Ο Ρόλος της Ακίνητης Περιουσίας στην επιλογή του Βέλτιστου Χαρτοφυλακίου.

- Ίρις Παπαμαστοράκη

- Επιβλέπων Καθηγητής:
  Δ. Μαλλιαρόπουλος

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1. Introduction

Since the pioneering work of Markowitz (1952), portfolio theorists have examined ways of expanding the range of assets to be included in wealth portfolios. Early empirical studies examined only combinations of domestic equities. As technology and reliable data availability began to improve, the selection of assets to be considered in the construction of optimal portfolios expanded significantly. These expansions have included domestic, non-equity financial and real assets and international financial assets.

Given the worldwide boom of real estate prices and the worries of the financial community about the potential implications of the likely burst of a bubble in real estate markets, it is important to study this asset class.

The value of real estate in the United States is comparable to that of the aggregate U.S. stock market, yet real estate has played a minor role in the empirical asset pricing literature. The omission of this important component of aggregate wealth has generally been attributed to data or measurement problems. However, the data available today capture enough of the risks associated with holding real estate assets to be reflected in the cross-section of asset returns. This suggests that at this stage, there are essentially no reasons for excluding real estate assets from the empirical asset pricing and optimal portfolio literature.

Portfolio choice in the presence of housing is especially important because owner-occupied housing is the single most important asset in many investors' portfolios. According to the 1998 Survey of Consumer Finances, two thirds of U.S. households surveyed owned their primary residence. The aggregate value of residential real estate is correspondingly large, comprising 40% to 60% of the tangible wealth of the U.S. economy. Case (2000) estimates that it was about $11.6 trillion at the end of 1999, $8.9 trillion of which consisted of owner-occupied housing.

Another aspect underlining the importance of real estate is that the dramatic increase in stock and residential housing prices during the recent economic expansion in the U.S. has led to renewed policy and scientific interest in the effects of household wealth upon consumption levels. To the extent that wealth plays an important role in determining household consumption, there are reasons to fear that declining house prices may exacerbate a slowdown in the economy by depressing the consumption spending of households and, thus, aggregate demand.

2. Project Outline

The main issues we will address in this study are the following:

We will investigate the effect of real estate on the risk-return characteristics of optimal portfolios. For this purpose, we will investigate the change in the risk-return tradeoff by including real estate in optimal portfolios consisting of bonds, stocks and Fama&French Portfolios formed on Size and on Book-to-Market. In order to assess the diversification benefits for investors of including real estate in their portfolios, we will use a number of well-known portfolio performance and efficiency tests. These tests are known in the literature as mean-variance intersection and spanning tests.

It is well known that the solution of the optimal portfolio differs according to whether the investor is allowed to sell assets short. Hence, it is important to compute optimal portfolios for both cases of short selling and no short selling.

The analysis will also take into account the illiquidity characteristic of the real estate asset class. Thus, we will also compute for portfolios where the investor is able to buy and sell all assets constantly and portfolios for which the investor is restricted in the housing asset only. This restriction implies a hedging demand for both bonds and stocks, implying that the investor will choose to change his position in real estate in order to protect his investment from the risk the real estate restriction brings in.

Another interesting aspect, which we won’t address in this analysis, is following:

The basic portfolio choice model of Markowitz assumes that the investor is myopic in the sense that he is only interested in a one-period investment. However, real estate is an asset that contrary to financial assets and due to its nature cannot be treated easily as a short-term investment. Households usually treat housing wealth as an investment for lifetime, or, at least, a long-term investment. As a result, the portfolio choice of a household, which includes housing wealth in its assets, is an intertemporal optimization problem. In this type of problems, the solution of the optimal portfolio usually consists of two parts: the standard Markowitz solution (the universal hedge portfolio) plus the hedging demand component which adjusts the universal portfolio by taking into account the effect of a change in the investment opportunity set on the optimal composition of the portfolio. So, the upcoming question is whether there is a hedging demand component for stocks and bonds induced by real estate. A hedging demand for stocks and bonds can arise in this case if for example changes in mortgage rates, which affect the refinancing costs of real estate are correlated with changes in the value of the stock, or bond component in the portfolio. If this is the case, then the fact that the investor owns a house means that he will choose a different optimal portfolio of stocks and bonds than would be the case if he would rent a house. The reason is that by owning these financial assets, the investor can hedge the risk of a change in the refinancing costs of real estate.

The empirical analysis will focus on the U.S. market, since there are more reliable monthly data available on residential house prices, than for other countries.

3. Literature Review

Financial literature so far has dealt with the real estate asset class, asking whether it is indeed optimal to include a substantial amount of residential real estate in an investment portfolio besides holding stocks and bonds, domestically or internationally, if this inclusion results in statistically significant gains in portfolio performance, diversification gains and the improvement of the empirical performance of well-known asset pricing models.

Also the issue of mean-variance efficiency of household portfolios when housing wealth is considered is addressed, and the hypothesis that observed portfolios are mean variance efficient is tested.

In his analysis, *De Roon, et al (2002)* concludes that the mean price returns on real estate are generally too low in order for investors to include this asset class in their portfolio. So in order to include real estate, the non-price increase or consumption benefit must be significant (10% for a 50% investment in real estate).
He argues that residential real estate offers significant diversification benefits: the global minimum variance portfolio containing long positions in all asset classes, adding real estate to a portfolio of stocks and bonds can reduce the standard deviation of the GMV portfolio by at least 50%. This suggests that real estate should be a serious part of an investment portfolio due to the large diversification benefit that comes with it.
According to *Rubens et al. (1998),* allocating funds to real and international assets does not result in significant gains. Adding such assets may result in additional returns, but the addition may not improve the mean-variance portfolio performance.
Hoesli, et al. (2003) considering annual data pertaining to stocks, bonds, direct real estate, indirect real estate (i.e. real estate securities) and cash for the U.S., the U.K., France, the Netherlands, Sweden, Switzerland and Australia, found that with unhedged returns, the optimal weight which should be allocated to real estate in mixed-asset portfolios is in the 5-15% range, and that the inclusion of real estate assets in such portfolios leads to a 5-10% reduction in the portfolios' risk level.
When international real estate investments are also considered, the risk reduction is increased to 10-20%, and so is the weight, which should be devoted to real estate in diversified portfolios.
Results using hedged returns are remarkably similar: adding real estate in mixed-asset portfolios makes it possible to reduce a portfolios' risk by 10 to 20%, when the optimal allocation to real estate is in the 15-25% range.
Thus, although the benefits form including real estate in a portfolio are very similar across countries, the way of gaining exposure to real estate varies across countries: in some countries the allocations tilted towards either domestic or international assets, while it is balanced in others.
The positive role of real estate in diversifying a portfolio is demonstrated, but the ways of achieving this vary according to the correlation of assets within

each country, and to the strategy that is considered as pertains to the management of currency risk.

Kullmann's (2001) paper provides strong evidence that including proxies for the return to real estate improves the performance of different empirical specifications of the CAPM. The findings are in general robust to the choice of assets being priced as well as the inclusion of additional factors, like the Fama – French HML and SMB portfolio returns.

Despite difficulties in quantifying the risks and returns associated with holding real estate assets, Kullmann shows that existing data capture enough risks associated with owning real estate to be reflected in asset prices.

Adding returns to both residential and commercial real estate does not weaken and generally improves the fit of the models being tested. More importantly, the findings of Kullmann suggest that housing- risk is priced in the equity market over and above market risk and risk related to firm-specific factors, such as size and value characteristics of companies, so there are no valid reasons for excluding the real estate asset from the empirical asset pricing literature.

Flavin and Yamashita (1999) point out that in the context of a reasonably general model with adjustment costs, an optimizing consumer will hold an asset portfolio, which is mean-variance efficient, if the return to housing is uncorrelated with the returns to financial assets.

The paper estimates the risk and return to financial assets and residential real estate, and confirms the absence of correlation between housing returns and asset returns. The absence of correlation between housing returns and financial asset returns implies that the inclusion of housing as an asset dramatically improves the unconstrained mean-variance frontier and confirms the popular notion that homeownership is a good investment.

The model used, shows that households, which are identical, in that they have identical preferences toward risk and identical perceptions of the risk and returns to different assets, will nevertheless hold quite different portfolios of financial assets because each household is optimizing their portfolio subject to a constraint on housing, which varies across households.

Compared to the mean-variance frontier, which corresponds to portfolios consisting solely of financial assets (stocks, bonds, T-bills), the introduction of housing and mortgages alters the risk and return trade-off in a direction, which pushes most households onto a binding nonnegative constraint on t-bills.

Another question raised in financial literature is whether the investment in housing affects the composition of an investor's portfolio, if it significantly affects the demand for stocks and bonds or even if house price risk and the illiquid nature of the housing investment leads investors to reduce their exposure to stocks.

As in *De Roon et al (2002),* when minimizing the variance of the total investment portfolio, the composition of the financial asset portfolio, consitsting of stocks and bonds is only slightly affected by the investment in real estate. This suggests that the issue of home ownership is relatively unimportant when determining the composition of the stock and bond portfolio, but is very important in the entire investment portfolio.

Cocco (2000) found that investment in housing has important implications for asset accumulation and portfolio choice among stocks and T-bills. Contrary to *Kullmann (2001)* and *Flavin and Yamashita (1999)*, *Cocco* found that early in life, and at low levels of financial net worth, housing investment keeps liquid assets low and reduces the benefits of equity market participation. House price risk and transaction costs of adjusting the level of housing crowd out stockholdings. The model used also proposes an explanation as to why in the data leverage and stockholdings tend to be positively correlated. For investors with a more leveraged portfolio capitalized labor income induces a shift in portfolio composition towards stocks so that for many of the parameterizations considered leverage and stock holdings tend to be positively correlated.

One last issue addressed is whether there is a link between increases in housing wealth, financial wealth, and consumer spending.

Due to *Case, et al (2001)*, there is evidence that, variations in housing market wealth have important effects upon consumption. The housing market appears to be more important than the stock market in influencing consumption in developed countries.

On the other hand testing whether households in general keep efficient portfolios, Pelizzon and Weber (2002) suggest that when only financial assets are considered, at most 7.52% are mean-variance efficient among the diversified portfolios. Taking a broader set of assets and liabilities, such as housing, mortgages and debt, many more households turn out to hold diversified portfolios. But no diversified household portfolio is found to be efficient.

Calculating the Sharpe index conditional on housing shows that around a third of all diversified portfolios are mean variance efficient; if the portfolio composed by risky financial assets, the risk free asset and the house are considered.

Compared to the efficiency results, which corresponds to portfolios consisting solely of financial assets such as stocks, bonds and short-term government Bonds (BOT), the introduction of housing and mortgage alters the risk and return trade-off in a direction, which pushes a higher number of household portfolios to be efficient.

4. Data

The data used are divided into two sets as following:

i. The first data set comprises out of: Stocks (S&P 500 Composite), Bonds (U.S. Government Bond, 5 Year, Yield Average, US) and Fama&French Portfolios formed on Size and on Book-to-Market.

   This set of data is called the “reduced portfolio”.

ii. While the second data set contains all the above assets plus the housing asset and is called the “expanded portfolio”.

In our study we use monthly data from January 1975 through October 2003 for United States stocks, bonds, real estate and the Fama& French Portfolios formed on Size and on Book-to-Market.

These data were obtained from EcoWin database and the Fama & French data library. All assets' returns are computed. Asset classes are described in detail below.

§ **S&P 500 Composite**

Standard and Poor's 500 Index is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all major industries. The index was developed with a base level of 10 for the 1941- 43 base period.

§ **Portfolios formed on Size and on Book-to-Market.**

These portfolios were taken from the Fama – French Data Library.

We used:

Value-weighted returns for portfolios formed on Size and on BE/ME on a monthly rate from January 1975 to October 2003.

(BE/ME is book equity at the last fiscal year end of the prior calendar year divided by ME at the end of December of the prior year).

The break points use Compustat firms plus the firms hand-collected from the Moodys Industrial, Utilities, Transportation, and Financial Manuals.

The portfolios include utilities.

§ **U.S. Government Bond, 5 Year, Yield Average, US**

United States bonds are bonds that are issued by the Government of the United States of America. When a purchaser buys US bonds he is purchasing the debt of the US Government and is in fact loaning money to the US. These types of investments are considered the safest of all investments. There is over $3 trillion in US bonds, bills, and notes on the market. US bonds are very safe and predictable and the most liquid of the debt market.

**All data are shown in Appendix A*

"U.S. Treasury securities offer the highest degree of creditworthiness available. That's why U.S. Treasuries should be the foundation of any well-diversified investment portfolio."

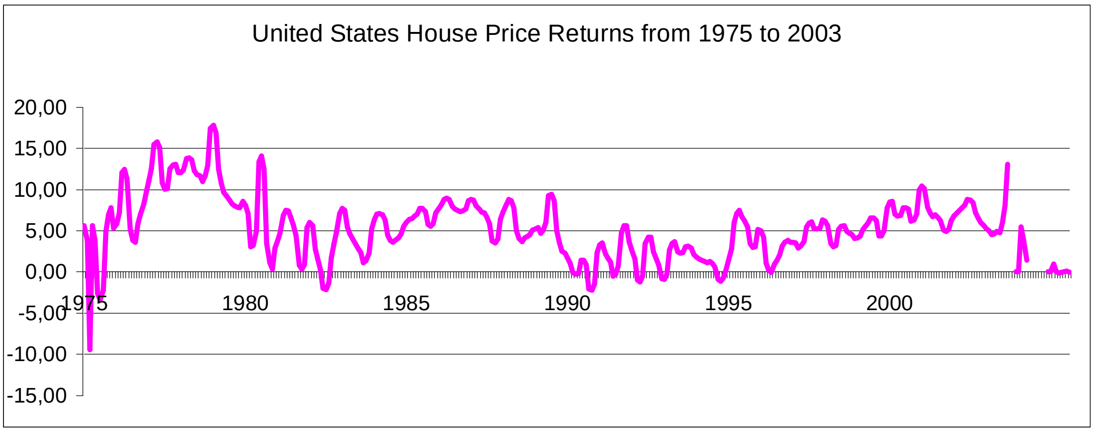
Merrill Lynch

§ **Housing**

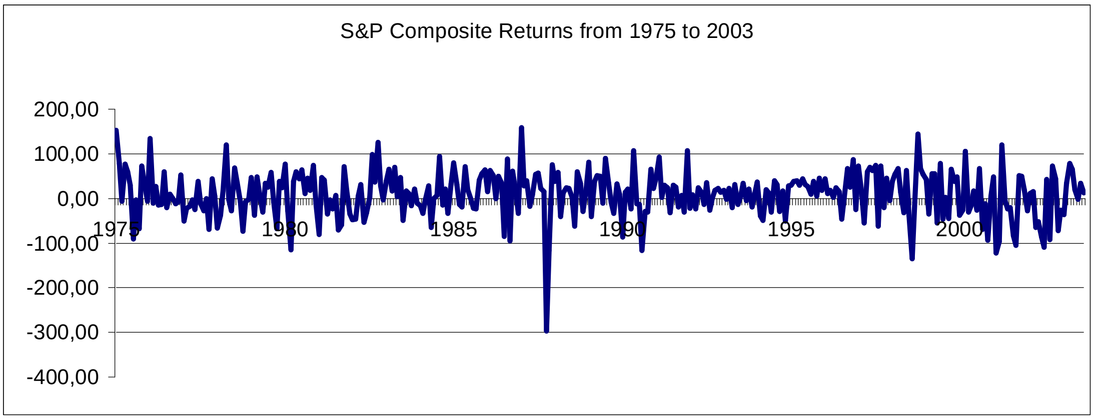
United States House Prices-National, from 1975 to 2003 taken from EcoWin.

4.1 A first look at the data

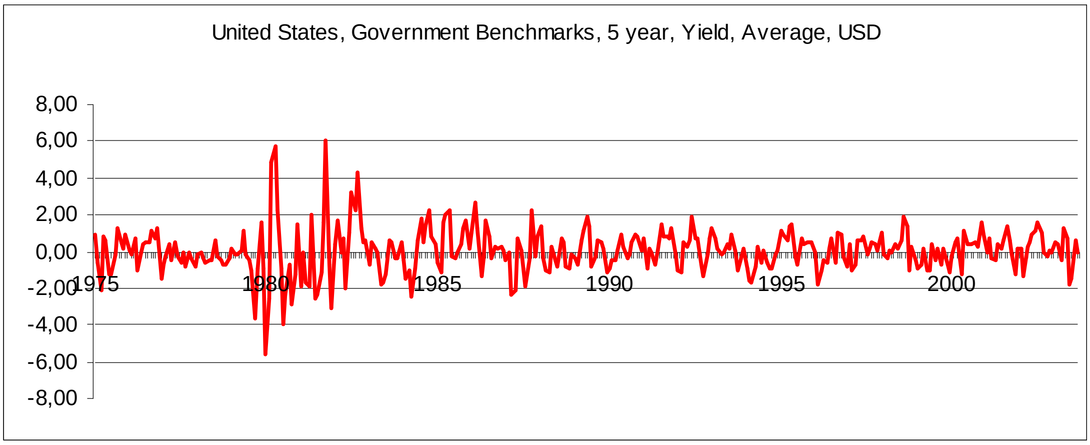
Following we can observe the data returns curve from 1975 through 2003, to get a first picture:



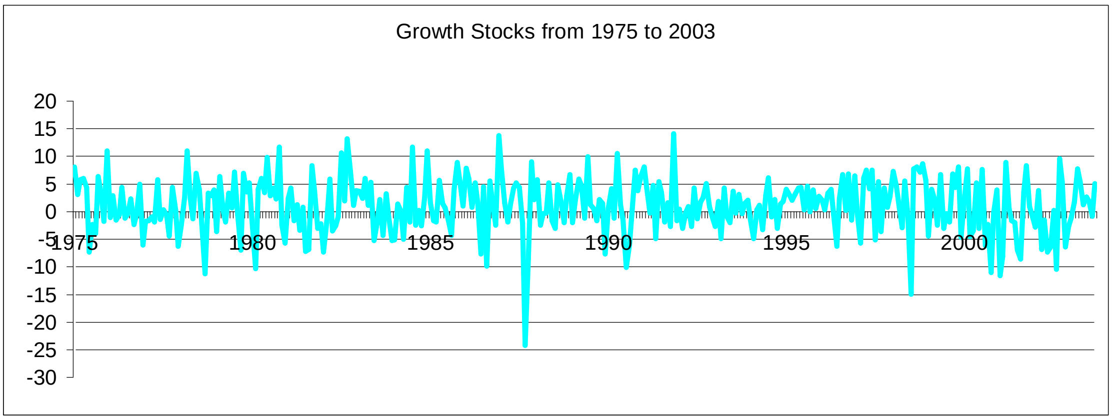
-Graph 1-



- Graph 2 –



- ***Graph 3 –***



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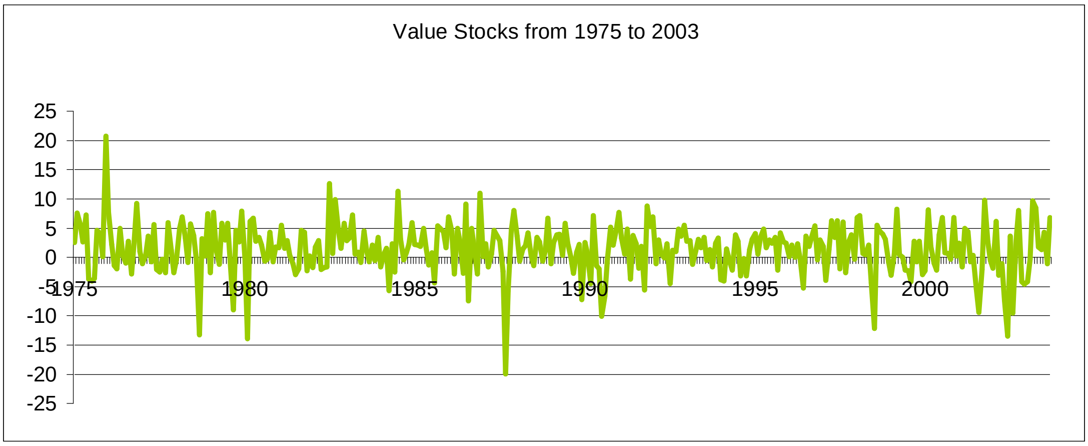
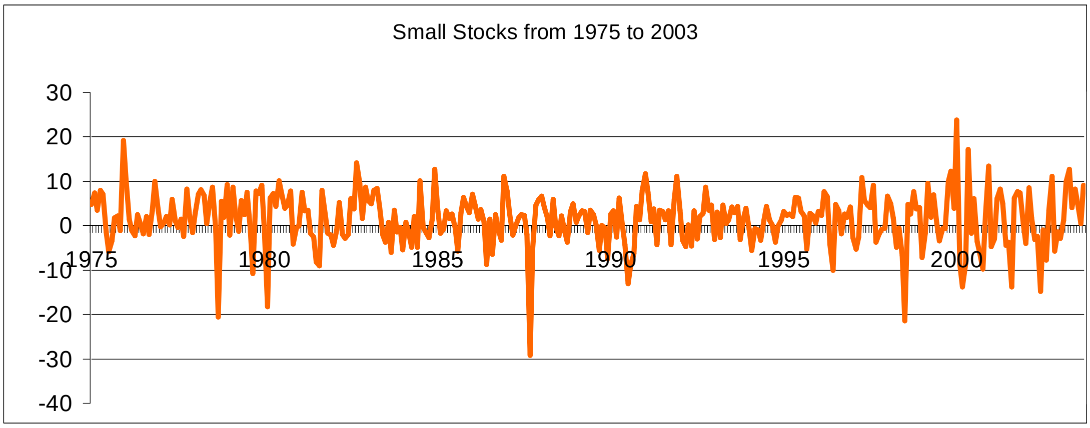
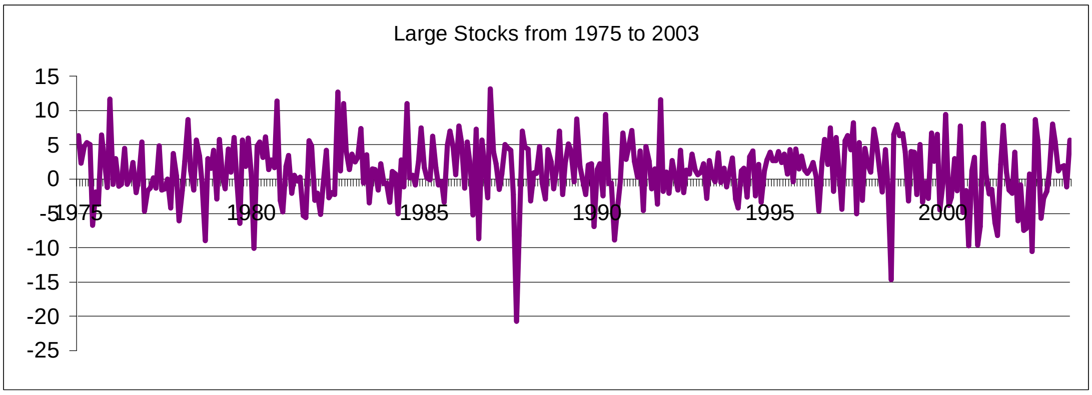


- Graph 5 -



- Graph 6 -



- Graph 7 –

Bellow mean returns, standard deviation and the correlations of the data returns are mentioned:

	Housing	S&P	5y	Growth Stocks	Value Stocks	Small Stocks	Large Stocks
Mean	5,45	9,38	0,04	1,10	1,40	1,46	1,13
Std.Dev.	3,82	51,72	1,21	4,98	4,32	5,89	4,44
Mean/Std.dev.	1,43	0,18	0,03	0,22	0,32	0,25	0,25

-Table 1-

- As we can see from *Table1*, the S&P has the highest mean return (9,38%), but also the highest Standard Deviation (51,72%); it is followed by the Housing Index with a 5,43% return, which in contrast has a lower Standard Deviation (3,82%).
- Looking at the Mean/Std. Deviation ratio (or high return - low risk-ratio), the best choice to invest in seems to be the Housing asset.

The investor of course is interested in investing in more than one asset, in other words in constructing a whole portfolio. So, among others, it is important to take a look at the assets' correlations that comprise the portfolio.

Correlation Matrix

	Housing	S&P	5y	Growth Stocks	Value Stocks	Small Stocks	Large Stocks
Housing	1,00	0,00	-0,11	-0,02	0,03	0,10	-0,03
Stocks S&P	0,00	1,00	-0,02	0,05	0,10	0,19	0,04
Bonds 5y	-0,11	-0,02	1,00	0,19	0,23	0,13	0,21
Growth Stocks	-0,02	0,05	0,19	1,00	0,79	0,78	0,97
Value Stocks	0,03	0,10	0,23	0,79	1,00	0,75	0,86
Small Stocks	0,10	0,19	0,13	0,78	0,75	1,00	0,74
Large Stocks	-0,03	0,04	0,21	0,97	0,86	0,74	1,00

-Table 2-

The average price returns on real estate are lower than the average returns on stocks, it also appears to be less risky than stocks and moreover it has low correlations with both stocks and bonds. This suggests that there may be benefits from including real estate in an investment portfolio.

Following we will analyze whether the mean returns on real estate are sufficiently high in order to include this asset as part of an investment portfolio.

5. Methodology

5.1 Optimal portfolios and Asset Pricing

Investors are interested in choosing a portfolio that maximizes their portfolios' expected return at a given level of variance or vice versa.

In order to achieve the above the investor must decide, which combination and what quantity of assets to choose. In other words he must solve the portfolio optimization problem.

Suppose the investor has already chosen to invest in bonds and stocks. The next step to take will be to choose how much money he will invest in bonds and how much in stocks so as to gain maximum profit at a given risk level.

Define:

μ: vector of expected returns of K assets (KX1)

w: vector of asset weights in portfolio (KX1)

Σ: variance covariance matrix of returns (KXK)

γ: degree of relative risk aversion (1X1)

The expected return of the portfolio is: $\mu_\rho = w'\mu$.

The variance of the portfolio return is: $Var[R_{\rho,t}] = \sigma_r^2 = w'\Sigma w$.

Utility (u) maximization by the investor implies that:

$$\max_w u = w'\mu - \frac{g}{2} w'\Sigma w + \eta(1 - w'\iota_K)$$

$$w'\imath_K=1$$

where η is the Langrange multiplier of the budget constraint. It can be interpreted as the zero-beta rate (the return of the portfolio that is not correlated with the optimal portfolio).

First order condition for a maximum gives the optimal portfolio.

Hence:

$$\partial q / \partial x = 0 \Rightarrow$$

$$w= \gamma^{-1}\Sigma^{-1}(\mu-\eta\imath_K)$$

This relation gives the optimal weights of the portfolio.

In order to find the portfolio on the efficient frontier with the lowest variance we solve following:

$$\min_w U = \frac{g}{2} w'\Sigma w + \eta(1-w'\imath_K)$$

Again from the first order condition we get:

$$w = \gamma^{-1}\Sigma^{-1}\eta\imath_K \quad \text{(A)}$$

In order to determine η, we pre-multiply this condition with $\imath_K$. (From the portfolio constraint we know that: $w'\imath_K = '\imath_K w= 1$).

$$\eta = \frac{g}{i'_k\Sigma^{-1}i_k}$$

So, substituting in (A), we get:

$$w = \frac{\Sigma^{-1}i_K}{i'_K\Sigma^{-1}i_K}$$

5.2 Derivation of the CAPM

Now we can show that if agents hold the market portfolio, CAPM holds.
We start from the optimal portfolio rule: $w = \gamma^{-1}\Sigma^{-1}(\mu-\eta\iota_K)$.

Solving for assets' returns, gives:

$$\mu-\eta\iota_K = \gamma\Sigma w. \quad (B)$$

In order to derive an equation for the market return, we pre-multiply with w':

$$w'(\mu-\eta\iota_K) = \gamma w'\Sigma w \quad or \quad m_m - h = g s_m^2$$

where: m_m and s_m^2 are the return and the variance of the market portfolio.

So: $\gamma = \frac{m_m - h}{s_m^2}$, substituting γ in (B) gives:

$$\mu-\eta\iota_K = \frac{m_m - h}{s_m^2}\Sigma x = \frac{m_m - h}{s_m^2}\begin{pmatrix} s_{im} \\ \cdot \\ s_{nm} \end{pmatrix}$$

This is the CAPM.

The first order condition of mean-variance optimization : $\mu-\eta\iota_K = \gamma\Sigma w$ imposes testable restrictions on returns if we specify the portfolio weights. We just derived that if the investor holds the market portfolio CAPM holds.

Similarly, we can test whether a specific portfolio is efficient.

5.3 Intersection and Spanning

Intersection (Figure: 1) and spanning (Figure: 2) refer to two important properties of portfolios. A portfolio A of N assets is said to span a smaller portfolio B of N_1 assets, if the efficient frontiers of the portfolios coincide. In other words, adding $N-N_1$ assets to portfolio B does not lead to a portfolio with superior mean-variance characteristics, i.e. higher mean returns per unit risk. This is the case when portfolio B can reproduce the characteristics of portfolio

A and happens when the N-N_1 assets included in portfolio A but not in portfolio B as a group are highly correlated with the N_1 assets of portfolio B. Portfolio B does not span portfolio A when adding N-N_1 assets to portfolio B leads to a portfolio with superior mean-variance characteristic, i.e. the efficient frontier of portfolio B cannot reproduce the characteristics of portfolio A.

Intersection is a concept similar to spanning except that it refers to portfolios of a specific investor, i.e. an investor with a given degree of risk aversion. If there is intersection between portfolio A and B for a specific degree of risk aversion, then investors with this degree of risk aversion are indifferent between portfolios A and B. In other words, adding N-N_1 assets to portfolio B does not lead to a portfolio with superior risk-return characteristics for this kind of investor.

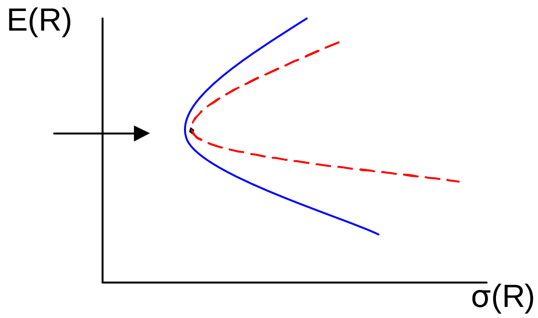


Figure: 1

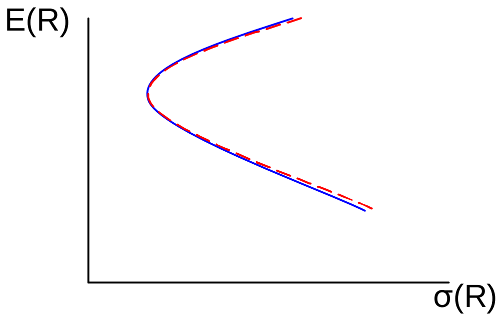


Figure: 2

____ : Mean variance frontier of risky asset set A

_ _ _ _ : Mean variance frontier of risky asset set B, (B<A)

Assume now that we have two different asset sets, asset set A and asset set B. Our portfolio comprises only out of asset set A and we wonder if it is optimal to include asset set B as well, as to improve our portfolios' performance.

For the two asset sets A and B the optimal portfolio is given by:

$$\begin{pmatrix} m_A - h \\ m_B - h \end{pmatrix} = g \begin{pmatrix} s^2{}_A & s_{BA} \\ s_{AB} & s^2{}_B \end{pmatrix} \begin{pmatrix} w_A \\ w_B \end{pmatrix}$$

Investing in B is not usefull if $w_B = 0$. So:

$$\begin{pmatrix} \mu_A - \eta \\ \mu_B - \eta \end{pmatrix} = \gamma \begin{pmatrix} \sigma^2_A & \sigma_{BA} \\ \sigma_{AB} & \sigma^2_B \end{pmatrix} \begin{pmatrix} w_A \\ 0 \end{pmatrix}$$

From the first row we can determine the optimal weight of A:

$$w_A = \frac{(\mu_A - \eta)}{\gamma \sigma^2_A}$$

From the second row we obtain:

$$\mu_B - \eta = \gamma \sigma_{A,B} w_A$$

and substituting for w_A , we obtain:

$$\mu_B = \eta \left(1 - \frac{\sigma_{A,B}}{\sigma^2_A} \right) + \frac{\sigma_{A,B}}{\sigma^2_A} \mu_A$$

Note that the ratio $\frac{\sigma_{A,B}}{\sigma^2_A}$ is the slope coefficient (beta) of a regression of the return of A on a constant and on the return of B.

Hence, the restriction $w_B = 0$, can be tested by running the regression:

$$r^B = \alpha + \beta r^A + u$$

and testing the restriction: $\alpha = \eta\ (1-\beta)$.

Concluding, we can test the restriction for either a specific η (or γ) – testing for intersection, whether a specific investor, adding asset set B to his portfolio will lead to a portfolio which is superior in terms of mean-variance characteristics, or for any η (or γ) – testing for spanning (independent of γ), whether for all investors adding asset set B to their portfolios will lead to a portfolio which is superior in terms of mean-variance characteristics.

Intersection:

Test whether for some specific η the condition: $a = \eta\ (1-\beta)$ holds.

Spanning:

Test whether for all η the condition $a=\eta(1-\beta)$ holds, if $\beta=1$ and $a=0$.

5.4 Testing

We will use several testing methods for mean-variance efficiency of portfolios. Such tests were initially developed out of the necessity to evaluate asset-pricing theories, like the CAPM. But since the testability of such theories is equivalent to testing whether the market portfolio is mean-variance efficient, as proven above, *the same methods can be used to test the efficiency of a single portfolio*.

1. Tests of Mean-Variance Efficiency and Intersection based on efficient set constants.

a. Jobson - Korkie

The testing procedures we are going to use, among others, are the Jobson - Korkie (JK) intersection and efficiency test statistics. The distinctive feature of their technique is its comparison of the maximum attainable Sharpe performance of an asset set with the potential performance of an asset subset. This technique can be used for the quantification of the performance contribution made by additional assets, the efficiency evaluation of a portfolio or market index, and for tests of multifactor asset pricing models.

JK developed a notional as well as a technical framework for testing the efficiency of portfolios, under two approaches:

- Test for intersection

  First they suppose a subset Γ_1 of N_1 assets from the population Γ of N assets, and denote the potential performance of asset sets Γ_1 and Γ as a_1 and a respectively. Testing for potential performance (intersection), they try to answer a first question of interest, whether the potential performances of the two asset sets are identical. This question can be answered through testing the hypothesis H_0: $a_1 = a$, a test for the comparative potential performance hypothesis. This hypothesis determines if the N_1 assets are jointly efficient with respect to the complete set of N assets, so rejecting H_0 implies that including the set of N_1 assets leads to a significant increase in the return per unit risk of the portfolio.

- Test for portfolio efficiency

  Under the second approach, testing for portfolio or index efficiency, JK construct a test in which the performance of a given portfolio (for example an index) is significantly different from the performance of the optimal portfolio that includes all the assets of the index. Hence, the null hypothesis in this test is that the weights of the indices' assets are equal to the weights of the same assets in the optimal portfolio. Rejection of the null hypothesis implies that the given portfolio is not efficient.

  The tests proposed by JK are the following:

  For intersection:

  1. The *Wald* statistic:

     Consider N and N_1 (the original population of assets is partitioned to two mutually exclusive and exhaustive subsets with $N-N_1$ and N_1 assets). We can partition the mean vector μ and the covariance matrix Σ as following:

     $\mu = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_2 & \Sigma_{22} \end{bmatrix}$

     The hypothesis H_0 can now be written as:

     $$H_0: a_1 = a \Rightarrow H_0: \mu_1'\Sigma_{11}^{-1}\mu_1 = \mu\Sigma^{-1}\mu$$

     or

     $$H_0: [\mu_2-\Sigma_{21}\Sigma_{11}^{-1}\mu_1]=0$$

     Let $\gamma = [[\mu_2-\Sigma_{21}\Sigma_{11}^{-1}\mu_1]]$ and $\hat{g}$ be its unbiased maximum likelihood estimator, with covariance matrix Ω. Since $\hat{g}$ is asymptotically normal, $\hat{g}'\Omega^{-1}\hat{g}$ will asymptotically follow a χ^2_{N-N1} distribution. Since Ω^{-1} is unknown, we can replace it by its unbiased estimator $\hat{\Omega}^{-1}$. The test statistic $\hat{g}'\hat{\Omega}^{-1}\hat{g}$ is a Wald statistic which is asymptotically χ^2_{N-N1} if the null hypothesis is true.

The Wald statistic can then be approximated by the following formula:

$$\Phi = (T\text{-}N)\left(\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}\right)s \sim \chi^2_{N\text{-}N1}$$

2. The *Likelihood Ratio* statistic:

   It can be shown that the likelihood ratio statistic for testing the H_0 hypothesis can be written as:

$$\Phi = (T\text{-}\frac{N}{2}-\frac{N_1}{2}\text{-}1)\ln\left[\frac{(1+\hat{a})}{1+\hat{a}_1}\right] \sim \chi^2_{N\text{-}N1}$$

   However, the distribution of the above mentioned statistic is known only asymptotically. In order to take into account small sample size effects, the following statistic is proposed:

$$\Phi = \left(\frac{T-N}{N-N_1}\right)\left(\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}\right) \sim F_{N\text{-}N1,T\text{-}N}$$

3. The Langrange Mulitplier or “score” statistic:

   Finally, we can derive the LM statistic for the null hypothesis $H_{0.}$ The statistic will again asymptotically follow the same $\chi^2_{N\text{-}N1}$ distribution as the previous statistics.

$$\Phi = T\frac{(\hat{a}-\hat{a}_1)}{(1+\hat{a})(1+\hat{a}_1)} \sim \chi^2_{N\text{-}N1}$$

For portfolio efficiency:

The tests applied for portfolio efficiency can be regarded as a special case of tests for intersection, since the potential performance of a single asset p equals its Sharpe performance. Thus, the statistics can be directly obtained from the above, for N_1=1 and $a_1=\frac{\hat{m}_p^2}{\hat{s}_p^2}$, where $\hat{m}_p^2$ and $\hat{s}_p^2$ are the sample mean and sample variance respectively of the portfolio of interest.

1. The *Wald* statistic:

$$\Phi = (T-N)\left(\frac{\hat{a}-\frac{\hat{m}_p^2}{\hat{s}_p^2}}{1+\frac{\hat{m}_p^2}{\hat{s}_p^2}}\right) \sim \chi^2_{N-N1}$$

2. The *Likelihood Ratio* statistic:

$$\Phi = \left(T-\frac{N}{2}-\frac{5}{2}\right)\ln\left[\frac{1+\hat{a}}{1+\frac{\hat{m}_p^2}{\hat{s}_p^2}}\right] \sim \chi^2_{N-N1}$$

and

$$\Phi = \left(\frac{T-N}{N-1}\right)\left(\frac{\hat{a}-\frac{\hat{m}_p^2}{\hat{s}_p^2}}{1+\frac{\hat{m}_p^2}{\hat{s}_p^2}}\right) \sim F_{N-1,T-N}$$

3. The Langrange Mulitplier or “score” statistic:

$$\Phi = T\,\frac{(\hat{a}-\frac{\hat{m}_p^2}{\hat{s}_p^2})}{(1+\hat{a})(1+\frac{\hat{m}_p^2}{\hat{s}_p^2})} \sim \chi^2_{N-1}$$

The intuition behind the formulas is that when the potential performance of the subset of assets or single portfolio is significantly below the potential performance of the full set of assets, the statistics take a rather large value and tend to reject the null hypothesis of efficiency of the subset of assets or portfolio.

b. GRS statistic (Gibbons, Ross and Shanken)

Another test statistic for portfolio intersection and portfolio efficiency is the GRS statistic. Under the common Normality and IID assumptions for asset returns, GRS construct a Wald statistic to test the hypothesis of zero intercepts (a=0) for an excess return regression model. But instead of an asymptotic χ^2, they derive an exact Hotelling T^2 statistic. This statistic can be used to test the null hypothesis of zero regression intercepts, which is equivalent to the hypothesis of efficiency of the examined portfolio. The GRS statistic is also applicable to test the potential performance of a subset of N_1 assets from a set of N assets.

$$GRS = \frac{T-N-1}{N}\frac{\left(\hat{a}_1^2-\hat{a}_2^2\right)}{\left(1+\hat{a}_2^2\right)} \sim F(N, T\text{-}N\text{-}1)$$

2. Tests based on Jensen's alphas.

Jensen's alpha is used as a performance measure and is defined in terms of one asset or portfolio relative to another. Jensen's alpha is the constant intercept in a regression of a set N_1 of asset excess returns on another set N of asset excess returns. It answers the question whether investors can improve the efficiency of their portfolio by expanding their portfolio of N assets to include the set N_1 of assets. In that case the hypothesis of spanning is equivalent to the hypothesis that: $a_j(\eta) = 0$, .η, [where $a_j(\eta)$ is Jensen's alpha].

It is common in the literature to define Jensen's alpha as the intercept of a regression of r_{t+1} in excess of the risk-free rate on the return of the market portfolio in excess of the risk-free rate.

$$r_{e\,t+1} = \alpha_j + \beta R_{et+1} + \varepsilon_t \qquad (\Gamma)$$

Jensen's alpha follows from a regression of r_{t+1} (returns of N_1 set of assets) on R_{t+1} (returns of N set of assets), and measures the performance of r_{t+1} relative to R_{t+1}. Stated differently, Jensen's alpha gives the potential improvement in performance when the additional assets are included in the portfolio

For a mean-variance investor with risk aversion γ the optimal portfolio of asset sets N and N1 is given by:

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = g^{-1} \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}^{-1} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$$

where:

μ_i= expected excess returns

Σ_{ij} = cov($r_{e\ t+1}$, $R_{e\ t+1}$)

The optimal weights w_2 can be derived as:

$$w_2 = \tilde{g}^{-1} \Sigma_{ee}^{-1} a_j$$

Where $\tilde{g}$ is the risk-aversion parameter associated with the extended portfolio (N + N_1) that has the same zero beta rate as the initial portfolio and $\Sigma_{\varepsilon\varepsilon}$ is the covariance matrix of the error terms in the Jensen regression (Γ). Note that $\tilde{g}$ can be computed from $\tilde{g}$ = $i'_{\kappa}\Sigma^{-1}\mu - \eta i'_{\kappa}\Sigma^{-1}i_{\kappa}$, where Σ is the covariance matrix of the extended portfolio and η is the zero beta rate of the benchmark portfolio. The optimal portfolio weights of the benchmark assets can be written as:

$$w_1 = \frac{g}{\tilde{g}} w_1^0 - \tilde{g}^{-1} \Sigma_{ee}^{-1} aj = \frac{g}{\tilde{g}} w_1^0 - B' w_2$$

where w_1^0 is the vector of optimal portfolio weights in the mean–variance efficient portfolio from the benchmark assets only, i.e.

$$w_1^0 = g^{-1} \Sigma_{cc}^{-1} m_c$$

The hypotheses of intersection and spanning imply that Jensen's alpha is zero for one or for all values of η, respectively, where η equals the expected return on the zero-beta portfolio of the given mean-variance efficient portfolio.

The test statistic for spanning can be written as:

$$x_W^{Span} = T\left(\frac{1+\hat{q}\left(\hat{h}_R^0\right)^2}{1+\hat{q}_R\left(\hat{h}_R^0\right)^2}-1\right)+T\left(\frac{\left(\hat{s}_R^{\,0}\right)^2}{\left(\hat{s}^{\,0}\right)^2}-1\right) \sim \chi^2_{2N}$$

where:

$\hat{h}_R^0$*:* the expected return on the global minimum variance portfolio of R_{t+1}

$\left(\hat{s}_R^{\,0}\right)^2$*:* the variance of the given portfolio

$\hat{s}^{\,0}$*:* is the global minimum variance of (R_{t+1}, r_{t+1}).

In this analysis we focus on the Likelihood Ratio Statistic, proposed by Jobson and Korkie, since it is appropriate for small sample sizes.

These tests are going to be applied in the following cases:

- § No Restriction on the Housing asset
    - a. Short-selling allowed
    - b. No short-selling allowed
- § Restriction on the Housing asset
    - c. Short-selling allowed
    - d. No short-selling allowed

6. Test results

Our analysis starts by investigating whether adding home equity to a stock and bond portfolio improves portfolio efficiency. We will test for intersection and spanning, examining this for the cases where short selling is allowed or not and for different restrictions concerning the real estate asset only. We also will submit these results for different values of *η* (or *γ)*.

Here we must explain that *η,* the Langrange multiplier – zero beta rate of the efficient portfolio, is related to *γ*, the risk aversion parameter of the investor, through the following formula:

$$h = \frac{g}{i'k\Sigma^{-1}ik}$$

Where:

Σ: is the variance-covariance matrix of returns (NXN)

ικ’: vector of ones (NX1)

So we can either test for a specific η or for a specific γ. We choose to test for η. Particularly for η= 0%, 1%, 2%, 3% and 4%.

6.1 Unrestricted

In this section we assume that all assets, bonds, stocks and the home asset can be sold and bought by the investor in one period, so there are no restrictions for any asset class.

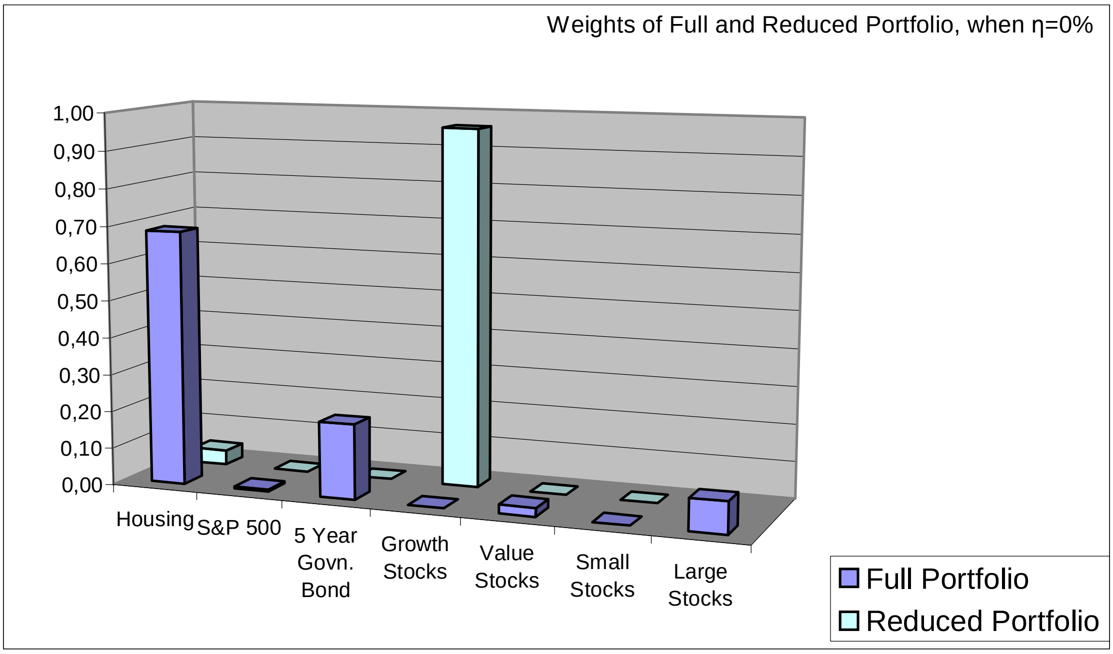
From Table set A, we can see that for all values of η, assets weights, for the components of the full and reduced portfolios, are given in the following order: Housing, S&P 500, 5-Year Bond US, Growth Stocks, Value Stocks, Small Stocks, Large Stocks and S&P 500, 5-Year Bond US, Growth Stocks, Value Stocks, Small Stocks, Large Stocks, respectively. All tables show these for both cases of short selling and no short selling.

Now we can take a first glance at changes in the asset weights and the test statistics, for all given values of η, when the housing asset is introduced.

From -*Table A1*- below and for a given level of η=0%, it is obvious that the investors behavior, concerning his asset allocation, changes when the real estate asset is introduced to his portfolio.

In particular, when short selling is not allowed, for the reduced portfolio his wealth is allocated at a 100% in stocks; when real estate is added, he allocates his wealth for 68% in real estate, 20% in bonds and only 12% in stocks, particularly in Large Stocks. (The above can easily be seen in -*Graph A1*-).

-Graph A1-



Again for η=0%, and when short selling is allowed, the investor changes his asset allocation when the full portfolio is introduced. In particular, while in case of the reduced portfolio he chooses to invest mostly in Stocks (S&P) and to sell Bonds short, now he invests in real estate and chooses to sell Stocks short (S&P). For the other components' weights of the portfolio (Growth Stocks, Value Stocks, Small Stocks, Large Stocks) the change also is significant.

Lets look at the following table (-*Table A2-).* Here, for a given level of η=1% and when short selling is disallowed, the investor chooses to invest all of his wealth in Stocks, mostly in Value Stocks and less in the S&P. When the real estate asset becomes available, he chooses to invest 95% of his wealth in real estate and only 5% in Stocks, this time mostly in Large Stocks.

When η=1%, and short selling is allowed, he allocates 365% of his wealth in housing while selling less bonds short, he invests less of his wealth in the S&P, and changes his position in the other portfolio components.

Given η =2%, the investor decides to invest even more of his wealth in real estate (99%), and a slight percentage in the S&P (1%), in contrast to investing all of his wealth in the S&P, as he did before real estate became available and while short selling is disallowed. In *Tables A4* and *A5*, again we conclude that the introduction of real estate into the investors' portfolio, for η=3 and 4% respectively, changes dramatically his asset allocation, driving him straightforward to the housing asset, leaving Stocks at a very low percentage.

From the statistics in the same tables, it is obvious that there is no intersection and no spanning, meaning that there is a diversification benefit for the investor when the real estate asset is introduced.

Test results *strongly imply* that an outward shift in the efficient frontier will occur when adding the home equity to the investors' portfolio, improving the investors' return per unit risk.

Excel Sheet No Restrictions

6.2 Restricted

In this second part of the analysis, we test again for intersection and spanning, while a restriction for the home asset is introduced. This means that, contrary to the above analysis, the investor cannot buy or sell the house he has invested in, in one time period; he is “stuck” with it. Contrary he is allowed to buy or sell any of the other assets (Stocks, Bonds and the Fama& French Portfolios formed on Size and on Book-to-Market.)
This assumption of course makes much more sense. Almost no investor is able to buy and sell a house every month, or every year. The percentage of the investors’ wealth that corresponds to the value of the real estate asset he owns, will be the restriction mentioned in the analysis that follows. This restriction moreover brings into question, whether a hedging demand for stocks and bonds is introduced, if the demand for stocks and bonds changes due to the fact that the housing asset cannot be altered.

- For a value of η = 0%, the investor keeps a high degree of risk aversion. As we can see from *Table sets B* and *C* and for restrictions 40, 50 and 60%, the investor chooses to invest in Government Bonds, which have a very low standard deviation. He only invests a very small amount in Value Stocks and Large Stocks, so as to achieve a small diversification benefit.

- When η becomes 1%, he again chooses Government Bonds, but since he is becoming more risky, he also chooses a slight amount of Stocks, in particular Value Stocks and Large Stocks, that have the lowest standard deviation (risk) above all other stock categories (Small Stocks, Growth Stocks and of course the S&P Comosite).

- As η takes a value of 2%, the investor lowers his exposure to Government Bonds dramatically, investing even more in the Value Stocks and Large Stocks; and starting to invest small amounts in the S&P Composite, which according to its standard deviation, is the riskiest asset of all mentioned. We can observe that the higher the restriction gets for the Housing asset, the less risky the investor becomes.

- For η=3%, he no longer insists in investing any money in Government Bonds, contrarily he insists in buying stocks. Particularly, for a restriction of 40% in housing, he decides to invest all of his remaining wealth in the S&P Composite. For both 50 and 60% restrictions, the investor becomes less risky than he was in the 40% restriction case, and decides to invest in Stocks again, but in Growth Stocks, that, as mentioned before are less risky than the S&P Composite.

- Finally for a value of η=4%, implying that the investor becomes even less risk averse, no matter the restriction parameter, he chooses to invest all of his remaining wealth in the S&P Composite, the most volatile asset.

To summarize, we found that, an investor with a high degree of risk aversion (low η) chooses mostly to invest in Bonds than in Stocks. Even when he chooses to invest in the stock market, he picks Value Stocks and Large Stocks that are less volatile than the remaining stock categories. As the investor becomes riskier, the Stocks/Bonds ratio gets higher; even within the Stock category, he more and more decides to invest in the riskier Stocks.

All these are true for all the restriction parameters, 40, 50 and 60%, of real estate. It must be said though that the higher the restriction, the slower the steps the investor takes towards the risky direction.

Accordingly it is easy to interpret the results given for the case where short selling is allowed.

From Table set B, we also conclude, that for all* levels of restriction and values of η, the inclusion of real estate is optimal. Due to the intersection and spanning tests, we find that real estate inclusion offers diversification benefits to the investor once again, raising the return per unit risk of his portfolio.

Excel Sheet Restrictions - Hedging

In Flavin M. and Yamashita T. (1999), we find a similar analysis. The authors interpret the different values of risk aversion and the housing restriction, as an age pattern. They suppose, that young investors are more risk averse than elder investors, since the proportion of wealth that is accumulated to housing is higher for them, than for elder investors, and that young investors have still to pay a great amount in their mortgage and therefore prefer to invest in riskless assets.
Their results conclude that the ratio of bonds to stocks increases with the levels of risk aversion. This is true for high levels of restriction, that imply a specific age group (young investors), for lower levels of restrictions (elder investors), contrarily to our analysis, this pattern does not exist.

6.2.1 The Hedging Demand Component

Above results imply, that a hedging demand for stocks and bonds is induced by the restricted housing asset, which means that investors change their demand for both bonds and stocks in order to protect themselves from the risk that comes with the illiquidity constraint of the housing asset.

DeRoon et al., find that stocks and bonds do not provide a good hedge for positions in real estate, implying that the relative demand for either is not significantly affected by home ownership.
They obtained these results through following methodology:

If there is a hedging demand for stocks and bonds induced by real estate, then the fact that investors own a house means that they will choose a different optimal portfolio of stocks and bonds than would be the case if they would rent a house.
To investigate this, let r be the vector of returns on the assets that an agent can invest in, let γ be his risk aversion parameter and η be the associated zero-beta rate on a mean-variance efficient portfolio. The return on the real

estate the investor is exposed to is r_E and the size of his exposure as a fraction of invested wealth is q. The optimal demand for assets is:

$$w= \gamma var[r]^{-1}\{E[r]-\eta i\} - q var[r]^{-1} cov[r, r_E]$$

The first part of this optimal demand is known as the speculative demand, and equals the standard Markowitz optimal portfolio demand. The second term is known as the hedging demand. Only this hedging demand depends on the exogenous exposure, in this case to real estate. It depends on the covariances whether or not there will be a hedging demand induced by the position in real estate. If the latter term is not equal to zero the optimal portfolio will deviate from the standard Markowitz solution, precisely because the investor wants to hedge the risk associated with owning a house.

Notice that the term var[r]-1cov[r, rE] equals the vector of slope coefficients in a regression of rE on the returns of the available investment securities r. To the extent that these slope coefficients are different from zero, the assets (like stocks and bonds) can serve, as a hedge for real estate and the investor will use this hedge property in optimizing his portfolio. Thus, the question whether or not the hedging demand for stocks and bonds is zero can be answered by testing whether the slope coefficients in the regression are zero.

$$rE = \alpha + \beta 1 rstock + \beta 2 rbond + \varepsilon$$

Contrary to DeRoon et al., we find strong evidence that a hedging demand for both stocks and bonds is induced by real estate. For all cases of η and the various holding restrictions of the real estate asset, the investors' allocation changes due to the restricted real estate component of the portfolio. As described above, it is obvious that the demand for bonds and all stock categories differs when the restriction is imposed.

These hedging demand components are illustrated bellow, for the case where short selling is not allowed:

Excel Sheet Restrictions - *Hedging*

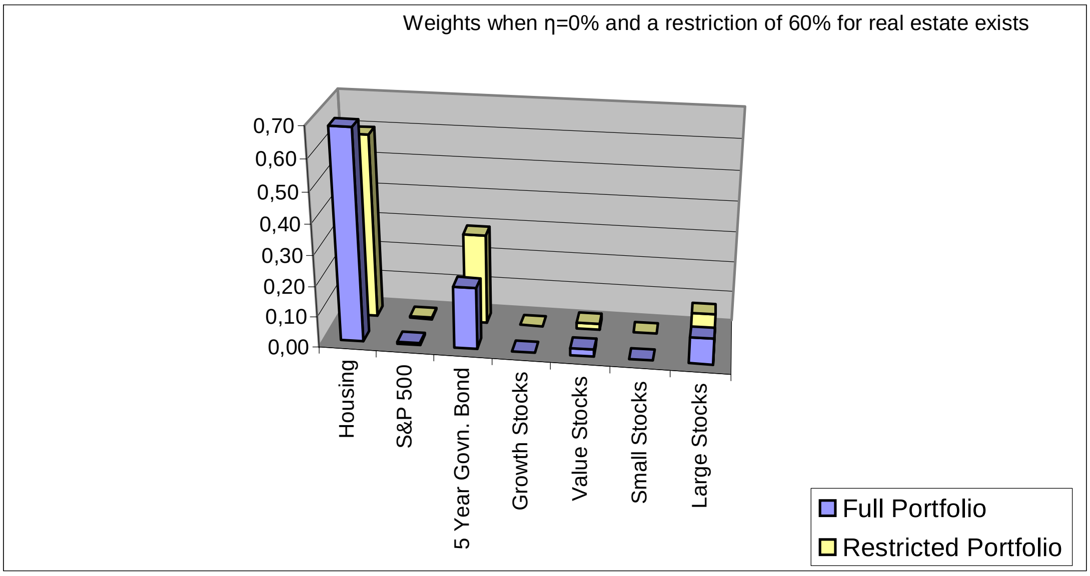
The differences in the test results can be drawn from the sample size that DeRoon et al. use in their research, which is much smaller than the sample size used in this analysis [(1980 – 1997) and (1975 – 2003), respectively]. A much more important reason for this difference, might be the fact, that they compute the betas of the portfolio components (shown in the above mentioned methodology) in order to figure for the existence of a hedging demand for stocks and bonds, not taking into account different values of γ (η) and different restrictions of the real estate asset class in the portfolio.

6.2.2 An Exception.

When η is 0% and the restriction is 60%, test results say, that *there is* intersection. This on the other hand is apparent, looking at -*Table B11*-.

We see that the investor in the 0% case, with or without the restriction, chooses to invest 60 to 68% of his wealth in real estate, which is not a remarkable difference. Therefore his allocation in bonds and stocks does not change significantly. This exception is true due to the fact, mentioned in Kullman (2001), that it is optimal and common for the investor to allocate around 60% of his wealth in real estate, especially for a value of η=0%, which implies a high level of risk aversion by the investor.

To see this more clearly, we can take a look at -*Graph B1-:*



-Graph B1-

7. Conculsion

In our analysis we find that, investors should include the real estate asset in their portfolio, due to the diversification benefit it introduces to their portfolios. We prove that a portfolio that includes real estate, bonds, stocks and the Fama&French Portfolios formed on Size and on Book-to-Market in comparison to a portfolio containing bonds, stocks and the Fama&French Portfolios formed on Size and on Book-to-Market only, is optimal, raising the investors' return per unit risk.

When a restriction on the housing asset is imposed, we find that, an investor with a high degree of risk aversion (low η) chooses mostly to invest in Bonds than in Stocks. Even when he chooses to invest in the stock market, he picks Value and Large Stocks that are less volatile than the remaining stock categories. As the investor becomes riskier, the Stocks/Bonds ratio gets higher; even within the Stock category, he more and more decides to invest in the riskier Stocks.

All these are true for all the restriction parameters introduced to real estate. It must be said though that the higher the restriction, the slower the steps the investor takes towards the risky direction.

Another aspect which is important, is the evidence we give, that the presence of the real estate asset implies a hedging demand for stocks and bonds, proving that the investor changes his allocation in stocks, bonds, and the Fama&French Portfolios formed on Size and on Book-to-Market when he is restricted in the housing asset, to protect his wealth form the risk that comes with this restriction.

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Appendix

	United Stat	S&P Compos	United States	Growth Stocks	Value Stocks	Small Stocks	Large Stocks	
1975:02:00	5.61	152.03	0.90	8.05	2.52	4.78	6.31	1975
1975:03:00	3.80	77.76	-0.57	3.06	7.58	7.32	2.37	
1975:04:00	-9.41	-5.65	-2.07	5.79	5.6	3.53	4.72	
1975:05:00	5.61	76.77	0.81	5.94	2.71	7.9	5.35	
1975:06:00	3.80	59.52	0.63	4.39	7.31	7.04	5	
1975:07:00	-2.63	29.07	-1.23	-7.33	-3.73	-1.37	-6.71	
1975:08:00	-3.39	-90.09	-1.23	-2.3	-4.01	-5.81	-1.89	
1975:09:00	-2.22	-3.55	-0.12	-3.98	-3.63	-3.37	-3.69	
1975:10:00	4.90	-67.52	1.20	6.33	4.62	1.76	6.43	
1975:11:00	6.91	72.43	0.51	2.92	4.03	2.21	3.16	
1975:12:00	7.82	34.64	0.12	-1.77	0.16	-1.11	-1.25	
1976:01:00	5.31	-6.37	0.90	11	20.71	19.13	11.69	
1976:02:00	5.89	134.30	0.03	-1.07	7.71	10.4	-0.71	1976
1976:03:00	7.17	-10.15	-0.12	2.9	2.21	1.44	3	
1976:04:00	12.07	26.34	0.72	-1.56	-1.39	-1.15	-1.04	
1976:05:00	12.42	-15.59	-1.02	-0.61	-1.9	-2.3	-0.79	
1976:06:00	11.27	-12.79	-0.06	4.33	4.92	2.53	4.51	
1976:07:00	5.16	60.12	0.36	-1.19	0.52	0.21	-0.75	
1976:08:00	3.80	-20.64	0.54	-0.5	-0.98	-1.82	0.16	
1976:09:00	3.61	10.07	0.54	2.26	2.72	2.01	2.44	
1976:10:00	5.89	1.27	1.14	-2.38	-2.86	-1.93	-2	
1976:11:00	6.98	-12.39	0.69	-0.49	2.68	3.35	0.1	
1976:12:00	8.21	-7.12	1.26	4.92	9.23	9.97	5.42	
1977:01:00	9.67	51.68	-1.44	-6.01	1.01	3.74	-4.7	
1977:02:00	11.07	-51.09	-0.75	-1.77	-1.12	-0.19	-1.65	1977
1977:03:00	12.51	-22.21	-0.30	-1.61	0.4	0.64	-1.33	
1977:04:00	15.50	-17.41	0.42	-0.98	3.68	1.99	0.17	
1977:05:00	15.82	-3.39	-0.45	-1.84	-0.72	0.2	-1.35	
1977:06:00	15.09	-24.51	0.54	5.72	5.64	5.99	4.86	
1977:07:00	10.85	38.62	-0.24	-1.46	-2.01	0.99	-1.55	
1977:08:00	10.05	-11.81	-0.57	0.27	-2.5	-0.37	-1.43	
1977:09:00	10.09	-28.05	-0.03	-0.55	-0.27	1.41	-0.03	
1977:10:00	12.53	-1.12	-0.84	-4.4	-2.63	-2.35	-4.23	
1977:11:00	12.99	-68.79	-0.06	4.33	5.91	8.17	3.75	
1977:12:00	13.09	43.09	-0.42	0.36	1.27	2.52	0.49	
1978:01:00	12.05	5.18	-0.87	-6.32	-2.64	-1.48	-6.03	
1978:02:00	12.08	-67.08	-0.18	-2.33	-0.4	2.71	-1.66	1978
1978:03:00	12.38	-37.13	-0.09	2.52	4.96	7.13	2.6	
1978:04:00	13.76	17.35	-0.36	10.97	6.9	8.15	8.64	
1978:05:00	13.88	118.85	-0.60	2.26	3.33	6.81	1.31	
1978:06:00	13.62	-3.94	-0.54	-1.35	-0.87	0.53	-1.63	
1978:07:00	12.27	-28.19	-0.54	6.95	5.69	5.48	5.67	
1978:08:00	11.87	68.31	0.63	4.03	3.77	8.68	3.35	
1978:09:00	11.66	35.47	-0.30	-2.49	-0.16	-0.65	-0.63	
1978:10:00	11.01	-8.36	-0.54	-11.22	-13.23	-20.5	-9.01	
1978:11:00	11.65	-73.41	-0.69	3.38	3.23	5.55	3.01	
1978:12:00	12.92	-7.33	-0.72	2.83	0.21	1.91	1.59	
1979:01:00	17.42	-1.87	-0.36	3.86	7.44	9.26	4.21	
1979:02:00	17.82	47.13	0.21	-3.66	-2.59	-2.06	-2.9	1979
1979:03:00	16.89	-37.31	-0.21	6.28	7.71	8.64	5.77	
1979:04:00	12.43	48.54	-0.15	0.27	1.34	2.46	0.24	
1979:05:00	10.80	9.24	0.03	-1.91	-1.17	-1.31	-1.38	
1979:06:00	9.63	-29.99	1.17	3.34	5.81	5.69	4.38	
1979:07:00	9.27	33.65	-0.15	0.69	2.95	2.48	1.06	
1979:08:00	8.70	25.38	-0.48	7.12	5.81	7.57	6.02	
1979:09:00	8.28	57.91	-1.05	-0.58	-0.74	-0.63	0.21	

1979:10:00	8.06	-8.37	-3.66	-6.95	-8.94	-10.79	-6.47	
1979:11:00	7.88	-68.11	-0.90	6.92	4.62	7.83	5.65	
1979:12:00	7.79	37.55	1.53	3.61	2.61	7.16	1.87	
1980:01:00	8.55	23.69	-0.96	5.22	7.9	9.14	5.97	
1980:02:00	8.07	77.28	-5.58	-2.66	-0.1	-1.04	0.44	1980
1980:03:00	7.13	-27.63	-2.61	-10.33	-13.95	-18.19	-10.13	
1980:04:00	3.04	-115.46	4.89	3.97	6.13	6.17	4.84	
1980:05:00	3.23	37.91	5.67	5.96	6.7	7.28	5.42	
1980:06:00	4.96	58.84	2.22	3.4	2.78	4.41	3.21	
1980:07:00	13.41	44.35	-0.96	9.82	3.43	10.07	6.17	
1980:08:00	14.11	63.84	-3.93	2.81	2.11	7.21	1.39	
1980:09:00	12.46	11.53	-2.34	4.2	-0.62	3.92	2.75	
1980:10:00	3.44	45.70	-0.72	2.3	-0.27	4.59	1.65	
1980:11:00	1.14	17.89	-2.91	11.63	4.35	7.75	11.34	
1980:12:00	0.35	73.67	-1.26	-2.5	-0.76	-4.15	-3.1	
1981:01:00	2.88	-12.75	1.44	-5.7	1.73	-0.58	-4.75	
1981:02:00	3.75	-80.89	-1.92	2.44	1.64	0.06	1.88	1981
1981:03:00	4.77	47.28	0.00	4.22	5.55	7.57	3.42	
1981:04:00	6.90	40.75	-1.74	-1.63	1.57	3.39	-2.03	
1981:05:00	7.46	-34.31	-1.92	1.23	2.89	3.42	0.6	
1981:06:00	7.44	-2.81	2.04	-3.42	0.34	-1.65	-0.27	
1981:07:00	6.57	-24.17	-2.52	0.82	-1.11	-2.56	0.3	
1981:08:00	5.64	6.55	-2.31	-7.19	-2.94	-8.1	-5.29	
1981:09:00	4.37	-70.65	-1.11	-6.83	-1.92	-9.02	-5.62	
1981:10:00	0.84	-59.39	1.56	8.31	4.66	7.92	5.64	
1981:11:00	0.33	70.84	6.09	3.79	4.27	2.63	4.89	
1981:12:00	0.91	18.22	-0.66	-3.05	-2.25	-1.68	-3.1	
1982:01:00	5.38	-34.27	-3.15	-2.26	0.24	-2.14	-2.07	
1982:02:00	6.00	-47.64	0.33	-7.36	-1.73	-4.42	-5.18	1982
1982:03:00	5.61	-46.43	1.68	-1.98	1.82	-1.03	-0.86	
1982:04:00	2.77	5.07	-0.06	5.85	2.84	5.25	4.17	
1982:05:00	1.48	31.43	0.75	-3.48	-2.11	-2.03	-2.7	
1982:06:00	0.28	-53.89	-2.04	-2.61	-1.71	-2.81	-2	
1982:07:00	-1.98	-32.34	1.08	-0.98	-1.66	-1.92	-2.25	
1982:08:00	-2.15	2.98	3.21	10.57	12.64	6.14	12.65	
1982:09:00	-1.38	97.96	2.25	1.99	0.73	3.75	1.21	
1982:10:00	1.73	37.17	4.35	13.13	9.85	14.08	10.99	
1982:11:00	3.34	125.97	1.26	7.37	4.59	9.52	4.13	
1982:12:00	4.84	28.45	0.48	1.14	1.51	1.63	1.36	
1983:01:00	7.11	-3.29	0.57	3.8	5.79	8.74	3.63	
1983:02:00	7.70	39.42	-0.69	3.72	2.88	5.68	2.53	1983
1983:03:00	7.52	64.70	0.54	2.43	3.35	4.93	3.19	
1983:04:00	5.45	16.43	0.18	6.01	7.22	7.98	7.35	
1983:05:00	4.64	69.72	-0.03	1.11	0.58	8.34	-0.57	
1983:06:00	3.94	3.25	-1.80	5.33	0.88	4.22	3.56	
1983:07:00	3.39	46.06	-1.74	-5.26	-0.8	-2.17	-3.44	
1983:08:00	2.85	-49.83	-1.26	-1.9	4.55	-3.7	1.45	
1983:09:00	2.37	16.11	0.60	2.2	1.12	0.71	1.39	
1983:10:00	1.13	11.42	0.45	-4.31	-0.72	-5.96	-1.59	
1983:11:00	1.36	-15.74	-0.39	3.19	2.12	3.42	2.2	
1983:12:00	2.26	20.72	-0.39	-1.11	-0.55	-1.45	-0.7	
1984:01:00	5.26	-11.30	0.51	-5.18	3.45	-0.44	-0.57	
1984:02:00	6.35	-16.04	-0.51	-5.1	-1.61	-5.41	-3.33	1984
1984:03:00	6.98	-34.03	-1.44	1.41	0.62	0.81	1.15	
1984:04:00	7.12	-1.59	-1.05	0.11	1.52	-1.24	0.78	
1984:05:00	6.93	27.78	-2.40	-4.96	-5.64	-4.88	-5.03	
1984:06:00	6.35	-64.34	-0.93	4.4	2.31	2.04	2.77	
1984:07:00	4.45	-0.31	0.63	-1.87	-2.53	-4.82	-1.14	
1984:08:00	3.85	6.87	1.77	11.62	11.37	10.06	11	
1984:09:00	3.60	94.32	0.45	-2.4	3.08	-0.4	0.17	
1984:10:00	3.91	-14.92	1.41	0.22	-0.46	-1.75	0.58	
1984:11:00	4.17	20.74	2.19	-2.62	0.83	-2.73	-0.84	

1984:12:00	4.59	-33.93	0.78	2.52	2.19	1.36	2.61	
1985:01:00	5.54	32.14	0.42	10.9	5.93	12.67	7.48	
1985:02:00	6.01	79.06	-0.60	1.59	2.23	3.15	1.72	1985
1985:03:00	6.38	30.64	-1.17	-1.7	2.1	-1.64	0.22	
1985:04:00	6.50	-13.04	1.53	-1.86	1.9	-0.74	-0.07	
1985:05:00	6.77	-19.35	2.01	5.64	4.92	3.3	6.23	
1985:06:00	7.04	71.49	2.22	1.47	1.52	1.64	1.64	
1985:07:00	7.73	19.55	-0.30	0.61	-1.32	2.68	-0.8	
1985:08:00	7.69	-2.00	-0.33	-1.56	0.77	-0.54	-0.4	
1985:09:00	7.35	-21.94	0.00	-4.34	-4.4	-5.68	-3.38	
1985:10:00	5.75	-22.86	0.36	3.97	5.42	2.88	4.97	
1985:11:00	5.57	40.86	1.23	8.84	4.97	6.31	6.97	
1985:12:00	5.82	55.00	1.65	5.57	4.39	4.15	4.69	
1986:01:00	7.17	63.08	0.15	1.01	1.68	2.89	0.68	
1986:02:00	7.73	15.13	1.02	7.85	6.93	7.05	7.74	1986
1986:03:00	8.21	62.61	2.64	5.98	4.52	4.69	5.43	
1986:04:00	8.82	50.66	1.23	0.79	-2.84	1.41	-1.34	
1986:05:00	8.93	0.10	-1.41	5.12	4.93	3.7	5.41	
1986:06:00	8.79	49.39	-0.36	1.61	2.53	0.93	1.79	
1986:07:00	7.95	33.80	1.74	-7.61	-2.74	-8.68	-5.21	
1986:08:00	7.68	-84.46	0.78	4.46	9.12	1.52	7.28	
1986:09:00	7.50	88.69	-0.36	-9.88	-7.41	-6.43	-8.67	
1986:10:00	7.30	-95.40	0.27	5.52	4.98	2.54	5.65	
1986:11:00	7.39	61.09	0.21	1.7	1.58	-0.73	2.1	
1986:12:00	7.66	15.76	0.27	-2.49	-2.85	-3.3	-2.69	
1987:01:00	8.69	-33.62	0.09	13.71	10.99	11.13	13.2	
1987:02:00	8.84	158.87	-0.45	7.64	0.18	7.74	3.98	1987
1987:03:00	8.73	28.10	0.00	1.62	2.34	2.57	2.22	
1987:04:00	8.01	39.17	-2.34	-1.92	-1.64	-2.08	-1.46	
1987:05:00	7.64	-18.03	-2.07	1.29	0.59	-0.26	0.7	
1987:06:00	7.28	7.48	0.72	4.27	4.58	1.79	5.02	
1987:07:00	7.17	53.09	0.03	5.15	3.7	2.49	4.54	
1987:08:00	6.61	56.60	-0.93	4.4	2.87	2.3	4.19	
1987:09:00	5.86	21.83	-1.86	-2.07	-2.42	-1.98	-2.23	
1987:10:00	3.72	14.53	-0.42	-24.19	-19.96	-29.19	-20.76	
1987:11:00	3.50	-296.21	2.19	-8.6	-5.66	-5.07	-8.21	
1987:12:00	3.97	-116.91	-0.30	9.02	4.74	4.58	6.95	
1988:01:00	6.37	75.57	0.81	2.14	8.01	5.8	4.67	
1988:02:00	7.27	38.05	1.41	5.73	3.64	6.62	4.4	1988
1988:03:00	7.92	58.23	-0.36	-2.45	-0.63	4.18	-3.19	
1988:04:00	8.84	-40.25	-1.08	-0.43	1.4	1.76	0.92	
1988:05:00	8.63	12.31	-1.17	-0.31	1.99	-2.31	0.82	
1988:06:00	7.81	23.31	0.27	5.14	4.2	5.92	4.78	
1988:07:00	4.98	22.69	-0.51	-1.66	0.23	-0.2	-0.55	
1988:08:00	4.08	1.90	-0.84	-3.07	-1.43	-2.22	-2.88	
1988:09:00	3.65	-62.71	0.75	4.78	3.44	2.14	4.26	
1988:10:00	4.19	59.05	0.54	1.79	2.61	-1.19	2.48	
1988:11:00	4.31	33.49	-0.84	-1.94	-0.68	-3.71	-1.44	
1988:12:00	4.52	-28.59	-0.90	2.91	1.11	3.17	1.82	
1989:01:00	5.06	22.81	-0.18	6.72	6.75	4.93	7	
1989:02:00	5.26	80.91	-0.36	-2.02	-1.07	0.7	-2.29	1989
1989:03:00	5.36	-41.00	-0.72	2.51	2.58	2.22	2.33	
1989:04:00	4.66	38.17	0.63	5.87	3.9	3.36	5.13	
1989:05:00	5.11	50.46	1.17	4.74	3.93	3.23	4.13	
1989:06:00	5.99	48.87	1.86	-1.18	0.31	-1.56	-0.38	
1989:07:00	9.23	-10.26	1.38	9.96	5.86	3.53	8.76	
1989:08:00	9.45	88.80	-0.78	1.22	2.48	2.42	1.87	
1989:09:00	8.63	34.34	-0.24	0.6	0.01	0.17	-0.04	
1989:10:00	5.00	-9.74	0.60	-1.7	-2.67	-5.61	-2.28	
1989:11:00	3.59	-33.54	0.48	2.21	0.88	0.01	2.02	
1989:12:00	2.55	32.72	0.18	1.53	2.22	-0.61	2.24	
1990:01:00	2.31	9.44	-1.11	-7.61	-7.19	-7.42	-6.96	

1990:02:00	1.65	-86.62	-0.90	0.67	2.51	2.67	1.44	1990
1990:03:00	1.01	14.33	-0.54	4.12	0.58	3.3	2.24	
1990:04:00	-0.06	21.30	-0.51	-1.19	-4.46	-2.5	-2.4	
1990:05:00	-0.31	-23.07	0.09	10.51	7.1	6.18	9.41	
1990:06:00	-0.19	106.75	0.93	1.28	-1.42	0.18	-0.64	
1990:07:00	1.46	-12.02	0.30	-1.59	-1.93	-4.24	-0.61	
1990:08:00	1.43	-13.49	-0.33	-10.08	-10.05	-13.1	-8.86	
1990:09:00	0.88	-116.75	-0.21	-6.06	-6.68	-8.88	-4.88	
1990:10:00	-2.06	-28.69	0.54	-0.06	-0.3	-6.32	-0.37	
1990:11:00	-2.21	-30.52	0.93	7.45	5.19	4.41	6.76	
1990:12:00	-1.46	64.97	0.87	3.81	2.07	1.37	2.85	
1991:01:00	2.36	22.41	0.09	6.48	4.81	8.02	4.62	
1991:02:00	3.31	45.78	0.69	8.11	7.74	11.75	7.05	1991
1991:03:00	3.55	92.24	-0.90	4.25	3.32	7.46	2.57	
1991:04:00	2.20	2.69	0.21	-0.31	0.62	0.86	0.27	
1991:05:00	1.70	28.74	0.00	4.67	4.83	3.84	4.08	
1991:06:00	1.16	24.27	-0.72	-4.87	-3.72	-4.35	-4.56	
1991:07:00	-0.53	-31.77	0.09	5.36	3.8	3.44	4.76	
1991:08:00	-0.31	28.86	1.44	3.47	2.36	3.19	2.61	
1991:09:00	0.70	25.49	0.87	-1.93	-1.8	1.31	-1.38	
1991:10:00	4.76	-19.06	0.81	1.64	1.91	3.32	1.52	
1991:11:00	5.64	6.52	0.75	-2.68	-5.59	-4.22	-3.62	
1991:12:00	5.62	-30.84	1.29	14.08	8.76	5.93	11.61	
1992:01:00	3.66	107.37	-0.15	-1.61	5.28	11.1	-1.82	
1992:02:00	2.66	-21.95	-1.02	0.45	6.9	3.46	1	1992
1992:03:00	1.58	8.53	-1.11	-3.04	-1.03	-3.31	-2.07	
1992:04:00	-0.98	-24.16	0.51	-0.17	3.03	-4.73	2.67	
1992:05:00	-1.22	24.39	0.27	0.84	0.62	0.16	0.61	
1992:06:00	-0.52	13.80	0.63	-2.69	-0.09	-4.56	-1.59	
1992:07:00	3.42	-12.78	1.92	4.2	2.34	3.31	4.15	
1992:08:00	4.24	34.97	0.72	-1.35	-4.52	-2.94	-1.97	
1992:09:00	4.25	-25.74	0.66	1.65	1.27	2.03	1.29	
1992:10:00	2.43	0.63	-0.66	2.56	1.06	2.82	0.73	
1992:11:00	1.66	18.48	-1.32	5.05	4.88	8.72	3.61	
1992:12:00	0.89	22.58	-0.12	0.89	3.65	3.46	1.34	
1993:01:00	-0.80	13.63	0.75	-0.88	5.54	4.63	0.66	
1993:02:00	-0.88	18.61	1.20	-2.7	2.79	-3.18	1.05	1993
1993:03:00	-0.27	-1.38	0.72	1.8	2.88	3.06	2.22	
1993:04:00	2.67	22.30	0.18	-4.9	-1.2	-2.69	-2.77	
1993:05:00	3.44	-21.08	-0.21	4.22	1.1	4.57	2.72	
1993:06:00	3.66	30.42	-0.06	-0.88	3.06	0.43	0.41	
1993:07:00	2.41	-12.76	0.39	-2.05	1.72	1.29	-0.35	
1993:08:00	2.29	3.02	0.18	3.68	3.46	4.23	3.82	
1993:09:00	2.34	34.14	0.90	-0.29	-0.49	2.88	-0.43	
1993:10:00	3.09	-4.80	0.06	3.08	1.29	4.32	1.57	
1993:11:00	3.11	20.15	-1.05	-0.37	-1.61	-3.1	-1.12	
1993:12:00	2.92	-18.59	-0.27	1.52	2.46	1.67	1.34	
1994:01:00	2.16	9.19	0.18	2	3.35	3.96	3.07	
1994:02:00	1.84	36.01	-0.93	-1.58	-3.82	-0.76	-2.89	1994
1994:03:00	1.58	-38.59	-1.62	-4.87	-4.02	-5.57	-4.21	
1994:04:00	1.42	-49.26	-1.74	0.27	1.49	-0.64	1.21	
1994:05:00	1.25	19.39	-0.78	1.12	-0.74	-0.97	1.61	
1994:06:00	1.12	12.12	0.24	-3.25	-2.21	-3.23	-2.58	
1994:07:00	1.31	-30.33	-0.63	2.92	3.84	1.18	3.25	
1994:08:00	1.04	39.16	0.09	6.06	2.72	4.35	4.11	
1994:09:00	0.58	31.27	-0.60	-0.95	-3.15	1.14	-2.41	
1994:10:00	-0.93	-29.34	-0.96	2.15	-0.14	0.2	2.16	
1994:11:00	-1.09	17.24	-0.96	-3.01	-3.14	-3.66	-3.35	
1994:12:00	-0.78	-51.00	-0.18	1.34	1.34	-0.04	1.24	
1995:01:00	0.20	27.33	0.06	1.86	3.05	1.2	2.84	
1995:02:00	1.34	28.76	1.17	3.96	4.08	3.24	3.91	1995
1995:03:00	2.82	38.24	0.96	2.96	0.61	2.33	2.68	

1995:04:00	5.98	39.38	0.57	2	3.4	2.57	2.71	
1995:05:00	7.08	29.31	1.35	3.26	4.88	2.08	3.98	
1995:06:00	7.50	44.05	1.44	4.22	1.64	6.41	2.43	
1995:07:00	6.71	30.21	-0.24	4.35	3.03	6.18	3.6	
1995:08:00	6.21	27.22	-0.69	0.3	2.49	3.25	0.76	
1995:09:00	5.47	8.95	0.72	4.65	3.46	2.16	4.32	
1995:10:00	3.45	37.48	0.42	-0.08	-2.16	-5.23	-0.41	
1995:11:00	2.98	5.15	0.51	3.93	4.18	2.75	4.42	
1995:12:00	3.03	45.86	0.54	0.55	2.62	2.27	1.47	
1996:01:00	5.16	17.55	0.45	2.73	2.42	0.57	3.39	
1996:02:00	5.02	43.13	-0.06	2.1	0.19	3.19	1.34	1996
1996:03:00	4.21	11.04	-1.77	0.36	2.12	2.39	0.81	
1996:04:00	1.04	17.34	-0.99	3.3	0.04	7.62	1.5	
1996:05:00	0.19	1.56	-0.54	3.98	2.29	6.49	2.45	
1996:06:00	-0.05	23.78	-0.63	-0.58	-0.35	-4.15	0.3	
1996:07:00	0.99	14.65	0.15	-6.24	-5.25	-9.98	-4.7	
1996:08:00	1.46	-46.81	0.75	2.45	3.64	4.85	2.48	
1996:09:00	2.03	3.63	-0.63	6.67	1.87	3.38	5.75	
1996:10:00	3.13	66.39	0.99	0.22	3.54	-1.79	2.1	
1996:11:00	3.58	25.31	0.90	6.82	5.38	2.63	7.47	
1996:12:00	3.81	86.80	-0.30	-1.53	-0.43	1.85	-1.81	
1997:01:00	3.61	-25.36	-0.78	6.41	3.01	4.24	6.08	
1997:02:00	3.59	72.28	0.39	-0.42	1.93	-2.75	0.54	1997
1997:03:00	3.53	13.02	-1.02	-5.71	-3.94	-5.22	-4.43	
1997:04:00	2.93	-55.06	-0.66	6.06	1.32	-2.58	5.62	
1997:05:00	3.16	59.91	0.57	7.5	6.23	10.86	6.34	
1997:06:00	3.70	69.81	0.57	4.15	3.41	5.67	4.27	
1997:07:00	5.46	61.72	0.79	7.53	6.24	4.65	8.23	
1997:08:00	5.96	73.28	-0.13	-5.11	-1.92	4.11	-5.08	
1997:09:00	6.11	-61.97	0.15	5.38	6.06	9.09	5.35	
1997:10:00	5.26	72.40	0.54	-3.65	-2.61	-3.73	-3.13	
1997:11:00	5.22	-20.80	0.39	4.22	2.78	-1.78	4.43	
1997:12:00	5.33	44.89	0.09	0.79	3.86	-0.88	2	
1998:01:00	6.29	-5.37	1.04	2.71	-0.35	-0.58	0.98	
1998:02:00	6.14	37.54	-0.21	7.27	6.88	6.66	7.3	1998
1998:03:00	5.59	54.39	-0.36	4.6	7.13	4.86	5.19	
1998:04:00	3.41	67.31	0.00	1.14	0.68	1.83	1.25	
1998:05:00	3.05	13.84	-0.06	-2.88	0.45	-4.79	-1.85	
1998:06:00	3.24	-32.57	0.33	5.51	2.14	-0.53	4.26	
1998:07:00	5.13	61.72	0.19	-0.82	-4.33	-6.74	-1.14	
1998:08:00	5.53	-38.34	0.56	-14.94	-12.18	-21.37	-14.67	
1998:09:00	5.61	-134.77	1.95	7.74	5.46	4.73	6.57	
1998:10:00	4.93	-9.54	1.32	8.1	4.48	2.64	7.91	
1998:11:00	4.72	143.40	-1.08	7.12	4	7.7	6.32	
1998:12:00	4.52	66.85	0.27	8.62	3.07	3.79	6.58	
1999:01:00	4.08	53.86	-0.45	5.61	-0.79	4.13	3.93	
1999:02:00	4.12	42.01	-0.95	-4.46	-3.05	-7.09	-3.22	1999
1999:03:00	4.37	-35.27	-0.67	4.05	0.59	-1.75	4.01	
1999:04:00	5.17	54.61	0.18	2.02	8.22	9.6	3.91	
1999:05:00	5.58	55.21	-1.08	-2.4	0.43	1.87	-2.2	
1999:06:00	5.92	-54.71	-1.07	6.71	0.03	6.96	4.99	
1999:07:00	6.56	77.81	0.35	-3.08	-2.13	0.43	-3.35	
1999:08:00	6.54	-46.88	-0.48	-0.33	-2.31	-3.46	-0.51	
1999:09:00	6.22	2.72	0.12	-1.88	-4.19	-0.74	-2.8	
1999:10:00	4.35	-44.31	-0.69	6.84	2.8	-0.74	6.72	
1999:11:00	4.36	64.93	0.18	4.36	-0.77	9.91	2.64	
1999:12:00	5.00	38.02	-0.66	8.08	2.77	12.26	6.54	
2000:01:00	7.83	48.38	-1.17	-4.6	-2.97	3.88	-4.44	
2000:02:00	8.47	-38.49	-0.30	2.07	-2.27	23.75	-0.37	2000
2000:03:00	8.55	-25.90	0.54	7.77	8.14	-8.7	9.38	
2000:04:00	7.00	105.53	0.72	-4.58	1.94	-13.76	-3.79	
2000:05:00	6.80	-30.44	-1.29	-3.47	-0.91	-8.54	-2.64	

2000:06:00	6.85	-15.99	1.17	5.22	-2.22	17.2	3.01	
2000:07:00	7.78	17.05	0.36	-3.08	3.89	-1.64	-1.69	
2000:08:00	7.82	-25.95	0.36	7.57	6.88	6.1	7.74	
2000:09:00	7.64	67.07	0.39	-6.21	0.75	-3.53	-4.9	
2000:10:00	6.17	-68.63	0.45	-2.33	0.74	-6.21	-1.67	
2000:11:00	6.33	-12.61	0.24	-11.02	-0.35	-9.69	-9.71	
2000:12:00	7.02	-93.00	1.59	-0.18	6.86	2.51	1.18	
2001:01:00	9.95	4.60	0.93	3.91	0.3	13.45	3.2	
2001:02:00	10.40	47.40	-0.09	-11.59	2.48	-4.73	-9.61	2001
2001:03:00	10.12	-121.49	0.75	-8.11	-1.64	-3.04	-6.91	
2001:04:00	7.88	-95.93	-0.36	8.91	5.01	6.12	8.1	
2001:05:00	7.18	120.05	-0.51	0.53	4.4	8.28	0.74	
2001:06:00	6.73	-5.48	0.36	-1.68	-0.74	5	-2.15	
2001:07:00	6.98	-23.02	0.15	-1.96	0.41	-4.38	-1.48	
2001:08:00	6.66	-20.34	0.57	-6.96	-5.3	-3.64	-6.38	
2001:09:00	6.24	-84.15	1.35	-8.6	-9.42	-13.82	-8.2	
2001:10:00	5.08	-105.07	0.63	3.53	-2.02	6.24	2.17	
2001:11:00	4.93	51.51	-0.18	8.24	9.77	7.59	7.86	
2001:12:00	5.14	49.65	-1.26	0.95	2.69	7.37	0.99	
2002:01:00	6.28	19.15	0.15	-1.03	-0.47	0.44	-1.6	
2002:02:00	6.77	-27.36	0.12	-2.75	-1.81	-3.93	-2.1	2002
2002:03:00	7.19	10.20	-1.32	3.79	6.21	8.58	3.92	
2002:04:00	7.46	15.55	0.27	-6.82	-3.09	1.7	-6.11	
2002:05:00	7.78	-64.59	0.48	-1.56	-1.06	-3.19	-0.83	
2002:06:00	8.09	-51.66	0.90	-7.35	-7.3	-2.47	-7.45	
2002:07:00	8.81	-86.07	1.14	-6.37	-13.45	-14.74	-7.23	
2002:08:00	8.76	-108.84	1.56	0.2	3.69	-0.96	0.73	
2002:09:00	8.38	41.87	1.05	-10.49	-9.55	-7.69	-10.53	
2002:10:00	7.15	-92.78	-0.03	9.54	2.8	3.44	8.66	
2002:11:00	6.55	72.82	-0.30	5.37	8.06	11.1	5.55	
2002:12:00	6.04	43.90	0.06	-6.38	-4.09	-5.75	-5.72	
2003:01:00	5.61	-72.39	-0.06	-2.66	-4.55	-1.38	-2.76	
2003:02:00	5.26	-26.90	0.45	-1.13	-4.1	-2.86	-1.79	2003
2003:03:00	4.99	-36.12	0.36	1.72	-0.25	1.12	1.08	
2003:04:00	4.57	33.55	-0.45	7.71	9.63	9.94	8.06	
2003:05:00	4.63	78.22	1.23	5.36	8.49	12.69	5.52	
2003:06:00	4.92	64.63	0.75	1.27	1.73	4.02	1.25	
2003:07:00	4.75	18.86	-1.80	2.67	1.31	8.17	1.78	
2003:08:00	6.05	-2.65	-1.50	1.79	4.27	4.94	1.94	
2003:09:00	8.07	33.63	0.57	-0.82	-1.08	0.53	-1.16	
2003:10:00	13.04	12.09	-0.03	5.11	6.87	9.05	5.65	

Covariance Matrix

	Housing (National)	S&P	5y	Growth Stocl	Value Stocks	Small Stocks	Largel Stocks
Mean	5.45	9.38	0.04	1.10	1.40	1.46	1.13
Std.Dev.	3.82	51.72	1.21	4.98	4.32	5.89	4.44
Mean/Std.d	1.43	0.18	0.03	0.22	0.32	0.25	0.25

Correlation Matrix

	Housing (National)	S&P	5y	Growth Stocl	Value Stocks	Small Stocks	Largel Stocks
Housing	1.00	0.00	-0.11	-0.02	0.03	0.10	-0.03
Stocks S&P	0.00	1.00	-0.02	0.05	0.10	0.19	0.04
Bonds 5y	-0.11	-0.02	1.00	0.19	0.23	0.13	0.21
Growth Sto	-0.02	0.05	0.19	1.00	0.79	0.78	0.97
Value Stock	0.03	0.10	0.23	0.79	1.00	0.75	0.86
Small Stock	0.10	0.19	0.13	0.78	0.75	1.00	0.74
Largel Stoc	-0.03	0.04	0.21	0.97	0.86	0.74	1.00

Covariance Matrix

	Housing (National)	S&P	5y	Growth Stocl	Value Stocks	Small Stocks	Largel Stocks
Housing	14.53	0.24	-0.51	-0.37	0.45	2.31	-0.44
Stocks S&P	0.24	2'667.00	-1.14	13.40	22.78	58.14	8.41
Bonds 5y	-0.51	-1.14	1.47	1.12	1.22	0.96	1.14
Growth Sto	-0.37	13.40	1.12	24.72	16.94	22.73	21.49
Value Stock	0.45	22.78	1.22	16.94	18.65	19.06	16.43
Small Stock	2.31	58.14	0.96	22.73	19.06	34.63	19.36
Largel Stoc	-0.44	8.41	1.14	21.49	16.43	19.36	19.68

Table: A1

1975-2003 **η=0%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Reduced Portfolio*		*Full Portfolio*	*Reduced Portfolio*
Sharpe Ratio	1.47	0.36	*Sharpe Ratio*	-0.10	0.37
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.68		*Housing*	230.66	
S&P 500	0.01	0.04	*S&P 500*	-361.26	0.66
5 Year Govn. Bond	0.20	0.00	*5 Year Govn. Bond*	-7.10	-0.90
Growth Stocks	0.00	0.00	*Growth Stocks*	30.73	-1.95
Value Stocks	0.02	0.96	*Value Stocks*	40.79	1.35
Small Stocks	0.00	0.00	*Small Stocks*	33.76	0.20
Large Stocks	0.09	0.00	*Large Stocks*	33.42	1.64
Spanning Test	1'837.70		*Spanning Test*	9.36	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	600.80		*Jobson Korkie Int. Test*	N/A	
P-val	1.00		*P-val*	N/A	
GRS Intersection Test	87.10		*GRS Intersection Test*	N/A	
P-val	1.00		*P-val*	N/A	

Table:A2

1975-2003 **η=1%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Reduced Portfolio*		*Full Portfolio*	*Reduced Portfolio*
Sharpe Ratio	1.18	0.18	*Sharpe Ratio*	1.35	0.35
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.95		*Housing*	3.65	
S&P 500	0.01	0.14	*S&P 500*	0.04	0.28
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-3.33	-6.03
Growth Stocks	0.00	0.00	*Growth Stocks*	-0.02	-0.96
Value Stocks	0.00	0.86	*Value Stocks*	0.90	5.63
Small Stocks	0.00	0.00	*Small Stocks*	-0.75	0.89
Large Stocks	0.04	0.00	*Large Stocks*	0.52	1.20
Spanning Test	432.66		*Spanning Test*	241.21	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int.Test	435.21		*Jobson Korkie Int. Test*	500.64	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	63.11		*GRS Intersection Test*	72.60	
P-val	1.00		*P-val*	1.00	

Table: A3

1975-2003 **η=2%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Reduced Portfolio*		*Full Portfolio*	*Reduced Portfolio*
Sharpe Ratio	0.92	0.14	*Sharpe Ratio*	1.36	0.20
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99		*Housing*	8.50	
S&P 500	0.01	1.00	*S&P 500*	0.09	1.70
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-9.24	-2.59
Growth Stocks	0.00	0.00	*Growth Stocks*	0.71	-0.05
Value Stocks	0.00	0.00	*Value Stocks*	1.47	0.96
Small Stocks	0.00	0.00	*Small Stocks*	-1.75	0.83
Large Stocks	0.00	0.00	*Large Stocks*	1.21	0.16
Spanning Test	5'720.20		*Spanning Test*	265.45	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	265.30		*Jobson Korkie Int.Test*	572.46	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	38.50		*GRS Intersection Test*	83.01	
P-val	1.00		*P-val*	1.00	

Table:A4

1975-2003 **η=3%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Reduced Portfolio*		*Full Portfolio*	*Reduced Portfolio*
Sharpe Ratio	0.65	0.12	*Sharpe Ratio*	1.36	0.19
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99		*Housing*	2.76	
S&P 500	0.01	1.00	*S&P 500*	0.02	2.61
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-2.06	-2.47
Growth Stocks	0.00	0.00	*Growth Stocks*	0.16	-0.16
Value Stocks	0.00	0.00	*Value Stocks*	0.21	0.59
Small Stocks	0.00	0.00	*Small Stocks*	-0.27	0.48
Large Stocks	0.00	0.00	*Large Stocks*	0.18	-0.04
Spanning Test	5'015.80		*Spanning Test*	266.41	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	134.50		*Jobson Korkie Int. Test*	575.34	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	19.50		*GRS Intersection Test*	83.43	
P-val	1.00		*P-val*	1.00	

Table: A5

1975-2003 **η=4%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Reduced Portfolio*		*Full Portfolio*	*Reduced Portfolio*
Sharpe Ratio	0.39	0.10	*Sharpe Ratio*	1.33	0.19
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.98		*Housing*	3.57	
S&P 500	0.02	1.00	*S&P 500*	0.03	3.50
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-2.86	-3.60
Growth Stocks	0.00	0.00	*Growth Stocks*	0.10	-0.26
Value Stocks	0.00	0.00	*Value Stocks*	0.07	0.87
Small Stocks	0.00	0.00	*Small Stocks*	0.06	0.61
Large Stocks	0.00	0.00	*Large Stocks*	0.02	-0.11
Spanning Test	4'489.40		*Spanning Test*	259.40	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	47.40		*Jobson Korkie Int. Test*	554.24	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	6.90		*GRS Intersection Test*	80.37	
P-val	1.00		*P-val*	1.00	

Table B1

RESTRICTION:0,4

1975-2003 **η=0%**

	NO SHORT SELLING ALLOWED		SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	1.47	1.41	*Sharpe Ratio*	-0.10	1.42
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.68	0.40	*Housing*	230.66	0.40
S&P 500	0.01	0.00	*S&P 500*	-361.26	0.01
5 Year Govn. Bond	0.20	0.52	*5 Year Govn. Bond*	-7.10	0.51
Growth Stocks	0.00	0.00	*Growth Stocks*	30.73	-0.08
Value Stocks	0.02	0.02	*Value Stocks*	40.79	0.03
Small Stocks	0.00	0.00	*Small Stocks*	33.76	-0.05
Large Stocks	0.09	0.06	*Large Stocks*	33.42	0.19
Spanning Test	11.25		*Spanning Test*	144.98	
P-val	1.00		*P-val*	0.00	
Jobson Korkie Int. Test	22.87		*Jobson Korkie Int. Test*	N/A	
P-val	1.00		*P-val*	N/A	
GRS Intersection Test	3.19		*GRS Intersection Test*	N/A	
P-val	1.00		*P-val*	N/A	

Table B2

RESTRICTION:0,4

1975-2003 **η=1%**

	NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED	
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	1.18	0.82	*Sharpe Ratio*	1.35	0.83
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.95	0.40	*Housing*	26'462.00	0.40
S&P 500	0.01	0.01	*S&P 500*	411.10	0.01
5 Year Govn. Bond	0.00	0.44	*5 Year Govn. Bond*	-33'342.00	0.43
Growth Stocks	0.00	0.00	*Growth Stocks*	-239.46	-0.16
Value Stocks	0.00	0.11	*Value Stocks*	8'962.10	0.10
Small Stocks	0.00	0.00	*Small Stocks*	-7'475.64	-0.05
Large Stocks	0.04	0.04	*Large Stocks*	5'222.90	0.27
Spanning Test	N/A		*Spanning Test*	100.81	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	147.41		*Jobson Korkie Int. Test*	231.16	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	20.57		*GRS Intersection Test*	32.26	
P-val	1.00		*P-val*	1.00	

Table B3

RESTRICTION:0,4

1975-2003 **η=2%**

	NO SHORT SELLING ALLOWED		SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.92	0.37	*Sharpe Ratio*	1.36	0.38
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99	0.40	*Housing*	7.95	0.40
S&P 500	0.01	0.03	*S&P 500*	0.10	0.04
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-9.03	-0.44
Growth Stocks	0.00	0.00	*Growth Stocks*	0.83	-1.03
Value Stocks	0.00	0.57	*Value Stocks*	1.51	0.82
Small Stocks	0.00	0.00	*Small Stocks*	-1.63	0.04
Large Stocks	0.00	0.00	*Large Stocks*	1.27	1.17
Spanning Test	N/A		*Spanning Test*	197.42	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	212.64		*Jobson Korkie Int. Test*	508.14	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	29.67		*GRS Intersection Test*	70.90	
P-val	1.00		*P-val*	1.00	

Table B4

RESTRICTION:0,4

1975-2003 **η=3%**

	NO SHORT SELLING ALLOWED		SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.65	0.16	*Sharpe Ratio*	1.36	0.32
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99	0.40	*Housing*	1.86	0.40
S&P 500	0.01	0.60	*S&P 500*	0.12	4'357.00
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-1.56	-86'183.00
Growth Stocks	0.00	0.00	*Growth Stocks*	0.26	7'726.60
Value Stocks	0.00	0.00	*Value Stocks*	0.31	34'076.00
Small Stocks	0.00	0.00	*Small Stocks*	-0.27	27'425.00
Large Stocks	0.00	0.00	*Large Stocks*	0.28	12'599.00
Spanning Test	62.21		*Spanning Test*	207.17	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	135.67		*Jobson Korkie Int. Test*	539.10	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	18.93		*GRS Intersection Test*	75.22	
P-val	1.00		*P-val*	1.00	

Table B5

RESTRICTION:0,4

1975-2003 **η=4%**

	NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED	
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.39	0.12	*Sharpe Ratio*	1.33	0.21
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.98	0.40	*Housing*	2.67	0.40
S&P 500	0.02	0.60	*S&P 500*	0.18	24'806.00
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-2.61	-54'170.00
Growth Stocks	0.00	0.00	*Growth Stocks*	0.30	257.00
Value Stocks	0.00	0.00	*Value Stocks*	0.17	14'403.00
Small Stocks	0.00	0.00	*Small Stocks*	0.16	12'133.00
Large Stocks	0.00	0.00	*Large Stocks*	0.12	2'571.60
Spanning Test	23.04		*Spanning Test*	216.51	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	47.62		*Jobson Korkie Int. Test*	569.30	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	6.65		*GRS Intersection Test*	79.44	
P-val	1.00		*P-val*	1.00	

Table B6

RESTRICTION:0,5

1975-2003 **η=0%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	1.47	1.45	*Sharpe Ratio*	-0.1049	1.47
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.68	0.50	*Housing*	229.80	0.50
S&P 500	0.01	0.00	*S&P 500*	-361.01	0.01
5 Year Govn. Bond	0.20	0.41	*5 Year Govn. Bond*	-7.30	0.40
Growth Stocks	0.00	0.00	*Growth Stocks*	30.00	-0.07
Value Stocks	0.02	0.02	*Value Stocks*	40.01	0.04
Small Stocks	0.00	0.00	*Small Stocks*	37.00	-0.07
Large Stocks	0.09	0.07	*Large Stocks*	32.50	0.20
Spanning Test	3.17		*Spanning Test*	N/A	
P-val	0.92		*P-val*	N/A	
Jobson Korkie Int. Test	6.38		*Jobson Korkie Int. Test*	N/A	
P-val	0.99		*P-val*	N/A	
GRS Intersection Test	0.89		*GRS Intersection Test*	N/A	
P-val	0.49		*P-val*	N/A	

Table B7

RESTRICTION:0,5

1975-2003 **η=1%**

	NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED	
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	1.18	0.96	*Sharpe Ratio*	1.35	0.97
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.95	0.50	*Housing*	3.37	0.50
S&P 500	0.01	0.01	*S&P 500*	0.07	0.01
5 Year Govn. Bond	0.00	0.35	*5 Year Govn. Bond*	-3.29	0.34
Growth Stocks	0.00	0.00	*Growth Stocks*	-0.02	-0.13
Value Stocks	0.00	0.09	*Value Stocks*	1.00	0.09
Small Stocks	0.00	0.00	*Small Stocks*	-0.65	-0.07
Large Stocks	0.04	0.06	*Large Stocks*	0.52	0.26
Spanning Test	N/A		*Spanning Test*	70.39	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	83.58		*Jobson Korkie Int. Test*	155.17	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	11.66		*GRS Intersection Test*	21.65	
P-val	1.00		*P-val*	1.00	

Table B8

RESTRICTION:0,5

1975-2003 **η=2%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.92	0.52	*Sharpe Ratio*	1.36	0.52
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99	0.50	*Housing*	8.50	0.50
S&P 500	0.01	0.02	*S&P 500*	0.09	0.02
5 Year Govn. Bond	0.00	0.13	*5 Year Govn. Bond*	-9.24	0.11
Growth Stocks	0.00	0.00	*Growth Stocks*	0.71	-0.36
Value Stocks	0.00	0.34	*Value Stocks*	1.47	0.28
Small Stocks	0.00	0.00	*Small Stocks*	-1.75	-0.04
Large Stocks	0.00	0.01	*Large Stocks*	1.21	0.50
Spanning Test	N/A		*Spanning Test*	169.28	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	155.25		*Jobson Korkie Int. Test*	421.87	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	21.66		*GRS Intersection Test*	58.87	
P-val	1.00		*P-val*	1.00	

Table B9

RESTRICTION:0,5

1975-2003 **η=3%**

	NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED	
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.65	0.21	*Sharpe Ratio*	1.36	0.34
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99	0.50	*Housing*	2.61	0.50
S&P 500	0.01	0.06	*S&P 500*	0.04	745.53
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-2.06	-20'893.00
Growth Stocks	0.00	0.00	*Growth Stocks*	0.18	-950.46
Value Stocks	0.00	0.44	*Value Stocks*	0.30	12'416.00
Small Stocks	0.00	0.00	*Small Stocks*	-0.27	6'406.01
Large Stocks	0.00	0.00	*Large Stocks*	0.20	2'276.42
Spanning Test	N/A		*Spanning Test*	204.97	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	125.95		*Jobson Korkie Int. Test*	532.08	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	17.57		*GRS Intersection Test*	74.24	
P-val	1.00		*P-val*	1.00	

Table B10

RESTRICTION:0,5

1975-2003 **η=4%**

NO SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*
Sharpe Ratio	0.39	0.13
Optimal Portfolio Weights		
Housing	0.98	0.50
S&P 500	0.02	0.50
5 Year Govn. Bond	0.00	0.00
Growth Stocks	0.00	0.00
Value Stocks	0.00	0.00
Small Stocks	0.00	0.00
Large Stocks	0.00	0.00
Spanning Test	22.62	
P-val	1.00	
Jobson Korkie Int. Test	46.72	
P-val	1.00	
GRS Intersection Test	6.52	
P-val	1.00	

SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*
Sharpe Ratio	1.33	0.23
Optimal Portfolio Weights		
Housing	3.00	0.50
S&P 500	0.03	26'939.00
5 Year Govn. Bond	-2.28	-88'809.00
Growth Stocks	0.10	3'553.20
Value Stocks	0.07	27'474.00
Small Stocks	0.06	23'328.00
Large Stocks	0.02	7'515.30
Spanning Test	214.03	
P-val	1.00	
Jobson Korkie Int. Test	561.21	
P-val	1.00	
GRS Intersection Test	78.31	
P-val	1.00	

Table B11

RESTRICTION:0,6

1975-2003 **η=0%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	1.47	1.47	*Sharpe Ratio*	-0.10	1.49
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.68	0.60	*Housing*	230.66	0.60
S&P 500	0.01	0.01	*S&P 500*	-361.26	0.01
5 Year Govn. Bond	0.20	0.30	*5 Year Govn. Bond*	-7.10	0.29
Growth Stocks	0.00	0.00	*Growth Stocks*	30.73	-0.07
Value Stocks	0.02	0.02	*Value Stocks*	40.79	0.05
Small Stocks	0.00	0.00	*Small Stocks*	33.76	-0.09
Large Stocks	0.09	0.08	*Large Stocks*	33.42	0.22
Spanning Test	0.47		*Spanning Test*	N/A	
P-val	0.50		*P-val*	N/A	
Jobson Korkie Int. Test	0.93		*Jobson Korkie Int. Test*	N/A	
P-val	0.67		*P-val*	N/A	
GRS Intersection Test	0.13		*GRS Intersection Test*	N/A	
P-val	0.00		*P-val*	N/A	

Table B12

RESTRICTION:0,6

1975-2003 **η=1%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	1.18	1.05	*Sharpe Ratio*	1.35	1.07
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.95	0.60	*Housing*	3.10	0.60
S&P 500	0.01	0.01	*S&P 500*	0.04	0.01
5 Year Govn. Bond	0.00	0.24	*5 Year Govn. Bond*	-3.33	0.23
Growth Stocks	0.00	0.00	*Growth Stocks*	-0.02	-0.12
Value Stocks	0.00	0.08	*Value Stocks*	1.14	0.10
Small Stocks	0.00	0.00	*Small Stocks*	-0.55	-0.09
Large Stocks	0.04	0.07	*Large Stocks*	0.62	0.27
Spanning Test	N/A		*Spanning Test*	51.75	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	46.66		*Jobson Korkie Int. Test*	111.29	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	6.51		*GRS Intersection Test*	15.53	
P-val	1.00		*P-val*	1.00	

Table B13

RESTRICTION:0,6

1975-2003 **η=2%**

NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.92	0.65	*Sharpe Ratio*	1.36	0.6651
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99	0.60	*Housing*	8.03	0.60
S&P 500	0.01	0.01	*S&P 500*	0.09	0.01
5 Year Govn. Bond	0.00	0.11	*5 Year Govn. Bond*	-8.77	0.09
Growth Stocks	0.00	0.00	*Growth Stocks*	0.71	-0.26
Value Stocks	0.00	0.24	*Value Stocks*	1.47	0.22
Small Stocks	0.00	0.00	*Small Stocks*	-1.75	-0.07
Large Stocks	0.00	0.04	*Large Stocks*	1.21	0.42
Spanning Test	N/A		*Spanning Test*	138.67	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	98.54		*Jobson Korkie Int. Test*	333.24	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	13.75		*GRS Intersection Test*	46.50	
P-val	1.00		*P-val*	1.00	

Table B14

RESTRICTION:0,6

1975-2003 **η=3%**

	NO SHORT SELLING ALLOWED			SHORT SELLING ALLOWED	
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.65	0.33	*Sharpe Ratio*	1.36	0.38
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.99	0.60	*Housing*	2.76	0.60
S&P 500	0.01	0.03	*S&P 500*	0.02	0.06
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-2.06	-1.15
Growth Stocks	0.00	0.00	*Growth Stocks*	0.16	-1.52
Value Stocks	0.00	0.37	*Value Stocks*	0.21	1.25
Small Stocks	0.00	0.00	*Small Stocks*	-0.27	0.05
Large Stocks	0.00	0.00	*Large Stocks*	0.18	1.71
Spanning Test	N/A		*Spanning Test*	197.67	
P-val	N/A		*P-val*	1.00	
Jobson Korkie Int. Test	100.19		*Jobson Korkie Int. Test*	508.93	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	13.98		*GRS Intersection Test*	71.01	
P-val	1.00		*P-val*	1.00	

Table B15

RESTRICTION:0,6

1975-2003 **η=4%**

	NO SHORT SELLING ALLOWED		SHORT SELLING ALLOWED		
	Full Portfolio	*Restricted Portfolio*		*Full Portfolio*	*Restricted Portfolio*
Sharpe Ratio	0.39	0.15	*Sharpe Ratio*	1.33	0.31
Optimal Portfolio Weights			*Optimal Portfolio Weights*		
Housing	0.98	0.60	*Housing*	3.57	0.60
S&P 500	0.02	0.40	*S&P 500*	0.03	19'004.00
5 Year Govn. Bond	0.00	0.00	*5 Year Govn. Bond*	-2.86	-244'360.60
Growth Stocks	0.00	0.00	*Growth Stocks*	0.10	22'323.00
Value Stocks	0.00	0.00	*Value Stocks*	0.07	91'159.00
Small Stocks	0.00	0.00	*Small Stocks*	0.06	77'941.00
Large Stocks	0.00	0.00	*Large Stocks*	0.02	33'934.00
Spanning Test	21.94		*Spanning Test*	202.16	
P-val	1.00		*P-val*	1.00	
Jobson Korkie Int. Test	45.28		*Jobson Korkie Int. Test*	523.12	
P-val	1.00		*P-val*	1.00	
GRS Intersection Test	6.32		*GRS Intersection Test*	72.99	
P-val	1.00		*P-val*	1.00	

Table C

Hedging demand components

	Restriction 0,4	Restriction 0,5	Restriction 0,6
η=0%			
Housing	-0.28	-0.18	-0.08
S&P 500	0.00	0.00	0.00
5 Year Govn. Bond	0.31	0.21	0.09
Small Stocks	0.00	0.00	0.00
Large Stocks	0.00	0.00	0.00
Small Stocks	0.00	0.00	0.00
Large Stocks	-0.03	-0.02	-0.01
η=1%			
Housing	-0.55	-0.45	-0.35
S&P 500	0.00	0.00	0.00
5 Year Govn. Bond	0.44	0.35	0.24
Small Stocks	0.00	0.00	0.00
Large Stocks	0.11	0.09	0.08
Small Stocks	0.00	0.00	0.00
Large Stocks	0.01	0.02	0.03
η=2%			
Housing	-0.59	-0.49	-0.39
S&P 500	0.02	0.00	0.00
5 Year Govn. Bond	0.00	0.13	0.11
Small Stocks	0.00	0.00	0.00
Large Stocks	0.57	0.34	0.24
Small Stocks	0.00	0.00	0.00
Large Stocks	0.00	0.01	0.04
η=3%			
Housing	-0.59	-0.49	-0.39
S&P 500	0.59	0.05	0.02
5 Year Govn. Bond	0.00	0.00	0.00
Small Stocks	0.00	0.00	0.00
Large Stocks	0.00	0.44	0.37
Small Stocks	0.00	0.00	0.00
Large Stocks	0.00	0.00	0.00
η=4%			
Housing	-0.58	-0.48	-0.38
S&P 500	0.58	0.48	0.38
5 Year Govn. Bond	0.00	0.00	0.00
Small Stocks	0.00	0.00	0.00
Large Stocks	0.00	0.00	0.00
Small Stocks	0.00	0.00	0.00
Large Stocks	0.00	0.00	0.00