscribed issuance. Similarly, the oligopolistic market power exercised by each investment manager affects equilibrium bids.

We have shown how investment mandates define the bidding strategy and the stop-out yield. If stringent mandates are followed with restricted budget limits, then the bidding will be reduced resulting in a lower cost of capital for the issuer, and the bond will be allocated to a greater number of investors.

The lending interest rates of other debt's sources are used as a benchmark to calculate a spread from the resale in the secondary market, and it seems that the bidding strategy follows the same course with this spread.

Contrary to the current practice for pricing corporate bonds, the uniform-price auction is a well-understood rule by all parties, and it is a mechanism already used for the pricing of Treasury bills. Investors reveal their valuations directly in their bid, so the final allocation and price reflect each bidding strategy.

2.5 Appendix

2.5.1 Proof of symmetric equilibrium

By Lemma 2.1 bidders directly reveal their types by announcing their budget limits. Thus, the expected profit from (2.5) can be rewritten as:

$$\begin{aligned}\mathbb{E}(\pi_i) &= \alpha_i \int_{c^\ell}^{\bar{c}} \hat{r}(b_i(c_i), b_{-i}(y)) f(y|c_i) dy - \alpha_i \mathbb{E}[r^s] \int_{c^\ell}^{\bar{c}} f(y|c_i) dy \\ &= \alpha_i \int_{c^\ell}^{\bar{c}} \hat{r}(b_i(c_i), b_{-i}(y)) f(y|c_i) dy - \alpha_i \mathbb{E}[r^s] [F(\bar{c}|c_i) - F(c^\ell|c_i)]\end{aligned}\quad (2.7)$$

We integrate by parts the integral $\int_{c^\ell}^{\bar{c}} \hat{r}(b_i(c_i), b_{-i}(y)) f(y|c_i) dy$, on the right hand side. By the continuity property of the distribution F , the probability of CDF for each bidder i to bid a budget, $c_i \leq c_i^\ell$, equals zero. Because none of the bidders will place a bid above their risk limit r_i^ℓ (Figure 2.1).

This means that when $b_{-i}(c^\ell, r^\ell) = 0$, the bond will not be issued, as none of the bidders can buy the whole issuance. In other words, the stop-out yield $\hat{r}(b_i(\tau_i), b_{-i}(c^\ell, r^\ell)) = \hat{r}(b_i(\tau_i), 0) = 0$.

Substituting in equation (2.7), the optimization problem is:

$$\max_{c_i} \mathbb{E}(\pi_i) = \alpha_i \left[\hat{r}(b_i(c_i), b_{-i}(\bar{c}))F(\bar{c}|c_i) - \int_{c^\ell}^{\bar{c}} \hat{r}'(b_i(c_i), b_{-i}(y))F(y|c_i)dy - \mathbb{E}[r^s]F(\bar{c}|c_i) \right]$$

$$\text{s.t. } \int_{c^\ell}^{\bar{c}} \hat{r}'(b_i(\tau_i), b_{-i}(y))F(y|c_i)dy \leq 0$$

To be a symmetric Bayesian Nash equilibrium, it is necessary that the first-order conditions to be zero. Ex ante, at the optimum, the expected stop-out yield can not be further diminished, hence the aforementioned constraint is satisfied with the equality.

Because of the symmetry, all bidders share the same type c^* . By Lemma 2.1 bidders maximize with respect to their budget limit.

$$\begin{aligned} 0 &= \left. \frac{\partial \mathbb{E}(\pi_i)}{\partial c_i} \right|_{(c_i=c^*)} \\ &= \left([\hat{r}(b_i(c_i), b_{-i}(\bar{c}))F(\bar{c}|c_i) - \mathbb{E}[r^s]F(\bar{c}|c_i)] \alpha_i(c_i) \right)' \\ &= \alpha_i'(c^*) \hat{r}(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*) - \alpha_i'(c^*) \mathbb{E}[r^s]F(\bar{c} = c^*|c^*) \\ &\quad + \alpha_i(c^*) \hat{r}'(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*) \end{aligned} \quad (2.8)$$

We substitute (2.4) to (2.8) and for simplicity reasons we denote $\rho(c^*) = \frac{\alpha_i'(c^*)}{\alpha_i(c^*)}$, which is the relative rate of change for the symmetric allocation $\alpha_i = \alpha^*$, and with b^* the symmetric bidding strategy. Thus,

$$\rho(c^*) [\Theta - n\theta b^*(c^*)] + [-\theta n b^{*'}(c^*)] - \rho(c^*) \mathbb{E}[r^s] = 0$$

Denominating with $(-\theta n)$ and by substitution of $\xi = \frac{\theta}{\Theta - \mathbb{E}[r^s]}$, where $\xi < \frac{1}{\lambda n}$, we result to a first-order non-homogeneous differential equation:

$$b^{*'}(c^*) + \rho(c^*) b^*(c^*) = \frac{\rho(c^*)}{\xi n}$$

The solution to the first-order differential equation is given by

$$\begin{aligned}
b^*(c^*) &= e^{-\int \rho(c^*) dc^*} \left(\int e^{\int \rho(c^*) dc^*} \frac{\rho(c^*)}{\xi n} dc^* + \Gamma \right) \\
&= e^{\ln \alpha^{-1}(c^*)} \left(\int e^{\ln \alpha(c^*)} \frac{\rho(c^*)}{\xi n} dc^* + \Gamma \right) \\
&= \frac{1}{\alpha(c^*)} \left(\int \frac{\alpha'(c^*)}{\xi n} dc^* + \Gamma \right) \\
&= \frac{1}{\alpha(c^*)} \left(\frac{\alpha(c^*)}{\xi n} + \Gamma \right)
\end{aligned}$$

where Γ is an arbitrary constant. Thus, we conclude

$$b^*(c^*) = \frac{1}{\xi n} + \frac{\Gamma}{\alpha(c^*)}, \text{ with } c^* \in [c^\ell, \bar{c}] \quad (2.9)$$

Now since $b(c^\ell) = \lambda$ is the initial condition of our differential equation, then the value of the constant $\Gamma = \alpha(c^\ell)[\lambda - \frac{1}{\xi n}]$, where $\alpha(c^\ell) \in (0, 1)$ and corresponds to the minimum allocation of the winning bidder. Thus, the solution of equation (2.9) is unique and can be re-written:

$$\begin{aligned}
b^*(c^*) &= \frac{1}{\xi n} + \frac{\alpha(c^\ell)[\lambda - \frac{1}{\xi n}]}{\alpha(c^*)} \\
&= \frac{1}{\xi n} + \frac{\alpha(c^\ell) \lambda}{\alpha(c^*)} - \frac{\alpha(c^\ell)}{\alpha(c^*) \xi n} \\
&= \lambda \frac{\alpha(c^\ell)}{\alpha(c^*)} + \frac{1}{\xi n} \left[1 - \frac{\alpha(c^\ell)}{\alpha(c^*)} \right].
\end{aligned}$$

□

2.5.2 Proof of Corollary 2.2

In the symmetric case, the symmetric equilibrium yield from (2.4) is:

$$\hat{r} = \Theta - \theta n b^*(c^*)$$

Substituting equation (2.6) in (2.4) we can rewrite equivalently:

$$\begin{aligned}\hat{r} &= \Theta - \theta n \left(\lambda \frac{a(c^\ell)}{a(c^*)} + \frac{1}{\xi n} \left[1 - \frac{a(c^\ell)}{a(c^*)} \right] \right) \\ &= \Theta - \frac{\theta n \lambda \alpha(c^\ell)}{\alpha(c^*)} - \Theta + \mathbb{E}[r^s] - (\Theta - \mathbb{E}[r^s]) \frac{\alpha(c^\ell)}{\alpha(c^*)} \\ &= \mathbb{E}[r^s] - (\Theta - \theta n \lambda - \mathbb{E}[r^s]) \frac{\alpha(c^\ell)}{\alpha(c^*)} \\ &= \mathbb{E}[r^s] - (\Theta - \theta n b^*(c^\ell) - \mathbb{E}[r^s]) \frac{\alpha(c^\ell)}{\alpha(c^*)}\end{aligned}$$

By substituting with the symmetric minimum bid $b^*(c^\ell) = \lambda$, we result in a equilibrium yield equal to the symmetric risk limit that is $r^{\ell^*} = \Theta - \theta n b^*(c^\ell)$. Thus, we conclude that

$$\hat{r} = \mathbb{E}[r^s] - (r^{\ell^*} - \mathbb{E}[r^s]) \frac{\alpha(c^\ell)}{\alpha(c^*)}.$$

□

Chapter 3

Blind portfolios' auctions in two-rounds

Abstract

This chapter proposes a two-stage sealed-bid model for the execution of portfolios. An asset manager auctions a portfolio of securities to a set of brokers who are unaware of the specific details about individual securities. We prove that our mechanism may reduce the costs of execution for the asset manager and may mitigate the “winner’s curse” for participating brokers.

Keywords - Two-round auctions, Affiliated signals, Principal blind bidding

JEL Classification - D44, D47, D53, G10

3.1 Introduction

One of the most challenging mechanisms in the modern investment trading strategies is the auction of portfolios. Here, we are concerned with the case of principal blind bidding, as it can reduce execution costs up to 48% relative to traditional channels [Kavajecz and Keim, 2005]. This type of strategy has reached about 11% of the average daily trading volume of NYSE in the last few years [Giannikos et al., 2012]. However, our proposed mechanism can be extended in all portfolios¹.

Under the rule of a first-price sealed-bid auction, an asset manager demands liquidity for a portfolio at an execution price and a group of brokers competes to undertake this execution for the lowest commission fee. Ex ante, brokers receive public information about the aggregate characteristics of the portfolio (e.g. the total value of the portfolio, sector exposure, volatility e.t.c.) and only the winning broker becomes aware of the actual securities included in the package. Also, brokers observe private signals regarding the expected value of the portfolio in case of winning.

Each broker's bid is the charged commission fee for executing the trades of securities upon the agreed price [Kavajecz and Keim, 2005; Padilla and Van Roy, 2012]. It is optimal for the asset manager to reveal portfolio's characteristics after market-closing and the agreed execution price to be the last closing price [Forsythe et al., 1989; Giannikos et al., 2012].

Asset manager's incentive to participate in principal blind auction is to reduce the costs of liquidation especially when there is an immediate need. Executing the portfolio in a mutually agreed price, mitigates and reduces asset manager's risk exposure, from any trading extensions.

Brokers' interest in the auction is not only to earn a commission fee, but likely to obtain a pool of reverse transactions which offset their own exposure or to open new positions. Brokers' challenge is to maximize profits by submitting the lowest possible bid (fee) to win the auction and, at the same time, to cover any risks associated with portfolio's acquisition.

¹For instance, in ICE bonds' portfolios the investment managers auction a portfolio of bonds on all-or-nothing basis to one or multiple bidders in a discrete, pre-determined period.

To our knowledge, the current literature for blind portfolios is limited due to data restrictions. The most important work is the one of Kavajecz and Keim [2005] who use historical data from an asset manager to model empirically the economic viability and performance of this trading mechanism. They find that broker's ability to cross shares with internal inventory and the longer trading out period reduce costs relative to traditional channels. Giannikos and Suen [2007a,b] and Giannikos et al. [2012] make one step further to explain the perception of brokers to risk exposure in the stock's spread. They use proxies to measure the two major categories of inventory risk and information asymmetry risk in order to predict the winning bids. Finally, Padilla and Van Roy [2012] use a theoretical model of a first-price auction to compute the impact of a trusted intermediary on transaction costs. They perform a comparative analysis on the number of participating brokers and broker's risk aversion.

Our work is the first study that discusses the "winner's curse" in principal blind bidding. We propose a two-stage auction under the frame of "affiliated values" by Milgrom and Weber [1982] and Perry et al. [2000]. Initially, brokers submit simultaneously their fees as a sealed-bid knowing only the aggregate characteristics of the portfolio. Only brokers with the two lowest fees move on the second stage. Then, ineligible bidding fees for the next stage are revealed. In the second stage, the two remaining brokers participate in a sealed second-price auction constrained by their first-round bidding fees.

This mechanism performs better than first-price auction, since information release for the losing bids at the interim can further lower costs for the asset manager. Additionally, the revelation of losing bids discloses others' private information about the change in securities' prices [Milgrom and Stokey, 1982] and addresses to the problem of "winner's curse" associated with portfolio's valuation. Yet, too much information can be detrimental [Kavajecz and Keim, 2005; Milgrom and Stokey, 1982]. If the bidders are aware of the actual securities included in the portfolio, they can quote the entire trading cost of the asset manager, while, if the trading costs of the bidders are revealed to the asset manager, the latter can extract the bidding surplus.

The chapter is organized as follows. Section 3.2 introduces concepts and notation used in the model and market's mechanism. Section 3.3 derives the intuitive form for

the conditions that enable the equilibrium analysis. Finally, section 3.4 concludes and discusses the results.

3.2 Preliminaries

An asset manager puts at auction $m > 2$ securities packaged in an indivisible portfolio $\theta \in \mathbb{R}^m$. We denote by $\theta^+ = \max\{0, \theta\}$ the long part of the portfolio and $\theta^- = \min\{0, \theta\}$ the *short* part. We assume that a set of risk-neutral brokers, $\mathbb{N} = \{1, \dots, n\}$, are competing² for θ .

The auction occurs in two rounds: $t = 1, 2$. Each broker $i \in \mathbb{N}$ submits consecutively a bid (fee) $\phi_i^t \in [0, 1]$. For arbitrary security k , brokers form expectations on the percentage change of its price, $\frac{\Delta p_k}{p_k^*}$, with $\Delta p_k = \mathbb{E}(p_k) - p_k^*$, where $p_k^* \in \mathbb{R}_+$ denotes the agreed exercise price and $\mathbb{E}(p_k) \in \mathbb{R}_+$ the anticipated price of the security when delivered, conditional on broker's private information.

Information is revealed in two stages. Initially, each broker i observes a private signal $s_i \in [-s, s] \subset \mathbb{R}$ for the random variables Δp_k and θ_k and a public signal $z \in \mathbb{Z}$ for the aggregate characteristics of portfolio. Denote by \mathbf{s} the vector of private signals for all brokers. Random variables (\mathbf{s}, z) are drawn from a continuous and symmetric joint distribution $F(\cdot)$, which is fully supported by the density $f(\cdot) > 0$ that satisfies the affiliation properties of Milgrom and Weber [1982]. After the completion of the first round, the asset manager updates public information. We denote respectively the order statistics Y_1, Y_2, \dots for the bids of the losing participants.

3.2.1 The Mechanism

The auction occurs in two rounds:

1st round. The asset manager puts up the portfolio for bidding in a first-price sealed-bid auction, where the two lowest bids are qualified for the second round. At the outset, broker i receives information (s_i, z) and submits a fee (in basis points) ϕ_i^1 to execute the portfolio trade at $p^* \in \mathbb{R}_+^m$. Each bidding strategy at the restricted

²Without loss of generality we assume no entry costs for participating brokers.

game ϕ_i^t is monotonically decreasing on the private signal s_i and $\mathbb{E}(p_k)$ is monotonically increasing.

2nd round. The asset manager updates the available public information by revealing Y_1, Y_2, \dots , which is related to the losing bids of the first-round participants. A sealed-bid second-price auction takes place for the two winners. Each winner $i \in \mathbb{N}$ submits a fee $\phi_i^2 \in [0, \phi_i^1]$. The auction design does not allow any bid to exceed the first-round bidding. The lowest bid wins the auction and the winning broker charges the second lowest bid.

The standard tie-breaking rule applies to both rounds. Figure 3.1 presents the two-round auction design comparatively to the existing auction format.

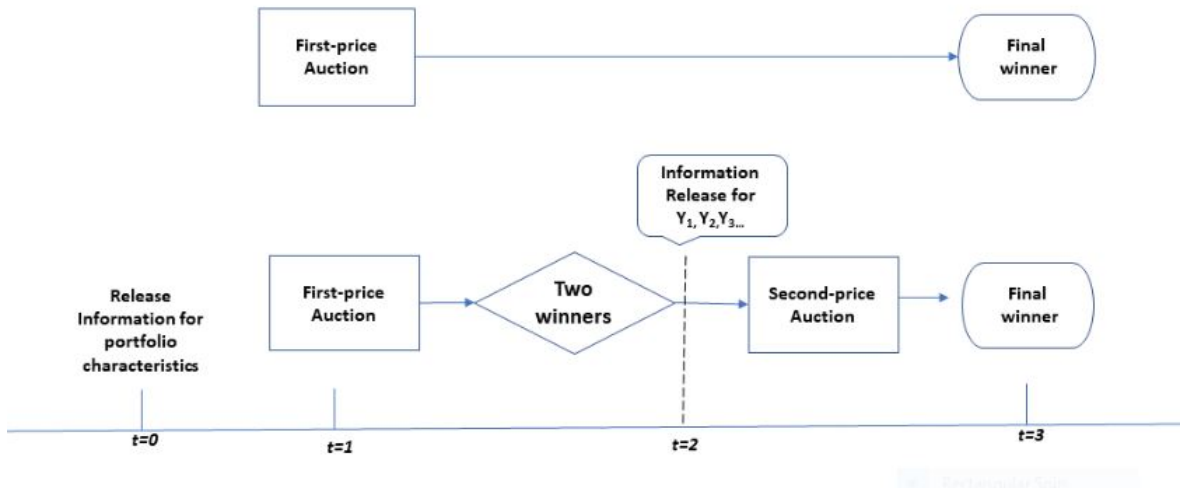


Figure 3.1: Two-round auction design.

The broker aims to submit a fee which neutralizes potential losses. The expected payoff of the selected winner i for security k is given by

$$\pi_{i,k}(\phi^2 | s_i, z) = \mathbb{E} [|\theta_k(s_i, z)| \cdot p_k^* \cdot \phi_j^2 - \theta_k(s_i, z) \cdot \Delta p_k(s_i, z)] \cdot \mathbf{1}_{\{\phi_i^2 < \phi_j^2\}} \quad (3.1)$$

where the last term is an indicator function for $\phi_i^2 < \phi_j^2$.

For each security k , the expected payoff of the broker is a fee received upon the value of the trade ($|\theta_k(s_i, z)| \cdot p^* \cdot \phi_j^2$) minus the potential losses from the price variation ($\theta_k(s_i, z) \cdot \Delta p_k(s_i, z)$), if exist. For the aggregate portfolio θ winner's payoff results immediately from (3.1) and follows the same rule. For portfolio $\theta \in \mathbb{R}^m$ and security prices $p^* \in \mathbb{R}_+^m$, the payoff becomes

$$\pi_i(\phi^2 | s_i, z) = \mathbb{E} [|\theta(s_i, z)| \cdot p^* \cdot \phi_j^2 - \theta(s_i, z) \cdot \Delta p(s_i, z)] \cdot \mathbf{1}_{\{\phi_i^2 < \phi_j^2\}}. \quad (3.2)$$

3.3 Equilibrium

Since fees come to countervail potential losses, competing fees will most likely tend to become zero. Two cases are legitimate, either when it is a *long* position (θ_k^+) and the expected price differential is negative, *i.e.*, $\mathbb{E}(p_k) < p_k^*$, or when it is a *short* position (θ_k^-) and the expected price differential is positive, *i.e.*, $\mathbb{E}(p_k) > p_k^*$. In both cases, the expected profit turns to be nonnegative for a nonnegative fee. The following proposition suggests this result for the symmetric case (*i.e.*, $s_i = s_j$ for all $i \neq j$).

PROPOSITION 3.1. *For any arbitrary security k , when brokers anticipate a long position θ_k^+ and $\Delta p_k(s_i, z) < 0$ or a short position θ_k^- and $\Delta p_k(s_i, z) > 0$, brokers submit a fee $\phi^1 = \mathbf{0}$, with $\phi^1 \in \mathbb{R}_+^n$.*

Proof. Under these assumptions, the expected profits are always nonnegative. We claim that in the first round they will bid a fee equal to zero. Suppose that this is not the case and in the first round brokers submit $\phi^1 > \mathbf{0}$. Then any arbitrary broker i has an incentive to submit a lower fee to ensure his participation in the second round. Thus, the bidding profile $\phi^1 > \mathbf{0}$ cannot be the first-round equilibrium. Necessarily, it is $\phi^1 = \mathbf{0}$. \square

By Proposition 3.1 an interesting corollary follows.

COROLLARY 3.1. *In the second round, $\phi^2 = \mathbf{0}$ for $\phi^2 \in \mathbb{R}_+^2$, is a weakly dominant strategy symmetric equilibrium.*

Proof. The proof follows directly from the assumption that $\phi_i^2 \in [0, \phi_i^1]$, with $i = 1, 2$ the winning broker of the first round. \square

Next, we prove the symmetric equilibrium for the remaining cases *i.e.*, θ_k^+ with $\Delta p_k(s_i, z) > 0$ and θ_k^- with $\Delta p_k(s_i, z) < 0$. In both cases, the expected profit can be negative. The ϕ is activated as a countervailing mechanism.

Now we will show the restricted second-round symmetric equilibrium.

PROPOSITION 3.2. *Suppose θ_k^+ with $\Delta p_k(s_i, z) > 0$ and θ_k^- with $\Delta p_k(s_i, z) < 0$, for all k . The symmetric equilibrium of the restricted second-round auction is*

$$\phi^2(s^*, s^*, z) = \min\left\{\phi_k^1(s^*, s^*, z), \frac{\Delta p_k(s^*, s^*, z)}{p_k^*}\right\}$$

and it is unique for monotonically decreasing bidding strategies.

Proof. We prove the result for arbitrary security k . Without loss of generality, suppose bidder j is the loser of the second-round auction. Generally in Vickrey auctions, bidding your valuation is a weakly dominant strategy equilibrium and bidders achieve no surplus.

In our case, it is $\mathbb{E} [|\theta_k(s_i, z)| \cdot p_k^* \cdot \phi_j^2 - \theta_k(s_i, z) \cdot \Delta p_k(s_i, z)] = 0$ or equally $\phi_i^2 = \frac{\Delta p_k(s_i, z)}{p_k^*}$. Following Milgrom and Weber [1982] (Theorem 6) for the symmetric equilibrium and since $\phi^2(s^*)$ is monotonically decreasing, the conditional expected payoff by (3.1) for the winning bidder i becomes:

$$\begin{aligned} & \mathbb{E} \left[(|\theta_k(s_i, Y_1, z)| p_k^* \phi_j^2 - \theta_k(s_i, Y_1, z) \Delta p_k(s_i, Y_1, z)) \cdot \mathbf{1}_{\{\phi_i^2 < \phi_j^2(Y_1)\}} \mid s_i = s^* \right] \\ &= \mathbb{E} \left[\mathbb{E} [(|\theta_k(s_i, Y_1, z)| p_k^* \phi_j^2(Y_1) - \theta_k(s_i, Y_1, z) \Delta p_k(s_i, Y_1, z)) \cdot \mathbf{1}_{\{\phi_i^2 < \phi_j^2(Y_1)\}} \right. \\ & \quad \left. \mid s_i = s^*, Y_1 = s^*, z \right] \mid s_i = s^* \right] \\ &= \int_{-s}^{s^*} [|\theta_k(s^*, \alpha, z)| p_k^* \phi_j^2(\alpha, \alpha, z) - \theta_k(s^*, \alpha, z) \Delta p_k(s^*, \alpha, z)] f_{Y_1}(\alpha | s^*, z) d\alpha. \end{aligned}$$

where f_{Y_1} denotes the conditional density of Y_1 given $s_i = s^*$ (with s^* to be the symmetric signal) and public information z . By substituting the symmetric bidding strategy where $\phi_j^2 = \phi_i^2 = \frac{\Delta p_k(s^*, s^*, z)}{p_k^*}$ we have

$$\begin{aligned} & \int_{-s}^{s^*} \left[|\theta_k(s^*, \alpha, z)| \frac{p_k^* \Delta p_k(\alpha, \alpha, z)}{p_k^*} - \theta_k(s^*, \alpha, z) \Delta p_k(s^*, \alpha, z) \right] f_{Y_1}(\alpha | s^*, z) d\alpha \\ &= \int_{-s}^{s^*} [|\theta_k(s^*, \alpha, z)| \Delta p_k(\alpha, \alpha, z) - \theta_k(s^*, \alpha, z) \Delta p_k(s^*, \alpha, z)] f_{Y_1}(\alpha | s^*, z) d\alpha \end{aligned}$$

Since Δp_k is increasing in the first argument, the integral is negative for $a < s^*$ and positive for $s^* < a$. Indeed, the integral is maximized by choosing $\phi_i^2(s) = \phi_j^2(s^*)$.

This proves that $\phi_i^2(s^*)$ is the best response for bidder i . *Mutadis mutandis* the result generalizes for the portfolio. \square

Similarly, in the restricted first-round game, the brokers' incentive is to participate in the auction without understating their valuations. At the same time, they want to avoid being excluded from the second round. The following lemma suggests that if there is a strict best response towards rivals' bidding, then brokers ask for a higher fee.

LEMMA 3.1. *Suppose a signal s and $\phi^1(s)$ the first-round bid function for all brokers. Then, $\tilde{\phi}_i^1(s)$ is the strictly best response to $\phi_{-i}^1(s)$, only if $\tilde{\phi}_i^1(s) > \phi_i^1(s)$.*

Proof. We prove this by contradiction. Suppose $\tilde{\phi}^1(s) < \phi^1(s)$, then this response qualifies broker i for the second round. Because bidding strategies are monotonically decreasing, it must be true that $s > Y_1$. Yet, Δp_k is increasing in the first argument with $\frac{\Delta p_k(s, Y_1, z)}{p_k^*} > \frac{\Delta p_k(Y_1, Y_1, z)}{p_k^*}$, for all k . If this is the case, the expected payoff turns to be negative.

In exposition, if $\pi_i(Y_1, Y_1, z) = 0$ then $\pi_i(s, Y_1, z) < 0$. Thus, it is not the best response for broker i to bid lower. \square

Now, we examine the first-round restricted game equilibrium.

PROPOSITION 3.3. *Suppose the symmetric case, where all receive signal s^* . Then, for an expected percentage change in prices $\frac{\Delta p(s^*, s^*, z)}{p^*} = \left(\frac{\Delta p_1}{p_1^*}, \dots, \frac{\Delta p_m}{p_m^*} \right)$, all strict monotone decreasing strategies $\phi^1(s^*)$ such that*

$$\phi^1(s^*) \geq \frac{\Delta p_k(s^*, s^*, z)}{p_k^*}$$

are symmetric equilibria, for each security k .

Proof. A qualified broker for the second round with signal s^* , never regrets bidding $\phi^1(s^*) > \frac{\Delta p_k(s^*, s^*, z)}{p_k^*}$ in the first round. By Lemma 3.1, a deviation from $\phi^1(s^*)$ must satisfy $\tilde{\phi}^1(s) > \phi^1(s^*)$ to be profitable. Under this condition, bidding $\tilde{\phi}^1(s)$ does not qualify him for the second round, and this deviation makes sense only when $s < s^*$. Thus, he forgoes a net cost $\frac{\Delta p_k(s, Y_1, z) - \Delta p_k(s^*, Y_1, z)}{p_k^*} < 0$ for not participating in the second round. It is evident that if the condition is satisfied with equality the symmetric equilibrium is strict. \square

3.3.1 Cost Effects

Next, we examine how the asset manager's decision in the second round to update brokers with credible public information may affect the auction's outcomes. Following the same analysis, the symmetric equilibrium given by *Proposition 3.2* holds.

PROPOSITION 3.4. *Information revelation at interim may induce lower execution costs for the asset manager.*

Proof. The asset manager's incentive is to minimize the execution costs. By design, the second-round fee is always bounded in $[0, \phi^1]$. Next, we prove that in some occasions, fees may be lower. At interim, public information $z \in \mathbb{Z}$ is updated by encompassing the order statistics of first-round losing participants, Y_1, Y_2, \dots . It turns out that random variable z takes a new value z' . First, suppose $z' > z$, and the broker i updates his private information to $s'_i > s_i$ by the affiliation property. Let an arbitrary security k in a "short" position (θ_k^-). This results in $\frac{\Delta p_k(s'_i, z')}{p_k^*} > \frac{\Delta p_k(s_i, z)}{p_k^*}$. In the second round, new information admits a lower bid by the difference of $\frac{\Delta p_k(s'_i, z') - \Delta p_k(s_i, z)}{p_k^*} > 0$. Hence, $\phi_i^2(s_i, z) \geq \phi_i^2(s'_i, z')$. If the security k is "long" (θ_k^+), the broker i incurs a loss, still $\phi^2 \in [0, \phi^1]$, i.e., the broker cannot make an upward correction. Second, $z' < z$ the broker updates information with $s'_i < s_i$. This results in $\frac{\Delta p_k(s'_i, z')}{p_k^*} < \frac{\Delta p_k(s_i, z)}{p_k^*}$. If the security is "long" (θ_k^+), an update in information allows a lower bid again by the difference $\frac{\Delta p_k(s'_i, z') - \Delta p_k(s_i, z)}{p_k^*}$. This means that $\phi_i^2(s_i, z) \geq \phi_i^2(s'_i, z')$ while if "short" (θ_k^-) broker i experiences a loss bounded by ϕ^1 . \square

Remark. By updating public information at the beginning of the second round, the asset manager apprises first-round winners of others' unwillingness to bid low. This information is unfavorable for the valuation of $\frac{\Delta p(s_i, z)}{p^*}$ and reveals the "winner's curse" that was induced by the first-round bidding. However, this information release can mitigate "winner's curse" from second-round bidding e.g. if the winner did not have information for the unfavorable valuation he could bid lower and he would have a worse payoff than anticipated. On the other side, the first-round bids constrain the asset manager's costs to any increase. Contrariwise, releasing favorable information for winners, curtails the asset manager's cost and mitigates "winner's curse".

3.4 Concluding Remarks

This research represents the first attempt to explore the “winner’s curse” phenomenon in blind portfolios as a two-stage sealed-bid trading mechanism. We have shown for both stages the existence of symmetric equilibrium. This approach elegantly sidesteps bidding complexity and any computational problems arising in practice, as it is a direct incentive-compatible mechanism [Perry et al., 2000]. It promotes competition among brokers and ensures that this auction format meets the requirements set by the asset manager.

Information disclosure after the first-round bidding, can further reduce asset manager’s costs. On top of that, it allows brokers to access spot market signals and to update their valuations. Thus, the broker can predict more accurately the true value of the portfolio. This mitigates “winner’s curse” effect, and in that sense improves brokers’ efficiency in blind portfolio auctions.

A possible extension of the model could be a package auction of a fully divisible blind portfolio. This can be conceptually viewed in the framework of Day and Milgrom [2008], in which a mechanism is designed to maximize the social welfare of all participants.

Chapter 4

A core-selecting auction for portfolio's packages

Abstract

We introduce the “local-global” approach for a divisible portfolio, and perform an equilibrium analysis for two variants of core-selecting auctions. Our main novelty is extending the Nearest-VCG pricing rule in a dynamic two-round setup, mitigating bidders’ free-riding incentives and further reducing the sellers’ costs. The two-round setup admits an information-revelation mechanism that may offset the “winner’s curse”, and it is in accord with the existing iterative procedure of combinatorial auctions. With portfolio trading becoming an increasingly important part of investment strategies, our mechanism contributes to increasing interest in portfolio auction protocols.

Keywords - Package auction · VCG payments · Portfolio Trading

JEL Classification - D44 · D47 · G11

4.1 Introduction

Portfolio Auctions (PA) have recently been in the limelight due to their rapid growth in fixed-income markets. The growth approximates 5% of total market trading volume based on the recent estimates of TRACE¹. The changes in the market behavior have been driven by the various developments in ETFs and the surge of algorithmic trading, which has facilitated the vertical slicing of portfolios². The automated execution protocol for portfolio trading by ICE Bonds Portfolio Auction and Tradeweb's portfolio trading platform are examples of the current investment strategies in using auctions for trading portfolios.

More commonly known as basket or program trading, it has been in the investment landscape for a while and has accounted for 50% to 60% of the total daily trading volume at the NYSE³. It operates in two formats called principal and agency. The portfolios are offered to a discrete group of broker-dealers who can offer bids fast for a bundle of trades executed as a single transaction.

In this chapter, we focus on *principal* trade in which a broker undertakes the risk price on behalf of the asset manager and executes the portfolio at an agreed price plus a commission fee. After acquiring the portfolio, the broker must attempt to minimize any deficit caused by the actual execution versus the agreed price with the asset manager (a price equal or better in the market).

The impact for asset managers is vast, especially in periods of high volatility where cost savings and speedy risk transfer are imperative. Even though the asset managers always have the option to execute those trades at their discretion, using a portfolio auction trading tool enables them to have access to multiple liquidity providers simultaneously, to reduce any information leakage and to guarantee execution's efficiency—an efficient way to deal with large and complex transactions.

From the broker's perspective, this strategy is an opportunity to gain access to new

¹<https://www.finra.org/filing-reporting/trace>

²When considering liquidity management a portfolio can be sliced horizontally or vertically. A slice of liquid assets is described as slicing the portfolio horizontally, while a proportional slice on the entire portfolio is considered as vertical slicing.

³In periods with high volatility, it can reach up to 90%. Since 2012, the NYSE no longer publishes weekly program trading reports.

order flows that might match their inventory or facilitate new business [Padilla and Van Roy, 2012]. The challenge is to bid low enough to win the auction and at the same time to cover the assumed price risk from the execution.

In the current form, the auction rule is a single round first-price sealed-bid, and the entire portfolio is awarded to one winning broker. However, it might be possible that some brokers have a higher valuation for a slice of the portfolio. In this case, the asset manager might receive a lower overall bid for the whole portfolio if he attributes the different slices to various brokers.

We divide the portfolio into packages for a discrete number of brokers with different valuations in this context. We characterize those who compete for the packages as “local” brokers and those who bid for the whole portfolio as “global” brokers. Each of the brokers is aware of the individual securities included in the portfolio, which takes long positions in all securities.

The auction occurs in two rounds. In the first round, the asset manager performs simultaneous sealed-bid first-price auctions (a) for “local” brokers who compete for each package separately, and (b) for “global” brokers who compete for the aggregate portfolio. The first round qualifies one global broker and one local broker for each package to participate in the second round. The asset manager releases information with the qualified bids for the discrete number of local brokers and the global broker. This attribute in the design provides a solution to brokers’ information asymmetries by revealing valuations.

In the second round, the local brokers jointly compete for the whole portfolio against the global, who values packages as perfect complements⁴. If the coalition of local brokers submits a bid lower than that of global, the coalition wins, and the fees are awarded to each local broker based on a pricing rule that ensures a *core* outcome.

The *core* is a set of payoff vectors that correspond to a set of allocations of packages, where no better outcome exists both for the asset manager and the local brokers. In other words, the coalition of local brokers is unblocked and feasible [Day and Milgrom, 2008]. Any payoff vector for the coalition of local brokers is obtained in the core is

⁴The utility of the whole portfolio has a higher utility than the sum of the utilities for the individual packages [Cramton et al., 2006].

bidder-optimal only if there is no other payoff vector which is *Pareto-optimal*. The set of such points is called the *bidder-optimal-frontier* of the core [Milgrom, 2004].

We perform an equilibrium analysis for two core-selecting auctions: the well-known Nearest-VCG rule [Day and Cramton, 2012] and a “Dynamic” version of Nearest-VCG rule that fits multiple round auctions that we are introducing here.

4.2 Related Literature

Most literature in package auctions (or combinatorial auctions) focuses on technicalities for developing fast heuristics to solve the complex winner’s determination problem [De Vries and Vohra, 2003; Rothkopf et al., 1998], while economists focus on specific properties by using simplified theoretical models.

The theory of package auctions can be traced to the seminal paper of Vickrey [1961], in which each bidder is asked to pay an amount equal to the externalities he exerts on the competing bidders. Vickrey showed that this payment rule motivates bidders to submit a “bid” according to the actual demand schedules, regardless of the bids made by others. After some years, Clarke [1971] and Groves [1973] generalized the Vickrey mechanism to other applications.

It is easy to think that a Vickrey auction could generate an efficient outcome for package auctions due to its appealing property of incentive compatibility. Nevertheless, in practice, the Vickrey auction is never used because this mechanism can lead to low payoffs for the auctioneer, even if bids are high enough [Milgrom, 2007]. Also, the Vickrey pricing is determined by a non-monotonic function of broker’s values in the sense that an increase in the number of brokers can reduce equilibrium revenues for the asset manager up to zero. Thus, brokers can use profitably “shill brokers” to increase competition in order to finally charge higher fees [Ausubel and Milgrom, 2002, 2006].

To mitigate the aforementioned shortcomings, the existing literature has proposed alternative procedures. Ausubel and Milgrom [2002] developed a mechanism called the ascending proxy auction, while, Day and Milgrom [2008] and Day and Cramton [2012] suggested a new cluster of payment rules for core-selecting auctions with respect to the reported values.

Recently, Ausubel and Baranov [2020] provide a theoretical justification for the use of core-selecting auctions. They propose an incomplete-information setting in which bidders' values are correlated and analyze the equilibrium under a "local-local-global" approach. They found that in environments with positive correlations, core-selecting auctions can be significantly closer to the true core than the VCG outcome⁵. To our knowledge, Krishna and Rosenthal [1996] were the first to explore an independent private value setting⁶ for the simultaneous sale of multiple items in the "local-global" setting.

Since package auctions have this cooperative flavor, the problem that arises when the coalition wins is the distribution of payments, when the VCG outcome is not in the core⁷. Then, the closer the bidder-optimal frontier gets to VCG pricing, the fewer incentives for misreporting [Ausubel and Milgrom, 2002; Day and Milgrom, 2008]. Day and Raghavan [2007] and Day and Cramton [2012] find alternative payment rules which minimize bidder's incentives for this strategic manipulation. The rule that they discuss is called the *Nearest-VCG*, which is a point in the bidder-optimal-frontier where the maximum deviation from VCG pricing is minimized.

Alternatively, Erdil and Klemperer [2010] have proposed a new class of pricing rules for core-selecting package auctions focusing on the marginal incentives to deviate from "truthful bidding". Those rules are called "reference rules" and are determined, as far as possible, independently by bidders' bids. The idea is to select a point in the bidder-optimal frontier that is close in a reference point. Motivated by their suggestion, we construct a new payment rule, the Dynamic-Nearest-VCG, using an endogenous reference point suggested by the brokers' own strategic bidding behavior.

One of the merits of an iterative combinatorial auction,⁸ is their ease of deployment. It allows bidders to learn about rivals' valuation, and it is the most popular combinatorial auction format used in practice. For example, the FCC has used only multi-round formats for its auction design [Pekeč and Rothkopf, 2003].

⁵Goeree and Lien [2016] showed that core-selecting auctions concerning true values do not exist.

⁶Rosenthal and Wang [1996] extended the setting with common values.

⁷If the VCG outcome is in the core, it is the unique bidder-optimal allocation.

⁸A multiple-round bidding process at which the auctioneer releases information regarding the provisional winners and the actual prices, at the end of each round. Bidders obtain information regarding the bids of their rivals and can modify their bids in the following rounds [Parkes, 2006].

Our chapter proceeds as follows. Section 4.3 presents the model and describes the mechanism for the two rounds. Section 4.4 derives to the intuitive form of the optimality conditions and analyses the equilibrium. Finally, section 4.5 discusses the results and concludes. All proofs can be found in the Appendix 4.6.

4.3 Model

An asset manager sells $m > 2$ securities packaged in a divisible portfolio $\Theta \in \mathbb{R}_+^m$. A set of risk-neutral brokers, $\mathbb{N} = \{1, \dots, n\}$ are competing⁹ for a portfolio Θ . There are two types of brokers who participate in the auction, a set of *local* brokers denoted by the generic element $\ell \in \mathbb{L}$, and a set of *global* brokers $g \in \mathbb{G}$, where $\mathbb{L}, \mathbb{G} \subset \mathbb{N}$ are disjoint and $\mathbb{L} \cup \mathbb{G} = \mathbb{N}$.

We assume that Θ is divided in a finite set of q packages θ_j , with $j = 1, \dots, q$, such as $\theta_j \in \mathbb{R}_+^m$ and $\Theta = \sum_{j=1}^q \theta_j$. Each broker $i \in \mathbb{N}$ observes a private signal $s_i \in \mathbb{S}$ about the value of the Θ or θ_j and a public signal $z \in \mathbb{Z}$ for the aggregate characteristics of portfolio. Information (\mathbf{s}, z) , where $\mathbf{s} = (s_i, s_{-i}, s_g) \forall i \in \mathbb{L}$ and $g \in \mathbb{G}$, is distributed according to a continuous i.i.d. function $F_\ell(\cdot)$, with the density function $f_\ell(\cdot) > 0$ for *local* brokers, and $F_g(\cdot)$, with $f_g(\cdot) > 0$ for *global* brokers respectively.

All brokers form expectations for the percentage change of the securities' prices given by the vector,

$$\mathbb{E}\left[\frac{\Delta p}{p^*} | s_i, z\right] = \left(\frac{p_k^* - \mathbb{E}[p_k | s_i, z]}{p_k^*} \right)_{k \leq m},$$

where the vector $p^* \in \mathbb{R}_+^m$ includes the agreed exercise price for m securities, and the vector $p \in \mathbb{R}_+^m$ the anticipated price of m securities when delivered. Both random vectors are conditional on the signal received and the available public information. Evidently, when $\mathbb{E}[\frac{\Delta p}{p^*} | s_i, z] > 0$ brokers anticipate to incur a loss. Also, we denote by $\omega_j = \frac{p^* \cdot \theta_j}{p^* \cdot \Theta}$, the weight of θ_j 's value over Θ with $\omega_j \in (0, 1]$.

⁹Without loss of generality, we assume no entry costs for participating brokers.

4.3.1 Mechanism

The auction takes place in two rounds $t = 1, 2$. Each broker $i \in \mathbb{N}$ submits consecutively a single¹⁰ bid (fee) in basis points $\phi_i^t \in [0, 1]$, with $\phi_i^1(s_i, z)$ to be the first-round bid and $\phi_i^2(s_i, z')$ be the second-round bid for z' an update in public information. The fee is calculated ad valorem on the portfolios value. The standard tie-breaking rule (in which the winner is selected at random) applies to both rounds.

First round. The asset manager initiates $q + 1$ simultaneous sealed first-price auctions for the q packages and the aggregate portfolio Θ . Each local $i \in \mathbb{L}$ competes for the package θ_j that he is interested in and receives no extra utility from owning more than one package. Accordingly, each global $g \in \mathbb{G}$ competes for the aggregate Θ and receives no utility from owning a single package. The qualified brokers for the next round are q local winners and one global winner with the lowest bids.

Second round. At the outset, the asset manager, updates the available public information to z' , by revealing the winning bids of the previous round. Then, the qualified “local” winners of the first round, defined as $\mathbb{Q} \subset \mathbb{L}$ with cardinality $|\mathbb{Q}| = q$ i.e. the number of local packages, jointly compete against the qualified “global” winner g .

In all cases, the second-round bids are bounded from above by the first-round bidding. This round follows the rules of core-selecting auctions with two possible outcomes¹¹: the global broker g wins all packages as Θ when $\phi_g^2 < \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$, and each local broker i wins one package θ_i if $\phi_g^2 > \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$.

The payoff of a local broker i , who wins a package θ_j with m securities for the charged commission $c_i \in \mathbb{R}_+$ is given by:

$$\mathbb{E}[\pi_i(\phi^2 | s_i, z)] = [(\theta_i \cdot p^*) \cdot c_i - \theta_i \cdot \mathbb{E}[\Delta p(s_i, z)]] \cdot \mathbf{1}_{\{\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 < \phi_g^2\}} \quad (4.1)$$

where the last term is an indicator function for $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 < \phi_g^2$, when the whole portfolio Θ is assigned to local brokers for execution.

For each package θ_i the expected payoff of each local broker i results from the

¹⁰For simplicity we restrict our analysis to this class of auctions.

¹¹Without loss of generality ties are resolved.

charged commission upon the trading value $(\theta_i \cdot p^* \cdot c_i)$ minus the potential losses from the price variation $(\theta_i \cdot \mathbb{E}[\Delta p(s_i, z)])$, if exists. We denote the private valuation of each local broker i for θ_i with $\alpha_i = \mathbb{E} \left[\frac{\theta_i \cdot \Delta p}{\theta_i \cdot p^*} | s_i, z \right]$, with $\alpha_i \in \mathbb{R}$.

On the other hand, if the global broker wins the aggregate portfolio Θ , for $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 > \phi_g^2$, he charges $C = \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$ and follows a similar payoff function. We denote the private valuation for Θ of each global broker g with $v = \mathbb{E} \left[\frac{\Theta \cdot \Delta p}{\Theta \cdot p^*} | s_g, z \right]$, with $v \in \mathbb{R}$. The broker-optimal-frontier, for any local broker i , is satisfied when $\sum_{i \in \mathbb{Q}} c_i = \phi_g^2$.

The VCG pricing function $c(\phi_\ell^2, \phi_g^2)$, where $\phi_\ell^2 = (\phi_1^2, \dots, \phi_q^2)$, is given by:

$$c(\phi_\ell^2, \phi_g^2) = \begin{cases} (c_1^V, \dots, c_q^V, 0) & \text{if } \phi_g^2 > \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2, \\ (0, \dots, 0, C) & \text{if } \phi_g^2 < \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2. \end{cases} \quad (4.2)$$

$$\text{where } c_i^V = \max \left\{ 0, \frac{\phi_g^2 - \sum_{j \neq i} \omega_j \phi_j^2}{\omega_i} \right\}.$$

Respectively, the core-selecting pricing rule is given by:

$$c(\phi_\ell^2, \phi_g^2) = \begin{cases} (c_1, \dots, c_q, 0) & \text{if } \phi_g^2 > \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2, \\ (0, \dots, 0, C) & \text{if } \phi_g^2 < \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2. \end{cases} \quad (4.3)$$

such that $c_i \in [\phi_i^2, c_i^V]$ with $\sum_{i \in \mathbb{Q}} c_i \leq \phi_g^2$ and $C \in [\phi_g^2, \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2]$.

If the VCG outcome is in the core, no broker has an incentive to deviate from his truthful preferences and it is the only selected Pareto-dominant outcome [Ausubel and Milgrom, 2002]. However, when the VCG is outside the core, a different pricing rule is necessary if we are to minimize the incentives for deviation [Day and Raghavan, 2007].

In the following, we present the two core-selecting pricing rules: the nearest-VCG rule [Day and Cramton, 2012] and the Dynamic-Nearest-VCG, a slight modification of the former that we introduce to accommodate our two-round set up. In both cases, the global broker receives a fee equal to $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$ upon winning. Whereas if local brokers

win, they apportion ϕ_g^2 as follows:

1) Nearest-VCG rule

This pricing approach was firstly introduced by Day and Raghavan [2007] and Day and Cramton [2012]. The fundamental notion is to select a point in the broker-optimal-frontier which will minimize the euclidean distance from the VCG outcome [Ausubel and Baranov, 2020]. For a finite set of locals the payments for the weighted-packages are divided into:

$$c_i(\phi_\ell^2, \phi_g^2) = (\omega_1 c_1^V - \Delta_1, \dots, \omega_q c_q^V - \Delta_q, 0), \quad (4.4)$$

where $\Delta_i = \left(\sum_{i \in \mathbb{Q}} \omega_i c_i^V - \phi_g^2 \right) \omega_i$ is the minimum downward correction on the VCG outcome that corresponds to each local broker i .

2) Dynamic-NVCG (D-NVCG) rule

This rule selects a vector of payments in the *broker-optimal-frontier* determined by local brokers' first-round bidding. The rationale is that overbidding incentives in the first round are penalized for deviating from the VCG pricing.

Suppose $\mathbb{Q} = \mathbb{Q}^u \cup \mathbb{Q}^d$ and $\mathbb{Q}^u \cap \mathbb{Q}^d = \emptyset$, where $\mathbb{Q}^u = \{j \in \mathbb{Q} | \phi_j^1 > c_j^V\}$ and $\mathbb{Q}^d = \{i \in \mathbb{Q} | \phi_i^1 \leq c_i^V\}$. Then, for any bidder i , the final fees of all bidders are readjusted by $\epsilon_i = \omega_i(\phi_i^1 - c_i^V)$.

$$c_i(\phi_\ell^2, \phi_g^2) = \begin{cases} [\omega_i c_i^V - \Delta_i] + \omega_i \frac{\sum_{j \in \mathbb{Q}^u} \epsilon_j}{\sum_{i \in \mathbb{Q}^d} \omega_i} & \text{if } \phi_i^1 \leq c_i^V \\ [\omega_i c_i^V - \Delta_i] - \epsilon_i & \text{if } \phi_i^1 > c_i^V \end{cases} \quad (4.5)$$

where $\Delta_i = \left(\sum_{i \in \mathbb{Q}} \omega_i c_i^V - \phi_g^2 \right) \omega_i$.

Each bidder with $\phi_i^1 > c_i^V$ will receive a downward adjustment on the nearest-VCG pricing equal to the deviation ϵ_i , while a bidder with $\phi_i^1 \leq c_i^V$ will be rewarded for his strategy in the first-round with an increase in the nearest-VCG fee.

4.3.2 Examples

We provide two examples for the implementation of the payment rules.

Example 1 Assume an asset manager demands liquidity for a portfolio Θ . He decides to divide the portfolio into two packages: the first package is θ_1 with a weighted-value $\omega_1 = 0.6$ over the nominal value of Θ , and the second package is θ_2 with a weighted-value $\omega_2 = 0.4$ over the nominal value of Θ . The qualified winners of the first round are: the local broker 1 for the package θ_1 , the local broker 2 for the package θ_2 and the global broker g for portfolio Θ .

In the second round, let's suppose that brokers submit the following bids:

Local 1		Local 2		Global
$\phi_{\ell_1}^1$	$\phi_{\ell_1}^2$	$\phi_{\ell_2}^1$	$\phi_{\ell_2}^2$	ϕ_g^2
27	25	19	10	22

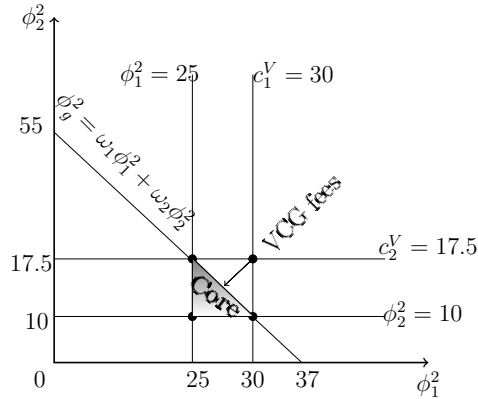
Since the aggregate bid of the local brokers equals to $\omega_1\phi_{\ell_1}^2 + \omega_2\phi_{\ell_2}^2 = 19$, they win. Thus, the asset manager assigns to the local brokers to execute the portfolio Θ jointly.

At this point, the question that arises is how much the asset manager will have to pay each local broker to execute each assigned package. If the asset manager applies the VCG pricing rule from equation (4.2), he will have to pay the local broker 1 with $c_1^V = 30$ and broker 2 with $c_2^V = 17.5$. However, for the asset manager their total payment $\omega_1c_1^V + \omega_2c_2^V$, would be higher than if the portfolio was assigned to the global broker. Thus, the total payment of the two locals must not exceed the bidding ϕ_g^2 of the global bidder (“second-price” rule).

Figure 4.1 maps the payoff vectors for which the coalition of locals is not blocked. Following equation (4.3), the core corresponds to $c_1 \in [25, 30]$ for local broker 1, $c_2 \in [10, 17.5]$ for local broker 2, and $C \in [19, 22]$ for the global.

One can readily notice that the constraints defining upwardly the core are simply the tie-breaking bids. Suppose that the local broker 1 bids $\phi_1^2 > 30$. This outcome would be blocked by the global's bid $\phi_g^2 = 22$ and broker 2's bid $\phi_2^2 = 10$. The same applies if the local broker 2 bids $\phi_2^2 > 17.5$. The lower bounds on the local brokers' pricing are their bids, consistent with the assumption of individual rationality.

Figure 4.1: Core point closest to VCG payments



Using the Nearest-VCG pricing rule from equation (4.4), we will minimize the distance from the VCG pricing rule to obtain an outcome that will be included in the core intervals. In the core interval, local brokers' payoff is maximized at *broker-optimal-frontier* where the tie-break occurs. Thus, the asset manager will pay broker 1 with a commission $c_1 = 16.2$ (that is $\omega_1 c_1^V = 18$ and $\Delta_1 = 1.8$) and broker 2 with a commission $c_2 = 5.8$ (that is $\omega_2 c_2^V = 7$ and $\Delta_2 = 1.2$).

One shortcoming of the Nearest-VCG pricing rule is that has been designed for single-round auctions without encompassing the bidding behavior of the previous round. With the Dynamic-Nearest-VCG, incentives for bidding close to truthful valuations in the first round are rewarded, while those who misreport are “punished” by receiving a lower commission fee when the auction ends. For instance, the local broker 2 submits $\phi_2^1 = 19$ in the first round. This bid qualifies him for the second round, yet in the second round, he has a larger interval $19 \leq \phi_2^2 \leq 10$ to reduce. By the information release at the interim, each broker is updated for the prices' estimates of others. According to equation (4.5) the asset manager will pay the Nearest-VCG prices minus any deviation between first-round bidding and the VCG outcome, $c_2 = 5.2$ for broker 2 and $c_1 = 16.8$ for broker 1.

In the next example, we illustrate the pricing rules for $\ell > 2$ local brokers:

Example 2 Assume that the qualified winners for the second round are 5 local brokers who compete against 1 global. Table 4.1 presents each local broker i 's bid for each package θ_i with a weight ω_i , respectively. Since $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 < \phi_g^2$, with $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 = 23$ and $\phi_g^2 = 25$, the asset manager assigns the portfolio's execution to local brokers. The VCG

Local	Weights	Bids		VCG	Core	Nearest	Dynamic
Brokers	ω_i	ϕ_i^1	ϕ_i^2	c_i^V	Interval	VCG	NVCG
1	0.15	34	20	40	[20, 40]	4.2	4.725
2	0.1	24	20	50	[20, 50]	3.8	4.15
3	0.4	24	15	22.5	[15, 22.5]	4.2	3.6
4	0.2	44	25	40	[25, 40]	5.6	4.8
5	0.15	48	40	60	[40, 60]	7.2	7.725
<i>Global</i>		28	25		[22, 25]		

Table 4.1: Example 2 with 5 local bidders and 1 global

outcome c_i^V is calculated for each local broker i based on equation (4.2) and presented in the relevant column.

Similarly to the previous example, for the local broker i , a fee higher than $\phi_i^2 > c_i^V$ is blocked by the coalition of locals given that others submit a bid equal to ϕ_{-i}^2 and the global's bid $\phi_g^2 = 25$. The broker-optimal-frontier is satisfied for $\sum_{i=1}^5 \omega_i \phi_i^2 = 25$ maximizing the local brokers' pay-offs, for every core interval defined by equation (4.3).

With Nearest-VCG pricing rule from equation (4.4) the asset managers pays a commission fee to the local broker 1 equal to 4.2 ($\omega_1 c_1^V = 6$, $\Delta_1 = 1.8$). This means that the local broker 1 sacrifices 1.8 basis points of the charged commission fee to win the package. For the local brokers 2,3,4,5, the Nearest-VCG fees are presented in Table 4.1.

In this example, we see that two local brokers have submitted a higher fee in the first round: local broker 3 with $\phi_3^1 > c_3^V$ and local broker 4 with $\phi_4^1 > c_4^V$, respectively. This results in high fees in the first round, while the anticipated bids for both brokers are low. The last pricing rule restricts brokers from manipulating the outcome of the auction.

Applying the Dynamic-Nearest-VCG pricing rule from equation (4.5) for the local brokers 3 and 4 who have bidden excessively in the first round, the asset manager will pay the local broker 3 a commission equal to 3.6 ($\epsilon_3 = 0.6$) and the local broker 4 a commission equal to 4.8 ($\epsilon_4 = 0.8$). Both local brokers 3 and 4 will bear an extra cost for their misreporting incentives in the first round.

Those local brokers who bid prudently in the first round will receive an increase

in the commission fee. Specifically, the local broker 1 will receive a fee equal to 4.725 ($\sum_{j \in \mathbb{Q}^u} \epsilon_j = 1.4$ and $\sum_{i \in \mathbb{Q}^d} \omega_i = 0.4$). The same rule works for local brokers 2 and 5.

4.4 Analysis

We start our analysis by characterizing our mechanism in the second round as pivotal. This means that the fee received by any local broker i is equal to the loss imposed on other locals by adjusting ϕ_i^2 to attribute i 's values. We define a bid ϕ_i^2 submitted by broker i as *pivotal*, if and only if $\phi_g^2 = \sum_{i \in Q} \omega_i \phi_i^2$ holds and if for any $\gamma > 0$, a bid $\phi_i^2 - \gamma$ attributes a non-empty package θ_i , while $\phi_i^2 + \gamma$ yields the null package.

If an auction satisfies the pivotal pricing property, then if the pivotal broker wins, he receives a commission equal to the bidding fee [Ausubel and Baranov, 2020]. In a local-global setting, this property is satisfied for any core-selecting auction.

LEMMA 4.1 (Ausubel and Baranov [2020]). *Every core selecting auction satisfies the pivotal pricing property.*

It is not hard to prove that the Dynamic-NVCG pricing rule results in allocations that belong to the broker-optimal-frontier and minimizes any misreporting incentives.

LEMMA 4.2. *Any auction with a Dynamic-NVCG pricing rule $c(\phi_i^2, \phi_g^2)$ is a core-selecting auction.*

Proof: See the Appendix 4.6.1. □

We do not disregard the Nearest-VCG, but instead, we are using it as a touchstone to improve incentive compatibility further and reduce the degree of manipulation freedom in a two-stage framework [Parkes et al., 2001]. The following Proposition explains why the Dynamic-NVCG rule is optimal for distributing the commission of local brokers when there are perverse incentives by some brokers in the first round.

PROPOSITION 4.1. *For any local bidder i bidding above $\mathbb{E}[c_i^V | s_i, z]$ in the first round is always a weakly dominated strategy.*

Proof: See the Appendix 4.6.2. □

The following Assumption 4.1 imposes continuity and monotonicity for all pricing rules, and this will simplify our analysis.

ASSUMPTION 4.1. *For any winning bidder i and any bidding vector $(\phi_1^2, \dots, \phi_q^2, \phi_g^2)$, the pricing function $c_i(\phi_1^2, \dots, \phi_q^2, \phi_g^2)$ is continuous in all bids, differentiable and non-decreasing in bidder's i bid.*

Bosshard et al. [2017] have proved that the N-VCG rule does not always satisfy the non-decreasing condition. This seems to be not the case in our mechanism. This can be easily confirmed from equation (4.5) and by Example 2 in Table 4.1. In particular, an increase of broker 1' bid from 3 to 3.3, increases the NVCG to 4.38 and the Dynamic-NVCG from 4.72 to 5.13.

In the VCG mechanism, brokers bid their valuation truthfully, and it is their weakly dominant strategy with no surplus. The following lemma suggests the global broker has a weakly dominant strategy in the second round.

LEMMA 4.3. *Suppose that Assumption 4.1 is satisfied. Then, for the restricted second-round auction and for $v(s_g, z') > 0$, $\phi_g^2 = \min\{\phi_g^1, v(s_g, z')\}$ is a weakly dominant strategy for the global broker.*

Proof: See the Appendix 4.6.3. □

Whereas for the local bidders, it is always worse off to bid lower than their expected losses, $\alpha_i(s_i, z') > 0$.

LEMMA 4.4. *Suppose that Assumption 4.1 and the pivotal pricing property are satisfied. Then, for each local broker i any bid $\phi_i^2 \in [0, \min\{\phi_i^1, \alpha_i(s_i, z')\})$ is a weakly dominated strategy.*

Proof: See the Appendix 4.6.4. □

Suppose now that all local brokers $j \neq i$ bid according to the profile $(\phi_j^{2*})_{j \neq i}$. Let $H_i \equiv H_i[\phi_i^2, \alpha_i(s_i, z')]$ be the probability of winning for a local bidder i who bids $\phi_i^2 \in [\alpha_i(s_i, z'), \phi_i^1]$, and its marginal probability $h_i \equiv h_i[\phi_i^2, \alpha_i(s_i, z')]$:

$$H_i = \Pr \left(\omega_i \phi_i^2 + \sum_{j \neq i} \omega_j \phi_j^{2*} \leq v(s_g, z') \mid \alpha_i(s_i, z') \right)$$

$$h_i = \frac{\partial H_i(\phi_i^2, \alpha_i(s_i, z'))}{\partial \phi_i^2} \quad (4.6)$$

Also, we denote the expected commission fee of each local bidder i with $c_i \equiv C_i(\phi_i^2, \alpha_i(s_i, z'))$ and with $MC_i \equiv MC_i(\phi_i^2, \alpha_i(s_i, z'))$ the expected marginal commission when each local broker i bids $\phi_i^2 \in [\alpha_i(s_i, z'), \phi_i^1]$.

$$C_i = \mathbb{E} \left[c_i(\phi_i^2, \sum_{j \neq i} \phi_j^{2*}, v(s_g, z')) \mid \alpha_i(s_i, z') \right] \quad (4.7)$$

$$MC_i = \mathbb{E} \left[\frac{\partial c_i(\phi_i^2, \sum_{j \neq i} \phi_j^{2*}, v(s_g, z'))}{\partial \phi_i^2} \mid \alpha_i(s_i, z') \right]$$

The expected marginal commission expresses any change in the expected commission arising by the incremental increase in the bidding ϕ_i^2 . For instance, if brokers anticipate a loss in the expected prices, they will counterbalance their payoff by moving their bid upwardly.

Next, we define the first-order optimality conditions for the local broker's maximization problem on the steps of Ausubel and Baranov [2020].

PROPOSITION 4.2. *Under Assumption 4.1 and the pivotal pricing property, the optimality condition for choosing $0 < \phi_i^2 \leq \phi_i^1$ for a local bidder i is given by:*

$$MC_i = \left(\alpha_i(s_i, z') - \phi_i^2 \right) h_i. \quad (4.8)$$

Proof: See the Appendix 4.6.5. □

Intuitively, if $MC_i < 0$ it means that $\alpha_i(s_i, z') < \phi_i^2$, and broker i is not included among the winners. Otherwise, if $\phi_i^2 < \alpha_i(s_i, z')$, broker i will have to increase his bidding fee to reach the optimal payoff where $MC_i = 0$.

THEOREM 4.1. *For each pricing rule, it exists an equilibrium where the bidding function of each broker is given by:*

(a) *for the NVCG rule*

$$\phi_i^2 = \begin{cases} \alpha_i(s_i, z') - \sigma_i \omega_i^2 (q - 1) & , \text{if } \alpha_i > 0 \\ 0 & , \text{if } \alpha_i \leq 0 \end{cases} \quad (4.9)$$

(b) *for the D-NVCG rule*

$$\phi_i^2 = \begin{cases} \alpha_i(s_i, z') - \sigma_i \omega_i^2 (q - 1) & , \text{if } \phi_i^1 > \phi_i^V \text{ and } \alpha_i > 0 \\ \alpha_i(s_i, z') - \sigma_i \omega_i^2 \left[\frac{\ell}{\sum_{i \in Q^d} \omega_i} + (q - 1) \right] & , \text{if } \phi_i^1 \leq \phi_i^V \text{ and } \alpha_i > 0 \\ 0 & , \text{if } \alpha_i \leq 0 \end{cases} \quad (4.10)$$

where $\frac{1}{\sigma_i} \equiv \frac{h_i}{H_i}$ is a reverse hazard rate and with ℓ to be the number of local bidders with $\phi_i^1 > \phi_i^V$.

Proof: See the Appendix 4.6.6. □

In Theorem 4.1 we proved the existence of equilibrium for nearest-VCG and the Dynamic-Nearest-VCG. In both cases, we have shown that when the portfolio is sliced in many packages, the equilibrium bid of a winning broker is negatively affected by the number of packages and the size of his package in the overall portfolio.

4.5 Conclusion

This research has studied a stylized model for a portfolio's auction, which is divided into packages. We design a "local-global" environment with a finite set of locals, each one interested in a single package. We introduce a dynamic setup for the Nearest-VCG pricing rule, which conforms to a multiple-round auction. This new pricing setup aligns brokers' incentives to lower bids mitigating the free-riding opportunities of the first round. Using an endogenous reference rule for the expected VCG pricing outcome, the brokers are motivated to submit bids close to their truthful valuations in the first round, squeezing execution costs downwardly.

Additionally, we proved that our mechanism allows the asset manager to engage many brokers in the auction process, resulting in lower transaction costs. The information update at the interim for others' valuations mitigates the "winner's curse" [Zarpala and Voliotis, 2021]. Also, it increases broker's trust in the sense that the auction's rules have been followed.

Finally, we propose a simple iterative mechanism that provides transparency in the auction process to eliminate the complexity of the winner's determination problem and keeps the brokers' problems manageable, incorporating their strategic incentives.

4.6 Appendix

4.6.1 Proof of Lemma 4.2

From equation (4.5), the total sum of local brokers' commission is:

$$\begin{aligned} & \sum_{i \in \mathbb{Q}^d} \omega_i \left[c_i^V + \frac{\sum_{j \in \mathbb{Q}^u} \epsilon_j}{\sum_{i \in \mathbb{Q}^d} \omega_i} \right] - \sum_{i \in \mathbb{Q}^d} \Delta_i + \sum_{j \in \mathbb{Q}^u} [\omega_j c_j^V - \epsilon_j] - \sum_{j \in \mathbb{Q}^u} \Delta_j \\ &= \sum_{i \in \mathbb{Q}} \omega_i c_i^V - \sum_{i \in \mathbb{Q}} \Delta_i = \phi_g^2 \end{aligned}$$

Consequently, the Dynamic-NVCG rule always lies on the broker-optimal frontier. \square

4.6.2 Proof of Proposition 4.1

Suppose not. Then for any broker i , a bidding strategy $\phi_i^{1'} \leq \mathbb{E}[c_i^V | s_i, z]$ is weakly dominated by $\phi_i^1 > \mathbb{E}[c_i^V | s_i, z]$. By substitution in equation (4.1) the pricing rule of (4.5) for $\pi_i(\phi_i^2 | s_i, z') \geq \pi_i(\phi_i^1 | s_i, z')$ we have:

$$\theta_i \cdot p^* (\omega_i c_i^V - \Delta_i - \epsilon_i) \geq \theta_i \cdot p^* \left(\omega_i c_i^V - \Delta_i + \omega_i \frac{\epsilon_i}{\sum_{i \in \mathbb{Q}^d} \omega_i} \right)$$

By solving this inequality we result $\phi_i^1 \leq c^V$. Thus, any bidding in the first round above VCG price is a weakly dominated strategy. \square

4.6.3 Proof of Lemma 4.3

By design no broker can bid in the second round higher than his first-round bid. For the VCG mechanism, it is a weakly dominant strategy for the global bidder to bid his “valuation”, which in our case equals to $v(s_g, z')$. The result follows directly. \square

4.6.4 Proof of Lemma 4.4

For an arbitrary broker i , let $\alpha_i(s_i, z') \geq \phi_i^1$. Then, for any strategy $\phi_i^2 \leq \phi_i^1$ is trivially weakly dominated and obtains negative surplus. Suppose now that for broker i , it is $\alpha_i(s_i, z') < \phi_i^1$. For the local broker i with $\hat{\phi}_i^2 = \alpha_i(s_i, z')$ and $\hat{\phi}_i^2 > \phi_i^2$, we prove that bidding ϕ_i^2 is weakly dominated. By *Assumption 4.1*, it will always result in $c_i(\phi_\ell^2, \phi_g^2) \leq c_i(\hat{\phi}_\ell^2, \hat{\phi}_g^2)$, and from the pivotal pricing property, it follows:

$$\begin{aligned} \mathbb{E}[\theta_i \cdot p^* \cdot c_i(\phi_\ell^2, \phi_g^2) - \theta_i \cdot \Delta p(s, z')] &\leq \mathbb{E}[\theta_i \cdot p^* \cdot c_i(\hat{\phi}_\ell^2, \hat{\phi}_g^2) - \theta_i \cdot \Delta p(s, z')] \\ &\leq \mathbb{E}[\theta_i \cdot p^* \cdot \frac{\theta_i \cdot \Delta p(s, z')}{\theta_i \cdot p^*} - \theta_i \cdot \Delta p(s, z')] = 0 \end{aligned}$$

Thus, any $\hat{\phi}_i^2 > \phi_i^2$ is weakly dominated. \square

4.6.5 Proof of Proposition 4.2

We apply the optimality condition on the expected payoff of equation (4.1) upon the probability of winning

$$E[\pi_i(\phi_i^2 | s_i, z')] = \left[\theta_i \cdot p^* \cdot c_i - \theta_i \cdot \mathbb{E}[\Delta p | s_i, z'] \right] H_i$$

with $0 \leq \phi_i^2 \leq \phi_i^1$

$$\begin{aligned} \frac{\partial E[\pi_i(\phi_i^2 | s_i, z')]}{\partial \phi_i^2} &= \left[\theta_i \cdot p^* \cdot \frac{\partial c_i}{\partial \phi_i^2} \right] \cdot H_i + \left[\theta_i \cdot p^* \cdot c_i - \theta_i \cdot \mathbb{E}[\Delta p | s_i, z'] \right] \cdot h_i \\ &= \theta_i \cdot p^* MC_i + \theta_i \cdot p^* \cdot c_i \cdot h_i - \theta_i \cdot \mathbb{E}[\Delta p | s_i, z'] \cdot h_i = 0 \end{aligned}$$

By Lemma 4.4, ϕ_i^2 is always nonnegative. Due to *Assumption 4.1* and the pivotal pricing property the following is in effect:

$$c_i \equiv c_i(\omega_i \phi_i^2, \sum_{j \neq i} \omega_j \phi_j^{2*}, \phi_i^2 + \sum_{j \neq i} \omega_j \phi_j^{2*}) = \phi_i^2$$

Thus, it is easily to conclude that:

$$\begin{aligned} MC_i &= \theta_i \cdot \mathbb{E}[\Delta p | s_i, z'] \cdot h_i - \theta_i \cdot p^* \cdot \phi_i^2 \cdot h_i \\ MC_i &= \left(\frac{\theta_i \cdot \mathbb{E}[\Delta p | s_i, z']}{\theta_i \cdot p^*} - \phi_i^2 \right) \cdot h_i \end{aligned}$$

□

4.6.6 Proof of Theorem 4.1

The optimality conditions are given by equation (4.8).

- (a) From equation (4.4) of the *Nearest-VCG rule*, the expected marginal commission of equation (4.7) for a broker i is:

$$\begin{aligned} MC_i &= [\omega_i c^V - \Delta_i]' H_i \\ &= \omega_i^2 (q - 1) H_i \end{aligned}$$

Replacing the above to (4.8) we result to the equilibrium bid:

$$\phi_i^2 = \frac{\theta_i \cdot \mathbb{E}[\Delta p | s_i, z']}{\theta_i \cdot p^*} - \omega_i^2 (q - 1) \frac{H_i}{h_i}$$

- (b) For the D-NVCG if $\phi_i^1 > \phi_i^V$ the equilibrium bidding is similar to the *Nearest-VCG* of equation (4.9). However, if there are ℓ number of locals with $\phi_j^1 > \phi_j^V$ and bidder's i bid in the first round is $\phi_i^1 \leq \phi_i^V$, then the expected marginal commission from equation (4.5) is given by:

$$MC_i = \omega_i^2 \left[\frac{\ell}{\sum_{i \in Q^d} \omega_i} + (q - 1) \right] H_i$$

By substitution in the optimality conditions of equations (4.8) we conclude:

$$\phi_i^2 = \frac{\theta_i \cdot \mathbb{E}[\Delta p | s_i, z']}{\theta_i \cdot p^*} - \omega_i^2 \left[\frac{\ell}{\sum_{i \in Q^d} \omega_i} + (q - 1) \right] \frac{H_i}{h_i}$$

□

Chapter 5

Summary and Conclusions

The objective of this research was to associate auction theory with corporate finance. In three separate essays, we investigated how the auction-theoretical tools (Chapter 1) can be applied under three financial aspects: (a) corporate bonds, (b) blind portfolios, and (c) divisible portfolios. While recognizing the limitations of our analysis, we believe that we have largely achieved our goal. Our contribution is presented in each chapter separately, here below we present a summary of our results.

In Chapter 2, we have attempted to apply auction theory in the pricing of corporate bonds under the presence of investment mandates, and to our knowledge, we are the first to address this issue in the current literature. We employed the risk limits imposed by the investment mandates because we wanted to adjust our design in the corporate bond market, where credit ratings play a crucial role. Also, we modeled a statistical independent secondary market to capture the value of the bond for each investment manager who participates in the auction.

Our market design is a uniform-price auction in which investment managers directly reveal their budget limits in their bidding strategies, bound by the risk limits set on the investment mandates. We have proven that under this setting, it exists a symmetric Bayes-Nash equilibrium.

The result shows that the symmetric equilibrium bidding strategy is inversely affected by the number of competitive bidders, which means that the investment managers will receive a smaller share of the bond (symmetric Cournot oligopoly) in an oversubscribed issuance. Similarly, the oligopolistic market power exercised by each investment manager affects equilibrium bids. Other factors like the lending interest rates of other debt sources are used as a benchmark to calculate a spread from the resale in the secondary market, and it seems that

the symmetric bidding strategy follows the same course with this spread.

The yield of the issuance in the symmetric equilibrium reflects that the degree of over-subscription and the market power of each participating investor is not among the yield's determinants, yet it is affected by the strictness of investment mandates that each bidder encounters, other things held constant. This means that investment mandates with low-risk acceptance and restricted budget limits can decrease the issuer's cost of capital. Inversely, an expectation for underpricing in the secondary market bounces the issuer's costs upwardly.

Also, a high demand guided by strict investment mandates would result in allocating the bond to a greater number of investors, since any bidder who participates in the issuance in the limit would bid the minimum.

Contrary to the current practice for the pricing of corporate bonds, the uniform-price auction is a well-understood rule by all parties, and it is a mechanism already used for the pricing of Treasury bills. Investors reveal their valuations directly in their bid, so the final allocation and the price represent each bidding strategy.

The second essay presented in Chapter 3 was about the auction of blind portfolios. We have investigated this investment strategy and proposed a two-stage sealed-bid market design with information release in the interim. Relatively to the current one-stage auction design, our proposal may eliminate the "winner's curse" for the participating brokers as it allows them to access spot market signals and update their valuations. Thus, the broker can predict the actual value of the portfolio more accurately and mitigate the "winner's curse" effect. In that sense, a two-stage design may improve brokers' efficiency in a blind portfolio auction. Additionally, we have shown that it can reduce the liquidation costs of the asset manager.

Our approach elegantly sidesteps bidding complexity and any computational problems arising in practice, as it is a direct incentive-compatible mechanism [Perry et al., 2000]. Also, it promotes competition among brokers and ensures that this auction format meets the requirements set by the asset manager. As a possible extension of the model mentioned above, we conceptualized the framework of Day and Milgrom [2008] and divided the portfolio into packages. We analyze this framework in the last essay of this thesis, where we auctioned a divisible portfolio into packages.

In Chapter 4, we have designed a "local-global" environment with a finite set of locals, each interested in a single package, and introduced a novel dynamic set-up for the Nearest-VCG pricing rule conforming to a multi-round auction. This new pricing-rule set up aligns brokers' incentives to lower bids mitigating the free-riding opportunities for investors on the the first

round. Using an endogenous reference rule for the expected VCG pricing outcome, the brokers are motivated to submit bids close to their truthful valuations in the first round squeezing execution costs downwardly.

We proved that our mechanism allows the asset manager to engage many brokers in the auction process, resulting in lower transaction costs. The information update in the interim for others' valuations mitigates the "winner's curse" [Zarpala and Voliotis, 2021]. Also, it increases broker's trust in the sense that the auction rules have been followed.

Finally, it is a simple iterative mechanism that provides transparency in the auction process to eliminate the complexity of the winner's determination problem and keeps the brokers' problems manageable, incorporating their strategic incentives.

Bibliography

- James J Anton and Dennis A Yao. Coordination in Split Award Auctions. *The Quarterly Journal of Economics*, 107(2):681–707, 1992.
- Ian R. Appel, Todd A. Gormley, and Donald B. Keim. Passive investors, not passive owners. *Journal of Financial Economics*, 121(1):111–141, 2016.
- Olivier Armantier and Erwann Sbaï. Estimation and comparison of treasury auction formats when bidders are asymmetric. *Journal of Applied Econometrics*, 21:745–779, 2006.
- Olivier Armantier and Erwann Sbaï. Comparison of Alternative Payment Mechanisms for French Treasury Auctions. *Annals of Economics and Statistics*, 93/94:135–160, 2009.
- Lawrence M Ausubel. An efficient ascending-bid auction for multiple objects. *American Economic Review*, 94(5):1452–1475, 2004.
- Lawrence M Ausubel and Oleg Baranov. Core-selecting auctions with incomplete information. *International Journal of Game Theory*, 49:251–273, 2020.
- Lawrence M Ausubel and Paul R Milgrom. Ascending Auctions with Package Bidding. *Frontiers of Theoretical Economics*, 1(1), 2002.
- Lawrence M Ausubel and Paul R Milgrom. *The lovely but lonely Vickrey Auction*. MIT Press, 2006.
- Lawrence M Ausubel, Peter Cramton, Marek Pycia, Marzena Rostek, and Marek Weretka. Demand reduction and inefficiency in multi-unit auctions. *Review of Economic Studies*, 81(4):1366–1400, 2014.
- Lawrence M Ausubel, Justin E Burkett, and Emel Filiz-Ozbay. An experiment on auctions with endogenous budget constraints. *Experimental Economics*, 20(4):973–1006, 2017.

- Kerry Back and Jaime F. Zender. Auctions of Divisible Goods: On the Rationale for the Treasury Experiment. *Review of Financial Studies*, 6(4):733–764, 1993.
- Ramin Baghai, Bo Becker, and Stefan Pitschner. The Private Use of Credit Ratings: Evidence from Investment Mandates. 2019.
- J.-P. Benoit and V Krishna. Multiple-Object Auctions with Budget Constrained Bidders. *The Review of Financial Studies*, 68(1):155–179, 2001.
- Hendrik Bessembinder, Chester Spatt, and Kumar Venkataraman. A Survey of the Microstructure of Fixed-Income Markets. *Journal of Financial and Quantitative Analysis*, 55(1):1–45, 2020.
- Martin Bichler. *Market Design*. Cambridge University Press, dec 2017.
- Sushil Bikhchandani and Chi-Fu Huang. Auctions with Resale Markets: An Exploratory Model of Treasury Bill Markets. *The Review of Financial Studies*, 2(3):311–339, 1989.
- Sushil Bikhchandani and Chi-Fu Huang. The Economics of Treasury Securities Markets. *Journal of Economic Perspectives*, 7(3):117–134, 1993.
- Ken Binmore and Joe Swierzbinski. Treasury auctions: Uniform or discriminatory? *Review of Economic Design*, 5:387–410, 2000.
- Tilman Borgers, Daniel Krahmer, and Roland Strausz. *An introduction to the theory of mechanism design*. Oxford University Press, 2015.
- Vitor Bosshard, Ye Wang, and Sven Seuken. Non-decreasing Payment Rules for Combinatorial Auctions. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence (IJCAI-18)*, pages 105–113, 2017.
- Vitor; Bosshard, Benedikt; Bünz, Benjamin; Lubin, and Sven Seuken. Computing Bayes-Nash Equilibria in Combinatorial Auctions with Verification. *Journal of Artificial Intelligence Research*, 69:531–570, 2020.
- Jeremy Bulow and Paul Klemperer. Auctions Versus Negotiations. *American Economic Review*, 86(1):180–194, 1996.
- Jeremy Bulow and John Roberts. The Simple Economics of Optimal Auctions. *Journal of Political Economy*, 97(5):1060–1090, 1989.

BIBLIOGRAPHY

- Justin Burkett and Kyle Woodward. Uniform price auctions with a last accepted bid pricing rule. *Journal of Economic Theory*, 185, 2020.
- Sara Castellanos and Marco Oviedo. Optimal Bidding in the Mexican Treasury Securities Primary Auctions: Results of a Structural Econometric Approach. *Latin American Journal of Economics-formerly Cuadernos de Economía*, 45(131):3–28, 2008.
- S. Çelik, G Demirta, and M Isaksson. Corporate Bond Market Trends , Emerging Risks and Monetary Policy. Technical report, OECD Capital Market Series, 2020.
- Yeon-Koo Che. Standard Auctions with Financially Constrained Bidders. *Review of Economic Studies*, 65(1):1–21, 1998.
- Edward H Clarke. Multipart Pricing of Public Goods. *Public Choice*, 11:17–33, 1971.
- Francesca Cornelli and David Goldreich. Bookbuilding and strategic allocation. *Journal of Finance*, 56(6):2337–2369, 2001.
- Peter Cramton and Jesse A. Schwartz. Collusive Bidding: Lessons from the FCC Spectrum Auctions. *Journal of Regulatory Economics*, 17(3):229–252, 2000.
- Peter Cramton, Yoav Shoham, and Richard Steinberg. *Combinatorial Auctions*. MIT Press, 2006.
- Jacques Crémer, Richard P Mclean, and Jacques Cre-Mer. Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are Interdependent. *Econometrica*, 53(2):345–361, 1985.
- Michele Dathan, Sergei Davydenko, and Joseph L Rotman. Debt Issuance in the Era of Passive Investment. 2020.
- Robert Day and Paul Milgrom. Core-selecting package auctions. *International Journal of Game Theory*, 36:393–407, 2008.
- Robert W Day and Peter Cramton. Quadratic core-selecting payment rules for combinatorial auctions. *Operations Research*, 60(3):588–603, 2012.
- Robert W Day and S Raghavan. Fair Payments for Efficient Allocations in Public Sector Combinatorial Auctions. *Management Science*, 53(9):1389–1406, 2007.
- Sven De Vries and Rakesh V Vohra. Combinatorial auctions: A survey. *Journal on Computing*, 15(3):284–309, 2003.

BIBLIOGRAPHY

- Shahar Dobzinski, Ron Lavi, and Noam Nisan. Multi-unit auctions with budget limits. *Games and Economic Behavior*, 74(2):486–503, 2012.
- Aytek Erdil and Paul Klemperer. A new payment rule for core-selecting package auctions. *Journal of the European Economic Association*, 8(2/3):537–547, 2010.
- Philippe Février, Raphaële Préget, and Michael Visser. Econometrics of Share Auctions *. 2004.
- Robert Forsythe, R. Mark Isaac, and Thomas R Palfrey. Theories and Tests of "Blind Bidding" in Sealed-bid auctions. *The RAND Journal of Economics*, 20(2):214–238, 1989.
- Milton Friedman. *A Program For Monetary Stability*. Fordham University Press, New York, 1960.
- Arieh Gavious, Benny Moldovanu, and Aner Sela. Bid Costs and Endogenous Bid Caps. *The RAND Journal of Economics*, 33(4):709–722, 2002.
- Christos Giannikos and Tin Shan Suen. Estimating two structural spread models for trading blind principal bids. In *European Financial Management*, pages 1–49. inproceedings, 2007a.
- Christos Giannikos and Tin Shan Suen. Pricing Determinants of Blind Principal Bidding and Liquidity Provider Behavior. In *European Financial Management Meeting*, pages 1–33, 2007b.
- Christos Giannikos, Hany Guirguis, and Tin Shan Suen. Modelling the Blind Principal Bid Basket Trading Cost. *European Financial Management*, 18(2):271–302, 2012.
- Jacob K Goeree and Yuanchuan Lien. On the impossibility of core-selecting auctions. *Theoretical Economics*, 11(1):41–52, 2016.
- Gautam Goswami, Thomas H Noe, and Michael J Rebello. Collusion in Uniform-price Auction: Experimental Evidence and Implications for Treasury Auctions. *The Review of Financial Studies*, 9(3):757–785, 1996.
- Theodore Groves. Incentives in Teams. *Econometrica*, 41(4):617, 1973.
- Michel A Habib and Alexandre Ziegler. Why government bonds are sold by auction and corporate bonds by posted-price selling. *Journal of Financial Intermediation*, 16(3):343–367, 2007.

BIBLIOGRAPHY

- Isa E Hafalir, R Ravi, and Amin Sayedi. A near Pareto optimal auction with budget constraints. *Games and Economic Behavior*, 74(2):699–708, 2012.
- John C Harsanyi. Games with Incomplete Information played by "Bayesian" Players. *Management Science*, 14(3):159–182, 1967.
- Ali Hortaçsu and David Mcadams. Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market. *Journal of Political Economy*, 188(5):833–865, 2010.
- Giuliano Iannotta. *Investment banking: A guide to underwriting and advisory services*. Springer International Publishing, 2010.
- Philippe Jehiel. Manipulative auction design. *Theoretical Economics*, 6(2):185–217, 2011.
- Tim Jenkinson and Howard Jones. Bids and Allocations in European IPO Bookbuilding. *Journal of Finance*, 59(5):2309–2338, 2004.
- John H Kagel and Dan Levin. The Winner's Curse and Public Information in Common Value Auctions. *The American Economic Review*, 76(5):894–920, 1986.
- John H Kagel and Jonathan Levin. Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions. *Econometrica*, 69(2):413–454, 2001.
- Boo-Sung Kang and Steven L. Puller. The Effect of Auction Format on efficiency and revenue in divisible goods auctions: A test using Korean Treasury Auctions. *The Journal of Industrial Economics*, 56(2):90–332, 2008.
- Kenneth A Kavajecz and Donald B Keim. Packaging Liquidity: Blind Auctions and Transaction Efficiencies. *Journal of Financial and Quantitative Analysis*, 40(3):465–492, 2005.
- Paul Klemperer. *Auctions: Theory and Practice*. Princeton University Press, 2003.
- Ilan Kremer and Kjell G Nyborg. Underpricing and market power in uniform price auctions. *The Review of Financial Studies*, 17(3):849–877, 2004.
- Ilan Kremer and Kjell G Nyborg. Divisible-Good Auctions: The Role of Allocation Rules. *The RAND Journal of Economics*, 35(1):147–159, 2007.
- Vijay Krishna. *Auction Theory*. Elsevier Inc, 2nd edition, 2010.

BIBLIOGRAPHY

- Vijay Krishna and Robert W Rosenthal. Simultaneous Auctions with Synergies. *Games and Economic Behavior*, 17(1):1–31, 1996.
- Daniel Marszalec. The Impact of Auction Choice on Revenue in Treasury Bill Auctions - An Empirical Evaluation. *International Journal of Industrial Organization*, 53:215–239, 2017.
- Eric Maskin and John Riley. Optimal Multi-Unit Auctions. In *The Economics of Missing Markets, Information, and Games*, pages 313–335. Oxford University Press, 1989.
- B Y Steven Matthews. Comparing Auctions for Risk Averse Buyers: A Buyer’s Point of View. *Econometrica*, 55(3):633–646, 1987.
- Steven A. Matthews. Information Acquisition in Discriminatory Auctions. In *Bayesian Models in Economic Theory*, pages 181–207. Elsevier Science Ltd, 1984.
- R Preston McAfee and John McMillan. Auctions with a stochastic number of bidders. *Journal of Economic Theory*, 43(1):1–19, 1987.
- John McMillan. Selling Spectrum Rights. *Journal of Economic Perspectives*, 8(3):145–162, 1994.
- Richard A McNeal. The Brides of Babylon: Herodotus 1.196. *Historia: Zeitschrift für Alte Geschichte*, 37(1):54–71, 1988.
- Flavio M. Menezes and Paulo K. Monteiro. *An Introduction to Auction Theory*. Oxford University Press, 2005.
- Michael J. Fishman. A Theory of Preemptive Takeover Bidding. *The RAND Journal of Economics*, 19(1):88–101, 1988.
- Paul Milgrom. Auctions and Bidding: A Primer. *Journal of Economic Perspectives*, 3(3):3–22, 1989.
- Paul Milgrom. Package auctions and exchanges. *Econometrica*, 75(4):935–965, 2007.
- Paul Milgrom and Nancy Stokey. Information, trade and common knowledge. *Journal of Economic Theory*, 26(1):17–27, 1982.
- Paul R. Milgrom. Putting Auction Theory to work. pages 255–295, 2004.
- Paul R.; Milgrom and Robert W. Day. Optimal Incentives in Core-Selecting Auctions. In *The Handbook of Market Design*, chapter 11, pages 282–298. Oxford University Press, 2013.

BIBLIOGRAPHY

- Paul R Milgrom and Robert J Weber. A Theory of Auctions and Competitive Bidding. *Econometrica*, 50(5):1089–1122, 1982.
- Asunción Mochón and Yago Sáez. *Understanding Auctions*. Springer International Publishing, 2015.
- Roger B Myerson. Optimal Auction Design. *Mathematics of Operational Research*, 6(1): 58–73, 1981.
- Stanislava Nikolova, Liying Wang, and Juan (Julie) Wu. Institutional allocations in the primary market for corporate bonds. *Journal of Financial Economics*, 137(2):470–490, 2020.
- Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay Vazirani. *Algorithmic Game Theory*. 2007.
- Kjell G Nyborg and Suresh Sundaresan. Discriminatory versus Uniform Treasury auctions: Evidence from when-issued transactions. *Financial Economics*, 42(1):63–104, 1996.
- Michael Padilla and Benjamin Van Roy. Intermediated blind portfolio auctions. *Management Science*, 58(9):1747–1760, 2012.
- David C. Parkes. Iterative combinatorial auctions. In *Combinatorial Auctions*, pages 41–78. MIT Press, 2006.
- David C Parkes, Jayant Kalagnanam, and Marta Eso. Achieving Budget-Balance with Vickrey-Based Payment Schemes in Exchanges. In *Proceedings of the Seventeenth International Joint Conference on Artificial Intelligence (IJCAI-01)*, pages 1161–1168, 2001.
- Aleksandar Pekeč and Michael H. Rothkopf. Combinatorial auction design. *Management Science*, 49(11):1485–1503, 2003.
- Motty Perry, Elmar Wolfstetter, and Shmuel Zamir. A sealed-bid auction that matches the english auction. *Games and Economic Behavior*, 33(2):265–273, 2000.
- Reichert Ortega. *Models for Competitive Bidding Under Uncertainty*. PhD thesis, Stanford University, 1968.
- John G Riley and William F Samuelson. Optimal Auctions. *American Economic Review*, 71 (3):381–392, 1981.

- Robert W Rosenthal and Ruqu Wang. Simultaneous auctions with synergies and common values. *Games and Economic Behavior*, 17(1):32–55, 1996.
- Michael H Rothkopf, Aleksandar Pekeč, and Ronald M Harstad. Computationally manageable combinatorial auctions. *Management Science*, 44(8):1131–1147, 1998.
- Yoav Shoham and Kevin Leyton-Brown. Protocols for Strategic Agents: Mechanism Design. In *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, pages 273–327. 2010.
- Rafael Tenorio. On Strategic Quantity Bidding in Multiple Unit Auctions. *The journal of Industrial Economics*, XLV(2):0022–1821, 1997.
- William Vickrey. Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of Finance*, 16(1):8–37, 1961.
- James J D Wang and Jaime F Zender. Auctioning divisible goods. *Economic Theory*, 19: 673–705, 2002.
- D Michael Whinston and B Douglas Bernheim. Menu Auctions, Resource Allocation, and Economic Influence. *The Quarterly Journal of Economics*, 101(1):1–32, 1986.
- Robert Wilson. Competitive Bidding with Disparate Information. *Management Science*, 15 (7):446–448, 1969.
- Robert Wilson. Auctions of Shares. *The Quarterly Journal of Economics*, 93(4):675–689, 1979.
- Lamprini Zarpala and Dimitris Voliotis. Blind portfolios’ auctions in two-rounds. *Annals of Finance*, 2021.
- Ping Zhang. Uniform price auctions and fixed price offerings in IPOs: an experimental comparison. *Experimental Economics*, 12:202–219, 2009.