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Implied convenience yield and its properties

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December 21, 2011

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Abstract

In this thesis we calculate a term structure of the convenience yield for a number of commodities using observed futures prices. This shows how the market values the convenience yield. Then we assume that the spot price and the convenience yield of a commodity follow a joint stochastic process of the CIR type and use the Kalman filter to make an approximation of the convenience yield. Finally we empirically test the storage theory by researching the relationship between basis, convenience yield and inventories.

Keywords: convenience yield, term structure, kalman filter, stochastic process, theory of storage, basis

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Chapter 1

Introduction

1.1 Facts about commodities

Commodity markets have experienced dramatic changes in recent years, concerning the trading volume, the variety of available derivative instruments and the range of underlying commodities. The volatility of supply and prices has increased as a result of political upheavals in some countries, economic mutation, new environmental regulation, a huge rise in the consumption of commodities in countries such as India, Brazil and China and other structural changes. Nowadays commodity price risk is an important element of the world economy, as it has an important effect on the economy of both developed and developing countries. This fact has made hedging activities (through derivative instruments) of great importance for many sectors of the economy. For all these reasons there is a widespread interest in models for pricing and hedging commodity-linked contingent claims. The fact that any transaction on commodities may be physical (delivery of the commodity) or financial (a cash flow from one party to the other at maturity and no exchange of the underlying good) is in sharp contrast to bonds and stock markets where all trades are financial. Instead physical and financial commodity markets present strong correlation. Price and volatility observed in financial transactions are related to the analogous quantities in the physical market, both because of the physical delivery that may take place at maturity of a futures contract and the existence of a relationship between spot and futures contracts. Besides, commodities offer a variety of empirical properties, which makes them different from stocks, bonds and other conventional financial assets. Their returns are supposed to show small or negative correlation with the returns of traditional financial assets, resulting in diversification benefits. They can also be used as a shield against inflation. Specifically commodity futures present a number of interesting properties:

1. Commodity futures prices are often in backwardation, that means they decline with time to delivery.
2. Spot and futures prices are mean-reverting for many commodities.
3. Commodity prices are strongly heteroscedastic and price volatility is correlated with the degree of backwardation.
4. Unlike financial assets, many commodities have pronounced seasonalities in both price levels and volatilities.

The major commodity markets in the US are the Chicago Board of Trade (CBOT), which is the world's oldest futures and options exchange, the Chicago Mercantile Exchange (CME), the New York Mercantile Exchange (NYMEX), which is the world's largest physical commodity futures exchange and the Commodity Exchange (COMEX), which is a division of NYMEX. These four markets have merged into CME Group since 2007. The London Metal Exchange (LME) is the world's largest market in options, and futures contracts on base and other metals.

1.2 Literature review

The theory of storage of Kaldor (1939), Working (1948, 1949) and Telser (1958) has been the foundation of the theoretical explorations of futures – forward prices and convenience yield, which is defined by Brennan (1991) as the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery. Based on this theory researchers have adopted two approaches to modelling commodity prices. The first approach is mainly statistical in nature and requires an exogenous specification of the process followed by the convenience yield for a commodity e.g., Brennan and Schwartz (1985), Gibson and Schwartz (1990) and Schwartz (1997). The second strand of the literature derives the price processes endogenously in an equilibrium valuation framework with competitive storage e.g., Williams and Wright (1991), Deaton and Laroque (1992, 1996), Routledge, Seppi, and Spatt (2000).

Gibson and Schwartz (1990) introduce a two-factor model for pricing financial and real assets contingent on the price of oil. The factors are the spot oil price and the convenience yield both of which are unobservable. The convenience yield is viewed as a net dividend yield that accrues to the physical owner of the commodity. It is assumed that the spot price and the convenience yield follow a joint stochastic process with constant correlation. More specifically the spot price follows a geometric Brownian motion and the convenience yield an Ornstein-Uhlenbeck (O – U) stochastic process with mean reverting property. This model performs remarkably well for valuing short term futures contracts, but is less accurate for valuing longer term contracts. It also has a number of shortcomings. First it does not prevent the convenience yield from taking negative values, a fact that allows for arbitrage opportunities. For the preclusion of arbitrage opportunities the discounted futures prices excluding carrying costs have to be lower than the contemporaneous spot prices, which is not the case when the convenience yield is negative. Secondly it assumes that the volatility of both the spot price and the convenience yield and the correlation between them is constant. This property opposes to the theory of storage which implies that the volatility of the spot price depends on the level of the spot price and convenience yield (Fama and French 1987).

Schwartz (1997) tests the performance of three models for commodity futures pricing. The first is a one-factor model similar to the one proposed by Ross (1995) and the second is based on the Gibson and Schwartz (1990) model that has already been presented. The third model extends the Gibson and Schwartz model by assuming that the interest rates are actually following a mean reverting stochastic process, instead of being constant. The second and third models outperform the first by a significant margin for both short and long term futures contracts pricing. Models 2 and 3 give similar results for short term contracts but for long term contracts model 3 fits the data marginally better. However the extended Gibson and Schwartz model (model 3) does not override the shortcomings of the basic Gibson and Schwartz model, which we have already been referred.

Ribeiro and Hodges (2004) develop a two-factor model for commodity futures valuation, which, similar to the Gibson and Schwartz model, assumes that the spot price and the instantaneous convenience yield follow a joint stochastic process with constant correlation. But this model presents two important additions to the Gibson and Schwartz model. First it is assumed that the convenience yield follows a Cox – Ingersoll – Ross (CIR) stochastic process. This way it is ensured that the convenience yield cannot become negative, thus precluding any arbitrage opportunities. Secondly the spot price and convenience yield volatility is proportional to the square root of the instantaneous convenience yield, thus taking into account the relation between the level of the spot price and convenience yield and their volatility as implied by the theory of storage. The Ribeiro and Hodges model fits the data slightly better than the Gibson and Schwartz model but both tend to become less accurate as the maturity of the futures contracts is increased.

Fama and French (1987) examine the theory of storage for valuing commodity futures. The theory suggests that the difference between futures and spot prices equals the interest foregone in storing the commodity, plus the marginal storage cost, minus the marginal convenience yield. According to the storage theory there is a negative relation between the convenience yield and the level of inventories. They define the basis as

$$b = \frac{F(t,T) - S(t)}{S(t)} \tag{1.1}$$

and calculate the standard deviation of the basis for various commodities. It is observed that for products which present strong seasonal variation in supply and demand, such as agricultural products, the standard deviation is high. Commodities which are subject to high storage costs, such as animal products, also present high standard deviation of the basis. Both these facts are consistent with the storage theory. Because data for inventories are not widely available and the convenience yield is unobservable the basis is regressed against the interest rate and monthly seasonal dummies as a way to capture any seasonal effects on the variance of the convenience yield. The storage theory implies that the basis should have a one – for – one relationship with the interest rate. All commodities tested have a positive relation between the basis and the interest rate but metals show the strongest evidence.

Deaton and Laroque (1992) suggest a relationship between the level of inventories and the future spot price variance. This is explained by the fact that inventories are viewed as buffer stocks that dampen the effects on spot prices caused by potential shocks in supply or demand. Gorton, Hayashi and Rouwenhorst (2008) extend the Deaton and Laroque model by assuming that the level of inventories is negatively related to the required risk premium of commodity futures. This happens because futures contracts are supposed to provide insurance against price volatility. The empirical findings show that the volatility of expected future spot prices is higher when the inventories are low, which results in higher futures risk premiums. It is also implied that the basis has a positive, non linear relationship with the level of inventories.

Brooks, Lazar, Prokopczuk and Symeonidis (2011) show that scarcity, which is defined as the inverse of inventory, is informative about the shape of the futures curve, which is approximated by the interest adjusted basis. According to the storage theory the convenience yield is negatively related to the interest adjusted basis.As a result it is concluded that the convenience yield is an increasing function of scarcity, a fact that is further supported by the empirical findings. There is also empirical evidence that the spot price volatility is positively related to scarcity for the majority of commodities examined in the paper.

Chapter 2

The dataset

The dataset consists of daily observations of the closing prices of commodity futures contracts, which are provided by Bloomberg. Data from January 1991 to November 2010 are used for this thesis, so we have a wide range of observations, including bullish and bearish periods, the 2005 – 2008 commodity market boom and the recent 2008 – 2009 credit market crisis. Inventory data for the energy sector are provided by the US department of energy. Bloomberg is the source of inventory data for metals.

Crude oil

Crude oil is a naturally occurring, unrefined petroleum product composed of hydrocarbon deposits. Crude oil can be refined to produce usable products such as gasoline, diesel and various forms of petrochemicals. Physical crude oil markets are highly fluid, global and volatile. For instance, the international trade flows on which they are based are radically different from those of precious metals (gold, silver, platinum) since oil is an asset destined to be consumed, not a store of value. The global crude oil supply, at 79.5 Mb/d in 2003 – of which 27.0 Mb/d is produced by OPEC – follows demand very closely, the adjustment being made through inventories. Crude oil futures are traded on the New York Mercantile Exchange (NYMEX) under the ticker symbol CL in US dollars per barrel. Crude Oil futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are CLF, CLG, CLH, CLJ, CLK, CLM, CLN, CLQ, CLU, CLV, CLX, CLZ. Each contract has a size of 1000 barrels (42000 gallons) of crude oil. The last trading day for each contract is the third business day prior to the 25th calendar day of the month preceding the delivery month. If the 25th calendar day of the month is a non-business day, trading shall cease on the third business day prior to the business day preceding the 25th calendar day.

Heating oil

Heating oil is a refined byproduct of crude oil. After crude oil is broken down during the refinement process, it is separated into heating oil. Similar to diesel oil, heating oil's official name is No. 2 fuel oil. The use of heating oil gained ground with the invention of the oil burner in the 1920s. Until then, homes were heated by coal. Heating oil is an important alternative energy source for homes that lack access to natural gas. This limited window of necessity has caused heating oil to become one of the most seasonally affected traded commodities available. As a byproduct of oil, heating oil takes lower priority than gasoline needs. While heating oil contracts are used to hedge against price fluctuation in jet fuel and diesel, they are essentially different products and adhere to their own quality standards. The price of heating oil depends mainly on the weather, the distribution of natural gas and the refinery capacity. Heating oil futures are traded on the New York Mercantile Exchange (NYMEX) under the ticker symbol HO in US dollars per barrel. Heating Oil futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are HOF, HOG, HOH, HOJ, HOK, HOM, HON, HOQ, HOU, HOV, HOX, HOZ. Each contract has a size of 1000 barrels (42000 gallons) of heating oil. The last trading day for each contract is the last business day of the month preceding the delivery month.

Brent oil

Brent Crude is the biggest of the many major classifications of crude oil consisting of Brent Crude, Brent Sweet Light Crude, Oseberg, Ekofisk, and Forties (BFOE). The other well – known classifications (also called references or benchmarks) are the OPEC Reference Basket, Dubai Crude and West Texas Intermediate. Brent Crude is sourced from the North Sea. The Brent Crude oil marker is also known as Brent Blend, London Brent and Brent petroleum. It is used to price two thirds of the world's internationally traded crude oil supplies. The name "Brent" comes from the naming policy of Shell UK Exploration and Production, operating on behalf of ExxonMobil and Royal Dutch Shell, which originally named all of its fields after birds. Petroleum production from Europe, Africa and the Middle East flowing west tends to be priced relative to this oil, i.e. it forms a benchmark. However, large parts of Europe now receive their oil from Russia. Brent oil futures are traded on the electronic Intercontinental Exchange, known as ICE, since 2005, under the ticker symbol CO in US dollars per barrel. Brent Oil futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are COF, COG, COH, COJ, COK, COM, CON, COQ, COU, COV, COX, COZ. Each contract has a size of 1000 barrels (42000 gallons) of brent oil. The last trading day for each contract is the business day prior to the 15th calendar day prior to the start of the delivery month. If the 15th calendar day is a non – business day in London, trading shall cease on the business day (in London) immediately preceding the day prior to the 15th calendar day.

Gasoline

Gasoline is a blended product that is characterized by its octane number and vapor pressure as well as its benzene, lead and oxygenate content. It is the refining product in highest relative demand, especially in the US. A process called cracking (thermal and catalytic) is therefore implemented in refineries in order to extract more gasoline from the other cuts (in particular, residues from the fractional distillation process), by breaking their longer, less valuable molecules. Gasoline futures were traded on the New York Mercantile Exchange (NYMEX) under the ticker symbol HU in US dollars per barrel. As from 2007 the ticker symbol has changed to XB. Gasoline futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are XBF, XBG, XBH, XBJ, XBK, XBM, XBN, XBQ, XBU, XBV, XBX, XBZ. Each contract has a size of 1000 barrels (42000 gallons) of gasoline. The last trading day for each contract is the last business day of the month preceding the delivery month.

Natural gas

Natural gas is used extensively to heat homes, and also has important applications in commercial and industrial settings. It is similar to what is referred to as biogas, which is methane produced from the breakdown of organic matter. Because it is a fossil fuel, it contains many secondary products that must be filtered out of the methane to render it commercially viable. Ethane, propane, butane, helium and hydrogen sulfide are removed and are considered a secondary income source for refiners. Once considered an ineffective byproduct of oil production, natural gas is steadily finding a foothold in today's world, as the world's crude oil reserves are slowly being depleted. Economical, environmentally friendly and efficient, natural gas is the cleanest-burning fossil fuel. As more efficient and inexpensive ways of capturing, transporting, distributing and liquefying are developed, it is steadily inching its way closer to becoming a viable fuel alternative for the future. Natural gas futures are traded on the New York Mercantile Exchange (NYMEX) under the ticker symbol NG in US dollars per barrel. Natural gas futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are NGF, NGG, NGH, NGJ, NGK, NGM, NGN, NGQ, NGU, NGV, NGX, NGZ. Each contract has a size of 10000 million British thermal units. The last trading day for each contract is three business days prior to the first calendar day of the delivery month.

Copper

Copper is a metal that has been known all around the world since ancient times. From West Africa to China to Europe to Central and South America, copper has been mined and worked continuously from as far back as 8700 BC. As one of the few independently occurring metals, copper has been used in a multitude of forms, from prehistoric pendants to modern-day piping and more. As a highly versatile material, copper can conduct electricity and is a necessary trace mineral in all living things. It also possesses the ability to destroy germs on contact. Copper is mined in large open pit mines, which also process molybdenum (an element used to strengthen steel) as a byproduct. The demand for copper in India and China plays a significant role in determining when, not if, copper reserves will be depleted. Current copper calculations suggest that the earth will run out of copper in as little as 61 years. As stated by NYMEX copper is the third most widely used metal in the world. Copper futures are traded on the New York Mercantile Exchange (NYMEX) under the ticker symbol HG in US cents per pound. Copper futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are HGF, HGG, HGH, HGJ, HGK, HGM, HGN, HGQ, HGU, HGV, HGX, HGZ. Each contract has a size of 25000 pounds of copper. The last trading day for each contract is the third to last business day of the maturing delivery month.

Gold

From the ancient times gold has been revered as a symbol of wealth and prosperity. Gold has also been used as currency and as a way to prop up the fiat money of various countries. One of the most significant moves to gold – and silver – backed currency occurred in 1792, which was the year that the US put the dollar on the gold and silver standard. Gold has developed widespread commercial use as a coating on electrical connectors. It can be found on various devices, from audio and video cables to computer and component cables and connectors. It is also used as an investment and in jewelry. As an investment, gold has cyclically come into and out of favor, and has experienced some of the most extreme pricing of any of the commodity markets. The International Monetary Fund (IMF) and the Washington Agreement on Gold (WAG) have very strict requirements in gold sales: less than 400 tons per year and members cannot use gold to back or replace their currency. China, South Africa, the US, Australia, Canada, Indonesia and Russia collectively represent the backbone of global gold production. Gold futures are traded on the New York Mercantile Exchange (NYMEX) under the ticker symbol GC in US dollars per troy ounce. Gold futures are delivered every year in January, February, March, April, May, June, July, August, September, October, November and December. The respective ticker symbols for each contract are GCF, GCG, GCH, GCJ, GCK, GCM, GCN, GCQ, GCU, GCV, GCX, GCZ. Each contract has a size of 100 troy ounces of gold. The last trading day for each contract is the third to last business day of the maturing delivery month.

Chapter 3

Implied convenience yield

3.1 Determination of convenience yield

Our aim is to construct a term structure of the implied convenience yield from observed futures prices. The evolution of the implied convenience yield through time shows the market's evaluation of the convenience yield. From theory the price of a commodity future is given by the equation

$$F = Se^{(r - y) \cdot T} \tag{3.1}$$

where F is the commodity future price, S is the spot price of the commodity, r is the annualized continuously compounded risk free interest rate, T is the time to maturity measured in years and y is the convenience yield. y is expressed as percentage. In reality there are not observable spot prices for commodities like crude oil, natural gas etc. In order to override this problem we approximate spot prices by the closest to maturity future contract. Equation (3.1) is transformed to:

$$F_1 = F_0 e^{(r_1 - y) \cdot T_1} \tag{3.2}$$

where F_0 is the price of the closest to maturity future contract – the approximation of the spot price – and F_1 the price of the second closest to maturity future contract. T_1 are the days to maturity of the second closest to maturity future contract. r_1 is the risk free interest rate for T_1 days. The price of the third closest to maturity future contract is given by the equation:

$$F_2 = F_0 e^{(r_2 - y) \cdot T_2} \tag{3.3}$$

For the calculation of the implied convenience yield between “today” and the next month we divide (3.3) by (3.2).

$$\frac{F_2}{F_1} = \frac{e^{(r_2 - y) \cdot T_2}}{e^{(r_1 - y) \cdot T_1}} \tag{3.4}$$

Finally we get the following equation for the calculation of the implied convenience yield.

$$y = \frac{r_1 T_1 - r_2 T_2}{T_1 - T_2} + \frac{\ln(\frac{F_2}{F_1})}{T_1 - T_2} \tag{3.5}$$

The risk free interest rate is assumed to be the Libor rate with maturity respective to the remaining days to maturity of the futures contract. As there is no Libor rate with maturity exactly matching the maturity of the future contract we have to interpolate between the two Libor rates which bracket the maturity of the future contract. In order to avoid expiration trading effects we filter the contracts with less than five business days to maturity. To avoid data noise from thin trading, we filter days with low volume of trading.

3.2 Facts about Libor

Libor stands for London InterBank Offered Rate. Libor is the primary benchmark for short term interest rates globally. It is used as a barometer to measure strain in money markets and often as a gauge of the market’s expectation of future central bank interest rates. It is produced for ten currencies with 15 maturities quoted for each, ranging from overnight to 12 Months producing 150 rates each business day. It is used as the basis for settlement of interest rate contracts on many of the world's major futures and options exchanges and has recently been used as a barometer by the media to measure the health of financial monetary markets. Independent research indicates that around $350 trillion of swaps and $10 trillion of loans are indexed to Libor. Libor is a benchmark; giving an indication of the average rate a leading bank, for a given currency, can obtain unsecured funding for a given period in a given currency. It therefore represents the lowest real – world cost of unsecured funding in the London market. Every contributor bank is asked to base their Libor submissions on the following question; "At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?" Therefore, submissions are based upon the lowest perceived rate that a bank on a certain currency panel could go into the inter-bank money market and obtain sizable funding, for a given maturity.

Every Libor rate produced is calculated using a trimmed arithmetic mean. Once each bank's submission is received by Thomson Reuters, they rank them in descending order and then exclude the highest and lowest submission or submissions – this is the trimming process. The remaining contributions are then arithmetically averaged to create a Libor quote. This is repeated for every currency and maturity producing 150 rates every business day. The banks that serve as contributors are chosen with the aim of reflecting the balance of the market for a given currency based on the following criteria.

1. scale of market activity
2. credit rating
3. perceived expertise in the currency concerned

For all currencies other than EUR and GBP the period between Fixing Date and Value Date will be two London business days after the Fixing Date. However, if that day is not both a London business day and a business day in the principal financial centre of the currency concerned, the next following day that is a business day in both centers shall be the Value Date. The day – count convention used for Libor calculation is actual days/360.

3.3 Example of Libor interpolation

We assume that today's date is Tuesday, 18 January 2011. We need to determine a Libor rate with a maturity of Tuesday, 8 March 2011, which is approximately 1.5 months from today. As there is no Libor quote available for the required maturity, it is necessary to estimate the unknown rate by using linear interpolation. The unknown Libor rate is denoted as R_x. The two closest maturity Libor rates which bracket the unknown rate from below and above are the one – month and two – month Libor rates, which are denoted as R_1 and R_2 respectively. The days to maturity for the unknown rate, the one – month and two – month Libor rates are denoted as t_x, t_1 and t_2 respectively. The following facts will be used for the calculation of the unknown rate.

Today's date	18 – Jan – 2011
Maturity date of unknown rate	08 – Mar – 2011
Days to maturity of unknown rate	49
Today's one – month Libor rate	0.26063%
Maturity date of one – month Libor rate	21 – Feb – 2011
Days to maturity of one – month Libor rate	34
Today's two – month Libor rate	0.28250%
Maturity date of two – month Libor rate	21 – Mar – 2011
Days to maturity of two – month Libor rate	62

The value date for Libor is two business days after the fixing date, which in our case is Thursday, 20 January 2011. The one – month and two – month Libor rates would normally mature on 20 February 2011 and 20 March 2011. But both these dates fall on Sunday, so according to the modified following business day convention that is used for Libor, the maturity date moves forward to the next "good" business day as long as it is in the same month. That means that the maturity dates for the one – month and two – month Libor rates become Monday, 21 February 2011 and Monday, 21 March 2011 respectively. In case the following business day lies in the next month the maturity date goes back to the last business day of the preceding month.

We assume that the unknown rate R_x lies on the straight line between the two known rates. From a basic formula for the slope of a line:

$$\frac{R_x - R_1}{t_x - t_1} = \frac{R_2 - R_1}{t_2 - t_1} \Rightarrow$$

$$R_x = R_1 + \frac{R_2 - R_1}{t_2 - t_1}(t_x - t_1) \tag{3.6}$$

Using (3.6) and the available data we can calculate the unknown rate R_x as follows.

$$R_x = 0.0026063 + \frac{0.0028250 - 0.0026063}{62 - 34}(49 - 34) = 0.0027235 = 0.27235\%$$

3.4 Term structure of the implied convenience yield

Using (3.5), the Libor interpolation formula and the available data for the future contract prices and maturities we calculate the implied convenience yield for each commodity. We start by calculating the convenience yield using “spot” commodity prices (in reality the nearby maturing futures contracts) and the second closest to maturity futures prices. Then we repeat the procedure using “spot” prices and the third closest to maturity futures prices. This process is repeated until we get to the 12 – month futures contracts. The result is a term structure of the convenience yield. This means that between January 1991 and November 2010 there are daily observations of the convenience yield from “today” to 12 months ahead. These observations can be shown in a surface diagram. The following graphs show the evolution of the implied convenience yield through time from daily observations.

For figures 3.1 – 3.6 axis “x” represents the time, which corresponds to the time period between Jan 91 and Nov 10. Axis “y” represents the month for which the implied convenience yield is calculated. For instance for y = 5 the implied convenience yield is calculated between the “spot” price and the 6 – month futures price. The y – axis values range between 0 (for "spot" convenience yield") and 11. Axis "z" represents the value of the implied convenience yield.

We observe that the implied convenience yield for all commodities follows a similar path through time. In general as we move from the "spot" to the 11 – month implied convenience yield its value increases. During the periods of the burst of the dot.com bubble and the financial crisis in 2008 the implied convenience yield shows a sudden and severe decrease. The period from July 2004 till the financial crisis in 2008 shows a steep increase for the implied convenience yield.

Figure 3.1: The evolution of the term structure of the implied convenience yield of crude oil through time.

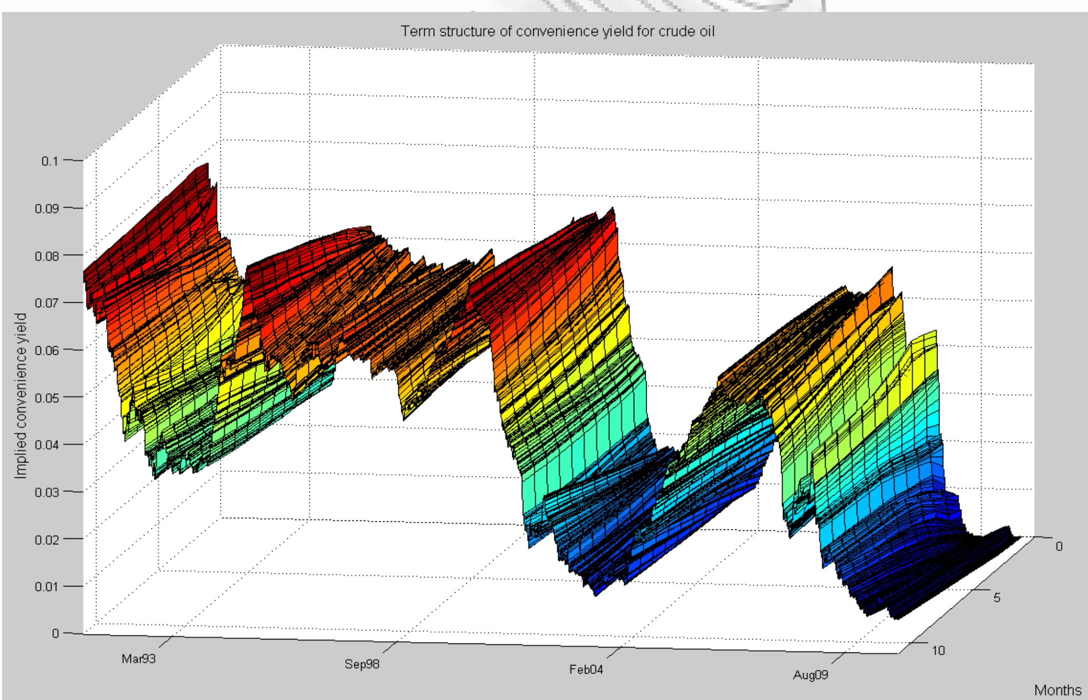


Figure 3.2: The evolution of the term structure of the implied convenience yield of heating oil through time.

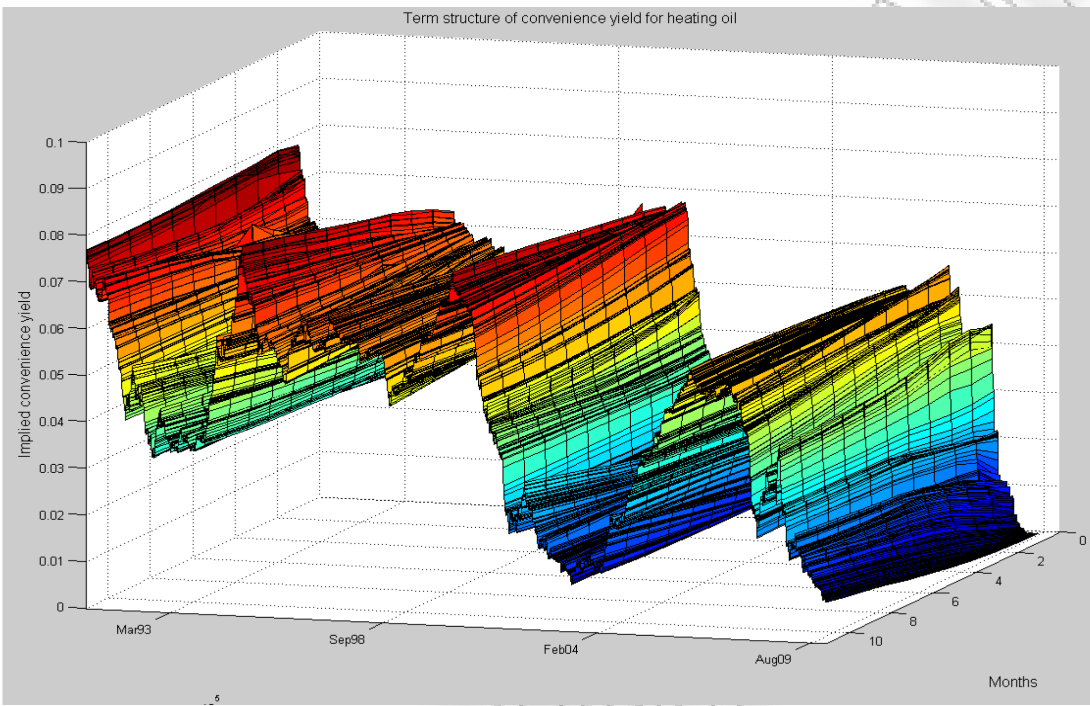


Figure 3.3: The evolution of the term structure of the implied convenience yield of natural gas through time.

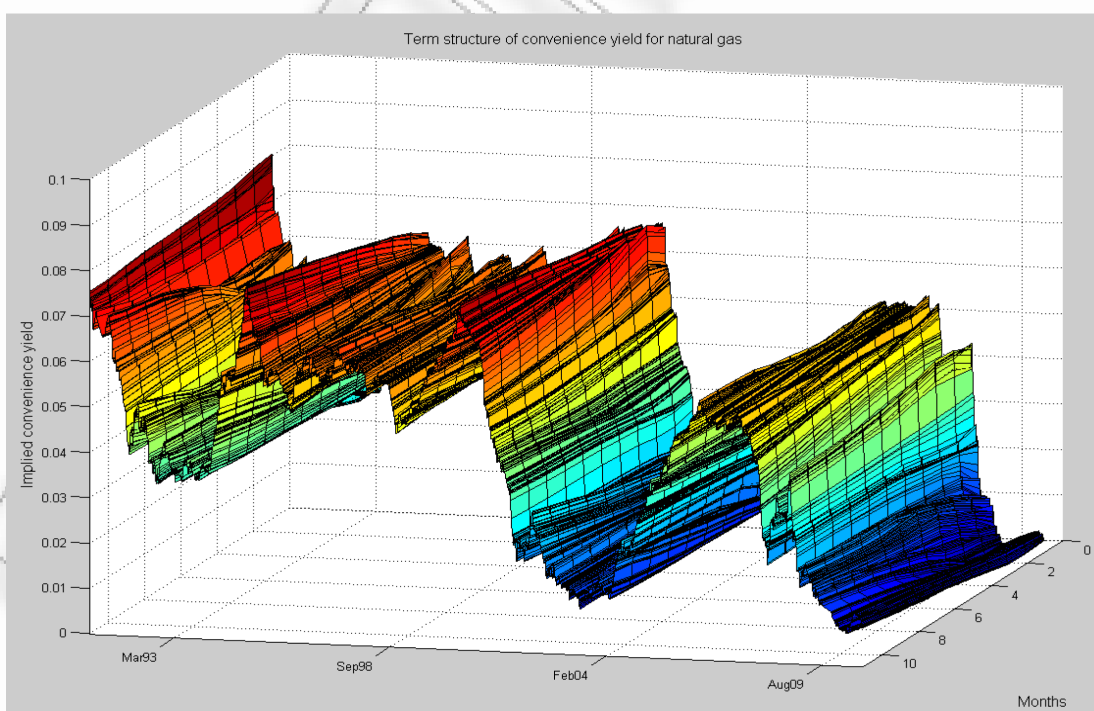


Figure 3.4: The evolution of the term structure of the implied convenience yield of gasoline through time.

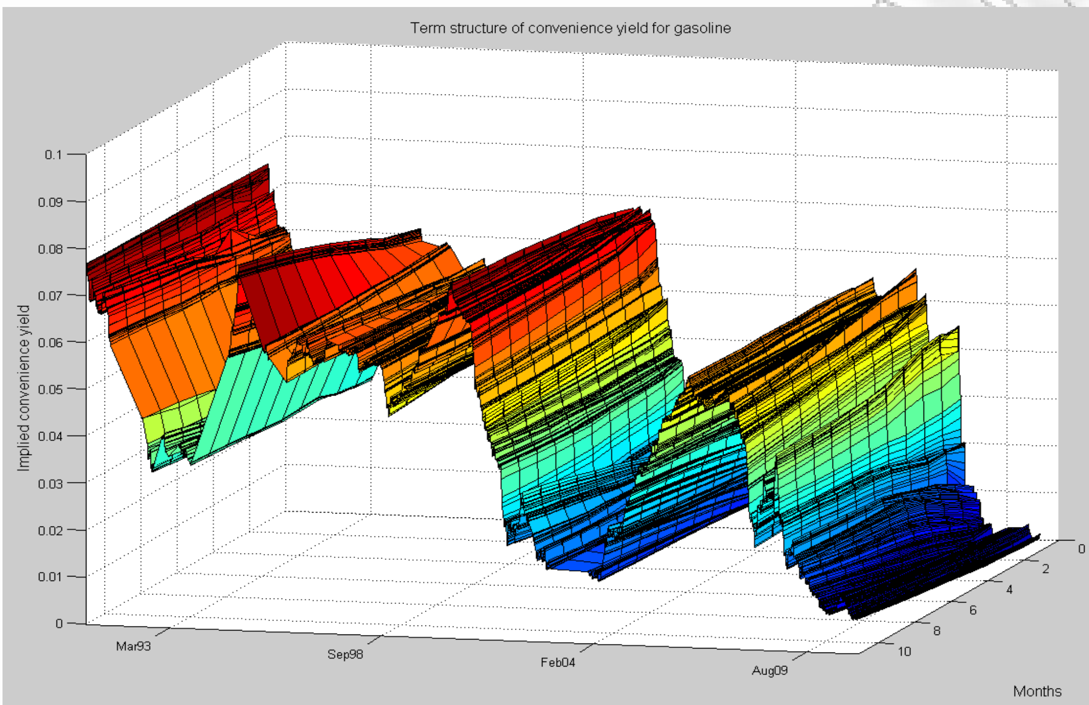


Figure 3.5: The evolution of the term structure of the implied convenience yield of oil brent through time.

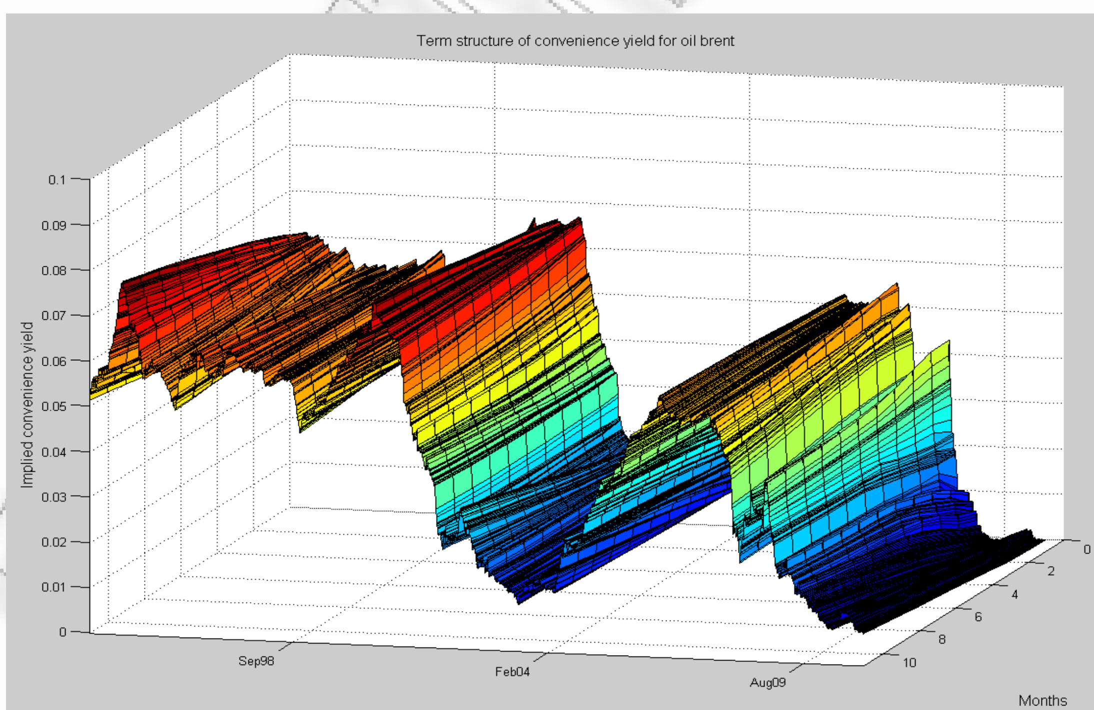
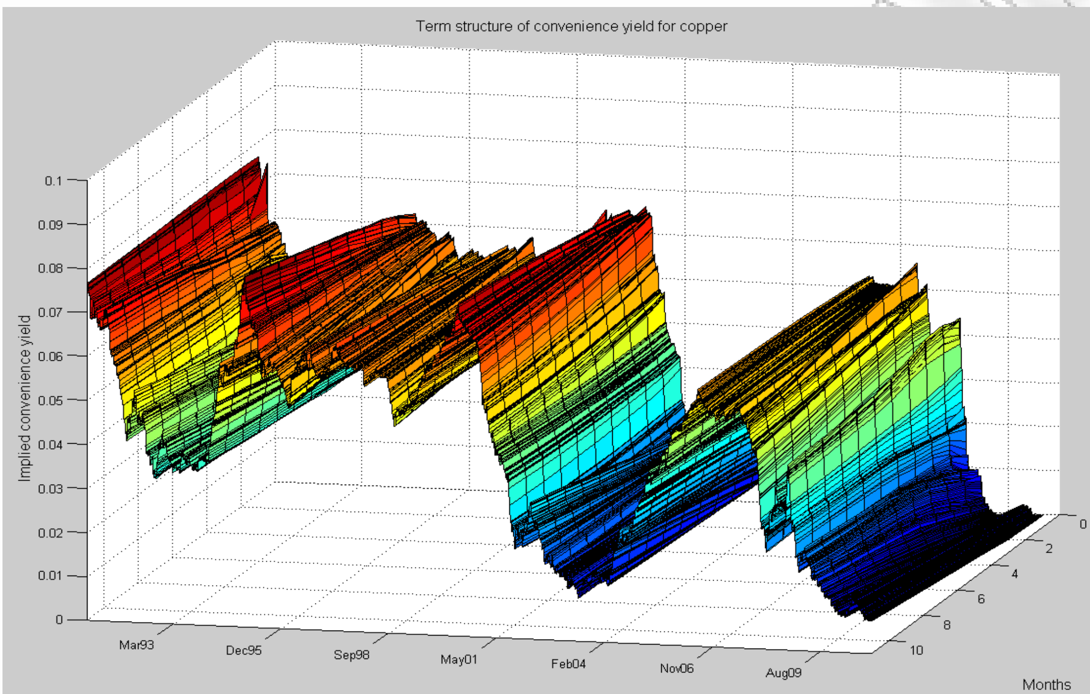


Figure 3.6: The evolution of the term structure of the implied convenience yield of copper through time.



Chapter 4

Estimation of convenience yield using Kalman filter

4.1 Description of the Kalman filter

The Kalman filter is a mathematical method named after Rudolf E. Kalman, which uses measurements observed over time, containing noise, to estimate the state x of a discrete time controlled process which follows a linear stochastic difference equation. The Kalman filter is a very powerful tool that is used extensively in many fields such as engineering, seismology, weather forecasting and finance. An important property of Kalman filter is that for estimating the current state of the system, it uses only the new measurement and the estimate from the previous time step and not all the previous data, a fact that makes its application simpler. We assume that the state x that Kalman filter tries to estimate follows the stochastic differential equation

$$x_t = A_t \cdot x_{t-1} + B_t \cdot u_{t-1} + w_{t-1} \qquad (4.1)$$

The measurement equation is assumed to be given by

$$z_t = H_t \cdot x_t + v_t \qquad (4.2)$$

where A_t is a [n x n] matrix, B_t is a [n x 1] matrix, H_t a [m x n] matrix. w_{t-1} and v_t are the process and measurement noise respectively and are assumed to be uncorrelated with normal probability distributions.

$$p(w) \sim N(0,Q) \qquad (4.3)$$

$$p(v) \sim N(0,R) \tag{4.4}$$

The Kalman filter algorithm first makes a prediction $\hat{x}_t^- \in R^n$, called “a priori”, about the state of the system. Then it uses the new measurement as the corrector to make a more accurate estimation $\hat{x}_t \in R^n$, called “a posteriori”, of the state. The a priori and a posteriori estimate errors are defined as e_t^- and e_t respectively.

$$e_t^- = x_t - \hat{x}_t^- \tag{4.5}$$

$$e_t = x_t - \hat{x}_t \tag{4.5}$$

The a priori and a posteriori estimate error covariance matrices are then defined as

$$P_t^- = E[e_k^- \cdot e_k^{-\,T}] \tag{4.6}$$

$$P_t = E[e_k \cdot e_k^T] \tag{4.7}$$

The Kalman algorithm is given in the following figure.

Predictor equations
$\hat{x}_t^- = A_t \cdot \hat{x}_{t-1} + B_t \cdot u_{t-1}$
$P_t^- = A_t \cdot P_{t-1} \cdot A_t' + Q_t$

$\uparrow \ \downarrow$

Correction equations
$K_t = P_t^- \cdot H_t' \cdot (H_t \cdot P_t^- \cdot H_t' + R_t)^{-1}$
$\hat{x}_t = \hat{x}_t^- + K_t \cdot (z_t - H_t \cdot \hat{x}_t^-)$
$P_t = (I - K_t \cdot H_t) \cdot P_t^-$

4.2 The Cox – Ingersoll – Ross (CIR) process

We assume that the spot price and the instantaneous convenience yield follow a Cox – Ross – Ingersoll joint stochastic process

$$dS = (\mu - \delta)\cdot S\cdot dt + \sigma_1 \cdot \sqrt{\delta}\cdot S\cdot dZ_1 \tag{4.8}$$

$$d\delta = \alpha\cdot(m - \delta)\cdot dt + \sigma_2 \cdot \sqrt{\delta}\cdot dZ_2 \tag{4.9}$$

where μ is the total expected return on the spot commodity price, S is the price of the underlying, δ is the convenience yield, σ_1 is the volatility term of dS, dZ_1 is a standard Brownian motion, α is the speed of adjustment, m is the long range mean to which δ tends to revert, σ_2 is the volatility term of $d\delta$ and dZ_2 is a standard Brownian motion. The CIR process ensures the non negativity of the convenience yield. Some sort of cash and carry arbitrage opportunities may emerge if the convenience yield becomes negative. The model assumes that there is constant correlation between the two Brownian motions with $dZ_1dZ_2 = \rho dt$. We take the natural logarithm of the spot price $x = \ln(S)$ and applying Ito's lemma in (4.8) the process followed by the log price is given by

$$dx = (\mu - (1 + 0.5\cdot\sigma_1^2)\cdot\delta)\cdot dt + \sigma_1\cdot\sqrt{\delta}\cdot dZ_1 \tag{4.10}$$

In a risk neutral world the expected growth rate of a commodity price is $\mu - b$, $b = \lambda_\rho\sigma$ where λ_ρ is the market price of risk of the commodity price. The expected growth rate of a commodity price in a risk neutral world can also be defines as $r + c + \delta$, where c is the marginal cost of storage, expressed as a proportion of the spot price. These two expressions have to be equal

$$r + c + \delta = \mu - \lambda_\rho\cdot\sigma_1\cdot\sqrt{\delta} \tag{4.11}$$

We substitute the drift rate μ in the real world process with the drift rate (r + c + δ) under the risk neutral measure. The convenience yield is not traded so the risk associated with it cannot be hedged. We assume a market price of risk λ related to the convenience yield and the drift rate of the convenience yield process in a risk neutral world becomes α(m – δ) – λ. Under the risk neutral measure the joint stochastic process followed by the spot price and the convenience yield becomes

$$dS = (r + c - \delta) \cdot S \cdot dt + \sigma_1 \cdot \sqrt{\delta} \cdot S \cdot d\tilde{Z}_1 \qquad (4.12)$$

$$d\delta = [\alpha \cdot (m - \delta) - \lambda] \cdot dt + \sigma_2 \cdot \sqrt{\delta} \cdot d\tilde{Z}_2 \qquad (4.13)$$

The two Brownian motions are again constantly correlated with $d\tilde{Z}_1 d\tilde{Z}_2 = \rho dt$. The process for the log price under the risk neutral measure becomes

$$dx = (r + c - (1 + 0.5 \cdot \sigma_1^2) \cdot \delta) \cdot dt + \sigma_1 \cdot \sqrt{\delta} \cdot d\tilde{Z}_1 \qquad (4.14)$$

The futures prices have to satisfy the partial differential equation (PDE)

$$\begin{cases} \frac{1}{2}\sigma_1^2 \delta S^2 F_{SS} + \frac{1}{2}\sigma_2^2 \delta F_{\delta\delta} + \rho\sigma_1\sigma_2\delta S F_{S\delta} + (r + c - \delta) S F_S + (\alpha(m - \delta) - \lambda) F_\delta - F_T = 0 \\ F(S, \delta, 0) = 0 \end{cases} \qquad (4.15)$$

This PDE suggests an exponential affine form solution

$$\begin{cases} F(S, \delta, \tau) = S e^{A(\tau) - B(\tau)\delta} \\ A(0) = 0 \\ B(0) = 0 \end{cases} \qquad (4.16)$$

By substituting (4.16) into (4.15) we have

$$\begin{cases} \frac{1}{2}\sigma_2^2 B^2 + (\alpha - \rho\sigma_1\sigma_2)B - 1 + B_\tau = 0 \\ r + c + (\lambda - \alpha m)B - A_\tau = 0 \end{cases} \tag{4.17}$$

The solution to the two ordinary differential equations (4.17) is given by

$$\begin{cases} A(\tau) = (r + c)\tau + (\lambda - \alpha m)\int_t^T B(q)dq \\ B(\tau) = \dfrac{2(1 - e^{-k_1\tau})}{k_1 + k_2 + (k_1 - k_2)e^{-k_1\tau}} \end{cases} \tag{4.18}$$

where

$$\begin{cases} \int_t^T B(q)dq = \dfrac{2}{k_1(k_1 + k_2)} \ln\left[\dfrac{(k_1 + k_2)e^{k_1\tau} + k_1 - k_2}{2k_1}\right] + \\ \qquad + \dfrac{2}{k_1(k_1 - k_2)} \ln\left[\dfrac{k_1 + k_2 + (k_1 - k_2)e^{-k_1\tau}}{2k_1}\right] \\ k_1 = \sqrt{k_2^2 + 2\sigma_2^2} \\ k_2 = \alpha - \rho\sigma_1\sigma_2 \end{cases} \tag{4.19}$$

Finally the solution to the equation (4.15) is given by (4.16) and (4.18).

4.3 State space formulation

The natural logarithm of the futures prices is derived by the equation (4.16)

$$\ln F(S, \delta, \tau) = \ln S + A(\tau) - B(\tau)\delta \tag{4.20}$$

Using the equation (4.20) we can write the measurement equation as follows

$$Y_t = d_t + Z_t[x_t, \delta_t]' + \varepsilon_t, \quad t = 1, 2, ..., NOBS_{\text{(number of observations)}} \tag{4.21}$$

where

- Y_t is a n x 1 vector containing the natural logarithms of the observed futures prices for the n different maturities
- $d_t = [A(\tau_i)]$ for i = 1,2,…,n is a n x 1 vector, which is derived from equation (4.18)
- $Z_t = [1, -B(\tau_i)]$ for i = 1,2,…,n is a n x 2 matrix, which is derived from equation (4.18)
- ε_t is a n x 1 vector of serially uncorrelated white noise with $E[\varepsilon_t] = 0$ and $Var[\varepsilon_t] = H_t$. The covariance matrix H_t is assumed to be diagonal to make computations easier. This vector accounts for possible errors in the data.

From equations (4.9) and (4.14) we derive the transition equation

$$\begin{bmatrix} x_t \\ \delta_t \end{bmatrix} = \begin{pmatrix} \mu\Delta t \\ \alpha m\Delta t \end{pmatrix} + \begin{pmatrix} 1 & -(1+\frac{1}{2}\sigma_1^2)\Delta t \\ 0 & 1-\alpha\Delta t \end{pmatrix} \begin{bmatrix} x_{t-\Delta t} \\ \delta_{t-\Delta t} \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_t^1 \\ \xi_t^2 \end{pmatrix} \tag{4.22}$$

which can be written for notational simplicity

$$a_t = c_t + Q_t a_{t-1} + R_t \xi_t, \quad t = 1,2,...,NOBS \tag{4.23}$$

ξ_t is a 2 x 1 vector of serially uncorrelated white noise with the following properties.

- $E[\xi_t] = 0$
- $Var[\xi_t] = V_t$

where

$$V_t = \begin{pmatrix} \sigma_1^2 \Delta t \delta_{t-\Delta t} & \rho\sigma_1\sqrt{\Delta t}\sqrt{\delta_{t-\Delta t}}\sqrt{Var[\delta_t \mid \delta_{t-1}]} \\ \rho\sigma_1\sqrt{\Delta t}\sqrt{\delta_{t-\Delta t}}\sqrt{Var[\delta_t \mid \delta_{t-1}]} & Var[\delta_t \mid \delta_{t-1}] \end{pmatrix} \tag{4.24}$$

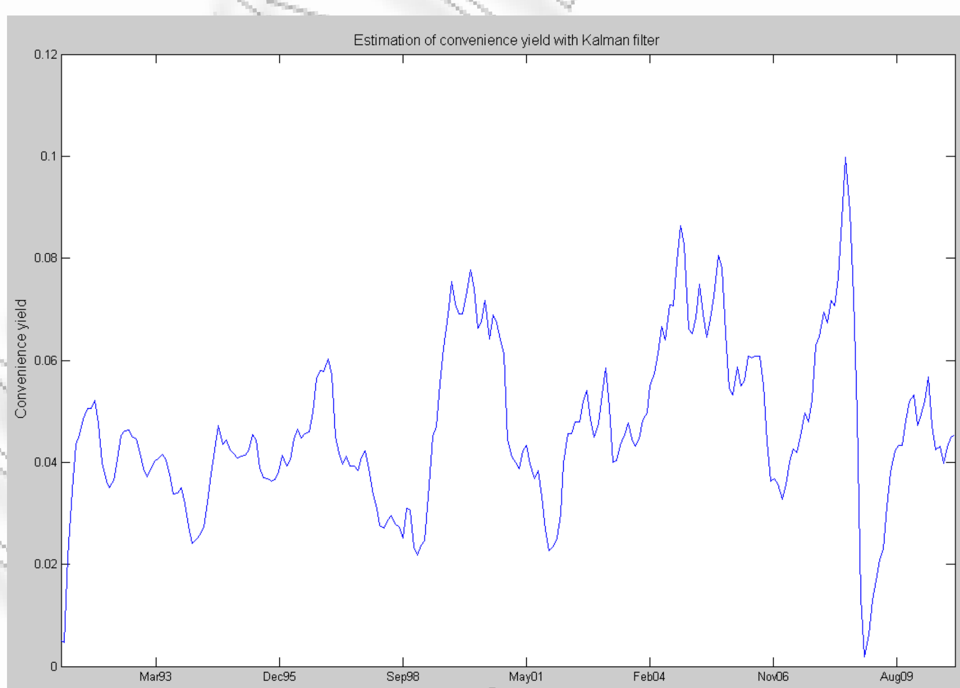
$$\text{Var}[\delta_t \mid \delta_{t-1}] = m\left(\frac{\sigma_2^2}{2\alpha}\right)(1-e^{-\alpha\Delta t})^2 + \delta_{t-1}\left(\frac{\sigma_2^2}{\alpha}\right)(e^{-\alpha\Delta t} - e^{-2\alpha\Delta t}) \qquad (4.25)$$

The matrices H_t, Z_t, Q_t, c_t, d_t and V_t, which are used by the Kalman filter are dependent on the unknown parameter set $\varphi = \{m, \mu, \lambda, \alpha, \sigma_1, \sigma_2, \rho, \varepsilon_t\}$. One of the purposes of the Kalman filter is to estimate these parameters. We start the iterative procedure by picking randomly a parameter set in order to calibrate the Kalman filter. An optimization routine is used in order to maximize the quasi likelihood function or equally the logarithm of it, which is:

$$\ln L(Y;\varphi) = -\frac{1}{2}\frac{n(t_T - t_0)}{\Delta t}\ln 2\pi - \frac{1}{2}\sum_t \ln\{\det(F_t)\} - \frac{1}{2}\sum_t v_t F_t^{-1} v_t \qquad (4.26)$$

Figure 4.1 shows the results from the application of the Kalman algorithm for the approximation of the evolution of the convenience yield of crude oil through time.

Figure 4.1: Estimation of the convenience yield using Kalman filter for crude oil



Chapter 5

An empirical test of the storage theory

5.1 The storage theory

According to the storage theory the difference between the spot price and the futures price of a commodity can be attributed to the interest foregone, the marginal storage cost and the convenience yield.

$$F(t,T) - S(t) = S(t)r(t,T) + c(t,T) - \delta(t,T) \tag{5.1}$$

where F(t,T) is the futures price with maturity T, S(t) is the spot price, r(t,T) is the risk free interest rate for the time period T – t, c(t,T) is the marginal storage cost per unit of inventory for the time period T – t and δ(t,T) is the marginal convenience yield per unit of storage. The theory of storage explains the behavior of commodity prices based on economic fundamentals. Specifically it predicts a negative relation between inventories and convenience yield. When inventories are high the benefit to be had for someone who has physical possession of the commodity is relatively low. But when inventories are low the possibility of a stock – out of the commodity increases, which means that the expected future spot price volatility also increases. As a result the benefit of holding the commodity is greater leading to a higher convenience yield. In other words the inventories act as a buffer, which dampens the effect on the spot prices, caused by shocks in supply and demand.

5.2 The interest adjusted futures basis

We divide (5.1) with S(t).

$$\frac{F(t,T)-S(t)}{S(t)}-r(t,T)=\frac{c(t,T)-\delta(t,T)}{S(t)} \tag{5.2}$$

The left part of equation (5.2) is called interest adjusted basis. It has been used by many papers as an approximation of inventories since especially in the past inventories data were not available (Fama and French, 1987). Specifically the sign of the futures basis can be indicative of the level of inventories. High (low) inventories for a commodity mean that the convenience yield is low (high) and from (5.1) the futures price tends to be higher (lower) than the spot price. In this case the forward curves are in "backwardation" ("contango") and from (5.2) the interest adjusted futures basis is positive (negative). Thus the theory of storage suggests a positive correlation between the interest adjusted futures basis and the level of inventories. Since there is a negative correlation between convenience yield and the level of inventories, while at the same time the interest adjusted futures basis is a monotonically increasing function of the level of inventories, we come to the conclusion that the interest adjusted futures basis and the convenience yield are positively related.

We empirically test these three implications. We use real inventories data where available and the implied convenience yield data from chapter 3. The interest adjusted futures basis is calculated for 6– and 12– month future contracts from daily observations. As an approximation of the spot price we take the price of the closest to maturity future contract. Similar to chapter 3 we filter the contracts with less than five business days to maturity and days with low volume of trading, in order to avoid expiration trading effects and data noise from thin trading. The interest adjusted futures basis is calculated as

$$b_{i,t}=\frac{F_{i,t,T_2}-F_{i,t,T_1}}{F_{i,t,T_1}}-r_{f,t}\frac{T_2-T_1}{365} \tag{5.3}$$

where $b_{i,t}$ is the daily interest adjusted futures basis, $F_{i,t,T2}$ is the futures price with T_2 days to maturity, $F_{i,t,T1}$ is the futures price of the closest to maturity contract maturing in T_1

days, $r_{f,t}$ is the risk free interest rate and $\frac{T_2 - T_1}{365}$ represents the time period in years for which the interest rate is calculated.

5.3 Empirical testing

First we test the relationship between the convenience yield and the interest adjusted futures basis. Specifically the 6 – month interest adjusted futures basis $b_{i,\tau}$ is regressed against the 6 – month convenience yield $\delta_{i,\tau}$ for each commodity i. To make the test more robust we repeat the same procedure between the 12 – month adjusted basis and the 12 – month convenience yield.

$$b_{i,\tau} = a_i + \beta_i \delta_{i,\tau} + e_{i,\tau} \tag{5.4}$$

Since we have daily data we calculate the average of the adjusted basis and the convenience yield for each month. The results of the regression are displayed in table 5.1.

<u>Table 5.1: Adjusted basis against convenience yield regression</u>

This table displays the co – efficient α and β from the regression of the adjusted basis against the implied convenience yield, the t – statistic of co – efficient β and the correlation co – efficient between the adjusted basis and the convenience yield.

	Crude oil	**Heating oil**	**Brent oil**	**Natural gas**	**Gasoline**	**Copper**	**Gold**
			6 month contracts				
α	0.039	0.041	0.039	0.042	0.039	0.034	0.035
β	-0.086	-0.043	-0.107	-0.021	-0.044	-0.273	-1.495
t statistic	-5.040	-2.826	-5.508	-3.131	-3.731	-8.743	-8.254
corr. co-efficient	-0.311	-0.181	-0.337	-0.193	-0.236	-0.494	-0.473

	Crude oil	Heating oil	Brent oil	Natural gas	Gasoline	Copper	Gold
				12 month contracts			
α	0.0401	0.041	0.039	0.044	0.039	0.035	0.031
β	-0.054	-0.045	-0.059	-0.031	-0.069	-0.138	-1.431
t statistic	-5.137	-4.132	-4.876	-5.215	-5.987	-9.027	-10.307
corr. co-efficient	-0.317	-0.264	-0.327	-0.329	-0.409	-0.506	-0.556

As implied by the theory of storage there is a negative relation between the interest adjusted futures basis and the implied convenience yield. This is shown both from the negative sign of the co – efficient β of the regression and from the negative correlation co – efficient. The co – efficient β is statistically significant at the 1% level for all commodities. The results from the regressions using the 12 – month contracts seem to support the storage theory slightly stronger than those from the 6 – month but overall they are similar. Metals (gold and copper) show a stronger negative relation between the adjusted basis and the convenience yield than the energy category.

Next we test the relationship between the implied convenience yield and inventories. For the inventories we have monthly observations, so we regress the average of the daily observations of the implied convenience yield for each month $\delta_{i,\tau}$ against the inventories data from the preceding month $I_{i,\tau-1}$.

$$\delta_{i,\tau} = \alpha_i + \beta_i I_{i,\tau-1} + e_{i,\tau} \tag{5.5}$$

The results from the regression are displayed in table 5.2.

Table 5.2: Convenience yield against inventories regression

This table displays the co – efficient α and β from the regression of the implied convenience yield against the inventories, the t – statistic of co – efficient β and the correlation co – efficient between the inventories and the convenience yield.

	Crude oil	**Heating oil**	**Natural gas**	**Gasoline**
		6 month contracts		
α	0.153	0.091	0.078	0.163
β	-0.112	-0.050	-0.038	-0.122
t statistic	-7.236	-5.338	-2.696	-3.991
corr. co-efficient	-0.425	-0.326	-0.173	-0.252
		12 month contracts		
α	0.151	0.093	0.085	0.211
β	-0.108	-0.049	-0.042	-0.168
t statistic	-7.155	-5.372	-3.124	-4.754
corr. co-efficient	-0.422	-0.327	-0.200	-0.319

The results of the regression confirm the negative relation between the convenience yield and the inventories suggested by the theory of storage. The slope (β) of the OLS line varies between -3.8% and -16.8%. The co – efficient β of the regression is statistically significant at the 1% level for all commodities. The results from the 6 – month and 12 – month implied convenience yield are nearly identical strengthening further the robustness of the test. The correlation co – efficient between the convenience yield and inventories is clearly negative for all commodities with values ranging between -0.173 and -0.425.

Finally we test the relationship between the interest adjusted basis and the level of inventories. The average of the daily observations of the interest adjusted basis for each month $b_{i,\tau}$ is regressed against the inventories data from the preceding month $I_{i,\tau-1}$.

$$b_{i,\tau} = \alpha_i + \beta_i I_{i,\tau-1} + e_{i,\tau} \tag{5.5}$$

The results from the regression are displayed in table 5.3.

Table 5.3: Adjusted basis against inventories regression

This table displays the co – efficient α and β from the regression of the interest adjusted futures basis against the inventories, the t – statistic of co – efficient β and the correlation co – efficient between the inventories and the adjusted basis.

	Crude oil	**Heating oil**	**Natural gas**	**Gasoline**
	6 month contracts			
α	-0.591	-0.133	0.182	-0.361
β	0.567	0.121	-0.134	0.333
t statistic	11.118	3.318	-1.256	2.515
corr. co-efficient	0.586	0.213	-0.081	0.162
	12 month contracts			
α	-0.986	-0.489	-0.426	-0.857
β	0.930	0.449	0.460	0.801
t statistic	11.784	10.820	4.169	4.874
corr. co-efficient	0.608	0.570	0.262	0.337

The results of the regression once again are compatible with the positive relation between the adjusted basis and inventories suggested by the theory of storage. The only exception is natural gas, which shows a negative correlation between the adjusted basis and inventories but only for the 6 – month basis. However in this case the co – efficient β is not statistically significant (t = -1.256), a fact that weakens the importance of the result. The co – efficient β in all the other cases is statistically significant at the 1% level. The results from the 12 – month adjusted basis imply a stronger support of the storage theory. Many papers have used the basis as an approximation for the level of inventories and the results in table 5.3 suggest that this is a reasonable hypothesis.

References

Brennan M.J., (1958), The supply of storage, American Economic Review, 48, 50-72.

Brennan M. J. and Schwartz E. S., (1985), Evaluating natural resource investments, Journal of Business 58, 133-155.

Brooks C., Lazar E., Prokopczuk M., Symeonidis L., (2011), Futures basis, scarcity and commodity price volatility: An empirical analysis, ICMA Centre discussion papers in finance DP2011 – 20.

Carmona R. and Ludkovski M., (2003), Spot convenience yield models for energy assets.

Carmona R. and Ludkovski M., (2004), Convenience yield model with partial observations and exponential utility.

Daskalaki C. and Skiadopoulos G., (2011), Should investors include commodities in their portfolios after all? New evidence.

Fama E.F. and French K.R., (1987), Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage, Journal of Business, 60, 55-73.

Fama E.F. and French K.R., (1988), Business cycles and the behavior of metal prices, Journal of Finance, 63, 1075-1093.

Gibson R. and Schwartz E. S., (1990), Stochastic convenience yield and the pricing of oil contingent claims, Journal of Finance 45, 959-976.

Gorton G., Hayashi F. and Rouwenhorst G., (2005), Commodity futures: A Japanese perspective.

Gorton G., Hayashi F. and Rouwenhorst G., (2008), The fundamentals of commodity futures returns.

Hamilton J.D., (1994), Time series analysis, Princeton Univ. Press.

Harvey A.C., (1994), Forecasting, structural time series models and the Kalman filter, Cambridge Univ. Press.

Lautier D., (2003), The informational value of crude oil futures prices.

Miltersen K.R. and Schwartz E.S., (1998), Pricing options on commodity futures with stochastic term structures of convenience yields and interest rates, Journal of Financial and Quantitative Analysis, 33, 33-59.

Ribeiro D.R. and Hodges S.D., (2004), A two factor model for commodity prices and futures valuation.

Rouwenhorst G. and Gorton G., (2004), Facts and Fantasies about Commodity Futures, Yale ICF Working Paper No. 04-20.

Schwartz E.S., (1997), The Stochastic Behavior of Commodity Prices : Implications for Valuation and Hedging, The Journal of Finance, vol. LII, n°3, July.

Schwartz E.S. and Smith J.E., (2000), Short-Term Variations and Long-Term Dynamics in Commodity Prices, Management Science 46, 893–911.

Welch G. and Bishop G., (2006), An introduction to the Kalman filter.

Working H., (1949), The theory of the price of storage, American Economic Review, 39, 1254-1262.