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Can the variance risk premium be predicted? Evidence from major U.S. indices

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Abstract

In this dissertation, we investigate whether the variance risk premium (VRP) in the three U.S indices, particularly S&P 500, Dow Jones and Nasdaq, can be predicted. To this end, we conduct both in-sample analysis and out-of-sample analysis. To quantify the VRP we use the realized variance and the squared index implied volatility as an approach for the variance swap rate. We find that in all three markets the average VRP is negative. A number of factors which are considered indicators of the trading activity and the stock market conditions can predict the VRP. This is confirmed by the in-sample analysis and the out-of-sample analysis.

Keywords: Variance swap, Variance risk premium, Realized variance, Index implied volatility, In-Sample analysis, Out-of-Sample analysis

Contents

"Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful"

G. Box

Introduction

Trading on volatility has increased tremendously in the past decade and it continues to become more and more popular. In 1993 the Chicago Board Options Exchange (CBOE) first introduces the VIX, an index of implied volatility of the S&P 100. In 2003 the CBOE changes the methodology of VIX calculation and therefore its definition, now the VIX is the index implied volatility of the S&P 500. CBOE also provide two other volatility indices, the VXD and the VXN, for the Dow Jones and the Nasdaq indices. This resulted in an increase of the volatility market because of the new products on the volatility, e.g. futures contracts on volatility and options on volatility. Volatility swaps and variance swaps, also, are two financial derivatives which are part of the expanding market volatility.

Volatility swaps and variance swaps are over-the-counter financial instruments which allow the investors to bet directly on volatility and on squared volatility, respectively. Variance swaps are slightly more preferred from the investors because their payoff function is convex in volatility. And in the theoretical background, again, they are more preferred because it is easier to replicate the fair value of the variance swaps (see e.g. Demeterfi et al. 1999). Variance swaps is a forward contract on realized variance. The buyer, at the expiry date of the contract, has to pay a fixed variance and she gets paid the realized variance. These are multiplied with the variance notional in order to be expressed in real money. The expectation of this payoff is the variance risk premium (VRP).

The question is, what drives the VRP and can we predict it? There are a few papers which investigate the contemporaneous relationship of different economic and market indicators with the VRP (see Carr and Wu, 2009, Bollerslev et al., 2011). However, the question we address in this dissertation is very similar with Konstantinidi and Skiadopoulos (2016). We try to answer whether the VRP of the three major US indices, S&P 500, Dow Jones and Nasdaq, can be predicted. And to do this, we first have to quantify the VRP.

Carr and Wu (2009) define the VRP as the difference between the expected realized variance under the physical measure and the expected realized variance under the risk-neutral measure. However, there are many methods proposed to quantify the VRP. In this dissertation, in line with a large number of papers (see e.g. Carr and Wu, 2006, Carr and Wu, 2009, Trolle and Schwartz, 2010, Konstantinidi and Skiadopoulos, 2016), we use the ex-post realized variance to quantify the expected realized variance under physical measure. And to approximate the expected realized variance under the risk-neutral measure we use the standard approximation in the literature (see e.g. Carr and Wu, 2006, Londono, 2011, Bollerslev et al. 2014), the implied index volatility of each stock index, respectively. In this way the VRP quantified, is an ex-post VRP.

We use this ex-post VRP to construct a parsimonious linear model with independent variables which are indicators of the trading activity and the stock market conditions. From the conditional expectation of this model, we extract the ex-ante VRP. First, we conduct in-sample analysis to establish a relationship between the predictor variables and the VRP. And secondly, we conduct an out-of-sample analysis to investigate whether the (ex-post) VRP can be predicted. For these analyses, we use daily data which span from 2001 to 2017 to construct the payoff of a variance swap contract with one month maturity.

This dissertation investigates whether is possible a parsimonious model can predict the ex-post VRP better than simple “guessing”, by using a naïve model, such as the random walk. And if it does how much better. To evaluate this, several out-of-sample statistical criteria are employed e.g. the out-of-sample R squared (Campbell and Thompson, 2008).

The possibility to predict the VRP is very important for both speculators and investors who just want to hedge their portfolios. Speculators, could just take a bet in volatility for a very short period of time with the only goal the profit. On the other hand, there are some types of portfolios which are sensitive to the volatility of the market, such as a portfolio of options. Investors who have in their possession these portfolios wish to hedge against future volatility using variance swap contracts. Thus, predicting the VRP is very important to them.

Konstantinidi and Skiadopoulos (2016) find that a number of predictor variable can explain the S&P 500 VRP, and they confirmed this by conducting both in-sample and out-of-sample analysis. The methodology and the steps we follow in this dissertation are very similar to their paper. Hence, we expect to have similar results in the three market VRPs, both when conducting the in-sample evaluation and the out-of-sample evaluation. And, as have been found generally in variance risk premium' literature, we expect that historically the variance swap payoff (ex-post VRP) to be negative for all the three markets (SPX, DJX and NDX).

Our results, indeed, confirm most of the above expectations. We find that the average VRP is negative for all the three market VRPs and statistically significant. To conduct the in-sample analysis we rely on simple ordinary least squares (OLS) regression. To compute the t-statistics we use three approaches, Newey-West, Hansen-Hodrick and White approaches. The majority of the predictor variables we use to construct the predictor model are statistically significant. This means that they have a predictor power in the VRP. The out-of-sample analysis results are that the multiple predictor models we constructed to predict the VRP outperform the naïve model in all the three market VRPs.

Literature Review: As mentioned above, this dissertation shares a lot of similarities with Konstantinidi and Skiadopoulos (2016). In that paper the authors investigate whether the S&P 500 variance risk premium can be predicted. The main differences are that they use actual swap rates instead of synthetic as we do here. Also, they analyze the ex-post VRP for different maturities and different investment horizons. The authors construct four different models to extract the VRP. These models are constructed with predictor variables related to the trading activity, stock market condition, economic conditions and the variation in the volatility of the SPX, respectively.

Another paper which this dissertation is similar, is from Carr and Wu (2009). However, they investigate the contemporaneous relationship of the VRP with the candidate variables expected to load it. They quantify the VRP using the ex-post realized variance from forward contract prices on the underlying asset and to approach the swap rate they use a portfolio of options similar to the VIX. Their analysis run with data from 1996 to 2003 and on a large range of stocks and indices. Among others, they find that neither the market excess returns (which is approached by SPX returns) or the Fama and French factors cannot explain the VRP.

There are plenty of papers that analyze the VRP in the literature, not whether it is predictable or not but from different interesting aspects.

For example, Ait-Sahalia et al. use a parametric stochastic volatility model and actual swap rates to investigate the term structure of the ex-ante VRP on the S&P 500. They find that the term structure of the VRP has a downward sloping. Using different variables in regression analysis they find that this downward slope is mainly a result of the investors' fear for downward jumps in the market.

Of course, the literature on VRP expands more than just the VRP on indices and stocks. They are a number of papers which analyze the VRP on bonds, energy commodities etc.

Trolle and Schwartz (2010) investigate the VRP on the crude oil and natural gas. The authors quantify the VRP calculating the ex-post realized variance on the commodities by using forward on the underlying assets. And to approximate the variance swap rate they use a modified formula which is very similar to the VIX. Their study is based on data which span from 1996 to 2006. Their main findings are that average VRP on both crude oil and natural gas are negative and statistically significant.

Choi et al. (2017) study the variance risk premium in the Treasury market. They approximate the variance swap rate using a portfolio of put and call options similar to the VIX. They calculate the realized variance, in line with Bondarenko (2014), using a mix formula of simple and log returns from treasury future contracts. The authors conclude to some interesting findings. They find that the excess returns on variance swaps are higher than the returns investing in straddle strategies, which is considered a typical investment on volatility. Using standard risk factors variables, they try to explain these returns. The results of their analysis were that they cannot. They also find that the bond variance risk premiums are negative, this last finding corresponds with the other similar findings in equity and commodities market.

The remainder of the dissertation is organized as follows. Section 1 is entirely dedicated to the variance risk premium. In this section we analyze the VRP, provide a brief analysis of the historical VRP and describe the methodology we use to quantify it. In Section 2 we provide the theoretical background of the predictor variables which is expected to drive the VRP. We explain why and how they could drive the VRP. Section 3 describes these variables, how are constructed, provides summary statistics, time series plots etc. In section 4 and 5, respectively, the in-sample and the out-of-sample analysis are conducted. In these sections, also, the results of the analysis are provided, followed by some short comments. In the Section 6 results of further robustness tests are discussed. And the last section concludes.

1. Variance swap and Variance Risk Premium

1.1. Variance Swap

A swap contract is an agreement between two parties to exchange future cash flows. Swap contracts were first introduced in the early 1980 and since then the market has grown tremendously comparing other derivatives. This is because, in a swap, principal payments are not exchanged so the risk has the investors to deal with is lower.

A variance swap is an instrument which allows investors to trade future realized volatility against current implied volatility (Bossu et al., 2005). The buyer of the swap agrees to pay the strike price of the swap, the variance swap rate and the seller agrees to pay the realized variance over the life of the contract. At maturity the payoff for the long party is:

$$\{RV_{t\to t+T} - VS_{t\to t+T}\}\, N, \qquad (1.1)$$

where $RV_{t\to t+T}$ is the realized annualized variance from time t to maturity T, $VS_{t\to t+T}$ is the variance swap rate and N is the notional amount of dollars which converts the difference between two variances to dollar payoff. If this difference is negative at maturity, the buyer has to pay the seller the difference multiplied by N amount of dollars, if it is positive the seller has to pay the buyer the difference multiplied by N amount of dollars.

The final realized variance ($RV_{t\to t+T}$) at time T is defined by the above equation:

$$RV_{t\to t+T} = \frac{252}{T} * \sum_{i=1}^{T} ln\left(\frac{S_{t+i}}{S_{t+i-1}}\right)^2, \tag{1.2}$$

where S_t is the closing price of the underlying at time t. Note that the average return which usually appears in statistics textbooks is missing from the formula. The reason is because the average daily return is close to zero, thus its impact is insignificant. Furthermore, its exclusion has the benefit of making the final realized variance additive.

Volatility swaps are very similar products to the variance swaps. They both trade volatility and in both cases, at the inception of the trade, the strike price is usually chosen such the fair value of the swap is equal to zero. However, there is a basic difference between them, volatility swaps use the square root of the realized variance. Thus, the payoff of a variance swap is convex to volatility and the volatility swaps linear to volatility as illustrated Figure 1. This means that an investor in a long position in a variance swap has more profit for an increase in volatility that an investor in volatility swap. Therefore, the strike price for the variance swap is slightly higher than the volatility swap.

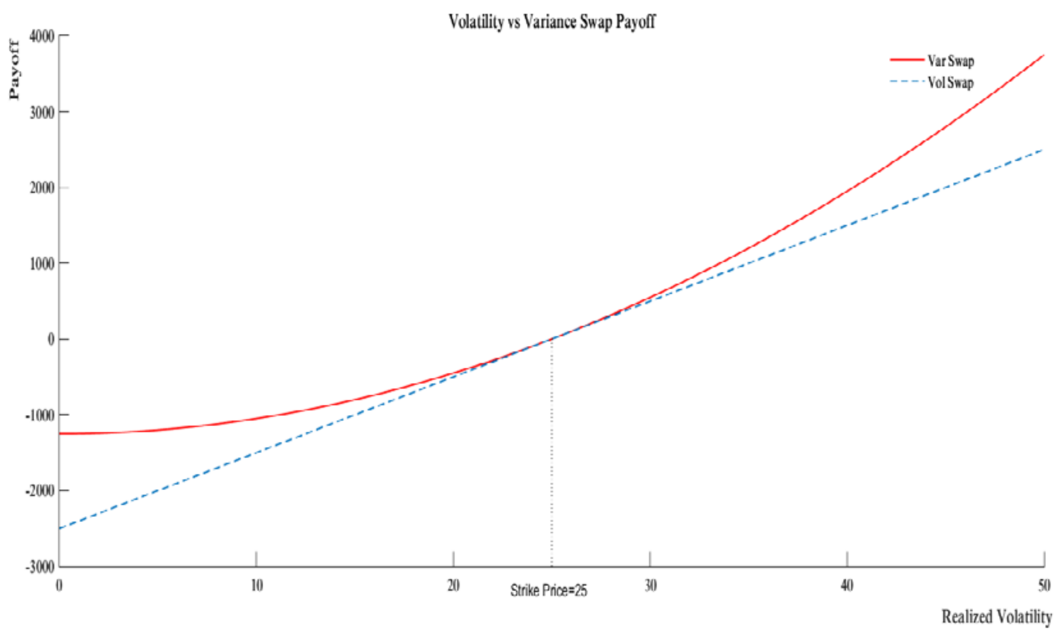

Figure 1: The payoff function plots of a volatility swap and a variance swap for the same strike price (K=25).

The market definitions for the payoffs of the volatility and variance swaps are:

$$Vol\frac{P}{L} = (Realized\ Volatility - Strike\ Price) \times Vega\ Notional, \tag{1.3}$$

$$Var\frac{P}{L} = (Realized\ Volatility^2 - Strike\ price^2) \times Var\ Notional. \tag{1.4}$$

The variance notional is usually expressed in volatility terms and is defined as:

$$Variance\ Notional = \frac{Vega\ Notional}{2 \times Strike\ Price}. \tag{1.5}$$

The definition in equation (1.4) is similar to the one in equation (1.1). In the case of equation (1.4) realized variance and swap rate are multiplied with 10000 in order to give as a result a whole number. For example, an investor enters in a long position with 100$ in volatility swap with a strike price K=25 and in a long position in a variance swap with a variance notional of 2$. If final realized volatility is 26, the payoff from the volatility swap is 100$ and from variance swap is $(26^2–25^2) \times 2=$ 102$. If the final realized volatility is 24, the investor has to pay 100$ for the volatility swap and 98$ for the variance swap.

As the interest rate swaps, variance swaps initially worth close to zero. Thus, in absence of arbitrage opportunities the variance swap rate is equal to the risk-neutral expectation of the realized variance at time T,

$$VS_{t\to t+T} = E_t^Q(RV_{t\to t+T}). \tag{1.6}$$

1.2. Variance Risk Premium

The variance risk premium is usually defined as:

$$VRP_{t\to t+T} = E_t^P(RV_{t\to t+22}) - E_t^Q(RV_{t\to t+T}), \qquad (1.7)$$

where $E_t^P(RV_{t\to t+22})$ and $E_t^Q(RV_{t\to t+T})$ are the physical and risk-neutral expectations of the realized variance, respectively. However, quantifying the VRP is not so obvious. Several methodologies are proposed in the literature. Since the variance swaps are out-of-the-country financial instruments and there are not historical data available these methodologies concern both the expected realized variance and the swap rate.

The monthly true realized variance is something which practically is impossible to observe, let alone the forecasting. Nevertheless, there are many econometric approaches in the literature on realized variance forecasting. Generally, the simplest way to forecast volatility is by using GARCH models or historical volatility. Of course, more complicated models are proposed in the literature. Bollerslev, Marrone, Xu and Zhou (2014) use bivariate VAR (1)-GARCH (1,1)-DCC model, Bekaert and Hoerova (2014) use a model which include several independent variables. In this way, the quantified VRP is an ex-ante VRP.

There are other methodologies proposed in the literature to quantify the VRP. A number of papers, Carr and Wu (2006), Trolle and Schwartz (2010), Konstantinidi and Skiadopoulos (2016) etc. calculate the realized variance as described above as the total sum of daily log returns from time t to t+22. In this way the realized variance computed is the ex-post realized variance and the VRP is an ex-post VRP. In this dissertation the ex-post VRP is used as a method to quantify the VRP. The equation employed to calculate the ex-post realized variance is the equation (1.2) as described in the previous section with T=22.

The second component which is needed to quantify the VRP is the variance swap rate. Konstantinidi and Skiadopoulos (2016) use actual variance swap quotes written on the S&P 500 But since, as mentioned above, variance swaps are over-the-counter financial products, quotes on variance swap rates are not publicly available. Thus, several methodologies have been proposed in the bibliography to approximate them. Carr and Wu (2009) use the value of a particular portfolio of options to approximate the variance swap rate. A considerable number of papers use the index implied volatility as an approach of the variance swap rate, more specifically they use VIX which is the index volatility of the S&P 500, e.g. Carr and Wu (2006), Trolle and Schwartz (2010), Bekaert and Hoerova (2014), Mueller, Sabtchevsky, Vedolin and Whelan (2016) etc. This is because the VIX squared approximates the conditional risk-neutral expectation of the realized thirty calendar day S&P 500 index variance. Londono (2011) use different index volatilities to quantify different index VRPs, e.g. VDAX index volatility for the DAX index, VXJ for the Nikkei 225 index, VFTSE for the FTSE etc. In this dissertation, a similar method is used. For the three stock indices S&P 500 (SPX), Dow Jones Industrial Average (DJX) and Nasdaq (NDX) the three respective volatility indices VIX, VXD, and VXN are used. The next section is entirely dedicated to VIX, which is the oldest and the most important index volatility of the three. We explain why it is considered a good approximation of the variance swap rate and we mention the main errors of this approximation.

1.3. The VIX

In 1993, the Chicago Board Options Exchange introduced the CBOE Volatility Index (VIX) which measured the 30-day implied volatility from at-the-money options of the S&P 100. In 2003 CBOE and Goldman Sachs updated the VIX. Now the new VIX is based on the S&P 500 index and estimates the expected volatility by averaging the weighted prices of S&P 500 puts and calls over a large range of strike prices and at two nearby maturities. CBOE used this methodology and back-calculated the VIX since 1990 using historical options price.

The CBOE Dow Jones Industrial Average volatility index (VXD) and CBOE Nasdaq-100 volatility index (VXN) were introduced at 1997 and 2001 respectively. The method to calculate the VXD and VXN is identical as the method used to calculate VIX with the only exception that

CBOE includes only “standard” option series in the calculation. As the new VIX, these two indices are back-calculated with the new methodology. The formula used to calculate the VIX, which as mentioned above is the same for VXD and VXN too, as provided from CBOE is as follows:

$$\sigma^2 = \frac{2}{T}\sum_{i=1}^{T}\frac{\Delta K_i}{K_i^2}\, e^{RT}\, Q(K_i) - 1/T\left[\frac{F}{K_0} - 1\right]^2, \tag{1.8}$$

where σ×100 is the VIX, T is time to expiration, F is the forward index level desired from index option price, K_0 is the first strike below the forward index level, K_i is the strike price of the i-th out-of-the-money option; a call if $K_i > K_0$; and a put if $K_i < K_0$; both put and call if $K_i = K_0$,

ΔK_i denotes the interval between strike prices-half the difference between the strike on either side of K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}, \tag{1.9}$$

R denote the risk-free interest rate expiration and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i.

At this point, it is crucial to clarify the difference between the historical volatility and the implied volatility. The historical volatility can be approached from the historical observations, e.g. the realized volatility (annualized) from day t-22 to t of S&P 500 can be approached by the sum of squared daily log returns:

$$RV_{t-22\to t} = \frac{252}{22}\sum_{i=1}^{22} ln\left(\frac{S_{t-i}}{S_{t-i-1}}\right)^2. \tag{1.10}$$

On the other hand, the implied volatility is something we can’t measure directly but can estimate it from the options’ price. The three volatility indices we use, VIX, VXD and VXN estimate the implied volatility of the SPX, DJX and NDX respectively.

The variance swap rate is the fair value of future realized variance, a concept first introduced from Demeterfi et al. (1999). They argue that the variance swap rate is equal to the expected realized variance from day 0 to the end of the contract in a risk-neutral world as follows:

$$VS = \frac{1}{T} E^{\mathbb{Q}}\left[\int_0^T \sigma^2(t, \dots) dt]\right]. \quad (1.11)$$

The authors demonstrate that the VS can be computed from the equation below:

$$VS = \frac{2}{T}\left\{rT - \left[\frac{S_0}{S_*} exp(rT) - 1\right] - ln\left(\frac{S_*}{S_0}\right)\right. \quad (1.12)$$

$$\left. + exp(rT)\int_0^{S_*} \frac{P(T,K)}{K^2} dK + exp(rT)\int_{S_*}^{\infty} \frac{C(T,K)}{K^2} dK\right\}.$$

Jiang and Tian (2007) demonstrate that the concept of the fair value as defined above and the model-free implied variance formulated by Britten-Jones and Neuberger are identical concepts. Britten-Jones and Neuberger found that implied volatility can be calculated not from option pricing model (e.g. BSM model) but using a set of options with the same maturity from the equation modified by Jiang and Tian (2007) as follows:

$$E^{\mathbb{Q}}(RV_{BN}) = \frac{2\,exp(rT)}{T}\left[\int_0^{F_0} \frac{P(T,K)}{K^2}dK + \int_{F_0}^{\infty} \frac{C(T,K)}{K^2}dK\ \right], \qquad (1.13)$$

where RV_{BN} is the realized variance from time 0 to T and F_0 is the forward price with maturity equal with that of the options. The fact that these two different methods of measuring the future realized volatility are similar is very important. Firstly, because it shown that the fair value of variance and the implied squared volatility are equivalent concepts and the implied volatility is a concept very familiar in the literature. Secondly, it explains why the VIX, except that it can be seen as the variance swap rate (squared VIX), it known as the 'investors' fear gauge". The proof on why the equation (1.13) is equivalent with the equation (1.12) is provided in the Appendix B.

As explained above the CBOE calculate the squared VIX using the equation (1.8) which is the discretization of the equation (1.12). In line with Jiang and Tian (2007), we discuss the approximations and the problems which occur from the CBOE formula. The equation (1.12) from Demeterfi et al. (1999) it assumes that there are strike prices (K) from 0 to infinity, something which of course it doesn't stand in the real world. Thus, the CBOE approximate the K=0 with the lower strike price (K_L) and the infinity with the higher strike price K_H. So, the sum of the two integrals from zero to infinity becomes:

$$\int_0^{K_0} \frac{P(T,K)}{K^2}dK + \int_{K_0}^{\infty} \frac{C(T,K)}{K^2}dK\ \approx \int_{K_L}^{K_0} \frac{P(T,K)}{K^2}dK + \int_{K_0}^{K_U} \frac{C(T,K)}{K^2}dK. \qquad (1.14)$$

This results in a truncation error which is defined as follows:

$$\varepsilon_{trunc} = -\frac{2}{T}e^{rT}\left[\int_0^{K_L} \frac{P(T,K)}{K^2}dK + \int_{K_U}^{\infty_0} \frac{C(T,K)}{K^2}dK\ \right] \qquad (1.15)$$

The CBOE uses the following expression to approximate the two integrals from equation (1.14):

$$\int_{K_L}^{K_0} \frac{P(T,K)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(T,K)}{K^2} dK \approx \sum_{i=1}^{T} \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i). \tag{1.16}$$

This has as a result a discretization error which is defined as:

$$\varepsilon_{disc} = \sum_{i=1}^{T} \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \int_{K_L}^{K_0} \frac{P(T,K)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(T,K)}{K^2} dK. \tag{1.17}$$

The CBOE approximates the term which precedes the two integrals in equation (1.12), using a Taylor expansion, as follows:

$$\frac{2}{T} \{ rT - \left[\frac{S_0}{S_*} exp(rT) - 1\right] - ln\left(\frac{S_*}{S_0}\right) \approx 1/T \left[\frac{F}{K_0} - 1\right]^2. \tag{1.18}$$

The error from the approximation with this expansion is defined as follows:

$$\varepsilon_{expan} = \frac{2}{T}\left\{ \left[\left(\frac{F_0}{K_0} - 1\right) - \frac{1}{2}\left(\frac{F_0}{K_0} - 1\right)^2 \right] - ln\left(\frac{F_0}{K_0}\right) \right\}. \tag{1.19}$$

The last approximation which CBOE use in its formula is about the maturity of the options. The VIX index approximates the model-free implied variance for 30 days maturity, this requires at any point in time the existence of options which expire exactly in 30 days. Thus, CBOE calculates the variance from options which expire in less than 30 days but is the closest maturity to the 30 days and also, calculate the variance from options with more than 30 days maturity but have the closest maturity to the 30 days. The VIX provided at the end from the CBOE is a result of the follow interpolation formula:

$$VIX_{30}^2 = \frac{1}{30}\ [\omega T_1 VIX^2(T_1) + (1-\omega)T_2 VIX^2(T_2)], \qquad (1.20)$$

where

$$\omega = \frac{T_2 - 30}{T_2 - T_1}, \qquad (1.21)$$

T_2 is the closest maturity more than 30 days and T_1 is the closest maturity less than 30 days. The error which results from this interpolation process is defined as:

$$\varepsilon_{interp} = \widehat{VIX}_{30}^2 - VIX_{30}^2, \qquad (1.22)$$

where $\widehat{VIX}_{30}^2$ is the estimated variance from the interpolation and VIX_{30}^2 is the "true" variance if could be calculated with no interpolation. This error occurs due to the fact that the model-free implied variance is not a linear function of maturity.

Another issue in the VIX formula is that assume no jumps in the underlying asset. The VIX's formula is based in equation (1.12) from Demeterfi et al. which assumes that the underlying asset follow an Ito's process. In presence of jumps the approximation of the risk-neutral expected volatility from the VIX results in another error, the error of jumps. Thus, taking into account that the best formula of VIX is the continuous one, the risk-neutral expectation of the realized variance if we allow jumps to the underlying assets becomes:

$$E^{\mathbb{Q}}(RV) = \frac{2\,exp(rT)}{T}\left[\int_0^{F_0} \frac{P(T,K)}{K^2}dK + \int_{F_0}^{\infty_0} \frac{C(T,K)}{K^2}dK \quad\right] + \varepsilon \implies \tag{1.23}$$

$$E^{\mathbb{Q}}(RV) = VIX^2 + \varepsilon, \tag{1.24}$$

where ε is the error of jumps and it zero if the process of the underlying is purely continuous. This error ε is defined (Carr and Wu, 2006) as:

$$\varepsilon = -\frac{2}{T-t}E^{\mathbb{Q}}\int_t^T \int_{\mathbb{R}^0} \left[e^x - 1 - x - \frac{x^2}{2}\right] v_s(x)dxds, \tag{1.25}$$

for more details on equation 1.25 see Carr and Wu (2006 and 2009).

Martin (2013) presents a new index to measure the risk-neutral expectation of the future realized variance, the SVIX. He argues that if in a swap contract the realized variance is defined be simple returns and not by the log returns then the strike price of the variance swap can be perfectly replicated by the SVIX. This new index corrects the problem of jumps and is provided in the equation below:

$$E^{\mathbb{Q}}(RV) = SVIX^2 = \frac{2e^{rT}}{T}\left[\int_0^{F_T} \frac{1}{S_0^2}\;P(K)dK + \int_{F_T}^{\infty} \frac{1}{S_0^2}\;C(K)dK\right], \tag{1.26}$$

where realized variance (RV) is the sum of simple returns and not the log returns. However, in short time horizon Martin argue that VIX^2 is an accurate approximation of the $SVIX^2$ hence the VIX^2 could be considered an accurate approximation of the variance swap rate.

Bondarenko (2014), use another method to deal with the jump error. He also changes the method of measuring the realized variance using the the following equation:

$$\widetilde{RV} = 2\sum_{i=1}^{n}\left(\frac{S_i}{S_{i-1}} - 1 - ln\frac{S_i}{S_{i-1}}\right). \tag{1.27}$$

Choi, Mueller and Vedolin (2017) use this approach to construct the payoff of variance swaps on Treasury Bonds. They find that this new measure of realized variance can be perfectly replicated for every price path and every partition. Thus, the strike price of the variance swap rate (VS) is defined as:

$$VS = E^{\mathbb{Q}}\left[\widetilde{RV}\right] = \frac{2\,exp(rT)}{T}\left[\int_0^{F_0}\frac{P(T,K)}{K^2}dK + \int_{F_0}^{\infty}\frac{C(T,K)}{K^2}dK\right]. \tag{1.28}$$

In the empirical ground, Carr and Wu (2009) use a very similar formula with the VIX formula to approximate the variance swap rates for different stock indices and stocks (e.g. SPX, NDX, Microsoft, Intel etc.) for 30 days maturity. Konstantinidi and Skiadopoulos (2016) compare this method with the actual VS with different maturities. They find that there is a statistically significant difference between the mean of the variance swaps' payoff calculated with Carr and Wu and the mean from the actual VS, this is for two months and above investment horizon. For one-month time horizon, they found that this difference is not statistically significant. Ait-Sahalia, Karaman and Mancini (2015) use actual VS with maturity 2, 3 and 6 months to compare them with the respective approximation from the VIX method. They find that the squared VIX and the actual VS have similar term structure characteristics qualitatively, but they sample statistics (mean, standard deviation etc.) are different. However, these differences are larger for the longer maturity (6 months) and become smaller for the shorter maturity (2 months).

Encouraged from the above results and the fact that the VIX formula is the most used in the industry to approximate the risk-neutral expectation of the volatility and, also, it is very easy to obtain data on it and the other two indices, we rely on the squared VIX as the approximation of the risk-free variance. And because the way the other two indices (VXD, VXN) are calculated are almost identical to the VIX, we employ these two indices as approximators of their respective VS.

Thus, the equation 1.7 becomes:

$$VRP_{t\to t+T} = E_t^P(RV_{t\to t+22}) - IV_t \qquad (1.29)$$

1.4. Historical Behavior of the three markets' VRP

At this point it is important to analyze the historical behavior of the VRPs. For this analyze all data available for the three VRPs from 2001 to 2017 is used. Summary statistics for all the three VRPs are provide in Table 1.1. They share a lot of similarities with previous literature. Historical average is negative statistically significant for the three VRPs. This indicates that investors who enter in a long position on variance swap contract are willing to accept a significantly negative average payoff. Accordingly, shorting variance swap contracts on these indices generate average positive payoffs. Of course, these positive payoffs do not come at zero risk, this is because generally there is a high volatility on VRPs. This explain and the fact that among the three VRPs we are investigating the highest payoff (for the shorting part) is from index VRP with the higher volatility (standard deviation) and the lower payoff come from the index VRP with the lower volatility. The distributions of VRPs exhibit heavy tails for all the three indices with kurtosis of S&P 500 VRP being the largest (62.45). There is a lack of symmetry in all the VRPs, they display positive skewness with S&P 500 having the highest. Using the Std. Deviation as an estimation of volatility it seems that the Nasdaq VRP has the higher volatility among the three VRPs. Also, there is a high pairwise correlation among the three VRPs with the correlation between SPX VRP and DJX VRP being close to 1.

Table 1.1: Summary statistics for the variance risk premiums

		SPX VRP	DJX VRP	NDX VRP
Mean		−0.0104	−0.0085	−0.0258
Max		0.653	0.561	0.590
Min		−0.324	−0.304	−0.374
Std. Dev		0.0494	0.0426	0.0618
Skew		5.64	5.26	1.38
Kurt		61.71	58.10	23.96
t-stat (NW)		−3.32	−3.12	−6.42
t-stat (HH)		−2.61	−2.45	−4.92
t-stat (W)		−13.54	−12.85	−26.89
95% CI	**Lower**	−0.0165	−0.0138	−0.0337
	Upper	−0.0043	−0.0032	−0.0179

Mean, Max, Min, Std. Dev, Skew, Kurt, t-stat and 95% CI report sample average, sample maximum and minimum values, sample standard deviation, sample skewness, sample kurtosis, t-statistics of the mean VRPs and 95% CI the 95% confidence interval of the mean of VRPs. NW, HH t-stat are the t-statistics calculated with Newey-West and Hansen-Hodrick approach, respectively. W t-stat stands for the White robustness t-statistics calculated from the OLS. Standard errors to construct 95% CI are computed with Newey-West(NW) approach using a lag-length equal to 22 trading day. Data to calculate these statistics span from 2001 to 2017.

Table 1.2: VRPs Correlation Matrix

	SPX VRP	DJX VRP	NDX VRP
SPX VRP	1	0.981	0.772
DJX VRP		1	0.745
NDX VRP			1

Table 1.2 displays the pairwise correlation between the three markets VRP for the period 2001-2017.

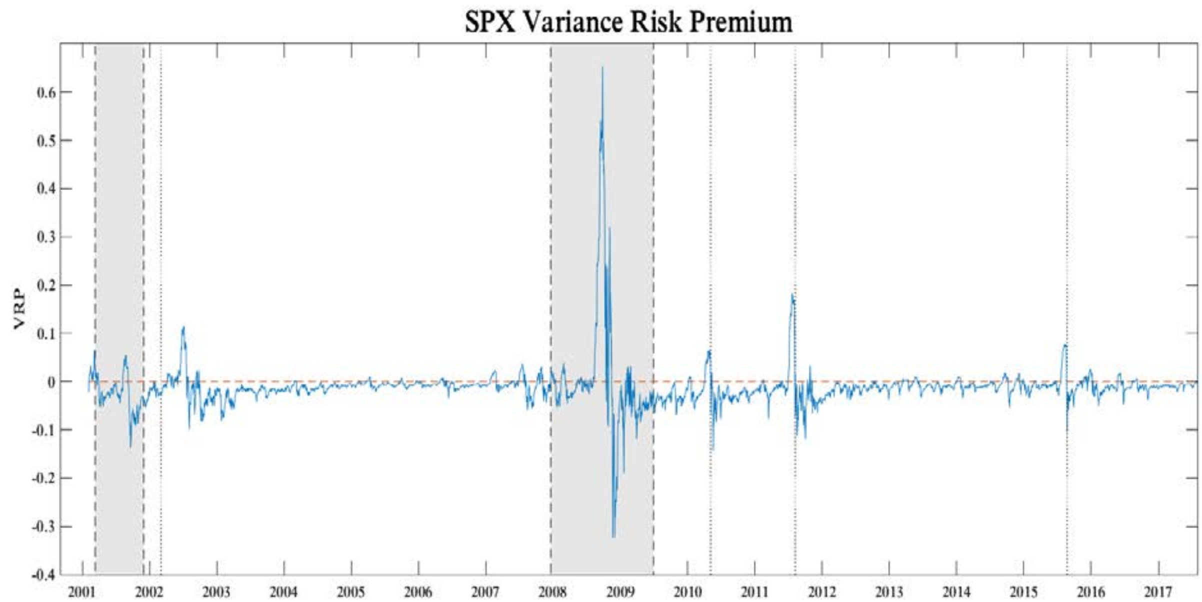

Figure 2: This figure plots the ex-post SPX VRP, which is defined as the difference between the ex-post realized SPX returns for 22 trading days variance and the squared VIX, from 2001 to 2017. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015.

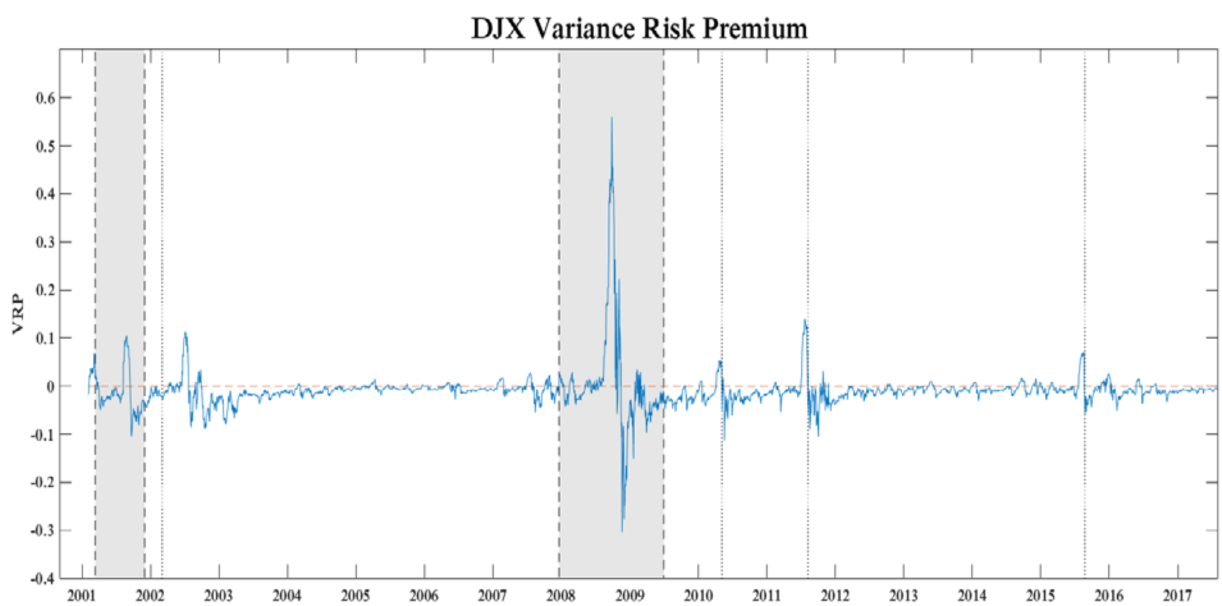

Figure 3: This figure plots the ex-post DJX VRP, which is defined as the difference between the ex-post realized variance of DJX returns for 22 trading days and the squared VXD, from 2001 to 2017.

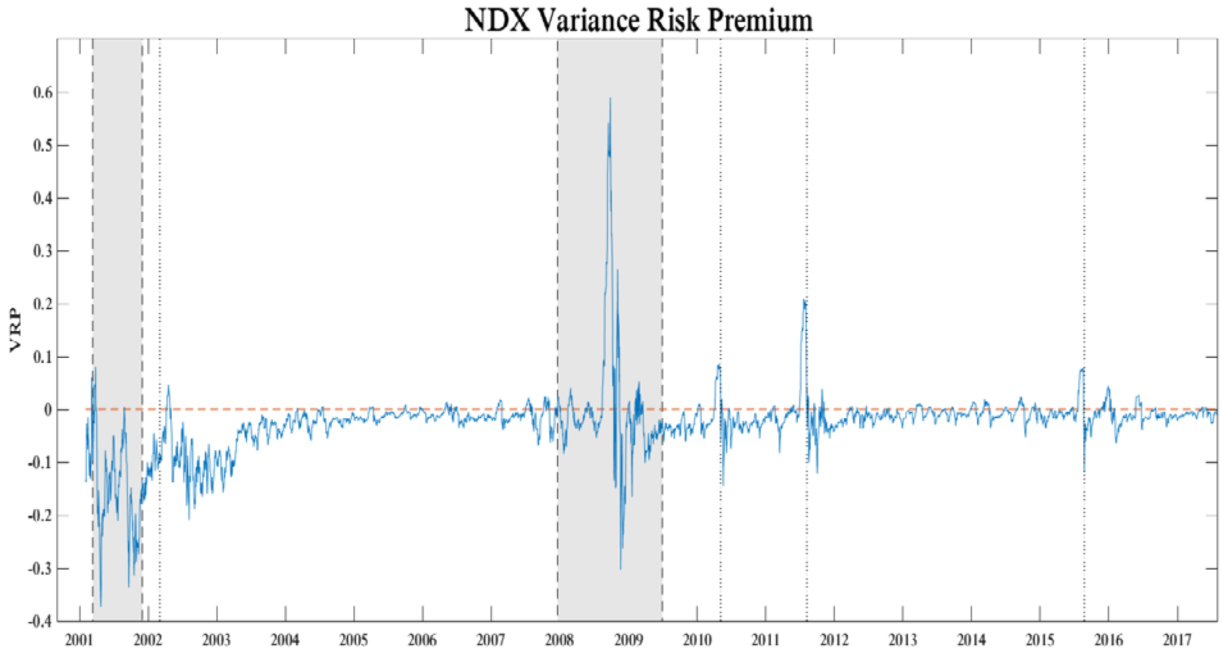

Figure 4: This figure plots the ex-post NDX VRP, which is defined as the difference between the ex-post realized variance of NDX returns for 22 trading days and the squared VXN, from 2001 to 2017.

Figures 2,3 and 4 display the variance risk premiums for the S&P 500, Dow Jones and Nasdaq from February 2001 to July 2017 employing the following equation.

$$VRP^{j}_{t\to t+22} = RV^{j}_{t\to t+22} - IV_t^{2(j)}, \quad (1.30)$$

where $\mathrm{IV}_t^{2(j)}$ stands for index volatility closing price at day t and j=SPX, DJX and NDX.

The three plots provided in the figures above confirm that variance risk premiums are stationary time series. From SPX VRP and DJX VRP plot one can easily confirm the high correlation, close to 1, between them. From 2001 to 2017 the two plots follow almost identical paths and all the three time series are very sensitive to financial and economic shocks.

At this point it is interesting to discuss about the three time series of the three variance risk premiums dividing them in subperiods. First one, is the first shaded area, the Early 2000s Recession, according to National Bureau of Economic Research the US economy was in recession from March 2001 to November 2001. The second one, is from January 2003 to November 2007, this was a quiet period for the stock market without any important financial or economic shock. The third subperiod is the Great Recession which it started about November 2007 and it ended about June 2009. Fourth one is from 6 May 2010 to December 2011, in this period two important financial events happened, Flash Crash (6/5/2010) and Black Monday (8/8/2011). The 2010 Flash Crash is referred to stock market crash in the US which lasted for a very short time and after that the stock market rebounds immediately. The 2011 Black Monday is referred to US stock market crash which followed the credit rating downgrade of the US sovereign debt from AAA (risk-free) to AA+. This day DJX made the 6th largest drop in his history and that was only one trading day after (4/8/2011) the 10th largest drop of the DJX. The last subperiod is from January 2012 to July 2017. Statistics for these periods are provided in the table below:

Table 1.3: Summary statistics for the VRPs in each subperiod

		3/2001-11/2001	***1/2003-11/2007***	***11/2007-6/2009***	***5/2010-12/2011***	***1/2012-8/2017***
SPX	*Mean*	-0.0294	-0.0110	0.0095	-0.0188	-0.0093
	Std. Dev	0.0360	0.0132	0.1367	0.0450	0.0139
DJX	*Mean*	-0.0184	-0.0097	0.0083	-0.0145	-0.0079
	Std. Dev	0.0436	0.0130	0.1161	0.0349	0.0121
NDX	*Mean*	-0.1566	-0.0231	0.0023	-0.0143	-0.0092
	Std. Dev	0.0890	0.0255	0.1216	0.0490	0.0159

Table 1.3 displays the sample mean and the sample standard deviation of the three market VRPs for the five different time periods: 3/2001-11/2001, 1/2003-11/2007, 11/2007-6/2009, 5/2010-12/2011 and 5/2010-12/2011

There are a few things worth to point out from the above analysis. Average VRPs become more negative during the Early 2000s Recession as expected but during the Great Regression of 2007-2009 the average VRPs are inexplicably positive. The reason(s) why is (are) very interesting to investigate but it is not the aim of this dissertation. Also, the average VRPs become more negative during the period 5/2011-12/2011 when the two financial shocks, Flash Crash and Black Monday happened. This again confirm the theory that the investors which enter in a long position on swap variance contract are willing to pay a negative payoff in order to hedge against high volatility during periods of regressions and financial shocks.

On the other hand, the high volatility in the two recession periods and during the period of two financial shocks mentioned above remind that positive payoffs for the short party in a variance swap contract do not come without any risk. During the Early 2000s Recession volatility on the three VRPs was much higher than the volatility during the period 1/2003-11/2007. The latter, of course, was a quiet period from financial and economic shocks. Thus, at this period the positive payoffs for the short party were much lower comparing to the period during the Early 2000s Recession. Similar results appear and for the period 5/2010-12/2011 compare to the period 1/2012-8/2017. More negative VRPs, which means positive payoffs for the short party, during the 5/2/2010-12/2011 than the period 1/2012-8/2017 and respectively higher volatility for the former compare to the latter. During the Great Recession the volatilities were very high for all the three indices which can be interpreted as high risk for the short party. This risk, converted in real losses after the positive average for the three VRPs. Which it is in contrast with negative averages of all subperiods discussed above and the negative historical average for all period from 2001 to 2017. This means that during this short period, parties which were short in variance swaps contracts had negative payoffs.

2. *Theoretical Background*

We want to investigate if the SPX VRP, DJX VRP and NDX VRP can be predicted. To do this, in line with Konstantinidi and Skiadopoulos (2016), we calculate the VRP as the conditional expectation of the profit and loss from a long position in a variance swap after 22 trading days:

$$VRP_{t\to t+22} = E_t^P[P\&L_{t\to t+22}] \qquad (2.1)$$

Next, we consider a number of variables which have been found that can drive the VRP in order to construct the following model:

$$E_t^P[P\&L_{t\to t+22}] = \beta_0 + \sum_{i=1}^{n} \beta_i x_{it}, \qquad (2.2)$$

where x_{it} stands for the i variable, β_i is the coefficient of the i variable and β_0 is the constant of the model. Note that we use the variables at time t to predict the ex-post VRP which is the $P\&L_{t\to t+22}$.

The variables we investigate to have a predictive power in the VRP are (i) the TED spread, (ii) the index return, (iii) the credit spread, (iv) the term spread, (v) the put-call ratio and (iv) the squared index volatility. Following a similar approach with Konstantinidi and Skiadopoulos (2016). We separate these variables into two groups. In the first one, are the variables related to the trading activity and in the second one, are the variables related to the stock market conditions.

2.1. Trading Activity

We consider the TED spread, the credit spread and the term spread as indicators of trading activity of the broker-dealers.

Broker-dealers when shorting a variance swap have some cost which could occur in two ways. Firstly, when they short a variance swap they should cover this position as they do when they sell a put/call option. To do so, they have to buy a portfolio of options which (not perfectly) replicate the payoff of a variance swap. And secondly, even if they not wish to cover this position the Net Capital Rule obligate them to have enough liquidity to cover their obligations. Hence, they have to borrow these funds in order to have the liquidity required by the US Securities and Exchange Commission.

The TED spread is considered as an indicator of investors' funding liquidity. That is, as the TED spread increases the lower funding liquidity, hence more difficult for the investor to keep funding her trading activity. We expect a negative relationship between the TED spread and the VRP, as the TED increases the VRP become more negative. This is because, as Gârleanu, Pedersen and Poteshman (2009) argue, broker-dealers are short in index options. Bakshi and Kapadia (2003) note that long positions in options are hedges against significant market declines. The authors also indicate that increased realized volatility coincides with a downside in market asset prices. Thus, the long side in variance swap are willing to pay a higher VRP in order to hedge against downward moves in the market. The broker-dealers who are short in options receive the VRP as a compensation to continue to keep this position (Konstantinidi and Skiadopoulos, 2016). But when broker-dealers have difficulties raising funds, it becomes harder for them to enter in a short position in variance swap contract. Thus, the investors which want to hedge by entering in a long position in a variance swap have to pay a greater VRP in order to make the broker-dealers enter in a short position.

We expect a similar negative relationship between the credit spread and the VRP. This is because the credit spread as an economic indicator not just follows the general state of the economy but can gauge also the sentiment of the market participants for the future of the economy, hence can forecast the future economic activity (see e.g. Gomes and Schmid, 2010). As the credit spread increases borrowing funds will be more expensive for the broker-dealers hence, lower the funding liquidity. Thus, similar in the case of the TED spread the broker-dealers which are short in variance swaps will ask a higher VRP and the buyer must offer a higher VRP to convince them to stay in a short position.

The idea of using the term spread (TS) as a predictor variable is similar with the TED spread and the credit spread. We expect a positive relationship between the term spread and the VRP, as the TS decreases the VRP become more negative. There are plenty of literature which empirically confirm that the slope of the yield curve can predict the future state of the economy (see Estrella and Hardouvelis, 1991). Franz (2013) states "*Examples for global risk factors are the TED spread or the term spread which can both be interpreted as indicators for the health of the economy and indicators for the expected performance of financial markets*". As the term spread decreases (slope of the yield curve become flatter or negative) again, borrowing funds for the broker dealers become more expensive and funding liquidity decreases. And as mentioned above, it will be more difficult for the broker-dealers to enter in a short position in variance swap hence, they will demand a higher VRP and the buyers will offer a higher VRP.

2.2. Stock Market Conditions

We consider the squared index volatility as a predictor variable of the VRP and we expect a negative relationship between them. Whaley (2008) argue that the VIX express mainly the fear of the investors for a downside in stock price. Also, Whaley (2000) found both a negative correlation and an asymmetric relationship between the VIX and the SPX returns. He found that an increase in the VIX will be followed by negative returns in the SPX, which will be higher in absolute value than the positive return followed by a decrease in VIX. In the case of the negative returns, the future realized volatility will be high. Thus, as the index volatility increases the investors are willing to pay a higher VRP to get advantage of this future high volatility. We consider in this analysis that a similar connection exists, also, between the other two stock indices and their respective index volatilities.

Note that the squared index volatility already appears in the calculation of the (ex-post) VRP as an approach to the variance swap rate. Thus, the general predictor model for each index VRP is:

$$VRP_{t\to t+22} = \beta_0 + \beta_1 IV_t^2 + \sum_{i=2}^{6} \beta_i x_{it} + \varepsilon_{t\to t+22}, \tag{2.3}$$

where $VRP_{t\to t+22}$ is the ex-post variance risk premium for each index as described in the previous section and it is defined as:

$$VRP_{t\to t+22} = RV_{t\to t+22} - IV_t^2, \tag{2.4}$$

$RV_{t\to t+22}$ is the realized variance, IV_t^2 is the squared index volatility, x_{it} stands for all the other variables, $\varepsilon_{t\to t+22}$ is the error of the model and β_0, β_i are the coefficients of the model.

The structural relation is really between $RV_{t\to t+22}$ and the other predictor variables, including of course the IV^2. Thus, the equation (2.3) is just a (valid) re-arrangement of the following equation:

$$RV_{t\to t+22} = \beta_0 + (1-\beta_1) IV_t^2 + \sum_{i=2}^{6} \beta_i x_{it} + \varepsilon_{t\to t+22}, \tag{2.5}$$

We also consider the index return as a predictor of the VRP. Stock returns have a negative relationship with both the realized volatility and the implied volatility due to the “leverage effect”. Thus, given also the negative relationship of the index implied volatility with the VRP, we expect a positive relationship between the stock return and the VRP. This is because as the index return decreases the index volatility will increase and the VRP will decrease.

Finally, we consider the put-call ratio as a predictor variable of the VRP and we anticipate that there is a negative relationship between them. Han (2008) found that a variety of proxies for the market sentiment are significantly related to the risk-neutral skewness of the S&P 500 index. In particular, he found that the risk-neutral skewness of the SPX returns become more negative when the market sentiment turns more bearish. Put-call ratio is considered as a measure of the market sentiment. When the put-call ratio is close to one or greater it indicates a bearish sentiment of the market, hence a more negative risk-neutral skewness. A negative risk-neutral skewness reflects investor's fears for downward jumps in the index price (Bakshi and Kapadia, 2003, Bates, 2000). In periods of downward jumps the realized variance is expected to be greater than the variance swap rate, meaning that the buyer of the swap will have profit. Thus, when risk-neutral skewness becomes more negative, put-call ratio increasing, the buyers of the variance swap are willing to pay a greater VRP and sellers are asking for a greater VRP. That is, as the PC increases the VRP become more negative because the long part on the variance swap is willing to pay a larger premium in order to get advantage of this expected high realized volatility.

3. Data Description

In the previous section we considered six variables which we expect to have a predictor power in the VRP. In this section we briefly describe these six variables, the TED spread, the index returns the credit spread (CS), the term spread, the put-call ratio and the squared index volatility. Time series plots of these variables are provided, summary statistics followed by some short comments, methodology used to calculate etc.

3.1. TED spread

TED spread, from now on just TED, is defined as:

$$TED = 3\text{-month LIBOR rate} - 3\text{-month T-Bill rate.} \qquad (3.1)$$

LIBOR stands for London Interbank Offered Rate. It is the average interest rate at which banks lend to each other in London market. LIBOR is based in five currencies, US dollar (USD), Euro (EUR), pound sterling (GBP), Japanese yen (JPY), Swiss franc (CHF) and seven maturities, overnight, one week, 1 month, 2 months, 3 months, 6 and 12 months. In our case the 3 months based in USD is employed. Generally, not just as a part of TED, LIBOR is a key benchmark rate. This is because not just reflect the cost at which a bank can borrows funds from another bank but also is used as the reference rate for a huge range of financial products. From interest rate swaps to a simple mortgage loan or a saving account. In times of financial crisis, the LIBOR increases because banks do not trust each other so they lend at a higher LIBOR. The idea is simple, the higher the risk the higher the rate.

T-Bill stands for Treasury Bill. A Treasury Bill is a debt security issued by the US government with maturity less than one year. It does not pay a coupon rate but is sold at a discount and its yield is the difference in which is purchased and its face value. As the LIBOR this is a very important rate which is closely watched by financial markets because it is considered the closest interest rate to the risk-free interest rate. The T-Bill interest rate affect a range of other interest e.g. the corporate bond interest rate.

The way TED is constructed it provide a measure of liquidity for the investors. Assuming that 3-month T-Bill interest rate is the risk-free interest rate an increase in TED spread it come more from an increase in 3-month LIBOR interest rate. This means that when TED spread becomes wider it becomes more expensive for investors to raise funds.

In Table C.1 in the Appendix C are provided some key statistics for the 3-month LIBOR, 3-month T-Bill and TED. Figure C.1 and C.2 provide timeseries plots of 3-month LIBOR, 3-month T-Bill and TED. Data to construct the summary statistics of the TED spread and its components and to provide the timeseries plots span from 2001 to 2017.

There are some interesting things to point out from these plots and statistics. TED spread as the VRPs is asymmetric with a skewness value of 3.79 and has fat tails this is presumed on the high value kurtosis. TED spread is the difference of two interest rate so is a stationary time series, this is confirmed from the plots too. TED reach its peak at 10/10/2008 less than a month after the bankruptcy of Lehman Brothers. On this day 3-month LIBOR was at 0.0482 and the T-Bill at 0.0024 making the TED to reach at 0.0458. Its lower value (0.0009) was at 15/3/2010. This was mainly an impact from FED politic on interest rate. The FED lowered its interest rate to all time lower on December 2008 and did not hike it again until December 2015.

Daily data from trading days on 3-month LIBOR and 3-month T-Bill to construct the TED spread are obtained from Thomson Reuters Datastream.

3.2. Index Return

Next, we consider SPX return (R_{SPX}), DJX return (R_{DJX}) and NDX return (R_{NDX}) as predictor variables for the SPX VRP, DJX VRP and DJX VRP respectively. There is a well-established negative relationship, which have been discussed in literature in both empirical and theoretical framework, between stock returns and both realized and implied volatility (Figlewski and Xiaozu, 2000). Usually a decrease in stock price results in an increase of the stock volatility and an increase of the stock price results in a decrease of the stock volatility. This phenomenon is called the “leverage effect”.

An earlier discussion about this phenomenon is provided by Black (1976). An intuitive explanation for this could be that when the leverage of a firm increases it means that its debt increases making the firm riskier, hence the volatility of firm’s equities increases.

A financial explanation which describe the relationship between stock returns’ volatility and the leverage of the firm is as follow. The firm value is:

$$V = E + L \tag{3.2}$$

where V is the total value of the firm, E defines the value of equities and L stand for the total value of liabilities of the firm. If we assume that equities are only the number (N) of outstanding shares (S) and liabilities are only risk-free debt (S) the above equation become:

$$V = NS + D. \tag{3.3}$$

Now, if there is a random change ΔV in the market value of the firm it will affect the value of the N shares and for each share by the follow equation:

$$\frac{\Delta S}{S} = \frac{\Delta V}{E}. \tag{3.4}$$

With simple algebra it can be easily shown that equation (3.4) can be written as:

$$\frac{\Delta S}{S} = \frac{\Delta V}{V}\left(1 + \frac{D}{E}\right). \tag{3.5}$$

This means that a percental returns on stocks are equal the percental returns on firm value multiplied by 1 and the leverage of the firm. Thus, the equation (3.5) can be written as:

$$R_S = R_V\, L. \tag{3.6}$$

From the equation (3.6) we can easily obtain the standard deviation of stock return:

$$\sigma_S = \sigma_V\, L. \tag{3.7}$$

Thus, ceteris paribus, a decrease in stock price of the firm will increase the leverage and the volatility of the stock returns.

A more mathematical approach (Geske, 1979) to show the negative relationship between the volatility of the stock's returns and its price is by following the Merton model for the valuation of a firm. Merton in 1974 under some assumptions showed that owing one firm's share (S) is equally of owing a call option with the underlying asset the value of the assets of the firm and strike price the face value of the debt. One of the results of Merton paper is that volatility of equities can be expressed as follows:

$$\sigma_E = \frac{\partial E}{\partial V}\frac{V}{E}\ \sigma_V, \tag{3.8}$$

where V is the value of assets, E the value of equities, and σ_V is the volatility of assets. Note that in this case the partial derivative $\frac{\partial \mathrm{E}}{\partial \mathrm{V}}$ is the delta of the equity (stock) and it can be proven that it is equal N (d_1).

Now if the partial derivative of equity standard deviation with respect to the value equity is taken we have the following result:

$$\frac{\partial \sigma_S}{\partial E} = -\frac{V}{S^2}\left(\frac{\partial S}{\partial V}\right)\sigma_V, \tag{3.9}$$

which it becomes:

$$\frac{\partial \sigma_S}{\partial E} = -\frac{V}{S^2}\ N(d_1)\ \sigma_V < 0. \tag{3.10}$$

The way the index returns are used as a predictor variable in this dissertation is similar with Konstantinidi and Skiadopoulos (2016). We use the annualized sum of the daily log returns of the index from day t back to day t-21, this is a total sum of 22 daily returns. Daily closing price for each index to construct this sum are obtained from Yahoo Finance. Equation (3.11) provided below describe the method used to calculate this sum:

$$R_j = \frac{252}{22}\sum_{i=0}^{22} ln\frac{S_{t-i}}{S_{t-i-1}}, \tag{3.11}$$

where j = SPX, DJX, NDX

In the Appendix C, Table C.1 is provided a panel with the summary statistics of the three index returns. Also, in this appendix the time series plots of three index returns are provided. Again, to provide these statistics and the plots all data available from 2001 to 2017 are employed.

There are some things worth to point out from the table D.1 about the index returns. The minimum returns, as we defined it, “coincide” to be at the same date for the three indices (10/27/2008) during the period of Great Recession as someone could easily anticipate. The maximum values are at 4/6/2009 for the SPX and DJX and at 5/7/2001 for the NDX. It can be easily observed that high average returns come with high risk (high volatility) for all the three indices.

3.3. Credit Spread

We define the credit spread (CS) as:

$$CS = Moody's\ Baa - Moody's\ Aaa\ , \tag{3.12}$$

where Moody’s Baa stands for the Moody’s Seasoned Baa Corporate Bond Yield and Moody’s Seasoned Aaa Corporate Bond Yield.

Moody’s gradations for the corporate bonds are indicated by nine symbols, Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C. Each symbol represents a category of bonds with similar characteristics and of course, they have an equal probability of default. Aaa is the highest rating which implies that the company has a high credit rating and a very small probability of default, generally Aaa corporate bond yield is considered as substitution of the 10-years US T-Bond yield. The lowest gradation is C which, according to Moody's, are typically under default process with low probability to recover. From Aa to C numbers are also used from Moody's as sub graduations e.g. Aaa, Aa1 Aa2, Aa3, A1 and so go on. Baa is considered a relatively high credit risk rate with moderate default risk and a worthy investment.

A widening credit spread usually indicates a slowing economy and it means that the creditworthiness in most of the corporates is low. This is because when economy is not doing well bond rates of corporations which have a low credit rate are considered more likely to default, hence the investors demand a higher yield. A narrowing credit spread usually indicate that economy is doing well. In this case investors are not concerned from a high probability of default in general and they demand returns from the Baa bonds which are closer to the Aaa bonds.

Credit spread as economic measure not just follows the general state of economy as mentioned above but can gauge also the sentiment of the market participants for the future of the economy, hence can forecast the future economic activity (see e.g. Gomes and Schmid, 2010). Thus, a high credit spread can be considered as a strong signal of upcoming recession which almost always come with high volatility. Corradi, Distaso and Mele (2013) develop a model in which 75% of this countercyclical stock volatility can be explained by macroeconomics factors.

This is not the first time macro-finance indicator are used to predict the VRP. Bollerslev, Gibson and Zhou (2011) use credit spread as a predictor variable and find that it is important to understand the time-variation of the variance risk premium. Konstantinidi and Skiadopoulos (2016) also use credit spread as a predictor variable for the VRP of the SPX and find that is statistically significant. On the other hand, Carr and Wu (2009) chose the credit spread as a predictor variable in their analysis of the variance risk premium of a number of stocks and indices include SPX VRP, DJX VRP and NDX VRP. They conclude that the credit risk does not have a significant power in predicting the VRP. Thus, it will be interesting in this dissertation to test these results.

Summary statistics and time series plots for the CS are provided in the Appendix C, table C.1. It is important before continuing to the next section to discuss the main statistics of the VRP and the information provided by the plots. Baa and Aaa corporate bond yields' distribution look very similar to the normal distribution with a skewness close to zero and kurtosis close to 3 (excess kurtosis close to 0), but they do not follow the normal distribution. This can be confirmed, among others, from their difference, the credit spread. CS has a very high kurtosis (13.0) comparing to the normal distribution's kurtosis (3) and it appear to be right-skewed. CS reached its maximum value (3.5%) at 3/12/2008 roughly in the middle of the financial crisis. This can be explained by the fact that around this period Baa corporate bond yield hits its maximum of 9.5% at 31/10/2008 and Aaa was moving between 5.3% and 6.3%. Though Aaa corporate bond yield reached its minimum at 7/8/2016. This coincided at the same day with the minimum of Baa yield. CS at 13/6/2014 reached its minimum value indicating a recovering US economy. However, FED starts raising its rate first time after financial crisis only on December 2015. Daily data on Baa seasoned corporate bond yield and Aaa seasoned corporate bond yield are obtained from on line database of Federal Bank of Saint Louis (FRED).

3.4. Term Spread

Term spread (TS) represents the difference of interest rates between a short-term debt instrument and a long-term debt instrument. Usually is defined as the difference between the 10-year T-Bond yield and the 3-month T-Bill yield. In this dissertation instead of the three-month T-Bill yield, the one-month LIBOR is used. The one-month LIBOR has a correlation close to one (0.976) with the 3-month T-Bill yield, using data from 2001 to 2017. The fact that LIBOR is the benchmark interest rate in a vast part of the financial market guide us to use the one-month LIBOR instead of the 3-month T-Bill. The equation to measure this term spread is as follows:

$$TS = \text{10-year T-Bond} - \text{one-month LIBOR} \tag{3.13}$$

LIBOR has been described in previous section where the 3-month LIBOR was employed to construct the TED spread. In this case, to construct the term spread, we use the same LIBOR (expressed in USD) but for one-month loan.

The US 10-year T-Bond is one of the most popular bond in the world. When an investor purchases a 10-year T-Bond is actually lending to the US government and since the probability of default for the US government is close to zero the probability of loan not been repaid is close to zero. Therefore, the 10-year T-Bond is considered as a risk-free investment and its yield is considered as the risk-free interest rate. As in all economic and financial markets the law of supply and demand make its appear to the bonds market too. When the investors feel optimistic for the future state of the US economy, ceteris paribus, they prefer to invest in assets with higher returns, hence the demand for the 10-year T-Bonds decreases and the yield increases. And when the investors are feeling pessimistic for the future state of the US economy, ceteris paribus, the demand for safe assets, such as in our case the 10-year T-Bond increases and the yield decreases.

One can think about term spread as a very similar concept to the yield curve. The yield curve can have three forms, normal yield curve, inverted yield curve and flat yield curve. The normal yield curve has a positive slope which it means that longer maturity yields are higher than shorter in our case it means a positive term spread. This is the reason why yield curve with positive slope is called normal yield curve, because investors usually demand higher yields for longer maturity. The inverted yield curve is the inverse of the normal one. Investors demand higher yields for short maturity bonds, in our case a negative term spread. An inverted yield curve usually is a strong signal for a forthcoming recession. A flat yield curve indicates that short-term and long-term yields are almost equal in our case this means a term spread close to zero. A flat yield curve is often interpreted that investors are worried about future trend of the economy and probably they anticipate a recession.

There are plenty of literature which empirically confirm that the slope of the yield curve can predict future state of the economy (see Estrella and Hardouvelis, 1991). Franz (2013) states "*Examples for global risk factors are the TED spread or the term spread which can both be interpreted as indicators for the health of the economy and indicators for the expected performance of financial markets*".

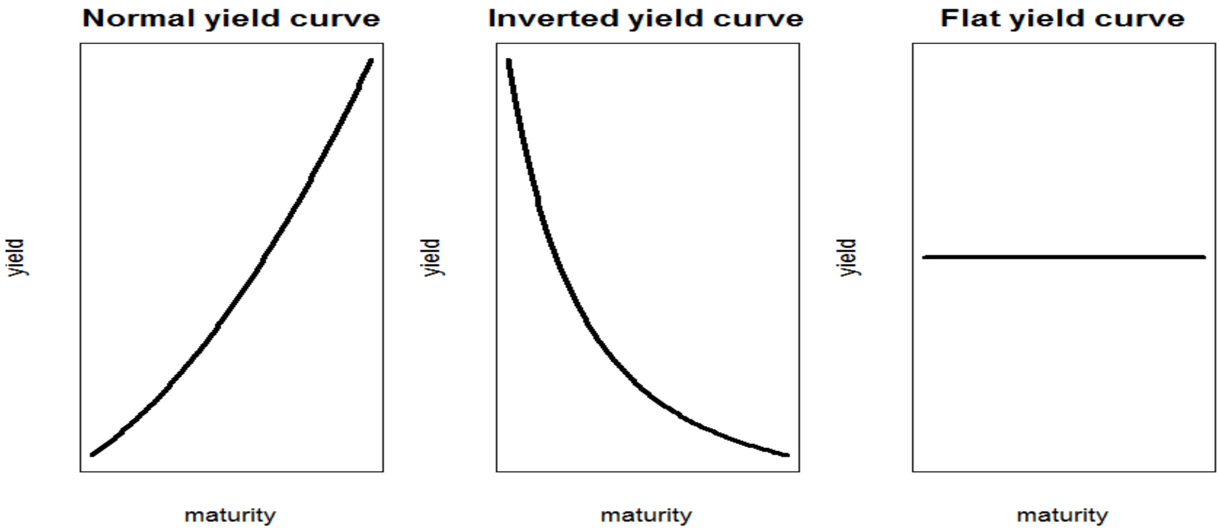

Figure 5: This figure presents the three main forms which the yield curve can takes.

Therefore, the reason why term spread is used as a predictor variable is similar with the credit spread. Both term spread, and credit spread are considered that can predict upcoming recession. Carr and Wu (2006) and Konstantinidi and Skiadopoulos (2016) also use the term spread as a variable which could have a predictive power on the VRP. The former conclude that the term spread does not have a predictive power on the large range of VRPs they analyze, including the SPX VRP, DJX VRP and NDX VRP. The latter, which analyze the SPX VRP for different maturities and different investing time horizons, have mixed results for the different maturities and investing time horizons. Sometimes the term spread is statistically significant and sometimes it is not, but in almost all cases they confirm the positive relationship mentioned above. Thus, our in-sample analysis is an opportunity to confirm these results in all the three VRPs.

In Appendix C, Table C.2, Panel D we provide the summary statistics and the timeseries plots of the term spread and its component as we have done for the other variables. All data from 2001 to 2017 are used for this. Daily data on one-month LIBOR and 10-year T-Bond to construct the term spread are obtained from Datastream.

3.5. The Put-Call Ratio

At any given point in time, the price of an option contends information about the risk-neutral distribution of its underlying asset. From this information, one can extract with different methods the moments of the distribution, for a more detailed analysis see Bakshi, Kapadia and Madan (2003). The time horizon of the risk-neutral probability density function which occurs is the same as the time expiration of the option.

The most often used and referred moment is, of course, the implied volatility. The implied volatility is a different concept from the historical volatility. For measuring the historical volatility of a stock, for example, the standard deviation of its past returns is used. The implied volatility expresses the sentiment of investors about future volatility, their expectation about the future volatility.

Another moment of the risk-neutral distribution is the 3rd moment, the skewness of the distribution. Skewness is a measure of the symmetry of a distribution, for example, the normal distribution has zero skewness. A distribution with skewness higher than zero (positive skewness) is defined as a right-skewed distribution and a distribution with skewness lower than zero (negative skewness) is defined as a left-skewed distribution.

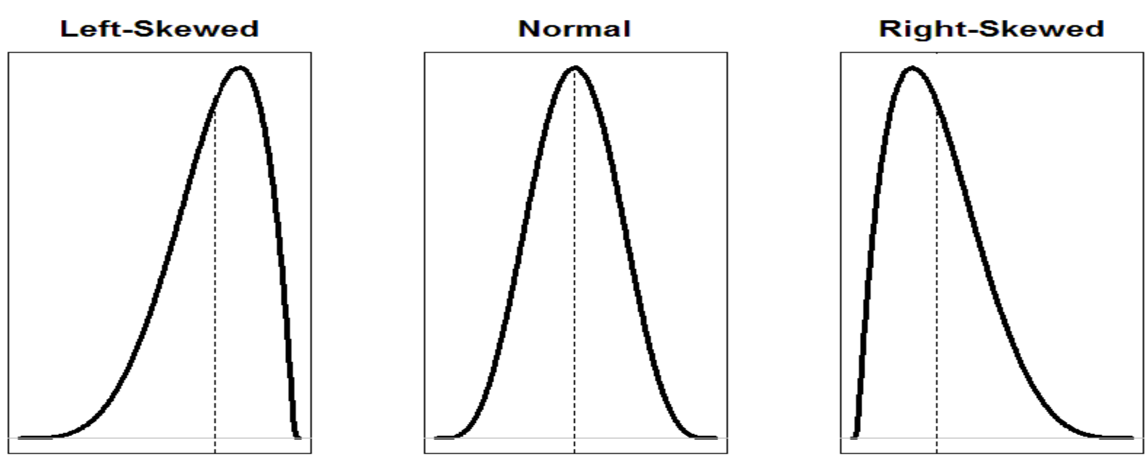

Figure 6: The figure shows three typical probability density function (pdf). From the left to the right, a pdf with negative skewness, with zero skewness and with a positive skewness.

A zero-skewed distribution of the stock returns means that the stock returns are symmetrically distributed around the mean of the distribution. For example, if the mean of returns is zero negative returns and positive are symmetrically distributed around zero, e.g. a -3% and a 3% return have the same probability. A right-skewed distribution, on the other hand, it means that negative returns could be lower (in absolute value) than the positive returns. Generally, this is preferred by investors because it means that extremes positive returns are higher than the negative returns expressed in absolute terms. The left-skewed distribution indicates that extreme negative returns are higher than positive returns. The left-skewed distribution is undesirable for the investors

The implied skewness extracted from the price of an option measure the risk-neutral expectation of the investors for the future skewness. As mentioned above they are several ways to extract the implied skewness (see Bakshi et al. ,2003 and Jackwerth, 2004). On Chicago Board of Options Exchange (CBOE) website, one can find data about SKEW index which measure the risk-neutral skewness of the SPX for the next 30 calendar days and is calculated using similar methodology with VIX calculation.

We obtain daily data of all traded put and call options on each index from Bloomberg to construct the respective put-call ratio as described in the following equation:

$$PC^i = \frac{P^i}{C^i}, \tag{3.14}$$

where P in the numerator stands for all daily traded index put options, C in the denominator stands for all daily traded index call options and i= SPX, DJX, NDX.

In the Appendix C are provided the summary statistics of the PC and the three time series plots for each index. The data to construct these span from 2001 to 2017.

All the three put-call ratios have positive skewness and very high kurtosis with the NDX PC having both them higher from the other two index PCs. This indicates that the distributions of the three PCs are far away from the normal distribution (Skewness=0 and Kurtosis=3). It appears from the that the SPX PC is more stable than the other two counterparts and the DJX the more volatile, with a standard deviation of 1.93. Minimum values for the SPX PC, DJX PC and NDX PC correspond to 30/7/2002, 16/9/2016 and 27/6/2001, respectively. And they reached their peak at 2/12/2002, 1/3/2012 and 16/1/2004, respectively.

3.6. Index Implied Volatility

The VIX, VXD and VXN are indices just as the SPX, DJX and NDX are. Their difference with stock indices is that their daily closing price display volatility instead of actual prices (for a more detailed analysis on VIX see Whaley, 2008). The VIX was the first index volatility published from the CBOE on-line, in 1993, followed by the VXD and the VXN. When the VIX was first launched was intended to serve for two purposes, first one as a benchmark index for the risk-neutral expected volatility in short-term (30 calendar days) and second one to give the opportunity to the investors to invest directly in volatility via futures and options on the VIX. Whale (2008) compare the implied index volatility to the yield to maturity of a bond. Like the yield to maturity, the implied volatility is not directly observed but can be computed from the current price of options on the stock index.

The methodology which VIX, VXD and VXN are calculated similar and described in previous section, hence we will not provide further analysis in this section about this. We obtain daily data on the three indices from the CBOE.

As with the other variables summary statistics and time series plots for the three squared index volatilities are provided in the Appendix C. There are not too much to comment about the summary statistics of the three IVs. All three of them are positively skewed with “fat tails”. In the three above graphs both index price and index volatility are displayed in order to have a “picture” of that negative correlation described before. In all the three indices in the same period when squared IV reached its peak the index prices drop at historical lowest (second lowest for the NDX) from 2001 to 2017. This confirm the negative correlation of the IV with its index price. And because all the three IV show similar correlation with their index price strengthens our idea to use all the three indices as a “fear gauge” for their respective index.

4. In-Sample Analysis

For the in-sample analysis our data span from 2/2/2001 to 31/8/2007 for the SPX VRP and the DJX VRP and from 20/3/2001 to 31/8/2007 for the NDX VRP because of non-available data issue. As a start we run three OLS including all the six predictor variables considered in previous section. However, the credit spread (CS) it strongly appears to be statistically insignificant for the SPX VRP and DJX VRP model indicating no predictive power at all. And the term spread (TS) is not statistically significant for the NDX VRP, with a very low t-statistic, indicting again no predictive power. Thus, we run five variables OLS, in which the CS is omitted from the SPX VRP and DJX VRP models and the TS is omitted from the NDX VRP model. In the model with the five variables the t-statistics are impressively improved. On the other hand, some criteria for the fitness of the model are slightly better for the six-variable model of the SPX VRP and the DJX VRP and are the same for the NDX VRP model. Nevertheless, these differences are so small that we could ignore them, e.g. for the SPX VRP model the akaiki information criterion is -5.05 for the five-variables model and -5.10 for the six-variables model. The results of the six-variables OLS are provided in the appendix, also a table with the fitness model criteria of the two models. Thus, to keep our models simple we choose to process with the parsimonious five-variables models. The three final models we use for this analysis are described from the three equations below:

$$VRP_{t\to t+22}^{SPX} = \beta_0 + \beta_1 TED_t + \beta_2 R_{t-22,t}^{SPX} + \beta_3 TS_t + \beta_4 PC_t^{SPX} + \beta_5 VIX_t^2 + \varepsilon_{t\to t+22} \quad (4.1)$$

$$VRP_{t\to t+22}^{DJX} = \beta_0 + \beta_1 TED_t + \beta_2 R_{t-22,t}^{DJX} + \beta_3 TS_t + \beta_4 PC_t^{DJX} + \beta_5 VXD_t^2 + \varepsilon_{t\to t+22} \quad (4.2)$$

$$VRP_{t\to t+22}^{NDX} = \beta_0 + \beta_1 TED_t + \beta_2 R_{t-22,t}^{NDX} + \beta_3 CS_t + \beta_4 PC_t^{NDX} + \beta_5 VXN_t^2 + \varepsilon_{t\to t+22} \quad (4.3)$$

Note that the conditional expectation of the above VRP (ex-post) is the ex-ante VRP.

We use OLS method to estimate the coefficients of the equations. The criteria we use to evaluate the in-sample results are the t-statistic and the adjusted R^2. Because the data we use are overlapping to calculate the t-statistics we use the Newey-West (1987) approach standard errors. However, because is very usual in the literature we also provide the t-statistics calculated with the Hansen and Hodrick (1980) approach and White approach. Hansen-Hodrick approach differs from the Newey-West approach because it assumes that the autocorrelation does not fade gradually over time, but it is constant until the of the period which it is assumed that exist. On the other hand, the white approach does not take into account the problem of the autocorrelation, only the heteroscedasticity. Before the presentation and the discussion of the in-sample results we briefly describe the method of the OLS and the criteria we use in order to provide an explanation why we use those. From the pairwise correlation matrices provided below, it can be concluded that there is no need to worry about the multicollinearity problem. Particularly, the highest (in absolute value) pairwise correlation between the predictor variables is the pairwise correlation between the TED and the term spread (TS), -0.7. Indicating that a decrease in TS is strongly expected to be followed by an increase in TED but cannot be considered as a multicollinearity problem. The second highest pairwise correlation is between the credit spread (CS) and the index volatility for each the three markets. The highest of these three correlations is the one between VXD and CS, 0.53, and again it cannot be considered a problem. However, this is an indicator that when at the period investigated an increase in CS it was followed by an increase in index volatility.

In Appendix C a table with pairwise correlation of all predictor variables from 2001 to 2017 is provided. The highest pairwise correlation are between the CS and the VIX^2 (0.75) and the CS and the VXD^2 (0.73). However, the CS is omitted from the OLS model for the SPX VRP and the DJX VRP. Third higher pairwise correlation is between TED and VIX^2 (0.55). Thus, the pairwise correlations for the predictor variables using all data in the sample confirm the assumption of no multicollinearity problem of the variables.

Table 4.1: Correlation Matrix

Panel A: SPX VRP Predictor Variables' Correlation Matrix						
	TED	R^{SPX}	***CS***	***TS***	PC^{SPX}	VIX^2
TED	1	−0.02	−0.32	−0.7	0.09	−0.22
R^{SPX}		1	−0.11	−0.02	0.01	−0.45
CS			1	0.36	−0.18	0.49
TS				1	−0.08	0.32
PC^{SPX}					1	−0.20
VIX^2						1
Panel B: DJX VRP Predictor Variables' Correlation Matrix						
	TED	R^{DJX}	***CS***	***TS***	PC^{DJX}	VXD^2
TED	1	−0.01	−0.32	−0.7	−0.02	−0.25
R^{DJX}		1	−0.06	−0.03	−0.05	−0.38
CS			1	0.36	0.13	0.53
TS				1	0.10	0.34
PC^{DJX}					1	0.26
VXD^2						1
Panel C: NDX VRP Predictor Variables' Correlation Matrix						
	TED	R^{NDX}	***CS***	***TS***	PC^{NDX}	VXN^2
TED	1	−0.03	−0.32	−0.7	−0.13	−0.22
R^{NDX}		1	−0.06	−0.01	0.04	−0.26
CS			1	0.36	0.03	0.41
TS				1	0.17	0.34
PC^{NDX}					1	−0.09
VXN^2						1

Table 4.1 displays the pairwise correlation of the predictor variables considered for each model. To calculate the pairwise correlation all data available from 2001 to 2007 are used.

In the following table (4.2) the OLS results for each final VRP model are provided.

Table 4.2: 5 Predictor Variable Model OLS Results

Panel A: SPX VRP 5 Predictor Model OLS Results							
	C	***TED***	***R^{SPX}***	***TS***	***PC^{SPX}***	***VIX^2***	***Adj. R^2***
Coeff.	0.0166	−2.0704	−0.0149	−0.2624	−0.0012	−0.3921	0.28
NW	(3.34)	(−2.80)	(−2.72)	(−2.03)	(−1.32)	(−5.13)	−
HH	(2.76)	(−2.55)	(−2.37)	(−1.75)	(−1.19)	(−4.38)	−
W	(8.48)	(−7.18)	(−9.24)	(−6.71)	(−2.06)	(−15.88)	−
Panel B: DJX VRP 5 Predictor Model OLS Results							
	C	***TED***	***R^{DJX}***	***TS***	***PC^{DJX}***	***VXD^2***	***Adj. R^2***
Coeff.	0.0162	−2.7243	−0.0144	−0.3747	0.0018	−0.3811	0.26
NW	(2.72)	(−2.46)	(−2.81)	(−2.34)	(2.37)	(−5.44)	−
HH	(2.37)	(−2.19)	(−2.59)	(−2.04)	(2.24)	(−5.20)	−
W	(8.42)	(−7.34)	(−8.87)	(−8.06)	(4.15)	(−16.63)	−
Panel C: VXD VRP 5 Predictor Model OLS Results							
	C	***TED***	***R^{NDX}***	***CS***	***PC_{NDX}***	***VXN^2***	***Adj. R^2***
Coeff.	−0.0161	1.2167	−0.0180	2.3815	−0.0007	−0.5717)	0.79
NW	(−1.30)	(0.86)	(−3.81)	(1.58)	(−1.05)	(−12.42)	−
HH	(−1.23)	(0.88)	(−3.49)	(1.51)	(−1.00)	(−12.67)	−
W	(−4.02)	(2.49)	(−11.11)	(4.93)	(−1.20)	(−35.87)	−

In the table 4.2 the in-sample (2001-2007) OLS results are provided. For the SPX VRP and the DJX VRP the credit spread (CS) is omitted and the term spread (TS) is omitted for the NDX VRP. Coeff. stands for the coefficients of the predictor variables. NW and HH are the t-statistics computed with Newey-West and Hansen-Hodrick approach respectively and W stands for the White robustness t-statistics. Adj. R^2 is the adjusted R^2.

The TED spread's coefficient for the SPX VRP and the DJX VRP has a negative sign as we expected (see section 2), and it is statistically significant. These results confirm the results of Konstandinidi and Skiadopoulos (2016), they also find a statistically significant negative relationship between the SPX VRP and the TED spread. We cannot say the same for the relationship between TED spread and the NDX VRP. In this case the TED is not statistically significant and interestingly it appears to have a positive relationship in the NDX VRP. Conducting the one tail hypothesis test that TED coefficient is equal to zero versus the alternative that is greater than zero the Newey-West t-statistics (0.86) is smaller than the critic value (1.282) for 90%

confidence value, hence we cannot reject the null hypothesis. However, this do not necessary means that it is zero. For example, if we run the test for 80% confidence level the critical value is 0.84, hence we reject the null hypothesis of the zero coefficient in favor that the TED coefficient is positive. If the hypothesis of the positive relationship between the TED spread and the NDX VRP stands true one explanation it could be the different nature of the NDX with the other two indices. NDX is based on high technology companies and sometimes the shocks that effect it, could be different from the other two. This is confirmed also from the pairwise correlation of the three indices for the period 2001-2007, SPX and DJX correlation is 0.97, SPX-NDX is 0.89 and DJX-NDX is 0.8, interesting is also the fact that the NDX average return in this period is slightly negative in contrast with the slightly positive one of the other two. This could be a reason why the NDX VRP and the other two VRPs differ in behavior (see tables 1.1 and 1.2). NDX VRP is more negative in this period, and the coefficient of variation ratio (CV) is the lowest (SPX VRP CV=6.99, DJX VRP CV=8.8, NDX VRP CV=5.2) as calculated with standard deviation from Newey-West approach. This indicates that for the shorter party selling variance swap is a very good investment. Thus, as the TED spread increases and the liquidity decreases the number of the investors who want to buy variance swaps increases but it seems that, because of the above, the number of short investors increases more, driving the VRP to increase.

The results for the three index volatilities are as expected. The sign of the coefficients confirms a number of papers (e.g. Egloff et al., 2010, Konstantinidi and Skiadopoulos, 2016) which argue that there is a negative relationship between the VRP and the implied volatility. The t-statistics in all the three models confirm also that they are statistically significant.

The index returns are statistically significant but their relationship with the VRP is not as we expected. We expected a positive relationship between the index returns and the VRP, because as the index returns decrease the implied volatility increases and given the expected negative relationship of the implied volatility with the VRP, the VRP would decreases too. The negative relationship of the VRP with the index volatility is confirmed from the results above, we also found a negative sample correlation between all index volatilities and their respective index returns, though the relationship between the index returns and the VRP is negative. The way we insert the index returns as a predictor variable in the model is by taking the past 22 trading days sum. A negative/positive sum has a direct effect in the index implied volatility, increasing/decreasing it.

Another indirect effect it could be from the predictive power of the variable. The variable as it is constructed it is as a random walk. If this random walk, in average can for short investment horizon predict better than the implied volatility the index returns. That is, that in average, lower returns will be followed by lower returns which implied volatility cannot predict correctly. This means that in average the future realized variance will be higher than index volatility, hence the negative relationship between the index returns and the VRP. Konstantinidi and Skiadopoulos (2016) found the same in their multiple predictor model, though not statistically significant, for variance swaps of one month investment horizon.

The results for the-put call ratio are mixed. For the SPX VRP and the NDX VRP the signs of the coefficients are as expected, though statistically insignificant. On the other hand, the relationship between the DJX VRP and its put-call ratio is positive and statistically significant. The put-call ratio is a measure which measure the psychology among the market participants, their sentiment about the future prices of the underlying asset. As described in section (2.2) the idea is that as the PC increases indicate the fear of the market participants for a decrease in underlying asset price which is accompanied with high realized variance, hence they are willing to pay a higher VRP to hedge against this. Most of the time this sentiment indicators tend to predict wrong or exaggerates, hence the more negative VRP. This is not the case for the DJX. A bearish sentiment in the market will have a positive effect in the variance swap rate, but as it can be observed from the results the realized variance is higher and the reason for this it could be that indeed asset price decreases, probably more than expected.

The sign of the credit spread (CS) coefficient also is not as expected for the NDX. Note that we expected for both TED and CS a negative relationship with the NDX VRP and they both appear to have a positive relationship with it. One can consider the CS a similar economic indicator with the TED, hence the explanation for this positive relationship could be the same as for the TED above.

The relationship of the term spread (TS) with the SPX VRP and DJX VRP is negative in contrast with our expectation. As the TS decreases it predicts a forthcoming recession, hence the investors in order to hedge again future high realized volatility are willing to pay a more negative VRP. That is a higher variance swap rate than usual. However, it seems that the realized volatility, in this period, go beyond their expectation, hence the negative relationship of the TS and the two VRPs.

It appears that some of the variables have not the relationship we expected because of previous literature. This is not unusual for this type of financial variables. For example, the correlation between stock returns and bond returns has changed sign several times (see e.g. Andersson et al., 2008).

We don't have much to say about the adjusted R^2. As it can be seen it is high enough for such model, especially for the NDX VRP model. However, this doesn't necessary mean that the model has good prediction power out-of-sample, this is something will be analyzed in the next section.

In the appendix D there two tables in which are provided the OLS results for the six variable model.

5. Out-of-Sample Analysis

For the out-of-sample analysis, we use an expanding window methodology. That is, we first run an OLS with the in-sample data to construct the predictor models. Then we go in the first out-of-sample observation use the models to make a prediction about VRPs. We save the residual and re-run the OLS with this new observation hence, the in-sample size increases by one observation. This is repeated for every out-of-sample observation. To evaluate the out-of-sample results we use the out of sample R^2 (Campbell and Thompson, 2008), the mean correct prediction (MCP), Theil's UI and UII, the root mean squared error (RMSE) and the mean absolute error (MAE). Before presenting the out-of-sample results these criteria are briefly described. The out-of-sample analysis is conducted for the same period for all the three VRPs, from 9/2007 to 07/2017.

5.1. The Out-of-Sample Criteria

The out-of-sample R^2 is defined as:

$$R_{OS}^2 = 1 - \frac{\sum_{t=1}^{T}\left(VRP_{t\to t+22} - \widehat{VRP}_{t\to t+22}\right)^2}{\sum_{t=1}^{T}\left(VRP_{t\to t+22} - \widehat{VRP}_{t\to t+22}^{b}\right)^2}, \tag{5.1}$$

where $\mathrm{VRP}_{t\to t+22}$ is the actual ex-post VRP observed, $\widehat{\mathrm{VRP}}_{t\to t+22}$ the ex-post VRP estimated from the model and $\widehat{\mathrm{VRP}}_{t\to t+22}^{b}$ the ex-post VRP estimated from the benchmark model. We chose as benchmark model a random walk (RW) which is defined as:

$$VRP_{t\to t+22} = VRP_{t-22\to t} + \varepsilon_{t+22}. \tag{5.2}$$

Note that, this is a very naïve model. It says that the expected future value of the VRP is the present value of the VRP.

When the $\mathrm{R}_{\mathrm{OS}}^2 > 0$, it means that the model predictions outperform the naïve model predictions. We test the null hypothesis $H_0: R_{OS}^2 > 0$ versus the alternative $H_A: R_{OS}^2 \leq 0$ we use the modified by Harvey et al. Diebold-Mariano test (DM), which is defined as:

$$MDM = \left(\frac{T + 1 - 2h + T^{-1}h(h-1)}{T}\right)^{1/2} DM, \tag{5.3}$$

where DM for one step ahead forecast (h=1) is defined as:

$$DM = \frac{\bar{d}}{\left(\widehat{Var}(d)\right)^{1/2}}. \tag{5.4}$$

To better understand the DM test let first assume two forecasting models, as in our case Y_t^M for the multivariable model and Y_t^b for the random walk model. Let Y_t be the actual observed value. Thus, the forecast error is defined as:

$$e_{jt} = Y_{At} - Y_{jt} \tag{5.5}$$

where j=M, b (predictor or benchmark model, respectively) and $\mathrm{Y_{At}}$ is the actual value observed. Now we have to define a loss function $g(\mathrm{e_{jt}})$ which is usually, depending on which statistic we want to test, defined as:

$$g(e_{jt}) = e_{jt}^2 \ or \ g(e_{jt}) = |e_{jt}| \tag{5.6}$$

Thus the $\bar{\mathrm{d}}$ can now be defined as:

$$\bar{d} = \frac{1}{T}\sum_{t=1}^{T} d_t \tag{5.7}$$

where $d_t = \mathrm{g(e_{Mt})} - \mathrm{g(e_{Bt})}$.

The idea of the DM test is to conduct the hypothesis test: $H_0: E(d_t) \geq 0$ versus $H_A: E(d_t) < 0$. Which is equivalent with the hypothesis test: $H_0: R_{OS} \leq 0$ versus $H_A: R_{OS} > 0$, recall equation (5.1) for the definition of the R_{OS}. If the absolute value of the MDM test is higher than the critical value of the t-student distribution with (T-1) degrees of freedom for a specific confidence level we reject the null hypothesis.

The errors of the prediction are correlated (remembered how we define the multiple variables predictor models from equation 4.1-4.3 and how the random walk model is defined from equation 5.2), hence we use the Newey-West approach with 22 lags to calculate the standard errors.

The mean correct prediction (MCP) measures the percentage of the correct prediction of actual sign $(-/+)$ of the VRP. That is, the number of predictions from the multiple variable model which appear to have the same sign as the actual VRPs observed divided with the total number of the predictions. We also perform a ratio test which is defined as:

$$RT = \frac{\hat{R} - R_0}{\left(\frac{R_0(1-R_0)}{T}\right)^{1/2}}, \tag{5.8}$$

for the null hypothesis $H_0: R \leq R_0$ against the alternative $H_A: R > R_0$. We reject the null hypothesis if the value of the test is higher than the critical value obtained from the standard normal distribution of a specific confidence interval. We perform the test for $\mathrm{R}_0 = 0.5$.

The next criteria we use to evaluate the out-of-sample results are the Theil's UI and UII which are defined as:

$$UI = \frac{\left[\frac{1}{T}\sum_{t=1}^{T}(A_t - P_t)^2\right]^{1/2}}{\left(\frac{1}{T}\sum_{t=1}^{T}A_t^2\right)^{1/2} + \left(\frac{1}{T}\sum_{t=1}^{T}P_t^2\right)^{1/2}}, \tag{5.9}$$

$$UII = \frac{\left[\sum_{t=1}^{T-1}\left(\frac{P_{t+1} - A_{t+1}}{A_t}\right)^2\right]^{1/2}}{\left[\sum_{t=1}^{T-1}\left(\frac{A_{t+1} - A_t}{A_t}\right)^2\right]^{1/2}}, \tag{5.10}$$

where P stands for the predicted value of the variable and A stands for the actual value observed of the variable.

Theil proposed the UI in *Economic Forecasts and Policy* (1958). UI can take values from 0 to 1, when the value of UI is zero means a perfect prediction, hence the lower the UI, the more accurate the prediction. UII has been proposed by Theil in *Applied Economic Forecasting (1966).* UII in contrast with UI it is not bounded and can take values higher than 1. When UII takes values lower than one it means that the predictor model is better than a naïve model or simple guessing. If UII takes the value one it means that the predictor model and the naïve model has the same accuracy. And if the UII takes value higher than one it means that the predictor model should not be used because just “guessing” with the naïve model is better.

Generally, only the UII is used because the UI has a number of drawbacks (see Bliemel, 1973). However, UI continues to be used because it gives a first intuitive picture of the quality of the prediction model, this is the reason why is also provided in this dissertation.

The last two criteria we use to evaluate the out-of-sample results are the root mean squared error (RMSE) and the mean absolute error (MAE). These two statistics measures are defined as:

$$RMSE = \left[\frac{1}{T}\sum_{t=1}^{T}(A_t - P_t)^2\right]^{1/2}, \qquad (5.11)$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|A_t - P_t|, \qquad (5.12)$$

where, again A stands for the actual value of the VRP observed and P stands for the value of the VRP predicted by the model (multiple variable model and random walk model).

Both RMSE and MAE are not bounded and theoretically could take values from 0 to ∞. The most important difference between RMSE and MAE is that the MAE calculate the sum of absolute errors that implies using the same weight for the errors (large or small). On the other hand, RMSE gives a higher weight to the large errors this is due to the fact that it calculates the sum of squared error. They cannot be compared together (RMSE with MAE) because, as it can be seen from the above equations, it stands that RMSE ≥ MAE. Thus, we construct the RMSE and MAE for the naïve model and the multiple predictor model. RMSE and MAE are negatively-oriented scores, hence lower values are better.

We test the null hypothesis $H_0\colon RMSE^M \geq RMSE^b$ against the alternative $H_0\colon RMSE^M < RMSE^b$ performing a MDM test as described above with loss function the squared errors. The $\mathrm{RMSE}^{\mathrm{M}}$ and $\mathrm{RMSE}^{\mathrm{b}}$ are the RMSE of the multiple predictor model and the random walk, respectively. To test the null hypothesis $H_0\colon MAE^M \geq MAE^b$ against $H_0\colon MAE^M < MAE^b$ performing a MDM test with loss function the absolute value of the error. $\mathrm{MAE}^{\mathrm{M}}$ and $\mathrm{MAE}^{\mathrm{b}}$ stand for the MAE of the multiple variable model and the random walk model, respectively. In particular the test we run are for the RMSE:

$$H_0\colon E(e_{Mt}^2 - e_{bt}^2) \geq 0$$

$$H_A\colon E(e_{Mt}^2 - e_{bt}^2) < 0$$

And for the MAE:

$$H_0\colon E(|e_{Mt}| - |e_{bt}|) \geq 0$$

$$H_A\colon E(|e_{Mt}| - |e_{bt}|) < 0$$

The e_{Mt} and e_{bt} are the predictor errors from the multiple variable model and the random walk respectively.

5.2. The Out-of-Sample Results

In this section, the out-of-sample results as they have been provided from the table 5.1 are discussed.

Table 5.1: The Out-of-Sample Evaluation Criteria

Panel A: SPX VRP Out-of-Sample Results						
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
SPX Model	0.439**	0.78***	0.62	0.88	0.0538**	0.0220***
SPX RW	–	–	–	–	0.0718	0.0304
Panel B: NDX VRP Out-of-Sample Results						
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
DJX Model	0.423**	0.76***	0.62	0.88	0.0454**	0.0191***
DJX RW	–	–	–	–	0.0600	0.0262
Panel C: VXD VRP Out-of-Sample Results						
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
NDX Model	*0.331***	*0.76****	*0.64*	*0.92*	*0.0519***	*0.0237***
NDX RW	–	–	–	–	*0.0634*	*0.0278*

Entries report the out-of-sample R^2, the Mean Correct Prediction (MCP), the Theil's UI and Theil's UII. One, two and three asterisks in the parenthesis under the R^2 denote the rejection of the null hypothesis H_0, the predictor model doesn't outperform the benchmark model against the alternative hypothesis that the predictor model outperforms the benchmark model at 10%, 5% and 1% respectively. One, two and three asterisks in the parenthesis under the MCP denote the rejection of the null hypothesis that the proportion ratio is equal or lower to 50% against the alternative that is higher than 50%. The root mean squared error (RMSE) and the mean absolute error (MAE) of the predictor model and the naïve model (random walk) are also provided. One, two and three asterisks denote the rejection of the null hypothesis H_0 the predictor model doesn't outperform the benchmark model against the alternative hypothesis that the predictor model outperforms the benchmark model at 10%, 5% and 1% respectively. The out-of-sample data span from 9/2007 to 7/2017.

The out-of-sample R^2 is positive for all the three VRPs, indicating that all the three multiple predictor models outperform the benchmark model. As it can be seen from the table these positive R^2 are significantly positive for a confidence level of 95%.

We have calculated the mean correct prediction (MCP) for the three multiple predictor models. The results provided above show that the MCP is significantly larger than 50% for the three VRPs. The MCP of the three predictor models have values larger than 75%, this implies that the three models can predict the correct sign of the actual VRP for more than 75%.

We have also calculated the Theil's UI and UII for the three multiple variable models. Theil's UIs are lower than one for the three models but not close to zero which the case of perfect prediction. Theil's UIIs are slightly lower than one and this implies that the models predict accuracy is slightly better than a naïve model. The RMSEs for the three VRPs we want to predict are lower in the case we use the multiple predictor model against the random walk (RW). Thus, this indicates that predicting future VRPs with the model is better than just using the naïve rule of forecasting with a random walk. We also conduct a modified Diebold-Mariano test, which confirm that at 95% confidence level the RMSE of the models is lower than the RMSE of the benchmark model for all the three models.

We had similar results when calculated the MAE of the models and the MAE of the random walk. Again, in the three VRPs the MAE of the models are significantly lower than the MAE calculated from the random walk prediction.

Generally, the conclusion from these results is that the multiple variable models we use to predict the VRPs outperform the benchmark model of the random walk.

6. Robustness Tests

In this section, we perform further tests to verify the robustness of the results from sections 4 and 5. We conduct two similar analysis as in these sections but with different sub-samples.

In the first test, the data for the in-sample analysis span from 2/2001 to 7/2009 for the SPX VRP and the DJX VRP and from 3/2001 to 7/2009 for NDX VRP. The rest of data from 8/2009 to 7/2017 are used for the out-of-sample analysis. Thus, we add to the original in-sample data (2001-2007) observations from the period of the US financial crisis. In this period as we analyzed in section 1.4, the VRP in all the three markets showed a very high volatility. In this period, they reached extremes values than usual, both negative and positive. From the financial market point of view, this period was very important and has led into a lot of changes. These changes include for example, the way banks are supervised (Basel III) or more temporary changes as the low interest policy of FED. A very important change results in the relationship among the credit entities. With the collapse of Lehman Brothers, collapsed also the theory of "too big to fail". Hence, the fear of default spiked and only a few government entities were considered risk-free.

In the second robustness test the data for the in-sample analysis span from 2/2001 to 12/2011 for the VRPs of SPX and DJX market and from 3/2001 to 12/2011 for the NDX VRP. The data for the out-of-sample analysis for all the three VRPs span from 1/2012 to 7/2017. Black Monday of 2011 is the reason why we chose to add the period from 2009 to 2011 to the previous in-sample dataset because.

On 5 August 2011, Standard and Poor downgraded the credit rating of the USA, for the first time in history, from AAA to AA+. Black Monday of 2011 refers to the Monday 8 August 2011 when the US stock markets crashed. The three US major indices SPX, DJX and NDX dropped 6.6%, 5.5% and 6.9%, respectively. For the Dow Jones (DJX) this was the 6th largest drop in its history. The VIX at this day jumped to 44%, the highest level since 2009 and VXD and VXN reached the high values of 40.5% and 44.7% respectively. Generally, especially the second half of 2011, was a period of turmoil for the stocks and the financial markets in the USA and in the major global markets. The effect of this period on the three market indices VRP can easily be observed from their respective time series plots and the summary statistic table in section 1.4. This year (2011) all the three VRP had a very high volatility. From earlier 2012 to the end of the period we analyze the three VRPs it has been a relatively quiet period for the VRPs.

From the pairwise correlation matrix in section 4 (2001-2007) and the one in appendix C (2001-2017), we conclude that there is no reason to concern about multicollinearity issues in the model regressions we run. In appendix D, in Tables D.1-4, are presented the in-sample OLS results and the out-of-sample criteria results.

All the evidence from the out-of-sample analysis suggest that the predictor model constructed from the five variables outperform the random walk model. For the first out-of-sample analysis, 2009-2017, the OS R^2 is statistically significant positive for the SPX VRP and the DJX VRP. Also, both the root mean squared error (RMSE) and the mean absolute error (MAE) of the predictor model are statistically significant lower than random walk ones, in SPX and DJX VRP models. Theil's UII values slightly lower than one for these two VRPs predictor model suggest that they are slightly better predictors than a naïve model. The mean correct prediction (MCP) is statistically significant greater than 50% for all the three predictor models. In the second out-of-sample analysis, 2012-2017, all the three OS R^2 are significantly positive. Also, the three RMSE of the three predictor models are statistically significant lower than the ones of the random walk. On the other, hand all the three predictor models' MAE are lower than MAE of the random walk, but only the SPX and DJX are significantly lower. Both Theil's UI and UII indicate that the predictor models are better predictors from a naïve model. Last, the MCP are statistically significant higher than 50% for the three predictor models.

However, the most interest results to discuss occur from the in-sample analysis. The adjusted R^2 remains at the same levels for SPX VRP and DJX VRP models and it is lower for the NDX VRP model. Both the index volatility and the index return continue to have a statistically significant negative relationship with the VRP, confirming the results of the original in-sample analysis (2001-2007). It appears from the results in the two in-sample analyses that the TED spread and the term spread (TS) change their relationship with the VRP after the financial crisis (2007-2009). For the TED the NW t-statistics for the two VRP models in the two analyses vary from 2.09 to 1.90 and respectively for the TS from 1.92 to 1.52. These t-statistics values are not negligible hence, this change on the signs deserves some short comments.

As it has been explained in section 2 the relationship between the predictor variables and the VRP is not a direct relationship, e.g. GDP and consumption, their connection is indirect. Values of the VRP in time, as has been described, depend mainly on the sentiment of the buyer of the variance swaps, on how do they feel about the future. The variables we chose to construct the predictor models try to explain the VRP by capturing the sentiments of the market participants. Hence, the way they explain/effect the VRP is based on the sentiments of the end-users of the variance swaps. The fact that the VRP is mainly based in the sentiments of the end-users has as a result the VRP to be very sensitive from different market shocks. Thus, the relationship between the VRP and the predictor variables, TED and TS, is not very stable. Average investors tend to exaggerate when they feel very enthusiast, or their fear level is very high, one can think about the “held behavior” theory. So, taking into account the radical changes among the credit entities during the 2007-2009, this change of the sign in TED and TS it does make sense. And also, as mentioned in previous section, this is not so unusual in financial variables, correlation between SPX returns and US bond returns have change sign multiple times in their history.

Conclusion

This dissertation investigates whether the variance risk premiums (VRP) on the three major US indices, S&P 500 (SPX), Dow Jones (DJX) and Nasdaq (NDX), can be predicted.

To do this, we employ a parsimonious model for each market VRP and from which the conditional expectation is the ex-ante VRP. These models include predictor variables which are considered to be indicators of the trading activity and the stock market conditions. To construct this model, first we quantify the (ex-post) VRP relying on the realized variance, which is measured as the sum of the 22 trading days log returns of the index, and on the index volatility.

Most of our predictor variables appear to be statistically significant despite the fact that some of them have a different effect on the VRP from what we first expected. However, the VRP is very sensitive because it defined via the sentiments of the end-users of the variance swaps. Thus, we find that some variables could have different effect in different time period on the VRP making that relationship with the VRP not stable in time.

For example, term spread has a statistically significant negative effect on the VRP of the SPX and the DJX instead of a negative first expected. But this relationship become positive after the financial crisis (2007-2009).

The sum of log returns from the past 22 trading days has also a statistically significant negative effect instead of a positive one as anticipated on all the three market VRPs.

On the other hand, the TED spread relationship with the SPX VRP and DJX VRP is negative as expected and statistically significant. But similar with the term spread it changes sign after the financial crisis.

Conducting the out-of-sample analysis we find that prediction of the three models we use to calculate the ex-ante VRP outperform the benchmark model' prediction. All the out-of-sample criteria we employ to our evaluation confirm these findings. These results are also confirmed by the further robustness tests conducted.

This dissertation attempts to investigate mainly whether the VRP on the three markets discussed can be better predicted by the model proposed than the naïve model of the random walk. Coming to the end of this research some other similar questions arrive. Can these models "beat" the random walk in longer horizons than a day? What if instead of the random walk the historical average is used as a benchmark model? It would be very interesting if we also expand this analysis in other markets, e.g. CAC 40, DAX and FTSE 100. All these issues could be the subjects for future dissertation researches.

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Appendix A: Extracting the fair value of the future realized variance

We provide the proof of the equation 1.12, in line with Demeterfi et al. (1999).

First, we assume that the S_t follows the below Itô process:

$$\frac{dS_t}{S_t} = \mu(S,t)dt + \sigma(S,t)dZ_t \quad (A.1)$$

where S_0 is the current price of the underlying asset, $C(T,K)$ and $P(T,K)$ denote the current fair value of a call and a put respectively, K is the strike price of the options, r is the risk-free interest rate and S_* is an arbitrary price for the asset which usually is set equal (or close) to the forward price.

By applying Itô lemma to the $\ln(S_t)$ we have:

$$d(lnS_t) = \left[S_t\mu_t\frac{1}{S_t} - \frac{1}{2}S_t^2\sigma_t^2\frac{1}{S^2}\right]dt + S_t\sigma_t\frac{1}{S_t}dZ_t \implies \quad (A.2)$$

$$d(lnS_t) = \left(\mu_t - \frac{1}{2}\sigma_t^2\right)dt + \sigma_t dZ_t, \quad (A.3)$$

where μ_t and σ_t^2 are the instantaneous mean and variance at time t.

Subtracting A.3 from A.1 we obtain:

$$\frac{dS_t}{S_t} - d(lnS_t) = \frac{1}{2}\sigma_t^2 \quad (A.4)$$

The average variance from t=0 to t=T is:

$$V_t = \frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\int_0^T \frac{dS_t}{S_t} - \int_0^T dlnS_t \implies \tag{A.5}$$

$$V_t = \frac{2}{T}\int_0^T \frac{dS_t}{S_t} - ln\frac{S_T}{S_0} \tag{A.6}$$

The strike price (K_{VAR}) of the variance swap is defined as the expectation of this variance under the risk-neutral measure:

$$K_{VAR} = E\left(\frac{2}{T}\int_0^T \frac{dS_t}{S_t} - ln\frac{S_T}{S_0}\right) \implies \tag{A.7}$$

$$K_{VAR} = \frac{2}{T}\left(rT - E\left[ln\left(\frac{S_T}{S_0}\right)\right]\right), \tag{A.8}$$

$E\left[ln\left(\frac{S_T}{S_0}\right)\right]$ is the expected payoff from a log contract and can be replicated by using a forward contract and a set of put and call options at the same expiration (see Carr and Madam, 1998). At this point Demeterfi et al. introduce a new arbitrary parameter S_* to define the boundary between the calls and the puts. Thus, the log payoff can be written as:

$$ln\left(\frac{S_T}{S_0}\right) = ln\left(\frac{S_T}{S_*}\right) - ln\left(\frac{S_*}{S_0}\right). \tag{A.9}$$

The second term of equation A.9 is a independent constant of the S_T hence we have to replicate only the first term as follows:

$$-ln\frac{S_T}{S_*} = -\frac{S_T - S_*}{S_*} + \int_0^{S_*} \frac{1}{K^2}(K - S_T)^+ dK + \int_{S_*}^{\infty} \frac{1}{K^2}(S_T - K)^+ dK \implies \tag{A.10}$$

$$-ln\frac{S_T}{S_*} = -\frac{S_T - S_*}{S_*} + \int_0^{S_*} \frac{1}{K^2}(K - S_T)^+ dK + \int_{S_*}^{\infty} \frac{1}{K^2}(S_T - K)^+ dK \implies \tag{A.11}$$

$$-E\left[ln\frac{S_T}{S_*}\right] = -ln\frac{S_*}{S_0} - \frac{E(S_T) - S_*}{S_*}\int_0^{S_*} \frac{1}{K^2}[E(K - S_T)^+]dK + \int_{S_*}^{\infty} \frac{1}{K^2}[E(S_T - K)^+]dK \implies \tag{A.12}$$

$$-E\left[ln\frac{S_T}{S_*}\right] = -ln\frac{S_*}{S_0} - \left(\frac{S_0}{S_*}e^{rT} - 1\right) + e^{rT}\left(\int_0^{S_*} \frac{1}{K^2}[E(K - S_T)^+]dK \quad + \int_{S_*}^{\infty} \frac{1}{K^2}[E(S_T - K)^+]dK\right) \tag{A.13}$$

Thus, the equation A.8 can be written as:

$$K_{VAR} = \frac{2}{T}\left[rT - \left(\frac{S_0}{S_*}e^{rT} - 1\right) - ln\left(\frac{S_*}{S_0}\right) + e^{rT}\left(\int_0^{S_*} \frac{1}{K^2}P(T,K)dK + \int_{S_*}^{\infty} \frac{1}{K^2}C(T,K)dK\right)\right] \tag{A.14}$$

This is the equation 26 from Demeterfi et al. and is the equation 1.12 of this dissertation.

Appendix B: Proof of the equivalence of the model-free implied variance and the fair value of the future variance

Following Jiang and Tian (2007) we provide a proof for the equivalence of the model-free implied variance and the fair value of the future variance. The model-free equation of the implied variance of Britten-Jones and Neuberger (2000) can be written as Jiang and Tian (2005) has shown:

$$V_{BN} = \frac{2}{T}\int_0^{\infty} \frac{e^{rT}C(T,K) - (S_0 e^{rT} - K)^+}{K^2} dK. \tag{B.1}$$

Partitioning the integral into two parts at $F_0 = S_0 e^{rT}$ the equation B.1 become:

$$V_{BN} = \frac{2e^{rT}}{T}\left[\int_0^{S_0 e^{rT}} \frac{C(T,K) - (S_0 - Ke^{-rT})^+}{K^2} dK + \int_{S_0 e^{rT}}^{\infty} \frac{C(T,K) - (S_0 - Ke^{-rT})^+}{K^2} dK\right] \Rightarrow \tag{B.2}$$

$$V_{BN} = \frac{2e^{rT}}{T}\left[\int_0^{F_0} \frac{C(T,K) - (S_0 - Ke^{-rT})^+}{K^2} dK + \int_{F_0}^{\infty} \frac{C(T,K)}{K^2} dK\right]. \tag{B.3}$$

Using the put-call parity ($C + Ke^{-rT} = P + S_0$) the equation B.3 can be written as:

$$V_{BN} = \frac{2e^{rT}}{T}\left[\int_0^{F_0} \frac{P(T,K)}{K^2} dK + \int_{F_0}^{\infty} \frac{C(T,K)}{K^2} dK\right], \tag{B.4}$$

which is equation 1.13.

The equation B.4 can be written using basic calculus as:

$$V_{BN} = \frac{2e^{rT}}{T}\left[\int_0^{S_*} \frac{P(T,K)}{K^2}dK + \int_{S_*}^{\infty} \frac{C(T,K)}{K^2}dK + \int_{S_*}^{F_0} \frac{P(T,K) - C(T,K)}{K^2}dK\right], \quad (B.5)$$

where $F_0 > S_*$.

Using the put-call parity again B.5 can be written as:

$$V_{BN} = \frac{2e^{rT}}{T}\left[\int_0^{S_*} \frac{P(T,K)}{K^2}dK + \int_{S_*}^{\infty} \frac{C(T,K)}{K^2}dK + \int_{S_*}^{F_0} \frac{Ke^{-rT} - S_0}{K^2}dK\right] \quad (B.6)$$

After integrating the third term inside the bracket, the equation B.7 is written as:

$$V_{BN} = \frac{2}{T}\left\{rT - \left[\frac{S_0}{S_*}e^{rT} - 1\right] - ln\left(\frac{S_*}{S_0}\right) + \; e^{rT}\left[\int_0^{S_*} \frac{P(T,K)}{K^2}dK + \int_{S_*}^{\infty} \frac{C(T,K)}{K^2}dK\}\right]\right\}, \quad (B.7)$$

this is equation is the same with equation 26 of Demeterfi et al. and with equation 1.12 of this dissertation.

Appendix C: Predictor Variables Summary Statistics and Time Series Plots

Table C.1: Summary statistics of the predictor variables and their components

	Mean	Max	Min	Std. Dev	Skew	Kurt	ADF
Panel A: TED Spread							
	Mean	***Max***	***Min***	***Std. Dev***	***Skew***	***Kurt***	***ADF***
3-month LIBOR	0.0175	0.0573	0.0022	0.0173	1.03	2.66	–
3-month T-Bill	0.0133	0.0505	−0.0002	0.0160	1.12	2.90	–
TED	0.0043	0.0458	0.0009	0.0043	3.79	22.63	–
Panel B: Index Returns							
	Mean	***Max***	***Min***	***Std. Dev***	***Skew***	***Kurt***	***ADF***
SPX	0.0368	2.32	−4.05	0.56	−1.12	7.09	–
DJX	0.0426	2.18	−3.42	0.53	−1.31	8.26	–
NDX	0.0522	3.23	−4.27	0.73	−1.09	6.65	–
Panel C: Credit Spread							
	Mean	***Max***	***Min***	***Std. Dev***	***Skew***	***Kurt***	***ADF***
Baa	0.061	0.095	0.042	0.011	0.351	2.4	–
Aaa	0.050	0.074	0.032	0.010	0.163	2.3	–
CS	0.011	0.035	0.005	0.005	2.857	13.0	–

The entries report summary statistics for the TED Spread, the Index Returns and the Credit Spread for the period from 2001 to 2017. Mean, Max, Min, Std. Dev, Skew, Kurt, report sample average, sample maximum, minimum values, sample standard deviation, sample skewness and sample kurtosis for the three variables, respectively. ADF is the value of Augmented Dickey Fuller test including in the equation both trend and intercept.

Table C.2: Summary statistics of the predictor variables and their components

	Mean	Max	Min	Std. Dev	Skew	Kurt	ADF
Panel D: Term Spread							
10-year T-Bond	*0.034*	*0.055*	*0.014*	*0.011*	*−0.041*	*1.68*	−
1-month LIBOR	*0.016*	*0.058*	*0.001*	*0.018*	*1.051*	*2.69*	−
TS	*0.018*	*0.038*	*−0.015*	*0.012*	*2.637*	*2.64*	−
Panel E: Put-Call Ratio							
	Mean	**Max**	**Min**	**Std. Dev**	**Skew**	**Kurt**	**ADF**
SPX	1.73	12.3	0.099	0.56	2.9	38	*−8.54*
DJX	1.68	48.0	0.046	1.93	8.7	145	*−17.39*
NDX	1.58	35.9	0.001	1.25	8.8	171	*−9.55*
Panel F: Squared Index Volatility							
	Mean	**Max**	**Min**	**Std. Dev**	**Skew**	**Kurt**	**ADF**
VIX^2	0.047	0.654	0.009	0.055	4.5	32.3	*−4.71*
VXD^2	0.041	0.557	0.006	0.046	4.3	30.6	*−4.69*
VXN^2	0.079	0.650	0.011	0.088	2.4	9.8	*−4.72*

The entries report summary statistics for the TED Spread, the Index Returns and the Credit Spread for the period from 2001 to 2017. Mean, Max, Min, Std. Dev, Skew, Kurt, report sample average, sample maximum, minimum values, sample standard deviation, sample skewness and sample kurtosis for the three variables, respectively. ADF is the value of Augmented Dickey Fuller test including in the equation both trend and intercept.

Table C.3: Correlation Matrix for all Period

Panel A: SPX VRP Predictor Variables' Correlation Matrix						
	TED	R^{SPX}	***CS***	***TS***	PC^{SPX}	VIX^2
TED	1	−0.31	0.50	−0.48	0.02	0.55
R^{SPX}		1	−0.18	0.11	−0.04	−0.50
CS			1	0.1	−0.12	0.75
TS				1	−0.08	0.07
PC^{SPX}					1	−0.08
VIX^2						1
Panel B: DJX VRP Predictor Variables' Correlation Matrix						
	TED	R^{DJX}	***CS***	***TS***	PC^{DJX}	VXD^2
TED	1	−0.28	0.50	−0.48	−0.02	0.53
R^{DJX}		1	−0.17	0.08	0.01	−0.45
CS			1	0.1	0.00	0.73
TS				1	0.04	0.07
PC^{DJX}					1	0.05
VXD^2						1
Panel C: NDX VRP Predictor Variables' Correlation Matrix						
	TED	R^{NDX}	***CS***	***TS***	PC^{NDX}	VXN^2
TED	1	−0.27	0.50	−0.48	−0.08	0.30
R^{NDX}		1	−0.13	0.11	0.02	−0.38
CS			1	0.10	−0.04	0.46
TS				1	0.11	0.10
PC^{NDX}					1	−0.08
VXN^2						1

This table displays the pairwise correlation of the predictor variables considered for each model. To calculate the pairwise correlation all data available from 2001 to 2017 are used.

TED Spread and its components time series plots

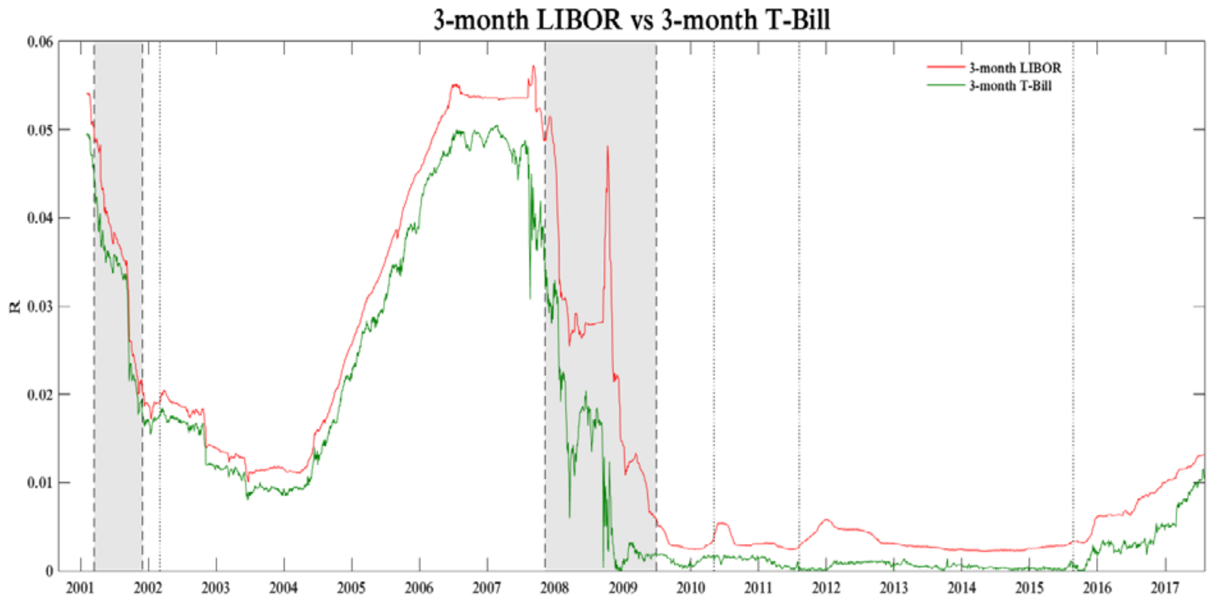

Figure C.1: The figure plots the 3-month LIBOR and the 3-month T-Bill from 2001 to 2017. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015. The plots are based on daily observations.

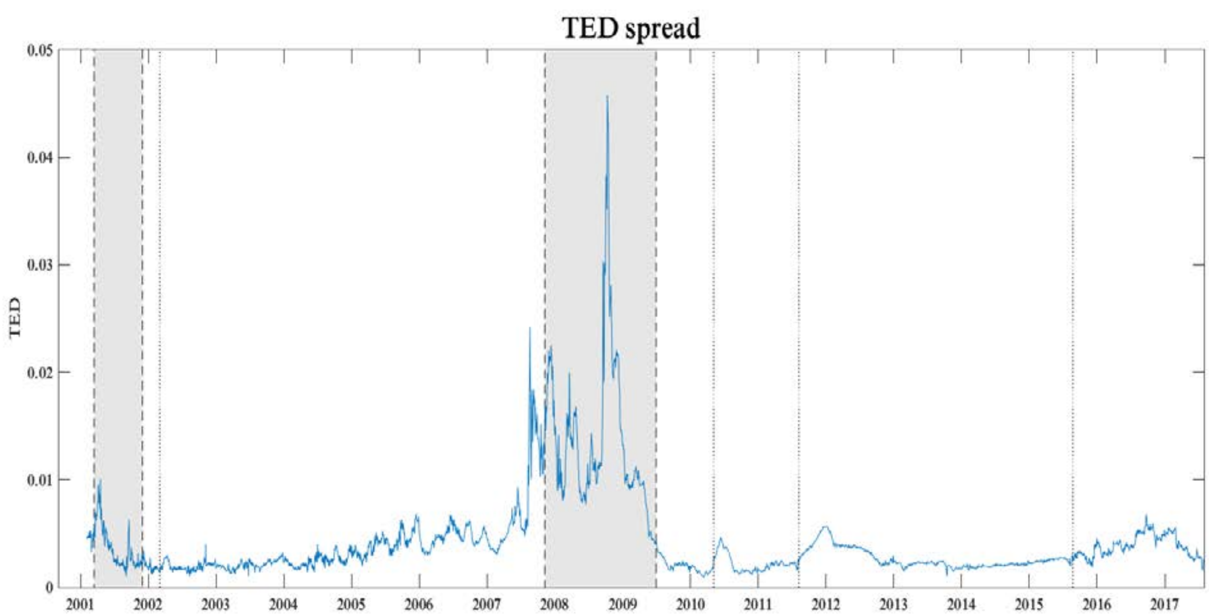

Figure C.2: The figure plots the TED spread, which is defined as the difference between the 3-month LIBOR and the 3-month T-Bill, from 2001 to 2017.

Time series plots of the three index returns

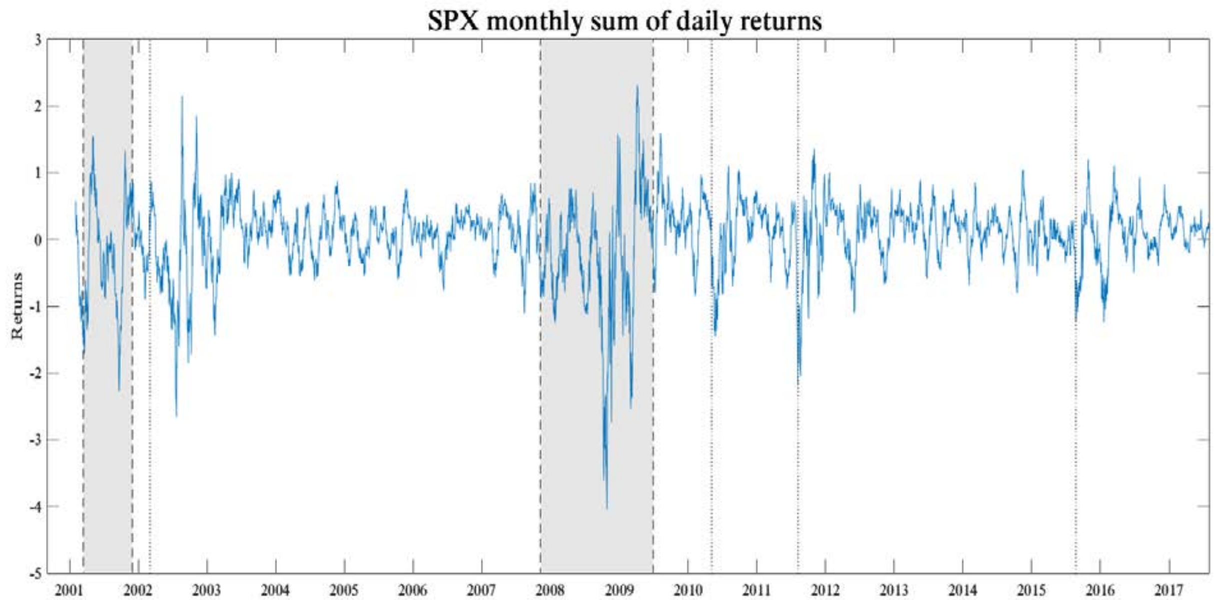

Figure C.3: This figure plots for each day t the sum of the SPX daily returns from the day t to t−21, from 2001 to 2017. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015.

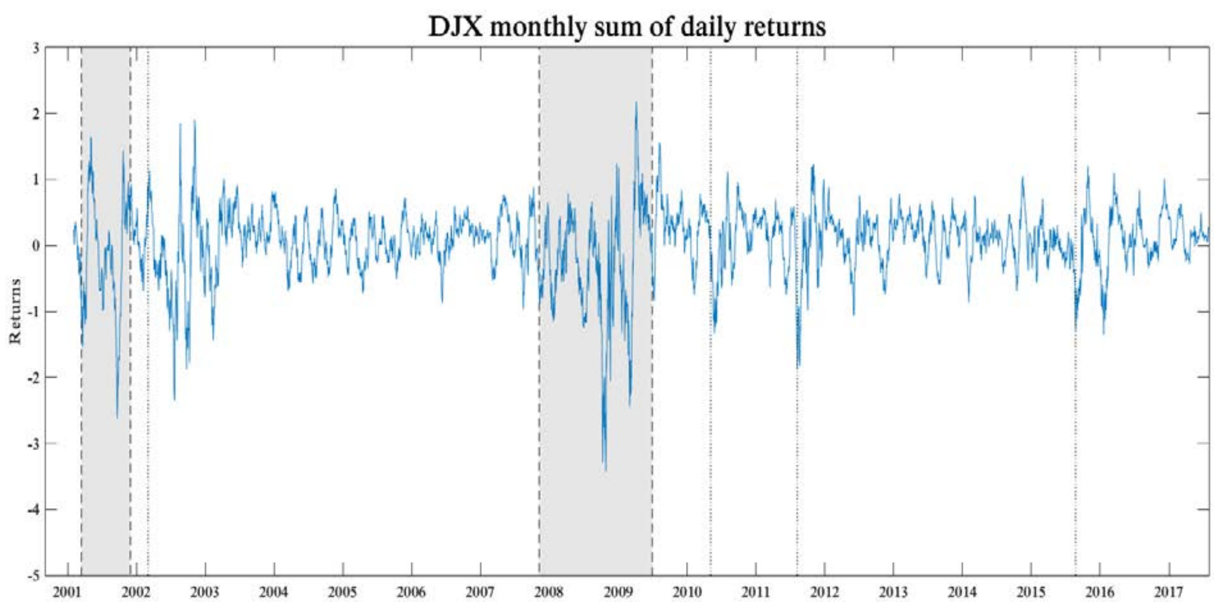

Figure C.4: This figure plots for each day t the sum of the DJX daily returns from the day t to t−21, from 2001 to 2017.

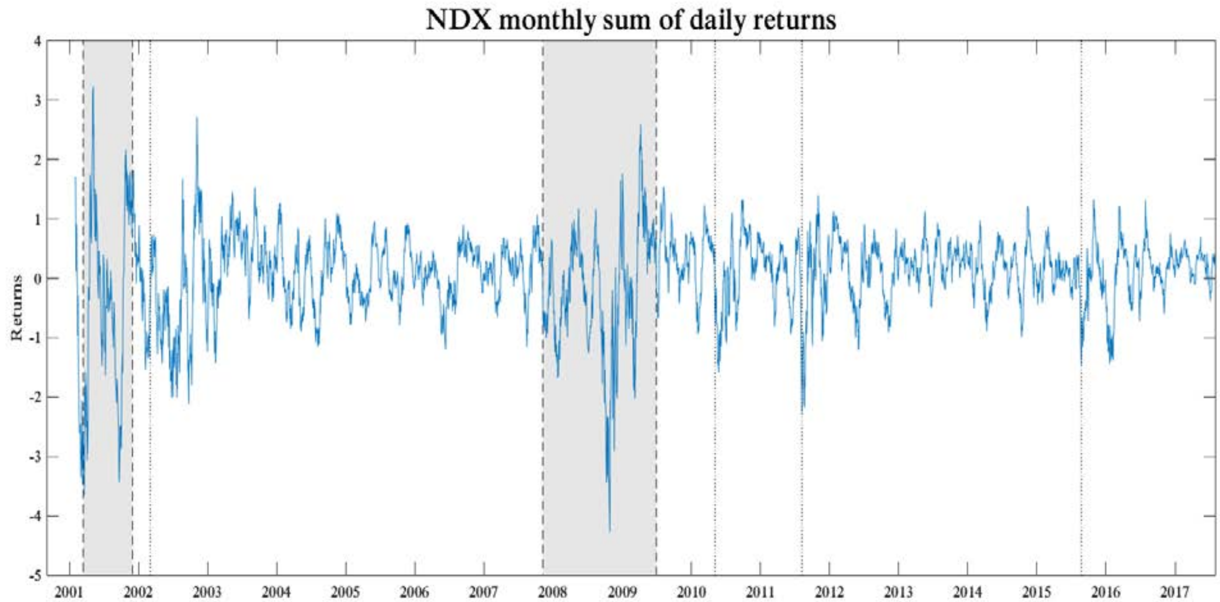

Figure C.5: This figure plots for each day t the sum of the NDX daily returns from the day t to t−21, from 2001 to 2017.

Credit Spread and its components time series plots

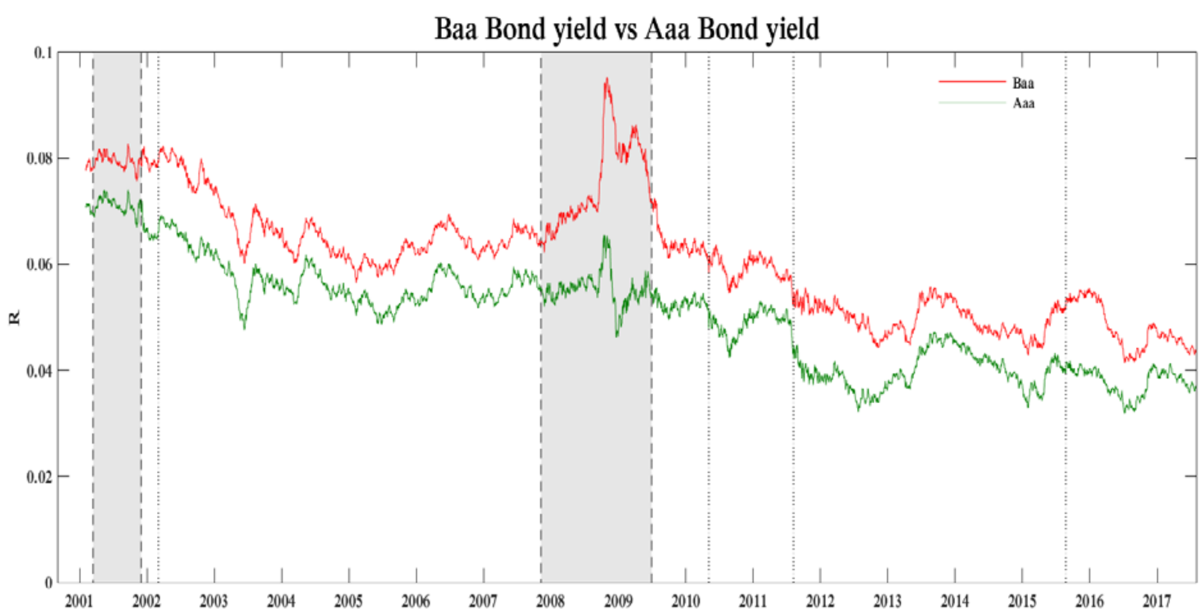

Figure C.6: The figure shows the time series plots of Moody's Seasoned Baa Corporate Bond Yield and Moody's Seasoned Aaa Corporate Bond Yield, from 2001-2017. The time *series plotted are constructed from daily data. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015.*

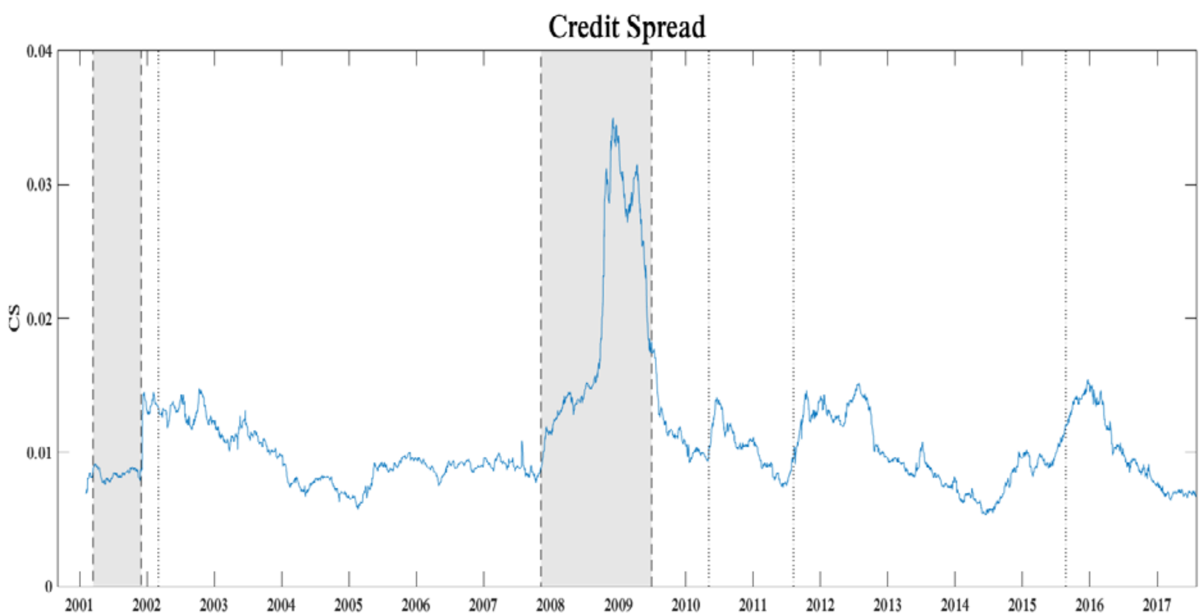

Figure C.7: This figure shows the time series plot of the credit spread, which is defined as the difference between Moody's Seasoned Baa Corporate Bond Yield and Moody's Seasoned Aaa Corporate Bond Yield, from 2001 to 2017 based on daily data.

<u>Term Spread and its components time series plots</u>

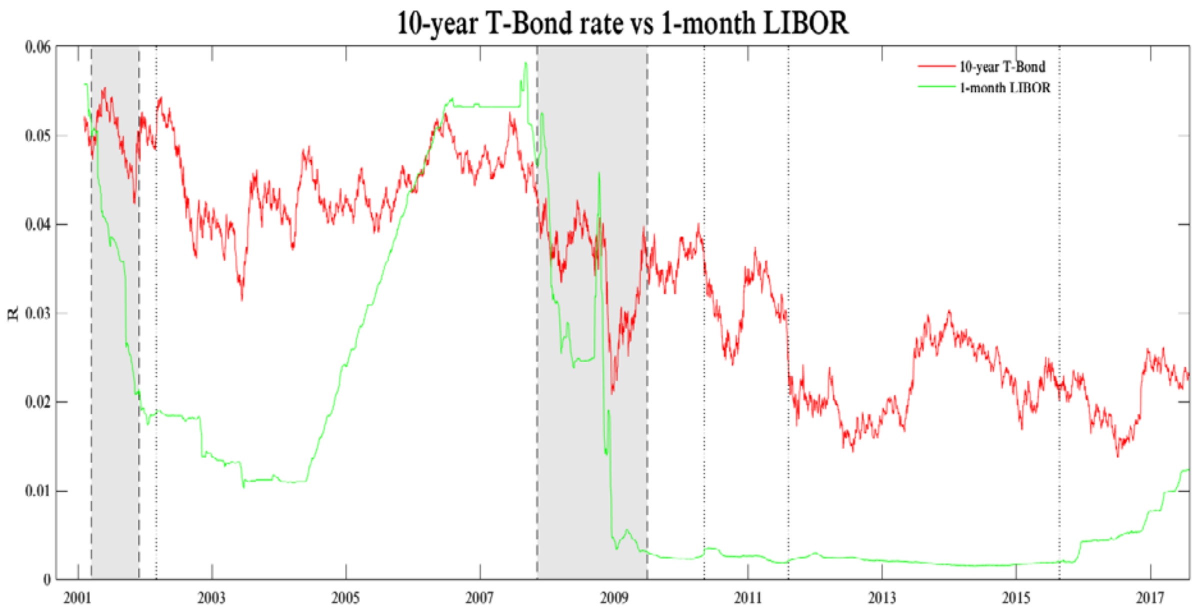

Figure C.8: This figure plots the 10-year T-Bond and the 1-month LIBOR based on daily observations from 2001 to 2017. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015.

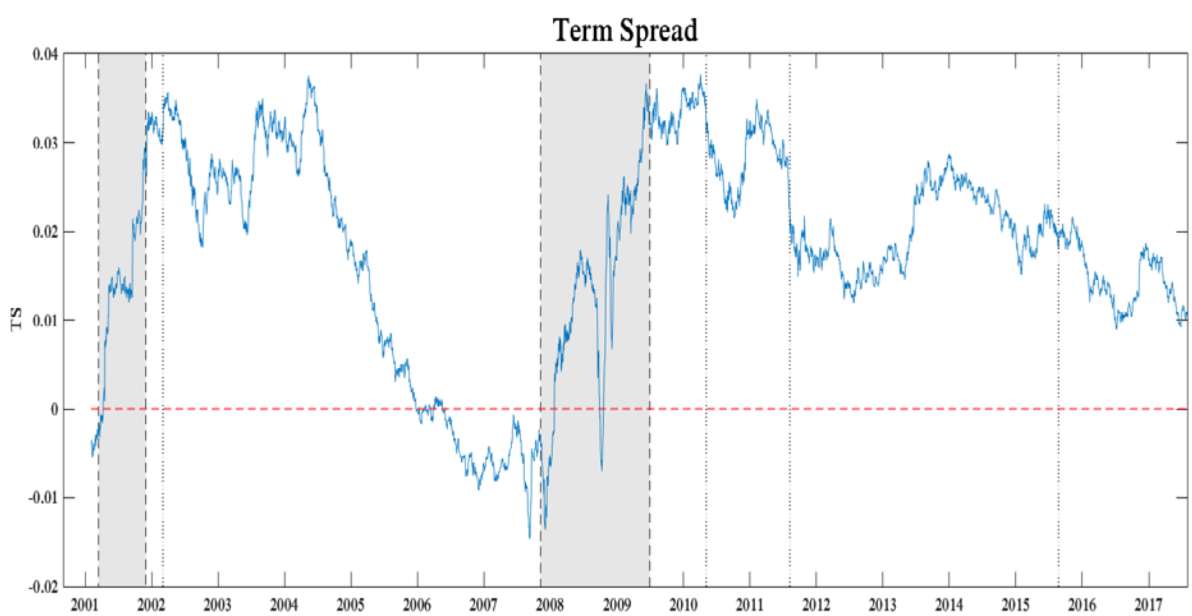

Figure C.9: The figure plots the difference between the 10-year T-Bond and the 1-month LIBOR, which defines the term spread, from 2001 to 2017 based on daily observations.

<u>Time series plots of the put-call ratio for the three indices</u>

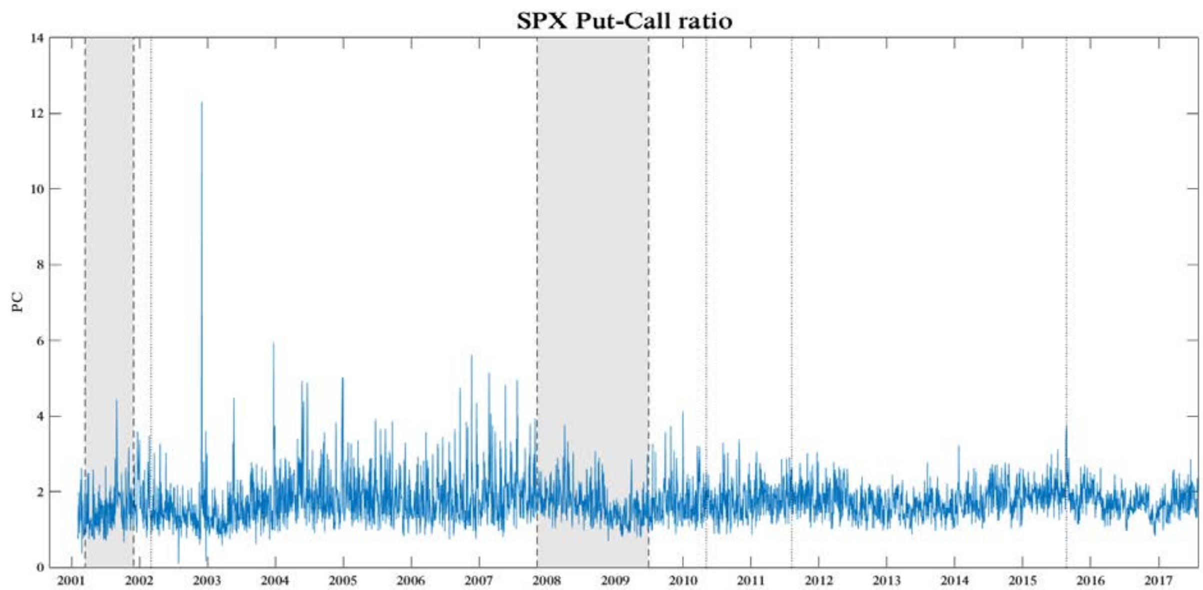

Figure C.10: This figure plots the SPX put-call ratio from 2001 to 2017 based on daily data. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015.

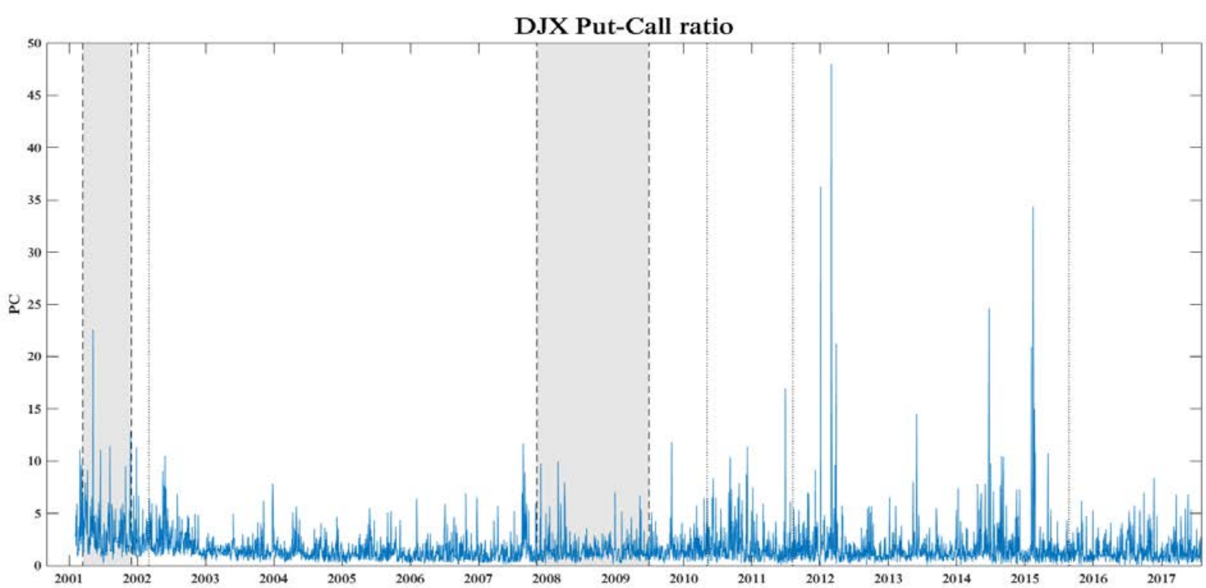

Figure C.11: The figure shows the plot of the DJX put-call ratio from 2001 to 2017 based on daily data. First and second shaded areas represent the Early 2000s Recession and the Great Recession, respectively.

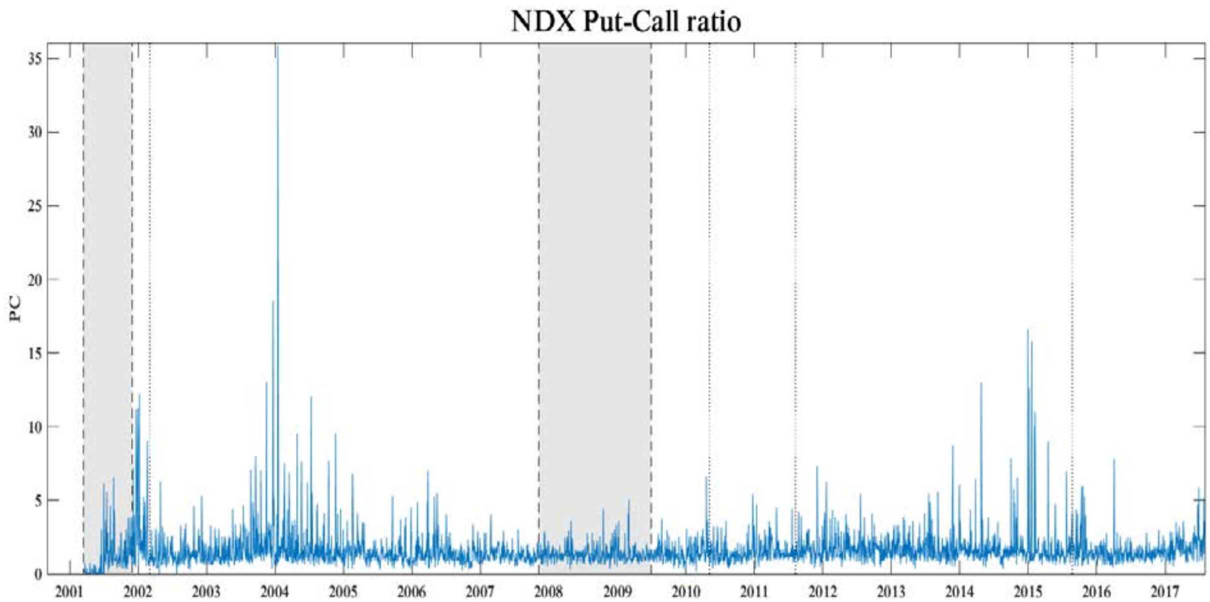

Figure C.12: The figure displays the time series plot of the NDX put-call ratio from 2001 to 2017 based on daily data.

<u>Time series plots for the three market index volatilities and the closing price of the three indices</u>

Figure C.13: The figure plots daily closing price on the SPX and daily squared closing price on the VIX from 2001 to 2017. First and second shaded areas represent the Early 2000s

Recession and the Great Recession, respectively. The four vertical dotted line represent, Stock Market Downturn of 2002 (4 March 2002), Flash Crash (6 May,2010), Black Monday (8 August 2011) and the stock market crash of the 24 August 2015.

Figure C.14: This figure plots daily closing price on the DJX and daily squared closing price on the VXD from 2001 to 2017.

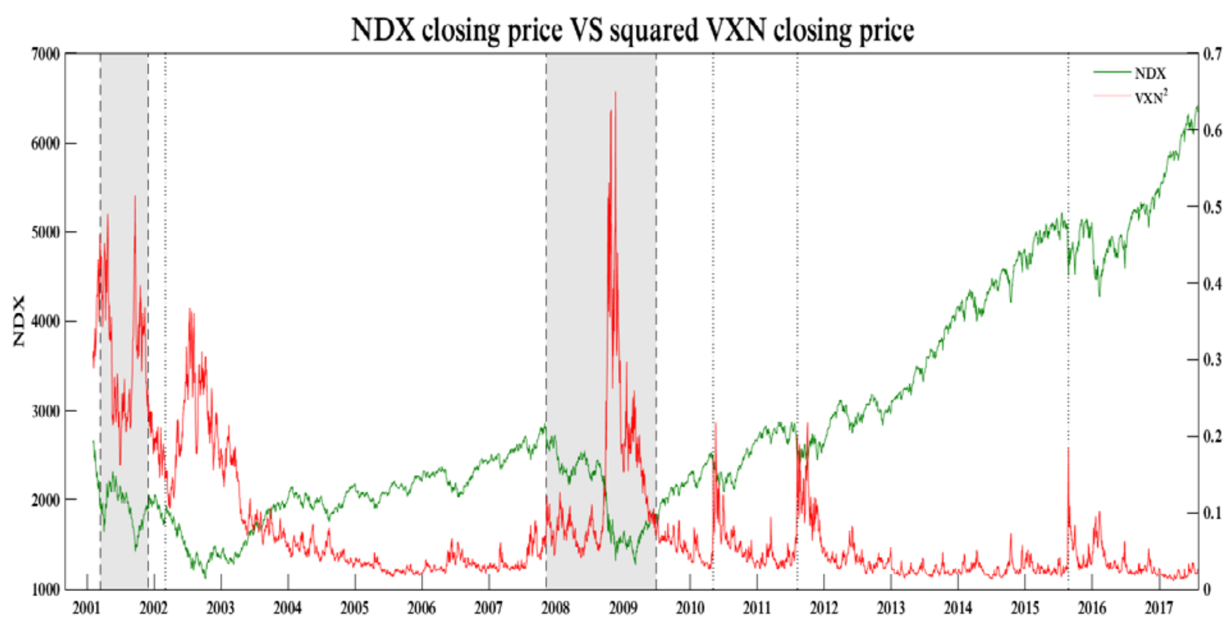

Figure C.15: The figure plots daily closing price on the NDX and daily squared closing price on the VXN from 2001 to 2017.

Appendix D: Alternative Predictor Model OLS Results

Table D.1: Six Predictor Variable Model OLS Results

Panel A: SPX VRP 6 Predictor Model Results								
	C	**TED**	**R^{SPX}**	**CS**	**TS**	**PC^{SPX}**	**VIX^2**	**Adj. R^2**
Coeff.	-0.0081	-1.7277	-0.0159	2.7610	-0.3102	-0.0007	-0.4677	0.33
NW	(-0.64)	(-2.39)	(-2.96)	(1.95)	(-2.50)	(-0.81)	(-5.51)	$-$
HH	(-0.55)	(-2.21)	(-2.56)	(1.62)	(-2.18)	(-0.73)	(-4.71)	$-$
W	(-2.17)	(-6.37)	(-10.21)	(7.11)	(-8.40)	(-1.24)	(-17.30)	$-$
Panel B: DJX VRP 6 Predictor Model Results								
	C	***TED***	***R^{DJX}***	***CS***	***TS***	***PC^{DJX}***	***VXD^2***	***Adj. R^2***
Coeff.	0.0006	-2.5417	-0.0153	1.8426	-0.4056	0.0019	-0.4409	0.27
NW	(0.04)	(-2.12)	(-2.94)	(1.21)	(-2.61)	(2.40)	(-5.06)	$-$
HH	(0.04)	(-1.98)	(-2.62)	(1.05)	(-2.26)	(2.25)	(-4.59)	$-$
W	(0.14)	(-6.80)	(-9.59)	(4.37)	(-9.14)	(4.33)	(-16.07)	$-$
Panel C: VXD VRP 6 Predictor Model Results								
	C	***TED***	***R^{NDX}***	***CS***	***TS***	***PC^{NDX}***	***VXN^2***	***Adj. R^2***
Coeff.	-0.0154	1.0753	-0.0179	2.3995	-0.0316	-0.0007	-0.5709	0.79
NW	(-1.03)	(0.68)	(-3.82)	(1.66)	(-0.15)	(-0.99)	(-11.71)	$-$
HH	(-0.97)	(0.65)	(-3.50)	(1.59)	(-0.14)	(-0.95)	(-11.95)	$-$
W	(-3.26)	(2.00)	(-11.15)	(5.18)	(-0.48)	(-1.15)	(-34.00)	$-$

In this table are provided the in-sample (2001-2007) OLS results from the models using all the six variables considered. Under the coefficients of the predictors, in the parenthesis, are provided the t-statistics NW, HH and W which are calculated using the Newey-West, Hansen-Hodrick and White approach respectively. The Adj. R^2 stands for the adjusted R^2 of the OLS regression.

Table D.2: OLS Model Fitting Criteria

	SE	SSR	F	AIC
Panel A: SPX VRP Model Fitting Criteria				
SPX 5V Model	0.0194	0.6139	130.6	−5.05
SPX 6V Model	0.0188	0.5796	131.3	−5.10
Panel B: DJX VRP Model Fitting Criteria				
	SE	**SSR**	**F**	**AIC**
DJX 5V Model	0.0210	0.7187	112.8	−4.89
DJX 6V Model	0.0208	0.7042	101.5	−4.91
Panel C: NDX VRP Model Fitting Criteria				
	SE	**SSR**	**F**	**AIC**
NDX 5V Model	0.0276	1.2214	1279	−4.34
NDX 6V Model	0.0276	1.2213	1065	−4.34

Table D.2 display the main statistics criteria used to evaluate the five variables and the six variables OLS models. SE stands for the standard error of the regression model, SSR is the sum of squared residuals, F is the value of the F-statistics and AIC stands for the Akaike Information Criterion.

Appendix E: Robustness Test Tables of In-Sample and Out-of-Sample Results

First robustness test: In-Sample from 2001 to 2009 and Out-of-Sample from 2009 to 2017

Table E.1: 5 Predictor Variable Model OLS Results

			Panel A: SPX VRP 5 Predictor Model Results				
	C	***TED***	***R^{SPX}***	***TS***	***PC^{SPX}***	***VIX^2***	***Adj. R^2***
Coeff.	−0.0282	7.4731	−0.0268	0.9496	−0.0026	−0.5605	0.26
NW	(−1.93)	(2.01)	(−3.30)	(1.72)	(−1.19)	(−2.96)	−
HH	(−1.65)	(1.63)	(−2.78)	(1.39)	(−1.06)	(−2.56)	−
W	(−5.83)	(7.78)	(−10.80)	(6.54)	(−1.59)	(−10.18)	−
			Panel B: DJX VRP 5 Predictor Model Results				
	C	***TED***	***R^{DJX}***	***TS***	***PC^{DJX}***	***VXD^2***	***Adj. R^2***
Coeff.	−0.0254	5.9522	−0.0234	0.6645	0.0008	−0.5178	0.25
NW	(−1.72)	(1.90)	(−3.57)	(1.52)	(0.76)	(−3.08)	−
HH	(−1.43)	(1.54)	(−3.07)	(1.24)	(0.69)	(−2.69)	−
W	(−6.33)	(7.43)	(−11.36)	(5.71)	(1.42)	(−10.43)	−
			Panel C: VXD VRP 5 Predictor Model Results				
	C	***TED***	***R^{NDX}***	***CS***	***PC_{NDX}***	***VXN^2***	***Adj. R^2***
Coeff.	−0.0291	5.4029	−0.0215	1.7212	0.0001	−0.5527	0.54
NW	(−3.12)	(2.41)	(−4.88)	(1.75)	(0.18)	(−14.70)	−
HH	(−3.69)	(2.06)	(−4.41)	(1.81)	(0.21)	(−16.12)	−
W	(−8.59)	(8.55)	(−13.82)	(5.29)	(0.24)	(−37.49)	−

In this table, the in-sample (2001-2009) OLS results are provided. For the SPX VRP and the DJX VRP the credit spread (CS) is omitted and the term spread (TS) is omitted for the NDX VRP. Coeff. stands for the coefficients of the predictor variables. NW and HH are the t-statistics computed with Newey-West and Hansen-Hodrick approach respectively and W stands for the White robustness t-statistics. Adj. R^2 is the adjusted R^2.

Table E.2: The Out-of-Sample Evaluation Criteria

Panel A: SPX VRP Out-of-Sample Results						
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
SPX Model	0.512**	0.79***	0.53	0.89	0.0253**	0.0146***
SPX RW	–	–	–	–	0.0362	0.0193
Panel B: NDX VRP Out-of-Sample Results						
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
DJX Model	0.500**	0.79***	0.54	0.90	0.0205**	0.0123***
DJX RW	–	–	–	–	0.0290	0.0161
Panel C: VXD VRP Out-of-Sample Results						
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
NDX Model	*0.138*	*0.76****	*0.57*	*0.99*	*0.0300*	*0.0170*
NDX RW	–	–	–	–	*0.0324*	*0.0176*

Entries report the out-of-sample R^2, the Mean Correct Prediction(MCP), the Theil's UI and Theil's UII. One, two and three asterisks in the parenthesis under the R^2 denote the rejection of the null hypothesis H_0, the predictor model doesn't outperform the benchmark model against the alternative hypothesis that the predictor model outperforms the benchmark model at 10%, 5% and 1% respectively. One, two and three asterisks in the parenthesis under the MPC denote the rejection of the null hypothesis that the proportion ratio is equal or lower to 50% against the alternative that is higher than 50%. The root mean squared error (RMSE) and the mean absolute error (MAE) of the predictor model and the naïve model (random walk) are also provided. One, two and three asterisks denote the rejection of the null hypothesis H_0 the predictor model doesn't outperform the benchmark model against the alternative hypothesis that the predictor model outperforms the benchmark model at 10%, 5% and 1% respectively. The out-of-sample data span from 8/2009 to 7/2017.

<u>Second robustness test: In-Sample from 2001 to 2011 and Out-of-Sample from 2012 to 2017</u>

Table E.3: 5 Predictor Variable Model OLS Results

			Panel A: SPX VRP 5 Predictor Model Results				
	C	***TED***	R^{SPX}	***TS***	PC^{SPX}	VIX^2	***Adj.*** R^2
Coeff.	−0.0312	7.4655	−0.0212	0.9841	−0.0009	−0.5455	0.24
NW	(−2.08)	(2.09)	(−2.99)	(1.92)	(−0.49)	(−3.27)	–
HH	(−1.75)	(1.68)	(−2.50)	(1.55)	(−0.44)	(−2.84)	–
W	(−6.61)	(8.14)	(−9.93)	(7.32)	(−0.67)	(−11.31)	–
			Panel B: DJX VRP 5 Predictor Model Results				
	C	***TED***	R^{DJX}	***TS***	PC^{DJX}	VXD^2	***Adj.*** R^2
Coeff.	−0.0252	6.01	−0.0190	0.7082	0.0004	−0.5137	0.24
NW	(−1.74)	(1.97)	(−3.26)	(1.72)	(0.52)	(−3.38)	–
HH	(−1.43)	(1.59)	(−2.77)	(1.41)	(0.47)	(−2.97)	–
W	(−6.49)	(7.74)	(−10.58)	(6.57)	(0.93)	(−11.54)	–
			Panel C: VXD VRP 5 Predictor Model Results				
	C	***TED***	R^{NDX}	***CS***	PC_{NDX}	VXN^2	***Adj.*** R^2
Coeff.	−0.0217	5.1057	−0.0184	1.6791	0.0001	−0.5637	0.49
NW	(−2.25)	(2.27)	(−4.61)	(1.72)	(0.12)	(−14.29)	–
HH	(−2.37)	(1.89)	(−4.11)	(1.74)	(0.14)	(−14.42)	–
W	(−6.72)	(8.36)	(−13.18)	(5.37)	(0.16)	(−39.74)	–

In this table, the in-sample (2001-2011) OLS results are provided. For the SPX VRP and the DJX VRP the credit spread (CS) is omitted and the term spread (TS) is omitted for the NDX VRP. Coeff. stands for the coefficients of the predictor variables. NW and HH are the t-statistics computed with Newey-West and Hansen-Hodrick approach respectively and W stands for the White robustness t-statistics. Adj. R^2 *is the adjusted* R^2.

Table E.4: The Out-of-Sample Evaluation Criteria

		Panel A: SPX VRP Out-of-Sample Results				
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
SPX Model	0.45*	0.76***	0.52	0.90	0.0151*	0.0109*
SPX RW	–	–	–	–	0.0203	0.0127
		Panel B: NDX VRP Out-of-Sample Results				
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
DJX Model	0.45*	0.77***	0.54	0.92	0.0134*	0.0097
DJX RW	–	–	–	–	0.0179	0.0111
		Panel C: VXD VRP Out-of-Sample Results				
	OS R^2	***MCP***	***UI***	***UII***	***RMSE***	***MAE***
NDX Model	*0.199**	*0.72****	*0.54*	*0.92*	*0.0169**	*0.0126*
NDX RW	–	–	–	–	*0.0189*	*0.0129*

Entries report the out-of-sample R^2, the Mean Correct Prediction(MCP), the Theil's UI and Theil's UII. One, two and three asterisks in the parenthesis under the R^2 denote the rejection of the null hypothesis H_0, the predictor model doesn't outperform the benchmark model against the alternative hypothesis that the predictor model outperforms the benchmark model at 10%, 5% and 1% respectively. One, two and three asterisks in the parenthesis under the MPC denote the rejection of the null hypothesis that the proportion ratio is equal or lower to 50% against the alternative that is higher than 50%. The root mean squared error (RMSE) and the mean absolute error (MAE) of the predictor model and the naïve model (random walk) are also provided. One, two and three asterisks denote the rejection of the null hypothesis H_0 the predictor model doesn't outperform the benchmark model against the alternative hypothesis that the predictor model outperforms the benchmark model at 10%, 5% and 1% respectively. The out-of-sample data span from 1/2012 to 7/2017.