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Spillover of Tail Risk: An Application to Major European and US Stock Indices

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Abstract

International financial crises occurred during the past two decades have turned academic researcher's attention to financial spillovers. Previous studies have dealt with and eventually detected return and volatility spillovers across countries during crises periods. However, no previous research has dealt with the spillover of tail risk. In this paper our objective is to detect, if there exists, spillover of tail risk using a well known measure for risk management purposes; Value at Risk. We calculate the 1 day, 95% and 99% Value at Risk for major European and US stock indices – CAC40 (France), DAX30 (Germany), FTSE100 (UK), EUROSTOXX50 (Europe) and DJ INDUSTRIALS, NASDAQ100 and S&P500 (USA). We follow the historical simulation and variance approach; variance is estimated as a moving average, an exponentially weighted moving average and a GARCH-type model. We also use Extreme Value Theory as an alternative method to calculate VaR. In order to investigate spillover effects, Granger causality and contemporaneous and lagged relationships across the changes in the VaR of the various indices are examined, since the VaR series in levels are observed as non stationary. We also attempt to capture the concept of cointegration of the non stationary series examined. The results of the current research indicate that spillovers of tail risk do exist; US indices causes European markets, but Europe as well – especially FTSE100 – has effects on USA. In addition, regional effects and also contemporaneous effects are observed. These results help the forecast of the behaviour of a market, when information for a negative shock in another foreign market exists.

1 Introduction

Financial spillovers have received great attention from academic researchers and business practitioners, particularly after the international financial crises occurred during the past two decades. Financial spillovers are multivariate effects that explain the chance of having a crisis at the home market today, when there was a negative shock to another market yesterday. Previous studies have dealt with and eventually detected return and volatility spillovers across countries during crises periods. In this paper we focus the risk of a given portfolio on the tails of its returns distribution. Furthermore, we approach the tail risk of a given portfolio through Value at Risk, a well known measure for risk management purposes. Our objective is to detect, if there exists, spillover of tail risk across major US and European stock indices – United States, United Kingdom, Germany and France – through the changes in the VaR of each stock market. If the information for a negative shock in a foreign market helps the forecast of the behaviour of another market, then we conclude that tail risk spills over and one market causes the other.

1.1 Review of Previous Studies

There are no previous studies examining the spillover of tail risk through the VaR measure, which is the point of our methodology. However, a large number of previous empirical studies are attempting to find out how financial shocks get transmitted across countries emphasizing on return and volatility spillovers. The result seems to be that the US stock market is the dominant capital market influencing other mature and developing markets. International capital markets are strongly correlated with the US market and past returns of the US market affect present returns on other markets. Malliaris and Urrutia (1992) provide statistical evidence regarding the international propagation of the stock market crash of October 1987; the questions of causality or lead-lag relationships are empirically investigated for several major equity markets – New York S&P 500, Tokyo Nikkei, London FT-30, Hong Kong Hang Seng, Singapore Straits Times, Australia All Ordinaries – by means of Granger methodology (1969). A dramatic increase in bidirectional and unidirectional causality is observed for the month of the crash. Tests of contemporaneous causality indicate also an increase of contemporaneous causality after the month of the crash. The increase in feedback and contemporaneous causality among the national stock markets during the month of the crash suggests that the crash probably started simultaneously in all the stock markets. Thus, the market crash of October 1987 seems to have been an international crisis of the equity markets.

Many researchers have pointed out volatility as the most crucial distributional characteristic of the asset returns. Volatility is the second moment of the distribution and points out the likelihood of extreme shifts of in assets value. There is a great amount of empirical work on understanding how stock returns and volatility are transmitted across countries. Many studies are based on ARCH models due to the fact that stock price volatility is time-varying and that high volatility events are usually characterised by a high correlation of stock market returns. Hamao, Masulis and Ng (1990) studied price changes and volatility spillovers across the New York, Tokyo and London stock markets using close-to-open and open-to-close daily stock returns and a (univariate) GARCH(1,1)-M model. They found volatility spillovers only in the period following the October 1987 crash and not in the pre-October 1987 period and identified an asymmetry, since empirical evidence of price volatility spillovers suggests London was affected by but did not have any effect on New York and Tokyo was affected by both London and New York but did not have any effect on the latter two stock markets.

Concerning the transmission of implied volatilities across markets, since it affects option prices and hedge ratios and may indicate changes in expected volatility, Gemmill and Kamiyama (2000) examine whether changes in the implied volatility and implied skewness of one market are quickly reflected in other markets, using daily data over the period 1985-1995 from the US, Japan and UK. They find that implied volatilities are correlated across time zones and changes in implied volatility are transmitted from one market to the next. By contrast, they find little tendency for changes in implied skewness in one market to spread to other nations' markets, which suggests that local factors must be the cause of daily shifts in skewness. Finally, Skiadopoulos (2004) constructs an implied volatility index (GVIX) for the Greek derivatives market. It is found that the underlying stock market can forecast the future movements of GVIX. However, the reverse relationship does not hold. Finally, a contemporaneous spillover between GVIX and the US volatility indices VXO and VXN is detected.

1.2 Objectives

The current paper focuses on the spillover of tail risk. Previous studies have approached the definition of risk of a portfolio through the volatility of the distribution of the assets returns. In this paper we move forward, since we shift our attention and concern from risk considered in terms of volatility to risk considered in terms of extreme losses with low probability of being exceeded; this kind of risk is defined as tail risk. More specifically, we not only are interested in the returns and the volatility of the distribution of returns, but we also use them so as to calculate another risk measure, Value-at-Risk (VaR). VaR is a measure of the extreme risk that is possible to be faced by an asset, a portfolio or a whole market. Nevertheless, VaR focuses on the extreme downside risk in contrast to the risk measure of volatility. The latter is a measure of possible up- or downside moves of the portfolio. Furthermore, VaR specifies in a single number the possible risk of an institution for the next day. And the logical question that arises is whether this kind of risk, the tail risk through the above approach, presents spillover effects. Spillover of tail risk is examined through Granger causality in order to reach a regression among the changes in the VaR of the various markets. We are also interested in the contemporaneous and lagged regressions across the various VaR series. If Granger causality is verified and such regressions is extracted, then spillover effects are present. Our study is an application across the major US and European stock indices. The indices under examination are Standard&Poor's 500, Dow Jones Industrials and NASDAQ 100 for the US, FTSE 100 for the UK, DAX 30 for Germany and finally CAC 40 for France. Our aim is to detect whether spillover of tail risk is present or not across these markets during the period of the last 10 years (1/1/1996-31/12/2005).

2 Calculating Value-at-Risk

Value at Risk (VaR) has become the standard measure that financial analysts use to quantify market risk. Market risk estimates the uncertainty of future earnings, due to the changes in market conditions and reflects the potential economic loss caused by the decrease in the market value of a portfolio. VaR is defined as the largest potential loss in value of a portfolio of financial instruments with a given probability if it is left undamaged over a certain holding period. VaR is the number that indicates how much a financial institution can lose with probability α% over a given time horizon. In other words, VaR is the loss in market value of a portfolio over the given time horizon that is exceeded with probability 1-α. Hence,

$$\Pr(\Delta_{\mathrm{T}}\Pi_t \le -VaR_{a,\mathrm{T}}) = a \qquad (2.1)$$

where $\Delta_{\mathrm{T}}\Pi_t = \Pi_{t+\mathrm{T}} - \Pi_t$ is the change (loss) in the portfolio value over the holding period T.

While VaR is a simple and intuitive concept, its measurement is very challenging. We will classify the existing models into three basic categories:

1. Nonparametric (Historical Simulation and Monte Carlo Simulation),
2. Parametric (RiskMetrics and GARCH),
3. Semiparametric (Extreme Value Theory).

Banks generally use one of two broad methodologies for measuring market risk exposure: historical simulation and variance/covariance matrix methodology.

2.1 Historical Simulation

Historical simulation is a simple, non-parametric approach that requires relatively few assumptions about the statistical distributions of the underlying portfolio. It assumes that the future distribution of portfolio returns is well approximated by the empirical distribution of the past (historical) observations. The historical changes construct the distribution of potential future portfolio profits and losses and then the value at risk is calculated as the lower a-percentile (a is the coinfidence level) of the distribution, namely the loss that is exceeded only a% of the time. By relying on actual prices, this method accommodates non-normal distributions and therefore it accounts for "fat tails" and non-zero skewness.

This approach is relatively simple and it does not require simulations or the development of an analytical model. Moreover it can easily incorporate non-linear instruments such as options.

2.2 Variance/Covariance Approach

The variance/covariance approach is a parametric method, based on the assumption that the returns of the underlying are normally distributed. Using this assumption, it is possible to determine the distribution of mark-to-market portfolio profits and losses, which is also Normal:

$$\Delta_{\mathrm{T}}\Pi_t \sim N(\mu_t, \sigma_t^2)$$

VaR is defined as:

$$VaR_{\alpha,\mathrm{T}} = -\mathrm{Z}_\alpha \sigma_t - \mu_t \qquad (2.2)$$

where

$$Z_t = \frac{\Delta_{\mathrm{T}}\Pi_t - \mu_t}{\sigma_t} \sim N(0,1) \qquad (2.3)$$

and hence

Z_α is the α^{th} percentile of the standard normal density.

The portfolio P&L has a standard deviation $\sigma_t = (p'Vp)^{1/2}$, where p is the vector of nominal amounts invested in each asset and V is the variance covariance matrix of P&L.

The above method is know as the **Delta-Normal (DN) method** and is suitable for linear portfolios of stocks and futures.

Historical data is used to measure the major parameters: mean, standard deviation, correlation. The overall distribution of the market parameters is constructed from this data. Therefore, the 5% quantile corresponding to VaR can be calculated at $1.65 \cdot \sigma$ below the mean (the 1% level can be calculated at $2.33 \cdot \sigma$).

The specific method relies heavily on the important assumption that all of the major market parameters are normally distributed. In fact, historical distributions of market returns are far from being normal. This problem is well known and is related to fat tails (kurtosis).

Volatility Models

Estimating standard deviations and correlations has been of great importance in finance and econometrics in recent years. Consequently researchers have come up with methods, which are often referred to as volatility models. The methods range from extreme value techniques (Parkinson, 1980) to more complicated nonlinear modelling such as GARCH (Bollerslev, 1986) and stochastic volatility (Harvey et. al, 1994). Among academics, and increasingly among practitioners, ARCH-type models of volatility and the related GARCH, EGARCH and TGARCH formulations have gained the most attention.

Historical Volatility (Equally Weighted Moving Average)

First of all, we refer to the simple equally weighted model, which uses the variance of a moving sample as forecast for the next period, hence

$$\hat{\sigma}_t^2 = \frac{\sum_{i=t-n}^{t-1} (r_i - \bar{r})^2}{n-1} \qquad (2.4)$$

where r_i are the returns at time i and $\bar{r} = \sum_{i=t-n}^{t-1} \frac{r_i}{n}$ is the estimated average return of the sample.

This estimation accepts that volatility changes. Instead of increasing the sample size as new information arrives, we use a n-period moving average (MA) model.

The RiskMetrics Model (Exponentially Weighted Moving Average)

J. P. Morgan (1996) proposed an exponentially weighted moving average model (EWMA). The volatility of the next period can be calculated as a weighted average of the current volatility and the squared return. This model is known as the RiskMetrics model:

$$\hat{\sigma}_t^2 = \lambda \hat{\sigma}_{t-1}^2 + (1-\lambda) r_{t-1}^2 \qquad (2.5).$$

The value of weight factor λ (exponential decay factor) determines the extent to which the most recent observation affects $\hat{\sigma}_t^2$. As an initial value for σ^2, we use the squared returns.

The above equation can be written as

$$\hat{\sigma}_t^2 = (1-\lambda) \sum_{i=1}^{\infty} \lambda^{i-1} r_{t-i}^2 \qquad (2.6).$$

EWMA takes into account the autocorrelation in squared return. This means that if r_{t-1}^2 is high, $\hat{\sigma}_t^2$ will be also high.

In order to estimate the parameter λ of the EWMA, the J. P. Morgan's RiskMetrics methodology assumes that the mean value of daily returns is zero and shows the relationship between the tolerance level, the decay factor, and the effective amount of data required by the EWMA. For example, setting a tolerance level to 1% and the decay factor to 0.94, we see the EWMA uses approximately 74 days of historical data.

The RiskMetrics approach assumes that volatility σ_t is changing with time (t). If we measure such volatility in terms of variance (or its square root, that is the standard deviation), then it is fair to think that variance changes with time. In statistics, changing variances are often denoted by the term heteroscedasticity, reflecting the clusters of large and small returns. This means that periods of large returns are clustered and distinct from periods of small returns, which are also clustered.

ARCH

The evidence that time series realizations of returns often exhibit time-dependent volatility and such models capture volatility persistence was first formalized in Engle's (1982) ARCH (Auto Regressive Conditional Heteroscedasticity) model. The basic idea of ARCH models is that the mean-corrected asset return r_t is serially uncorrelated, but dependent, and the dependence of r_t can be described by a simple quadratic function of its lagged values.

The equation that also holds in these models is the conditional mean equation [ARMA*(m,s)*]:

$$r_t = \mu_t + \varepsilon_t \qquad (2.7)$$

where

$$\mu_t = \phi_0 + \sum_{i=1}^{m} \phi_i r_{t-i} + \sum_{i=1}^{s} \theta_i \varepsilon_{t-i} \qquad (2.8)$$

and $\varepsilon_t \sim N(0, \sigma_t^2)$.

The ARCH(q) model that Engle (1982) introduced expresses the conditional variance as a linear function of the past q squared innovations:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \qquad (2.9)$$

The parameters must satisfy $\alpha_0 > 0$ and $\alpha_i \geq 0$ for i = *1,...,q* for the conditional variance to be positive.

GARCH

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of an asset return. Bollerslev (1986) proposed a useful extension known as the generalized ARCH (GARCH) model. For a log return series r_t, we assume that the mean equation of the process can be adequately described by an ARMA model. Let $\varepsilon_t = r_t - \mu_t$ be the mean-corrected log return. Bollerslev proposed a generalization of the ARCH model, the GARCH(p,q) model:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \qquad (2.10)$$

where in normal GARCH we assume that $\varepsilon_t \sim N(0,\sigma_t^2)$ (unconditional leptokurtic distribution) , $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{\max(p,q)} (\alpha_i \beta_i) < 1$.

Finally, the unconditional variance of GARCH(1,1) model is given by:

$$\sigma^2 = \frac{\alpha_0}{1-\alpha_1-\beta_1} \qquad (2.11).$$

The GARCH(*p,q*) model successfully captures several characteristics of financial time series, such as thick tailed returns and volatility clustering, as "large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes " (Mandelbrot, 1963).

However, the ARCH and GARCH models have several drawbacks. For instance, the error term for returns models often has heavier tails than the Normal distribution. Therefore, we can assume the t-distribution is used as conditional distribution for the residuals (t-GARCH model).

2.3 Extreme Value Theory

The extreme value theory (EVT) states that the extreme tail of a wide range of distributions can approximately be described by a relative simple distribution, the generalized Pareto distribution. Virtually all results in EVT assume that returns are IID. Asset returns are also appear to approach normality at long horizons, thus EVT is more important at short horizons, such as daily. Unfortunately, at short horizons the IID assumption does not hold due to the time-varying variance patterns. We therefore need to get rid of the variance dynamics before applying EVT. The standardized portfolio returns are given from the following formula:

$$z_t = \frac{r_t}{\sigma_t} \sim iidD(0,1) \qquad (2.12)$$

where $D(0,\sigma_t^2)$ is the distribution of returns. We assume that the mean of returns distribution $\mu_t = 0$, because the returns are daily.

The standardized returns are assumed to be IID, thus we will proceed to apply EVT to them and then combine EVT with the variance models.

Let a threshold *u* being bellow a value *x* given that the standardized return *z* is beyond the threshold *u*. Then:

$$F_u(x) \equiv \Pr\{z-u \leq x | z > u\} = \frac{F(x+u)-F)(u)}{1-F(u)} \qquad (2.13)$$

where $x > u$ and

$F(.)$ is the distribution of the standardized returns.

As *u* gets large, $F_u(x)$ converges to the generalized Pareto distribution:

$$G(x;\xi,\beta) = \begin{cases} 1-\left(1+\frac{\xi x}{\beta}\right)^{-1/\xi}, & \xi \neq 0 \\ 1-\exp\left(-\frac{x}{\beta}\right), & \xi = 0 \end{cases} \qquad (2.14)$$

with $\beta > 0$ and

$$\begin{cases} x \geq u, & \xi \geq 0 \\ u \leq x \leq u - \dfrac{\beta}{\xi}, & \xi < 0 \end{cases} \qquad (2.15)$$

Financial interest is covered from the above distribution, since when the tail parameter ξ is positive, then the returns distribution is fat tailed.

Let

$$y = x + u \qquad (2.16)$$

Then, ξ is estimated by the simple Hill estimator as [1]

$$\xi = \frac{1}{T_u} \sum_{i=1}^{T_u} \ln(y_i / u) \qquad (2.17)$$

The loss quantile is defined as:

$$F_{1-\alpha}^{-1} = u \left(\frac{\alpha}{T_u / T} \right)^{-\xi} \qquad (2.18)$$

where T is the total sample size,

T_u is the number of observations beyond the threshold and hence

T_u / T denotes the proportion of data point beyond the threshold.

The VaR from the EVT combined with the variance model is calculated as:

$$VaR_{\alpha,\mathrm{T}} = \sigma_t F_{\alpha,\mathrm{T}}^{-1} = \sigma_t u \left(\frac{\alpha}{T_u / T} \right)^{-\xi} \qquad (2.19)$$

The quantile such that $(1-\alpha)$% of losses is the same as minus the quantile such that α% of returns.

2.4 Back Testing

Back testing is an important reality check of the method used for calculating VaR. It involves testing how well the VaR estimates would have performed in the past.

Unconditional Coverage Testing (Kupiec Test)

Suppose that we are calculating a 1-day $(1-\alpha\%)$ VaR, where $(1-\alpha\%)$ is the confidence level. Unconditional coverage back testing involves checking if the fraction of violations obtained for the particular VaR model is significantly different from the fraction α%. To test this, we look at how often the loss in a day exceeds the 1-day $(1-\alpha\%)$ VaR that has been calculated for this day. If this happens on about α% of the days, we can feel reasonably comfortable with the methodology for calculating VaR. If it happens more often than α% of the days, then the methodology is suspect.

More formally, we calculate the following likelihood ratio so as to check the unconditional coverage hypothesis:

[1] For more information, see Christoffersen, P., Elements of Financial Management.

$$LR_{UC} = -2\ln\left[(1-\alpha)^{T-N}\alpha^{N}\right] + 2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] \sim \chi^2(1) \qquad (2.20)$$

where α is the confidence level,

T is the total number of observations of the sample,

N is the number of days that a violation is observed.

Asymptotically – when the number of observations T goes to infinity – the test is distributed as a χ^2 with one degree of freedom, this is $\chi^2(1)$.

Choosing a significance level for the test, we will have a critical value from the $\chi^2(1)$ distribution. If the critical value is lower than the LR_{uc} test value, then we reject the VaR model at the specific significance level.

Conditional Coverage Testing (Christoffersen Test)

We also want to test whether the violations of the VaR models are clustered in time or not. If the VaR violations occur around the same time, then the risk of bankruptcy would be much higher than if the violations occurred randomly through time.

If we are interested in simultaneously testing if the VaR violations are independent (present no clustering) and the average number of violations is correct, then we should perform the conditional coverage test:

$$LR_{CC} = -2\ln\left[(1-\alpha)^{T-N}\alpha^{N}\right] + 2\ln\left[(1-p_{01})^{n_{00}} p_{01}^{n_{01}} (1-p_{11})^{n_{10}} p_{11}^{n_{11}}\right] \sim \chi^2(2) \qquad (2.21)$$

where α is the confidence level,

n_{ij} is the number of observations with value *i* followed by *j*,

$p_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ are the corresponding probabilities,

while *i, j* = 1 denotes that an exception has been made and *i, j* = 0 indicates the opposite.

Asymptotically – when the number of observations T goes to infinity – the test is distributed as a χ^2 with two degree of freedom, this is $\chi^2(2)$.

Choosing a significance level for the test, we will have a critical value from the $\chi^2(2)$ distribution. If the critical value is lower than the LR_{uc} test value, then we reject the VaR model at the specific significance level.

3 Spillovers: Methodology

3.1 Unit Root Tests

In order to proceed to the implementation of our methodology and examine the spillover effects, our series must be stationary.

A process X_t is stationary if the following conditions hold:

1. $E(X_t) = \mu < \infty$ (constant mean)
2. $Cov(X_t;X_{t-s}) = \gamma_s < \infty$ (depends on s but not on t)

For $s = 0$ the second condition implies that a stationary process has constant variance, $Var(X_t) = \sigma^2$.

A non stationary process arises when one of the conditions for stationarity does not hold.

Testing for unit-roots means testing the hypothesis:

$$H_0 : \varphi = 1 \qquad (3.1)$$
$$vs.\ H_1 : |\varphi| < 1 \qquad (3.2)$$

in the following general model:

$$X_t = m + \varphi X_{t-1} + dt + \varepsilon_t \qquad (3.3)$$

where m is the intercept, d is the trend and ε_t is a white noise process.

A series Y_t is integrated of order *d* (denoted *I(d)*) if it must be differenced at least d times in order to make it stationary.

In order to detect the non-stationarity of a series, we perform the Augmented Dickey-Fuller (1981) test and the Philips-Perron (1988) test.

If our series are found to be non stationary, then we should test for integration of the first differences of our series. If the new series our found to be stationary, then we use these series for Granger causality purposes and the examination of lagged and contemporaneous spillover effects.

3.2 Granger Causality

Granger causality is a technique for determining whether one time series is useful in forecasting another. Testing Granger causality involves using *F*-tests to test whether lagged information on a variable *Y* provides any statistically significant information about a variable *X* in the presence of lagged *X*. *X* is said to be Granger-caused by *Y* if *Y* helps in the prediction of *X*, or equivalently if the coefficients on the lagged *Y* are statistically significant. It is important to note that the statement '*Y* Granger causes *X'* does not imply that *X* is the effect or the result of *Y*. Granger causality measures precedence and information content but does not by itself indicate causality in the more common use of the term.

In order to test for Granger causality across two variables - X_t and Y_t - we run bivariate regressions with a lag length set as *k*. These are called unrestricted regressions:

$$X_t = \alpha_X + \sum_{i=1}^{k} \beta_i X_{t-i} + \sum_{i=1}^{k} \gamma_i Y_{t-i} + \varepsilon_t \qquad (3.1)$$

The Granger causality is examined by testing the null hypothesis whether all γ_i are equal to zero

$$H_0\!: \gamma_1 = \ldots = \gamma_k = 0 \qquad (3.2)$$

that is we perform a Wald test with Wald statistic:

$$W = \frac{\left(SSR_R - SSR_{UR}\right)}{SSR_{UR} / \left(n - 2k - 1\right)} \qquad (3.3)$$

which is asymptotically distributed as χ^2 under H_0.

If we further assume that the errors ε_t are independent and identically normally distributed, we have an exact, finite sample *F*-statistic:
using a standard *F*-test:

$$F = \frac{W}{q} = \frac{\left(SSR_R - SSR_{UR}\right)/k}{SSR_{UR}/(n - 2k - 1)} \qquad (3.4)$$

where SSR_{UR} is the residual sum of squares of the unrestricted regression above,

SSR_R is the residual sum of squares of the restricted regression, which is the regression without the lags of Y_t.

The finite sample *F*-statistic is for standard linear regression; In non-standard settings, the reported F-statistic does not possess the desired finite-sample properties.

In these cases, while asymptotically valid, the F-statistic results should be viewed as illustrative and for comparison purposes only.

If the ADF and PP unit root tests have verified that the series on levels are non-stationary and the first differences are stationary (first integrated), then the Granger causality tests are performed across the first differences of the series:

$$\Delta X_t = \alpha_x + \sum_{i=1}^{k} \beta_{x,i} \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_{X,i} \Delta Y_{t-i} + \varepsilon_{X,t} \quad (3.5)$$

3.3 Spillover Effects

Finally, in order to identify the spillover effects of tail risk across the under examination indices, we perform the following regressions:

$$X_t = \alpha_0 + \sum_{i=1}^{k} \sum_{f=1}^{m} \alpha_{f,i} Y_{f,t-i} + \varepsilon_t \quad \text{(Lagged relationship)} \quad (3.6)$$

$$X_t = \beta_0 + \sum_{f=1}^{m-1} \beta_f Y_{f,t} + u_t \quad \text{(Contemporaneous relationship)} \quad (3.7)$$

where X_t is the dependent series at time t,

$Y_{f,t}$ is the independent series corresponding to index f at time t,

α_0 and β_0 are the respective constants of each model,

$\alpha_{f,i}$ is the coefficient of the independent series corresponding to index f in the lagged regression,

β_f is the coefficient of the independent series corresponding to index f in the contemporaneous regression,

m is the total number of indices examined

k is the total number of lags used in the model,

ε_t and u_t are the residuals of the models.

The first equation checks whether the value of the independent series lead the values of the dependent series. The second equation tests whether there is a contemporaneous relationship between the dependent and the independent series. Contemporaneous refers to the same calendar date t.

4 Value-at-Risk: Results and Discussion

4.1 Data Series

In our research we study seven of the major US and European stock indices: The Standard & Poor's 500 (S&P 500), the Dow Jones Industrial Average (DJINDUS) and the NASDAQ 100 for the United States, the EURO STOXX 50 for Europe, the Financial Times 100 Share Index (FTSE 100) for the United Kingdom, the DAX 30 for Germany and the CAC 40 for France.

In order to investigate the daily dynamics and spillover effects of tail risk, we use daily stock market closing prices. In total we use the daily data of the last 15 years (1/1/1991-30/12/2005). In order to estimate the parameters of the VaR calculation, which are the variance estimators and the GARCH parameters, we use the first 5 years (1/1/1991-31/12/1995). The period that is examined for causality and spillovers across the indices is the last 10 years (1/1/1996-31/12/2005).

Using the daily index prices we calculate the daily continuously compounded returns of each index using the formula:

$$r_t = \ln \frac{S_t}{S_{t-1}} \qquad (4.1)$$

where r_t is the continuously compounded return between day $t-1$ and t,

S_t is the index price at day t.

In the following table the descriptive statistics for the daily log returns of each index are presented.

	CAC40	DAX30	DJINDUS	EUROSTOXX50	FTSE100	NASDAQ100	SP500
Mean	0.000290	0.000346	0.000359	0.000365	0.000246	0.000538	0.000340
Median	0.000000	0.000394	0.000146	0.000590	0.000035	0.000659	0.000087
Maximum	0.070023	0.075527	0.061547	0.070781	0.059026	0.172030	0.055732
Minimum	-0.076780	-0.098710	-0.074550	-0.073550	-0.058850	-0.103780	-0.071130
Std.deviation	0.013148	0.014102	0.009825	0.012587	0.010166	0.019318	0.009969
Skewness	-0.100220	-0.284930	-0.218070	-0.169890	-0.114990	0.130186	-0.096110
Kurtosis	6.098662	7.351774	8.086439	7.390425	6.424191	7.801949	7.224627
Jarque-Bera	1572.028*	3140.620*	4249.204*	3161.586*	1920.301*	3770.584*	2915.906*
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 1: Descriptive Statistics for the daily log returns of the indices for the period 1/1/1991-30/12/2006. * denotes rejection of the hypothesis of Normality of the returns distribution at 5% significance level.

As the descriptive statistics for the daily log returns demonstrate, none of the indices follows the normal distribution of returns. The indices demonstrate leptokurtosis and hence fat tails, which is a sign of volatility clustering, since large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes. Most of the returns distributions are negatively skewed. The negatively skewed returns confirm the leverage effect – the negative correlation between the changes in volatility and the changes in the stock prices – observed in the asset returns distribution. However, NASDAQ100 present positive skewness, which is although slightly above zero.

4.2 Value-at-Risk Calculation

During a crisis period, extreme returns do happen, so we are interested in focusing on the tails of the distribution of returns. In our methodology, the definition of tail risk is approached through the measure of Value-at-Risk (VaR).

Two methods for the estimation of VaR are used, the historical simulation and the variance-covariance (Delta Normal) method. In the historical simulation method, we assume that the 1-day VaR is the 1% or 5% worst return occurred the past 100 and 250 days. In the variance approach, we attempt to estimate the variance of the following day using a moving average of the past observations (10, 30, 60, 74 and 130) and an exponentially moving average (EWMA) of the past 74 observations (for λ=0.94 following the Risk Metrics methodology). Since variance has been estimated, we calculate VaR as the 1% and 5% percentiles of Normal distribution and those according to the Extreme Value Theory. Finally, an alternative estimation of daily variance is produced: a GARCH-type model assuming that the errors follow both Normal and t-Student distribution. The GARCH models accepted are for all indices

GARCH(1,1), because these models produce the lowest Akaike and Schwarz criterion and the squared residuals of the respective mean equation have no autocorrelation; finally, these GARCH models produce the best VaR series according to back testing techniques.

First we use the method of historical simulation. We begin with the assumption that future distribution of asset returns is well approximated by the empirical distribution of the historical observations. This assumption accounts for non-normality of the implied distribution, namely fat tails and non-zero skewness. In order to perform the historical simulation method, we use two sizes of past observations; 100 and 250. Under our initial assumption, the returns corresponding to the last 100 and 250 observations respectively constitute two future distributions of index returns. The "historical" $VaR_{a,T}$ is calculated as the lower 1% and 5% (confidence level) of this distribution. This process is repeated for each of the following days, using the rolling "window" of the same number of observations (100 and 250) and hence we calculate the daily VaR for each day and for each index.

The second method for the estimation of VaR is the Variance/Covariance method (Delta Normal approach). In the variance approach, we attempt to estimate the variance of the following day using a moving average of the past observations (10, 30, 60, 74 and 130) using the following formula:

$$\sigma_t^2 = \frac{1}{n-1}\sum_{i=1}^{n} r_{t-i}^2 \qquad (4.2)$$

where r_t is the continuously compounded return between day $t-1$ and t,

n the number of observations

and the mean of the returns distribution equals 0.

We also estimate the variance as an exponentially moving average (EWMA). According to the RiskMetrics methodology, we can set $\lambda = 0.94$. If the required tolerance level is set to 1%, then the EWMA uses approximately 74 days of historical data. The RiskMetrics formula is:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \qquad (4.3)$$

or equally

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{n} \lambda^{i-1} r_{t-i}^2 \qquad (4.4)$$

where λ is the weight factor (exponential decay factor). As an initial value for σ^2 we use the average of the squared returns observed during the first five years (1991-1995).

Since variance has been estimated, we calculate VaR as the 1% and 5% percentiles of Normal distribution and those according to the Extreme Value Theory.

4.3 GARCH

Finally, we can use a GARCH-type model for the variance estimation. If we perform the Ljung-Box-Pierce Q-test in the squared returns of each index, we observe that significant serial correlation exists. In addition, Engle's ARCH test verifies that heteroscedasticity exists, since it shows significant evidence in support of GARCH effects. The above results are the same for all indices; hence we estimate a GARCH-type model for the variance estimation. This model consists of two equations. The first one is the conditional mean equation:

$$r_t = \mu_t + \varepsilon_t \qquad (4.5)$$

where r_t is the continuously compounded return between day $t-1$ and t,

μ_t is the mean of the returns distribution,

$\varepsilon_t \sim N(0, \sigma_t^2)$.

In our methodology we assume that the mean of the returns distribution μ_t equals 0. According to Figlewski's results (2004), it typically increases forecast accuracy to compute volatility around an assumed mean of zero rather than around the realized mean in the data sample. Hence, we assume that

$$r_t = \varepsilon_t \qquad (4.6)$$

The second equation is the conditional variance equation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \qquad (4.7)$$

where r_t is the continuously compounded return between day $t-1$ and t,

σ_t^2 is the variance of the distribution of day t,

α_i are the coefficients that measure the persistence of returns and β_i are the coefficients that measure the persistence of variance with $\alpha_0 > 0, \alpha_i \geq 0, \beta_i \geq 0$ and $\sum_{i=1}^{\max(p,q)} (\alpha_i \beta_i) < 1$.

In the appendix we can observe the GARCH estimation outputs. Before reaching the following results, higher models have been estimated. In order to select the most appropriate GARCH-type model, we have compared the Schwarz statistics produced by each model for the same index. Finally we have selected that model that produces the smallest Schwarz statistic. Definitely we are also interested in the significance of the parameters of our model. We choose 95% as the significance level in order to accept or reject the estimated coefficients (or 5% in order to reject or accept respectively the null hypothesis of the coefficients are equal to zero). As we have already mentioned, for the estimation of the GARCH models we use the data of the first five years (1/1/1991-31/12/1996). Finally, if we perform the Ljung-Box-Pierce Q-test and Engle's ARCH test in the squared standardized innovations, we observe that neither serial correlation exists nor heteroscedasticity respectively exists.

We estimate two GARCH models for each index; one under the assumption that the errors of the mean equation follow the Normal distribution and the second under the assumption that the errors follow the Student's t distribution. Since we have assumed that the mean of the returns distribution is constant and equals 0, the errors equals the realized daily returns. As the previous figures presented, the distributions of returns of the indices are leptokurtic with heavy tails, a distribution that is closer to Student's t distribution rather than to Normal one. This fact has a result that the following estimations of the daily VaR is better for GARCH models assuming Student's t distribution for the errors of the mean equations than assuming the Normal distribution.

As we observe in every case the best GARCH model accepted is a GARCH(1,1) model. In the case of NASDAQ100, however, there is a significant difference. The model accepted for NASDAQ100 and assuming that the errors follow both Normal and Student's t distribution is an AR(1)-GARCH(1,1) model; we have inserted an autoregressive coefficient in the mean equation in order to eliminate the autocorrelation observed without it in the mean equation.

We should notice here that since the under examination coefficients are always positive, then we produce one-sided tests. This means that we are interested in the half of the probabilities that correspond to the respective z-Stat. and are presented below. As a result, all the coefficients below are significant at 5% probability level.

$$\text{Mean equation: } r_t = \varepsilon_t \quad (4.8)$$

$$\text{or (in case of NASDAQ100) } r_t = \rho_1 r_{t-1} + \varepsilon_t \quad (4.9)$$

$$\text{Variance equation: } \sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (4.10)$$

	Coef.	*t-Stat.*		*Coef.*	*t-Stat.*		*Coef.*	*t-Stat.*
Error Distribution: Normal								
CAC 40			DAX 30			DJINDUS		
α_0	1.5E-05	2.730	α_0	7.4E-06	3.898	α_0	2.0E-07	2.820
α_1	0.052	2.997	α_1	0.871	28.864	α_1	0.014	5.286
β_1	0.823	13.328	β_1	0.052	4.692	β_1	0.981	314.406
EURO STOXX 50			FTSE 100			S&P 500		
α_0	6.1E-06	4.223	α_0	2.5E-06	3.130	α_0	2.5E-07	3.012
α_1	0.056	4.430	α_1	0.064	5.881	α_1	0.020	5.094
β_1	0.848	25.101	β_1	0.897	42.134	β_1	0.973	205.092
Error Distribution: Student's t								
CAC 40			DAX 30			DJINDUS		
α_0	1.2E-05	1.788	α_0	2.7E-06	2.467	α_0	3.8E-07	1.491
α_1	0.046	2.353	α_1	0.060	3.585	α_1	0.019	2.670
β_1	0.854	12.301	β_1	0.910	38.620	β_1	0.972	90.369
EURO STOXX 50			FTSE 100			S&P 500		
α_0	4.5E-06	2.522	α_0	1.4E-06	1.767	α_0	2.0E-07	1.223
α_1	0.065	2.956	α_1	0.040	3.100	α_1	0.021	3.000
β_1	0.859	19.154	β_1	0.935	39.418	β_1	0.974	111.082

Table 2: Coefficient estimates of the GARCH(1,1) model for the indices with mean equation: $r_t = \varepsilon_t$, $\varepsilon_t \sim D(0,\sigma_t^2)$ and variance equation: $\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. The dataset used is the period 1/1/1991-31/12/1995. Two different distributions for the residuals is assumed; Normal and Student's t. A t-statistic greater than 2 in magnitude corresponds to a significant coefficient at approximately 5% probability level.

	Coef.	*t-Stat.*		*Coef.*	*t-Stat.*
Error Distribution:			Error Distribution: Student's t		
NASDAQ 100					
ρ_1	0.083	3.008	ρ_1	0.082	2.978
α_0	8.8E-07	1.592	α_0	9.1E-07	1.046
α_1	0.021	4.673	α_1	0.021	2.806
β_1	0.973	143.305	β_1	0.973	84.181

Table 3: Coefficient estimates of the AR(1)GARCH(1,1) model for the NASDAQ 100 index with mean equation: $r_t = \rho_1 r_{t-1} + \varepsilon_t$, $\varepsilon_t \sim D(0,\sigma_t^2)$ and variance equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. The dataset used is the period 1/1/1991-31/12/1995. Two different distributions for the residuals is assumed; Normal and Student's t. A t-statistic greater than 2 in magnitude corresponds to approximately a 5% significance level.

4.4 Extreme Value Theory

In extreme value theory the threshold *u* that we use plays a very important role. In our methodology we use *u*=5% for the 1 day, 95% VaR series and *u*=1% for the a day, 99% VaR series, because we want to capture the probability of the VaR being exceeded. As a result, we calculate the parameter ξ according to the simple Hill estimator (2.24):

$$\xi = \frac{1}{T_u}\sum_{i=1}^{T_u}\ln(y_i/u)$$

where $y = x + u$ and

T_u is the number of observations beyond the threshold.

The estimators calculated are all positive for every index, something consistent with the theory of financial series. Finally, we calculate VaR according to the formula (2.26):

$$VaR_{\alpha,\mathrm{T}} = \sigma_t F_{\alpha,\mathrm{T}}^{-1} = \sigma_t u\left(\frac{\alpha}{T_u/T}\right)^{-\xi},$$

where T is the total sample size,

$\sigma_t = \sqrt{\sigma_t^2}$ is the standard deviation of the sample, since $\sigma_t{}^2$ is the variance estimator calculated by the variance models used before.

4.5 Back Testing Results

Finally, the VaR estimations are controlled by the back testing technique in order to find the more accurate estimation of tail risk. We perform the Kupiec and Christoffersen tests in order to accept or reject each VaR model. The first test counts for number of times that the loss in a day exceeds the 1-day $(1-\alpha\%)$ VaR that has been calculated for this day. If this happens on about α% of the days, then our methodology proves to be right and the calculated VaR a good measure for the estimation of the tail risk of each index. If it happens more often than α% of the days, then the methodology is suspect. The second tests counts for the number of times the actual loss exceeded the VaR of that day and also the clustered exceptions observed in following days. The backtesting period is the last 10 years (1/1/1996 – 30/12/2005), a total of 2610 observations. We back-test the 1 day, 95% VaR methods at 5% probability level, which corresponds to 3.841 and 5.991 critical values of Kupiec and Christoffersen tests respectively.

After the back testing procedure, among the accepted methods we prefer the one that produces the lowest average VaR, in order to capture the least capital. In the following tables the different methods of VaR are presented and back tested.

CAC 40	1 DAY, 95% VaR				1 DAY, 99% VaR			
	No of Exceptions	LR_{UC}	LR_{CC}	Average VaR	No of Exceptions	LR_{UC}	LR_{CC}	Average VaR
	Historical Simulation							
HS100	152	3.542*	5.612*	**-0.0210**	56	26.050	30.410	-0.0308
HS250	139	0.564*	3.410*	-0.0216	41	7.320	9.264	-0.0340
	Variance Approach							
MA130	137	0.328*	3.482*	-0.0217	54	23.024	30.873	-0.0308
EWMA74	151	3.227*	3.967*	-0.0212	44	10.283	10.401	-0.0300

	EVT							
MA30	182	19.150	20.229	-0.0200	33	1.700*	2.329*	**-0.0333**
EWMA74	180	17.759	18.494	-0.0201	29	0.314*	1.278*	-0.0335
	GARCH							
GARCH(1,1) *(ε~N)*	186	22.079	23.363	-0.0193	72	55.141	56.887	-0.0273
GARCH(1,1) *(ε~t)*	135	0.162*	1.561*	-0.0220	20	1.566*	***	-0.0344

Table 4: Back-testing the 1 day, 95% and 99% VaR methods for CAC 40. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. * indicates no calculation for the LR_{cc} due to zero clustered exceptions. The preferred model that produces the minimum average VaR is indicated with bold.**

DAX 30	1 DAY, 95% VaR				1 DAY, 99% VaR			
	No of Exceptions	LR_{UC}	LR_{CC}	Average VaR	No of Exceptions	LR_{UC}	LR_{CC}	Average VaR
	Historical Simulation							
HS100	163	7.923	10.354	-0.0233	62	35.986	41.624	-0.0334
HS250	148	2.372*	10.382	-0.0242	33	1.700*	4.985*	-0.0381
	Variance Approach							
MA10	187	22.833	23.163	-0.0228	69	49.076	49.147	**-0.0322**
MA74	149	2.645*	5.099*	**-0.0237**	48	14.876	24.765	-0.0336
EWMA74	153	3.872	4.486*	-0.0234	41	7.320	9.264	-0.0331
	EVT							
MA10	239	**	**	-0.0200	36	3.392*	3.827*	-0.0380
EWMA74	210	**	**	-0.0206	20	1.566*	3.712*	-0.0391
	GARCH							
GARCH(1,1) *(ε~N)*	221	**	**	-0.0197	87	89.138	91.963	-0.0279
GARCH(1,1) *(ε~t)*	108	4.327	4.930*	-0.0258	9	15.148	***	-0.0425

Table 5: Back-testing the 1 day, 95% and 99% VaR methods for DAX 30. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. ** indicates that the respective LR is too high, due to the excess number of exceptions. * indicates no calculation for the LR_{cc} due to zero clustered exceptions. The preferred model that produces the minimum average VaR is indicated with bold.**

DJINDUS	1 DAY, 95% VaR				1 DAY, 99% VaR			
	No of Exceptions	**LR_{UC}**	**LR_{CC}**	**Average VaR**	**No of Exceptions**	**LR_{UC}**	**LR_{CC}**	**Average VaR**
	Historical Simulation							
HS100	161	7.007	16.406	-0.0161	55	24.518	32.062	-0.0239
HS260	145	1.640*	12.324	-0.0166	37	4.071	4.451*	-0.0258
	Variance Approach							
MA60	137	0.336*	3.482*	-0.0171	46	12.491	13.822	-0.0242
EWMA74	141	0.868*	4.559*	**-0.0169**	42	8.260	8.432	-0.0239
	EVT							
MA30	182	19.156	24.687	-0.0153	29	0.314*	1.278*	**-0.0279**
EWMA74	172	12.684	17.882	-0.0154	24	0.175*	***	-0.0280
	GARCH							
GARCH(1,1) (ε~N)	161	7.007	13.304	-0.0158	50	17.431	18.374	-0.0224

GARCH(1,1) (ε~t)	50	8.133	13.818	-0.0186	16	4.580	***	-0.0304

Table 6: Back-testing the 1 day, 95% and 99% VaR methods for DJINDUS. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. * indicates no calculation for the LR_{cc} due to zero clustered exceptions. The preferred model that produces the minimum average VaR is indicated with bold.**

EUROSTOXX50	1 DAY, 95% VaR				1 DAY, 99% VaR			
	No of observations	**LR_{UC}**	**LR_{CC}**	**Average VaR**	**No of observations**	**LR_{UC}**	**LR_{CC}**	**Average VaR**
	Historical Simulation							
HS100	176	15.127	18.473	-0.0206	62	35.986	39.187	-0.0303
HS260	158	5.726	12.750	-0.0215	41	7.320	9.264	-0.0340
	Variance Approach							
MA60	146	1.869*	4.753*	-0.0212	51	18.770	21.577	-0.0300
EWMA74	156	4.943	5.086*	-0.0211	44	10.283	11.841	-0.0298
	EVT							
MA30	209	**	**	-0.0187	26	0.000*	1.283*	-0.0354
EWMA74	206	**	**	-0.0188	21	1.079*	3.056*	-0.0356
	GARCH							
GARCH(1,1) (ε~N)	242	**	**	-0.0168	105	136.959	137.468	-0.0238
GARCH(1,1) (ε~t)	144	1.424*	2.117*	**-0.0207**	23	0.387*	**	**-0.0337**

Table 7: Back-testing the 1 day, 95% and 99% VaR methods for EURO STOXX 50. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. ** indicates that the respective LR is too high, due to the excess number of exceptions. The preferred model that produces the minimum average VaR is indicated with bold.

FTSE 100	1 DAY, 95% VaR				1 DAY, 99% VaR			
	No of observations	**LR_{UC}**	**LR_{CC}**	**Average VaR**	**No of observations**	**LR_{UC}**	**LR_{CC}**	**Average VaR**
	Historical Simulation							
HS100	159	6.144	24.108	-0.0166	58	29.222	33.169	-0.0236
HS250	141	0.868*	17.255	-0.0172	39	5.591	7.825	-0.02672
	Variance Approach							
MA74	152	3.549*	24.660	-0.0168	56	26.050	28.106	-0.02373
EWMA74	155	4.579	10.838	-0.0166	49	16.132	16.177	-0.02346
	EVT							
MA10	227	**	**	-0.0148	65	41.407	41.745	-0.02303
MA30	206	**	**	-0.0152	56	26.050	26.131	-0.02359
MA60	189	24.392	42.243	-0.0154	55	24.518	26.712	-0.02388
MA74	186	22.079	39.203	-0.0154	54	23.024	23.658	-0.02398
MA130	175	14.498	32.144	-0.0156	54	23.024	27.825	-0.02428
EWMA74	192	26.809	32.568	-0.0152	48	14.876	14.928	-0.02371
	GARCH							
GARCH(1,1) (ε~N)	170	11.537	14.852	-0.0155	60	32.536	32.840	-0.0219
GARCH(1,1) (ε~t)	126	0.165*	5.340*	**-0.171**	22	0.687*	***	**-0.0263**

Table 8: Back-testing the 1 day, 95% and 99% VaR methods for FTSE 100. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by

equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. ** indicates that the respective LR is too high, due to the excess number of exceptions. * indicates no calculation for the LR_{cc} due to zero clustered exceptions. The preferred model that produces the minimum average VaR is indicated with bold.**

NASDAQ 100	1 DAY, 95% VaR				1 DAY, 99% VaR			
	No of observations	LR_{UC}	LR_{CC}	Average VaR	No of observations	LR_{UC}	LR_{CC}	Average VaR
	Historical Simulation							
HS100	151	3.234*	3.768*	-0.0315	49	16.132	***	-0.0420
HS250	131	0.002*	0.135*	-0.0330	32	1.257*	***	-0.0466
	Variance Approach							
MA30	144	1.424*	1.669*	-0.0327	36	3.392*	3.827*	-0.0463
EWMA74	137	0.336*	0.449*	-0.0328	26	0.000*	***	-0.0464
	EVT							
MA10	201	**	**	-0.0304	29	0.314*	1.278*	-0.0509
MA60	152	3.549*	4.826*	-0.0316	13	8.145	***	-0.0528
EWMA74	164	8.400	8.727	-0.0313	11	11.279	***	-0.0523
	GARCH							
GARCH(1,1) *(ε~N)*	138	0.446*	1.452*	**-0.0315**	29	0.314*	***	**-0.0437**
GARCH(1,1) *(ε~t)*	83	20.784	20.899	-0.0358	9	8.145	***	-0.0526

Table 9: Back-testing the 1 day, 95% and 99% VaR methods for NASDAQ 100. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. ** indicates that the respective LR is too high, due to the excess number of exceptions. * indicates no calculation for the LR_{cc} due to zero clustered exceptions. The preferred model that produces the minimum average VaR is indicated with bold.**

S&P 500	1 DAY, 95% VaR				1 DAY, 95% VaR			
	No of observations	**LR_{UC}**	**LR_{CC}**	**Average VaR**	**No of observations**	**LR_{UC}**	**LR_{CC}**	**Average VaR**
	Historical Simulation							
HS100	154	4.222	10.715	-0.0166	59	30.862	34.613	-0.0235
HS250	139	0.571*	2.400*	-0.0170	40	6.430	11.511	-0.0253
	Variance Approach							
MA30	147	2.113*	5.984*	-0.0173	54	23.024	27.825	-0.0245
EWMA74	139	0.571*	1.042*	-0.0174	49	16.132	17.165	-0.0246
	EVT							
MA30	165	8.891	17.163	-0.0165	31	0.877*	***	**-0.0272**
EWMA74	160	6.569	7.775	-0.0165	28	0.136*	1.199*	-0.0273
	GARCH							
GARCH(1,1) (ε~N)	159	6.144	8.212	-0.0161	60	32.536	36.097	-0.0228
GARCH(1,1) (ε~t)	81	22.720	26.567	-0.0205	11	11.279	***	-0.0343

Table 10: Back-testing the 1 day, 95% and 99% VaR methods for S&P 500. LR_{UC} and LR_{CC} are reffered to Kupiec (unconditional) and Christoffersen (conditional) tests statistics calculated by equations (2.27) and (2.28) respectively. Results are reported with critical values 3.841 and 5.991 respectively at 5% probability level. The backtesting period is 10 years (2610 observations). * indicates that the model is accepted by the specific back-testing technique. ** indicates that the respective LR is too high, due to the excess number of exceptions. * indicates no calculation for**

the LR_{cc} due to zero clustered exceptions. The preferred model that produces the minimum average VaR is indicated with bold.

Having estimated both the 95% and the 99% VaR, we reach the following conclusions about the different VaR approaches. First of all, it is obvious that Extreme Value Theory produces better results for 99% VaR than for 95% VaR. This facts is logical, since the EVT refers to the tails of the distribution of returns. Hence, since 99% VaR corresponds to higher tail risk, EVT suits better to 99% VaR. In addition, we observe that MA approach produces the lowest average VaR, but it is not always accepted. Moreover, GARCH models with t distribution for the residuals have better performance than the GARCH models with Normal distribution for the residuals.

In the following tables the dicriptive statistics of the selected VaR for each index are presented.

1 DAY, 95% VaR							
	CAC40	DAX30	DJINDUS	EUROSTOXX50	FTSE100	NASDAQ100	SP500
Mean	-0.0210	-0.0237	-0.0169	-0.0207	-0.0171	-0.0315	-0.0169
Median	-0.0195	-0.0210	-0.0157	-0.0184	-0.0156	-0.0273	-0.0173
Max	-0.0068	-0.0092	-0.0071	-0.0119	-0.0105	-0.0139	-0.0077
Min	-0.0513	-0.0588	-0.0421	-0.0554	-0.0445	-0.0765	-0.0312
Std.dev.	0.0097	0.0107	0.0066	0.0079	0.0061	0.0128	0.0055
Skewness	-1.0488	-1.0794	-1.2834	-1.7856	-1.5903	-0.9576	-0.4127
Kurtosis	3.6537	3.6341	4.5996	6.2287	5.7167	3.3468	2.4234
Jarque-Bera	524.944*	550.518*	994.730*	2520.564*	1902.791*	411.995*	110.258*
Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 11: Descriptive Statistics for the 1 day, 95% VaR series of the indices for the period 1/1/1996-30/12/2006. * denotes rejection of the hypothesis of Normality of the VaR series distribution at 5% significance level.

1 DAY, 99% VaR							
	CAC40	DAX30	DJINDUS	EUROSTOXX50	FTSE100	NASDAQ100	SP500
Mean	-0.0333	-0.0380	-0.0279	-0.0337	-0.0263	-0.0445	-0.0272
Median	-0.0300	-0.0327	-0.0257	-0.0300	-0.0239	-0.0386	-0.0248
Max	-0.0131	-0.0086	-0.0107	-0.0193	-0.0160	-0.0197	-0.0113
Min	-0.0913	-0.1300	-0.0698	-0.0903	-0.0682	-0.1082	-0.0690
Std.dev.	0.0153	0.0210	0.0114	0.0129	0.0093	0.0181	0.0109
Skewness	-1.5165	-1.5238	-1.3644	-1.7856	-1.5903	-0.9576	-1.1779
Kurtosis	5.3427	5.5326	4.7145	6.2287	5.7165	3.3468	4.2866
Jarque-Bera	1597.196*	1707.574*	1129.488*	2520.534*	1902.625*	411.995*	783.521*
Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 12: Descriptive Statistics for the 1 day, 99% VaR series of the indices for the period 1/1/1996-30/12/2006. * denotes rejection of the hypothesis of Normality of the VaR series distribution at 5% significance level.

5 Spillovers: Results and Discussion

5.1 Unit Root Test

In order to proceed to the causality and spillover examination, we should first examine whether the VaR series are stationary or not. This fact is examined by Augmented Dickey-Fuller test and Phillips-Perron test.

	ADF	ADF	PP	PP
1 day, 95% VaR				
CAC40	0.088*	0.255*	0.215*	0.485*
DAX30	0.062*	0.222*	0.181*	0.475*
DJINDUS	0.001	0.003	0.001	0.003
EUROSTOXX50	0.000	0.001	0.000	0.001
FTSE100	0.002	0.010	0.007	0.032
NASDAQ100	0.424*	0.698*	0.460*	0.721*
SP500	0.158*	0.373*	0.372*	0.665*
1 day, 99% VaR				
CAC40	0.004	0.022	0.007	0.035
DAX30	0.001	0.005	0.000	0.000
DJINDUS	0.000	0.000	0.000	0.001
EUROSTOXX50	0.001	0.001	0.000	0.001
FTSE100	0.002	0.010	0.007	0.032
NASDAQ100	0.425*	0.698*	0.460*	0.721*
SP500	0.000	0.002	0.001	0.003

Table 13: Augmented Dickey-Fuller and Phillips-Perron tests for Unit Root in the 1 day, 95% and 99% VaR series of the various indices. The probabilities reported correspond to t-statistics and support the null hypothesis that a series has a unit root. * indicates that the null hypothesis of the existence of a unit root is significant at 5% probability level.

Implementing the Augmented Dickey-Fuller and the Phillips-Perron unit root tests, we reach the following results. The indices that present a stationary 1 day, 95% VaR are DJ INDUSTRIALS, EURO STOXX50 and FTSE100. On the other hand, CAC40, DAX30, NASDAQ100 and S&P500 are characterized as non-stationary since they have a unit root. Moreover, the non-stationary VaR series are integrated of order 1, which means that their lagged differences are stationary.

However, following Gemmill and Kamiyama (2000), if we test for autocorrelation of the stationary VaR series, the Q-statistics reject the insignificance of the autocorrelation coefficients and the correlograms display high degree of persistence. We see that the first autoregressive coefficient is nearly 1; it is 0.988 DJ INDUSTRIALS, 0.978 EURO STOXX50 and 0.993 FTSE100. These statistics prove that there exists a high degree of autocorrelation.

Examining stationarity of 99% VaR series, we conclude that all indices have no unit root except for NASDAQ 100. However, these stationary series present high first order autocorrelation, 0.995 for CAC 40, 0.978 for DAX 30, 0.992 for DJ INDUSTRIALS, 0.978 for EUROSTOXX 50, 0.993 for FTSE100 and 0.992 for S&P 500 and a high degree of autocorrelation.

Due to the high degree of autocorrelation, an analysis in levels would give a highly autocorrelated error term (and significant DW statistic). We think that a change in VaR of one index is very likely to lead to a change in the VaR of the other.

The above results let us use the first differenced series (changes) of VaR for both stationary and non-stationary series. The ADF and PP tests verify that the first differences have no unit root. By using the changes of VaR of each index, we manage to cope with new series that are stationary.

In the following tables the descriptive statistics of the daily changes of the 1 day, 95% and 99% VaR series of the various induces are presented.

1 DAY, 95% VaR							
	CAC40	DAX30	DJINDUS	EUROSTOXX50	FTSE100	NASDAQ100	SP500
Mean	1.86E-06	7.80E-07	1.04E-06	1.05E-06	1.13E-06	2.04E-06	-6.49E-07
Median	0	5.46E-07	2.57E-04	3.10E-04	1.50E-04	1.55E-04	1.54E-07
Std.dev.	0.0007	0.0004	0.0010	0.0016	0.0007	0.0007	0.0002
Skewness	-0.4016	-0.5579	-6.8123	-2.8201	-3.7355	-6.1325	-0.8067
Kurtosis	35.5505	26.5038	90.9538	17.5695	30.5964	75.1848	54.1399
Jarque-Bera	115251*	60189*	861133*	26534*	88856*	582793*	284587*
Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 14: Descriptive Statistics for the daily changes (first differences) in the 1 day, 95% VaR series of the indices for the period 1/1/1996-30/12/2006. * denotes rejection of the hypothesis of Normality of the VaR series distribution at 5% significance level.

1 DAY, 99% VaR							
	CAC40	DAX30	DJINDUS	EUROSTOXX50	FTSE100	NASDAQ100	SP500
Mean	6.09E-06	2.77E-06	2.07E-06	1.82E-06	1.80E-06	2.88E-06	1.24E-06
Median	2.49E-06	-7.19E-06	3.20E-07	5.07E-04	2.30E-04	2.19E-04	-2.15E-06
Std.dev.	0.0013	0.0043	0.0013	0.0027	0.0010	0.0010	0.0012
Skewness	-0.3051	0.0453	-1.2888	-2.8196	-3.7349	-6.1325	-0.6599
Kurtosis	18.5794	12.5440	48.7847	17.5663	30.5880	75.1848	36.3531
Jarque-Bera	26426*	9903*	228601*	26522*	88803*	582793*	121119*
Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 15: Descriptive Statistics for the daily changes (first differences) in the 1 day, 99% VaR series of the indices for the period 1/1/1996-30/12/2006. * denotes rejection of the hypothesis of Normality of the VaR series distribution at 5% significance level.

5.2 Granger Causality

The first way to detect spillover of tail risk across the seven indices is by examining the bivariate Granger causality. As it was mentioned earlier, the VaR series present non-stationarity; hence, we use the changes of VaR, in order our series to become stationary. With Granger causality we examine if the information about the tail risk that an index faced the previous days causes the tail risk of another index the next day. We perform the Granger causality test by running bivariate regressions for all possible pairs (X, Y):

$$\Delta X_t = \alpha_0 + \sum_{i=1}^{k} \beta_i \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \varepsilon_t \qquad (5.1)$$

$$\Delta Y_t = \alpha_0 + \sum_{i=1}^{k} \beta_i \Delta Y_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta X_{t-i} + \varepsilon_t \qquad (5.2)$$

where the null hypothesis is $H_0 : \gamma_1 = ... = \gamma_k = 0$,

ΔX_t and ΔY_t is the daily change in the 1 day, 95% or 99% VaR from time *t-1* to time *t* of the index *X* and *Y* respectively.

ε_t are the residuals.

The null hypothesis is that *X* does not Granger-cause *Y* in the first regression and that *Y* does not Granger-cause *X* in the second regression.

This means that we carry out pairwise Granger causality tests and test whether an endogenous variable can be treated as exogenous. For each equation, we calculate χ^2-statistics (Wald test) to test the null joint hypothesis.

$$W = \frac{\left(SSR_R - SSR_{UR}\right)}{SSR_{UR}/\left(n-2k-1\right)} \sim \chi^2(q) \qquad (5.3)$$

where SSR_{UR} is the residual sum of squares of the unrestricted regression above,

SSR_R is the residual sum of squares of the restricted regression, which is the regression without the lags of Y_t,

k is the number of lags used,

n is the number of observations.

The bivariate lagged regressions are performed in a VAR model. A crucial point in the construction of a VAR model is the definition of the number of lags taken into account in the model. We choose the appropriate number of lags according to Schwarz criterion; we accept the VAR model that produces the lowest Schwarz statistic. We prefer Schwarz criterion versus Akaike information criterion, because the first imposes a larger penalty for additional coefficients than the latter; besides, it is logical to assume that the most recent information has greater effect than older information.

Bivariate Granger Causality Tests for daily changes of 1 day, 95% VaR			
Direction of Causality	χ^2 -Stat.	Probability	lags
Δ(DAX30) ⇨ Δ(CAC40)*	18.334	0.001	4
Δ(CAC40) ⇨ Δ(DAX30)*	22.838	0.000	
Δ(DJINDUS) ⇨ Δ(CAC40)	2.890	0.089	1
Δ(CAC40) ⇨ Δ(DJINDUS)	3.382	0.066	
Δ(EUROSTOXX50) ⇨ Δ(CAC40)	2.147	0.143	1
Δ(CAC40) ⇨ Δ(EUROSTOXX50)	0.751	0.386	
Δ(FTSE100) ⇨ Δ(CAC40)	0.731	0.393	1
Δ(CAC40) ⇨ Δ(FTSE100)**	4.585	0.032	
Δ(NASDAQ100) ⇨ Δ(CAC40)	0.345	0.557	1
Δ(CAC40) ⇨ Δ(NASDAQ100)*	7.131	0.008	
Δ(SP500) ⇨ Δ(CAC40)*	21.691	0.000	3
Δ(CAC40) ⇨ Δ(SP500)*	34.993	0.000	
Δ(DJINDUS) ⇨ Δ(DAX30)*	59.615	0.000	4
Δ(DAX30) ⇨ Δ(DJINDUS)*	40.721	0.000	
Δ(EUROSTOXX50) ⇨ Δ(DAX30)**	11.916	0.018	4
Δ(DAX30) ⇨ Δ(EUROSTOXX50)	4.974	0.290	
Δ(FTSE100) ⇨ Δ(DAX30)*	16.953	0.002	4

Δ(DAX30) ⇨ Δ(FTSE100)	11.155	0.025	
Δ(NASDAQ100) ⇨ Δ(DAX30)**	9.755	0.045	4
Δ(DAX30) ⇨ Δ(NASDAQ100)*	15.783	0.003	
Δ(SP500) ⇨ Δ(DAX30)*	43.455	0.000	4
Δ(DAX30) ⇨ Δ(SP 500)*	27.347	0.000	
Δ(EUROSTOXX50) ⇨ Δ(DJINDUS)**	4.369	0.037	1
Δ(DJINDUS) ⇨ Δ(EUROSTOXX50)*	15.209	0.000	
Δ(FTSE100) ⇨ Δ(DJINDUS)*	81.924	0.000	4
Δ(DJINDUS) ⇨ Δ(FTSE100)*	13.491	0.009	
Δ(NASDAQ100) ⇨ Δ(DJINDUS)	0.296	0.586	1
Δ(DJINDUS) ⇨ Δ(NASDAQ100)*	19.688	0.000	
Δ(SP500) ⇨ Δ(DJINDUS)*	13.484	0.004	3
Δ(DJINDUS) ⇨ Δ(SP500)*	19.528	0.000	
Δ(FTSE100) ⇨ Δ(EUROSTOXX50)*	24.734	0.000	3
Δ(EUROSTOXX50) ⇨ Δ(FTSE100)	4.602	0.203	
Δ(NASDAQ100) ⇨ Δ(EUROSTOXX50)**	4.589	0.032	1
Δ(EUROSTOXX50) ⇨ Δ(NASDAQ100)	0.077	0.782	
Δ(SP500) ⇨ Δ(EUROSTOXX50)*	10.083	0.006	2
Δ(EUROSTOXX50) ⇨ Δ(SP500)	4.551	0.103	
Δ(NASDAQ100) ⇨ Δ(FTSE100)**	7.088	0.029	2
Δ(FTSE100) ⇨ Δ(NASDAQ100)**	8.465	0.015	
Δ(SP500) ⇨ Δ(FTSE100)	4.854	0.088	2
Δ(FTSE100) ⇨ Δ(SP500)*	9.777	0.008	
Δ(SP500) ⇨ Δ(NASDAQ100)*	17.061	0.000	2
Δ(NASDAQ100) ⇨ Δ(SP500)*	9.372	0.009	

Table 16: Bivariate Granger Causality tests between the daily changes (first differences) of the 1 day, 95% VaR of the various indices. The model examined is

$$\Delta X_t = \alpha_0 + \sum_{i=1}^{k} \beta_i \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \varepsilon_t$$

$$\Delta Y_t = \alpha_0 + \sum_{i=1}^{k} \beta_i \Delta Y_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta X_{t-i} + \varepsilon_t$$

where the null hypothesis is $H_0 : \gamma_1 = ... = \gamma_k = 0$ and ΔX and ΔY represent the daily changes of 1 day, 99% VaR of indices *X* and *Y* respectively. The *k* lags used in each model are specified by Schwarz criterion. χ^2-statistic (4.14) and the respective probability correspond to the null hypothesis, which means that Δ*Y* does not Granger cause Δ*X*. * indicates rejection of the null hypothesis and significant Granger causality at 1% probability level and ** indicates rejection of the null and significant Granger causality at 5% probability level.

Bivariate Granger Causality Tests for daily changes of 1 day, 99% VaR			
Direction of Causality	χ^2 -Stat.	Probability	lags
Δ(DAX30) ⇨ Δ(CAC40)	0.724	0.395	1
Δ(CAC40) ⇨ Δ(DAX30)	0.492	0.483	
Δ(DJINDUS) ⇨ Δ(CAC40)*	50.379	0.000	4
Δ(CAC40) ⇨ Δ(DJINDUS)*	67.221	0.000	
Δ(EUROSTOXX50) ⇨ Δ(CAC40)	1.856	0.603	3

Δ(CAC40) ⇨ Δ(EUROSTOXX50)	7.385	0.061	
Δ(FTSE100) ⇨ Δ(CAC40)	3.783	0.052	1
Δ(CAC40) ⇨ Δ(FTSE100)	0.253	0.615	
Δ(NASDAQ100) ⇨ Δ(CAC40)*	15.089	0.000	1
Δ(CAC40) ⇨ Δ(NASDAQ100)	3.679	0.055	
Δ(SP500) ⇨ Δ(CAC40)*	65.809	0.000	2
Δ(CAC40) ⇨ Δ(SP500)*	17.446	0.000	
Δ(DJINDUS) ⇨ Δ(DAX30)*	12.608	0.000	1
Δ(DAX30) ⇨ Δ(DJINDUS)	0.419	0.517	
Δ(EUROSTOXX50) ⇨ Δ(DAX30)**	5.107	0.024	1
Δ(DAX30) ⇨ Δ(EUROSTOXX50)	0.823	0.364	
Δ(FTSE100) ⇨ Δ(DAX30)*	6.745	0.009	1
Δ(DAX30) ⇨ Δ(FTSE100)	0.291	0.590	
Δ(NASDAQ100) ⇨ Δ(DAX30)	3.503	0.061	1
Δ(DAX30) ⇨ Δ(NASDAQ100)	0.122	0.727	
Δ(SP500) ⇨ Δ(DAX30)*	12.742	0.000	1
Δ(DAX30) ⇨ Δ(SP 500)	0.101	0.751	
Δ(EUROSTOXX50) ⇨ Δ(DJINDUS)	2.978	0.084	1
Δ(DJINDUS) ⇨ Δ(EUROSTOXX50)*	13.772	0.000	
Δ(FTSE100) ⇨ Δ(DJINDUS)*	24.659	0.000	3
Δ(DJINDUS) ⇨ Δ(FTSE100)*	19.241	0.000	
Δ(NASDAQ100) ⇨ Δ(DJINDUS)	0.601	0.741	2
Δ(DJINDUS) ⇨ Δ(NASDAQ100)*	15.065	0.001	
Δ(SP500) ⇨ Δ(DJINDUS)	0.768	0.381	1
Δ(DJINDUS) ⇨ Δ(SP500)	3.772	0.052	
Δ(FTSE100) ⇨ Δ(EUROSTOXX50)**	4.592	0.032	1
Δ(EUROSTOXX50) ⇨ Δ(FTSE100)	0.078	0.780	
Δ(NASDAQ100) ⇨ Δ(EUROSTOXX50)**	4.592	0.032	1
Δ(EUROSTOXX50) ⇨ Δ(NASDAQ100)	0.078	0.780	
Δ(SP500) ⇨ Δ(EUROSTOXX50)*	17.769	0.000	1
Δ(EUROSTOXX50) ⇨ Δ(SP500)	1.464	0.226	
Δ(NASDAQ100) ⇨ Δ(FTSE100)**	7.096	0.029	2
Δ(FTSE100) ⇨ Δ(NASDAQ100)**	8.435	0.015	
Δ(SP500) ⇨ Δ(FTSE100)*	22.083	0.000	2
Δ(FTSE100) ⇨ Δ(SP500)**	8.265	0.016	
Δ(SP500) ⇨ Δ(NASDAQ100)*	11.625	0.003	2
Δ(NASDAQ100) ⇨ Δ(SP500)	0.583	0.747	

Table 17: Bivariate Granger Causality tests between the daily changes (first differences) of the 1 day, 99% VaR of the various indices. The model examined is

$$\Delta X_t = \alpha_0 + \sum_{i=1}^{k} \beta_i \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \varepsilon_t$$

$$\Delta Y_t = \alpha_0 + \sum_{i=1}^{k} \beta_i \Delta Y_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta X_{t-i} + \varepsilon_t$$

where the null hypothesis is $H_0 : \gamma_1 = ... = \gamma_k = 0$ and ΔX and ΔY represent the daily changes of 1 day, 99% VaR of indices *X* and *Y* respectively. The *k* lags used in each model are specified by

Schwarz criterion. χ^2-statistic (4.14) and the respective probability correspond to the null hypothesis, which means that *Y* does not Granger cause *X*. * indicates rejection of the null hypothesis and significant Granger causality at 1% probability level and ** indicates rejection of the null and significant Granger causality at 5% probability level.

5.3 Spillover effetcs

Finally, in order to identify the spillover effects of tail risk across the under examination indices, we perform the following regressions:

$$\Delta VaR_{d,t} = \alpha_{d,0} + \sum_{i=1}^{k}\sum_{f=1}^{m} \alpha_{d,f,i} \Delta VaR_{f,t-i} + \varepsilon_{d,t} \quad \text{(Lagged relationship)} \qquad (5.4)$$

$$\Delta VaR_{d,t} = \beta_{d,0} + \sum_{f=1}^{m-1} \beta_{d,f} \Delta VaR_{f,t} + u_{d,t} \quad \text{(Contemporaneous relationship)} \qquad (5.5)$$

where $\Delta VaR_{d,t}$ is the change in the 1 day VaR between time *t-1* and *t* of the dependent index *d*,

$\Delta VaR_{f,t}$ is the change in the 1 day VaR between time *t-1* and *t* of the independent index *f*,

$\alpha_{d,0}$ and $\beta_{d,o}$ are the constants of the regressions of the dependent index *d*,

$\alpha_{d,f,i}$ are the coefficients of the lagged regression of the dependent index *d* corresponding to independent index *f* with lag *i*,

$\beta_{d,f}$ are the coefficients of the contemporaneous regression of the dependent index *d* corresponding to independent index *f*,

m is the total number of indices examined

k is the total number of lags used in the model,

$\varepsilon_{d,t}$ and $u_{d,t}$ are the residuals of the models.

The first equation checks whether the changes in the independent indices and the same domestic index lead the dependent index. The second equation tests whether there is a contemporaneous relationship between the changes in the VaR of the dependent index and the changes in the VaR of the independent indices. Contemporaneous refers to the same calendar date *t*.

Lagged relationship

The next step of our methodology consists of examining for multivariate lagged relationship across the changes of the various VaR series. We approach this subject by Vector Autoregression (VAR) models.

The VAR model examined consists of 7 equations of the form:

$$\Delta X_t = c^X + \sum_{i=1}^{k} \alpha^1_{X,i} \Delta CAC40_{t-i} + \sum_{i=1}^{k} \alpha^2_{X,i} \Delta DAX30_{t-i} + \sum_{i=1}^{k} \alpha^3_{X,i} \Delta DJINDUS_{t-i} + \sum_{i=1}^{k} \alpha^4_{X,i} \Delta EUROSTOXX50_{t-i} +$$
$$+ \sum_{i=1}^{k} \alpha^4_{X,i} \Delta FTSE100_{t-i} + \sum_{i=1}^{k} \alpha^5_{X,i} \Delta NASDAQ100_{t-i} + \sum_{i=1}^{k} \alpha^6_{X,i} \Delta SP500_{t-i} + \varepsilon^X_t \qquad (5.6)$$

where ΔX is the daily change (first difference) of the 1 day, 95% or 99% VaR of dependent index *X*,

X is each one of the seven indices,

k is the total number of lags used in the VAR model.

Each one of the seven VaR series is set us dependent variable and thus, the VAR model consists of a total number of seven equations.

In the following tables we test for lagged spillover effects across the changes of 1 day, 95% VaR of the seven indices. Following Schwarz criterion, we select that model, which produces the lowest Schwarz statistic. For both the 95% and 99% VaR Schwarz criterion suggests 1 lag.

LAGGED RELATIONSHIP ACROSS 1 DAY, 95% VaR							
	Δ(CAC40)	Δ(DAX30)	Δ(DJINDUS)	Δ(EUROSTOXX50)	Δ(FTSE100)	Δ(NASDAQ100)	Δ(SP500)
α_0	1.9E-06	7.4-07	8.8E-07	10.0E-07	7.7E-07	1.8E-06	-5.4E-07
α_1	0.012	0.049*	0.037	0.047	0.053*	0.054*	0.021*
α_2	0.093*	0.170*	0.044	-0.059	0.064	0.045	0.033*
α_3	0.020	0.028*	-0.003	0.110*	0.056*	0.055*	-0.022*
α_4	-0.026*	-0.045*	-0.004	-0.116*	-0.037*	-0.024	-0.011*
α_5	-0.006	0.018	0.078*	-0.034	0.023	0.029	0.010
α_6	-0.017	-0.010	-0.031	0.002	0.019	0.028	-0.006
α_7	0.110	0.223*	0.107	0.208	-0.051	0.179*	0.218*
R^2	0.006	0.052	0.005	0.018	0.012	0.022	0.042
SC	-11.797	-12.651	-11.039	-9.993	-11.757	-11.716	-14.251

Table 18: Estimation of coefficients included in the model

$$\Delta X_t = \alpha_0^X + \alpha_1^X \Delta CAC40_{t-1} + \alpha_2^X \Delta DAX30_{t-1} + \alpha_3^X \Delta DJINDUS_{t-1} + \alpha_4^X \Delta EUROSTOXX50_{t-1} + \alpha_5^X \Delta FTSE100_{t-1} + \alpha_6^X \Delta NASDAQ100_{t-1} + \alpha_7^X \Delta SP500_{t-1} + \varepsilon_t^X$$

that displays the lagged relationship across the daily changes in the 1 day, 95% VaR of the various indices with *X* being the dependent VaR series. * indicates a significance coefficient at 5% probability level. *SC* corresponds to the Schwarz statistic.

Examining the lagged relationship across the changes of the 1 day, 99% VaR series of the various indices, we reach the following results.

LAGGED RELATIONSHIP ACROSS 1 DAY, 99% VaR							
	Δ(CAC40)	Δ(DAX30)	Δ(DJINDUS)	Δ(EUROSTOXX50)	Δ(FTSE100)	Δ(NASDAQ100)	Δ(SP500)
α_0	5.7E-06	3.1E-06	1.6E-06	2.4E-06	1.5E-06	2.3E-06	7.5E-07
α_1	0.025	-0.245*	0.047	-0.155*	0.005	0.023	0.028
α_2	0.007	-0.058*	-0.003	0.008	0.006	0.000	-0.003
α_3	-0.060	0.107	0.048	-0.028	0.026	0.068	0.087
α_4	-0.039*	0.098	-0.002	-0.082*	-0.027*	-0.013	0.001
α_5	0.084*	0.178	0.033	-0.016	0.025	0.024	0.018
α_6	-0.005	-0.012	0.008	-0.010	0.010	0.064*	0.012
α_7	0.231*	0.177	0.025	0.244*	0.053	-0.014	0.007
R^2	0.031	0.011	0.011	0.022	0.011	0.016	0.014
SC	-10.393	-8.040	-10.417	-9.018	-10.901	-11.016	-10.581

Table 19: Estimation of coefficients included in the model

$$\Delta X_t = \alpha_0^X + \alpha_1^X \Delta CAC40_{t-1} + \alpha_2^X \Delta DAX30_{t-1} + \alpha_3^X \Delta DJINDUS_{t-1} + \alpha_4^X \Delta EUROSTOXX50_{t-1} + \alpha_5^X \Delta FTSE100_{t-1} + \alpha_6^X \Delta NASDAQ100_{t-1} + \alpha_7^X \Delta SP500_{t-1} + \varepsilon_t^X$$

that displays the lagged relationship across the daily changes in the 1 day, 99% VaR of the various indices with *X* being the dependent VaR series. The t-statistics of each estimated coefficient are presented in the brackets. A t-statistic greater than 2 in magnitude corresponds to a significant coefficient at approximately 5% probability level. * indicates a significance coefficient at 5% probability level. *SC* corresponds to Schwarz criterion.

Contemporaneous relationship

We continue with performing contemporaneous regressions across the daily changes of the VaR series of the various indices.

$$\Delta CAC40_t = \beta_0 + \beta_1 \Delta DAX30_t + \beta_2 \Delta DJINDUS_t + \beta_3 \Delta EUROSTOXX50_t + \beta_4 \Delta FTSE100_t + \beta_5 \Delta NASDAQ100_t + \beta_6 \Delta SP500_t + \varepsilon_t \quad (5.7)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	1.9E-06	0.152	0.879	β_0	5.3E-06	0.254	0.799
β_1**	0.068	2.057	0.040	β_1	-0.002	-0.360	0.719
β_2	-0.005	-0.267	0.790	β_2	-0.010	-0.264	0.792
β_3*	0.099	9.090	0.000	β_3*	0.235	19.964	0.000
β_4	0.005	0.192	0.848	β_4*	0.171	6.343	0.000
β_5**	-0.056	-2.540	0.011	β_5*	-0.076	-2.990	0.003
β_6	0.114	1.368	0.171	β_6*	0.235	5.207	0.000
R^2	0.073			R^2	0.006		
Schwarz	-11.870			Schwarz	-10.829		

Table 20: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with CAC 40 being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

$$\Delta DAX30_t = \beta_0 + \beta_1 \Delta CAC40_t + \beta_2 \Delta DJINDUS_t + \beta_3 \Delta EUROSTOXX50_t + \beta_4 \Delta FTSE100_t + \beta_5 \Delta NASDAQ100_t + \beta_6 \Delta SP500_t + \varepsilon_t \quad (5.8)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	6.4E-07	0.087	0.931	β_0	4.0E-07	0.006	0.995
β_1**	0.024	2.057	0.040	β_1	-0.023	-0.360	0.719
β_2	-0.013	-1.244	0.214	β_2	0.066	0.511	0.609
β_3*	0.114	18.520	0.000	β_3*	0.921	24.522	0.000
β_4*	0.055	3.772	0.000	β_4	-0.031	-0.349	0.727
β_5*	0.038	2.910	0.004	β_5*	0.231	2.745	0.006
β_6*	0.224	4.585	0.000	β_6	0.079	0.528	0.598
R^2	0.275			R^2	0.338		
Schwarz	-12.923			Schwarz	-8.444		

Table 21: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with DAX 30 being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

$$\Delta DJINDUS_t = \beta_0 + \beta_1 \Delta CAC40_t + \beta_2 \Delta DAX30_t + \beta_3 \Delta EUROSTOXX50_t + \beta_4 \Delta FTSE100_t + \beta_5 \Delta NASDAQ100_t + \beta_6 \Delta SP500_t + \varepsilon_t \quad (5.9)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	1.5E-06	0.112	0.911	β_0	1.1E-06	0.106	0.915
β_1	-0.006	-0.267	0.790	β_1	-0.003	-0.264	0.792

β₂	-0.045	-1.244	0.214	***β₂***	0.002	0.511	0.609
β_3*	0.056	4.606	0.000	β_3	0.004	0.643	0.520
β_4*	0.116	4.292	0.000	β_4	0.013	0.980	0.327
β_5*	0.398	17.293	0.000	β_5*	-0.115	-9.157	0.000
β_6*	2.224	27.692	0.000	β_6*	1.031	100.129	0.000
R^2	0.473			R^2	0.836		
Schwarz	-11.678			**Schwarz**	-12.575		

Table 22: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with DJINDUS being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

$$\Delta EUROSTOXX50_t = \beta_0 + \beta_1 \Delta CAC40_t + \beta_2 \Delta DAX30_t + \beta_3 \Delta DJINDUS_t + \beta_4 \Delta FTSE100_t + \beta_5 \Delta NASDAQ100_t + \beta_6 \Delta SP500_t + \varepsilon_t \quad (5.10)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	-2.1E-06	-0.095	0.924	β_0	-4.3E-06	-0.132	0.895
β_1*	0.311	9.090	0.000	β_1*	0.565	19.964	0.000
β_2*	1.025	18.520	0.000	β_2*	0.204	24.522	0.000
β_3*	0.145	4.606	0.000	β_3	0.039	0.643	0.520
β_4*	1.229	33.749	0.000	β_4*	1.033	28.020	0.000
β_5	0.036	0.915	0.360	β_5*	0.126	3.180	0.002
β_6	-0.244	-1.655	0.098	β_6**	-0.168	-2.382	0.017
R^2	0.525			R^2	0.614		
Schwarz	-10.723			**Schwarz**	-9.952		

Table 23: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with EURO STOXX 50 being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

$$\Delta FTSE100_t = \beta_0 + \beta_1 \Delta CAC40_t + \beta_2 \Delta DAX30_t + \beta_3 \Delta DJINDUS_t + \beta_4 \Delta EUROSTOXX50_t + \beta_5 \Delta NASDAQ100_t + \beta_6 \Delta SP500_t + \varepsilon_t \quad (5.11)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	5.6E-07	0.057	0.955	β_0	6.9E-07	0.045	0.964
β_1	0.003	0.192	0.848	β_1*	0.089	6.343	0.000
β_2*	0.099	3.772	0.000	β_2	-0.002	-0.349	0.727
β_3*	0.061	4.292	0.000	β_3	0.028	0.980	0.327
β_4*	0.248	33.749	0.000	β_4*	0.224	28.020	0.000
β_5	0.029	1.636	0.102	β_5**	0.038	2.051	0.040
β_6**	-0.165	-2.496	0.013	β_6	0.002	0.070	0.944
R^2	0.438			R^2	0.443		
Schwarz	-12.325			**Schwarz**	-11.478		

Table 24: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with FTSE 100 being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

$$\Delta NASDAQ100_t = \beta_0 + \beta_1 \Delta CAC40_t + \beta_2 \Delta DAX30_t + \beta_3 \Delta DJINDUS_t + \beta_4 \Delta EUROSTOXX50_t + \beta_5 \Delta FTSE100_t + \beta_6 \Delta SP500_t + \varepsilon_t \quad (5.12)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	2.3E-06	0.209	0.834	β_0	2.7E-06	0.170	0.865
β_1**	-0.044	-2.540	0.011	β_1*	-0.045	-2.990	0.003
β_2*	0.086	2.910	0.004	β_2*	0.013	2.745	0.006
β_3*	0.259	17.293	0.000	β_3*	-0.272	-9.157	0.000
β_4	0.009	0.915	0.360	β_4*	0.031	3.180	0.002
β_5	0.036	1.636	0.102	β_5**	0.043	2.051	0.040
β_6*	0.886	12.371	0.000	β_6*	0.667	20.616	0.000
R^2	0.337			R^2	0.298		
Schwarz	-12.108			Schwarz	-11.358		

Table 25: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with NASDAQ 100 being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

$$\Delta SP500_t = \beta_0 + \beta_1 \Delta CAC40_t + \beta_2 \Delta DAX30_t + \beta_3 \Delta DJINDUS_t + \beta_4 \Delta EUROSTOXX50_t + \beta_5 \Delta FTSE100_t + \beta_6 \Delta NASDAQ100_t + \varepsilon_t \quad (5.13)$$

1 DAY, 95% VaR				1 DAY, 99% VaR			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
β_0	-9.0E-07	-0.308	0.758	β_0	-1.2E-06	-0.135	0.893
β_1	0.006	1.368	0.171	β_1*	0.044	5.207	0.000
β_2*	0.036	4.585	0.000	β_2	0.001	0.528	0.598
β_3*	0.102	27.692	0.000	β_3*	0.770	100.129	0.000
β_4	-0.004	-1.655	0.098	β_4**	-0.013	-2.382	0.017
β_5**	-0.014	-2.496	0.013	β_5	0.001	0.070	0.944
β_6*	0.063	12.371	0.000	β_6*	0.211	20.616	0.000
R^2	0.420			R^2	0.856		
Schwarz	-14.757			Schwarz	-12.510		

Table 26: Coefficient estimation of the contemporaneous regressions across the 1 day, 95% and 99% VaR of the various indices with S&P 500 being the dependent VaR series. * indicates significant coefficient at 1% probability level and ** indicates significant coefficient at 5% probability level.

5.4 Comments on the results

The implementation of the above methodology has produced several interesting results regarding the spillover effects across the seven under examination indices. The results that we have reached indicate that spillover effects do exists among the major European and US stock indices. Both Europe and USA effect on each other. Especially, it is obvious that the US indices Granger cause the European indices in a great degree. In particular, if we use the 1 day, 95% VaR as the risk measure of tail risk, then DJ INDUSTRIALS seems to be the index that has the greater spillover effect on the other indices; the specific US index Granger causes and also has a lead effect on DAX30, EUROSTOXX50, FTSE100 (the greater European indices) and also on the US NASDAQ100 and S&P500. Moreover, DJ INDUSTRIALS has contemporaneous effects on almost every index – EUROSTOXX50 nad FTSE from Europe and NASDAQ100 and SP500 from the USA. In addition, S&P500 seems to be another index with great influence, since it Granger causes every index apart from FTSE100. It also lead DAX30 and NASDAQ100 and contemporaneously spills over DAX, DJ INDUSTRIALS, FTSE100 and NASDAQ100. Finally, NASDAQ100 is mainly caused by other indices – from every index except for EUROSTOXX50 – and also led and contemporaneously affected by the US and European indices.

Among the European indices, FTSE100 present significant spillover effects on the other indices, since it Granger causes many US ones – DJ INDUSTRIALS, NASDAQ100 and S&P500 and the European DAX30 and EUROSTOXX50. However, this causality seems to have extension greater than 1 day, since FTSE100 has a lagged effect (for 1 day lag) only to DJ INDUSTRIALS. Contemporaneously, however, FTSE100 has effect on DAX, DJ INDUSTRIALS, EUROSTOXX50 and S&P500. Furthermore, it is worth noticing the CAC40 index. This index Granger causes every index apart from DJ INDUSTRIALS and EUROSTOXX50. The lead effects of CAC40 are also significant, since its effects are not only on the European DAX30 and FTSE100, but also on the US NASDAQ100 and S&P500. DAX30 index causes and is caused by many indices. It is caused by every index and causes all apart from EUROSTOXX50. It is mainly led more than it leads and contemporaneously it has relationship with 5 indices, all except from DJ INDUSTRIALS. EUROSTOXX50 finally, though it has only low Granger causality effects – only to DAX30 and DJ INDUSTRIALS – has interesting lead spillover effects on many indices, since it affects on the change of their VaR the next day; lagged relationship is observed from EUROSTOXX50 to CAC40, DAX30, FTSE100 and S&P500. Contemporaneously is affected by DJ INDUSTRIALS. This spillover is reasonable, since the specific index consists of the greater European stocks. In general, we have to point that it is obvious that the contemporaneous effects across all indices are significant and in a great degree. This fact shows that the changes in tail risk across the indices take part in the same day rather than 1 day after.

If we use the 1 day, 99% VaR as the appropriate risk measure, then we also reach interesting results. In this situation, we observe that the effects across indices are restricted. DJ INDUSTRIALS still Granger causes many indices for US and Europe – CAC40, DAX30, EUROSTOXX50, FTSE100, NASDAQ100. Though, no lagged relationship from DJ INDUSTRIALS is observed to any index. Concerning the lagged relationship in general, the US indices appear to be affected by no index. Contemporaneous effects are also present in this situation, but, as we observe, DJ INDUSTRIALS is affected only by the US indices NASDAQ100 and S&P500. S&P500 presents great causality effects since it causes 5 indices – CAC40, DAX30, EUROSTOXX50, FTSE100 and NASDAQ100. This facts displays the great effect that S&P500 has on all European indices under examination. NASDAQ100 finally causes CAC40, EUROSTOXX50 and FTSE100, all European again. It has no lagged effects, however, to other indices, but it presents extreme significant contemporaneous effects, since it has effects on every index.

Regarding the European indices, FTSE100 has significant causality effects on five indices – DAX30, DJ INDUSTRIALS, EUROSTOXX50, NASDAQ100 and S&P500. That is it affects both the European and the US indices. FTSE100 has lagged effect only on CAC40 but contemporaneously it affects on CAC40, EUROSTOXX50 and NASDAQ100. CAC40 also causes the US indices but has no lagged effect on them with lag 1; it has effect on DAX30 and EUROSTOXX50. Contemporaneously it has effects on EUROSTOXX50, FTSE100 and NASDAQ100. DAX30 has no granger causality effect and also no lagged effects on any index. Contemporaneously it is related only with European indices – EUROSTOXX50 and NASDAQ100. EUROSTOXX50 also, as it is constructed by European stocks, Granger causes DAX30, it leads CAC40 and FTSE100. Only contemporaneously it is related with both European and US indices.

The above results indicate the major influence of US indices on the European ones. The US indices, especially DJ INDUSTRIALS and S&P500 influence CAC40, DAX30, EUROSTOXX50 and FTSE100. On the other hand, among the European indices the greater influence is presented by FTSE100, which influences the markets inside Europe and the US markets. In addition, European indices in general are related to each other due to their vinicity and the economic convergence of one European country to each other. The above results verify the objective of our research; the tail risk do spill over the major European and US indices.

6 Taking Cointegration into Account

The Augmented Dickey-Fuller and the Phillips-Perron unit root tests of the levels of the VaR series have indicated that CAC40, DAX0, NASDAQ100 and S&P500 for the 95% VaR and NASDAQ100 for the 99% VaR are non stationary, since they have a unit root. All these non stationary series are I(1), that is the series in first differences have no unit root. Since the series are of the same degree of integration, we should examine whether the VaR series are cointegrated or not. Two series Yt and Xt are said to be cointegrated if:

1. Both are I(d), $d \neq 0$ and the same for both series.
2. There is a linear combination of them that is I(0), that is, there exist $a = (a_1; a_2)$ non zero such that $a_1 Y_t + a_2 X_t$ is I(0).

The vector a is called the cointegrating vector.

Cointegration implies that two time series are bound together by a stable long-term relationship. It is examined by Johansen test among series that have the same order of integration.

According to Sander and Kleimeier, 2000, if the series are found to be I(1) and not cointegrated, causality tests on the first differences of the VaR series should be applied:

$$\Delta X_t = \alpha_x + \sum_{i=1}^{k} \beta_{x,i} \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_{X,i} \Delta Y_{t-i} + \varepsilon_{X,t} \qquad (6.1)$$

These tests are the same as the precedent ones.

If the series are found to be I(1) and cointegrated, then an error correction term (ECT) is introduced and thus causality tests should be applied on the following equations:

$$\Delta X_t = \alpha_Y + \sum_{i=1}^{k} \beta_{X,i} \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_{X,i} \Delta Y_{t-i} + \phi_Y ECT_{X,Y} + \varepsilon_{Y,t} \qquad (6.2)$$

$$\Delta Y_t = \alpha_Y + \sum_{i=1}^{k} \beta_{Y,i} \Delta Y_{t-i} + \sum_{i=1}^{k} \gamma_{Y,i} \Delta X_{t-i} + \phi_Y ECT_{X,Y} + \varepsilon_{Y,t} \qquad (6.3)$$

where X_t and Y_t are the VaR series examined and ΔX_t and ΔY_t is the change of VaR from time $t-1$ to time t,

$\varepsilon_{X,t}$ and $\varepsilon_{Y,t}$ are the residuals of the models.

ECT is the error correction term, which consists of the cointegrating equation which has the form:

$$ECT_{X,Y} = a_1 Y_{t-1} + a_2 X_{t-1} + c \qquad (6.4)$$

where a_1 and a_2 consist the cointegrated refered above – $a_1\ Y_t + a_2\ X_t$ is I(0).

We start with the examination if bivariate cointegration exists in the non stationary series. We follow the Engle-Granger residual approach. We first estimate a "long-run relationship":

$$Y_t = a + bX_t + \varepsilon_t \qquad (6.1)$$

by OLS. Note that if the series are cointegrated and that if $\hat{a}$ and $\hat{b}$ are good estimates of the cointegrating coefficients, then $Y_t - \hat{a} - \hat{b}X_t$ should be I(0). We continue with testing for unit root in the residuals ε_t of the model. If the residuals series prove to have a unit root, then we reject cointegration between *X* and *Y*.

In the following tables we run the biavariate regressions across the non stationary series and then we test the residuals of each model for unit root:

$CAC40_t = a + bDAX30_t + \varepsilon_t$ (6.2)				$CAC40_t = a + bNASDAQ_t + \varepsilon_t$ (6.3)				$CAC40_t = a + bSP500_t + \varepsilon_t$ (6.4)			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
b	0.834	119.836	0.000	*b*	0.423	34.435	0.000	*b*	1.557	96.379	0.000
a	-0.001	-6.379	0.000	*a*	-0.008	-18.306	0.000	*a*	0.005	18.449	0.000
ADF* for ε_t		t-Stat. -4.971	Prob. 0.000	ADF** for ε_t		t-Stat. -2.272	Prob. 0.449	ADF* for ε_t		t-Stat. -4.269	Prob. 0.004
$DAX30_t = a + bNASDAQ_t + \varepsilon_t$ (6.5)				$DAX30_t = a + bSP500_t + \varepsilon_t$ (6.6)				$NASDAQ_t = a + bSP500_t + \varepsilon_t$ (6.7)			
	Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.		Coef.	t-Stat.	Prob.
b	0.412	28.969	0.000	*b*	1.649	82.008	0.000	*b*	1.732	56.852	0.000
a	-0.011	-22.319	0.000	*a*	0.004	11.391	0.000	*a*	-0.002	-4.232	0.000
ADF** for ε_t		t-Stat. -2.252	Prob. 0.460	ADF** for ε_t		t-Stat. -3.761	Prob. 0.019	ADF* for ε_t		t-Stat. -2.732	Prob. 0.006

Table 27: Bivariate regressions between the non stationary 1 day, 95% VaR series (in levels) and test for unit root in the residuals series of the above regressions. * indicates no unit root at probability level 1% and thus accept cointegration of the level series and ** indicates existence of unit root at 1% probability level and rejection of cointegration of the levell series.

We obsere that the bivariate regression between the non stationary series produce residuals series with unit root and lead to rejection of cointegration for cases of CAC40 and NASDAQ100, DAX30 and NASDAQ100, DAX30 and SP500, while rejection of unit root and thus existence of cointegration is observed in the cases of CAC40 and DAX30, CAC40 and SP500, NASDAQ100 and SP500.

In the next step we will construct a vector error correction model (VEC). A vector error correction (VEC) model is a restricted VAR designed for use with nonstationary series that are known to be cointegrated. The VEC has cointegration relations built into the specification so that it restricts the long-run behavior of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics. The cointegration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

In order to set up a VEC model, it is first needed to specify the appropriate number of lags for the first differenced series that will be used in the model. We follow the Schwarz criterion. It suggests 4 lags for the relationship CAC40, DAX30, 3 lags for CAC40, SP500, 2 lags for NASDAQ100, SP500. Also, it is needed to specify the deterministic trend assumption of test. If we observe carefully the 95% VaR series, we detect no trend in our series, there is an intercept, however. Of course the number of cointegrating equations is 1, since we are working on a bivariate model. This fact is also verified by both trace and maximum eigenvalue tests. Finally, the new bivariate relationships between the above cointegrated series form the following Granger causality results:

Bivariate Granger Causality Tests for daily changes of 1 day, 95% VaR			
Direction of Causality	χ^2 -Stat.	Probability	lags
Δ(DAX30) ⇨ Δ(CAC40)**	11.632	0.020	4
Δ(CAC40) ⇨ Δ(DAX30)*	21.529	0.000	
Δ(SP500) ⇨ Δ(CAC40)*	22.742	0.000	4
Δ(CAC40) ⇨ Δ(SP500)*	24.771	0.000	
Δ(SP500) ⇨ Δ(NASDAQ100)*	16.875	0.000	3
Δ(NASDAQ100) ⇨ Δ(SP500)*	9.828	0.007	

Table 28: Bivariate Granger Causality tests between the daily changes (first differences) of the 1 day, 95% VaR of the various indices. The model examined is

$$\Delta X_t = \alpha_x + \sum_{i=1}^{k} \beta_{X,i} \Delta X_{t-i} + \sum_{i=1}^{k} \gamma_{X,i} \Delta Y_{t-i} + \phi_X ECT_{X,Y} + \varepsilon_{X,t}$$

$$\Delta Y_t = \alpha_Y + \sum_{i=1}^{k} \beta_{Y,i} \Delta Y_{t-i} + \sum_{i=1}^{k} \gamma_{Y,i} \Delta X_{t-i} + \phi_Y ECT_{X,Y} + \varepsilon_{Y,t}$$

where the null hypothesis is $H_0 : \gamma_1 = ... = \gamma_k = 0$ and ΔX and ΔY represent the daily changes of 1 day, 95% VaR of indices *X* and *Y* respectively and ECT is the error correction term. The *k* lags used in each model are specified by Schwarz criterion. χ^2-statistic (4.14) and the respective probability correspond to the null hypothesis, which means that *Y* does not Granger cause *X*. * indicates rejection of the null hypothesis and significant Granger causality at 1% probability level and ** indicates rejection of the null and significant Granger causality at 5% probability level.

The results obtained including the cointegration of the above non stationary series are the same with the results obtained with not taking cointegration into consideration. The only difference is the significance of causality from DAX30 to CAC40; without cointegration it is significant at 1% probability level, while with cointegration it is significant at 5% level.

Regarding the 1 day, 99% VaR series, the Augmented Dickey-Fuller and Phillips Perron tests have displayed that only one series (NASDAQ100) has a unit root. This fact creates no need for cointegration estimation, since no other series is integrated.

We continue with the examination of cointegration in a multivariate model. Initially, we examine the 1 day, 95% VaR series. Cointegration refers to the long run behaviour of the series. Although the non cointegrating series present a high degree of autocorrelation, as the correlograms of DJ INDUSTRIALS, EUROSTOXX50 and FTSE100 display, asymptotically the mean of the changes of the above VaR series is neither zero nor stable. Hence, we cannot assume that our series are stationary.

Though, these series are of crucial interest, since our objective is to indicate the spillovers across the seven indices under examination. Therefore, we search for cointegration among the four non stationary series and we set the stationary series as exogenous. In order to proceed to the cointegration test, we should also specify the number of lag intervals for the differenced series. As we mentioned before, the Scwarz criterion for the VAR of the first differenced VaR series suggests that we should use 1 lag. Also, it is needed to specify the deterministic trend assumption of test. If we observe carefully the 95% VaR series, we detect no trend in our series but an intercept. We continue with the implementation of the Johansen cointegration test across the seven 95% VaR series; both trace and maximum eigenvalue tests indicate acception of the null hypothesis of no cointegration at the 5% critical level. Thus, we have to insert no error correction equation in our model.

We should also point that according to Lütkepohl and Reimers (1992) multivariate nonstationary cointegrated systems the standard Wald or likelihood ratio tests for linear restrictions may not have the usual asymptotic χ^2 distributions. However, this problem does not arise in testing for Granger causality in bivariate VAR processes.

7 Conclusion

In our research we have coped with spillover of tail risk. We have approached tail risk through the risk measure of Value-at-Risk and performed an application on seven major European and US stock indices; CAC40, DAX30, FTSE100, EUROSTOXX50, DJ INDUSTRIALS, NASDAQ100 and S&P500. We have used as dataset the daily closing prices of each stock index for the last 15 years and we have calculated VaR with various methods; the historical simulation of the last 100 and 250 observations and the variance approach. The latter needs a variance estimation to be specified. For this reason we have estimated variance as a moving average, an exponentially weighted moving average and a GARCH model for each index. In addition, we have also introduced extreme value theory in order to capture the non-Normality of the distribution of the stock returns. Having estimated variance, we have calculated various 1 day, 95% and 99% VaR series for each index. Back-testing has followed so as to accept or reject the VaR methods produced. Since the various methods have been backtested, we have finally accepted for each index that one, which produces the minimum average VaR. The next step have been the spillovers research. In order to use the VaR series, we have first checked them for unit roots and stationarity through Augmented Dickey-Fuller and Phillips-Perron tests. Due to the fact that our series have proved to be not stationary, we have used the first differences in our following research. We have performed bivariate Granger causality tests for the daily changes of 95% VaR and 99% VaR. Causality have observed significant across various indices. The next step has been the estimation of lagged and contemporaneous relationships across the markets. This step has been performed by running lagged and contemporaneous regressions across the changes of the VaR series. Spillover effects have been verified through these techniques and the implementation of our methodology. Finally, we have taken cointegration into account and calculated again the bivariate causality across the non stationary series.

Our results are in accordance to the general fact observed by other studies on spillovers; US indices and in particular DJ INDUSTRIALS and S&P500 have the greatest effect across the indices, both contemporaneous and lead verified also by Granger causality tests. Various studies have observed that US pays a dominant role among stock markets; for example, Malliaris, Urrutia (1992) testing returns spillovers and Gemmill, Kamiyama (2000) searching for option volatility and skewness spillovers. Another interesting result is that FTSE100 and also FTSE100 plays also a significant role, since it leads many other markets. Malliaris and Urrutia (1992) have also observed spillover effects from the UK market to other markets including USA. Moreover, Aboura (2003) has found a causal relationship from the French index to US and Germany concerning the changes of the implied volatilities of each index. Finally, it is interesting to notice that there exists significant contemporaneous effect across the markets.

Comparing the two different levels of risk (95% and 99% VaR), we observe that the 95% VaR has as a result more spillover effects to be observed across the markets. On the other hand, 99% VaR is refered to the risk that is less probable to be exceeded. This level of risk has as a result the restriction of the spillover effects and causalities, but it indicates the great impact that the US indices have on the European markets. As we mentioned before, the influence of FTSE100 is verified as well and the same are the contemporaneous and regional effects.

Further research on spillovers of tail risk should focus on the Conditional Value at Risk (CVaR) that is the expected loss that would occur if the VaR was exceeded.

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