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Efficiency Measurement of 6 Major Container Ports in West Africa with Data Envelopment Analysis and Stochastic Frontier Analysis

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Abstract

The aim of this dissertation is to apply the DEA and SFA methods to evaluate efficiencies of 6 major ports out of 12 in total, in West Africa and to understand if these ports can become the main hubs of container transport to African inland in the future and how they can evolve through the time. The selection of 6 West African ports based on their container throughput levels which is over 100,000 TEU's per year. The DEA and SFA methods were used to determine their relative efficiencies and their efficiencies over time through window analysis for the period 2006-2012.The DEA and SFA methods were applied to a number of inputs such as total quay length, total terminal area, number of quayside cranes, number of gantry cranes and number of reach stackers and a single output which is the total TEU's throughput. It was determined via DEA method that the Port of Tema in Ghana with the Port of Lomé in Togo was the most efficient West African ports under study. On the Contrary the Port of Cotonou in Benin was found to be the least efficient port obtaining the lowest average efficiency rating over the period 2006-2012 via DEA method.In most cases, West African ports could be said to exhibit high levels of efficiency considering that four out of six ports had an average efficiency score of 76% or higher for the period under study. Through SFA method the results were much different from DEA, as only three ports had average efficiency scores over 76%. Port of Dakar was the most efficient of West African ports and Port of Lagos was determined as the most inefficient port.

Keywords

Data Envelopment Analysis (DEA), Stochastic Frontier Analysis (SFA), Port Efficiency, West-Africa, Window Analysis.

1. Introduction

The efficiency of port operation is an important indicator of economic development since more than 80% of the global international trade is conducted by way of maritime transportation. In order to assist the container ports to identify their own strengths, weakness, and the potentially existent threats and opportunities in a competitive environment, it is essentially necessary and critically important to select a set of impartial and objective measures for introducing the efficiency evaluation (Lin, L. C. & Tseng, L. A., 2005).

This dissertation is aiming to evaluate efficiencies via the application of DEA and SFA method to 6 major ports out of 12 in total, in West Africa and to understand if these ports can become the main hubs of container transport to Africa inland in the future and how they can evolve through the time and to contribute via comparison of SFA and DEA method to the previous observations made in the same sample of ports by Kobina G. van Dyck in 2015.

Container transport and containerization has led to increased competition between ports worldwide. These days, hinterlands have become more shared due to better efficiency of ports and increased hinterland connectivity facilitated by containerization and multi-modality. The result of this intense inter-port competition in the container port sector is the interest in efficiency analysis by port operators and port users (Cullinane, K. & Wang T. F., 2007).

Efficiency analysis provides port operators and port authorities with a means of making more informed decisions with regards to port planning and operations whiles it provides port users with a means of assessing the relative competiveness of ports in order to make informed decisions on port utilization to maximize its efficiency and productivity.

Ports in Western-Europe, North-America and East-Asia have, for many years, utilized efficiency analysis to improve operations by minimizing the use of resources for production. This led to port growth and massive investment in port related activities. The port industry, in West Africa has seen major growth in recent times. The last 20 years, a number of West African ports have undergone restructuring and reform processes. These processes targets on allowing more private sector involvement in the port sector to generate investment for port development and to increase capacities, efficiencies and productivities of ports. Lately, port development in West Africa has been directed towards attaining hub port status (Kobina G. van Dyck, 2015).

Competition of international ports is at its highest level and private sector investment in port facilities continues to rise in the region. Nowadays, ports that play a regional role in West Africa are generally viewed as the leading potential hub port contenders, including ports in Ghana, Togo, Ivory Coast, Benin and Senegal, which provide transit services for landlocked countries in West Africa.

However, the global ranking of these ports except the Port of Lagos in Nigeria is lower than top 100 container ports due to the small amount of TEUs.

In the list with the busiest ports in Africa only the Port of Lagos in Nigeria and the Port of Abidjan in Ivory Coast are included.

The largest economy in West Africa and which has some of the largest ports in the region, Nigeria, however does not play a significant regional role as the distance between its ports and the landlocked countries in the region is great.

Additionally, in the past, Nigeria's large domestic demand has been the government's priority. Recently however, the Nigerian government is looking to play a more regional role in shipping and is directing its port development efforts to that effect. There are several examples of port development projects in West Africa that have regional focus and are directed at attaining regional hub status.

For example, in Nigeria, the Lekki Port project pursue to create a multi-purpose deep water port in the Lagos free trade zone area with a projected capacity of 2.5 million TEU's (twenty-foot equivalent units) per annum. The port will include container, dry bulk and liquid bulk berths with a 14-metre draught and 670 metres turning cycle to accommodate larger ships. (http://lekkiport.com/theport/key-facilities.html).

Similarly, the Ghana Ports and Harbours Authority (GPHA) has secured $1.5 billion for the expansion of the Port of Tema.

The project involves the construction of four (4) deep water berths and an access channel to accommodate larger vessels with high capacity equipment.

The aim of the project is to create the largest cargo port in West Africa with a capacity of 3.5 million TEU per annum once complete in 2018 (Port Finance International, 2014).

The Port of Lomé has constructed a $640 million berth in Togo. The new quay has double docking capacity and measures 450 metres able to accommodate vessels of more than 7000 TEU capacity (AFDB, 2010).

Similar port development projects can be found in other West African countries, as there is no exclusivity in the selection of a hub by shipping lines. The selection of a port to act as hub depends on a number of factors. In latest surveys, major shipping lines calling at West African ports were required to rank factors influential to the selection of a hub for the region (Kobina G. van Dyck, 2015).

High port efficiency and performance were ranked first amongst a list of 20 factors. West African ports have been noted to be highly congested and inefficient as compared with ports in Europe and Asia (Cullinane, K. P. B. & Wang, T. F., 2006).

However, the aim of this dissertation is to empirically assess the efficiencies of ports in West Africa utilizing the DEA and SFA methods. Measurement and analysis of port efficiency in West Africa allow port users to make efficiency comparisons and provide regional and national port operators with an important management tool for making informed decisions on port planning and operations (Kobina G. van Dyck, 2015).

This dissertation is organized as follows:

- Section 2 discusses the literatures for port operating efficiency.
- Section 3 presents the methodologies of DEA, SFA and their differences.
- Section 4 assesses the efficiency ratings and ranks of 6 West African container ports.
- Section 5 presents the conclusions and the proposals.

Source: Google Maps

Figure 1.Map of the six (6) West African port positions

2. Background Literature Review

Previous literature about the port operating efficiency is relatively humble in comparison to the literature available on other infrastructure activities (Estache et al., 2002).

The main reason is that procedures of port production are complex because they are including pilotage, towage, berthing, cargo and container handling, warehousing, and logistics. The improvement for port operating efficiency could include: improvement in efficiency through private sector management skills, enhancement of service quality through improved commercial responsiveness, reduction in the fiscal burden of loss making public enterprises, a reduction in the financial demands on central and local government through access to private sector capital, and additional revenue streams (McDonagh, 1999).

From the point of view of container terminal productivity, each port's player has his own self-interest and his own definition of productivity, proposed by Dowd and Leschine (1990).

As most port operations have been privatized, private operators aimed to maximize output, namely, container throughput and operating efficiencies (Heaver et al., 2000).

The operating efficiency of a container port or a container terminal is a mixture of multiple inputs and multiple outputs, which is in conformity with the characteristics of Data Envelopment Analysis (DEA). The Data Envelopment Analysis (DEA) with mathematical programming techniques has applied to the measurement of port efficiency for hypothetical port data by Roll and Hayuth (1993).

There are numerous papers that have extended and applied alternative models of the Data Envelopment Analysis (DEA) methodology, including BCC, Additive, FDH (Free Disposal Hull), etc, such as an application of BCC model to check global efficiencies of 26 Spain ports using 5 observations for each port from 1993 to 1997 and to examine efficiency evolution of individual port (Martínez et al., 1999).

Utilization of CCR and additive models to make an international comparison of technical efficiencies in 4 Australian and 12 other international container ports in 1996 was proposed by Tongzon (2001).

The CCR, BCC, and FDH models also used by Wang, Song, and Cullinane (2003) to evaluate production efficiencies of 57 terminals within 28 container ports for year 2001, and find that the FDH model is the best model of port efficiency measurement.

Valentine & Gray (2001) further applied CCR model to calculate relative efficiencies of 31 global container ports in 2001, and follow cluster analysis to determine whether there is a particular type of ownership and organizational arrangement that leads to higher efficiency rating. However, privatization of container port operation has been prevail in recent years, and private terminal operators targeting to maximize profit, which is in abidance with the characteristics of stochastic frontier analysis (SFA).

SFA method stays on the quantitative economy theory that has been applied to the measurement of technical efficiency for 28 Britain ports during 1983-1990, by Liu (1993).

SFA with Cobb-Douglas and Translog production function for the half-normal and truncated-normal distributions to estimate production efficiencies of 11 Mexico container ports with two inputs labour and capital and one output, volume of merchandise handled from 1996 to 1999 was applied by Estache et al (2002).

Also, SFA method with Cobb-Douglas production function for the half-normal, exponential, and truncated-normal distributions to estimate production efficiencies of 15 Asian container ports and terminals with unbalanced-panel data between 1989 and 1998) was used by Cullinane, Song, and Gray (2002).

DEA and SFA methods also applied both to estimate the relative productive efficiency for 74 railway systems in 1999, and use the two-stage method of DEA with CCR and BCC models and the SFA method with Translog production function for the half-normal and truncated-normal distributions by Lan and Lin (2003).

The research on operating efficiency at ports by applying DEA or SFA is summarized as shown on Table Additionally, both DEA and SFA methods are also applied together in transport industry.

Author	Data Description	Model Evaluation	Input/output Variables	Efficiency Concept
Roll and Hayuth (1993)	20 ports in the world Cross-section Hypothetical port data	DEA	Input: manpower, capital, Cargo uniformity. Output: total cargo throughput, level of service, users' satisfaction, ship calls.	Technical efficiency, Sensitivity.
Martínez, Diaz, Navarro, and Ravelo (1999)	26 Spanish ports Panel data in 1993-1997	DEA with BCC Model	Input: labour expenditures, depreciation charges, other expenditures Output: Total cargo throughput, revenue for the rent of port facilities	Global efficiency, Slack analysis
Tongzon [2001]	4 Australian and 12 other international container ports Cross-section data 1996	DEA with CCR and Additive DEA models (constant returns to scale and variable returns to scale)	Input: number of cranes, number of container berths, number of tugs, terminal area, delay time, and labour Output: annual container throughput, and ship working rate	Technical efficiency, Slack analysis
Valentine and Gray (2001)	31 world ports Cross-section data 1998	DEA with CCR Model	Input: total length of berth, and container berth length Output: container throughput, total cargo throughput	Technical efficiency
Wang, Song, and Cullinane (2003)	28 world ports with 57 container terminals Cross-section data 2001	DEA with CCR, BCC, and FDH models	Input: quay length, terminal area, and number of quayside gantry, yard gantry, and straddle carrier Output: container throughput	Technical efficiency
Liu (1995)	28 UK ports Panel data 1983-1990	SFA with stochastic Translog frontier production function (SPF)	Input: labour by total wage Payments, and capital by the net-book value of fix asset Output: total turnover	Technical efficiency
Cullinane, Song, and Gray (2002)	15 Asian container ports Panel data 1989~1998	SFA with Cobb-Douglas production function for the half-normal, exponential, and truncated-normal distributions	Input: terminal quay length, terminal area, and number of cargo handling equipment Output: annual container throughput	Productive Efficiency
Estache, Gonzalez, and Trujillo (2002)	11 Mexico ports Panel data 1996-1999	SFA with Cobb-Douglas and Translog production function for the half-normal and truncated-normal distributions	Input: the number of workers, length of docks Output: the volume of Handling merchandise	Technical efficiency

Figure 2.The Applications of DEA and SFA Methods on Ports Operating Efficiency

The slack analysis of DEA supply observation to increase or decrease input resources to improve efficiency scores on the other hand the SFA method focuses on the economic justification and hypothesis testing. A mixture of both DEA and SFA support management helps to have a more comprehensive understanding of the operating efficiency of container ports and terminals and to identify the causes of efficiency and causes of inefficiency.

Furthermore, both two methods are frontier function to measure efficiencies of all firms with cross-section and panel data, and many container port's and terminal operations may have characteristics of consistency for DEA and SFA.

However, we would adopt both DEA and SFA methods to evaluate container port's operating efficiency.

Therefore, previous research on port's and terminals efficiency usually adopts either DEA or SFA method, but not both of them (Lin, L. C. & Tseng, L. A., 2005).

This dissertation is intended to measure the relative operating efficiencies of the 6 West African container ports from 2006 to 2012 by first applying Data Envelopment Analysis (DEA) with DEAP 2.1 and secondly SFA with Cobb-Douglas production function with Frontier 4.1 for the truncated-normal distribution.

Previous evaluation to the West African container ports was proposed by Kobin G.van Dyck (2015) using DEA method.

3. Methodology

3.1 DATA ENVELOPMENT ANALYSIS (DEA)

Data envelopment analysis (DEA) is a non-parametric mathematical programming approach to frontier estimation. These models which are presented here is brief, with relatively little technical detail. Detailed methodology presented by Seiford and Thrall (1990), Lovell (1993), Ali and Seiford (1993), Lovell (1994), Charnes et al (1995) and Seiford (1996).The piecewise-linear convex hull approach to frontier estimation, proposed by Farrell (1957), was considered by only a handful of authors in the two decades following Farrell's paper. Authors such as Boles (1966) and Afriat (1972) suggested mathematical programming methods, which could achieve the task, but the method did not receive wide attention until a paper by Charnes, Cooper and Rhodes (1978), which coined the term data envelopment analysis (DEA). There is large number of papers, which have extended and applied the DEA methodology.A model proposed by Charnes, Cooper and Rhodes (1978), which had an input orientation and assumed constant returns to scale (CRS)[1].Following papers have considered alternative sets of assumptions, such as Banker, Charnes and Cooper (1984) who proposed a variable returns to scale (VRS) model. The following discussion of DEA begins with a description of the input-orientated CRS model in section 3.1, because this model was the first to be widely applied Data Envelopment Analysis (DEA) made for the purpose of calculating efficiencies in production. In the program are implemented the methods which are based on the work of Rolf Fare and Shawna Grosskopf.

[1] *The constant return to scale assumption allows one to represent the technology using a unit isoquant. Furthermore, Farrell also discussed the extension of his method so as to accommodate more than two inputs, multiple outputs, and non-constant returns to scale.*

In the program are available three options:

- The first involves the standard CRS and VRS DEA models that involve the calculation of technical and scale efficiencies which are outlined in Fare, Grosskopf and Lovell (1994).
- The second option considers the extension of these models to account for cost and allocative efficiencies. These methods are also outlined in Fare et al (1994).
- The third option considers the application of Malmquist DEA methods to panel data to calculate indices of total factor productivity (TFP) change, technological change, technical efficiency change and scale efficiency change. These latter methods are discussed in Fare, Grosskopf, Norris and Zhang (1994).

In this thesis we will apply only the first and the third method because we want to calculate the technical efficiencies with CRS model (input oriented) and the total factor productivity (TFP) change, the technological change, the technical efficiency change and at least the scale efficiency change. An input or an output orientation is available in all methods (with the exception of the cost efficiencies option). The output from the program contains, where applicable, technical, scale, allocative and cost efficiency estimates, residual slacks, peers, total factor of productivity and technological change indices.

3.1.1 Input-Orientated Measures

Farrell has shown his ideas using a simple example with firms which use two inputs (x_1 and x_2) to produce a single output (y), under the assumption of constant returns to scale.

Knowledge of the isoquant of the fully efficient firm, represented by SS' in ***Figure 3*** permits the measurement of technical efficiency. If a given firm uses number of inputs, defined by the point P, to produce a unit of output, the distance QP can define the technical inefficiency of that firm, which is the amount by which all inputs could be proportionally reduced without reduction in output. This is usually expressed in percentage terms by the ratio ${QP}/{0P}$, which represents the percentage by which all inputs could be reduced.

The technical efficiency (TE) of a firm is most commonly measured by the ratio:

$$TE_I = {0Q}/{0P} \qquad (1)$$

This is equal to one minus ${QP}/{0P}$.

It will take a value between zero and one, and hence provides an indicator of the degree of technical inefficiency of the firm. A value of one indicates the firm is fully technically efficient.

The point Q is technically efficient because it lies on the efficient isoquant.

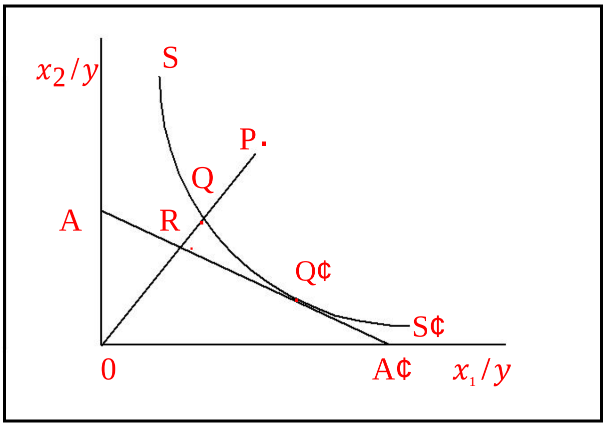

***Figure 3.**Technical and Allocative Efficiencies*

If the input price ratio, represented by the line AA' in ***Figure 3***, is also known, allocative efficiency may also be calculated. The allocative efficiency (AE) of the firm operating at P is defined to be the ratio

$$AE_I = {}^{0R}/_{0Q} \qquad (2)$$

Since the distance RQ represents the reduction in production costs that would occur if production were to occur at allocative (and technically) efficiency at point Q', instead of the technically efficiency, but allocative inefficient, point Q.

The total economic efficiency (EE) is defined to be the ratio

$$EE_I = {}^{0R}/_{0P} \tag{3}$$

The distance RP can be explained in terms of a cost reduction.

The product of technical and allocative efficiency provides the overall economic efficiency.

$$TE_I \times AE_I = (0Q/0P)\times(0R/0Q) = (0R/0P) = EE_I \tag{4}$$

(All three measures are bounded by zero and one)

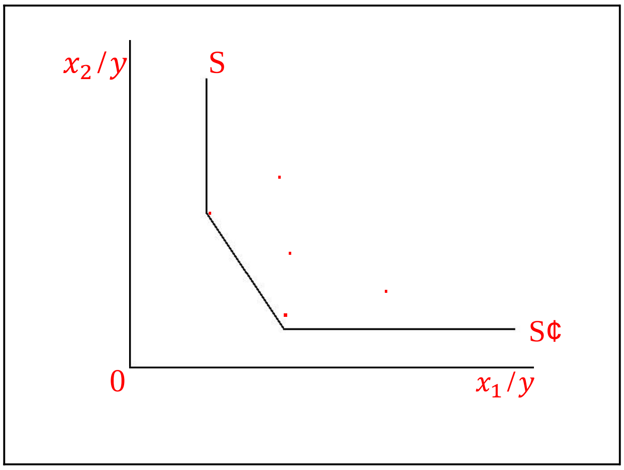

Figure 4.*Piecewise Linear Convex Isoquant*

The efficient isoquant must be estimated from the sample data. Suggested by Farrell the use of either (a) a non-parametric piecewise-linear convex isoquant constructed such that no observed point should lie to the left or below it (refer to ***Figure 4***), or (b) a parametric function, such as the Cobb-Douglas form, fitted to the data, again such that no observed point should lie to the left or below it.

Farrell has shown an illustration of his methods using agricultural data for the 48 continental states of the US.

3.1.2 Output-Orientated Measures

The aforementioned input-orientated technical efficiency measure answers the question; by how much can input quantities be proportionally minimized without changing the output quantities produced. The alternative question is how much can output quantities be proportionally maximized without altering the input quantities used.

Difference between Input and Output oriented measures

The difference between the output and input orientated measures can be illustrated using a simple example involving one input and one output. This is depicted in Figure 3(a) where we have decreasing returns to scale technology represented by $f(x)$, and an inefficient firm operating at the point P. The Farrell input orientated measure of technical efficiency (TE) would be equal to the ratio AB/AP, while the output-orientated measure of technical efficiency (TE) would be CP/CD.

The output and input orientated measures will only provide equivalent measures of technical efficiency when constant returns to scale exist, but will be unequal when increasing or decreasing returns to scale are present (Fare and Lovell 1978).

The constant returns to scale case is depicted in ***Figure 5(b)*** where we observe that

$AB/AP = CP/CD$, for any inefficient point P we care to choose.

Furthermore output-orientated measures can be explained further by considering the case where production involves two outputs (y_1 and y_2) and a single input (x_1). If we again assume constant returns to scale, we can represent the technology by a unit production possibility curve in two dimensions. This example is depicted in ***Figure 6*** where the line ZZ' is the unit production possibility curve and the point A corresponds to an inefficient firm. Note that the inefficient point A, lies below the curve in this case because ZZ' represents the upper bound of production possibilities.

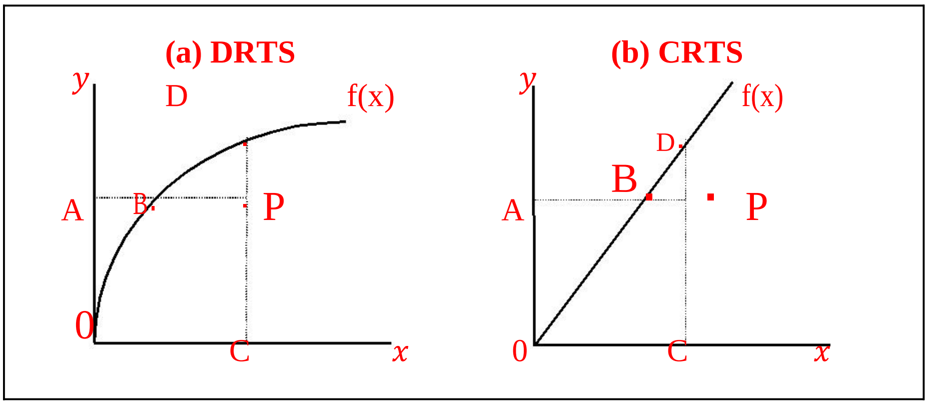

Figure 5.*Input and Output Orientated Technical Efficiency Measures*

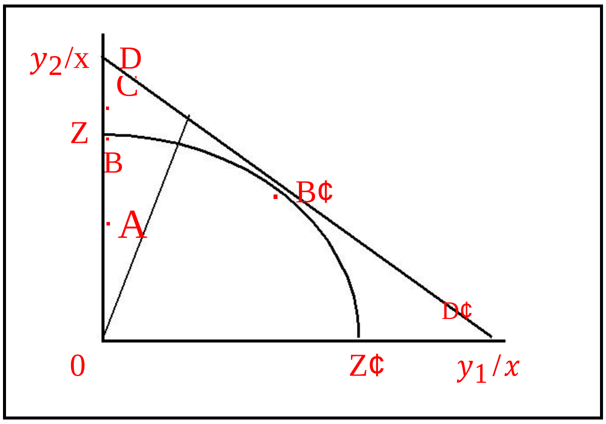

Figure 6.*Technical and Allocative Efficiencies from an output orientation*

Output-orientated Farrell's efficiency measures would be defined as follows:

In ***Figure 6*** the distance AB represents technical inefficiency and that is, the amount by which outputs could be increased without requiring extra inputs.

A measure of output-orientated technical efficiency is the ratio

$$TE_O = {}^{0A}/_{0B} \qquad (5)$$

Having a price information then we can draw the is revenue line DD', and define the allocative efficiency to be,

$$AE_O = {}^{0B}/_{0C} \qquad (6)$$

Which, has an increasing interpretation (similar to the cost reducing interpretation of allocative inefficiency in the input-orientated case). Also, one can define overall economic efficiency as the product of these two measures:

$$EE_O = \left({}^{0A}/_{0C}\right) = \left({}^{0A}/_{0B}\right) \times \left({}^{0B}/_{0C}\right) = TE_O \times AE_O \qquad (7)$$

**(All three measures are bounded by zero and one)*

Two more points should be made, regarding the six efficiency measures that we have defined:

1) All of them are measured along a ray from the origin to the observed production point. Therefore they hold the relative proportions of inputs or outputs constant. One advantage of these radial efficiency measures is that they are units invariant. That is, changing the units of measurement (e.g. measuring quantity of labour in person hours instead of person years) will not change the value of the efficiency measure. A non-radial measure, such as the shortest distance from the production point to the production surface, may be argued for, but this measure will not be invariant to the units of measurement chosen. Changing the units of measurement in this case could result in the identification of a different “nearest” point. This issue is discussed further regarding the treatment of slacks in DEA.
2) Input and output by Farrell orientated technical efficiency measures can be shown to be equal to the input and output distance functions discussed in Shepherd (1970). This observation becomes important when we discuss the use of Data Envelopment Analysis methods in calculating Malmquist indices of TFP change.

3.1.3 The Constant Returns to Scale Model (CRS)

By defining some notation we assume there are data on K inputs and M outputs on each of N firms or DMU's[2] as they tend to be called in the DEA literature.

For i-th DMU these are represented by the vectors x_i and y_i, respectively. The $K \times N$ input matrix, X, and the $M \times N$ output matrix, Y, represent the data of all N DMU's. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all observed points lie on or below the production frontier.

For the simple example of an industry where one output is produced using two inputs, it can be visualised as a number of intersecting planes forming a tight fitting cover over a scatter of points in three-dimensional space. Given the CRS assumption, this can also be represented by a unit isoquant in input/input space (refer to ***Figure 4***).

The best way to introduce DEA is via the ratio form. For each DMU we would like to obtain a measure of the ratio of all outputs over all inputs, such as $(u'yi)/(v'xi)$, where u is a $M \times 1$ vector of output weights and v is a $K \times 1$ vector of input weights. To select optimal weights we specify the mathematical programming problem:

$$\max_{u,v} \left(\frac{u'yi}{v'xi}\right),$$

$$st \quad \frac{u'yj}{v'xj} \leq 1, \quad j = 1, 2, \ldots, N,$$

$$u, v \geq 0 \qquad (8)$$

[2] *DMU stands for "decision making unit". It is a more appropriate term than "firm" when, for example, a bank is studying the performance of its branches or an education district is studying the performance of its schools.*

This involves finding values for u and v, such that the efficiency measure of the i-th DMU is maximised, subject to the constraint that all efficiency measures must be less than or equal to one. One problem with this particular ratio formulation is that it has an infinite number of solutions[3].

To avoid this one can impose the constraint, $v'xi = 1$, which provides:

$$\begin{aligned} \max_{\mu,\nu} \ & (\mu' yi), \\ st \quad & \nu' xi = 1, \\ & yj - \nu' xj \leq 0, j = 1, 2, \dots, N, \\ & \mu, \nu \geq 0 \end{aligned} \qquad (9)$$

When, the notation change from u and v to μ and ν, the transformation is reflected.

This form is known as the multiplier form of the linear programming problem.

The duality used in linear programming, one can derive an equivalent envelopment form of this problem:

$$\begin{aligned} & \min_{\vartheta,\lambda} \vartheta \\ & -yi + Y\lambda \geq 0, \\ & \vartheta xi - X\lambda \geq 0, \\ & \lambda \geq 0 \end{aligned} \qquad (10)$$

[3]*That is, if (u*, v*) is a solution, then (αu*, αv*) is another solution, etc.*

Where, ϑ is a scalar and λ is a $N \times 1$ vector of constants. This envelopment form involves fewer constraints than the multiplier form ($K + M < N + 1$), and also is generally the preferred form to solve[4] Value of ϑ obtained will be the efficiency score for the i-th DMU. It will satisfy $\vartheta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient DMU, according to the Farrell (1957) definition.

Note that the linear programming problem must be solved N times, once for each DMU in the sample. A value of θ is then obtained for each DMU.

[4]*The forms defined by equations 8 and 9 are introduced here for expository purposes.*

3.1.4 Slacks

The piecewise linear form of the non-parametric frontier in DEA can cause a few difficulties in efficiency measurement. The problem arises because of the sections of the piecewise linear frontier which run parallel to the axes (refer ***Figure 4***) which do not occur in most parametric functions (refer ***Figure 3***). To illustrate the problem, refer to ***Figure 7*** where the DMU's using input combinations C and D are the two efficient DMU's which define the frontier, and DMU's A and B are inefficient DMU's. The Farrell (1957) measure of technical efficiency gives the efficiency of DMU's A and B as OA'/OA and OB'/OB, respectively.
However, it is questionable as to whether the point A' is an efficient point since one could reduce the amount of input x_2 used (by the amount CA') and still produce the same output. This is known as input slack in the literature[5]. Once one considers a case involving more inputs and/or multiple outputs, the diagrams are no longer as simple, and the possibility of the related concept of output slack also occurs[6]. Thus it could be argued that both the Farrell measure of technical efficiency and any non-zero input or output slacks should be reported to provide an accurate indication of technical efficiency of a DMU in a DEA analysis[7].
Note that for i-th DMU the output slacks will be equal to zero only if $Y\lambda - yi = 0$, while the input slacks will be equal to zero only if $\vartheta xi - X\lambda = 0$ (for the given optimal values of ϑ and λ).

[5] *Some authors use the term input excess.*
[6] *Output slack is illustrated later in these notes.*
[7] *Koopman's (1951) definition of technical efficiency was stricter than the Farrell (1957) definition. The former is equivalent to stating that a firm is only technically efficient if it operates on the frontier and furthermore that all associated slacks are zero.*

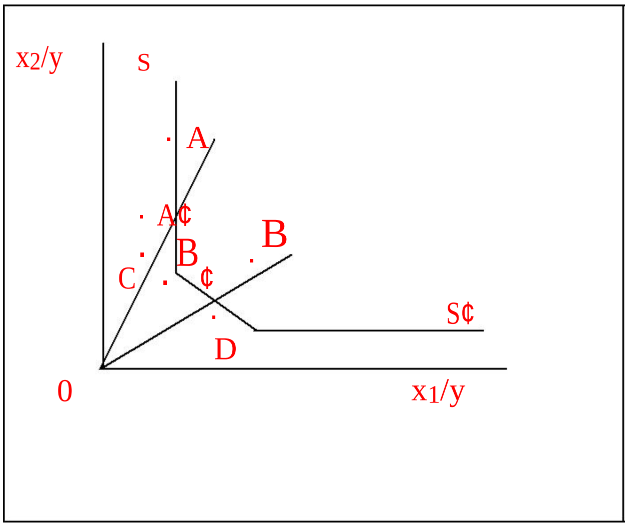

Figure 7. *Efficiency Measurement and Input Slacks*

In ***Figure 7*** the input slack connected with the point A' is CA' of input x_2. When more inputs and outputs than considered in this simple example, the identification of the "nearest" efficient frontier point (such as C), and therefore the following calculation of slacks, is not a trivial task. Some authors such as Ali and Seiford (1993) have suggested the solution of a second-stage linear programming problem to move to an efficient frontier point by MAXIMISING the sum of slacks required to move from an inefficient frontier point (such as A' in ***Figure 7***) to an efficient frontier point (such as point C). This second stage linear programming problem may be defined by:

$$\min_{\lambda} OS, IS - (M1'OS + K1'IS),$$

$$st, \quad -yi + Y\lambda - OS = 0,$$

$$\theta xi - X\lambda - IS = 0,$$

$$\lambda \geq 0, OS \geq 0, IS \geq 0 \qquad (11)$$

Where OS is a $M \times 1$ vector of output slacks, IS is a $K \times 1$ vector of input slacks, and $M1$ and $K1$ are $M \times 1$ and $K \times 1$ vectors of ones, respectively. Note that in this second stage linear program, ϑ is not a variable; its value is taken from the first-stage results. Furthermore, note that this second-stage linear program must also be solved for each of the N DMU's involved[8]

There are three choices in the DEAP software regarding the treatment of slacks.

- The one-stage DEA, in which we conduct the LP in equation 10 which calculate, slacks residually.
- The two-stage DEA, where we conduct the LP's in equations 10 and 11 and
- The multi-stage DEA, where we conduct a sequence of radial LP's to identify the efficient projected point.

The multi-stage DEA method is more computationally demanding that the other two methods and in this thesis we used only this.

The benefits of this approach are that it identifies efficient projected points which have input and output mixes which are as similar as possible to those of the inefficient points, and that it is also invariant to units of measurement. Therefore we would recommend the use of the multi-stage method over the other two alternatives.

[8] *This method is used by all the popular DEA software such as Warwick DEA and IDEAS.*

Slacks may be viewed as being an artefact of the frontier construction method chosen (DEA) and the use of finite sample sizes. If an infinite sample size were available and/or if an alternative frontier construction method was used, which involved a smooth function surface, the slack issue would disappear. In addition to this observation it also seems quite reasonable to accept the arguments of Ferrier and Lovell (1990) that slacks may essentially be viewed as allocative inefficiency.

Hence we believe that an analysis of technical efficiency can reasonably concentrate upon the radial efficiency score provided in the first stage DEA LP (refer to equation 10).

However if one insists on identifying Koopmans-efficient projected points then we would strongly recommend the use of the multi-stage method in preference to the two-stage method for the reasons outlined above[9].

[9] *However is also included the 2-stage option in this software because it is the method used in other popular DEA software packages such as Warwick DEA and IDEAS.*

3.1.5 The Variable Returns to Scale Model (VRS) and Scale Efficiencies

The CRS assumption is only appropriate when all DMU’s are operating at an optimal scale (i.e one corresponding to the flat portion of the LRAC curve). Imperfect competition, constraints on finance, etc. may cause a DMU to be not operating at optimal scale. Banker, Charnes and Cooper (1984) suggested an extension of the CRS DEA model to account for variable returns to scale (VRS) situations. The use of the CRS specification when not all DMU’s are operating at the optimal scale will result in measures of TE, which are confounded by scale efficiencies (SE). The use of the VRS specification will permit the calculation of TE devoid of these SE effects.

The CRS linear programming problem can be easily modified to account for VRS by adding the convexity constraint: $N1' = 1$ to (10) to provide:

$$\min_{\vartheta,\lambda} \vartheta$$

$$st \quad -y_i + Y\lambda \geq 0,$$

$$N1'\lambda = 1$$

$$\lambda \geq 0 \qquad (12)$$

Where, $N1$ is a $N \times 1$ vector of ones.

This approach forms a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are greater than or equal to those obtained using the CRS model. The VRS specification has been the most commonly used specification in the 1990's.

Calculation of scale efficiencies in many studies have decomposed the TE scores obtained from a CRS DEA into two components, one due to scale inefficiency and one due to "pure" technical inefficiency.
This may be done by conducting both a CRS and a VRS DEA upon the same data. If there is a difference in the two TE scores for a particular DMU, then this indicates that the DMU has scale inefficiency, and that the scale inefficiency can be calculated from the difference between the VRS TE score and the CRS TE score.

Figure 8 attempts to illustrate this. In this figure we have a one-input one-output example and have drawn the CRS and VRS DEA frontiers. Under CRS the input orientated technical inefficiency of the point P is the distance PP_C, while under VRS the technical inefficiency would only be PPV.

The difference between these two, $P_C P_V$, is put down to scale inefficiency.

One can also express all of this in ratio efficiency measures as:

$$TE_{I,CRS} = APC/AP$$

$$TE_{I,VRS} = APV/AP$$

$$SE_I = APC/APV$$

Where, all of these measures will be bounded by zero and one.

We also note that,

$TE_I, CRS = TE_I, VRS \times SE_I$

Because,

$$APC/_{AP} = (APV/_{AP}) \times (APC/_{APV}).$$

The CRS technical efficiency measure is decomposed into “pure” technical efficiency and scale efficiency.

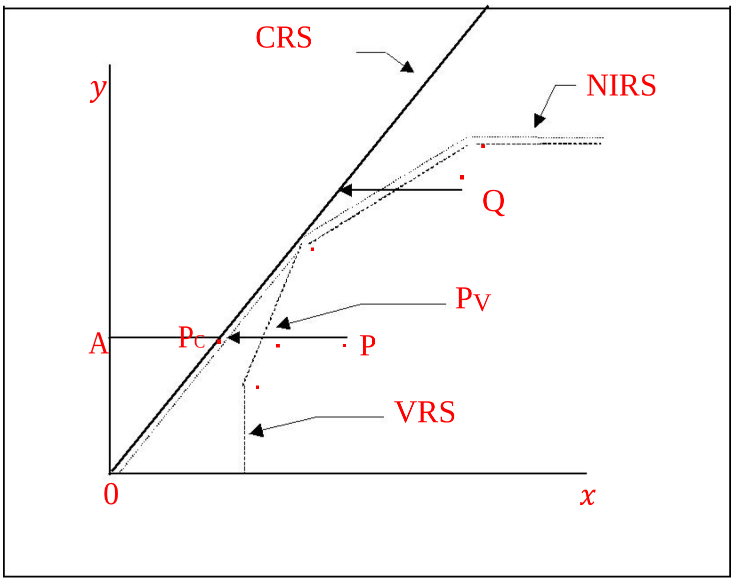

Figure 8.*Calculation of Scale Economies in DEA*

One shortcoming of this measure of scale efficiency is that the value does not indicate whether the DMU is operating in an area of increasing or the decreasing returns to scale. This may be determined by running an addition DEA problem with non-increasing returns to scale (NIRS) imposed.

This can be done by altering the DEA model in equation 12 by substituting the $N1'\lambda = 1$ restriction with $N1'\lambda \leq 1$, to provide:

$$\begin{aligned} &\min_{\vartheta,\lambda} \vartheta \\ st \quad &-yi + Y\lambda \geq 0, \\ &\Theta xi - X\lambda \geq 0, \\ &N1'\lambda = 1 \\ &\lambda \geq 0 \end{aligned} \tag{13}$$

The NIRS DEA frontier is also plotted in ***Figure 8***.The nature of the scale inefficiencies (i.e. due to increasing or decreasing returns to scale) for a particular DMU can be determined by seeing whether the NIRS TE score is equal to the VRS TE score. If they are unequal (as will be the case for the point P in ***Figure 8***) then increasing returns to scale exist for that DMU. If they are equal (as is the case for point Q in ***Figure 8***) then decreasing returns to scale apply. An example of this approach applied to international airlines is provided in BIE (1994).

3.1.6 The Malmquist Index

When one has panel data, one may use DEA-like linear programs and a (input- or output-based) Malmquist TFP index to measure productivity change, and to decompose this productivity change into technical change and technical efficiency change.

Fare et al (1994) specifies an output-based Malmquist productivity change index[10] as:

$$\boldsymbol{m}_{o}(y_{t+1}, x_{t+1}, y_t, x_t) = \left[\frac{do^{t}(x_{t+1}, y_{t+1})}{do^{t}(x_t, y_t)} \times \frac{do^{t+1}(x_{t+1}, y_{t+1})}{do^{t+1}(x_t, y_t)}\right]^{1/2} \quad \textit{(14)}$$

This represents the productivity of the production point (x_{t+1}, y_{t+1}) relative to the production point (x_t, y_t). A value greater than one will indicate positive TFP growth from period t to period $t + 1$. This index is, in fact, the geometric mean of two output-based Malmquist TFP indices. One index uses period t technology and the other period $t + 1$ technology. To calculate ***equation 14*** we must calculate the four component distance functions, which will involve four LP problems (similar to those conducted in calculating Farrell technical efficiency (TE) measures).

---

[10] *The subscript "o" has been introduced to remind that these are output-orientated measures. Note that input-orientated Malmquist TFP indices can also be defined in a similar way to the output-orientated measures.*

We begin by assuming CRS technology (we conduct a further decomposition later to look at scale efficiency questions). The CRS output-orientated LP used to calculate ${d_o}^t(x^t, y^t)$ is identical to ***equation 15***, except that the convexity (VRS) restriction has been removed and time subscripts have been included.

That is,

$$
\begin{aligned}
[{d_o}^t(x_t, y_t)]^{-1} &= \max_{\phi,\lambda} \phi, \\
st \quad -\phi y_{it} + Y_t\lambda &\geq 0, \\
x_{it} - X_t\lambda &\geq 0, \\
\lambda &\geq 0,
\end{aligned}
\qquad (15)
$$

The remaining three (3) LP problems are simple variants of this:

$$
\begin{aligned}
[{d_o}^{t+1}(x_{t+1}, y_{t+1})]^{-1} &= \max_{\phi,\lambda} \phi, \\
st \quad -\phi y_{i,t+1} + Y_t\lambda &\geq 0, \\
x_{i,t+1} - X_{t+1}\lambda &\geq 0, \\
\lambda &\geq 0
\end{aligned}
\qquad (16)
$$

$$
\begin{aligned}
[{d_o}^{t}(x_{t+1}, y_{t+1})]^{-1} &= \max_{\phi,\lambda} \phi, \\
st \quad -\phi y_{i,t+1} + Y_t\lambda &\geq 0, \\
x_{i,t+1} - X_t\lambda &\geq 0, \\
\lambda &\geq 0
\end{aligned}
\qquad (17)
$$

$$\left[d_o^{t+1}(x_t, y_t)\right]^{-1} = \max_{\phi,\lambda} \phi,$$

$$st \quad -\phi y_{it} + Y_{t+1}\lambda \geq 0,$$

$$x_{it} - X_{t+1}\lambda \geq 0,$$

$$\lambda \geq 0 \qquad (18)$$

In LP's 17 and 18, where production points are compared to technologies from different time periods, the φ parameter need not be ≥ 1, as it must be when calculating Farrell efficiencies. The point could lie above the feasible production set. This will most likely occur in LP 17 where a production point from period $t + 1$ is compared to technology in period t. If technical progress has occurred, then a value of $\varphi < 1$ is possible. Note that it could also possibly occur in LP 17 if technical regress has occurred, but this is less likely.

Some points to keep in mind are that the φ and λ's are likely to take different values in the above four LP's. Furthermore, note that the above four LP's must be calculated for each firm in the sample.

So, for example if you have 20 firms and 2 time periods you must calculate 80 LP's. Also, note that as you add extra time periods, you must calculate an extra three LP's for each firm (to construct a chained index). If you have T time periods, you must calculate $(3T - 2)$ LP's for each firm in the sample. Furthermore, if you have N firms, you will need to calculate $N \times (3T - 2)$ LP's. For example, with $N = 20$ firms and $T = 10$ time periods, this would provide $20 \times (3 \times 10 - 2) = 560$ LP's.On each and every firm result for each and every adjacent pair of time periods can be tabulated, and/or summary measures across time and/or space can be presented.

3.1.7 Scale Efficiency

The previous approach can be extended by decomposing the (CRS) technical efficiency change into scale efficiency and “pure” (VRS) technical efficiency components. This will involve calculating two additional LP’s (when comparing two production points). These would involve repeating LP’s 17 and 18 with the convexity restriction ($N1\,\lambda = 1$) added to each. That is, one would calculate the distance functions relative to a VRS (instead of a CRS) technology.

One can then use the CRS and VRS values to calculate the scale efficiency effect residually, using the methods outlined in section 3.2. In the case of N firms and T time periods, this would increase the number of LP’s from $N \times (3T - 2)$ to $N \times (4T - 2)$.

3.2 Stochastic Frontier Analysis (SFA)

3.2.1 SFA and FRONTIER 4.1

The stochastic frontier models can contain panel data and accept firm effects that are distributed as abbreviated normal random variables.

Two primary model specifications considered in the program are:

- Error components specification with time-varying efficiencies permitted (Battese and Coelli, 1992), which was estimated by FRONTIER Version 2.0.
- A model specification in which the firm effects are directly influenced by a number of variables (Battese and Coelli, 1995).

The program also allows the estimation of many other models. FRONTIER Version 4.1, the program we worked on, is to provide maximum likelihood, estimates of a wide variety of stochastic frontier production and cost functions.

3.2.2 Model specifications

The stochastic frontier production function was independently proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). The original specification involved a production function specified for cross-sectional data which had an error term which had two components:

- One to account for random effects
- Another to account the technical inefficiency.

This model can be expressed in the following form:

$$(1)\ Y_i = x_{i\beta} + (V_i - U_i) \qquad ,i = 1, \ldots, N,$$

Y_i is the production (or the logarithm of the production) of the i-th firm.

Where,
x_i is a $k \times 1$ vector of (transformations of the) input quantities of the i-th firm

β is an vector of unknown parameters;
V_i are random variables which are assumed to be iid.

$N(0, \sigma_{V^2})$, and independent of the be iid.

This specification has been used in a vast number of empirical applications over the past two decades. It has also been altered and extended in a number of ways and extensions include the specification of more general distributional assumptions for the U_i, such as the abbreviated normal or two-parameter gamma distributions, the consideration of panel data and time-varying technical efficiencies, the extension of the methodology to cost functions and also to the estimation of systems of equations, etc.

A big number of completed reviews of this literature are available, such as Forsund, Lovell and Schmidt (1980), Schmidt (1986), Bauer (1990) and Greene (1993). The FRONTIER Version 4.1 as a computer program can be used to obtain maximum likelihood estimates of a subset of the stochastic frontier production and cost functions which have been proposed in the literature.

The computer program was written to estimate the model specifications detailed in Battese and Coelli (1988, 1992 and 1995) and Battese, Coelli and Colby (1989).

3.2.3 Model 1: The Battese and Coelli (1992) Specification

Battese and Coelli (1992) designed a stochastic frontier production function for panel data, which has firm effects, which are assumed to be distributed as abbreviated normal random variables, which are also permitted to vary systematically with time. The model may be expressed as:

$$(2) \quad Y_{it} = x_{it}\beta + (V_{it} - U_{it}) \qquad , i = 1...N, t = 1...T,$$

, where Y_{it} is (the logarithm of) the production of the i-th firm in the t-th time period; X_{it} *is a* $k \times 1$ *vector of (transformations of the) input quantities of the* i*-th firm in the* t*-th time period;*

β *is as defined earlier*

Where,

the V_{it} are random variables which are assumed to be iid $N\ (0, \sigma_{V^2})$and independent of the $U_{it} = (U_i exp(-\eta(t - T))$, where the U_i are non-negative random variables which are assumed to account for technical inefficiency in production and are assumed to be iid as truncations at zero of the $N(\sigma, \sigma_{U^2})$ distribution,

η is a parameter to be estimated and the panel of data need not be complete (i.e. Unbalanced panel data).

Utilising the parameterization of Battese and Corra (1977) who replace σ_{V^2} and σ_{U^2} with $\sigma^2 = \sigma_{V^2} + \sigma_{U^2}$ and $\sigma = \sigma_{U^2} / (\sigma_{V^2} + \sigma_{U^2})$. This is done with the calculation of the maximum likelihood estimates in mind. The parameter, γ, must lie between 0 and 1 and thus this range can be searched to provide a good starting value for use in an iterative maximization process such as the Davidon-Fletcher-Powell (DFP) algorithm. The log-likelihood function of this model is presented in the appendix in Battese and Coelli (1992).

The insert of one or more restrictions upon this model formulation can provide a number of the special cases of this particular model which have appeared in the literature. Setting η to be zero provides the time-invariant model set out in Battese, Coelli and Colby (1989). Furthermore, restricting the formulation to a full (balanced) panel of data gives the production function assumed in Battese and Coelli (1988). The additional restriction of μ equal to zero reduces the model-to-model One in Pitt and Lee (1981).

One may add a fourth restriction of $T = 1$ to return to the original cross-sectional, half-normal formulation of Aigner, Lovell and Schmidt (1977). Obviously a large number of permutations exist. For example, if all these restrictions excepting $\mu = 0$ are imposed, the model suggested by Stevenson (1980) results. Furthermore, if the cost function option is selected, we can estimate the model specification in Hughes (1988) and the Schmidt and Lovell (1979) specification, which assumed allocative efficiency. These latter two specifications are the cost function analogues of the production functions in Battese and Coelli (1988) and Aigner, Lovell and Schmidt (1977), respectively.

There are a large number of model choices that could be considered for any particular application. For example, one can assume a half-normal distribution for the inefficiency effects or the more general abbreviated normal distribution. If panel data is available, one could assume time-invariant or time-varying efficiencies. When such choices are available, it is recommended that a number of the alternative models be estimated and that a preferred model be selected using likelihood ratio tests.

Someone can also test whether any form of stochastic frontier production function is required at all by testing the significance of the γ parameter[11]. If the null hypothesis, that γ equals zero, is accepted, this would indicate that σ_{U^2} is zero and hence that the U_{it} term should be removed from the model, leaving a specification with parameters that can be consistently estimated using ordinary least squares.

[11] *It should be noted that any likelihood ratio test statistic involving a null hypothesis which includes the restriction that γ is zero does not have a chi-square distribution because the restriction defines a point on the boundary of the parameter space. In this case the likelihood ratio statistic has been shown to have a mixed chi-square distribution. For more on this point see Lee (1993) and Coelli (1993, 1994).*

3.2.4 Model 2: The Battese and Coelli (1995) Specification

After empirical studies (Pitt and Lee, 1981) have estimated stochastic frontiers and predicted firm-level efficiencies using these estimated functions, and then regressed the predicted efficiencies upon firm-specific variables, such as managerial experience, ownership characteristics, etc, in an attempt to identify some of the reasons for differences in predicted efficiencies between firms in an industry. This has been recognised as a useful exercise, but the two-stage estimation procedure has also been recognised as one, which is inconsistent in its assumptions regarding the independence of the inefficiency effects in the two estimation stages. The two-stage estimation procedure is unlikely to provide estimates, which are as efficient as those that could be obtained using a single-stage estimation procedure.

The issue was addressed by Kumbhakar, Ghosh and McGukin (1991) and Reifschneider and Stevenson (1991) who propose stochastic frontier models in which the inefficiency effects (U_i) are expressed as an explicit function of a vector of firm-specific variables and a random error. Battese and Coelli (1995) propose a model, which is equivalent to the Kumbhakar, Ghosh and McGukin (1991) specification, with the exceptions that allocative efficiency is imposed, the first-order profit maximising conditions removed, and panel data is permitted.

The Battese and Coelli (1995) model specification may be expressed as:

(3) $Y_{it} = x_{it}\beta + (V_{it} - U_{it})$ $,i = 1,\ldots,N, t = 1,\ldots,T,$

Where,

Y_{it}, x_{it}, and $\beta$are as defined earlier,

V_{it} are random variables which are assumed to be iid. $N(0,\sigma_{V^2})$, and independent of the U_{it} which are non-negative random variables which are assumed to account for technical inefficiency in production and are assumed to be independently distributed as truncations at zero of the $N(m_{it}, x_{U^2})$ distribution; where:

(4) $m_{it} = z_{it}\delta$

Where z_{it} is a $p \times 1$ vector of variables which may influence the efficiency of a firm, the δ is a $1 \times p$ vector of parameters to be estimated.

Once again it was used the parameterisation from Battese and Corra (1977), replacing σ_{V^2} and σ_{U^2} with and $\sigma = \sigma_{V^2} / (\sigma_{V^2} + \sigma_{U^2})$.

The log-likelihood function of this model is presented in the appendix in the working paper Battese and Coelli (1993).

This model specification also has a number of other model specifications as special cases. If we set $T = 1$ and z_{it} contains the value one and no other variables (i.e. only a constant term), then the model reduces to the truncated normal specification in Stevenson (1980), where δ_0 (the only element in δ) will have the same interpretation as the μ parameter in Stevenson (1980). It should be noted, however, that the model defined by (3) and (4) does not have the model defined by (2) as a special case and neither does the converse apply. Thus these two model specifications are non-nested and hence no set of restrictions can be defined to permit a test of one specification versus the other.

3.2.5 Efficiency Predictions[12]

The program calculates predictions of individual firm technical efficiencies from estimated stochastic production frontiers, and predictions of individual firm cost efficiencies from estimated stochastic cost frontiers. The measures of technical efficiency relative to the production frontier (1) and of cost efficiency relative to the cost frontier (5) are both defined as:

$$EFF_i = E\,(Y_i \times |U_i, X_i) / \,E\,(Y_i \times |U_i = 0, X_i) \qquad (6)$$

Where Y_i^* is the production (or cost) of the i-th firm, which will be equal to Y_i,

when the dependent variable is in original units and will be equal to $exp(Y_i)$,

when the dependent variable is in logs.

[12] *The discussion here will again be in terms of the cross-sectional models. Extension to panel data case is straightforward.*

In the case of a production frontier, EFF_i takes a value between zero and one, while it will take a value between one and infinity in the cost function case. The efficiency measures can be defined as:

Cost or Production	***Logged Dependent Variable.***	***Efficiency (EFF_i)***
production	**yes**	$\boldsymbol{exp(-U_i)}$
cost	**yes**	$\boldsymbol{exp(U_i)}$
production	**no**	$\boldsymbol{(x_i\beta - U_i)/(x_i\beta)}$
cost	**no**	$\boldsymbol{(x_i\beta + U_i)/(x_i\beta)}$

Figure 9.*Cost or Production, Efficiency (EFFi)*

The above expressions for EFF_i all rely upon the value of the unobservable U_i being predicted. This is achieved by deriving expressions for the conditional expectation of these functions of the U_i, conditional upon the observed value of $(V_i - U_i)$.

The resulting expressions are generalizations of the results in Jondrow et al (1982) and Battese and Coelli (1988). The relevant expressions for the production function cases are provided in Battese and Coelli (1992) and in Battese and Coelli (1993, 1995), and the expressions for the cost efficiencies relative to a cost frontier, have been obtained by minor alterations of the technical efficiency expressions in these papers.

3.3 Comparison of SFA and DEA Methods

Despite both SFA and DEA methods are efficiency frontier analysis methods and are originally introduced to the efficiency concepts developed by Farrell (1957), there are essential differences between the econometric approach and mathematical programming methods to construction of a production frontier and calculation of efficiency relative to the frontier as shown in ***Figure 10***.

DEA is a non-parametric approach and is suited to measure efficiencies of Proceedings of the Eastern Asia Society for Transportation Studies, Vol. 5, pp. 592-607, 2005 597 deterministic industries for multiple inputs/outputs information. DEA has been applied to assess performance of non-profit organizations or branches, such as school, hospitals, universities, courts, public sector, agriculture, etc (Doyle & Green, 1994; Coelli, 1996). But in recent years, more and more scholars have applied it to evaluate performance of profit organizations.

On the other hand, SFA is a parametric approach, and is suited to measure efficiencies of stochastic industry for input/output information. SFA needs to assume a production function of the usual regression form and a distribution type of error item which is equal to the sum of two components, the first part is symmetric and captures statistical noise such as weather, luck, machine breakdown and other events beyond the control of firms, and the second part represents technical inefficiency of firms. SFA has been applied to measure performance of profit organizations.

Source: Coelli et.al. (1997, Lan et.al. (2003)	**Stochastic Frontier Analysis (SFA)**	**Data Envelopment Analysis (DEA)**
Consistency	***Both DEA and SFA methods are used for calculating efficiency with frontier analysis, and are similar in that they determine a frontier and inefficiency based on that frontier.***	
Characteristic	Parametric method	Non-Parametric method
Efficiency measurement	Technical efficiency, scale elasticity, scale efficiency, allocative efficiencies, technical change and TFP change.	Technical efficiency, scale elasticity, scale efficiency, allocative efficiencies, congestion efficiencies, technical change and TFP change.
Strengths	1. It doesn't assume that all firms are efficient in advance. 2. SFA makes an accommodation for statistical noise such as random variables of weather, luck, machine breakdown and other events beyond the control of firms, and measures error. 3. It doesn't need to price information available. 4. It is capable to hypothesis test. 5. To estimate the best technical efficiencies of firm, rather than average technical efficiencies of a firm.	1. It doesn't assume that all firms are efficient in advance. 2. It could handle with efficiency measurement of multiple inputs and multiple outputs. 3. It doesn't need to price information available. 4. It does not need to assume function type and distribution type. 5. While sample size is small, it is compared with relative efficiency. 6. Both the CCR and BCC models have nature of unit invariance.
Weaknesses	1. It needs to assume functional form and distribution type in advance. 2. It needs enough samples to avoid lack of degree of freedom. 3. The assumed distribution type is sensitive to assessing efficiency scores.	1. It doesn't make accommodation for statistical noise such as measure error. 2. It isn't capable to hypothesis test. 3. When the newly added DMU is an outlier, it could affect the efficiency measurement.
Application	It has applied to measure performance of profit organizations.	It has applied to assess performance of non-profit organizations or branches of firm.

Figure 10. *The Comparison of SFA and DEA Methods*

Data Envelopment Analysis via Deap 2.1 programme and Stochastic Frontier Analysis via Frontier 4.1 programme were selected in this dissertation because the efficiency of the container port industry has been variously studied under DEA and SFA and the majority of papers and dissertations use these two programmes/methods to evaluate and measure the efficiency of ports globally. In view of the strengths and weaknesses associated with these two approaches, the efficiency estimates and scale properties come from these analysis are not always convincing. This dissertation applies both approaches to the same set of container port data for the six major container ports in West Africa and compares the results obtained. Specifically in DEA method selection was used the input oriented results because the analysis aims to measure the input indicators to be compared to the output.

4. Empirical application to 6 Container Port Terminals in West Africa

For this case study, the container port terminals, which were selected in West Africa, are above 100,000 TEUs per year, for the period 2006-2012.

This case study selects six West African container ports in six different countries with total throughput over 100,000 TEUs per year as shown in ***Table 1***.Specifically the ports selected in this thesis are:

- Tema Port in Ghana
- Abidjan Port in Cote D'Ivoire
- Dakar Port in Senegal
- Lomé Port in Togo
- Cotonou Port in Benin
- Lagos Complex in Nigeria

These port data are collected mainly from (Kobina G. van Dyck, 2015).

The majority of West African ports have both dedicated container berths/terminals and multi-purpose berths.

For this case study, the container terminals were used for the analysis in order to have equality in comparison and analysis of the data. Furthermore, the container terminals were the main terminals for the handling of containerized cargo at the ports. The ports analysed can be found in ***Table 1***.

Port	Terminal	Container Throughput (TEUs)						
Years		*2006*	*2007*	*2008*	*2009*	*2010*	*2011*	*2012*
Tema	**MPS Terminal**	**425,408**	**489,147**	**555,009**	**525,694**	**590,147**	**756,899**	**824,238**
Abidjan	**SETV Terminal**	**507,100**	**531,809**	**652,358**	**610,185**	**561,535**	**546,417**	**633,917**
Dakar	**DP World Terminal**	**375,876**	**424,457**	**347,483**	**331,076**	**349,231**	**369,137**	**383,903**
Lomé	**Bollore Africa Logistics**	**215,892**	**237,891**	**296,109**	**354,480**	**339,853**	**352,695**	**288,481**
Cotonou	**Bollore Africa Logistics**	**140,500**	**167,791**	**193,745**	**272,820**	**316,744**	**334,798**	**348,190**
Lagos Complex	**APM Terminals-Apapa**	**587,600**	**711,100**	**947,400**	**710,800**	**1,128,171**	**1,413,27**	**1,623,141**

Table 1.*Container throughput 2006-2012 (Kobina G. van Dyck, 2015)*

This case study initially selected five inputs of container port infrastructures, including:

- **Total Quay Length**
- **Terminal Area**
- **Number of Quayside Cranes**
- **Number of Container Gantry Cranes**
- **Number of Reach Stackers** and single output,
- **Container Throughput** as shown in ***Table 1.***

These indicators were selected to be in compliance with characteristic of consistency for both DEA and SFA.

DATA	
INPUTS	**OUTPUTS**
Total Quay Length (m)	**Container throughput (TEUs)**
Terminal area (ha)	
Number of Quayside Cranes	
Number of Yard Gantry Cranes	
Number of Reach Stackers	

Figure 11.*Data and analysis indicators*

Container throughput trend for the period 2006-2012 is shown on ***Figure 12***. The difference between Lagos Complex Port and the other ports is clear. However we can see that it suffers from throughput fluctuations over time. Insignificant fluctuations on throughput noticed in all the other ports with exception of the port of Cotonou, which increased its throughput levels since 2006.

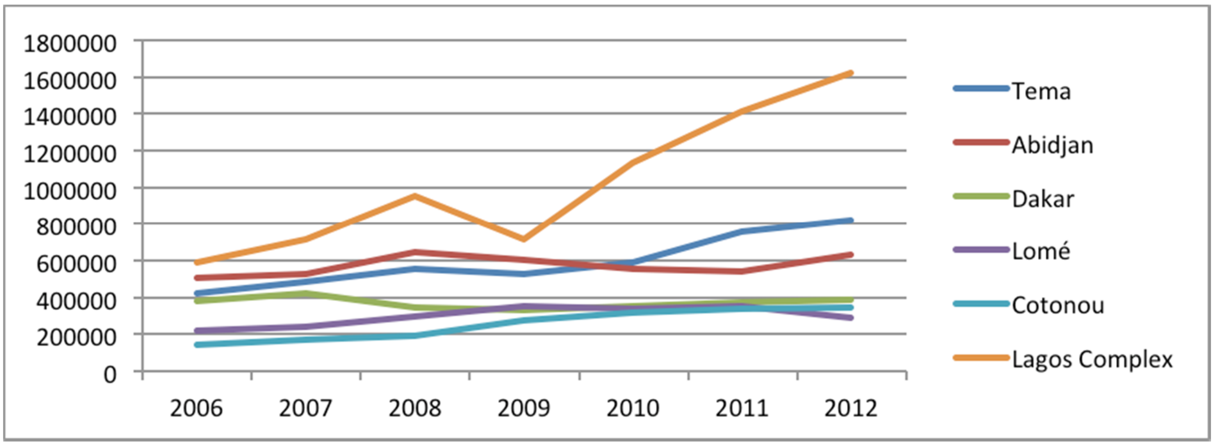

Figure 12.*Container throughput trend 2006-2012*

The selection of input and output variables is very important to the analysis of efficiency of ports and container terminals.

Unclear variables can lead to false conclusions about port efficiency (Cullinane and Wang. 2006). Input and output variables should explain container port production as much as possible (Cullinane K.P.B. et al., 2004).

Container throughput (output data) used in the efficiency analysis as primary basis upon which container ports are compared. As a container terminals and ports depends on the efficient use of land, labour and capital (equipment), the input data includes the quay length (m), the terminal area (ha), the number of quayside cranes, the number of yard gantry cranes, and the number of reach stackers used in each port over the period 2006-2012.

Specifically the quay length indicator is important in evaluating the efficiency of ports and container terminals. The quay length is also one important indicator as to the turn and around time that can be achieved by ports, since it shows the size of a ship that can be allocated a berth at a particular point in time.

Berth availability as a function of quay length can influence the efficiency of shipping lines. Furthermore, the number of quay-side cranes is an important measure of productivity.

This input directly changes the speed with which container ships may be served for example if a container terminal has more cranes, this may increase the number of containers handled per hour, and effects the turn-around time as well.

The number of quayside cranes increases the agility of the port by handling more vessels at the same time (Pjevčević, D., 2012). The berth length and number of quay-side cranes accordingly influence the berth-side productivity. Likewise, the terminal area, the number of yard gantry cranes, and the number of reach-stackers reflect yard-side productivity. In this case study, the number of yard gantry cranes and reach stackers is used in the evaluation because of their common use within terminals and ports in particular.

The input and output data have been collected from (World Bank, 2014 & MLTC/CATRAM, 2013).

In ***Table 2*** are shown the summary statistics of the data used.

	Container throughput (TEUs)	Total Quay Length (m)	Terminal area (ha)	Number of Quayside Cranes	Number of Yard Gantry Cranes	Number of Reach Stackers
Mean	**683645.00**	**701.50**	**27.67**	**5.17**	**8.67**	**20.33**
Standard deviation	**502677.21**	**244.54**	**17.07**	**2.40**	**5.75**	**6.02**
Minimum	**288481.00**	**430.00**	**10.00**	**4.00**	**0.00**	**15.00**
Maximum	**1623141.00**	**1005.00**	**55.00**	**10.00**	**16.00**	**31.00**

Table 2.*Sum statistics for sample of 6 West African ports*

In ***Tables 3, 4, 5, 6, 7 and 8*** are shown the inputs and the output data for the period 2006-2012 more precisely.

Port	*Variables*	*2006*	*2007*	*2008*	*2009*	*2010*	*2011*	*2012*
Port of Tema	***Container throughput***	***425,408***	***489,147***	***555,009***	***525,694***	***590,147***	***756,899***	***824,238***
	Total quay length (m)	***574***	***574***	***574***	***574***	***574***	***574***	***574***
	Terminal area (ha)	***10***	***10***	***10***	***10***	***10***	***10***	***10***
	Number of quayside cranes	***6***	***6***	***6***	***6***	***6***	***6***	***8***
	Number of yard gantry cranes	***4***	***4***	***4***	***4***	***4***	***4***	***13***
	Number of reach stackers	***0***	***4***	***4***	***10***	***10***	***10***	***23***

Table 3.*Input and output variables for port of Tema*

Port	Variables	2006	2007	2008	2009	2010	2011	2012
Port of Abidjan	Container throughput	507,100	531,809	652,358	610,185	561,535	546,417	633,917
	Total quay length (m)	1000	1000	1000	1000	1000	1000	1000
	Terminal area (ha)	34	34	34	34	34	34	34
	number of quayside cranes	3	3	3	3	3	3	4
	number of yard gantry cranes	16	16	16	16	16	16	16
	number of Reach stackers	19	19	19	19	19	19	19

Table 4. Input and output variables for port of Abidjan

Port	Variables	2006	2007	2008	2009	2010	2011	2012
Port of Dakar	Container throughput	375,876	424,457	347,483	331,076	349,231	369,137	383,903
	Total quay length (m)	660	660	660	660	660	660	660
	Terminal area (ha)	35	35	35	35	35	35	35
	number of quayside cranes	4	4	4	4	4	4	4
	number of yard gantry cranes	8	8	8	8	10	10	10
	number of reach stackers	15	15	15	15	15	15	15

Table 5. Input and output variables for port of Dakar

Port	Variables	2006	2007	2008	2009	2010	2011	2012
Port of Lomé	Container throughput	215,892	237,891	296,109	354,480	339,853	352,695	288,481
	Total quay length (m)	430	430	430	430	430	430	430
	Terminal area (ha)	12	12	12	12	12	12	12
	number of quayside cranes	4	4	4	4	4	4	6
	number of yard gantry cranes	0	0	0	0	0	0	0
	number of reach stackers	19	19	19	19	19	19	19

Table 6. *Input and output variables for port of Lomé*

Port	Variables	2006	2007	2008	2009	2010	2011	2012
Port of Cotonou	Container throughput	140,500	167,791	193,745	272,820	316,744	334,798	348,190
	Total quay length (m)	540	540	540	540	540	540	540
	Terminal area (ha)	20	20	20	20	20	20	20
	number of quayside cranes	4	4	4	4	4	4	8
	number of yard gantry cranes	10	10	10	10	10	10	10
	number of reach stackers	15	15	15	15	15	15	15

Table 7. *Input and output variables for port of Cotonou*

Port	Variables	2006	2007	2008	2009	2010	2011	2012
Lagos Port Complex	Container throughput	587,600	711,100	947,400	710,800	1,128,17	1,413,27	1,623,14
	Total quay length (m)	1005	1005	1005	1005	1005	1005	1005
	Terminal area (ha)	55	55	55	55	55	55	55
	number of quayside cranes	10	10	10	10	10	10	10
	number of yard gantry cranes	12	12	12	12	12	12	12
	number of reach stackers	31	31	31	31	31	31	31

Table 8. *Input and output variables for port of Lagos Port Complex*

4.1 DEAP application results CRS model

4.1.1 CRS Technical Efficiency results input oriented with DEAP 2.1

The most efficient West African ports are found to be the Port of Tema in Ghana and the Port of Lomé in Togo which both exhibits an average relative efficiency score of 100% for the period of analysis.

The Port of Tema and the Port of Lomé achieves 100% efficiency in all 7 years while the port of Abidjan achieves 99.66% efficiency.

Due to the world financial crisis on trade in 2008 as it shown in ***Table 9*** we notice a shortfall in years 2008 and 2009 but after 2009 efficiency scores start to rise again, except the port of Dakar in Senegal which shows a reduction in efficiency through the next 3 years as well.

Amongst the ports in this case study, the Port of Tema is one of the smallest ports in terms size (terminal area and berth length) but one of the largest in terms of throughput in West Africa.

On the other hand, the Port of Cotonou is relatively the least efficient port amongst the sample taken with an efficiency score of 52%, which indicates the port could have achieved efficiency with 52% of its inputs.

The port has excessive capacity in relation to its inputs and therefore there exists a lot of waste in production.

PERIOD 2006-2012

PORTS	2006	2007	2008	2009	2010	2011	2012	Mean
TEMA	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
ABIDJAN	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	97.60%	99.66%
DAKAR	95.00%	97.50%	67.80%	68.30%	66.60%	61.50%	59.10%	73.69%
LOME	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
COTONOU	40.30%	43.10%	42.80%	63.90%	66.50%	58.90%	49.50%	52.14%
LAGOS	80.60%	85.20%	99.70%	79.00%	100.00%	100.00%	100.00%	92.07%
Mean	85.98%	87.63%	85.05%	85.20%	88.85%	86.73%	84.37%	

Table 9.CRS Technical Efficiency scores for the period 2006-2012 and Means

Cotonou never achieves efficiency levels higher than 67% in the period 2006-2012.

In size, Cotonou is similar to Tema Port but achieves significantly lower output than Tema.

As a solution to increase its efficiency, there are two ways

- Put in measures to attract more containerized cargo
- Reduce its use of inputs.

Lagos Port Complex is the largest port in terms of size and throughput amongst the ports under this case study. The port is located in Nigeria, Africa’s largest economy and most populous nation.

Due to the analysis, the port achieves an average efficiency rating of 92.07%.

Lagos Port Complex achieves its lowest rating during 2009, as a result of the effects of the world financial crisis on trade.

The Port of Dakar exhibits quite an average performance throughout the period 2006-2012. The lowest efficiency score throughout the period was 49%, also in 2009 as with other ports in the sample. The port however only manages to achieve a high efficiency score of 82%, averaging 62% efficiency over the seven-year period under study.

Table 10 shows the ranking of West African ports according to their relative efficiency.

Port of Tema, in Ghana with the Port of Lomé in Togo, are the most efficient ports amongst the sample with the Port of Abidjan closely following with a little difference. On the other hand, the Port of Cotonou is the least efficient and exhibited substantial waste in production throughout the period under study as we can see in the chart in ***Figure 13*** and ***Figure 14*** as well which shows the port means for all years.

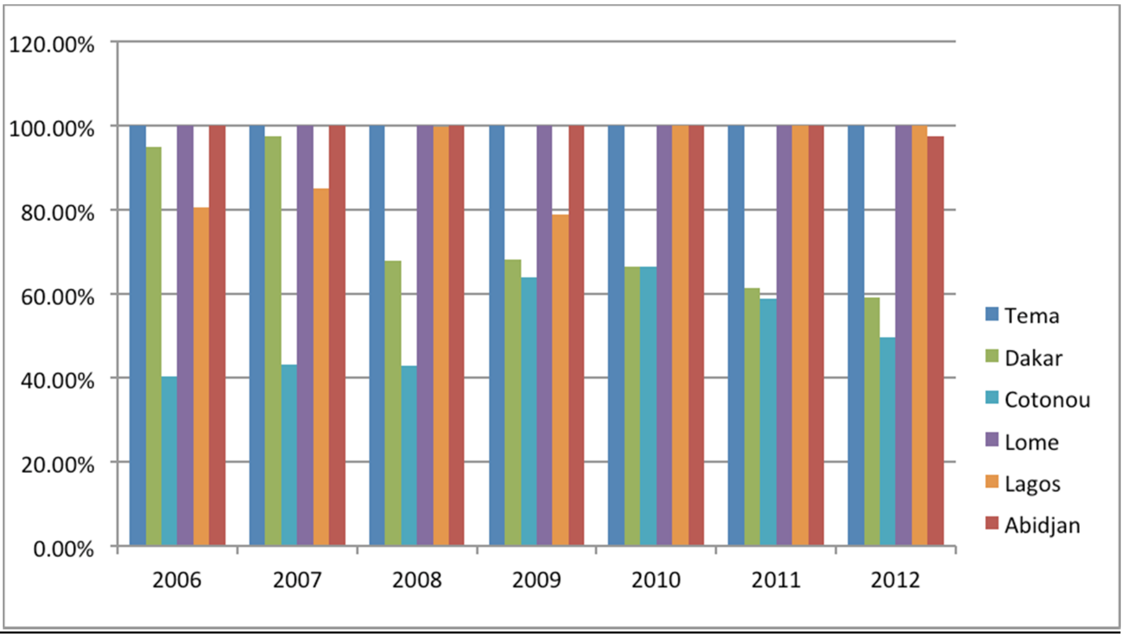

Figure 13.*Technical Efficiencies for each port for the period 2006-2012*

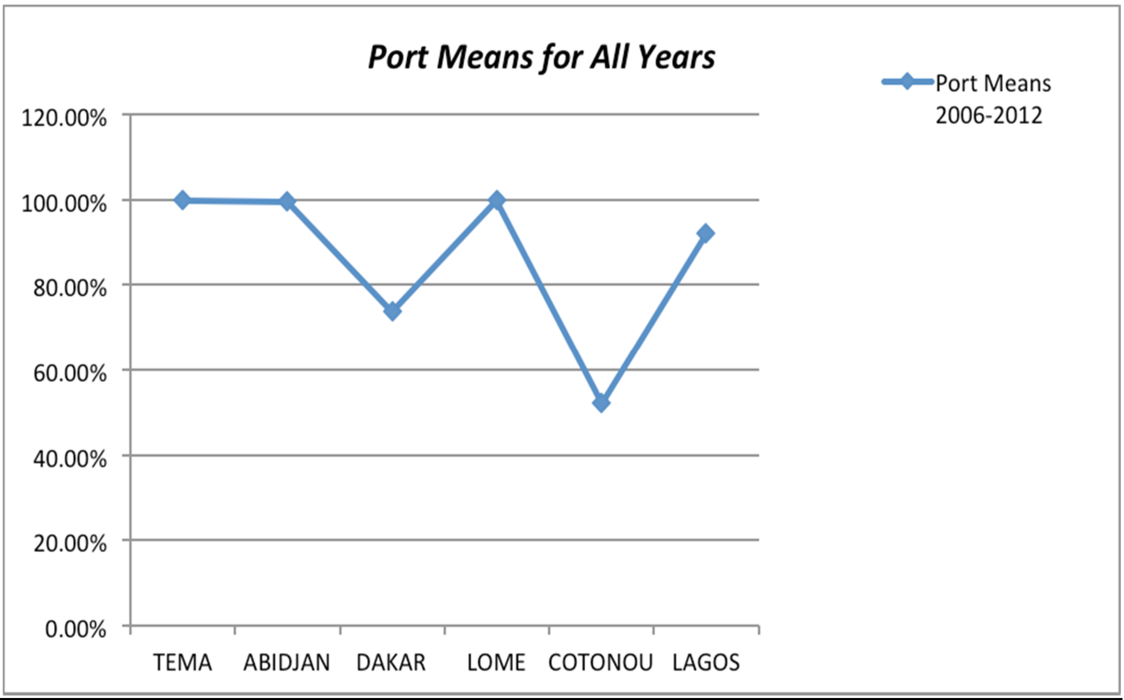

Figure 14.*Port Average Technical Efficiency scores for the period 2006-2012 (Means)*

In ***Figure 15*** we can notice that all ports mean shown a reduction due to the global crisis in years 2008 and 2009.

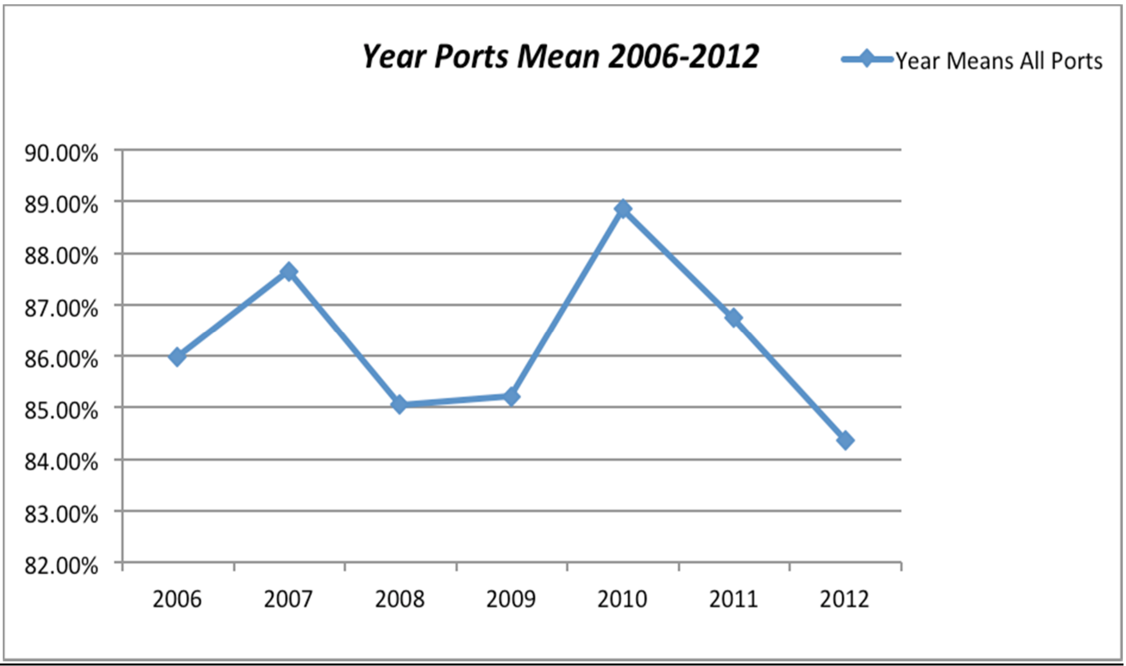

Figure 15.*Average Technical scores for all ports for the period 2006-2012 (Means)*

The ranking via Deap 2.1 is shown in ***Table 10*** and in the chart in ***Figure 16*** as well.

PORTS	TECHNICAL EFFICIENCY	RANK
TEMA	100.00%	1st
LOME	100.00%	2nd
ABIDJAN	99.66%	3rd
LAGOS	92.07%	4th
DAKAR	73.69%	5th
COTONOU	52.14%	6th

Table 10.*DEAP Port Ranking 2006-2012*

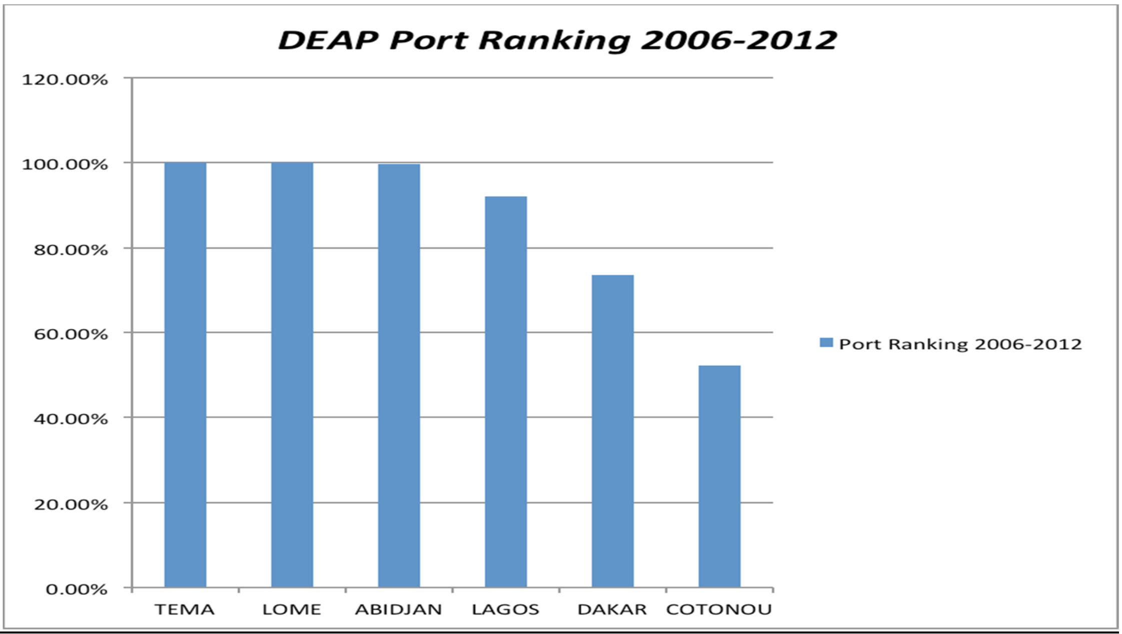

Figure 16.*DEAP Port Ranking 2006-2016*

4.2 SFA application results

4.2.1 Technical Efficiency results with Frontier 4.1

SFA is a parametrical method, which is used to estimate the efficiencies on port sector but as well as in other sectors too. SFA is more accurate, as it makes accommodation for statistical noise such as random variables of weather, luck, machine breakdown and other events beyond the control of firms, and measures error that's why the technical efficiency scores are reduced compared to Deap 2.1 results as we observe in ***Table 11*** and in the chart in ***Figure 17*** .

PERIOD 2006-2012

PORTS	2006	2007	2008	2009	2010	2011	2012	Mean
TEMA	79.50%	73.40%	83.30%	68.20%	76.60%	98.20%	91.10%	81.47%
ABIDJAN	73.90%	77.50%	95.10%	88.90%	81.90%	79.60%	73.50%	81.49%
DAKAR	88.40%	99.80%	81.70%	77.90%	85.30%	90.20%	93.80%	88.16%
LOME	50.80%	56.00%	69.70%	83.50%	80.00%	83.70%	49.00%	67.53%
COTONOU	40.30%	48.10%	55.60%	78.30%	90.90%	96.10%	57.50%	66.69%
LAGOS	34.40%	41.60%	55.50%	41.60%	66.10%	82.80%	95.10%	59.59%
Mean	61.22%	66.07%	73.48%	73.07%	80.13%	88.43%	76.67%	

Table 11.*Technical Efficiency scores for the period 2006-2012 and Means*

Port of Dakar in Senegal is the most efficient port amongst the sample with the Port of Abidjan closely following with a little difference. On the other hand, the Port of Lagos Complex in Nigeria is the least efficient and exhibited substantial waste in production throughout the period under study as we can see in the chart in ***Table 11***, ***Figure 17*** and in ***Figure 18*** which shows the technical efficiencies for each port for the period 2006-2012 and the port average technical efficiency scores for the period 2006-2012 respectively.

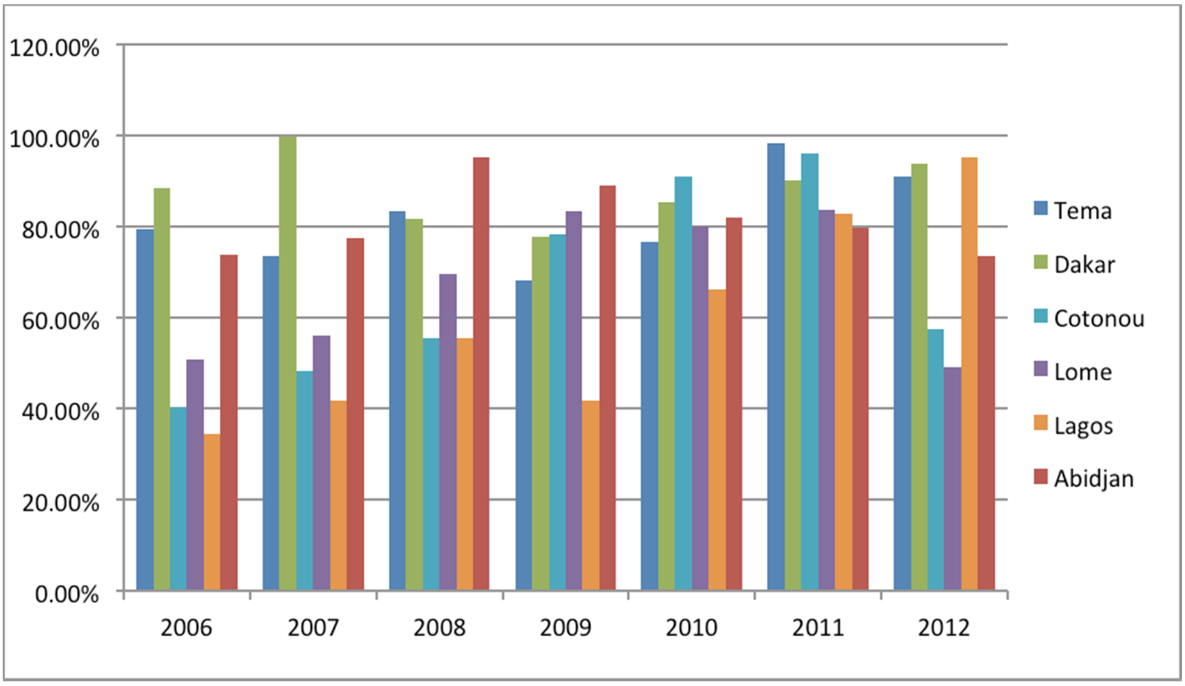

Figure 17.*Technical Efficiencies for each port for the period 2006-2012*

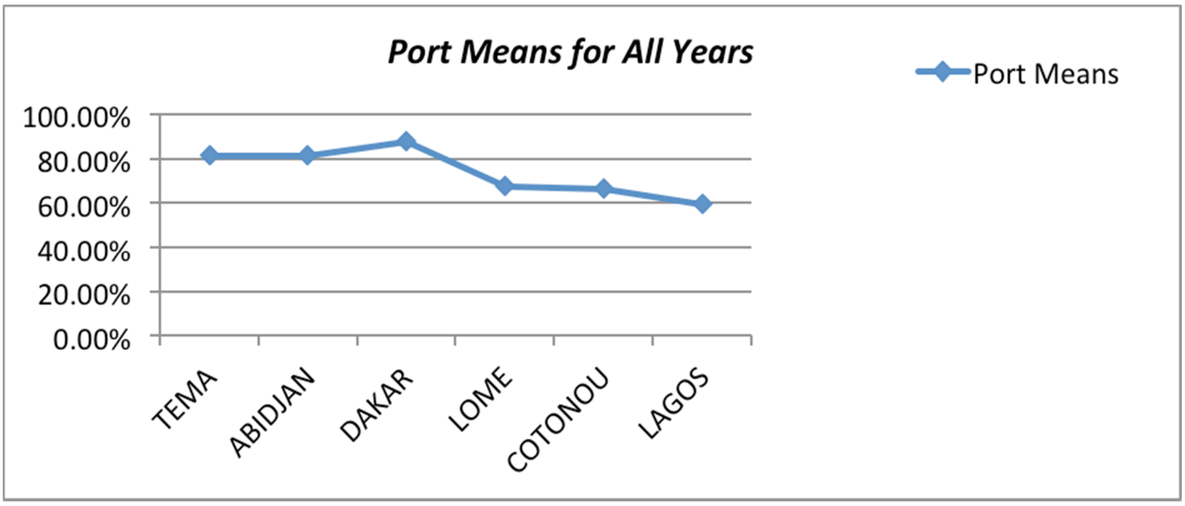

Figure 18.Port Average Technical Efficiency scores for the period 2006-2012 (Means)

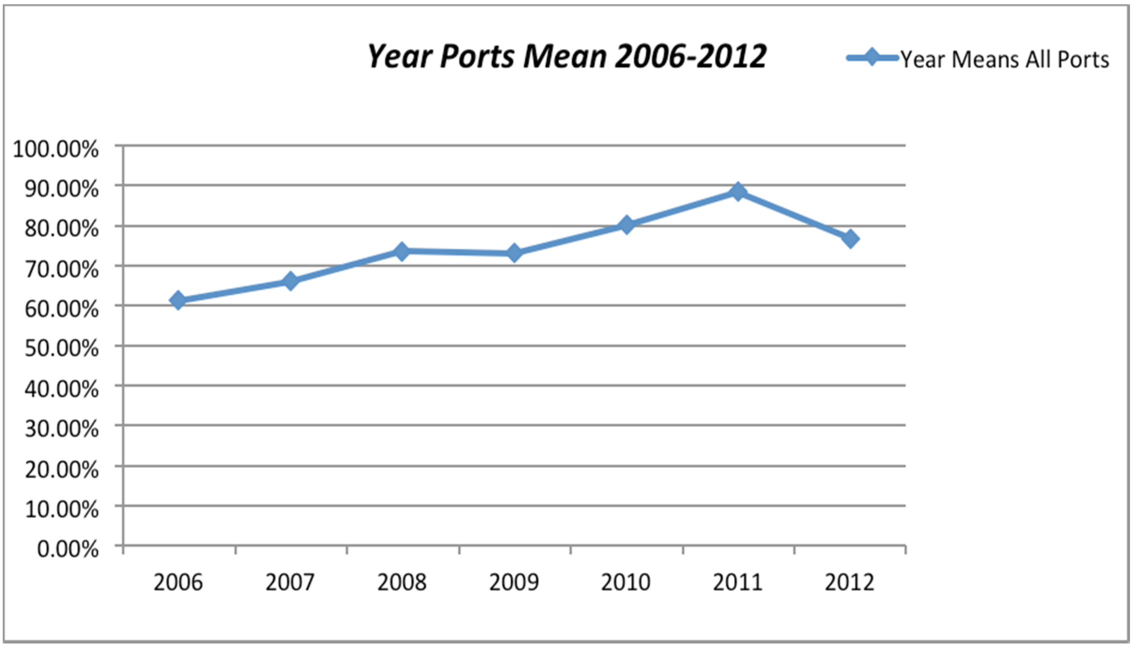

Figure 19.Average Technical scores for all ports for the period 2006-2012 (Means)

Due to the results under study via SFAcd Frontier 4.1 method it is noticed the reduced technical efficiency for all ports for the period 2006-2012 and the change in ranking compared to Deap 2.1 method because of the parametrical approach of SFA as shown in ***Table 12*** and in the chart in ***Figure 20***.

PORTS	TECHNICAL EFFICIENCY	RANK
DAKAR	88.16%	1st
ABIDJAN	81.49%	2nd
TEMA	81.47%	3rd
LOME	67.53%	4th
COTONOU	66.69%	5th
LAGOS	59.59%	6th

Table 12.*SFAcd Port Ranking 2006-2012*

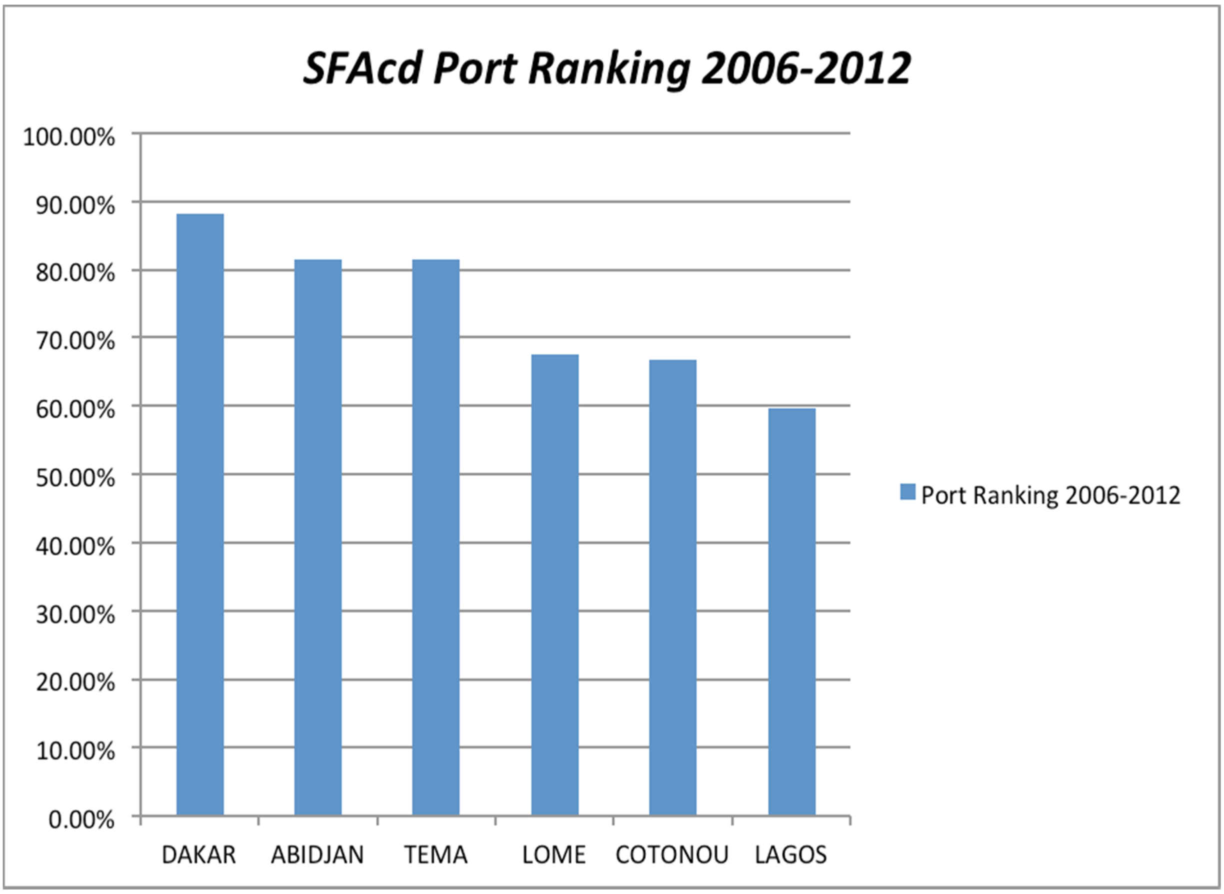

Figure 20.SFAcd Port Ranking 2006-2012

4.3 DEAP vs SFAcd

The biggest difference observed between the two efficiency methods are in the ports of Dakar in Senegal and in Port of Lagos complex in Nigeria as shown in ***Table 13*** and in the chart in ***Figure 21***.

DEAP Port Ranking			SFAcd Port Ranking	
PORTS	TECHNICAL EFFICIENCY	RANK	TECHNICAL EFFICIENCY	RANK
TEMA	100.00%	1st	81.47%	3rd
LOME	100.00%	2nd	67.53%	4th
ABIDJAN	99.66%	3rd	81.49%	2nd
LAGOS	92.07%	4th	59.59%	6th
DAKAR	73.69%	5th	88.16%	1st
COTONOU	52.14%	6th	66.69%	5th

Table 13.*DEAP and SFAcd Port Rankings*

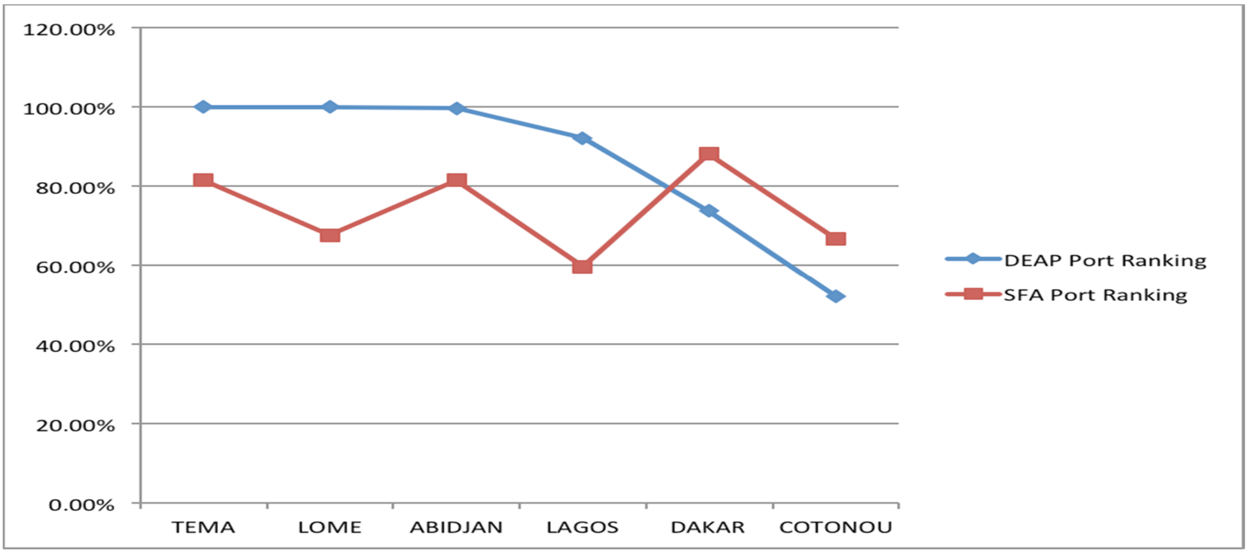

Figure 21.*DEAP and SFAcd Port Rankings*

5. Conclusions and proposals

This dissertation has proposed groundwork for a comparative evaluation of 6 major West African container ports and the rational use of their operational activities.

The analysis was based on two methods, the DEA and SFA method and the selection of ports based on their container throughput levels (up to 100,000 TEU's).

Results provided to improve the operations of poorly performing container ports and become a tool for them to solve inefficiencies. The general conclusion is that via Data Envelopment Analysis the majority of ports are efficient considering that four out of six ports had an average efficiency score of 76% or higher for the period under study but due to its non-parametrical approach as well, except the Port of Cotonou which is obviously has managerial issues due to the throughput levels observed.

Through SFA method the results were much different from DEA, as only three ports had average efficiency scores over 76%, including the Port of Lagos in the list of inefficient ports with Port of Cotonou. The performance of the Lagos Port Complex adds to literature a doubt on the notion that larger ports are more efficient. The Lagos Port Complex, which is the largest amongst the sample in terms of size and throughput, achieves an average efficiency score of 92% via DEA but only 60% via SFA which is a result of some inefficiency in its operations probably and of parametrical approach and accuracy of SFA method, which makes an accommodation for statistical noise such as random variables of weather, luck, machine breakdown and other events beyond the control of firms, and measures error and for a large port as Port of Lagos the possibility is increased for these variables to happen.

The adoption of only Data Envelopment Analysis method point to the need to use the Stochastic Frontier Analysis as well, in order to provide more accurate benchmark analysis and to improve the validity and accuracy of the results in measuring the container ports efficiency.

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Appendix A

Pictures of Ports

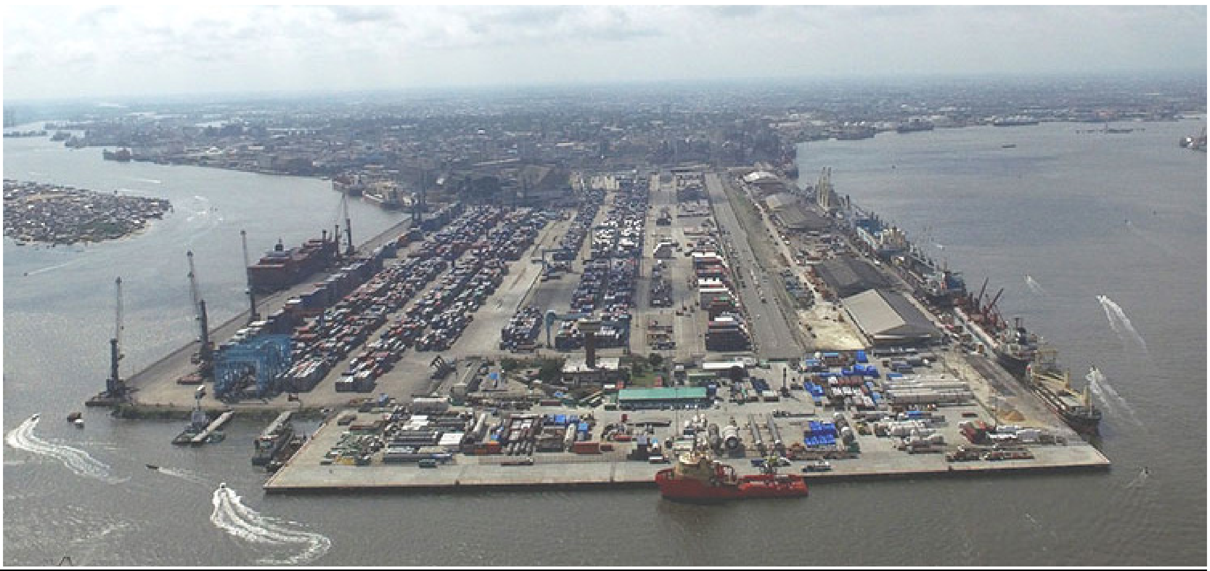
1. Tema Port, Ghana

2. Abidjan Port, Ivory Coast

3. <u>Dakar, Senegal</u>

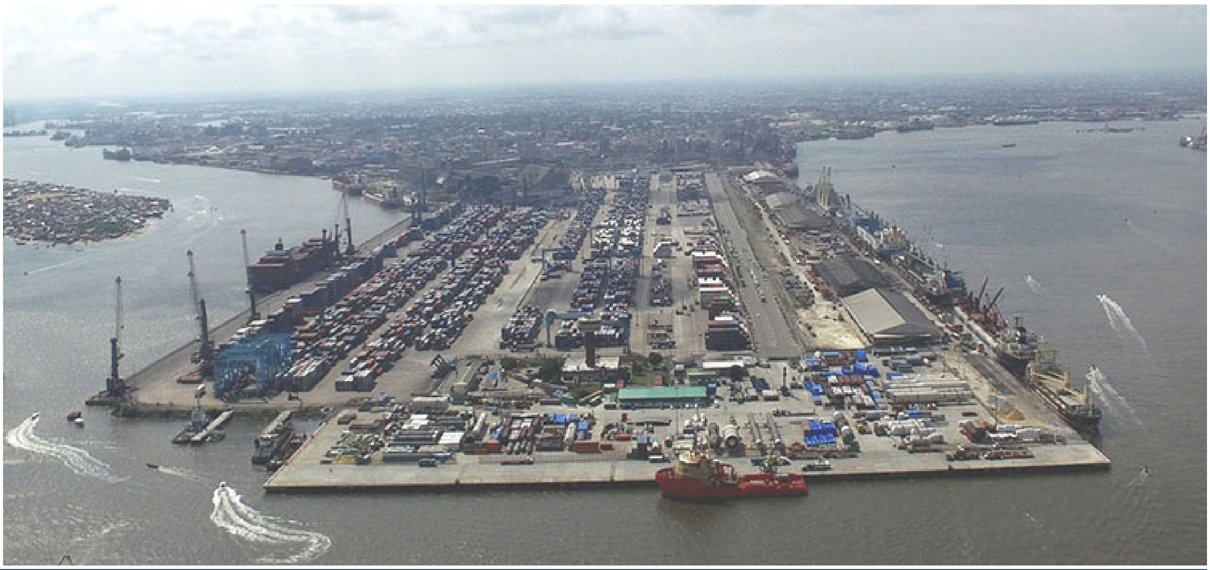
4. <u>Lomé, Togo</u>

5. Cotonou, Benin



6. Lagos Port Complex, Nigeria

Pictures of Input Indicators used



1. ***Lagos Quay Length 1005(m)***

2. ***Port Terminal Area (ha)***

3. Quayside Crane

4. Yard Gantry Crane

5. ***Reach Stackers***

Appendix B

Efficiency Summary DEAP Version 2.1

Year 2006

Firm	Technical Efficiency
1	1.000
2	1.000
3	0.950
4	1.000
5	0.403
6	0.806
mean	0.860

Year 2007

Firm	Technical Efficiency
1	1.000
2	1.000
3	0.975
4	1.000
5	0.431
6	0.852
mean	0.876

Year 2008

Firm	Technical Efficiency
1	1.000
2	1.000
3	0.678
4	1.000
5	0.428
6	0.997
mean	0.850

Year 2009

Firm	Technical Efficiency
1	1.000
2	1.000
3	0.683
4	1.000
5	0.639
6	0.790
mean	0.852

Year 2010

Firm	Technical Efficiency
1	1.000
2	1.000
3	0.666
4	1.000
5	0.665
6	1.000
mean	0.888

Year 2011

Firm	Technical Efficiency
1	1.000
2	1.000
3	0.615
4	1.000
5	0.589
6	1.000
mean	0.867

Year 2012

Firm	Technical Efficiency
1	1.000
2	0.976
3	0.591
4	1.000
5	0.495
6	1.000
mean	0.844

Efficiency Summary SFAcd (Frontier 4.1)

Year 2006

Firm	Technical Efficiency
1	0.795
2	0.739
3	0.884
4	0.508
5	0.403
6	0.344
mean	0.622

Year 2007

Firm	Technical Efficiency
1	0.734
2	0.775
3	0.998
4	0.560
5	0.481
6	0.416
mean	0.660

Year 2008

Firm	Technical Efficiency
1	0.833
2	0.951
3	0.817
4	0.697
5	0.556
6	0.555
mean	0.734

Year 2009

Firm	Technical Efficiency
1	0.682
2	0.889
3	0.779
4	0.835
5	0.783
6	0.416
mean	0.730

Year 2010

Firm	Technical Efficiency
1	0.766
2	0.819
3	0.853
4	0.800
5	0.909
6	0.661
mean	0.801

Year 2011

Firm	Technical Efficiency
1	0.982
2	0.796
3	0.902
4	0.837
5	0.961
6	0.828
mean	0.884

Year 2012

Firm	Technical Efficiency
1	0.911
2	0.735
3	0.938
4	0.490
5	0.575
6	0.951
mean	0.766