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Abstract

In this paper we seek to systematically explore the solution space of a solvable nuclear war model proposed by Norman C. Dalkey, for the various parameters that generate varying opposing side profiles. In the process, we attempt to identify optimal warfighting strategies as these derive from the solution space, examine the model's basic premise that a mixed counterforce/countervalue targeting strategy is viable, while proposing a slight variation of the model, one better reflecting the emerging multi-actor global geopolitical domain.

keywords: nuclear war, simulation, game theory, strategic stability, deterrence

Dedicated to the memory of my godfather, John Gabriel Skylodemos, to whom I owe everything.

Στην μνήμη του νονού μου, Ιωάννη Γαβριήλ Σκυλοδήμου, στον οποίο οφείλω τα πάντα.

Contents

1 Introduction

Simulating nuclear war is a notoriously difficult undertaking, since neither available historical data exist, nor is it possible to credibly extrapolate from the results of conventional exchanges in order to build an appropriate model. Even describing the level of the exchange, amongst a spectrum of escalating states of conflict [Davis and Stan, 1984] (Table 1), involving warfighting capabilities that may be ill-defined as strategic or sub-strategic, is a major challenge. Further complications arise from the recent introduction of weapons that blur the border between tactical, operational-tactical, and strategic-operational, while simultaneously undermining long-established concepts of deterrence [Michael C. Horowitz, Paul Scharre and Alexander-Velez Green, 2019].

Conflict Level
General Strategic Nuclear
Counterforce Strategic Nuclear
Demonstrative Strategic Nuclear
General Tactical Nuclear
Demonstrative Tactical Nuclear
Biological
Chemical
General Conventional
Demonstrative Conventional
Regional
Crisis

Table 1. Various states of conflict. The shaded ones represent phases of nuclear escalation. [Davis and Stan, 1984]

More so, the interest on the subject of modeling nuclear war has subsided significantly since the end of the Cold War, along with the perceived likelihood of the threat of actual nuclear war materializing. Given the reemergence of great power-competition amongst peer and near-peer competitors, this complacency seems unfounded and the need for robust simulation as a decision support tool for policy makers and planners becomes of prime importance.

To this end, we revisit a model of nuclear war proposed by Norman C. Dalkey [Dalkey, 1965], that aims to address a key issue, that of viewing any such exchange as having a single possible outcome-mutual destruction of the two sides. The basic, but untested, premise, when considering a war game between two nuclear armed opponents that possess more than enough destructive power to annihilate each other, is that they will chose to use it to its full extent resulting in mutual assured destruction. This has been conceptualized as the M.A.D. (Mutual Assured Destruction) doctrine, a lynchpin of deterrence theory assumptions during the Cold War.

To discuss this premise in the context of game theory, we first provide the definition of the two strategies available to players waging nuclear war. These are counterforce and countervalue. Counterforce refers to the targeting of enemy strategic assets, weapons and means of delivery, while countervalue refers to the targeting of civilian-urban centers. The M.A.D. doctrine assumes that given two opponents of roughly the same capabilities in terms of weapon effectiveness and warhead numbers, who both have excess destructive potential above the one required to destroy the enemy, the preferred strategy would be to choose a pure countervalue targeting.

The rationale is that if one choses counterforce and the other countervalue, the side choosing counterforce receives unacceptable damage and the one choosing countervalue does not-i.e., that side wins. If both chose counterforce, no one comes out as a clear winner either. By choosing countervalue each is assured of a result at least as good as the one of the opponent, hence countervalue is always a dominant strategy. This is in line with what is to be expected of a highly non-cooperative, purely antagonistic form of gaming, where every player assumes the other is out to do him as much damage as possible [Shubik, 1987], thus aiming at the “least bad”/’best of the worst” of all possible outcomes (minimax principle).

This analysis, which during the Cold War was the main argument in “proving” that nuclear war is highly unlikely, can be demonstrated in the payoff matrix of Table 2, but leaves much to be desired regarding its validity. The very definition of nuclear war as a zero-sum game is in question [Shubik, 1987]. Also doubts arise when dealing with opponents of uneven force levels, either numerically or pertaining to the effectiveness of their weapons. To explore outcomes of such potential matches, we need a model that views nuclear war as potentially winnable, or, if a clear win is unattainable, at least as an exchange that does not de facto lead to the destruction of all players.

Blue \ Red	**Counterforce**	**Countervalue**
Counterforce	**(0,0)**	**(-∞,W)**
Countervalue	**(W, -∞)**	**(-∞, −∞)**

Table 2. Payoff matrix when nuclear war is viewed as a zero-sum game.

2 A Solvable Model

2.1 A Single Weapon Two-Player Model

Such a model is provided by Norman C. Dalkey’s work [Dalkey, 1965] and to the best of our knowledge, remains the most concise and thorough published treatment on the subject. It is based on the assumption of increasing concern, which states that if for each side there exists a critical level, a threshold above which further received damage becomes unacceptable, then the significance of further damage as we approach this level rises sharply, while the amount of damage that must be inflicted to the other side to compensate also rises exponentially. This implies that the game might end prior to mutual annihilation, with both players reaching an equilibrium where “enough” damage has been respectively achieved, so nuclear war is neither unwinnable, nor an event with very low probability as the M.A.D. doctrine suggests. While we focus mainly on the results we can obtain from the model regarding how best to conduct the game, we do not ignore the otherwise very significant political and philosophical ramifications of this postulate. Assessing the statistical likelihood that mutual destruction occurs even when taking into account the presence of equilibria is by itself of great value to the decision maker. But first, we present the mathematical formulation of the model.

Dalkey introduces a payoff function in the form of:

$$P_B = D_R - D_B - A/(C_B - D_B)$$

where P_B: Payoff to Blue, D_R: Damage to Red value targets, D_B: Damage to Blue value targets, C_B: Critical level of Blue and A is a scaling constant.

For the Red player, the function takes the symmetric form (by interchanging subscripts and introducing the respective scaling constant B) of:

$$P_R = D_B - D_R - B/(C_R - D_R)$$

Now we need to introduce a number of parameters that differentiate between the players, creating unique profiles for each, seeking to best model the differences in the number of weapons available and the effectiveness of each side's weapons.

These parameters are: M: number of offensive weapons available to Blue, N: Number of offensive weapons available to Red, u: Blue player counterforce effectiveness, v: Red player counterforce effectiveness.

By counterforce effectiveness we mean the chance that a Blue (respectively, a Red) weapon that survives a Red attack, finds and destroys a Red weapon (respectively, a Blue one). So, u is the average probability ($u \in (0,1)$) of destroying a Red weapon at its site before it has a chance to launch.

The model assumes a single type of weapon available for each player and that each such weapon targets a single enemy target, either of the counterforce or countervalue category. As such, for given player profiles, it is a weapon allocation problem, where each side has to choose a strategy ranging between a pure counterforce to a pure countervalue mix of enemy targets. By x we denote the percentage ($x \in [0,1]$) of forces the Blue player allocates to counterforce and by y the one the Red player does. The remaining weapons are considered to be allocated to countervalue targets where they have a probability of one ($u = v = 1$) to destroy their targets.

For policy purposes, that is if the problem is more comprehensively viewed as a resource allocation problem where an estimate of a prospective Red player profile (M, v, C_R) is available (via intelligence analysis, or in the case of number of weapons, capped through treaties), we seek a respective Blue player profile (N, u, C_B) best suited to match the opponent's. Thus, it is reduced to an optimization problem. We further assume that the game occurs in a single round (a one-shot game).

The complete mathematical formulation of the one-weapon model is as follows:

$$P_B(x) = \frac{M - Nvy}{1 - uvxy}(1 - x) - \frac{N - Mux}{1 - uvxy}(1 - y) - \frac{A}{C_B - \frac{N - Mux}{1 - uvxy}(1 - y)}$$

$$P_R(y) = \frac{N - Mux}{1 - uvxy}(1 - y) - \frac{M - Nvy}{1 - uvxy}(1 - x) - \frac{B}{C_R - \frac{M - Nvy}{1 - uvxy}(1 - x)}$$

By taking the partial derivatives we obtain the system (S):

$$\theta P_B/\theta x = \big(1 - u + uy(1 - v)\big)[C_B\,(1 - uvxy) - (N - Mux)(1 - y)]^2 - Au(1 - y)(1 - uvxy)^2 = 0$$

$$\theta P_R/\theta y = \big(1 - v + vx(1 - u)\big)[C_2(1 - uvxy) - (M - Nvy)(1 - x)]^2 - Bv(1 - x)(1 - uvxy)^2 = 0$$

At a maximum we have $\theta P_B/\theta x = \theta P_R/\theta y = 0$ and at these points we seek the (x, y) solutions.

Furthermore, one can refine this search to identify amongst these pairs, the ones satisfying the Nash equilibrium, that is (x^*, y^*) such that:

$P_B(x^*, y^*) \geq P_B(x, y^*)$ and $P_R(x^*, y^*) \geq P_R(x^*, y)$

The derivation of these equations can be found on Dalkey's original work.

We summarize the parameters and variables involved in the next table.

We take the liberty in setting the Sensitivity parameter equal to the multiple of ten times the Critical Level, that is $A = 10 * C_B, B = 10 * C_R$. Simulations show that the results obtained are virtually identical and this is in line with viewing it as the adjusted uncertainty each player has in regard to his Critical Level.

	Blue	**Red**
Number of weapons	M	N
Counterforce effectiveness	u	v
Proportion of forces allocated to counterforce	x	y
Critical Level of damage	C_B	C_R
Sensitivity	A	B

Table 3. Parameters and variables of the model.

2.2 Limitations of the Model

The model has some intrinsic limitations. These involve a number of issues, that are as relevant today as they were when the model was proposed.

- The countervalue effectiveness of all players is assumed to be 1. That is, whichever countervalue target is allocated a weapon, is considered destroyed.
- It assumes that a single weapon (missile) carries a single warhead, thus each weapon can only be allocated to a single target.
- It makes no distinction between the value of countervalue targets.

The above limitations, though not critical when assessing the validity of the model, are compounded today by ones which in our view will require a more thorough treatment given expected changes on the geopolitical landscape and innovations in the field of strategic warfare. These are:

- The model does not take into account the conventional counterforce capability of the players, a potentially highly destabilizing factor.
- It does not consider the presence of other players that might be involved in the exchange. It is strictly a two-player game.

2.3 A Game of Three Players

Introducing more players to the model is a necessity stemming from the changing geopolitical environment, where-in contrast to the Cold War-more powers possess strategic nuclear weapons. We assume that another player, Yellow, with comparable capabilities in terms of number of weapons and counterforce effectiveness exists. If a nuclear exchange between Blue and Red is taking place, we can make the following hypothesis.

As more damage is received by Blue and Red, the concern of what Yellow will do once the exchange is over begins to mount. Nuclear war between two players is a highly non-cooperative game, where each side seeks the other's destruction, using the entirety of its arsenal. But this means that even a victorious outcome leaves the winner with a depleted arsenal and high levels of damage received. This translates to a victor that can easily fall prey to a side that was not involved in the initial exchange and has retained both its arsenal and its civilian targets intact. Questions of morality regarding the willingness of the non-involved player to destroy both the victor and the defeated side are inconsequential. What we consider of great interest by following this line of argument, is that what was initially a highly non-cooperative game between Blue and Red, as the game progresses and damage mounts on both sides, evolves into a cooperative game where Blue and Red must show mutual restraint in order to retain enough potential to withstand a possible attack by Yellow.

2.4 Towards an Enhanced Model

We will briefly discuss the formulation of an enhanced model, though this will not be part of the simulations and wargaming we will run later on.

We propose a few minor adaptations of the current model, integrating the aforementioned concerns while sketching the outline of a new model.

The number of weapons available can be updated to a new value prior to running the simulation, based on the effectiveness of the opponent's conventional counterforce capabilities. Thus, we have:

$$M_{re} = M - K_R,\ N_{re} = N - K_B$$

Where $M_{re},\ N_{re}$ are the updated reduced values of M, N respectively and K_B, K_R are the number of weapons destroyed by Blue and Red's conventional counterforce attack, respectively.

In order to incorporate the presence of a third player who does not take part in the exchange directly, on the current model, we may add more parameters. These would involve the number of weapons Blue and Red believe will have to be retained in order to deter Yellow as well as a reduction of the critical level of Blue and Red in order to ensure capacity to receive damage when facing Yellow. The simplest formulation would be:

$$C_B' = C_B - L_{BY},\ C_R' = C_R - L_{RY}$$

where $L_{BY,}\ L_{RY}$ represent the reduction of Blue and Red's critical level in fear of Yellow respectively.

Thus, we have

$$M' = M - K_R - F_{BY}, \qquad N' = N - K_B - F_{RY}$$

where $F_{BY},\ F_{RY}$ represent the number of weapons Blue and Red store in order to confront Yellow respectively. Implementing these on the current problem formulation and obtaining results for the new parameters is fairly easy since we are essentially dealing with the same model.

A more detailed approach would involve a new formulation of the game, building a new model. We briefly provide an outline of it.

So, let's assume that three players, Blue, Red and Yellow face off on a single-shot game. Now, each player has to allocate weapons towards two players, both for counterforce and countervalue purposes. Taking as an example Blue, x_1 of his nuclear weapons (as a percentage over total available) are allocated to target Red's weapons and x_2 are allocated to target Yellow's weapons. Thus $1-(x_1+x_2)$ weapons remain, of which x_3 are allocated on a countervalue role against Red and finally $1-x_3$ are allocated on a countervalue role against Yellow. Similarly, for Red and Yellow we have three more variables for each, for a total of nine variables. It is easy to see that for the hypothetical case of an N-by-N game we would have $N(2N-3)$different variables.

Indeed, for N players, each player allocates weapons to counterforce using one variable for each other player, thus $N-1$ variables; then for the first $N-2$ players, allocates weapons to counterforce using one variable for each, while the remaining are allocated by default to the $(N-1)^{th}$ player. In total we have $N(N-1+N-2)=N(2N-3)$ variables.

If we take into account conventional counterforce capacity, assuming that each player has an amount of non-nuclear weapons capable of targeting the opponents' nuclear ones, we would have to add three more variables, one for each, corresponding to how these are allocated amongst the other two. Besides these variables we can retain the same parameters of the original model regarding counterforce effectiveness, number of weapons and critical levels, while incorporating more to take into account conventional counterforce as well. The payoff functions will be modified accordingly. As we can see, the complexity of the resulting system would be orders of magnitude greater than the two-player game.

A mathematical expression of this new model is beyond the scope of this thesis, but still, we believe it is a course worth exploring on future work. For now, we return to the study of (S), discussing the notion of a solution before proceeding with our simulations.

3 The Notion of a Solution

As described till now, our task seems pretty straightforward. For a given set of parameters we can solve (S), evaluate the solutions, and see if amongst them exists a noncooperative Nash equilibrium. Dalkey demonstrates the presence of such equilibria when the system is of a symmetric form, where the parameters have the same values for both sides; That is, for the case where $M=N, u=v, C_B=C_A, A=B$. Solving (S) in general and seeking a closed form of x, y in respect to the parameters appears to be a dead end due to the inherent complexity of the system. But a strategy of solving (S) for a large enough number of parameter values can grant us a robust insight on the solution space.

Stepping away for a moment from the mathematical formulation, the questions of how each opponent can have an accurate model not only of the adversary but of his own capabilities when measured against the opponent, must be addressed. In a real-world scenario, knowing with certainty the parameters that define the problem appears to be a highly improbable task. We need only to focus on the counterforce effectiveness of each side (u, v) and the critical damage level (C_B, C_R) to realize the fallacy of any notion of absolute accuracy regarding their values. Counterforce effectiveness attempts to quantify the average probability that an enemy weapon will be destroyed prior to launch. This is an aggregate of multiple parameters pertaining to technical aspects regarding the weapon, the timing of the launch, which is itself directly linked to prompt launch warning, also an aggregate of multiple factors. And this, without taking into consideration the chance that a weapon site is either hit but the target is not destroyed due to insufficient warhead yield, CEP (Circular error probable) of the weapon, or the target is reached in time but enemy ABM (Anti-ballistic missile) defenses intercept it, or the role conventionally armed weapons such as hypersonic and cruise missiles, swarm drones enhanced by AI and remote sensing increasingly play in counterforce scenarios [Lieber and Press, 2017]. Of course, one could argue that after a nuclear exchange has taken place, one could accurately calculate such values for u and v, but prior to it only a rough estimate is achievable.

To an even greater degree this holds true for the two sides' critical levels (C_B and C_R), that conceptualize the threshold of damage acceptable to each. We can think of them in the sense of the practical question of "how many cities" is each side willing to lose until the exchange ends. Some of the factors at play here are the character of each side's leadership, political and military, the cultural perceptions regarding national pride, the willingness of each side to achieve strategic goals that presumably led to the confrontation and the degree by which these were fulfilled prior to escalation. It is extremely difficult to place exact values to such abstract notions and yet, if any strategic game modelling is to have an impact on decision support, it must take them into consideration. Moreover, the practice of seeking pure strategies of non-cooperative Nash equilibria in applications of national defense, especially when many stalemate positions occur, has been criticized as potentially misleading [Shubik, 1987].

Thus, a more comprehensive, but at the same time abstract, notion of the solution of (S) must be pursued, one that seeks to identify optimal overall strategies given varied opponent profiles and based more on probability/statistical modeling. Such an approach will better inform the political and military leadership in order to plan ahead for the event of nuclear war. Practical interpretation of the exploration of the solution space should come in the form of high-level abstractions regarding the juxtaposition of capabilities and resulting prospects for defeat or victory.

As an example, describing an opponent profile as "*Red is in possession of robust counterforce capabilities* (so v is "high"), *displays considerable aversion to risk and values the status quo* (so presumably C_R is "low")", "*our best approach would be to maximize the number of weapons we can obtain* (so allocate resources towards a high M number) *while in the event of nuclear war be willing to sacrifice more than he is* (so we have a higher C_B than his C_R) ". These abstractions might seem to be in contrast to the need for precision, but actually are in line with what is expected in high-level decision making and wargaming [Davis, 1988].

4 War Gaming with Varied Opponent Profiles

We solve the system (S) for various combinations of parameters, in order to get as precise an understanding as possible of how these interact. Moreover, we juxtapose and study the behavior of a single parameter while setting fixed values to the other ones. A sufficient comprehension of the systems' behavior can be obtained given a relatively detailed partition of each parameter's value range and thus our approach is only limited by computational requirements. The code used for our simulations was implemented on MATLAB 2021a.

Inspection indicates that when local maxima do exist, so do equilibrium points, not only for symmetric opponent profiles-i.e., opponents with identical parameter values-but for varied opponent profiles as well.

We assign the descriptors "Low", "Medium", "High" on the parameters depending on their value range and simulate ("wargame") varied opponent profiles. This produces a total of $6^3 = 216$ (counting the duplicates) scenarios. Of course, this assignment is arbitrary in nature and can be modified/enriched per the wishes of the analyst.

	Low	**Medium**	**High**
$\boldsymbol{u}, \boldsymbol{v}$	0.1-0.3	0.3-0.6	0.6-0.9
$\boldsymbol{M}, \boldsymbol{N}$	100-150	150-200	200-250
$\boldsymbol{C_B}, \boldsymbol{C_R}$	85-90	90-95	95-100

Table 4. Descriptors of the parameters.

Below we provide the general outline of a table we use to summarize the results of a given simulation, on the first six rows the parameters and their relevant descriptors and on the following rows the statistical results of the simulation. These involve the percentage of wins for each player, the percentage of wins where a counterforce strategy was adopted-this we define as an allocation of forces on a counterforce role greater than 25% of all available forces-the percentage of outcomes where both survive/are destroyed, as well as the percentage of times where no equilibrium exists. This latter part, the non-presence of an equilibrium point is worth examining in more detail regarding its implications, but here we chose to incorporate the outcomes it entails as part of our statistical analysis. It is easy to exclude these and focus solely on the cases where an equilibrium does exist, but inspection shows that this, the presence of an equilibrium, is usually not the case.

For each such wargame, we also present various graphs that help get a better understanding of how the solutions, including but not limited to the equilibrium points, relate with the parameters, the payoff values, and the game outcomes. Since we are dealing with a multivariate/multidimensional problem, these are quite useful in granting a better understanding of its solution space.

Simulation	
u	
v	
M	
N	
C_B	
C_R	
Blue Wins	
Red Wins	
Blue Wins Counterforce	
Red Wins Counterforce	
Mutual Assured Destruction	
Both Survive	
No equilibrium reached	

Table 5. Summarizing the results of a single simulation.

4.1 Examples of Simulations

Simulation 1

In this example Blue has a clear superiority on the number of weapons, suffers a disadvantage on his counterforce capability and has a lower critical level. Running the simulation, we obtain the following results:

Simulation 1	
u	Low
v	Medium
M	High
N	Low
C_B	Low
C_R	Medium
Blue Wins	64.3
Red Wins	32.4
Blue Wins Counterforce	99.5
Red Wins Counterforce	46.5
Mutual Assured Destruction	3.1
Both Survive	30.1
No equilibrium reached	66.2

Table 1.1. Summarizing the results of Simulation 1.

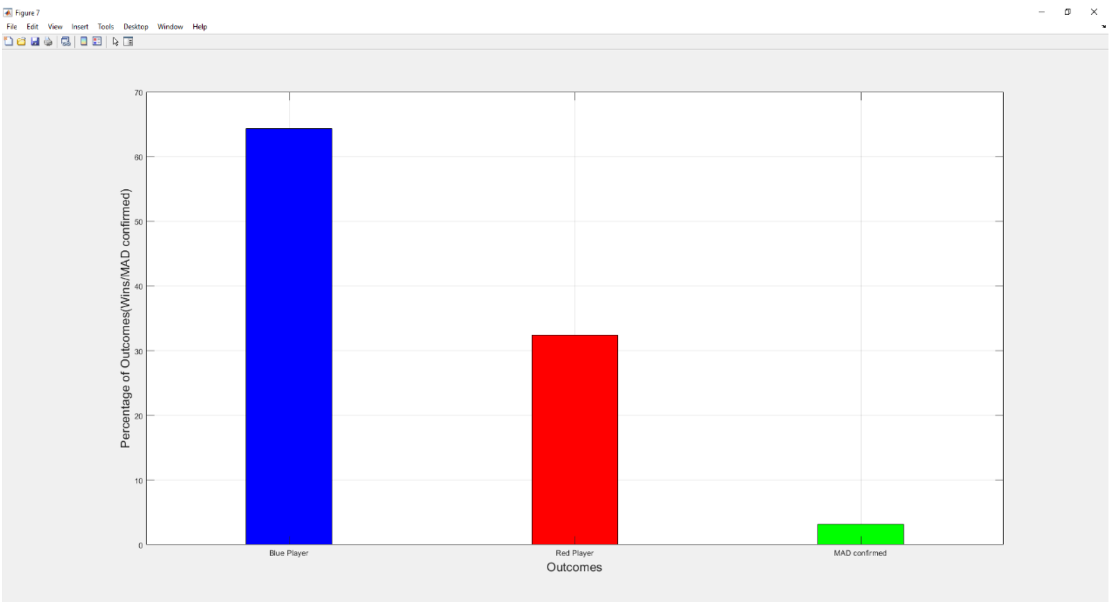

Figure 1.1. 1st-simulation. Percentage of Wins.

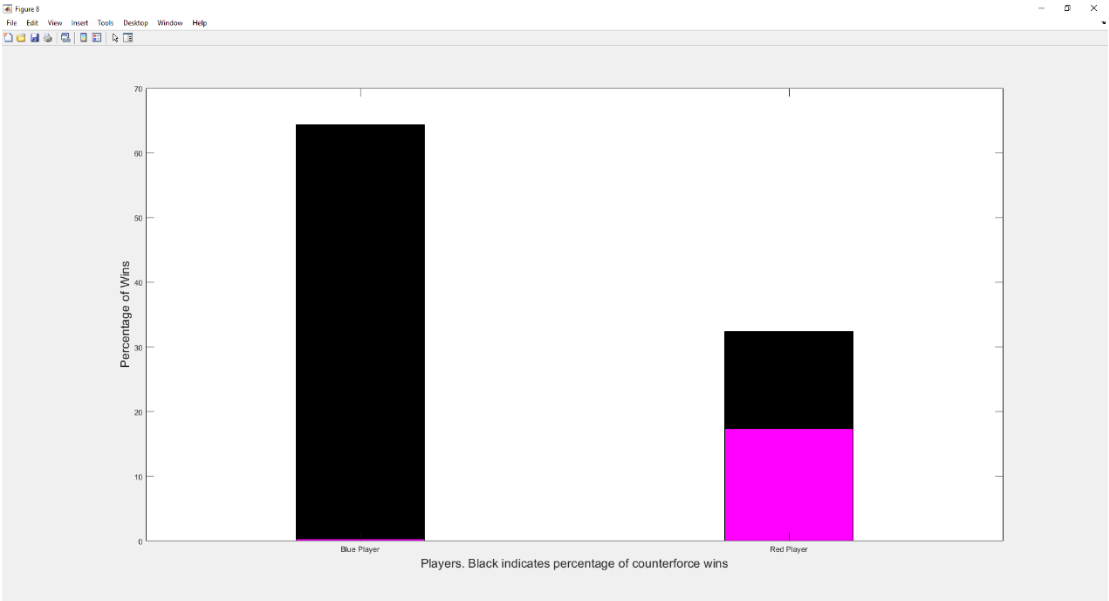

Figure 1.2. 1st-simulation. Percentage of Wins Counterforce.

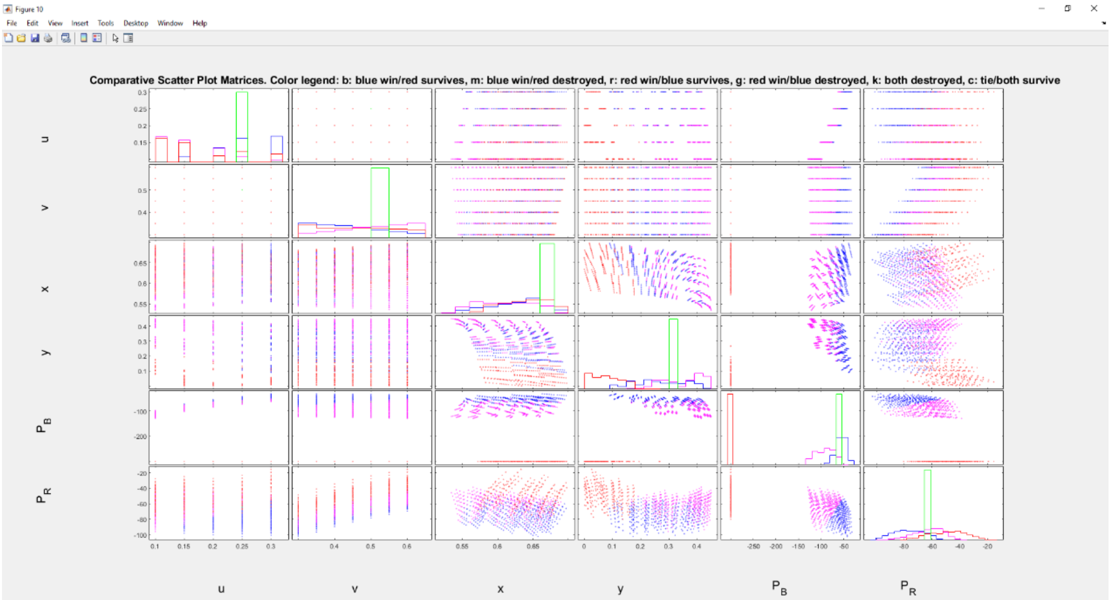

Figure 1.3. 1st-simulation. Comparative Scatter Plot Matrices.

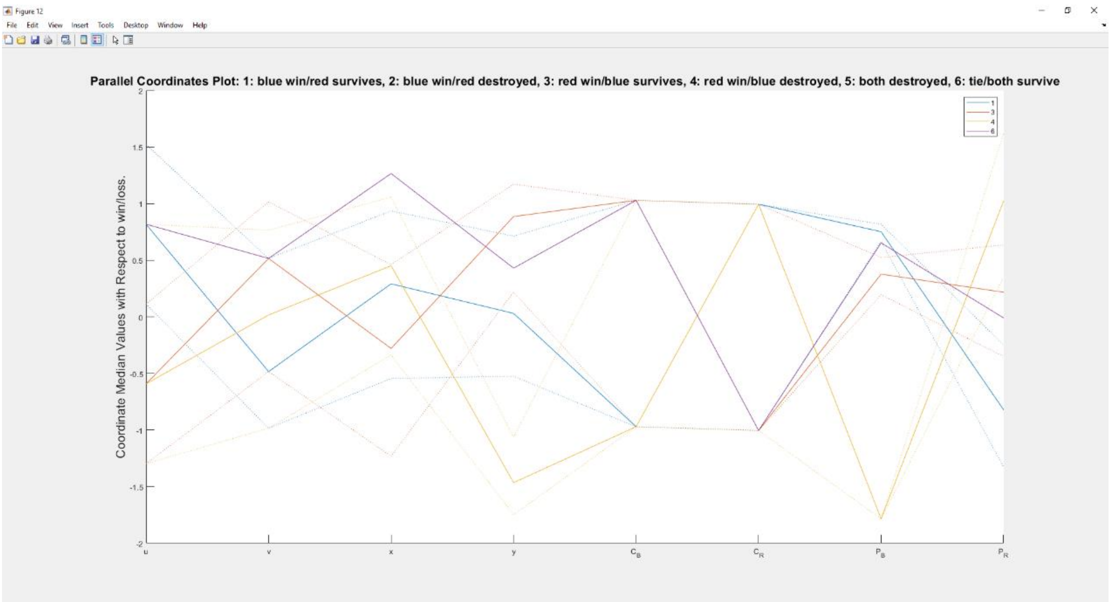

Figure 1.4. 1st-simulation. Parallel Coordinates Plot.

Simulation 2

In this second simulation, we change only the Critical Level of Blue from Low to High while holding all other parameters same as on the 1st. Perhaps counterintuitively, this leads to a significant decrease in the percent of wins of Blue (43.7% compared to 64.3%) but also produces a system, where the number of cases where no equilibrium is reached is lessened to 58.4% from 66.2% and the number of cases where both are destroyed rises to 10.7% from 3.1%. A low Critical Level (signifying a greater fear of being inflicted damage) seems to lead to a greater propensity for choosing purer Countervalue targeting strategies but for that we shall get a better insight by the following simulation (3).

Simulation 2	
u	**Low**
v	**Medium**
M	**High**
N	**Low**
C_B	**High**
C_R	**Medium**
Blue Wins	**43.7**
Red Wins	**45.6**
Blue Wins Counterforce	**100**
Red Wins Counterforce	**26.7**
Mutual Assured Destruction	**10.7**
Both Survive	**29**
No equilibrium reached	**58.4**

Table 2.1. Summarizing the results of Simulation 2.

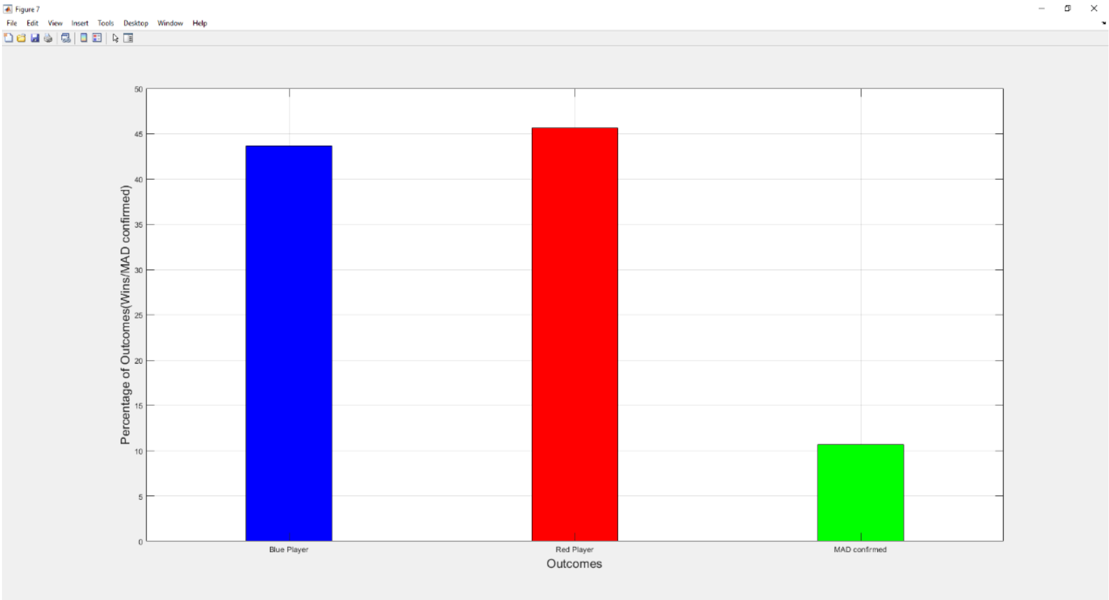

Figure 2.1. 2nd-simulation. Percentage of Wins.

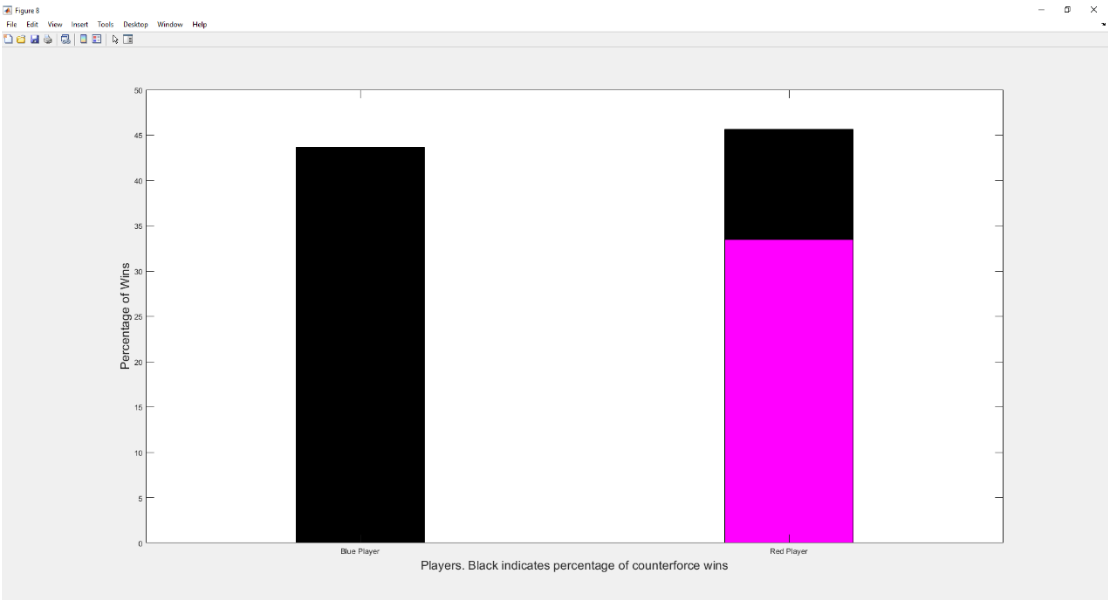

Figure 2.2. 2nd-simulation. Percentage of Wins Counterforce.

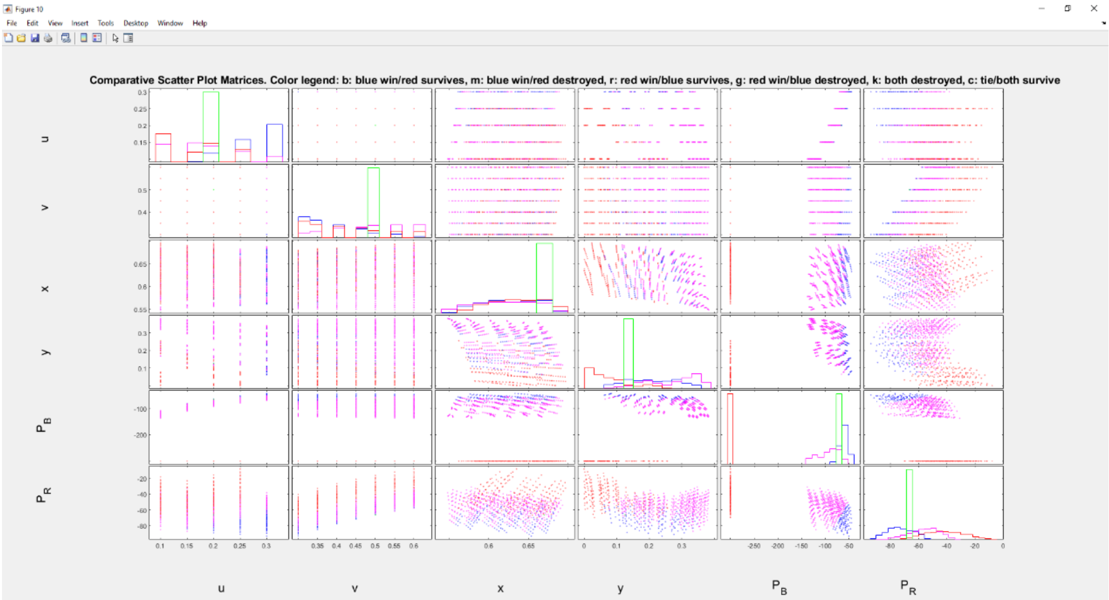

Figure 2.3. 2nd-simulation. Comparative Scatter Plot Matrices.

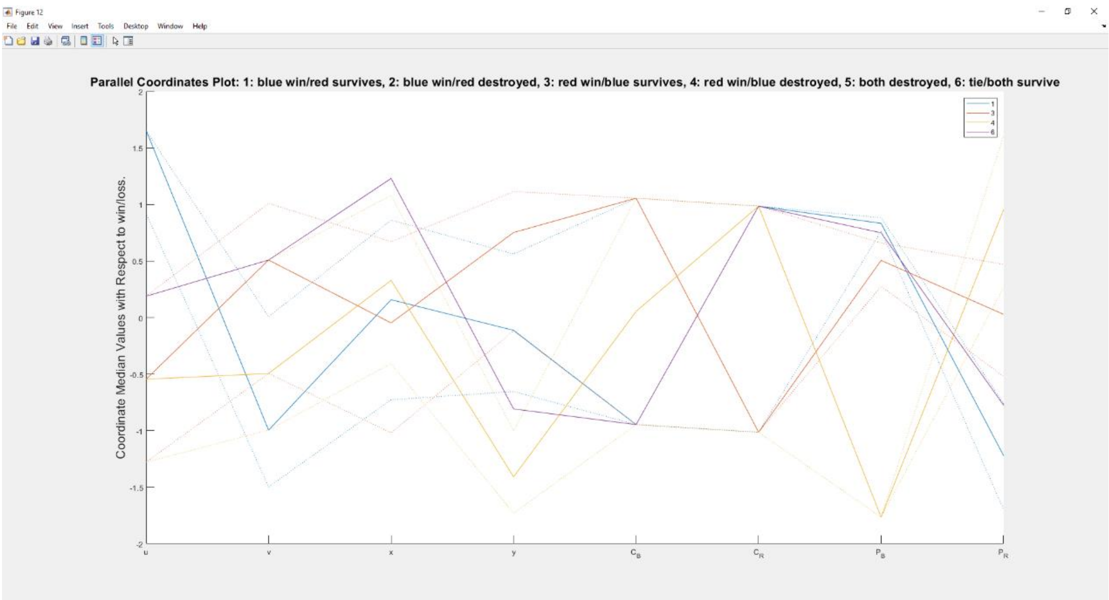

Figure 2.4. 2nd -simulation. Parallel Coordinates Plot.

Simulation 3

Here we retain all parameters of the 1st simulation similar except the number of weapons for the Blue player. Whereas in the first example Blue enjoyed a significant advantage (High vs Low), now both players have a Low number of weapons available. We observe a reversal of the win ratio as compared to Simulation 1 and an overwhelming decrease in the percentage of Blue Counterforce wins (from virtually 100% to < 20%). Also, the cases where both survive has risen to 45.6% from 30.1% and the percent of cases where an equilibrium is reached is more than half the times.

Simulation 3	
u	**Low**
v	**Medium**
M	**High**
N	**Low**
C_B	**Low**
C_R	**Medium**
Blue Wins	**26.7**
Red Wins	**71.5**
Blue Wins Counterforce	**19**
Red Wins Counterforce	**60.3**
Mutual Assured Destruction	**1.6**
Both Survive	**45.6**
No equilibrium reached	**49.1**

Table 3.1. Summarizing the results of Simulation 3.

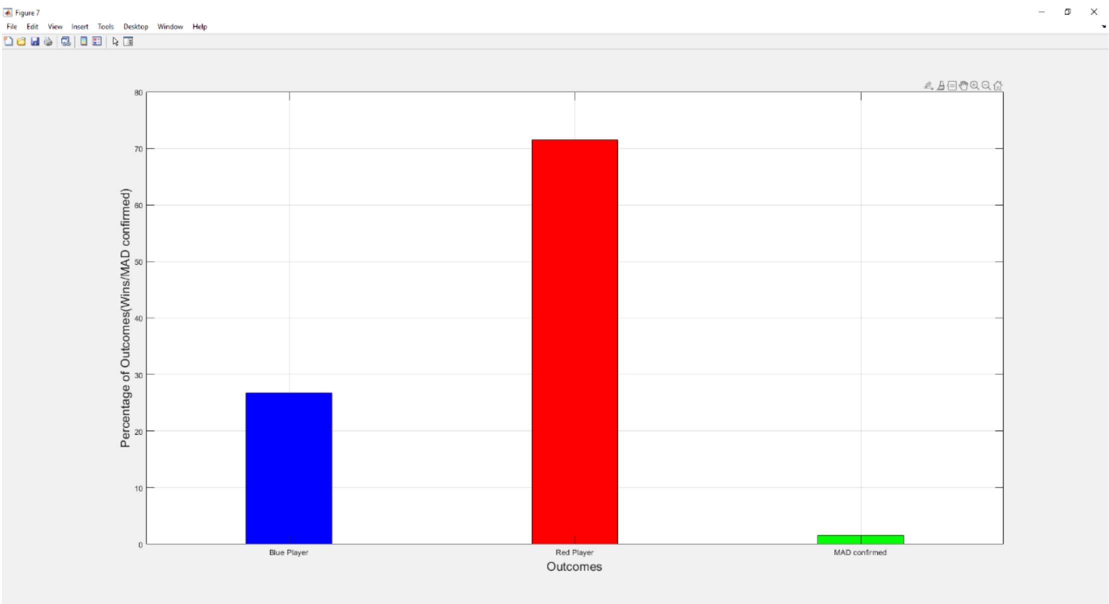

Figure 3.1. 3rd-simulation. Percentage of Wins.

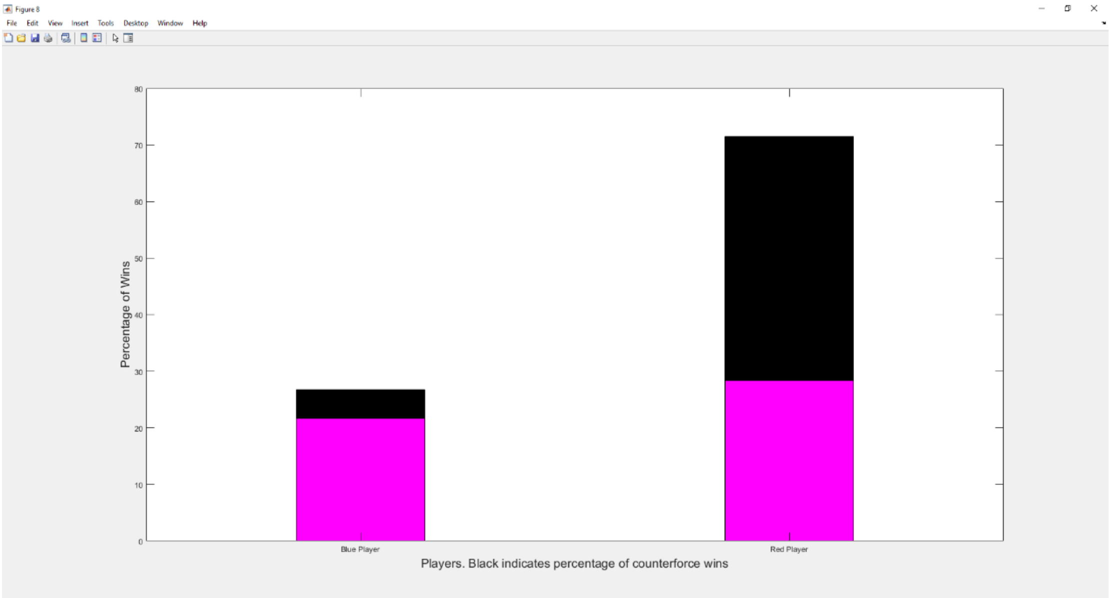

Figure 3.2. 3rd-simulation. Percentage of Wins Counterforce.

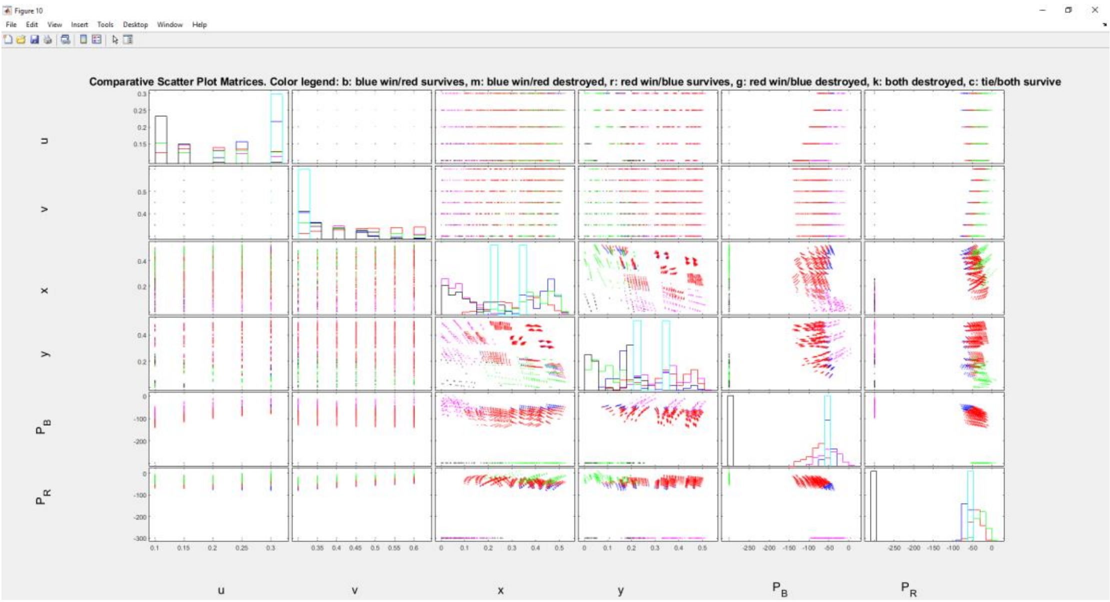

Figure 3.3. 3rd-simulation. Comparative Scatter Plot Matrices.

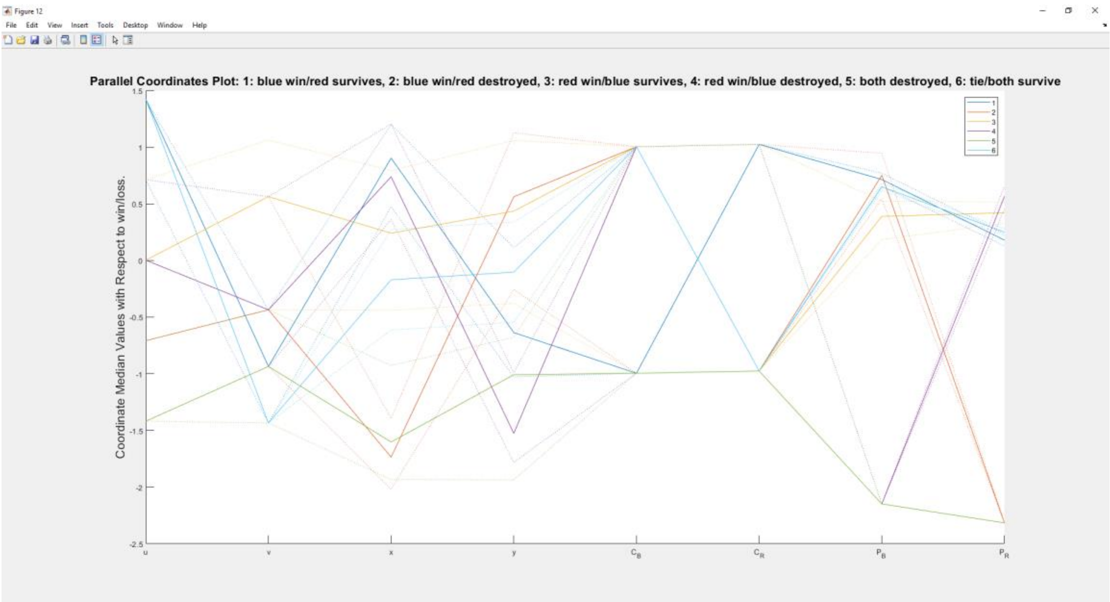

Figure 3.4. 3rd-simulation. Parallel Coordinates Plot.

Simulation 4

In this example we have two opponents with similar profiles, resulting in-as expected-similar outcomes. Both enjoy high counterforce capability, have a low number of weapons and a medium critical level. What is of interest is the percent of games where an equilibrium does exist, which is over 97%. We seek to explore whether this is due to the relative symmetry of the player profiles or due to the high counterforce capabilities of the players. To that end, we run another simulation (Simulation 5) where we change only the counterforce values to Low from High.

Simulation 4	
u	**High**
v	**High**
M	**Low**
N	**Low**
C_B	**Medium**
C_R	**Medium**
Blue Wins	**42**
Red Wins	**42**
Blue Wins Counterforce	**94.6**
Red Wins Counterforce	**94.6**
Mutual Assured Destruction	**14.2**
Both Survive	**84.6**
No equilibrium reached	**2.5**

Table 4.1. Summarizing the results of Simulation 4.

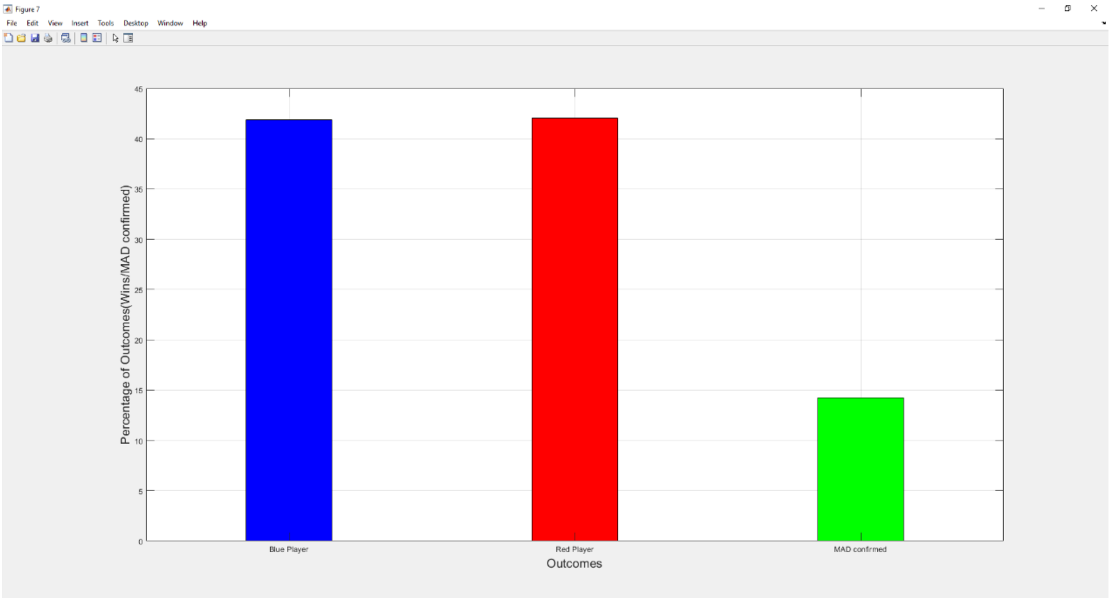

Figure 4.1. 4th-simulation. Percentage of Wins.

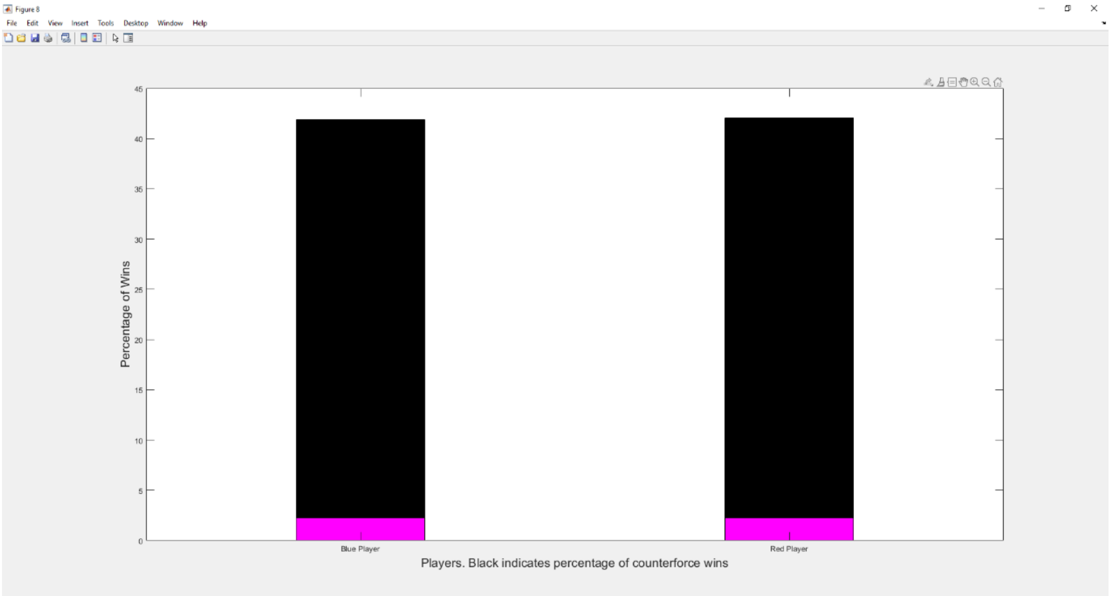

Figure 4.2. 4th-simulation. Percentage of Wins Counterforce.

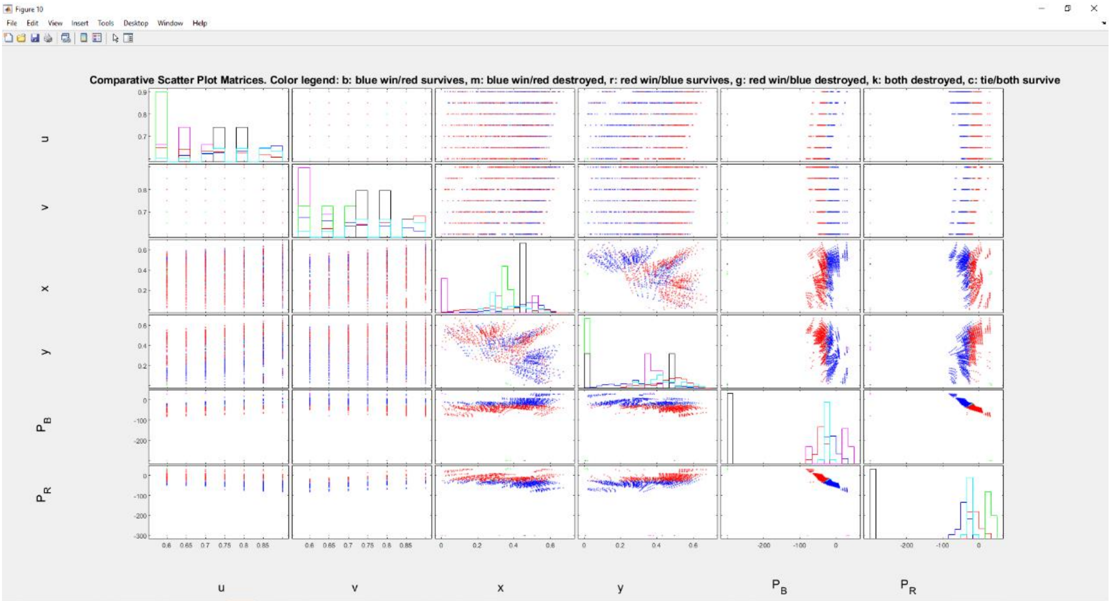

Figure 4.3. 4th-simulation. Comparative Scatter Plot Matrices.

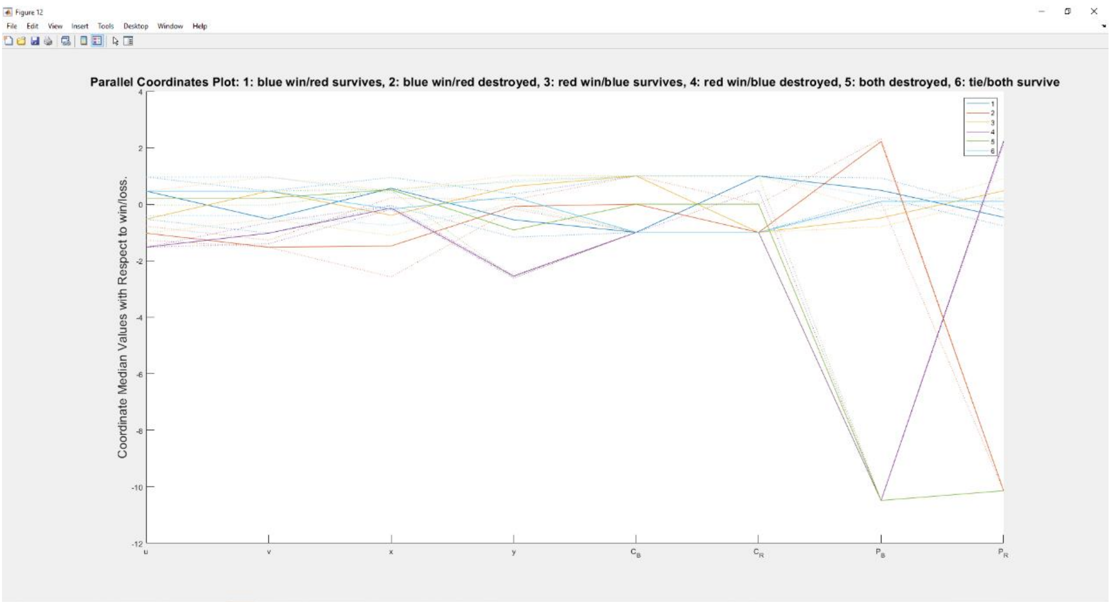

Figure 4.4. 4th-simulation. Parallel Coordinates Plot.

Simulation 5

Compared to Simulation 4, by leaving all other parameters similar, besides the counterforce potential of both Blue and Red, we obtain the following results, showing that the almost 100% (97.5%) success in reaching equilibrium points on the former simulation was due to the high values of u and v.

Simulation 5	
u	Low
v	Low
M	Low
N	Low
C_B	Medium
C_R	Medium
Blue Wins	45.9
Red Wins	45.9
Blue Wins Counterforce	51.4
Red Wins Counterforce	51.4
Mutual Assured Destruction	7.6
Both Survive	38.2
No equilibrium reached	58.3

Table 5.1. Summarizing the results of Simulation 5.

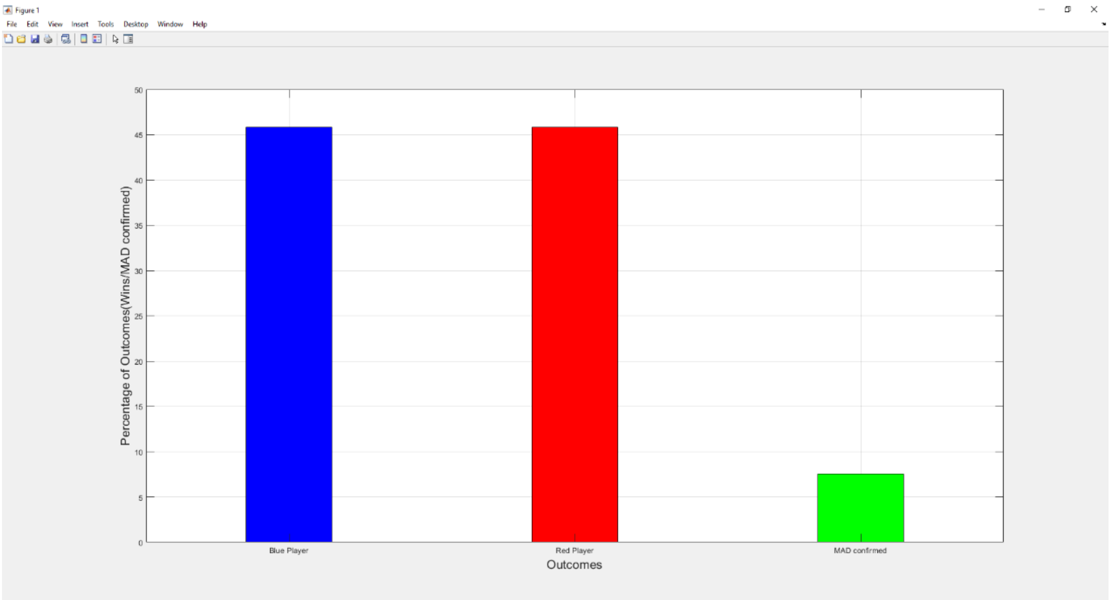

Figure 5.1. 5th-simulation. Percentage of Wins.

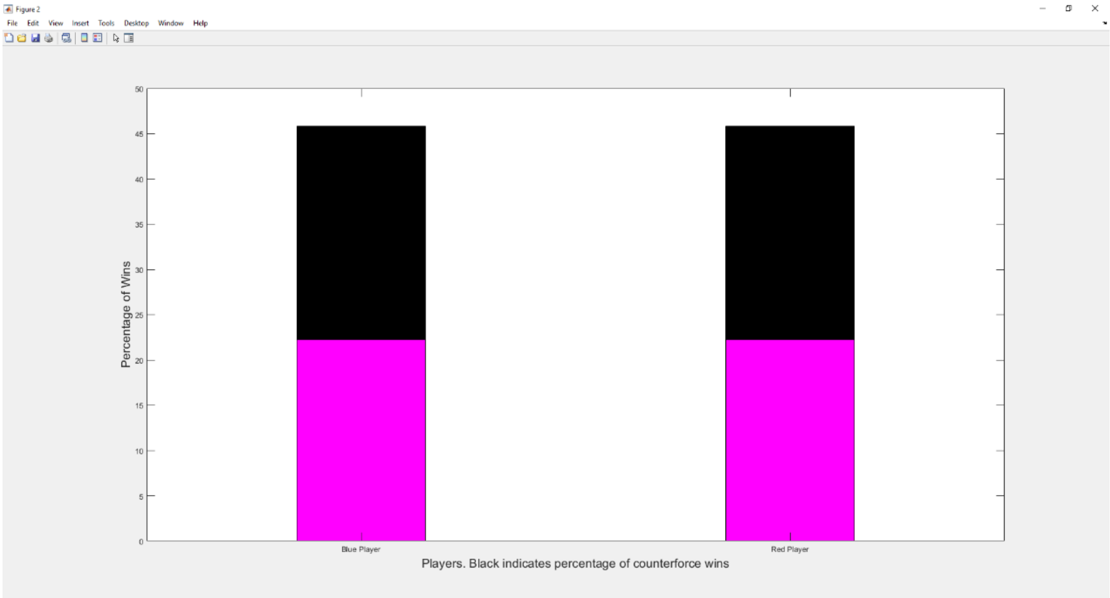

Figure 5.2. 5th-simulation. Percentage of Wins Counterforce.

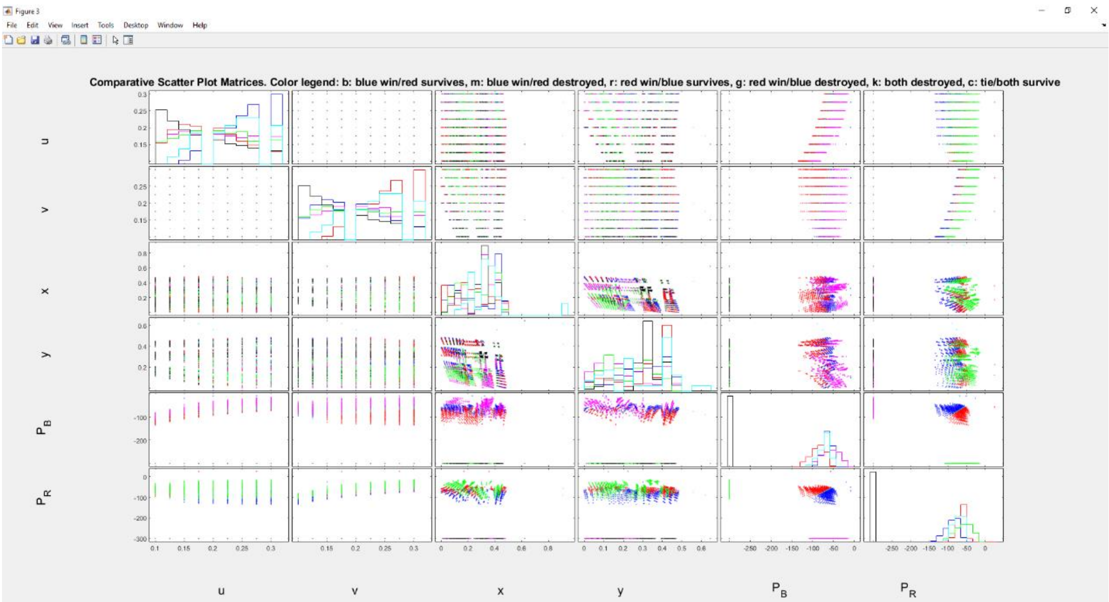

Figure 5.3. 5th-simulation. Comparative Scatter Plot Matrices.

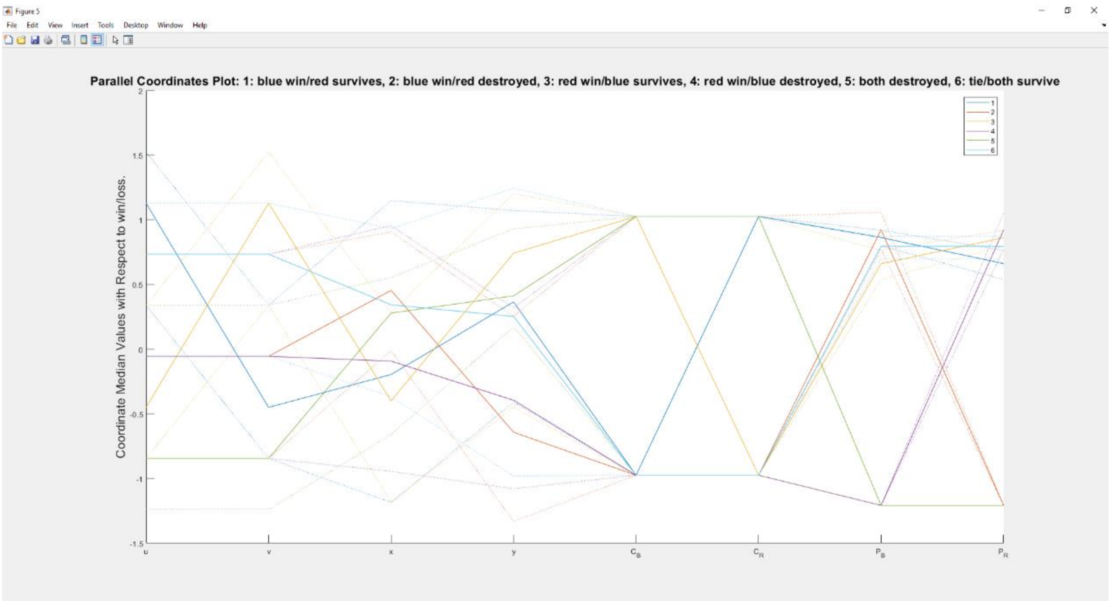

Figure 5.4. 5th-simulation. Parallel Coordinates Plot.

Simulation 6

Further trying to discern the factors that contribute to the stability of the solutions-that is, the percent of cases where an equilibrium is reached-we change the number of weapons for both players, from Low to High. Such values, above 200, practically mean that both have very significant excess damage potential-they can destroy each other two times over and "then some". We can see that the number of wins for each has slightly risen, while-counterintuitively-the percent of cases where both are destroyed has fallen considerably to less than 0.5%. But a very high number of cases result in unstable solutions with the no equilibrium percent rising to almost 75%.

Simulation 6	
u	**Low**
v	**Low**
M	**High**
N	**High**
C_B	**Medium**
C_R	**Medium**
Blue Wins	**49.5**
Red Wins	**49.5**
Blue Wins Counterforce	**100**
Red Wins Counterforce	**100**
Mutual Assured Destruction	**0.48**
Both Survive	**25.2**
No equilibrium reached	**74.3**

Table 6.1. Summarizing the results of Simulation 6.

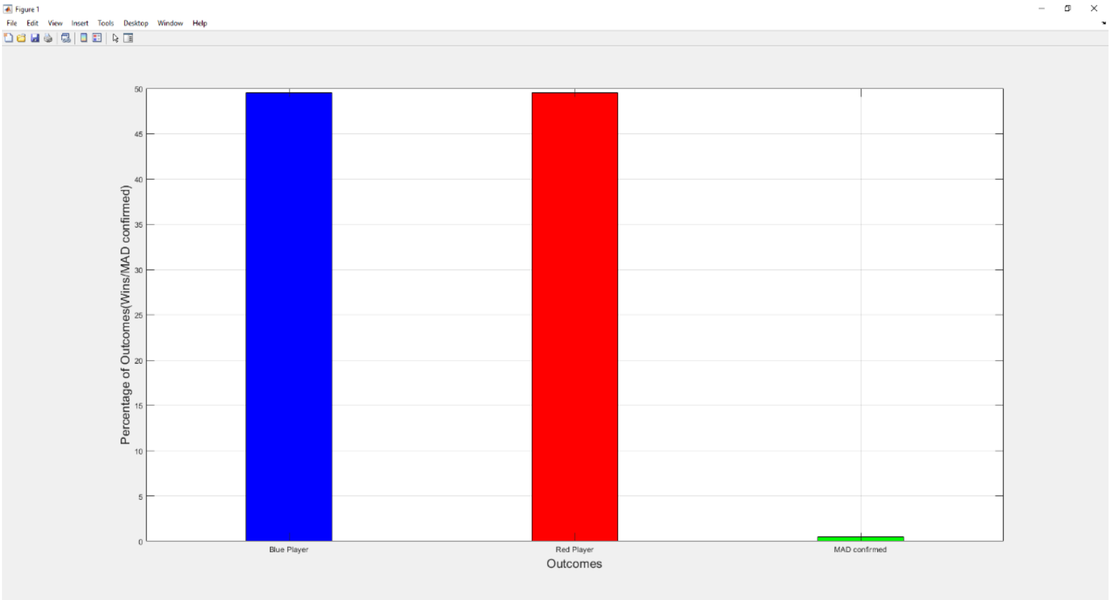

Figure 6.1. 6th-simulation. Percentage of Wins.

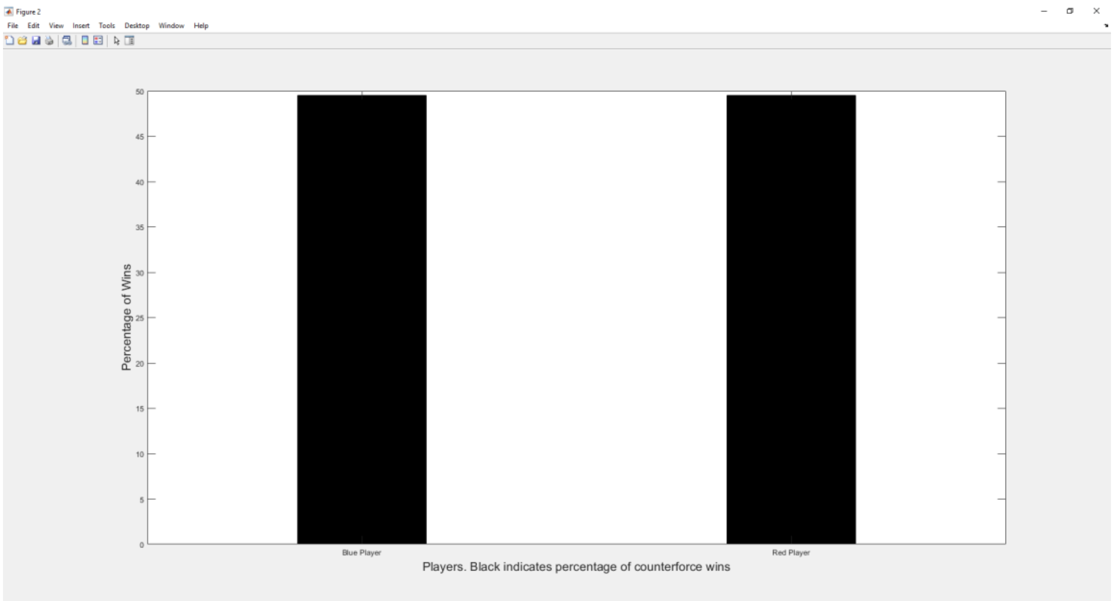

Figure 6.2. 6th-simulation. Percentage of Wins Counterforce.

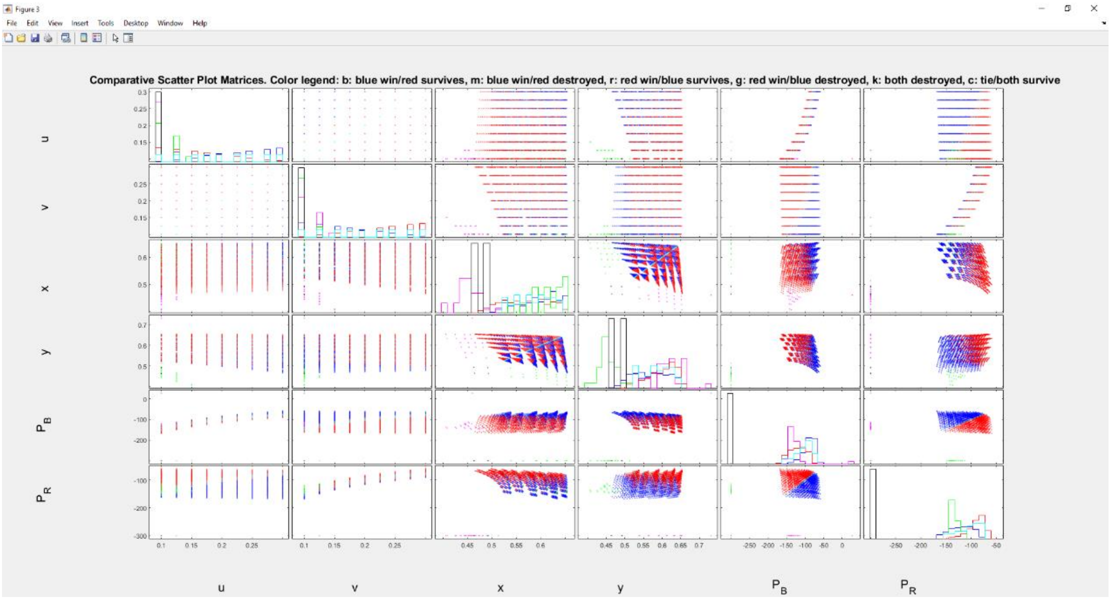

Figure 6.3. 6th-simulation. Comparative Scatter Plot Matrices.

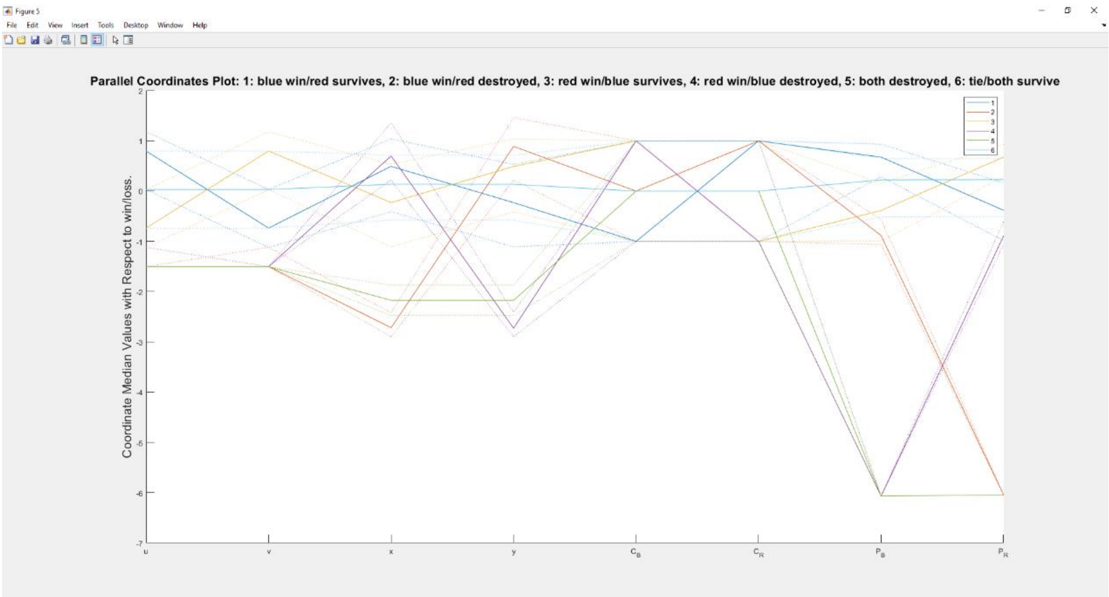

Figure 6.4. 6th-simulation. Parallel Coordinates Plot.

Simulation 7

On this final example, we return to non "symmetric" opponents, with Blue having Low counterforce capabilities, High number of weapons and a High critical level, facing off Red who enjoys High counterforce potential, lacks in the number of weapons, having Low capabilities but a similar High critical level.

Simulation 6	
u	Low
v	High
M	High
N	Low
C_B	High
C_R	High
Blue Wins	47.2
Red Wins	44.5
Blue Wins Counterforce	81
Red Wins Counterforce	32
Mutual Assured Destruction	8.1
Both Survive	30
No equilibrium reached	59.3

Table 7.1. Summarizing the results of Simulation 7.

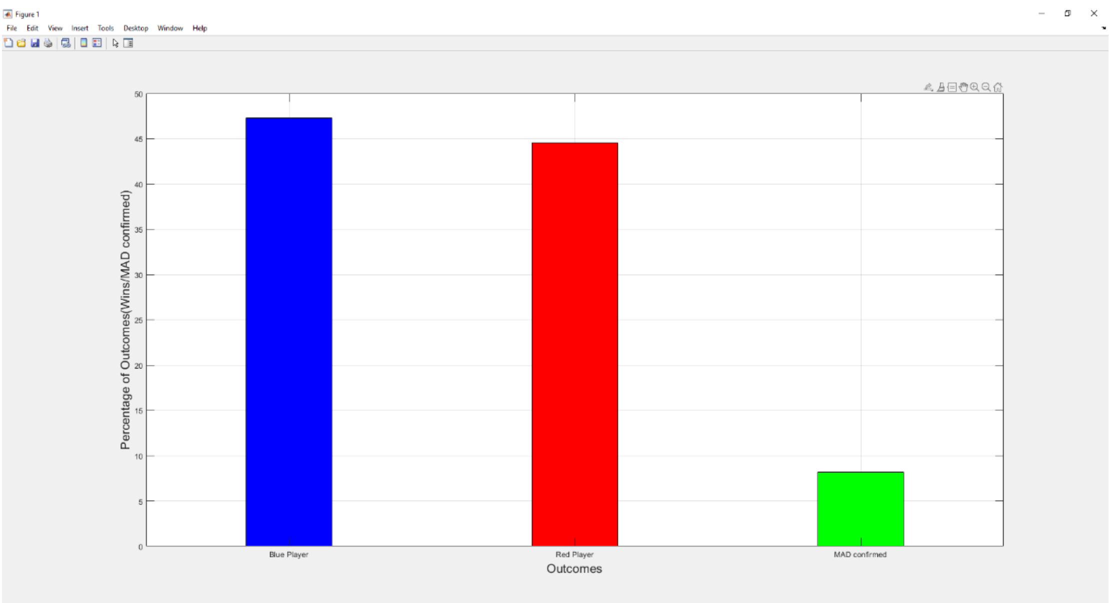

Figure 7.1. 7th-simulation. Percentage of Wins.

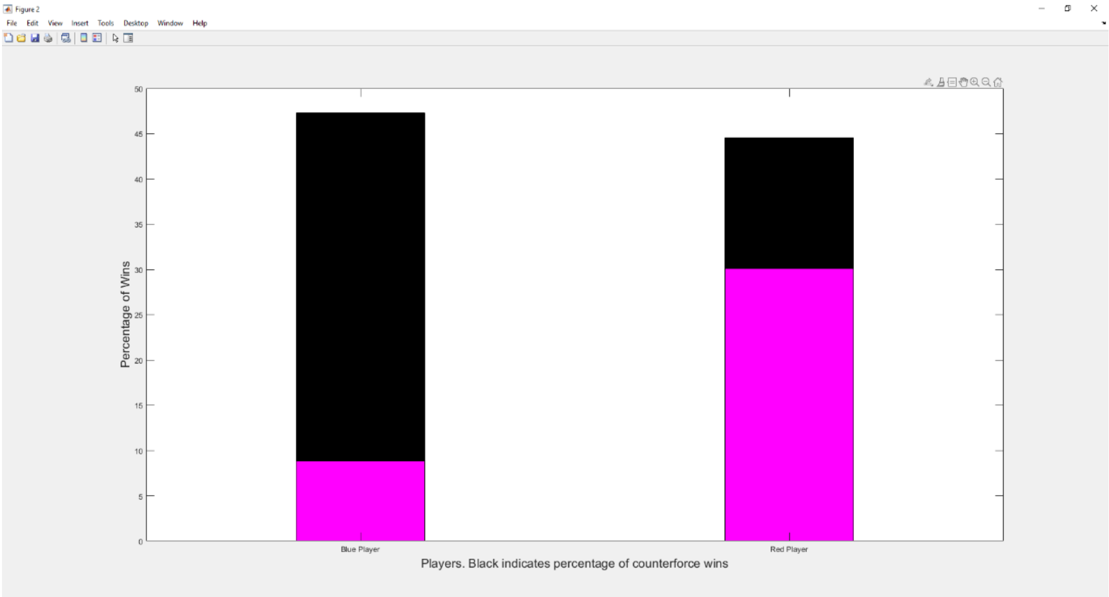

Figure 7.2. 7^{th}-simulation. Percentage of Wins Counterforce.

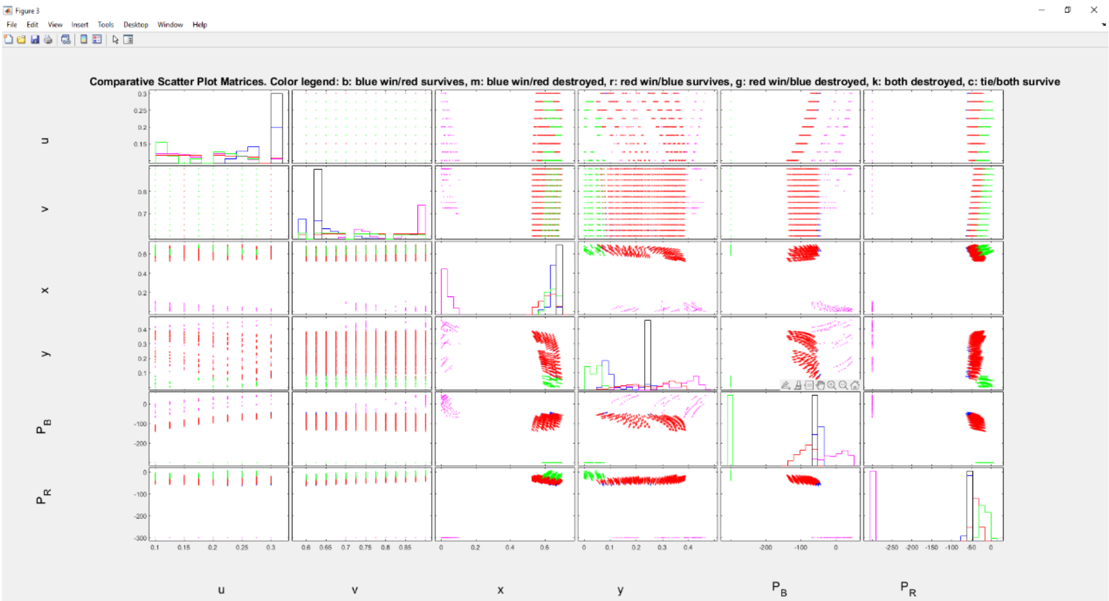

Figure 7.3. 7^{th}-simulation. Comparative Scatter Plot Matrices.

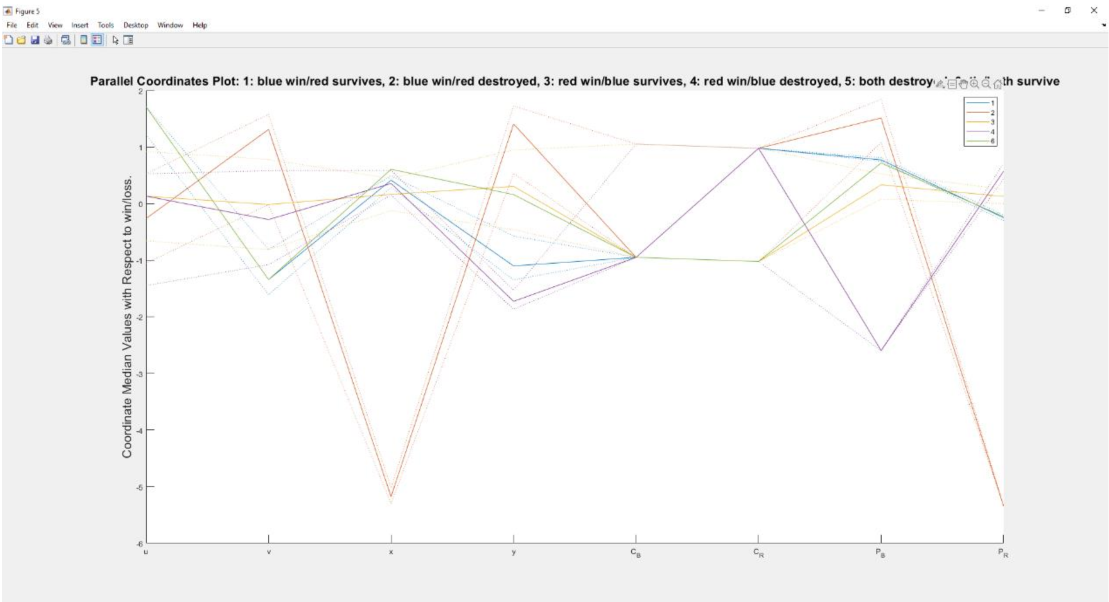

Figure 7.4. 7th-simulation. Parallel Coordinates Plot.

5 Behavior of the Solutions while a Single Parameter Varies

In this chapter we attempt to discern the behavior of the system solutions, as each of the three main types of parameters (u, M, C_B) takes different values while all others remain stable. Since the case for varying the (v, N, C_R) is symmetric, we focus on Blue player's parameter values. We plot the graph of the solutions (x,y) with respect to the parameter we study on each example, simultaneously providing for each point all values of importance to the analyst, as shown on the examples below. We are interested solely on solutions where equilibria do exist (the second-degree partial derivatives of the payoff functions are both < 0), even if they are not unique.

Example 1

Here we allow u to take values from 0.1 to 0.9 with a step of 0.01 while the rest of the parameters are as follows:

Example 1	
u	0.1:0.01:0.9
v	0.3
M	150
N	150
C_B	100
C_R	100

Table 8.1. Parameter values of Example 1.

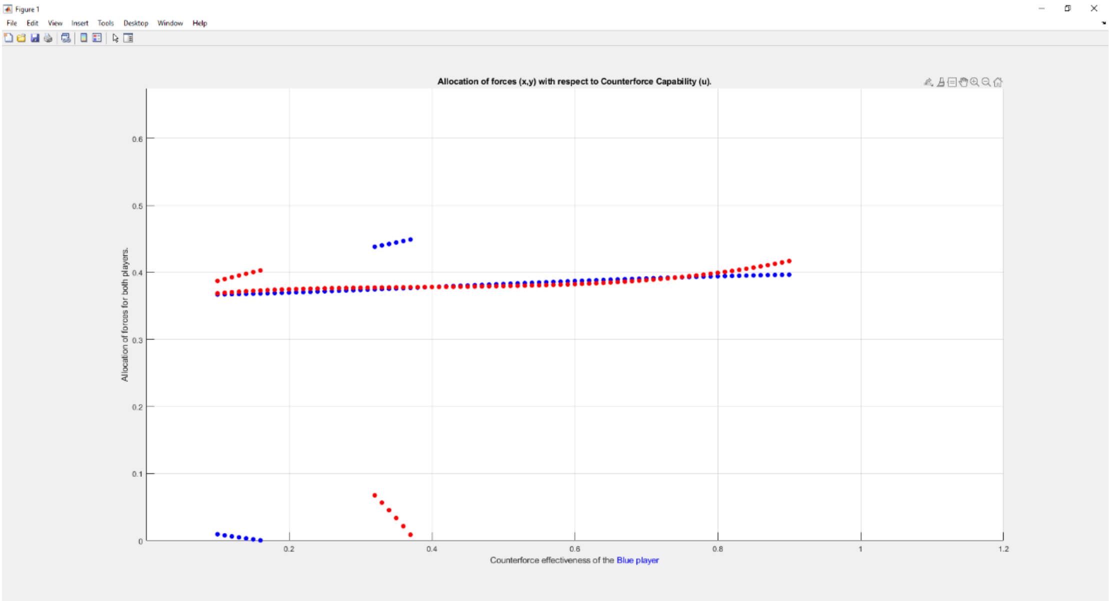

Figure 8.1. 1st-example. Allocation of forces with respect to counterforce capability.

We readily observe that on all solutions belonging to the two main curves spanning in the middle and across the spectrum of values for u, we obtain solutions where both players survive the engagement, while the outliers located on the nonadjacent lines represent games where at least one of the players is destroyed. We shall confirm the prevalence of this pattern with further examples.

Furthermore, we can see that as the counterforce effectiveness of Blues increases, Red allocates more weapons on a counterforce role as well, and even for the maximum value of u, 0.9, Red, enjoying a not insignificant counterforce value of $v = 0.4$ himself, can still reach relatively acceptable outcomes, where Blue receives damage almost equal to 60 (59.9) and Red equal to 80. For critical levels equal to 100 for both players, this roughly translates to the winner Blue losing 60 out of 100 civilian targets and Red losing 80.

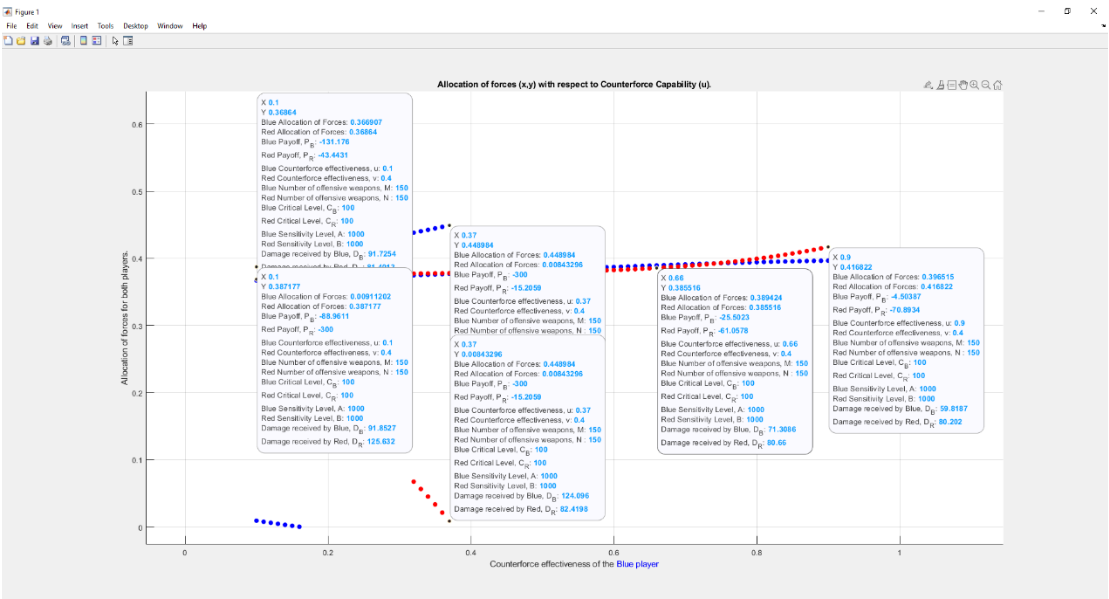

Figure 8.2. 1st-example. Allocation of forces with respect to counterforce capability. Legend of selected solutions.

Example 2

Here, as in Example 1, we allow u to take values from 0.1 to 0.9 with a step of 0.01, but change Red's counterforce capability to a very significant value of 0.8:

Example 2	
u	**0.1:0.01:0.9**
v	**0.8**
M	**150**
N	**150**
C_B	**100**
C_R	**100**

Table 9.1. Parameter values of Example 2.

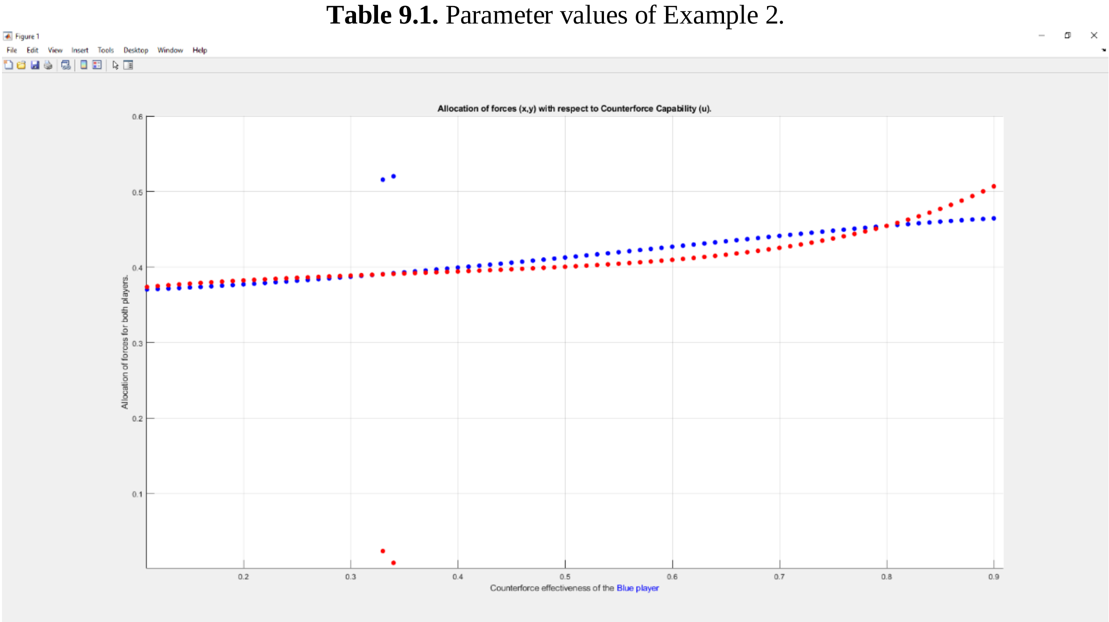

Figure 9.1. 2nd-example. Allocation of forces with respect to counterforce capability.

Again, we observe the outlier solutions representing cases where one of the two players is destroyed, a relatively smooth central pair of graphs, representing solutions where both survive, and a steady-but rising more steeply around the 0.7 value- increase on Red's allocation of forces to counterforce role, as Blue's counterforce capabilities grow. Also worth noticing is that here we get no solution where Red is destroyed, most likely due to his high counterforce capability.

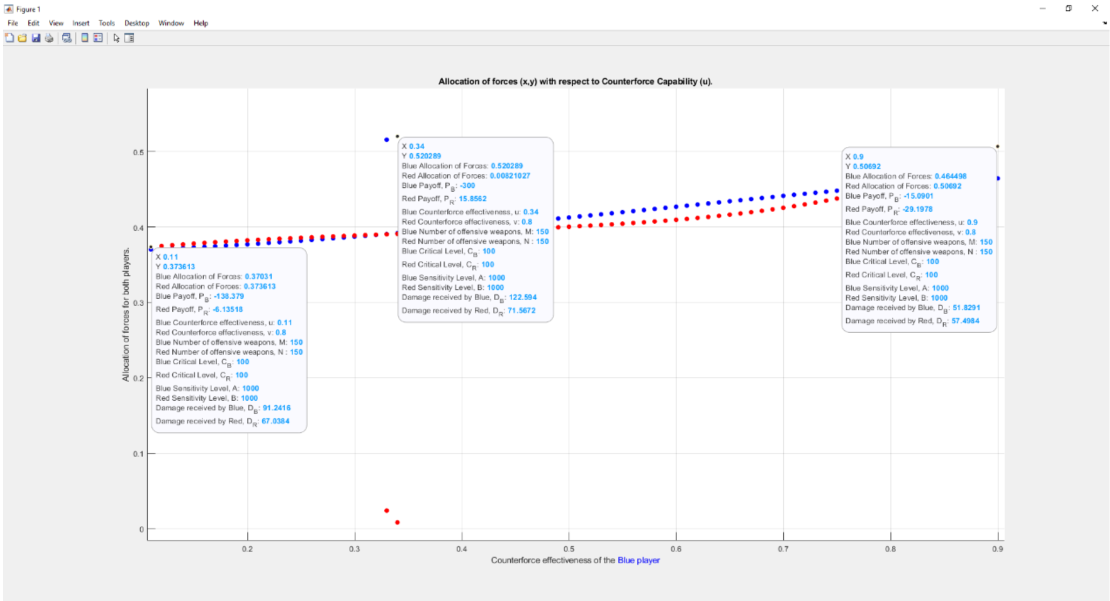

Figure 9.2. 2nd-example. Allocation of forces with respect to counterforce capability. Legend of selected solutions.

Example 3

Like before, we let u take values from 0.1 to 0.9 with a step of 0.01, but now we change Red's counterforce capability to a quite low value of 0.2:

Example 3	
$\boldsymbol{u}$	**0.1:0.01:0.9**
$\boldsymbol{v}$	**0.2**
$\boldsymbol{M}$	**150**
$\boldsymbol{N}$	**150**
$\boldsymbol{C_B}$	**100**
$\boldsymbol{C_R}$	**100**

Table 10.1. Parameter values of Example 3.

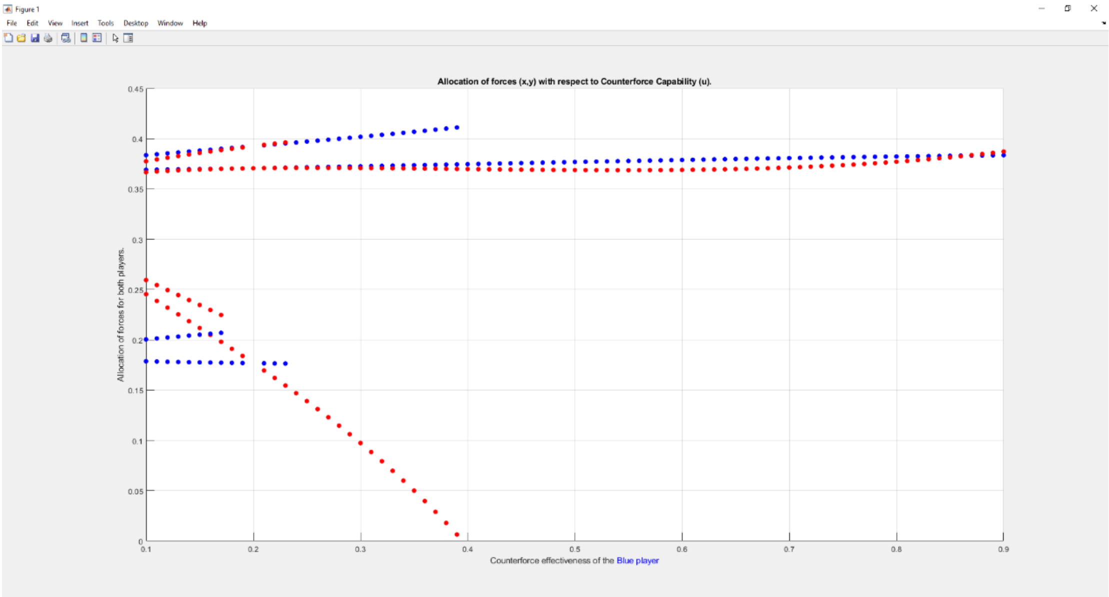

Figure 10.1. 3rd-example. Allocation of forces with respect to counterforce capability.

The situation here considerably differs regarding the outlier values. We have solutions that lead to mutual assured destruction when counterforce values are equally small for both players, even if they allocate an amount of forces $> 20\%$ to counterforce role. The downward moving outlier red curve and its "mirror" upward moving blue curve, beginning at values of 0.1 up to 0.4, indicate solutions where the Blue player is destroyed when Red chooses a pure countervalue strategy. But given a robust counterforce capability for Blue ($u > 0.4$), the solutions stabilize across the central intersecting curves where both players survive. Moreover, these solutions-along the central curves in instances where more than 2 solutions exist, represent dominant strategies.

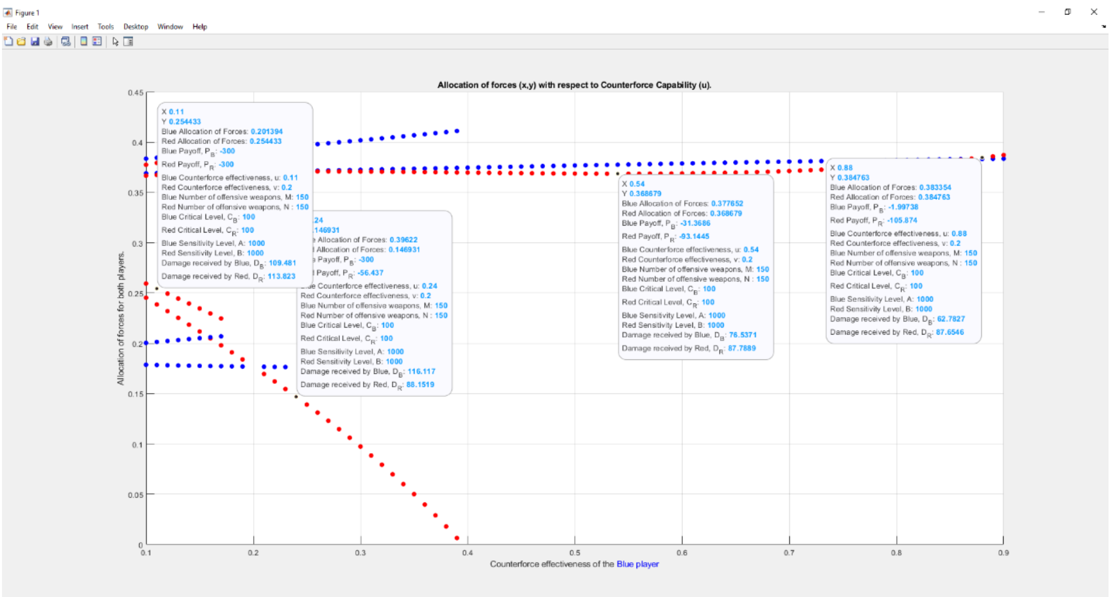

Figure 10.2. 3rd-example. Allocation of forces with respect to counterforce capability. Legend of selected solutions.

Example 4

We provide a final example with u varying, where both players have a large number of weapons ($M = N = 250$) available and Red's counterforce capability is set at $v = 0.5$.

Example 4	
u	**0.1:001:0.9**
v	**0.5**
M	**250**
N	**250**
C_B	**100**
C_R	**100**

Table 11.1. Parameter values of Example 4.

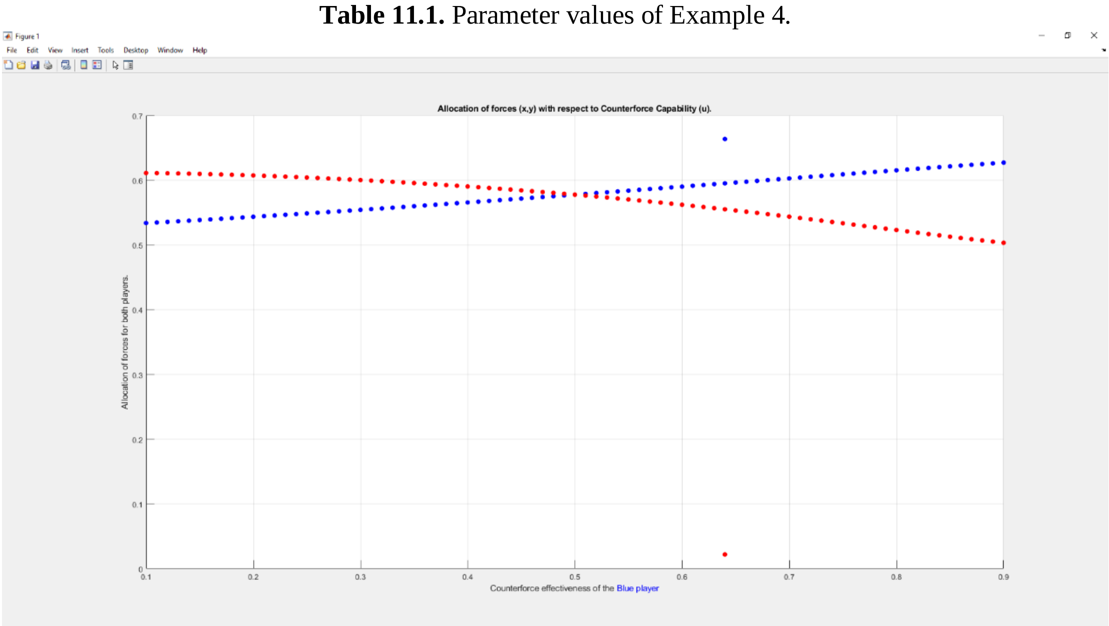

Figure 11.1. 4th-example. Allocation of forces with respect to counterforce capability.

Somewhat counterintuitively, the large number of weapons ensuring excess damage potential (each can destroy the other more than two times over), does not lead to mutual destruction and presents a remarkably stable solution space. Only one outlier solution exists (the two mirroring points at a value of $u = 0.64$) where Red goes for a pure countervalue strategy and Blue is destroyed.

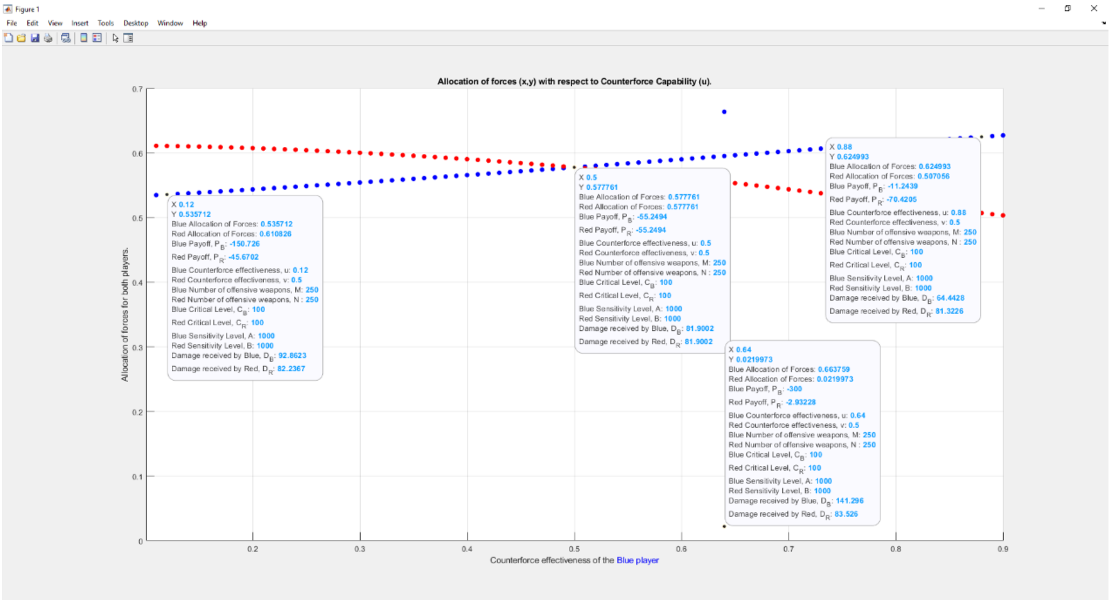

Figure 11.2. 4th -example. Allocation of forces with respect to counterforce capability. Legend of selected solutions.

Example 5

We now turn our attention to an example where M is varying. As we can see, for values of $M < 107$, the system does not produce any equilibrium points. While it does, their behavior falls into patterns seen previously.

Example 5	
u	**0.4**
v	**0.4**
M	**50:1:250**
N	**150**
C_B	**100**
C_R	**100**

Table 12.1. Parameter values of Example 5.

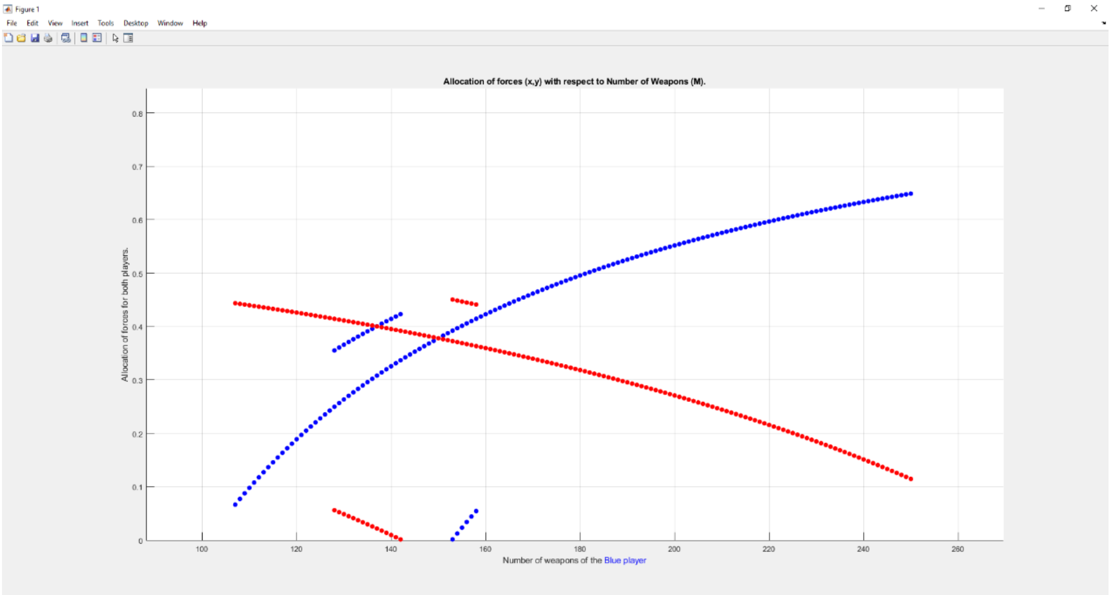

Figure 12.1. 5th-example. Allocation of forces with respect to number of weapons.

With M rising, Blue is allocating more and more weapons to a counterforce role and concurrently Red choses an increasingly countervalue strategy. Outlier values where either one is destroyed appear in two groups, for values of M between 128 and 142 and 153 and 158.

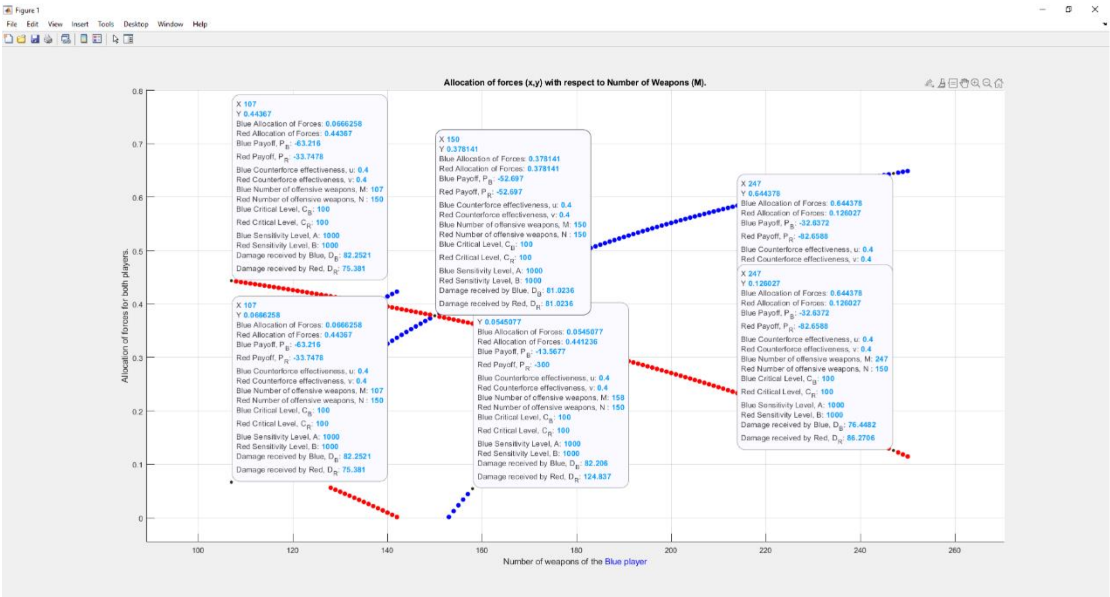

Figure 12.2. 5th -example. Allocation of forces with respect to number of weapons. Legend of selected solutions.

Example 6

Here we again allow M to vary and we pit uneven opponents, with Blue having a quite low counterforce potential ($u = 0.2$) while Red a very substantial ($v = 0.8$) one. Critical levels of the two are also uneven, with Blue set on 80 and Red on 100.

Example 6	
u	**0.2**
v	**0.2**
M	**50:1:250**
N	**150**
C_B	**80**
C_R	**100**

Table 13.1. Parameter values of Example 6.

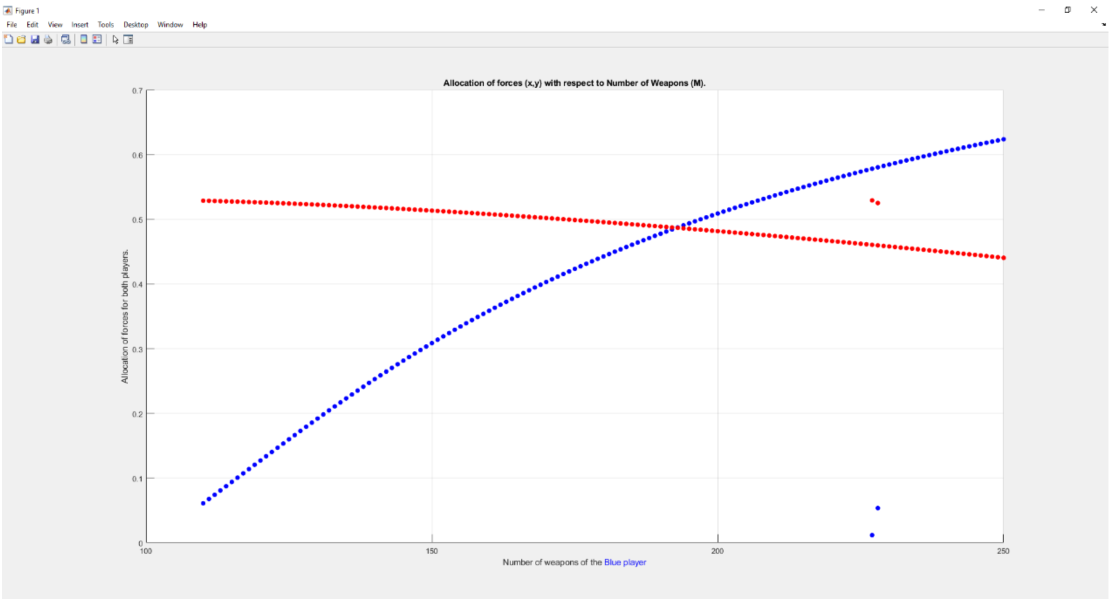

Figure 13.1. 6th-example. Allocation of forces with respect to number of weapons.

The number of outliers has been reduced to just two, but otherwise the behavior of the solutions is very similar, again producing no solutions for low values of M (for $M < 111$).

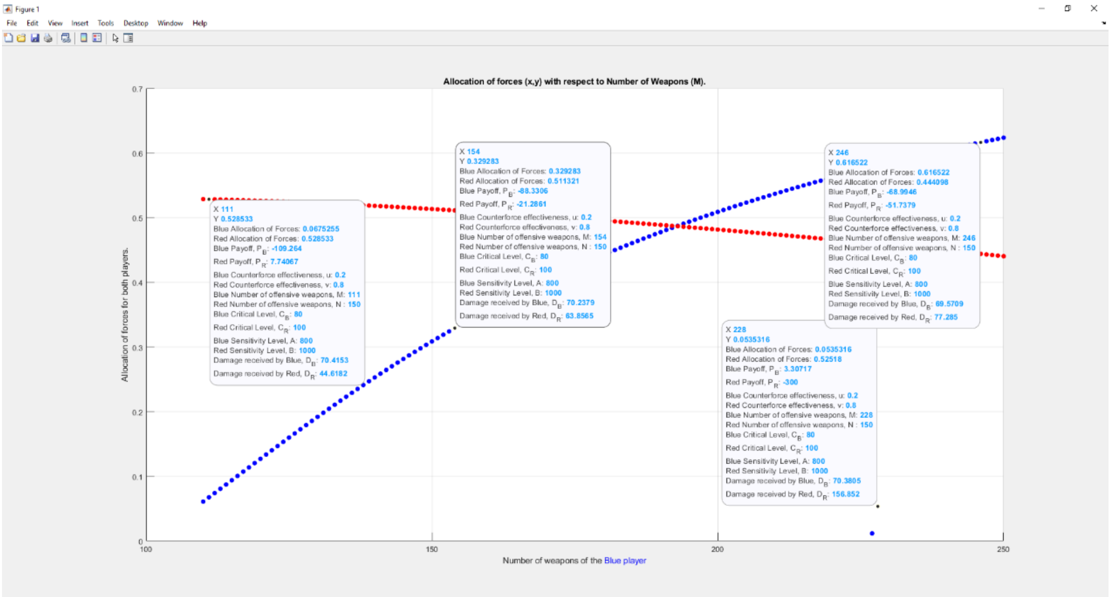

Figure 13.2. 6th -example. Allocation of forces with respect to number of weapons. Legend of selected solutions.

Example 7

Finally, we turn our attention to studying the behavior of the system when the critical level varies.

Example 7	
u	**0.3**
v	**0.3**
M	**100**
N	**100**
C_B	**50:1:100**
C_R	**100**

Table 14.1. Parameter values of Example 7.

We observe that for values of C_B less than 67, the system produces no solutions, while for values up to 76, all solutions lead to the destruction of Blue. But as we move to greater values of C_B, both players survive the exchange. As Blue's critical level increases (his capacity to endure damage), so does his allocation of weapons to counterforce while for the Red player we observe the reversed trend.

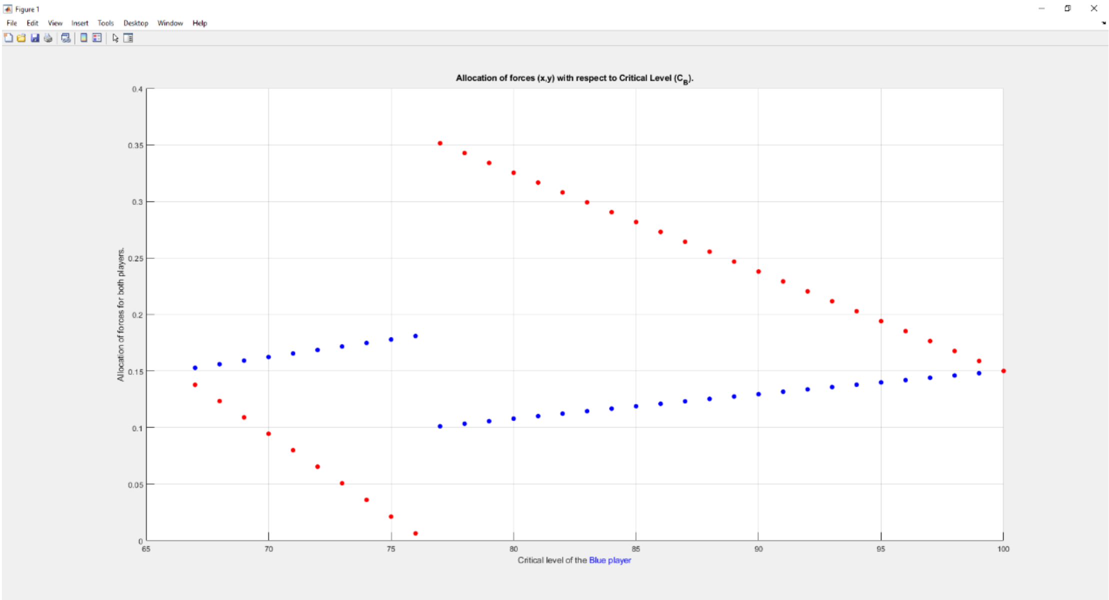

Figure 14.1. 7th-example. Allocation of forces with respect to number of weapons.

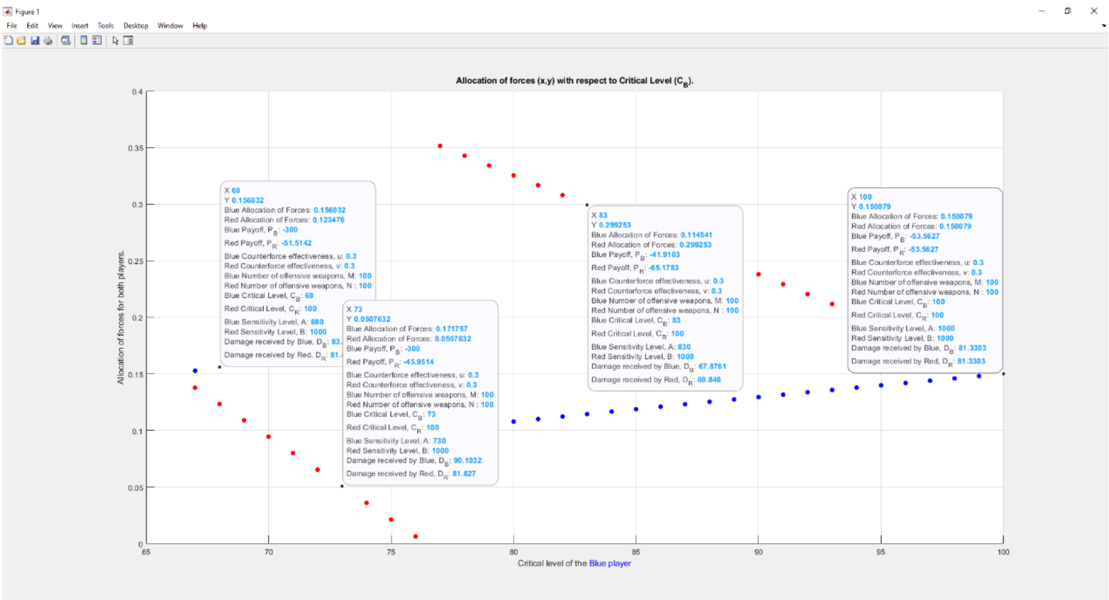

Figure 14.2. 7th -example. Allocation of forces with respect to number of weapons. Legend of selected solutions.

Example 8

Here we again pit two uneven opponents, with Blue having a clear disadvantage on counterforce ($u = 0.2, v = 0.8$) and Red a clear disadvantage on the number of weapons ($M = 200, N = 110$).

Example 8	
u	0.2
v	0.8
M	200
N	110
C_B	50:1:100
C_R	100

Table 15.1. Parameter values of Example 8.

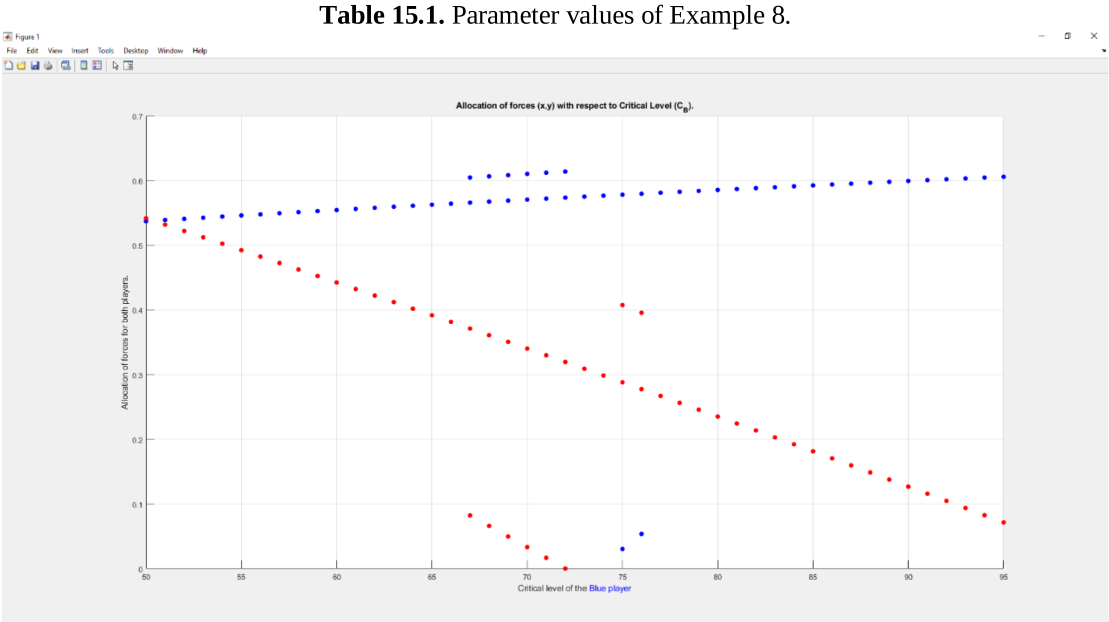

Figure 15.1. 8th-example. Allocation of forces with respect to number of weapons.

Interestingly, we obtain solutions for the entire run of the values of C_B up to and including $C_B = 95$ but not for the last five values. As C_B increases, Blue allocates more weapons on counterforce and Red on countervalue, but in a very uneven fashion. Even a very high fixed counterforce value of 0.8 for Red is not enough to fully compensate for the difference in available weapons, nor does it prevent solutions where Blue, opting for a pure countervalue strategy, destroys him outright.

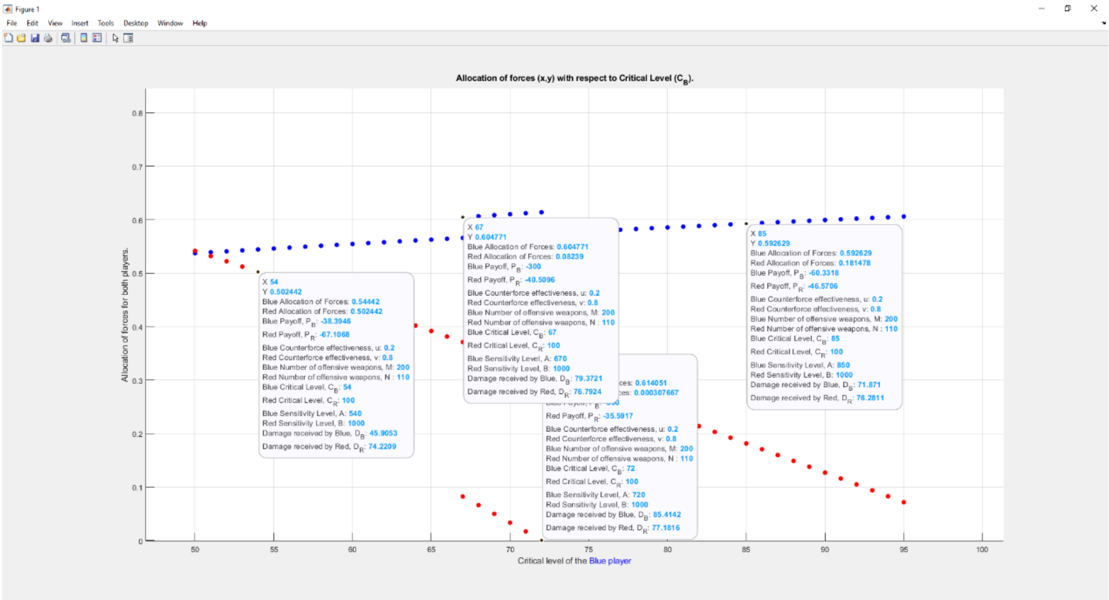

Figure 15.2. 8th -example. Allocation of forces with respect to number of weapons. Legend of selected solutions.

6 Conclusion

6.1 Discussion of Results

By expanding our research beyond the symmetric case studied by Dalkey, where both opponents have identical parameter values, we have demonstrated that the model can indeed produce solutions in the form of pure equilibria, but the proportion of these to cases where only local maxima or non-dominating solutions are reached, varies considerably. Can we say that the validity of the model is confirmed? That is, if the assumption of increasing concern is assumed to hold, are mixed counterforce/countervalue strategies viable?

We answer on the affirmative, though this entails the much broader question of what the interpretation of the cases is where no equilibria are reached. As we recorded from the simulations we run, with the exception of a select few, we observe a high instance of such cases-where no pure equilibrium is reached. Even so, we can say with a significant degree of confidence, that if the assumption of increasing concern upon which the model is based is correct, then, in line with the hypothesis, a nuclear exchange indeed does not assuredly lead to the mutual destruction of the two opponents, including cases where the two sides considerably differ on counterforce effectiveness, number of weapons and critical levels.

What is the direct practical result of this work? Despite the inherent difficulties of accurately knowing the parameters related to counterforce effectiveness and critical levels of both the friendly side and the adversary's (the number of available weapons is known for the friendly side and to a very high level of certainty for the opponent's side as well), an estimate can be made upon which to base simulations that will help inform the leadership and support a cost benefit analysis of the decision-making process.

By running all combinations of opposing player profiles, we can obtain a detailed mapping of outcomes, ideally suited for decision support on high-level, extremely time-constrained crises scenarios. Moreover, the results of this work can be employed by viewing the problem as an allocation of resources between the number of weapons and counterforce capability, against fixed opponent profiles.

6.2 Future Work

As we mentioned on the end of Paragraph 2.4, a formulation of an enhanced model where the number of players is increased, as well as the conventional counterforce capabilities of each player are taken into account, is essential to adapt it to the emerging geopolitical environment and new disruptive technologies entering the realm of strategic warfare. To that end, we hope to revisit this subject providing such a model.

Of particular interest we consider the way the game potentially changes from a highly uncooperative one, to one of a cooperative form when more than two players are considered. Whether this signifies a more stable strategic environment on a multi-polar world, or the instability of the system rises is a question worth pursuing through modeling the interactions of a nuclear war taking place within such a system.

Beyond that, tackling the challenges of getting a working prediction of the parameter values of an opponent's potential and critical levels, though it falls outside the scope of this work, is an absolute necessity in and on itself, if any robust simulation is to take place.

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