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University of Piraeus

Department of Banking and Financial Management

Master of Science in Banking and Financial Management

Bill To:	Ship To:

Volatility Clustering in Monthly Stock Returns
And
Temporal Aggregation of Simulated Data.

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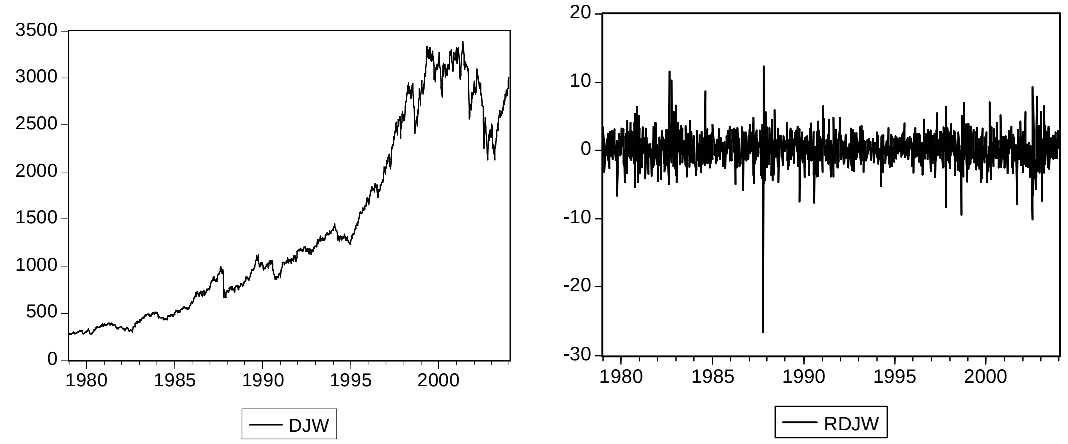

[The Dow Jones Composite Price Index 65 and its returns from 1979 to 2003]

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Total

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Volatility Clustering in Monthly Stock Returns
And
Temporal Aggregation of Simulated Data.

Abstract:

Financial markets display pronounced volatility clustering. Over the last decade financial economists have began to seriously model the temporal dependencies in return volatilities. This paper investigates the volatility clustering using econometric models based on the methodology of temporal aggregation for GARCH processes. We initially derive low frequency volatility models first from low frequency data and then from high frequency data using a Monte Carlo simulation. We compare the different models and finally we try to find out whose model's parameters are more efficient. The one's that were produced directly from low frequency data or the other's that was produced through temporal aggregation from high frequency data?

The paper is divided in three parts. In the first part we summarize the theoretical background on which this empirical survey is based. In the second part we make use of real data and more precisely, data of the Dow Jones Industrial 65 composite price index and the Standard and Poor´s 500 composite price index. In the third part we produce, through a Monte Carlo simulation, low frequency volatility models from high frequency data using the methodology of temporal aggregation. The initial values for our simulation are taken from the analysis of the real data and our effort is to replicate the behaviour of Dow Jones Index's volatility.

Introduction:

The stock return volatility in a period of time is called the conditional variance of these returns in the same period. The volatility of flows of an investment is very important and the prediction of this volatility is a powerful financial tool for investors, bankers, traders etc. During the last years more and more papers are dealt with the behaviour of volatility. This is not something strange since volatility is to some degree predictable contrary to returns whose future behaviour is more difficult to fit in a model and more difficult to be predicted.

The econometric models had initially focused on the study of conditional mean. The expansion of econometric theory and the need of the market for further understanding of the risk drove the study of higher order moments. The models that where produced are called ARCH-type models (Autoregressive Conditional Heteroskedasticity Models).

The purpose of each study determines what frequency will be used in each case. For instance if the forecasts of future volatility will be used by a stock trader the forecasts will be meaningful if they predict next minute's or hour's volatility. In such a study we are obliged to use high frequency data (every second, minute, hour, day).

On the other hand predictions that are going to be used by an investment manager will be useful only if they are referred to next month's or next year's volatility. In this case low frequency data has to be used (weekly, bi-weekly or monthly data).

PART 1

1.1 Stylised facts about stock returns volatility:

The behaviour of stock returns volatility has some special characteristics that have been confirmed in numerous studies:

1. The distribution of returns is leptokurtic e.g. contrary to normal distribution it has more extreme values. This phenomenon is referred in bibliography as thick tails. It is well established that the unconditional distribution of asset returns has heavy tails. Typical Kurtosis estimates range from 4 to 50 indicating very extreme non-normality. This is a feature that should be incorporated in any volatility model. The relation between the conditional density of returns and the unconditional density partially reveals the source of heavy tails. If the conditional density is Gaussian, then the unconditional density will have excess kurtosis due simply to the mixture of Gaussian densities with different volatilities. However there is no reason to assume that the conditional density is itself fat tailed, generating still greater kurtosis in the unconditional density.
2. Stock returns exhibit volatility clustering. In other words the autocorrelation of first lags is statistically significant. This phenomenon is related to the thick tales phenomenon that was mentioned above. What we know is that in high frequency data the volatility-clustering phenomenon is very intense. As we move to lower frequency data the phenomenon is gradually becoming less and less intense. Mandelbrot (1963) and Fama (1965) both reported evidence that large changes in the price of an asset are often followed by other large changes; this behaviour has been reported by numerous other studies, such as Bailie et al. (1996), Chou (1998) and Schwert (1989). The implication of such

volatility clustering is that volatility shocks today will influence the expectation of volatility many periods in the future.

3. Volatility clustering implies that volatility comes and goes. Thus a period of high volatility will eventually give way to more "normal" volatility and similarly, a period of low volatility will be followed by rise. Mean reversion in volatility is generally interpreted as meaning that there is "normal" level of volatility to which volatility will eventually return. Very long forecasts of volatility should all converge to this same normal level of volatility, no matter when they are made. There is a controversy over the "normal" level of volatility and whether it is constant over all time and institutional changes.
4. The asymmetry effect that a "good news shock" and a "bad news shock" have on the stock returns is another characteristic of stock return volatility. More precisely it has been found that the volatility is negatively related to equity returns. [Black (1976), Christie (1982), Nelson (1991), Glosten et al (1993)] This asymmetry is sometimes ascribed to a leverage affect and sometimes to a risk premium effect. In the former theory, as the price of a stock falls, its debt-to-equity ratio rises, increasing the volatility of returns to equity holders. In the latter story, news of increasing volatility reduces the demand for a stock because of risk aversion. The consequent decline in stock value is followed by the increased volatility as forecast by the news.
5. Another characteristic of stock returns behaviour is the exaltation of the volatility just before a predictable event of great importance like the opening and the closing in every trading day in the stock market or the public announcement of the corporate earnings.
6. Another very interesting characteristic of the volatility is its simultaneously raise and fall in different interactive markets such as the stock market as a whole, the market for a single stock, the T-bills market and more general the world money markets.

7. According to Glosten et al. (1993) there is a strong positive relation between the stocks return volatility and interest rates volatility. On the other hand only a weak link was detected between macroeconomic uncertainty and volatility.

1.2 Definitions and Notation:

In order to define the models properly let $\{e_t, t \in \mathrm{Z}\}$ be a sequence of stationary errors with finite fourth moments. Define operators

$$A(L) = 1 + \sum_{i=1}^{q} a_i L^i \text{ and } B(L) = 1 - \sum_{i=1}^{p} b_i L^i \text{ and}$$

let the sequence $\{h_t, t \in Z\}$ be defined as the stationary solution of:

$$B(L)h_t = y \ + \{\mathrm{A}(L) - 1\} e_t^2$$

We assume that *B(L)* and *B(L)+1-A(L)* have roots outside the unit circle and hence are invertible. Three definition of GARCH will be used:

Strong GARCH:

The sequence $\{e_t, t \in \mathrm{Z}\}$ is defined to be generated by a strong GARCH (p,q) process if ψ, *A(L)* and *B(L)* can be chosen such that :

$$x_t = e_t / \sqrt{h_t} \sim i.i.d.\ D(0,1)$$

where D(0,1) specifies a distribution with mean zero and unit variance.

Semi-strong GARCH:

The sequence $\{e_t, t \in Z\}$ is defined to be generated by a semi-strong GARCH (p,q) process if ψ*, A(L)* and *B(L)* can be chosen such that :

$$E[e_t / e_{t-1}, e_{t-2,\ldots}] = 0 \quad \text{and}$$

$$E[e_t^2 / e_{t-1}, e_{t-2}, \ldots] = h_t$$

Weak GARCH:

The sequence $\{e_t, t \in Z\}$ is defined to be generated by a weak GARCH (p,q) process if ψ*, A(L)* and *B(L)* can be chosen such that :

$$P[e_t / e_{t-1}, e_{t-2}, \ldots] = 0 \quad \text{and}$$
$$P[e_t^2 / e_{t-1}, e_{t-2}, \ldots] = h_t$$

where $P[x_t / e_{t-1}, e_{t-2}, \ldots]$ denotes the best linear predictor of x_t in terms of $1, e_{t-1,} e_{t-2}, \ldots, e_{t-1}^2, e_{t-2}^2, \ldots, i.e.$

$$E(x_t - P[x_t / e_{t-1}, e_{t-2}, \ldots])e_{t-i}^r = 0 \text{ for } i \geq 1 \text{ and } r = 0,1,2.$$

All these GARCH definitions require $\sum_{i=1}^{p} b_i + \sum_{i=1}^{q} a_i < 1$. The most popular distributions are normal and t distributions. A strong GARCH process will also be semi-strong GARCH. On the other hand a semi-strong GARCH process with time – varying higher order conditional moments of the rescaled innovations $x_t = e_t / \sqrt{h_t}$ is not strong GARCH. Finally the requirements for weak GARCH are met both by strong and semi-strong GARCH processes. The weak GARCH definition is quite general and captures the characterizing features of the other GARCH formulations. It is possible to obtain strongly consistent estimators of the GARCH parameters in this general formulation.

The most general model used is the ARMA model with GARCH errors:

$$\Gamma(L) y_t = \Theta(L) e_t \text{ where :}$$

$$\Gamma(L) = \Pi_{i=1}^{P} (1 - g_i L) \text{ and } \Theta(L) = \Pi_{i=1}^{Q} (1 - q_i L).$$

It is assumed that all standard regularity conditions are fulfilled. This implies that the roots of *Γ(L)* and *Θ(L)* are all outside the unit circle and that no roots of *Γ(L)* coincides with roots of *Θ(L)*.

High frequency observations are assumed to be on $y_t (t = 1, \mathbf{K}, T)$. If y_t is stock variable low frequency observations are assumed to be on $y_t (t = m, 2m, \mathbf{K}, T)$ where *m* is some known integer. We suppose that T is a multiple of m.

1.3 Temporal Aggregation:

When high frequency data is available we can calculate a low frequency model using high frequency data through temporal aggregation. The methodology for temporal aggregation that will be followed in this paper was initially introduced by Drost and Nijman (1993). This methodology enables us to compare the parameters of the two low frequency models. The one that will be produced by low frequency data and the other that will be produced by high frequency data. It's interesting to see the behavior of conditional variance and how it reacts to frequency changing.

It is feasible to estimate strongly consistent estimators of the low frequency model. Simulations have proven that the quasi-maximum likelihood estimator is very close to the real values of the parameters.

This analysis will be focused on GARCH (1,1) model. This choice is not random. Many studies have proven that the simple GARCH (1,1) model has better forecasting ability in relation to all other models that are usually more complicated.

The temporal aggregation has also bothered Diebold and Nelson. In 1988 Diebold showed that the conditional heteroskedasticity disappears as time sampling interval tends to infinity. In the case of flow variables the implied low frequency distribution tends to normal distribution. In 1990 Nelson studied the effects of the increasing frequency of the sample.

1.4 Aggregation of GARCH (1,1):

In the case of stock variables the class of symmetric weak GARCH (1,1) models is closed against temporal aggregation. The high frequency parameters y , b , a and k_y determine the corresponding low frequency parameters.

More precisely, if $\{y_t, t \in \mathrm{Z}\}$ is weak GARCH (1,1) with symmetric marginal distributions and $h_t = y + bh_{t-1} + ay_{t-1}^2$ then $\{y_{tm} \in \mathrm{Z}\}$ is symmetric weak GARCH (1,1) with:

$$h_{(m)tm} = y_{(m)} + b_{(m)}h_{(m)tm-m} + a_{(m)}y_{tm-m}^2 \text{ where:}$$

$$y_{(m)} = y\,\frac{1-(b+a)^{(m)}}{1-(b+a)},\; a_{(m)} = (b+a)^m - b_{(m)},\text{ and}$$

$b_{(m)} \in (0,1)$ is the solution of the quadratic equation :

$$\frac{b_{(m)}}{1+b_{(m)}^2} = \frac{b(b+a)^{m-1}}{1+a^2\dfrac{1-(b+a)^{2m-2}}{1-(b+a)^2}+b^2(b+a)^{2m-2}}.$$

What is observed is that $b_{(m)} + a_{(m)} = (b+a)^m$ tends to zero as m tends to infinity. Hence conditional heteroskedasticity disappears in the limit if the process is aggregated more and more.

We can derive strongly consistent estimators of the high and low frequency parameters only based upon low frequency data. Let us consider the low frequency ARMA (1,1) model that is known to generate the squared observations:

$$y_{tm}^2 = y_{(m)} + (b_{(m)} + a_{(m)})y_{tm-m}^2 + h_{tm} - b_{(m)}h_{tm-m}$$

where $h_{tm}^2 - h_{(m)tm}$. The h_{tm} are uncorrelated. Assume that $\{h_{tm}\}$ is ergodic. As a result the vector process $\{(y_{tm}^2, y_{tm-m}^2, y_{tm-2m}^2)\}$ is also ergodic. This implies that the sample mean and the first two sample autocorrelations converge almost surely and a simple one-one relation between these limits and the GARCH parameters determines the required estimators. The assumption of ergodicity of the low frequency process is e.g. trivially satisfied if there exists an underlying high frequency strong GARCH process. Then the low frequency GARCH parameters can be consistently estimated using low frequency data. This implies that consistent estimation of the high frequency parameters, based on low frequency data only, is possible. The high frequency parameters are uniquely determined by the corresponding low frequency ones.

The problem of multiple high frequency models that are consistent with the low frequency evidence that can arise in ARMA (1,1) models is absent in the GARCH (1,1) model because of the restriction that all parameters are nonnegative. Drost and Nijman proved that GARCH (1,1) are close to ARCH (1) models if the sampling interval is large and the conditional heteroskedasticity disappears when the sampling period is large. Furthermore they showed that highly aggregated models with nontrivial variance parameters are generated by a DGM close to integration in variance models.

PART 2

2.1 Data description and Variable Construction:

Daily, weekly and monthly data was downloaded from DATASTREAM database for the period: January 1979 to May 2004. We initially examine two indices: The Dow Jones Industrial Composite 65 price index (from now on DJ) and the Standard and Poor's 500 Composite price index (from now on SP). These indices are chosen because they are large enough, with available data for a long period of time, economically important and because such indices are minimally impacted by isolated corporate developments contrary to indices of small countries. From now on the returns of these two indices are RDJ and RSP respectively. If the data is at the daily, weekly or monthly frequency the abbreviation will have the prefix D~ for daily data, W~ for weekly data and M~ for monthly data respectively (DRDJ, WRDJ, MRDJ, etc).

In order to examine the stochastic process over the study period we employed models of conditional variances using the generalized autoregressive conditional heteroskedasticity (GARCH) formulation. The autoregressive conditional heteroskedasticity (ARCH) model was introduced by Engle (1982) and allows the variance of the error term to vary over time, in contrast to the standard time series regression models that assume a constant variance. The generalized ARCH models, i.e. the GARCH models, have been found to be valuable in modeling the time series behavior of stock returns (Akgiray, 1989; French et al. 1987;). Bollerslev (1986) allows the conditional variance to be a function of prior period's squared errors as well as of its past conditional variances. The GARCH model has the advantage of incorporating heteroskedasticity into the estimation procedure. All GARCH models are martingale difference implying that all expectations are unbiased. The GARCH models are capable of capturing the tendency for volatility clustering in financial data. Volatility clustering in stock returns implies that large (small) price changes follow large (small) price changes of either sign. Engle et al. (1987) provide an extension of the GARCH model where the conditional mean is an explicit function of the conditional variance. Such a model is known as the GARCH in the mean model.

The persistence of shocks to volatility depends on the sum of the $\alpha+\beta$ parameters. Values of the sum lower than unity imply a tendency for the volatility response to decay over time. In contrast, values of the sum equal or greater than unity imply indefinite or increasing volatility persistence to shocks over time. However a significant impact of volatility on the stock prices can only take place if shocks to volatility persist over a long time.

Figure (1): Dow Jones Industrial 65 Composite Price Index and Standard & Poor’s 500 Composite Price Index.

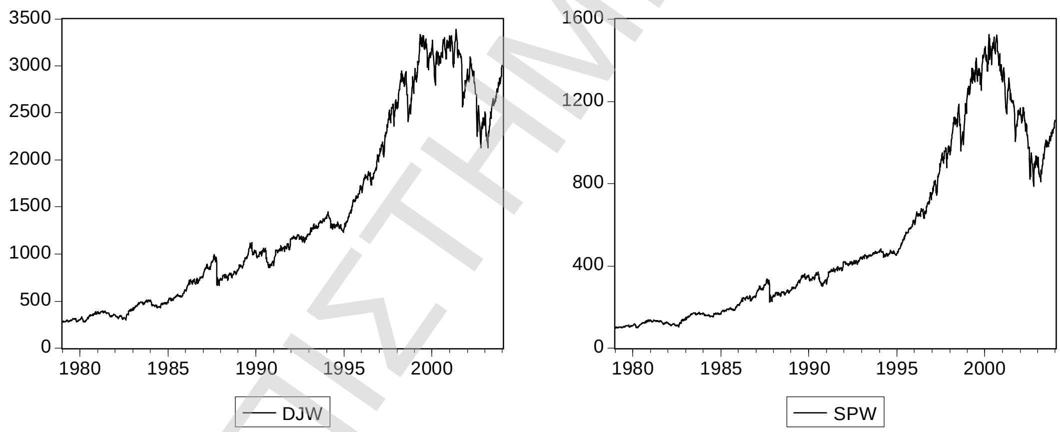
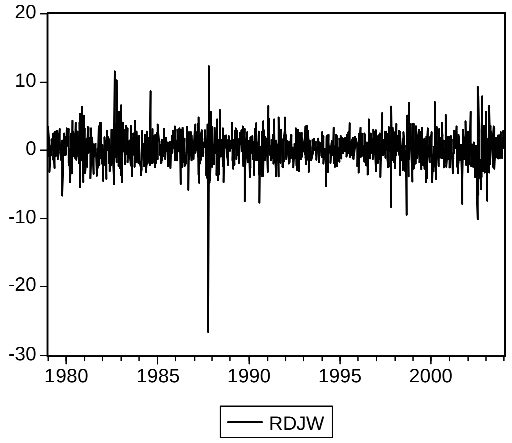
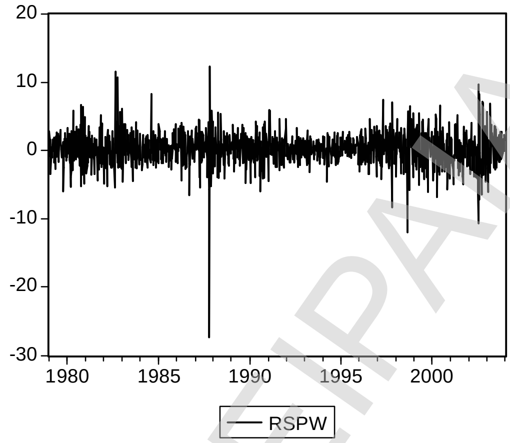


Figure (2): Returns of Dow Jones Industrial 65 Composite Price Index and Standard & Poor’s 500 Composite Price Index.





As we notice volatility clustering is present in both time series. It appears as clusters of high and low volatility. During the 90's volatility is not as intense as it is during the first years of the 00's or the 80's.

Some summary statistics of the data are presented in the tables below:

(1) Returns of Dow Jones Industrial 65 Composite Price Index:

	Daily Data	Weekly Data	Monthly Data
Number of observations	6523	1304	300
Mean	0.044825	0.212353	0.954055
Std. Dev.	1.049377	2.341541	4.354195
Skewness	-1.558261	-1.049756	-0.599395
Kurtosis	41.22057	17.92316	6.269915

(2) Returns of Standard & Poor's 500 Composite Price Index:

	Daily Data	Weekly Data	Monthly Data
Number of observations	6523	1304	300
Mean	0.042898	0.218016	0.91189
Std. Dev.	1.034637	2.436149	4.262545
Skewness	-1.224119	-1.021674	-0.619104
Kurtosis	30.78527	17.06107	5.468054

We use the following measure for skewness:

$$s = \frac{1}{T}\left(\frac{r_i - \bar{r}}{\hat{s}}\right)^3 .$$

The skewness of a symmetric distribution, such as the normal distribution, is zero. Positive skewness means that the distribution has a long right tail and negative skewness implies that the distribution has a long left tail. At the daily, weekly and monthly frequency the series seem to be negatively skewed.

The series also exhibit high kurtosis. Kurtosis measures the peakedness or flatness of the distribution of the series. Kurtosis is computed as:

$$k = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_i - \hat{r}}{s^2}\right)^4$$

The level of kurtosis increases as data frequency increases. On a daily basis it equals 41.22057 for the Dow Jones 65. In the presence of volatility clustering the squared returns should be highly autocorrelated. An analysis of the correlogram of the returns indicates no dependence in the mean of the series and so we will assume a constant conditional mean.

2.2 Econometric Analysis of Real Data

The results of the "real" data analysis will be presented in the next few pages. The analysis was performed with the E-views programme. The ARCH (1) parameter is the parameter α of the GARCH (1,1) model, which was presented analytically in the first part of this paper. The GARCH (1) parameter is the parameter β of the GARCH (1,1) model.

The behavior of the volatility is examined, as data frequency switches from daily to weekly and from weekly to bi-weekly and from bi-weekly to monthly frequency for both indices. The final results are presented in a separate table at the end of this analysis.

Dependent Variable: RDJD
Method: ML - ARCH (Marquardt)
Date: 04/26/04 Time: 16:01
Sample (adjusted): 2 6524
Included observations: 6523 after adjusting endpoints
Convergence achieved after 42 iterations
Variance backcast: ON

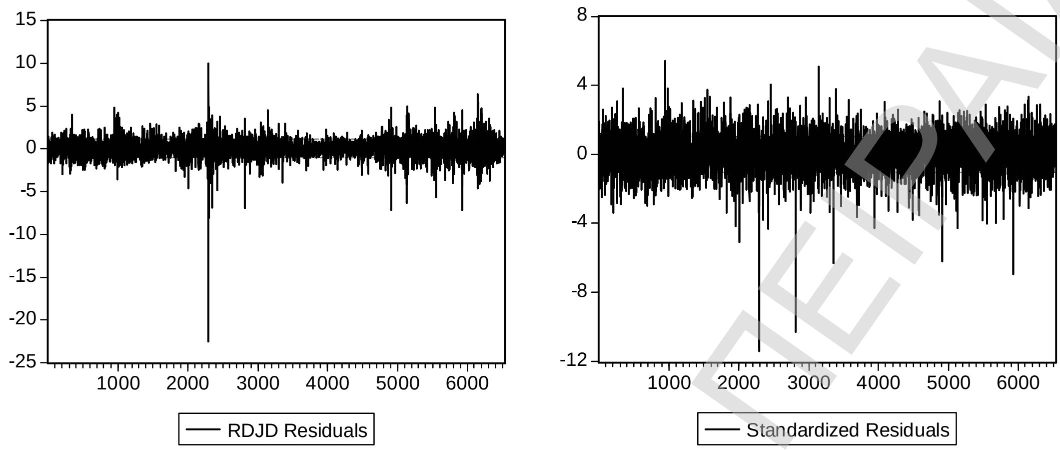
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.062681	0.010555	5.938273	0.0000
		Variance Equation		
C	0.013842	0.001471	9.408010	0.0000
ARCH(1)	0.065156	0.001577	41.32581	0.0000
GARCH(1)	0.923652	0.003039	303.8919	0.0000

R-squared	-0.000290	Mean dependent var	0.044825
Adjusted R-squared	-0.000750	S.D. dependent var	1.049377
S.E. of regression	1.049770	Akaike info criterion	2.694861
Sum squared resid	7184.054	Schwarz criterion	2.699021
Log likelihood	-8785.289	Durbin-Watson stat	1.976462

Estimation Command:
=====================
ARCH RDJD C

Estimation Equation:
=====================
RDJD = C(1)

Substituted Coefficients:
=====================
RDJD = 0.06268101107



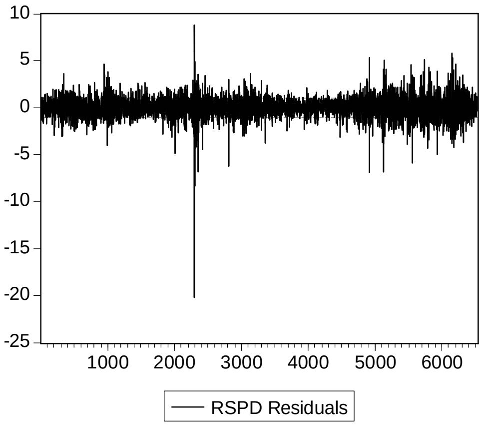
Dependent Variable: RSPD
Method: ML - ARCH (Marquardt)
Date: 04/26/04 Time: 16:04
Sample(adjusted): 2 6524
Included observations: 6523 after adjusting endpoints
Convergence achieved after 33 iterations
Variance backcast: ON

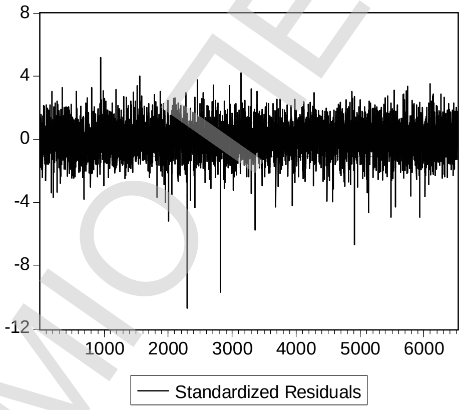
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.058845	0.010227	5.753839	0.0000
	Variance Equation			
C	0.009647	0.001160	8.317874	0.0000
ARCH(1)	0.062214	0.001539	40.42083	0.0000
GARCH(1)	0.930327	0.002631	353.6221	0.0000
R-squared	-0.000238	Mean dependent var		0.042898
Adjusted R-squared	-0.000698	S.D. dependent var		1.034637
S.E. of regression	1.034998	Akaike info criterion		2.652259
Sum squared resid	6983.285	Schwarz criterion		2.656418
Log likelihood	-8646.343	Durbin-Watson stat		1.959678

Estimation Command:
=====================
ARCH RSPD C

Estimation Equation
=====================
RSPD = C(1)

Substituted Coefficients:
=====================
RSPD = 0.05884517401





Dependent Variable: RDJW
Method: ML - ARCH (Marquardt)
Date: 04/26/04 Time: 16:05
Sample(adjusted): 1/08/1979 12/29/2003
Included observations: 1304 after adjusting endpoints
Convergence achieved after 17 iterations
Variance backcast: ON

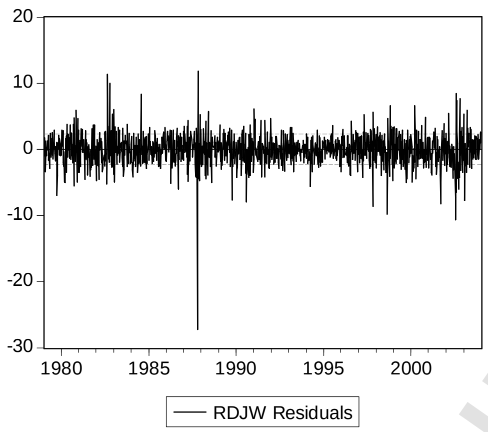
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.305436	0.057408	5.320457	0.0000
	Variance Equation			
C	0.634838	0.126391	5.022817	0.0000
ARCH(1)	0.181574	0.011712	15.50371	0.0000
GARCH(1)	0.714808	0.031308	22.83176	0.0000

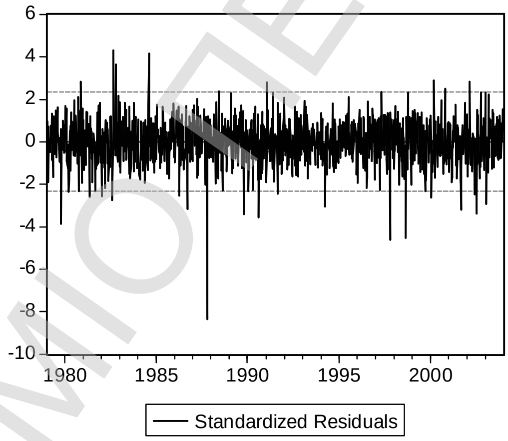
R-squared	-0.001582	Mean dependent var	0.212353
Adjusted R-squared	-0.003893	S.D. dependent var	2.341541
S.E. of regression	2.346095	Akaike info criterion	4.422333
Sum squared resid	7155.407	Schwarz criterion	4.438202
Log likelihood	-2879.361	Durbin-Watson stat	2.102479

Estimation Command:
=====================
ARCH RDJW C

Estimation Equation:
=====================
RDJW = C(1)

Substituted Coefficients:
=====================
RDJW = 0.3054363252





Dependent Variable: RSPW
Method: ML - ARCH (Marquardt)
Date: 04/26/04 Time: 16:17
Sample(adjusted): 1/15/1979 12/29/2003
Included observations: 1303 after adjusting endpoints
Convergence achieved after 41 iterations
Variance backcast: ON

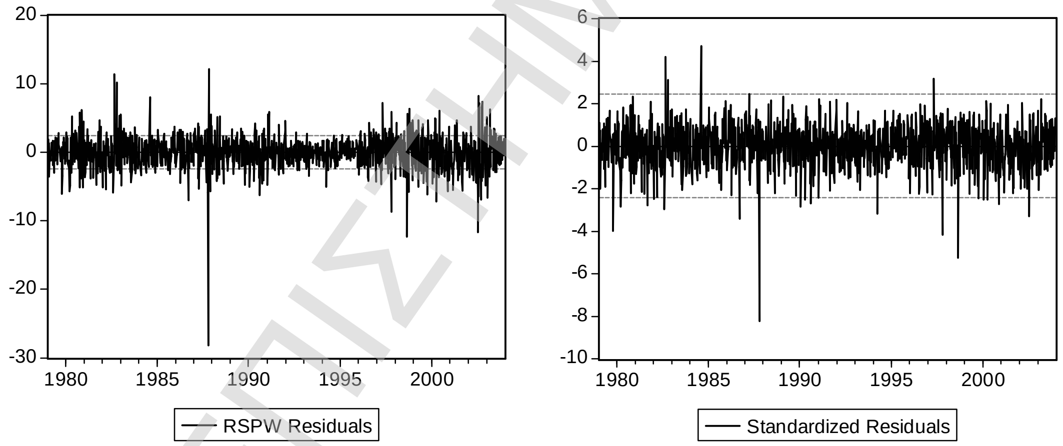
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.282851	0.053520	5.284984	0.0000
RSPW(-1)	-0.099358	0.030662	-3.240387	0.0012
	Variance Equation			
C	0.257329	0.065714	3.915880	0.0001
ARCH(1)	0.161614	0.011831	13.66067	0.0000
GARCH(1)	0.809945	0.021012	38.54631	0.0000
R-squared	0.008305	Mean dependent var		0.216036

Adjusted R-squared	0.005249	S.D. dependent var	2.436033
S.E. of regression	2.429632	Akaike info criterion	4.451290
Sum squared resid	7662.237	Schwarz criterion	4.471138
Log likelihood	-2895.016	F-statistic	2.717567
Durbin-Watson stat	1.996498	Prob(F-statistic)	0.028502

Estimation Command:
=====================
ARCH RSPW C RSPW(-1)

Estimation Equation:
=====================
RSPW = C(1) + C(2)*RSPW(-1)

Substituted Coefficients:
=====================
RSPW = 0.2828508197 - 0.09935765588*RSPW(-1)



Dependent Variable: RDJM
Method: ML - ARCH (Marquardt)
Date: 04/26/04 Time: 16:14
Sample(adjusted): 1979:02 2004:01
Included observations: 300 after adjusting endpoints
Convergence achieved after 42 iterations

Variance backcast: ON

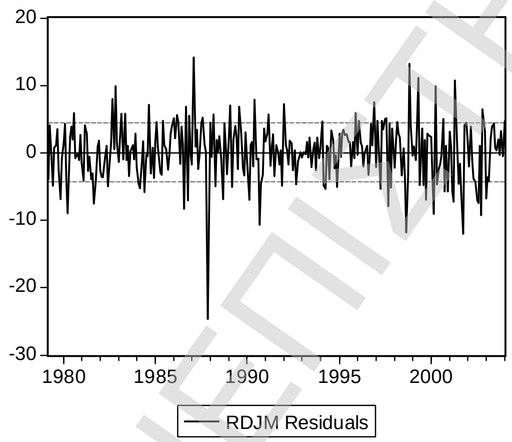
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.902588	0.228763	3.945518	0.0001
Variance Equation				
C	0.571728	0.503837	1.134749	0.2565
ARCH(1)	0.068681	0.035054	1.959308	0.0501
GARCH(1)	0.805582	0.048971	18.49237	0.0000

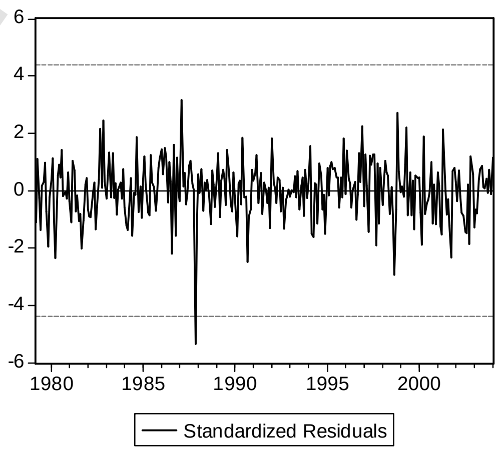
R-squared	-0.000140	Mean dependent var	0.954055
Adjusted R-squared	-0.010277	S.D. dependent var	4.354195
S.E. of regression	4.376511	Akaike info criterion	5.766413
Sum squared resid	5669.539	Schwarz criterion	5.815797
Log likelihood	-860.9619	Durbin-Watson stat	1.990198

Estimation Command:
=====================
ARCH RDJM C

Estimation Equation:
=====================
RDJM = C(1)

Substituted Coefficients:
=====================
RDJM = 0.9025879876





Dependent Variable: RSPM
Method: ML - ARCH (Marquardt)
Date: 04/26/04 Time: 16:09
Sample(adjusted): 1979:02 2004:01
Included observations: 300 after adjusting endpoints
Convergence achieved after 23 iterations
Variance backcast: ON

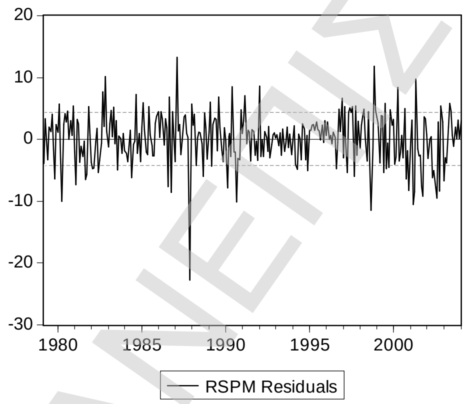
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.896568	0.215841	4.153845	0.0000
		Variance Equation		
C	0.472939	0.367377	1.287337	0.1980
ARCH(1)	0.088983	0.037161	2.394535	0.0166
GARCH(1)	0.791873	0.039289	22.70054	0.0000

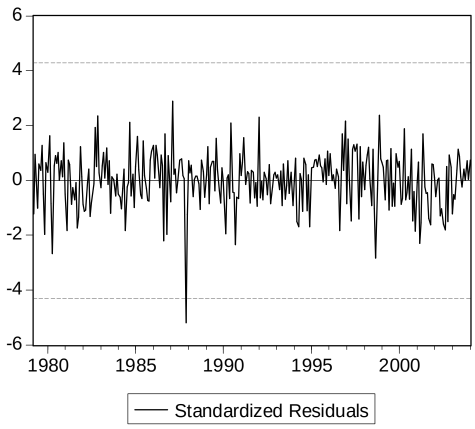
R-squared	-0.000012	Mean dependent var	0.911189
Adjusted R-squared	-0.010147	S.D. dependent var	4.262545
S.E. of regression	4.284117	Akaike info criterion	5.713813
Sum squared resid	5432.682	Schwarz criterion	5.763197
Log likelihood	-853.0719	Durbin-Watson stat	1.954979

Estimation Command:
=====================
ARCH RSPM C

Estimation Equation:
=====================
RSPM = C(1)

Substituted Coefficients:
=====================
RSPM = 0.89656841





The results of the analysis of the real data are presented in the next paragraph.

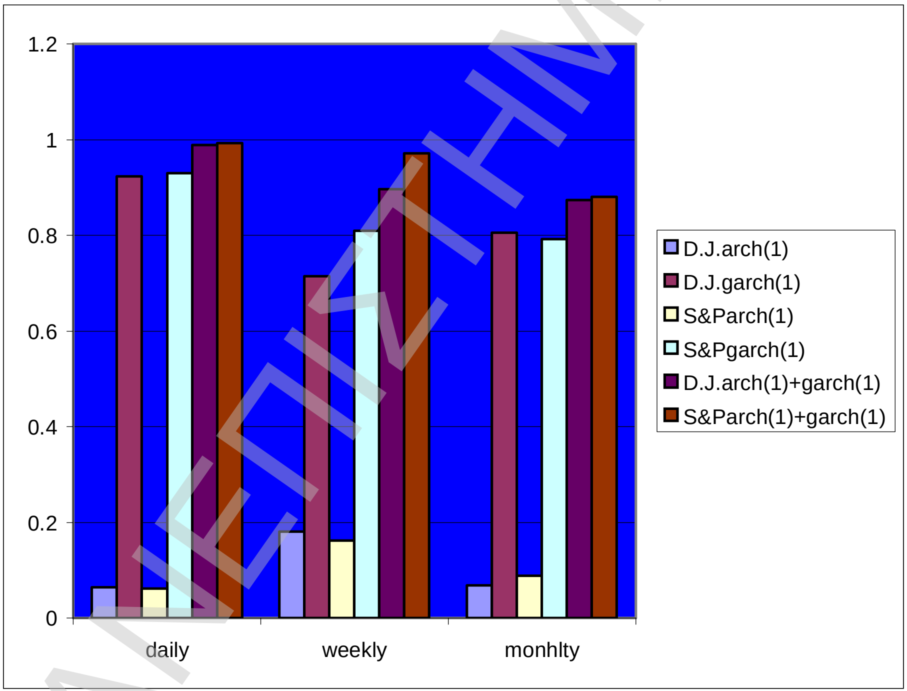
2.3 Results of the Econometric Analysis

The final results are presented in the next table:

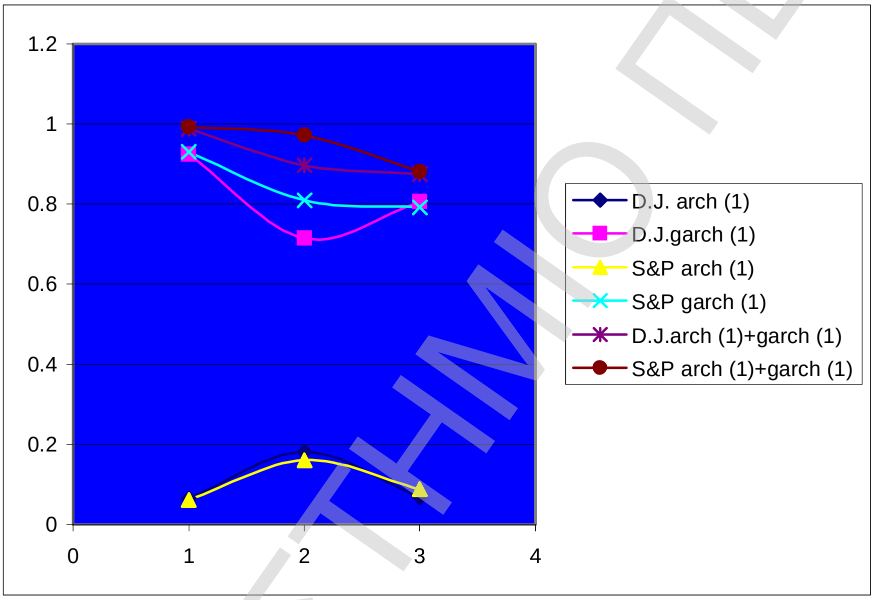
DOW JONES	D.J. arch (1)	D.J.garch (1)	D.J.arch (1)+garch (1)
Daily	0.065156	0.923652	0.988808
Weekly	0.181574	0.714808	0.896382
Monthly	0.068681	0.805582	0.874263

S&P	S&P arch (1)	S&P garch (1)	S&P arch (1)+garch (1)
Daily	0.062214	0.930327	0.992541
Weekly	0.161614	0.809945	0.971559
Monthly	0.088983	0.791873	0.880856

As we can see volatility persistence lowers as the data frequency lowers. This is something we expected to happen. It is attributed to a number of reasons, which will be analysed in the third part of this paper.



In this figure it is clear that the sum of α and β parameters if falling as the data frequency becomes lower. The sum of the two parameters is presented with the purple line for the Dow Jones time series and with the brown line for the Standard and Poor time series. From this analysis of the real data we will obtain the initial values for the Monte Carlo simulation which will be performed in the next part of the paper.



Part 3

3.1 Monte Carlo Simulation Program

The Monte Carlo Simulation program is written in EViews 4.0, a forecasting and analysis package for Windows-based computers and is analytically presented in appendix B at the end of this paper.

The program is analytically presented with extended comments between each step. The key points of the process are the following:

The purpose of this simulation is to create data and more precisely simulated stock returns that have some characteristics that resemble with those of the real world. These stock returns are created by a variance equation that is GARCH (1,1) and a mean equation that is a standard term. The model that was used is the following:

*h(!i)=!g+!b*h(!i-1)+!a*t(!i-1)^2+e(!i)*
y(!i)=!c

The initial values that are needed for this process are taken from the analysis that took place earlier. The data that is created is supposed to be the daily stock returns. The weekly, bi-weekly and monthly stock returns are taken from the daily stock returns series and are series in descending frequency. The program estimates the variance equation 100 times for each frequency and calculates the average of the repetitions and presents it as the direct estimation of each estimator. Then the program estimates the parameters not directly from the generated data but using the equations for the temporal aggregation with the daily data. At the end of the simulation a table is created where you can find two different estimations for each parameter. The first estimation is calculated by the correspondent frequency data and the latter using only high frequency (daily) data.

3.2 Results of the Monte – Carlo Simulation

These are the results of the Monte – Carlo simulation program. The average estimated parameters are those that were estimated directly from the correspondent data frequency. For example the daily average estimated parameters were estimated from daily data, the weekly parameters from weekly data etc. In fact the so-called daily data is nothing more than a high frequency data series. The weekly data is a series with lower frequency etc. I use this terminology (daily, weekly, bi-weekly etc.) for reason of better comprehension. The latter estimators which are called "formula" estimators are those that were estimated using the equations of temporal aggregation. More specifically the weekly, bi – weekly and monthly estimators were estimated from high frequency data (the so - called daily data). That 's why these estimators cannot estimate parameters for the daily frequency.

	Daily	**Weekly**	**Bi-weekly**	**Monthly**
Average estimated g	**0.7415078**	**0.7193845**	**0.8378596**	**0.7869354**
Average estimated b	**0.0631654**	**0.195431**	**0.0302438**	**0.0634974**
Average estimated a	**0.9168478**	**0.7614882**	**0.676073**	**0.944330**
g formula		**0.6330264**	**0.7867453**	**0.7908663**
b formula		**0.17503646**	**0.028047**	**0.074**
a formula		**0.782237**	**0.619**	**0.9022**

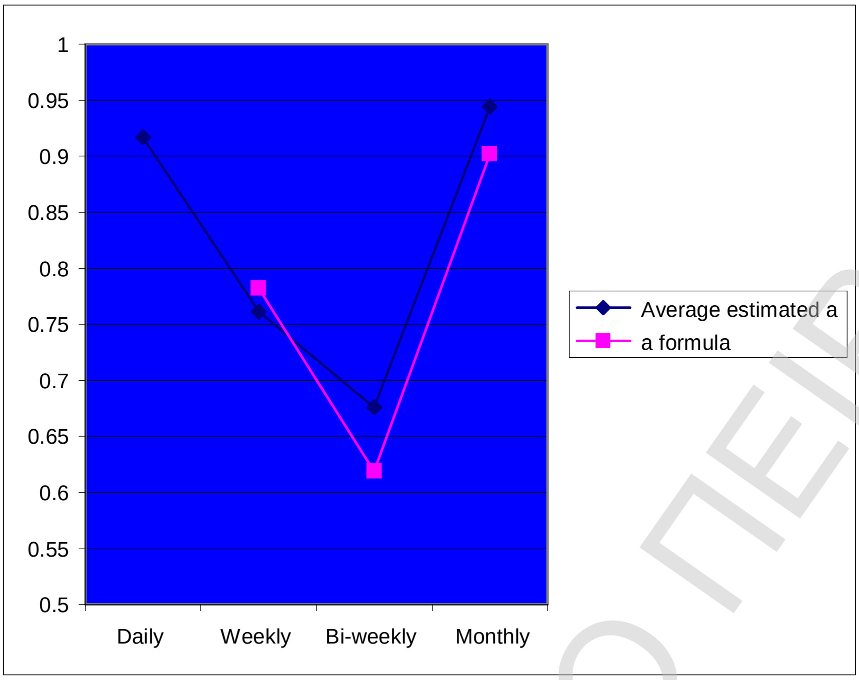


Fig. 1: Direct and implied estimations of parameter α.

In the above figure we observe the behavior of the direct (deep blue) and implied (pink) estimators for the parameter α. As we can see the two parameters are very close. In the weekly frequency the implied estimator (temporal aggregated estimator) is higher than the direct estimator. The direct estimator seems to be higher though in the bi-weekly and in the monthly frequency.

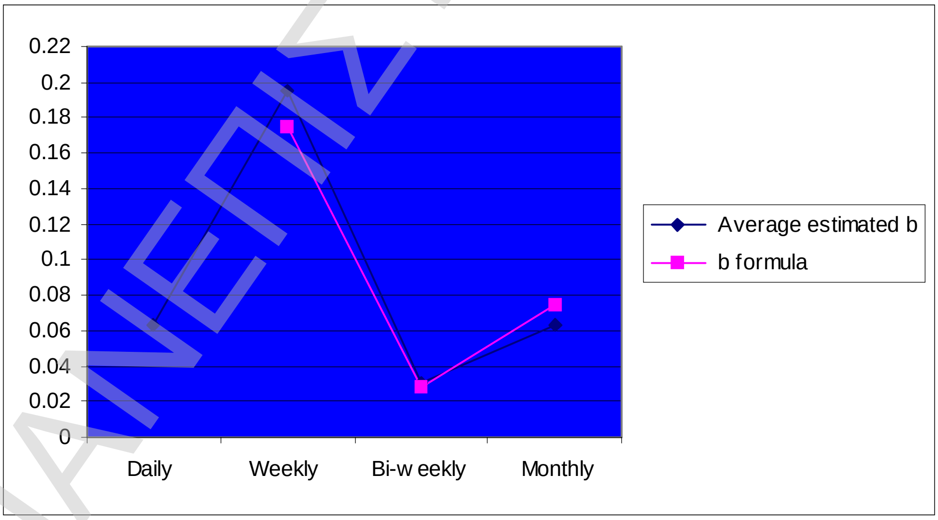


Fig. 2: Direct and implied estimations of parameter β.

In this figure we can see that the implied estimators (pink) are close to the direct estimators (deep blue). The implied estimators are calculated with high frequency data. The direct estimators are calculated with weekly, bi- weekly and monthly data.

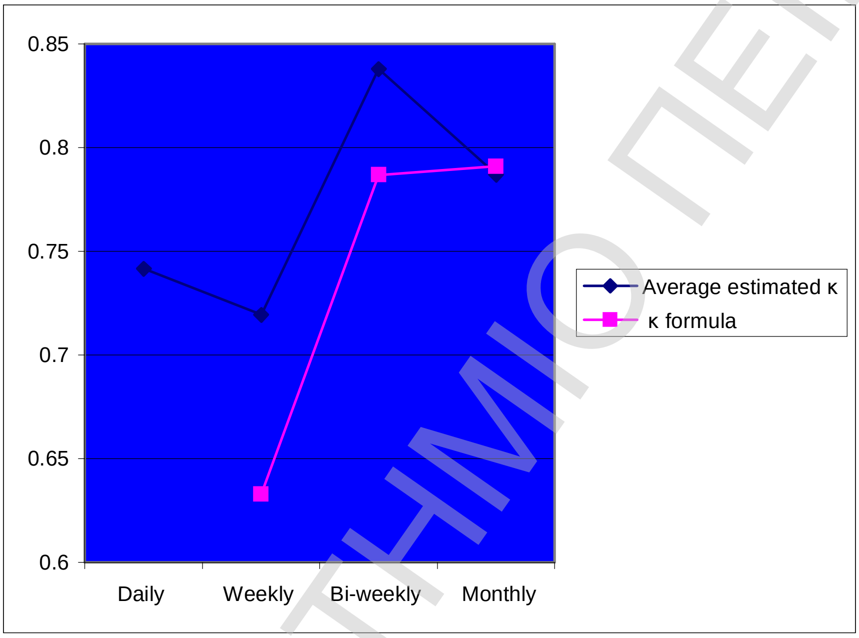


Fig. 3: Direct and implied estimations of parameter κ.

In the above figure we can see the behavior of the two different estimators for the parameter γ. The implied estimator (pink) that was estimated with high frequency data is lower than the direct estimators (deep blue) for the weekly and bi-weekly frequency. For the monthly data the two estimators seem to coincide.

The parameter κ (or g) is the kurtosis coefficient. This parameter is used for the calculation of parameter b (or β).

3.2 Results on the efficiency of the estimators for the parameters:

In this part we try to find out whether implied estimators based on high frequency data are more efficiency than direct estimators based on low frequency. Drost and Nijman suggest that implied estimators might be more efficient than the direct estimators. A gain of efficiency should then be caused by the fact that on higher frequency more data are available than on lower frequency and this is in fact a gain in efficiency.

We address this issue by calculating the bias between the true parameter and the implied one for each replication in the Monte – Carlo simulation. The moving average of the sum of the biases is the standard deviation of the parameter*. We calculated the standard deviation for each frequency (weekly, bi-weekly and monthly). As we can see temporal aggregation by itself leads to larger standard deviation when the frequency decreases. For the Dow Jones Composite Price Index parameter α_1 increases from 0.015 to 0.052 and from 0.025 to 0.058 for β_1. Temporal aggregation leads to larger (implied) standard deviation when the frequency decreases. This increase is more than offset by the gain from the use of the additional information. To verify these results we performed the Monte Carlo simulation. In our simulation we generated 100 time series for a simulated GARCH (1,1) process with a length of 5000 observations based on the direct parameter estimates for the Dow Jones at the daily frequency. Similar to the procedure for the stock return series we then directly estimated the parameters at four different frequencies, which we denote daily, weekly, bi-weekly and monthly. Based on these direct estimates we calculated the implied estimates at the lower frequencies.

---

*We calculated the differences: $\bar{a}_{implied} - a_{true}$ and $\bar{a}_{direct} - a_{true}$ for parameter α and $\bar{b}_{implied} - b_{true}$ and $\bar{b}_{direct} - b_{true}$.

In the next figure we plotted the direct estimates of α_1 and β_1 at the daily frequency (the cloud at the right) and the implied estimates at the lower frequencies (the cloud on the left shows the monthly implied estimates by the daily direct estimates). This plot shows some interesting results. The direct estimates at the daily frequency for α_1 tend to be slightly higher than the true parameter α_1. The average of the direct estimates equals 0.105 with a standard deviation of 0.046. Even though the true parameter lies within 1.96 standard deviations of the estimates, the direct estimates for α1 seem marginally biased. The direct estimate of β_1 does not seem to exhibit any bias. The average estimate for this parameter almost equals the true parameter value (0.879 and 0.878 respectively). The standard error for β1 equals 0.0069.

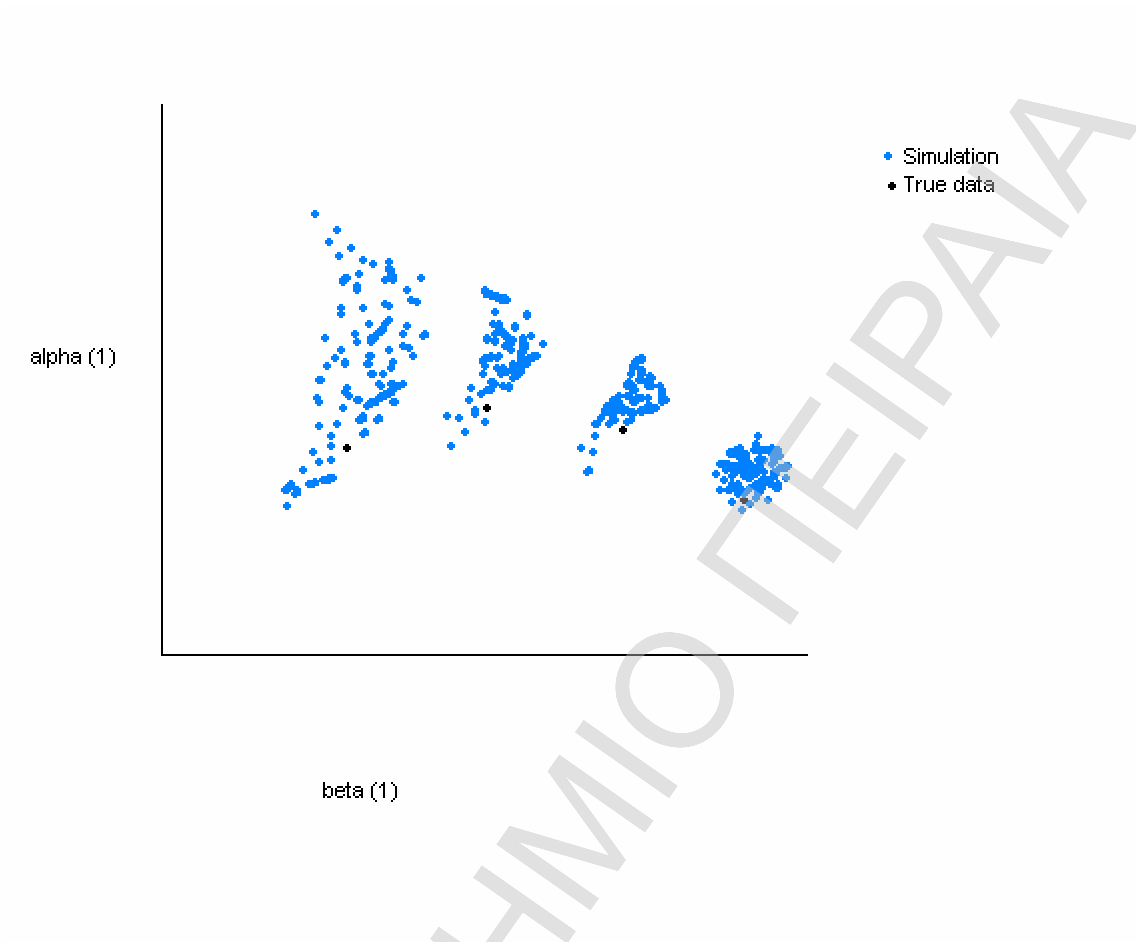
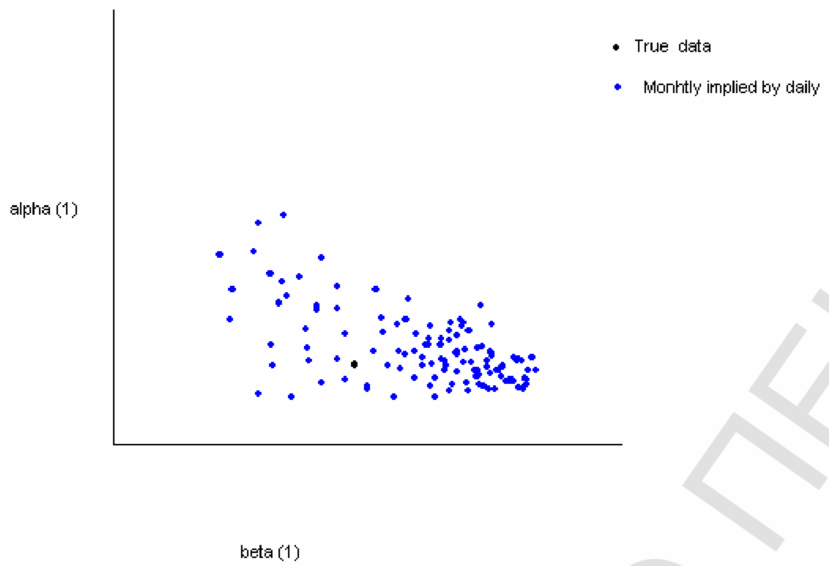


Fig. 1: Temporal aggregation results of 100 simulated time series. Most right cloud is the direct estimates at the daily frequency, moving to the left we find the implied weekly estimates, implied bi weekly estimates and implied monthly estimates (all implied by direct estimates at the daily frequency)

The increase in the standard errors caused by the temporal aggregation is smaller than the decrease in efficiency due to increase in the number of observations when we derive the estimates of the lower frequency from the high frequency estimates. This can be seen in the next figure.



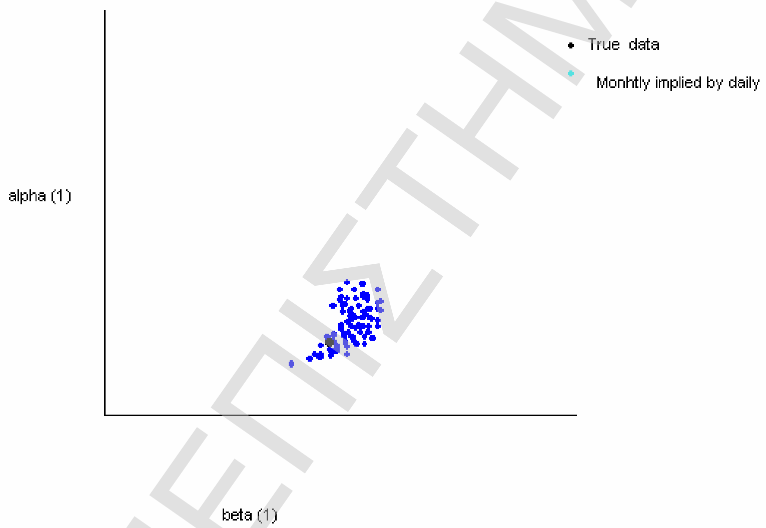


Fig 2: Monthly estimates: direct (1st figure) and implied by daily estimates (2nd figure).

In the above figures we compare the monthly direct estimates with the implied estimates of the daily frequency. The implied estimates are much more dense. This confirms our earlier result that the implied estimators are more efficient than the direct ones.

In the next figure we consider the efficiency increase in estimators α_1 and β_1. We measure the efficiency increase by the empirical standard errors derived from the 100 generated series.

When we compare the standard errors of the direct estimators with those implied estimators we find that for α_1 the efficiency increases from 0.111 to 0.049. For β_1 the increase is even higher.

	Monthly parameter estimates are determined using:			
	Monthly direct estimates	Implied estimates based on direct bi-weekly estimates	Direct weekly estimates	Direct daily estimates
Standard Deviation for α_1	0.111	0.068	0.057	0.049
Standard Deviation for β_1	0.187	0.112	0.062	0.046

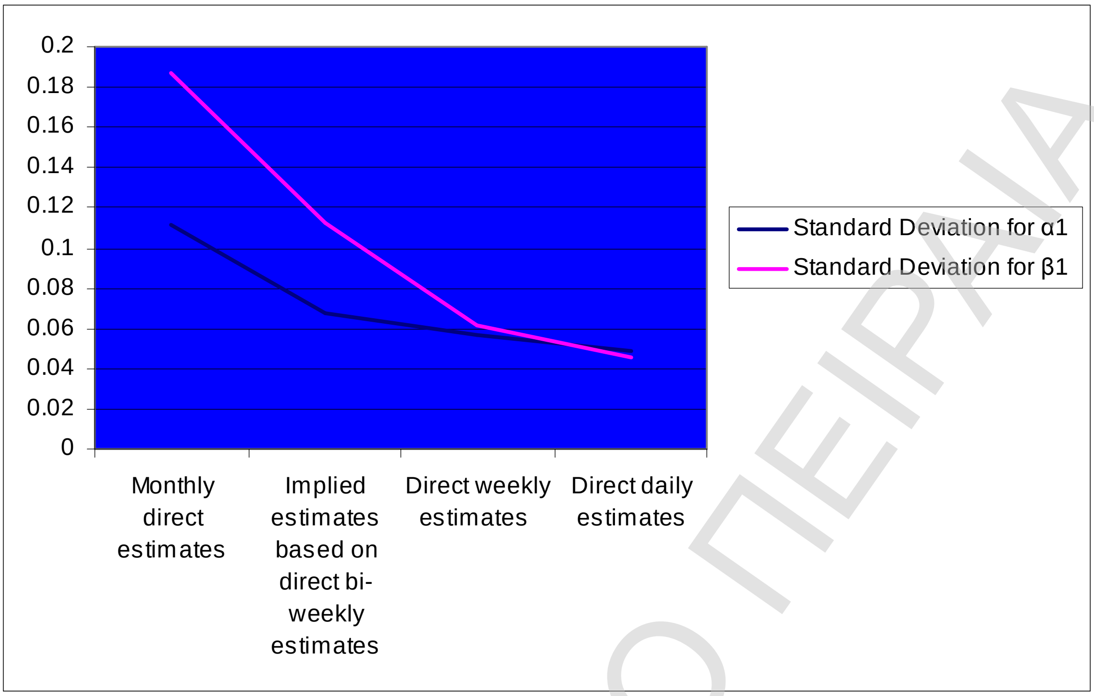


Fig. 3: Efficiency increase in estimators measured by standard deviation when monthly parameter estimates are determined using direct monthly estimates implied estimates based on direct bi-weekly estimates, direct weekly estimates and finally direct daily estimates.

There are plenty of other reasons why we make use of the implied estimators instead of the direct estimators, which are easier to be estimated.

Another reason why we prefer the implied estimations is that the structure of long time series, like Dow Jones' Price Index, is usually subject to radical changes that may set the econometric analysis difficult or even trivial.

A final reason why we prefer the implied estimators is mainly econometric. The implied estimators, as Drost and Nijman proved, are more efficient that the direct estimators since they incorporate more data.

The estimators, that where estimated through the temporal aggregation, are closer to the true parameters in comparison with the ones that were estimated directly from the correspondent data frequency. The use of daily data produced more effective estimators than the use of data of lower frequency.

Summarizing we find that **if one has the possibility to estimate low frequency models using high frequency data, implied estimation using temporal aggregation seems to be preferred to direct estimation using the data at the lower frequency.**

CONCLUSIONS

In this empirical analysis we proved that for long time series (like the Dow Jones Price Index) the homoskedasticity assumption can be rejected. We estimated low frequency parameters from high frequency data. The implied parameters are relatively more efficient from the directly estimated parameters. Our Monte Carlo simulation confirms that these implied estimations based on higher frequencies are more efficient than the direct estimators. Our results suggest that temporal aggregation is extremely useful to estimate low frequency GARCH models using high frequency data but that temporal aggregation also offers the possibility to reduce parameter estimation uncertainty.

Appendix A

The case of flow variables:

In their paper Drost and Nijman showed that the class of ARMA models with weak GARCH errors is closed under temporal aggregation. Weak GARCH is defined as a GARCH model in which the projections of the conditional variance are considered. Then, the following conditions hold:

$$LP\ [\ \varepsilon_t / E_{t-1}\ ] = 0$$

$$LP\ [\ \varepsilon^2_t / E_{t-1}\ ] = h_t$$

Here LP $[\varepsilon_t / E_{t-1}] = 0$ and LP $[\varepsilon 2t / E_{t-1}] = h_t$ are the best linear projections of ε_t and ε^2_t in terms of ε_{t-s} and ε^2_{t-s} and 1 for s≥1.

If the disturbances ε_t follow a weak GARCH (1,1) process with unconditional kurtosis κ_ε, then the aggregated disturbances $\varepsilon_t^{(m)}$ over m periods (t=1,2…) are also weak GARCH (1,1) distributed. In the case of flow variables upon aggregating it then follows:

$$e_t^m = \sum_{s=0}^{m-1} e_{tm-s}$$

For the aggregated series, the conditional variance is equal to:

$$h_t^{(m)} = a_0^{(m)} + a_1^{(m)} e_{(t-1)}^{(m)2} + b_1^{(m)} h_{(t-1)}^{(m)}$$

The corresponding parameters are equal to:

$$a_0^{(m)} = m a_0 \frac{1-(a_1 - b_1)^m}{1-(a_1 - b_1)}$$

$$a_1^{(m)} = (a_1 + b_1)^m - b_1^{(m)}$$

and $b_1^{(m)}$ is the solution of

$$\frac{b_1^{(m)}}{1+b_1^{(m)2}} = \frac{a(a_1, b_1, k_e, m)(a_1 + b_1)^m - b(a_1, b_1, m)}{a(a_1, b_1, k_e, m)(1+(a_1 + b_1)^{2m}) - 2b(a_1, b_1, m)}$$

Here the functions a and b are defined as follows:

$$a(a_1, b_1, k_e, m) = m(1-b_1)^2 + 2m(m-1)\frac{(1-a_1-b_1)^2(1-b_t^2-2a_1 b_1)}{(k_e-1)(1-(a_1+b_1)^2)} +$$

$$4\frac{(m-1-m(a_1+b_1)+(a_1+b_1)^m)(a_1-a_1 b_1(a_1+b_1))}{1-(a_1+b_1)^2}$$

$$b(a_1, b_1, m) = (a_1 - a_1 b_1(a_1+b_1))\frac{1-(a_1+b_1)^{2m}}{1-(a_1+b_1)^2}$$

The kurtosis of the aggregated series is equal to

$$k_t^{(m)} = 3 + \frac{k_e - 3}{m} + 6(k_e - 1) * \frac{(m - 1 - m(a_1 + b_1) + (a_1 + b_1)^m)(a_1 - a_1 b_1 (a_1 + b_1(a_1 + b_1))}{m^2 (1 - a_1 - b_1)^2 (1 - b_1^2 - 2a_1 b_1)}$$

Appendix B

(1) I define the size of the sample.

```
' SIZE OF THE SAMPLE'
!size=5000
```

(2) I define the size of the replications.

```
' NUMBER OF REPLICATIONS'
!rep=100
```

(3) I define the result file whose name is GARCH with the size defined above. Then I define the initial values of the coefficients as estimated from daily stock returns.

```
'RESULT FILE'
create garch u 1  !size
'DEFINITION OF COEFS'

' initial values as estimated from daily stock returns with a GARCH(1,1) , AR(1) model'

!f=0.011429
!c=0.044197
!a=0.9258808
!g=0.01336
!b=0.063525

!j=5 'step in weekly data'
!l=10 ' step in bi-weekly data'
!m=20 ' step in monthly data'
```

(4) Then I proceed to the simulation generating processes for daily returns with conditional variance GARCH (1,1). The series y represents daily returns which is an AR (1) model. The series h represents the conditional variance, which is a GARCH (1,1). The yw series represents weekly returns, the ybm represents biweekly series and the ym represents the monthly returns series.

```
' SIMULATION STEP 1  - Generate processes for daily returns and conditional variance GARCH (1,1)'
series y
series h

series yw

series ybw
series ym
```

The series e is a pseudo random draw from a uniform distribution (0,1). The series t generates pseudo random draws from a normal distribution with zero mean and unit variance.

```
series e=rnd
series t=nrnd


for !i=2 to !size
```

t(1), h(1) and y(1) are the initial values for the garch(1,1) model. The h(!i)is the variance equation and the y(!i) is the daily returns equation.

```
t(1)=0.5
h(1)=0.3
h(!i)=!g+!b*h(!i-1)+!a*t(!i-1)^2+e(!i)
y(1)=0.5
y(!i)=!c+!f*y(!i-1)
next
```

(5) Then I create aggregated processes for weekly, bi-weekly and monthly returns. For the weekly frequency I will choose the fifth, the tenth, the fifteenth value etc, for the bi-weekly frequency I will choose the tenth, the twentieth, the thirtieth value etc

and for the monthly frequency I will choose the twentieth, the fortieth, the sixtieth values etc.

```
' SIMULATION STEP 2 - Generate processes for weekly, bi-weekly and monthly returns'

for !i=1 to (!size-1)/!j

'step= the number of days that we aggregate stock returns- for a week , step=5'
series yw

yw(1)=y(1)
yw(!i+1)=y(!i+1+4*!i)
next

for !i=2 to (!size-1)/!l


'step= the number of days that we aggregate  stock returns- for 2 weeks , step=10'

ybw(1)=yw(1)
ybw(!i)=yw(2*!i-1)


next

for !i=2 to (!size-1)/!m

'step= the number of days that we aggregate  stock returns- for month , step=20'

ym(1)=ybw(1)
ym(!i)=ybw(2*!i-1)
next
```

(6) Then I estimate the variance equations for daily, weekly, bi-weekly and monthly return which follow a GARCH (1,1). The simulation will be replicated 100 times. I create matrices for the results.

' SIMULATION STEP 3 - Estimate variance equations for daily, weekly, bi-weekly and monthly returns - GARCH (1,1)- repeat simulation n times'

```
matrix(!rep , 3) bd
matrix(!rep , 3) bw
matrix(!rep , 3) bbw
matrix(!rep , 3) bm

for !n=1 to !rep
equation arcy!n.arch(1,1)   y(!n)
equation arcyw!n.arch(1,1) yw(!n)
equation arcybw!n.arch(1,1) ybw(!n)
equation arcym!n.arch(1,1) ym(!n)

freeze(resultsd!n) arcy!n
freeze(resultsw!n) arcyw!n
freeze(resultsbw!n) arcybw!n
freeze(resultsm!n) arcym!n

bd(!n,1)=resultsd!n(13,2)
bd(!n,2)=resultsd!n(14,2)
bd(!n,3)=resultsd!n(15,2)

bw(!n,1)=resultsw!n(13,2)
bw(!n,2)=resultsw!n(14,2)
bw(!n,3)=resultsw!n(15,2)

bbw(!n,1)=resultsbw!n(13,2)
bbw(!n,2)=resultsbw!n(14,2)
bbw(!n,3)=resultsbw!n(15,2)

bm(!n,1)=resultsm!n(13,2)
bm(!n,2)=resultsm!n(14,2)



bm(!n,3)=resultsm!n(15,2)
next

for !j=1 to 3

vector akd!j=@columnextract(bd,!j)
```

```
vector akw!j=@columnextract(bw,!j)
vector akbw!j=@columnextract(bbw,!j)
vector akm!j=@columnextract(bm,!j)
next

for !h=1 to 3
series akfd!h
series akfw!h
series akfbw!h
series akfm!h
next
for !p=1 to 3
for !k=1 to !rep
akfd!p(!k)=akd!p(!k)
akfw!p(!k)=akw!p(!k)
akfbw!p(!k)=akbw!p(!k)
akfm!p(!k)=akm!p(!k)
next
next
```

(7) In this step I will compute the mean of the estimated variance equations, which were estimated in the previous step. The mean will be placed on a matrix.

```
' SIMULATION STEP 4 - From n-times estimated variance equations find the mean of the empirical distribution of estimated parameters'
scalar gdsim2
scalar bdsim2
scalar adsim2
gdsim2=@mean(akfd1)
bdsim2=@mean(akfd2)
adsim2=@mean(akfd3)


!gdsim=gdsim2
!bdsim=bdsim2
!adsim=adsim2
```

(8) Then I will solve the quadratic equation as presented in Drost-Nijman paper for temporal aggregation.

' SIMULATION STEP 5 - Solve the quadratic equation for aggregate b parameter for weekly, bi-weekly and monthly and calculate coefficients from formulas'

```
scalar cw
scalar cbw
scalar cm


cw=(!bdsim*((!bdsim+!adsim)^(!j-1)))/(1+((!adsim^2)*(1-(!bdsim+!adsim)^(2*!j-2))/(1-(!bdsim+!adsim)^2))+(!bdsim)^2*(!bdsim+!adsim)^(2*!j-2))
!cw=cw

scalar dcw

dcw=((-1)^2)-4*!cw*!cw

!dcw=dcw

scalar estbw1
scalar estbw2
estbw1=(1-@sqrt(!dcw))/2*!cw
estbw2=(1+@sqrt(!dcw))/2*!cw

cbw=(!bdsim*((!bdsim+!adsim)^(!l-1)))/(1+((!adsim^2)*(1-(!bdsim+!adsim)^(2*!l-2))/(1-(!bdsim+!adsim)^2))+(!bdsim)^2*(!bdsim+!adsim)^(2*!l-2))
!cbw=cbw

scalar dcbw

dcbw=((-1)^2)-4*!cbw*!cbw

!dcbw=dcbw

scalar estbbw1
scalar estbbw2
estbbw1=(1-@sqrt(!dcbw))/2*!cbw
estbbw2=(1+@sqrt(!dcbw))/2*!cbw
```

```
cm=(!bdsim*((!bdsim+!adsim)^(!m-1)))/(1+((!adsim^2)*(1-
(!bdsim+!adsim)^(2*!m-2))/(1-
(!bdsim+!adsim)^2))+(!bdsim)^2*(!bdsim+!adsim)^(2*!m-2))
!cm=cm

scalar dcm




dcm=((-1)^2)-4*!cm*!cm

!dcm=dcm

scalar estbm1
scalar estbm2
estbm1=(1-@sqrt(!dcm))/2*!cm
estbm2=(1+@sqrt(!dcbw))/2*!cm
```

(9) The model is run and the results are presented in a table.

```
' FINAL STEP - Show results'

table(18,6) finalresults
finalresults(2,3)="weekly "
finalresults(2,4)="bi-weekly "
finalresults(2,5)=" monthly"
finalresults(3,1)="average estimated g "
finalresults(4,1)=" average estimated b"
finalresults(5,1)=" average estimated a"
finalresults(7,1)=" g formula "
finalresults(8,1)=" b formula"
finalresults(9,1)=" a formula"


finalresults(11,1)=" estimated gd"
finalresults(12,1)=" estimated bd  "
finalresults(13,1)=" estimated ad "

finalresults(15,1)=" initial g"
finalresults(16,1)=" initial b "
```

```
finalresults(17,1)=" initial a"


finalresults(3,3)=@mean(akfw1)
finalresults(4,3)=@mean(akfw2)
finalresults(5,3)=@mean(akfw3)

finalresults(3,4)=@mean(akfbw1)
finalresults(4,4)=@mean(akfbw2)
finalresults(5,4)=@mean(akfbw3)

finalresults(3,5)=@mean(akfm1)
finalresults(4,5)=@mean(akfm2)
finalresults(5,5)=@mean(akfm3)



finalresults(11,3)=@mean(akfd1)
finalresults(12,3)=@mean(akfd2)
finalresults(13,3)=@mean(akfd3)


finalresults (7,3)=!gdsim*(1-(!bdsim+!adsim)^(!j))/(1-(!bdsim+!adsim))
finalresults (7,4)=!gdsim*(1-(!bdsim+!adsim)^(!l))/(1-(!bdsim+!adsim))
finalresults (7,5)=!gdsim*(1-(!bdsim+!adsim)^(!m))/(1-(!bdsim+!adsim))

finalresults (8,3)=estbw2
finalresults (8,4)=estbbw2
finalresults (8,5)=estbm2

finalresults (9,3)=(!adsim+!bdsim)^(!j)-estbw2
finalresults (9,4)=(!adsim+!bdsim)^(!l)-estbbw2
finalresults (9,5)=(!adsim+!bdsim)^(!m)-estbm2


finalresults (15,3)=!g*(1-(!b+!a)^(!j))/(1-(!b+!a))
finalresults (15,4)=!g*(1-(!b+!a)^(!l))/(1-(!b+!a))
finalresults (15,5)=!g*(1-(!b+!a)^(!m))/(1-(!b+!a))

finalresults (17,3)=(!a+!b)^(!j)-estbw2
finalresults (17,4)=(!a+!b)^(!l)-estbbw2
```

finalresults (17,5)=(!a+!b)^(!m)-estbm2

Show final results

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Volatility Clustering in Monthly Stock Returns And Temporal Aggregation of Simulated Data.

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University of Piraeus

M.Sc. in Financial Management and Banking

July 2004

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Σκοπός της εργασίας είναι η μελέτη της δεσμευμένης διακύμανσης των αποδόσεων των χρηματιστηριακών μετοχών για διάφορες συχνότητες δεδομένων. Επίσης η κατασκευή μοντέλου χαμηλής συχνότητας με χρήση δεδομένων υψηλής συχνότητας.

Εισαγωγικοί Ορισμοί:

•Διακύμανση είναι η μεταβλητότητα των τιμών των μετοχών σε μία περίοδο χρόνου.

•Η διακύμανση είναι μέτρου κινδύνου και διαχρονικά μεταβάλλεται.

Σε αυτήν την εργασία όταν αναφερόμαστε σε :

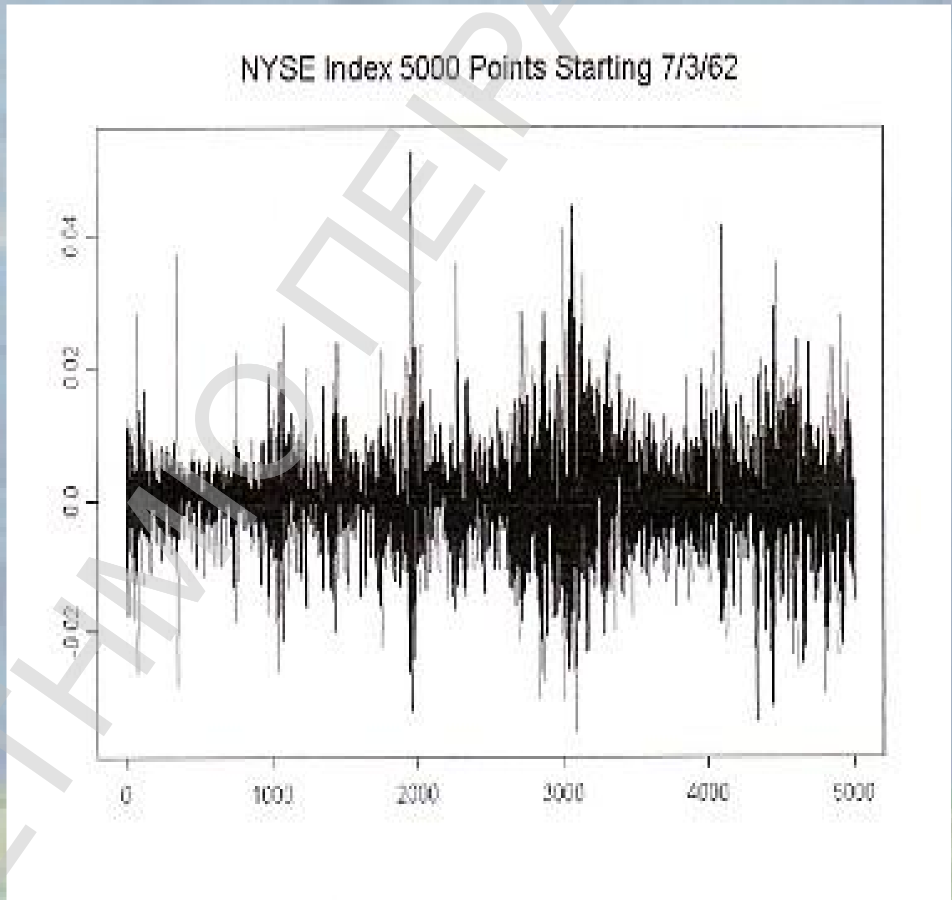
•<u>*High frequency data*</u> μιλάμε για <u>*daily returns*</u>

•<u>*Low frequency data*</u> για <u>*weekly*</u> , <u>*bi-weekly*</u> , <u>*monthly returns*</u>.

- Stylized facts about volatility:

1) Η κατανομές των αποδόσεων είναι λεπτόκυρτη.
2) Οι αποδόσεις των μετοχών επιδεικνύουν το λεγόμενο volatility clustering.
3) Οι αποδόσεις έχουν τάση για γρήγορη επιστροφή στο μέσο (mean reversion).
4) Η μεταβλητότητα που παρατηρείται ως συνέπεια κακών ειδήσεων είναι πιο έντονη από αυτή που παρατηρείται ως συνέπεια καλών ειδήσεων.
5) Πριν από σημαντικά αλλά προβλέψιμα γεγονότα όπως η ανακοίνωση των ετήσιων αποτελεσμάτων η μεταβλητότητα αυξάνεται.
6) Η έξαρση της διακύμανσης σε μία αγορά επεκτείνεται και σε όλες τις παρεμφερείς της.

- Το volatility clustering εμφανίζεται ως διαδοχικές περίοδοι αυξημένης διακύμανσης στις αποδόσεις των μετοχών που ακολουθούνται από περιόδους σχετικά μειωμένης διακύμανσης.



Volatility Clustering in Monthly Stock Returns And Temporal Aggregation of Simulated Data.

Υποθέτοντας ότι οι αποδόσεις r_t ακολουθούν ένα στάσιμο ARMA (p,q) παίρνουμε το μοντέλο:

$$r_t = \mu_t + \varepsilon_t$$

$$\mu_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i r_{t-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

$$r_t, p, q \geq 0$$

Το γραμμικό **ARCH (q)** μοντέλο εμφανίσθηκε στην βιβλιογραφία το 1982 από τον Rob Engle. Το μοντέλο αυτό θεωρεί ότι η δεσμευμένη διακύμανση είναι γραμμικός συνδυασμός των παρελθόντων ε_t.

$$\varepsilon_t = \sigma_t \eta_t \quad , \quad \sigma_t^2 = a_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 = a_0 + a(L)\varepsilon_t^2$$

$$a_0 > 0 \quad , \quad a_i \geq 0 \quad , \quad i > 0$$

Στην γενική τους μορφή το **GARCH (p,q)** γράφεται ως εξής:

$\varepsilon_t = \sigma_t \eta_t$, $$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_l \sigma_{t-i}^2 = a_0 + a(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

$a_0 > 0$, $a_i \geq 0$, $\beta_i \geq 0$ και $\sum_{i=1}^{max(p,q)} (a_i + \beta_i) < 1$ ώστε να διασφαλιστεί ότι η αδέσμευτη διακύμανση υπάρχει και είναι πραγματικός αριθμός.

Temporal Aggregation : η διαδικασία δημιουργίας οικονομετρικών μοντέλων για μία συγκεκριμένη συχνότητα χρησιμοποιώντας δεδομένα διαφορετικής συχνότητας. Συνήθως με δεδομένα υψηλής συχνότητας κατασκευάζουμε χαμηλής συχνότητας μοντέλα.

Aggregation of GARCH (1,1):

$$y_t$$

$$h_t = y + bh_{t-1} + ay_{t-1}^2$$

$$y_{tm}$$

$$h_{(m)tm} = y_{(m)} + b_{(m)}h_{(m)tm-m} + a_{(m)}y_{tm-m}^2$$

Aggregation of GARCH (1,1):

$$\boxed{y_{(m)} = y\,\frac{1-(b+a)^{(m)}}{1-(b+a)},\ a_{(m)} = (b+a)^{m} - b_{(m)},}$$

$$b_{(m)} \in (0,1)$$

$$\frac{b_{(m)}}{1+b_{(m)}^{2}} = \frac{b(b+a)^{m-1}}{1+a^{2}\dfrac{1-(b+a)^{2m-2}}{1-(b+a)^{2}}+b^{2}(b+a)^{2m-2}}.$$

$$\boxed{y_{tm}^{2} = y_{(m)} + (b_{(m)} + a_{(m)})y_{tm-m}^{2} + h_{tm} - b_{(m)}h_{tm-m}}$$

Βάση δεδομένων : DATASTREAM

Χρονική διάρκεια δεδομένων: 1/1/1979-1/1/2004

Συχνότητα δεδομένων:Ημερήσια , Εβδομαδιαία και Μηνιαία

Περιγραφικά Στατιστικά Δεδομένων

	Daily Data	**Weekly Data**	**Monthly Data**
Number of observations	**6523**	**1304**	**300**
Mean	**0.044825**	**0.212353**	**0.954055**
Std. Dev.	**1.049377**	**2.341541**	**4.354195**

Υπολογισμός GARCH (1,1) για τις διάφορες συχνότητες δεδομένων:

DOW JONES	ARCH (1)	GARCH (1)	ARCH (1)+GARCH (1)
Daily	0.065156	0.923652	0.988808
Weekly	0.181574	0.714808	0.896382
Monthly	0.068681	0.805582	0.874263

Monte Carlo Simulation :

Το πρόγραμμα γράφτηκε στο οικονομετρικό πακέτο E-VIEWS

Το μοντέλο που χρησιμοποιήσαμε στην προσομοίωση μας ήταν το ακόλουθο:

h(!i)=!g+!b*h(!i-1)+!a*t(!i-1)^2+e(!i)
y(!i)=!c

Results of the Monte – Carlo Simulation

	Daily	Weekly	Bi-weekly	Monthly
Average estimated g	0.7415078	0.7193845	0.8378596	0.7869354
Average estimated b	0.0631654	0.195431	0.0302438	0.0634974
Average estimated a	0.9168478	0.7614882	0.676073	0.944330
g formula		0.6330264	0.7867453	0.7908663
b formula		0.17503646	0.028047	0.074
a formula		0.782237	0.619	0.9022

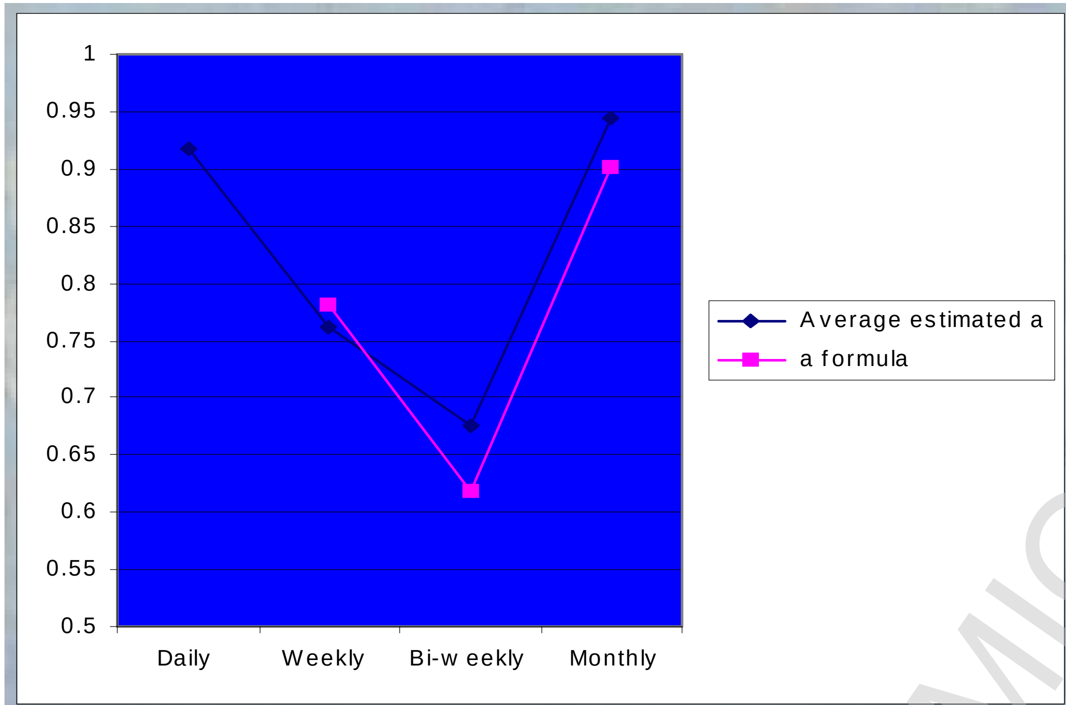
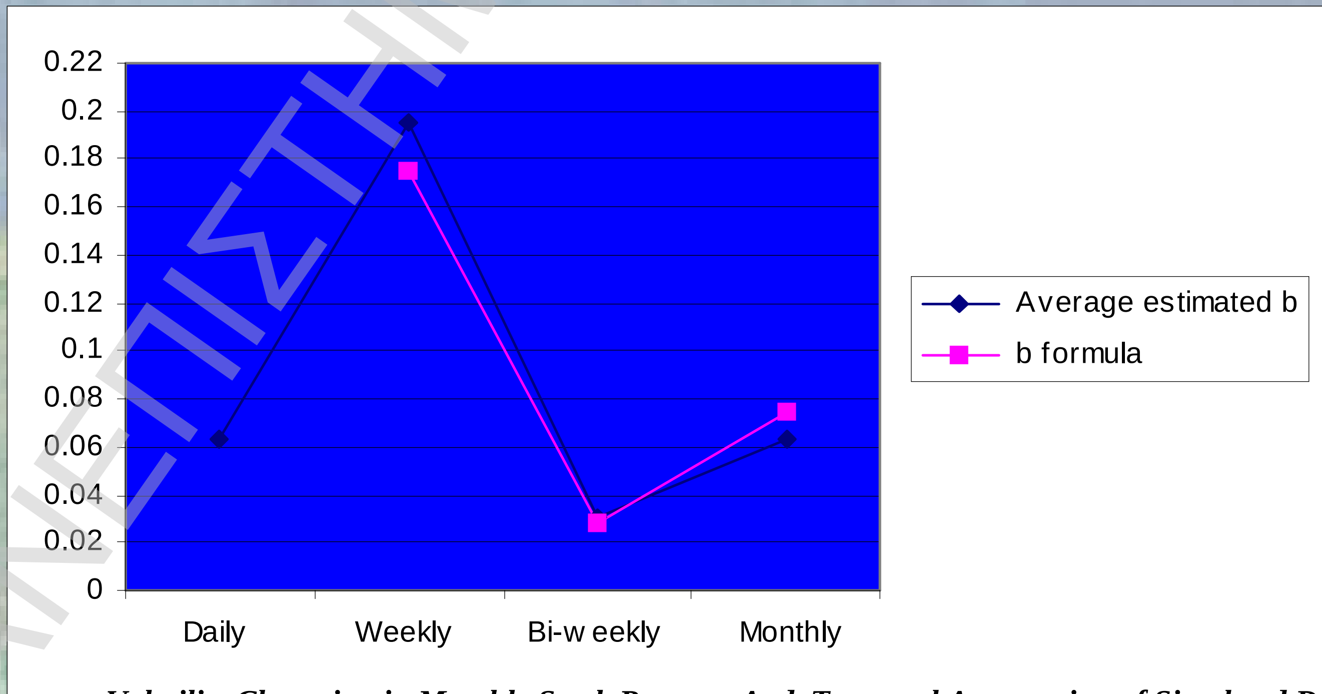


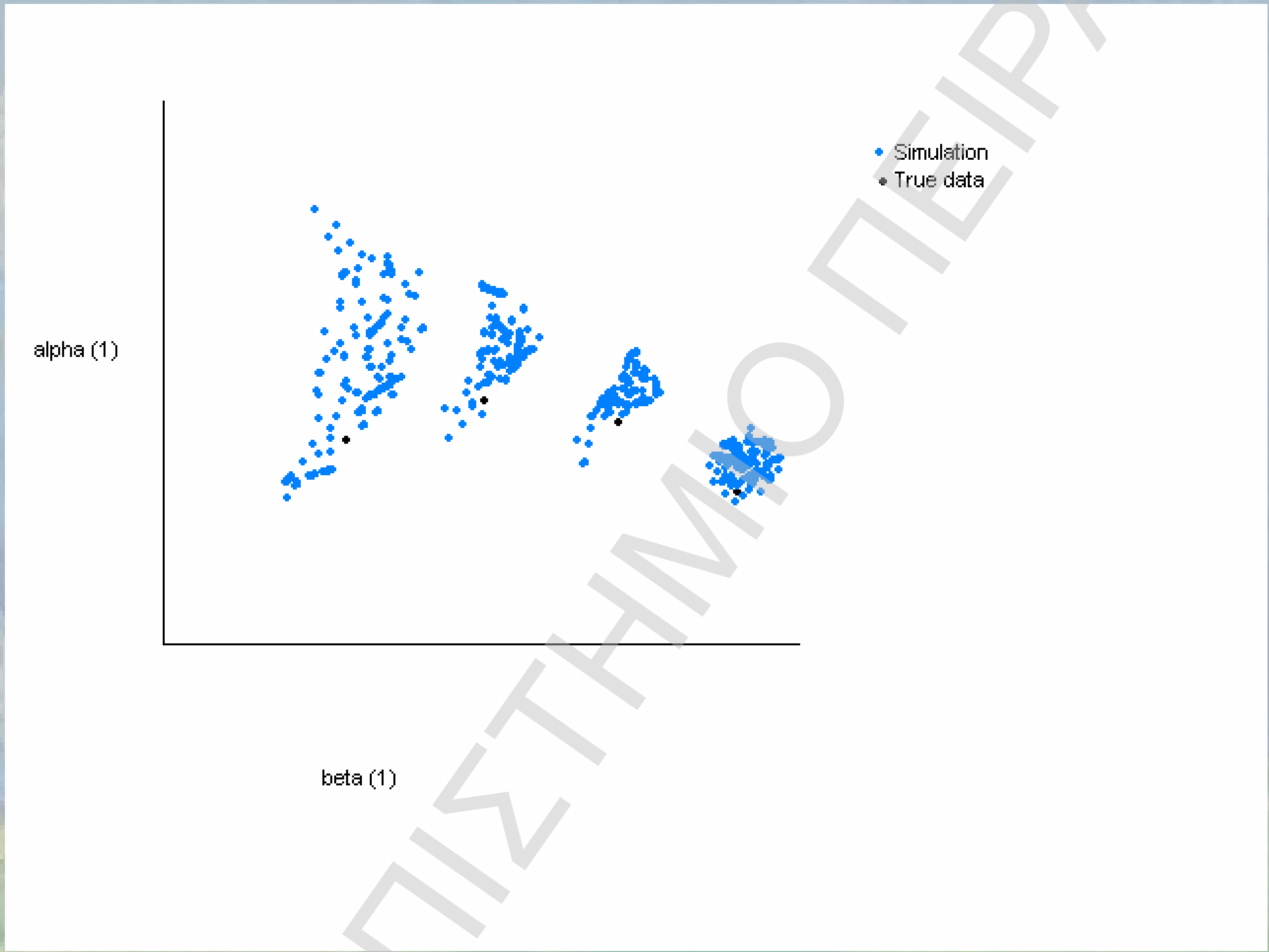
Fig. 1: Direct and implied estimations of parameter α.

Fig. 2: Direct and implied estimations of parameter β.

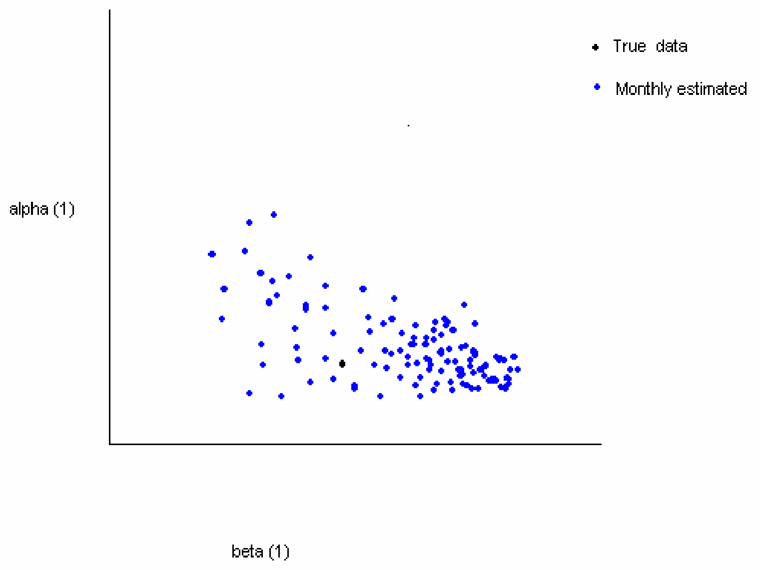


Γιατί να προτιμήσω τους implied estimators αντί των direct estimators?

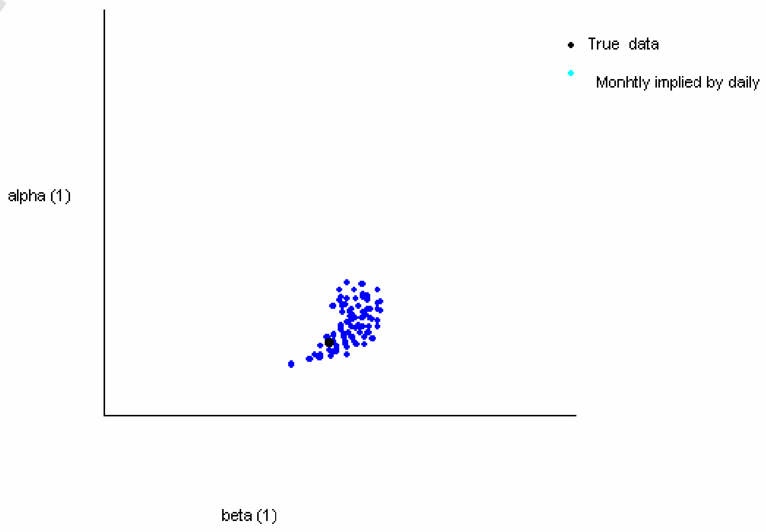
1) Γιατί οι implied estimators είναι πολύ efficient από τους direct.
2) Η δομή της χρονοσειράς όσο πιο πίσω πάω τόσο πιο επικίνδυνο είναι να έχει υποστεί η δομή της αλλοίωση και να είναι δύσκολη η μελέτη της.



Temporal aggregation results of 100 simulated time series.



Monthly estimates: direct (1st figure) and implied by daily estimates (2nd figure).



if one has the possibility to estimate low frequency models using high frequency data, implied estimation using temporal aggregation seems to be preferred to direct estimation using the data at the lower frequency.


Ευχαριστώ πολύ και
Καλό Καλοκαίρι
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