"Value-at-Risk and Market Risk: Overview and an Application to the Athens Stock Exchange"

M.Sc in Financial Analysis for Executives

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Abstract

This paper provides an explanation of the significance of market risk for institutions and provides information on Value-at-Risk (VaR) methodologies as proposed by the Basel Committee on Banking Supervision and as implemented by institutions in order to measure this type of risk. It also analyzes the application of four VaR models, Historical Simulation with 100 observations, Historical Simulation with 250 observations and Variance-Covariance using Exponentially Weighted Moving Average and Simple Moving Average to forecast variance. These models are used to forecast the one-day ahead Value-at-Risk (VaR) of five one-asset portfolios based on data from the Athens Stock Exchange (ASE). By applying bactesting techniques as proposed by Christoffersen (1998) we confirm the basic weaknesses of these methods as stated in the related bibliography.

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Introduction

The importance of effective risk management has become very crucial nowadays, as the outcome of several significant factors. During the past few decades we have witnessed tremendous volatility in exchange rates, interest rates and commodity prices. The accelerating trend towards the use of derivatives has offered advantages in hedging financial risks and has provided speculative opportunities although under certain circumstances the use of them may generate huge losses. Growth in foreign trade and increasing international financial links among companies has led to portfolios including large numbers of cash and derivative instruments as well as equities from emerging economies. The complexity of these portfolios has led to the demand for a portfolio-level quantitative measure of market risk that can be reported to senior management and shareholders. Financial disasters of the last decades such as the German Metallgesellschaft firm, Daiwa bank and the Barings bank are only a few examples of how billions of dollars can be lost due to poor management of financial risks. In order to ensure financial stability, it is imperative to develop a way of measuring market risk and implement sound risk management.

In this paper we will focus on market risk which is one of the broad categories of financial risks, which stems from volatility in the market prices. The dissertation is divided into two main parts. The first one, aims to provide an understanding of the importance in the Value-at-Risk statistic. We will review the basic guidelines for its computation and analyse the various methods to compute it. In the second part, we will use the simple methods proposed by the bibliography for the computation of Value-at-Risk in the Athens Stock Exchange.

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1. Market Risk and Value-at-Risk

1.1. Types of Risk

The chance that an investment's actual return will be different than expected is called risk. This includes the possibility of losing some or all of the original investment Firms are exposed to various risks, due to business cycles, inflation, changes in government policies, wars or even natural phenomena such as earthquakes.

Business risks are those which the corporation willingly assumes in order to create a competitive advantage and add value to its shareholders. It may be quantifiable, such as business cycles, or non quantifiable, such as changes in competitive behavior.

Market Risk is the risk to a financial portfolio due to movements in market prices, such as equity prices, foreign exchange rates, interest rates and commodity prices. Market risk can be directional or nondirectional. Directional involves exposures to the direction of movements in financial variables, such as stock prices and it is measured by linear approximations such as betas for stocks, duration for interest rates and delta for options. Nondirectional risk involves nonlinear exposures, such as volatility changes, namely volatility risk, which is the exposure to movements in actual or implied volatility.

Credit risk is the risk that the counterparty may be unwilling or unable to fulfill in part or in full its contractual obligations. Commercial banks traditionally take on large amounts of credit risk through their loan portfolios. A change in credit ratings or in the market's perception of default may create changes in market prices causing credit and market risk to overlap.

Liquidity risk emerges by conducting transactions in markets with low liquidity, low trading volume and large bid-ask spreads. In the event of early liquidation the attempt to sell may push asset prices down. The owner of the assets may in this case be forced to sell at less than fair market value or even within a time frame that is longer than expected.

Operational risk arises from a technical failure and human error in the operation of a firm, including fraud, failure of management, process errors, or even physical catastrophe. Operational risk can lead to market and credit risk. Examples of losses due to failures in measuring and controlling this type of risk are the Natwest Capital Markets case and the Orange County case. This type of risk can be managed using self-insurance, or third-party

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insurance and one can be protected by having clear separation of responsibilities and strong internal controls.

The above types of risk should be regarded on the aggregate level because they interact. In their book, Crouhy, Galai and Mark (2001), analyze all the above types of risk and among other things they propose tools in order to control and manage them. If an institution focuses on one category of risk it may be moving away from this specific category and going into other types of risk that may be even more difficult to evaluate. For example, banks that were subject to capital constraints for credit risk transferred some of this credit risk to market risk with a lower capital requirement. We can also state the example of the widespread use of marking-to-market which decreases credit risk but creates a need for day-to day handling of cash-flow payments, which increases operational risk.

1.2. Value-at-Risk (VaR)

Portfolio theory employs various measures in its attempt to describe market risk in a single numerical figure. In order to quantify risk, it is necessary to forecast the distribution of future returns and select a suitable measure of risk. One of the modern risk-measuring techniques, financial institutions and regulators have turned to is Value at Risk (VaR).

Value at Risk (VaR) is an easy to understand method for quantifying market risk. It measures the worst expected loss over a given horizon, under normal market conditions and at a given confidence level. If for example a financial institution states that the daily VaR is € 20 millions at a 99% confidence level, there is 1 in a 100 chance, under normal market conditions for a loss to be greater than the above amount. Losses greater than the estimated VaR should only occur with the probability α (e.g. 5%, 1%) Thus, if managers and shareholder don't like the reported number, they can modify the portfolio's structure so as to be less risky.

VaR provides an aggregate view of the portfolio's risk taking into account leverage and correlations. It is a forward-looking risk measure. It is attractive because it is easy to understand and it provides an estimate of the amount of capital that is needed to support a certain level of risk.

VaR can encourage fund managers to think of the portfolio as a set of assets that are exposed to several sources of risk. As Aragones (1999) suggests, once these exposures are identified and quantified, it becomes possible to analyze their interaction in a way such as to determine which trades are a hedge to the portfolio and which are the largest sources of risk to the institution.

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The need for the use of VaR has emerged from: 1) the pressure of regulators for better control over a financial crisis, 2) the globalization of financial markets which has led to exposure to more sources of risk, 3) technological advances which have made enterprise-wide risk management a not-so-distant reality. Ideally, as Hendricks and Hirtle (1997) argue "the information generated by the models will allow supervisors and financial market participants to compare risk exposures over time and across institutions."

VaR is nowadays used in active management risk. It helps to allocate capital across traders, business units, products and generally the whole institution. Risk-based capital charges can guide an institution toward a better risk/return profile. Portfolio managers can make better decisions by understanding the impact of a trade on portfolio risk. All of the above can help to create greater shareholder value added (SVA).

A good and adequate risk-management model is necessary because, on the one hand, regulators are tempted to set high capital-adequacy levels, just to be safe, but on the other hand, shareholders must earn a competitive rate of return. We need to set a level using a proper form of risk-management systems in order to set the appropriate level of capital to support losses suffered while at the same time avoid being very strict, because that capital is iddle, thus restraints the comparative advantage of the institution.

One of the benefits of VaR is that in the process of computing it, it provides a methodology for critically thinking about risk. One is forced to confront its exposure to financial risks. One of the main advantages of VaR, probably the main reason why it is becoming an essential tool for conveying trading risks to management and shareholders is the fact that it summarizes in a single, easy to understand number, the downside risk of an institution.

VaR can be reported in either of two ways: as an amount denominated in the local currency (e.g. Euro VaR of portfolio € 500.000) or as a percentage, which is given by the euro VaR divided by the total value of the portfolio (e.g. 5.8% of today's portfolio value).

Other VaR measures are the marginal VaR and the incremental VaR. The marginal VaR of a position with respect to a portfolio is the amount of risk that the position is adding to the portfolio. Marginal VaR depends heavily on the correlation of the position in accordance to the rest of the portfolio. If the correlation coefficient $\rho = -1$ then the position behaves in the exact opposite of the portfolio, thus decreases the risk of the portfolio by an amount equal to its stand-alone VaR If $\rho=1$ the position behaves in the exact way the portfolio does and hence its contribution to the risk of the portfolio is equal to its individual VaR. Incremental VaR is useful because sometimes a portfolio rebalancing is needed. In this case where one may wish to sell a small amount and not the entire position incremental VaR is computed. In an article concerning the Asian equity market, Aragones (1999) points out the use of incremental VaR (also called diversified VaR) well as the use of component VaR which allows the break down of the diversified VaR into its main sources and identifies the trades that act as a hedge with respect to total portfolio risk

1.3. The Basle Committee on Banking Supervision Guidelines

1.3.1. History and Methods to use

Economic risk capital is the amount of capital that institutions would devote to support their financial activities in the absence of regulatory constraints after consideration of the risk-return trade-offs involved. One could argue that shareholders should be left alone to decide on where to allocate their capital and take the risk of their decision. But, regulation is necessary when free market appears to be unable to allocate resources efficiently. This can make sure that externalities - which is what can happen when an institution's failure affects other firms - are avoided.

Commercial banks, security houses and insurance institutions are required to carry enough capital to provide a cushion against unexpected losses. Since firms are exposed to multiple sources of risk, VaR as a risk measure technique has become a standard benchmark.

After the incident with the German bank Herstatt that was forced into liquidation leaving counterparty banks with released payments that were not paid the G-10 (actually eleven countries) formed a committee under the auspices of the bank for International Supervision (BIS). The committee was called the Basle Committee on Banking Supervision and comprised from representatives from central banks and regulatory authorities. The G-10 countries were bound to implement the recommendations made by the Committee as national laws.

On July 15, 1988 the Central Bankers from the Group of ten countries (G-10-), concluded on the Basel Accord which was a financial agreement for the regulation of commercial banks. The main purposes of the accord were to strengthen the soundness and stability of the international banking system, by providing a minimum standard for capital requirements and also to harmonize global regulations. The Basel Capital Accord of 1988 provided the first step toward tighter risk management by setting minimum capital requirements that must should be met by commercial banks to guard against credit risks.

The 1988 Basel Accord was criticized due to several issues that it didn't cover, and due to the fact that is was a very general approach. One of its main deficiencies was the non-recognition of market risk. In 1996, the Basel Committee amended the Basel Capital Accord to incorporate market risk. This added a capital charge for market risk based on either one of two approaches: the standardized method or the internal model method. Banks were given the choice between the two methods mentioned above because, during the implementation process, Central banks recognized the fact that risk management models used by major banks were far more advanced than anything they could propose. Thus, banks were given the option to use their own VaR risk-management models as the basis for required capital ratios. Banks had long realized that good risk –management systems would allow them to use their capital more efficiently, thus providing a competitive advantage.

1. Standardized Method

In the standardized method, the capital charge for market risk is a simple sum of measured risk for all factors (such as debt/equity/FX/commodities/options) The bank's market risk is first computed for portfolios exposed to interest rate risk, exchange rate risk, equity risk and commodity risk using specific guidelines. For interest rate risk the duration and maturity are used. Currency and equity risk have a market risk capital charge 8% of the net position and commodities 15%. Especially for equities, 8% is applied without regarding for their actual return volatilities. Diversification effects within broad categories of assets -fixed income, equity, FX and commodities- can be recognized. Low correlation implies that the risk of the portfolio is less than that of the sum of individual component risks, still, the Basel Committee is assuming the worst case scenario.

2. Internal Method

In April 1995 the Basel Committee allowed to banks the use of their own risk measurement models to determine their capital charge. Banks must meet certain qualitative and quantitative standards.

a) Qualitative:

- Existence of an independent risk control unit with active involvement of senior management
- The model must be closely integrated into day-to day risk management and should be used in conjunction with internal trading and exposure limits

- Programs of stress testing and back-testing should exist
- A routine for ensuring compliance and an independent review of both risk management and risk measurement should be carried out at regular intervals

b) Quantitative:

The capital charge for the internal model is the higher of the previous day's VaR or the average of daily VaR on each of the preceding 60 days times a multiplication factor, symbolized with k and sometimes called “hysteria factor”, which has a minimum of 3. It is determined by the number of times losses exceed the day's VaR figures. The increase in the multiplication factor is designed to scale up the confidence level as is implied by the number of exceptions that is found to the desired confidence level, which for the Basle Committee is 99%. A plus factor or penalty that is directly related to the ex-post performance of the model (backtesting), ranging from 0 to 1, is added.

$$MRC_t^{IMA} = \max\left(k\frac{1}{60}\sum_{i=1}^{60} VAR_{t-i,}\ VAR_{t-i,}\right) + SRC_t \qquad (1)$$

Thus, VaR is computed daily using a 99th percentile, one-tailed confidence interval and a minimum holding period of 10 trading days. Banks are allowed to scale up their 1-day VaR measure by the square root of 10. Historical observation period is subject to a minimum length of 1 year and the data should be updated at least once a quarter. An institution must maintain at all times its capital above a minimum level equal to the sum of the charge to cover general market risk plus a charge to cover credit risk.

Since most financial institutions would be tempted to underestimate and report lower VaR numbers in order to maintain lower levels of capital, the Basle Committee proposed backtesting, where regulators evaluate on a quarterly basis the frequency of exceptions, which is the number of daily losses that exceed the reported VaR.

The market risk charge of the internal model approach (IMA) on any day t is:

Banking executives prefer the internal model method, as studies have shown that the standardize approach is inefficient, especially due to the fact that it requires seven times more capital then a 10-day VaR. New guidelines were proposed by the Basel Committee in June 1999, known as Basel II. One of the main changes is the use of external credit ratings for risk weights. Those ratings are provided by credit rating agencies (such as Standard and Poor's).

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1.3.2. Criticism on the implementation

Although the internal model approach was preferred compared to the standardize approach, its implementation has been criticized. Domenico Cuoco and Hong Liu (2003) state that the choice of the multiplier k has been thought of as being too high, thus representing a drawback to the accurate report of the risk in the trading portfolio of an institution. As institutions are allowed to use the square root of ten rule to transform their daily VaR to the 10-day VaR that the Basle Committee requires, it can be thought as imprecise since this rule is valid only if the return distribution is normal. Although backtesting is performed, there still can be limitations in detecting inaccurate risk estimates. Finally, capital requirements to cover general market risk are estimated separately from those to cover credit risk. Although there are many advantages in the adoption of Basel II, as Herrings (2005) states, the costs of implementing its specific guidelines might outweigh the gains.

1.3.3. Bactesting according to the Basle Committee

Backtesting compares the daily profits and losses with model –generated risk measures to assess the quality and accuracy of the institution’s risk measurement systems. It tests whether the observed percentage of outcomes covered by the risk measure is consistent with a 99% level of confidence. To backtest, a 1-day holding period is applied. The framework adopted by the Basel Committee calculates the number of times that the trading outcomes are not covered by the risk measures – the exceptions- on a quarterly basis using the most recent 12 months of data. The boundaries are based on a sample of 250 observations and it has a range of possible responses which are classified into 3 zones:

- **Green Zone**-the backtesting results do not suggest a problem with the quality or accuracy of the bank’s model. Only four exceptions are allowed.
- **Yellow Zone-** backtesting results raise questions- up to 9 exceptions are allowed. Outcomes of this range are plausible for both accurate and inaccurate models. The number of exceptions will guide the size of potential supervisory increases in a firm’s capital requirements.

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- **Red Zone-** the backtesting results indicate a problem with the bank's risk model – 10 or more exceptions. In this case the supervisor will increase the multiplication factor by one and investigate on the model.

	No of exceptions (in 250 days)	Multiplication factor
Green Zone	0-4	3.00
	5	3.40
	6	3.50
Yellow Zone	7	3.65
	8	3.75
	9	3.85
Red Zone	>10	4.00

Table 1: The 3 zones of possible responses to backtesting and their classification criteria according to the BIS

Table 1 shows that if the number of violations, that is the number of times the VaR is exceeded is not more than 4, the institution is in the Green Zone and the multiplicative factor is 3. This means that the capital requirement is 3 times the VaR of the institution's portfolio. As the number of violations reported increases the institution passes to the Yellow Zone or even to the Red Zone with a multiplication factor of 4.

2. Computation of Value-at-Risk (VaR)

2.1. An introduction

Market risk is usually measured in terms of percentiles, also referred to as quantiles of a portfolio's return distribution and are defined as the cutoff values "q" such that the area to their right or left represents a given probability.

The advantage of using this term is that a percentile corresponds to both a magnitude (the euro amount of risk) and an exact probability. For example, the 5th percentile of a distribution of returns is defined as the value that exceeds 5% of the returns and is often denoted as α.

Using the percentiles of the standard distribution we know that for e.g. 95% confidence level, there is a 5% probability that an observed return is less than -1.65 times the standard deviation plus its mean (Probability ($r_t<-1.65\sigma_t + \mu_t$) = 5%). Since we can assume that the

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mean is equal to zero, we are left only with the standard deviation. We only use the probability of the return being less than -1.65 because we are interested in one –tail, the left area under the probability curve.

To compute VaR we have to take the following steps:

1) Mark-to market the current portfolio
2) Measure the variability of the risk factors
3) Set the time-horizon (holding period)
4) Set the confidence level
5) Report the worst loss by processing the above information

There are various models and much literature on how to compute VaR, but still no final agreement on what VaR model should be used. Most of the studies, show that the VaR model that should be used is a function of the specified portfolio and the data set used to estimate the parameters.

The holding period is the first factor according to Minnich (1999) that must be set in order to compute VaR, and should be determined by the entity's horizon. Firms with actively trading portfolios may use 1-day holding period, while regulators use 10-day holding period (which is usually scaled up from the 1-day holding period results), but one could use a 5-day, or a 2-day holding period as well. The basic rule that one should keep in mind when setting the rules to compute VaR is that the current portfolio will remain unchanged throughout the holding horizon, that is also one of the defaults of VaR. Generally, the holding period should correspond to the longest period needed for orderly portfolio liquidation. Thus, the horizon should be related to the liquidity of the securities. For instance, a bank's portfolio will be much easier to close out than a portfolio invested in stocks from emerging markets. For an investment manager the horizon may be a month or a quarter, for regulators as said earlier the required horizon is 2 weeks (or 10 days), viewed as the necessary period to force bank compliance.

In order to compute VaR the second factor one must decide on is the confidence level. The probability used as the cutoff can be set by either the user of VaR, thus the risk –manager, or it can be set, as it is by the Basle Committee from the outside of the firm. Depending on its use, the confidence level can be 95% or 99%. It is important to state what is an abnormal loss: one that occurs with a probability of 5% or one that occurs with the probability of 1%? JP Morgan at its RiskMetrics system uses 5%, whereas regulators demand 1%.

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Concerning the choice of the holding horizon and the confidence level, one must take under consideration that as the horizon or the confidence level increase, VaR will increase. That is VaR for 1 day is less than VaR for 10 days, and VaR 95% confidence level is less than VaR for 99% confidence level. The choice of the horizon and the confidence level is very important, especially when VaR figures are used to set capital requirements for an institution. A loss exceeding VaR could, in this case, lead to bankruptcy. The choice of those factors will also affect backtesting. To backtest, one will compare VaR with the realized P&L, in order to catch biases in the reported VaR figures. The shorter the horizon the powerful the test. That is why Basel Committee performs backtesting over a 1-day horizon, even though the horizon for capital adequacy purposes is 10 business days.

To compute the VaR of a portfolio, one must decompose the instruments into the portfolio and identify the basic market rates and prices that affect its value, which are the market factors. Especially when the portfolio contains complex instruments, such as swaps or loans, it is essential to express the instruments' values in terms of basic market factors. After this decomposition, a key part of the problem of quantifying market risk is finished. The next step is to determine or estimate the statistical distribution of the potential future values of the market factors. This will help determine potential future changes in the values of the various positions that compose the portfolio, and then aggregate across positions in order to determine the potential future changes in portfolio value. VaR is a measure of these potential changes in portfolio value.

There are two analytical approaches to measuring Value-at –Risk: simple VaR for linear instruments, and delta-gamma VaR for nonlinear instruments. In the former case, the relative change in a position's value is a linear function of the underlying return. In the latter case, the distribution of the portfolio's relative change is not normal. That is, we cannot define VaR with the same scaling factor as "α" (e.g. 1.65, 2.33- assuming normality) times the standard deviation. In this case one would calculate the first four moments, that is, the mean, the standard deviation, the skewness and the kurtosis, then find a distribution that has the same four moments and calculate the corresponding percentile of this distribution. A portfolio is a set of positions, each of which is composed of some underlying security. To calculate the risk of the portfolio, one must be able to compute the risk of the individual positions that compose the portfolio. Doing this requires an understanding of how a position's value changes when the value of its underlying security changes. This is the point when we need to make the necessary break-down into linear (e.g. equities) and non-linear positions (e.g. stock option).

The beginning of VaR goes back to Markowitz's (1952) work. He first pointed out to the utility and need to examine the risk compared to the return. He proposed the standard deviation as a measure of dispersion.

There are 4 types of financial market risk: interest rate risk, exchange rate risk, equity risk and commodity risk. Risk is measured by the standard deviation of unexpected outcomes, or volatility. As said earlier in this text, market risk is caused by movements in asset prices or asset returns. Variance, which is a measure of the dispersion of data points around the mean is used in order to have an indication of the variability of values computed as:

$$s^2 = \sum \frac{(x-m)^2}{N} \qquad (2)$$

that is, the average of the square of the distance of each data point from the mean. By calculating the square root of variance from equation (2) one can find standard deviation, denoted as σ. Standard deviation can then be used to compute volatility, which refers to the amount of uncertainty, or risk about the size of changes in a security's value. Volatility is the annualized standard deviation and is given by:

$$\mathrm{v} = \frac{s}{\sqrt{\Delta_t}} \qquad (3)$$

where Δt is the time unit that one measures the returns, e.g. for daily data, Δt is set 1/252. Risk can be better measured by short-term volatility.

A loss can occur due to volatility in the financial variables or through the direct exposure to the source of risk. VaR which states how bad things can get under normal market conditions with a certain probability captures the combined effect of volatility and exposure.

2.2. Stylized facts about returns

In order to implement the various methods for computing VaR it is a common method to use the returns of the asset. It is preferable to use "log" returns on an asset as:

$$\mathrm{R}_{t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right) \qquad (4)$$

which is close to the arithmetic return given by:

$$r_{t+1} = \frac{S_{t+1} - S_t}{S_t} \tag{5}$$

but has the advantage that through log returns one can easily calculate the compounded return at the k-day horizon simply as the sum of the daily returns.

Daily returns are stochastic and have very little autocorrelation, that is returns are almost impossible to predict from their own past. This assumption is consistent with efficient markets theory, where the current prices include all relevant information about a particular asset. Thus, prices follow a random walk. The expected return and the variance increase linearly with time, while volatility grows with the square root of time.

It is also true that, especially for equities there is negative correlation between variance and returns. This is the so-called leverage effect, that is that a drop in a stock price increases the leverage of the firm, as debt stays constant. So a price drop increases variance as the stock becomes more risky.

Correlation between assets is time-varying. It appears to increase in highly volatile down markets. It is generally accepted to use a mean equal to zero, especially for daily data, since the estimation of the mean creates large estimation errors. Figlewski, in his research has found that forecast accuracy is increased when computed around an assumed mean of zero rather than around the realized mean in the data sample, except for very long time periods in relatively low volatility markets.

Jorion (2001) states that the distribution of rates of return is sometimes estimated over a number of previous periods assuming that all observations are identically and independently distributed (i.i.d.). For daily intervals the squared average return is very small versus its variance, therefore we can ignore the mean in the estimation of daily risk measures and put it equal to zero.

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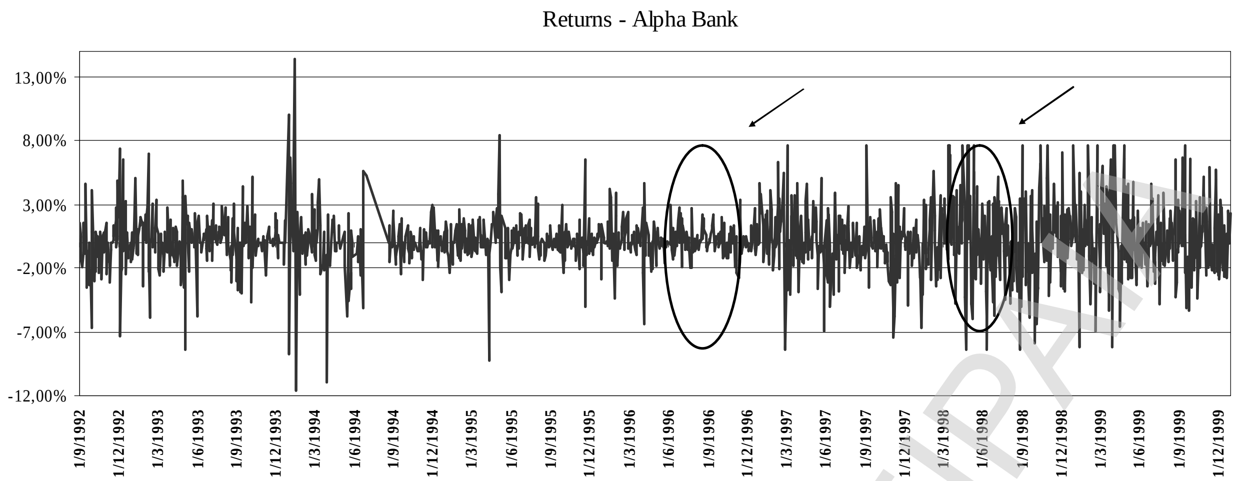

Figure 1: Daily logarithmic returns of Alpha Bank stock over the period 1992-1999. Clustering of returns, large changes are followed by large changes of either sign and small changes are followed by small changes.

If we examine the daily returns as presented in figure 1 we can observe periods of large returns that are clustered and distinct from periods of small returns which are also clustered. This is clear evidence of volatility clustering. If we measured volatility in terms of variance we would think that variance changes with time, reflecting clusters of large and small returns. That means that σ^2 is changing with time, which is described by the term heteroskedacity. The above shows a violation of the identically distributed assumption about returns.

To test for the assumption of independence of returns, we would first start with the observation of volatility clusters which shows some evidence of autocorrelation in variances. Thus to test whether returns are independent we would start by testing if they are autocorrelated. The autocorrelation coefficient measures the correlation of returns across time. For a time series of observations, Rt, t=1,2…..n, the kth order autocorrelation coefficient ρk is defined as:

$$r_{\mathrm{k}} = \frac{s^{2}_{\mathrm{t,t-k}}}{s^{2}_{\mathrm{t}}} \tag{6}$$

If a time series is not autocorrelated, then ρk will not be significantly different from 0. JP Morgan in its RiskMetrics Technical Document (1996) uses the Box-Ljung (BL) test statistic testing the null hypothesis that a time series is not autocorrelated. By applying this test to the USD/DEM, the S&P returns and daily log price changes of a selected series of commodity futures contracts they found little evidence of autocorrelation. According to previous research

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it is often found that financial returns over the short-run are autocorrelated but the magnitudes of the autocorrelation are too small to be economically significant. According to Fama and French (1988), for longer return horizons there is evidence of significant negative autocorrelation.

The above fact doesn't prove that squared returns are uncorrelated too. This could mean that variances are autocorrelated. By using again the Box-Ljung (BL) statistics, RiskMetrics rejects the null hypothesis that the variances of the daily returns are not autocorrelated for both the USD/DEM and S&P 500 squared returns.

The fact that returns have been thought as to follow a normal distribution and the problems that arise from this assumption have been studied several times. Since the early work of Mandelbrot (1963) and Fama (1965) researchers have documented certain stylized facts about the statistical properties of financial returns, as more recently were stated by Christoffersen (1996). Their conclusions can be summarized:

- Daily returns have very little autocorrelation, that means that returns are almost impossible to predict from their own past.
- The unconditional distribution of daily returns exhibits fatter tails than the normal distribution. This means a higher probability of large losses. Fat tails are measured by kurtosis, which in the normal distribution is equal to 3.
- The peak of the return distribution is higher and narrower than that predicted by the normal distribution. This characteristic along with fat tails characterizes a leptokurtotic distribution.
- The return distribution is asymmetric and negatively skewed, especially for stock market data that tend to exhibit very large drops but not equally large up-moves.
- For short horizons, variance measured by squared returns has positive correlation with the past.
- At short horizons, such as daily, the mean of returns can be set equal to zero. It is statistically impossible to reject a zero mean return.
- In equity data we can detect the leverage effect, that is a negative correlation between variance and returns. That means, that a drop in a stock price will increase the leverage of the firm as long as debt stays constant.
- As said earlier, correlation between assets appears to be time varying. It increases in highly volatile down markets and especially during market crashes.
- As the return-horizon increases, the unconditional return distribution changes and looks like the normal distribution.

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In order to overcome the failure of the normal distribution to accurately model return, researchers have established alternative modeling methods that fall in either of two classes: unconditional (time-independent) and conditional (time-dependent) distribution of returns. Models in the second category are the GARCH and Stochastic Volatility models. Those models treat volatility as a time-dependent, persistent process and they account for volatility clustering, mentioned earlier. Brooks et al. (2000) found that the use of GARCH-type models produced some inaccuracies by overstating the persistence in return volatility but proposed a simple modification in order to overcome this problem.

2.3. The three basic methods to compute VaR

The three basic methods for calculating VaR are: 1. the historical simulation, 2. the variance-covariance approach and 3. the Monte Carlo simulation.

2.3.1. Historical Simulation

Historical simulation (HS) is powerful because of its simplicity and because of the relatively few assumptions needed to make about the statistical distributions of the underlying market factors. It is a non-parametric method in which it is assumed that the distribution of the relevant market returns is constant over the sample period. To calculate VaR numbers, the returns of the risk factors for each day within the sample period are viewed as a possible scenario for future returns. The distribution of profits and losses is constructed by taking the current portfolio and subjecting it to the actual changes in the market factors experienced during each of the last n periods –days. In this way, sets of hypothetical market factors are constructed, by using their current values and the changes experienced during the last n periods. Then the hypothetical P&L is computed. The VaR number for a specific confidence interval is derived as the corresponding quantile of the empirical historical distribution. For example, one can select the loss which is equaled or exceeded 5% of the time. This method, specifically assumes that the future distribution of returns is well described by the empirical historical one. VaR is calculated as the 100p'th percentile of the returns.

By relying on actual prices, this method does not need the assumption of normal distribution of return, thus, it accounts for "fat tails" and non-zero skewness. It can also handle portfolios including non-linear instruments.

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Often the 1-day VaR is scaled by the square root of 10, especially for the shake of capital allocation. This extrapolation is valid under the assumption of i.i.d. returns, but one of the main advantages of the HS simulation method is that it doesn't rely on the assumption of i.i.d., therefore, making the extrapolation seems curious. On the other hand, the method is data-intensive and requires much computer power. In practice, one could also be faced with the problem that historical asset prices for the assets held today may not be available. Such difficulty may arise in the case of some derivatives. In this case one should use the historical prices of similar assets and construct pseudo asset prices. Also, its results are affected by the choice of the sample length, that is the choice of the window length of historical data. If n is too large, the most recent observations, that probably describe better the future distribution, carry the same weight with the older returns. If n is too small, either too few or insufficient extreme events will be observed. This last remark was confirmed by Goorbergh and Vlaar (1999), as they argued that the VaR estimates for Dutch equity index were extremely sensitive to the sample length. If for example a time window of 250 days is used for a 99% confidence level VaR than, through the way historical simulation VaR is computed, only the second and third smallest returns will matter for the calculation. In the case of a crash in the sample, which will be the smallest return, the VaR number may not change very much if the new second and third return are similar to the previous one.

The historical simulation method makes no explicit assumption about the distribution of asset returns. It lets historical data dictate the shape of the distribution. Under this method, portfolios are valued under a number of different historical time windows which are user defined. By using actual historical data instead of simulated returns from a predetermined distribution we can capture the fat tails that are often found in many risk factors' return distributions. Two main sources of fat tails are: "jumps", that is, significant unexpected discontinuous changes in prices and "stochastic volatility", meaning volatility that changes at random over time, usually with some persistence. Persistence means that high recent volatility will lead to high forecast of volatility in the near future and recent low volatility will lead to a prediction of lower volatility on the near future. Thus, the dynamics of volatility are not captured by the historical simulation method. Hull et al. (1998), recognize the accelerating use of historical simulation by institutions and propose an approach in order to incorporate volatility updating into the method.

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2.3.2. *Variance Covariance Method*

This approach incorporates conditionality and is based on the assumption that the underlying market factors have a univariate normal distribution. This implies that the changes in the value of a linear portfolio are normally distributed. By assuming that daily revenues are i.i.d VaR at the 95% confidence level from the 5% left-side "losing tail" of a histogram can be derived. The computation of VaR can be simplified if the distribution of returns belongs to a parametric family, like the normal distribution. In this case, VaR can be derived from the portfolio's standard deviation using a multiplicative factor (e.g. 1.65), depending on the confidence level.

In the different methods used to find VaR the normal distribution for returns is often assumed and thus used in the calculation of VaR, because it describes many populations and has convenient properties, although, daily returns have a distribution with fatter tails than the normal distribution. That means that there is a higher probability of large losses than what the normal distribution would suggest. Especially the stock market occasionally exhibits very large drops but not equally large up-moves. This causes the return distribution to be asymmetric, or negatively skewed.

It is well known that the normal distribution is fully described by its first two moments, the standard deviation and the mean. As said earlier, the mean is usually set equal to zero (especially for daily data), thus, the VaR of a portfolio is essentially a multiple of the standard deviation. Under the variance-covariance approach, the VaR is given by:

$$\mathrm{VaR} = -a \ \sqrt{\mathrm{w}^{'}} \, \Sigma \, \mathrm{w} \tag{7}$$

where w is a vector of absolute portfolio weights and w' is its transpose. Σ, denotes a variance-covariance matrix, while α, is a scaling factor, set equal to 1.65 for a 95% confidence level and 2.33 for a 99% confidence level.

For this method, an estimation of the parameters of asset return distribution must be made. It is necessary to estimate the mean, the variance and correlation of returns especially between assets. The covariance matrix of the risk factors is needed, where the variances and covariances are usually estimated from daily historical series of the returns of the relevant risk factors using equally weighted moving averages. In order to use the variance-covariance approach we must establish a variance forecasting model for each of the assets of the portfolio.

The advantages of this approach are that it is fast and relatively easy to implement. On the other hand, the method reveals problems if the portfolio contains a significant amount of nonlinear financial instruments, such as options, because then, the resulting P& L distributions of the underlying risk factors are not normal. Another important defect of this approach is that the resulting VaR it is very contingent on the method used to estimate the variance –covariance matrix.

2.3.3. Monte Carlo Simulation

This method has many similarities to the historical simulation. The main difference is that rather than carrying out the simulation using the observed historical changes in the market factors, one chooses a statistical distribution that is believed to adequately capture or approximate the possible changes in the market factors (e.g bivariate t-distribution). Then, a pseudo-random number generator is used to generate thousands of hypothetical changes in the market factors. These are then used to construct thousands of hypothetical portfolio P&Ls on the current portfolio and the distribution of possible portfolio profit or loss. The main idea is to produce a large number of future price scenarios using the volatility and correlation estimates for the underlying assets on the portfolio and then to value the portfolio for each of these scenarios. After generating a number of scenarios and valuing the portfolio under each of them, VaR can be reported by ordering the portfolio's return scenarios and choosing the corresponding lowest absolute return in the same way as in the historical simulation approach.

This third approach has the advantage that it produces a distribution of P&L changes and it provides a good level of control over price volatility. It accounts for a wide range of exposures and risks, including nonlinear price risk and volatility risk. It incorporates time variation in volatility, fat tails and extreme scenarios.

On the other hand, this method is mathematically intensive and is expensive to implement in terms of systems infrastructure and intellectual development. It also requires assumptions about the distribution and the correlation.

2.4. Forecasting Variance – Volatility

In order to use a variance forecasting model we need to assume normality in returns. We then need to establish a model for forecasting tomorrow's variance (σ^2_{t+1}). Thus, in order

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to use this method, one will need an estimation of tomorrow's variance. There are many approaches concerning this forecast. In its simplest form, the variance of tomorrow will be the simple average of the most recent n observations, such that σ^2_{t+1} is equal to the sum of n past squared returns, divided by n.

As simple as it is, this model has certain drawbacks, since it puts equal weights in the past n observations, such that variance when plotted over time exhibits box-shaped patterns. This points out the significance in the choice of n, as a high n will lead to an excessively smooth variance, and a low n will lead to jagged pattern of variance over time.

Since volatility changes over time, if one uses a fixed-volatility estimation approach, it may be appropriate to limit the amount of past data used. The tradeoff will be between losing accuracy because throwing away old data makes the sample size smaller than it might be, and losing accuracy by including old data that may contain relatively little information about the current state of the system. To overcome this problem, one solution would be to use a weighted average of historical observations with weights that decline with the age of each data point. That is the main key in the method adopted by RiskMetrics.

The method followed by Risk Metrics (first released in October 1994) on how returns are generated over time is based on the following:

- Return variances are heteroskedastic (change over time) and autocorrelated
- Return covariances are autocorrelated and possess dynamic features.
- Returns are normally distributed

This last assumption is useful because:

1. only the mean and variance are required to describe the entire shape of the distribution
2. The sum of multivariate normal returns is normally distributed which facilitates the description of portfolio returns

The forecasts of RiskMetrics (1996) are based again on historical price data. According to this method, tomorrow's variance can be seen as a weighted average of today's variance and today's squared return. Thus, this model assumes that returns follow a conditional normal distribution, conditional on the standard deviation.

As opposed to the equally weighted method, the resulting VaR estimates incorporate effects such as the well-known volatility clustering of time-dependent variances. This approach seems to have 2 important advantages compared to the SMA model. First, volatility reacts faster to shocks in the market as recent data carry more weight than data in the distant past. Second, after a shock (such as a large return), volatility declines exponentially as the weight of the shock observation falls.

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This method weights current observations more then past observations in calculating conditional variances given by:

$$\sigma_t^2 = \lambda \sigma_t^2 + (1-\lambda)R_t^2 \tag{8}$$

where the parameter "λ", sometimes termed the "decay factor", determines the exponentially declining weighting scheme of the observations.

Thus, recent returns matter more for tomorrow's variance than distant returns. The RiskMetrics approach, proposed by JP Morgan uses 0.94 as the value for lambda (although it is sometimes considered as being set to high). One must take under consideration that the persistence of the estimated variances depends on the chosen lambda. The smaller the decay factor, the greater the weight given to recent events. If the decay factor is set equal to one, the model becomes an equally weighted average of squared returns.

Although this method may be better than the simple average forecast we mentioned prior, it has certain disadvantages, such as the fact that it does not allow for the leverage effect (which is exhibited especially in equity data).

The principal reason for preferring to use standard deviations (volatility) as input in calculating VaR is the strong evidence that the volatility of financial returns is predictable, thus it makes sense to make forecasts of it to predict future values of the return distribution.

Variance, measured by squared returns, exhibits autocorrelation, such that if the period examined is one of high variance, then the next day is likely to be a day of high variance as well. Each underlying return in an efficient market follows a random walk, that is

$$r_t = \mu_t + \sigma\varepsilon_t, \text{ where } \varepsilon_t \sim \text{IID N}(0,1) \tag{9}$$

The zero mean random disturbance ε_t is independent of all past and future ε_t's. The lack of serial correlation in the random ε's is the defining characteristic of efficient market pricing. That is that past price movements give no information about the sign of the random component of return in period t. The logical extension of the random walk model was adopted by Black and Scholes given by the following equation:

$$\frac{dP}{P} = \mu d_t + \sigma d_z \tag{10}$$

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This is also known as the lognormal diffusion model where d_z is a time independent random disturbance, again with mean 0 and variance 1. In the above equation "σ" is volatility (can also be used with the letter "v"), that is the standard deviation of the annual return. This model produces returns that follow a normal distribution, where asset prices have a lognormal distribution (e.g. stocks). In this model volatility is constant and the standard deviation of returns over a holding period increases with the square root of the length of the period. This model, and subsequent extensions of it, has become the standard way to model asset price behavior, in financial applications generally, including derivatives pricing. The use of a simple moving average leads to relatively abrupt changes in the standard deviation once the shock falls out of the measurement sample.

Comparing, the EWMA and SMA in volatility forecasting one may expect that after a turmoil, the former will reflect the change in volatility more rapidly than the SMA method. This may suggest that EWMA incorporates external shocks better than equally weighted moving averages, thus providing a more realistic measure of current volatility. Again in this method the mean is set equal to zero. This model can also be used to construct covariance and correlation forecasts. While modeling the variances and covariances the model assumes that the variance process is nonstationary, in other words not mean-reverting.

The above model is not a perfect one since it assumes that volatility is constant. If this was the case, than there would be no need to forecast. According to Figlewski (1997) there is time variation in the returns distribution, that is, volatility changes randomly over time. Price movements are not perfectly uncorrelated over time, especially at very short intervals. He again notices that the distribution of returns demonstrates fat tails, there is more weight in the tails of the actual return distribution than in a lognormal distribution with the same variance. Occasionally prices "jump" from one level to another without trading at the prices in between to occur (as in the case of a devaluation of an exchange rate). Finally he states that there is mean reversion both in volatility and in the price level.

Figlewski points out, that especially for an institution that needs to calculate VaR, the volatility parameter can be estimated from a sample of historical price data. The sampling error that may seem as a problem in this case may be overcome by using a large number of data points even if they are only observed over very short intervals. The major problem in the case of using past historical data to forecast, is that, since volatility changes over time, old data become obsolete. The accuracy may be improved by eliminating points from the sample data when they get old. Using historical volatility as a forecast of future volatility, assumes that volatility is a constant parameter, even though there is a great deal of evidence that it is not.

Volatility is also characterized by seasonality effects. As Darrell Duffie and Jun Pan (1997) state, “there are day-of-the-week effects in volatility that reflect institutional market features, including the desire of market makers to close out their positions over weekends”. Figlewski, in his studies, has concluded that the most accurate estimates of future volatility is made from longest samples of past prices forecasting longer horizons.

Other methods to estimate standard deviations and correlations have been found. Those methods range from extreme value techniques (Parkinson, 1980) to nonlinear modeling such as GARCH (Bollerslev, 1986) and stochastic volatility (Harvy at. Al., 1994). GARCH-type models have gained the most attention.

A GARCH model explains variance by two distributed lags, one on past squared residuals to capture high frequency effects, and the second on lagged values of the variance itself, to capture longer term influences. The simplest, and most commonly used member of the GARCH family is the GARCH (1,1) model. There are similarities between the GARCH (1,1) model and the EWMA due to the fact of estimations from daily return data in which the EWMA mimics the product of the forecast made from the IGARCH model. GARCH (1,1) and RiskMetrics might yield the same results for short horizons but for longer ones, the RiskMetrics model will predict that all future days will be of high – variance if a shock has happened in the near past while GARCH model will more realistically assume that in the future variance will revert to the average value. This model although simple, captures the time-variation of variances. The main problem of this model is that it restricts the impact of a shock to be independent of its sign. This is a major disadvantage for the estimation of assets, such as stocks, where there is strong evidence of an asymmetric response to shocks. Stock return volatility increases following a sharp price drop but has not the same response in the case of sharp price rise, where volatility may become lower.

The main shortcomings of the ARCH-type models are that they need a large number of data points for robust estimation, they have a relatively complicated estimation procedure and they are mainly good in producing one step ahead forecasts and not for longer horizons.

West and Cho (1995) found that GARCH models did not significantly outperform the standard deviation estimates of the SMA model except for very short time horizons.

As forecasts for variances, implied volatilities are also used, as many practitioners look to the market to provide them with an indication of future potential return distributions. Implied volatility, as it can be extracted from a particular option pricing model is the market’s

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forecast of future volatility. This method is not yet used globally, since it would require observable options prices on all instruments that may compose a portfolio. It is generally used in conjunction with foreign exchange instruments, since only exchange traded options are reliable sources of prices. Another problem with implied volatility is that most option pricing model assume that the standard deviation is constant. Even according to academic research there isn’t clear evidence of which method is better, as it appears that each one shows superiority depending on the time series considered. Xu and Taylor (1995) note that “volatility predictors calculated from options prices are better predictors of future volatility than standard deviations calculated from historical asset price data”. On the other hand, Kroner, Kneafsey and Claessens (1995) note that researchers are beginning to conclude that GARCH (historical based) forecasts outperform implied volatility forecasts.

If we wanted to measure the return of a portfolio with more than one asset, we could just multiply the return of each asset with the corresponding weight of that asset, as:

$$R_p = w_1R_1 + w_2R_2 + \ldots + w_nR_n \qquad (11)$$

where w_i is the weight of each asset in the total value of the portfolio and R_i is the logarithmic return (continuously compounded returns) of each individual asset. The above formula is used to compute the portfolio return according to a property of the normal distribution that the sum of normal random variables is itself normally distributed. Thus portfolio returns are the weighted sum of individual security returns. The VaR of the portfolio can then be easily calculated. However, this aggregate VaR method depends on portfolio allocation, that is the weights, and would require to repeat the volatility modeling, done in some methods, every time the portfolio is changed.

When we have more than one asset in the portfolio we must account for their movements relative to one another. These movements are captured by correlations and covariances between the different assets. In large portfolios the estimation of correlation may be very difficult and time-consuming. To overcome this problem and reduce the dimensionality of the portfolio, a simple way is to use observed market returns as factors. An example one may consider the case of equities in which when the stock portfolio is well diversified, one may use the relative to the stock index to equal the variance of the portfolio. This is also the method proposed by RiskMetrics. Especially for equity, it is sometimes common practice to map equity positions to their respective indices. That is, changes in the value of a position can be described by sensitivity to changes in an index. This practice is based on the principles of single-index models that relate the return of a stock to the return of a stock index in order to forecast the correlation structure between securities. One such model is the Capital Asset Pricing Model (CAPM). In this case, the return of a stock, R_t, is defined as:

$$R_t = \beta_t r_{m,t} + \alpha_t + \varepsilon_t \tag{12}$$

The β coefficient is defined as the risk of the stock due to market changes, also referred as systematic risk. The α coefficient is the expected value of a stock's return that is firm-specific. Through diversification of the portfolio, the firm-specific component can be dropped-out and the computation of VaR can be thus defined as:

$$VaR_t = V_t \cdot \beta_t \cdot 1.65\,\sigma_{m,t,} \tag{13}$$

where $\sigma_{m,t}$ is the standard deviation of the index. The beta coefficient measures the covariation between the return on the individual stock and the return on the market index. One problem that may occur is that this method is correct in the case of well-diversified portfolios in which non-systematic risk (the risk of the firm) is negligible. Another problem is that betas do not remain unchanged over time, especially if equity from emerging markets are included.

The variance of the portfolio returns is given by:

$$\sigma^2_{PF,t+1} = \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j \sigma_{i,t+1} \sigma_{j,t+1} \rho_{ij,t+1} \tag{14}$$

3. Backtesting and evaluation of the methods

3.1. Backtesting

The VaR models that are used need to be tested as to their performance in predicting accurately the desired number or percentage of loss under each confidence level and holding period desired. One of the measures of model performance is a count of the number of times that the VaR estimates "underpredict" future losses. If for example we assume that there is a 5% chance that the observed loss exceeds the VaR forecast and our time horizon is 20 days,

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the expected number of VaR violations would be 20x0.05 = 1. That means that we expect one violation in 20 days.

To evaluate the different VaR methods, the most widely known methods are presented below as proposed by Christoffersen(1998). Those are:

a) The likelihood ratio test of unconditional coverage (LRuc)[1]

b) The Likelihood ratio test of independence (LRind)

c) The joint test of coverage and independence (LRcc)

One can make two types of errors when testing a hypothesis: (i) Type I errors occur when a model that is correct is rejected and (ii) Type II errors occur when one accepts an incorrect model. Type II errors are considered more serious in risk management, therefore specific attention should be paid in order to avoid them.

To test if the VaR as reported is the one promised we would want to test with a 99% confidence level whether in 100 observations there is only one exception. We can do that by examining a time series of past ex ante VaR forecasts and past ex post returns. Then we put 1 if the negative return is less then the VaR and 0 if the return is greater than the VaR. This is called the hit sequence of violations and it should be completely unpredictable in the time the forecast of VaR is made and independently distributed over time.

The hit sequence is:

$$I_{t+1} = \begin{cases} 1, \text{ if } r_{t+1} < -\mathrm{VaR}_{t+1} \\ 0, \text{ if } r_{t+1} > -\mathrm{VaR}_{t+1} \end{cases} \tag{15}$$

To test if the fraction of violations that one has obtained from a particular risk model, let's symbolize it as π, is different from the fraction α (e.g. 5% or 1%) one can use the unconditional coverage test. Thus the null hypothesis Ho: $\pi = \alpha$ versus H1: $\pi \neq \alpha$. The expected value of π is: $\frac{T_1}{T}$, where T_1 is the number of violation in the sample T. The likelihood ratio test used is:

$$LR_{uc} = -2\ln\left(\frac{(1-a)^{T_0} a^{T_1}}{\left\{ \left(1-\frac{T_1}{T}\right)^{T_0} \left(\frac{T_1}{T}\right)^{T_1} \right\}} \right) \sim c_1^2 \tag{16}$$

[1] This likelihood ratio test was first proposed by Kupiec (1995)

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The test will be distributed as a χ^2 with one degree of freedom. We choose a significance level of 10% for the test which will have a critical value of 2.7055. If the LR_{uc} test value is greater than the above value then we reject the null hypothesis at the 10% level. We choose a high significance level in order to avoid Type II errors as is the common practice in risk management, although this high significance test may lead us to reject a correct model, that is make a Type I error.

Even if the model has a correct unconditional coverage it still presents a problem if all the violations are happening around the same time. The concentration of risk in a very short period could lead to bankruptcy much easier than if the violations were scattered randomly through time. A test that can show us if violations are clustered in time is the independence test. The LR test of independence tests the hypothesis of independence against a first order Markov chain. Independence would mean that the days on which the actual losses are larger than the estimated value at risk are independent from each other. If today is a nonviolation day, that is a 0, the probability of tomorrow being a violation, that is a 1, is π_{01}. Then π_{11} is the probability of tomorrow being a violation given that today is also a violation day. Based on a first-order Markov property, the basic assumption is that only the outcome from today will matter for tomorrow's outcome. The entire distribution is described by the two probabilities π_{01} and π_{11}. The probability of a nonviolation following a nonviolation is 1- π_{01} and the probability of a nonviolation following a violation is 1- π_{11}. We let $T_{i,j,}$ where i,j=0,1 be the number of observations with a j following an i.

$$\hat{\Pi} = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix} = \begin{pmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{pmatrix} = \begin{pmatrix} \dfrac{T_{00}}{T_{00}+T_{01}} & \dfrac{T_{01}}{T_{00}+T_{01}} \\ \dfrac{T_{10}}{T_{10}+T_{11}} & \dfrac{T_{11}}{T_{10}+T_{11}} \end{pmatrix} \tag{17}$$

What we basically want to test is that the probability of a violation tomorrow does not depend on today being a violation or not. The null hypothesis to test is Ho: $\pi_{01} = \pi_{11}$ versus H_1: $\pi_{01} \neq \pi_{11}$ and the likelihood ratio of this test is:

$$LR_{ind} = -2\left(\ln\left(1-\pi_{01}\right)^{T_{00}} \pi_{01}^{T_{01}} \left(1-\pi_{11}\right)^{T_{10}} \pi_{11}^{T_{11}} - \ln\left(\left(1-\pi_{0}\right)^{T_{00}+T_{01}} \pi_{0}^{T_{01}+T_{11}}\right)\right) \sim \chi_1^2 \tag{18}$$

In large samples the distribution follows as well a χ^2 distribution with one degree of freedom.

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Last test will be the Conditional Coverage Test in which we can simultaneously test if the VaR violations are independent and the average number of violations is correct. The three tests are related by the following equation:

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{19}$$

The null hypothesis is Ho: $\pi_{01}=\pi_{11}=\alpha$. This test can be calculated by summing the two individual tests for unconditional coverage and independence.

Other methods to use in order to backtest have been proposed and implemented, such as the Loss Function proposed by Lopez (1999), used in order to support the 3 tests mentioned above. This specific method can also be tailored in order to evaluate specific interests that regulators might have.

3.2. Evaluation of the methods

Historical simulation and Monte Carlo work well even in the presence of options and option-like instruments in a portfolio. In contrary the variance-covariance method is less able to capture the risks of this kind of portfolios since it replaces the nonlinear functions with linear approximations. This problem is least severe when the holding period is one day, since large changes in the underlying rates or prices are unlikely over such short periods.

The historical simulation method is easy to implement, especially for portfolios for which data on the past values of the basic market factors are available, such as equities and currencies. With the Monte Carlo simulation method, computation may be more difficult requiring not only computational skills but also expertise and judgment since one needs to select a distribution and estimate the parameters. Concerning ease of communication to senior management, the historical simulation method is the easier and the Monte Carlo the hardest. All methods rely on historical data, but the historical simulation method is the only that relies so directly on them. The above characteristics of each method are analyzed in detail by Linsmeier et al. (2000)

Concerning the estimation of volatility, Figlewski (1997) compared the different historical methods and concluded that specifically for equity markets the GARCH (1,1) forecasts are better, presenting a smaller RMSE. This performance, he found, is not very well replicated for other markets. On the other hand Christoffersen et al. (2001) by comparing VaR measures computed with various methods found that at the 99% confidence level the RiskMetrics approach outperforms GARCH methods.

In favor of more dynamic models to estimate portfolio level returns are Andersen et al. (2005) who in their study state that Berkowitz and O'Brien (2002) found evidence that the risk models adopted by banks perform poorly and that they are outperformed by a simple GARCH model. They found that the VaR reported by banks underestimate on average risk when compared with the ex post P&Ls. Although this finding could be due to the reported P&L being "dirty", that is containing non-risk income from fees, commissions and intraday trading profits, Berkowitz and O'Brien (2002) found that the VaR violations which occur tend to cluster in time. After such event as the 1998 Russia default, market volatility increases persistently and according to their study, bank models tend to ignore this or at least react to with considerable delay.

3.3. Criticism on VaR

Artzner et al (1999) pointed out 4 desirable properties of risk measures for capital adequacy purposes. A measure for risk can be expressed as a function of the distribution of a portfolio value W, which can be summarized into a single number ρ(W). Below is Artzner et al's definition of a coherent measure of risk.

1. Monotonicity: if $W_1 \leq W_2$, $\rho(W_1) \geq \rho(W_2)$, or if a portfolio has systematically lower returns than another for all states of the world, its risk must by greater.

2. Translation invariance: $\rho(W_1 + k) = \rho(W_1) - k$, or adding cash k to a portfolio should reduce its risk by k.

3. Homogeneity: $\rho(bW) = b\rho(W)$, or increasing the size of a portfolio by b should simply scale its risk by the same factor.

4. Subadditivity: $\rho(W_1 + W_2) \leq \rho(W_1) + \rho(W_2)$, or merging portfolios cannot increase risk.

Artzner et al (1999) show that the last property is not satisfied by the quantile-based VaR. When returns are normally distributed, however, the standard deviation-based VaR satisfies the property of subadditivity:

$$\sigma(W_1 + W_2) <= \sigma(W_1) + \sigma(W_2) \tag{20}$$

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As Markowitz had shown, the volatility of a portfolio is less than the sum of volatilities.

Important limitations of VaR are that it assumes that the portfolio is left unmanaged over the holding period and also, according to Figlewski, computing VaR for 10 days using ideally 10-day nonoverlapping past returns would mean that one would need 10 times as many past daily returns, which creates a problem in the case of most of the assets.

Another criticism about VaR is that it does not provide an estimate for the size of losses if the VaR level is exceeded, therefore it does not state by how much actual losses will exceed the VaR figure.

3.4. Other methods to compute VaR

VaR measures do not give the worst potential loss. The behavior in the tail of the distribution can be analyzed through stress testing techniques which must be viewed as a complement to VaR as Mina J. et al. (2001) suggest. If the reported value at risk is exceeded one cannot know how large the loss will be. Stress testing can answer this question by performing a set of scenario analyses to investigate the effects of extreme market conditions.

It usually begins with a set of hypothetical extreme market scenarios. The effect of these scenarios on the prices of all instruments in a portfolio and the impact on portfolio value is the next step. This second step is the portfolio revaluation and resembles to the process followed in the Monte Carlo and historical simulation case. The most important part of stress tests is the selection of scenarios. One way is to select a period in which a financial crisis occurred, such as Black Monday (1987) or the Russian crisis (1998) and another way is to use scenarios that are user–defined, that is, the user changes some of the risk factors setting them to a specific value.

The advantage of stress test is that it is not based on statistical assumptions about the distribution of risk factor returns. Thus it is a complement to any statistically based risk measure.

Another method is the Expected Shortfall which is a sub-additive risk statistic that describes how large losses are on average when they exceed the VaR level, hence provides further information about the tail of the P&L distribution. Mathematically is can be defined as the conditional expectation of the portfolio losses given that they are greater than VaR.

$$ES^p_{t+1} = -E_t\left[R_{t+1} \middle| R_{t+1} < -VaR^p_{t+1}\right] \tag{21}$$

The expected shortfall states the expected value of tomorrow's return, conditional on the fact that this return is worse than the VaR. This method has been proposed as an alternative measure of risk by Delbaen (1998) and Artzner (1997). Expected Shortfall allows the direct comparison of the tails of two distributions and combined with VaR provides a measure of the cost of insuring portfolio losses beyond VaR at an α confidence level. McNeil (1999), proposes Extreme Value Theory (EVT) in order to measure extreme risks and in order to overcome the problem of assuming normality in returns for different risk measures.

If a company has portfolios that are consisted of relatively simple instruments it may find it easier to use other alternatives to value at risk, to overcome the difficulty of implementing its system and calculations. One alternative is Sensitivity analysis, in which case, hypothetical changes in the value of each market factor (or instrument) are made and then, through pricing models, the new value of the portfolio is computed. Thus the change in the portfolio that arises from the change in the market factor is calculated. An effort must be made to cover the range of likely changes in the market factors. These sorts of computations provide a good picture of the risks of portfolios with exposures to only a few market factors but in the case of larger portfolios it would be difficult to handle very large data of this sort.

4. Empirical Investigation

4.1. An Introduction

The VaR of an equity position is defined as the market value of the investment in that stock (V_t), multiplied by the price volatility estimate of that stock's returns and the corresponding "α" factor, the choice of which depends on the chosen confidence level (e.g. 95%).

In this part of the paper, we will use the historical simulation approach and the variance-covariance method in order to estimate VaR for equities. Each model will be used to generate daily VaR estimates on the assumption of a one-day holding period at the 95% and the 99% confidence interval.

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4.2. Data and Methodology

We will use the historical simulation approach with a time window of 100 and 250 historical observations respectively (HS-100, HS-250). Due to the equal weighting of each historical scenario, these models do not discriminate between recent scenarios and those back in time. All of them carry the same probability of occurrence. In the case of historical simulation, if markets are not very volatile at the moment while the sample used for simulating VaR contains data stemming from a volatile period we will end by overestimating VaR. On the other hand, if we are in a highly volatile period and we use data based on a low volatility period, we will end by underestimating the VaR number.

The variance-covariance approach is computed using daily variances estimated by means of two methods:

1. Equally weighted moving averages, which we will call Simple Moving Average (SMA) using a window length of 250 actual trading days.
2. Exponentially weighted moving averages (EWMA) which incorporates volatility clustering of time dependent variances. Lambda (λ) is set at 0.94 for estimating the daily variances, in accordance with J.P. Morgan and computed according to equation (8).

The two methods are applied to 5 single-stock portfolios consisting of 4 different equities from the banking sector that are listed on the Athens Stock Exchange in the FTSE/ASE 20 index, Eurobank, National Bank, Alpha Bank and Piraeus Bank and 1 index, the FTSE/ASE 20. The assumption is that the investor holds a certain amount of euros in these stocks. The amount invested in each portfolio is 1,000,000 €. Financial institutions typically hold portfolios comprising of a multitude of assets, in this paper however, we have chosen a simple linear portfolio structure to avoid complicated issues concerning the valuation and mapping of complex financial instruments that would add extra noise to the comparisons. The portfolio is assumed to remain unchanged throughout the period. The sample is split into two parts. The one that is used each time to calculate the required inputs, such as variances, and the remaining which will be used for backtesting. All calculations rest on the assumption that the means of the daily return series are zero.

	Equities	Amount Invested
Portfolio 1	Alpha Bank	1,000,000
Portfolio 2	Piraeus Bank	1,000,000
Portfolio 3	National Bank	1,000,000
Portfolio 4	Eurobank	1,000,000
Portfolio 5	FTSE/ASE 20	1,000,000

Table 2: Description of each portfolios' synthesis and amount (in €)

Equity data were obtained from Bloomberg for the period of April 14th 1999 to February 16th 2007 that is 1,962 observations. We computed log returns for all portfolios. We focused on long positions which means that we have bought the asset at a given price and we are concerned with the case that the price of this asset falls resulting in losses.

We have chosen to use both HS-100 and HS-250 in order to observe the effect of the historical period used on VaR calculations. In a study in September 2003[1] concerning the computation of VaR using data from the Greek stock market, the authors state that “several papers have found that the performance of the method depends on the sample size. For example, Hendricks (1996) and Vlaar (2000) found that historical simulation tends to yield more accurate estimates as the sample size increases”. On the contrary, concerning this specific topic, Angelidis and Benos (2004) mention that Hoppe (1998) proposed the use of a smaller sample size in order to accommodate the structural changes of the trading behavior.

From the diagram below we can see that HS-100 observation versus HS-250 demonstrates box-shaped patterns arising from the sudden inclusion and exclusion of large losses in the moving sample. That is one evidence of how crucial it is to select the right number of observations to perform the statistic.

[1] See Skiadopoulos et al. (2003)

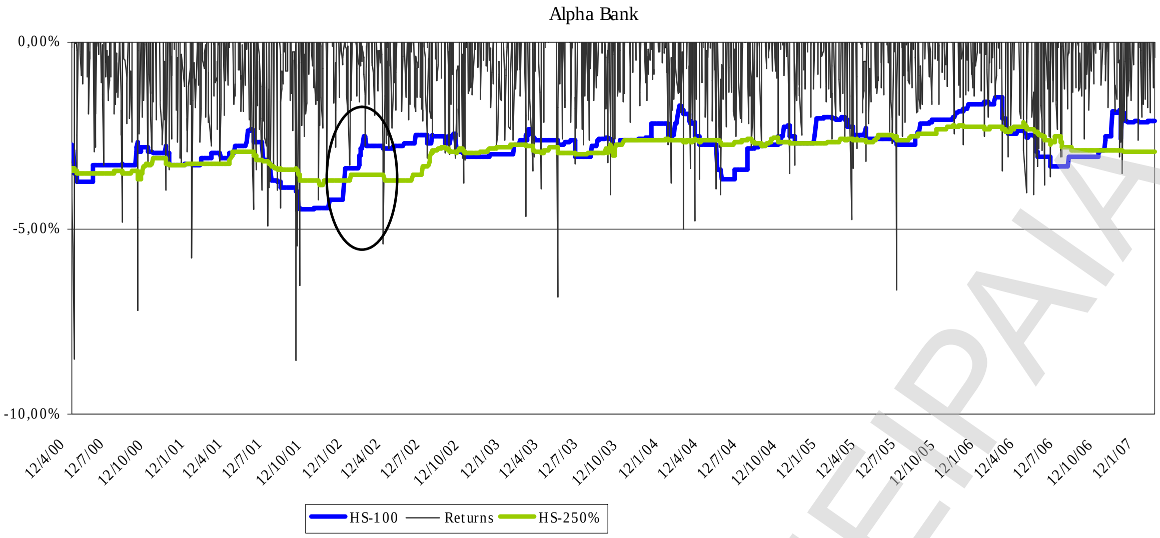

Figure 2: Alpha Bank logarithmic returns over the period 2000-2007 and VaR computed using Historical Simulation with a time window of 100 and 250 observations respectively for the 95% confidence level. Box-shaped pattern.

There is a continuous debate on whether the correlations between different financial prices are stable enough to be relied upon when quantifying risk and also on how to model the behavior of volatility in market prices. Since some VaR methodologies, such as the variance-covariance approach, assume that financial returns are normally distributed, we would like to see whether this hypothesis is correct for our sample of equity series, thus the method yields correct results.

A basic feature for equity returns is the fact that they are characterized by skewness, a measure of the degree to which positive deviations from the mean are larger than negative deviations from the mean. So if one holds long positions, then negative skewness creates a problem for value at risk, since it implies that large negative returns are more likely than large positive returns. Skewness characterizes the asymmetry of a distribution around its mean and is 0 for the normal distribution. It is computed as:

$$s^3 = E[(r_t - m)^3] \qquad (22)$$

On the other hand kurtosis measures the relative peakedness or flatness of a given distribution and is equal to 3 for the normal distribution, computed as:

$$s^4 = E[(r_t - m)^4] \qquad (23)$$

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4.3. Results

The test statistics for the 5 portfolios are presented in table 3. If skewness is equal to 0 than the tails on the distribution follow a normal distribution. If skewness is negative than the distribution has thinner tails than the normal and if skewness is positive, as in our case, than the distribution has fatter tails. The mean is equal to zero which supports the fact that we compute volatility using a zero mean. The presence of extreme events is supported by the fact that the maximum and minimum values are quite large in absolute terms.

Summary Statistics					
	Alpha Bank	*Piraeus Bank*	*National Bank*	*Eurobank*	*FTSE/ASE 20*
Mean	0.01%	0.01%	0.01%	0.00%	0.00%
Standard Deviation	1.98%	1.94%	2.00%	1.85%	1.50%
Sample Variance	0.04%	0.04%	0.04%	0.03%	0.02%
Kurtosis	2.3332	3.3737	2.3100	3.0467	3.9605
Skewness	0.4184	0.3333	0.2074	0.3459	0.1497
Minimum	-8.56%	-11.39%	-9.82%	-8.34%	-9.60%
Maximum	11.14%	9.50%	11.06%	9.31%	8.68%
Observations	1962	1962	1962	1962	1962

Table 3: Descriptive Statistics of daily log returns of the prices of the 5 portfolios for the period of 14/4/1999 to 16/2/2007

Table 4 presents the VaR estimates from the analysis of the 5 portfolios under the four VaR methods used in the research (HS-100, HS-250, EWMA and SMA).

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Average VaR - 95%								
	HS-100		HS-250		EWMA		SMA	
	percentage (%)	Amount (in €)	percentage (%)	Amount (in €)	percentage (%)	Amount (in €)	percentage (%)	Amount (in €)
Alpha Bank	-2.81%	-28,074	-2.94%	-29,382	-3.04%	-30,401	-3.18%	-31,801
Piraeus Bank	-2.64%	-26,402	-2.79%	-27,938	-2.80%	-28,025	-3.02%	-30,159
National Bank	-3.02%	-30,179	-3.19%	-31,877	-3.09%	-30,937	-3.22%	-32,206
Eurobank	-2.41%	-24,140	-2.56%	-25,617	-2.61%	-26,122	-2.80%	-28,018
FTSE/ASE 20	-2.11%	-21,099	-2.21%	-22,097	-2.17%	-21,679	-2.33%	-23,256

Average VaR - 99%								
	HS-100		HS-250		EWMA		SMA	
	percentage (%)	Amount (in €)	percentage (%)	Amount (in €)	percentage (%)	Amount (in €)	percentage (%)	Amount (in €)
Alpha Bank	-3.95%	-39,481	-4.29%	-42,937	-4.30%	-42,996	-4.50%	-44,977
Piraeus Bank	-3.82%	-38,157	-4.25%	-42,468	-3.96%	-39,636	-4.27%	-42,654
National Bank	-4.47%	-44,679	-4.83%	-48,261	-4.38%	-43,755	-4.55%	-45,549
Eurobank	-3.43%	-34,278	-3.92%	-39,204	-3.69%	-36,945	-3.96%	-39,626
FTSE/ASE 20	-3.10%	-30,963	-3.42%	-34,158	-3.07%	-30,662	-3.29%	-32,891

Table 4: Average VaR results both as a percentage and an amount in euros for the 5 portfolios at the 95% and 99% confidence level respectively. VaR is computed by Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA). Data included 12/4/2000-16/2/2007.

Comparing the average VaR numbers, we cannot conclude with certainty that in both cases, that is 95% and 99% confidence level there is one and only method that is the most conservative. In the 95% confidence level the SMA method produces the highest average VaR numbers while in the 99% case both the HS-250 and the SMA exhibit high VaR figures. In general the SMA method appears to be conservative enough compared with the rest of the methods. In the 99% confidence level, the results seem to be in accordance with a study by Skiadopoulos et al. (2003) concerning the Greek Stock Exchange where the higher VaR figure is produced by the Historical Simulation method with 250 observations. Adversely, Cassidy and Gizycky (1997) found that concerning foreign exchange portfolios, the lowest VaR figure among the three basic methods (Historical Simulation, Variance-Covariance and Monte Carlo) is demonstrated from the Historical Simulation approach.

As said earlier, the method that is used to compute VaR affects the capital that is held by institutions to cover losses related with market risk. If for example bank (A) uses the

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historical simulation method with 250 observations and bank (B) uses the variance-covariance approach with the EWMA estimator of variance, for the same exactly portfolios bank (A) will have to keep higher capital coverage in order to meet the BIS requirements at the 99% confidence level. If one did not know that the two banks hold the exact same portfolios, the assumption that the assets held by bank (A) are more risky than those held by bank (B) could easily be made.

	Exceptions - 95%							
	HS-100		*HS-250*		*EWMA*		*SMA*	
	No	*%*	**No**	*%*	**No**	*%*	**No**	*%*
Alpha Bank	99	5.79%	89	5.20%	79	4.62%	64	3.74%
Piraeus Bank	102	5.96%	84	5.91%	86	5.03%	61	3.57%
National Bank	112	6.55%	89	5.20%	94	5.49%	88	5.14%
Eurobank	106	6.20%	83	4.85%	84	4.91%	70	4.09%
FTSE / ASE 20	104	6.08%	84	4.91%	89	5.20%	66	3.86%
Average	*104.6*	*6.11%*	*85.8*	*5.01%*	*86.4*	*5.05%*	*69.8*	*4.08%*
	Exceptions - 99%							
	No	*%*	**No**	*%*	**No**	*%*	**No**	*%*
Alpha Bank	40	2.34%	23	1.34%	23	1.34%	17	0.99%
Piraeus Bank	42	2.45%	25	1.46%	22	1.29%	19	1.11%
National Bank	44	2.57%	27	1.58%	23	1.34%	30	1.75%
Eurobank	41	2.40%	20	1.17%	24	1.40%	15	0.88%
FTSE / ASE 20	39	2.28%	24	1.40%	28	1.64%	21	1.23%
Average	*41.2*	*2.41%*	*23.8*	*1.39%*	*24*	*1.40%*	*20.4*	*1.19%*

Table 5: No of violations, that is the number of times the calculated VaR is exceeded in the sample size, as well as the frequency of exceptions for the 5 portfolios respectively. Results presented include the 95% and 99% confidence level. The methods used to compute VaR are: Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA). Data included cover the period from 12/4/2000 to 16/2/2007

An adequate model for the computation of VaR would yield an average exception rate equal to the "α" value, that is 5% for the 95% confidence level and 1% for the 99% confidence level respectively. According to table 5, on average, for the 95% confidence level the HS-250 and the EWMA seem to yield the best results. On the other hand the HS-100 method seems to underestimate VaR, while the SMA method overestimates VaR, which is in accordance with the fact that this method presented the higher average VaR amount (Table 4). For the 99% confidence level on the other hand, no model seem to perform adequately. HS-100 exhibits the same characteristic as in the 95% case underestimating VaR, while SMA seems to be the more accurate in this case as the values produced by this method are on average closer to the desired level of exceptions, that is 1%. Raaji et al. 1998, calculated VaR on foreign exchange portfolios using similar methods and found that at the 99% confidence

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level the SMA method shows the weakest performance. The difference of results may be due to the differences in the assets used in the two researches.

Figures 3 to 7 provide a visual sense of the VaR calculation results for the 5 portfolios, under each of the four VaR methods used (HS-100, HS-250, EWMA and SMA), for the 95% and 99% percentile risk measures respectively as well as the actual losses of the portfolios during the backtesting period. Although the patterns for the different VaR methods present differences, they seem to be basically similar for the two confidence levels. The VaR that is obtained in the 95% confidence level under the HS-100 method is closer to the VaR computed by the rest of the measures compared with the 99% case which is again in accordance with previous research for the VaR of equities in the Greek Stock Exchange[1].

As expected both HS-100 and HS-250 exhibit box-shaped patterns that are much more visible in the historical simulation method with 100 observations. What can be said is that VaR computed by the historical simulation method stays constant for long periods and than changes in an abrupt way, which is more obvious in the case of 100 observations. Extreme events that may create large negative returns influence VaR numbers for long periods. This fact can be seen clearly in figure 5, that shows VaR calculations for portfolio 3 – National Bank and specifically at the 99% confidence level where in the period from September 2001 to September 2002 a negative return of -9.82% influences the VaR number computed by the Historical Simulation methods remarkably and for a long period.

[1] See Skiadopoulos et al. (2003)

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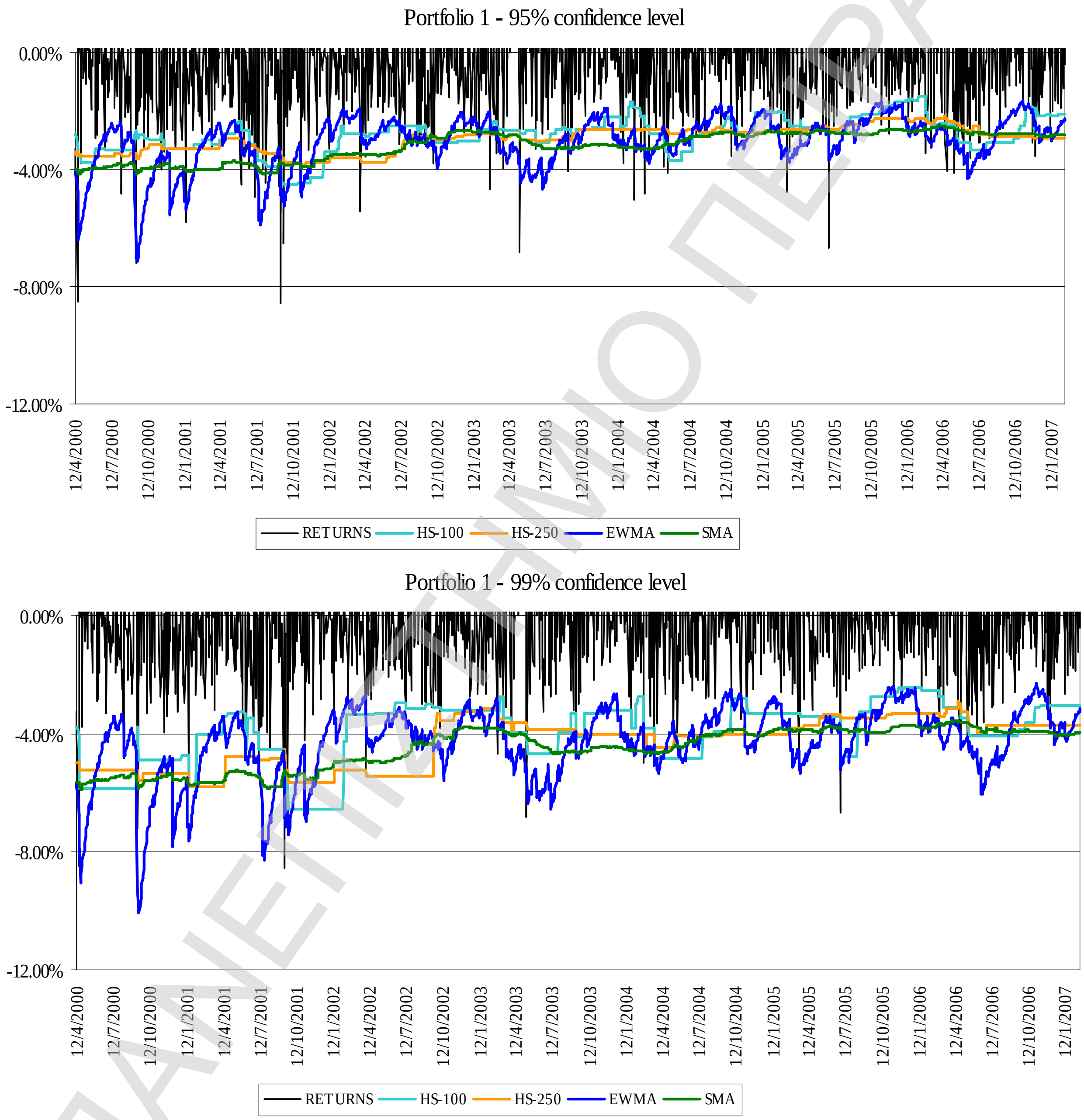

Figure 3: Log returns and the 95% and 99% VaR for portfolio 1 - Alpha Bank as calculated by the following methods: Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250) Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA) for the period 12/4/2000-16/2/2007.

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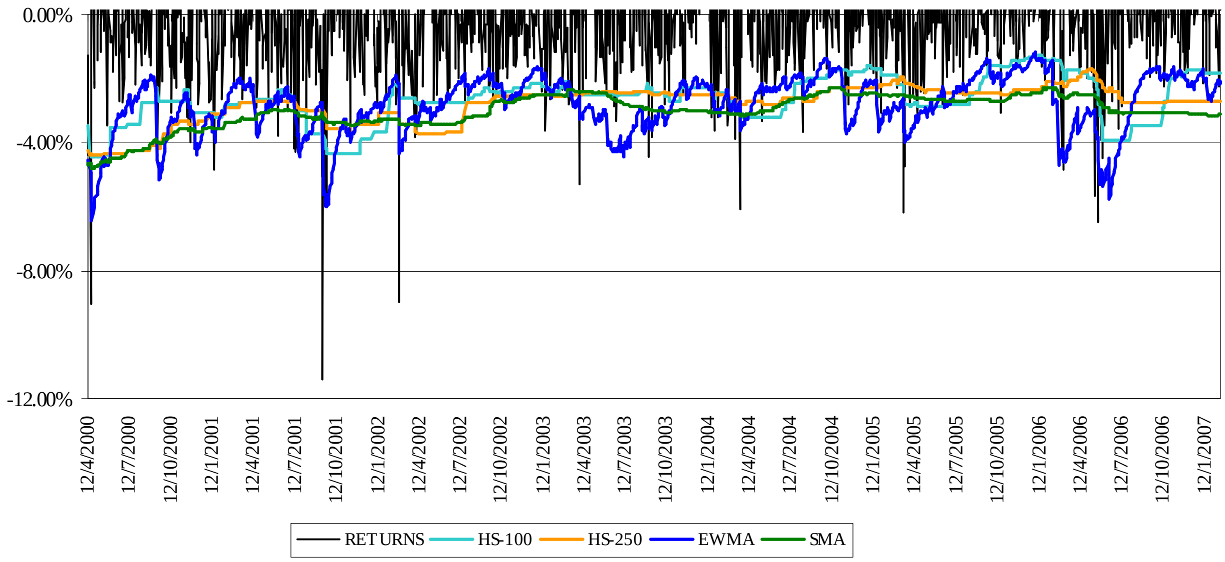
Portfolio 2 - 95% confidence level



Portfolio 2 - 99% confidence level

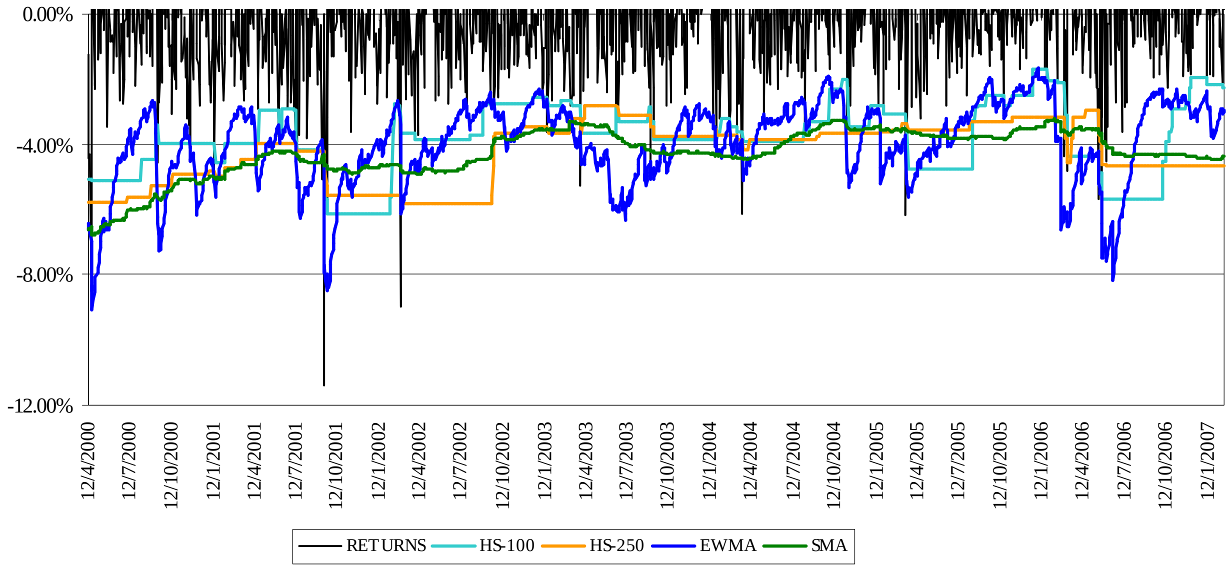
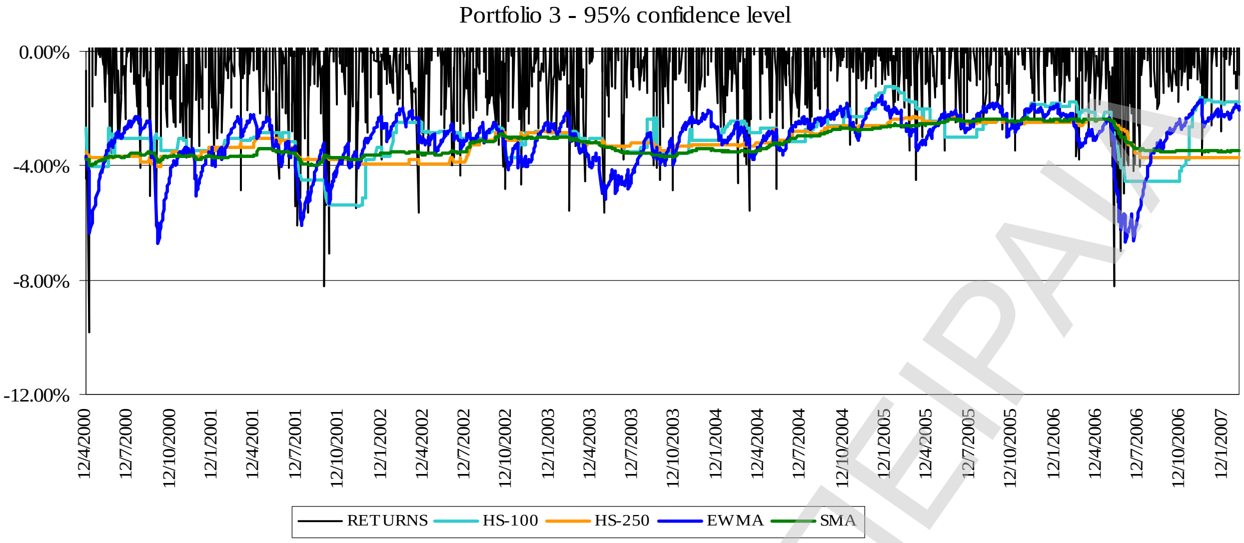


Figure 4: Log returns and the 95% and 99% VaR for portfolio 2 - Piraeus Bank, as calculated by the following methods: Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250) Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA) for the period 12/4/2000-16/2/2007.



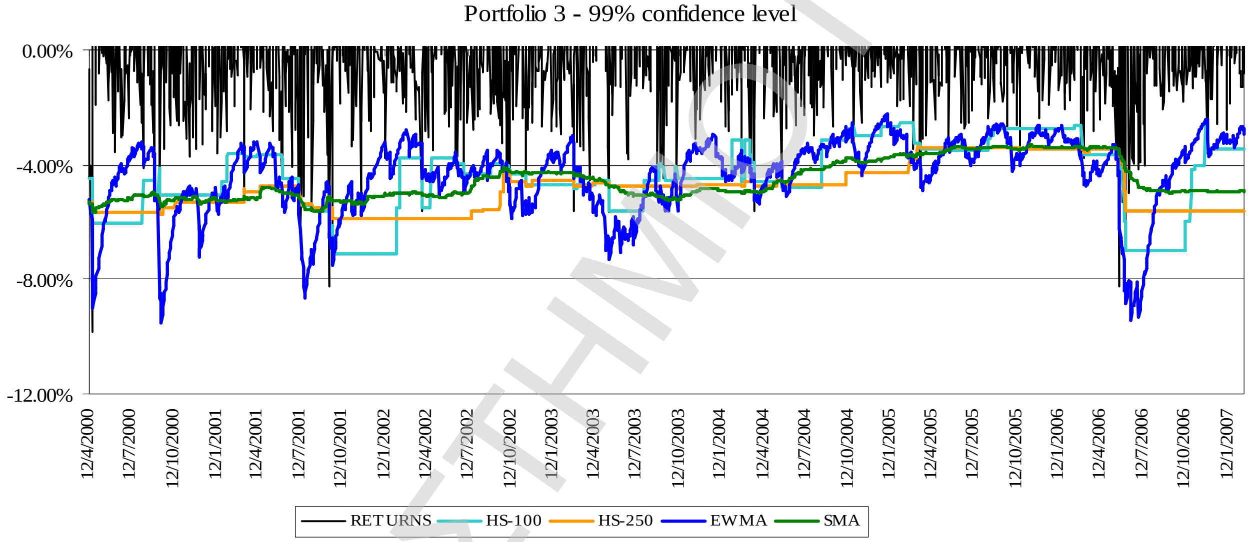


Figure 5: Log returns and the 95% and 99% VaR for portfolio 3 - National Bank, as calculated by the following methods: Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250) Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA) for the period 12/4/2000-16/2/2007.

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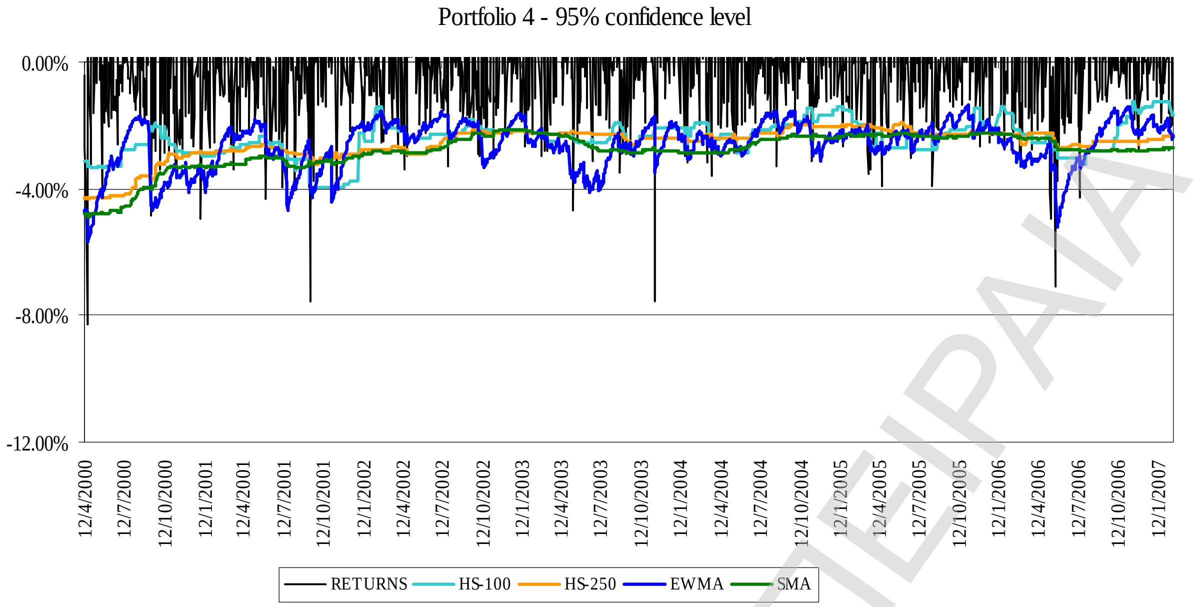

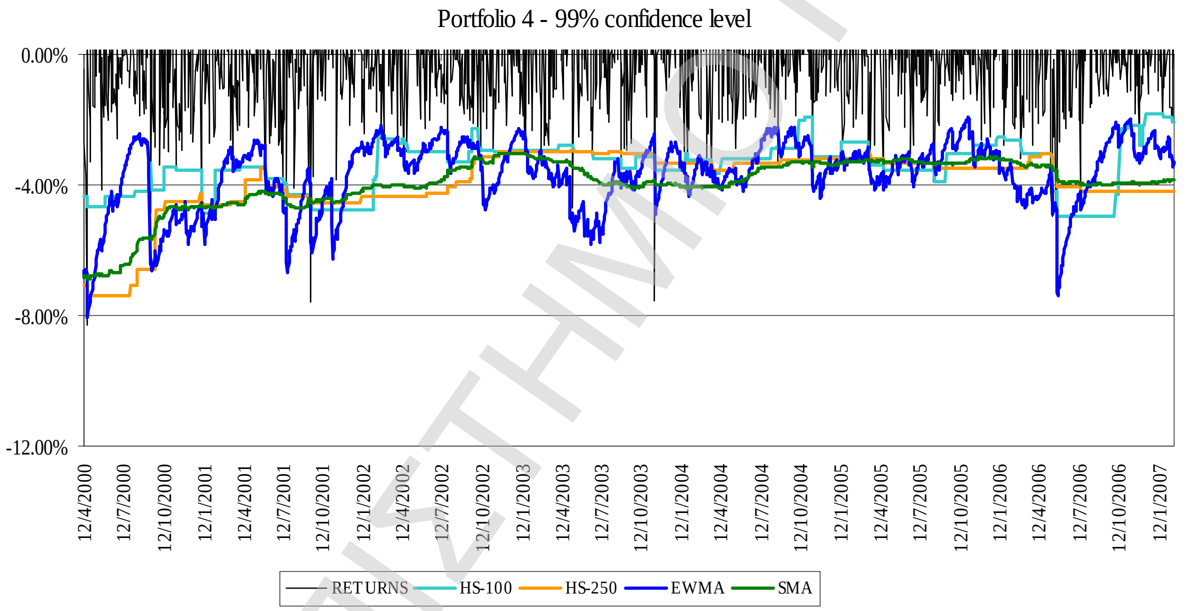

Figure 6: Log returns and the 95% and 99% VaR for portfolio 4 - Eurobank, as calculated by the following methods: Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250) Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA) for the period 12/4/2000-16/2/2007.

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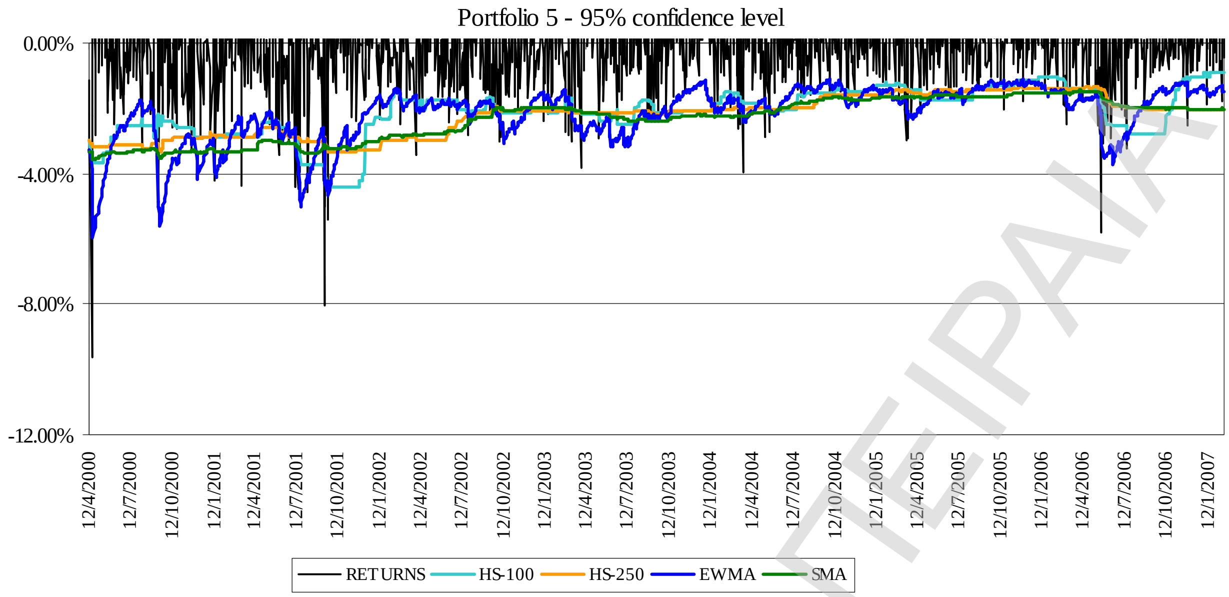

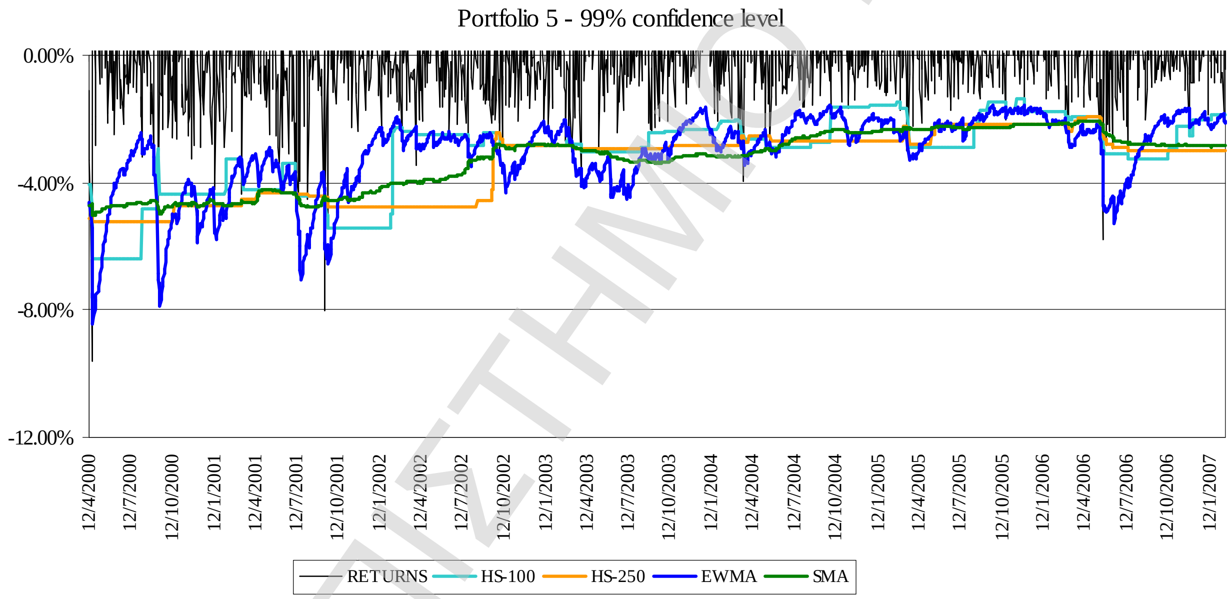

Figure 7: Log returns and the 95% and 99% VaR for portfolio 5 - FTSE/ASE 20 as calculated by the following methods: Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250) Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA) for the period 12/4/2000-16/2/2007.

The calculations of VaR in variance-covariance approaches are influenced by the methods used to estimate daily variance. The simple moving average (SMA) method produces a smoother VaR series than the exponentially weighted moving average (EWMA) method due to the fact that VaR series that are calculated with the SMA method react more slowly to changes in market volatility. Figure 8 presents a diagram of the volatility forecast, under the two methods used (EWMA, SMA), for portfolio 3 – National Bank.

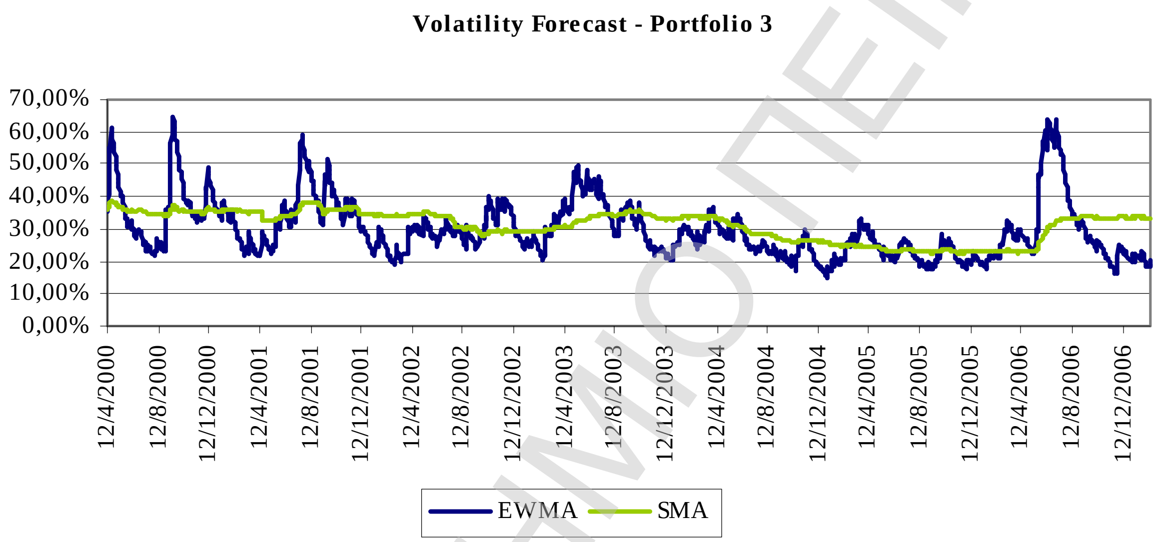

Figure 8: Volatility forecast for portfolio 3 - National Bank under the Exponentially Weighted Moving Average (EWMA) and the Simple Moving Average (SMA) methods for the period from 12/4/2000 to 16/2/2007.

4.4. Backtesting

After presenting the different characteristics between the various VaR methods we will compare the methods by backtesting them using the three tests proposed by Christoffersen (1998). The first one, the unconditional coverage (LR_{uc}), tests whether the VaR violations are statistically equal to the expected ones, otherwise the institution is not using its capital efficiently. The second test, which is the independence test (LR_{ind}), is about the independence of VaR violations and the third one which tests jointly for independence and correct coverage is the conditional coverage test (LR_{cc}). The results are presented separately for the 5 portfolios under the different VaR methods used for both 95% and 99% confidence level from table 6 to table 10.

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Portfolio 1-Alpha Bank								
Bactesting - 95% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	2.1233		0.1446		0.5412		6.2366	*
LRind	4.4370	*	2.2913		8.5940	*	2.3808	
LRcc	6.5603	*	2.4359		9.1351	*	8.6174	*
Bactesting - 99% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	22.4680	*	1.8487		1.8487		0.0007	
LRind	3.0892	*	1.0088		1.0088		1.9785	
LRcc	25.5573	*	2.8576		2.8576		1.9792	

Table 6: Portfolio's 1- Alpha Bank values of Christoffersen's backtesting tests are presented for VaR computed with Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA), both for the 95% and 99% confidence level respectively. The null hypothesis tested with LRuc is that the average number of VaR violations is correct. The null hypothesis tested with LRind is that the VaR violations are independent. The null hypothesis tested with LRcc is that the average number of VaR violations is correct and that the VaR violations are independent. Data included 12/4/2000-16/2/2007

*The null hypothesis is rejected at the 10% significance level

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Portfolio2- Piraeus Bank								
Bactesting - 95% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	3.1449	*	0.0297		0.0025		8.2052	*
LRind	8.8984	*	6.8724	*	6.2488	*	5.0455	*
LRcc	12.0433	*	6.9021	*	6.2513	*	13.2507	*
Bactesting - 99% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	26.0201	*	3.2174	*	1.2948		0.2036	
LRind	0.7593		0.7780		1.1394		1.6015	
LRcc	26.7794	*	3.9954		1.1394		1.8051	

Table 7: Portfolio's 2 - Piraeus Bank values of Christoffersen's backtesting tests are presented for VaR computed with Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA), both for the 95% and 99% confidence level respectively. The null hypothesis tested with LRuc is that the average number of VaR violations is correct. The null hypothesis tested with LRind is that the VaR violations are independent. The null hypothesis tested with LRcc is that the average number of VaR violations is correct and that the VaR violations are independent. Data included 12/4/2000-16/2/2007

*The null hypothesis is rejected at the 10% significance level

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Portfolio 3 - National Bank								
Bactesting - 95% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	7.8779	*	0.1446		0.8524		0.0732	
LRind	5.6739	*	14.4643	*	1.5197		12.3133	*
LRcc	13.5517	*	14.6089	*	2.3721		12.3865	*
Bactesting - 99% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	29.7675	*	4.9112	*	1.8487		8.0104	*
LRind	0.5773		0.5836		0.6268		1.0708	
LRcc	30.3448	*	5.4949	*	2.4755		9.0812	*

Table 8: Portfolio's 3 - National Bank values of Christoffersen's backtesting tests are presented for VaR computed with Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA), both for the 95% and 99% confidence level respectively. The null hypothesis tested with LRuc is that the average number of VaR violations is correct. The null hypothesis tested with LRind is that the VaR violations are independent. The null hypothesis tested with LRcc is that the average number of VaR violations is correct and that the VaR violations are independent. Data included 12/4/2000-16/2/2007

*The null hypothesis is rejected at the 10% significance level

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Portfolio 4 - Eurobank								
Bactesting - 95% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	4.7980	*	0.0808		0.0297		3.1635	*
LRind	11.5395	*	7.1978	*	1.8703		9.5203	*
LRcc	16.3375	*	7.2785	*	1.9001		12.6837	*
Bactesting - 99% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	24.2191	*	0.4677		2.4908		0.2742	
LRind	5.6693	*	5.3810	*	4.0420	*	2.4244	
LRcc	29.8884	*	5.8487	*	6.5328	*	2.6987	

Table 9: Portfolio's 4 - Eurobank values of Christoffersen's backtesting tests are presented for VaR computed with Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA), both for the 95% and 99% confidence level respectively. The null hypothesis tested with LRuc is that the average number of VaR violations is correct. The null hypothesis tested with LRind is that the VaR violations are independent. The null hypothesis tested with LRcc is that the average number of VaR violations is correct and that the VaR violations are independent. Data included 12/4/2000-16/2/2007

*The null hypothesis is rejected at the 10% significance level

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Portfolio 5 - FTSE/ASE 20								
Bactesting - 95% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	3.9305	*	0.0297		0.1446		5.0873	*
LRind	1.1549		6.8724	*	3.6983	*	5.9048	*
LRcc	5.0854	*	6.9021	*	3.8429		10.9921	*
Bactesting - 99% confidence level								
	HS-100		***HS-250***		***EWMA***		***SMA***	
LRuc	20.7682	*	2.4908		5.8725	*	0.8330	
LRind	6.3382	*	0.6829		0.9317		7.3702	*
LRcc	27.1064	*	3.1737		6.8042	*	8.2032	*

Table 10: Portfolio's 5 - FTSE/ASE 20 values of Christoffersen's backtesting tests are presented for VaR computed with Historical Simulation 100 observations (HS-100), Historical Simulation 250 observations (HS-250), Exponentially Weighted Moving Average (EWMA) and Simple Moving Average (SMA), both for the 95% and 99% confidence level respectively. The null hypothesis tested with LRuc is that the average number of VaR violations is correct. The null hypothesis tested with LRind is that the VaR violations are independent. The null hypothesis tested with LRcc is that the average number of VaR violations is correct and that the VaR violations are independent. Data included 12/4/2000-16/2/2007

According to the backtesting results the HS-100 method fails to satisfy adequately all 3 tests for both confidence levels. In the 95% case the EWMA and HS-250 methods satisfy the unconditional coverage test, while they fail to satisfy the other two tests. In the 99% confidence level the EWMA and SMA methods present similar results performing better compared with the other two methods. The first two tests, unconditional coverage and independence test are satisfied 80% of the time, that is in 4 out of the 5 portfolios, while the conditional coverage test is satisfied 60% of the time for the two methods. The SMA method performs much better in the 99% confidence level, compared with the 95% case. According to the results of the independence test in 3 out of the 4 methods used in this paper there is evidence that there are not many clustered violations in the 99% confidence level. In the 95% case it seems that if a violation occurs, then the probability of observing an exceedence the next day is high.

*The null hypothesis is rejected at the 10% significance level

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Berkowitz et al. (1995) conducted similar research in order to evaluate historical simulation methods with data taken from 4 business lines of a large international bank. In his research he did not reject the VaR figures for 3 out 4 business lines.

There is extensive literature proposing the use of GARCH models, Monte-Carlo simulation and Filtered Historical Simulation (FHS) to calculate VaR for the Greek Stock Exchange such as the work of Diamandis et al. (2006) and Angelidis et al. (2006). Other papers presenting the calculation of the different VaR methods for data obtained from the Athens Stock Exchange are those of Skiadopoulos et al. (2003) in which the historical simulation approach for both confidence levels is accepted as far as the failure of exception results (computed for 100 and 252 observations). The difference with the results that we presented in this paper may be due to different sample sizes used. Angelidis and Benos (2007), analyze various methods to compute VaR and estimate volatility at the 99% and 97.5% confidence level for Greek equities. According to their research the historical volatility method satisfies the first test of Christoffersen, while the variance-covariance with SMA and the EWMA underestimate VaR and are rejected by the backtesting techniques. Among the various measures used in their study they propose the use of Filtered Historical Simulation (FHS) and the Extreme Value Theory (EVT).

5. Conclusion and Further Research

5.1. Conclusion

Value-at-Risk (VaR) is a risk measure that enables an institution to determine how much the value of the portfolio could decline over a defined time horizon with a given probability as a result of adverse changes in market conditions.

This paper analyzed the Value at Risk (VaR) approach to estimate market risk as proposed by the Basle Committee and as it is implemented by institutions. We have presented and examined the 3 methods proposed to compute VaR as well as both the advantages and criticism on this method as documented in various studies.

In the second section we focused on the computation of one-day VaR for the 95% and 99% confidence level respectively, for portfolios consisting of equities from the Athens Stock Exchange. We have chosen to use simple methods that, to our knowledge, are used by financial institutions to compute VaR, for linear portfolios. Of the four methods used, two of them were based on the historical simulation approach with different historical period lengths

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and the other two were based on the variance-covariance approach with different variance estimators, the equally and the exponentially weighted average.

In accordance to previous research on the same methods to compute VaR, such as that of Beder (1995), although the portfolios are the same, the different methodologies may yield quite different VaR figures and the choice of the confidence level has a great effect on the performance of VaR approaches. This points out the fact that VaR numbers computed with different methodologies across different institutions should not be directly compared because the results may be quite misleading. Comparing the different methods used, although it seems that HS-100 computes on average the less amount of capital coverage, it fails to satisfy the backtesting criteria therefore it should not be used. On the other hand, the SMA method, although it presents better backtesting results it yields the most conservative average VaR figures. Out of the 4 methods used the one that seems to perform better, in the 99% confidence level, based on the criteria of a considerably low average VaR figure,an adequate exception rate and average acceptance (accepted in 3 out of 5 portfolios used) in most of the tests, is the EWMA. In the 95% case the results are mixed since even the EWMA fails to satisfy 2 out of the three tests.

The purpose of our analysis was to use simple methods under the constraints that most risk management practitioners work, following the research undertaken for foreign exchange portfolios by Hendricks (1996) and that of Pritsker (2001) on equities and extend them to the Athens Stock Exchange. The reason for choosing those methods was based on the statement made by Andersen et al. (2005) that although GARCH models perform well in modeling the different features of returns such as volatility mean-reversion, they are too parameterized in order to be used in realistic problems.

The results presented earlier in this paper seem to indicate that even for simple linear portfolios the simple methods of historical simulation to calculate VaR seem to perform poorly. These results do not conclude to the fact that these methods are not useful but they rather indicate that their performance should be examined carefully for regulatory purposes.

There is no certain conclusion in the different papers on which is the best method to use since the methods seem to yield different results according to the asset under consideration We should keep in mind that an institution's portfolio may contain several complex instruments and that in these cases especially, the institution should not expect with such certainty that the firm will not lose more than 1% at the 99% confidence level since VaR changes significantly based on the time horizon, the sample size, correlation assumptions and quantitative techniques. VaR should therefore provide an expectation of outcomes based on a

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specific set of assumptions. VaR should be also supplemented with stress testing, procedures and controls in order to attain the appropriate level of risk control.

5.2. Further Research

The EWMA method, proposed by RiskMetrics, is widely used to forecast the variance of the conditional distribution of asset returns. It is appropriate when the asset returns are drawn from a normal distribution. However, there is considerable evidence that the distribution of most financial returns is not well approximated by normal distribution, even conditionally, since it is typically found to be leptokurtic and has fatter tails than the normal distribution.

Liu et al. (2004) propose the use of the power exponential distribution, as proposed by Guerma & Harris (2002), to construct a serial of EWMA family estimators. Testing the results for equities, they found that the EWMA estimators based on power exponential distribution rather than normal distribution offer a superior coverage for the extreme risk over the RiskMetrics estimator and that these estimators demonstrate excellent accuracy in VaR estimation. According to this research we would propose to use this methodology in the case of the Athens Stock Exchange and evaluate the results to test the superiority of the forecast.

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