



**University of Piraeus  
MSc in Banking and Finance  
Department of Banking and Financial Management  
July 2007**

**Master thesis:**

**“LIQUIDITY AND STOCK PRICE VOLATILITY:  
EVIDENCE FROM THE GREEK STOCK MARKET”**

**by  
VASILEIOS ANDRIKOPOULOS  
MXRH/0501**



**Committee members:**

**CHRISTOU CHRISTINA (SUPERVISOR)  
TSAGARAKIS NIKOLAOS  
SKIADOPOULOS GEORGIOS**

**University of Piraeus  
MSc in Banking and Finance  
Department of Banking and Financial Management  
June 2007**

**Master thesis:**

**“LIQUIDITY AND STOCK PRICE VOLATILITY:  
EVIDENCE FROM THE GREEK STOCK MARKET”**

**by  
VASILEIOS ANDRIKOPOULOS  
MXRH/0501**

### **Acknowledgments**

I would like to thank my supervisor Dr Christou Christina for her continuous support and encouragement throughout this project. I would also like to thank all the professors of the Department of Banking and Financial Management for the knowledge and inspiration that they provided me with during the two years I attended this post-graduate course in Banking and Finance.

This dissertation is dedicated in its entirety to my family for their unconditional support and encouragement, as well as for their belief in my potential during the preparation and completion of this study.

## CONTENTS

## Pages

Abstract.....	7
1. Introduction.....	8
2. Literature related to "Liquidity and Asset Pricing Theory" .....	9
3. Market liquidity proxies.....	20
3.1 Bid-Ask Spread.....	20
3.2 Stock Turnover.....	21
3.3 Illiquidity Ratio.....	21
3.4 Liquidity Ratio.....	22
3.5 Return Reversal.....	23
3.6 Standardized Turnover.....	23
4. Literature related to "Trading Activity and Stock Price Volatility".....	24
5. Measures of volatility.....	37
5.1 Conditional Heteroscedastic Models.....	38
5.1.1 The ARCH model.....	38
5.1.2 The GARCH model.....	38
5.1.3 The GARCH-M model.....	38
5.1.4 The Integrated GARCH model.....	39
5.1.5 The Exponential GARCH model.....	40
5.1.6 The Stochastic Volatility Model.....	41
5.1.7 The Long-Memory Stochastic Volatility Model.....	41
5.2 Realized Volatility.....	42
5.2.1 Intraday Returns.....	42
5.2.2 Historical volatility.....	42
5.2.3 Alternative measures of volatility.....	43
6. Data.....	43
7. Methodology.....	46
7.1 The Heteroskedastic Mixture Model and ARCH.....	46
7.2 GMM estimation.....	47
8. Empirical results.....	48
8.1 The Heteroskedastic Mixture Model and ARCH.....	48
8.2 GMM estimation.....	66
8.3 Conclusions.....	72
9. Summary.....	75
References.....	76

## LIST OF TABLES

**Table1** Companies of the FTSE-20 index included in the sample and period of quotation.

**Table2** Companies of the MIDCAP 40 index included in the sample and period of quotation.

**Table3** Companies of the SMALLCAP 80 index included in the sample and period of quotation.

**Table4** Maximum Likelihood Estimates of GARCH (1,1) Model without Liquidity for the FTSE-20 stocks.

**Table5** Maximum Likelihood Estimates of GARCH (1,1) Model without Liquidity for the MIDCAP 40 stocks.

**Table6** Maximum Likelihood Estimates of GARCH (1,1) Model without Liquidity for the SMALLCAP 80 stocks.

**Table7** Maximum Likelihood Estimates of GARCH (1,1) Model with the illiquidity ratio liquidity measure for the FTSE-20 stocks.

**Table8** Maximum Likelihood Estimates of GARCH (1,1) Model with the illiquidity ratio liquidity measure for the MIDCAP 40 stocks.

**Table9** Maximum Likelihood Estimates of GARCH (1,1) Model with the illiquidity ratio liquidity measure for the SMALLCAP 80 stocks.

**Table10** Maximum Likelihood Estimates of GARCH (1,1) Model with the return reversal  $\gamma$  liquidity measure for the FTSE-20 stocks.

**Table11** Maximum Likelihood Estimates of GARCH (1,1) Model with the return reversal  $\gamma$  liquidity measure for the MIDCAP 40 stocks.

**Table12** Maximum Likelihood Estimates of GARCH (1,1) Model with the return reversal  $\gamma$  liquidity measure for the SMALLCAP 80 stocks.

**Table13** Maximum Likelihood Estimates of GARCH (1,1) Model with the stock turnover liquidity measure for the FTSE-20 stocks.

**Table14** Maximum Likelihood Estimates of GARCH (1,1) Model with the stock turnover liquidity measure for the MIDCAP 40 stocks.

**Table15** Maximum Likelihood Estimates of GARCH (1,1) Model with the stock turnover liquidity measure for the SMALLCAP 80 stocks.

**Table16** Maximum Likelihood Estimates of GARCH (1,1) Model with the standardized turnover LM1 liquidity measure for the FTSE-20 stocks.

**Table17** Maximum Likelihood Estimates of GARCH (1,1) Model with the standardized turnover LM1 liquidity measure for the MIDCAP 40 stocks.

**Table18** Maximum Likelihood Estimates of GARCH (1,1) Model with the standardized turnover LM1 liquidity measure for the SMALLCAP 80 stocks.

**Table19** Maximum Likelihood Estimates of GARCH (1,1) Model with the standardized turnover LM12 liquidity measure for the FTSE-20 stocks.

**Table20** Maximum Likelihood Estimates of GARCH (1,1) Model with the standardized turnover LM12 liquidity measure for the MIDCAP 40 stocks.

**Table21** Maximum Likelihood Estimates of GARCH (1,1) Model with the standardized turnover LM12 liquidity measure for the SMALLCAP 80 stocks.

**Table 22** Regression of historical volatility and the illiquidity ratio, return reversal and stock turnover liquidity measures for the FTSE-20 stocks.

**Table 23** Regression of historical volatility and the standardized turnover LM1 and LM12 liquidity measures for the FTSE-20 stocks.

**Table 24** Regression of historical volatility and the illiquidity ratio, return reversal and stock turnover liquidity measures for the MIDCAP 40 stocks.

**Table 25** Regression of historical volatility and the standardized turnover LM1 and LM12 liquidity measures for the MIDCAP 40 stocks.

**Table 26** Regression of historical volatility and the illiquidity ratio, return reversal and stock turnover liquidity measures for the SMALLCAP 80 stocks.

**Table 27** Regression of historical volatility and the standardized turnover LM1 and LM12 liquidity measures for the SMALLCAP 80 stocks.

**Table 28** Number of shares per index for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model.

**Table 29** Percentage of shares per index for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model.

**Table 30** Total number of shares for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model.

**Table 31** Total percentage of shares for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model.

**Table 32** Number of shares per index for which every liquidity measure is statistical significant when included in the GMM Estimation Model.

**Table 33** Percentage of shares per index for which every liquidity measure is statistical significant when included in the GMM Estimation Model.

**Table 34** Total number of shares for which every liquidity measure is statistical significant when included in the GMM Estimation Model.

**Table 35** Total percentage of shares for which every liquidity measure is statistical significant when included in the GMM Estimation Model.

## **Abstract**

The main purpose of this paper is to examine the relationship between liquidity and stock return volatility in the Greek stock market. The motivation for this study was provided by the growing interest in liquidity that has emerged in the asset pricing literature over recent years. We use five measures of liquidity in order to investigate the relation between liquidity and the volatility of share prices. The one proposed by *Pastor and Stambaugh (2001)* is associated with the strength of volume-related return reversals, the second is the illiquidity ratio, as employed by *Amihud (2002)*, which is defined as the daily ratio of absolute stock return to its dollar volume, averaged over some period, the third is the turnover rate proposed by *Datar, Naik and Radcliffe (1998)*, which is defined as the number of shares traded divided by the number of shares outstanding in the stock and last is the standardized turnover LM1 and LM12 liquidity measure proposed by *Liu (2006)*. Then we test how stock return volatility is influenced when each of the five liquidity proxies is included in the conditional variance equation of the GARCH model and in the linear statistical model of the GMM estimation method.

## 1.Introduction

In asset pricing theory, various models have been developed to describe the cross-section of expected returns. Sharpe (1964), Lintner (1965) and Black (1972) proposed the traditional Capital Asset Pricing Model (CAPM) which argues that market beta is the only risk factor to explain the cross-sectional variation of expected stock returns, and it was successfully proved in empirical work because every investment strategy which seemed to provide a high average, turned out to also have a high beta. Later, Fama and French (1992) claimed that the CAPM has no explanatory power regarding the cross-sectional expected returns, while size and book-to-market ratio have an important role. In this sense, Fama and French (1993) argued that the apparent superior returns of the size portfolios and book-to-market portfolios represent compensation for extra-market risk. As a result, they proposed a three-factor model in which the three factors are (i) the excess return on a broad market portfolio; (ii) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks; (iii) the difference between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks.

In recent financial literature, the question that has been widely documented is whether liquidity determines expected returns. In standard asset pricing theory, it is generally accepted that expected stock returns are related cross-sectionally to return' sensitivities to state variables with pervasive effects on investors' overall welfare. Liquidity appears to be a good candidate for a priced state variable. Financial researchers like Pastor and Stambaugh (2003) have developed liquidity-adjusted asset pricing models that include the three factors of Fama and French (1993) and a liquidity factor, in order to examine the relationship between liquidity and expected stock returns. Their results show that liquidity plays a significant role in asset pricing.

Pastor and Stambaugh (2003) describe liquidity as a broad and elusive concept that generally denotes the ability to trade large quantities quickly, at low cost, and without moving the price. Liu (2006) points out that this description highlights four dimensions to liquidity, namely, trading quantity, trading speed, trading cost, and price impact.

Liquidity is an important feature of the investment environment and macroeconomy. It varies over time both for individual stocks and for the market as a whole and the possibility that might disappear from a market, and so not be available when it is needed, is a big source of risk to investors. It seems reasonable that since investors care about holding period returns net of trading costs, less liquid (and more costly to trade) assets need to provide higher gross returns compared to more liquid assets. Liquidation is costlier when liquidity is lower, and those greater costs are especially unwelcome to an investor whose wealth has already dropped and who thus has higher marginal utility of wealth. Unless the investor expects higher returns from holding these assets, he would prefer assets less likely to require liquidation when liquidity is low, even if the latter assets are just as likely to require liquidation on average.



In recent years, there has also been a renewed interest in the relation between trading activity and stock price volatility. In a market with asymmetrically informed agents, trades convey information and cause a persistent impact on security price. By observing trading activity, the market maker gradually learns the information held by informed traders and adjusts prices to reflect his expectation of the security value conditional on all available information including prior trades. Price dynamics are therefore driven by the mechanism of information learning.

Many researchers, using volume as a proxy for information arrival, have developed models in order to investigate the relation between information arrival and return volatility. Clark (1973) suggests the mixture of distribution hypothesis (MDH) model where return and trading volume are driven by the same underlying latent news arrival, or information flow, variable so that the arrival of unexpected 'good news' results in a price increase, whereas the arrival of 'bad news' results in a price decrease and concludes that trading volume and return volatility are positive correlated. Lamoureux and Lastrapes (1990) using trading volume as a proxy for daily information arrival, find that volatility persistence vanishes under the presence of trading volume series in the conditional variance equation of the GARCH model, while Huang and Masulis (2003) use the GMM estimation method to examine if price volatility is strongly impacted by trade frequency and by trade size.

Our purpose in this study is to make a combination of these two very important issues. Specifically, we investigate the role of liquidity in the process that generates stock return volatility in the FTSE-20, MIDCAP 40 and SMALLCAP 80 index of the Greek Stock Market. For this purpose we construct five liquidity measures and include them in the conditional variance equation of the GARCH model and in the linear statistical model of the GMM estimation method. Consequently, we obtain the significance of the various liquidity measures and define their relationship with return volatility.

The remainder of the study is organized as follows. Section 2 contains a brief overview of the existing literature related to "liquidity and asset pricing theory". Section 3 refers to the various liquidity measures proposed by financial researchers. Section 4 contains a brief overview of the existing literature related to "trading activity and stock price volatility". Section 5 explains how volatility can be modelled or measured. Section 6 describes the data set. In Section 7 the models used in the paper are specified. Section 8 presents the empirical results on the liquidity-return volatility for various liquidity measures and provides a discussion of the findings and their implications. Finally, section 9 contains the summary of the study.

## **2.Literature related to "Liquidity and Asset Pricing Theory"**

One of the first researches that examine the relationship between liquidity and asset pricing is the paper by **Amihud and Mendelson (1986)**. In their paper they study the effect of the bid-ask spread on asset pricing. They

analyze a model in which investors with different expected holding periods trade assets with different relative spreads. They mention that illiquidity can be measured by the cost of immediate execution. An investor willing to transact faces a tradeoff: He may either wait to transact at a favourable price or insist on immediate execution at the current bid or ask price. The quoted ask (offer) price includes a premium for immediate buying, and the bid price similarly reflects a concession required for immediate sale. Thus, a natural measure of illiquidity is the spread between the bid and ask prices, which is the sum of the buying premium and the selling concession. Indeed, the relative spread on stocks has been found to be negatively correlated with liquidity characteristics such as the trading volume, the number of shareholders, the number of market makers trading the stock and the stock price continuity.

They suggest that expected asset returns are increasing in the (relative) bid-ask spread. They first model the effects of the spread on asset returns. Their model predicts that higher-spread assets yield higher expected returns, and that there is a clientele effect whereby investors with longer holding periods select assets with higher spreads. The resulting testable hypothesis is that asset returns are an increasing and concave function of the spread. Their model also predicts that expected returns net of trading costs increase with the holding period, and consequently higher-spread assets yield higher net returns to their holders. Hence, an investor expecting a long holding period can gain by holding high-spread assets.

Their data consist of monthly securities returns provided by the Center for research in Security Prices and relative bid-ask spreads collected for the NYSE stocks from *Fitch's Stock Quotations on the NYSE*. The relative spread is the dollar spread divided by the average of the bid and ask prices at year end. The actual spread variable is the average of the beginning and end-of-year relative spreads for each of the years 1960-1979. The relationship between stock returns, relative risk and spread is tested over the period 1961-1980 and they find that expected stock return increases with the bid-ask spread (positive relationship between expected stock return and illiquidity). However their model does not examine the existence of monthly seasonality in the relationship between expected returns and bid-ask spreads.

**Eleswarapu and Reinganum (1993)** investigate the seasonal behaviour of the liquidity premium in asset pricing. The purpose of their paper is two fold: 1) to investigate the relation between average returns and bid-ask spreads in January and in non-January months, and 2) to determine if Amihud and Mendelson's (1986) empirical results are sensitive to their restrictive portfolio selection criteria.

They test the cross-sectional relation between monthly returns, betas, and the relative bid-ask spread over the period 1961-90 using NYSE firms. They obtain monthly NYSE stock returns from tapes provided by the Center for Research in Security Prices. The relative spread of a stock is the dollar bid-ask spread divided by the average of the bid and the ask prices. As in Amihud and Mendelson's (1986) the average spread is the average of the beginning and end-of-year relative spreads. For 1960-79, the relative spread data are

provided by Stoll and Whaley (1983); for the 1980-89 period, the year-end spread data are obtained from Fitch Investors Service, Inc.

Their results suggest a strong seasonal component. In the 1961-90 period, the liquidity premium is reliably positive only during the month of January. For the non-January months, one cannot detect a positive liquidity premium. That is, the impact of the relative bid-ask spread on asset pricing in non-January months cannot be reliably distinguished from zero. The evidence in their paper, unlike the original Amihud and Mendelson (1986) study, suggests a significant size effect even after controlling for spreads and beta. The restrictive sample selection criteria of Amihud and Mendelson (1986) tend to systematically exclude smaller firms and hence bias the results against finding a size effect. By modifying the portfolio formation technique, Eleswarapu and Reinganum (1993) increase the number of firms included in the analysis by 45%.

**Brennan and Subrahmanyam (1996)** bring together diverse empirical techniques from asset pricing and market microstructure research to examine the return-illiquidity relation. Specifically, they estimate measures of illiquidity from intraday transactions data and use the Fama and French (1993) factors to adjust for risk. The use of transactions data enables them to estimate both the variable (trade-size-dependent) and the fixed costs of transacting. By empirically examining the effects of both variable and fixed components of illiquidity on asset returns they are able to shed light on the importance of the empirical measures of adverse selection in influencing asset returns. Moreover, since there is evidence that the activities of brokerage house analysts increase liquidity (Brennan and Subrahmanyam 1995a), their findings have implications for the social value of security analysis.

They use intraday data from the Institute for the Study of Securities Markets for the years 1984 and 1988 and the methods of Glosten and Harris (1988) and Hasbrouck (1991) to decompose estimated trading costs into variable and fixed components.

They find that estimates of both the variable and the fixed components of the proportional cost of transacting are also significantly positively related to excess returns. The coefficient of the proportional spread, however, is negative, both when it is the only trading cost variable in the regression and when it is included along with our transaction cost variables. The sign of the spread coefficient is inconsistent with the role of this variable as a measure of the cost of transacting. They hypothesize that the spread is proxying for a risk variable associated with price level or firm size that is not captured by the Fama-French three-factor model. Their findings indicate that the explanatory power of the bid-ask spreads appears largely to be due to the effect of (the reciprocal of) the price level. Indeed, the coefficient of the spread is not significant in the presence of the price level variable and the cost of illiquidity variables.

Finally they address the issue of seasonality raised by Eleswarapu and Reinganum (1993). A likelihood ratio test of seasonality leads them to conclude that there are no significant monthly seasonal components in the compensation for their transaction cost measures, the bid-ask spread, or the

inverse price level variable, after allowing for the effect of the Fama-French risk factors.

**Brennan, Chordia and Subrahmanyam (1998)** examine the relation between stock returns, measures of risk, and several non-risk security characteristics, including the book-to-market ratio, firm size, the stock price, the dividend yield, and lagged returns.

Their approach differs from that of Fama and French (1996) in three principal ways. First, rather than specifying the risk factors a priori, they follow the intuition of the APT, that the risk factors should be those which capture the variation of returns in large well-diversified portfolios, and use the principal components approach of Connor and Korajczyk (1988) to estimate risk factors. They then repeat the analysis using the Fama and French factors. Secondly, rather than limiting themselves to the set of firm characteristics that Fama and French have found to be associated with average returns, notably size and book-to-market ratio, they estimate simultaneously the marginal effects of eight firm characteristics, including dividend yield, and measures of market liquidity such as share price and trading volume, as well as lagged returns. Thirdly instead of examining the returns on portfolios, they examine the risk-adjusted returns on individual securities.

When they use only size, book-to-market, and lagged returns as the explanatory variables, they find that these variables are significantly related to expected returns even after risk-adjustment using the Connor and Korajczyk factors. When the analysis is repeated using the Fama and French portfolios as factors, the size and book-to-market effects are attenuated by a factor of about 1/3, and their significance is weakened as well. Expanding the set of explanatory variables, they find that a return-momentum effect persists, and also that there is a negative and significant relation between returns and trading volume, regardless of whether the risk-adjustment is done with the Connor and Korajczyk factors or the Fama and French factors. In addition, the introduction of the trading volume makes the coefficient of the firm size variable positive and significant. The dividend yield variable is significant with the Connor and Korajczyk factors but not with the Fama and French factors.

The basic data consist of monthly returns and other characteristics for a sample of the common stock of companies from NYSE/AMEX and NASDAQ for the period January 1966 to December 1995.

**Jacoby, Fowler and Gottesman (2000)** derive a liquidity-adjusted version of the CAPM based on returns calculated after taking into account the effect of the bid-ask spread. Their model demonstrates that the measure of systematic risk should incorporate liquidity costs (the bid-ask spread).

The contribution of their paper is to demonstrate that beta and liquidity are inseparable. They develop a CAPM-based model by adopting Amihud and Mendelson's (1989) conclusion that the bid-ask spread is the true reason for the existence of the size effect. Their model shows that the true measure of systematic risk, when considering liquidity costs, has to be the one based on net after-spread returns. This theoretical conclusion anticipates that the beta measure and the spread effect are inseparable. By identifying a significant size effect, described by Fama and French (1992), with the spread effect,

they suggest that an after-spread beta may produce significant results for the same period (1963-1990).

The after-spread beta measure they derive is non-linear in the traditional beta. The non-linear specification indicates that rejection of the traditional CAPM is expected, especially when the liquidity effect is significant. This point allows them to contrast the early empirical success of the CAPM obtained by Black et al. (1972), and Fama and MacBeth (1973) against the Fama and French (1992) study. The earlier studies only used data from the high liquid NYSE, while the data used by Fama and French (1992) also include securities from the less liquid AMEX and NASDAQ.

This supposition is further supported by another important result obtained by Kothari et al. (1995), who claims that when betas are estimated annually, a significant relationship is found for the periods 1927-1990, and 1941-1990. This result contradicts Fama and French's (1992) rejection of the CAPM for the same period (1941-1990) based on monthly estimation of the betas. These contradictory results can be explained by the fact that liquidity costs proxied by the bid-ask spread are more prominent for shorter (monthly) holding periods, while their relative importance weakens for longer (annual) holding periods.

They further examine the relationship between the expected return and the future spread cost within the CAPM framework. This positive relationship in their model is found to be convex. This finding differs from Amihud and Mendelson's (1986) concave relationship, but it agrees with empirical evidence obtained by Brennan and Subrahmanyam (1996).

Many investigators have tried to study the relation between liquidity and expected stock returns using alternative liquidity measures.

**Datar, Naik and Radcliffe (1998)** attempt to shed light on the relation between liquidity and asset returns using a proxy for liquidity that is different from the bid-ask spread measure widely used by researchers. The reason for proposing a new proxy for liquidity is two fold. First, the data on bid-ask spread is hard to obtain on a monthly basis over long periods of time (Amihud & Mendelson (1986), and Eleswarapu & Reinganum (1993) use the average of the bid-ask spread at the beginning and at the end of the year as a proxy for the liquidity of a stock through that year). Second, Peterson and Fialkowski (1994) show that the quoted spread is a poor proxy for the actual transactions costs faced by investors and call for an alternative proxy which may do a better job of capturing the liquidity of an asset.

In their paper, they propose the turnover rate of an asset as a proxy for its liquidity. Using the turnover rate as a proxy for liquidity they examine whether stock returns are negatively related to liquidity as predicted by Amihud & Mendelson's model. They investigate if this relation persists after controlling for the firm size, book to market ratio and the firm beta.

Their results support the predictions of Amihud & Mendelson's model. They find that the stock returns are a decreasing function of the turnover rates. The turnover rate is significantly negatively related to stock returns and the negative sign on the turnover variable confirms that illiquid stocks offer higher average returns than liquid stocks. This relation persists after controlling for the firm size, book to market ratio and the firm beta. In

contrast to the findings of Eleswarapu & Reinganum, they don't observe any evidence of January seasonality. In particular, they find that the stock returns are strongly related to the turnover rates throughout the year. Finally, when subdivide their dataset into two halves, they observe that the liquidity effect is significant in the first as well as in the second half.

Their dataset consists of all non-financial firms on the NYSE from July 31, 1962 through December 31, 1991. Monthly returns are collected from the Center of Research in Securities Prices (CRSP) and the book value is extracted from the COMPUSTAT tapes. They calculate the monthly return as a percentage change in the value of one dollar of investment in that stock during month  $t$ . In their dataset, on average there are about 880 stocks in each month.

**Chordia, Subrahmanyam and Anshuman (2001)** document a negative and surprisingly strong relation between average returns and both the level as well as the variability of trading activity, after controlling for the well-known size, book-to-market ratio, and momentum effects, as well as the price level and dividend yield. This negative relation is statistically and economically significant.

Their analysis of the effect of volatility of trading activity on expected returns is motivated by a very plausible reason for the variability of liquidity to be priced, namely, that agents are risk averse and dislike variability in liquidity, so that stocks with greater variability should command higher expected returns. They find that the data does not support this hypothesis. There is reliable evidence that stocks with high variability in trading activity command lower expected returns.

They find that their negative relationship between average returns and the coefficients of variation of both dollar trading volume and share turnover persists after a number of robustness checks. These checks include different definitions of variability in liquidity, performing separate regressions for NYSE, Amex, and NASDAQ stocks, accounting for the Pontiff and Shall (1998) predictor variables, and testing whether their effect serves as a proxy for non-linearities in the relation between the level of liquidity and asset returns.

In their empirical investigation, they use the Brennan, Chordia and Subrahmanyam (1998) methodology to relate expected returns to the volatility of liquidity. Since they do not have data on bid-ask spreads for a length of time sufficient to allow a reliable calculation of standard deviation, they proxy for liquidity by two measures of trading activity: dollar trading volume and share turnover.

Their basic data consist of monthly returns and other characteristics for a sample of the common stock of NYSE and AMEX-listed companies for the period January 1966 to December 1995.

**Amihud (2002)** examines return-illiquidity relationship over time. He proposes that *over time, the ex ante stock excess return is increasing in the expected illiquidity of the stock market.*

The illiquidity measure employed by Amihud, called ILLIQ, is the daily ratio of absolute stock return to its dollar volume, averaged over some period. This measure is interpreted as the daily stock price reaction to a dollar of trading volume. While finer and better measures of illiquidity are available

from market microstructure data on transactions and quotes, ILLIQ can be easily obtained from databases that contain daily data on stock return and volume. This makes ILLIQ available for most stock markets and enables to construct a time series of illiquidity over a long period of time, which is necessary for the study of the effects of illiquidity over time.

His results show that both across stocks and over time, expected stock returns are an increasing function of expected illiquidity. Across NYSE stocks during 1964-1997, ILLIQ has a positive and highly significant effect on expected return. His new tests of the effects of illiquidity over time show that expected market illiquidity has a positive and significant effect on ex ante stock excess return (stock return in excess of the Treasury bill rate), and unexpected illiquidity has a negative and significant effect on contemporaneous stock return. Market illiquidity is the average ILLIQ across stocks in each period, and expected illiquidity is obtained from an autoregressive model. The negative effect of unexpected illiquidity is because higher realized illiquidity raises expected illiquidity, which in turn leads to higher stock expected return. Then, stock prices should decline to make the expected return rise (assuming that corporate cash flows are unaffected by market liquidity). The effects of illiquidity on stock excess return remain significant after including in the model two variables that are known to affect expected stock returns: the default yield premium on low-rated corporate bonds and the term yield premium on long-term Treasury bonds.

The effects over time of illiquidity on stock excess return differ across stocks by their liquidity or size: the effects of both expected and unexpected illiquidity are stronger on the returns of small stock portfolios. This suggests that the variations over time in the "small firm effect"-the excess return on small firms' stock- is partially due to changes in market illiquidity. This is because in times of dire liquidity, there is a "flight to liquidity" that makes large stocks relatively more attractive. The greater sensitivity of small stocks to illiquidity means that these stocks are subject to greater illiquidity risk which, if priced, should result in higher illiquidity risk premium.

**Pastor and Stambaugh (2003)** investigate whether expected returns are related to systematic liquidity risk in returns, as opposed to the level of liquidity per se. Instead of investigating the level of liquidity as a characteristic that is relevant for pricing, their study entertains market-wide liquidity as a state variable that affects expected stock returns because its innovations have effects that are pervasive across common stocks. Their paper focuses on systematic liquidity risk in returns and finds that stocks whose returns are more exposed to market-wide liquidity fluctuations command higher expected returns. Stocks that are more sensitive to aggregate liquidity have substantially higher expected returns, even after accounting for exposures to the market return as well as size, value, and momentum factors.

Liquidity has many dimensions. Their study focuses on a dimension associated with temporary price changes accompanying order flow. They construct a measure of market liquidity in a given month as the equally weighted average of the liquidity measures of the individual stocks on the NYSE and AMEX, using daily data within the month. Their liquidity measure is

also characterized by significant commonality across stocks, supporting the notion of aggregate liquidity as a priced state variable. Smaller stocks are less liquid, according to their measure, and the smallest stocks have high sensitivities to aggregate liquidity.

**Chan and Faff (2003)** examine whether cross-sectional variations in individual stock returns can be explained by differences in liquidity (proxied by share turnover), in the context of the Fama-French variables of size, book-to-market and stock beta for the Australian equity market. They apply the basic framework of Datar et al. (1998)-ensuring comparability with US evidence, and they conduct some robustness checking which addresses two main issues: (a) the role of momentum effects; and (b) the impact of potential nonlinearities.

Their analysis is performed at the monthly level for the period from January 1989 to December 1999, and all returns are continuously compounded. Their data come from two main sources. From the IRESS financial database, they collect for all currently listed companies the volume of shares traded per month, the balance date and the end of financial year balance sheet numbers to calculate a book value for each company. Companies without both a book value and trading activity data on IRESS are deleted from their sample. The remaining companies are matched with the same companies recorded in the Australian Graduate School of Management (AGSM) price relative file. From the AGSM price relative file, they extract the company price relative, the value-weighted market price relatives, the risk-free price relative, the market capitalisation and the number of shares on issue for each company in each month of their sample period.

Their main findings all relate to the asset pricing role of turnover/liquidity and can be summarised as follows. First and foremost, they find for the full sample period, for the two sub-periods, for all months and for the turnover-augmented Fama-French model that stock returns are strongly negatively related to turnover. Second, they find that while the role of turnover may be weakened by January and/or July seasonality, it is not seriously so. Third, they find that the importance of turnover is robust to the inclusion of a momentum factor. Fourth, they find that the role of turnover is not greatly affected by modelling the potential for nonlinear relationships. Fifth, they find that the size effect is not evident in the Australian market over the 1990s, thereby providing an important out-of-sample confirmation of a similar finding in US markets.

**Jun, Marathe and Shawky (2003)** investigate the time-series variation in aggregate liquidity for several emerging equity markets and also examine the cross-sectional behaviour of liquidity across countries.

The primary source for their data is the Emerging Market Database, part of the International Financial Statistics, originally compiled and maintained by the World Bank. Beginning with 1998, the Emerging markets database is being maintained by Standard & Poor's. They use monthly data for 27 emerging equity markets covering the period January 1992 through December 1999. They obtain monthly returns on US equity indices from CRSP. They also use regional Morgan Stanley World Index (MSCI), as a proxy



for the returns on the world market index. For comparability purposes, all their return data are in terms of US dollars.

They find that stock returns in emerging countries are positively correlated with market liquidity as measured by turnover ratio, trading value as well as turnover-volatility multiple. The results hold in both cross-sectional and time-series analysis, and are quite robust even after they control for world market beta, market capitalization and price-to-book ratio. The positive correlation between stock returns and market liquidity in a time-series analysis is consistent with the findings in developed markets.

**Gibson and Mougeot (2004)** focus on a broader definition of systematic liquidity in order to examine whether long term – in their case, monthly – random movements in market liquidity affect stock prices to the extent that their returns covary with changes in market liquidity. They examine the significance and magnitude of systematic liquidity risk pricing for an actively traded well-diversified US stock portfolio, which is the S&P 500 stock market index.

Two important difficulties are related with the concept of aggregate market liquidity risk. First, they need to define a proxy for the state variable describing aggregate market liquidity and second to specify a joint stochastic process for the latter and the excess returns of the market portfolio. They also need a proxy for longer horizons market-wide liquidity shocks. For that purpose, they choose to define the market liquidity as the number of traded shares in the S&P 500 Index during a month. The changes in the state variable are represented by the monthly relative changes in the number of traded shares in the S&P 500 Index.

They further assume that the market excess returns and the liquidity state variable jointly follow a bivariate Garch (1,1) -in- mean process with possibly time-varying unitary market and liquidity risk premia in the general specification of the model. In the latter, the unitary liquidity and market risk premia are driven by a set of instrumental variables that capture business cycles effects on investors' risk aversion.

They use monthly data covering the period from January 1973 – December 1997, for a total of 300 observations. The market excess return is calculated as the difference between the continuously return of the Standard and Poor's 500 composite stock index (S&P 500) and the yield on a one-month treasury bill.

The results suggest that liquidity risk is indeed priced during the entire as well as over sub-periods in the US. The sign of the liquidity risk premium is significantly negative and time-varying. Furthermore, according to these preliminary results, the unitary market risk premium becomes insignificant within the general bivariate Garch (1,1) -in- mean model with constant risk premia. According to their results, systematic liquidity risk dominates market risk and is insensitive to the introduction of extreme liquidity events such as the October'87 Crash.

**Acharya and Pedersen (2005)** present a simple theoretical model that helps explain how asset prices are affected by liquidity risk and commonality in liquidity. In their model, risk-averse agents in an overlapping generations economy trade securities whose liquidity varies randomly over time. They

solve the model explicitly and derive a liquidity-adjusted capital asset pricing model (CAPM). In the liquidity-adjusted CAPM the expected return of a security is increasing in its expected illiquidity and its "net beta", which is proportional to the covariance of its return,  $r^i$ , net of its exogenous illiquidity costs,  $c^i$ , with the market portfolio's net return,  $r^M - c^M$ . The net beta can be decomposed into the standard market beta and three betas representing different forms of liquidity risk. These liquidity risks are associated with: (i) commonality in liquidity with the market liquidity,  $\text{cov}(c^i, c^M)$ ; (ii) return sensitivity to market liquidity,  $\text{cov}(r^i, c^M)$ ; and, (iii) liquidity sensitivity to market returns,  $\text{cov}(c^i, r^M)$ .

They use the illiquidity measure of Amihud (2002) to proxy for  $c^i$ . They employ daily return and volume data from CRSP from July 1<sup>st</sup>, 1962 until December 31<sup>st</sup>, 1999 for all common shares listed on NYSE and AMEX. To keep their liquidity measure consistent across stocks, they do not include Nasdaq since the volume data includes interdealer trades (and only starts in 1982). Also, they use book-to-market data based on the COMPUSTAT measure of book value.

Their model shows that the CAPM applies for returns net of illiquidity costs. This implies that investors should worry about a security's performance and tradability both in market downturns and when liquidity "dries up". Said differently, the required return of a security  $i$  is increasing in the covariance between its illiquidity and the market illiquidity,  $\text{cov}_t(c_{t+1}^i, c_{t+1}^M)$ , decreasing in the covariance between the security's return and the market illiquidity,  $\text{cov}_t(r_{t+1}^i, c_{t+1}^M)$ , and decreasing in the covariance between its illiquidity and market returns,  $\text{cov}_t(c_{t+1}^i, r_{t+1}^M)$ . The model further shows that positive shocks to illiquidity, if persistent, are associated with a low contemporaneous returns and high predicted future returns.

They find that the liquidity-adjusted CAPM fares better than the standard CAPM in terms of  $R^2$  for cross-sectional returns and p-values in specification tests, even though both models employ exactly one degree of freedom. Further, they find weak evidence that liquidity risk is important over and above the effects of market risk and the level of liquidity. The model has a reasonably good fit for portfolios sorted by liquidity, liquidity variation, and size, but it fails to explain the book-to-market effect.

Their model also provides a framework in which they can study the economic significance of liquidity risk. They find that liquidity risk explains about 1.1% of cross-sectional returns when the effect of average liquidity is calibrated to the typical holding period in the data and the model restriction of a single risk premium is imposed. About 80% of this effect is due to the liquidity sensitivity to the market return,  $\text{cov}_t(c_{t+1}^i, c_{t+1}^M)$ , an effect not previously studied in the literature. Freeing up risk premia leads to larger estimates of the liquidity risk premium, but these results are estimated imprecisely because of collinearity between liquidity and liquidity risk.

**Martinez, Nieto, Rubio and Tapia (2005)** in their empirical work analyze whether the Spanish expected returns during the 1990s are associated cross sectionally with betas estimated relative to three competing liquidity risk factors. In particular, they propose a new market-wide liquidity factor that is defined as the difference between the returns of stocks highly

sensitive to changes in the relative bid-ask spread and the returns from stocks with low sensitivities to those changes. They argue that stocks with positive covariability between returns and this factor are assets whose returns tend to go down when aggregate liquidity is low and, hence, do not hedge a potential liquidity crisis. Consequently, investors will require a premium to hold these assets.

Their empirical results show that the liquidity risk factor proposed by Pastor and Stambaugh (2003), which should be associated with the strength of volume-related return reversals because order flow induces greater return reversals when liquidity is lower, does not carry a premium in the Spanish stock market. Furthermore for the liquidity risk factor suggested by Amihud (2002), which is defined for individual stock as the ratio of the daily absolute return to the euro trading volume on that day, they find, both in time series and in the traditional cross-sectional framework, evidence consistent with market-wide liquidity risk being priced. Therefore, given an adequate illiquidity risk factor, it seems that the stochastic discount factor should be linearly related not only to the aggregate wealth return and state variables predicting future returns, but also to aggregate illiquidity risk.

Their data consist of individual daily and monthly returns for all stocks traded on the Spanish continuous market from January 1991 through December 2000. The return of the market is an equally weighted portfolio comprised of all stocks available either in a given month or on a particular day in the sample. The monthly Treasury bill rate observed in the secondary market is used as the risk-free rate when monthly data are needed. All individual stocks are employed to construct three alternative liquidity-based 10 sorted portfolios, and also the traditional 10 portfolios formed according to market value. Data from portfolios are always monthly returns. For the same set of common stocks, they also have daily data on the relative bid-ask spread, depth, and both the number of shares traded and the euro trading volume.

**Marcelo and Quiros (2006)** examine the asset-pricing role of illiquidity in the Spanish stock market. They consider that systematic liquidity shocks should affect the optimal behaviour of agents in financial markets. Indeed, fluctuations in various measures of liquidity are significantly correlated across common stocks. Accordingly their paper empirically analyzes whether Spanish expected returns vary in relation to a liquidity risk factor constructed employing the aggregate ratio of absolute stock returns on euro volume as suggested by Amihud (2002). In particular, illiquidity is defined for each individual stock as the ratio of the daily absolute return on the euro trading volume on that day.

They generate a mimicking portfolio for illiquidity by extending the approximately orthogonalizing procedure of Fama and French (1993) and use it as an augmenting variable in their three-factor model and the standard CAPM. The advantage of this construction is that each factor is formed while controlling for the effects of the other ones.

Their results for the Spanish stock market indicate that time varying expected excess asset returns can be explained by the two asset-pricing models considered when they include the illiquidity risk factor as an

augmenting variable. However, their cross-sectional empirical results show the payment for assuming higher illiquidity risk is mainly limited to the month of January.

Their basic data consist of individual daily and monthly returns for stocks traded on the Spanish Continuous market from January 1994 to December 2002. They also include companies that belong to a high-technology sector and traded on the Spanish "Nuevo Mercado" from January 2000. The number of stocks in the sample range from 140 to 159 during the period analyzed, beginning with 140 stocks in January 1994 and concluding with 146 in December 2002. For the same set of common stocks, they also have daily data on the trading volume (2016 average daily observations per security). This daily data is employed for the monthly calculation of firms' illiquidity ratios.

**Liu (2006)** proposes a new liquidity measure for individual stocks, which he defines as the standardized turnover-adjusted number of zero daily trading volumes over the prior 12 months. This measure captures multiple dimensions of liquidity such as trading speed, trading quantity, and trading cost, with particular emphasis on trading speed, that is, the continuity of trading and the potential delay or difficulty in executing an order. He also develops a liquidity-augmented asset pricing model, a two-factor augmented CAPM, that comprises both market and liquidity factors. Finally he explores the role that liquidity risk plays in explaining the various pricing anomalies documented in the finance literature.

His sample comprises all NYSE/AMEX/NASDAQ ordinary common stocks over the period January 1960 to December 2003. Because trading volumes for NASDAQ stocks are inflated relative to NYSE/AMEX stocks due to interdealer trades, he examines the liquidity effect separately for NYSE/AMEX stocks and NASDAQ stocks, with a comprehensive examination of liquidity based on NYSE/AMEX stocks. Daily trading volume, number of shares outstanding, bid and ask spreads, monthly return, market value, and annual accounting data for calculating the book-to-market, cash flow to price, and earnings to price ratio come from the CRSP/COMPUSTAT merged (CCM) database.

Using the new measure of liquidity he shows that illiquid stocks tend to be small-value and low-turnover stocks with large bid-ask spreads and large absolute return -to-volume ratios, consistent with the intuitive properties of illiquid stocks. The two-factor (market and liquidity) model he develops successfully describes the cross-section of stock returns. It not only captures the liquidity risk that the CAPM and the Fama-French three-factor model fail to explain, but it also provides evidence supporting a liquidity risk-based explanation of various established market anomalies.

### **3. Market Liquidity Proxies**

#### **3.1 Bid-Ask Spread**

The proportional quoted bid-ask spread, typically calculated as the difference between the bid and ask price divided by the bid-ask midpoint, is a

widely used measure of market liquidity. It directly measures the cost of executing a small trade.

$$PQSPR_{it} = \frac{\sum_{d=1}^{D_{it}} (p_{idt}^A - p_{idt}^B) / (0,5 p_{idt}^A + 0,5 p_{idt}^B)}{D_{it}}$$

where  $p_{idt}^A$  and  $p_{idt}^B$  are the ask and bid prices for stock  $i$  on day  $d$  in month  $t$   
 $D_{it}$  is the number of days for stock  $i$  in month  $t$ .

The market-wide proportional quoted bid-ask spread is taken to be the cross sectional average of these stock's monthly proportional quoted spreads.

### 3.2 Stock Turnover

Datar, Naik and Radcliffe (1998) propose the turnover rate of an asset as a proxy for its liquidity. They define the turnover rate of a stock as the number of shares traded divided by the number of shares outstanding in that stock and think of it as an intuitive metric of the liquidity of the stock. The advantage of using the turnover rate as a proxy for liquidity is two-fold. First, it has strong theoretical approach. Amihud and Mendelson (1986) prove that in equilibrium liquidity is correlated with trading frequency. So, if one cannot observe liquidity directly but can observe the turnover rate, then one can use the latter as a proxy for liquidity. Second, the data on turnover rates is relatively easy to obtain (it can be constructed on a monthly basis). This enables us to capture month by month variation in the liquidity of assets and allows the examination of liquidity effects across a large number of stocks over a long period of time.

The monthly turnover measure is the average of daily share turnover:

$$stov_{it} = \frac{\sum_{d=1}^{D_{it}} \{vol_{idt} / no_{idt}\}}{D_{it}}$$

where  $vol_{idt}$  is the euro value of shares traded-volume (or the number of shares traded) of stock  $i$  on day  $d$  in month  $t$

$no_{idt}$  is the number of shares outstanding

$D_{it}$  is the number of observations for stock  $i$  in month  $t$

The market-wide stock turnover liquidity measure is calculated as the cross-sectional of the stocks' monthly stock turnover:

$$stov_t = \frac{\sum_{i=1}^{N_t} \{stov_{it}\}}{N_t}$$

### 3.3 Illiquidity Ratio

The illiquidity measure employed by Amihud (2002) is the daily ratio of absolute stock return to its dollar volume, averaged over some period. It can

be interpreted as the daily price response associated with one dollar of trading volume, thus serving as a rough measure of price impact. There are measures of illiquidity such as the bid ask spread or the probability of information-based trading, which require a lot of microstructure data that are not available in many stock markets. And, even when available, the data do not cover very long periods of time. The illiquidity ratio can be easily obtained from databases that contain daily data on stock return and volume and enables to construct long time series of illiquidity that are necessary to test the effects over time of illiquidity across a large number of stocks. The illiquidity ratio of stock  $i$  in month  $t$  is calculated as:

$$ILLIQ_{it} = \left\{ \sum_{d=1}^{D_{it}} |R_{itd}| / V_{itd} \right\} / D_{it}$$

where  $R_{itd}$  and  $V_{itd}$  are, respectively, the return and volume (euro or share) on day  $d$  in month  $t$ , and  $D_{it}$  is the number of valid observation days in month  $t$  for stock  $i$ . The intuition behind this illiquidity measure is as follows. A stock is illiquid, that is, it has a high value of  $ILLIQ_{it}$  if the stock's price moves a lot in response to little volume.

The market-wide illiquidity ratio is calculated as the cross-sectional of the stocks' monthly illiquidity ratios:

$$ILLIQ_t = \left\{ \sum_{i=1}^{N_t} ILLIQ_{it} \right\} / N_t$$

### 3.4 Liquidity Ratio

The illiquidity ratio is strongly related to the liquidity ratio known as the Amivest measure, the ratio of the sum of the daily volume to the sum of the absolute return. Amihud (1997) and Berkman and Eleswarapu (1998) used the liquidity ratio to study the effect of changes in liquidity on the value of stocks that were subject to changes in their trading methods. The liquidity ratio, however, does not have the intuitive interpretation of measuring the average daily association between a unit volume and the price change, as does  $ILLIQ$ .

The liquidity ratio of stock  $i$  in month  $t$  is calculated as:

$$L_{it} = \left\{ \sum_{d=1}^{D_{it}} V_{itd} / |R_{itd}| \right\} / D_{it}$$

where  $R_{itd}$  and  $V_{itd}$  are, respectively, the return and volume (euro or share) on day  $d$  in month  $t$ , and  $D_{it}$  is the number of valid observation days in month  $t$  for stock  $i$ . The average is taken over all days in the sample where  $R_{itd} \neq 0$ .

### 3.5 Return Reversal

Pastor and Stambaugh (2002) propose a liquidity measure associated with the strength of volume-related return reversals. This liquidity measure for stock  $i$  in month  $t$  is the ordinary-least-squares (OLS) estimate of  $\gamma_{i,t}$  in the regression:

$$R_{i,d+1,t}^e = \alpha_{i,t} + \beta_{i,t} R_{i,d,t} + \gamma_{i,t} \text{sign}(R_{i,d,t}^e) \text{vol}_{i,d,t} + u_{i,d+1,t}$$

where quantities are defined as follows:

$R_{i,d+1,t}^e$  is the excess return with respect to the value weighted market return for stock  $i$  on day  $t+1$  in month  $t$

$R_{i,d,t}$  is the return for stock  $i$  on day  $d$  in month  $t$ , and

$\text{vol}_{i,d,t}$  the dollar volume for stock  $i$  on day  $d$  in month  $t$

A stock's liquidity is computed in a given month only if there are more than 15 observations with which to estimate the above regression ( $D > 15$ ). The basic idea is that "order flow", constructed here simply as volume signed by the contemporaneous return on the stock in excess of the market, should be accompanied by a return that one expects to be partially reversed in the future if the stock is not perfectly liquid. The greater is that expected reversal for a given euro volume, the lower is the stock's liquidity. That is, one would expect  $\gamma_{i,t}$  to be negative in general and larger in absolute magnitude when liquidity is lower. The market-wide return-reversal measure in a given month is the equally weighted average of the return-reversal of individual stocks.

### 3.6 Standardized Turnover

Liu (2006) defines the liquidity measure of a security,  $LMx$ , as the standardized turnover-adjusted number of zero daily trading volumes over the prior  $x$  months ( $x=1,6,12$ ), that is,

$$LMx = \left\{ \text{Number of zero daily volumes in prior } x \text{ months} + \left[ \frac{1}{(x\text{-month turnover})} \right] / \text{Deflator} \right\} \times (21x / \text{NoTD}) \quad (1)$$

where  $x\text{-month turnover}$  is turnover over the prior  $x$  months, calculated as the sum of daily turnover over the prior  $x$  months, daily turnover is the ratio of the number of shares traded on a day to the number of shares outstanding at the end of the day,  $\text{NoTD}$  is the total number of trading days in the market over the prior  $x$  months, and  $\text{Deflator}$  is chosen such that

$$0 < \left[ \frac{1}{(x\text{-month turnover})} \right] / \text{Deflator} < 1 \text{ for all sample stocks.}$$

Given the turnover adjustment (the second term in the elbows of Eq(1)), two stocks with the same integer number of zero daily trading volumes can be distinguished: the one with the larger turnover is most liquid. Thus, the turnover adjustment acts as a tie-breaker when sorting stocks based on the

number of zero daily trading volumes over the prior  $x$  months. Because the number of trading days in a month can vary from 15 to 23, multiplication by the factor  $21x/NoTD$  standardizes the number of trading days in a month to 21, which makes the liquidity measure comparable over time.

The liquidity measure given by Eq.(1) captures multiple dimensions of liquidity, placing particular emphasis on trading speed, which existing research largely ignores. First, the number of zero daily trading volumes over the prior  $x$  months captures the continuity of trading and the potential delay or difficulty in executing an order. In other words, the absence of trade in a security indicates its degree of illiquidity: the more frequent the absence of trade, the less liquid the security. In extreme cases of zero trading volumes the measure captures "lock-in risk", the danger that assets cannot be sold. Second, the turnover adjustment enables the new liquidity measure to capture the dimension of trading quantity. Specifically, conditional on the number of zero trading volume days, stocks with high(low) turnover are more(less) liquid. Third, the new liquidity measure reflects the trading cost dimension of liquidity: the more liquid the stock, the less costly it is for investors to trade. Finally the new liquidity measure is highly correlated with the return-to-volume measure of *Amihud (2002)*, which *Amihud* proposes to capture the price-impact dimension of liquidity.

#### **4.Literature related to "Trading Activity and Stock Price Volatility"**

One of the first studies that attempt to relate price and volume in the stock market is the paper by **Granger and Morgenstern (1963)**. After applying spectral analysis to weekly price and volume data for the period 1939-1961 from the NYSE, they find that there is no connection between the price series and the corresponding volume series.

**Godfrey, Granger and Morgenstern (1964)** extend their previous investigation including daily and transaction data for individual stocks. Although they find that the volume series tend to be a quarter cycle out of phase with the series of lows, the corresponding coherence is too low to attach any significance to this result. Again they conclude that the changes in the price of a stock are not correlated with the volume of transactions.

**Ying (1966)** applies a series of chi-squared tests, analysis of variance, and cross-spectral methods to uncover the relation between stock prices and volume of sales. The data he chooses for his investigation consists of Standard and Poor's 500 composite stocks daily closing price indexes and daily volumes of stock sales on the NYSE from January, 1957 to December, 1962. Ying's significant results are:

- A small volume is usually accompanied by a fall in price.
- A large volume is usually accompanied by a rise in price.
- A large increase in volume is usually accompanied by either a large rise in price or a large fall in price.



- A large volume is usually followed by a rise in price.
- If the volume has been decreasing consecutively for a period of five trading days, then there will be a tendency for the price to fall over the next four trading days.
- If the volume has been increasing consecutively for a period of five trading days, then there will be a tendency for the price to rise over the next four trading days.

**Clark (1973)** examines the relationship between the variability of the daily price change and the volume of trading. His explanation, which is secondary to his effort to explain why the probability distribution of the daily price change is leptokurtic, emphasizes randomness in the number of within-day transactions. In his model the daily price change is the sum of a random number of within-day price changes and thus the variance of the daily price change is a random variable with a mean proportional to the mean number of daily transactions. Finally he argues that the trading volume is related positively to the number of within-day transactions, and so the trading volume is related positively to the variability of the price change.

The data on price, transactions, and volume for cotton futures are in daily form for the years 1945-1958 in *Trade in Cotton Futures*. Except for a brief period during the Korean War when trading was suspended (January 26, 1951 to March 23, 1951) due to price controls, his series are recorded daily and they represent two periods of 1000 observations each. Sample 1 is from January 17, 1947 to August 31, 1950 while sample 2 is from March 24, 1951 to February 10, 1955.

**Copeland (1976)** analyzes asset trading in a world with sequential information arrival. The equilibrium adjustment process is unlike *tatonnement* because it examines the many possible incomplete equilibria between the initial and final equilibria where individuals have identical sets of information. In a world with sequential information arrival the price change between the initial and final equilibria is known with certainty. However, the price adjustment paths as well as the total volume of trading are shown to be random variables. The model he develops uses probability theory to express the expected number of trades generated by a given piece of new information. The conclusions of his sequential information arrival model are:

- There is a positive correlation between the absolute value of price changes and the expected value of trading volume with high values occurring when traders have unanimous opinions about new information and low values occurring where they disagree.
- Trading volume is a logarithmically increasing function of the number of traders, and of the strength of new information.
- If the short sales constraint is binding they observe positive skewness in the distribution of volume with the degree of skewness to increase with the strength of information.
- Trading volume is identical when all traders are optimists or pessimists.

**Epps and Epps (1976)** examine the mechanisms of within-day trading. The change in the market price on each within-day transaction or market clearing is the average of the changes in all of the traders' reservation prices. They assume there is a positive relationship between the extent to which

traders disagree when they revise their reservation prices and the absolute value of the change in the market price. That is, an increase in the extent to which traders disagree is associated with a larger absolute price change. The price variability-volume relationship arises, then, because the volume of trading is positively related to the extent to which traders disagree when they revise their reservation prices. Their results show that the variance of the change in log price depends on volume.

Their data are obtained from "Stocks Sales on the New York Stock Exchange" during the month of January, 1971. They select 20 stocks randomly from a population of 83 NYSE stocks with "asked" quotations of at least \$50 and with at least ten million shares outstanding as of January 29, 1971.

**Jennings, Starks and Fellingham (1981)** develop a model describing the adjustment of an asset market to new information via changes in investors' expectations. They emphasize on the information's impact on asset prices and trading volume. Their model differs from Copeland's sequential information arrival process, which is extended by an equilibrium model that includes a margin requirement as a realistic restriction on short sales. Specifically in their information arrival model the market adjustment process is formulated in an equilibrium analysis derived from a market where each investor maximizes expected utility of terminal wealth under uncertainty. With margin requirements their model predicts a rather complex relationship between price change, volume, and the factors which influence these two variables. Both variables are shown to be sensitive to the number of investors, the mix between optimists, pessimists and uninformed, the costs of the margin requirement, and the actual level of the expectations of each class of investors.

**Tauchen and Pitts (1983)** derive and estimate a more general model of the price change and the trading volume on speculative markets than the model of Clark (1973) and Epps and Epps (1976). The Clark (1973) and Epps and Epps (1976) models are complementary and give considerable insight into the behaviour of speculative markets. Yet the two models provide a description of speculative markets that is incomplete and can be extended in two directions. First, both models work with the conditional distribution of the square of the price change over a short interval of time,  $\Delta P^2$ , given the volume of trading,  $V$ , for the same interval of time. Application of either model requires the investigator to specify in advance or discover by nonlinear regression the functional form of the conditional expectation  $E[\Delta P^2 | V]$ . Second, neither model considers growth in the size of speculative markets such as that experienced by many of the new financial futures markets.

Like the Epps and Epps model, **Tauchen and Pitts (1983)** model begins with an equilibrium theory of within-day price determination. Instead of Epps and Epps's assumption, which gives them a nearly exact positive relationship between the absolute value of the change in the market price and the trading volume on each within-day market clearing, they use a variance-components scheme to model the within-day revisions of traders' reservation prices. This allows them to derive the joint probability distribution of the price change and the trading volume for each within-day market clearing. Adding the random

number of within-day price changes and volumes gives the daily values of each variable. The result is a bivariate normal mixture model with a likelihood function that depends only on a few easily interpreted parameters.

If the number of traders is fixed then their model predicts that the distribution of the daily price change is leptokurtic and that the square of the daily price change is positively related to the daily trading volume. If the numbers of traders is growing then their model predicts that the mean trading volume increases linearly with the numbers of traders. The reason is that the trading volume is one-half of the sum of the absolute changes in the traders' positions; another trader contributes another term to the sum. Their model also predicts that the variance of the price change decreases with more traders. The reason for this is that the market price change during a simple market clearing is the average of the changes in the traders' reservation prices. More terms in the average tend to wash out the effects of inter-trader differences.

Their data are 876 daily observations on price change and trading volume for the 90-day T-bill futures contracts traded at the Chicago Merchantile Exchange. The contracts call for the delivery of a \$1 million face-value U.S. T-bill. Their sample begins on the first day of trading, January 6, 1976, and ends on June 30, 1979. Weekends are treated like overnight periods. They use the exchange's formulas to convert the quoted interest rates into prices. They also use Clark's (1973) method to aggregate the prices for different delivery dates into a price for a simple composite contract. Finally their price data, which are for the composite contract are expressed in thousands of dollars and trading volume, which is the total for all contracts, is expressed in thousand of contracts.

**Karpoff (1987)** investigates the price-volume relation in financial markets and implies the following empirical propositions:

- The correlation between volume and positive price changes is positive.
- The correlation between volume and negative price changes is negative.
- Test using data on volume and the absolute value of price changes will yield positive correlations and heteroskedastic error terms.
- Test using data on volume and price changes per se will yield positive correlations. When ranked by the price change, the residuals from a linear regression of volume on price changes will be autocorrelated.

**Easley and O'Hara (1987)** attempt to develop a formal model of the effect of information on the price-trade size relationship. They show that quantity matters because it is correlated with private information about a security's true value. In particular, they show that an adverse selection problem arises because, given that they wish to trade, informed traders prefer to trade larger amounts at any given price. Since uninformed traders do not share this quantity bias, the larger the trade size, the more likely it is that the market maker is trading with an informed trader. This information effect dictates that the market maker's optimal pricing strategy also depends on quantity, with large trade prices reflecting this increased probability of

information-based trading. In their model, trade size affects security prices because it changes perceptions of the value of the underlying asset.

They also show that the possibility of information-based trading need not always result in a bid-ask spread. Depending on market conditions, such as width (the ratio of large to small trade size) or depth (the fraction of large trades made by the uninformed), informed traders may choose to trade only large quantities, leaving the price for small trades unaffected.

Finally their work identifies a second important effect of information on the price-quantity relation. Although the market maker faces uncertainty about whether any individual trader is informed, there is also uncertainty about whether any new information even exists. This latter uncertainty dictates that both the size and the sequence of trades matter in trading the price-quantity relationship.

**Jain and Joh (1988)** provide empirical evidence on the intraday joint distributions of hourly common stock index returns and trading volume. They use a five-year period to study the day of the week and the hour of the day effects in both returns and trading volume. They also examine the relation between trading volume and the absolute value of returns.

Their data consist of hourly NYSE common stock trading volume and returns for the years 1979 to 1983, comprising 1263 trading days. Since the NYSE exchange was open six hours per day (from 10 a.m. to 4 p.m.) during these years, they obtain a total of 7578 hourly observations. The hourly trading volume data for the entire NYSE are obtained from the Wall Street Journal. Standard & Poor's (S&P) 500 index returns are used as an index of market returns. The hourly returns data are obtained from the Standard & Poor's corporation.

Their results show that the average trading volumes across six trading hours of the day differ significantly. Average volume is highest during the first hour, declines monotonically until the fourth hour, but increases again on the fifth and sixth hours. The average volume across days of the week (for each hour) are also significantly different. Average daily trading volume is lowest on Monday, increases monotonically from Monday to Wednesday, and then declines monotonically on Thursday and Friday.

They also show that common stock returns differ across trading hours of the day. On average, largest stock returns occur during the first (except on Monday) and the last trading hours. The lowest average return is earned in the fifth hour of the day. In particular, average stock returns are significantly negative only during the first hour of Monday.

They also find that there is a positive correlation between contemporaneous trading volume and absolute value of returns (or square of returns). Their result also indicates that the relation between volume and absolute return is significantly different when returns are positive than when returns are nonpositive.

Finally their results show that the hourly trading volume is caused by returns in that up to four lagged prewhitened returns (residuals from ARIMA models) are correlated with prewhitened trading volume. In contrast, there is only weak evidence of causality from volume to returns, as prewhitened volume is weakly correlated with one hour leading prewhitened returns.

**Admati and Pfleiderer (1988)** develop a theory in which concentrated-trading patterns arise endogenously as a result of the strategic behavior of liquidity traders and informed traders. Their results provide a partial explanation for some of the recent empirical findings concerning the patterns of volume and price variability in intraday transaction data. Some of the conclusions of their theory are these:

- In equilibrium, discretionary liquidity trading is typically concentrated.
- If discretionary liquidity traders can allocate their trades across different periods, then in equilibrium their trading is relatively more concentrated in periods closer to the realization of their demands.
- Informed traders trade more actively in periods when liquidity trading is concentrated.
- If information acquisition is endogenous, then in equilibrium more traders become privately informed in periods of concentrated liquidity trading, and prices are more informative in those periods.

**Lamoureux and Lastrapes (1990)** provide empirical support for the notion that Autoregressive Conditional Heteroskedasticity (ARCH) in daily stock return data reflects time dependence in the process generating information flow to the market. They exploit the implication of the mixture model that the variance of daily price increments is heteroskedastic-specifically, positively related to the rate of daily information arrival using daily trading volume as a proxy for the mixing variable.

Their data set comprises daily return and volume data for 20 actively traded stocks. Actively traded stocks are most likely to have a sufficient large number of information arrivals per day to satisfy the conditions for the CLT. Their sample is chosen from a population of stocks for which options trade on the CBOE. They obtain daily stock returns from the 1986 version of the GRSP database, based upon the last daily transaction price of the security. Finally they take daily transactions volume (number of shares traded during the day) for each stock from Standard and Poor's Daily Stock Price Records.

Their results show that daily trading volume have significant explanatory power regarding the variance of daily returns, which is an implication of the assumption that daily returns are subordinated to intraday equilibrium returns. Furthermore, ARCH effects tend to disappear when volume is included in the variance equation.

**Gallant, Rossi and Tauchen (1992)** undertake an empirical investigation of the dynamic interrelationships among price and volume movements on the stock market. They organize their effort around the tasks of estimating and interpreting the conditional one-step-ahead density of joint price change and volume process.

Their data consist of the daily closing value of the S&P composite stock index and the daily volume of shares traded on the NYSE. Price index data for the period from 1928 to 1985 are generously supplied to them by Robert Stambaugh. They extend the price data through 1987. Their volume data are from the Standard and Poor's Security Price Index Record.

Their examination of the fitted conditional density reveals four major findings regarding the interactions between stock prices and volume:

- The daily trading volume is positively and nonlinearly related to the magnitude of the daily price change. This association is a characteristic of both the unconditional distribution of price changes and volume and the conditional distribution given past price changes and volume constant.
- Price changes lead to volume movements. The effect is fairly symmetric, with large price declines having nearly the same impact on subsequent volume as large price increases.
- If volume is excluded from their analysis, then the conditional variance function of the price change given the lagged price change is found to be symmetric over most of the range of the data, but asymmetric in the extreme tails (outermost 10 percent of the data). When volume is introduced in their analysis, the asymmetric response of volatility is found to be mainly a feature of large price movements accompanied by high volume.
- For bivariate price-volume estimation, there is evidence for a positive association between the conditional mean and the conditional variance of daily stock returns.

**Barclay and Warner (1993)** examine the proportion of the cumulative stock-price change that occurs in each trade-size category for a sample of NYSE stocks. Their central hypothesis is that if privately informed traders concentrate their trades in medium sizes, and if stock-price movements are due mainly to private information revealed through these investors' trades, then most of a stock's cumulative price change will take place on medium-size trades. Their tests focus on a sample of tender-offer targets. These firms have large abnormal price increases, on average, before the initial tender-offer announcement. In addition, they believe that some traders have valuable private information during the preannouncement period.

Their sample consists of all NYSE firms that were tender-offer targets between 1981 and 1984. There are 108 tender offers involving 105 different target firms. They find that most of the sample securities' preannouncement cumulative stock-price change occurs on medium-size trades. This evidence is consistent with their hypothesis that informed traders are concentrated in medium sizes and that price movements are due mainly to informed traders' private information. Their results appear more general because they also apply to nonevent period long before the sample securities experience systematic unusual behavior, and to a sample of all NYSE securities.

**Blume, Easley and O'Hara (1994)** investigate the informational role of volume. Their goal in their paper is to determine how the statistical properties of volume relate to the underlying value of the asset and to the behavior of market prices. For this purpose they develop an alternative equilibrium approach for studying the behavior of security markets. Their model is standard in that some fundamental is unknown to all traders and traders receive signals that are informative of the asset fundamental. However, in their model aggregate supply is fixed. The source of noise is the quality of the information; specifically the precision of the signal distribution.

Their results show that volume provides information about the quality of traders' information that cannot be deduced from the price statistic. They also

show that sequences of volume and prices can be informative, and demonstrate that traders who use information contained in the market statistic will do "better" than traders who do not. Their model also demonstrates that volume and the absolute value of price changes are positively correlated, and provides interesting comparative static prediction of the effects of information precision and dispersion on the price-volume relationship. Finally, they show that although traders will learn the asset's value, and prices will thus converge to the full information or strong form efficient price, volume does not converge to zero. In fact, volume's distribution is nondegenerate.

**Hiemstra and Jones (1994)** use linear and nonlinear Granger causality tests to examine the dynamic relation between daily aggregate stock prices and trading volume. They also examine the extent to which the nonlinear predictive power of trading volume for stock returns detected by the modified Baek and Brock (1992) test can be attributed to volume serving as a proxy for the daily flow of new information into the market.

They compute stock returns from daily closing prices for the Dow Jones Price Index. For the period 1915 to 1940, stock returns are based on the Dow Jones Industrial Average. For the period 1941 to 1990, stock returns are based on the Dow Jones 65 Composite Index. The trading volume series is total daily trading volume on the NYSE. The daily stock returns are continuous rates of return, computed as 100 times the first difference of the natural logarithm of the daily stock price,  $P_t$ , in successive time periods; that is,  $100 \cdot \ln(P_t / P_{t-1})$ . Finally they apply the tests for two sample periods (1915 to 1946 and 1947 to 1990).

Their test provides evidence of significant bidirectional nonlinear Granger causality between stock returns and trading volume in both sample periods. After controlling for simply volatility effects, their test continues to provide evidence of significant nonlinear Granger causality from trading volume to stock returns when volume is served as a proxy for daily information flow in the stochastic process generating stock return variance.

**Jones, Kaul and Lipson (1994)** test whether number of transactions per se, or their size (or volume), generates price volatility. For their test they use daily data of NASDAQ-NMS securities over the period 1986-1991 and they calculate the returns from the average of closing bid-ask quotes, rather than transactions prices.

Their evidence show that the volatility-volume relation typically disappears when they control for the relation between volatility and number of transactions. Specifically, daily volatility is significantly positively related to both average daily trade size and number of daily transactions. However, in regressions of volatility on average trade size and number of transactions, the volatility-volume relation is rendered statistically insignificant while the relation between volatility and number of transactions remains virtually unaltered. Average size of trades has a statistically significant positive relation with volatility only for small firms, but on average even this statistical relation seems to be of little economic significance. Thus, their evidence strongly suggests that the occurrence of transactions per se contains all the information pertinent to the pricing of securities.

**Foster and Viswanathan (1995)** use a theoretical model of speculative trading to undertake a detailed examination of the statistical relation between trading volume and price volatility. Their model predicts conditional heteroskedasticity in trading volume and the variance of price changes and positive autocorrelation in trading volume.

To test their model, they use the time series of half-hourly trading volume and quote midpoint changes for IBM in 1988. With 6 and a half hour of trading a day and 253 trading days in 1988, they have 3,289 observations of price and volume data. Their data are taken from the ISSM tapes. For transactions in each half-hour interval, they compute the trading volume and quote midpoint. Then they compute the sum, on a half-hourly basis, of the absolute trading volume and changes in the quote midpoints. Finally they use quotes that are least five seconds older than the transaction to determine which bid-ask quote is available for each transaction.

They test their model using the simulated method of moments (SMM). From their results it appears that many informed traders pay little to receive relatively imprecise information and that the bulk of trading comes due to intense competition between these informed traders. Moreover, it appears that their model is unable to explain the relation between current trading volume and lags of trading volume and squared volume's (and its lag's) relation to squared price changes. After scaling these values by their standard errors it is less clear that these moment conditions are responsible for the model's demise.

**Andersen (1996)** develops a model of the daily return-volume relationship by integrating the market microstructure setting of Glosten and Milgrom (1985) with the stochastic volatility, information flow perspective of the "Mixture of Distribution Hypothesis" (MDH). At first the joint distribution is derived via weak conditions on the information arrival process. Subsequently, the model is expanded into a full dynamic representation by providing a specific stochastic volatility process for the information arrivals. Both representations are estimated and tested for five major individual common stocks on the NYSE over the period 1973-1991.

The main contributions of his article are as follows. First, he develops modifications to the standard MDH that arise naturally from the microstructure setting. Second, he reinforces the recent empirical findings by resoundingly rejecting the restrictions that the standard MDH imposes on contemporaneous return-volume observations, while controlling for the trend in volume and using a long sample. In contrast, his alternative version of the MDH provides an overall acceptable characterization of these features of the data, so the general framework of the MDH may yet provide a useful basis for structural modeling of the interaction of market variables in response to information flows and, ultimately, the sources of return volatility. Third, he demonstrates that a stochastic volatility representation of the information arrival process that generalizes the popular GARCH (1,1) results in a dynamic specification of the joint system that is consistent with the main contemporaneous as well as dynamic features of the data. Fourth, he documents that, in spite of the overall satisfactory fit, the simultaneous incorporation of returns and volume data results in a significant reduction in



the estimated volatility persistence relative to the usual results obtained from univariate return series.

**Suominen (2001)** examines the informational role of trading volume. In his paper, he develops a theoretical model of stock markets that is consistent with several stylized facts on the stock return trading volume relationship, and in which trading volume plays an important role in traders' learning. In his model, new private information about equity returns is available in any given period only with some probability. In addition, this probability changes stochastically over time as the source of uncertainty in equity return changes. The public information arrival is also probabilistic, but, for simplicity, its arrival rate is constant. There are two types of traders: informed speculators and liquidity traders. Both types of traders act competitively and estimate the availability of private information using past periods' trading volume and use this information to adjust their strategies. Finally the market is organized as a limit order market.

His model generates several results related to the stock price variability trading volume relation. First, he shows that there is a positive correlation between price variability and volume and autocorrelation in price variability. Positive correlation between price variability and trading volume arises because trading by informed traders reveals private information to markets and affects prices. The expected price variability depends on the availability of private information, and inherits any autocorrelation in the process that determines it. Moreover his model predicts that the expected price variability, conditional on the public information set, is autocorrelated and mean reverting. In fact, he derives a closed-form solution to conditional variance of price changes that looks very similar to a GARCH model. In contrast to most GARCH models the evolution of conditional variance in his model depends on trading volume. Another result is that the expected trading volume can be either positively or negatively correlated with the expected price variability. Finally, his model provides predictions on the limit and market order placement strategies of traders.

**Lee and Rui (2002)** examine the dynamic relations – causal relations and the sign and magnitude of dynamic effects – between stock market trading volume and returns (and volatility) for both domestic and cross-country markets by using data of the three largest stock markets: New York, Tokyo, and London.

Their data set comprises daily market price index and trading volume series. For the US Stock Exchange, they use the S&P 500 index. The index covers the period of 2 January 1973 -1 December 1999, and consists of 6784 observations for each series. For the Tokyo Stock Exchange, they use the Tokyo Stock Exchange price Index (TOPIX). The index covers the period of 7 January 1974 -1 December 1999, and consists of 6525 observations. For London, they use the Financial Times-Stock Exchange (FT-SE) 100 index. The index covers the period of 27 October 1986 -1 December 1999, and consists of 3310 observations for each variable. They collect their data from DataStream database and they express stock returns in percent. Their major findings are the follows:

- Trading volume does not Granger-cause stock market returns on each of three stock markets.
- There exists a positive feedback relationship between trading volume and return volatility in all three markets.
- Regarding the cross-country causal relationship, US financial market variables such as returns, volatility and trading volume have an extensive predictive power for UK and Japanese financial market variables. In particular, US trading volume contains information about Japanese and UK financial variables.
- Sub-sample analyses show evidence of stronger spillover effects after the 1987 market crash and an increased importance of trading volume as an information variable after the introduction of options in the US and Japan.

**Huang and Masulis (2003)** examine the generality of the Jones, Kaul and Lipson (1994) conclusion that stock price volatility is strongly impacted by trade frequency (the number of shares), but not by trade size. They also investigate whether the results observed for the London market are consistent with strategic trading by information-motivated investors or liquidity traders seeking to exploit the guaranteed maximum quoted depth.

They analyze the stocks comprising the dominant market index on the LSE, the Financial Times Stock exchange (FTSE-100) index in 1995, before the 1997 adoption of the stock exchange trading system. This index is composed of the 100 largest domestic stocks based on equity capitalization, which in recent years has presented about 70% of the total equity capitalization of all UK stocks. They analyze both daytime and hourly data. They obtain these data from monthly CD-ROM files produced by the LSE, which they combine into an annual file and then extensively check for data errors.

To explore the relation between trading activity and price volatility, they begin by decomposing share volume into its number of trades and average trade size and then use these two variables as regressors in their model of stock price volatility. As JKL observed, these two trading activity measures have the attractive properties of being weakly correlated with each other, while being strongly, positively correlated with share volume. They use Jones et al (1994) linear specification in their statistical model:

$$V_{it} = \alpha + \beta A_{it} + \gamma N_{it} + \varepsilon_{it}$$

where  $V_{it}$  represents price volatility,  $A_{it}$  represents average trade size and  $N_{it}$  represents the number of trades, in each case for stock  $i$  over the day  $t$ . They estimate this equation using Hansen's (1982) generalized method of moments (GMM) method. The GMM estimation method imposes weak distribution assumptions on the observable variables and endogenously adjusts the estimates to account for general forms of conditional heteroskedasticity and/or serial correlation that may be present in the error structure. Serial correlation in stock price volatility is a particular concern, given the widely documented strong positive serial correlation found in squared stock returns.

They show that, for overall sample, price volatility is directly related to trade frequency and more weakly, but positively related to trade size. Their results also show that trades in a small category are the only ones that consistently have a significant impact on price volatility. For small trades, they find significant impacts on price volatility from both trade size and trading frequency, particularly when they move from daytime to hourly data. In examining whether this relation varies across stocks categorized by equity capitalization or trading volume, they find no evidence of significant differences, which indicates that trade size is not acting as a proxy for equity capitalization or stock liquidity. Another important finding of their study is that the impact on price volatility of trading activity is concentrated in trades close to one normal market size. This evidence is consistent with strategic trading behavior of informed investors being concentrated in orders of a particular size. Informed traders have incentives to purposely break up large block trades so as to execute trades at the existing quotes. Trades of one normal market size accomplish this objective. Trades of one normal market size can also be attractive to large liquidity traders who break up their trades, seeking execution at guaranteed quote levels, or to dealers adjusting their inventory position following large block trades.

**Darrat, Rahman and Zhong (2003)** examine the contemporaneous correlation as well as the lead-lag relation between trading volume and return volatility. For this purpose they use intraday data and utilize the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model to measure return volatility.

Their sample consists of transaction prices and trading volumes from April 1, 1998 to June 30, 1998 on the 30 stocks of the DJIA. They obtain intraday transaction data (trades and quotes) from the NYSE Trade and Quote database. They divide each trading day into 78 successive 5-minute intervals when the market is open at 9:30 a.m. through 4:00 p.m., Eastern Standard Time. From the data, they compute 5-minute interval return and trading volumes. They generate the 5-minute return series for each stock by taking the log of the ratio of transaction prices in successive intervals. Because stock returns are computed within each day using only intraday prices, they exclude overnight returns from the series.

Their results suggest that only three stocks show a positive and significant contemporaneous correlation between trading volume and return volatility. The vast majority of the DJIA stocks (27) show no significant positive contemporaneous correlation between volume and volatility. Their results also suggest that there exists significant causality flowing from trading volume to return volatility in at least 12 stocks. The calculated statistics for the reverse causality from volatility to volume are generally much smaller, but achieve statistical significance in two cases. Therefore, almost half of the DJIA stocks show robust evidence of significant causality between volume and volatility in one way or another.

**Wang, Wang and Liu (2005)** investigate the relationship between information flow and return volatility of market portfolios and individuals stocks on two Chinese Stock Exchanges. Employing trading volume as a proxy

of information flows, they test with a GARCH (1,1) model the relevance of the MDH in explaining the conditional heteroskedasticity of returns.

The data sets used in their empirical study consist of the bivariate daily return and trading volume series for 22 actively traded stocks listed on the Chinese Stock Exchange and four market portfolios, proxied by four market indices, namely Shanghai A and Shenzhen A shares and Shanghai B and Shenzhen B shares. For the individual stocks they first choose 100 top companies according to the market capitalization, and then they select those companies whose data are available since 1995:01:02. There are 22 companies meeting their criterion. Their data sets end at 2002:12:31, yielding 2087 observations in total for each series. The same period is also applied to the market return and trading volumes. Daily closing prices and trading volumes are retrieved from Datastream.

They find that the inclusion of trading volume as a proxy of information arrivals in the GARCH specification reduces the persistence of the conditional variance dramatically for the individual stocks. Consistent with their analysis of the institutional and ownership structure of listed Chinese companies, which differentiate between the A share market and the B share market, they find that while trading volume acts, to a lower extent, as a proxy of information arrivals for the two B share portfolios, trading volume does not play a role for the two A share portfolios.

**Xu, Chen and Wu (2006)** examine the comovements of return volatility and volume using a duration-based model where market activity is measured by time duration between trades. They propose a time-consistent VAR model of return volatility and volume that generalizes the traditional MDH model. Volume and volatility are both adjusted for time duration between trades and modeled simultaneously. This formulation permits them to study the interactions between price volatility and the information content of trades using transaction data.

They obtain their data for price, size, time and date for each stock transaction of Dow Jones 30 stocks from the TAQ (Trades and Quotes) database of the NYSE over the period April 1 to June 30, 1995. Their data include trades and quotes from the NYSE, and exclude the overnight return and the opening trade. Their model is specified in "transaction time" and so their data are indexed by trading time. The midquote immediately before transaction  $t$  is indexed as midquote  $t-1$ , while the prevailing midquote after transaction  $t$  and prior to transaction  $t+1$  is indexed as midquote  $t$ . Volatility at time  $t$  is computed as the absolute percentage change in midquotes from  $t-1$  to  $t$ .

They find that volatility and volume (per unit of time) are highly correlated with past volatility and volume. Furthermore, they show that time duration between trades has a negative effect on both the price adjustment to trades and the correlation between current and past trading volume. Their result suggests that time duration affects the dynamic volatility-volume relationship and this effect appears to be quite stable over time. Finally they find that the informed component of price variations varies across stocks and that the bid-ask spreads have a significant positive relation with the informed component of volatility.

**Ane and Ureche-Rangau (2006)** focusing on the long memory properties of power transformations of absolute returns and trading volume, investigate, in a non-parametric setting, to which extent the temporal dependence of volatility and volume of speculative assets is compatible with the bivariate mixture model (BMM). The results of their investigations suggest that although the variables do share common short-term movements, they have fundamentally different long-term behavior. Some direct implications for market participants are:

- Volume could be an interesting variable to take into consideration for agents, like traders, with a very short-term investment horizon while it has little role to play in improving the volatility forecasts for agents, like most portfolio managers, that have a medium – to long – term investment horizon. The inclusion of lagged volume could lead to very erratic forecasts when the investment horizon increases.
- The effect on out-of-sample forecasting performance of including measures of lagged volumes in equations for forecasting volatility makes sense in short-memory models (like ARMA and GARCH models) but not in long-memory models (like ARFIMA, FIGARCH and *multi*-fractal models).
- The investigation of the explanatory content of different powers of the trading volume suggests that the effect is essentially linear and consequently that when it is appropriate to insert volume in a volatility equation, the traditional models are also the most powerful.

## **5. Measures of Volatility**

Volatility is a basic feature of the financial markets. It is an important factor in options trading, where volatility means the conditional variance of the underlying asset return, and in risk management since volatility modeling provides a simple approach to calculating value at risk of a financial position. Modeling the volatility of a time series can improve the efficiency in parameter estimation and the accuracy in interval forecast.

Volatility is not directly observable but it has some characteristics that are commonly seen in asset returns. First, there exists volatility cluster (volatility may be high for certain time periods and low for others). Second, volatility evolves over time in a continuous manner- that is, volatility jumps are rare. Third, volatility does not diverge to infinity- that is, volatility varies within some fixed range. Finally, volatility reacts differently to a big price increase or a big price drop. These properties play an important role in the development of volatility models.

## **5.1 Conditional Heteroscedastic Models**

### **5.1.1 The ARCH model**

The family of ARCH (autoregressive conditionally heteroskedastic) models introduced by Engle (1982). ARCH models make the conditional variance of the time  $t$  prediction error a function of time, system parameters, exogenous and lagged endogenous variables, and past prediction errors. For each integer  $t$ , let  $\xi_t$  be a model's (scalar) prediction error,  $b$  a vector of parameters,  $x_t$  a vector of predetermined variables and  $\sigma_t^2$  the variance of  $\xi_t$  given information at time  $t$ . A univariate ARCH model based on Engle (1982) equations 1-5 sets

- (1)  $\xi_t = \sigma_t z_t$ ,
- (2)  $z_t \sim \text{i.i.d}$  with  
 $E(z_t) = 0$ ,  $\text{Var}(z_t) = 1$ , and
- (3)  $\sigma_t^2 = \sigma^2(\xi_{t-1}, \xi_{t-2}, \dots, t, x_t, b)$   
 $= \sigma^2(\sigma_{t-1} z_{t-1}, \sigma_{t-2} z_{t-2}, \dots, t, x_t, b)$

The system (1)-(3) can easily be given a multivariate interpretation, in which case  $z_t$  is an  $n$  by one vector and  $\sigma_t^2$  is an  $n$  by  $n$  matrix. Any of the form (1)-(3) whether univariate or multivariate is an ARCH model.

A widely used specification for  $\sigma^2(\cdot, \dots, \cdot)$  is the linear ARCH model introduced by Engle (1982) which makes  $\sigma_t^2$  linear in lagged values of  $\xi_t^2 = \sigma_t^2 z_t^2$  by defining

$$(4) \quad \sigma_t^2 = \omega + \sum_{j=1}^p \alpha_j z_{t-j}^2 \sigma_{t-j}^2$$

where  $\omega$  and  $\alpha_j$  are nonnegative.

### **5.1.2 The GARCH model**

The most widely used specification for  $\sigma^2(\cdot, \dots, \cdot)$  is the linear GARCH model (generalized autoregressive conditionally heteroskedastic) introduced by Bollerslev (1986) which also makes  $\sigma_t^2$  linear in lagged values of  $\xi_t^2 = \sigma_t^2 z_t^2$  by defining

$$(5) \quad \sigma_t^2 = \omega + \sum_{i=1}^q \beta_i \sigma_{t-i}^2 + \sum_{j=1}^p \alpha_j z_{t-j}^2 \sigma_{t-j}^2$$

where  $\omega$ ,  $\alpha_j$  and  $\beta_i$  are nonnegative.

### **5.1.3 The GARCH-M model**

The GARCH-M model of Engle and Bollerslev (1986a), adds another equation

$$(6) \quad R_t = a + b\sigma_t^2 + \xi_t$$

in which  $\sigma_t^2$ , the conditional variance of  $R_t$ , enters the conditional mean of  $R_t$  as well. For example if  $R_t$  is the return on a portfolio at time  $t$ , its required rate of return may be linear in its risk as measured by  $\sigma_t^2$ .

Substituting for the  $\beta_i\sigma_{t-i}^2$  terms equation (5) can be written as

$$(7) \quad \sigma_t^2 = \omega^* + \sum_{k=1}^{\infty} \phi_k z_{t-k}^2 \sigma_{t-k}^2$$

It is readily verified that if  $\omega$ ,  $\alpha_j$  and  $\beta_i$  are nonnegative,  $\omega^*$  and the  $\phi_k$  are also nonnegative. By setting conditional variance equal to a constant plus a weighted average (with positive weights) of past squared residuals, GARCH models elegantly capture the volatility clustering in asset returns. This feature of GARCH models accounts for both their theoretical appeal and their empirical success.

#### 5.1.4 The Integrated GARCH model

The IGARCH model of Engle and Bollerslev (1986a) sets

$$(8) \quad \sigma_t^2 = \omega + \sum_{i=1}^q (1-\alpha_i) \sigma_{t-i}^2 + \sum_{j=1}^p \alpha_j z_{t-j}^2 \sigma_{t-j}^2$$

where  $0 < \alpha \leq 1$  for every  $i, j$ .

#### Limitations of GARCH models

Researchers have found that stocks returns are negatively correlated with changes in return volatility – i.e., volatility tends to rise in response to “bad news” (excess returns lower than expected) and to fall in response to “good news” (excess returns higher than expected). GARCH models, however, assume that only the magnitude and not the positivity or negativity of unanticipated excess returns determines feature  $\sigma_t^2$ . If the distribution of  $z_t$  is symmetric, the change in variance tomorrow is conditionally uncorrelated with excess returns today. In (4)-(5),  $\sigma_t^2$  is a function of lagged  $\sigma_t^2$  and lagged  $z_t^2$ , and so is invariant to changes in the algebraic sign of the  $z_t$ 's – i.e., only the size, not the sign, of lagged residuals determines conditional variance. This suggests that a model in which  $\sigma_t^2$  responds asymmetrically to positive and negative residuals might be preferable for asset pricing applications.

Another limitation of GARCH models results from the nonnegativity constraints on  $\omega^*$  and the  $\phi_k$  in (7), which are imposed to ensure that  $\sigma_t^2$  remains nonnegative for all  $t$  with probability one. These constraints imply

that increasing  $z_t^2$  in any period increases  $\sigma_{t+m}^2$  for all  $m \geq 1$ , ruling out random oscillatory behavior in the  $\sigma_t^2$  process. Furthermore, these nonnegativity constraints can create difficulties in estimating GARCH models.

A third drawback of GARCH modeling concerns the interpretation of the "persistence" of shocks to conditional variance. If volatility shocks persist indefinitely, they may move the whole term structure of risk premia, and are therefore likely to have a significant impact on investment in long-lived capital goods (Poterba and Summers (1986)).

### 5.1.5 The Exponential GARCH model

If  $\sigma_t^2$  is to be the conditional variance of  $\xi_t$  given information at time  $t$ , it clearly must be nonnegative with probability one. GARCH models ensure this by making  $\sigma_t^2$  a linear combination (with positive weights) of positive random variables. Nelson (1991) adopts another natural device for ensuring that  $\sigma_t^2$  remains nonnegative, by making  $\ln(\sigma_t^2)$  linear in some function of time and lagged  $z_t$ 's. That is, for some suitable function  $g$ :

$$(9) \quad \ln(\sigma_t^2) = a_t + \sum_{k=1}^{\infty} \beta_k g(z_{t-k}), \quad \beta_1 \equiv 1,$$

where  $\{a_t\}_{t=-\infty, \infty}$  and  $\{\beta_k\}_{k=1, \infty}$  are real, nonstochastic, scalar sequences. To accommodate the asymmetric relation between stock return and volatility changes, the value of  $g(z_t)$  must be a function of both the magnitude and the sign of  $z_t$ . One choice, that in certain important cases turns out to give  $\sigma_t^2$  well-behaved moments, is to make  $g(z_t)$  a linear combination of  $z_t$  and  $|z_t|$ :

$$(10) \quad g(z_t) \equiv \theta z_t + \gamma [ |z_t| - E |z_t| ]$$

where  $\theta$  and  $\gamma$  are real constants.

By construction,  $\{g(z_t)\}_{t=-\infty, \infty}$  is a zero-mean, i.i.d. random sequence. The two components of  $g(z_t)$  are  $\theta z_t$  and  $\gamma [ |z_t| - E |z_t| ]$ , each with mean zero. If the distribution of  $z_t$  is symmetric, the two components are orthogonal, though of course they are not independent. Over the range  $0 < z_t < \infty$ ,  $g(z_t)$  is linear in  $z_t$  with slope  $\theta + \gamma$ , and over the range  $-\infty < z_t \leq 0$ ,  $g(z_t)$  is linear with slope  $\theta - \gamma$ . Thus,  $g(z_t)$  allows the conditional variance process  $\{\sigma_t^2\}$  to respond asymmetrically to rises and falls in stock price.

To see that the term  $\gamma [ |z_t| - E |z_t| ]$  represents a magnitude effect in the spirit of the GARCH models, Nelson assumes that  $\gamma > 0$  and  $\theta = 0$ . The innovation in  $\ln(\sigma_{t+1}^2)$  is then positive (negative) when the magnitude of  $z_t$  is larger (smaller) than its expected value. If  $\gamma = 0$  and  $\theta < 0$  then the innovation in conditional variance is positive (negative) when returns innovations are negative (positive).



One limitation of the GARCH models is that their dynamics are unduly restrictive and they impose inequality constraints that are frequently violated by estimated coefficients. But in equations (9)-(10) there are no inequality constraints whatever, and that cycling is permitted, since the  $\beta_k$  terms can be negative or positive.

The final criticism of GARCH models is that it is difficult to evaluate whether shocks to variance "persist" or not. In exponential GARCH, however,  $\ln(\sigma_t^2)$  is a linear process, and its stationarity (covariance or strict) and ergodicity are easily checked. If the shocks to  $\{\ln(\sigma_t^2)\}$  die out quickly enough, and if we remove the deterministic, possibly time-varying component  $\{\alpha_t\}$ , then  $\{\ln(\sigma_t^2)\}$  is strictly stationary and ergodic.

### 5.1.6 The Stochastic Volatility Model

An alternative model for estimating stochastic model is a simple model in which the conditional variance of a series  $\{y_t\}$  follows a log-AR(1) process. **Jacquier, Polson and Rossi (1993)** consider priors and methods for the general multivariate case:

$$y_t = \sqrt{h_t} u_t, \quad \ln h_t = \alpha + \delta \ln h_{t-1} + \sigma_v v_t$$

where  $(u_t, v_t) \sim$  independent  $N(0,1)$  and the correlation between  $u_t$  and  $v_t$  is assumed to be 0. In their Bayesian simulation framework, they introduce exogenous regressors into the mean equation and accommodate an AR(p) process for the log variance.

Although the preceding model is quite parsimonious, it is capable of exhibiting a wide range of behavior. Like ARCH/GARCH models, the model can give rise to a high persistence in volatility (sometimes referred to as "volatility clustering"). Even if  $\delta=0$ , the model is a variance mixture that will give rise to excess kurtosis in the marginal distribution of the data. In ARCH/GARCH models with normal errors, the degree of kurtosis is tied to the roots of the variance equation; as the variances become more autocorrelated, the degree of mixing also increases. In the ARCH/GARCH literature, it has become common (e.g., Nelson 1991) to use nonnormal innovation densities to accommodate the high kurtosis of various financial time series. In the stochastic volatility model, the  $\sigma_v$  parameter governs the degree of mixing independently of the degree of smoothness in the variance evolution.

### 5.1.7 The Long-Memory Stochastic Volatility Model

A large body of research suggests that the conditional volatility of asset prices displays long memory or long-range persistence. **Breidt, Crato and De Lima (1998)** propose a new time series representation of persistence in

conditional volatility that they call a long memory stochastic volatility model (LMSV).

Their stochastic volatility model is defined by

$$y_t = \sigma_t \xi_t, \quad \sigma_t = \sigma \exp(u_t), \quad (1-B)^d u_t = \eta_t$$

where  $\{u_t\}$  is independent of  $\{\xi_t\}$ ,  $\{\xi_t\}$  is independent and identically distributed (i.i.d.) with mean zero and variance one,  $\{\eta_t\}$  is independent and identically distributed  $N(0, \sigma_\eta^2)$  and  $d \in (-0.5, 0.5)$ . The long memory stochastic volatility model (LMSV) is a stationary long-memory process and is easily fitted and analyzed using standard tools for weakly stationary process.

## **5.2 Realized Volatility**

### **5.2.1 Intraday Returns**

**Andersen, Bollerslev, Diebold and Ebens (2001)** using continuously recorded transaction prices, construct estimates of ex-post realized daily volatilities by summing squares and cross-products of intraday high-frequency returns. Volatility estimates so constructed are model-free, and as the sampling frequency of the returns approaches infinity, they are free from measurement error.

They assume that the logarithmic  $N \times 1$  vector price process,  $p_t$ , follows a multivariate continuous-time stochastic volatility diffusion,

$$dp_t = \mu_t dt + \Omega_t dW_t$$

where  $W_t$  denotes a standard  $N$ -dimensional Brownian motion, the process for the  $N \times N$  positive definite diffusion matrix,  $\Omega_t$ , is strictly stationary, and they normalize the unit time interval, or  $h=1$ , to represent one trading day. Conditional on the sample path realization of  $\mu_t$  and  $\Omega_t$ , the distribution of the continuously compounded  $h$ -period returns,  $r_{t+h,h} \equiv p_{t+h} - p_t$ , is then

$$r_{t+h,h} \mid \sigma\{\mu_{t+\tau}, \Omega_{t+\tau}\}_{\tau=0}^h \sim N\left(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Omega_{t+\tau} d\tau\right),$$

where  $\sigma\{\mu_{t+\tau}, \Omega_{t+\tau}\}_{\tau=0}^h$  denotes the  $\sigma$ -field generated by the sample paths of  $\mu_{t+\tau}$  and  $\Omega_{t+\tau}$  for  $0 \leq \tau \leq h$ . The integrated diffusion matrix thus provides a natural measure of the true latent  $h$ -period volatility.

### **5.2.2 Historical volatility**

The volatility  $\sigma$  of a stock is a measure of uncertainty about the returns provided by the stock. The volatility of a stock price can be defined as the

standard deviation of the return provided by the stock in 1 year when the return is expressed using continuous compounding.

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time (e.g., every day, week, or month). Define :

$n+1$ : Number of observations

$S_i$ : Stock price at the end of  $i$ th interval, with  $i=0,1,\dots,n$

$\tau$ : Length of time interval in years

and  $u_i = \ln(S_i/S_{i-1})$  for  $i=1,2,\dots,n$ .

The usual estimate,  $s$ , of the standard deviation of the  $u_i$  is given by

$$s^2 = \frac{\sum_{i=1}^n (u_i - u^*)^2}{(n-1)}$$

where  $u^*$  is the mean of the  $u_i$ . The standard deviation of the  $u_i$  is  $\sigma\sqrt{T}$ . The variable  $s$  is therefore an estimate of  $\sigma\sqrt{T}$ .

### 5.2.3 Alternative measures

Volatility is alternatively measured as absolute value of closing price minus opening price (open to close), absolute value of closing price minus opening price measured in natural logs (return), absolute value of the error term from an OLS regression of return on indicator variables for turn of the year, end of tax year, triple witching days and first trading day following weekends and holidays (filtered return), absolute value of closing price minus lagged closing price (close to close), squared returns and high-low price range.

Although squared returns provide model-free unbiased estimates of the ex-post realized volatility, they are also a very noisy volatility indicator and hence do not allow for reliable inference regarding the true underlying latent volatility.

## 6.Data

Our sample uses data for the period January 1993-December 2005 and includes companies listed in the FTSE-20, MIDCAP 40 and SMALLCAP 80 of the Greek Stock Market on May 3, 2007. Tables 1,2,3 present the companies of our sample and the estimation period for each company.

**Table 1**

<b>FTSE-20</b>	<b>Companies</b>	<b>Period</b>
<b>1</b>	ALPHA BANK	01/1993-12/2005
<b>2</b>	BANK OF PIRAEUS	01/1993-12/2005
<b>3</b>	COCA-COLA HLC.BT.	01/1993-12/2005
<b>4</b>	EFG EUROBANK ERGASIAS	05/1999-12/2005
<b>5</b>	EMPORIKI BK.OF GREECE	01/1993-12/2005
<b>6</b>	FOLLI-FOLLIE	12/1997-12/2005
<b>7</b>	HELLENIC PETROLEUM	08/1998-12/2005
<b>8</b>	HELLENIC TECHNODOMIKI	07/1996-12/2005
<b>9</b>	INTRALOT INTGRTD.SYS.& SVS.	01/2000-12/2005
<b>10</b>	NATIONAL BK.OF GREECE	01/1993-12/2005
<b>11</b>	OTE-HELLENIC TELC.	06/1996-12/2005
<b>12</b>	TITAN CEMENT CR	01/1993-12/2005
<b>13</b>	VIOHALCO CB	09/1995-12/2005

**Table 2**

<b>MIDCAP 40</b>	<b>Companies</b>	<b>Period</b>
<b>14</b>	ATHENS MEDICAL	01/1993-12/2005
<b>15</b>	ATTICA HOLDINGS	01/1993-12/2005
<b>16</b>	BLUE STAR MARITIME	08/1994-12/2005
<b>17</b>	ELVAL	09/1996-12/2005
<b>18</b>	FOURLIS HOLDING	01/1998-12/2005
<b>19</b>	FRIGOGLASS	01/2000-12/2005
<b>20</b>	GEK GROUP OF COMPANIES	07/1999-12/2005
<b>21</b>	GENERAL HELLENIC BANK	02/1993-12/2005
<b>22</b>	GR SARANTIS	09/1994-12/2005
<b>23</b>	HALCOR METAL PROC.	06/1997-12/2005
<b>24</b>	HERACLES	01/1993-12/2005
<b>25</b>	INFO QUEST CR	02/1999-12/2005
<b>26</b>	INTRACOM	07/1996-12/2005
<b>27</b>	J & P AVAX	06/1996-12/2005
<b>28</b>	JUMBO	08/1997-12/2005
<b>29</b>	LAMDA DEVELOPMENT	10/1998-12/2005
<b>30</b>	M J MAILIS	08/1994-12/2005
<b>31</b>	METKA	07/1996-12/2005
<b>32</b>	MYTILINEOS HLDGS	10/1995-12/2005
<b>33</b>	SIDENOR METAL PROC.	09/1996-12/2005
<b>34</b>	S&B INDUSTRIAL MRLS.	02/1995-12/2005
<b>35</b>	TECHNICAL OLYMPIC	01/1999-12/2005
<b>36</b>	TELETYPOS	10/1994-12/2005
<b>37</b>	TERNA	07/1996-12/2005

**Table 3**

<b>SMALLCAP 80</b>	<b>Companies</b>	<b>Period</b>
<b>38</b>	AGROTIKI INSURANCE	04/1999-12/2005
<b>39</b>	ALCO HELLAS ALUMINUM	05/1997-12/2005
<b>40</b>	ALLATINI	01/1993-12/2005
<b>41</b>	ALPHA REAL ESTATE	08/1999-12/2005
<b>42</b>	ALUMIL MILONAS CR	03/1998-12/2005
<b>43</b>	ANEK LINES CR	03/1999-12/2005
<b>44</b>	ASPIS PRONIA GEN INS	09/1994-12/2005
<b>45</b>	ATHENA	07/1996-12/2005
<b>46</b>	ATTI-KAT	06/1996-12/2005
<b>47</b>	AXON HOLDINGS	08/1994-12/2005
<b>48</b>	BANK OF ATTICA	01/1993-12/2005
<b>49</b>	BYTE COMPUTER	04/2000-12/2005
<b>50</b>	DOMIKI KRITIS	04/2000-12/2005
<b>51</b>	DRUCKFARBEN HELLAS	02/1999-12/2005
<b>52</b>	EDRASIS PSALLIDAS	06/1996-12/2005
<b>53</b>	ELGEKA CR	10/1999-12/2005
<b>54</b>	ELTRAK CR	01/1993-12/2005
<b>55</b>	ETEM	08/1994-12/2005
<b>56</b>	EVEREST HOLDINGS&INVS.	10/1999-12/2005
<b>57</b>	FLEXOPACK	12/1996-12/2005
<b>58</b>	HELLENIC CABLES	07/1996-12/2005
<b>59</b>	HELLENIC FABRICS	01/1999-12/2005
<b>60</b>	HELLENIC SUGAR IND.	11/1993-12/2005
<b>61</b>	ILEKTRONIKI ATHINON	02/2000-12/2005
<b>62</b>	INFORM P LYKOS	09/1994-12/2005
<b>63</b>	INTERTECH	04/1997-12/2005
<b>64</b>	KALPINIS SIMOS	12/1997-12/2005
<b>65</b>	KATSELIS SONS CR	05/1995-12/2005
<b>66</b>	KEKROPS	03/1999-12/2005
<b>67</b>	KLEEMAN HELLAS	06/1999-12/2005
<b>68</b>	KOUMBAS HOLDINGS CR	05/1996-12/2005
<b>69</b>	LAN-NET	04/1998-12/2005
<b>70</b>	LAVIPHARM CR	01/1996-12/2005
<b>71</b>	LOULIS MILLS	01/1993-12/2005
<b>72</b>	NEORION HOLDINGS	02/2000-12/2005
<b>73</b>	PETROPOULOS	01/2000-12/2005
<b>74</b>	REDS	10/1997-12/2005
<b>75</b>	SANYO HELLAS	01/1993-12/2005
<b>76</b>	SATO	12/1997-12/2005
<b>77</b>	SELONDA AQUACULTURE	08/1994-12/2005
<b>78</b>	SHELMAN	05/1995-12/2005
<b>79</b>	THRACE PLASTICS	08/1995-12/2005
<b>80</b>	UNISYSTEMS INFO.SYSTEMS	02/2000-12/2005
<b>81</b>	VIOTER	07/1996-12/2005

Daily and monthly prices are obtained from Datastream. Stock returns are calculated from monthly stock prices at close, adjusted for dividend payouts and stock splits as:  $R_t = (P_t - P_{t-1}) / P_{t-1}$ , where  $P_t$  and  $P_{t-1}$  are the adjusted closing price for month  $t$  and  $t-1$  respectively. The return of the market is an equally-weighted portfolio comprised of all stocks available either in a given month or on a particular day in the sample. The Greek three month Treasury-bill rate is used as the risk-free rate. We also obtain from Effect Finance Database daily data on volume, defined as the euro value of shares traded. Finally we calculate price volatility from historical data (historical volatility).

We construct the liquidity measures under two basic conditions. First each stock should be traded for at least 15 days per month ( $D > 15$ ). Second each liquidity measure should be calculated, under the first condition, the latest April 2000. The result of these two conditions is the diminution of the number of our sample stocks. Specifically 13 stocks of the FTSE-20, 24 stocks of the MIDCAP 40 and 44 stocks of the SMALLCAP 80 satisfy the two conditions. Finally we use a deflator of 50 in constructing LM12, and a deflator of 2000 for LM1.

## 7. Methodology

### 7.1 The Heteroskedastic Mixture Model and ARCH

Our empirical study is based on the Generalized ARCH (GARCH) model of **Bollerslev (1986)**, which restricts the conditional variance of a time series to depend upon past squared residuals of the process. We first compute the variance of returns for all our sample stocks using the GARCH (1,1) model:

$$(1) \quad R_t = a_0 + a_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t$$

$$(2) \quad \varepsilon_t | (\varepsilon_{t-1}, \varepsilon_{t-2}, \dots) \sim N(0, h_t)$$

$$(3) \quad h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$$

where  $R_t$  represents the rate of return,  $\beta_0 > 0$  is a constant term,  $\beta_1$  is the parameter of the squared residuals (ARCH term) and  $\beta_2$  is the parameter of the conditional variance (GARCH term), lagged by one period. The parameters of the model are estimated by means of the Maximum Likelihood method. If the parameters  $\beta_1$  and  $\beta_2$  are positive, then shocks to volatility persist over time and the degree of persistence is determined by the magnitude of these parameters.

The focus of our empirical tests is to examine the relationship between the estimated conditional variance of returns and the various liquidity measures. We consider that liquidity measures such as the return reversal, illiquidity ratio, turnover rate and standardized turnover are likely to contain

the amount of monthly information that flows into the market and thus we estimate for all our sample stocks the following model:

$$(1) \quad R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t$$

$$(2') \quad \varepsilon_t | (LM_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots) \sim N(0, h_t)$$

$$(3') \quad h_t = \beta_0 + \beta_1 \varepsilon_{t-1} + \beta_2 h_{t-1} + \beta_3 LM_t$$

where LM represents the liquidity measure

A succinct measure of the persistence of variance as measured by GARCH is the sum  $(\beta_1 + \beta_2)$ : as the sum approaches unity, the greater is the persistence of shocks to volatility. If the parameter  $\beta_3$  is positive and statistically significant ( $\beta_3 > 0$ ) then there exists a positive relationship between the return volatility and the liquidity measure and the persistence of volatility as measured by  $(\beta_1 + \beta_2)$  should become negligible.

## 7.2 GMM estimation

In our empirical study, besides the GARCH model, we use the GMM estimation method to examine the relationship between price volatility and liquidity. We follow **Huang and Masulis (2003)** procedure, which is based on the **Jones, Kaul and Lipson (1994)** model.

**Huang and Masulis (2003)** in order to explore the relation between trading activity and price volatility begin by decomposing share volume into its number of trades and average trade size and then use these two variables as regressors in their model of stock price volatility. As JKL observed, these two trading activity measures have the attractive properties of being weakly correlated with each other, while being strongly, positively correlated with share volume. They use JKL's linear specification in their statistical model:

$$V_{it} = \alpha + \beta A_{it} + \gamma N_{it} + \varepsilon_{it}$$

where  $V_{it}$  represents price volatility,  $A_{it}$  represents average trade size and  $N_{it}$  represents the number of trades, in each case for stock  $i$  over the day  $t$ . They estimate this equation using Hansen's (1982) generalized method of moments (GMM) method. The GMM estimation method imposes weak distribution assumptions on the observable variables and endogenously adjusts the estimates to account for general forms of conditional heteroskedasticity and/or serial correlation that may be present in the error structure. Serial correlation in stock price volatility is a particular concern, given the widely documented strong positive serial correlation found in squared stock returns.

Because we are interest about the relation between price volatility and liquidity we estimate the following model:

$$V_{it} = \alpha + \beta LM_{it} + \varepsilon_{it}$$

where  $V_{it}$  represents price volatility and  $LM_{it}$  represents the liquidity measure in each case for stock  $i$  over the month  $t$ . If the parameter  $\beta$  is positive and statistically significant  $\beta > 0$  then there exists a positive relationship between price volatility and liquidity.

## 8. Empirical results

### 8.1 The Heteroskedastic Mixture Model and ARCH

We first estimate a GARCH (1,1) model for all stocks of the three indices without any liquidity measure being included in the variance equation. The number of lags in mean equation varies in each stock accordingly with the type of autocorrelation that characterizes the returns of each stock. The parameters are estimated jointly using numerical techniques to maximize the likelihood function. Tables 4,5 and 6 report the estimated coefficients, the asymptotic t-statistics and the p-values for the FTSE-20, the MIDCAP and SMALLCAP shares respectively and provide strong evidence that monthly stock returns can be characterized by the GARCH model when liquidity is excluded from the variance equation.

**Table 4**  
**Maximum Likelihood Estimates of GARCH (1,1) Model without Liquidity**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \quad \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), \quad h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}, \quad R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_1 + \beta_2$   persistence
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	
1	0.100043	1.857451	0.0632	0.666469*	2.985930	0.0028	0.766512
2	0.389202*	5.016236	0.0000	0.618875*	8.820000	0.0000	1.008077
3	0.104511	1.349599	0.1771	0.785441*	5.327188	0.0000	0.889952
4	0.565482*	2.144755	0.0320	0.088793	0.539605	0.5895	0.654275
5	0.141555*	2.478128	0.0132	0.817633*	10.89858	0.0000	0.959188
6	0.055497	1.153448	0.2487	0.868147*	15.97002	0.0000	0.923644
7	0.535959*	3.569761	0.0004	-0.070906	-0.938465	0.3480	0.465053
8	0.268481*	2.362618	0.0181	0.682824*	6.363904	0.0000	0.951305
9	-0.102742*	-4.534526	0.0000	1.050030*	152.8365	0.0000	0.947288
10	0.234953*	2.441647	0.0146	0.678941*	4.874045	0.0000	0.913894
11	0.005990	0.126870	0.8990	0.853618*	5.792894	0.0000	0.859608
12	0.075659	1.089594	0.2759	0.882503*	10.31515	0.0000	0.958162
13	0.174918	1.739135	0.0820	0.438880	1.573880	0.1155	0.613798
<b>Mean</b>	0.196116			0.643173			0.839289

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.



In Table 4 we see that the average coefficient for the previous shock,  $\beta_1$ , is 0.196116 and for the lagged variance,  $\beta_2$ , is 0.643173 which means that the lagged variance affects conditional variance more than the lagged error term. Furthermore, the absolute value  $|\beta_1 + \beta_2|$  is, except for Co. 1, 4, 7 and 13, very high and greater than 0.85 which means that great shocks to volatility persist over time.

**Table 5**  
**Maximum Likelihood Estimates of GARCH (1,1) Model without Liquidity**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$ \beta_1 + \beta_2 $ persistence
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	
14	0.109303	1.826608	0.0678	0.819254*	8.901690	0.0000	0.928557
15	0.162308	1.744586	0.0811	0.572343*	2.133925	0.0328	0.734651
16	0.225289*	4.184245	0.0000	0.762485*	16.79750	0.0000	0.987774
17	0.426741*	3.055037	0.0023	-0.168360	-1.151335	0.2496	0.258381
18	-0.022927	-1.631302	0.1028	1.031368*	54.35660	0.0000	1.008441
19	0.027170	0.285303	0.7754	0.579178*	2.911807	0.0036	0.606348
20	-0.109966*	-3.222168	0.0013	0.770811*	11.54102	0.0000	0.660845
21	-0.000537	-0.006806	0.9946	0.445992	0.046716	0.9627	0.445455
22	0.137504	1.540517	0.1234	0.791022*	6.000283	0.0000	0.928526
23	0.229405	1.897136	0.0578	0.747035*	7.526414	0.0000	0.976440
24	0.035883	1.069103	0.2850	0.927528*	16.69972	0.0000	0.963411
25	0.283688	1.197758	0.2310	0.398938	0.796232	0.4259	0.682626
26	-0.056425	-0.830368	0.4063	0.154929	0.221646	0.8246	0.098504
27	0.295854*	2.615042	0.0089	0.736569*	9.497557	0.0000	1.032423
28	0.049485*	2.607500	0.0091	0.933091*	26.45468	0.0000	0.982576
29	0.321286*	2.420417	0.0155	0.671307*	6.226192	0.0000	0.992593
30	0.181803	1.330812	0.1833	0.746480*	6.279633	0.0000	0.928283
31	0.194082*	2.132215	0.0330	0.743692*	5.269284	0.0000	0.937774
32	-0.047130*	-2.127619	0.0334	0.970011*	17.92952	0.0000	0.922881
33	0.537933*	3.503580	0.0005	0.044945	0.236883	0.8127	0.582878
34	0.079011	0.593766	0.5527	-0.255402	-0.589455	0.5556	0.176391
35	-0.055347	-0.922521	0.3563	1.008582*	15.22061	0.0000	0.953235
36	0.044528	0.986800	0.3237	0.898968*	6.047335	0.0000	0.943496
37	0.035049	1.296380	0.1948	0.898174*	11.93782	0.0000	0.933223
<b>Mean</b>	0.128497			0.634539			0.777738

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

In Table 5 we see that the average coefficient for the previous shock,  $\beta_1$ , is 0.128497 and for the lagged variance,  $\beta_2$ , is 0.634539 which means that the lagged variance affects conditional variance more than the lagged error term. Furthermore the absolute value  $|\beta_1 + \beta_2|$  is, except for Co. 15, 17, 19, 20, 21, 25, 26, 33 and 34, very high and greater than 0.92 which means that great shocks to volatility persist over time. The sum of the two coefficients  $\beta_1, \beta_2$ , is close to 1 for most of the stocks and greater than 1 for two stocks which indicates that the GARCH (1,1) model is integrated.

**Table 6**  
**Maximum Likelihood Estimates of GARCH (1,1) Model without Liquidity**

$$R_t = a_0 + a_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_1 + \beta_2$   persistence
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	
38	-0.153782*	-3.219365	0.0013	1.078136*	18.88573	0.0000	0.924354
39	-0.082074*	-9.886178	0.0000	1.034428*	1792.198	0.0000	0.952384
40	0.145000*	3.517714	0.0004	0.830067*	19.79757	0.0000	0.975067
41	0.565530*	2.157117	0.0310	0.316545*	2.086959	0.0369	0.882075
42	0.401798*	2.576146	0.0100	0.423015*	2.320608	0.0203	0.824813
43	0.061047	-1.863035	0.0625	1.059861*	32.98426	0.0000	0.998814
44	0.048445	0.821581	0.4113	0.797407*	11.68837	0.0000	0.845852
45	0.159060*	2.091884	0.0364	0.777838*	9.335102	0.0000	0.936898
46	0.000612	0.065385	0.9479	0.910155*	13.79900	0.0000	0.910767
47	-0.070388*	-24.34024	0.0000	1.019422*	714.7148	0.0000	0.949034
48	0.105365*	2.358723	0.0183	0.850678*	15.52427	0.0000	0.956043
49	-0.079764*	-5.705310	0.0000	1.064157*	565.6806	0.0000	0.984393
50	-0.212835*	-10.01533	0.0000	1.052110*	122.9630	0.0000	0.839275
51	0.180818*	2.707908	0.0068	0.804723*	13.56257	0.0000	0.985541
52	0.132065*	2.281604	0.0225	0.770244*	6.266431	0.0000	0.902309
53	0.195121	0.823243	0.4104	0.465409	0.912480	0.3615	0.660530
54	0.125832*	2.239771	0.0251	0.773926*	8.109714	0.0000	0.899758
55	0.217773	1.709359	0.0874	0.458330	1.848083	0.0646	0.676103
56	0.242463	1.409661	0.1586	0.630436*	2.849189	0.0044	0.872899
57	0.383245*	2.514854	0.0119	-0.157511	-0.626342	0.5311	0.225734
58	0.080170	0.889715	0.3736	0.757247*	2.917825	0.0035	0.837417
59	0.626049*	3.557314	0.0004	0.384545*	2.655502	0.0079	1.010594
60	0.400794*	5.072958	0.0000	0.459603*	4.862091	0.0000	0.860397
61	0.417437	1.356427	0.1750	0.438038	1.327525	0.1843	0.855475
62	-0.060906*	-5.461491	0.0000	0.982677*	51.07766	0.0000	0.921771
63	0.561034*	2.434858	0.0149	0.479939*	2.431411	0.0150	1.040973
64	0.117027	1.861315	0.0627	0.871885*	12.81576	0.0000	0.988912
65	0.181546*	2.455128	0.0141	0.777165*	7.624509	0.0000	0.958711
66	0.865326*	5.633345	0.0000	0.281516*	3.103531	0.0019	1.146842
67	0.601598*	2.264273	0.0236	0.514004*	3.858469	0.0001	1.115602
68	1.959008*	11.07273	0.0000	0.000324	0.010168	0.9919	1.959332
69	-0.033134*	-2.543793	0.0110	0.945918*	12.30458	0.0000	0.912784
70	0.044045	1.402198	0.1609	0.886340*	13.04484	0.0000	0.930385
71	0.096521	1.549309	0.1213	0.809678*	7.521634	0.0000	0.906199
72	0.051024	-1.500235	0.1336	0.759906*	2.016172	0.0438	0.708882
73	-0.041867*	-7.013937	0.0000	1.006968*	226.9127	0.0000	0.965101
74	0.269557	1.432164	0.1521	0.366693	1.108124	0.2678	0.636250
75	0.334836*	2.608501	0.0091	0.681993*	7.413995	0.0000	1.016829
76	0.701524*	2.664787	0.0077	-0.146620	-1.440854	0.1496	0.554904
77	0.604442*	3.576599	0.0003	0.419376*	4.993283	0.0000	1.023818
78	0.251731*	2.211251	0.0270	0.619146*	3.719478	0.0002	0.870877
79	1.233004*	6.935783	0.0000	0.188400	1.929875	0.0536	1.421404
80	0.213633*	1.988713	0.0467	0.307778	1.836974	0.0662	0.521411
81	0.111163*	2.761476	0.0058	0.854267*	19.31715	0.0000	0.965430
<b>Mean</b>	0.271611			0.650140			0.916657

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

In Table 6 we see that the average coefficient for the previous shock,  $\beta_1$ , is 0.271611 and for the lagged variance,  $\beta_2$ , is 0.650140 which means that the lagged variance affects conditional variance more than the lagged error term. Furthermore, the absolute value  $|\beta_1 + \beta_2|$  is, except for Co. 53, 55, 57, 72, 74, 76 and 80, very high and greater than 0.82, which means that great shocks to volatility persist over time. The sum of the two coefficients is close to 1 for most of the stocks and greater than 1 for eight stocks, which indicates that the GARCH (1,1) model is integrated.

We then estimate a GARCH (1,1) model with various liquidity measures being included in the variance equation. Our purpose is to examine whether the inclusion of the liquidity measure produces a reduction in the significance of coefficients  $\beta_1, \beta_2$ , and thus a reduction in the persistence of conditional volatility.

Tables 7,8 and 9 report the estimated coefficients, the asymptotic t-statistics and the p-values for the FTSE-20, the MIDCAP and the SMALLCAP shares respectively, when the illiquidity ratio is included as an exogenous variable in the variance equation. The results in Table 7 suggest that the inclusion of the illiquidity ratio does not eliminate the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  remain statistically significant for seven and ten stocks respectively, as in the restricted model, while the average  $\beta_1$  is reduced from 0.196116 to 0.123250 and the average  $\beta_2$  is increased from 0.643173 to 0.683433 respectively. The absolute value  $|\beta_1 + \beta_2|$  remains, except for Co. 4, 7, 11 and 13, very high (greater than 0.88) and the average persistence is reduced from 0.839289 to 0.806683, which means that volatility clustering still exists. The coefficient for the illiquidity ratio  $\beta_3$  is statistically significant only for two stocks, negative for Co. 5 and positive for Co. 6.

**Table 7**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with ILLIQ ratio**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{ILLIQ}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
<b>1</b>	0.057912	1.923339	0.0544	0.850144*	13.28745	0.0000	-0.008214	-1.299447	0.1938	0.908056
<b>2</b>	0.388677*	5.006446	0.0000	0.619675*	8.825176	0.0000	-0.000657	-0.052949	0.9578	1.008352
<b>3</b>	0.104792	1.302083	0.1929	0.783776*	5.144232	0.0000	-0.000558	-0.093654	0.9254	0.888568
<b>4</b>	-0.100272*	-2.014560	0.0440	0.775916*	3.102119	0.0019	0.285636	1.416757	0.1566	0.675644
<b>5</b>	0.110117*	2.536501	0.0112	0.857534*	14.98867	0.0000	-0.003274*	-2.293811	0.0218	0.967651
<b>6</b>	-0.129687*	-11.40644	0.0000	1.019919*	991.3248	0.0000	0.054779*	4.917243	0.0000	0.890232
<b>7</b>	0.575086*	3.373941	0.0007	-0.071887	-0.901955	0.3671	-0.113359	-1.687728	0.0915	0.503199
<b>8</b>	0.223860*	2.545934	0.0109	0.746092*	8.192382	0.0000	0.010618	0.803811	0.4215	0.969952
<b>9</b>	-0.106206	-1.706920	0.0878	1.050948*	9.823490	0.0000	0.073912	0.481586	0.6301	0.944742
<b>10</b>	0.235065*	2.377364	0.0174	0.678246*	4.822868	0.0000	-0.000179	-0.031854	0.9746	0.913311
<b>11</b>	-0.009497	-0.141460	0.8875	0.373820	0.759444	0.4476	-0.257962	-0.744640	0.4565	0.364323
<b>12</b>	0.072561	1.039674	0.2985	0.869552*	10.32349	0.0000	-0.007422	-1.302475	0.1928	0.942113
<b>13</b>	0.179839	1.714045	0.0865	0.330895	0.733455	0.4633	0.036982	0.364032	0.7158	0.510734
<b>Mean</b>	0.123250			0.683433			0.005408			0.806683

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

Our results for the FTSE-20 index imply that when the illiquidity ratio is included in the variance equation, the liquidity effect is negligible and the degree of persistence measured by  $|\beta_1 + \beta_2|$  remains almost at the same level.

The results in Table 8 suggest that the inclusion of the illiquidity ratio reduces, but does not eliminate, the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for ten and seventeen stocks respectively and the average  $\beta_1$  is reduced from 0.128497 to 0.073484 and the average  $\beta_2$  from 0.634539 to 0.560968. The absolute value  $|\beta_1 + \beta_2|$  remains high and greater than 0.88 for thirteen companies, while the average persistence is reduced from 0.777738 to 0.725356, which means that volatility clustering still exists. The coefficient for the illiquidity ratio  $\beta_3$  is statistically significant for ten stocks, negative for Co. 17, 23, 30, 34, 36 and 37, and positive for Co. 18, 19, 25 and 28. Our results for the MIDCAP index shares imply that when the illiquidity ratio is significant in the variance equation, the liquidity effect reduces the degree of persistence, measured by  $|\beta_1 + \beta_2|$ , for Co. 18, 23, 28, 30, 36 and 37, and increases the degree of persistence for Co. 17, 19, 25 and 34.

**Table 8**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with ILLIQ ratio**

$$R_t = a_0 + a_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{ILLIQ}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $ persistence
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	
14	0.110044	1.686198	0.0918	0.800099*	7.228406	0.0000	-0.001034	-0.659379	0.5097	0.910143
15	0.167279	1.750884	0.0800	0.536189*	2.035947	0.0418	-0.004602	-0.692724	0.4885	0.703468
16	0.224335*	3.916512	0.0001	0.728057*	13.97470	0.0000	-0.190855	-1.899038	0.0576	0.952392
17	0.010717	0.066578	0.9469	0.501011	1.225910	0.2202	-0.431633*	-4.608133	0.0000	0.511728
18	0.040248	0.792853	0.4279	0.861382*	12.59240	0.0000	0.173183*	2.661136	0.0078	0.901630
19	-0.163769*	-9.746322	0.0000	0.911009*	23.11600	0.0000	0.086548*	4.541685	0.0000	0.747240
20	-0.100624*	-2.254386	0.0242	0.760764*	10.05616	0.0000	0.087120	0.676490	0.4987	0.660140
21	0.093227	0.881120	0.3783	-0.482700	-0.921720	0.3567	0.014037	0.418860	0.6753	0.389473
22	0.149801	1.466158	0.1426	0.767139*	5.012246	0.0000	-0.003210	-0.241838	0.8089	0.916940
23	-0.044166*	-4.058247	0.0000	0.945750*	33.05301	0.0000	-0.181158*	-6.854324	0.0000	0.901584
24	0.026239	0.758982	0.4479	0.922349*	12.97311	0.0000	-0.001796	-0.478419	0.6324	0.948588
25	-0.114365*	-4.636834	0.0000	1.076121*	259.9890	0.0000	0.061529*	2.486318	0.0129	0.961756
26	-0.057498	-0.948176	0.3430	0.113493	0.126569	0.8993	0.013710	0.105460	0.9160	0.055995
27	0.229685*	2.879915	0.0040	0.752189*	9.431415	0.0000	0.050927	1.618186	0.1056	0.981874
28	0.072949*	2.365110	0.0180	-0.774425*	-2.863696	0.0042	0.139717*	2.102466	0.0355	0.701476
29	0.299242*	2.129364	0.0332	0.669751*	5.483247	0.0000	0.061791	0.708647	0.4785	0.968993
30	0.070244	1.022557	0.3065	0.780730*	8.694572	0.0000	-0.108796*	-4.762275	0.0000	0.850974
31	0.175462	1.784295	0.0744	0.733063*	4.772063	0.0000	-0.008255	-0.693894	0.4877	0.908525
32	-0.036964	-0.506818	0.6123	0.553983	1.080260	0.2800	-0.008347	-0.830474	0.4063	0.517019
33	0.498562*	3.277392	0.0010	0.040944	0.214818	0.8299	0.022280	0.522693	0.6012	0.539506
34	-0.013970*	-3.195746	0.0014	0.978642*	88.04955	0.0000	-0.046400*	-6.032705	0.0000	0.964672
35	0.030458	0.393170	0.6942	0.784769*	6.259826	0.0000	0.183625	1.652795	0.0984	0.815227
36	0.107855	0.924387	0.3553	0.089784	0.186907	0.8517	-0.347436*	-2.924709	0.0034	0.197639
37	-0.011366	-0.190452	0.8490	0.413137	1.100721	0.2710	-0.148145*	-2.556808	0.0106	0.401771
Mean	0.073484			0.560968			-0.02447			0.725365

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

**Table 9**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with ILLIQ ratio**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{ILLIQ}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
38	-0.121879*	-2.467816	0.0136	0.448314	0.973054	0.3305	-1.811133	-0.948472	0.3429	0.326435
39	-0.070450*	-4.805393	0.0000	0.554803*	5.093296	0.0000	-0.137060*	-2.855121	0.0043	0.484353
40	0.230462*	2.089857	0.0366	0.552747*	3.421421	0.0006	-0.088594*	-2.568410	0.0102	0.783209
41	0.009989	0.077254	0.9384	0.434896	1.858911	0.0630	-3.001395*	-3.924872	0.0001	0.444885
42	0.109364	1.100598	0.2711	0.504019	1.887172	0.0591	-1.490185	-0.483445	0.6288	0.613383
43	-0.081335*	-2.021730	0.0432	-0.407597	-0.745829	0.4558	0.994352	0.658538	0.5102	0.488932
44	0.048516	0.760854	0.4467	0.796903*	11.49628	0.0000	0.199281	0.720112	0.4715	0.845419
45	0.061291	0.430256	0.6670	0.478749	0.600783	0.5480	-0.013425*	-4.208323	0.0000	0.540040
46	-0.035873	-0.639300	0.5226	0.129583	0.268672	0.7882	-0.083063	-0.061438	0.9510	0.093710
47	-0.020410	-0.342070	0.7323	0.134322	0.241611	0.8091	-0.121844*	-1.995878	0.0459	0.113912
48	-0.026785*	-2.131780	0.0330	0.500822	0.896399	0.3700	-0.007029	-0.607274	0.5437	0.474037
49	-0.033390	-0.471291	0.6374	0.877994*	7.473336	0.0000	-0.072354	-1.627044	0.1037	0.844604
50	-0.216940*	-27.66043	0.0000	1.048792*	510.5876	0.0000	-0.008541	-0.235031	0.8142	0.831852
51	0.180784*	2.407939	0.0160	0.804852*	11.75448	0.0000	0.000273	0.007698	0.9939	0.985636
52	0.141198*	2.268644	0.0233	0.750189*	6.489340	0.0000	0.584535	0.843635	0.3989	0.891387
53	0.052634	0.247440	0.8046	-0.133929	-0.102376	0.9185	-0.045070	-0.747561	0.4547	0.081295
54	0.102359*	2.318812	0.0204	0.791636*	9.143957	0.0000	-0.008225*	-2.624422	0.0087	0.893995
55	0.054111	0.751024	0.4526	0.690615*	2.917207	0.0035	-0.035984*	-3.590639	0.0003	0.744726
56	-0.061820	-0.764275	0.4447	1.059993*	9.957322	0.0000	0.006352	1.895533	0.0580	0.998173
57	0.407002*	2.344586	0.0190	-0.138185	-0.605001	0.5452	-0.398233*	-2.681415	0.0073	0.268817
58	0.169825	1.068929	0.2851	0.334899	0.974079	0.3300	-0.114991	-0.619749	0.5354	0.504724
59	0.597619*	2.505730	0.0122	0.338235*	1.967112	0.0492	-0.058742	-1.422512	0.1549	0.935854
60	0.091512	0.790077	0.4295	0.511474*	2.205383	0.0274	-0.130163	-1.339876	0.1803	0.602986
61	-0.126962*	-7.069856	0.0000	-0.138474	-0.270419	0.7868	-2.607916*	-3.099125	0.0019	0.265436
62	-0.012105	-0.143365	0.8860	0.528175	0.949351	0.3424	-0.044320	-0.467670	0.6400	0.516070
63	0.386713	1.371308	0.1703	0.298001	0.854082	0.3931	-0.035941*	-3.063294	0.0022	0.684714
64	-0.006210	-0.509372	0.6105	0.999676*	56.76517	0.0000	0.045578*	2.968884	0.0030	0.993466
65	0.178197*	2.372597	0.0177	0.771029*	7.563338	0.0000	-0.018318	-1.283204	0.1994	0.949226
66	0.857632*	5.653087	0.0000	0.280935*	3.062216	0.0022	-0.029501	-0.219656	0.8261	1.138567
67	0.367119	1.614017	0.1065	0.313996	1.445578	0.1483	-5.227236	-1.914930	0.0555	0.681115
68	2.056322*	10.49840	0.0000	0.000852	0.019431	0.9845	-0.022364	-1.124142	0.2610	2.057174
69	-0.025310	-0.883784	0.3768	0.754243*	4.824513	0.0000	-0.198812*	-2.175001	0.0296	0.728933
70	0.047473	0.866700	0.3861	0.639275*	3.194867	0.0014	-0.694599	-1.825471	0.0679	0.686748
71	0.101936	1.594067	0.1109	0.803782*	7.544594	0.0000	0.011795	0.702136	0.4826	0.905718
72	-0.111766*	-3.400519	0.0007	0.938622*	10.59446	0.0000	0.019456*	3.074060	0.0021	0.826856
73	0.099728	1.225025	0.2206	0.844876*	13.73862	0.0000	0.003800*	2.053266	0.0400	0.944604
74	0.218936	1.122968	0.2615	0.383540	1.083156	0.2787	-0.054024	-0.671529	0.5019	0.602476
75	0.133570	1.198418	0.2308	0.556943*	2.115191	0.0344	-0.003401	-1.085118	0.2779	0.690513
76	0.513952	1.856569	0.0634	0.548955*	2.634260	0.0084	0.016824	0.594908	0.5519	1.062907
77	0.012185	0.135874	0.8919	0.165414	0.802988	0.4220	-0.074248*	-3.437767	0.0006	0.177599
78	0.132321	1.589998	0.1118	0.795601*	7.580098	0.0000	-0.313808*	-2.434880	0.0149	0.927922
79	1.172663*	7.088760	0.0000	0.213466*	1.997159	0.0458	-0.038015	-1.661778	0.0966	1.386129
80	0.183234	1.797318	0.0723	0.346127	1.953006	0.0508	-0.695132	-1.249582	0.2115	0.529361
81	0.011027	0.600665	0.5481	0.995829*	55.15584	0.0000	0.011888*	6.228876	0.0000	1.006856
Mean	0.176783			0.502386			-0.35885			0.717154

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

The results in Table 9 suggest that the inclusion of the illiquidity ratio reduces the GARCH effects substantially. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for seventeen and twenty five stocks respectively, while in the restricted model they are significant for thirty one and thirty five, and the average  $\beta_1$  is reduced from 0.271611 to 0.176783 and  $\beta_2$  from 0.650140 to 0.502386. The absolute value  $|\beta_1+\beta_2|$  is reduced for thirty three companies and in most cases this reduction is very high. The average persistence is reduced from 0.916657 to 0.717154, which means that volatility clustering exists but is lower. The coefficient for the illiquidity ratio  $\beta_3$  is statistically significant for seventeen stocks, negative for Co. 39, 40, 41, 45, 47, 54, 55, 57, 61, 63, 69, 77 and 78 and positive for Co. 64, 72, 73 and 81. Our results for the SMALLCAP index imply that when the illiquidity ratio is significant in the variance equation, the liquidity effect reduces the degree of persistence, except for Co. 55, 57, 64, 72, 78, 81 where volatility persistence is increased.

Tables 10,11 and 12 report the estimated coefficients, the asymptotic t-statistics and the p-values for the FTSE-20, the MIDCAP and the SMALLCAP shares respectively, when return reversal is included as an exogenous variable in the variance equation. The results in Table 10 propose that the inclusion of the return reversal does not eliminate the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  remain statistically significant for seven and ten stocks respectively, as in the restricted model, while the average  $\beta_1$  is reduced from 0.196116 to 0.129004 and the average  $\beta_2$  is reduced substantially from 0.643173 to 0.208465. The absolute value  $|\beta_1+ \beta_2|$  remains , except for Co. 3, 4, 7, 9 and 13, very high (greater than 0.85) and the average persistence is reduced from 0.839289 to 0.791667, which means that volatility clustering still exists. The coefficient for return reversal  $\beta_3$  is statistically significant for five stocks, negative for Co. 10 and positive for Co. 1, 3, 5 and 7.

**Table 10**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with  $\gamma$**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \gamma_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $ persistence
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	
1	0.014625	1.459701	0.1444	-0.951751*	-44.96899	0.0000	0.001644*	6.414410	0.0000	0.937126
2	0.387254*	5.005061	0.0000	0.621150*	8.805283	0.0000	0.000136	0.098295	0.9217	1.008404
3	0.062381	1.610179	0.1074	-0.710918*	-5.913938	0.0000	0.001407*	3.034304	0.0024	0.648537
4	-0.085863	-1.736754	0.0824	0.843995*	3.260352	0.0011	-0.014837	-1.060813	0.2888	0.758132
5	0.119075*	2.286540	0.0222	0.790546*	9.742235	0.0000	0.000126*	2.287118	0.0222	0.909621
6	-0.058198*	-339.1270	0.0000	1.014655*	107.3664	0.0000	-0.001212	-1.750525	0.0800	0.956457
7	0.563439*	3.027868	0.0025	-0.080973	-1.145070	0.2522	0.000681*	4.212423	0.0000	0.482466
8	0.263136*	2.254320	0.0242	0.682367*	6.264945	0.0000	-0.003559	-0.310305	0.7563	0.945503
9	-0.034923	-0.170095	0.8649	-0.319096	-0.783169	0.4335	-0.044908	-1.775624	0.0758	0.354019
10	0.245131*	2.581017	0.0099	0.607903*	4.448457	0.0000	-0.001112*	-5.368744	0.0000	0.853034
11	-0.001515	-0.034241	0.9727	0.856107*	5.101823	0.0000	0.000124	0.326312	0.7442	0.854592
12	0.013426*	6.320582	0.0000	-1.026031*	-94.85359	0.0000	-0.000969	-0.400316	0.6889	1.012605
13	0.189085	1.959559	0.0500	0.382093	1.721649	0.0851	-0.011654	-0.832312	0.4052	0.571178
Mean	0.129004			0.208465			-0.005702			0.791667

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

Our results for FTSE-20 index imply that when return reversal is significant in the variance equation the liquidity effect reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$  for Co. 3, 5 and 10.

The results in Table 11 are similar with those in Table 8. The coefficient for return reversal  $\beta_3$  is statistically significant for eleven stocks, negative for Co. 14, 16, 18, 22 and 30, and positive for Co. 15, 24, 26, 28 and 36. The Garch effects remain significant since both estimated parameters  $\beta_1$  and  $\beta_2$  are statistically significant for eleven and eighteen stocks respectively. The average coefficients  $\beta_1$  and  $\beta_2$  are reduced from 0.128497 to 0.092839 and from 0.634539 to 0.597403 respectively. The absolute value  $|\beta_1 + \beta_2|$  is reduced for thirteen companies, while the average persistence is increased from 0.777738 to 0.792660 with the presence of the return reversal in the model. Our results for the MIDCAP index imply that when return reversal is included in the variance equation, the liquidity effect increases the degree of persistence.

**Table 11**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with  $\gamma$**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \gamma_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
14	0.143024*	2.192419	0.0283	0.788984*	9.235096	0.0000	-0.001226*	-2.878005	0.0040	0.932008
15	0.078175*	2.298200	0.0216	0.893512*	22.92836	0.0000	0.011094*	5.054815	0.0000	0.971687
16	0.214147*	4.054613	0.0001	0.766585*	17.38840	0.0000	-0.001859*	-2.796556	0.0052	0.980732
17	0.446433*	2.940891	0.0033	-0.167608	-0.927305	0.3538	-0.012199	-0.194591	0.8457	0.278825
18	-0.054011	-1.938106	0.0526	1.027807*	26.90168	0.0000	-0.013282*	-2.178379	0.0294	0.973796
19	0.030482	0.309745	0.7568	0.574743*	2.835070	0.0046	0.000733	0.057114	0.9545	0.605225
20	-0.108775*	-3.122984	0.0018	0.769030*	11.37445	0.0000	-0.004928	-0.081167	0.9353	0.660255
21	0.099264	0.965152	0.3345	-0.516237	-1.139938	0.2543	-0.004018	-0.611014	0.5412	0.416973
22	0.174798	1.499695	0.1337	0.706923*	5.516790	0.0000	-0.003114*	-2.552885	0.0107	0.881721
23	-0.031487	-0.815943	0.4145	0.655413	1.726761	0.0842	0.033248	0.919311	0.3579	0.623926
24	0.009761	0.198575	0.8426	0.747338*	12.99456	0.0000	0.001974*	5.386018	0.0000	0.757099
25	0.225398	1.050763	0.2934	0.440696	0.952308	0.3409	0.005413	1.485922	0.1373	0.666094
26	-0.037965	-0.582400	0.5603	-0.774070*	-3.969792	0.0001	0.056226*	3.285498	0.0010	0.812035
27	0.270537*	2.245278	0.0248	0.614377*	4.977624	0.0000	-0.017525	-1.556176	0.1197	0.884914
28	0.011526	1.587751	0.1123	1.022960*	550.6008	0.0000	0.003017*	3.311266	0.0009	1.034486
29	0.329858*	2.216687	0.0266	0.669820*	5.471019	0.0000	-0.009337	-0.662108	0.5079	0.999678
30	0.066614	1.169041	0.2424	0.786477*	8.082824	0.0000	-0.006083*	-4.891707	0.0000	0.853091
31	0.220057*	2.051088	0.0403	0.731457*	5.095649	0.0000	0.002488	0.476318	0.6338	0.951514
32	-0.084610*	-88.28778	0.0000	1.042494*	30.07432	0.0000	-0.009671*	-2.560990	0.0104	0.957884
33	0.357091*	4.163890	0.0000	0.090395	0.253657	0.7998	-0.000274	-0.976132	0.3290	0.447486
34	0.001412	0.148947	0.8816	0.930005*	8.620563	0.0000	0.002015	1.893914	0.0582	0.931417
35	-0.063194*	-21.37586	0.0000	1.020128*	118.8728	0.0000	0.001841	1.226927	0.2199	0.956934
36	-0.027611	-1.500112	0.1336	1.029403*	267.9270	0.0000	0.021861*	2.932595	0.0034	1.001792
37	-0.042765	-1.307080	0.1912	0.487044	1.057323	0.2904	0.055961	0.684231	0.4938	0.444279
Mean	0.092839			0.597403			0.004681			0.792660

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

**Table 12**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with  $\gamma$**

$$R_t = a_0 + a_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \gamma_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
38	-0.125738*	-4.826177	0.0000	0.557914	1.640760	0.1008	0.095686	1.179553	0.2382	0.432176
39	-0.064161*	-2.264336	0.0236	0.246635*	2.001830	0.0453	-0.076564	-1.417752	0.1563	0.182474
40	0.082377*	2.512598	0.0120	0.866571*	27.63630	0.0000	-0.009857*	-4.590011	0.0000	0.948948
41	0.545318*	2.218462	0.0265	0.333254*	2.202777	0.0276	0.052589	0.425354	0.6706	0.878572
42	0.395291*	2.449969	0.0143	0.421907*	2.320057	0.0203	0.010329	0.852545	0.3939	0.817198
43	-0.079909	-1.602585	0.1090	-0.366110	-0.633197	0.5266	0.016285	0.741094	0.4586	0.446019
44	-0.039710	-1.803926	0.0712	0.567772*	9.971051	0.0000	-3.64E-05*	-9.947534	0.0000	0.528062
45	0.142191*	1.978645	0.0479	0.786524*	9.440437	0.0000	-0.023202	-1.439151	0.1501	0.928715
46	0.032614	0.681750	0.4954	0.667260*	3.316702	0.0009	-0.009014*	-2.817372	0.0048	0.699874
47	-0.036612*	-4.962115	0.0000	0.486704	0.969551	0.3323	-0.041617*	-2.486924	0.0129	0.450092
48	-0.004951	-0.075478	0.9398	0.526531*	5.164707	0.0000	-0.003237*	-4.166434	0.0000	0.521580
49	-0.015250	-0.124530	0.9009	0.577547	1.392476	0.1638	-0.010540	-1.467872	0.1421	0.562297
50	-0.129698*	-3.315921	0.0009	0.462845	0.937927	0.3483	0.007808	0.245109	0.8064	0.333147
51	-0.004508	-0.152845	0.8785	0.898047*	26.20495	0.0000	-0.002082*	-7.526989	0.0000	0.893539
52	0.135895*	2.164710	0.0304	0.761612*	5.541800	0.0000	-0.057122	-1.081725	0.2794	0.897507
53	0.211328	0.911438	0.3621	0.537445	1.469590	0.1417	-0.005904	-0.280675	0.7790	0.748773
54	0.115736*	2.018366	0.0436	0.463053*	2.231339	0.0257	0.000811*	6.957156	0.0000	0.578789
55	-0.077703*	-4.032001	0.0001	1.014645*	81.11153	0.0000	0.005404*	4.653305	0.0000	0.936942
56	-0.093571*	-2.434031	0.0149	1.060980*	16.26091	0.0000	-0.002746	-0.604111	0.5458	0.967409
57	0.201497	1.873219	0.0610	0.686585*	5.584038	0.0000	-0.011617	-1.601653	0.1092	0.888082
58	-0.001148	-0.045854	0.9634	0.897497*	11.94515	0.0000	-0.051184*	-2.038036	0.0415	0.896349
59	0.626894*	3.305788	0.0009	0.385368*	2.646850	0.0081	0.000682	0.583609	0.5595	1.012262
60	0.379865*	5.066967	0.0000	0.468425*	4.850245	0.0000	0.008799	0.625475	0.5317	0.848290
61	-0.122282*	-5.746381	0.0000	0.364904	1.582240	0.1136	-0.217776*	-3.461231	0.0005	0.242622
62	-0.043624	-1.244027	0.2135	0.464166*	3.753485	0.0002	-0.008764*	-5.290076	0.0000	0.420542
63	0.561139*	2.430266	0.0151	0.479885*	2.429248	0.0151	-8.80E-06	-0.001082	0.9991	1.041024
64	0.122255	1.242449	0.2141	0.845272*	8.139867	0.0000	0.000722*	2.306229	0.0211	0.967527
65	0.100737	1.147605	0.2511	0.429027	1.352716	0.1761	0.005271*	6.279279	0.0000	0.529764
66	0.272815*	2.909701	0.0036	0.432993*	2.438018	0.0148	0.136890*	4.112377	0.0000	0.705808
67	0.334602	1.605660	0.1083	0.162763	0.914614	0.3604	3.004051*	3.017998	0.0025	0.497365
68	2.121138*	10.49148	0.0000	0.000893	0.020148	0.9839	-0.007062	-0.389610	0.6968	2.122031
69	-0.033084*	-2.489543	0.0128	0.945810*	11.05819	0.0000	-0.000237	-0.011940	0.9905	0.912726
70	0.043741	1.392916	0.1636	0.886999*	12.56326	0.0000	0.001581	0.054132	0.9568	0.930740
71	0.092309	1.450591	0.1469	0.802138*	7.255093	0.0000	-0.001685	-0.664460	0.5064	0.894447
72	-0.053858*	-4.109708	0.0000	0.396774	1.044529	0.2962	-0.168119	-0.851390	0.3946	0.342916
73	-0.042257*	-202.7750	0.0000	1.003050*	143.1213	0.0000	0.001760	0.704888	0.4809	0.960793
74	0.305563	1.496408	0.1345	0.278830	0.956843	0.3386	-0.000359	-0.826210	0.4087	0.584393
75	0.093475*	2.094966	0.0362	0.821809*	11.16016	0.0000	-0.002155*	-3.262492	0.0011	0.915284
76	0.181525	0.620570	0.5349	0.324009	0.415817	0.6775	0.062924	0.531556	0.5950	0.505534
77	0.596899*	3.421049	0.0006	0.424988*	4.980987	0.0000	-0.000657	-0.480920	0.6306	1.021887
78	0.243647*	2.245386	0.0247	0.641981*	4.860674	0.0000	-0.041175	-1.666388	0.0956	0.885628
79	1.032586*	7.959065	0.0000	0.233278*	2.931217	0.0034	-0.023292*	-7.228422	0.0000	1.265864
80	0.218896*	1.967521	0.0491	0.327797	1.923775	0.0544	0.349899	0.925119	0.3549	0.546693
81	0.110398*	3.160028	0.0016	0.871279*	22.04984	0.0000	0.015306*	2.763049	0.0057	0.981677
Mean	0.189378			0.555629			0.068199			0.765281

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.



Table 12 shows that the inclusion of the return reversal  $\gamma$  in the variance equation does not eliminate the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for twenty seven and thirty stocks respectively and the average  $\beta_1$  is reduced from 0.271611 to 0.189378 and  $\beta_2$  from 0.650140 to 0.555629. The absolute value  $|\beta_1 + \beta_2|$  is reduced for thirty two companies and in most cases this reduction is very high. However the average persistence is reduced from 0.916657 to 0.765281, which means that volatility clustering exists but is lower. The coefficient for the return reversal  $\beta_3$  is statistically significant for eighteen stocks, negative for Co. 40, 44, 46, 47, 48, 51, 58, 61, 62, 75, 79, and positive for Co. 54, 55, 64, 65, 66, 67, 81. Our results for the SMALLCAP index imply that when the  $\gamma$  is significant in the variance equation, the liquidity effect is small and the degree of persistence is increased.

Tables 13,14 and15 report the estimated coefficients, the asymptotic t-statistics and the p-values for the FTSE-20, the MIDCAP and the SMALLCAP shares respectively, when stock turnover is included as an exogenous variable in the variance equation. The results in Table 13 propose that the inclusion of the stock turnover reduces the GARCH effects substantially. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for four and eight stocks while the average  $\beta_1$  is reduced from 0.196116 to 0.109305 and the average  $\beta_2$  is reduced considerably from 0.643173 to 0.332897. The absolute value  $|\beta_1 + \beta_2|$  is decreased for all stocks and the average persistence is reduced from 0.839289 to 0.445581, which means that shocks to volatility tend to be vanished. The coefficient for the stock turnover  $\beta_3$  is positive for all stocks except for Co. 4 and statistically significant for eight stocks, namely 1, 2, 3, 6, 7, 8, 10 and 12, which implies a positive and significant relation between return volatility and the turnover ratio.

**Table 13**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with stock turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{STOV}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
1	0.056211	0.708918	0.4784	0.467835*	2.439206	0.0147	3.352741*	3.136370	0.0017	0.524046
2	0.379588*	5.671759	0.0000	-0.057778*	-3.343542	0.0008	4.094083*	3.461258	0.0005	0.321810
3	0.030421	0.922878	0.3561	0.588366*	6.151400	0.0000	7.898705*	6.216268	0.0000	0.618787
4	-0.091887*	-2.105337	0.0353	0.866162*	5.106789	0.0000	-0.904875	-1.770527	0.0766	0.774275
5	0.149068*	2.072722	0.0382	0.528089*	2.806043	0.0050	3.084242	1.709471	0.0874	0.677157
6	0.233467	1.293590	0.1958	0.292212	1.128817	0.2590	3.011745*	2.079666	0.0376	0.525679
7	0.429569*	6.337557	0.0000	-0.189569*	-8.986562	0.0000	6.652038*	6.452254	0.0000	0.240000
8	0.096773	1.230399	0.2185	0.505798*	2.664995	0.0077	3.184676*	2.109131	0.0349	0.602571
9	-0.114613	-1.481157	0.1386	0.092648	0.114660	0.9087	2.679147	1.118462	0.2634	0.021965
10	0.043054	0.581369	0.5610	0.193207	0.694274	0.4875	6.393787*	2.974886	0.0029	0.236261
11	0.003710	0.055396	0.9558	0.153742	0.284127	0.7763	2.233254	1.789486	0.0735	0.157452
12	0.016582	0.225516	0.8216	0.376579	1.435609	0.1511	3.751442*	3.472890	0.0005	0.393161
13	0.189026	1.759259	0.0785	0.510369*	1.987374	0.0469	0.786406	1.355588	0.1752	0.699395
Mean	0.109305			0.332897			3.555183			0.445581

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

Our results for FTSE-20 index imply that when the turnover ratio is included in the variance equation, the liquidity effect is big and reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

The results in Table 14 suggest that the inclusion of the stock turnover in the variance equation reduces considerably the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant only for four and nine stocks respectively, while the average  $\beta_1$  is reduced from 0.128497 to 0.047682 and the average  $\beta_2$  is reduced substantially from 0.634539 to 0.271083. The absolute value  $|\beta_1 + \beta_2|$  is decreased for all stocks, except for Co. 19, and the average persistence is reduced from 0.777738 to 0.417349, which means that shocks to volatility tend to be vanished. The coefficient for the stock turnover  $\beta_3$  is positive for all stocks except for Co. 20 and 22 and statistically significant for thirteen stocks, namely 14, 16, 17, 20, 21, 23, 26, 27, 29, 30, 34, 35, 37, which implies a positive and significant relation between return volatility and turnover ratio. Our results for the MIDCAP index imply that when the turnover ratio is included in the variance equation, the liquidity effect is big, and reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

**Table 14**  
**Maximum Likelihood Estimates of GARCH (1,1) Model with stock turnover**

$$R_t = a_0 + a_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{STOV}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
14	0.108837	1.633919	0.1023	0.743741*	7.501439	0.0000	1.019953*	2.038183	0.0415	0.852578
15	0.090457	0.842399	0.3996	0.177758	0.491623	0.6230	4.642702	1.883708	0.0596	0.268215
16	0.059347	1.010742	0.3121	-0.198269	0.816562	0.4142	8.004310*	2.736857	0.0062	0.257616
17	-0.011330	-0.249707	0.8028	-0.048007*	-2.288182	0.0221	9.093877*	3.389950	0.0007	0.059337
18	-0.037952	-0.506339	0.6126	0.263502	0.386607	0.6990	9.665209	1.760640	0.0783	0.225550
19	0.009928	0.112370	0.9105	0.613764*	3.342151	0.0008	-0.359610	-0.323022	0.7467	0.623692
20	-0.080487	-1.260770	0.2074	0.640242*	3.526028	0.0004	3.243405*	2.271849	0.0231	0.559755
21	0.023130	0.330963	0.7407	0.196263	0.710186	0.4776	6.026336*	2.077077	0.0378	0.219393
22	-0.009690	-0.340270	0.7337	-0.909207*	-13.98375	0.0000	-0.308277	-0.946010	0.3441	0.918897
23	0.092772*	2.513479	0.0120	-0.284481	-1.886459	0.0592	3.610234*	5.052054	0.0000	0.191709
24	-0.022426	-0.389796	0.6967	0.442616	1.214477	0.2246	3.426168	1.425472	0.1540	0.420190
25	0.303667	1.154702	0.2482	0.184014	0.403545	0.6865	6.090139	0.664105	0.5066	0.487681
26	-0.032757	-0.646137	0.5182	0.553012*	2.135596	0.0327	5.963424*	2.113194	0.0346	0.520255
27	-0.002129	-0.035843	0.9714	0.734574*	5.729779	0.0000	2.791144*	3.078446	0.0021	0.732445
28	0.013804	0.096750	0.9229	0.255857	0.719823	0.4716	5.312210	1.531302	0.1257	0.269661
29	0.025454	0.207693	0.8355	-0.038518	-0.203915	0.8384	12.97236*	2.699391	0.0069	0.013064
30	0.103386	1.040827	0.2980	0.284580	1.514691	0.1299	4.983244*	4.214248	0.0000	0.387966
31	-0.093993*	-3.935142	0.0001	0.543988	1.246175	0.2127	1.977953	1.519013	0.1288	0.449995
32	0.011272	0.119419	0.9049	0.152277	0.173040	0.8626	2.291719	1.087992	0.2766	0.163549
33	0.521510*	3.390997	0.0007	0.043859	0.204684	0.8378	0.390179	0.215134	0.8297	0.565369
34	0.140567	0.884736	0.3763	-0.043758	-0.130714	0.8960	5.395417*	3.336054	0.0008	0.096809
35	-0.115478*	-2.851721	0.0043	1.000334*	23.30598	0.0000	1.118303*	24.64952	0.0000	0.884856
36	0.098855	0.987650	0.3233	0.270683	0.769410	0.4416	1.771866	1.384883	0.1661	0.369538
37	-0.052370	-1.238369	0.2156	0.530635*	2.274604	0.0229	6.654501*	2.243064	0.0249	0.478265
Mean	0.047682			0.271083			4.407365			0.417349

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

**Table 15**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stock turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{STOV}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
38	-0.138555*	-5.765041	0.0000	0.615248*	6.743557	0.0000	-0.596403	-1.832421	0.0669	0.476693
39	-0.055067*	-3.049422	0.0023	0.473747*	4.246801	0.0000	7.534395*	3.464644	0.0005	0.418680
40	-0.055518*	-2.034192	0.0419	0.487776*	2.339233	0.0193	3.554755*	3.200498	0.0014	0.432258
41	0.265725*	2.067004	0.0387	0.248302*	2.091857	0.0365	8.778468*	2.439789	0.0147	0.514027
42	0.227475	1.661532	0.0966	-0.149836	-0.638363	0.5232	13.50687*	2.502314	0.0123	0.077639
43	-0.085142	-1.643401	0.1003	-0.248531	-0.406502	0.6844	-0.561940	-0.741618	0.4583	0.333673
44	-0.054111*	-2.023528	0.0430	0.605291*	8.959357	0.0000	0.796057	1.083961	0.2784	0.551180
45	0.095660	0.842788	0.3993	0.459702*	2.564358	0.0103	8.066835*	2.304604	0.0212	0.555362
46	-0.054505	-1.679333	0.0931	0.312796	1.155302	0.2480	10.40189*	3.866014	0.0001	0.258291
47	-0.016083	-1.070192	0.2845	0.378777	1.665027	0.0959	6.971395*	2.340717	0.0192	0.362694
48	0.486478*	3.905106	0.0001	0.261515*	2.993412	0.0028	5.367934*	4.905362	0.0000	0.747993
49	0.041478	0.268294	0.7885	-0.024838	-0.092042	0.9267	23.64963*	2.052162	0.0402	0.016640
50	-0.216304*	-5.043670	0.0000	1.067087*	8.549955	0.0000	-0.429765*	-2.227999	0.0259	0.850783
51	-0.094693*	-2.830661	0.0046	1.013784*	22.39232	0.0000	0.657452	1.150514	0.2499	0.919091
52	0.084725	1.359853	0.1739	0.671109*	2.751894	0.0059	1.306728	1.428009	0.1533	0.755834
53	0.196386	0.794781	0.4267	0.444646	0.789164	0.4300	0.139319	0.097308	0.9225	0.641032
54	-0.005686	-0.145544	0.8843	0.613611*	5.988897	0.0000	5.964288*	2.480016	0.0131	0.607925
55	0.148738	1.425204	0.1541	0.451372*	2.401011	0.0163	3.739862*	2.445855	0.0145	0.600110
56	0.233145	1.389967	0.1645	0.684800*	2.992487	0.0028	-0.201809	-0.451832	0.6514	0.917945
57	0.142025	1.024103	0.3058	0.387150	0.999868	0.3174	3.369633	1.487858	0.1368	0.529175
58	0.055801	0.694947	0.4871	-0.040768	-0.163148	0.8704	4.906929*	2.433855	0.0149	0.015033
59	0.208724	1.222207	0.2216	0.475631	1.811730	0.0700	2.262391*	2.164802	0.0304	0.684355
60	0.254311*	2.595691	0.0094	0.449318*	4.548922	0.0000	1.586358*	2.315964	0.0206	0.703629
61	-0.126522*	-5.363732	0.0000	0.539664*	2.626039	0.0086	4.263473*	2.007779	0.0447	0.413142
62	0.133679	1.250257	0.2112	0.100890	0.327592	0.7432	3.283011*	2.058597	0.0395	0.234569
63	0.546810*	2.338768	0.0193	0.477438*	2.363497	0.0181	0.119094	0.346660	0.7288	1.024248
64	0.164705	0.869352	0.3847	0.241330	1.319388	0.1870	5.934881*	2.668375	0.0076	0.406035
65	0.018906	0.274198	0.7839	0.801297*	5.846121	0.0000	0.859820*	2.360693	0.0182	0.820203
66	0.640987*	5.057265	0.0000	-0.041678	-1.495988	0.1347	20.22156*	2.019900	0.0434	0.599309
67	0.051498	0.790102	0.4295	-0.853245*	-4.419293	0.0000	1.015232	1.151598	0.2495	0.801747
68	1.389151*	10.28304	0.0000	0.001113	0.015234	0.9878	2.981228*	3.251657	0.0011	1.390264
69	-0.055328	-0.981703	0.3262	0.209435	1.018251	0.3086	5.498641*	4.313106	0.0000	0.154107
70	-0.141651*	-4.136173	0.0000	1.044073*	19.36378	0.0000	0.923734*	3.543886	0.0004	0.902422
71	-0.027680	-0.617874	0.5367	0.322998	0.791565	0.4286	1.946294	1.685428	0.0919	0.295318
72	-0.065282	-1.579140	0.1143	0.648019	1.611102	0.1072	4.264055	0.838557	0.4017	0.582737
73	-0.083439*	-15.30938	0.0000	1.020728*	45.13476	0.0000	0.410500*	2.193300	0.0283	0.937289
74	-0.051258	-0.573989	0.5660	0.508626	1.242778	0.2139	3.354420	1.268161	0.2047	0.457368
75	0.088444	0.821720	0.4112	0.273686	0.952825	0.3407	6.808885*	2.776596	0.0055	0.362130
76	-0.048978	-0.712035	0.4764	0.420932	0.331247	0.7405	2.921008	0.551867	0.5810	0.371954
77	-0.063675	-1.041628	0.2976	0.329289	1.445995	0.1482	3.977125*	2.852942	0.0043	0.265614
78	-0.098984*	-7.400007	0.0000	0.975423*	264.2873	0.0000	2.477824*	9.476979	0.0000	0.876439
79	-0.034118	-0.758329	0.4483	0.239211	1.052006	0.2928	9.371017*	3.985378	0.0001	0.205093
80	-0.060659	-1.281769	0.1999	0.416638*	3.228795	0.0012	17.12827*	3.069494	0.0021	0.355979
81	-0.037473	-1.113871	0.2653	0.502373*	2.211752	0.0270	4.400567*	2.896543	0.0038	0.464900
Mean	0.086457			0.404907			4.839362			0.542975

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

The results in Table 15 are similar to those in Tables 13 and 14. They show that the inclusion of the stock turnover in the variance equation reduces the GARCH effects substantially. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for sixteen and twenty three stocks respectively, while the average  $\beta_1$  is reduced from 0.271611 to 0.086457 and the average  $\beta_2$  is reduced from 0.650140 to 0.404907. The absolute value  $|\beta_1 + \beta_2|$  is decreased for all stocks, except for Co. 56, 57 and 78 and the average persistence is reduced from 0.916657 to 0.542975, which means that shocks to volatility tend to be vanished. The coefficient for the stock turnover  $\beta_3$  is positive for all stocks, except for Co. 38, 43 and 50, and statistically significant for thirty stocks, namely 39, 40, 41, 42, 45, 46, 47, 48, 49, 54, 55, 58, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 73, 75, 77, 78, 79, 80 and 81, which implies a positive and significant relation between return volatility and turnover ratio. Our results for the SMALLCAP index imply that when the turnover ratio is included in the variance equation, the liquidity effect is big, and reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

Tables 16,17 and 18 report the estimated coefficients, the asymptotic t-statistics and the p-values for the FTSE-20, the MIDCAP and SMALLCAP shares respectively, when the standardized turnover LM1 is included as an exogenous variable in the variance equation. The results in Table 16 suggest that the inclusion of the LM1 reduces the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for six and seven stocks respectively, while the average  $\beta_1$  is reduced from 0.196116 to 0.120451 and the average  $\beta_2$  is reduced from 0.643173 to 0.495781. The absolute value  $|\beta_1 + \beta_2|$  is decreased for nine stocks and the average persistence is reduced from 0.839289 to 0.677938, which means fewer shocks to volatility.

**Table 16**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stdr-turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 LM1_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
1	0.116746	1.674838	0.0940	0.522890*	1.991342	0.0464	-0.004513	-0.656580	0.5115	0.639636
2	0.411369*	5.137903	0.0000	0.581743*	7.785851	0.0000	-0.001034	-0.516676	0.6054	0.993112
3	-0.022244	-1.403127	0.1606	0.997026*	66.71621	0.0000	0.006914*	3.265803	0.0011	0.974782
4	-0.090002	-1.432537	0.1520	-0.311090	-0.632640	0.5270	-0.106584	-1.859357	0.0630	0.401092
5	-0.007061	-1.212377	0.2254	1.024628*	430.6339	0.0000	0.012088*	4.617134	0.0000	1.017567
6	-0.080615*	-12.89835	0.0000	1.037353*	168.8284	0.0000	0.001756*	2.073704	0.0381	0.956738
7	0.418454*	3.262637	0.0011	-0.108029	-0.871261	0.3836	-0.124147*	-3.075102	0.0021	0.310425
8	0.228561*	2.631279	0.0085	0.744770*	8.504453	0.0000	0.002618	0.788194	0.4306	0.973331
9	-0.064944	-0.368991	0.7121	0.271493	0.186827	0.8518	-0.084481	-0.508721	0.6109	0.206549
10	0.273062*	2.482912	0.0130	0.638815*	4.838666	0.0000	-0.023452*	-2.795942	0.0052	0.911877
11	-0.002074	-0.031490	0.9749	0.617760	1.422159	0.1550	-0.003354	-0.619485	0.5356	0.615686
12	0.040330	0.257215	0.7970	0.215804	0.399986	0.6892	-0.004593	-0.965323	0.3344	0.256134
13	0.344279*	2.784483	0.0054	0.211989	1.341912	0.1796	0.057093	1.691383	0.0908	0.556268
Mean	0.120451			0.495781			-0.020899			0.677938

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

The coefficient for the standardized turnover  $\beta_3$  is statistically significant for five stocks, negative for Co. 7, 10, and positive for Co. 3, 5 and 6. Our results for the FTSE-20 index imply that although the standardized turnover LM1 is significant only for five stocks when it is included in the variance equation, the liquidity effect reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

The results in Table 17 show that the inclusion of the LM1 reduces the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for six and ten stocks respectively, while the average  $\beta_1$  is reduced from 0.128497 to 0.088226 and the average  $\beta_2$  is reduced from 0.634539 to 0.557418. The absolute value  $|\beta_1 + \beta_2|$  is decreased for nineteen stocks and the average persistence is reduced from 0.777738 to 0.645644, which means that shocks to volatility are lower. The coefficient of the standardized turnover  $\beta_3$  is negative and statistically significant for seven stocks, namely 16, 17, 20, 24, 25, 29, and 31, which implies a negative relationship between LM1 and return volatility. Our results for the MIDCAP index imply that when the standardized turnover LM1 is included in the variance equation, the liquidity effect reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

**Table 17**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stdr-turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 LM1_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $ persistence
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	
14	-0.006919	-0.084830	0.9324	0.470847	0.847019	0.3970	-0.006284	-0.698785	0.4847	0.463928
15	0.132369	1.386238	0.1657	0.393632	1.101499	0.2707	-0.004670	-0.712200	0.4763	0.526001
16	0.209080*	3.953279	0.0001	0.757548*	15.93890	0.0000	-0.003859*	-1.986315	0.0470	0.966628
17	0.240218	1.937797	0.0526	0.390220	1.471380	0.1412	-0.004629*	-2.122486	0.0338	0.630438
18	-0.022389*	-10.17576	0.0000	0.994457*	29.44162	0.0000	0.004729	0.934445	0.3501	0.972068
19	0.001158	0.006259	0.9950	0.441693	1.414104	0.1573	-0.137186	-0.998167	0.3182	0.442851
20	-0.208202*	-40.26409	0.0000	0.979067*	64.40410	0.0000	-0.006224*	-5.088170	0.0000	0.770865
21	-0.013834	-0.164764	0.8691	0.369346	0.424550	0.6712	-0.002890	-1.407501	0.1593	0.355512
22	0.113230	1.388702	0.1649	0.790314*	6.809255	0.0000	-0.001380	-1.382225	0.1669	0.903544
23	-0.040239	-0.845760	0.3977	0.586681	1.537924	0.1241	-0.024329	-1.662014	0.0965	0.546442
24	0.081675	0.973078	0.3305	0.497795	1.516268	0.1295	-0.006514*	-4.286052	0.0000	0.579470
25	0.191405	0.824972	0.4094	0.261100	0.599512	0.5488	-0.645787*	-2.103289	0.0354	0.452505
26	-0.062372	-0.984725	0.3248	0.085345	0.118714	0.9055	-0.007098	-0.174370	0.8616	0.022973
27	0.285581*	2.565994	0.0103	0.720193*	8.310019	0.0000	-0.003375	-1.249659	0.2114	1.005774
28	0.031894	0.370531	0.7110	0.509248	0.406968	0.6840	-0.010070	-1.114594	0.2650	0.541142
29	0.376819	1.280792	0.2003	0.343353*	2.056295	0.0398	-0.012375*	-9.723318	0.0000	0.720172
30	0.155848	1.288236	0.1977	0.744462*	6.391287	0.0000	-0.001762	-1.068672	0.2852	0.900310
31	0.104749	0.888430	0.3743	0.484558	1.265045	0.2059	-0.009769*	-2.235615	0.0254	0.589307
32	-0.045854	-1.641319	0.1007	0.969809*	18.24157	0.0000	0.000173	0.064384	0.9487	0.923955
33	0.537040*	3.498157	0.0005	0.045168	0.236445	0.8131	-0.000142	-0.011913	0.9905	0.582208
34	0.048401	0.615435	0.5383	0.165881	0.302147	0.7625	-0.003772	-1.127304	0.2596	0.214282
35	-0.039067*	-11.74192	0.0000	1.010178*	57.25281	0.0000	-0.011334	-1.376748	0.1686	0.971111
36	0.059562	0.927860	0.3535	0.845863*	3.844941	0.0001	-0.003726	-0.734911	0.4624	0.905425
37	-0.012730	-0.195681	0.8449	0.521270	0.730719	0.4650	-0.014815	-0.607366	0.5436	0.508540
Mean	0.088226			0.557418			-0.038212			0.645644

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

**Table 18**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stdr-turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 LM1_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
38	-0.116232	-1.047458	0.2949	0.454986	0.669859	0.5029	0.003021	0.403960	0.6862	0.338754
39	-0.099103*	-6.578841	0.0000	1.042180*	888.3767	0.0000	-0.001286	-0.164825	0.8691	0.943077
40	0.065561	1.850945	0.0642	0.874611*	22.03283	0.0000	-0.002479*	-3.659610	0.0003	0.940172
41	0.020122	0.149575	0.8811	0.411461	1.668226	0.0953	-0.357701*	-2.194223	0.0282	0.431583
42	0.376357*	2.341134	0.0192	0.413066*	2.300938	0.0214	-0.002037	-0.794407	0.4270	0.789423
43	0.157507	1.022849	0.3064	0.575083	1.740364	0.0818	-0.207852	-1.521523	0.1281	0.732590
44	-0.034714	-1.259548	0.2078	0.536593*	48.79801	0.0000	-0.005496	-0.646732	0.5178	0.501879
45	0.181736*	2.113291	0.0346	0.724017*	8.082211	0.0000	-0.005316	-1.801591	0.0716	0.905753
46	-0.004612	-0.404314	0.6860	0.880525*	13.46312	0.0000	-0.013511	-1.798151	0.0722	0.875913
47	-0.083508*	-76.94855	0.0000	1.033869*	292.6525	0.0000	-0.009156*	-53.07106	0.0000	0.950361
48	-0.009587	-0.147830	0.8825	0.544540*	2.600164	0.0093	-0.027776	-1.473994	0.1405	0.534953
49	0.092442	0.431281	0.6663	-0.445302	-0.885272	0.3760	-0.089887*	-3.618386	0.0003	0.352860
50	-0.121423*	-3.213658	0.0013	0.484407*	2.531838	0.0113	-0.891543*	-7.352511	0.0000	0.362984
51	-0.043081*	-6.244567	0.0000	0.999425*	1139.031	0.0000	0.000143	0.142012	0.8871	0.956344
52	0.138281*	2.025581	0.0428	0.735712*	4.460719	0.0000	-0.001224	-0.636611	0.5244	0.873993
53	0.204234	0.854404	0.3929	0.545599	1.306040	0.1915	-0.004579	-0.698450	0.4849	0.749833
54	0.191827*	2.439358	0.0147	0.634949*	6.512311	0.0000	-0.003526*	-4.798003	0.0000	0.826776
55	0.000788	0.008637	0.9931	0.491425	1.347304	0.1779	-0.005483*	-7.718244	0.0000	0.492213
56	0.209998	1.321349	0.1864	0.678817*	3.287130	0.0010	0.010639	0.476763	0.6335	0.888815
57	0.225757*	2.050748	0.0403	0.159316	0.359967	0.7189	-0.004428	-1.579166	0.1143	0.385073
58	0.236252	1.311069	0.1898	0.259332	1.050315	0.2936	-0.002306	-1.847879	0.0646	0.495584
59	0.643567*	3.525046	0.0004	0.381770*	2.535931	0.0112	-0.006344	-0.329177	0.7420	1.025337
60	0.371602*	4.970794	0.0000	0.480117*	4.877947	0.0000	-0.001762	-0.410591	0.6814	0.851719
61	-0.013218	-0.107498	0.9144	0.296283	1.544925	0.1224	-0.119989*	-5.133861	0.0000	0.283065
62	-0.090508*	-6.556840	0.0000	0.970646*	44.89304	0.0000	-0.007996*	-5.712674	0.0000	0.880138
63	0.634865*	2.369308	0.0178	0.385842	1.886713	0.0592	-0.001610	-0.715469	0.4743	1.020707
64	-0.034590	-0.709022	0.4783	1.004231*	19.72402	0.0000	0.005819*	10.92432	0.0000	0.969641
65	0.182644*	2.473339	0.0134	0.780235*	7.623876	0.0000	0.000232	0.397462	0.6910	0.962879
66	0.854908*	5.655758	0.0000	0.269612*	3.197060	0.0014	-0.007075	-0.819518	0.4125	1.124520
67	0.037653	0.315365	0.7525	0.457187	0.824887	0.4094	-0.108107	-1.258957	0.2080	0.494840
68	0.802335*	4.153271	0.0000	0.034691	1.046653	0.2953	-0.019998*	-2.635359	0.0084	0.837026
69	-0.044507	-0.494574	0.6209	0.540929	0.708881	0.4784	-0.049804*	-2.256875	0.0240	0.496422
70	0.069655	1.275095	0.2023	0.678494*	6.176191	0.0000	-0.004061*	-3.913843	0.0001	0.748149
71	0.078088	1.306180	0.1915	0.803728*	7.646229	0.0000	-0.001998	-1.369301	0.1709	0.881816
72	-0.054685	-1.557023	0.1195	0.762180*	2.098846	0.0358	-0.006985	-0.488791	0.6250	0.707495
73	0.121218	0.891212	0.3728	0.481353	1.186455	0.2354	-0.005452*	-12.27715	0.0000	0.602571
74	0.044593	0.725493	0.4681	0.613912*	4.092406	0.0000	-0.261937*	-2.761329	0.0058	0.658505
75	0.302383*	2.288766	0.0221	0.664474*	7.117806	0.0000	-0.002776*	-2.722996	0.0065	0.966857
76	0.041594	0.282744	0.7774	0.452185*	2.078711	0.0376	-0.030424*	-2.722910	0.0065	0.493779
77	-0.020066	-0.355163	0.7225	0.445504	0.663508	0.5070	-0.009627	-0.917135	0.3591	0.425438
78	0.201572	1.819046	0.0689	0.624094*	3.068919	0.0021	-0.001267	-1.741253	0.0816	0.825666
79	0.967314*	7.932142	0.0000	0.144952	1.276297	0.2019	-0.002858	-0.260346	0.7946	1.112266
80	0.022890	0.136797	0.8912	0.433877	1.672066	0.0945	-0.348722*	-3.641543	0.0003	0.456767
81	0.106850*	2.764302	0.0057	0.866067*	18.91976	0.0000	0.001950	1.107003	0.2683	0.972917
Mean	0.154881			0.559249			-0.059377			0.730169

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

The results in Table 18 show that the inclusion of the LM1 reduces the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for nineteen and twenty seven stocks respectively, while the average  $\beta_1$  is reduced from 0.271611 to 0.154881 and the average  $\beta_2$  is reduced from 0.650140 to 0.559249. The absolute value  $|\beta_1 + \beta_2|$  is decreased for thirty six stocks and the average persistence is reduced from 0.916657 to 0.730169, which means fewer shocks to volatility. The coefficient for the standardized turnover  $\beta_3$  is statistically significant for eighteen stocks and negative for thirty eight stocks. The results reveal a negative relationship between LM1 and return volatility for Co. 40, 41, 47, 49, 50, 54, 55, 61, 62, 68, 69, 70, 73, 74, 75, 76 and 80. Our results for the SMALLCAP index imply that when the standardized turnover LM1 is included in the variance equation, the liquidity effect reduces the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

Tables 19,20 and 21 report the estimated coefficients, the asymptotic t-statistics and the p-values for the FTSE-20, the MIDCAP and SMALLCAP shares respectively, when the standardized turnover LM12 is included as an exogenous variable in the variance equation. The results in Table 19 suggest that the inclusion of the LM12 does not eliminate the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for seven and eleven stocks respectively, while the average  $\beta_1$  is reduced from 0.196116 to 0.106014 and the average  $\beta_2$  is reduced from 0.643173 to 0.618348. The absolute value  $|\beta_1 + \beta_2|$  is decreased for nine stocks and the average persistence is increased from 0.839289 to 0.884752, which means that shocks to volatility persist over time. The coefficient for standardized turnover  $\beta_3$  is statistically significant for seven stocks, negative for Co. 7 and positive for Co. 1, 3, 5, 11, 12 and 13.

**Table 19**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stdr-turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{LM12}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
<b>1</b>	-0.041526*	-4.144984	0.0000	1.019341*	234.0237	0.0000	0.001267*	4.442474	0.0000	0.977815
<b>2</b>	0.406327*	4.785555	0.0000	0.620881*	8.297706	0.0000	0.000134	0.549802	0.5825	1.027208
<b>3</b>	0.058665	1.351378	0.1766	0.878527*	18.92842	0.0000	0.000976*	2.143740	0.0321	0.937192
<b>4</b>	0.033742	1.385567	0.1659	-1.076275*	-10.19917	0.0000	-0.000106	-0.657840	0.5106	1.042533
<b>5</b>	-0.048323*	-3.912167	0.0001	1.030830*	131.7698	0.0000	0.001300*	4.308615	0.0000	0.982507
<b>6</b>	0.267924	1.654001	0.0981	0.675947*	4.175358	0.0000	-0.000597	-1.782138	0.0747	0.943871
<b>7</b>	0.406633*	3.253208	0.0011	-0.106549	-1.021926	0.3068	-0.048405*	-2.400323	0.0164	0.300084
<b>8</b>	0.235616*	2.662177	0.0078	0.739543*	8.770636	0.0000	0.000119	1.179350	0.2383	0.975159
<b>9</b>	0.152307	0.540402	0.5889	0.357183	0.524861	0.5997	0.088215	0.806773	0.4198	0.509490
<b>10</b>	0.087883	1.786464	0.0740	0.792505*	9.934041	0.0000	0.005612	1.865940	0.0620	0.880388
<b>11</b>	-0.079849*	-2.584461	0.0098	1.049712*	493.3715	0.0000	0.000412*	2.332257	0.0197	0.969863
<b>12</b>	-0.053762*	-23.78772	0.0000	1.031923*	164.9923	0.0000	0.000335*	48.43822	0.0000	0.978161
<b>13</b>	-0.047452	-1.417572	0.1563	1.024954*	30.51155	0.0000	0.000489*	3.363028	0.0008	0.977502
<b>Mean</b>	0.106014			0.618348			0.003827			0.884752

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

Our results for the FTSE-20 index imply that when the standardized turnover LM12 is included in the variance equation, the liquidity effect increases the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

The results in Table 20 suggest that the inclusion of the LM12 does not eliminate the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for seven and sixteen stocks respectively, while the average  $\beta_1$  is reduced from 0.128497 to 0.077859 and the average  $\beta_2$  is reduced from 0.634539 to 0.531284. The absolute value  $|\beta_1 + \beta_2|$  is decreased for ten stocks and the average persistence is reduced from 0.777738 to 0.755363, which means that shocks to volatility persist over time. The coefficient for standardized turnover  $\beta_3$  is statistically significant for ten stocks, negative for Co. 16, 20, 23 and 37, and positive for Co. 14, 21, 24, 29, 30 and 34. Our results for the MIDCAP index imply that when the standardized turnover LM12 is included in the variance equation, the liquidity effect decreases the degree of persistence measured by  $|\beta_1 + \beta_2|$ .

**Table 20**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stdr-turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \text{LM12}_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$ \beta_1 + \beta_2 $
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	persistence
14	0.016925	1.825398	0.0679	1.001460*	1648.694	0.0000	0.000788*	4.931088	0.0000	1.018385
15	0.127477	1.538737	0.1239	0.607430*	3.291152	0.0010	2.24E-05	0.046812	0.9627	0.734907
16	0.180699	1.826174	0.0678	-0.280327	-1.065820	0.2865	-0.006438*	-3.272772	0.0011	0.099628
17	0.453645*	2.615061	0.0089	-0.163829	-1.203018	0.2290	0.000460	0.114665	0.9087	0.289816
18	0.101985	0.855428	0.3923	-0.418102	-0.774236	0.4388	0.005814	1.818776	0.0689	0.316117
19	0.033713	0.407446	0.6837	0.689446	1.146358	0.2516	0.021363	0.422903	0.6724	0.723159
20	-0.195407*	-4.649523	0.0000	1.003793*	13.66446	0.0000	-0.000641*	-3.061439	0.0022	0.808386
21	-0.036421	-1.176229	0.2395	1.047225*	27.75014	0.0000	7.59E-05*	4.112331	0.0000	1.010804
22	0.150603	1.473136	0.1407	0.771967*	5.487424	0.0000	6.26E-05	0.363503	0.7162	0.922570
23	-0.051676	-1.400295	0.1614	0.533917*	2.797322	0.0052	-0.041507*	-2.840562	0.0045	0.482241
24	-0.051847*	-5.051270	0.0000	1.030859*	137.0604	0.0000	0.000717*	5.030921	0.0000	0.979012
25	0.282578	1.108161	0.2678	0.415600	0.756334	0.4494	0.020219	0.169525	0.8654	0.698178
26	-0.044234	-0.779702	0.4356	-0.555451	-0.687543	0.4917	0.001589	1.139721	0.2544	0.599685
27	0.301112*	2.566511	0.0103	0.737619*	9.017124	0.0000	1.40E-05	0.222787	0.8237	1.038731
28	-0.081814*	-4.458254	0.0000	1.007920*	476.5011	0.0000	0.000170	0.239262	0.8109	0.926106
29	-0.098763*	-3.450870	0.0006	1.017850*	27.36861	0.0000	0.000501*	2.394598	0.0166	0.919087
30	0.017213	1.032021	0.3021	0.927759*	34.70807	0.0000	0.001067*	4.062510	0.0000	0.944972
31	0.177054	1.809550	0.0704	0.721527*	4.307064	0.0000	-7.84E-05	-0.891117	0.3729	0.898581
32	-0.085890	-1.904923	0.0568	0.900139*	9.104331	0.0000	-0.000259	-0.696108	0.4864	0.814249
33	0.546883*	3.338513	0.0008	0.045161	0.240650	0.8098	-4.64E-05	-0.113924	0.9093	0.592044
34	0.008676	0.464329	0.6424	0.882059*	25.97881	0.0000	0.001184*	3.516007	0.0004	0.890735
35	0.055956	0.509474	0.6104	0.742516*	3.754674	0.0002	0.001046	0.875484	0.3813	0.798472
36	0.062197	0.949692	0.3423	0.821455*	3.594597	0.0003	-0.000223	-0.398161	0.6905	0.883652
37	-0.002025	-0.062831	0.9499	-0.737181	-1.408777	0.1589	-0.002656*	-2.527324	0.0115	0.739206
Mean	0.077859			0.531284			0.000135			0.755363

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.



**Table 21**  
**Maximum Likelihood Estimates of GARCH(1,1) Model with stdr-turnover**

$$R_t = \alpha_0 + \alpha_1 \sum_{i=1}^k R_{t-i} + \varepsilon_t, \varepsilon_t | (\varepsilon_{t-1}, \dots) \sim N(0, h_t), h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 LM12_t, R_t \text{ is monthly return}$$

*If conditional returns are not normal then estimated standard errors may be biased.*

Co	$\beta_1$			$\beta_2$			$\beta_3$			$\beta_1 + \beta_2$   persistence
	Coefficient	z-stat.	Prob.	Coefficient	z-stat.	Prob.	Coefficient	z-stat.	Prob.	
38	-0.166270*	-2.926844	0.0034	1.019671*	14.14834	0.0000	-0.000495	-1.159701	0.2462	0.853401
39	0.197107*	2.129806	0.0332	0.759902*	6.436464	0.0000	0.001032	0.635885	0.5249	0.957009
40	0.085514*	2.126995	0.0334	0.830348*	18.20539	0.0000	-0.000303*	-3.122666	0.0018	0.915862
41	0.505862	1.948547	0.0513	0.371648	1.773102	0.0762	-0.007277	-0.229992	0.8181	0.877510
42	-0.028323*	-2.644964	0.0082	1.012323*	91.70455	0.0000	0.000117	1.155608	0.2478	0.984000
43	0.051134*	3.397645	0.0007	-1.063999*	-43.02001	0.0000	0.004771	0.047066	0.9625	1.012865
44	0.014956	0.732155	0.4641	0.999491*	2430.455	0.0000	-0.000457*	-8.874901	0.0000	1.014447
45	0.412115*	2.672590	0.0075	-0.184056*	-2.948466	0.0032	-0.003689*	-2.760019	0.0058	0.228059
46	-0.010080	-1.223677	0.2211	-0.959695*	-51.62516	0.0000	-0.007847*	-1.974744	0.0483	0.969775
47	0.363896*	2.822830	0.0048	-0.014153	-0.066063	0.9473	-0.002677*	-2.479707	0.0131	0.349743
48	0.092650*	2.491763	0.0127	0.851011*	19.80554	0.0000	0.008296*	2.469847	0.0135	0.943661
49	-0.085128	-0.785741	0.4320	0.983069*	8.107198	0.0000	0.011836*	3.859578	0.0001	0.897941
50	-0.186911	-1.929925	0.0536	1.062295*	86.71742	0.0000	0.024984	0.090178	0.9281	0.875384
51	-0.027267	-0.370807	0.7108	0.472926	1.483126	0.1380	-0.003063*	-2.401376	0.0163	0.445659
52	0.146850	1.793829	0.0728	0.707946*	3.325507	0.0009	-5.95E-05	-0.550726	0.5818	0.854796
53	0.091168	0.402723	0.6872	0.001413	0.000677	0.9995	-0.001376	-0.423508	0.6719	0.092581
54	0.167997*	2.081117	0.0374	0.662638*	5.533904	0.0000	-0.000351*	-2.888736	0.0039	0.830635
55	0.250925*	2.306119	0.0211	0.557749*	3.089618	0.0020	0.000342	1.341968	0.1796	0.808674
56	0.352202	1.763975	0.0777	-0.346400	-1.029935	0.3030	0.046646	0.761552	0.4463	0.005802
57	0.350995*	2.177715	0.0294	-0.185664	-0.576984	0.5639	-0.000879*	-2.482593	0.0130	0.165331
58	0.122479	1.320697	0.1866	0.776321*	4.711724	0.0000	0.000103	1.217774	0.2233	0.898800
59	0.593796*	3.485440	0.0005	0.345403*	2.009450	0.0445	0.000976	0.558108	0.5768	0.939199
60	0.636003*	4.930553	0.0000	0.149003	0.979235	0.3275	-0.000497	-0.583390	0.5596	0.785006
61	-0.067386*	-5.365901	0.0000	1.001726*	353.2006	0.0000	0.036630*	3.952289	0.0001	0.934340
62	0.020548	0.545325	0.5855	0.662368*	3.783787	0.0002	-0.000352	-0.555474	0.5786	0.682916
63	0.587615*	2.016338	0.0438	0.264155	1.386474	0.1656	-0.001438*	-3.528204	0.0004	0.851770
64	-0.046013*	-5.418969	0.0000	0.997124*	204.7758	0.0000	0.000288*	7.470969	0.0000	0.951111
65	0.171119*	2.518653	0.0118	0.781491*	8.861135	0.0000	-5.64E-05	-1.234346	0.2171	0.952610
66	-0.127530*	-2.322775	0.0202	0.779896*	12.45699	0.0000	-0.002719*	-131.9158	0.0000	0.652366
67	0.182680	0.990456	0.3220	-0.197245	-0.917550	0.3589	-0.257140*	-18.54671	0.0000	0.014565
68	-0.012646	-0.229686	0.8183	-0.642868*	-2.526724	0.0115	0.053678*	6.896509	0.0000	0.655514
69	-0.030384*	-3.979698	0.0001	1.035433*	126.2778	0.0000	-0.002112*	-2.958528	0.0031	1.005049
70	0.070466	0.571558	0.5676	0.605081	0.892174	0.3723	-0.000479	-0.517547	0.6048	0.675547
71	0.111549	1.849089	0.0644	0.819441*	7.831024	0.0000	0.000321	1.500685	0.1334	0.930990
72	-0.001925	-0.057470	0.9542	-1.016068*	-27.42743	0.0000	0.002463	0.209427	0.8341	1.017993
73	-0.048157	-0.787429	0.4310	0.505395*	5.361977	0.0000	-0.000648*	-8.871770	0.0000	0.457238
74	0.120193*	2.451634	0.0142	-0.957546*	-13.47125	0.0000	-0.559640*	-4.451233	0.0000	0.837353
75	0.296114*	2.251250	0.0244	0.694786*	7.346661	0.0000	-0.000207	-0.822023	0.4111	0.990900
76	-0.056229*	-4.346720	0.0000	1.034800*	238.0090	0.0000	0.000926*	5.705388	0.0000	0.978571
77	-0.066473*	-2.447926	0.0144	0.010833	0.011779	0.9906	-0.004645	-1.087038	0.2770	0.055640
78	0.334944*	1.972750	0.0485	-0.199503	-0.752193	0.4519	-0.000637*	-2.510523	0.0121	0.135441
79	1.258313*	6.515540	0.0000	0.014593	0.220989	0.8251	-0.001818	-1.585406	0.1129	1.272906
80	0.228119	1.565552	0.1175	-0.244164	-0.314092	0.7535	-0.077611	-1.064132	0.2873	0.016045
81	0.112179*	2.671370	0.0076	0.846857*	19.17472	0.0000	-7.66E-05*	-2.727349	0.0064	0.959036
Mean	0.158381			0.354677			-0.016935			0.721228

\* Statistically significant at 5% assuming that returns are conditionally normally distributed.

The results in Table 21 show that the inclusion of the LM12 reduces the GARCH effects. The coefficients  $\beta_1$  and  $\beta_2$  are statistically significant for twenty six and thirty stocks respectively, while the average  $\beta_1$  is reduced from 0.271611 to 0.158381 and the average  $\beta_2$  is reduced from 0.650140 to 0.354677. The absolute value  $|\beta_1+\beta_2|$  is decreased for thirty stocks and the average persistence is reduced from 0.916657 to 0.721228, which means that shocks to volatility persist over time. The coefficient for standardized turnover  $\beta_3$  is statistically significant for twenty two stocks, negative for Co. 40, 44, 45, 46, 47, 51, 54, 57, 63, 66, 67, 69, 73, 74, 78 and 81, and positive for Co. 48, 49, 61, 64, 68 and 76. Our results for the SMALLCAP index imply that when the standardized turnover LM12 is included in the variance equation, the liquidity effect decreases the degree of persistence measured by  $|\beta_1+ \beta_2|$ .

## 8.2 GMM estimation

Tables 22 and 23 report the GMM estimates of the relation between price volatility, measured from historical data (historical volatility), and the five liquidity measures for the FTSE-20 individual stocks. In Table 22 we observe that the coefficient of the illiquidity ratio is negative for eight stocks and statistically significant only for Co. 11 and 12, which implies a negative relationship between the illiquidity ratio and the price volatility for these two stocks. Furthermore, the coefficient of the return reversal  $\gamma$  liquidity measure is positive for nine companies and statistically insignificant for all FTSE-20 stocks. Finally, the coefficient of the stock turnover liquidity measure is positive for twelve companies and statistically significant for nine, Co. 1, 2, 3, 5, 6, 7, 8, 10 and 12, which implies a positive relationship between the stock turnover and price volatility.

**Table 22**  
**Regression of price volatility and various liquidity measures**

Co	ILLIQ			$\gamma$			STOV		
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.
<b>1</b>	-0.274872	-1.536392	0.1265	0.069898	0.137677	0.8907	20.583591*	8.007574	0.0000
<b>2</b>	0.226439	0.396331	0.6924	-0.014513	-0.780567	0.4363	16.792972*	7.577604	0.0000
<b>3</b>	-0.079746	-0.660906	0.5097	-0.257852	-0.047845	0.9619	63.146060*	3.170681	0.0018
<b>4</b>	0.043406	0.401776	0.6890	1.045446	0.070168	0.9442	9.398890	0.315043	0.7536
<b>5</b>	-0.100551	-0.417530	0.6769	0.013500	0.167859	0.8669	16.512573*	10.17534	0.0000
<b>6</b>	0.359696	1.835893	0.0695	0.002342	0.077602	0.9383	26.419483*	5.244286	0.0000
<b>7</b>	-1.036478	-1.693638	0.0940	0.002433	0.702832	0.4841	32.493775*	3.101270	0.0026
<b>8</b>	-0.185842	-1.793072	0.0757	-11.692066	-0.020761	0.9835	15.383133*	5.438125	0.0000
<b>9</b>	0.225357	0.904144	0.3691	-0.004143	-0.050463	0.9599	12.775405	1.715661	0.0907
<b>10</b>	-0.152962	-1.464955	0.1450	0.027596	0.293927	0.7692	11.399258*	3.008685	0.0031
<b>11</b>	-0.166512*	-1.995060	0.0485	0.003895	0.454621	0.6503	-2.274992	-0.095610	0.9240
<b>12</b>	-0.992645*	-2.148341	0.0333	0.078284	1.141393	0.2555	36.223336*	7.686388	0.0000
<b>13</b>	0.023275	0.774928	0.4399	0.041720	0.926432	0.3561	0.304810	0.099096	0.9212

\* Statistically significant at 5%.

The following estimation is estimated by GMM for each of the 13 stocks of FTSE-20 index,  $V_{it} = \alpha + \beta LM_{it} + \varepsilon_{it}$ , where  $V_{it}$  is the volatility measure for stock  $i$  on month  $t$  and  $LM_{it}$  is the liquidity measure for stock  $i$  on month  $t$ . Volatility is measured from historical data (historical volatility) and the liquidity measures used are the illiquidity ratio, the  $\gamma$  return reversal and the stock turnover.

In Table 23 we see that the coefficient of the standardized turnover LM1 liquidity measure is negative for ten stocks and statistically significant only for Co. 7, which implies a negative relationship between the standardized turnover LM1 and price volatility. Furthermore, the coefficient of the standardized turnover LM12 liquidity measure is positive in most cases (8 out of 13 companies) and statistically significant for four stocks, namely Co. 3, 4, 7 and 13.. The relationship between the standardized turnover LM12 and price volatility is positive for Co. 3, 4 and 13, and negative for Co. 7.

**Table 23**  
**Regression of price volatility and various liquidity measures**

Co	LM1			LM12		
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.
1	1.828723	0.850107	0.3966	0.009243	1.209759	0.2284
2	0.795792	0.995692	0.3210	-0.000559	-0.099324	0.9210
3	-0.088714	-0.355113	0.7230	0.007291*	2.575929	0.0110
4	-0.247033	-0.902021	0.3699	0.003217*	2.860321	0.0054
5	-3.116303	-0.545649	0.5861	0.008125	1.532012	0.1277
6	0.118386	0.548503	0.5846	-0.003533	-1.574310	0.1192
7	-0.945326*	-3.715040	0.0004	-0.258237*	-2.754427	0.0074
8	-0.136749	-1.045824	0.2979	-0.000219	-0.334938	0.7383
9	-0.557457	-0.755874	0.4523	-0.193027	-0.752276	0.4549
10	-0.955133	-0.534442	0.5938	0.003652	0.673551	0.5017
11	-0.092632	-1.425448	0.1568	0.002805	0.514726	0.6079
12	-0.046421	-1.567419	0.1191	0.004780	1.660658	0.0990
13	-0.010205	-1.898935	0.0600	0.002768*	2.387409	0.0187

\* Statistically significant at 5%.

The following estimation is estimated by GMM for each of the 13 stocks of FTSE-20 index,  $V_{it} = \alpha + \beta LM_{it} + \varepsilon_{it}$ , where  $V_{it}$  is the volatility measure for stock  $i$  on month  $t$  and  $LM_{it}$  is the liquidity measure for stock  $i$  on month  $t$ . Volatility is measured from historical data (historical volatility) and the liquidity measures used are the standardized turnover LM1 and LM12.

Tables 24 and 25 report the GMM estimates of the relation between price volatility, measured from historical data (historical volatility), and the five liquidity measures for the MIDCAP 40 individual stocks. In Table 24 we see that the coefficient of the illiquidity ratio is negative for most of the stocks and statistically significant for eight stocks. The results reveal that the relationship between the illiquidity ratio and price volatility is negative for Co. 16, 21, 30, 31, 32 and 34 and positive for Co. 29 and 35. Furthermore, the coefficient of the return reversal  $\gamma$  liquidity measure is negative for the majority of the stocks and statistically insignificant for all MIDCAP 40 stocks.

Finally the coefficient of the stock turnover liquidity measure is positive for all the MIDCAP 40 companies and statistically significant for seventeen, Co. 14, 15, 16, 18, 20, 21, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36 and 37, which implies a positive relationship between the stock turnover and price volatility.

**Table 24**  
**Regression of price volatility and various liquidity measures**

Co	ILLIQ			γ			STOV		
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.
14	-0.098507	-1.835991	0.0683	-0.003428	-0.647535	0.5183	14.034206*	3.251297	0.0014
15	-0.010797	-0.144645	0.8852	-0.012515	-0.768186	0.4436	14.701241*	2.290692	0.0233
16	-0.675623*	-4.096605	0.0001	0.027550	0.332990	0.7397	7.969891*	8.018675	0.0000
17	-0.207889	-1.754861	0.0821	-0.041257	-1.716665	0.0889	10.406387	0.938484	0.3501
18	0.324084	1.892377	0.0616	0.043420	0.934994	0.3522	11.269758*	2.498048	0.0142
19	0.120398	0.530429	0.5975	-0.027037	-0.818024	0.4162	4.491506	0.069934	0.9444
20	-0.859833	-1.178943	0.2421	0.107168	0.129826	0.8971	20.335037*	4.348027	0.0000
21	-0.287978*	-2.811311	0.0056	0.010673	1.472112	0.1431	12.009006*	3.021095	0.0030
22	-0.167131	-1.384067	0.1687	-0.243335	-0.341092	0.7336	11.651959	1.776286	0.0780
23	-0.674793	-1.938580	0.0554	-0.002103	-0.232541	0.8166	0.456787	0.340948	0.7339
24	-0.135763	-0.758851	0.4491	-0.025947	-1.013063	0.3126	37.995445	1.213325	0.2269
25	0.589791	1.290583	0.2006	0.012565	0.129953	0.8969	22.081260	1.490705	0.1400
26	0.012217	0.198787	0.8428	0.500995	0.723678	0.4708	17.112290*	2.570860	0.0115
27	0.239027	1.309374	0.1931	-0.008817	-0.030229	0.9759	11.502906*	3.101227	0.0024
28	0.240585	0.786590	0.4334	-0.036166	-0.427542	0.6699	16.882873*	4.163586	0.0001
29	0.534411*	2.218778	0.0292	0.029975	0.488818	0.6262	8.181469*	4.331657	0.0000
30	-0.955591*	-2.140585	0.0341	-0.000442	-0.011915	0.9905	13.980169*	4.545123	0.0000
31	-0.257321*	-2.483423	0.0145	-0.100702	-1.146818	0.2539	5.409405*	3.903392	0.0002
32	-0.175884*	-2.675634	0.0085	-0.047205	-1.122051	0.2641	6.326456*	4.739050	0.0000
33	0.243564	0.859467	0.3920	0.001728	0.842408	0.4014	51.641732	1.718742	0.0885
34	-0.924232*	-3.086988	0.0025	0.026533	0.267603	0.7894	2.605834*	7.498597	0.0000
35	0.470356*	5.506748	0.0000	-0.039039	-0.553289	0.5816	15.690697*	3.637155	0.0005
36	-1.413289	-1.287468	0.2002	1.063011	0.960687	0.3385	5.932790*	5.072341	0.0000
37	-0.012103	-0.070277	0.9441	-0.043814	-0.251545	0.8019	15.042740*	5.224847	0.0000

\* Statistically significant at 5%.

The following estimation is estimated by GMM for each of the 24 stocks of MIDCAP 40 index,  $V_{it} = \alpha + \beta LM_{it} + \varepsilon_{it}$  where  $V_{it}$  is the volatility measure for stock  $i$  on month  $t$  and  $LM_{it}$  is the liquidity measure for stock  $i$  on month  $t$ . Volatility is measured from historical data (historical volatility) and the liquidity measures used are the illiquidity ratio, the  $\gamma$  return reversal and the stock turnover.

In Table 25 we see that the coefficient of the standardized turnover LM1 liquidity measure is positive for thirteen stocks and statistically significant only for three stocks. The results reveal that the relationship between the standardized turnover LM1 and price volatility is negative for Co. 16, 23 and 25. Furthermore, the coefficient of the standardized turnover LM12 liquidity measure is positive in most of the cases (19 out of 24) and statistically significant for ten stocks. The relationship between the standardized turnover LM12 and price volatility is positive for Co. 14, 18, 19, 20, 23, 28, 29 and 35, and negative for Co. 16 and 37.

**Table 25**  
**Regression of price volatility and various liquidity measures**

Co	LM1			LM12		
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.
14	1.507076	1.093241	0.2760	0.005458*	2.312146	0.0222
15	1.017272	1.134540	0.2583	0.000199	0.081872	0.9349
16	-0.750053*	-3.545432	0.0005	-0.008001*	-2.342146	0.0208
17	-0.047109	-1.788740	0.0764	0.001434	0.399513	0.6904
18	0.151472	0.806491	0.4220	0.001531*	2.315277	0.0228
19	-0.138114	-0.547720	0.5857	0.234555*	2.737857	0.0082
20	0.260707	1.007244	0.3171	0.001338*	5.232862	0.0000
21	-0.034012	-1.742672	0.0834	-0.000362	-0.272603	0.7856
22	-0.054161	-1.031916	0.3040	0.001265	0.590815	0.5557
23	-0.499350*	-2.752972	0.0070	0.004418*	2.024854	0.0457
24	-0.355929	-0.713568	0.4766	0.005912	0.992582	0.3226
25	-2.114872*	-2.570007	0.0120	-0.176278	-0.363194	0.7176
26	1.994397	0.823716	0.4119	0.000260	0.741421	0.4600
27	0.763883	0.832720	0.4068	0.000165	0.185024	0.8535
28	-2.411988	-1.462898	0.1467	0.021955*	3.651919	0.0004
29	0.031707	1.910810	0.0594	0.001677*	4.554159	0.0000
30	0.503263	1.137102	0.2575	0.002264	0.719861	0.4730
31	1.121081	1.347933	0.1804	-0.000779	-1.748985	0.0831
32	-0.075264	-0.358789	0.7204	0.003350	1.361855	0.1760
33	0.082312	0.447767	0.6552	0.003634	1.265354	0.2084
34	0.032674	0.696145	0.4876	0.002965	1.105978	0.2710
35	0.034314	0.932181	0.3540	0.003432*	6.396610	0.0000
36	-0.604593	-0.468472	0.6402	0.004803	0.869058	0.3865
37	7.111176	0.304901	0.7610	-0.001549*	-2.235454	0.0274

\* Statistically significant at 5%.

The following estimation is estimated by GMM for each of the 24 stocks of MIDCAP 40 index,  $V_{it} = \alpha + \beta LM_{it} + \varepsilon_{it}$  where  $V_{it}$  is the volatility measure for stock  $i$  on month  $t$  and  $LM_{it}$  is the liquidity measure for stock  $i$  on month  $t$ . Volatility is measured from historical data (historical volatility) and the liquidity measures used are the standardized turnover LM1 and LM12.

Tables 26 and 27 report the GMM estimates of the relation between price volatility, measured from historical data (historical volatility), and the five liquidity measures for the SMALLCAP 80 individual stocks. In Table 26 we obtain that the coefficient of the illiquidity ratio is negative for most of the stocks and statistically significant for fifteen stocks. The results reveal that the relationship between the illiquidity ratio and price volatility is negative for Co. 40, 41, 47, 49, 51, 52, 67, 68, 69, 70, 74, 79, 80 and 81, and positive only for Co. 72. Furthermore, the coefficient of the return reversal  $\gamma$  liquidity measure is negative for the majority of the stocks and statistically significant only for Co. 38, 39 and 72. Finally, the coefficient of the stock turnover liquidity measure is, except for Co. 44 and 50, positive for all SMALLCAP 80 stocks and statistically significant for thirty three stocks. The results imply a positive relationship between the stock turnover and price volatility for Co. 39, 40, 41, 42, 43, 45, 46, 47, 48, 51, 52, 54, 56, 57, 59, 60, 61, 64, 65, 66, 67, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80 and 81, and negative relationship for Co. 50.

**Table 26**  
**Regression of price volatility and various liquidity measures**

Co	ILLIQ			Y			STOV		
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.
38	1.587335*	2.770811	0.0070	-0.142788*	-2.062507	0.0425	3.136104	0.926014	0.3573
39	-0.466964	-1.268029	0.2077	-0.392145*	-2.028023	0.0452	11.281806*	2.894593	0.0047
40	-1.023747*	-2.924925	0.0040	0.461071	0.411819	0.6810	8.479969*	5.631763	0.0000
41	-12.912424*	-2.462625	0.0161	-0.233271	-0.344509	0.7314	16.814295*	4.711502	0.0000
42	0.371789	0.531919	0.5961	0.138696	0.229254	0.8192	11.328585*	3.973786	0.0001
43	-0.679477	-1.124532	0.2642	0.078797	0.323833	0.7469	10.254017*	2.987031	0.0038
44	0.579609	1.494821	0.1373	-0.000012	-0.535062	0.5935	-4.321144	-1.205752	0.2301
45	-0.061851	-0.953185	0.3426	0.076928	0.891663	0.3745	8.860011*	2.282164	0.0244
46	-0.366698	-1.416168	0.1595	-0.230027	-0.655818	0.5133	13.480535*	3.828735	0.0002
47	-0.409930*	-3.754183	0.0003	-0.054304	-0.303195	0.7622	5.457691*	2.915489	0.0042
48	-0.984447	-1.321619	0.1883	-0.001063	-0.220265	0.8260	9.899105*	2.808171	0.0056
49	-0.999313*	-2.069447	0.0424	-0.105395	-0.684149	0.4963	54.717191	0.799506	0.4269
50	0.971566	1.925344	0.0585	-0.045739	-1.399878	0.1662	-6.147745*	-2.415591	0.0185
51	-2.768604*	-2.240820	0.0278	-0.032688	-0.187359	0.8519	13.307955*	2.902544	0.0048
52	-1.171664*	-2.153228	0.0334	-1.348382	-0.033216	0.9736	15.850639*	5.323032	0.0000
53	-0.096207	-1.476020	0.1443	0.118817	0.075927	0.9397	2.838970	0.534649	0.5945
54	-0.124623	-1.859964	0.0648	-0.000852	-0.200776	0.8411	14.840011*	5.814242	0.0000
55	-0.647573	-1.865369	0.0643	-3.952724	-0.109967	0.9126	4.335671	0.996225	0.3209
56	-0.128440	-1.490113	0.1406	-0.016768	-0.562657	0.5754	7.537003*	3.592749	0.0006
57	-1.259745	-1.466416	0.1455	-0.606632	-1.391408	0.1670	9.284027*	4.212224	0.0001
58	-0.071273	-1.859764	0.0656	-0.028604	-0.733176	0.4650	2.672586	0.164338	0.8698
59	0.748211	0.520213	0.6043	-0.013271	-0.582520	0.5618	7.143213*	2.622807	0.0104
60	-0.212980	-1.453478	0.1483	0.390119	0.311369	0.7560	9.602934*	5.897088	0.0000
61	-3.002494	-0.703023	0.4844	2.406187	0.147113	0.8835	23.980343*	3.316989	0.0015
62	-0.304375	-1.619718	0.1077	0.013363	0.147407	0.8830	2.925988	1.693121	0.0928
63	-0.175719	-1.291759	0.1994	0.018876	0.110311	0.9124	6.202227	1.225996	0.2230
64	-0.035254	-0.166831	0.8679	-0.088361	-0.132295	0.8950	14.076100*	5.244061	0.0000
65	-0.338396	-0.485321	0.6283	0.417624	0.793701	0.4289	12.856457*	5.233264	0.0000
66	-0.738193	-1.118334	0.2668	0.142825	0.872288	0.3857	9.716572*	2.784642	0.0067
67	-61.117262*	-4.013575	0.0001	-52.313951	-0.213206	0.8317	28.199754*	3.190840	0.0021
68	-0.173648*	-2.850321	0.0052	-0.137280	-0.656553	0.5128	3.878907	1.968957	0.0514
69	-0.727722*	-2.666529	0.0091	0.000246	0.004295	0.9966	0.776036	0.626874	0.5323
70	-11.025635*	-3.497851	0.0007	0.243477	1.182313	0.2395	10.473683*	4.323076	0.0000
71	-1.021548	-1.856988	0.0652	-0.035382	-0.677678	0.4990	9.453739*	5.181266	0.0000
72	0.081460*	2.452676	0.0168	-0.655033*	-3.473660	0.0009	10.724565	0.481804	0.6315
73	-1.003754	-1.952152	0.0550	-0.034791	-0.196515	0.8448	23.512879*	2.564179	0.0125
74	-1.847827*	-3.849497	0.0002	-0.003487	-0.758092	0.4503	13.813042*	5.454250	0.0000
75	-0.804425	-0.733981	0.4641	0.006603	0.327587	0.7437	6.222134*	2.777907	0.0062
76	0.175710	1.675037	0.0973	-0.594085	-1.034900	0.3034	10.171869*	3.747803	0.0003
77	0.048795	0.368999	0.7127	-0.002475	-0.953241	0.3422	2.636855*	2.180523	0.0310
78	-2.205882	-1.340912	0.1824	-0.867523	-0.676754	0.4998	16.438638*	3.002314	0.0032
79	-0.708892*	-2.918330	0.0042	0.268924	0.319706	0.7497	9.391787*	3.310308	0.0012
80	-7.951960*	-4.359304	0.0000	2.539920	0.893335	0.3748	30.051101*	2.225150	0.0294
81	-0.102549	-1.334398	0.1848	0.037545	1.437372	0.1534	9.392337*	4.101091	0.0001

\* Statistically significant at 5%.

The following estimation is estimated by GMM for each of the 44 stocks of SMALLCAP 80 index,  $V_{it} = a + \beta LM_{it} + \varepsilon_{it}$  where  $V_{it}$  is the volatility measure for stock  $i$  on month  $t$  and  $LM_{it}$  is the liquidity measure for stock  $i$  on month  $t$ . Volatility is

measured from historical data (historical volatility) and the liquidity measures used are the illiquidity ratio, the  $\gamma$  return reversal and the stock turnover.

**Table 27**  
**Regression of price volatility and various liquidity measures**

Co	LM1			LM12		
	Coefficient	t-stat.	Prob.	Coefficient	t-stat.	Prob.
38	-0.996655	-0.488176	0.6268	-0.004858	-1.400341	0.1660
39	-0.338180	-1.170885	0.2444	-0.002190	-0.503339	0.6159
40	-0.157934	-1.488716	0.1386	-0.007543*	-4.658679	0.0000
41	-2.122415*	-6.470423	0.0000	-0.628904*	-2.489192	0.0154
42	-0.039002	-1.466948	0.1458	-0.001141	-0.523235	0.6022
43	-2.701603*	-2.435707	0.0171	-0.613282	-1.409391	0.1632
44	-0.133266	-0.528562	0.5980	-0.020218*	-5.705666	0.0000
45	-0.327847	-0.589332	0.5568	-0.001320*	-2.839346	0.0054
46	0.438158	0.666582	0.5064	-0.002058	-1.813792	0.0724
47	6.966899	0.344239	0.7312	-0.007873*	-2.058092	0.0417
48	2.406837	0.429132	0.6684	0.016351*	2.890584	0.0044
49	-0.483902	-1.268192	0.2092	0.066566	0.569634	0.5712
50	3.736325	1.967823	0.0533	2.983449*	3.071964	0.0033
51	-0.549607	-1.098803	0.2751	-0.019109*	-4.664901	0.0000
52	1.147019	0.740414	0.4606	-0.001411	-1.765858	0.0801
53	0.260519	0.268396	0.7892	0.000368	0.106089	0.9159
54	-0.041940*	-2.183359	0.0305	-0.004601*	-2.914997	0.0041
55	-0.025287	-1.566317	0.1196	0.000587	0.301590	0.7635
56	-0.333346	-1.974010	0.0522	-0.138536	-1.201933	0.2340
57	-0.022368	-0.626833	0.5321	-0.000605	-0.594687	0.5534
58	-0.015576	-1.646913	0.1024	-0.000587	-1.117920	0.2660
59	0.084077	0.211289	0.8332	0.007240*	4.120191	0.0001
60	0.618561	0.632388	0.5281	0.004376	1.030054	0.3048
61	-0.417189	-1.627256	0.1083	0.271815*	2.206006	0.0314
62	0.655276	1.135848	0.2581	0.004166	1.635917	0.1044
63	-0.011131	-0.513128	0.6090	-0.001132*	-2.008540	0.0473
64	0.024531	1.102241	0.2732	0.002021*	2.970213	0.0038
65	0.326733	0.607224	0.5448	-0.002067*	-2.341744	0.0208
66	-0.080447	-1.073085	0.2865	0.000706*	2.349904	0.0213
67	-1.672503*	-3.442895	0.0009	-0.685265*	-3.061379	0.0032
68	-0.036390	-0.428487	0.6691	-0.003199	-0.714970	0.4762
69	0.712742	1.076192	0.2847	-0.001870	-0.220927	0.8257
70	-0.055248	-1.551145	0.1236	-0.003804*	-2.019372	0.0459
71	-0.646794	-0.503215	0.6155	-0.006833*	-2.111368	0.0365
72	-1.517547	-0.317486	0.7518	0.008290	0.501877	0.6177
73	-0.309035	-1.336426	0.1858	-0.011227*	-3.528036	0.0008
74	-2.837279*	-6.694565	0.0000	-1.004812*	-5.311314	0.0000
75	-0.048497*	-2.371603	0.0190	-0.002676	-0.980456	0.3285
76	0.009995	0.454847	0.6503	0.005357*	2.474435	0.0151
77	1.983944	0.801498	0.4243	-0.003950	-0.953558	0.3422
78	-0.013871*	-2.085142	0.0391	-0.000017	-0.059026	0.9530
79	0.708622	1.423946	0.1570	-0.006599	-1.867763	0.0644
80	-0.928141*	-5.814167	0.0000	0.098177	0.624065	0.5351
81	-0.101655*	-2.511386	0.0135	-0.001361*	-2.107596	0.0373

\* Statistically significant at 5%.

The following estimation is estimated by GMM for each of the 44 stocks of SMALLCAP 80 index,  $V_{it} = a + \beta LM_{it} + \varepsilon_{it}$  where  $V_{it}$  is the volatility measure for stock  $i$  on

month  $t$  and  $LM_{it}$  is the liquidity measure for stock  $i$  on month  $t$ . Volatility is measured from historical data (historical volatility) and the liquidity measures used are the standardized turnover LM1 and LM12.

In Table 27 we observe that the coefficient of the standardized turnover LM1 liquidity measure is negative for most of the stocks and statistically significant for nine stocks. The results reveal that a negative relationship between the standardized turnover LM1 and price volatility exists for Co. 41, 43, 54, 67, 68, 74, 75, 78, 80 and 81. Furthermore, the coefficient of the standardized turnover LM12 liquidity measure is negative in most of the cases and statistically significant for twenty two stocks. The results indicate a negative relationship between the standardized turnover LM12 and price volatility for Co. 40, 41, 44, 45, 47, 51, 54, 63, 65, 67, 70, 71, 73, 74 and 81, and a positive relationship for Co. 48, 50, 59, 61, 64, 66 and 76.

### 8.3 Conclusions

Following, we present the tables with the number and the percentage of the stocks for each index, for which every liquidity measure is statistically significant.

**Table 28**  
Number of shares per index for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model

#	ILLIQ	$\gamma$	STOV	LM1	LM12
FTSE20 (13)	2	5	8	5	7
MIDCAP40 (24)	10	11	13	7	10
SMALLCAP80 (44)	17	18	30	18	22

**Table 29**  
Percentage of shares per index for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model

%	ILLIQ	$\gamma$	STOV	LM1	LM12
FTSE20 (13)	15,384	38,461	61,538	38,461	53,846
MIDCAP40 (24)	41,667	45,833	54,167	29,167	41,667
SMALLCAP80 (44)	38,636	40,909	68,182	40,909	50



**Table 30**

**Total number of shares for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model**

#	ILLIQ	γ	STOV	LM1	LM12
<b>Total (81)</b>	29	34	51	30	39

**Table 31**

**Total percentage of shares for which every liquidity measure is statistical significant when included in the variance equation of the GARCH (1,1) Model**

%	ILLIQ	γ	STOV	LM1	LM12
<b>Total (81)</b>	35,802	41,975	62,963	37,037	48,148

**Table 32**

**Number of shares per index for which every liquidity measure is statistical significant when included in the GMM Estimation Model**

#	ILLIQ	γ	STOV	LM1	LM12
<b>FTSE20 (13)</b>	2	0	9	1	4
<b>MIDCAP40 (24)</b>	8	0	17	3	10
<b>SMALLCAP80 (44)</b>	15	3	33	9	22

**Table 33**

**Percentage of shares per index for which every liquidity measure is statistical significant when included in the GMM Estimation Model**

%	ILLIQ	γ	STOV	LM1	LM12
<b>FTSE20 (13)</b>	15,384	0	69,23	7,692	30,769
<b>MIDCAP40 (24)</b>	33,333	0	70,833	12,5	41,667
<b>SMALLCAP80 (44)</b>	34,091	6,818	75	20,454	50

**Table 34**

**Total number of shares for which every liquidity measure is statistical significant when included in the GMM Estimation Model**

#	ILLIQ	γ	STOV	LM1	LM12
<b>Total (81)</b>	25	3	59	13	36

**Table 35**

**Total percentage of shares for which every liquidity measure is statistical significant when included in the GMM Estimation Model**

%	ILLIQ	γ	STOV	LM1	LM12
<b>Total (81)</b>	30,864	3,703	72,839	16,049	44,444

The first conclusion that arises is that the liquidity measure that appears statistically significant for the biggest number of shares, and has a positive relationship with their return volatility, is the stock turnover. Specifically, the stock turnover influences the volatility of 61,538% of the FTSE-20 shares, 54,167% of the MIDCAP shares and 68,182% of the SMALLCAP shares, when it is included in the variance equation of the GARCH model, and the volatility of 69,23% of the FTSE-20 shares, 70,833% of the MIDCAP shares and 75% of the SMALLCAP shares, when it is included in the GMM estimation model.

The stock turnover is a measure that captures the trading quantity dimension of liquidity and is strongly correlated with volume, since it is defined as the ratio volume/ number of shares outstanding. When investors examine a company for the liquidity of its shares, the easiest for them is to observe the number of its shares that are being traded on a daily basis and based on this to decide for purchase or sale. Thus, it is logical that the stock turnover affects the volatility of most stocks, since it is easy observable from the investors and reflects their expectations and decisions. Indeed, the stock turnover's influence is positive, which means that the bigger the volume of a share, and thereafter its stock turnover (that means a very liquid stock), the higher the increase of its return volatility. Moreover, stock turnover appears to be statistically significant for approximately the same number of shares per index, for both models of volatility valuation.

The second conclusion that arises is that the twelve month standardized turnover LM12 is a better liquidity measure than the one month standardized turnover LM1, and statistically significant for approximately half of the shares. Nevertheless, even though theoretically it captures all four dimensions of liquidity, it affects the volatility of fewer stocks in comparison with the stock turnover that captures only the trading quantity dimension.

Additionally, the standardized turnover LM12 appears statistically significant for almost the same number of stocks per index for both models of volatility estimation. Specifically, it affects the volatility of seven of the FTSE-20 shares, when it is included in the variance equation of the GARCH model, and the volatility of four of the FTSE-20 shares, when it is included in the GMM estimation model, while it influences the volatility of ten of the MIDCAP shares and of 22 of the SMALLCAP shares for both models. On the contrary, even though based on the GARCH model, the standardized turnover LM1 seems to influence the volatility of a satisfactory number of shares (30), based on the GMM estimation model its significance diminishes notably and precisely at thirteen stocks.

Thirdly, the illiquidity ratio, which is a measure that captures the price impact dimension, appears to be statistically significant for 35,802% of the total companies, when it is included in the GARCH model, and for 30,864% when is included in the GMM estimation model. However, even though for the MIDCAP and SMALLCAP indexes it affects almost the same percentage of stocks, its influence at the volatility of the FTSE-20 shares is a lot smaller and differs significantly in relation to the other two indexes.

Fourthly, the liquidity measure return reversal  $\gamma$ , which also captures the price impact dimension, is statistically significant for more stocks than the

illiquidity ratio when the GARCH model is taken into account, and specifically significant for 34 stocks (41,975%). However, when it is included in the GMM estimation model, extreme phenomena arise, since it affects the volatility of only three stocks of the SMALLCAP index.

## 9. Summary

In this study we examined the relationship between liquidity and stock return volatility. The motivation for our study was provided by the growing interest in liquidity that has emerged in financial literature over recent years. We constructed five liquidity measures, the  $\gamma$  return reversal proposed by Pastor and Stambaugh (2001), the illiquidity ratio employed by Amihud (2002), the turnover rate proposed by Datar, Naik and Radcliffe (1998), and the standardized turnovers LM1 and LM12 proposed by Liu (2006) and then we tested how stock return volatility is influenced when each of the five liquidity proxies is included in the conditional variance equation of the GARCH model and in the linear statistical model of the GMM estimation method.

From our results we conclude that the liquidity measure which is statistically significant for most of our sample stocks and generates a positive relationship with their return volatility is the stock turnover. This result is confirmed by both approaches used to test this relation: a GARCH (1,1) specification with the stock turnover being included as an exogenous variable in the variance equation and a GMM estimation of the volatility equation with the stock turnover being the regressor.

Of course, the results should be interpreted with care given the short period of time covered by this research. We are forced to use monthly data only from 1993 to 2005 in our test of the estimation models, and this may be considered to be short for a study of this kind, but it wasn't feasible to find data for a longer period.

The empirical results of this study are indicative of further empirical work. In particular, it would be interesting to examine how liquidity affects the stock return volatility, using longer series of data and alternative liquidity and volatility measures.

## References

Acharya Viral V. and Pedersen Lasse Heje. "Asset pricing with liquidity risk". *Journal of Financial Economics* 77 (2005) 375-410.

Admati Anat R. and Pfleiderer Paul. "A Theory of Intraday Patterns: Volume and Price Variability". *The Review of Financial Studies* (1988), Vol. 1, No. 1, pp. 3-40.

Amihud Yakov. "Illiquidity and stock returns: cross-section and time-series effects". *Journal of Financial Markets* 5 (2002) 31-56.

Amihud Yakov and Mendelson Haim. "Asset pricing and the bid-ask spread". *Journal of Financial Economics* 17 (1986) 223-249 North-Holland.

Andersen Torben G.. "Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility". *The Journal of Finance*, Vol. 51, No. 1. (Mar.,1996), pp. 169-204.

Andersen Torben G., Bollerslev Tim, Diebold Francis X., Ebens Heiko. "The distribution of stock return volatility". *Journal of Financial Economics* 61 (2001) 43-76.

Ane Thierry and Ureche-Rangau Loredana. "Does trading volume really explain stock returns volatility?". *Journal of International Financial Markets, Institutions and Money* (2006).

Arago Vicent and Nieto Luisa. "Heteroskedasticity in the returns of the main world stock exchange indices: volume versus GARCH effects". *Journal of International Financial Markets, Institutions and Money* 15 (2005) 271-284.

Barclay Michael J. and Warner Jerold B.. "Stealth trading and volatility. Which trades move prices?". *Journal of Financial Economics* 34 (1993) 281-305 North-Holland.

Blume Lawrence, Easley David, O'Hara Maureen. "Market Statistics and Technical Analysis: The Role of Volume". *The Journal of Finance*, Vol. 49, No. 1. (Mar.,1994), pp. 153-181.

Bollerslev Tim. "Generalized Autoregressive Conditional Heteroskedasticity". *Journal of Econometrics* 31 (1986) 307-327 North-Holland.

Brailsford Timothy J.. "The empirical relationship between trading volume, returns and volatility". Research Paper 94-01, December 1994.

Breidt Jay F., Crato Nuno and De Lima Pedro. "The detection and estimation of long memory in stochastic volatility". *Journal of Econometrics* 83 (1998) 325-348.

Brennan Michael J. and Subrahmanyam Avaniidhar. "Market microstructure and asset pricing: On the compensation for illiquidity in stock returns". *Journal of Financial Economics* 41 (1996) 441-464.

Brennan Michael J., Chordia Tarun, Subrahmanyam Avaniidhar. "Alternative factor specifications, security characteristics, and the cross-section of expected stock returns". *Journal of Financial Economics* 49 (1998) 345-373.

Chan Howard W. and Faff Robert W.. "An investigation into the role of liquidity in asset pricing: Australian evidence". *Pacific-Basin Finance Journal* 11 (2003) 555-572.

Chordia Tarun, Subrahmanyam Avaniidhar, Anshuman V. Ravi. "Trading activity and expected stock returns". *Journal of Financial Economics* 59 (2001) 3-32.

Chordia Tarun, Roll Richard, Subrahmanyam Avaniidhar. "Commonality in liquidity". *Journal of Financial Economics* 56 (2000) 3-28.

Chordia Tarun, Roll Richard, Subrahmanyam Avaniidhar. "Market Liquidity and Trading Activity". *The Journal of Finance*, Vol LVI, No. 2, April 2001 501-530.

Chordia Tarun, Roll Richard, Subrahmanyam Avaniidhar. "Order imbalance, liquidity, and market returns". *Journal of Financial Economics* 65 (2002) 111-130.

Clark Peter K.. "A subordinated Stochastic Process Model with Finite Variance for Speculative Prices". *Econometrica*, Vol 41, No. 1. (Jan., 1973), pp. 135-155.

Copeland Thomas E.. "A Model of Asset Trading Under the Assumption of Sequential information Arrival". *The Journal of Finance*, Vol 31, No.4. (Sep.,1976), pp 1149-1168.

Darrat Ali F., Rahman Shafiqur, Zhong Maosen. "Intraday trading volume and return volatility of the DJIA stocks: A note". *Journal of Banking & Finance* 27 (2003) 2035-2043.

Datar Vinay T., Naik Narayan Y., Radcliffe Robert. "Liquidity and stock returns: An alternative test". *Journal of Financial Markets* 1 (1998) 203-219.

Diaz Antonio, Merrick John J. Jr., Navarro Eliseo. "Spanish Treasury bond market liquidity and volatility pre- and post-European Monetary Union". *Journal of Banking & Finance* 30 (2006) 1309-1332.

Ding Zhuixin, Granger Clive W.J. and Engle Robert F.. "A long memory property of stock market returns and a new model". *Journal of Empirical Finance* 1 (1993) 83-106 North-Holland.

Domowitz Ian, Hansch Oliver, Wang Xiaoxin. "Liquidity commonality and return co-movement". *Journal of Financial Markets* 8 (2005) 351-376.

Easley David and O'Hara Maureen. "Price, trade size, and information in securities markets". *Journal of Financial Economics* 19 (1987) 69-90. North-Holland.

Eleswarapu Venkat R. and Reinganum Marc R.. "The seasonal behaviour of the liquidity premium in asset pricing". *Journal of Financial Economics* 34 (1993) 373-386 North-Holland.

Engle Robert F.. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation". *Econometrica*, Vol 50, No. 4. (Jul., 1982), pp. 987-1007.

Epps Thomas W. and Epps Mary Lee. "The Stochastic Dependence of Security Price Changes and Transactions Volumes: Implications for the Mixture-of-Distributions Hypothesis". *Econometrica*, Vol 44, No. 2. (Mar., 1976), pp. 305-321.

Fama Eugene F. and French Kenneth R. "Common risk factors in the returns on stocks and bonds". *Journal of Financial Economics* 33 (1993) 3-56 North-Holland.

Fernando Chitru S.. "Commonality in liquidity: transmission of liquidity shocks across investors and securities". *Journal of Financial Intermediation* 12 (2003) 233-254.

Foster Douglas F. and Viswanathan S.. "Can Speculative Trading Explain the Volume-Volatility Relation?". *Journal of Business & Economic Statistics*, Vol. 13, No. 4. (Oct., 1995), pp. 379-396.

Gallant A. Ronald, Rossi Peter E., Tauchen George. "Stock Prices and Volume". *The Review of Financial Studies* (1992), Vol 5, No 2, pp.199-242.

Gibson Rajna, Mougeot Nicolas. "The pricing of systematic liquidity: Empirical evidence from the US stock market". *Journal of Banking & Finance* 28 (2004) 157-178.

Glosten Lawrence R. and Milgrom Paul R.. "Bid, ask and transaction prices in a specialist market with heterogeneously informed traders". *Journal of Financial Economics* 14 (1985) 71-100 North-Holland.

Hasbrouck Joel, Seppi Duane J.. "Common factors in prices, order flows, and liquidity". *Journal of Financial Economics* 59 (2001) 383-411.

Hiemstra Craig and Jones Jonathan D.. "Testing for Linear and Nonlinear Granger Causality in the Stock Price-Volume relation". *The Journal of Finance*, Vol. 49, No. 5. (Dec.,1994), pp. 1639-1664.

Ho Yan-Ki, Cheung Yan-Leung, Draper Paul, Pope Peter. "Return volatilities and trading activities on an emerging Asian Market". *Economics Letters* 39 (1992) 91-94 North-Holland.

Huang Roger D. and Masulis Ronald W.. "Trading activity and stock price volatility: evidence from the London Stock Exchange". *Journal of Empirical Finance* 10 (2003) 249-269.

Jacoby Gady, Fowler David J., Gottesman Aron A.. "The capital asset pricing model and the liquidity effect: A theoretical approach". *Journal of Financial Markets* 3 (2000) 69-81.

Jacquier Eric, Polson Nicholas G. and Rossi Peter E.. "Bayesian Analysis of Stochastic Volatility Models". *Journal of Business & Economic Statistics*, Vol. 12, No. 4. (Oct., 1994), pp. 371-389.

Jain Prem C. and Joh Gun-Ho. "The Dependence between Hourly Prices and Trading Volume". *The Journal of Financial and Quantitative Analysis*, Vol. 23, No. 3. (Sep., 1988), pp. 269-283.

Jennings Robert H., Starks Laura T., Fellingham John C.. "An Equilibrium Model of Asset Trading with Sequential Information Arrival". *The Journal of Finance*, Vol 36, No. 1. (Mar.,1981), pp 143-161.

Jones Charles M., Kaul Gautam, Lipson Marc L.. "Transactions, Volume, and Volatility". *The Review of Financial Studies* Winter (1994) Vol. 7, No. 4, pp. 631-651.

Jun Sang-Gyung, Marathe Achla, Shawky Hany A.. "Liquidity and stock returns in emerging equity markets". *Emerging Markets Review* 4 (2003) 1-24.

Karpoff Jonatahn M.. "The Relation Between Price Changes and Trading Volume: A Survey". *The Journal of Financial and Quantitative Analysis*, Vol. 22, No. 1. (Mar., 1987), pp. 109-126.

Lamoureux Christopher G., Lastrapes William D.. "Heteroskedasticity in Stock Return Data: Volume versus GARCH effects". *The Journal of Finance*, Vol 45, No. 1. (Mar.,1990), pp 221-229.

Lang Larry H. P., Litzenberger Robert H., Madrigal Vicente. "Testing Financial Market Equilibrium under Asymmetric Information". *The Journal of Political Economy*, Vol. 100, No. 2. (Apr.,1992), pp. 317-348.

Lee Bong-Soo and Rui Oliver M.. "The dynamic relationship between stock returns and trading volume: Domestic and cross-country evidence". *Journal of Banking & Finance* 26 (2002) 51-78.

Liu Weimin. "A liquidity-augmented capital asset pricing model". *Journal of Financial Economics* 82 (2006) 631-671.

Marcelo Jose Luis Miralles, Quiros Maria del Mar Miralles. "The role of an illiquidity risk factor in asset pricing: Empirical evidence from the Spanish stock market". *The Quarterly Review of Economics and Finance* 46 (2006) 254-267.

Martinez Miguel A., Nieto Belen, Rubio Gonzalo, Tapia Mikel. "Asset pricing and systematic liquidity risk: An empirical Investigation of the Spanish stock market". *International Review of Economics and Finance* 14 (2005) 81-103.

Melino Angelo and Turnbull Stuart M.. "Pricing foreign currency options with stochastic volatility". *Journal of Econometrics* 45 (1990) 239-265 North-Holland.

Naidu G. N. and Rozeff Michael S.. "Volume, volatility, liquidity and efficiency of the Singapore Stock Exchange before and after automation". *Pacific-Basin Finance Journal* 2 (1994) 23-42 North-Holland.

Nelson Daniel B.. "Conditional Heteroskedasticity in Asset Returns: A New Approach". *Econometrica*, Vol. 59, No. 2. (Mar., 1991), pp. 347-370.

Osborne M.F.M.. "Brownian Motion in the Stock Market". *Operations Research*, Vol. 7, No. 2 (Mar.-Apr.,1959), pp. 145-173.

Pagano Marco. "Trading Volume and Asset Liquidity". *The Quarterly Journal of Economics*, Vol. 104, No. 2. (May 1989), pp 255-274.

Pastor Lubos and Stambaugh Robert F.. "Liquidity Risk and Expected Stock Returns". *Journal of Political Economy* 111 (2003) 642-685.

Sadka Ronnie. "Momentum and post-earnings-announcement drift anomalies: The role of liquidity risk". *Journal of Financial Economics* 80 (2006) 309-349.

Stoll Hans R. and Whaley Robert E.. "Stock Market Structure and Volatility". *The Review of Financial Studies* 1990, Vol 3, No. 1, pp. 37-71.



Suominen Matti. "Trading volume and Information Revelation in Stock Markets". *The Journal of Financial and Quantitative Analysis*, Vol. 36, No. 4. (Dec., 2001), pp. 545-565.

Tauchen George E. and Pitts Mark. "The Price Variability-Volume Relationship on Speculative Markets". *Econometrica*, Vol. 51, No. 2. (Mar., 1983), pp. 485-506.

Wang Ping, Wang Peijie and Liu Aying. "Stock Return Volatility and Trading Volume: Evidence from the Chinese Stock Market". *Journal of Chinese Economic and Business Studies* Vol. 3, No. 1, 39-54, January 2005

Xu Xiaoqing Eleanor, Chen Peter and Wu Chunchi. "Time and dynamic volume-volatility relation". *Journal of Banking & Finance* 30 (2006) 1535-1558.

Ying Charles C.. "Stock Market Prices and Volumes of Sales". *Econometrica*, Vol. 34, No. 3. (Jul., 1966), pp. 676-685.